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Interactions of High Energy Particles With Nuclei

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Interactions of High Energy Particles With Nuclei

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Preface

This monograph is based on lectures given by Dr. Wiesław Czyż at the University of Virginia during the spring semester of 1973. They cover selected topics in the field of high energy diffractive scattering and production processes. In addition to reviewing some well-known material there is much here that is new, both in content and form of presentation.

The material presented here is also part of a program of research and cooperation between the author and the National Bureau of Standards that was begun in 1967 when the author was a National Science Foundation Senior Foreign Scientist Fellow at The American University and a Guest Worker at the National Bureau of Standards. This cooperation was continued on an informal basis during his several visits to this country between 1968 and 1971. Since July 1972 this research has been the subject of a grant by the National Bureau of Standards to the Institute of Nuclear Physics, Krakow, Poland under the PL-480 program. The sponsorship of this program has greatly facilitated this effort which has produced a series of articles written jointly by the author and NBS staff. The result of this cooperation is reflected, however, not only in published papers, but in numerous unpublished notes as well as in these lectures themselves.

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Interactions of High Energy Particles with Nuclei*

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Elastic scattering and diffractive production processes induced in nuclear targets by high energy projectiles are discussed in this article. Special attention is paid to the interaction of high energy hadrons and photons. Interactions of high energy electrons and neutrinos are briefly mentioned. The common features of all these processes are emphasized throughout the article: The multiple scattering and shadowing processes inside of the target nuclei. An effort is made to develop a unified way of treating nuclear interactions of particles which are either hadrons or exhibit some hadronic components in such interactions.

This article is divided into seven sections: (1) Introduction, (2) Description of multiple scattering, (3) Elastic scattering of hadrons from nuclei, (4) Diffractive dissociation and diffractive excitation, (5) Diffractive production of hadrons in hadron-nucleon collisions, (6) Shadowing effects in inelastic electron-nucleus scattering, (7) Shadowing effects in neutrino reactions on nuclei.

Key words: Diffractive production; diffractive scattering; Glauber model; hadronic components of photons; high energy scattering; multiple scattering; neutrino-nucleus interactions; shadowing effects.

1. Introduction

Let us start by giving a few motives for discussing this subject:

(a) It is well known that nuclear targets are of considerable importance in high energy physics. Work on vector meson production on nuclei or 3π (5π) coherent diffractive production (compare refs. [S3, S4, S5])[1] is a good example of the role of nuclear targets. One observed also excitations of specific nuclear levels by high energy hadrons [1, 2]. This opens a possibility [2] of selecting diffractive productions with nuclear levels as their analyzers.

(b) The very high energy incident particles *may* also be important for physicists working on nuclear structure—although this point does not seem to be well established (presumably due to poor energy resolutions of high energy beams). Nevertheless, one still hopes to be able to learn something new, for example about short range nucleon correlation functions in target nuclei or about the presence of resonances in nuclear ground states [3, 4]—just to name two problems.

(c) One may also hope that, in the cases where the scattering from a nucleus cannot be reduced to the "elementary amplitudes" of the incident particle—target nucleon, some new physical situations may occur which stem from the complexity of the target. For example, in the case of π-nucleus scattering in the region of the (3, 3) resonance [5, 6] one may hope to learn something about the nature of the (3, 3) resonance because the exclusion principle (due to the many particle structure of the target) may distort the resonance and this distortion may depend on its internal structure.

In these notes the interactions of various different particles with nuclear targets are to be considered. Of course, we cannot cover all problems related to the interactions of hadrons, photons, and electrons (virtual photons) with nuclei—we must choose a certain point of view which unifies all these problems. The *common denominator* which we shall emphasize is the existence (or lack thereof) of "shadow effects" (which occur mostly for forward scattering and production processes). Such effects are very well established in the case of hadron scattering and photon-nucleus interactions; they are not well known in the case of neutrino reactions and very virtual photons (see refs. [S3, S5]).

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* Based on a series of lectures given at Department of Physics, University of Virginia, during the spring semester 1973.

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[1] Figures in brackets indicate the literature references at the end of this paper.

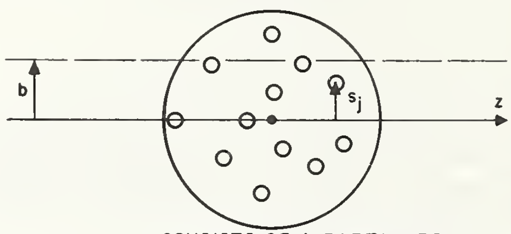
(Nonetheless there exists a motivation for belief in their existence: as in the case of photons interacting with nucleons—where the strongly interacting vector mesons seem to dominate—in the case of *neutrino* interactions—as was pointed out by Adler [7]—π-mesons should be important). In other words, in all the processes we are going to discuss, strongly interacting particles appear (as real or virtual particles) which may produce shadowing effects.

So, the crucial point in our discussion is an understanding of a multiple scattering process of strongly interacting particles inside of nuclear matter (or more generally: just a multiple scattering process with forces strong enough to insure the existence of multiple scattering). Hence we shall start with the very successful model of such processes: the Glauber model.

2. Description of Multiple Scattering

2.1. General Remarks

To construct the relevant formulae for the theory of multiple scattering one can employ various models of potential scattering. First let me quote the well-known formulae: one particle scatters from a collection of A particles at very small angles (in the Glauber model [S1]).



This is to a very good approximation a two dimensional process. The individual amplitude

$$f_j(\delta) = \frac{ik}{2\pi} \int d^2b \exp (i\boldsymbol{\Delta}\cdot\mathbf{b}) (1-e^{i\chi_j(b)}),$$

is shifted to the position of the jth nucleon:

$$f_j(\delta) = \frac{ik}{2\pi} \int d^2b \exp (i\boldsymbol{\Delta}\cdot\mathbf{b}) \{1- \exp [i\chi_j(\mathbf{b}-\mathbf{s}_j)]\},$$

where k is the momentum of the incident particle in laboratory frame

$\boldsymbol{\Delta}$ is the two-dimensional momentum transfer
b is the impact parameter
$\chi_j(b)$ is the phase shift which characterizes the incident particle—jth nucleon elastic scattering amplitude.

The expression

$$1-e^{i\chi_j(b)} = \gamma_j(b)$$

is called the profile of the jth nucleon, incident particle collision. Assuming

$$\chi(\mathbf{b}) = \sum_{j=1}^{A} \chi_j(\mathbf{b}-\mathbf{s}_j)$$

and assuming that the particle goes through the target so fast that all the nucleons are 'frozen' at certain positions, we get for the amplitude

$$\mathfrak{M}_{fi} = \frac{ik}{2\pi}\int d^3r_1 \ldots d^3r_A \Psi_f^*(r_1 \ldots r_A) \int d^2b \exp(i\mathbf{\Delta}\cdot\mathbf{b})$$

$$\times\{1-\exp[\sum_j \chi_j(\mathbf{b}-\mathbf{j}_j)]\}\Psi_i(\mathbf{r}_1\ldots\mathbf{r}_A)$$

$$= \frac{ik}{2\pi}\int d^2b \exp(i\mathbf{\Delta}\cdot\mathbf{b}) \int d^3r_1 \ldots d^3r_A \Psi_f^* \left[1-\prod_{j=1}^{A}(1-\gamma_j(\mathbf{b}-\mathbf{s}_j))\right]\Psi_i, \tag{2.1}$$

where Ψ_i and Ψ_f are the initial and final wave functions of the target nucleus.

One can produce many arguments which make this *important* formula plausible. One can use, e.g., an optical description of attenuation of a wave penetrating a medium. One can also use some arguments based on approximate solutions of the wave equation of the incident particle interacting through potentials with the target particles.

For instance, in the case of the Schrödinger equation

$$E\psi = \left(\frac{p^2}{2m}+V\right)\psi$$

in the limit $E\to\infty$,[2] and for the incident particle moving along the z axis, we present the solution in the form

$$\psi_k(x, y, z) = e^{ikz}\varphi(x, y, z).$$

If the potential is smooth enough (so that second derivatives of φ can be neglected), one can show that φ satisfies the *approximate equation*[3]

$$\frac{\partial\varphi(x, y, z)}{\partial z} = -\frac{i}{v}V(x, y, z)\varphi(x, y, z),$$

which gives

$$\psi_k \approx e^{ikz} - \frac{i}{v}\int_{-\infty}^{z} dz' V(x, y, z').$$

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[2] Notice that to have scattering in the limit $E\to\infty$ we have to have $V\sim EV'$ where V' is energy independent. Otherwise the high energy solution of the Schrödinger equation reduces to the Born approximation.

[3] $\Delta\psi + k^2\psi = \frac{2m}{\hbar^2}V\psi, \qquad \psi = e^{ikz}\varphi(x, y, z),$

$$\Delta\psi = e^{ikz}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\varphi(x, y, z) - k^2e^{ikz}\varphi + 2ike^{ikz}\frac{\partial\varphi}{\partial z} + e^{ikz}\frac{\partial^2\varphi}{\partial z^2},$$

hence, neglecting second derivatives of φ, we obtain the following equation for φ:

$$2ike^{ikz}\frac{\partial\varphi}{\partial z} = \frac{2m}{\hbar^2}Ve^{ikz}\varphi,$$

$$\frac{\partial\varphi}{\partial z} = -\frac{i}{v\hbar}V\varphi,$$

where we have used

$$p = k\hbar, \qquad v = \frac{p}{m}.$$

The amplitude for the particle to scatter from $\mathbf{k}$ to $\mathbf{k}'$ is:

$$\mathfrak{M}(\mathbf{k}', \mathbf{k}) = -\frac{m}{2\pi}\int d^3r \exp(-i\mathbf{k}'\cdot\mathbf{r})\, V(\mathbf{r})\psi_k(\mathbf{r})$$

$$\approx \frac{m}{2\pi}\int d^2b \exp(i\mathbf{\Delta}\cdot\mathbf{b}) \int_{-\infty}^{+\infty} dze^{ikz}V(\mathbf{b}, z) \exp\left(-\frac{i}{v}\int_{-\infty}^{z} dzV(\mathbf{b}, z')\right)$$

$$= \frac{ik}{2\pi}\int d^2b \exp(i\mathbf{\Delta}\cdot\mathbf{b}) \left[1-\exp\left(-\frac{i}{v}\int_{-\infty}^{+\infty} dz'V(\mathbf{b}, z')\right)\right], \qquad \mathbf{\Delta}=\mathbf{k}-\mathbf{k}'.$$

We can see that from the additivity of the potentials

$$V=\sum_{j=1}^{A} V_j,$$

we recover additivity of phase shifts.

There are many simplifications made in obtaining the fundamental formula (2.1); the reliability of this formula is of primary importance. The most complete analysis one can perform is presumably to employ the Watson multiple scattering theory, but we shall not present it here.

In fact it is amazing that (2.1) works so well. Even in the conceptually simplest cases of relativistic potential scattering one can give examples in which it breaks down.

Examples

Example 1. Dirac particle with anomalous magnetic moment in a given electromagnetic static field (notation from Bjorken and Drell [S7]):

$$\left(i\nabla\!\!\!/-eA\!\!\!/+\frac{\kappa e}{4m}\sigma_{\mu\nu}F^{\mu\nu}-m\right)\psi=0,$$

where $A\!\!\!/=\gamma^0A^0-\boldsymbol{\gamma}\cdot\boldsymbol{A}$, $\quad\sigma_{\mu\nu}=\frac{i}{2}(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)$. Denote $K=\frac{\kappa e}{2m}$

$$(i\nabla\!\!\!/-eA\!\!\!/+\tfrac{1}{2}K\sigma_{\mu\nu}F^{\mu\nu}-m)\psi=0.\quad \text{Take}$$

$$\left(i\gamma^0\frac{\partial}{\partial t}+i\boldsymbol{\gamma}\cdot\nabla-eV\gamma^0+\tfrac{1}{2}K\sigma_{\mu\nu}F^{\mu\nu}-m\right)\psi=0$$

and multiply it by γ^0: $\quad \gamma^0=\beta=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}, \qquad \boldsymbol{\alpha}=\begin{bmatrix}0 & \boldsymbol{\sigma}\\ \boldsymbol{\sigma} & 0\end{bmatrix}$ to get

$$\left(i\frac{\partial}{\partial t}+i\boldsymbol{\alpha}\cdot\nabla-eV+\beta\tfrac{1}{2}K\sigma_{\mu\nu}F^{\mu\nu}-\beta m\right)\psi=0.$$

This equation was worked out in ref. [8].

We introduce the electric and magnetic fields $(\mathbf{E}, \mathbf{B})$ in terms of which

$$\begin{aligned}\tfrac{1}{2}K\sigma_{\mu\nu}F^{\mu\nu} &= -2\tfrac{1}{2}K(\sigma_{01}E_x+\sigma_{02}E_y+\sigma_{03}E_z)\\ &\quad +2\tfrac{1}{2}K(\sigma_{23}B_x+\sigma_{31}B_y+\sigma_{12}B_z)\\ &= -Ki\boldsymbol{\alpha}\cdot\mathbf{E}+K\boldsymbol{\Sigma}\cdot\mathbf{B},\end{aligned}$$

where $\boldsymbol{\Sigma}=(\sigma_{23},\quad \sigma_{31},\quad \sigma_{12})$

$$\sigma_{23}=\begin{bmatrix}\sigma_x & 0\\ 0 & \sigma_x\end{bmatrix}, \qquad \sigma_{31}=\begin{bmatrix}\sigma_y & 0\\ 0 & \sigma_y\end{bmatrix}, \qquad \sigma_{12}=\begin{bmatrix}\sigma_z & 0\\ 0 & \sigma_z\end{bmatrix}.$$

(The sign 'minus' in front of $Ki\,\boldsymbol{\alpha}\cdot\mathbf{E}$ is because

$$\sigma_{0i}=g_{0\nu}g_{i\mu}\sigma^{\nu\mu}=g_{00}g_{ii}\sigma^{0i}=-\sigma^{0i}).$$

We get finally

$$i\frac{\partial}{\partial t}\psi=[-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}+eV-K\beta(\boldsymbol{\Sigma}\cdot\mathbf{B}-i\boldsymbol{\alpha}\cdot\mathbf{E})+\beta m]\psi. \tag{2.2}$$

The time dependence of $\psi\sim e^{-iEt}$ implies $i\dfrac{\partial}{\partial t}\to E$ and we have

$$E\psi=[-\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}+\beta m-K\beta(\boldsymbol{\Sigma}\cdot\mathbf{B}-i\boldsymbol{\alpha}\cdot\mathbf{E})+eV]\psi$$

$$\psi=e^{ikz}\varphi=\exp\,(iz\sqrt{E^2-m^2})\,\varphi\to e^{iEz}\varphi,\qquad (E\to\infty).$$

Inserting this into the Dirac equation and noting

$$-i\frac{\partial}{\partial z}e^{iEz}\varphi=Ee^{iEz}\varphi+e^{iEz}\left(-i\frac{\partial}{\partial z}\right)\varphi \text{ we get}$$

$$E(1-\alpha_3)\varphi=[-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}'+\beta m-K\beta(\boldsymbol{\Sigma}\cdot\mathbf{B}-i\boldsymbol{\alpha}\;\mathbf{E})+eV]\varphi. \tag{2.3}$$

Hence, in the limit $E\to\infty$ we have to have

$$(1-\alpha_3)\varphi\to 0,\qquad (1+\alpha_3)\varphi\to 2\varphi.$$

This is because the right-hand side of (2.3) does not contain the energy, E.

We multiply eq (2.3) from the left by $\frac{1}{2}(1+\alpha_3)$ and get (note that $(1+\alpha_3)(1-\alpha_3)=\alpha_3{}^2=0$)

$$0=\Big[-i\tfrac{1}{2}(1+\alpha_3)\,\boldsymbol{\alpha}_\perp\cdot\boldsymbol{\nabla}_\perp-i\tfrac{1}{2}(1+\alpha_3)\,\alpha_3\frac{\partial}{\partial z}+\tfrac{1}{2}(1+\alpha_3)\beta m$$

$$-K\tfrac{1}{2}(1+\alpha_3)\beta(\boldsymbol{\Sigma}\cdot\mathbf{B}-i\boldsymbol{\alpha}\cdot\mathbf{E})+eV\tfrac{1}{2}(1+\alpha_3)\Big]\varphi.$$

But in the limit $E\to\infty$,

$$(1+\alpha_3)\,\boldsymbol{\alpha}_\perp\varphi=\boldsymbol{\alpha}_\perp(1-\alpha_3)\varphi\to 0$$

$$(1+\alpha_3)\beta\varphi=\beta(1-\alpha_3)\varphi\to 0$$

$$(1+\alpha_3)\beta\Sigma_3\varphi=\beta\Sigma_3(1-\alpha_3)\varphi\to 0$$

$$(1+\alpha_3)\beta\alpha_3\varphi=\beta\alpha_3(1-\alpha_3)\varphi\to 0,$$

where the transverse components (in x, y plane) are marked $\perp$. Thus, finally, we find

$$\left[-i\frac{\partial}{\partial z}+eV-K\beta(\boldsymbol{\Sigma}_\perp\cdot\mathbf{B}_\perp-i\boldsymbol{\alpha}_\perp\cdot\mathbf{E}_\perp)\right]\varphi=0. \tag{2.4}$$

So, if the anamalous magnetic moment $K=0$, we end up with an expression which is virtually the same as in the case of the Schrödinger equation:

$$\left(-i\frac{\partial}{\partial z}+eV\right)\varphi=0$$

whose solution

$$\varphi=u(\mathbf{k})\,\exp\left(-ie\int_{-\infty}^{z}dz'V(\mathbf{b},z')\right)$$

gives

$$\psi = u(\mathbf{k}) \exp\left(ikz - ie\int_{-\infty}^{z} dz' V(\mathbf{b}, z')\right),$$

where $u(\mathbf{k})$ is a four-spinor. As

$$\mathfrak{M}(\mathbf{k}', \mathbf{k}) = -\frac{m}{2\pi}\int d^3r \bar{\psi}_f \gamma_0 e V(\mathbf{b}, z)\psi,$$

where $\psi_f = u(\mathbf{k}') \exp(iEz + i\mathbf{\Delta}\cdot\mathbf{b})$, we get

$$\mathfrak{M}(\mathbf{k}', \mathbf{k}) = \frac{m}{2\pi}\bar{u}(\mathbf{k}')\gamma_0 u(\mathbf{k}) i\int d^2b \exp(i\mathbf{\Delta}\cdot\mathbf{b})\left[1 - \exp\left(-ie\int_{-\infty}^{+\infty} dz\, V(\mathbf{b}, z)\right)\right].$$

So, in this case we also have additivity of phase shifts—hence the Glauber model: But when $K \neq 0$ the principle of additivity of phase shifts breaks down. Let us consider this case in more detail.

From the equation $\left[-i\dfrac{\partial}{\partial z} + eV - K\beta(\mathbf{\Sigma}_\perp\cdot\mathbf{B}_\perp - i\boldsymbol{\alpha}_\perp\cdot\mathbf{E}_\perp)\right]\varphi = 0$ we can eliminate the 'trivial' dependence on V by substituting

$$\varphi = F\exp\left(-ie\int_{-\infty}^{z} dz' V(x, y, z')\right),$$

$$-i\frac{\partial}{\partial z}\varphi = \exp\left(-ie\int_{-\infty}^{z} dz' \ldots\right)\left(-i\frac{\partial}{\partial z}F\right) - eVF\exp\left(-ie\int_{-\infty}^{z} dz' \ldots\right)$$

and we get the following equation for F:

$$-i\frac{\partial}{\partial z}F = K\beta(\mathbf{\Sigma}_\perp\cdot\mathbf{B}_\perp - i\boldsymbol{\alpha}_\perp\cdot\mathbf{E}_\perp)F.$$

F is a four spinor but we can reduce it to an equation for a two component spinor because F has to satisfy the relation

$$(1-\alpha_3)F = 0, \qquad \alpha_3 = \begin{bmatrix} 0 & \sigma_z \\ \sigma_z & 0 \end{bmatrix}$$

So, F can be taken in the form

$$F = \begin{bmatrix} \chi \\ \sigma_z\chi \end{bmatrix}$$

$$-i\frac{\partial}{\partial z}F = K\begin{bmatrix} \boldsymbol{\sigma}_\perp\cdot\mathbf{B}_\perp & -i\boldsymbol{\sigma}_\perp\cdot\mathbf{E}_\perp \\ i\boldsymbol{\sigma}_\perp\cdot\mathbf{E}_\perp & -\boldsymbol{\sigma}_\perp\cdot\mathbf{B}_\perp \end{bmatrix}F$$

or

$$-i\frac{\partial}{\partial z}\chi = K(\boldsymbol{\sigma}_\perp\cdot\mathbf{B}_\perp - i\boldsymbol{\sigma}_\perp\cdot\mathbf{E}_\perp\sigma_z)\chi, \tag{2.5}$$

which is in fact a system of first order differential equations for two unknown functions (the two components of the spinor χ). Call

$$\mathfrak{A}(x, y, z) = K(\boldsymbol{\sigma}_\perp\cdot\mathbf{B}_\perp - i\boldsymbol{\sigma}_\perp\cdot\mathbf{E}_\perp\sigma_z).$$

In general

$$[\mathfrak{A}(x, y, z), \mathfrak{A}(x, y, z')] \neq 0.$$

Hence we have to use a z-ordered product to express χ in a compact form:

$$\chi = \left\{ \exp\left(i \int_{-\infty}^{z} dz' \mathcal{A}(x, y, z') \right) \right\}_{+} \chi_i.$$

Each infinitesimal step

$$\chi(z+\Delta z) - \chi(z) = i\Delta z K(\boldsymbol{\sigma}_\perp \cdot \mathbf{B}_\perp - i\boldsymbol{\sigma}_\perp \cdot \mathbf{E}_\perp \sigma_z)\chi(z),$$

$$\chi(z+\Delta z) = e^{i\Delta z K(\cdots)}\chi(z),$$

should be applied in order of increasing z's. That is what $\{\ldots\}_+$ means.

In any case, the additivity principle is violated: $\mathcal{A}_1$ and $\mathcal{A}_2$ generated by two sources of the electromagnetic field (at two different positions) are, in general, noncommuting operators and there is no way of adding phase shifts (or, equivalently, multiplying profiles). We can also see that the physical reason for this phenomenon is the coupling between different spin states produced by the term $K(\boldsymbol{\sigma}_\perp \cdot \mathbf{B}_\perp - i\boldsymbol{\sigma}_\perp \cdot \mathbf{E}_\perp \sigma_z)$. So, we have to deal with a coupled channels problem. We can also make the following remark: sometimes coupled channels can be decoupled by diagonalization.

A remark about "decoupling" channels through a diagonalization procedure

Start with a generalization (to N channels) of the eq (2.5)

$$-i \frac{\partial}{\partial z} \chi_n = \sum_{m=1}^{N} \mathcal{A}_{nm} \chi_m.$$

Note that "compositeness" of the incident particle is responsible for the existence of more than one channel. For instance, the presence of an anomalous magnetic moment can be looked upon as a mark of "compositeness." Suppose

$$\mathcal{A}_{nm}(\mathbf{r}, \mathbf{r}_1, \ldots, \mathbf{r}_A) = \sum_{j=1}^{A} \mathcal{A}_{nm}^{(j)}(\mathbf{r} - \mathbf{r}_j).$$

Diagonalization should produce a diagonal matrix of the form:

$$S^{-1}\mathcal{A}S = \begin{pmatrix} \sum_j^A \lambda_1^{(j)}(\mathbf{r}-\mathbf{r}_j) & 0 & 0 & \cdots \\ 0 & \sum_j^A \lambda_2^{(j)}(\mathbf{r}-\mathbf{r}_j) & 0 & \cdots \\ 0 & 0 & & \\ \vdots & \vdots & & \end{pmatrix}$$

$$= \sum_{j=1}^{A} \begin{pmatrix} \lambda_1^{(j)}(\mathbf{r}-\mathbf{r}_j) & 0 & 0 & \cdots \\ 0 & \lambda_2^{(j)}(\mathbf{r}-\mathbf{r}_j) & 0 & \cdots \\ 0 & 0 & & \\ \vdots & \vdots & & \end{pmatrix}$$

One achieves additivity if the same S diagonalizes all $\mathcal{A}^{(j)}_{nm}(\mathbf{r}-\mathbf{r}_j)$ *simultaneously*. If it does not, there is no context in which one could talk about additivity of phase shifts. In general the additivity does not occur. Take, e.g., pure Coulomb scattering ($B=0$, $V=$Coulomb potential) in eq (2.5): $-i\dfrac{\partial\chi}{\partial z}=\mathcal{A}\chi$. We have to diagonalize the matrix of eq (2.5) (compare ref. [8]):

$$S^{-1}\begin{pmatrix} 0 & -i(x-iy) \\ i(x+iy) & 0 \end{pmatrix} S=\begin{pmatrix} -(x^2+y^2)^{1/2} & 0 \\ 0 & (x^2+y^2)^{1/2} \end{pmatrix},$$

where

$$S=\begin{pmatrix} (x-iy)^{1/2} & (x-iy)^{1/2} \\ -i(x+iy)^{1/2} & i(x+iy)^{1/2} \end{pmatrix}$$

$$S^{-1}=\begin{pmatrix} i(x+iy)^{1/2} & -(x-iy)^{1/2} \\ i(x+iy)^{1/2} & (x-iy)^{1/2} \end{pmatrix}\frac{1}{2i(x^2+y^2)^{1/2}}.$$

Even this soluble case cannot be diagonalized for more than one scattering center if the Dirac particle has an anomalous magnetic moment, $K\neq 0$.

Without going into any details of the calculation let us quote the results. In the case when only one Coulomb potential is present (hence $B=0$, but $E\neq 0$), we have

$$\mathfrak{M}(\Delta)\sim i\chi_f^+\left\{\int d^2b\,\exp\,(i\boldsymbol{\Delta}\cdot\mathbf{b})\left[1-\exp\left(-ie\int_{-\infty}^{+\infty}dz V(\mathbf{b},z)+i\int_{-\infty}^{+\infty}dz\mathcal{A}(\mathbf{b},z)\right)\right]\right\}\chi_i,$$

where

$$\mathcal{A}(\mathbf{b},z)=K\frac{V'(r)}{r}\begin{pmatrix} 0, & -i(x-iy) \\ i(x+iy), & 0 \end{pmatrix}.$$

Note that since the z dependence is outside the spinor matrix this *does* commute at different z's: $[\mathcal{A}(b,z),\ \mathcal{A}(b,z')]=0$. Suppose, however, we have *two* sources of Coulomb field at two different points. Then

$$\mathcal{A}=\mathcal{A}_1+\mathcal{A}_2=K\frac{V_1'(|\mathbf{r}-\mathbf{r}_1|)}{|\mathbf{r}-\mathbf{r}_1|}\begin{pmatrix} 0, & -i[(x-x_1)-i(y-iy_1)] \\ i[(x-x_1)+i(y-y_1)], & 0 \end{pmatrix}$$
$$+K\frac{V_2'(|\mathbf{r}-\mathbf{r}_2|)}{|\mathbf{r}-\mathbf{r}_2|}\begin{pmatrix} 0, & -i[(x-x_2)-i(y-iy_2)] \\ i[(x-x_2)+i(y-y_2)], & 0 \end{pmatrix}.$$

Now, however, we do not have the Glauber model any more!

$$\mathfrak{M}(\Delta)\sim i\chi_f^+\left(\int d^2b\,\exp\,(i\boldsymbol{\Delta}\cdot\mathbf{b})\left[1-\exp\left(-ie\int_{-\infty}^{+\infty}dz\,(V_1+V_2)\right)\left\{\exp\left(i\int_{-\infty}^{+\infty}dz\,(\mathcal{A}_1+\mathcal{A}_2)\right)\right\}_+\right]\right)\chi_i$$

where $\{\ldots\}_+$ denotes the z-ordered product.
Because

$$\begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}\begin{pmatrix} 0 & c \\ d & 0 \end{pmatrix}\neq\begin{pmatrix} 0 & c \\ d & 0 \end{pmatrix}\begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}$$

we have

$$[\mathcal{A}(x,y,z),\quad \mathcal{A}(x,y,z')]\neq 0.$$

One could argue that the coupling to the anomalous moment is weak and hence not very relevant.

This is true, but one can give some other—though much more complex—examples of scattering from a classical external field in which the "principle of additivity of phase shifts" is also violated.

Let us consider a vector particle (hence a very relevant kind of particle to our further analysis).

Example 2. Scattering of a charged vector meson in a static field (we shall quote the results, for more details see refs. [9, 10]).

Let us allow for our vector particle to have an arbitrary magnetic moment and define the magnetic moment operator

$$\mathbf{M} = (1+\tilde{\kappa})\,\frac{e}{2m}\,\mathbf{S}, \tag{2.6}$$

($\mathbf{S}$—spin operator), where $\tilde{\kappa}$ determines the value of the magnetic moment. When $\tilde{\kappa}=0$ the equations of motion of such a particle are the so-called Proca equations. If, however, $\tilde{\kappa}\neq 0$ some additional terms appear (as in the case of the Dirac equation with anomalous magnetic moment). With $\tilde{\kappa}\neq 0$ we have (in the pseudo-euclidean metric, $\mu, \nu = 1, 2, 3, 4$), (compare ref. [11]),

$$\partial_\mu G_{\mu\nu} - m^2\varphi_\nu + ie\tilde{\kappa}\varphi_\mu F_{\mu\nu} = 0$$

where

$$G_{\mu\nu} = \partial_\mu\varphi_\nu - \partial_\nu\varphi_\mu, \qquad \partial_\mu = \frac{\partial}{\partial x_\mu} - ieA_\mu$$

$$F_{\mu\nu} = \frac{\partial A_\nu}{\partial x_\mu} - \frac{\partial A_\mu}{\partial x_\nu}.$$

We shall choose $A_\mu = i\delta_{\mu 4}V(\mathbf{r})$ (just the static Coulomb field). The results of a long and involved analysis [9, 10] are as follows:

1. We recover the principle of additivity of phase shifts only in the case $\tilde{\kappa}=1$.
2. In all the other cases (including $\tilde{\kappa}=0$), there is no additivity of phase shifts.

A general comment on Examples 1 and 2 is in order here. First: terminology. Many authors call $\tilde{\kappa}$ the anomalous magnetic moment of the vector meson [9, 10]. This is presumably so because when one starts with the free vector meson field equations

$$\frac{\partial}{\partial x_\mu}\left(\frac{\partial}{\partial x_\mu}\,\varphi_\nu - \frac{\partial}{\partial x_\nu}\,\varphi_\mu\right) - m^2\varphi_\nu = 0$$

and then introduces the electromagnetic field in the standard way $\left(\partial_\mu \equiv \dfrac{\partial}{\partial x_\mu} - ieA_\mu\right)$ one obtains the $\tilde{\kappa}=0$ case. (Similarly, if one starts with the free particle Dirac equation and introduces A_μ in the same way, one obtains the $\kappa=0$ case of Example 1). So, from this point of view, the cases $\kappa=0$ of Example 1 and $\tilde{\kappa}=0$ of Example 2 are analogous, and, as we know, in the first case the spin channels decouple in the high energy limit whereas in the second case they do not.

One may ask oneself a question: is there any simple way of telling which values of κ and $\tilde{\kappa}$ result in decoupling of various spin states in the high energy limit? The answer seems to be: yes. It is enough to observe that in Example 1 for $\kappa=0$, the relation between the magnetic moment $\mathbf{M}$ and spin $\mathbf{S}$ is

$$\mathbf{M} = \frac{e}{m}\,\mathbf{S} \tag{2.7}$$

where e is the charge and m the mass of the particle. Note that eq (2.6) gives the same relation between magnetic moment and spin when $\tilde{\kappa}=1$! So, in both Examples, *when κ and $\tilde{\kappa}$ are chosen to make eq* (2.7) *valid, the spin states decouple in the high energy limit.*

In order to make the condition (2.7) more plausible, let us consider a charged particle with spin **S** and magnetic moment **M** given by (2.7) moving in an almost uniform magnetic field B. This particle follows a circular trajectory with frequency

$$\omega = \frac{e}{m} B.$$

Its magnetic moment (hence its spin) precesses with frequency

$$\omega_p = \frac{M}{S} B = \frac{e}{m} B.$$

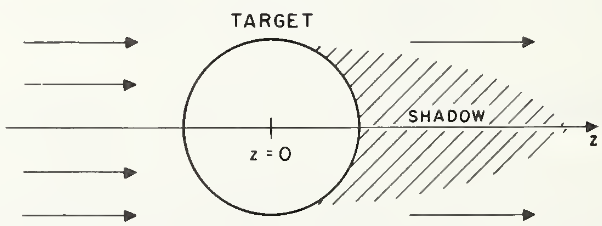
So eq (2.7) makes these two frequencies equal. But that means that *the projection of the spin on the direction of particle velocity is a constant of the motion.* Hence in this case all helicity spin states are decoupled.

Although we have considered a very special case of nonrelativistic motion in a constant magnetic field, the condition (2.7) for the decoupling of spin channels turns out to be very general: The relevant relativistic formulae for precession of the polarization of particles with arbitrary magnetic moments and spins in a slowly varying (in space) electromagnetic field were given in ref. [50]. Their immediate consequence is [51] that in the high energy limit and for the gyromagnetic ratio $g=2$ (hence when (2.7) is valid because the definition of g is through the equation $M=g\,(e/2m)\,S$) the projection of the polarization on the direction of motion is constant, and hence there is no coupling between various spin channels.

To conclude this section we may say that Examples 1 and 2 warn us that if the strong interactions are mediated through vector fields (analogous to the electromagnetic field) one can expect the "principle of additivity of phase shifts" to be violated.

3. Elastic Scattering of Hadrons from Nuclei

Let us go back for a moment to scattering of incident particles whose internal structure one can neglect (in particular the internal quantum numbers can be neglected). Let us start with just one scatterer:



The incident wave: e^{ikz}. The wave immediately behind the scatterer: $\approx e^{ikz} - \gamma(b) e^{ikz}$, $b \equiv (x,y)$. The shape of the shadow is given by $\gamma(b)$:

$$\gamma(b) = \frac{1}{2\pi ik} \int d^2\delta \exp\,(i\boldsymbol{\delta}\cdot\mathbf{b}) f(\delta),$$

where $f(\delta)$ is the elastic scattering amplitude as shown below,

$$k^2 = k_x^2 + k_y^2 + k_z^2, \qquad k_z = \sqrt{k^2 - \delta^2}, \qquad k_z z \cong kz - \frac{\delta^2 z}{2k^2}.$$

As long as $\delta^2 z/2k^2 \lesssim 1$, the z-dependence of the second term in the wave immediately behind the scatterer is given to a good approximation by e^{ikz}. Otherwise one should realize that k_z depends on δ which sits in the Fourier transform of the shadow. Hence, away from the scatterer, one would guess the following shape of the wave (compare D. R. Yennie article in [S3]):

$$e^{ikz} - \frac{1}{2\pi ik}\int d^2\delta \exp\left(iz\sqrt{k^2-\delta^2}\right) \exp\left(i\boldsymbol{\delta}\cdot\mathbf{b}\right) f(\delta) \rightarrow e^{ikz} - \gamma(b) e^{ikz} \qquad \text{(for small } z\text{'s).} \quad (3.1)$$

If the size of $\gamma(b)$, and hence of the scatterer, is a, the representative transverse momentum transfer is $\delta \approx a^{-1}$. We can then estimate the "healing" length, L, of the shadow:

$$\frac{L}{2ka^2} \lesssim 1, \qquad L \lesssim 2ka^2;$$

for $a = 1$ fm, $k = 10$ GeV we obtain

$$2ka^2 = 2 \times 10 \text{ GeV} \times 25 \text{ GeV}^{-2} = 100 \text{ fm}.$$

Note that (3.1) gives, as $r \rightarrow \infty$, (compare D. R. Yennie article in [S3])

$$\psi(\mathbf{r}) = e^{ikz} + [f(k\mathbf{e}_\perp)/r] \exp(ikr), \qquad (\mathbf{e}_\perp = \text{component of } \mathbf{r} \perp \mathbf{e}_z),$$

with $f(k\mathbf{e}_\perp)$ correctly given by the inverse of $\gamma(b)$. One can see this by shifting the origin of integration to $k\mathbf{e}_\perp$:

$$\boldsymbol{\delta} = k\mathbf{e}_\perp + \mathbf{q} \qquad \text{(we introduce a new variable } \mathbf{q}\text{).}$$

Then

$$z\sqrt{k^2-\delta^2} + \boldsymbol{\delta}\cdot\mathbf{b} \approx kr - \frac{1}{2}\frac{q^2 r}{k},$$

and

$$-\frac{1}{2\pi ik} e^{ikr} \int d^2q \exp\left(-\frac{i}{2}\frac{q^2 r}{k}\right) f(k\mathbf{e}_\perp + \mathbf{q}) \approx \frac{1}{r} f(k\mathbf{e}_\perp) e^{ikr}.$$

Remark:

When the incident wave already has a profile different from unity we get:

incident wave: $g(x, y) e^{ikz}$
transmitted wave: $g(x, y) e^{ikz}(1 - \gamma(x, y))$
(this is all under the assumption $z \ll L$).

Let us construct the "shape of the shadow" for a collection of scatterers (nucleons in a nucleus; see e.g., fig. 1):

The incident wave: e^{ikz}
after the first collision: $[1 - \gamma_1(\mathbf{b} - \mathbf{s}_1)] e^{ikz}$
after the second collision: $[1 - \gamma_2(\mathbf{b} - \mathbf{s}_2)][1 - \gamma_1(\mathbf{b} - \mathbf{s}_1)] e^{ikz}$
etc.

The 'shape of the shadow' for the whole collection is then

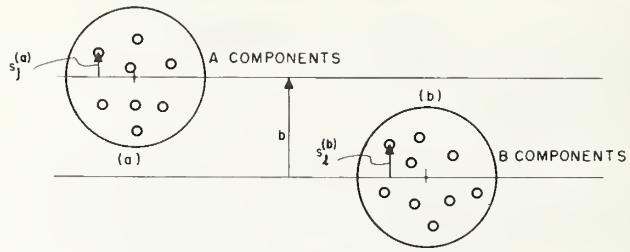
$$[1 - \Gamma(\mathbf{b}, \mathbf{s}_1 \ldots \mathbf{s}_A)] e^{ikz} = \prod_{j=1}^{A} [1 - \gamma_j(\mathbf{b} - \mathbf{s}_j)] e^{ikz}$$

$$\Gamma(\mathbf{b}, \mathbf{s}_1 \ldots \mathbf{s}_A) = 1 - \prod_{j=1}^{A} [1 - \gamma_j(\mathbf{b} - \mathbf{s}_j)].$$

So, we get again the formulae of section 2.

The previous case dealt with an elementary object scattering from a composite object. We already saw in the examples of scattering of relativistic particles from external electromagnetic fields that "internal structure" (in these cases the internal spin quantum numbers + anomalous magnetic moment) breaks down the "ansatz" of additivity of phase shifts. We can also have a look at this problem from the point of view of a Glauber-like description of scattering of two composite objects. The formulae given below are interesting also because they may be used to analyze high energy nucleus-nucleus collisions (which is not an academic problem because there are experimental projects under way).

The geometry of the process is shown in figure 3.



The profile describing the collision of two elements is:

$$\gamma_{jk}(\mathbf{b}-\mathbf{s}_l^{(b)}+\mathbf{s}_j^{(a)}).$$

For the sake of simplicity let us take the wave functions of (a) and (b) in the form of products of single particle wave functions. Let us assume also that all particles have the same single particle wave functions. The ground state wave functions are:

$$\Psi^{(a)}=\prod_j^A \varphi_0(r_j^{(a)}), \qquad \Psi^{(b)}=\prod_l^B \varphi_0(r_l^{(b)}),$$

and the elastic scattering amplitude is, therefore:

$$\mathfrak{M}=\frac{ik}{2\pi}\int d^2b \exp(i\boldsymbol{\Delta}\cdot\mathbf{b})\int d^3r_1^{(a)}\ldots d^3r_A^{(a)}\int d^3r_1^{(b)}\ldots d^3r_B^{(b)}\prod_j^A\prod_l^B \varphi_0^{(a)*}(r_j^{(a)})\varphi_0^{(b)*}(r_l^{(b)})$$

$$\times\left\{1-\prod_j^A\prod_l^B[1-\gamma_{jl}(\mathbf{b}-\mathbf{s}_l^{(b)}+\mathbf{s}_j^{(a)})]\right\}\varphi_0^{(a)}(r_j^{(a)})\varphi_0^{(b)}(r_l^{(b)}). \tag{3.2}$$

What kind of formulae would we have if the "ansatz" of additivity of phase shifts of the composite system (b) colliding with nucleons of (a) were valid? Let us look at the profile of the jth nucleon:

$$\Gamma_j(b)=\int d^2s_1^{(b)}\ldots d^2s_B^{(b)}\prod_l^B\rho^{(b)}(s_l)\left[1-\prod_j^B(1-\gamma_{jl}(\mathbf{b}-\mathbf{s}_l))\right]$$

$$=1-(1-\tilde{\gamma}_j(b))^B,$$

where we assumed all profiles to be identical and introduced $\tilde{\gamma}$

$$\tilde{\gamma}_j(b)=\int d^3r_l\,\varphi_0^{(b)*}(r_l^{(b)})\varphi_0^{(b)}(r_l^{(b)})\gamma_{jl}(\mathbf{b}-\mathbf{s}_l^{(b)})$$

which is the profile for elastic scattering of (b) from the jth nucleon of (a) and a two-dimensional density

$$\rho(s)=\int_{-\infty}^{+\infty}dz\,\varphi_0^*(\mathbf{r})\varphi_0(\mathbf{r}).$$

Then the additivity of phase shifts gives us the formula:

$$\mathfrak{M}=\frac{ik}{2\pi}\int d^2b\,\exp\,(i\boldsymbol{\Delta}\cdot\mathbf{b})\int d^2s_1{}^{(a)}\ldots d^2s_A{}^{(a)}\prod_j^A\rho^{(a)}(s_j{}^{(a)})\left\{1-\prod_j^A\left(1-\Gamma_j(\mathbf{b}-\mathbf{s}_j{}^{(a)})\right)\right\}. \tag{3.3}$$

This is different from (3.2). What is the difference? First let us note that (3.2) is a sum rule. For instance, we can extract from (3.2) the following contribution of the second order

$$\int d^3r_l{}^{(b)}\varphi_0{}^{(b)*}(r_l{}^{(b)})\,\gamma_{1l}(\mathbf{b}-\mathbf{s}_l{}^{(b)}+\mathbf{s}_1{}^{(a)})\,\gamma_{2l}(\mathbf{b}-\mathbf{s}_l{}^{(b)}+\mathbf{s}_2{}^{(a)})\,\varphi_0{}^{(b)}(r_l)$$

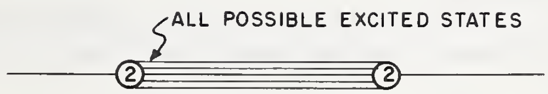
$$\equiv\int d^3r_l{}^{(b)}d^3r_l{}^{(b)\prime}\varphi_0{}^{(b)*}(r_l)\,\gamma_{1l}(\mathbf{b}-\mathbf{s}_l{}^{(b)}+\mathbf{s}_1{}^{(a)})\sum_n\varphi_n{}^{(b)}(r_l{}^{(b)})\,\varphi_n{}^{(b)*}(r_l{}^{(b)\prime})$$
$$\times\gamma_{2l}(\mathbf{b}-\mathbf{s}_l{}^{(b)\prime}+\mathbf{s}_2{}^{(a)})\,\varphi_0{}^{(b)}(r_l{}^{(b)\prime})$$

$$=\sum_n\int d^3r_l{}^{(b)}\varphi_0{}^{(b)*}(r_l{}^{(b)})\,\gamma_{1l}(\mathbf{b}-\mathbf{s}_l{}^{(b)}+\mathbf{s}_1{}^{(a)})\,\varphi_n{}^{(b)}(r_l{}^{(b)})\int d^3r_l{}^{(b)\prime}\varphi_n{}^{(b)*}(r_l{}^{(b)\prime})$$
$$\times\gamma_{2l}(\mathbf{b}-\mathbf{s}_l{}^{(b)\prime}+\mathbf{s}_2{}^{(a)})\,\varphi_0{}^{(b)}(r_l{}^{(b)}),$$

because

$$\sum\varphi_n{}^{(b)}(r_l{}^{(b)})\,\varphi_n{}^{(b)*}(r_l{}^{(b)\prime})=\delta^{(3)}(r_l{}^{(b)}-r_l{}^{(b)\prime}).$$

Hence the formula (3.2) sums over all intermediate excited states. For instance, the above contribution gives:



Suppose we reject the intermediate *excited states* and take only the *ground state* as a possible intermediate state (this is the way to eliminate all channels but one). Then each γ_{jl} can be averaged over $r_l{}^{(b)}$:

$$\int d^3r_l{}^{(b)}\,|\,\varphi_0{}^{(b)}(r_l{}^{(b)}\,|^2\,\gamma_{jl}(\mathbf{b}-\mathbf{s}_l{}^{(b)}+\mathbf{s}_j{}^{(a)})$$

$$=\int d^2s_l{}^{(b)}\rho^{(b)}(s_l)\,\gamma_{jl}(\mathbf{b}-\mathbf{s}_l{}^{(b)}+\mathbf{s}_j{}^{(a)})$$ (it does not depend on l when all nucleons are "equivalent")

$$=\tilde{\gamma}_{jl}(\mathbf{b}+\mathbf{s}_j{}^{(a)})$$

Then the formula (3.2) reduces to (3.3):

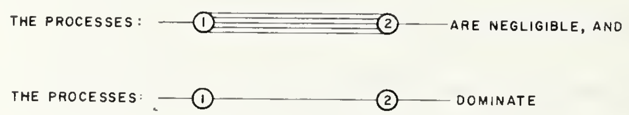
$$\mathfrak{M}=\frac{ik}{2\pi}\int d^2b\,\exp\,(i\boldsymbol{\Delta}\cdot\mathbf{b})\int d^2s_1{}^{(a)}\ldots d^2s_A{}^{(a)}\prod_j^A\rho^{(a)}(s_j{}^{(a)})\left\{1-\prod_j^A\left(1-\tilde{\gamma}_{jl}(\mathbf{b}+\mathbf{s}_j{}^{(a)})\right)^B\right\}$$

$$=\frac{ik}{2\pi}\int d^2b\,\exp\,(i\boldsymbol{\Delta}\cdot\mathbf{b})\int d^2s_1{}^{(a)}\ldots d^2s_A{}^{(a)}\left\{1-\prod_j^A\left(1-\Gamma_j(\mathbf{b}+\mathbf{s}_j{}^{(a)})\right)\right\}\prod_j^A\rho^{(a)}(s_j{}^{(a)})$$

(to make it identical to (3.3) we should substitute $\mathbf{s}_j{}^{(a)}\rightarrow-\mathbf{s}_j{}^{(a)}$). Hence we get a formula which follows from additivity of phase shifts.

It would seem, therefore, that indeed "compositeness" of the incident particle is decisive in destroying or satisfying additivity. The other "moral" is that if we know the structure of the composite body (b) we may still use a generalized Glauber model with additivity of all possible phase shifts of the pairs of components of (a) and (b).

Let us consider some limiting cases of eq (3.2) (compare ref. [12]). Let the radii of the two composite objects be R_a and R_b. The calculations of ref. [12] show that the smaller is R_b the nearer we are to the additivity of (b)-nucleon phase shifts. But that means that this additivity improves with increase of the binding of (b). Of course for $R_b \to 0$ the additivity becomes exact. One can see this explicitly by replacing for (b), $\rho^{(b)}(s) \sim \delta^{(2)}(s)$ (then (b) is a point-like object). When $R_b \to 0$ we in fact remove all the intermediate excited states already mentioned: In this case



Having written down the formula (3.2) this is a good place to discuss it a little further. As we have already said, it would be very interesting to test formulae of the type (3.2) against some experimental data. There is, however, very little data in existence to analyze. To the best of my knowledge, only deuteron-deuteron scattering data are available, but reliable calculations are very difficult because of the high spins involved. Nevertheless, there exist some calculations [S1] and there seems to be reasonable agreement between theory and experiment. But we shall talk about comparison with experiment at other occasions.

Some special cases of formula (3.2) were also employed to describe hadron-hadron scattering in the high energy limit. For example, the limit when A and B become very large was considered [13] (compare also [12]):

$$\lim_{A,B\to\infty} \mathfrak{M} = \frac{ik}{2\pi} \int d^2b \exp\,(i\mathbf{\Delta}\cdot\mathbf{b}) \left[1 - \exp\left(-AB \int d^2s^{(a)} d^2s^{(b)} \rho^{(a)}(s^{(a)}) \gamma(\mathbf{b} - \mathbf{s}^{(b)} - \mathbf{s}^{(a)}) \rho^{(b)}(s^{(b)})\right)\right] \tag{3.4}$$

where the ρ's were defined before and we assume that all γ_{jl}'s are the same. One gets this formula trivially from

$$\mathfrak{M} \cong \frac{ik}{2\pi} \int d^2b \exp\,(i\mathbf{\Delta}\cdot\mathbf{b}) \left[1 - \left(1 - \int d^2s^{(a)} d^2s^{(b)} \rho^{(a)}(s^{(a)}) \gamma(\mathbf{b} - \mathbf{s}^{(b)} + \mathbf{s}^{(a)}) \rho^{(b)}(s^{(b)})\right)^{AB}\right]$$

as $AB \to \infty$. But this formula has no intermediate excited states, neither of (b) nor of (a). So, the Chou & Yang [13] limit $A, B \to \infty$ looses all excited state contributions and becomes (3.4). *Equation (3.4) gives the well-known "droplet model" [13] elastic scattering amplitude of two composite objects whose hadronic matter distributions are given by $\rho^{(a)}(s^{(a)})$ and $\rho^{(b)}(s^{(b)})$.*

If one assumes that $\gamma(b)$ is a very narrow function of b (hence the components of the two hadrons are very small) we can write

$$\int d^2s^{(a)} d^2s^{(b)} \rho^{(a)}(\mathbf{s}^{(a)}) \gamma(\mathbf{b} - \mathbf{s}^{(b)} + \mathbf{s}^{(a)}) \rho^{(b)}(\mathbf{s}^{(b)}) \cong \kappa \int d^2s \rho^{(a)}(\mathbf{s}) \rho^{(b)}(\mathbf{b} + \mathbf{s})$$

$$= \frac{1}{(2\pi)^2} \int d^2q \exp\,(-i\mathbf{q}\cdot\mathbf{b}) F_{(a)}(q) F_{(b)}(q),$$

where κ is a free parameter.

If we accept that the densities of hadronic matter are the same as charge densities, $F_{(a)}$, $F_{(b)}$ are the charge form factors of the colliding hadrons. This formula was used successfully to:

(i) reproduce the proton charge form factors from elastic scattering hadron-hadron cross sections.
(ii) predict diffractive structure (e.g., diffractive minima) of the high energy hadron-hadron collisions.

The very recent measurements of p-p elastic collisions confirm the existence of such a structure (CERN-Serpukhov experiment).

In the form given above, the droplet model is very crude and I do not want to go beyond this qualitative description. One should perhaps mention at this point that the amplitude (3.4) contains the geometric shape of the colliding objects (e.g., their transverse density distributions). If these geometric characteristics do not depend on energy, one gets the total cross section (from the optical theorem) which is energy independent. So, it seems to be difficult to reconcile this model with the recent evidence for the increase of the total hadron-proton cross sections at very high energies (compare the data e.g., analyzed in ref. [14]).

Selection of formulae taken from a standard partial wave expansion [S6]

$$f_{\mathrm{el}}(k,\theta)=\frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)(\eta_l-1)P_l(\cos\theta),$$

$$\sigma_{\mathrm{el}}(k)=\pi\lambda\!\!\!^{-}{}^2\sum_{l=0}^{\infty}(2l+1)\,|\eta_l-1|^2,$$

$$\sigma_{\mathrm{TOT}}(k)=\frac{4\pi}{k}\,\mathrm{Im}\,f_{\mathrm{el}}(k,0),\qquad \sigma_{\mathrm{TOT}}(k)=\sigma_{\mathrm{el}}(k)+\sigma_{\mathrm{inel}}(k),$$

$$\sigma_{\mathrm{TOT}}(k)=2\pi\lambda\!\!\!^{-}{}^2\sum_{l=0}^{\infty}(2l+1)(1-\mathrm{Re}\,\eta_l)$$

$$\sigma_{\mathrm{inel}}(k)=\pi\lambda\!\!\!^{-}{}^2\sum_{l=0}^{\infty}(2l+1)(1-|\eta_l|^2)$$

$$P_l(\cos\theta)\cong J_0(2(l+\tfrac{1}{2})\sin\tfrac{1}{2}\theta),\qquad \sin^2\tfrac{1}{2}\theta\ll 1,\qquad l\gg 1,\tag{3.5}$$

$$f_{\mathrm{el}}(k,\theta)\cong\frac{1}{2ik}\int_0^{\infty}dl\,(l+\tfrac{1}{2})J_0(2(l+\tfrac{1}{2})\sin\tfrac{1}{2}\theta)(\eta_l-1),$$

$$bk=l+\tfrac{1}{2},\qquad k\,db=dl,$$

$$f_{\mathrm{el}}(k,\theta)\cong -ik\int_0^{\infty}db\,bJ_0(\Delta b)[\eta(b)-1],$$

$$\Delta=2k\sin\tfrac{1}{2}\theta,\qquad \eta\left(\frac{l}{k}\right)=\eta_l.$$

Optical theorem and unitarity

Let us first consider the "elementary" collisions (whose scattering amplitude is determined by the profile $\gamma(b)$). As the wave passes a scatterer it gets modified by a factor $1-\gamma(b)$. Hence, the probability that the particle gets removed from the incident beam is $1-|1-\gamma(b)|^2=2\,\mathrm{Re}\,\gamma(b)-|\gamma(b)|^2$ (at the impact parameter b). Notice that here we use the same expression as in the following paragraphs: we identify $1-\eta$ with γ, and $1-|\eta|^2=1-|1-\gamma|^2$. Hence,

$$\sigma_{\text{inel}}=\int d^2b[2\,\text{Re}\,\gamma(b)-|\gamma(b)|^2].$$

As $\sigma_{\text{el}}=\int d^2b\,|\gamma(b)|^2$, (see *Remark* below) we have

$$\sigma_{\text{TOT}}=\int d^2b\,|\gamma(b)|^2+\int d^2b[2\,\text{Re}\,\gamma(b)-|\gamma(b)|^2]$$

$$=\int d^2b\,2\,\text{Re}\,\gamma(b).$$

We have, however,

$$f(\boldsymbol{\delta})=\frac{ik}{2\pi}\int d^2b\,\exp\,(i\boldsymbol{\delta}\cdot\mathbf{b})\gamma(b)$$

$$f(0)=\frac{ik}{2\pi}\int d^2b\gamma(b).$$

So,

$$\frac{4\pi}{k}\,\text{Im}\,f(0)=\int d^2b\,2\,\text{Re}\,\gamma(b)=\sigma_{\text{TOT}}.$$

Hence we do have the optical theorem built into our model.

Remark:

$$\frac{d\sigma}{d\Omega}=|f(\delta)|^2,\qquad \sigma_{\text{el}}=\int d\Omega\,|f|^2$$

$$=\int d\theta\,\sin\theta\,d\phi\,|f|^2\approx\frac{1}{(2\pi)^2}\int d^2\delta\int d^2b\,\exp\,(i\boldsymbol{\delta}\cdot\mathbf{b})\gamma(b)\int d^2b'\,\exp\,(-i\boldsymbol{\delta}\cdot\mathbf{b}')\gamma^*(b')$$

For small θ we have:

$$d^2\delta=d\delta\,\delta\,d\phi\cong k^2\theta\,d\theta\,d\phi,$$

hence

$$\sigma_{\text{el}}=\int d^2b\,|\gamma(b)|^2.$$

Now let us go over to composite targets.

Consider the case when one "elementary" particle scatters from a "composite" nucleus. In this case the profile is

$$\langle\Gamma\rangle=\langle\Psi_0\,|\,\Gamma(\mathbf{b};\mathbf{s}_1\ldots\mathbf{s}_A)\,|\,\Psi_0\rangle,$$

and we can write the same relations as before:

$$\sigma_{\text{TOT}}=\int d^2b\,|\langle\Gamma\rangle|^2+\int d^2b[2\,\text{Re}\,\langle\Gamma\rangle-|\langle\Gamma\rangle|^2]$$

because, due to the same arguments as before, $1-|1-\langle\Gamma\rangle|^2$ gives the probability (at the impact parameter $\mathbf{b}$) of losing the incident particle from the elastic channel. It is convenient however to split the second term into two physically different contributions:

$$\int d^2b[2\,\mathrm{Re}\,\langle\Gamma\rangle - |\langle\Gamma\rangle|^2] = \int d^2b[\langle\Gamma^+\Gamma\rangle - |\langle\Gamma\rangle|^2] \qquad (=\sigma_{\mathrm{DT}})$$

$$+\int d^2b[2\,\mathrm{Re}\,\langle\Gamma\rangle - \langle\Gamma^+\Gamma\rangle] \qquad (=\sigma_{\mathrm{PROD}})$$

Interpretation

The first contribution (σ_{DT}) comes from processes during which the target gets dissociated—*without producing any new particles*:

$$\sigma_{\mathrm{DT}} = \int d^2b[\sum_n \langle 0|\Gamma^+|n\rangle\langle n|\Gamma|0\rangle - |\langle 0|\Gamma|0\rangle|^2]$$

$$= \int d^2b[\sum_n |\langle n|\Gamma|0\rangle|^2 - |\langle 0|\Gamma|0\rangle|^2],$$

—the second contribution (σ_{PROD}) takes care of production processes coming from the *nucleons of the target nucleus:*

$$\sigma_{\mathrm{PROD}} = \left\langle \int d^2b[1-|\eta|^2]\right\rangle = \int d^2b[1-\langle|1-\Gamma|^2\rangle]$$

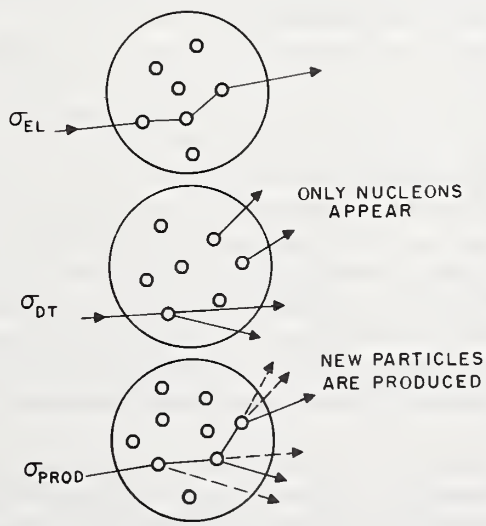
$$= \int d^2b[2\,\mathrm{Re}\,\langle\Gamma\rangle - \langle\Gamma^+\Gamma\rangle]$$

where the "reflection coefficient" $\eta(\mathbf{b}, \mathbf{s}_1 \ldots \mathbf{s}_A)$ (compare formulae (3.5)) is related to $\Gamma(\mathbf{b}, \mathbf{s}_1 \ldots \mathbf{s}_A)$ as follows:

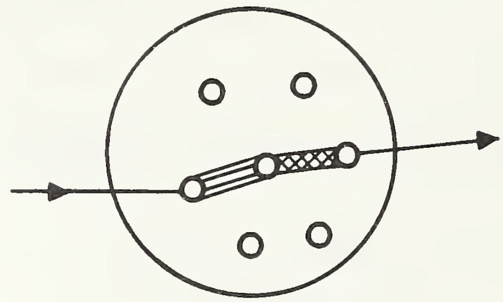
$$1-\eta(\mathbf{b}, \mathbf{s}_1 \ldots \mathbf{s}_A) = \Gamma(\mathbf{b}, \mathbf{s}_1 \ldots \mathbf{s}_A).$$

Hence $1-|\eta(\mathbf{b}, \mathbf{s}_1 \ldots \mathbf{s}_A)|^2$ gives (compare the formulae (3.5) of the standard partial wave analysis) the production cross section at the impact parameter $b \approx (l+\frac{1}{2})/k$ with all nucleons frozen at the positions $\mathbf{s}_1, \ldots \mathbf{s}_A$.

So, in our model there are three different contributions.



But as long as we construct the profiles of the target nucleus from profiles of elastic scattering, the processes like the one shown in figure 7 (with excited states of the projectile present at intermediate steps) are excluded.



They are the source of the so-called inelastic screening (or inelastic shadowing) phenomenon [47]. In order to include them we have to ascribe some kind of structure to the incident particle. Earlier in these notes we gave some examples of such cases.

To analyze this problem in more detail, one has to link it with diffractive production processes and we shall postpone such a discussion until our analysis of such processes. Here, let us make only the following points:

(i) Diffractive production processes are presumably weak (at least at energies of a few GeV) compared to elastic scattering processes (the cross section is $\sim 1/10$ of elastic cross section). (In fact this is one of the very important questions to be answered by the very high energy experiments of the future: how much cross section goes into diffractive production processes.)

(ii) Nondiffractive processes are presumably not contributing to the inelastic shadow—because the whole configuration of the target would eventually have to go back to the initial one—a very complex process in which the whole of the nucleus must take part (hence it occurs with small probability).

(iii) Hence "inelastic shadowing" stands a good chance to contribute little (a few percent) to the elastic cross section.

If this is so, then the three contributions to σ_{TOT} discussed above do approximately exhaust the list of processes contributing to elastic scattering.

From our discussion of the components of $\sigma_{\mathrm{TOT}}(\sigma_{\mathrm{el}}, \sigma_{\mathrm{DT}}, \sigma_{\mathrm{PROD}})$ it follows that the measurements of σ_{TOT} may be a good way of finding out whether the inelastic shadowing (or inelastic screening) corrections are important at very high energies: If one computed σ_{TOT} from the Glauber model (including all possible effects which the model allows for) and then found a definite discrepancy with experimentally measured σ_{TOT}—it would very strongly suggest the existence of inelastic shadowing phenomena described above. In fact such an analysis has recently been done for π-d scattering and seems to indicate the existence of such a discrepancy for energies above ~ 40 GeV [48].

The remaining important corrections to be discussed (although they are, in principle, included in the algorithm presented above) are: (i) the Coulomb corrections which play an important role in elastic scattering from nuclei of charged hadrons, and (ii) the corrections for the c.m. motion which are important for light nuclei but unimportant for heavy ones.

Let us consider first the Coulomb corrections for heavy nuclei. One can, in principle, use the individual amplitudes which have Coulomb interactions built into them (this very tedious calculation has been done, e.g., in refs. [16, 17], but we shall consider the effects produced by the average Coulomb potential produced by the whole nucleus [15] which produces almost identical results [16, 17].

We shall assume that, in the high energy limit, the total phase shift is the sum of the Coulomb phase shift (χ_c, the phase shift one would get if the strong interactions were switched off) and the strong interaction phase shift (χ_s, the phase shift we would get if the Coulomb interactions were switched off; for χ_s we have the expression $\chi_s = \sum_j \chi_{js}$). This assumption is, of course, obvious in potential scattering.

$$\chi(b) = -\frac{1}{v}\int_{-\infty}^{+\infty} dz[V_s(\mathbf{b}, z) + V_c(\mathbf{b}, z)] = \chi_s(b) + \chi_c(b)$$

$$\mathfrak{M} = ik\int_0^\infty db\, bJ_0(\Delta b)\{1 - \exp[i(\chi_c(b) + \chi_s(b))]\}$$

χ_c is purely real but χ_s is not

$$\chi_s(b) = i\xi(b) + \bar{\xi}(b),$$

$$\mathrm{Im}\,\mathfrak{M} = k\int_0^\infty db\, bJ_0(\Delta b)\{1 - e^{-\xi(b)}\cos(\chi_c(b) + \bar{\xi}(b))$$

$$\mathrm{Re}\,\mathfrak{M} = k\int_0^\infty db\, bJ_0(\Delta b)e^{-\xi(b)}\sin(\chi_c(b) + \bar{\xi}(b)). \tag{3.6}$$

Hence we do not add amplitudes, we add phase shifts. The cross section is

$$\frac{d\sigma_{\mathrm{el}}}{d\Omega} = |\,\mathrm{Im}\,\mathfrak{M}\,|^2 + |\,\mathrm{Re}\,\mathfrak{M}\,|^2.$$

The amplitude (3.6) has some simple properties which show that the Coulomb interaction may help us in learning about the real part of the strong interaction phase shift, $\bar{\xi}(b)$ (which is, as a rule, not well known: we know pretty well the absorption, which is given by $\xi(b)$ because this is the dominating process, but not $\bar{\xi}(b)$).

If the Coulomb interaction is absent ($\chi_c = 0$) the elastic cross section is invariant against the change of sign of $\bar{\xi}$. If, however, $\chi_c \neq 0$ some drastic changes may be introduced by changing the sign of $\bar{\xi}$ *which is equivalent to changing the charge of the incident beam of particles.* For isospin zero targets (^{4}He, ^{16}O), if one finds no difference between the elastic cross section for π^+ and π^- it implies that there is no real part in the $\pi^\pm$-nucleus elastic scattering strong interaction phase shift.

When $\chi_c = 0$, $1 - e^{-\xi(b)}\cos\bar{\xi}(b)$ and $e^{-\xi(b)}\sin\bar{\xi}(b)$ go to zero for $b > R$ (R is the radius of the target). They have, in general, quite different shapes, however—hence Im $\mathfrak{M}$ and Re $\mathfrak{M}$ oscillate differently. They are out of phase and since $\bar{\xi}(b)$ is small in general, $|\,\mathrm{Im}\,\mathfrak{M}\,| > |\,\mathrm{Re}\,\mathfrak{M}\,|$.

If, however, $\chi_c \neq 0$, the situation may change dramatically: $\chi_c(b)$ may 'stabilize' the arguments of cos (...) and sin (...): $\bar{\xi}(b)$ decreases; however, the Coulomb phase shift $\chi_c^P(b) \sim (Ze^2/v)\ln(kb)$ increases with b. This last expression is the Coulomb phase shift produced by a point charge. If $\chi_c + \bar{\xi}$ varies around $n\pi$, the situation is more or less the same as in the case $\chi_c = 0$ (scattering of neutral particles). If, however, $\chi_c + \bar{\xi}$ stabilizes around $(n + \frac{1}{2})\pi$, the roles of real and imaginary parts may be interchanged: Re $\mathfrak{M}$ may become large and Im $\mathfrak{M}$ small.

The Conclusion: The Coulomb interactions for large nuclei are, in general, important for all angles and momentum transfers.

In order to compute the amplitude one has to bear in mind that at large b, $\chi_c(b)$ behaves like a Coulomb phase shift produced by a point charge and hence diverges logarithmically. But we do know the analytic expression for the Coulomb scattering amplitude of point-like charges:

$$\mathfrak{M}_c^{(P)} = ik \int d^2b \exp\ (i\boldsymbol{\Delta}\cdot\mathbf{b})[1-e^{i\chi_c^P}]$$

$$= -\frac{n}{2k \sin^2 (\frac{1}{2}\theta)} \exp\ [-2in \ln \sin\ (\tfrac{1}{2}\theta) + 2i\sigma_0]$$

$$= -\frac{n}{\Delta^2/2k} \exp\ [-in \ln\ (\Delta^2/4k^2)]e^{2i\sigma_0}$$

where

$$n = Ze^2/v, \qquad \sigma_0 = \arg\Gamma(1+in).$$

and hence we get the convergent expression for the complete amplitude by adding and subtracting a Coulomb point charge amplitude:

$$\mathfrak{M} = ik \int_0^\infty db\, bJ_0(\Delta b)[1-\exp\ (i\chi_c^P(b))] + ik\int_0^\infty db\, bJ_0(\Delta b) \exp\ [i\chi_c^P(b) + i\chi_s(b)]$$

$$= \mathfrak{M}_c^{(P)} + ik \int_0^\infty db\, bJ_0(\Delta b)[\exp(i\chi_c^P(b)) - \exp\ (i\chi_c(b))(1-\Gamma(b))].$$

This last integral has no divergences anymore (although $\chi_c^P(b)$ and $\chi_c(b)$ both diverge logarithmically at large b). In general $\chi_c(b)$ has to be computed numerically

$$\chi_c(b) = -\frac{Ze^2}{v}\int_{-\infty}^{+\infty} dz \int d^3r' \frac{\rho_A(r')}{|\mathbf{r}-\mathbf{r}'|}$$

where $\mathbf{r} = (\mathbf{b}, z)$. Note that

$$\lim_{b\to\infty} \chi_c(b) = -\frac{Ze^2}{v}\int_{-\infty}^{+\infty} \frac{dz}{r} \int d^3r' \rho_A(r')$$

Hence, for large b, $\chi_c(b) \to \chi_c^P(b)$ and the integral for $\mathfrak{M}$ converges.

Let us construct $\chi_s(b)$ in the case of A large (a large target nucleus). We assume (for the sake of simplicity) the independent particle model wave function of the nucleus:

$$1 - \exp\ (i\chi_s(b)) = \int d^2s_1 \ldots d^2s_A \prod_{j=1}^{A} \rho(\mathbf{s}_j) \left\{1 - \prod_{j=1}^{A} (1-\gamma(\mathbf{b}-\mathbf{s}_j)\right\},$$

$$\exp\ [i\chi_s(b)] = \left(1 - \int d^2s\rho(\mathbf{s})\gamma(\mathbf{b}-\mathbf{s})\right)^A \xrightarrow[A\ \text{large}]{} \exp\left[-A\int d^2s\ \rho(\mathbf{s})\gamma(\mathbf{b}-\mathbf{s})\right].$$

(In order to perform a careful limiting procedure $A\to\infty$, one should keep $e^{i\chi_s(b)}$ under the integral sign of the expression for $\mathfrak{M}$ [12]. Generalizing slightly (allowing for different neutron and proton profiles and densities) we have (N number of neutrons, Z—number of protons):

$$i\chi_s(b) \approx -N\gamma_n(0)\rho_n(b) - Z\gamma_p(0)\rho_p(b).$$

When $\gamma_n(b)$ and $\gamma_p(b)$ are very sharp compared with $\rho_n(s)$ and $\rho_p(s)$ we have

$$i\chi_s(b) \approx -N\gamma_n(0)\rho_n(b) - Z\gamma_p(0)\rho_p(b).$$

As $f(\delta) = \frac{ik}{2\pi}\int d^2b \exp\ (i\boldsymbol{\delta}\cdot\mathbf{b})\gamma(b)$, when $\gamma(b)$ is very sharp compared to $1/\delta$ (hence we limit ourselves to forward scattering processes) we can approximate $\gamma(b) \cong \gamma(0)\delta^{(2)}(b)$, hence

$$f(0) = \frac{ik}{2\pi}\gamma(0), \qquad \text{and}$$

$$i\chi_s(b) \approx -N \frac{2\pi}{ik} f_n(0)\rho_n(b) - Z \frac{2\pi}{ik} f_p(0)\rho_p(b)$$

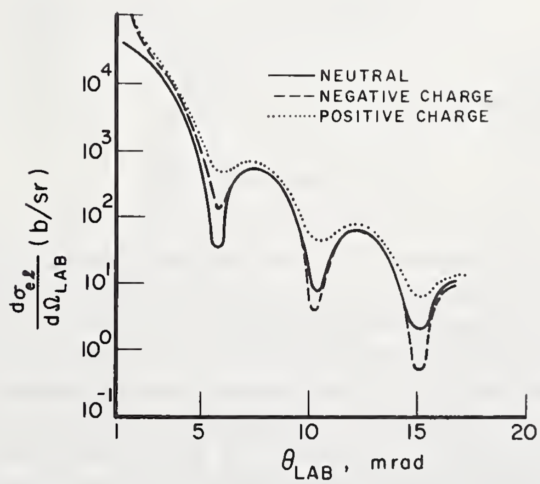
$$= -N\tfrac{1}{2}\sigma_n(1-i\alpha_n)\rho_n(b) - Z\tfrac{1}{2}\sigma_p(1-i\alpha_p)\rho_p(b). \qquad (3.7)$$

where α_n and α_p (the ratio of the real to the imaginary part of the forward forward scattering amplitude) are defined by $f_{n,p}(0) = (i+\alpha_{n,p})k\sigma_{n,p}/4\pi$ where $\sigma_{n,p}$ are the total cross section for scattering on either neutron or proton. (Incidentally, one can define an optical potential $-(1/v)\int_{-\infty}^{\infty} dz\ V_{\mathrm{opt}}(\mathbf{b},z) = \chi_s(b)$ which is equivalent to our multiple scattering description).

From this expression (3.7) one can see that the interplay of $\chi_c(b)$ and $\xi(b)$ is, in this optical limit, determined by the size and sign of α_n and α_p. Some calculations were done [15] with $\rho(r) = \rho_0(1-\exp[(r-R)/c])^{-1}$. For ^{208}Pb, $R=6.5$ fm and $c=0.523$ fm. The densities $\rho_n(b)$, $\rho_p(b)$ were obtained by integrating $\rho(r)$ over z. The parameters of $\chi_s(b)$ were taken from proton-nucleon scattering cross sections. For $\alpha_n=\alpha_p=-0.33$, $\sigma_n=\sigma_p=38.9$ mb (these parameters are resonable for ~20 GeV protons), one gets the following table

$\chi_c(b)+\xi(b)$ (rad)	b (fm)
8.43	0.0050
8.45	1.57
8.41	3.28
8.42	4.19
8.32	5.24
8.15	6.15
7.99	7.33
8.15	9.16

Note that $\tfrac{5}{2}\pi=7.85$, $\tfrac{6}{2}\pi=9.42$. This means that near to the nuclear boundary sin $(\chi_c(b)+\xi(b))$ is large. It is amusing that numerically Im $\mathfrak{M}$ with proper χ_c is approximately the same as Re $\mathfrak{M}$ without χ_c! This is true for ^{208}Pb. In general one gets all kinds of intermediate situations. In any case, the influence of the Coulomb interaction is very important "everywhere" as the figure below (see [15]) for a ^{208}Pb target and incident neutral-, positive-, and negative- particles which interact strongly as 20 GeV nucleons.



For heavy nuclei there are virtually no experiments with good enough resolution to have only pure elastic scattering (in which the target nucleus stays in the ground state). In order to have a genuine elastic scattering one would have to have an energy resolution ΔE a fraction of an MeV, which for $E\sim 20$ GeV is still inaccessible. Most experiments (e.g., CERN series-compare [S1]) have poor energy resolution of the incident and outgoing beam ($\sim$50 MeV), hence they sum over all nuclear excitations (without producing mesons, however). The cross section for such "inclusive" processes is

$$\frac{d\sigma_{\mathrm{sc}}}{d\Omega}=\sum_n\left|\frac{ik}{2\pi}\int d^2b\,\exp\,(i\boldsymbol{\Delta}\cdot\mathbf{b})\langle\Psi_n\mid\Gamma(\mathbf{b};\mathbf{s}_1\ldots\mathbf{s}_A)\mid\Psi_0\rangle\right|^2$$

$$=\frac{k^2}{(2\pi)^2}\int d^2b\,d^2b'\,\exp\,[i\boldsymbol{\Delta}\cdot(\mathbf{b}-\mathbf{b}')]\langle\Psi_0\mid\Gamma^+\Gamma\mid\Psi_0\rangle.$$

This cross section includes, of course, the elastic cross section. The cross section which, upon integration, gives σ_{DT} is

$$\frac{d\sigma_{\mathrm{DT}}}{d\Omega}=\frac{d\sigma_{\mathrm{sc}}}{d\Omega}-\frac{d\sigma_{\mathrm{el}}}{d\Omega}=\left(\frac{k}{2\pi}\right)^2\int d^2b\,d^2b'\,\exp\,[i\boldsymbol{\Delta}\cdot(\mathbf{b}-\mathbf{b}')]$$

$$\times[\langle\Psi_0\mid\Gamma^+(b)\,\Gamma(b)\mid\Psi_0\rangle-\langle\Psi_0\mid\Gamma^+(b)\mid\Psi_0\rangle\langle\Psi_0\mid\Gamma(b)\mid\Psi_0\rangle].$$

It is an interesting fact that while $d\sigma_{\mathrm{el}}/d\Omega$ is *very strongly* influenced by Coulomb interactions (as we have seen), $d\sigma_{\mathrm{DT}}/d\Omega$ is influenced *very little.* In order to make this fact more plausible, let us consider a collection of neutrons and protons which do not screen each other. Then, we would have

$$\left.\frac{d\sigma_{\mathrm{DT}}}{d\Omega}\right|_{\text{no screening}}\approx N\mid f_n(\Delta)\mid^2+Z\mid f_p(\Delta)\mid^2.$$

One may suspect the following:

If we introduce screening there will be, on the average, a certain fraction of nucleons inaccessible to the incident hadron. Hence the above formula can be applied to a certain "effective" number of nucleons. Indeed one can show (compare ref. [15]—the calculation was done with the Coulomb interactions present) that to a good approximation (note that since this formula does not exhibit a forward dip, it is not valid for small Δ)

$$\frac{d\sigma_{\mathrm{DT}}}{d\Omega}=\mathcal{A}\left[\frac{N}{A}\mid f_n(\Delta)\mid^2+\frac{Z}{A}\mid f_p(\Delta)\mid^2\right],$$

where the "effective number of nucleons" is

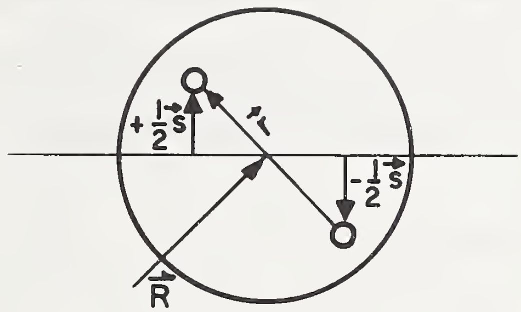
$$\mathcal{A}\cong A\int d^2b\,\rho(b)e^{-\sigma A\rho(b)},$$

where σ is an average total hadron-nucleon cross section. If indeed $\sigma_{\mathrm{DT}}/d\Omega$ has such a form, the only place where Coulomb interactions enter are in the individual proton amplitudes, $\mid f_p(\Delta)\mid^2$. But there we know, e.g., from the proton-proton elastic cross section, that Coulomb interactions are important for very, very small momentum transfers only. In any case, they enter *incoherently into* $d\sigma_{\mathrm{DT}}/d\Omega$. These two factors make Coulomb corrections insignificant in $d\sigma_{\mathrm{DT}}/d\Omega$.

How important are the details of the target nucleus wave function? *Not very important.* The most important are general characteristics: density distributions (hence possible deformations) *but not internal correlations.* From the published analyses of hadron-nucleus scattering (see e.g., [S2], [15], [3]) one may conclude that:

(i) the shapes of target nuclei are the most important factors determining the cross sections
(ii) the internal correlations of nucleons in the nucleus are unimportant for $d\sigma_{\rm sc}/d\Omega$ or $d\sigma_{\rm el}\,d\Omega$. They are of some importance for $d\sigma_{\rm DT}/d\Omega$ (especially at small momentum transfers [15], [3]). *The confrontation with experiment is impressive.* (Compare, e.g., the review article by R. J. Glauber in ref. [S2]).

When we want to discuss light nuclei we have to consider carefully the motion of the center of mass. Take, for example, a deuteron: here taking into account the c.m. motion is trivially accomplished by using the wave functions of the relative motion, $\phi(r)$.



For example, the elastic scattering amplitude is

$$\mathfrak{M}(\Delta) = \frac{ik}{2\pi}\int d^2b \exp\,(i\mathbf{\Delta}\cdot\mathbf{b}) \int d^3r \mid \phi(\mathbf{r}) \mid^2 [\gamma_p(\mathbf{b}-\tfrac{1}{2}\mathbf{s})$$

$$+\gamma_n(\mathbf{b}+\tfrac{1}{2}\mathbf{s})-\gamma_p(\mathbf{b}-\tfrac{1}{2}\mathbf{s})\gamma_n(\mathbf{b}+\tfrac{1}{2}\mathbf{s})].$$

In the case of more complicated targets the situation is much more involved and often leads to some serious computational problems. Let us introduce the transverse component of the c.m. vector

$$\mathbf{r}=\frac{1}{A}\sum_j^A \mathbf{s}_j,$$

and the relative coordinates

$$\mathbf{s}_j'=\mathbf{s}_j-\mathbf{r}, \qquad \mathbf{s}_j=\mathbf{s}_j'+\mathbf{r}$$

which are not independent any more:

$$\sum_j^A \mathbf{s}_j'=0.$$

The operator

$$\mathfrak{M}(\mathbf{\Delta};\mathbf{s}_1\ldots\mathbf{s}_A) = \frac{ik}{2\pi}\int d^2b \exp\,(i\mathbf{\Delta}\cdot\mathbf{b})\,\Gamma(\mathbf{b};\mathbf{s}_1\ldots\mathbf{s}_A)$$

has the following property (see below for the proof):

$$\mathfrak{M}(\mathbf{\Delta};\mathbf{s}_1\ldots\mathbf{s}_A) = \exp\,(i\mathbf{\Delta}\cdot\mathbf{r})\,\mathfrak{M}'(\mathbf{\Delta};\mathbf{s}_1'\ldots\mathbf{s}_A') \tag{3.8}$$

Then we can compute the correction factor to $\mathfrak{M}=\langle\mathfrak{M}(\mathbf{\Delta};\mathbf{s}_1\ldots\mathbf{s}_A)\rangle$ assuming the wave function to be in the form of a product of the c.m. wave function and the internal wave function.

$$\mathfrak{M}=\langle\mathcal{R}(\mathbf{r}) \mid \exp\,(i\mathbf{\Delta}\cdot\mathbf{r}) \mid \mathcal{R}(\mathbf{r})\rangle\underbrace{\langle\Phi_0(\mathbf{r}_1'\ldots\mathbf{r}_A') \mid \mathfrak{M}' \mid \Phi_0(\mathbf{r}_1'\ldots\mathbf{r}_A')\rangle}_{\text{This is the corrected amplitude.}} \tag{3.9}$$

Hence if we can factor out the c.m. wave function from the product $\Psi_0 = \prod_j \phi_j(r_j)$ we can stick to calculating $\mathfrak{M}$ with Ψ_0 but we have to multiply it by a correction factor:

$$\langle \mathcal{R}(\mathbf{r}) \mid \exp(i\boldsymbol{\Delta}\cdot\mathbf{r}) \mid \mathcal{R}(\mathbf{r})\rangle^{-1}.$$

This can be done explicitly in the case of oscillator potential wave functions (this is partly the reason why they are so popular!). There

$$\mathcal{R}(\mathbf{r}) = (A/\pi^3 \mathrm{R}^6)^{1/4} \exp(-A\mathrm{r}^2/2\mathrm{R}^2)$$

where R is the size parameter in the Gaussian factor in harmonic oscillator wave functions: $\exp(-r^2/2\mathrm{R}^2)$. Then

$$\langle \mathcal{R}(\mathbf{r}) \mid \exp(i\boldsymbol{\Delta}\cdot\mathbf{r}) \mid \mathcal{R}(\mathbf{r})\rangle^{-1} = \exp(\Delta^2\mathrm{R}^2/4A).$$

When one cannot do this factorization the computations become quite involved (s_j' are not independent!).

The proof of (3.8):

In

$$\mathfrak{M}(\boldsymbol{\Delta}; \mathbf{s}_1 \ldots \mathbf{s}_A) = \frac{ik}{2\pi} \int d^2b \exp(i\boldsymbol{\Delta}\cdot\mathbf{b}) \left\{1 - \prod_{j=1}^{A} [1-\gamma_j(\mathbf{b}-\mathbf{s}_j)]\right\}$$

replace

$$\mathbf{s}_j = \mathbf{r} + \mathbf{s}_j,$$

then we shall have $\mathbf{b}-\mathbf{r}$ instead of $\mathbf{b}$. After changing the variable $\mathbf{b} = \mathbf{r} + \mathbf{b}'$ we get the factor $\exp(i\boldsymbol{\Delta}\cdot\mathbf{r})$ in front, and the formula (3.9) follows.

One can write the general formula which takes into account the interdependence of internal coordinates by introducing a Dirac δ function into the amplitudes:

$$\mathfrak{M}(\Delta) = \frac{ik}{2\pi} \int d^2b \exp(i\boldsymbol{\Delta}\cdot\mathbf{b}) \int d^3r_1 \ldots d^3r_A \Psi_0^*(\mathbf{r}_1, \ldots \mathbf{r}_A) \left\{1 - \prod_{j=1}^{A} [1-\gamma_j(\mathbf{b}-\mathbf{s}_j)]\right\}$$

$$\times \Psi_0(\mathbf{r}_1, \ldots \mathbf{r}_A)\, \delta^{(3)}\left(\frac{1}{A} \sum_{j=1}^{A} \mathbf{r}_j\right).$$

This $\delta^{(3)}$ function eliminates redundant excitations of the system of A nucleons. When one cannot factorize the c.m. coordinate and one has to use the above formula the numerical calculations become much more involved (from trivial—they become difficult [18]).

An illustrative example: the ground state wave function is a Gaussian [19]. The ground state densities and the elementary amplitudes are taken in the form

$$|\Psi_0|^2 = \prod_{j=1}^{A} \rho(r_j), \qquad \rho(r) = \rho_0 \exp(-r_j^2/\mathrm{R}^2)$$

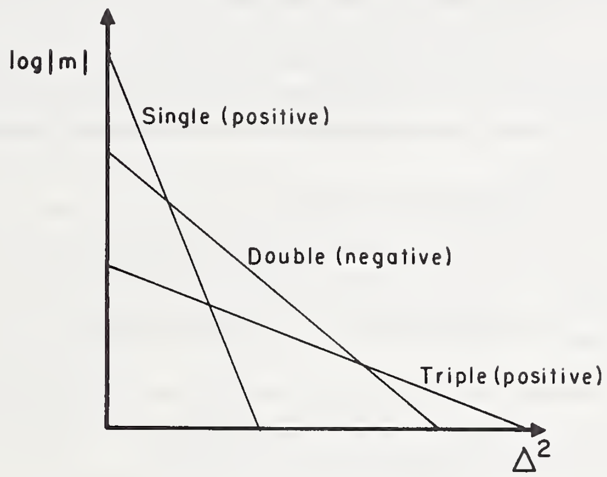
$$f(k) = \frac{(i+\alpha)k\sigma}{4\pi} \exp(-\tfrac{1}{2}a\delta^2).$$

Then the elastic scattering amplitude (with the c.m. motion correction included) reads

$$\mathfrak{M}(\Delta) = ik(\mathrm{R}^2+2a) \exp\left(\frac{\mathrm{R}^2\Delta^2}{4A}\right) \sum_{j=1}^{A} \binom{A}{j} (-1)^{j+1} \frac{1}{j} \left[\frac{\sigma(1-i\alpha)}{2\pi(\mathrm{R}^2+2a)}\right]^j \exp\left[-\frac{1}{4j}(\mathrm{R}^2+2a)\Delta^2\right]$$

Many general features of the multiple scattering are included in this formula:

(i) If we neglect α, the amplitude becomes purely imaginary (absorptive). A geometrical picture of single-, double-, etc. scattering contributions is as follows:



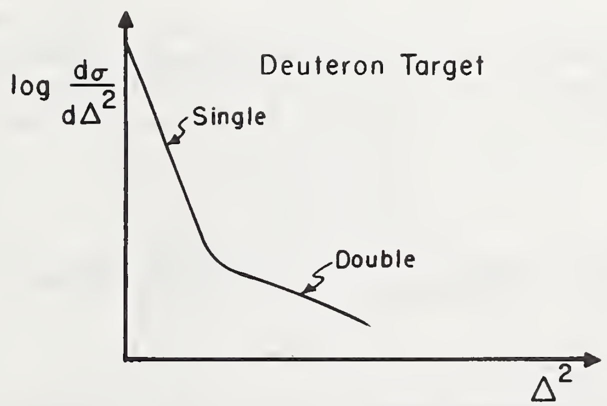
(ii) With this picture it is easy to establish the existence of diffractive minima, which are filled by the real part of $\mathfrak{M}$. (In order to have Re $\mathfrak{M}\neq 0$ we have to have $\alpha\neq 0$).

(iii) The importance of the c.m. motion correction can be seen from the factor exp $(R^2\Delta^2/4A)$. For small A (say $A=2$, 3 or 4) it can be a correction of as much as 2 orders of magnitude for $\Delta^2\approx 0.3$ GeV^2.

A few concluding remarks about the deuteron target.

A lot of attention was concentrated on the deuteron because it is a very important testing ground for multiple scattering theories (or models).

(i) In experiments (compare deuteron data contained in [S1]), one can clearly see the single and double scattering.



Remark: In fact, this clear distinction between single and double scattering was used to extract the ρ-nucleon total cross section in γ-ρ production experiments on deuterons (see section 4.2).

(ii) One can also see (again, compare [S1]) how important the deformation of the target is (existence of the D-state in the deuteron ground state). Let us discuss this effect in more detail.

The ground state deuteron wave function is:

$$\phi_m(\mathbf{r}) = (4\pi)^{-1/2} r^{-1} [u(r) + 8^{-1/2} S_{12} w(r)] \chi_{1,m} \tag{3.10}$$

where $u(r)$ and $w(r)$ are the radial S and D functions and

$$S_{12} = [3(\boldsymbol{\sigma}_1 \cdot \mathbf{r})(\boldsymbol{\sigma}_2 \cdot \mathbf{r}) - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2].$$

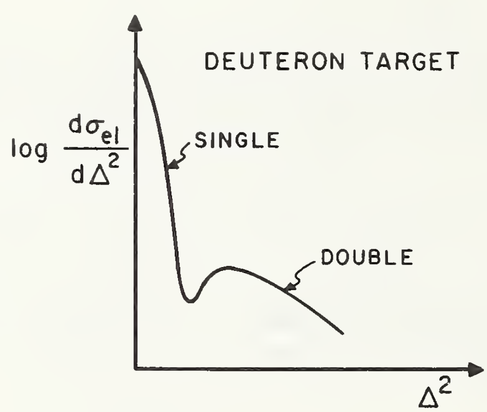
$\boldsymbol{\sigma}_1$, $\boldsymbol{\sigma}_2$ are the Pauli spin operators, and $\mathbf{r}$ is the neutron-proton relative coordinate. $\chi_{1,m}$ is the spin function for spin 1 with the magnetic quantum number m. The elastic cross-section is then

$$\frac{d\sigma_{\text{el}}}{d\Omega} = \frac{1}{3} \sum_{m,m'} |\langle m \mid \mathfrak{M}(\boldsymbol{\Delta}, \mathbf{s}) \mid m' \rangle|^2$$

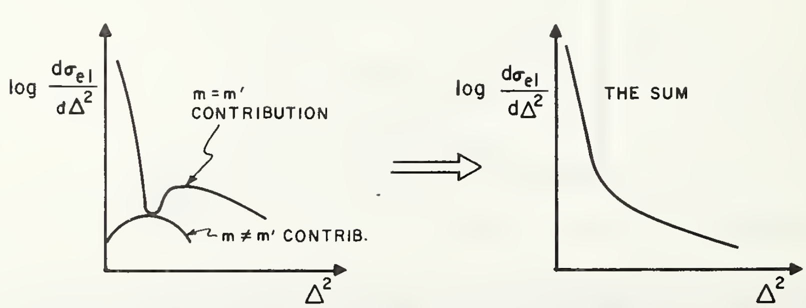
with

$$\mathfrak{M}(\boldsymbol{\Delta}, \mathbf{s}) = \frac{ik}{2\pi} \int d^2b \exp(i\boldsymbol{\Delta} \cdot \mathbf{b}) \{1 - \exp[i\chi_p(\mathbf{b} - \tfrac{1}{2}\mathbf{s}) + i\chi_n(\mathbf{b} + \tfrac{1}{2}\mathbf{s})]\},$$

which operator, in this approximation, does not depend on spins. So, if not for the S_{12} term in (3.10), we would have $\langle m \mid \mathfrak{M} \mid m' \rangle = 0$ for $m \neq m'$. In fact the $\langle m \mid \mathfrak{M} \mid m' \rangle$ contributions are indeed the most important but they always lead to a sharp diffractive minimum:



But $\langle m \mid S_{12} \mid m' \rangle \neq 0$ in general (also $\langle m \mid S_{12} S_{12} \mid m' \rangle \neq 0$). This matrix element enters $\langle m \mid \mathfrak{M}(\Delta, s) \mid m' \rangle$ and results in spin-flip transitions (classically: rotation of the deuteron spin) which have completely different "profiles" than $\langle m \mid \mathfrak{M} \mid m \rangle$, thus resulting in oscillations which are out of phase with oscillations of $\langle m \mid \mathfrak{M} \mid m \rangle$ and fill the diffractive minimum:



Incidentally, *only the spin of the deuteron as a whole is essential.* The qualitative effect is independent of the spin of the incident particle (the $\mathfrak{M}(\mathbf{\Delta}, \mathbf{s})$ operator does not act on spin quantum numbers.) All the other spin effects are presumably not important.

(iii) Calculations such as the one above, as well as more sophisticated calculations, have always produced cross sections in excellent agreement with experiment. (We are not considering here backward scattering, where the above model does not apply (see also [48])). There is only one exception: the experiment performed at CERN by Bradamante et al. [20]. In this experiment the discrepancy with theory occurs at a fairly large momentum transfer ($\Delta^2 \approx 2$ GeV^2). What is the cause? Perhaps some relativistic effects? There is no good answer, so far. Without going into any explanation of this discrepancy, let us emphasize the following point:

It is important to realize that when we use the same internal wave function in the initial and final states, we exclude, by doing this, any possible relativistic deformations of the recoiling target (we are still discussing only elastic processes). For large momentum transfers ($\Delta^2/M_T^2 \sim 1$) this is probably not a good approximation. Take the deuteron example. In the standard Glauber model, it is enough to have $\rho(s) = \int_{-\infty}^{\infty} dz \phi_0^*(\mathbf{s}, z)\phi_0(\mathbf{s}, z)$ to compute the cross section. Suppose there is some deformation in the final state:

$$\phi_0^*(\mathbf{s}, z) \to \phi_0'^*(\mathbf{\Delta}, \mathbf{s}, z)$$

(one can assume that the deformation is defined by the momentum transfer Δ). Then we should replace

$$\rho(s) \to I(\mathbf{\Delta}, \mathbf{s}) = \int_{-\infty}^{+\infty} dz\, \phi_0'^*(\mathbf{\Delta}, \mathbf{s}, z)\phi_0(\mathbf{s}, z)$$

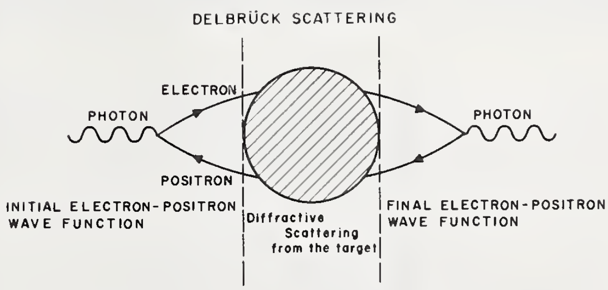
and the amplitude is

$$\mathfrak{M}(\Delta) = \frac{ik}{2\pi} \int d^2b\, d^2s\, e^{i\Delta\cdot \mathbf{b}} I(\mathbf{\Delta}, \mathbf{s}) \{1 - \exp[i\chi_p(\mathbf{b} - \tfrac{1}{2}\mathbf{s}) + i\chi_n(\mathbf{b} + \tfrac{1}{2}\mathbf{s})]\}.$$

The interesting fact is that in exactly the same form one can write the Delbrück amplitude

$$\mathfrak{M}^{\text{Delbrück}}(\Delta) \sim \int d^2b \exp(i\mathbf{\Delta}\cdot\mathbf{b}) I^\gamma(\mathbf{\Delta}, \mathbf{s}) \{1 - \exp[i\chi_c^-(\mathbf{b} - \tfrac{1}{2}\mathbf{s}) + i\chi_c^+(\mathbf{b} + \tfrac{1}{2}\mathbf{s})]\}$$

where $\chi_c^\pm$ are the Coulomb phase shifts of the electron-positron pair and $I^\gamma(\mathbf{\Delta}, \mathbf{s})$ is constructed from



the "relativistic wave functions" in an analogous way to that shown above in the case of the deuteron. Here the possibility of a well-defined procedure of introducing relativistic deformations occurs—modeled on QED! These and other related problems have been discussed in a series of papers by Cheng and Wu [21, 22] (see also [30], [S4]).

4. Diffractive Dissociation and Diffractive Excitation

Diffractive processes—a brief characterization.

(i) they do not vanish in the limit $E \to \infty$

(ii) the target plays a passive role (except in double diffraction, but in any case: *no quantum numbers are exchanged*).

Examples:

in QED; elastic electron (positron) scattering from a Coulomb field, Delbrück scattering, Compton scattering, etc.

in hadron physics; all kinds of elastic hadron-hadron scattering

$$\frac{d\sigma}{d\Delta^2} = f(\Delta^2)$$

(experiments seem to indicate that the differential cross sections depend weakly on energy)

inelastic processes:

$$\left.\begin{array}{l} \gamma \to \rho \\ \pi \to 3\pi \\ \pi \to 5\pi \\ p \to n + \pi \\ K \to K\pi\pi,\ K\pi\pi\pi \\ \text{etc.} \end{array}\right\} \text{on nucleons or nuclei.}$$

The nucleon and nuclear targets supplement each other because the nuclear medium amplifies the scattering of the produced objects.

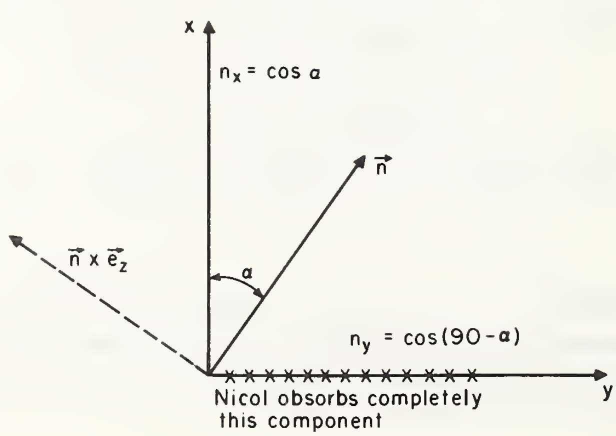
The model of diffractive processes described below is based on: M. L. Good and W. D. Walker (1960) [23]. The article which discusses some very early papers on the subject is: E. L. Feinberg and I. Pomerančuk (1956) [24]. For more recent discussions of many experimental and theoretical aspects of diffractive processes in hadron physics see refs. [S4] and the article by A. Białas in [25]. We shall describe diffractive production processes in very close analogy to diffractive dissociation phenomena which are well known in the case of systems where degeneracy exists.

Let us start with an example taken from optics. Consider the absorption of polarized light by an anisotropic absorber. The incident wave is polarized in the direction $\mathbf{n}$ (perpendicular to the z direction).

$$\mathbf{n} = (n_x, n_y)$$

$$\Psi_n = n_x \Psi_x + n_y \Psi_y$$

where Ψ_x is the wave polarized in the x direction and Ψ_y is the wave polarized in the y direction.



Suppose the target is a Nicol prism oriented in such a way that it stops all light polarized in the y direction. Hence, the only component which goes through is $n_x\Psi_x$. But it can be decomposed into $\mathbf{n}$ and $\mathbf{n}\times\mathbf{e}_z$ components. Hence due to the process of absorption, a new object is created: the wave which is polarized in the direction $\mathbf{n}\times\mathbf{e}_z$.

Let us compute the elastic and inelastic scattering amplitudes. Since the transmitted wave is $\varphi=n_x\Psi_x$, the wave which goes into scattering and production is

$$\Psi_n-\varphi=n_y\Psi_y=\lambda_{\mathrm{el}}\Psi_n+\lambda_{\mathrm{inel}}\Psi_{\mathbf{n}\times\mathbf{e}_z}$$

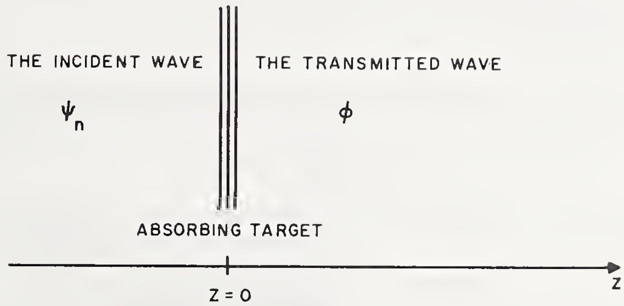
But

$$\Psi_y=n_y\Psi_n-n_x\Psi_{\mathbf{n}\times\mathbf{e}_z};$$

hence

$$\lambda_{\mathrm{el}}=n_y^2, \qquad \lambda_{\mathrm{inel}}=-n_xn_y.$$

Geometrical picture:



The exact solution is:

$$\Psi=\begin{cases}\Psi_n, & z<0\\ \varphi, & z>0.\end{cases}$$

The "undisturbed" wave is Ψ_n everywhere, hence the scattered wave is

$$\Psi_n-\Psi=\begin{cases}0, & z<0\\ \Psi_n-\varphi, & z>0.\end{cases}$$

Actually it is more important for our purposes to introduce *partial* absorption (in general different for the two components (x, y)).

The incident wave:	$\Psi_n=n_x\Psi_x+n_y\Psi_y$
The transmitted wave:	$\varphi=\eta_xn_x\Psi_x+\eta_yn_y\Psi_y$
The scattered wave:	$\Psi_n-\varphi=(1-\eta_x)n_x\Psi_x+(1-\eta_y)\eta_y\Psi_y$
	$=\lambda_{\mathrm{el}}\Psi_n+\lambda_{\mathrm{inel}}\Psi_{\mathbf{n}\times\mathbf{e}_z},$

and we get

$$\lambda_{\mathrm{el}}=n_x^2(1-\eta_x)+n_y^2(1-\eta_y)$$

$$\lambda_{\mathrm{inel}}=n_xn_y(\eta_y-\eta_x).$$

This formula shows that we always produce inelastic scattering, except in two cases:

(i) when the incident wave is polarized either along the x or y axis (hence either $n_x=0$ or $n_y=0$)
(ii) when the absorption coefficients are equal (the absorber is isotropic).

We shall extend this description to diffractive production processes of hadronic systems. We consider the incident hadron to be a superposition of some states which get eaten up at different rates during the passage through the target; the new combination emerging from the collision then contains, in general, a new particle (or a collection of new particles).

First we introduce the physical states of the system, $|\tilde{\lambda}_i\rangle$ (which are analogous to the states ψ_n and $\psi_{\mathbf{n}\times\mathbf{e}_z}$ of the photon). We want to compute $\langle\tilde{\lambda}_i|T|\tilde{\lambda}_i\rangle$. We expand $|\tilde{\lambda}_i\rangle$ into a set of states $|\lambda_i\rangle$ whose scattering and absorption in the target we assume known:

$$\left.\begin{aligned} |\tilde{\lambda}_i\rangle &= \sum_j d_{ij}\,|\lambda_j\rangle \\ |\lambda_i\rangle &= \sum_j c_{ij}\,|\tilde{\lambda}_j\rangle \end{aligned}\right\} \quad \begin{array}{l} |\tilde{\lambda}_i\rangle, |\lambda_i\rangle \text{ form} \\ \text{orthonormal sets} \\ \text{of states.} \end{array} \tag{4.1}$$

$$|\lambda_i\rangle = \sum_l \sum_j c_{ij}\,d_{jl}\,|\lambda_l\rangle \equiv |\lambda_i\rangle, \qquad \text{hence}$$

$$\sum_j c_{ij}\,d_{jl} = \delta_{il}.$$

The states $|\lambda_i\rangle$ are assumed to be eigenstates of T in the following sense:

$$T\,|\lambda_j\rangle = (1-\eta_j)\,|\lambda_j\rangle + \underbrace{\sum_k \gamma_{jk}\,|\mu_k\rangle}_{\substack{\text{This is the ``heat'' produced}\\ \text{in the absorber. All these}\\ \text{states are orthogonal to all}\\ \text{states which appear in}\\ \text{diffractive scattering.}}}$$

$$\begin{aligned} \langle\tilde{\lambda}_i|T|\tilde{\lambda}_j\rangle &= \langle\tilde{\lambda}_i|\,T\sum_n d_{jn}(1-\eta_n)\,|\lambda_n\rangle \\ &= \langle\tilde{\lambda}_i|\sum_n d_{jn}(1-\eta_n)\sum_l c_{nl}\,|\tilde{\lambda}_l\rangle \\ &= \langle\tilde{\lambda}_i|\sum_n d_{jn}(1-\eta_j+\eta_j-\eta_n)\sum_l c_{nl}\,|\tilde{\lambda}_l\rangle \\ &= \langle\tilde{\lambda}_i|\,(1-\eta_j)\sum_l\sum_n d_{jn}c_{nl}\,|\tilde{\lambda}_l\rangle - \sum_n\sum_l d_{jn}c_{nl}(\eta_n-\eta_j)\,\langle\tilde{\lambda}_i|\tilde{\lambda}_l\rangle \\ &= (1-\eta_j)\sum_l\sum_n d_{jn}c_{nl}\delta_{il} - \sum_n\sum_l d_{jn}c_{nl}(\eta_n-\eta_j)\,\delta_{il} \end{aligned}$$

So,

$$\langle\tilde{\lambda}_i|T|\tilde{\lambda}_j\rangle = (1-\eta_j)\delta_{ij} - \sum_n d_{jn}c_{ni}(\eta_n-\eta_j). \tag{4.2}$$

We have obtained the result completely analogous to the one obtained for the optical diffractive production: for $i\neq j$ the production amplitude is proportional to the difference in absorptions of the i, j components. This is a very general property. All specific models of diffractive production processes I know of exhibit this property. Otherwise the formula is so general that it has virtually no predictive power.[3]

The difficulty in applying it to any realistic process is the determination of absorption parameters η_i because the states $|\lambda_i\rangle$ are not observed in scattering experiments. (The process of $K_L{}^0\to K_s{}^0$ regeneration given below is an example where we know η_i's however!) The situation changes when

[3] One must, however, keep in mind that in the case when the coefficients d_{ij} are zero or of the same order of magnitude one does predict some characteristics of production processes from the knowledge of elastic scattering. Compare the end of this section.

we accept that diffractive production processes are weak compared to elastic scattering. This may mean that the transformation from $|\bar{\lambda}_i\rangle$ to $|\lambda_i\rangle$ differs little from unity:

$$\left.\begin{aligned} c_{ij}&=\delta_{ij}+\epsilon_{ij} \\ d_{ij}&=\delta_{ij}-\epsilon_{ij} \end{aligned}\right\} \quad \begin{aligned}&\epsilon_{ij} \text{ small, hence } \epsilon^2 \text{ terms} \\ &\text{can be neglected.}\end{aligned} \tag{4.3}$$

Remark: The minus sign guarantees the property

$$\delta_{il}=\sum_j c_{ij}\, d_{jl}=\sum_j (\delta_{ij}+\epsilon_{ij})(\delta_{jl}-\epsilon_{jl})$$

$$=\delta_{il}+\epsilon_{il}-\epsilon_{il}=\delta_{il}.$$

Then our basic formula takes the form:

$$\langle\bar{\lambda}_i\,|\,T\,|\,\bar{\lambda}_j\rangle=(1-\eta_j)\delta_{ij}-\sum_n(\eta_n-\eta_j)(\delta_{jn}-\epsilon_{jn})(\delta_{ni}+\epsilon_{ni})$$

$$=(1-\eta_j)\delta_{ij}-\underbrace{(\eta_i-\eta_j)\delta_{ij}}_{\text{zero}}+\epsilon_{ji}(\eta_i-\eta_j)-\epsilon_{ji}\underbrace{(\eta_j-\eta_j)}_{\text{zero}}$$

$$=(1-\eta_j)\delta_{ij}+\epsilon_{ji}(1-\eta_j)-\epsilon_{ji}(1-\eta_i).$$

In this approximation

$$\langle\bar{\lambda}_i\,|\,T\,|\,\bar{\lambda}_i\rangle=1-\eta_i.$$

Hence the absorption parameters η_i are determined by elastic scattering of real particles. The inelastic amplitude

$$\langle\bar{\lambda}_i\,|\,T\,|\,\bar{\lambda}_j\rangle=(1-\eta_i)\epsilon_{ij}-(1-\eta_j)\epsilon_{ij} \tag{4.4}$$

is proportional to the difference between the absorption of the produced particle and the absorption of the incident particle.

One still faces the problem of specifying the absorption parameters η_i and the coefficients ϵ_{ij}. The coefficient η_i of the incident particle is, as a rule, easy because this is a well-known particle which can form a beam and its scattering (elastic) properties are known reasonably well. The trouble is with the outgoing objects; e.g., when 3π are produced in the $\pi\to 3\pi$ reaction: are then its η's given by the absorption of 3π in the target? *In fact, one usually determines them experimentally* (see the end of this section). So far as the ϵ_{ij} are concerned, they are small—hence some perturbation theory can be used to compute them. We shall give some examples further in the text.

How does one implement this program? There are strongly interacting particles which realize precisely the above outlined scheme and we even know d_{ij}'s and η_j's: the neutral K mesons. Because of the relation

$$\begin{array}{ccccccc} \text{charge} & & \text{baryon no.} & & \text{strangeness} & & \text{isospin} \\ Q & = & \tfrac{1}{2}N_B & + & \tfrac{1}{2}S & + & T_3 \end{array}$$

the partners of K^+, K^0 particles are K^-, $\bar{K}^0$ antiparticles. Hence there are two different neutral K mesons which can be produced in the collision of strongly interacting particles: K^0 and $\bar{K}^0$ (they are different because they have opposite strangeness, unlike pions where π^0 are identical to $\bar{\pi}^0$!) which have the same masses, and thus can be considered as a two component degenerate system.
When left in empty space, however, both K^0 and $\bar{K}^0$, decay weakly with two different lifetimes as if they were made up of two different particles, which is indeed the case. These two particles are the following superpositions of $|K^0\rangle$ and $|\bar{K}^0\rangle$ states

$$|K_s^0\rangle=\frac{1}{\sqrt{2(1+|\delta|^2)}}[(1+\delta)\,|K^0\rangle-(1-\delta)\,|\bar{K}^0\rangle]\approx\frac{1}{\sqrt{2}}(|K^0\rangle-|\bar{K}^0\rangle)=|K_1^0\rangle$$

$$|K_L^0\rangle=\frac{1}{\sqrt{2(1+|\delta|^2)}}[(1+\delta)\,|K^0\rangle+(1-\delta)\,|\bar{K}^0\rangle]\approx\frac{1}{\sqrt{2}}(|K^0\rangle+|\bar{K}^0\rangle)=|K_2^0\rangle,$$

where δ is a small complex number, $|\delta| \sim 10^{-3} \ll 1$, which gives a measure of CP nonconservation. $|K_1^0\rangle$ and $|K_2^0\rangle$ are eigenstates of CP. Indeed

$$P|K^0\rangle = -|K^0\rangle, \qquad P|\bar{K}^0\rangle = -|\bar{K}^0\rangle.$$

$$C|K^0\rangle = |\bar{K}^0\rangle, \qquad C|\bar{K}^0\rangle = |K^0\rangle$$

$$CP|K^0\rangle = -|\bar{K}^0\rangle, \qquad CP|\bar{K}^0\rangle = -|K^0\rangle$$

but

$$CP\frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle) = -\frac{1}{\sqrt{2}}(|\bar{K}^0\rangle - |K^0\rangle) = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle).$$

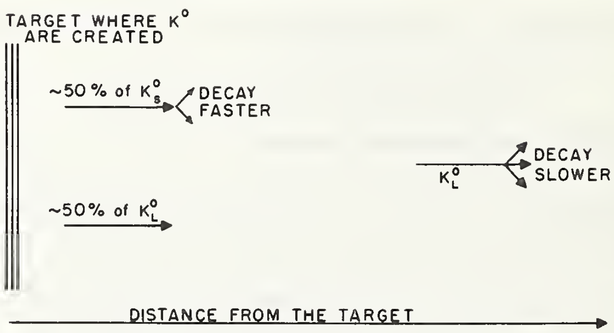
Similarly

$$CP|K_2^0\rangle = -|K_2^0\rangle.$$

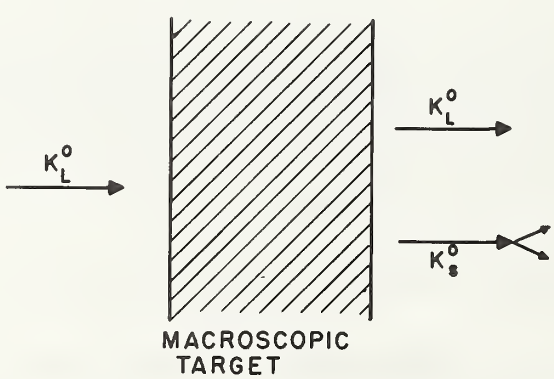
The particles K_s^0 and K_L^0 have the following lifetimes

$$\tau_s \approx 10^{-10}\text{ s}, \qquad \tau_L \approx 10^{-8}\text{ s}.$$

We observe these two different decays by looking at decaying K mesons in the beam.



So, after some distance only the K_L^0 beam is in existence. When we let the beam hit another target we can regenerate K_s^0 mesons because K^0 and $\bar{K}^0$ interact differently with matter, they are absorbed differently. (e.g., $\bar{K}^0(S=-1)+p\rightarrow\Lambda^0(S=-1)+\pi^+$ while $K^0(S=+1)$ cannot produce Λ^0). In figure 18 the so-called transmission regeneration is sketched.



One can, however, also observe diffractive production of K_s^0 from K_L^0 *on individual nuclei*. Cross sections for such diffractive coherent production processes were recently measured for copper and lead nuclei [26]. In order to measure these cross sections, one has to get off the forward direction where the transmission regeneration (which comes from a coherent process whose coherence extends over the whole block of matter) dominates. The amplitude (neglecting δ) is

$$\mathfrak{M}_{K_L^0 \to K_s^0} = \tfrac{1}{2}\langle K^0 \mid T \mid K^0 \rangle - \tfrac{1}{2}\langle \bar{K}^0 \mid T \mid \bar{K}^0 \rangle.$$

Hence $\mathfrak{M}_{K_L^0 \to K_s^0}$ is given by the elastic scattering amplitudes of K^0 and $\bar{K}^0$ from the nucleus

$$\langle K^0 \mid T \mid K^0 \rangle = \mathfrak{M}_{K^0}(\Delta^2) = \frac{ik}{2\pi}\int d^2b \exp\,(i\mathbf{\Delta}\cdot\mathbf{b})\{1-\exp\,[i\chi_{K^0}(b)]\}$$

$$\langle \bar{K}^0 \mid T \mid \bar{K}^0 \rangle = \mathfrak{M}_{\bar{K}^0}(\Delta^2) = \frac{ik}{2\pi}\int d^2b \exp\,(i\mathbf{\Delta}\cdot\mathbf{b})\{1-\exp\,[i\chi_{\bar{K}^0}(b)]\},$$

where, as was already shown for copper and lead target nuclei, it is enough to take the large-A approximation:

$$i\chi_{K^0}(b) = -N\frac{2\pi}{ik} f_{\bar{K}^0 n}(0)\,\rho_n(b) - Z\frac{2\pi}{ik} f_{K^0 p}(0)\,\rho_p(b),$$

and an analogous expression for $\bar{K}^0$, where

$$\rho_{n,p}(b) = \rho_0 \int_{-\infty}^{+\infty} dz \left[1+\exp\left(\frac{r-R_{n,p}}{C_{n,p}}\right)\right]^{-1}.$$

The final formula is

$$\mathfrak{M}_{K_L^0 \to K_S^0} = \frac{1}{2}\frac{ik}{2\pi}\int d^2b \exp\,(i\mathbf{\Delta}\cdot\mathbf{b})\{\exp\,[i\chi_{\bar{K}^0}(b)] - \exp\,[i\chi_{K^0}(b)]\}.$$

The elementary amplitudes can be gotten from $K^{\pm}$–nucleon scattering amplitudes assuming isospin symmetry:

$$f_{K^0 n}(0) = f_{K^+ p}(0), \qquad f_{\bar{K}^0 n}(0) = f_{K^- p}(0),$$

$$f_{K^0 p}(0) = f_{K^+ n}(0), \qquad f_{\bar{K}^0 p}(0) = f_{K^- n}(0).$$

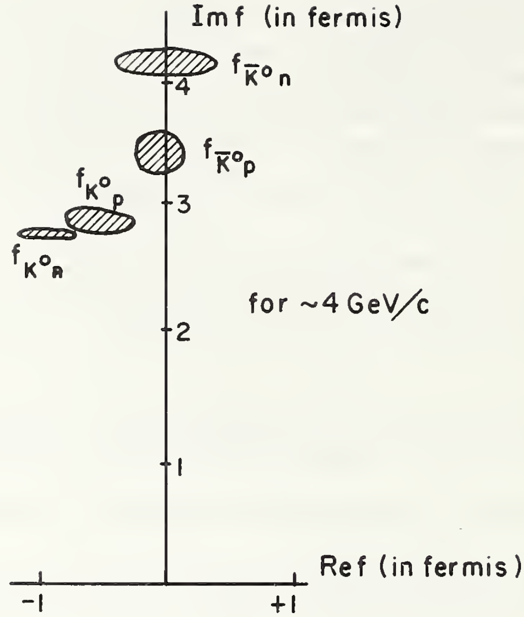
The standard way of calculating these amplitudes is:

(i) the imaginary parts are obtained from the optical theorem, e.g.

$$\operatorname{Im} f_{K^{\pm} p} = \frac{\sigma_{K^{\pm} p}}{4\pi}\,k.$$

(ii) the real parts from dispersion relations (for more details compare [26]). The amplitudes for neutrons are then obtained using some further acrobatics, as referred to in [26]. In any case, our knowledge of these amplitudes is rather poor.

The real and imaginary parts of the elastic K^0 and $\bar{K}^0$—nuclear amplitudes are sketched below. The uncertainties of our knowledge of these amplitudes are also shown [26].



From this picture it is clear that neutrons are much more effective in regeneration than protons (the difference in absorption of $\bar{K}^0$ and K^0 is much bigger in the case of neutrons).

One can get excellent fits to the differential cross sections $d\sigma_{K_L \to K_S}/d\Delta^2$ by making the neutron and proton distributions different. One gets the following nuclear parameters from the best fits [26].

	Pb	Cu	
R_p	6.60 fm	4.23 fm	these are well known from elastic scattering
C_p	0.50 fm	0.57 fm	
$R_n(C_n = C_p)$	(7.29±0.13) fm	(4.86±0.10) fm	
$C_n(R_n = R_p)$	(0.68±0.04) fm	(0.74±0.03) fm	

Discussion of this example:

1. Assuming that we can trust the input data (structure of K_s^0 and K_L^0, $f_{K^0 n}$, $f_{\bar{K}^0 n}$, $f_{K^0 p}$, $f_{\bar{K}^0 p}$ amplitudes) we obtained a very important piece of information about the target nuclei: the neutron distribution. This is so because neutrons are more effective in regeneration than protons. In this case we have not, however, obtained any new information about the elementary processes and the structure of K_s^0 and K_L^0.

2. *The main feature of the K regeneration process seems to be very general, however*: The process of diffractive production consists in rearrangement of the "components" (understood in a very broad sense) of the incident particle (system): *but components undergo only elastic scattering*. This description is common to many models of diffractive dissociation (and excitation).

3. In describing the regeneration process $K_L^0 \to K_s^0$ we drew heavily on the known structure of K_L^0 and K_s^0: they are superpositions of K^0 and $\bar{K}^0$, whose elastic scattering from nucleons is reasonably well known.

4.1. Generalizing to Other Diffractive Production (and Excitation) Processes

First of all, the components of the incident and the produced states are, in general, not degenerate: their invariant masses differ. This fact may introduce some important corrections at low energies. But in the limit of very high energies and small momentum transfer, all such effects disappear. Let us take, e.g., two such states and give them the same momentum p. Then their energies differ:

$$E-E^*=\sqrt{p^2+M^2}-\sqrt{p^2+M^{*2}} \xrightarrow[p\ \text{large}]{} \frac{M^2-M^{*2}}{2p}\,.$$

(The only important thing in these approximations is to have very large longitudinal momenta in the initial and final states.) As long as the time of the passage through (or the interaction with) the target is

$$\tau \ll \frac{2p}{M^2-M^{*2}}\,,$$

we can consider the states to be degenerate because their relative phase factor during the collision is very small and we have $\exp\left[-i(E-E^*)\tau\right]\approx 1$ to very good accuracy.

In fact the same argument shows that the incident state and the produced state can also be considered degenerate in the limit $p\to\infty$ (the fact that p will also change slightly during the collision does not change this conclusion). *Note that the degeneracy appears in the laboratory system, where the incident and produced systems move fast.*

One can also show that in this limit ($p\to\infty$) the longitudinal momentum transfer can also be neglected. So, in the limit $p\to 0$ all the states taking part in diffractive production processes can be considered degenerate and the longitudinal momentum transfer neglected. As was said, however, for low p's some important corrections may appear.

Summing up:

Our procedure for evaluating diffractive production in the limit of very high energy consists of two steps:

(i) Find a "plausible" model and identify the components of the incident and outgoing systems.

(ii) Compute the transition matrix element by making the components scatter elastically from the target.

If one can implement such a program, one can treat diffractive production processes on nucleons (elementary targets) and nuclei (composite targets) on the same footing: the only difference is that the components scatter elastically from a nucleon in the first and from a nucleus in the second case.

We shall start discussing the approach outlined at the beginning of section 4 with restrictions (4.3) because this scheme contains a large class of known models of diffractive production, including diffractive processes in QED [21], [22], [27], [28].

Without going into any details let me sketch an example of such an approach in the case of the process of proton dissociation $p\to n+\pi^+$. As in QED (compare e.g., lectures by W. Czyż in ref. [S4]) we express the states of the physical proton and the physical neutron–pion pair through the "bare" states of the proton ($\tilde{p}$) and the neutron–pion pair ($\tilde{\pi}\tilde{n}$).

Initial state: $$p = \underset{\tilde{p}}{\text{―――――}} + \underbrace{\overset{\tilde{\pi}}{\underset{\tilde{n}}{\text{- - - - - -}}}}_{\text{small admixture}} = d_1^p\,|\,\tilde{p}\rangle + d_2^p\,|\,\tilde{\pi}\tilde{n}\rangle$$

$$\text{Final state:}\quad n+\pi=\frac{\tilde{\pi}}{\tilde{n}}+\underbrace{\overset{\tilde{p}}{\overline{}}}_{\substack{\text{small}\\ \text{admixture}}}=d_1^{\pi n}\mid\tilde{\pi}\tilde{n}\rangle+d_2^{\pi n}\mid\tilde{p}\rangle.$$

In the sense given earlier in these notes, *for the purpose of describing elastic scattering* we have approximately (compare eq (4.4))

$$\tilde{p}\approx p,\qquad \tilde{\pi}+\tilde{p}\approx\pi+p.$$

The production amplitude is:

$$\mathfrak{M}_{fi}=\langle\tilde{p}\mid T\mid\tilde{p}\rangle\, d_1^{p}\, d_2^{\pi n}+\langle\tilde{\pi}\tilde{n}\mid T\mid\tilde{\pi}\tilde{n}\rangle\, d_2^{p}\, d_1^{\pi n}.$$

Suppose the target is a nucleon, then $\langle\tilde{p}\mid T\mid\tilde{p}\rangle$ is taken to be the proton-nucleon elastic scattering amplitude, and $\langle\tilde{\pi}\tilde{n}\mid T\mid\tilde{\pi}\tilde{n}\rangle$ is constructed (e.g., à la Glauber) from $\langle\tilde{\pi}\mid T\mid\tilde{\pi}\rangle$ and $\langle\tilde{n}\mid T\mid\tilde{n}\rangle$ pion-nucleon and neutron-nucleon elastic scattering amplitudes [29].

Suppose the target is a nucleus: everything goes the same way except that elastic scattering amplitudes are taken to be with the nucleus (not a nucleon as before). Again they can be computed à la Glauber [29], [S4].

The standard noncovariant perturbation theory is an "obvious" tool to construct such states. Let H' be the interaction Hamiltonian which couples the states $\mid\tilde{p}\rangle$ and $\mid\tilde{n}\tilde{\pi}\rangle$. Let this coupling be weak so that it is sufficient to take only the lowest order corrections:

$$\mid\Psi_i(k_p)\,\rangle=Z\left[\mid k_p,\,0,\,0\rangle+\sum_{k_nk_\pi}\frac{\langle k_nk_\pi\mid H'\mid k_p\rangle}{E_{k_p}-E_{k_n}-E_{k_\pi}}\mid 0,\,k_\pi,\,k_n\rangle+\cdots\right]$$

$$\mid\Psi_f(p_n,\,p_\pi)\,\rangle=Z\left[\mid 0,\,p_n,\,p_\pi\rangle-\sum_{pp}\frac{\langle p_p\mid H'\mid p_np_\pi\rangle}{E_{p_p}-E_{p_n}-E_{p_\pi}}\mid p_p,\,0,\,0\rangle+\cdots\right],\tag{4.6}$$

where Z is the renormalization constant,

$\mid k_p,\,0,\,0\rangle\equiv\mid\tilde{p}\rangle$ is a state of a bare proton

$\mid 0,\,k_\pi,\,k_n\rangle\equiv\mid\tilde{n}\tilde{\pi}\rangle$ is a state of a bare neutron-pion pair.

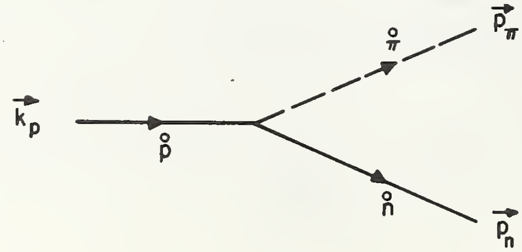
Note that the sign of the second term in $\mid\Psi_f\rangle$ is "minus" because the order of energies in the denominator was changed. These two states should be orthogonal. Indeed

$$\langle\Psi_f\mid\Psi_i\rangle=Z^2\left\{\frac{\langle p_np_\pi\mid H'\mid k_p\rangle}{E_{k_p}-E_{p_n}-E_{p_\pi}}-\frac{\langle k_p\mid H'\mid p_np_\pi\rangle^*}{E_{k_p}-E_{p_n}-E_{p_\pi}}\right\}=0$$

because H' is hermitian, hence $\langle k_p\mid H'\mid p_np_\pi\rangle^*=\langle p_np_\pi\mid H'\mid k_p\rangle$. Note also that if H' is small, $Z\approx 1$ to first order. So, we have:

$$d_1^{p}=d_1^{\pi n}=1\qquad\text{and}\qquad d_2^{p}=-(d_2^{\pi n})^*=\frac{\langle p_np_\pi\mid H'\mid k_p\rangle}{E_{k_p}-E_{p_n}-E_{p_\pi}}.\tag{4.7}$$

Graphically, d_2^{p} corresponds to the vertex:



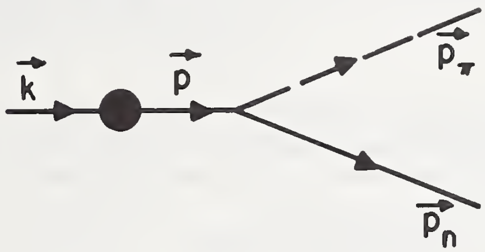
If we accept our "diffractive elastic scattering" operator T to be constructed à la Glauber (although, in principle, one can take something else for it, we prefer to use the Glauber model prescription because it works so well for elastic diffractive scattering) we have

$$T = t_p(\mathbf{p}_\perp - \mathbf{k}_\perp) + t_n(\mathbf{p}_{n\perp} - \mathbf{k}_{n\perp}) + t_\pi(\mathbf{p}_{\pi\perp} - \mathbf{k}_{\pi\perp}) - t_n(\mathbf{p}_{n\perp} - \mathbf{k}_{n\perp})t_\pi(\mathbf{p}_{\pi\perp} - \mathbf{k}_{\pi\perp}).$$

These four pieces of T produce the following contributions:

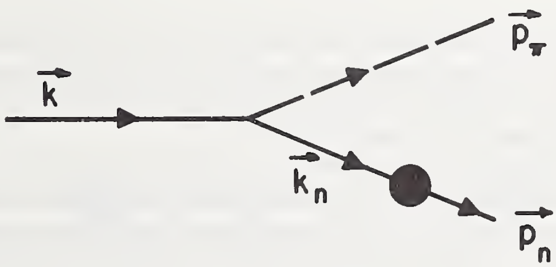
(i) $$\langle \Psi_f \mid t_p \mid \Psi_i \rangle = -\sum_p \frac{\langle p_n p_\pi \mid H' \mid p \rangle}{E_p - E_{p_n} - E_{p_\pi}} t_p(\mathbf{p}_\perp - \mathbf{k}_\perp),$$

graphically



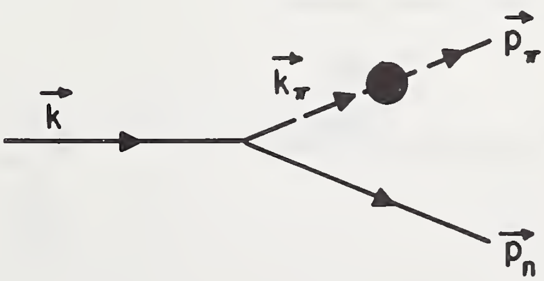
(ii) $$\langle \Psi_f \mid t_n \mid \Psi_i \rangle = \sum_{k_n} t_n(\mathbf{p}_{n\perp} - \mathbf{k}_{n\perp}) \frac{\langle k_n p_\pi \mid H' \mid k \rangle}{E_k - E_{k_n} - E_{p_n}},$$

graphically



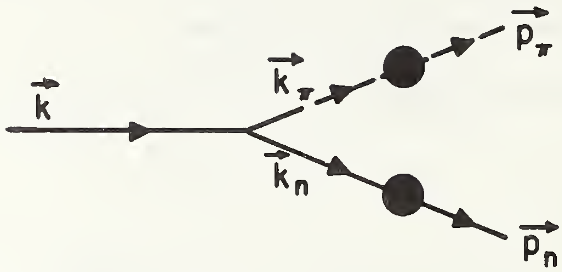
(iii) $$\langle \Psi_f \mid t_\pi \mid \Psi_i \rangle = \sum_{k_\pi} t_\pi(\mathbf{p}_{\pi\perp} - \mathbf{k}_{\pi\perp}) \frac{\langle p_n k_\pi \mid H' \mid k \rangle}{E_k - E_{p_n} - E_{k_\pi}}$$

graphically

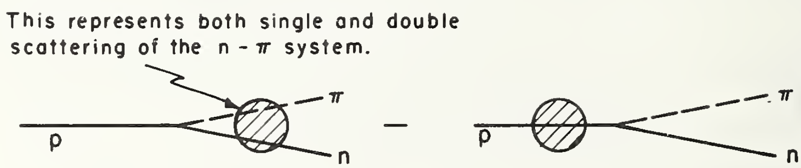


(iv) $\langle \Psi_f \mid t_n t_\pi \mid \Psi_i \rangle = \sum_{k_\pi k_n} t_\pi(\mathbf{p}_{\pi\perp} - \mathbf{k}_{\pi\perp}) t_n(\mathbf{p}_{n\perp} - \mathbf{k}_{n\perp}) \dfrac{\langle k_n k_\pi \mid H' \mid k \rangle}{E_k - E_{k_n} - E_{k_\pi}}$

graphically



One can summarize the situation as follows: the coefficients $d_2{}^p$ (or $d_2{}^{\pi n}$) give the amplitude for the neutron-pion fluctuation of the incoming proton. The total production amplitude is the difference of the two main contributions. This represents both single and double scattering of the $n-\pi$ system



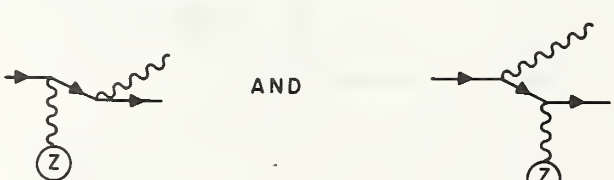
in which the diffractive elastic scattering occurs either before or after the fluctuation takes place.

Note that in these considerations we can have any target we want! The target is specified through the scattering operators t_π *and* t_n. Hence one can use the same technique to describe the processes on simple and composite (e.g., nuclei) targets [S4].

Relation of the above model to some well established techniques of describing diffractive dissociation. First of all, our description is quite similar to that applied by Cheng and Wu [21], Bjorken, Kogut, and Soper [27], and Jaroszewicz [30], for high energy bremsstrahlung and pair production processes in QED (see also [S4]). It is also very closely related to some one particle exchange models of diffractive production processes first suggested by Drell and Hiida [31] and continued by Deck [32], and M. Ross and Y. Y. Yam [33]. Our description also contains, as a special case, the so-called vector meson dominance models which are employed to describe interactions with hadronic targets at high energy, a process which will be discussed later.

Example 1: *The processes of QED*:

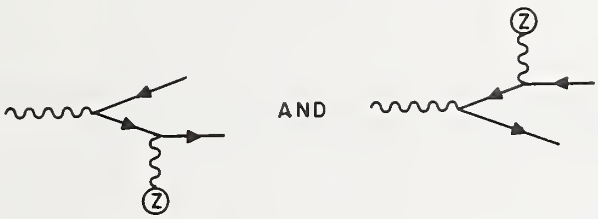
(a) Bremsstrahlung: the Feynman diagrams



go over to

where the dot ● means that complete (to all orders) elastic scattering amplitudes are to be inserted and the vertices are given by d_{ij} coefficients analogous to the ones in eqs (4.5), (4.6), and (4.7).

(b) Pair production: the Feynman diagrams



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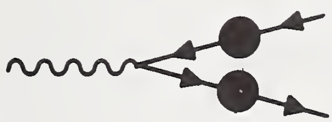
and in our formulation a double scattering process should be added:

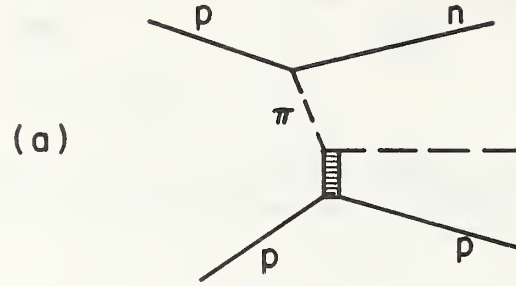


which makes our description different from the above two Feynman diagrams (in fact, more complete) but in total agreement with the Bethe and Maximon formulae [34] for the high energy limit of the pair production cross section in a strong Coulomb field [35].

Example 2: One particle exchange process:

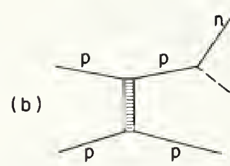
Drell and Hiida [31] started with the diagram

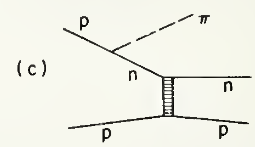
(a)



In such a model, the exchanged virtual pion scatters elastically (diffractively) off the target proton. Later (see e.g., Ross and Yam [33]), two more diagrams were added:

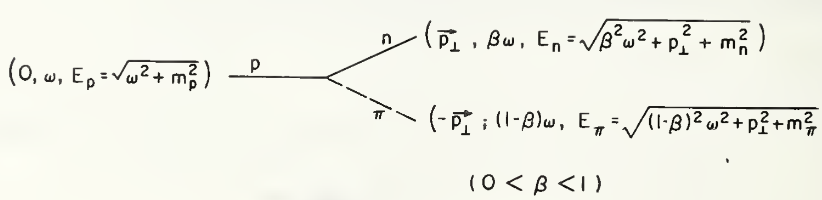
(b)





These diagrams (except for the "diffractive" vertex, they are just Feynman diagrams) are intimately related to our description. In order to see it, one should do some kinematics.

Let us work out some kinematical expressions associated with the vertex $d_2{}^p$ of the processes (i)–(iv):



where the four-vectors are denoted $(\mathbf{p}_\perp, p_z, p_0)$ and we employed conservation of the three momentum at the vertex. (Note that since we are using noncovariant perturbation theory the energy is not conserved at the vertex.)

Let us evaluate all expressions in the limit $\omega \to \infty \cdot$ (longitudinal component of the incident momentum very large). There are *two* independent variables which may be chosen as $\mathbf{p}_\perp$, β. (Here β is an arbitrary parameter, $0 < \beta < 1$).

$$\frac{1}{E_p-E_n-E_\pi}=\frac{1}{\sqrt{\omega^2+m_p{}^2}-\sqrt{\beta^2\omega^2+p_\perp{}^2+m_n{}^2}-\sqrt{(1-\beta)^2\omega^2+p_\perp{}^2+m_\pi{}^2}}$$

$$\xrightarrow[\omega\to\infty]{}\frac{2\omega\beta(1-\beta)}{[\beta(1-\beta)m_p{}^2-(1-\beta)m_n{}^2-\beta m_\pi{}^2-p_\perp{}^2]}$$

One can compute similarly the invariant mass of the $n-\pi$ system:

$$M_{n\pi}{}^{*2}=(\sqrt{\beta^2\omega^2+p_\perp{}^2+m_n{}^2}+\sqrt{(1-\beta)^2\omega^2+p_\perp{}^2+m_\pi{}^2})^2-(\mathbf{p}_\perp-\mathbf{p}_\perp)^2-(\beta\omega+(1-\beta)\omega)^2$$

$$\xrightarrow[\omega\to\infty]{}\frac{1}{\beta(1-\beta)}[p_\perp{}^2+m_n{}^2(1-\beta)+m_\pi{}^2\beta],$$

and the four-momentum transfers:

$$-t_{p\pi}=-m_n{}^2+\beta(M_{\pi n}{}^{*2}-m_p{}^2)$$

$$-t_{pn}=-m_\pi{}^2+(1-\beta)(M_{\pi n}{}^{*2}-m_p{}^2).$$

Hence, the following relations are valid

$$\frac{\omega}{M_{n\pi}{}^{*2}-m_p{}^2}=\frac{(1-\beta)\omega}{t_{pn}-m_\pi{}^2}=\frac{\beta\omega}{t_{p\pi}-m_\pi{}^2}=\tfrac{1}{2}\frac{1}{E_n+E_\pi-E_p}.$$

Since the Feynman diagrams (a)–(c) give the following contributions

$$\mathfrak{M}_{(a)}\sim A^{\pi p}(s_{p\pi})\frac{V}{t_{pn}-m_\pi{}^2},$$

$$\mathfrak{M}_{(b)}\sim A^{pp}(s_{pp})\frac{V}{m_p{}^2-M_{\pi n}{}^{*2}},$$

$$\mathfrak{M}_{(c)}\sim A^{np}(s_{np})\frac{V}{t_{p\pi}-m_n{}^2},$$

where A's are elastic scattering amplitudes and V's are the vertex functions, we can see, using the relation

$$A\sim\omega\sigma_{\text{tot}}$$

and above proven equalities, that (assuming the vertices identical, which is the case for forward amplitudes) $\mathfrak{M}_{(b)}\cong-\mathfrak{M}_{(c)}$, and that they cancel to a large extent (they cancel exactly in the forward direction if $\sigma_{pp}=\sigma_{pn}$).

We can also see the correspondence between our diagrams and the "one pion exchange" diagrams:

$$\text{(i)}\rightleftarrows\text{(b)},\qquad\text{(ii)}\rightleftarrows\text{(c)},\qquad\text{(iii)}\rightleftarrows\text{(a)}.$$

However, our diagram (iv) has no analogue in the Drell-Hiida-Ross-Yam-Deck model. The other difference is the lack of four-momentum conservation in (i)–(iv) (only three-momentum conservation). This last difference may sometimes be relevant (see e.g. A. Białas, W. Czyż, and A. Kotański [29]).

Example 3: *Photoprocesses and vector meson dominance*:

It is now a well established fact that, at high energy, photons exhibit shadowing. The total photo cross sections for complex nuclei vary approximately as $A^{0.9}$ in the few GeV energy range.

It is very suggestive then to accept that photons have in them strongly interacting components. Again, we can use a perturbation expansion, where the interaction Hamiltonian gives the photon-hadron interaction. We can write the physical photon state as follows:

$$| \Psi(k_\gamma) \rangle = Z \left\{ | k_\gamma, 0 \rangle + \sum_n \frac{\langle n | H' | k_\gamma \rangle}{E_\gamma - E_n} | 0, n \rangle + \cdots \right\},$$

where 'n' labels all possible hadronic states which can be coupled to a photon. We can also write, as before, the expansion of a hadronic state n

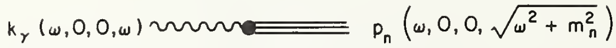
$$| \Psi(n) \rangle = Z \left\{ | 0, n \rangle - \sum_{k_\gamma} \frac{\langle k_\gamma | H' | n \rangle}{E_\gamma - E_n} | k_\gamma, 0 \rangle + \cdots \right\}.$$

Note that here the state $| 0, n \rangle$ is "almost physical." More precisely: as far as strong interactions go, it is physical. The situation is very similar to that in the $K_L \to K_s$ regeneration problem: we are expanding our physical states into states which are "physical in their strong interactions."

Since we know strongly interacting vector mesons with the same quantum numbers as the photon, the simplest possible assumption one can make is to accept that the $| 0, n \rangle$ states are dominated by a vector meson (or mesons). In fact, there is a well-known model of photo-hadronic interactions (Vector Meson Dominance model) which assumes just that.

Some important consequences can be inferred from the above expansions even without specifying the nature of the hadronic components. Let us call m_n the invariant mass of the hadronic component $| n \rangle$. Then

$$| E_\gamma - E_n | = | \omega - \sqrt{\omega^2 + m_n^2} | \approx \frac{m_n^2}{2\omega} \xrightarrow[\omega \to \infty]{} 0.$$



The time during which the hadronic vacuum fluctuation lives is $\Delta t \approx | (E_\gamma - E_n) |^{-1}$ and this also gives the distance it travels ($l = c\Delta t$). Hence, when $l \approx 2\omega/m_n^2 \gg R$ (R = nuclear radius) we shall have shadowing fully developed. *We can see, therefore, that full shadowing occurs at high enough energies.* At low photon energies $| E_\gamma - E_n |$ is large and the corresponding fluctuation cannot interact with the whole nucleus.

How can one test the VMD hypothesis? Let us denote the vector mesons by the letter V. Then (note that $Z = 1$ in our approximation)

$$| \Psi_i(k_\gamma) \rangle = | k_\gamma, 0 \rangle + \sum_V \frac{\langle k_v | H' | k_\gamma \rangle}{E_\gamma - E_V} | 0, k_V \rangle,$$

$$| \Psi_f(k_V) \rangle = | 0, k_V \rangle - \frac{\langle k_\gamma | H' | k_V \rangle}{E_\gamma - E_V} | k_\gamma, 0 \rangle.$$

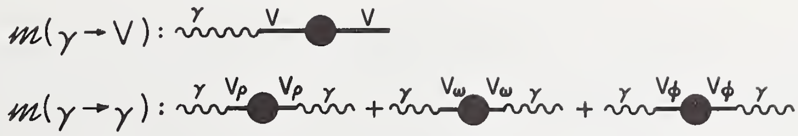
Then the photoproduction of V (on a hadronic target) has the forward amplitude

$$\mathfrak{M}(\gamma \to V) = \langle 0, k_V | t_V | 0, k_V \rangle \frac{\langle k_V | H' | k_\gamma \rangle}{E_\gamma - E_V}.$$

Similarly, the elastic scattering of a high energy photon (Compton scattering) from a hadronic target has the forward amplitude[4]

$$\mathfrak{M}(\gamma\to\gamma) = \sum_V \langle 0, k_V \mid t_V \mid 0, k_V \rangle \frac{\langle k_\gamma \mid H' \mid k_V \rangle \langle k_V \mid H' \mid k_\gamma \rangle}{(E_\gamma - E_V)(E_\gamma - E_V)} .$$

Graphically



(There are three known vector mesons: ρ, ω, ϕ with masses

$$m_\rho \cong 765 \text{ MeV}, \qquad m_\omega \cong 784 \text{ MeV}, \qquad m_\phi \cong 1020 \text{ MeV}).$$

One usually writes (α is the fine structure constant):

$$\frac{\langle k_V \mid H' \mid k_\gamma \rangle}{E_\gamma - E_V} \xrightarrow[\omega\to\infty]{} \frac{\sqrt{\pi\alpha}}{\gamma_V} .$$

Then

$$\mathfrak{M}(\gamma\to\gamma) = \sum_V \mathfrak{M}(\gamma\to V) \frac{\sqrt{\pi\alpha}}{\gamma_V} .$$

But

$$\mathfrak{M}(\gamma\to V) = \mathfrak{M}(V\to V) \frac{\sqrt{\pi\alpha}}{\gamma_V} ,$$

---

[4] Note that restricting the high energy photon-nucleus interaction to only vector meson interactions (VMD model) is a very drastic step. There are other possible interactions of the same order of magnitude ($\sim e^4$) whose role one should discuss: for instance all kinds of such Compton-like processes which do not belong to the VMD model, where the photon is absorbed by the target (or part of it) and re-emitted. E.g., when the total Compton amplitude is a *sum* of individual (photon-nucleon) Compton amplitudes

$$\mathfrak{M}_{\text{COMPTON}}(q) = \sum_{j=1}^{A} \exp(i\mathbf{q}\cdot\mathbf{r}_j)\mathfrak{M}_{\text{COMPTON}}^{(j)}(q)$$

where $\mathbf{q}$ is the three momentum transfer and $\mathbf{r}_j$ are position vectors of the nucleons in the target nucleus, it gives the following contribution to the total photon-nucleus cross section (which is proportional to A):

$$\frac{4\pi}{k} \operatorname{Im} \mathfrak{M}_{\text{COMPTON}}(0) = A\left[\frac{Z}{A}\sigma_T(\gamma p) + \frac{N}{A}\sigma_T(\gamma n)\right],$$

where $\sigma_T(\gamma p)$ and $\sigma_T(\gamma n)$ are the total photon-proton and photon-neutron cross sections, respectively. When the VMD model does reproduce the correct total photon-*nucleon* cross section the contribution given above is just a single scattering contribution of the multiple scattering of vector mesons and is *properly taken care of by the VMD model description of photon-nucleus* interactions. If, however, the VMD model fails to reproduce the total photon-*nucleon* cross section, the balance between the single and multiple scattering contributions given by the VMD model is disturbed and an additional contribution to the total cross section appears which is not screened ($\sim A$) (compare eq (4.10)). In fact, it is very likely that something like that may indeed take place. For instance in ref. [36] the authors working with the parton model find a *not screened* ($\sim A$) contribution amounting to $\sim$20 percent of $\sigma_T(\gamma Pb)$.

and hence

$$\mathfrak{M}(\gamma\rightarrow\gamma) = \sum_V \frac{\pi\alpha}{\gamma_V^2} \mathfrak{M}(V\rightarrow V),$$

$$\sigma_T(\gamma, \text{hadron}) = \frac{4\pi}{k_\gamma} \operatorname{Im} \mathfrak{M}(\gamma\rightarrow\gamma)$$

$$= \frac{\pi}{k_\gamma} \sum \frac{4\pi\alpha}{\gamma_V^2} \operatorname{Im} \mathfrak{M}(V\rightarrow V)\,|_0,$$

where $(\ldots)\,|_0$ denotes the forward value.

$$\sigma_T(\gamma, \text{hadron}) = \frac{\pi}{k_\gamma} \sum_V \frac{4\pi\alpha}{\gamma_V^2} \frac{\gamma_V}{\sqrt{\pi\alpha}} \operatorname{Im} \mathfrak{M}(\gamma\rightarrow V)\,|_0$$

$$|\,\mathfrak{M}(\gamma\rightarrow V)\,|_0\,|^2 = (1+\eta_V^2)\,(\operatorname{Im} \mathfrak{M}(\gamma\rightarrow V)\,|_0)^2 = \left.\frac{d\sigma_V}{d\Omega}\right|_0,$$

where η_V is the ratio of the real to the imaginary part of $\mathfrak{M}(\gamma\rightarrow V)$. So, finally, the relation which can be tested is as follows:

$$\sigma_T(\gamma, \text{hadron}) = \frac{\pi}{k_\gamma} \sum_V \frac{4\sqrt{\pi\alpha}}{\gamma_V} (1+\eta_V^2)^{-1/2} \left(\left.\frac{d\sigma_V}{d\Omega}\right|_0\right)^{1/2}. \tag{4.8}$$

By measuring independently the forward vector meson production cross sections and the total photo cross section one can check the internal consistency of the VMD. There are some other tests but the above equation was used in the analysis published recently by D. O. Caldwell et al. [37]. Another possible test is, e.g., the equation

$$\sigma_T(\gamma, \text{hadron}) = \frac{\pi}{k_\gamma} \sum_V \frac{4\pi\alpha}{\gamma_V^2} \operatorname{Im} \mathfrak{M}(V\rightarrow V).$$

Assuming $\mathfrak{M}(V\rightarrow V)$ purely imaginary (in the high energy limit) this becomes

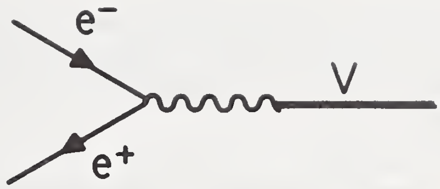
$$\sigma_T(\gamma, \text{hadron}) \cong \sum_V \frac{\pi\alpha}{\gamma_V^2} \sigma_{VN}. \tag{4.9}$$

First, let us discuss the formula (4.9) which is cruder and contains some nondirectly measurable parameters. The table below gives an idea of accuracy with which it is possible to test it (see K. Gottfried report in [S5]). At 7 GeV incident energy the left hand side is $\sigma_T(\gamma, \text{nucleon}) = 118\pm4\ \mu$b.

V	σ_{VN}(mb)	$\gamma_V^2/4\pi$	$\frac{\alpha\pi}{\gamma_V^2}\sigma_{VN}(\mu\text{b})$	energy (GeV)
ρ	27.5 ± 2	$0.62\ \ \pm0.05$	81 ± 14	6
ω	25.8 ± 8	$4.8\ \pm\pm0.5$	$10\pm\ 4$	7
ϕ	$13\ \ \pm3$	$2.8\ \ \pm0.2$	$8\pm\ 3$	23
			sum $=99\pm21\ \mu$b	

The sum $99\pm21\ \mu$b checks reasonably well with the value given above, $118\pm4\ \mu$b.

All these parameters are obtained from a host of various experiments: $\gamma_V^2/4\pi$ from e^+e^- storage rings where the following process is observed:



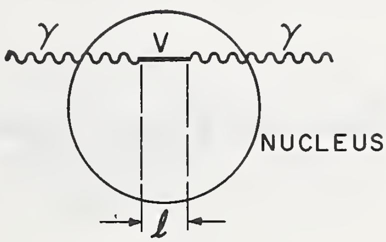
σ_{VN} and η_{VN} (not shown above) from the A dependence of V photoproduction, η_{VN} was also extracted from Compton scattering and leptonic decays. From the table above one can also see that the ρ meson contribution is more important than the contribution of the other two vector mesons.

Let us go back to formula (4.8). It had been checked both for nucleon and nuclear targets (nuclear targets: Pb, Cu, C) [37].

Nucleon targets: The authors assumed the ω and ϕ contributions to $\sigma_T(\gamma, \text{nucleon})$ to be 20 ± 2 μb (remember: this is just a small contribution). Then, by measuring $\sigma_T(\gamma, \text{nucleon})$ one can determine $(d\sigma/d\Omega)(\gamma p \to \rho^0 p)|_0$ for which *one gets much too high a value.* One may get agreement if one reduces $\gamma_\rho^2/4\pi$ from 0.62 to 0.37! But $\gamma_\rho^2/4\pi$ is well known from colliding beam experiments. So, it is unlikely that one should reduce it by a factor of almost 2! (Unless there is a strong dependence on the invariant mass of a virtual photon).

One can also get the correct answer when one accepts that there is a contribution from a Compton-like process (not given by the VMD model!) which does not show any screening and is proportional to A (see footnote 4).

Nuclear target: One could test (4.8) directly against experimental data for nuclear targets if the energy were high enough. Remember, however, that in order to have VMD active in its full strength one has to have $l \approx 2\omega/m_V^2 \gg R$, where R is the nuclear radius, which condition is not well satisfied at existing photon energies: e.g., $l_\rho = 2.1$ fm at 6 GeV. Hence one has to use a more sophisticated description which in fact allows for the hadronic fluctuation to fold back into a photon inside of the nucleus:



We shall come back to this point later. For the moment let us simply state the results of such a "sophisticated" description (the low energy version, in which $\Delta_{||} \neq 0$, is given below in eq (4.10)) which was presented in the paper by Caldwell et al. [37]. By investigating the A dependence of the $\sigma_T(\gamma A)$ cross section they found less screening than demanded by VMD but more than demanded by purely electromagnetic interactions. Hence the discrepancy is related to a partial breakdown of VMD, rather than to smaller γ_ρ (which would not change the screening). Indeed it seems that a Compton-like (non-VMD) contribution $\sim A$ which would give ~ 20 percent of σ_T could make the theory and experiment agree. In fact one does not need a purely electromagnetic interaction to obtain a contribution $\sim A$. It could come from a heavy vector boson whose

$$l \approx \frac{2\omega}{M_V^2} \ll R$$

is still short because its rest mass is large.

Conclusion: The VMD model is only approximately correct. There is no commonly accepted explanation of the discrepancies described above. Perhaps the Compton-like contributions to $\mathfrak{M}(\gamma \rightarrow \gamma)$ (as suggested by Brodsky, Close and Gunion [36]) should be added to VMD to explain the recent photoabsorption data.

4.2. Photoproduction of Vector Bosons

The breakdown of VMD which one sees from the results of Caldwell et al. [37] does not eliminate the possibility that the previously worked out relations between $\mathfrak{M}(\gamma \rightarrow \rho)$ and $\mathfrak{M}(\rho \rightarrow \rho)$ are good approximations. It is in fact commonly accepted that they form a sound basis for analysis of production of vector mesons on various nuclear targets and, since it is comparatively well documented experimentally, it is instructive to outline it here.

The production amplitude of a vector boson is (in the limit $\omega \rightarrow \infty$)

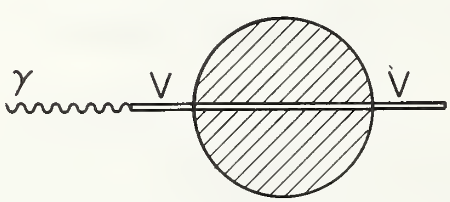
$$\mathfrak{M}(\gamma \rightarrow V) = \frac{\sqrt{\pi\alpha}}{\gamma_V} \langle 0, \mathbf{k}_V \mid t_V \mid 0, \mathbf{k}_V' = \mathbf{k}_\gamma \rangle \tag{4.9}$$

where $\langle \mid t_V \mid \rangle$ is just the elastic scattering amplitude of the vector meson V from the target nucleus (which we may take over from our previous discussion of hadronic elastic scattering from nuclei). So, in the high energy limit

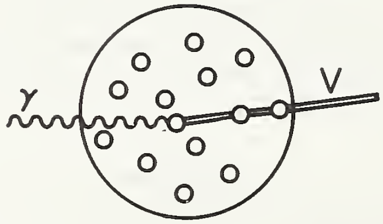
$$\langle 0, k_V \mid t_V \mid 0, k_V' \rangle = \frac{ik_V}{2\pi} \int d^2b \exp [i(\mathbf{k}_V - \mathbf{k}_V') \cdot \mathbf{b}]\{1 - \exp [i\chi_V(b)]\}$$

$$= \frac{ik_V}{2\pi} \int d^2b \exp (i\mathbf{\Delta} \cdot \mathbf{b})[1 - (1 - \langle V \mid \Gamma \mid V \rangle)^A].$$

$\mathfrak{M}(\gamma \rightarrow V)$ is given graphically below



In the high energy limit ($\omega \rightarrow \infty$) one can also describe this process as follows: the photon penetrates the nucleus up to a certain point where it converts into a V meson which scatters elastically from the other nucleons and then leaves the nucleus. Graphically



In computing the amplitude we have to sum over all nucleons of the target nucleus because the conversion of γ into V can occur on any one of them.

So, the production amplitude is in this case:

$$\mathfrak{M}(\gamma\rightarrow V)=\frac{ik_V}{2\pi}\int d^2b\ \exp\ (i\mathbf{\Delta}\cdot\mathbf{b})\langle V\mid\Gamma\mid\gamma\rangle\sum_{l=1}^{A}(1-\langle V\mid\Gamma\mid V\rangle)^{l-1}.$$

Summing the geometric series we then obtain

$$\mathfrak{M}(\gamma\rightarrow V)=\frac{ik_V}{2\pi}\int d^2b\ \exp\ (i\mathbf{\Delta}\cdot\mathbf{b})\frac{\langle V\mid\Gamma\mid\gamma\rangle}{\langle V\mid\Gamma\mid V\rangle}[1-(1-\langle V\mid\Gamma\mid V\rangle)^A].$$

But

$$\frac{\langle V\mid\Gamma\mid\gamma\rangle}{\langle V\mid\Gamma\mid V\rangle}=\frac{\sqrt{\pi\alpha}}{\gamma_V},$$

because, on one nucleon

$$\mathfrak{M}_{\text{nucleon}}(\gamma\rightarrow V)=\frac{\sqrt{\pi\alpha}}{\gamma_V}\mathfrak{M}_{\text{nucleon}}(V\rightarrow V)$$

and

$$\mathfrak{M}_{\text{nucleon}}=\frac{ik_V}{2\pi}\int d^2b\ \exp\ (i\boldsymbol{\delta}\cdot\mathbf{b})\,\Gamma_{\text{nucleon}}(b)$$

for both the $\gamma\rightarrow V$ and $V\rightarrow V$ processes. So, we get again the formula (4.9).

In all the formulae above we have used the profiles of nucleons smeared over the interior of the nucleus with the single nucleon density functions $\rho(r)$:

$$\Gamma(b)=\int d^3r\,\rho(\mathbf{r})\gamma(\mathbf{b}-\mathbf{s}),$$

where $\mathbf{r}=(\mathbf{s},z)$, as always.

The conclusion of the above discussion: The two seemingly different pictures give the same results *in the high energy limit.*

As we saw in the example of our discussion of $\sigma_T(\gamma, A)$, present experiments are not quite in this limit (at several GeV $l<R$). Hence if we want to analyze the existing experimental data we have to keep the longitudinal momentum transfer different from zero:

$$\Delta_{||}\approx\frac{m_V^2}{2\omega}.$$

Then the graphical description of figure 38 is not valid any more. One has to allow the amplitude $\mathfrak{M}_{\text{nucleon}}(\gamma\rightarrow V)$ to oscillate with the function

$$e^{i\Delta_{||}z}.$$

This factor partly destroys the coherence of the process of producing V's over the whole nuclear volume. We construct the production amplitude as the (previously introduced) picture of multiple scattering tells us to. For the independent particle ground state wave function we have

$$\mathfrak{M}(\gamma\rightarrow V)=\frac{ik_V}{2\pi}\int d^2b\ \exp\ (i\mathbf{\Delta}\cdot\mathbf{b})\int d^3r_1\ldots d^3r_A\prod_{j=1}^{A}\rho(r_j)$$
$$\times\sum_i\prod_{j\neq i}[1-\gamma_V(\mathbf{b}-\mathbf{s}_j)\Theta(z_j-z_i)]\gamma_{V\gamma}(\mathbf{b}-\mathbf{s}_i)\ \exp\ (i\Delta_{||}z_i)$$

Note that this expression satisfies the correct "weak interaction" limit: when the V interaction is negligible the amplitude reduces to

$$\mathfrak{M}(\gamma\to V)=\frac{ik_V}{2\pi}\sum_j\int d^3r_j\,\rho(r_j)\gamma_{V\gamma}(\mathbf{b}-\mathbf{s}_j)\exp(i\boldsymbol{\Delta}\cdot\mathbf{b}+i\Delta_{||}z)$$

For large A and the profiles γ much narrower than the density ρ, we have

$$\mathfrak{M}\to f_{V\gamma}(0)\int d^2b\exp(i\boldsymbol{\Delta}\cdot\mathbf{b})\int_{-\infty}^{\infty}dz\,\rho(\mathbf{b},z)e^{i\Delta_{||}z}$$

$$\times\exp\left[-\tfrac{1}{2}\sigma_{VN}A(1-\eta_{VN})\int_z^{\infty}dz'\rho(\mathbf{b},z')\right],$$

where $f_{V\gamma}(0)$ is the forward production amplitude on one nucleon corresponding to the profile $\gamma_{V\gamma}$.

The above formulae can also be obtained from the expression[5]

$$\mathfrak{M}(\gamma\to V)=\int d^3r_1\ldots d^3r_A\,|\Psi(\mathbf{r}_1,\ldots\mathbf{r}_A)|^2\sum_j\mathfrak{M}_j^{\gamma\to V}(q)\exp(i\mathbf{q}\cdot\mathbf{r}_i)$$

where $\mathbf{q}=(\boldsymbol{\Delta},\Delta_{||})$, and $\mathfrak{M}_j^{\gamma\to V}$ are the amplitudes for the production of a V meson on the jth nucleon *with the screening of the other nucleons taken into account.*

For a given configuration of the nucleons we have[6]

---

[5] When we want to shift the scatterer by the distance $\mathbf{r}$, the amplitude acquires the phase $e^{i\mathbf{q}\cdot\mathbf{r}}$. In other words, we have to transform the amplitudes as follows:

$$\mathfrak{M}(q)\to e^{i\mathbf{q}\cdot\mathbf{r}}\mathfrak{M}(q),$$

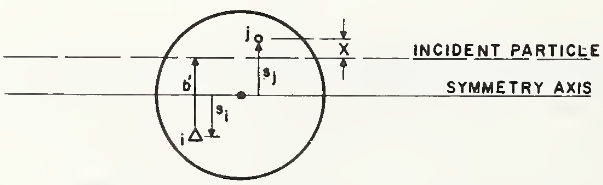
where $\mathbf{q}$ is the three-momentum transfer vector. As long as there is only one scattering center this phase factor is irrelevant, but when there are more scattering centers we obtain

$$\mathfrak{M}(q)=\sum_j e^{i\mathbf{q}\cdot\mathbf{r}_j}\mathfrak{M}j(q),$$

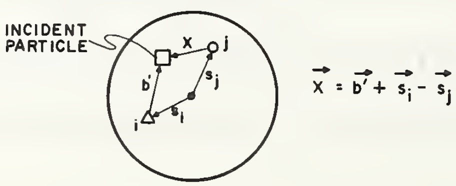
which is the formula used here.

[6] The argument in γ_V, which gives the elastic scattering vector meson-nucleon profiles, should be the transverse distance between a target nucleon and the incident particle. The geometry below shows that the argument of γ_V in the expression for $\mathfrak{M}(\gamma\to V)$ should be: $\mathbf{b}'+\mathbf{s}_i-\mathbf{s}_j$.

PROJECTION ON THE SCATTERING PLANE



PROJECTION ON A PLANE PERPENDICULAR TO THE INCIDENT PARTICLE DIRECTION



$$\sum_i \mathfrak{M}_i^{\gamma\to V}(q)\exp(i\mathbf{q}\cdot\mathbf{r}_i)=\frac{ik_V}{2\pi}\int d^2b\exp(i\mathbf{\Delta}\cdot\mathbf{b})\sum_i\prod_{j\neq i}[1-\gamma_V(\mathbf{b}-\mathbf{s}_j)\Theta(z_j-z_i)]\gamma_{V\gamma}(\mathbf{b}-\mathbf{s}_i)e^{i\Delta_{||}z_i}$$

$$\sum_i \exp(i\mathbf{\Delta}\cdot\mathbf{s}_i)e^{i\Delta_{||}z_i}\frac{ik_V}{2\pi}\int d^2b$$

$$\times\exp(i\mathbf{\Delta}\cdot\mathbf{b})\prod_{j\neq i}[1-\gamma_V(\mathbf{b}'+\mathbf{s}_i-\mathbf{s}_j)\Theta(z_j-z_i)]\gamma_{V\gamma}(b')$$

which, upon averaging over the ground state

$$|\Psi(\mathbf{r}_1\ldots\mathbf{r}_A)|^2\cong\prod_{j=1}^{A}\rho(\mathbf{r}_j)$$

becomes the formula given above. Note that the same reasoning gives, in the limit of large A,

$$\mathfrak{M}(\gamma\to V)=A\int d^2b\,dz\exp(i\mathbf{\Delta}\cdot\mathbf{b}+i\Delta_{||}z)\rho(b,z)f_{V\gamma}(0)\exp[-\tfrac{1}{2}\sigma_{VN}(1-i\eta_{VN})T_f(b,z)]$$

with

$$T_f(b,z)=A\int_z^{+\infty}dz'\rho(b,z').$$

This is again the same formula as before.

Note that for nucleons at different positions z, the attenuation of the outgoing vector meson beam is different. So, in the last two expressions for $\mathfrak{M}(\gamma\to V)$ our $\mathfrak{M}_i^{\gamma\to V}(q)$ *depend on the position of all the other nucleons.*

These formulae are now adapted to take care of nonnegligible $\Delta_{||}$. They are being used in all standard analyses of photoproduction of vector bosons on nuclei [S3].

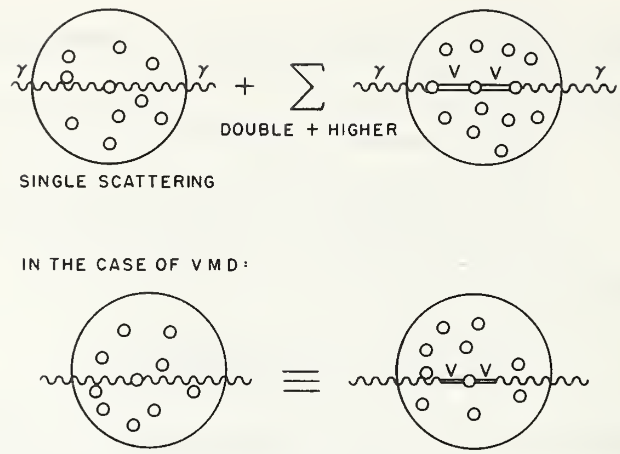
Let us also quote, for the sake of completeness (without giving derivation), the amplitude for elastic scattering of photons from a nucleus derived from the multiple scattering model, with $\Delta_{||}$ nonnegligible, in the limit of large A and VMD assumed:

$$\mathfrak{M}(\gamma\to\gamma)=\frac{ik}{2\pi}\int d^2b\exp(i\mathbf{\Delta}\cdot\mathbf{b})\left\{A\int d^3r\,\rho(r)\gamma_{\gamma\gamma}(\mathbf{b}-\mathbf{s})-\sum_V\left(\frac{2\pi}{ik}\right)^2\right.$$

$$\times f_{\gamma V}(0)f_{V\gamma}(0)\int dz_1\,dz_2A\rho(b,z_2)e^{-i\Delta_{||}z_2}\Theta(z_2-z_1)$$

$$\left.\times\exp\left([-\tfrac{1}{2}\sigma_{VN}(1-i\eta_{VN})]A\int_{z_1}^{z_2}dz'\rho(\mathbf{b},z')\right)A\rho(\mathbf{b},z_1)e^{i\Delta_{||}z_1}\right\}.\qquad(4.10)$$

In this formula the single scattering is separated out and one should set $\gamma_{\gamma\gamma}=\sum_V(\pi\alpha/\gamma_V^2)\gamma_{VV}$ if we apply VMD for the single scattering too. These two contributions in $\mathfrak{M}(\gamma\to\gamma)$ can be sketched as follows:



Note that the single scattering contribution always goes as A (just as any multiple scattering process) but it becomes progressively less important as $\Delta_{||} \to 0$ with increasing energy. Recall that the experiments discussed above [37] had shown that when $\sigma_T(\gamma A)$ is computed from (4.10), the single scattering contribution is not the one given by the VMD model (compare also footnote 1).

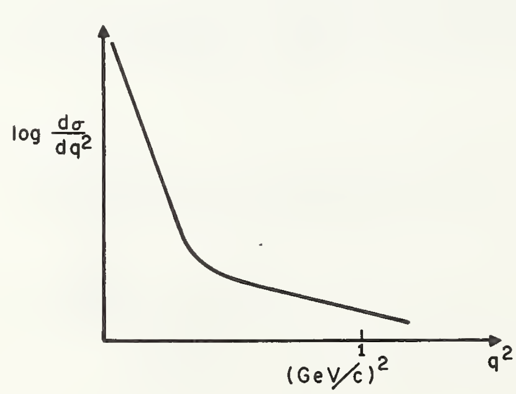
Now let us go back to photoproduction of vector mesons. The formulae discussed above were applied to analyze a multitude of experimental data. The first suggestion that one can obtain some important information on the properties of V-nucleon interactions came from S. D. Drell and J. S. Trefil [38]. Then a flood of papers followed. The references can be found in the review articles we referred to at the very beginning of these notes [S1, S3, S5].

A few general comments can be made by inspecting the formulae for photoproduction of vector mesons:

(i) They depend very strongly on A (in the optical limit the dependence on A is exponential).
(ii) There may be a very important interplay between the "phase factors"

$$e^{i\Delta_{||}z} \quad \text{and} \quad \exp\left[i\sigma_{VN}\eta_{VN}\tfrac{1}{2}T(b, z)\right].$$

So, the differential cross section for photoproduction of vector mesons should, in general, be sensitive to both σ_{VN} and η_{VN}. This is very important because these quantities cannot be obtained directly from any other experiments because there are no vector meson beams available due to their short lifetime. Here such indirect "measurement" is possible because the vector mesons interact with nucleons before they decay. The differential photoproduction cross sections look very much like elastic hadron-nucleus cross sections. They exhibit a steep slope at small q^2 and then a flat part at large q^2 with, possibly, some diffractive minima.



Just to give some idea of how the results look, let us describe briefly the results of ρ^0 photoproduction on deuterium (R. L. Anderson et al. [39]) and a DESY–MIT experiment (H. Alvensleben et al. [40]) of ρ^0 photoproduction on light, medium and heavy nuclei.

First, the deuterium target. We can use our formulae derived above after specifying them for $A=2$. We get

$$\mathfrak{M}_{\text{Deut.}}(\gamma\to\rho^0)$$

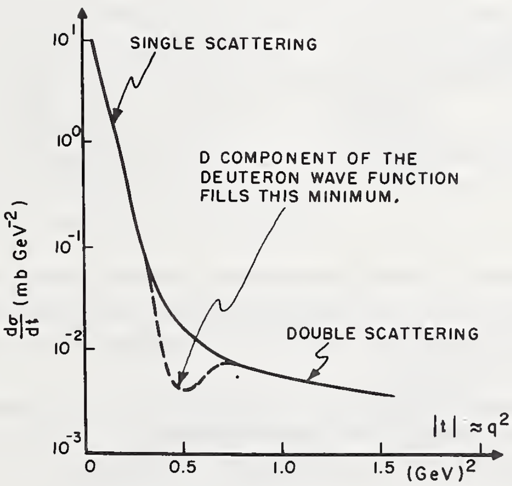
$$= \frac{ik}{2\pi}\int d^2b \exp(i\mathbf{\Delta}\cdot\mathbf{b})\int d^3r\, \rho(r)\{[1-\gamma_{\rho\rho}^{(p)}(\mathbf{b}-\tfrac{1}{2}\mathbf{s})\Theta(z)]\gamma_{\rho\gamma}^{(n)}(\mathbf{b}+\tfrac{1}{2}\mathbf{s})\exp(i\Delta_{||}\tfrac{1}{2}z)$$

$$+[1-\gamma_{\rho}^{(n)}(\mathbf{b}+\tfrac{1}{2}\mathbf{s})\Theta(-z)]\gamma_{\rho\gamma}^{(p)}(\mathbf{b}-\tfrac{1}{2}\mathbf{s})\exp(-i\Delta_{||}\tfrac{1}{2}z)\}$$

$$\left.\begin{aligned}&= \frac{ik}{2\pi}\int d^2b \exp(i\mathbf{\Delta}\cdot\mathbf{b})\int d^3r\, \rho(r)\gamma_{\rho\gamma}^{(n)}(\mathbf{b}+\tfrac{1}{2}\mathbf{s})\exp(i\Delta_{||}\tfrac{1}{2}z)\\ &+ \frac{ik}{2\pi}\int d^2b \exp(i\mathbf{\Delta}\cdot\mathbf{b})\int d^3r\, \rho(r)\gamma_{\rho\gamma}^{(p)}(\mathbf{b}-\tfrac{1}{2}\mathbf{s})\exp(-i\Delta_{||}\tfrac{1}{2}z)\end{aligned}\right\}\ \text{single scattering contributions}$$

$$\left.\begin{aligned}&- \frac{ik}{2\pi}\int d^2b \exp(i\mathbf{\Delta}\cdot\mathbf{b})\int d^3r\, \rho(r)\gamma_{\rho\rho}^{(p)}(\mathbf{b}-\tfrac{1}{2}\mathbf{s})\Theta(z)\gamma_{\rho\gamma}^{(n)}(\mathbf{b}+\tfrac{1}{2}\mathbf{s})\\ &\times \exp(i\Delta_{||}\tfrac{1}{2}z)\\ &- \frac{ik}{2\pi}\int d^2b \exp(i\mathbf{\Delta}\cdot\mathbf{b})\int d^3r\, \rho(r)\gamma_{\rho\rho}^{(n)}(\mathbf{b}+\tfrac{1}{2}\mathbf{s})\Theta(-z)\gamma_{\rho\gamma}^{(p)}(\mathbf{b}-\tfrac{1}{2}\mathbf{s})\\ &\times \exp(-i\Delta_{||}\tfrac{1}{2}z)\end{aligned}\right\}\ \text{double scattering contributions}$$

The cross section experimentally measured looks as follows:



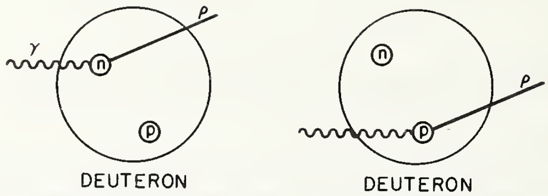
From the above formulae we can see that at small momentum transfers we are essentially measuring the amplitudes for photoproduction of ρ^0 on neutrons and protons, modulated by the deuteron form factor F. For example,

$$\frac{ik}{2\pi}\int d^2b \exp\,(i\mathbf{\Delta}\cdot\mathbf{b})\int d^3r\,\rho(r)\gamma_{\rho\gamma}{}^{(n)}(\mathbf{b}+\tfrac{1}{2}\mathbf{s})\exp\,(i\Delta_{||}\tfrac{1}{2}z)$$

$$=\int d^2b \exp\,(i\mathbf{\Delta}\cdot\mathbf{b})\int d^2s\,dz\,\rho(\mathbf{s},z)\int d^2\delta \exp\,[-i\boldsymbol{\delta}\cdot(\mathbf{b}+\tfrac{1}{2}\mathbf{s})]f_{\rho\gamma}{}^{(n)}(\delta)\exp\,(i\Delta_{||}\tfrac{1}{2}z)$$

$$=\int d^2\delta\,\delta^{(2)}(\mathbf{\Delta}-\boldsymbol{\delta})\int d^2s\,dz\,\rho(s,z)\exp\,(-i\boldsymbol{\delta}\cdot\tfrac{1}{2}\mathbf{s})\exp\,(i\Delta_{||}\tfrac{1}{2}z)f_{\rho\gamma}{}^{(n)}(\delta)$$

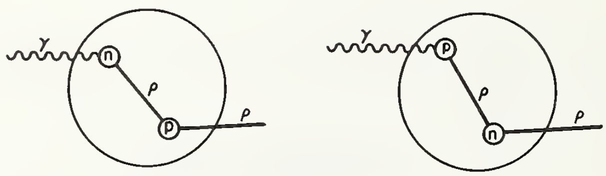
$$=F\left(\frac{\mathbf{\Delta}}{2},\frac{\Delta_{||}}{2}\right)f_{\rho\gamma}{}^{(n)}(\Delta)$$



and the analogous expression for the proton (fig. 44). Hence this part of the cross section, where these contributions dominate, can only confirm what we know from ρ^0 photoproduction on free nucleons.

At large momentum transfers, however, where the double scattering dominates, the amplitude is given approximately by (see also fig. 45)

$$\approx\int d^2\delta F(\delta)f_{\rho\rho}{}^{(p)}(\tfrac{1}{2}\mathbf{\Delta}+\boldsymbol{\delta})f_{\rho\gamma}{}^{(n)}(\tfrac{1}{2}\mathbf{\Delta}-\boldsymbol{\delta})+\int d^2\delta F(\delta)f_{\rho\rho}{}^{(n)}(\tfrac{1}{2}\mathbf{\Delta}-\boldsymbol{\delta})f_{\rho\gamma}{}^{(p)}(\tfrac{1}{2}\mathbf{\Delta}+\boldsymbol{\delta}).$$



So, by measuring the cross section at large momentum transfers *we can extract* $f_{\rho\rho}{}^{\text{nucleon}}$ *because we know* $f_{\rho\gamma}{}^{\text{nucleon}}$ *quite well.* In fact, since $F(\delta)$ is much steeper than $f_{\rho\rho}$ and $f_{\rho\gamma}$, it acts as a Dirac $\delta^{(2)}$ function, and *the measured differential cross section is proportional* (*the proportionality factor being known*) *to the elastic* ρ*-nucleon cross section.*

In this experiment [39], the recoiling deuteron was detected. This guaranteed the coherence of the process and excluded any excitations of the target nucleus.

There are a few points which should be stressed again at this stage:

(i) This is the most direct measurement of the elastic ρ-nucleon cross section in existence.
(ii) If one wants to extract $\sigma_{\rho N}$ or $\gamma_\rho^2/4\pi$, one can do it very safely because the analysis, at large enough q^2, depends insensitively on η_ρ. First one extracts $(d\sigma/dt)\,|_{\rho N\to\rho N}$ and then by taking the ratio

$$\frac{\gamma_\rho^2}{4\pi} = \frac{\alpha}{4} \frac{\left.\frac{d\sigma}{dt}\right|_{\rho N\to\rho N}}{\left.\frac{d\sigma}{dt}\right|_{\gamma N\to\rho N}}$$

one determines the coupling γ_ρ. (Here α is the fine structure constant.)

(iii) The results of this analysis check beautifully with the results obtained from ρ^0 photoproduction on larger nuclei (whose cross sections, incidentally, do depend sensitively on η_ρ as we have already indicated before).

(iv) The extraction of $(d\sigma/dt)\,|_{\rho N\to\rho N}$ does not depend on the VMD hypothesis. However, it assumes the multiple scattering model of Glauber completely and literally. For more details we refer to ref. [39].

Let us now go briefly to photoproduction of ρ^0 on various nuclei and take as an example the DESY–MIT experiment mentioned already [40]. Here, the target was not detected and the measured cross section contained contributions from nonelastic processes. The reaction measured was

$$\gamma+A\to\rho^0+A$$

$$\rho^0\to\pi^++\pi^-.$$

The following targets were used: H_2, Be, C, Al, Ti, Cu, Ag, Cd, In, Ta, W, Au, Pb, U. The parameters obtained from fitting the differential cross sections to their very extensive numerical data are [40],

$$\sigma_{\rho N}=27.7\pm1.7 \text{ mb}, \qquad \gamma_\rho^2/4\pi=0.59\pm0.08,$$

in good agreement with the more recent numbers already quoted. In analysis they assumed the ratio of the real to the imaginary part to be $\eta_\rho=0.2$ (in agreement with the dispersion relations calculations of the total photon cross sections.)

From the above description and the inspection of the data shown in ref [40] one sees that there are several points which bring about uncertainties in such an analysis. These uncertainties arise because one has to make some corrections in order to obtain the coherent ρ-production cross sections:

(i) nuclear excitations should be removed
(ii) processes which lead to $\pi^+\pi^-$ production (other than ρ–production) should be subtracted,
(iii) one has to decide which invariant masses of $\pi^+\pi^-$ are ρ^0's and which belong to some kind of background (compare the data). The problems of interference between $\pi^+\pi^-$ and ρ productions are also relevant (see T. H. Bauer in ref. [25]).

(i) and (ii) can be reliably estimated. Take (i). The process can be computed in the same manner as the poor energy resolution cross-section. The table below shows some illustrative calculations by Yennie (see K. Gottfried report in [S5]).

$(\sigma_{\text{inel}}/\sigma_{\text{coherent}})_{\theta=0}$

	No correlations in the target	With correlations in the target
C	0.060	0.013
Mg	.034	.009
Cu	.014	.003
U	.006	.001

So, as long as we measure small angle ρ^0 photoproduction we can neglect nucleus excitations (this won't produce more than a few percent error). The contribution (ii) was also estimated not to exceed $\sim$10 percent. The point (iii) makes the results of our analysis model dependent but again the extensive work of many experimental groups analysing their experiments seems to show that this model dependence produces $\sim$10 percent uncertainty. This is how far one can trust such numbers as quoted above for $\sigma_{\rho N}$ and $\gamma_\rho^2/4\pi$.

Photoproduction of ϕ and ω vector mesons

The data on these two mesons are much poorer than on ρ. There are in existence, however, several experiments in which they were photoproduced, both on protons and nuclei. The parameters (σ, $\gamma^2/4\pi$, η) were already given earlier in these notes. We shall not go into any details of these experiments. Let us stress only two points:

(i) ϕ is narrow, hence it is much safer to treat it as a well-defined particle.
(ii) ω has, however, a mass very near to m_ρ and these two mesons may "mix."

One can, in fact, suspect that in the "elastic" scattering after a vector meson is produced, some kind of superposition of the vector mesons propagates through the nuclear matter (C. Rogers and Colin Wilkin [41]). We shall not go into these problems now.

A summary of the picture of high energy photon interactions with hadronic targets (which include nuclei).

1. We considered the high energy limit of elastic scattering of photons by nucleons and nuclei. We discussed the evidence for the existence, in the physical high energy photon, of some strongly interacting components. The condition for the applicability of the high energy limit was that the characteristic length, l, defined by the incident energy and the lowest available hadronic invariant mass,

$$l = \frac{2\omega}{m^2} \gg R,$$

be much larger than the target radius, R. At presently available energies this condition is satisfied for nucleon targets but it is not satisfied for nuclear targets.

2. The consequence of this fact is that one cannot use the high energy limit description in interpreting the present experiments of photon interactions with nuclear targets (nucleon targets are OK). We derived the formulae corrected for a non-negligible longitudinal momentum transfer for the case of vector meson production (the formula for elastic photon scattering was also given without derivation).

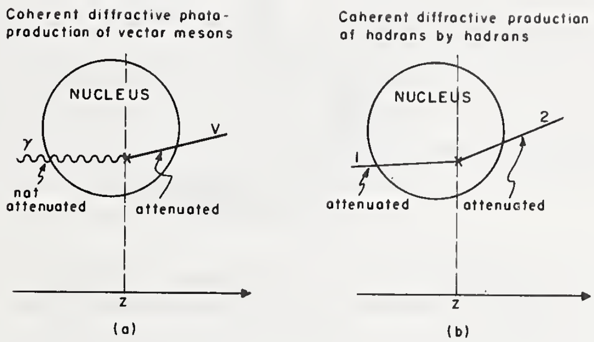
3. The Vector Meson Dominance model was briefly discussed and some recent experiments which seem to show its incompleteness described.

4. Photoproduction of vector mesons from deuterium, and from light, medium, and heavy nuclei was also analyzed.

5. Diffractive Production of Hadrons in Hadron–Nucleus Collisions

The Standard Analysis

We have already discussed some general features of diffractive production processes by hadrons (in the limit of very high energy). Let us now look into a few details of such processes with special emphasis on coherent production processes on nuclei. As in the case of vector meson photoproduction, in the existing experiments the incident energy of hadrons was too low to neglect the longitudinal momentum transfer and the high energy limit description is, at these energies, not applicable. However, with our experience in photoproduction of vector mesons on nuclear targets we can easily remedy this situation. So, the only change we should introduce into the formulae given for vector meson photoproduction is an attenuation of the incident beam of hadrons (the incident photons were not attenuated due to the weakness of electromagnetic interactions). Let us make this extension explicitly and obtain the so-called "one step" model for diffractive production of hadrons in hadron-nucleus collisions. In this "one step" process, the production takes place on *a nucleon* inside the nucleus.



Let us take the large A limit formulae for

(a) coherent diffractive photoproduction of ρ mesons:

$$\mathfrak{M}(\gamma\to\rho)=f_{\rho\gamma}(0)A\int d^2b\,dz\,\exp\,(i\mathbf{\Delta}\cdot\mathbf{b}+i\Delta_{||}z)\rho(\mathbf{b},z)\,\exp\left(-\tfrac{1}{2}\sigma_{VN}(1-i\eta_{VN})A\int_z^\infty dz'\rho(\mathbf{b},z')\right)$$

(b) coherent diffractive production of hadrons by hadrons:

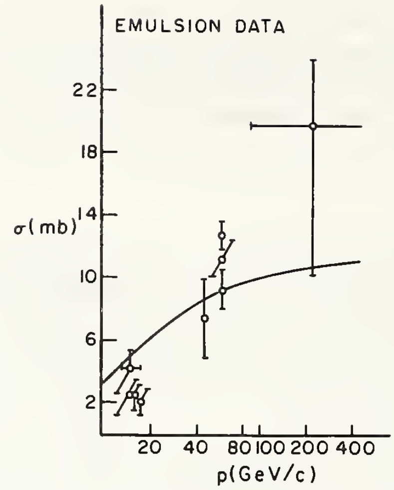
$$\mathfrak{M}(1\to2)=f_{21}(0)A\int d^2b\,dz\,\exp\,(i\mathbf{\Delta}\cdot\mathbf{b}+i\Delta_{||}z)\rho(b,z)\,\exp\left(-\tfrac{1}{2}\sigma_1(1-i\alpha_1)\int_{-\infty}^z dz'\rho(\mathbf{b},z')\right)$$

$$\times\exp\left(-\tfrac{1}{2}\sigma_2(1-i\alpha_2)\int_z^{+\infty} dz''\rho(b,z'')\right). \tag{5.1}$$

In the forward direction production

$$\Delta_{||}\approx\frac{M^{*2}-M^2}{2\omega},$$

hence the oscillating factor under the integral sign, $e^{i\Delta_{||}z}\to1$ as the energy increases. The only other energy dependence in our expression is possibly in f_{21}. When a diffractive production mechanism is effective, this amplitude is experimentally observed to produce a nucleon cross section which is approximately energy independent. Hence one should expect an increase of the $d\sigma_{21}/d\Delta^2$ cross section (obtained from the $\mathfrak{M}(1\to2)$ amplitude given above) as the incident energy increases and $e^{i\Delta_{||}z}\to1$. (Note the difference with vector meson photoproduction: there, due to an intricate interference with $\tfrac{1}{2}i\sigma_{VN}\eta_{VN}A\int_z^\infty dz'\rho$, it is hard to predict what the limit $\Delta_{||}\to0$ will produce.) The existing experimental data on 3π production by pions seem to support this conclusion (H. Leśniak and L. Leśniak [42]).



In computing the curve shown above, they assumed $\sigma_1=\sigma_2=25$ mb and a realistic density distributions for the nuclear targets. In addition, the cross section was weighted with an invariant mass distribution $W(m)$ obtained from coherent production of 3π on a nucleon.

There are many uncertainties in this calculation but the result seems to indicate that one gets to the limit $\Delta_{||}=0$ rather slowly and with presently available experimental data one has to take $\Delta_{||}$ into account. The point of taking the total 3π-nucleon cross section to be 25 mb, equal to the one π cross section, needs some explaining and that will be done below. Here, let us say only a few words about the high energy limit $(\Delta_{||}=0)$ and connect it with our earlier discussion. When $\Delta_{||}=0$ we can perform the integration over z $(\sigma'=\sigma(1-i\alpha))$:

$$A\int_{-\infty}^{+\infty} dz\,\rho(b,z)\exp\left(-\tfrac{1}{2}\sigma_1'A\int_{-\infty}^{z} dz'\rho(b,z')\right)\exp\left(-\tfrac{1}{2}\sigma_2'A\int_{z}^{\infty} dz''\rho(b,z'')\right)$$

$$=\frac{2}{\sigma_2'-\sigma_1'}\int_{-\infty}^{+\infty} dz\,\frac{d}{dz}\left[\exp\left(-\tfrac{1}{2}\sigma_1'A\int_{-\infty}^{z} dz'\rho(b,z')\right)\exp\left(-\tfrac{1}{2}\sigma_2'A\int_{z}^{+\infty} dz''\rho(b,z'')\right)\right]$$

$$=\frac{2}{\sigma_2'-\sigma_1'}\left[\exp\left(-\tfrac{1}{2}\sigma_1'A\int_{-\infty}^{+\infty} dz'\rho(b,z')\right)-\exp\left(-\tfrac{1}{2}\sigma_2'A\int_{-\infty}^{+\infty} dz''\rho(b,z'')\right)\right].$$

Hence, in the high energy $(\Delta_{||}=0)$ limit the expression for $\mathfrak{M}(1\rightarrow2)$ is

$$\mathfrak{M}(1\rightarrow2)=\frac{2f_{21}}{\sigma_2'-\sigma_1'}\int d^2b\exp\,(i\mathbf{\Delta}\cdot\mathbf{b})\left[\exp\left(-\tfrac{1}{2}\sigma_1'A\int_{-\infty}^{+\infty} dz'\rho(b,z')\right)\right.$$

$$\left.-\exp\left(-\tfrac{1}{2}\sigma_2'A\int_{-\infty}^{+\infty} dz''\rho(b,z'')\right)\right]. \tag{5.2}$$

Note that this is precisely the same expression that one would get from our model of the high energy diffractive process which assumes a weak transition between the initial (one pion) and the final (3π states). Let us repeat the arguments again.

$$\text{Initial state} = \frac{1}{\rule{6em}{0.4pt}} + \frac{2}{\text{small admixture}} = |\,\tilde{1}\rangle+d\,|\,\tilde{2}\rangle$$

$$\text{Final state} = \frac{2}{\rule{6em}{0.4pt}} + \frac{1}{\text{small admixture}} = |\,\tilde{2}\rangle-d^*\,|\,\tilde{1}\rangle$$

(the orthogonality condition:

$$(-\langle\tilde{1}\mid d+\langle\tilde{2}\mid)(\mid\tilde{1}\rangle+d\mid\tilde{2}\rangle)=-d+d=0).$$

Then the transition amplitude is

$$\mathfrak{M}(1\rightarrow 2)=[-\langle\tilde{1}\mid T\mid\tilde{1}\rangle+\langle\tilde{2}\mid T\mid\tilde{2}\rangle]\,d.$$

The results of these experiments are very puzzling and still very poorly understood. Let us discuss them briefly. The formula for $\mathfrak{M}(1\rightarrow 2)$ with $\Delta_{||}\neq 0$ was used to interpret the results. σ_1 was taken as the well-known pion-nucleon total cross section, σ_2 was a free parameter used to fit the coherent production cross section. The energy resolutions were too poor to have pure coherent processes but the incoherent processes were subtracted reasonably reliably because (as we stated many times) they are rather unimportant at small momentum transfers (where coherent processes are important). Realistic nuclear densities were used:

$$\rho(r)=\frac{\rho_0}{1+\exp\left(\dfrac{r-c}{a}\right)},\qquad \int d^3r\,\rho(r)=1,$$

$c=1.12A^{1/3}$ fm, $a=0.545$ fm. To compute the integrated coherent production cross section for a given bin of the invariant masses of the produced systems we use the formula

$$\sigma_{\text{coh}}(A,M_1,M_2)=\int_{M_1}^{M_2}dM^*\int_{q^2_{min}}^{q_c^2}dq^2\,\frac{d^2\sigma(\text{coh})}{dM^*\,dq^2},$$

where q_c^2 is a cut-off at the first maximum of the distribution. $\sigma_{\text{coh}}(A,M_1,M_2)$ *depends very critically on A and this fact enables one to extract* σ_2 (in complete analogy with photoproduction of vector-mesons). The amazing result was that σ_2 (both for 3π production and for 5π production came out to be very small). The tables below give some of the numbers obtained from the CERN experiment [S4]. The same equation is valid for nucleon targets. Hence we can eliminate d and get

$$\mathfrak{M}(1\rightarrow 2)=\frac{f_{21}}{\langle\tilde{2}\mid T^{(N)}\mid\tilde{2}\rangle-\langle\tilde{1}\mid T^{(N)}\mid\tilde{1}\rangle}[\langle\tilde{2}\mid T^{(A)}\mid\tilde{2}\rangle-\langle\tilde{1}\mid T^{(A)}\mid\tilde{1}\rangle],$$

which gives the production amplitude on the nucleus in terms of the production amplitude on a nucleon and the elastic scattering amplitudes of the objects 1 and 2 from the target nucleus

$$\langle\tilde{2}\mid T^{(A)}\mid\tilde{2}\rangle=\frac{ik}{2\pi}\int d^2b\,\exp(i\mathbf{\Delta}\cdot\mathbf{b})\{1-\exp[-\tfrac{1}{2}\sigma_2'T(b)]\},$$

$$\langle\tilde{1}\mid T^{(A)}\mid\tilde{1}\rangle=\frac{ik}{2\pi}\int d^2b\,\exp(i\mathbf{\Delta}\cdot\mathbf{b})\{1-\exp[-\tfrac{1}{2}\sigma_1'T(b)]\},$$

where

$$T(b)=A\int_{-\infty}^{+\infty}dz\,\rho(b,z),\qquad \sigma'=\sigma(1-i\alpha).$$

Let us use some standard parametrization of the nucleon amplitudes, e.g.,

$$\langle\tilde{2}\mid T^{(N)}\mid\tilde{2}\rangle=\frac{(i+\alpha_2)k\sigma_2}{4\pi}e^{-a_2q^2},\qquad \langle\tilde{1}\mid T^{(N)}\mid\tilde{1}\rangle=\frac{(i+\alpha_1)k\sigma_1}{4\pi}e^{-a_1q^2}.$$

We get, neglecting the q^2 dependence of the elementary amplitudes (which vary much slower with q^2 than $\langle \tilde{2} \mid T^{(A)} \mid \tilde{2} \rangle$ or $\langle \tilde{1} \mid T^{(A)} \mid \tilde{1} \rangle$):

$$\mathfrak{M}(1 \rightarrow 2) = \frac{2f_{21}}{\sigma_2' - \sigma_1'} \int d^2b \exp\ (i\mathbf{\Delta}\cdot\mathbf{b}) \{\exp\left[-\tfrac{1}{2}\sigma_1'T(b) - \tfrac{1}{2}\sigma_2'T(b)\right]\},$$

which is precisely the formula (5.2).

All these formulae are very useful (because of their simplicity) in discussing some effects which do not depend dramatically on $\Delta_{||} = 0$.

Let us go back, however, to the case $\Delta_{||} = 0$ and discuss some recent experiments in which $\pi \rightarrow 3\pi$ and $\pi \rightarrow 5\pi$ processes were measured on various nuclear targets. (A very rich literature on this subject, both from experimental and theoretical points of view, can be found in the Proceedings of the XII Cracow School of Theoretical Physics, June 1972 [S4].)

Production of the 3π System

Mass bin (GeV)	σ_2(mb) 9 GeV/c	σ_2(mb) 15 GeV/c
0.9–1.1	29±2	28±2
1.1–1.3	22±2	21±2
1.3–1.5	5^{+3}_{-2}	21±3
1.5–1.7	—	23±4
1.7–1.9	—	13±8

Production of the 5π System

Mass bin (GeV)	σ_2(mb) 15 GeV/c
1.5–1.7	10±7
1.7–1.9	13±10

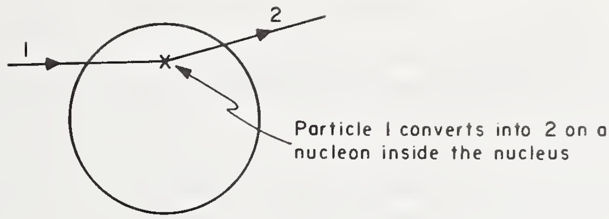
These results are very puzzling because the 3π and 5π systems do not seem to form well defined particles. For example, at 9 GeV/c the time required by a system of 1.2 GeV mass to cross one-half of the thickness of the Pb nucleus (6.5 fm) is $\tau \approx 2.9 \times 10^{-24}$ s or $\Gamma \approx 230$ MeV (relativistic dilation included). The observed distribution is wider than 230 MeV (perhaps even as much as 500 MeV). Besides, σ_2 should be smaller at 15 GeV than at 9 GeV (due to time dilation), which is not the case. In fact, it is hard to accept that these systems are resonances—they look more like a group of 3π or 5π! But then their cross section should be $3\sigma_{\pi N}$ and $5\sigma_{\pi N}$, respectively!

5.1. Discussion of the Anomalously Small Absorption of 3π and 5π Systems in Nuclear Matter

First, let us stress that there is no satisfactory explanation of this phenomenon available. One can, nevertheless, make a few points which may advance a bit our understanding of the process.

Let us start with an analysis of the process of diffractive production in the high energy limit in the language of multiple scattering. (We choose the high energy limit because it is simple and we believe that the finite longitudinal momentum has little to do with absorption properties of the outgoing systems.)

First, the "one step" mechanism of production:



Let us introduce the profiles of individual nucleons smeared over the volume of the target nucleus:

$$\langle 1 \mid \Gamma \mid 1\rangle \cong \frac{2\pi}{ik} f_{11}(0) \int_{-\infty}^{+\infty} dz\, \rho(b, z) = \frac{2\pi}{ik} f_{11}(0)\, T(b)$$

$$\langle 2 \mid \Gamma \mid 2\rangle \cong \frac{2\pi}{ik} f_{22}(0)\, T(b),$$

where we took only the forward amplitudes $f_{11}(0)$ and $f_{22}(0)$. We proceed exactly the same way as we did in obtaining the high energy limit of the $\gamma-\rho$ amplitude

$$\mathfrak{M}(1\to 2) = \frac{ik}{2\pi} \int d^2b \exp(i\mathbf{\Delta}\cdot\mathbf{b}) \sum_{c=1}^{A} \overbrace{(1-\langle 2 \mid \Gamma \mid 2\rangle) \ldots (1-\langle 2 \mid \Gamma \mid 2\rangle)}^{(c-1)\text{ factors}} \langle 2 \mid \Gamma^{(c)} \mid 1\rangle$$
$$\underbrace{\times (1-\langle 1 \mid \Gamma \mid 1\rangle) \ldots (1-\langle 1 \mid \Gamma \mid 1\rangle).}_{(A-c)\text{ factors}}$$

After summing the geometric series we get

$$\mathfrak{M}(1\to 2) = \frac{ik}{2\pi} \int d^2b \exp(i\mathbf{\Delta}\cdot\mathbf{b}) \langle 2 \mid \Gamma \mid 1\rangle \frac{(1-\langle 1 \mid \Gamma \mid 1\rangle^{A} - (1-\langle 2 \mid \Gamma \mid 2\rangle)^{A}}{\langle 1 \mid \Gamma \mid 1\rangle - \langle 2 \mid \Gamma \mid 2\rangle}$$

$$\xrightarrow[A\text{ large}]{} \frac{ik}{2\pi} \int d^2b \exp(i\mathbf{\Delta}\cdot\mathbf{b}) \langle 2 \mid \Gamma \mid 1\rangle [\langle 1 \mid \Gamma \mid 1\rangle - \langle 2 \mid \Gamma \mid 2\rangle]^{-1}$$
$$\times [\exp(-A\langle 1 \mid \Gamma \mid 1\rangle) - \exp(-A\langle 2 \mid \Gamma \mid 2\rangle)]$$
$$= \frac{2f_{21}}{\sigma_2' - \sigma_1'} \int d^2b \exp(i\mathbf{\Delta}\cdot\mathbf{b}) \{\exp[-\tfrac{1}{2}\sigma_1' T(b)] - \exp[-\tfrac{1}{2}\sigma_2' T(b)]\},$$

which is the same formula (5.2) which was obtained before.

As we have said before, the "weakness" of the production process makes the "one step" description plausible. Let us point out, however, that $K_L \to K_s$ regeneration on one nucleon is weak; nevertheless, this process cannot be described correctly as a one-step process. Indeed, from the "one step" formula (5.2) we get

$$\mathfrak{M}(K_L - K_s) = \frac{2f_{K_L\to K_s}}{\sigma_{K_s}' - \sigma_{K_L}'} \int d^2b \exp(i\mathbf{\Delta}\cdot\mathbf{b}) \{\exp[-\tfrac{1}{2}\sigma_{K_L}' T(b)] - \exp[-\tfrac{1}{2}\sigma_{K_s}' T(b)]\}, \quad (5.3)$$

which is wrong! We know the correct answer because we know the composition of $\mid K_L\rangle = 1/\sqrt{2}(\mid K^0\rangle + \mid \bar{K}^0\rangle)$ *and* $\mid K_s\rangle = 1/\sqrt{2}(\mid K^0\rangle - \mid \bar{K}^0\rangle)$ *which is nonperturbative, hence fundamentally different from the* $\gamma - V$

structure of the photon. In fact, accepting the fact that K^0 and $\bar{K}^0$ scatter only elastically from the nucleus we get the formula which *is correct*:

$$\mathfrak{M}(K_L \to K_s) = \frac{ik}{2\pi} \int d^2b \exp(i\mathbf{\Delta}\cdot\mathbf{b})\{\exp[-\tfrac{1}{2}\sigma_{K^0}{}'T(b)] - \exp[-\tfrac{1}{2}\sigma_{K^0}{}'T(b)]\}. \tag{5.4}$$

Since $f_{K_LK_L} = f_{K_sK_s} = \tfrac{1}{2}(f_{K^0K^0} + f_{\bar{K}^0\bar{K}^0})$, we can see from the numerical values given below that the profiles in (5.3) and (5.4) are different.

To complete the point we are making, let us compute $f_{K_LK_L} = f_{K_sK_s}$ and compare it with $f_{K_L\to K_s} = \tfrac{1}{2}(f_{K^0K^0} - f_{\bar{K}^0\bar{K}^0})$. We use the data of Foeth et al. [26]:

At 4 GeV/c incident momentum we have:

for neutrons:

$$\operatorname{Im} f^{(n)}_{K^0K^0} \approx 2.8 \text{ fm}, \qquad \operatorname{Re} f^{(n)}_{K^0K^0} \approx -1 \text{ fm}$$

$$\operatorname{Im} f^{(n)}_{\bar{K}^0\bar{K}^0} \approx 4.1 \text{ fm}, \qquad \operatorname{Re} f^{(n)}_{\bar{K}^0\bar{K}^0} \approx 0 \text{ fm}$$

for protons:

$$\operatorname{Im} f^{(p)}_{K^0K^0} \approx 2.9 \text{ fm}, \qquad \operatorname{Re} f^{(p)}_{K^0K^0} \approx -0.5 \text{ fm}$$

$$\operatorname{Im} f^{(p)}_{\bar{K}^0\bar{K}^0} \approx 3.4 \text{ fm}, \qquad \operatorname{Re} f^{(p)}_{\bar{K}^0\bar{K}^0} \approx 0 \text{ fm}.$$

and we get

$$\frac{\sigma^{(n)}_{K_L\to K_s}}{\sigma^{(n)}_{K_L}} \approx 0.055, \qquad \frac{\sigma^{(p)}_{K_L\to K_s}}{\sigma^{(p)}_{K_L}} \approx 0.012.$$

So, the production process on one nucleon is indeed weak! But as we now see clearly (comparing (5.3) and (5.4)), that is not enough to apply the "one step" formula; *one has to know the internal structure of the objects which undergo diffractive scattering.*

Remark:

Let us note that we may write the amplitude (5.4) so that it has the $K_L \to K_s$ regeneration amplitude on a single nucleon as a factor similarly to that appearing in the "one step" description, (5.3).

For a nucleon target we have:

$$f_{K_L\to K_s} = \tfrac{1}{2}(f_{K^0K^0} - f_{\bar{K}^0\bar{K}^0}).$$

For a nucleus target we have:

$$\mathfrak{M}(K_L \to K_s) = \frac{1}{2}\frac{ik}{2\pi} \int d^2b \exp(i\mathbf{\Delta}\cdot\mathbf{b})\{\exp[-\tfrac{1}{2}\sigma_{\bar{K}^0}T(b)] - \exp[-\tfrac{1}{2}\sigma_{K^0}T(b)]\}.$$

Thus, using the optical theorem we can write

$$\mathfrak{M}(K_L \to K_s) = \frac{2f_{K_L\to K_s}}{\sigma_{K^0}{}' - \sigma_{\bar{K}^0}{}'} \int d^2b \exp(i\mathbf{\Delta}\cdot\mathbf{b})\{\exp[-\tfrac{1}{2}\sigma_{\bar{K}^0}T(b)] - \exp[-\tfrac{1}{2}\sigma_{K^0}T(b)]\}. \tag{5.5}$$

But, *although this formula does seem to have the production taking place on one nucleon, in fact nothing like that takes place*: there is only a "smooth scattering" of the two components (K^0 and $\bar{K}^0$). No one nucleon along the path plays any role distinguished from the others. (From (5.5) one can see again that the "one step" formula (5.3) is incapable of describing $K_L \to K_s$ regeneration on nuclei).

Let us go back to the general case of incoming particle 1 and outgoing particle 2. From the above discussion we can see that (within the framework of the Good and Walker description of the diffractive production processes) we have to have *weak coupling* between $|\tilde{1}\rangle$ and $|\tilde{2}\rangle$ states in order to have a "one step" description, since, when the coupling is weak, we can replace

$$\langle\tilde{1} \mid T^{(N)} \mid \tilde{1}\rangle \approx \langle 1 \mid T^{(N)} \mid 1\rangle$$

and

$$\langle\tilde{2} \mid T^{(N)} \mid \tilde{2}\rangle \approx \langle 2 \mid T^{(N)} \mid 2\rangle.$$

Perhaps at this stage one should point out that one could easily explain the small absorption cross section by abandoning the "one step" description. The price to pay for it would be the loss of the interpretation of the states $|\tilde{1}\rangle$ and $|\tilde{2}\rangle$ as "almost" a one physical pion state and "almost" a three physical pion state, respectively.

For instance, the following scheme (which imitates $K_L \to K_s$ regeneration) would explain the observed effects. (Note that we are working in the high energy limit, hence our arguments are based on the assumption that the absorption properties of the objects produced are the same in this limit as at the experimentally available energies.) Let us accept that

$$\sigma_{\tilde{1}} \approx \sigma_\pi = \sigma_1 \qquad \text{and} \qquad \sigma_{\tilde{1}} - \sigma_{\tilde{2}} \ll \sigma_\pi \qquad \text{(and positive)},$$

and

$$|\pi\rangle = \frac{1}{\sqrt{2}} (|\tilde{1}\rangle + |\tilde{2}\rangle) = |1\rangle$$

$$|3\pi\rangle = \frac{1}{\sqrt{2}} (|\tilde{1}\rangle - |\tilde{2}\rangle) = |2\rangle. \tag{5.6}$$

(Note that the values of the σ's and the coefficients of the transformation (5.6) are independent quantities.) Then the production amplitude (as in the $K_L \to K_s$ regeneration) is

$$\mathfrak{M}(1\to 2) = \frac{2f_{21}}{\sigma_{\tilde{2}} - \sigma_{\tilde{1}}} \int d^2b \exp(i\mathbf{\Delta}\cdot\mathbf{b}) \{\exp[-\tfrac{1}{2}\sigma_{\tilde{1}} T(b)] - \exp[-\tfrac{1}{2}\sigma_{\tilde{2}} T(b)]\} \tag{5.7}$$

where

$$f_{21} = \tfrac{1}{2}[\langle\tilde{1} \mid T^{(N)} \mid \tilde{1}\rangle - \langle\tilde{2} \mid T^{(N)} \mid \tilde{2}\rangle].$$

The formula (5.7) contains the correct attenuation of the outgoing object (compare the discussion of experimental results given previously). Then the elastic scattering amplitude of $|1\rangle (\equiv |\pi\rangle)$ is totally determined

$$\mathfrak{M}(1\to 1) = \tfrac{1}{2}[\langle\tilde{1} \mid T^{(A)} \mid \tilde{1}\rangle + \langle\tilde{2} \mid T^{(A)} \mid \tilde{2}\rangle]$$

$$= \frac{ik}{2\pi} \int d^2b \exp(i\mathbf{\Delta}\cdot\mathbf{b}) \tfrac{1}{2}\{1 - \exp[-\tfrac{1}{2}\sigma_{\tilde{1}} T(b)] + 1 - \exp[-\tfrac{1}{2}\sigma_{\tilde{2}} T(b)]\}$$

which is, to a very good approximation,

$$\approx \frac{ik}{2\pi} \int d^2b \exp(i\mathbf{\Delta}\cdot\mathbf{b}) \{1 - \exp[-\tfrac{1}{2}\sigma_\pi T(b)]\} \tag{5.8}$$

hence the correct π-nucleus elastic scattering amplitude.

Then, one has to worry about the fact that $\mathfrak{M}(2\to 2) = \mathfrak{M}(1\to 1)$, as implied by (5.6). One can argue that $\mathfrak{M}(2\to 2)$ is not measurable since $|2\rangle$ decays into 3π and hence is not available as an initial beam, thus making this concern irrevelant.

This example shows that one can easily explain the "anomalously" low absorption of diffractively produced objects if one assumes that the initial (one pion) and the final (something decaying into 3π) states are just the different configurations of the same components. (In the weak coupling case" it is physically more accurate to describe the initial and final states as composed, approximately, of one and three particles, respectively.)

Note the following amusing point: Assuming $|\pi\rangle$ to be in the form of a superposition of two scattering eigenstates we determine uniquely the exponentials (the total cross sections). Once we have that, the production cross section is completely determined and comes out right!

In fact one can give a more general version of the above remark [43]: The transformation (5.6) may be replaced by a general transformation which preserves normalization and orthogonality

$$|i\rangle = \sum_l a_{il} | \tilde{l} \rangle,$$

$$\sum_s a_{is}^* a_{ls} = \delta_{il} \tag{5.9}$$

and the *assumption* made that all nonzero a_{il} are of the same order of magnitude. It introduces "democracy" among all physical states and treats the ground state of the incident particle on the same footing as the excited states. This makes good sense if one believes (following Good and Walker [23]) that in the high energy limit the ground and excited states of the incident system (e.g., a pion) are considered to be approximately degenerate.

The formula which gives all diffractive elastic and production amplitudes is

$$\langle f | T^{(A)} | i \rangle = \frac{ik}{2\pi} \sum_l a_{fl}^* a_{il} \int d^2b \exp(i\mathbf{\Delta}\cdot\mathbf{b}) \{1 - \exp[-\tfrac{1}{2}\sigma_l A T(b)]\}, \tag{5.10}$$

where the "total cross sections" σ_l are free parameters. In order to obtain (5.8), e.g., we have to make all σ_l approximately equal to σ_π (they cannot be exactly equal to σ_π because we would have no production!). When we do that, we have approximately (due to (5.9)) the *elastic amplitude* of the form (5.8), and the production amplitudes become approximately (again due to (5.9); for an explicit example see (5.7)),

$$\langle f | T^{(A)} | i \rangle \approx \frac{2f_{fi}}{\sigma_f - \sigma_i} \int d^2b \exp(i\mathbf{\Delta}\cdot\mathbf{b}) \{\exp[-\tfrac{1}{2}\sigma_i T(b)] - \exp[-\tfrac{1}{2}\sigma_f T(b)]\}, \quad f \neq i.$$

Since $\sigma_i \approx \sigma_f \approx \sigma_\pi$ we obtain the result obtained in experiment [S4].

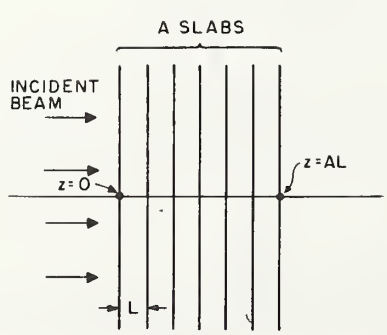
Let us make two comments to close this "strong coupling" description of diffractive production which, in contrast to the "one step" description, is *not a perturbative approach*:

(i) In the "strong coupling" model there is an internal relation between elastic and production processes, whereas in the "one step" picture there is none. To put it differently, *a definite relation between attenuations in the entrance and exit channels exists in this model.*

(ii) The "strong coupling" description implies the following general prediction: *In all coherent diffractive production processes where there are strongly interacting particles in the entrance and exit channels, and diffractive production processes are much weaker than elastic scattering amplitudes, one should see comparable attenuations in the entrance and all-exit channels.*

Before indicating other possibilities of interpretation of the low absorption effect, let us consider an example of a two component system (such as was described before, e.g., by eq (5.6)) penetrating a piece of nuclear matter.

The following remark about penetration through a sequence of thin-slabs of nuclear matter is in order here.



Assume that ρ (the density) does not depend on the transverse coordinate b (each slab extends to infinity in the **b** plane). The profiles of the elastic scattering amplitudes for one slab are

$$\langle \tilde{1} \mid \Gamma \mid \tilde{1} \rangle = \frac{2\pi}{ik} f_{\tilde{1}\tilde{1}} \rho L = \frac{2\pi}{ik} \frac{i\sigma_{\tilde{1}} k}{4\pi} \rho L = \tfrac{1}{2}\sigma_{\tilde{1}} \rho L \quad = \overbrace{\tfrac{1}{2}\sigma_{\tilde{1}} \rho \frac{z}{A}}^{\text{dimensionless}}$$

$$\langle \tilde{2} \mid \Gamma \mid \tilde{2} \rangle = \tfrac{1}{2}\sigma_{\tilde{2}} \rho \frac{z}{A} .$$

Let us consider the attenuation of the incident wave, e^{ikz}: After A slabs we have

$$\text{(i)} \quad e^{ikz}\{\tfrac{1}{2}(1-\langle \tilde{1} \mid \Gamma \mid \tilde{1} \rangle)^A + \tfrac{1}{2}(1-\langle \tilde{1} \mid \Gamma \mid \tilde{1} \rangle)^A\}$$

$$= \tfrac{1}{2} e^{ikz}[\exp(-\tfrac{1}{2}\sigma_{\tilde{1}}\rho z) + \exp(-\tfrac{1}{2}\sigma_{\tilde{2}}\rho z)].$$

The exponentials give the attenuation. This is the attenuation in the case of strong coupling between $| 1 \rangle$ and $| 2 \rangle$. Then, let us take the case where $| 1 \rangle$ scatters only elastically (hence we forbid the intermediate states $| 2 \rangle$). Then the attenuation of the incident wave is

$$\text{(ii)} \quad e^{ikz}(1-\langle 1 \mid \Gamma \mid 1 \rangle)^A = e^{ikz}[1-\tfrac{1}{2}(\langle \tilde{1} \mid \Gamma \mid \tilde{1} \rangle + \langle \tilde{2} \mid \Gamma \mid \tilde{2} \rangle)]^A$$

$$= e^{ikz} \exp[-\tfrac{1}{4}(\sigma_{\tilde{1}}+\sigma_{\tilde{2}})\rho z].$$

When $\sigma_{\tilde{1}} = \sigma_{\tilde{2}}$, the attenuation in case (i) is the same as in case (ii), but in the case $\sigma_{\tilde{1}} \neq \sigma_{\tilde{2}}$ the situation changes. Take, e.g., $\sigma_{\tilde{1}} = \tfrac{3}{4}\sigma$, $\sigma_{\tilde{2}} = \tfrac{1}{4}\sigma$. Then the attenuation factor in (i) is

$$\text{(i)} \quad \tfrac{1}{2}(e^{-(3/8)\sigma T} + e^{-(1/8)\sigma T}).$$

Instead, in case (ii) it is

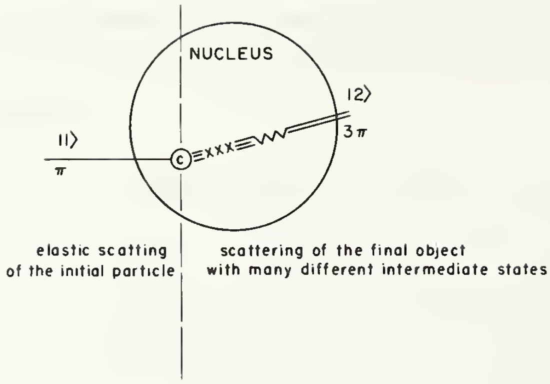
$$\text{(ii)} \quad e^{-(1/4)\sigma T}.$$

Hence, for large enough T, the attenuation in (ii) is stronger than the attenuation in (i). A very important conclusion follows from this observation: *by introducing strong coupling between the initial state of the incident particle and some other states one may reduce considerably the absorption of the initial state in nuclear matter.*

One can put it differently: the effect of allowing intermediate states to occur during the multiple scattering process may be an increased penetrability (or decreased absorption). Hence the so-called inelastic shadowing effect increases the penetrability of a specific component.

We have considered only the 2×2 case but one can consider much more involved systems (which contain more than two components). Such a scheme was developed by L. Van Hove ([44], see also [41]).

There is, therefore, still another possibility of explaining low absorption of the 3π (5π) systems produced coherently on nuclei: to consider it a "one step" process but to assume that the produced object, 3π (5π), is a superposition of several strongly coupled channels. This coupling may reduce the absorption of a 3π system as we have seen on the example quoted above. Graphically such process would look as follows:



This is, at the moment, the most commonly considered model of coherent diffractive production. In order to have a satisfactory solution, one would have to know precisely the superposition of states which form the produced 3π (5π) system and their interactions with the nucleons of the target. In other words, one has to specify the structure of the produced object. No one has, so far, proposed a model detailed enough and convincing enough. We shall give a few more details after making the following remark.

Remark:

One may also look at the problem of the penetration of a many component system through nuclear matter as a problem of diagonalization of the "profile-matrix." Let us again assume (for simplicity sake) that just two states are operative. Then the single scattering profile matrix is

$$\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}$$

Let us assume the following form for it

$$\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix} = \begin{pmatrix} \Gamma & \Gamma' \\ \Gamma' & \Gamma \end{pmatrix}$$

and diagonalize it:

$$\begin{vmatrix} \Gamma-\lambda, & \Gamma' \\ \Gamma', & \Gamma-\lambda \end{vmatrix} = 0, \qquad (\Gamma-\lambda)^2 = \Gamma'^2, \qquad \lambda = \Gamma \pm \Gamma'.$$

Take $\Gamma = \Gamma_{\tilde{1}\tilde{1}} + \Gamma_{\tilde{2}\tilde{2}}$, $\Gamma' = \Gamma_{\tilde{1}\tilde{1}} - \Gamma_{\tilde{2}\tilde{2}}$ and we get two eigenvalues:

$$\lambda = \begin{cases} 2\Gamma_{\tilde{1}\tilde{1}} \\ 2\Gamma_{\tilde{2}\tilde{2}} \end{cases}.$$

Hence the following transformation of the physical states diagonalizes the interaction

$$|1\rangle = \frac{1}{\sqrt{2}} (|\tilde{1}\rangle + |\tilde{2}\rangle)$$

$$|2\rangle = \frac{1}{\sqrt{2}} (|\tilde{1}\rangle - |\tilde{2}\rangle).$$

Hence the matrix

$$\begin{pmatrix} \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \\ \\ \dfrac{1}{\sqrt{2}} & -\dfrac{1}{\sqrt{2}} \end{pmatrix}$$

diagonalizes

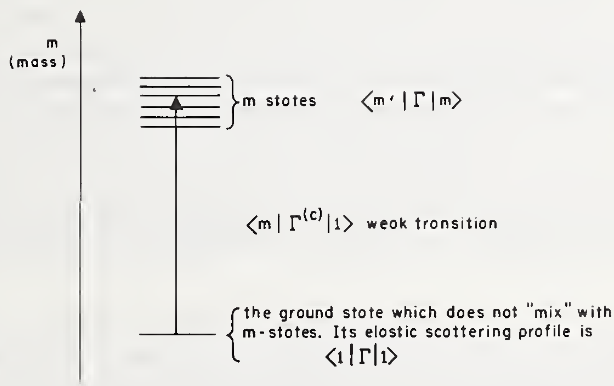
$$\begin{pmatrix} \Gamma, & \Gamma' \\ \\ \Gamma', & \Gamma \end{pmatrix}$$

Note that $|\,\tilde{1}\rangle$ and $|\,\tilde{2}\rangle$ are the "eigenstates" of the scattering; they scatter only elastically.

Note also that even if the transition between $1 \rightarrow 2$ (or $2 \rightarrow 1$) is very weak we have a superposition of these physical states propagating through the nuclear matter, a superposition in which both these states are equally important.

Now let us go back and close our discussion of the coherent diffractive production by pions of 3π and 5π systems on nuclei by formulating more completely the "one step" description which allows for reduction of absorption for the produced composite systems (L. Van Hove [44], C. Rogers and C. Wilkin [41], A. Białas and K. Zalewski [45]). It is a direct generalization of the multiple scattering we have given above for the standard "one step" amplitude.

We want to keep the "one step" production model, hence we assume the incident particle ground state $|\,1\rangle$ to be "well separated" from a set of excited states $|\,m\rangle$ from which the system $|\,2\rangle$ emerges. On the mass scale



Then, the transition amplitude is

$$\mathfrak{M}(1 \rightarrow 2) = \frac{ik}{2\pi} \int d^2b \exp\,(i\boldsymbol{\Delta} \cdot \mathbf{b})\; S \sum_{c} \,(1 - \langle 2\,|\,\Gamma\,|\,m\rangle)\,(1 - \langle m\,|\,\Gamma\,|\,m'\rangle)\,\ldots$$

all possible
"histories"

$$\ldots\,\langle m_c\,|\,\Gamma^{(c)}\,|\,1\rangle(1 - \langle 1\,|\,\Gamma\,|\,1\rangle)\,\ldots\,(1 - \langle 1\,|\,\Gamma\,|\,1\rangle).$$

A "history" is a sequence of intermediate states

$$|\,m_c\rangle, \ldots |\,m''\rangle, |\,m'\rangle, |\,m\rangle.$$

One can perform the sum S by diagonalizing $\langle m' \mid \Gamma \mid m \rangle$. In an abstract form:

$$\Gamma \mid \tilde{\alpha} \rangle = \lambda_\alpha \mid \tilde{\alpha} \rangle$$

$$\mid \tilde{\alpha} \rangle = \sum_m C(m, \alpha) \mid m \rangle. \tag{5.11}$$

The states $\mid m \rangle$ are physical states. The states $\mid \tilde{\alpha} \rangle$ are their linear combinations. (E.g., in the process of $K_L \rightarrow K_s$ regeneration they are degenerate and identical with $\mid K^0 \rangle$ and $\mid \bar{K}^0 \rangle$). In general they do not have a well defined energy. The states $\mid \tilde{\alpha} \rangle$ are decoupled, that is to say only diagonal elements $\langle \tilde{\alpha} \mid \Gamma \mid \tilde{\alpha} \rangle$ are different from zero. Thus

$$\underset{\text{“histories”}}{\underset{\text{all}}{S}} = \sum_\alpha \sum_{c=1}^{A} \langle 2 \mid \tilde{\alpha} \rangle (1-\lambda_\alpha)^{c-1} \langle \tilde{\alpha} \mid \Gamma^{(c)} \mid 1 \rangle (1 - \langle 1 \mid \Gamma \mid 1 \rangle)^{A-c}$$

$$= \sum_\alpha \frac{(1 - \langle 1 \mid \Gamma \mid 1 \rangle)^A - (1-\lambda_\alpha)^A}{\langle 1 \mid \Gamma \mid 1 \rangle - \lambda_\alpha} \langle 2 \mid \tilde{\alpha} \rangle \langle \tilde{\alpha} \mid \Gamma \mid 1 \rangle.$$

In the limit of large A ($(1-X)^A \rightarrow e^{-AX}$) we get

$$\mathfrak{M}(1 \rightarrow 2) = \frac{ik}{2\pi} \int d^2b \exp(i\boldsymbol{\Delta} \cdot \mathbf{b}) \sum_\alpha \langle 2 \mid \tilde{\alpha} \rangle \langle \tilde{\alpha} \mid \Gamma \mid 1 \rangle$$

$$\times \frac{\exp(-A \langle 1 \mid \Gamma \mid 1 \rangle) - \exp(-A\lambda_\alpha)}{\langle 1 \mid \Gamma \mid 1 \rangle - \lambda_\alpha}.$$

The attenuation of the outgoing beam is small when the λ_α's are small. So, in this approach the problem is to construct a physically plausible matrix $\langle m' \mid \Gamma \mid m \rangle$ which produces the correct eigenvalues λ_α. Hence one has to go deeply into the structure of the produced object. There is no commonly accepted model of such a structure but there are many examples ([41], [44], [45], [S4]) which show that one may obtain the desired low absorption in many ways.

6. Shadowing Effects in Inelastic Electron–Nucleus Scattering

Experiments performed to see shadowing effects in inelastic electron-nucleus scattering failed to show it—see H. Kendall in ref. [S5]. Inelastic electron scattering cross sections were measured at: $\theta = 6°$, the incident electron energies were 4.5, 7, 10, 13.5, 16, 19.5 GeV and the energy losses, ν, were 0.1 Gev $< \nu <$ 17 GeV. The targets were Be, Cu, and Au nuclei. They plotted

$$S = \frac{\sigma_{\text{nucleus}}(\text{exp})}{Z\sigma_p + N\sigma_n}$$

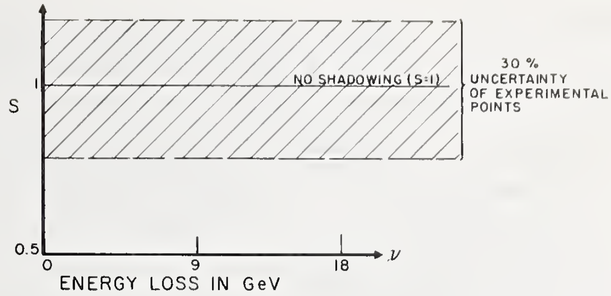
versus energy loss for two bins of the four-momentum squared

$$0.25 < q^2 < 0.75\,(\text{GeV/c})^2$$

and

$$0.75 < q^2 < 1.50\,(\text{GeV/c})^2.$$

All data points were consistent with $S = 1$ for all momentum transfers and energy losses (although the errors were quite large).



One may say that:

1. There is a definite disagreement with vector-meson dominance.
2. This is not very surprising because in the case of real photons, total cross sections do not exactly follow VMD. How this break-down depends on the "off-shellness" of the photon is not known at all. (Remember that this inelastic electron scattering experiment sums over all possible interactions of the virtual photons consistent with the kinematics of the experiment, hence a total cross section for virtual photons is being measured.)
3. Theoretical analysis of this effect is in a very preliminary stage.

7. Shadowing Effects in Neutrino Reactions on Nuclei

In the following experiment (see K. Borer et al. [46] and also J. S. Bell [S3]) no shadowing was observed:

The incident beam was: the CERN neutrino beam which has a very broad spectrum, so the incident energy was poorly defined but, just to give some idea, the average neutrino energy was about 1.5–2.0 GeV and the width of the spectrum was about 1.5 GeV.

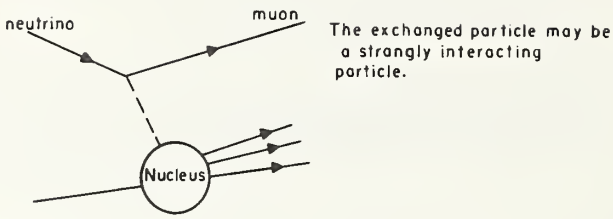
In a spark chamber set-up one could see muons produced, one could also see in which material the reaction took place, and one could make a rough measurement of the muon momentum.

The following results were obtained (θ is the angle of the outgoing μ)

	Ratio of event rates per nucleon in various pairs of nuclei	Expected ratio if no shadowing
$\theta<29°$		
Pb/C	0.97±0.04	1
Fe/C	0.89±0.08	1
Pb/Al	0.93±0.07	1
$\theta<5°$		
Pb/C	0.92±0.15	1

Hence no shadowing was observed.

1. In principle, one may expect some shadowing because



2. Again, for reasons somewhat similar to the case of electron scattering a lack of shadowing is not surprising (for more details of the theoretical analysis, see the lecture by J. S. Bell in ref. [S3]).

3. The theoretical analysis is at a very primitive stage and when the data improves a lot will remain to be done.

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16. ABSTRACT (*A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.*)

Elastic scattering and diffractive production processes induced in nuclear targets by high energy projectiles are discussed in this article. Special attention is paid to the interaction of high energy hadrons and photons. Interactions of high energy electrons and neutrinos are briefly mentioned. The common features of all these processes are emphasized throughout the article: The multiple scattering and shadowing processes inside of the target nuclei. An effort is made to develop a unified way of treating nuclear interactions of particles which are either hadrons or exhibit some hadronic components in such interactions.

This article is divided into five sections: 1) Introduction, 2) Description of multiple scattering, 3) Elastic scattering of hadrons from nuclei, 4) Diffractive dissociation and diffractive excitation, 5) Diffractive production of hadrons in hadron-nucleon collisions.

17. KEY WORDS (*six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons*)

Diffractive production; diffractive scattering; Glauber model; hadronic components of photons; high energy scattering; multiple scattering; neutrino-nucleus interactions; shadowing effects.

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