A UNITED STATES DEPARTMENT OF COMMERCE PUBLICATION

NBS MONOGRAPH 133

Mass and Mass Values

U.S. DEPARTMENT OF COMMERCE
National Bureau of Standards

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Issued January 1974

Library of Congress Catalog Number: 73-600054

National Bureau of Standards Monograph 133

Nat. Bur. Stand. (U.S.), Monogr. 133, 39 pages (Jan. 1974)

CODEN: NBSMA6

U.S. GOVERNMENT PRINTING OFFICE
WASHINGTON: 1974

For sale by the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402. (Order by SD Catalog No. C13.44:133). Price 70 cents.
Stock Number 0303-01178

Foreword

This monograph has been prepared for the benefit of metrologists and others concerned with the measurement of mass. It is addressed to two related but distinct subjects: the history and general philosophy of mass measurement and the specific problem of the buoyancy correction as applied to the weighing process.

The opening section is a discussion of the role of mass measurement in our society; its development and its significance. Underlying this section are some observations on the philosophy of measurement as developed by Mr. Pontius during his years as chief of the NBS section responsible for mass measurement.

In the second section, Mr. Pontius presents in detail the problem of air buoyancy corrections to the weighing process. The formulae required for the application of this correction are presented, together with an appendix of helpful tables and some typical examples.

We hope that you will find these sections thought-provoking with respect to measurement problems in general and helpful in their presentation of the procedures for correctly applying the correction for air buoyancy in precision weighing.

Dr. K. G. Kessler
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Paul E. Pontius

There are several bases for assigning mass values to weights to be used as mass standards. As a consequence a given weight may have several assigned mass values depending on the basis used. In many cases, the differences between these assigned values, although easily detectable with precise weighing equipment, are of no practical concern. However, in some instances these differences may be crucial. The first part of this paper is a historical summary of weighing, standards, and the assignment of value; and the interfacing of mass measurements with civilization. The second part of this paper discusses in detail the methods of assigning mass values. Ways to convert from values on one basis to values on another basis are discussed. Sample problems relating to the buoyant effect of the air are presented in the appendices.

Key words: Apparent mass; buoyancy corrections; mass comparison; mass value; true mass; weighing.

Introduction

Measurement—A specific sequence of operations which are performed with the intent of establishing a useful ordering.

Ordering differs from counting in that ordering is concerned with the uniqueness of a body in terms of its properties while in counting, all objects with nearly similar properties are treated as being identical [1].* Coins are counted to determine a total money value. One can order coins, however, according to weight, metal content, color and the like.[1] By ordering a sufficient number of properties, each coin would be uniquely identified. Ordering in some form or other has been a part of life from the earliest man. Measurement, a formalized ordering, consists of four elements: a circumstance which makes ordering necessary or desirable; a basis for the ordering; a sequence of operations, usually centered on a construct[2] which serves to aid the senses in discerning otherwise undetectable differences; and, since the sequence of operations involves a construct, a means of verifying the usefulness of the end result.

The use of common measurements, such as weighing, length measurement and volume measurement, undoubtedly started in the Neolithic Age during a period marking the end of a hunting way of life and the beginning of a full metal-using economy [2]. In the beginning, for those items which could be compared by these common measurements, relative value, or worth, was established by the physical size of some arbitrary embodiment of a measurement unit—the heft of a particular stone, the capacity of a particular pot, or the length of an arm or foot. The conversion to conventional measurement standards was well established by the time the written record begins [10]. In the second millennium, taxes in Egypt were paid in kind, but tax records were on the basis of equivalent labor [3]. In Mesopotamia, a recognized standard of exchange was one mina of silver being equivalent to 60 gur of grain [4], the gur being a volumetric measure [5]. An early mina

*Figures in brackets indicate the literature references on page 21.

[1] Stevens, Ref. [8], suggests counting as pairing a group of objects of a given class with the names of the numbers, the name of the last number in the last pair formed being the number of the objects in the group. In discussing ordering, he refers to the postulates of order:

(1) If $a \neq b$, then either $a < b$ or $b < a$
(2) If $a < b$, then $a \neq b$
(3) If $a < b$ and $b < c$, then $a < c$

where a, b and c are some particular attribute, $\neq$ means is different from and $<$ is not completely defined. $<$ could stand for less than, precedes, older than, etc. In the limit

$$\underset{a \to c}{L} \ (a < b < c), \qquad a = b = c$$

In this case, if the number A has been assigned to represent the attribute of a, A also compares to the same attribute of b and c. Further, if an additive operation can be defined for a particular attribute, this same logic can be used to develop an extensive magnitude measurement concept as suggested by Carnap, Ref. [9].

[2] Construct is used in the sense that a measurement is made with a collection of procedures and instruments which have been accepted more or less by tradition and which are designed to have optimum response to particular properties or phenomena. The response may, or may not, relate to the desired information. For example, one can determine the density of an object; however, extrapolating to obtain the density of a material depends upon the characteristics of the object. Determining a "contained" or "delivered" volume with water will establish product uniformity for a particular type of glassware but the result may not be applicable for other types of fluid.

weight (ca. 2400 B.C.) was marked "one mina of wages in wool [6]."

These same common measurements, refined somewhat, but fundamentally unchanged, remain with us today. For most of us, our first introduction to measurement occurred at an early age—learning simple measurement procedures and the relationship of customary measurement units such as the cup, the foot, and the pound. Understanding, however, requires something more than names for measurement units, a hierarchy of standards and elementary measurement procedures.

Youden [7] observed that people who must rely on measurement data are frequently so absorbed in the broader aspects of a particular problem that they give little, if any, attention to measurement detail. This observation is true of scientist and layman alike. Many become highly incensed when challenged on matters concerning measurement and measurement philosophy. It is a fact, however, that most of our measurement knowledge has been acquired haphazardly, that prevalent philosophies have come from a past which was severely limited by communications problems, and that in more cases than we would care to admit, precise measurements are specified because we don't know what else to do. If, in a given undertaking, we are successful, the success blesses all that has gone on before. If troubles are encountered, inevitably the procedural details of the measurements are the first items to be scrutinized, a fertile area for deception.

When all else fails, the questions which should have been considered in depth at the very beginning may come to the fore:

(1) Is the contemplated, or requested, measurement relevant?
(2) Is the estimate of the acceptable limit of error realistic?
(3) Is the data generated by the measurement process really what it is supposed to be?

When the stakes are not large, the backwards approach might be rationalized by accepting the costs of a certain amount of meaningless measurement effort and an occasional disaster as a part of the cost of success. However, the scope and cost, both in dollars and resources, of most undertakings today are such that these questions must be considered thoroughly in the early stages of a project, and decisions must be continuously monitored as the project develops. There are countless examples of the dire consequences when one fails to do so.

It is not difficult to group into three categories the reasons why most measurements are made. These categories are:

(1) Measurements to assure equity in trade and commerce;
(2) Measurements in which the judgment of the result is related to a particular function rather than the ability to measure, such as in the development of products or procedures and the establishment of the uniformity thereof; and
(3) Measurements to assist in understanding the laws of nature and the universe.

Complexity within and diversity of goals between the three categories creates almost insurmountable psychological barriers between the champions of various measurement philosophies. It is hard to separate the operations which are "necessary and sufficient" for success from the practices that are used for political and economic reasons, or merely for convenience. Philosophies and the nature of problems within each category change with time. Opportunities for the enterprising to employ diversions to their own benefit have always been present. The fact remains, however, that one measurement system must serve all, and that like measurements generally employ the same equipment and techniques regardless of the end usage; thus, it is important to understand why each of these categories is unique.

The importance of measurements in today's society has been influenced by political and economic factors associated with changes from an agrarian to a commercial society, the industrial growth following the dark ages, the scientific and industrial explosion of the last two centuries and the present electronic era of computers and rapid communications. Ultimately, regardless of the simplicity or complexity of the measurement, the result and its cost must be evaluated relative to its usefulness in accomplishing the task at hand. The work of the metrologist, if his role is to endure, lies in his contribution toward obtaining proper measurement data in the most economical way. Removing measurement from this supporting role creates a diversion, directing attention from important matters to what may well be trivial operational detail.

Part 1. The Roles of Mass Metrology in Civilization

1.1. The Role of Mass Measurement in Commerce

A. Prior to the Metric System of Measurement Units

The existence of deliberate alloys of copper with lead for small ornaments and alloys of copper with varying amounts of tin for a wide variety of bronzes implies an ability to make accurate measurements with a weighing device ca. 3000 B.C. and perhaps earlier [11]. That trade routes existed between Babylonia and India, and perhaps the Persian Gulf and Red Sea countries, at about the same time implies a development of commercial enterprise beyond barter [12]. Economic records were the earliest documents and these in turn influenced both the development of the written language and the development of numbering systems [13,14]. The transition between the tradition of an illiterate craftsman working with metals and a universally accepted commercial practice is largely conjecture.

The impartial judgment of the weighing operation was well known ca. 2000 B.C. as evidenced by the adoption of the balance as a symbol of social justice [15], a practice which continues today. Then, as now, the weighing operation will dispense equal value in the form of equal quantities of the same commodity. It was, and still is, easy to demonstrate that the comparison, or weighing out, has been accomplished within the practical limit of plus or minus a small weight or a few suitably small objects such as grains of wheat or barley. In the beginning there would have been no requirement that a standard quantity of one commodity should have any relation to the standard quantity of another commodity. The small weight or object used to verify the exactness of comparison could have been accepted by custom. Wealthy families, early rulers, or governments may have fostered the development of ordered weight sets in order to account for and protect their wealth. Measurement practices associated with collecting taxes in kind would likely be adopted in all other transactions.

Ordered sets of weights were in use ca. 2000 B.C. [16]. In these sets, each weight is related to the next larger weight by some fixed ratio. To develop such a set was a substantial undertaking. Individual weights were adjusted by trial and error until both the one-to-one and summation equalities were satisfied within the precision of the comparison process. Ratios between weights varied with preference to numbers which had many factors [17,18]. For example, if $12B$ was to be equivalent to A, then in addition to intercomparing the $12\,B$'s with A, the B's could be intercompared one by one, two by two, three by three, four by four and six by six. Once established, it was not difficult to verify that the ratios were proper, nor was it difficult to duplicate the set.

Precious metals were used for exchange from the earliest times [24]. "To weigh" meant payment in metal and "to measure" meant payment in grain [19]. Simple barter had become in essence sales. Goods of one sort being exchanged for goods of another sort were separately valued to a common standard, and these values brought to a common total [20]. Overseas trade involved capitalization, letters of credit, consignment, and payment of accounts on demand [21]. There is evidence that a mina weight ca. 2100 B.C. was propagated by duplication over a period of 1500 years (to ca. 600 B.C.) [22].

Maspero [23] gives the following description of an Egyptian market transaction:

"Exchanging commodities for metal necessitated two or three operations not required in ordinary barter. The rings or thin bent strips of metal which formed the "tabnû" and its multiples did not always contain the regulation amount of gold or silver, and were often of light weight. They had to be weighed at every fresh transaction in order to estimate their true value, and the interested parties never missed this excellent opportunity for a heated discussion: after having declared for a quarter of an hour that the scales were out of order, that the weighing had been carelessly performed, and that it should be done over again, they at last came to terms, exhausted with wrangling, and then went their way fairly satisfied with one another. It sometimes happened that a clever and unscrupulous dealer would alloy the rings, and mix with the precious metal as much of a baser sort as would be possible without danger of detection. The honest merchant who thought he was receiving in payment for some article, say eight tabnû of fine gold, and who had handed to him eight tabnû of some alloy resembling gold, but containing one-third of silver, lost in a single transaction, without suspecting it, almost one-third of his goods. The fear of such counterfeits was instrumental in restraining the use of tabnû for a long time among the people, and restricted the buying and selling in the markets to exchange in natural products or manufactured objects."

The impact of coinage guaranteed by the government (ca. 500 B.C.) was profound and is still with us today [25,26]. One normally thinks that measurements associated with the exchange of goods in commerce are ordering worth. This is only partly true from the viewpoint of the ultimate consumer. The establishment of a monetary system permitted a third party to enter the transaction without the difficulty of physically handling the material to be traded. Assigning a money value to a unit measure of a commodity permitted the establishment of a much broader market which was not generally concerned with each local transaction but which, nonetheless, established in part the money value for each commodity in the local market. The customer, then as now, must pay the asked price, the measurement process merely determining how much the total transaction will be.

Commerce thrives on the variation of commodity values with time and location [27]. This variation, coupled with confusion and perhaps a willful lack of communication on matters concerning money value and measurement units, is a happy situation for the enterprising entrepreneur. As far as the normal customer is concerned, the only element he has in common with the seller is the measurement process and perhaps some preferential treatment associated with social status, profession, or some other factor totally unrelated to the value of the commodity. Emphasis on the exactness of the measurement can mask more important factors such as the quality of the product offered for sale.

Uniform weights and measures, and common coinage were introduced throughout the Roman Empire [28,29,30]. Yet, perhaps with the exception of doing business with the government, it was not until the early part of the 18th century that the first real efforts toward a mandatory usage of uniform measures was started. Many leaders through the ages have made profound statements relating to the need for uniform measures. Little, however, was done except in the control of the quality of the coinage. No one ruler had been powerful enough to change the customary measures and practices of his land. This was changed in France with the establishment of the metric system of measurement units.

B. The Kilogram and the Pound[3]

It is not generally emphasized that the prime motivation for establishing the metric system of measurements was the utter chaos of the French marketplace.[4] It was not that the conditions in the French marketplace were any different than in any other marketplace, but it was these conditions coupled with two other factors which eventually brought about the reform. These factors were the French Revolution whose great objective was the elimination of all traces of the feudal system and royalty, and the influence of the natural philosophers of the time who realized the international importance of such a forward step in creating a common scientific language. Other powerful influences objected vigorously to the mandatory standards plan. After the new standards had been completed they were not readily accepted. Severe penalties were necessary to enforce their usage in the common measurements of the time. On the other hand, the metric system of measurements almost immediately became the measurement language of all science.

As with all previous artifacts which eventually reached the status of measurement standards, the choice for the basis of the metric standards was arbitrary. With the idea of constancy and reproducibility in mind, the choice for the length unit finally came down to either a ten-millionth part of the length of a quadrant of the earth's meridian, or the length of a pendulum with a specified period. The nonconcurrence of most of the important foreign powers who had been invited to participate in establishing the measurement system left the French to proceed alone.

From the measurements of a segment of a meridian between points near Barcelona and Dunkirk, it was determined by computation that the meridianal distance between the pole and the equator was

[3] This section is essentially an abstract of two papers. The Moreau paper [32] is an excellent general paper on the development of the metric standards and the work of the International Bureau of Weights and Measures. The Miller paper [31] is a comprehensive work describing the reconstruction of the Imperial Standard Pound. Reference to specific passages are made in this section.

[4] At that time there was no shortcoming in the ability to make measurements as evidenced by the use of existing equipment and measurement techniques to establish the new standards. A comprehensive study of density, hydrometry and hydrostatic weighing had been published in the 12th century [38]. Instructions for adjusting weights for use in assay work published in 1580 are just outlines, implying that the techniques of weighing and the precision of the equipment are common knowledge amongst assayers [39].

5130740 toises, from which the ten-millionth part, or the meter, was 3 pieds 11.296 lignes. A unit for mass was defined in terms of length and the density of water. The concept of mass was relatively new to science, and completely new in the history of weighing which had heretofore been concerned with quantities of material rather than the properties of matter. With the meter established in customary units, using hydrostatic weighings of carefully measured cylinders, it was determined that a mass of one kilogram was 18827.15 grains with respect to the weights of the Pile of Charlemagne. With these relationships defined in terms of customary units of measurement, it was then possible to proceed with the construction and adjustment of new standards for the metric units.

The first task was the construction of provisional metric standards. The construction of the kilogram and the meter of the Archives followed, the kilogram of the Archives no doubt being adjusted[5] with the same weights used to adjust the provisional kilogram. The kilogram of the Archives, as it was later discovered, had been adjusted prior to a precise determination of its displacement volume. This important measurement was not made after adjustment because of the fear that the water in a hydrostatic weighing would leach out some of the inclusions which were typical of the platinum of the time. While the technical developments were going on, the Treaty of the Meter was consummated, and the General Conference of Weights and Measures was established to review and finally accept the work.

Techniques were developed prior to the construction of the prototype standards which resulted in more homogeneous material (introduction of the oil fired furnace and the use of cold working). From a small group of kilograms made from the new material and adjusted in the same manner as the kilogram of the Archives, the one which was most nearly identical to the kilogram of the Archives, as deduced from the data resulting from direct comparisons, was chosen to be the prototype standard defined to embody a mass of exactly one kilogram. (This standard is now generally called the international prototype kilogram, designated by $\mathscr{K}$, to differentiate it from other prototype kilograms which are designated by number or letter-number combinations and used as transfer standards.) The task of manufacturing, adjusting, and establishing the mass values of the prototype standards for distribution to the nations who were participating in the metric convention was long and tedious. The survey to determine the length of the arc of the meridian had been started in June 1792. The General Conference[6] formally sanctioned the prototype meter and kilogram and the standards for distribution in September of 1889.

A second major effort in the construction of standards for measurement was going on within this same period. In 1834 all of England's standards of volumetric measure and weight were either totally destroyed or damaged by fire in the House of Parliament to such an extent that they were no longer suitable for use as standards. The Imperial standard troy pound was never recovered from the ruins. A commission, appointed to consider the steps to be taken for the restoration of the standards, concluded that while the law provided for reconstructing the standard of length on the basis of the length of a pendulum of specified period and for the reconstruction of the standard of weight on the basis of the weight of water, neither method would maintain the continuity of the unit.

In the case of length, there were difficulties in carrying out the specified experiment. In the case of weight, differences based on the best determinations of the weight of water by French, Austrian, Swedish and Russian scientists amounted to a difference on the order of one-thousandth of the whole weight. whereas the weighing operation could be performed with a precision smaller than one-millionth of the whole weight. Therefore, it was recommended that the reconstruction could best be accomplished by comparison with other weights and length measures which had previously been carefully compared with the destroyed standards. It was further recommended that the new standard should be the avoirdupois pound in common usage rather than the destroyed troy pound. In 1843 a committee was appointed to superintend the construction of the new standards.

This work resulted in the construction of a platinum avoirdupois pound standard and four copies, the copies to be deposited in such a manner that it would be unlikely that all of them would be lost or damaged simultaneously. It was decreed that "the Commissioners of Her Majesty's Treasury may

[5] Adjusting a weight is adding or removing material from a weight to establish a one-to-one relationship with an accepted standard. In the case of one-piece weights, such as the prototype kilogram, the weight to be adjusted is usually initially heavier than the standard. Material is carefully removed until the one-to-one relationship is established, or until the difference is some small part of the on-scale range of the instrument being used.

[6] The General Conference of Weights and Measures (CGPM) assisted by the International Committee of Weights and Measures (CIPM) and the Consultative Committee for Units (CCU), makes decisions and promulgates resolutions, recommendations and declarations for the International Bureau of Weights and Measures (BIPM). Reference [37] reproduces in chronological order the decisions promulgated since 1889.

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cause the same to be restored by reference to or adoption of any of the copies so deposited [33]." Careful work determined the relationship between the avoirdupois pound and the kilogram. While it was not until 1959 that the English speaking nations adopted an exact relation between the pound and the kilogram, this work provided the basis for coexistence of the two sets of measurement units [34]. The relationship adopted differed only slightly from that established as a part of the reconstruction program. (It was in this work that it was discovered that the displacement volume of the kilogram [35] of the Archives had not been precisely determined before final adjustment.)

The entire reconstruction was based on the existence of weights RS and SP of known displacement volume which had been compared with U. The average air temperature and barometric pressure for several hundred comparisons (used in the above definition) established a standard air density ρ_0. Knowing the displacement volume of the weight, T, used to construct the new standard, from comparisons with RS and SP in air of known density, one can compute the weight that T would appear to have if it were possible to compare it with U in air of density ρ_0 without knowing the density of U. In like manner, W above is a fictitious weight of 7000 grains of the same density as U, the lost Imperial standard; thus, the displacement volumes of weights must be known in order to compute values relative to the commercial pound, W.

This work included the construction and distribution of brass avoirdupois pound standards to approximately 30 countries, including the countries of the British Empire. Recognizing the practical difficulties which would arise because of the platinum defining standard and the brass standards for normal use, the platinum standard was defined to be one pound "in a vacuum"[7] and a commercial standard pound was defined as follows [36]:

"The commercial standard lb is a brass weight which in air (temperature 18.7 °C, barometric pressure 755.64 mm)... appears to weigh as much as W. ... For in air having the above mentioned temperature and pressure, the apparent weight of such a lb would be 7000/5760 of that of the lost standard."

The density of each of the new standards, both platinum and brass, was carefully determined. The assigned values, as computed from the comparison data, were expressed in the form of corrections, or deviations from a nominal value of 1, both on the basis as if compared with PS "in a vacuum," and as if compared with W in air of the defined density. For example, the correction for PS in a vacuum was expressed as 0.00000 since under this condition it is defined as 1 pound; however, because of its small displacement volume, if compared with W in air of specific gravity log delta = 7.07832 − 10 (air density approximately 1.1977 g/cm^3), it would appear to be 0.63407 grain heavy, thus on this basis the assigned correction was +0.63407 grain. This action firmly established two bases for stating values, one used to verify values assigned to standards with reference to the defining standard, and one to maintain the continuity of established commercial practices.

C. In the Early United States

In 1828, the Congress of the United States enacted legislation to the effect that the troy pound obtained from England in 1827 be the standard to be used in establishing the conformity of the coinage of the United States [40]. Apparently it was declared by Captain Kater, who had made the comparison with the Imperial pound standard which was later destroyed, to be an "exact" copy [41]. It is assumed that it was given the assigned value of 1 troy pound, the uncertainty of the comparison, or the announced correction, if any, being considered negligible. In 1830, the Senate directed the Secretary of the Treasury to study the weights and measures used at the principal Customhouses [44]. As a result of this study, the Treasury Department set out on its own to bring about uniformity in the standards of the Customhouses.

As a part of this work, Hassler constructed, along with other standards, a 7,000 grain avoirdupois pound based on the troy pound of the mint. It was reported later [42,43] that Hassler's pound agreed very well with the copy of the standard pound furnished to the United States by England, as mentioned earlier. Eventually, this program was expanded by resolution [45] of Congress to include equipping the states with weights, measures, and balances. In 1866 the Congress enacted [46] that "no contract or dealing, or pleading in any court shall be deemed invalid or liable to objection because the weights or measures expressed or referred to therein are weights or measures of the metric system." In due course the States were also furnished metric standards.

[7] Weighings are not actually made in a vacuum. By properly accounting for the buoyant forces acting on the objects being compared, the data can be adjusted to obtain the result expected if the weighing had been made in a vacuum. One can also include in the weighing a small weight which is nearly equivalent to the difference in buoyant forces acting on the objects being compared.

Gross changes in the form of the economy of the United States have occurred. America has been profoundly influenced by the nearness of the people to the soil and the leadership that an agrarian society develops [52]. As late as 1830 approximately 70 percent of the working population of the United States was involved in agriculture and other forms of food production, and in producing raw materials. Only about 20 percent were involved in manufacturing.[8] In such an environment weights and measures had a meaning in the value structure somewhat similar to that of ancient times. Now, something on the order of 30 percent are all that are involved in the area which includes producing food, raw materials, the manufacturing of both durable and nondurable goods, and construction. Thus, the number of items in which weights and measures have any relation to the value structure is very few, the major cost to the consumer being associated with value added rather than quantity.

The normal consumer can only choose from those products offered, selecting on the basis of asking price. The products offered, because of the high cost associated with establishing a large scale production, are only those which have a high probability of being desirable to the buying public. While measurement may be necessary to establish the price to the customer, there is no meaningful relationship between the weights and measures and the unit price one must pay to acquire the item. One does not weigh automobiles or television sets. Where measurements are a part of the transaction, they are, in essence, merely counting operations similar in nature to counting out a dozen where items are priced by the dozen. Under these circumstances, the virtues of precise measurement and the exactness of the standard do not guarantee equity in the marketplace.

D. Summary

In retrospect at this point, it seems clear that both the construction of the kilogram and the reconstruction of the pound were essentially scientific efforts directed toward assuring the longevity of the respective mass units. Both efforts required precise definitions and detail work far beyond that usually associated with the previous history of weighing. Having established platinum standards, the assignment of values to weights of other materials (mostly brass) required as much as, if not more, attention to procedural detail.

The above two efforts, establishment and maintenance of the unit and calibration, together with normal usage has, in effect, polarized activities into separate groups: one group which works with defining mass standards, one group which works with practical everyday weighings, and in the middle a group which ostensibly translates the scientific into the practical. The degree to which such a hierarchy can be effective is related to the extent to which a specific end use can be characterized. If a measurement process requirement can be completely specified, one can devise a plan which will reduce a complex measurement to a simple operational routine. Such an engineered system, however, is not always adequate and may be completely misleading in other areas of usage.

The intellectual elegance of the metric system was lost almost from the start. A careful redetermination of the density of water created a situation in which, according to the original definition, the value assigned to the prototype kilogram would be in error by about 28 parts in a million. To change the value of the prototype and all of its copies was unthinkable, therefore a new "volume" unit was proposed to replace the cubic centimeter. By conference action in 1901 (3d CGPM, 1901), the unit of volume, for high-accuracy determinations, was defined as the volume occupied by a mass of 1 kilogram of pure water at its maximum density and at standard pressure, this volume being called the liter. While it is doubtful that the discrepancy was at all significant in common measurement, the liter has been accepted almost universally. This caused no end of problems concerning both volume and density measurement. The circle has been complete for in 1961 (CIPM, 1961) the cubic decimeter was declared the unit for precise volume measurement, relegating the liter to the realm of customary units which still prevail.

Quite apart from the use of weights in commerce, various technologies over the centuries used weights as a convenient way to generate forces. The use of suspended or stacked weights to measure the draw of a bow, the ability of a structure to support a given load, and to characterize the strength of various materials has been prevalent throught history and continues today. This led to an ambiguity in both the names assigned to the units and to the comparison operations. In 1901 (3d CGPM, 1901), the General Conference considered it necessary to take action to

[8] The percentages have been estimated from various census reports. Because of the different classifications used over the years, they are only approximate. They are, however, valid indicators of a shift from an agrarian to an urban society in a very short time span.

put an end to the "ambiguity which in current practice still subsists on the meaning of the word weight, used sometimes for mass and sometimes for mechanical force."

The Conference declared: "The kilogram is the unit of mass, it is equal to the mass of the international prototype kilogram. The word *weight* denotes a quantity of the same nature as force, the weight of a body is the product of its mass and the acceleration due to gravity, in particular, the *standard weight* of a body is the product of its mass and the standard acceleration due to gravity."

This did not end the confusion [47,48]. Such a statement made no sense at all to those who were concerned with commercial weighing. To officially sanction such a definition of weight is to refuse to recognize that at some time the use of a standard acceleration of gravity in lieu of the appropriate local acceleration of gravity would introduce significant systematic errors in many measurements [49].

The situation has been rectified by including the Newton as an accepted unit for force in the supplementary units of the International System of Units, known as the SI system (11th CGPM, 1960). By this action, the meaning of the words weight and weighing could revert to more general meanings, for example: weight—an object which embodies a mass or mass related property of interest; weighing—to make a quantitative comparison.[9] While this action may in time discourage practices such as introducing the term "massing" [51] as meaning to make a mass measurement, universal acceptance may never be achieved because of the natural tendency of the literature to propagate what has gone on before.

1.2. The Role of Measurement in Technology

The innovative instinct of the earliest man, which was the basis of his survival, provided the start of technological development. The emergence of the arts and crafts represents the cumulative experience resulting from progressive novel action and invention which for the most part was based on perception rather than thought [53]. Having conceived a novel action of sufficient functional importance, such as the use of a flint blade in lieu of a sharpened stick, successive refinements follow, i.e., the bronze blade, the iron blade and the steel blade, as long as the function remains important to society. Flint was a trade item in the Neolithic age [54,55]. Most of the primary crafts were highly developed and widely diffused before the development of historical records [56]. Essential elements in the transformation to an urban civilization were the conversion of luxuries into necessities and the increase, and concentration, of social surplus [57]. While the gourd was functionally satisfactory, the clay pot was nicer; thus the potter shifted from a part time operation to full time, and ultimately to the pottery factory and a brisk international trade in pots, as existed ca. 400 B.C. [58].

It is difficult to trace the details of the various crafts. The Sumerians, for example, thought that all knowledge came from the gods, therefore it was sacred and could not be communicated. The priest passed on instructions orally being careful to limit instructions to the exact steps to be followed [59]. For the craftsman, his knowledge was his livelihood. Traditions were passed from father to son. Families became noted for their particular crafts. Later, where products and trades were concerned, to divulge details was to invite economic disaster from competition. The impressive state of development reached, however, can be observed in the artifacts produced and the longevity of some of the techniques. An example of the latter is the "touchstone" tests for purity of gold and silver alloys which made possible the issuance of coinage. Agricola described in 1556 essentially the same tests, indicating a longevity in excess of 2000 years [60].

In terms of the development of the crafts and the dissemination of the products, the Roman Empire was remarkable. While somewhat short on invention, the Romans perfected masonry, tiling, road building, surveying, molded pottery, blown glass, watermill, and a host of others [61]. The use of glass, for example, in a wide variety of applications including commercial packaging reached a scale unmatched before the 19th century [62]. That these could not be accomplished without measurement clearly emphasizes the fact that, where function is the main concern, all measurements are relative. Things work because relative geometry, proportion, or properties of materials are correct, not because of any particular choice of measurement units. Mortar, for example, lasts through the ages because the ingredients have the right properties and are combined in the right proportions. Machinery works because each part has the right characteristics and the rela-

[9] A facsimile of the first edition of Webster's Dictionary [50] gives the following definitions:

mass —a lump
weight—a mass by which bodies are weighed
weigh —to try the weight, consider, examine, judge . . . . etc.

tive dimensions are correct. Each craft had to develop its own methods for determining and describing the parameters which were critical to its particular trade or profession.

Early crafts encompassed the entire operation from raw material to finished products. As the demand for finished products increased, the time the craftsmen could afford to spend in making ready raw materials lessened. In some instances, the materials in a product came from several distant sources. These situations led to the development of early industries concerned basically with raw materials such as charcoal and metallic ores, and with quarrying, lumbering, and weaving. This action was the first breach in the tight security of the craft system. Craft Guilds appeared during the Medieval Age, and the resulting "codes" were probably more directed toward protection from competition than convincing the possible clients of the perfection of the product. For example, in 1454 the penalty for divulging the secrets of Venetian glass was death [64]. Craft mysteries persisted until the Industrial Revolution ca. 1750 [65]. The inventions of the 18th and 19th centuries brought about changes which are considered to be the Industrial Revolution. These changes can be summarized as follows: (1) a shift from animal and wind power to coal and steam, (2) the effects of this shift on the iron and textile industries [66], and (3) the change from working for a livelihood to working for a profit [67].

The forerunners of industry as we know it today stem from the military. The first large-scale demand for standardized goods was the provision of uniforms for large standing armies [68]. The use of interchangeable parts in the assembly of muskets and rifles was demonstrated by LeBlanc in France, and Whitney in the United States [69]. Through the years, the dividing line between raw material supply and preprocessing, such as the production of pig iron, steel and cloth, and product manufacturing has become more prominent, with the preprocessed materials becoming more like other commercial commodities. Most items which are procured today, either by the individual or by the government, are the results of the combined efforts of many throughout the world. Industrial subdivision, or compartmentalization with its large economic benefits, has created a special role for measurement. The material or preprocessed material supplier enlarges his market by resolving small differences in requirements among his customers. In time, the terminology of the supplier must be accepted by all who use his material; hence measurements become wed to marketing requirements rather than functional requirements.

Subdivision of a task requires a detailed delineation of what is to be done by each sub-unit. This can take the form of organization charts, specifications, detailed drawings, samples, and the like. Many ways are used depending upon the nature of the item and its function in the overall task. If someone else is to provide the service, some limits must be established for judging that the offered product will perform as intended in the overall endeavor. Determining the dividing line between success and failure is not always easy. These limits, once established and regardless of whether they were established by lengthy experiments, good engineering judgment, or by sheer guess, become fixed restraints on the next element of the subdivision. The effect is a dilution of the ability to make function related judgments. In complex situations, no one person knows the full scope of the task, therefore no one can instigate changes of any sort without fear of jeopardizing the entire venture.

It is a tendency for tolerances to be tightened by each organizational element through which the task must pass. In the procurement-production stage, the product must comply (within the tolerance) to the specification or drawing. Compliance is defined by a set of procedures, usually measurements, which supposedly will assure the buyer of the suitability of the product for its intended use. The net result is that the most precise measurement processes are frequently used to differentiate between scrap and acceptable parts in order to consummate a particular contract, the sorting limits in many cases having little relation to the function the parts must perform. Troubles are merely transferred to the gage if the measurements are differences between the part in question and a pseudo standard or gage. Difficult problems occur when a specification attempts to describe a complex part completely by dimensions or specification verbiage.

The mechanism for verifying specification compliance is created for the most part by those who do not fully understand either the measurement or the function. Many procedures rely on ritualistic documentation with little attention given to the characteristics of the measurement processes which are used. In many instances the status of the source of the documentation becomes more important than problems relating to the environment in which the required measurements may be valid and the environment in which the measurements of the product are to be made. It is not unusual to find that a prerequisite for doing business is the possession of such documents and precise measurement facilities which often do not relate to the completion of the task at hand.

However, in those cases where measurement data are really critical, the most important measurement is that on the production floor. The part or assembly will either operate properly or not regardless of the supporting hierarchy. The most precise measurements could, if necessary, be moved directly to the production floor to achieve the desired function.

Today, there is little doubt that the solutions of the most difficult and challenging measurement problems are being carried out in an environment of strict industrial security. This is similar to development in the days of the guilds. However, now external communications are necessary. The present economic facts of life make it necessary to know what is going on in related science and industry so that each new task is not a "re-invention of the wheel." A recent report suggests that innovations important to one industry may come from a completely nonrelated industry [70]. On the other hand, to divulge certain information at the developmental level is almost certain to result in an economic setback, perhaps even a catastrophe in the raw materials market, the product market, or in the capital market, sometimes in all three.

1.3. The Role of Measurement in Science

In sharp contrast to both previous areas of discussion, the advancement of science depends completely upon a free and open exchange of information [71]. Thus, having agreed to accept an arbitrary set of measurement units, it is imperative that the continuity of the units be maintained. By constructing a minimal set of units and constants from which all measurement quantities of interest can be derived, ambiguities are removed. By defining a means to realize each unit, in principle one can construct the units he needs without introducing ambiguity into the measurement system. What happens in practice is, of course, another story.

Most defining experiments are complex and tedious and not always related to the problems of measuring things or describing phenomena. Having established a definition of the unit of time based on an atomic phenomenon, and having constructed the hardware to realize the unit, the ease by which the unit can be disseminated by broadcast makes it highly unlikely that more than a few would seriously consider duplicating the effort. Mass, on the other hand, is and will no doubt for some time be embodied in a prototype standard to be disseminated by methods which are in essence many thousands of years old.

By international agreement, the SI-defined measurement units together with a substantial group of auxiliary units have replaced and augmented the original three—length, mass and volume—of the metric system. Having accepted the structure of the SI, the definition, or redefinition, of the measurement units, insofar as possible, must maintain the continuity of the original arbitrary units. Further, the uncertainty of the unit as realized must be compatible with the exploratory experiments in which the unit may be used.

One requirement for a phenomenon to be considered in redefining a unit is that under the contemplated definition, the newly defined unit would be more stable than the unit under the current definition [72]. Having verified that this would be the case, the next task is to determine the unit in terms of the new phenomenon to a degree such that the uncertainty of the unit as expressed by the new phenomenon is within the uncertainty limits associated with the unit as expressed by the old phenomenon. The important point is this action relates only to the definition of the unit, and may not be extendible in any form to the manner in which the unit is used to make other kinds of measurement. Because all units are candidates for redefinition, and because one is now able to evaluate the performance characteristics of a wide variety of measurement processes [73], a new definition for the "best" measurement process must be established.

In the distant past, a weight was attested, or certified, to be an exact copy of another by the reputation or position of the person making the comparison and by his stamp on the weight. Having obtained such a verification, one was free to use the marked weight as he wished. The report of calibration from a currently existing measurement facility is in essence no different. Throughout history, the status of the standard with which the unknown was compared and the status of the facility doing the comparison established the quality of the work. Since all methods of comparison were essentially the same, to refute all criticism one might decide to pay more and wait longer in order to utilize the highest status facility of the land. Little attention was given to the consistency of the measurements at operating level because there was no way to manipulate the masses of data required to evaluate a single measurement process, let alone a whole series of interconnected processes. One was paying for a judgment.

It has been well known from the beginning of precise measurement that repeated measurements often produce different numbers. The man who put his mark on the weight was in effect saying that it is close enough to some standard to be considered as an exact replica. The report of calibration says "call it this number," the number sometimes being accompanied by an uncertainty which is ridiculously small with reference to any practical usage, or when stated as a deviation from some nominal value, the deviation or the number being so small that the user may consider the item as exactly the nominal value.

It is now possible to look in detail at the performance characteristics of a measurement process [74] and at the consistency of measurement at any point in the entire system [75]. Further, the cost of relating a measurement to the manner in which the unit is defined may be prohibitive if indeed it is at all possible. Under these circumstances, the definition of the best process must start from the end use rather than the defining standard. Having first established that a particular measurement is necessary to the success of the venture at hand, the best process is that which produces these results in the most economical manner, based on verification by demonstration. This applies equally to the most complex scientific study or the simplest measurement. As a point of departure, it is necessary to make it clear to all the basis on which certain mass values are stated.

Part 2. The Basis for Stating Mass Values

2.1. Statement of the Problem

The nonrelativistic physical laws concerning the mass properties of objects were formulated by Newton [76], thus relative to the history of weighing, the concept of mass as a property of an object is a recent one. Because of the size of the solar system, Newton could consider the planets and their satellites as mass points in the vacuum of space [77]. On the other hand, most weighing devices are force comparators which operate in the normal environment of the surface of the earth.

The typical weighing instrument responds to a vertical force vector. The principal component of this vector is gravitational force. Next in order of magnitude is the buoyant force of the atmosphere. There are other vertical components which affect the instrument indication such as might be associated with air currents, electrostatic forces, magnetic forces, etc. [78]. However, if the object under study is sufficiently stable in mass, these other forces can usually be controlled with careful design and operation so that their effect on the weighing instrument performance is insignificant.

The buoyant force of the atmosphere precludes a mass point assumption (a mass with no displacement volume). This force is proportional to the displacement volume of the object under study and the density of the immediate environment. If ignored, it is frequently the source of the largest measurement discrepancy in precise mass measurement. If not understood, much time can be wasted on detailed computation with no tangible benefits to the work at hand [79].

The response of the weighing instrument to an applied force system is a number indication or observation, O_1, expressed symbolically as:

$$g(\mathscr{S}-\rho V_{\mathscr{S}}) \simeq O_1 \tag{1}$$

where $\mathscr{S}$[10] represents the mass property of one object; ρ the density of the surrounding environment; $V_{\mathscr{S}}$ the displaced volume; g the local acceleration of gravity; and O_1 the observation. The symbol $\simeq$ means that the observation O_1 results from the operation of putting the object whose mass is $\mathscr{S}$ on the balance pan. (The symbol O_j, $j=1$ to n, represents the number reading produced by the instrument. The sequence is repeated for each group of interrelated comparisons.)

One would hope that there would be an exact one-to-one relationship between the observation, O_1, and the mass of the object, $\mathscr{S}$, so that no further effort need be spent on the measurement. That this is not the case can be shown easily by following the same procedure with a second object which has the mass $\mathscr{X}$ to obtain observation O_2:

$$g(\mathscr{X} - \rho V_x) \simeq O_2 \tag{2}$$

Subtracting (2) from (1), and introducing a factor, K,[11] to convert the observed numbers to mass units.

$$(\mathscr{S}-\rho V_{\mathscr{S}})-(\mathscr{X}-\rho V_{\mathscr{X}})=K(O_1-O_2)$$

or

$$(\mathscr{S}-\mathscr{X})=K(O_1-O_2)+\rho(V_{\mathscr{S}}-V_{\mathscr{X}}) \tag{3}$$

In a normal environment, equality in observation, $(O_1 - O_2)=O$, means equality in mass, if, and only if, the displaced volumes are equal. In most cases a true estimate of $(\mathscr{S}-\mathscr{X})$ cannot be determined directly from the observations. In order to illustrate more clearly the nature of the difficulties this situation causes, one must examine the traditional method of number assignment.

If there exists a "master" artifact with a defined mass value N (the named or nominal value), and if our first weight is considered to be a copy of this artifact, the value S is usually expressed by two numbers; the "defined" value of the artifact, N, and a correction Cr_s such that $S=N+Cr_s$.

The value, X, to be assigned to a second weight, would also be expressed as a nominal value, N, plus a correction Cr_x, where Cr_x would be computed from

[10] Script letters are used to denote the mass property embodied in the objects of interest. Italic letters are used to denote the numbers assigned to express the mass property quantitatively relative to a standard of defined mass. For example, given an object with mass defined as one unit, $\mathscr{N}=1=N$, and an unknown mass $\mathscr{X}$, the mass properties $\mathscr{N}$ and $\mathscr{X}$ determine in part the mass difference as observed. The number assigned to the unknown, X, is deduced from the data produced by the measurement process.

[11] The operations required to establish a numerical value for K introduce g on the right hand side of the equation. As shown, K includes the factor $(1/g)$. Later this factor becomes (g/g) or 1 if all of the weighings required in the sequence have been done under the influence of the same g. The details of this operation are not discussed in this paper.

Cr_s and the measured mass difference, $(\mathscr{S}-\mathscr{X})$, as determined in (3) above.

$$X = S-(\mathscr{S}-\mathscr{X})$$
$$= N + Cr_s - (\mathscr{S}-\mathscr{X})$$
$$= N + Cr_x$$

where

$$Cr_x = Cr_s - (\mathscr{S}-\mathscr{X})$$

Corrections can be used in two ways. A number, say 0.98, can be expressed with reference to a nominal number, say 1, by the relation (nominal + correction) = (1 − 0.02) = 0.98. On the other hand, one might view the absolute value of the correction, |−0.02|, as a means of judging the closeness of 0.98 to 1. The latter usage is by far the most prevalent. In many cases it is assumed that $(\mathscr{S}-\mathscr{X}) = K(O_1 - O_2)$, thus adjustments of $\mathscr{X}$ to obtain a small Cr_x may be illusory if the magnitude of $\rho(V_{\mathscr{S}} - V_{\mathscr{X}})$ is large. The fact that every object displaces a volume of air must be treated in some way in all practical mass measurements.

2.2. Implicit and Explicit Treatment of Displacement Volumes

The term implicit, as applied to the treatment of displacement, is used to characterize one method for determining a number to represent the mass of an object. The mass values assigned by this method, while somewhat arbitrary, are consistent within prescribed limits and are suitable for many end use requirements. Detailed treatment of displacement is not generally required in comparing weights since it is considered in formulating the method. To illustrate, given an object that has an assigned mass value S, consider the task of assigning a value, X^*, to another object.

The implicit treatment of displacement volumes accepts the observed difference, $K(O_1 - O_2)$, as an estimate of $S - X$. Thus

$$X^* = S - K(O_1 - O_2) \tag{4}$$
$$\approx S - (\mathscr{S}-\mathscr{X})$$

In such a procedure, X^* is the mass that the unknown, $\mathscr{X}$, would appear to have when compared with an accepted standard. Such a procedure would certainly be convenient, and is indeed the case when S is a valid estimate of $\mathscr{S}$, and when the displacement volumes for $\mathscr{S}$ and $\mathscr{X}$ are identical. When these conditions do not apply, as is the case in many mass measurements, X^* is not necessarily a valid estimate of the mass, $\mathscr{X}$, in terms of the laws of physics.[12] While the consistency of the numbers assigned on this basis can be demonstrated, $|X^* - X|$ may be large compared to the precision of the measurement process.

The explicit treatment of displacement volumes strives to establish X as a valid estimate of $\mathscr{X}$. In such a system the number assignments are consistent and the values are valid estimates in terms of the laws of physics. In this case, from (3)

$$X = S - (\mathscr{S}-\mathscr{X})$$
$$X = S - K(O_1 - O_2) - \rho(V_{\mathscr{S}} - V_{\mathscr{X}}) \tag{5}$$

One must determine the air density ρ, at the time of each weighing, and one must have reasonable estimates for the displacement volumes, $V_{\mathscr{S}}$ and $V_{\mathscr{X}}$.

Historically, values have been assigned by a procedure in which displacement volumes have been treated implicitly. For weighings which are made primarily to establish equality of goods in trade, the buoyant force is, in many cases, substantially smaller than other accepted variabilities inherent to the material being weighed, i.e., least count, evaporation, smallest accounting unit, etc. On the other hand, faith in the system by the populace depends largely on the existence of good stable standards and the ability to demonstrate a consistency beyond that needed under normal circumstances. The demonstration should be simple, direct and within the capabilities of most practical weighing processes.

With the advent of the metric system and its standards, it became necessary to treat the problem of displacement volume explicitly—at least to the point of establishing the kilogram and the mass of the customary standards then in use relative to the kilogram. In this process, the customary standard was not changed—only the ratio of mass between it and the kilogram was established. This permitted a conversion between customary units and the kilogram but left the practical measurement system unchanged.

While on the surface the assignment of mass values appears to be perfectly straightforward, in fact it is not. There are many subtleties involved. Troubles occur in several ways. For example, the characteristics of the defining standard used in the

[12] In force measurements, for example, the use of "apparent mass" values without modification can introduce significant systematic error. Using an apparent mass value and an assumed density of 8.4g/cm^3 can leave a residual systematic error which may be significant in the most precise force and pressure measurement processes [82].

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implicit volume treatment are different in various countries. In such a situation, the value assignment for a given object is not unique, but differs by measurable amounts depending on whose standard is used. Further, the value assignment may differ from the actual mass by measurable amounts.

The implicit volume basis for stating the mass value, while specifically defined at the level of the highest calibration laboratories, is not generally understood by all who must make precise mass measurements. Detailed adjustment of values stated on one basis to values appropriate to another basis is frequently tedious. "Apparent mass" or "weight in air" are frequently used in situations where an explicit treatment would be more appropriate. These points are not necessarily faults of the implicit treatment but they emphasize a need to understand the details of such a system and the limiting boundary conditions associated with its general usage.

2.3. Values on the Basis of an Explicit Treatment of Displacement Volumes

The explicit treatment of displacement volumes requires a detailed treatment of the term $\rho(V_{\mathscr{S}} - V_{\mathscr{X}})$ in eq (3). The magnitude of this term can be illustrated by considering a kilogram weight made from platinum-iridium alloy (a material far too dense and far too expensive for everyday use) and a kilogram stainless steel weight. Inserting approximate values:

$$\rho(V_{ss} - V_p) = 1.2 \text{ mg/cm}^3 (125 - 46.5)\text{cm}^3 = 94.2 \text{ mg}$$

For a mass measurement process which has a standard deviation on the order of .050 mg (not uncommon), it is apparent that daily fluctuations in air density on the order of 0.1 percent can be a significant source of variability in a sequence of repeated measurements. While not treated in this paper, it should also be noted that the buoyant effect must be considered in establishing K, the constant used to relate instrument indication to mass units, as shown in many of the equations.

The density of the environment, ρ, is a function of the composition and of the pressure, temperature, and relative humidity at any given instant. The composition does not change rapidly, but the other three continually change in a cyclic manner. This causes a local variation in air density of perhaps as much as 3 percent. Variability in air density between various localities, because of changes in pressure with change in elevation, may be as much as 20 percent. Tables in appendix 1 show the approximate average air density in various locations throughout the United States.

Data on atmospheric pressure, temperature and relative humidity are usually taken before and after a given series of weighings. The National Bureau of Standards mass laboratory uses the formula given in appendix 2 to compute the average air density over the elapsed time interval. In less precise measurement processes the average air density from appendix 1 may be adequate.

The displacement volume depends upon the characteristics of the material and the manner in which the weight or object has been constructed. It is also a temperature dependent quantity. In critical cases, the displacement volume at a specific temperature, t_0, can usually be determined by a separate experiment in which the object is weighed in two mediums to obtain two relations:

$$g(\mathscr{S} - \rho_A V_{\mathscr{S}}^0) \simeq O_1$$

$$g(\mathscr{S} - \rho_F V_{\mathscr{S}}^0) \simeq O_2$$

from which

$$(\mathscr{S} - \rho_A V_{\mathscr{S}}^0) - (\mathscr{S} - \rho_F V_{\mathscr{S}}^0) = K(O_1 - O_2)$$

or, reduced to temperature t_0;

$$V_{\mathscr{S}}^0 = \frac{K(O_1 - O_2)}{(\rho_F - \rho_A)} \tag{6}$$

where ρ_A is the known density of a light medium such as air, and ρ_F is the density of a heavy medium such as water.[13]

Sometimes a suitable estimate for displacement volumes can be computed from published values for the density of material used in the construction of the weights. In some cases, particularly where large weights are involved, displacement volume estimates can be computed from physical measurements.

A stated volume, $V_{\mathscr{S}}^0$, is appropriate at only one temperature, t_0, thus to determine the volume, $V_{\mathscr{S}}$, at another temperature, t, the coefficient of expansion, $\alpha_{\mathscr{S}}$, must be known or estimated.

$$V_{\mathscr{S}} = V_{\mathscr{S}}^0[1 + \alpha_{\mathscr{S}}(t - t_0)]$$

From equations 1 and 2, with the term $K(O_1 - O_2)$ expressed in mass units, the value X, based on a "known" value S for the standard, X, can be com-

[13] In the case of determining displacement volumes for materials which cannot be submerged in a fluid, reasonable estimates can be obtained from sequences of weighings at various locations where the average air density is markedly different.

puted from one of the following two formulas or combinations thereof:

$$S - \rho V_{\mathscr{S}}^{0}[1+\alpha_{\mathscr{S}}(t - t_0)] - X+\rho V_{\mathscr{X}}^{0}[1+\alpha_{\mathscr{X}}(t - t_0)] = K(O_1 - O_2) \qquad (7)$$

or

$$S\left[1 - \frac{\rho[1+\alpha_{\mathscr{S}}(t-t_0)]}{d^0_{\mathscr{S}}}\right] - X\left[1 - \frac{\rho[1+\alpha_{\mathscr{X}}(t-t_0)]}{d^0_{\mathscr{X}}}\right] = K(O_1 - O_2) \qquad (8)$$

Detailed calculations to obtain X can range from simple hand computations to extensive computer solutions of incomplete block comparison designs. In any case, the basic relations are as shown in (7) and (8) above. Appendix 3 lists the material densities and coefficients of expansion used by the National Bureau of Standards in those cases where more specific data are not available.

Explicit treatment of displacement volumes, while detailed, has an advantage in that any time a more appropriate estimate of displaced volume or density is obtained, previous data can be reanalysed using the new volume estimates. This, in turn, changes mass measurements from a series of isolated comparisons from which one can only accept the last value into a related sequence of comparisons each of which contributes to establishing the appropriate value estimate.

An assessment of the adequacy of an explicit treatment of displacement volumes cannot be made on the basis of one measurement. Verification of the validity of the numbers assigned is in the consistency of the results of experiments which involve mass along with other measurement parameters, and in the ability to demonstrate that the values assigned over a wide range of conditions agree within the statistical limits for the measurement process.

2.4. Values on the Basis of an Implicit Treatment of Displacement Volumes

Stating values on the basis of an implicit treatment of displacement volumes stems from a practice dating far back in history. The adjustment of one weight relative to another to obtain an "exact" copy implies a "zero" observed difference, or at least a difference within some prescribed limits. If the adjusting process reduces the magnitude of the term $K(O_1 - O_2)$ from eq (3) to the point that it can be ignored, the value assigned to the unknown could be assumed to be the same as the accepted value for the known. That is, if:

$$K(O_1 - O_2) \rightarrow O$$

then, from eq (3),

$$(\mathscr{S} - \mathscr{X}) - \rho(V_{\mathscr{S}} - V_{\mathscr{X}}) \rightarrow O.$$

Ignoring the term $\rho(V_{\mathscr{S}} - V_{\mathscr{X}})$, it would seem that $(\mathscr{S} - \mathscr{X}) \rightarrow O$ and the apparent mass of $\mathscr{X}$ would be:

$$X^* = S + \epsilon = S - [(\mathscr{S} - \mathscr{X}) - \rho(V_{\mathscr{S}} - V_{\mathscr{X}})]$$

Expressed in terms of material density:

$$X^* = S + \epsilon = S - \left[\mathscr{S}\left(1 - \frac{\rho}{d_{\mathscr{S}}}\right) - \mathscr{X}\left(1 - \frac{\rho}{d_{\mathscr{X}}}\right)\right]$$

Establishing an "ideal" material of density d^* to replace $d_{\mathscr{S}}$, and limiting the density, $d_{\mathscr{X}}$, of materials to be used in the manufacturing of weights, ϵ can be held to predictable limits over some range of air density, ρ.

The apparent mass can be stated another way. The apparent mass, X^*, is the amount of any specified material which will "exactly" balance the unknown in a specified atmosphere, $\rho_0 = 1.2$ mg/cm^3, at the specified temperature, $t = 20$ °C.

For a standard with defined mass value N, made from material of density d_N, the computed apparent mass value is:

$$N^* = \frac{N\left(1 - \frac{\rho_0}{d_N}\right)}{\left(1 - \frac{\rho_0}{d^*}\right)} \qquad (9)$$

where ρ_0 is the defined air density and d^* is the density of the appropriate "ideal" material.

One can designate a practical mass standard to be one or more weights which are made from material which is essentially identical to the "ideal" material and which are closely adjusted to nominal value on the basis of an explicit treatment of displacement volumes. From this point on, other weights can be adjusted, or one can establish values for other weights, directly from the comparisons with the practical mass standard. Following a definite procedure, the mass values are assigned to weights without detailed corrections for the buoyant effect of the environment and these values can be verified within an impressively small limit. The simplicity of such an approach is lost, however, when, in the next step, the density of the object in question is not within the range normally specified for weights.

It has been traditional in the United States to use "normal brass" as the "ideal" material. NBS Circular 3 [87], in 1918, established the precise basis for stating values as:

". . . it is necessary to establish the values of commercial weights on a definite basis, and to this end all corrections and tolerances apply to the apparent mass as determined in air having a density of 1.2 mg per milliliter, against (brass) standards having a density of 8.4 g per cubic centimeter at 0 °C, whose coefficient of cubical expansion is 0.000 054 per degree centigrade and whose values are based on their true mass or weight in vacuo."

Circular 547 issued in 1954 [87], no doubt formalizing a prior practice, expanded the definition as follows:

" 'Apparent mass *vs* brass' values are those that the weights would be assigned on the basis of a comparison at 20 °C in normal air against normal brass standards . . . no corrections would be applied for the buoyant effect of the 'normal air' (defined as having a density of 1.2 mg/cm^3), . . . (normal brass standards being defined as standards composed of brass having a density of 8.4 g/cm^3 at 0 °C and a coefficient of cubical expansion of 0.000 054 per degree Celsius (centigrade))."

Circular 547 also required for each new set of weights a manufacturer's statement as to the density and composition of the material used to make the weights, the nature of the surface protection if used, and the type of construction.

The above specification has been the basis for consistent practical mass measurements in the United States for some time. These statements imply a direct comparison with a brass standard, a procedure which is suitable for many purposes. While the apparent mass value assigned is normally used in a manner which treats the displacement volume implicitly, many laboratories, including the National Bureau of Standards, assign such values on the basis of explicit treatment of displacement volumes.

Material density, like displacement volume, is temperature dependent. At the present time, most density data is given at $t=20$ °C, the specified temperature at the time of "comparison." The density of the "ideal" material, 8.4 g/cm^3 is stated at 0 °C. While this "ideal" material is called density 8.4 g/cm^3, the effective density at 20 °C is 8.3909 g/cm^3,[14] the above equation being

$$N^*=\frac{N\left(1-\frac{0.0012}{dN}\right)}{\left(1-\frac{0.0012}{8.3909}\right)} \tag{10}$$

"Apparent mass" values have interesting properties. Consider two weights with mass $\mathscr{B}$ and $\mathscr{A}$ and with assigned apparent mass values A^* and B^*. The observed mass difference between these weights when compared in an environment of density ρ_1 is:

$$K(O_1-O_2)=\mathscr{A}\left(1-\frac{\rho_1}{d_{\mathscr{A}}}\right)-\mathscr{B}\left(1-\frac{\rho_1}{d_{\mathscr{B}}}\right)$$

The assigned number difference, however, is:

$$A^*-B^*=\frac{A\left(1-\frac{\rho_0}{d_{\mathscr{A}}}\right)-B\left(1-\frac{\rho_0}{d_{\mathscr{B}}}\right)}{\left(1-\frac{\rho_0}{d^*}\right)}$$

Subtracting the one equation from the other and simplifying:

$$K(O_1-O_2)-(A^*-B^*)$$
$$\approx(\rho_0-\rho_1)\left(\frac{A}{d_{\mathscr{A}}}-\frac{B}{d_{\mathscr{B}}}\right)-\frac{\rho_0}{d^*}(A-B)$$

If the two weights are well adjusted and made from the same material, the observed difference and the assigned number difference in essence agree exactly. Minor variations, due to slight differences in the density of material and in the adjustment could easily be smaller than the claimed level of instrument performance.

If $d_{\mathscr{A}}$ and $d_{\mathscr{B}}$ are approximately equal to d_s, as is the case for most normal brasses or stainless steels, then:

$$(\rho_0-\rho_1)\left(\frac{A}{d_{\mathscr{A}}}-\frac{B}{d_{\mathscr{B}}}\right)=\Delta\rho\Delta V$$

[14] From the definition of density as mass per unit volume:

$$m=d_0V_0=D_{20}V_{20}$$

or

$$d_{20}=\frac{d_0V_0}{V_{20}}=\frac{d_0V_0}{V_0[1+\alpha(t-t)]}$$

For brass, defined to be of density 8.4 g/cm^3 at 0°C, with a defined coefficient of cubical expansion 0.000 054:

$$d_{20}=\frac{8.4}{[1+0.000054(20)]}\simeq 8.3909 \text{ g/cm}^3$$

The magnitude of $\Delta\rho$ is not likely to exceed the range of air density over the geographic area in which apparent mass values are used. The magnitude of ΔV is restricted by the range of material densities permitted in the construction of weights. The product, $\Delta\rho\Delta V=\epsilon$, is usually small relative to use requirements so that the observed difference is a reasonable estimate of an assigned number difference based on apparent mass, thus:

$$K(O_1 - O_2)=(A^* - B^*)+\epsilon \tag{11}$$

It is not a universal practice to state apparent mass values with reference to an 8.4 g/cm^3 material. The choice of "8.4" material as an ideal material undoubtedly dates far back in history. It is now a practice in some areas to state apparent mass values with reference to a material of density 8.0 g/cm^3 at 20 °C [80, 81, 91]. (There may also be other bases for stating values.) The use of material of density 8.0 g/cm^3 as the basis for stating values is particularly attractive to those who are interested in the refinement of the intercomparison of modern weights without the bother of added computational work. This can be achieved if good, stable artifact standards can be obtained in which the density is the same as that of the ideal material. A further treatment of the comparison between "8.4" and "8.0" material is presented in section 2.5 and appendix 6.

While the practice of treating displacement volumes implicitly is rather complex, it is possible, as has been demonstrated over a number of years, to obtain a high degree of consistency. The ability to demonstrate with ease the consistency of assigned values to well designed weights which have been carefully calibrated with proper treatment of displaced volume is undoubtedly an important factor in the development of modern, precise, direct indicating weighing equipment. It has, however, created a situation in which different numbers are assigned to represent the same property:

$X=$ the mass of the object, or
$X^*=$ the apparent mass versus brass of an object, or
$X^{**}=$ the apparent mass of the object versus density 8.0 g/cm^3 material.

The differences between these numbers can be significant in certain situations.

2.5. Conversion, One Basis to Another

In many practical weighings the small inconsistencies between mass values assigned on different bases are of academic interest only. In precise weighing, however, it may be unexpected to find that a mass value assigned to a particular weight can be verified on one direct reading instrument, but on another instrument of the same type, the assigned value appears to be in error. Such discrepancies are typical when weights with mass values assigned on different bases are intercompared. This section presents some of the methods for converting values on one basis to equivalent values on another basis.

A. Given the mass X of weight $\mathscr{X}$, find the defined apparent mass versus brass value, X^*, and the defined apparent mass versus density 8.0 g/cm^3 material, X^{**}. The density of the $\mathscr{X}$ weight at 20°C, $d_x{}^0$, must be known.

$$X^*=\frac{X\left(1-\dfrac{0.0012}{d_x^0}\right)}{\left(1-\dfrac{0.0012}{8.3909}\right)} \tag{12}$$

$$X^{**}=\frac{X\left(1-\dfrac{0.0012}{d_x^0}\right)}{\left(1-\dfrac{0.0012}{8.0}\right)} \tag{13}$$

B. Given the apparent mass value, X^* or X^{**}, find the mass value of the $\mathscr{X}=$ weight. The density of the $\mathscr{X}$ weight, d_x^0, must be known.

$$X=\frac{X^*\left(1-\dfrac{0.0012}{8.3909}\right)}{\left(1-\dfrac{0.0012}{d_x^0}\right)}=\frac{X^{**}\left(1-\dfrac{0.0012}{8.0}\right)}{\left(1-\dfrac{0.0012}{d_x^0}\right)} \tag{14}$$

C. Given the apparent mass value X^* find X^{**}, or given apparent mass value X^{**} find X^*

$$X^{**}=\frac{X^*\left(1-\dfrac{0.0012}{8.3909}\right)}{\left(1-\dfrac{0.0012}{8.0}\right)}\approx X^*(1+0.000\,007) \tag{15}$$

$$X^*=\frac{X^{**}\left(1-\dfrac{0.0012}{8.0}\right)}{\left(1-\dfrac{0.0012}{8.3909}\right)}\approx X^{**}(1-0.000\,007) \tag{16}$$

The above formulas apply to summations of weights as well as to single weights provided the material density is the same for all weights in the summation. (Other forms of these formulas appear in the literature [83,84].) Where weights of different material densities are included in a summation, for exactness a separate computation must be made for each material density. It is prudent however to make some preliminary computations to verify that the effort is worthwhile. Gross material density changes usually occur with the smaller weights, in which case, the difference between the values computed by the various methods may not be detectable in the measurement process.

D. The observation on a direct reading weighing instrument is an estimate of the apparent mass of the object under test. The success of these devices depends in part on careful material control of the built-in weight materials, and a careful adjustment of the built-in weights. In normal operation, the weighing can be accomplished in a minimum time. In simplified form, the comparison can be shown as:

$$\left(\sum_{i=1}^{n} \mathscr{A}_j - \rho V_{\Sigma a}\right) + \left(\sum_{j=0}^{m=0} \mathscr{B} - \rho V_{\Sigma b}\right) \rightarrow O_1 \equiv 0 \quad (17)$$

$$\left(\sum_{i=1}^{n} \mathscr{A} - \rho V_{\Sigma a}\right) - \left(\sum_{j=1}^{m} \mathscr{B} - \rho V_{\Sigma b}\right) + (\mathscr{X} - \rho V_{\mathscr{X}}) \rightarrow O_2 \quad (18)$$

so that, subtracting (17) from (18)

$$(\mathscr{X} - \rho V_{\mathscr{X}}) = K(O_2) + \left(\sum_{j=1}^{m} \mathscr{B}_j - \rho V_{\Sigma b}\right) \quad (19)$$

Initially $m = 0$ and $\Sigma\mathscr{A}$ includes everything which is suspended from the beam and internal to the instrument. Dialing off the weights constructs $\Sigma\mathscr{B}$ which cannot differ from $\Sigma\mathscr{X}$ in excess of the on-scale range of the instrument. The unknown, X, is compared with $\Sigma\mathscr{B}$, the dial reading, R^*, being the nominal summation of nominal values assigned to the weights on some specific basis and the observed difference being a subdivision of some small weight internal to the balance. Under the defining conditions of the basis for the values, X can be computed as follows, provided the density of the $\mathscr{X}$ weight, $d_x{}^0$, is known at the defining conditions.

$$X = \frac{[R^* + K(O_2)]\left(1 - \frac{0.0012}{8.3909}\right)}{\left(1 - \frac{0.0012}{d_x^0}\right)} + \epsilon \quad (20)$$

or

$$X = \frac{[R^* + K(O_2)]\left(1 - \frac{0.0012}{8.0}\right)}{\left(1 - \frac{0.0012}{d_x^0}\right)} + \epsilon \quad (21)$$

where ϵ represents the sum of the deviations from nominal values of the weights in $\Sigma\mathscr{B}$.

If the density, $d_x{}^0$, is very nearly the same as the internal weights and the ideal material density on which the values are based, the following can be assumed in many cases with negligible error:

$$X^* = R^* + K(O_2) + \epsilon \quad (22)$$

or

$$X^{**} = R^{**} + K(O_2) + \epsilon \quad (23)$$

It is possible to use more sophisticated procedures to establish the mass of an unknown relative to the balance indications, however these procedures are tedious and time consuming unless the balance manufacturer can furnish a table of mass equivalents for each dial position. In some instances it may be expedient to use the instrument as a comparator, comparing the unknown with weights for which all the appropriate parameters are known.

Other forms of these relations appear in the literature [41,42], depending upon how the problem has been formulated and the number of terms retained in series expansion. Many of these relations have been established for specific purposes, thus a careful evaluation would be in order before using in other circumstances. In all cases, one must first evaluate the task at hand to verify that a consideration beyond the implicit treatment of displacement volumes is really justified. If, in fact, explicit treatment of displacement volumes is required, one must start with a set of weights with assigned values, the uncertainty of the values being comparable with the precision of the process. One must know the characteristics of the material to be weighed and the characteristics of the measurement process. One must compute the air density for each weighing. If any step is omitted, confidence in the end result is illusory.

2.6. The Mass Measurement System and the NBS Mass Measurement Service

In order for the mass measurement system to function effectively, one must be able to make consistent mass measurements which are compatible with a variety of requirements [85]. One service of the Bureau of Standards provides in part a means by which all mass measurements within the system can be related. This service emphasizes the characterization of the object to be used as a mass standard, e.g., stability, volume, coefficient of expansion, etc. and the characterization of the measurement process used to relate the value assigned to an unknown to the accepted value of a standard.

The NBS report is in most cases a comprehensive report which includes a complete description of the measurement process and methods, all of the data, and the analysis of this data including that concerning the monitoring of the state of control of the NBS measurement process. The mass value, the uncertainty of the value, and the displacement volume at 20 °C is reported. The uncertainty figure is an expression of the overall uncertainty using three standard deviations as a limit to the effect of random errors of measurement associated with the NBS measurement processes. The magnitude of systematic errors from sources other than the use of accepted values for certain starting standards are considered negligible on the basis of separate studies of the NBS process. The errors associated with the use of certain NBS standards other than at the 1 kg level are included in the uncertainty statement. It should be noted that the magnitude of the uncertainty reflects the performance of an NBS measurement process. The mass unit, as realized in another measurement process, will be uncertain by an amount which is a combination of the uncertainties of the two processes, the NBS process and the process in which the calibrated standards are used.

In addition, corrections to nominal values are reported in which the displacement volume has been treated implicitly. Corrections are furnished on the basis of "apparent mass versus brass," as previously defined, and on the basis of "apparent mass" versus a material of density 8.0 g/cm^3 in air of density 1.2 mg/cm^3 at 20 °C. An example of the use of the "apparent mass versus brass" corrections in the adjustment of weights of different materials is presented in appendix 7.

The uncertainties associated with the assigned values are descriptive of the NBS mass measurement process and in no way reflect the effectiveness of the transfer of the value from NBS to another facility, or the output of another facility. A complete assessment of the uncertainty at any point in the system requires a characterization of all of the measurement processes involved.

On the other hand, the requirements within the mass measurement system are primarily functional, that is, the variability associated with mass measurement is only one of many sources of variability which must be controlled to accomplish a particular mission. In many cases, the variability of the mass measurement is by far the smallest in magnitude. In these cases, the number produced by the measurement process is considered to be "exact" as long as the area of doubt associated with that number is small with respect to the magnitude of the effects from other sources of variability. If an acceptable area of doubt is still large relative to the ability to measure, simplified practices should be used. For example, the details of the "basis for the value" and corrections for the buoyant effect of the air have no meaning relative to bulk weighing of highway materials or weighings on a pricing scale in which an acceptable variability is 2 cents on 5 dollars at unit prices of $0.26 per pound through $1.25 per pound [86].

Relating to a functional requirement does not negate the need for measurement process characterization. Appropriate characterization of any measurement process is fundamental to verifying that the results are produced in the most economical way and are consistent with the end use requirements. In addition, the specific characteristics of a particular process determine the need for further interest in the details of measurement. In the absence of a study of the characteristics of a particular process relative to its functional requirement, one can frequently become involved in details which are of no real concern.

It is important to note, in conclusion, that the preciseness of any defined basis for stating mass values cannot be realized beyond the performance characteristics of the measurement process in which the mass standards are being utilized. The use of National Bureau of Standards reports of assignment of mass values to document the validity of the output of an uncharacterized measurement process is illusory.

A paper such as this is largely a review of material from many sources and conversations with many people. Devoting a large portion of the paper to history is an attempt to provide a background which will be helpful in the interpretation of a rather complex existing situation. I want to thank those who contributed their thoughts and ideas. In particular I want to thank Dr. J. A. Simpson and Mr. J. M. Cameron for their many constructive comments, Dr. C. Eisenhart for his help in assembling the reference material, and the staff of the Mass, Length and Volume Section including Mr. H. E. Almer and Mr. Lloyd Macurdy, both now retired, who reviewed the many drafts. A portion of a paper by Mr. H. A. Bowman and Mr. R. M. Schoonover, Reference 90, is reprinted in part as Appendix 2.

References

[1] Hogben, Lancelot, F.R.S., Mathematics for the Million, pp. 51-52 (Pocket Books, Inc., New York, 1965).

[2] Hawkes, Jacquetta, Prehistory, Vol. I, Part 1, History of Mankind, Cultural and Scientific Development, p. 304 (New American Library (Mentor Books), New York, 1965).

[3] Maspero, Gaston, The Dawn of Civilization, Egypt and Chaldaea, pp. 283-284 (MacMillan, New York, 1922).

[4] Woolley, Sir Leonard, The Beginnings of Civilization, Vol. I, Part 2, History of Mankind, Cultural and Scientific Development, p. 277 (New American Library (Mentor Books), New York).

[5] Berriman, A. E., Historical Metrology, pp. 66-67 (E. P. Dutton, New York).

[6] Berriman, A. E., see [5] p. 55.

[7] Youden, W. J., Experimentation and Measurement, Preface, (Scholastic Book Services, New York, 1962).

[8] Stevens, S. S., Handbook of Experimental Psychology, pp. 9-14 (John Wiley & Sons, New York, 1951).

[9] Carnap, Rudolf, Extensive Magnitudes, Ch. 7, Philosophical Foundations of Physics, pp. 70-77, (Basic Books, New York).

[10] Childe, Gordon, V., The Prehistory of Science: Archaeological Documents, Part 1, The Evolution of Science, Readings from the History of Mankind, edited by Metraux, Guy S. and Crouzet, Francois, pp. 72-75 (New American Library (Mentor Books), New York, 1963).

[11] Childe, Gordon, V., see [10] pp. 66-71.

[12] Gandz, Solomon, and Neugebauer, O. E., Ancient Science, Vol. I, Toward Modern Science, edited by Palter, Robert M., pp. 8-9, 20-21 (Farrar, Straus & Cudahy (Noonday Press), New York, 1961).

[13] Neugebauer, O., Exact Sciences in Antiquity, pp. 18-19, (Harper & Brothers (Harper Torchbooks), New York, 1962).

[14] Woolley, Sir Leonard, see [4] pp. 362-364.

[15] Breasted, James Henry, The Dawn of Conscience, pp. 188-189, (Charles Scribner's Sons, New York).

[16] Chadwick, John, Life in Mycenaean Greece, Scientific American, 227, No. 4, pp. 37-44 (Oct. 1972).

[17] Durant, Will, Our Oriental Heritage, Part 1, The Story of Civilization, p. 79 (Simon and Schuster, New York, 1954).

[18] Maspero, Gaston, see [3] pp. 772-774.

[19] Woolley, Sir Leonard, see [4] pp. 748-751.

[20] Woolley, Sir Leonard, see [4] pp. 330-333.

[21] Woolley, Sir Leonard, see [4] pp. 340-341.

[22] Berriman, A. A., see [5] pp. 58-59.

[23] Maspero, Gaston, see [3] pp. 323-326.

[24] Forbes, R. J., Metals and Early Science, see [12] pp. 30-31.

[25] Durant, Will, see [17] p. 289.

[26] Childe, V. Gordon, What Happened in History, pp. 192-193 (Pelican Books, 1942).

[27] Maspero, Gaston, see [3] pp. 752-753.

[28] Kisch, Bruno, Scales and Weights, p. 11 (Yale University Press, New Haven, 1965).

[29] Durant, Will, Caesar and Christ, Part III, The Story of Civilization, pp. 78-79 (Simon and Schuster, New York).

[30] Pareti, Luigi, The Ancient World, Vol. II, History of Mankind, p. 138 (Harper & Row, New York, 1965).

[31] Miller, W. H., On the Construction of the New Imperial Standard Pound, and its Copies of Platinum; and on the Comparison of the Imperial Standard Pound with the Kilogramme des Archives, Phil. Trans. Roy. Soc. London, 146, Part III, p. 753 (1856).

[32] Moreau, Henry, The Genesis of the Metric System and the Work of the International Bureau of Weights and Measures, J. Chem. Educ., 30, p. 3 (Jan. 1953).

[33] Miller, W. H., see [31] pp. 943-945.

[34] Astin, A. V., Refinement of Values for the Yard and the Pound (Federal Register, Washington, D.C., July 1, 1959).

[35] Miller, W. H., see [31] p. 875.

[36] Miller, W. H., see [31] p. 862.

[37] Page, Chester W. and Vigoureux, Paul, Editors, The International System of Units (SI), Nat. Bur. Stand. (U.S.), Spec. Publ. 330, 42 pages (Jan. 1971).

[38] Winter, H. J. J., Muslim Mechanics and Mechanical Appliances, Endeavor, XV, 57, pp. 25-28 (1956).

[39] Sisco, Anneliese G. and Smith, Cyril S., Lazarus Ercker's Treatise on Ores and Assaying, pp. 89-91, 210 (University of Chicago Press, Chicago, 1951).

[40] Tittmann, O. H., The National Prototypes of the Standard Metre and Kilogramme, Appendix No. 18-Report for 1890, pp. 736-737, Coast and Geodetic Survey (U.S.) (1891).

[41] Tittmann, O. H., see [40] pp. 738-739.

[42] Tittmann, O. H., see [40] p. 739.

[43] Miller, W. H., see [31] pp. 753, 945.

[44] Fischer, Louis A., History of the Standard Weights and Measures of the United States, Bur. Stand. (U.S.), Misc. Publ. No. 64, pp. 5, 7 (1925).

[45] Fischer, Louis A., see [44] pp. 10, 12.

[46] Smith, Ralph W., The Federal Basis for Weights and Measures, Nat. Bur. Stand. (U.S), Circ. 593, pp. 12-13 (1958).

[47] Huntington, E. V., Agreed Upon Units in Mechanics, Bulletin of the Society for the Promotion of Engineering Education, 11, No. 4, p. 171 (1920).

[48] Huntington, E. V., Bibliographical Note on the Use of the Word Mass in Current Textbooks, The American Mathematical Monthly, 25, No. 1 (Jan., 1918).

[49] Tate, D. R., Gravity Measurements and the Standards Laboratory, Nat. Bur. Stand. (U.S.), Tech. Note 491, 10 pages (1969).

[50] Webster, Noah, A Compendious Dictionary of the English Language, A facsimile of the first (1806) edition (Crown Publishers, Inc. (Bounty Books), New York, 1970).

[51] Marcuccio, Phyllis, Editor, Science and Children, private communication to author Dec. 1970.

[52] Wright, Louis B., The Cultural Life of the American Colonies 1607-1763, pp. 1–3 (Harper & Row (Harper Torchbooks), New York, 1962).

[53] Usher, Abbott P., A History of Mechanical Inventions, pp. 56-59 (Beacon Press (Beacon Paperback), Boston, 1959).
[54] Hawkes, Jacquetta, see [2] pp. 431-435.
[55] Childe, V. Gordon, Early Forms of Society, A History of Technology, edited by Charles Singer, E. J. Holmyard and A. R. Hall, Vol. I, Part I, p. 44 (Oxford University Press, New York, 1954).
[56] Usher, Abbott P., see [53], pp. 50-51.
[57] Childe, V. Gordon, see [55] p. 45.
[58] Scott, Sir Lindsay, Pottery, see [55], Part IV, pp. 408-409.
[59] Contenau, Georges, Mediterranean Antiquity, Ch. 6, A History of Technology & Invention, edited by Maurice Daumas, Vol. I, Part II, p. 116 (Crown Publishers, New York, 1969).
[60] Agricola, Georgius, De Re Metallica, translated from the first Latin Edition of 1556 by Herbert Clark Hoover and Lou Henry Hoover, pp. 252-260 (Dover Publications, New York, 1950).
[61] Duval, Paul-Marie, The Roman Contribution to Technology, Ch. 9, see [59], pp. 256-257.
[62] Encyclopaedia Britannica, Vol. 10, Game to Gun Metal, Glass in Rome, p. 410 (William Benton, Chicago, 1959).
[63] Gille, Bertrand, The Medieval Age of the West (Fifth Century to 1350), Part 6, Ch. 21, see [59], pp. 562, 564, 566.
[64] Durant, Will, The Renaissance, The Story of Civilization, Part V, p. 313 (Simon and Schuster, New York, 1953).
[65] Ubbelohde, A. R. J. P., Change from Craft Mystery to Science, A History of Technology, Volume IV, see [55], pp. 663-669.
[66] Encyclopaedia Britannica, Vol. 12, Hydrozoa to Jeremy, Epistle of, Industrial Revolution, pp. 307-310 (William Benton, Chicago).
[67] Mumford, Lewis, The Paleotechnic Phase, Ch. IV, Technics and Civilization, pp. 151-159 (Harcourt, Brace and Co., New York, 1946).
[68] Mumford, Lewis, Agents of Mechanization, see [67], p. 92.
[69] Usher, Abbott P., see [53], pp. 378-380.
[70] Technological Innovation: Its Environment and Management, Dept. Commerce (U.S.), Panel Report, p. 7 (Jan. 1967).
[71] Astin, Allen V., Standards of Measurement, Scientific American, 218, No 6, p. 5, (June 1968).
[72] Huntoon, R. D., Status of the National Standards for Physical Measurement, Science, 50, p. 169, (Oct. 1965).
[73] Eisenhart, Churchill, Realistic Evaluation of the Precision and Accuracy of Instrument Calibration Systems, J. Res. Nat. Bur. Stand. (U.S.), 67C (Eng. and Instr.), No. 2, 161-187 (Apr.-June 1962).
[74] Pontius, P. E. and Cameron, J. M., Realistic Uncertainties and the Mass Measurement Process, Nat. Bur. Stand. (U.S.), Monogr. 103, 17 pages (1967).
[75] Pontius, P. E., Measurement Philosophy of the Pilot Program for Mass Calibration, Nat. Bur. Stand. (U.S.), Tech. Note 288, 39 pages (1968).
[76] Cajori, Florian, Translation and explanatory appendix, Sir Isaac Newton's Mathematical Principles of Natural Philosophy and His System of the World, pp. 38-639 (University of California Press, Berkeley, 1947).
[77] Rutherford, D. E., Classical Mechanics, Ch. 2, (Interscience Publishers, Inc., New York, 1951).
[78] Macurdy, L. B., Response of Highly Precise Balances to Thermal Gradients, J. Res. Nat. Bur. Stand. (U.S.), 67C, No. 3, 135-138 (July-Sept. 1964).
[79] Campbell, Norman R., An Account of the Principles of Measurement and Calculation (Longmans, Green and Co. Ltd., New York 1928).
[80] Smirnova, N. A. and Moskvin, N. E., Single Nominal Density for Weights, Izmeritel'naya Tekhnika, No. 2, pp. 25-28 (USSR), (Feb. 1963).
[81] Bigg, P. H., Weight-in-Air Basis of Adjustment of Precision Weights, J. Sci. Instr., 36, 359-361 (Aug. 1959).
[82] Cross, J. L., Reduction of Data for Piston Gage Pressure Measurements, Nat. Bur. Stand. (U.S.), Monogr. 65, p. 3, 9 pages (1963).
[83] Macurdy, L. B., Measurement of Mass, Ch. 72, Treatise on Analytical Chemistry, Vol. 7, Part I, pp. 4247-4277 (John Wiley & Sons, Inc., New York, 1967) Mettler Reprint No. 11 (Mettler Instrument Corp., Hyattstown, N.J.).
[84] Benedetti-Pichler, A. A., Essentials of Quantitative Analysis (Ronald Press, New York, 1956).
[85] Astin, A. V., Some Problems of Improving Accuracy of Measurement, IRE International Convention Record, Part 9, 1962.
[86] Specifications, Tolerances and Other Technical Requirements for Commercial Weighing and Measuring Devices, Nat. Bur. Stand. (U.S.), Handb. 44, 4th Edition, p. 3 (1971).
[87] Macurdy, L. B. and Lashof, T. W., Precision Laboratory Standards of Mass and Laboratory Weights, Nat. Bur. Stand. (U.S.), Circ. 547 (superceding NBS Circ. 3) (1954). This circular is out of print but its contents have been incorporated in NBS Handbook 77, Precision Measurement and Calibration, Volume III, Optics, Metrology and Radiation, pp. 588-706. This handbook is available for reference or circulation in most Government Depository Libraries throughout the United States.
[88] Standard Methods of Testing Single-Arm Balances, ASTM E319-68, Book of ASTM Standards, Part 30 (American Society for Testing and Materials, Philadelphia, 1968).
[89] Stabler, Thomas M., Specifications and Tolerances for Field Standard Weights, Nat. Bur. Stand. (U.S.), Handb. 105-1, 23 pages (1972).
[90] Bowman, H. A. and Schoonover, R. M. with Appendix by Jones, M. W., Procedure for High Precision Density Determinations by Hydrostatic Weighing, J. Res. Nat. Bur. Stand. (U.S.), 71C (Eng. and Instr.), No. 3, pp. 179-198 (July-Aug. 1967).
[91] Weights of Precision Class $E_1E_2F_1F_2M_1$ from 50 kg to 1 mg. International Recommendation No. 20, International Organization of Legal Metrology, Bureau International de Métrologie Légale, Paris, France (1973).

Appendix 1. Average Air Densities for a Group of Selected Cities

Listed below are the approximate average Winter (January) and Summer (June-July) air densities for a group of selected cities throughout the continental United States. These densities are for an assumed temperature of 23 °C and are based on information about the barometric pressure and relative humidity supplied by the Weather Bureau.

It should be remembered that these are average air densities and that the actual air density at a given time and place may differ from that given by as much as 3 percent in either direction.

Place		Air density	
State	City	Winter	Summer
		(mg/cm^3)	(mg/cm^3)
Alabama	Birmingham	1.16	1.16
	Montgomery	1.18	1.18
Arizona	Flagstaff	0.92	0.93
	Phoenix	1.15	1.14
	Tucson	1.08	1.08
	Yuma	1.19	1.18
Arkansas	Little Rock	1.18	1.18
California	Los Angeles	1.17	1.16
	Oakland	1.19	1.16
	San Diego	1.18	1.18
	San Francisco	1.18	1.18
Colorado	Denver	0.98	0.98
	Grand Junction	1.00	1.00
	Pueblo	1.00	1.00
Connecticut	Hartford	1.18	1.18
District of Columbia.	Washington	1.19	1.18
Florida	Jacksonville	1.19	1.18
	Key West	1.19	1.18
	Miami	1.19	1.18
	Tampa	1.19	1.18
Georgia	Atlanta	1.15	1.14
	Augusta (Aiken, S.C.)	1.18	1.18
	Savannah	1.19	1.18
Idaho	Boise	1.07	1.07
Illinois	Cairo	1.18	1.17
	Chicago	1.16	1.16
	Moline	1.17	1.17
	Springfield	1.16	1.16
Indiana	Fort Wayne	1.15	1.15
	Indianapolis	1.15	1.15
Iowa	Burlington	1.16	1.15
	Des Moines	1.15	1.15
	Dubuque	1.16	1.15
	Sioux City	1.15	1.13
Kansas	Concordia	1.13	1.12
	Dodge City	1.08	1.08
	Topeka	1.15	1.15
	Wichita	1.14	1.13
Kentucky	Louisville	1.17	1.17
Louisiana	New Orleans	1.19	1.18
	Shreveport	1.18	1.18
Maine	Eastport	1.18	1.18
Massachusetts	Boston	1.18	1.18
Michigan	Alpena	1.16	1.16
	Detroit	1.16	1.15
	Grand Rapids	1.15	1.15
	Marquette	1.15	1.15
	Sault Ste. Marie	1.16	1.16
Minnesota	Duluth	1.14	1.14
	Minneapolis	1.15	1.15
	St. Paul (Airport)	1.15	1.15
Mississippi	Vicksburg	1.18	1.18
Missouri	Kansas City	1.15	1.15
	St. Louis	1.17	1.16
	Springfield	1.14	1.13
Montana	Havre	1.08	1.08
	Helena	1.02	1.02
	Kalispell	1.03	1.03

Place		Air density	
State	City	Winter	Summer
		(mg/cm^3)	(mg/cm^3)
Nebraska	Lincoln	1.14	1.14
	North Platte	1.07	1.07
Nevada	Ely	0.95	0.96
	Las Vegas	1.11	1.11
New Hampshire	Concord	1.17	1.17
New Jersey	Newark	1.19	1.19
New Mexico	Albuquerque	1.00	1.00
New York	Albany	1.18	1.18
	Buffalo	1.16	1.15
North Carolina	Hatteras	1.18	1.17
	Raleigh	1.18	1.17
North Dakota	Bismarck	1.11	1.11
	Devils Lake	1.12	1.12
	Fargo	1.15	1.15
	Williston	1.11	1.11
Ohio	Cincinnati	1.16	1.16
	Dayton	1.15	1.15
	Cleveland	1.15	1.15
Oklahoma	Oklahoma City	1.14	1.14
Oregon	Baker	1.04	1.04
	Medford	1.13	1.14
	Portland	1.18	1.19
	Roseburg	1.17	1.17
Pennsylvania	Philadelphia	1.18	1.18
Rhode Island	Providence	1.18	1.18
South Carolina	Charleston	1.19	1.18
	Columbia	1.18	1.17
South Dakota	Huron	1.14	1.13
	Rapid City	1.05	1.06
Tennessee	Knoxville (Oak Ridge).	1.16	1.15
	Memphis	1.18	1.17
	Nashville	1.17	1.17
Texas	Abilene	1.12	1.11
	Amarillo	1.04	1.04
	Austin	1.17	1.16
	Brownsville	1.19	1.18
	Fort Worth	1.16	1.16
	El Paso	1.04	1.04
	Houston	1.18	1.18
	San Antonio	1.17	1.17
Utah	Salt Lake City	1.02	1.02
Vermont	Burlington	1.17	1.17
Virginia	Norfolk	1.19	1.18
	Richmond	1.18	1.18
Washington	Seattle	1.18	1.17
	Spokane	1.19	1.18
	Walla Walla	1.15	1.13
West Virginia	Elkins	1.11	1.11
	Parkersburg	1.16	1.16
Wisconsin	Green Bay	1.16	1.16
	Madison	1.15	1.15
	Milwaukee	1.16	1.15
Wyoming	Casper	0.97	0.97
	Cheyenne	0.94	0.96
	Sheridan	1.03	1.04

Appendix 2. Density of Air[1]

Major changes in the density of air, ρ_A, occur due to variation in the temperature, pressure, and relative humidity, and each of the parameters should be carefully observed. The instrumentation used for monitoring these ambient parameters need not be state-of-the-art type apparatus because it is necessary only to assure that the standard deviation in ρ_A is held to a few micrograms per cubic centimeter. Apparatus should be chosen which is convenient to use and maintain.

A. *Temperature.* A mercury-in-glass thermometer divided into 1/10 °C intervals over the range of 20 to 30 °C would be adequate.

B. *Relative humidity* can be measured on an electric hygrometer of the type discussed by Wexler[2] with its sensing element placed in the weighing chamber of the balance. This instrument usually contains a built-in thermistor type thermometer which is used to monitor the temperature in the immediate vicinity of the relative humidity sensing element. This should not be used as a replacement for the mercury thermometer. Although it is quite adequate for the use intended by the manufacturer (and it should be used in connection with relative humidity measurements) it will introduce serious uncertainties if used as a replacement for the mercury instrument.

Sling and aspiration psychrometers are unsatisfactory in this application. Their required airflow would cause unacceptable disturbances inside a balance, and they lack the required sensitivity and speed of response. Wexler discusses electric hygrometers in detail and we believe that their convenience, speed of response and sensitivity are such that no other type instrument currently available should be considered. Wexler estimates their drift at about 2 percent per year, and feels that semi-annual calibration is adequate.

Calibration can be performed by the manufacturer, NBS, or the experimenter himself using various techniques leading to a calibration accuracy of about 1 1/2 percent using the methods of Wexler and his associates[3].

C. *Barometric pressure* is most easily observed on an aneroid barometer. Such an instrument should be checked against a mercury barometer. We recommend specifying an antiparallax mirror scale for instruments used in this work.

D. *The basis for air density calculations.* If we have a mass, m_G, of dry gas at pressure p and absolute temperature θ, whose molecular weight is M_G, then from the ideal gas law

$$pV = nR\theta = \left(\frac{m_G}{M_G}\right) R\theta$$

and

$$m_G = \frac{pVM_G}{R\theta}$$

and

$$\frac{M_G}{R\theta} = \frac{m_G}{V} \cdot \frac{1}{p} = \frac{\rho_G}{p} \qquad (2)$$

where n is the number of moles involved and ρ_G the density of the dry gas.

If we have a mass, m_w, of water vapor at pressure e and temperature θ occupying volume V, then a similar argument gives

$$m_W = \frac{eVM_W}{R\theta}$$

If the above masses of gas and vapor are mixed in volume V, the barometric pressure B, will be (by Dalton's Law of partial pressures) equal to $p+e$ and $p = B - e$. The density, ρ_A, of the mixture is

$$\rho_A = \frac{m_G + m_W}{V} = \frac{M_G}{R\theta}\left[B - e\left(1 - \frac{M_W}{M_G}\right)\right].$$

Using $M_G/R\theta$ from eq (2)

$$\rho_A = \frac{\rho_G}{P}\left[B - \left(e1 - \frac{M_W}{M_G}\right)\right].$$

If the density of dry gas, ρ_G, is measured to be ρ_0, at some reference temperature θ_0, and reference

[1] Adapted from "Procedure for High Precision Density Determinations by Hydrostatic Weighing" [90]. The nomenclature of the original article has been retained. ρ_A, the density of air, is identical to ρ as used in this paper.

[2] Wexler, A., NBS Circ. 586 (1957).

[3] Wexler, A., Brombacher, W. G., NBS Circ. 512 (1951).

pressure ρ_0, then its density at any temperature θ and pressure p is

$$\rho_G = \rho_0 \left(\frac{\theta_0}{\theta}\right)\left(\frac{p}{p_0}\right)$$

and using this value of ρ_G in the above equation for ρ_A gives

$$\rho_A = \left(\frac{\rho_0\theta_0}{\theta p_0}\right)\left[B - e\left(1 - \frac{M_W}{M_G}\right)\right].$$

On the basis of the natural scale of atomic weights $M_w = 18.0160$ and $M_G = 28.966$.[4] If $\rho_0 = 1.29304 \times 10^{-3}$ g/cm³ when $\theta_0 = 273.16$ °K and $p_0 = 760$ mmHg,[5] then

$$\rho_A = 0.46475 \left(\frac{B - (0.0037803)(e_s)(H)}{t_A + 273.16}\right)$$

where t_A is temperature in °C, e_s the tabulated saturation vapor pressure of water at t_A and H the relative humidity expressed as a percentage. This formula assumes moist air behaves in accordance with the ideal gas law, under which circumstances compressibility (defined as $pV/R\theta$) is unity. Goff and Gratch[6] have measured the compressibility of moist air and have found that over the range of temperature, pressure and relative humidity ordinarily existing in the laboratory it varies between 0.9995 and 0.9997, so the above formula provides values of ρ_A too high by this factor. We feel justified in multiplying its right member by an average value of 0.9996, so that

$$\rho_A = 0.46456 \left[\frac{B - (0.0037803)(e_s)(H)}{t_A + 273.16}\right]. \quad (3)$$

[4] Harrison, L, Pt. 1, pp. 15,16. Humidity and Moisture. 3. edited by Wexler, A. and Wildhack, W. (Reinhold, New York, 1965.)
[5] NBS Circular 564, table 2A, p. 25 (1955).
[6] Goff, J. A. and Gratch, S., Smithsonian Meterological Tables, 6th Edition, Smithsonian Institution (1958).

Equation (3) may be used by the experimenter for calculation of air density based upon observed values of temperature, barometric pressure, relative humidity and tabulated values of e_s, the vapor pressure of water at saturation. In this equation ρ_A is a very weak function of e_s, changing by only 0.24×10^{-6} g/cm³ for a 1 mmHg change in e_s. Therefore it is possible to eliminate the inconvenience of using tables by making a very crude approximation for e_s in terms of the observed temperature without causing unacceptably large errors in ρ_A. Most weighings are conducted between 20 and 30 °C, and within this range we may assume

$$e_s = 1.435t_A - 11.72$$

under which circumstances

$$\rho_A = \frac{0.464554B - H(0.00252t_A - 0.020582)}{t_A + 273.16} \quad (4)$$

where ρ_A is in milligrams per cubic centimeter with adequately small error.

On the rare occasions when it is necessary to perform weighings outside of this temperature range, some adjustment in the approximation is called for. Over the temperature of 15 to 50 °C, air density may be approximated by

$$\rho_A = \frac{464.56B - H(0.085594t_A^2 - 1.8504t_A + 34.47)}{t_A + 273.16} \quad (4A)$$

where ρ_A is in micrograms per cubic centimeter. Although these two approximations result in very large errors in e_s, errors in ρ_A associated with their use do not exceed one microgram per cubic centimeter which is quite adequate for weighing work.

Appendix 3. Typical Weight Materials, Densities, and Coefficients of Volumetric Expansion

Where specific statements of displacement volume or material density and coefficient of cubical expansion are missing, the appropriate data from the following table is assumed to apply. The data is approximate, having been taken from various handbooks, or having been obtained from the weight manufacturers. The use of this data provides a specific basis for determining mass values, presuming that the manufacturers exercise some form of material control. For the most precise work, it may be necessary to determine the volume or density of the weight material by other means.

Material	Density at 20 °C	Coefficient of cubical expansion
American Balance Co. Stainless Steel	7.92	0.000045
Brass (Normal) (or Bronze)	8.3909	.000054
Stainless Steel	7.8	.000045
Platinum	21.5	.000026
Tantalum	16.6	.000020
Aluminum	2.7	.000069
Nichrome V	8.5	.000039
Nichrome	8.39	.000039
"Brunton" Metal (Ainsworth Stainless Steel)	7.89	.000045
Ainsworth Stainless Steel	7.85	.000045
Naval Brass (Ainsworth)	8.4	.000054
(W. & L. E. Gurley Stainless Steel)	7.916	.000045
Fisher "Permas" Stainless Steel	7.8	.000045
Troemner Stainless Steel	7.84	.000045
Voland Stainless Steel	7.8	.000045
"Beckerloy" Stainless Steel Torsion Balance Co	8.0	.000045
Gold	18.0	.000043
Quartz (Crystal)	2.65	.000033
Mettler Instrument Corp. Stainless Steel	7.76	.000045
Check Weight 8 "State Weight 8" Stainless Steel	8.0	.000045

Appendix 4. Variability Associated with Volume Differences and Changes in Air Density

Equation (3) can be shown as:

$$K(O_1-O_2)=(\mathcal{S}-\mathcal{X})-\rho(V_{\mathcal{S}}-V_{\mathcal{X}})$$

$K(O_1-O_2)$ is a number which is computed from the indications of the weighing device as $\mathcal{S}$ and $\mathcal{X}$ are compared. Since the air density, ρ, is continuously variable, no unique number will be obtained. The air density, ρ, changes in a cyclic manner, therefore the average observed number, $K(O_1-O_2)$, stable and offset from $(\mathcal{S}-\mathcal{X})$ by the amount $\bar{\rho}(V_{\mathcal{S}}-V_{\mathcal{X}})$. It is of interest to know the magnitude of the variability associated with $\rho(V_{\mathcal{S}}-V_{\mathcal{X}})$ and the magnitude of the offset in order to determine the necessity for a detailed adjustment of the observed data. Normally it is not difficult to determine the appropriate air density, but it may not be possible to determine precise displacement volumes in all cases.

Every measurement process has an inherent variability. For example, a sequence of comparisons of $\mathcal{S}$ with itself would produce a sequence of numbers, N_k, as follows:

$$N_k=(\mathcal{S}-\mathcal{X})-\rho_j(V_{\mathcal{S}}-V_{\mathcal{X}})=\epsilon_i$$

With $\mathcal{S}=\mathcal{X}$ and $V_{\mathcal{S}}=V_{\mathcal{S}}$, the variability of the observed difference is identical to the variability of the measurement process, which is represented by the random variable, ϵ_i, from a probability distribution with mean zero and standard deviation, σ_0.

Considering the air density to act like a random variable, such that:

$$\rho_j=\bar{\rho}+\eta_i$$

where ρ is some average air density and the η_i have standard deviation σ_1, then, in the case where $V_{\mathcal{S}}\neq V_{\mathcal{X}}$:

$$\text{obs diff}_i=\mathcal{S}-\mathcal{X}-\bar{\rho}(V_{\mathcal{S}}-V_{\mathcal{X}})+\epsilon_i-\eta_i(V_{\mathcal{S}}-V_{\mathcal{X}})$$

Thus the variability of the observed difference is a combination of two random variables.

One does not normally have long sequences[1] of comparisons to establish estimates of both σ_1 and σ_0. One can, however, compute the appropriate air density for each comparison, and test for correlation between the observed difference and the air density. If the air density determination has been done carefully, correlation indicates a significant volume difference which in turn implies that one or both of the assigned displacement volumes are in error. The absence of correlation does not mean that the volumes have been assigned correctly but that the variability from this source is much smaller than other variabilities which affect the process.

In a given location, with a 1 kg process of precision $s=0.050$ mg, and a normal air density variation over a range of 0.05 mg/cm^3, one would be hard pressed to detect a volume difference error of 1 cm^3. On the other hand, if comparisons were made at different locations, subject to a range of air densities of 0.25 mg/cm^3, correlation would be evident with a much smaller number of intercomparisons.

The significance of the magnitude of offset, $\bar{\rho}(V_{\mathcal{S}}-V_{\mathcal{X}})$, is also established relative to the process precision. For the above example, with $\bar{\rho}=1.2$ mg/cm^3 and $(V_{\mathcal{S}}-V_{\mathcal{X}})=1$ cm^3, the offset is significant with respect to the process precision. An adjustment of the observed data is required, particularly if the weights are to be used in other locations.

The same logic applies when volume differences are so large that data adjustment must be made. In this case

$$N_i+\rho_j(V_{\mathcal{S}}-V_{\mathcal{X}})=(\mathcal{S}-\mathcal{X})+\epsilon_i-\Delta\rho\Delta V$$

where $\Delta\rho$ is the difference between the actual air density and the computed air density ρ_j. ΔV is, in like manner, the difference between the actual $(V_{\mathcal{S}}-V_{\mathcal{X}})$ and the number assigned to be $(V_{\mathcal{S}}-V_{\mathcal{X}})$. If, after corrective action has been taken, a correlation is still evident, further refinements are necessary. It still remains, however, that one can only verify that the magnitude of the variability from this source has been reduced to some fraction of the residual variability from other sources.

[1] Long sequences are necessary to detect small changes in the standard deviation. For example, to be reasonably sure (probability of 0.95) of detecting a difference of 20 percent in the ratio σ_1/σ_2 with the usual statistical test (0.05 level of the F test) one would need 165 measurements in each group; for a 10 percent difference, 60; and for a 50 percent change, 35 measurements.

Appendix 5. Displacement Volume and the Assignment of Mass Values

The implicit treatment of displacement volumes in essence lumps the gravitational effect and the buoyant effect of the air to produce an effective mass value. The explicit treatment separates the two effects, that is, the stated mass value must always be used with the appropriate displacement volume. Two quantities are involved, the density of the air and the volume, or material density, of the weight under study. The air density, determined by measured parameters and formula, can be sufficiently accurate for most applications. The displacement volume, however, must frequently be determined from approximate data or rough estimates of material density. It is of interest to know how this condition affects the assignment of mass values. The following hypothetical situation will illustrate the problem.

A well characterized weight, $\mathscr{A}$, of known density, $d_{\mathscr{A}}$, and with assigned value, A, is used to establish a value for weight $\mathscr{B}$. It is assumed that weight $\mathscr{B}$ has a density of $d_{\mathscr{B}}$. Having assigned a value B relative to A, weight $\mathscr{B}$ is, in turn, used to establish a value, C, for weight $\mathscr{C}$, which is also a well characterized weight with density $d_{\mathscr{C}}$. After some time, it is discovered that the assumed density, $d_{\mathscr{B}}$, is in error. New evidence establishes a more appropriate value for $d_{\mathscr{B}}$. One must now decide whether the current value, C, is still appropriate or whether the sequence of measurements must be repeated to establish a new C.

Assuming $\mathscr{A}$, $\mathscr{B}$ and $\mathscr{C}$ to be nominally 1 kg weights with $d_{\mathscr{A}}$ known to be 7.8 g/cm^3 and $d_{\mathscr{B}}$ assumed to be 8.4 g/cm^3, the first comparison ($\mathscr{A}-\mathscr{B}$) might indicate an apparent difference of 5 mg. The value B would be computed as follows:

$$\left(A-\rho_1\frac{A}{7.8}\right)-\left(B-\rho_1\frac{B}{8.4}\right)=5\text{mg}$$

or

$$B_{8.4}=A-5\text{mg}-\rho_1\left(\frac{A}{7.8}-\frac{B}{8.4}\right)$$

The second comparison ($\mathscr{B}-\mathscr{C}$), in air of density ρ_2, might indicate an apparent difference of -7 mg. The value C would be computed as follows:

$$C=B_{8.4}+7\text{mg}-\rho_2\left(\frac{B}{8.4}-\frac{C}{d_{\mathscr{C}}}\right)$$

At this point it is discovered that $d_{\mathscr{B}}$ should be 8.0 g/cm^3 rather than 8.4 g/cm^3. Recomputation gives a new value, B:

$$B_{8.0}=A-5\text{mg}-\rho_1\left(\frac{A}{7.8}-\frac{B}{8.0}\right)$$

from which:

$$B_{8.4}-B_{8.0}=\rho_1\left[\frac{B}{8.4}-\frac{B}{8.0}\right]$$

For $B\approx 1$ kg and $\rho_1=1.2$ mg/cm^3

$$\begin{aligned}B_{8.4}-B_{8.0}&\approx(1.2)(119-125)\text{mg}\\&\approx-7.2\text{ mg}\end{aligned}$$

This is indeed a significant change in value assignment and perhaps would be cause for great concern. However, a mass value is never used alone but always in conjunction with a specific displacement volume, or density. A recomputation of C, based on the new mass value and density for $\mathscr{C}$, gives:

$$C_{\text{new}}=\left[A-5\text{mg}-\rho_1\left(\frac{A}{7.8}-\frac{B}{8.0}\right)+7\text{mg}\right.$$
$$\left.-\rho_2\left(\frac{B}{8.0}-\frac{C}{d_{\mathscr{C}}}\right)\right]$$

so that

$$C_{\text{new}}-C_{\text{old}}=\rho_1\left(\frac{B}{8.0}-\frac{B}{8.4}\right)-\rho_2\left(\frac{B}{8.0}-\frac{B}{8.4}\right)$$

$$=(\rho_1-\rho_2)\left(\frac{B}{8.0}-\frac{B}{8.4}\right)$$

If $\rho_1=\rho_2$, the value assignment, C, is not affected by the change in value B. If $(\rho_1-\rho_2)$ is sufficiently small, $(C_{old}-C_{new})$ will also be small, in which case it would be immaterial as to which value is retained. If all terms are large, and the measurements sufficiently documented, a new value for C could be computed from the existing measurement data. If, however, sufficient documentation has not been established, the feature of being able to update by computations based on current data is lost and the whole procedure must be repeated.

Appendix 6. On the Selection of an "Ideal" Weight Material

Selecting an ideal weight material is of interest where the implicit treatment of displacement volume method is used to determine mass values. A judicious choice of "ideal" material, together with restrictions on the materials to be used in the construction of weights, results in a system whereby the consistency of the numbers assigned can be verified by routine procedures. The relation for the discrepancy, D, between the observed difference between two weights and the number assignment difference as developed in Part 2, Section 2.4, can be written as:

$$D=\rho_0\left[A\left(\frac{1}{d_a}-\frac{1}{d^*}\right)-B\left(\frac{1}{d_b}-\frac{1}{d^*}\right)\right]-\rho_1\left[\frac{A}{d_a}-\frac{B}{d_b}\right]$$

Examining the above relation, $D=0$ if $A=B$ and if all of the weights are made from the same material, $d_a=d_b=d^*$. These conditions were close to reality in the past when most weights were made from brass or bronze. It may be the case in the future when most weights will be made from stainless steel.

The largest term in the relation is

$$\rho_1\left(\frac{A}{d_a}-\frac{B}{d_b}\right)=\rho_1(V_A-V_B)$$

The magnitude of this term is a function of the local air density, ρ_1, which cannot be specified. In this country ρ varies from approximately 1.2 mg/cm^3 in the coastal regions to approximately 0.9 mg/cm^3 in the mountain regions. Since it is not hard to adjust so that $A \approx B$, clearly the biggest gain in reducing the discrepancy is in the selection of materials so that $d_a \approx d_b$.

The magnitude of the first term is determined by the selection of a "standard" air density, ρ_0, and a material of density d^* for the "ideal" weight. The use of $\rho_0=0.0012$ g/cm^3 seems to be widely accepted. If d^* is selected to be about the average density of normal weight material, the range of magnitude is not changed but the sign is sometimes positive and sometimes negative—a situation suggesting a percentage improvement which may be largely illusory relative to the normal precision of measurement or the permitted deviation from nominal value allowed in the adjustment tolerance.[1]

[1] The limits for the systematic error $\rho_1(V_A-V_B)$ are established by (1) mass of A and B and (2) the range of densities of materials which are used in constructing A and B. This limit is not affected by the choice of "ideal" material. Selecting an "ideal" material at either extreme is like establishing a unidirectional tolerance (i.e., 1.000 − 0.000/+0.005) which expressed as a percentage with respect to the nominal value 1.000, is twice as large as the percentage error associated with a bidirectional tolerance (i.e., 1.000+0.0025/-0.0025). The choice is academic when the acceptable tolerance limits are large (i.e., 1.00+0.10/-0.10) or when the process precision is large (i.e., $\sigma=0.05>0.005>0.0025$).

Appendix 7. Examples Illustrating the Use of Apparent Mass Values

There is a generally widespread belief that the use of apparent mass values eliminates the need for detailed attention to the buoyant effect of the air. In certain circumstances, this belief is well founded; however, it may not be true in all cases. In the examples that follow, the conditions have been chosen to show that a confidence based on such a belief may be illusory. If apparent mass values are to be used in a particular situation, it is important to establish the magnitude of the discrepancy introduced by so doing, and to verify that such usage will not affect the desired result. The methods used in these sample problems can be used for such an evaluation. The examples are applied to all types of weighing devices, the difference from one to another being only in the determination of the term $K(O_1 - O_2)$.

1. Test for, or adjust to obtain, compliance with specified weight adjustment tolerances.

There are established tolerances, or limits, for the permitted deviation from nominal value for various kinds of weights.[1] There are no universally accepted rules for judging a weight to be out of tolerance. This does not cause difficulty when the tolerance limits are large and when the weight is adjusted to be very close to a nominal value. The situation can be quite confusing when the tolerance limits are small or when the values of the weights are close to these limits. Two factors are involved in the judgment to accept or reject: the random error of the measurement process, that is, the standard deviation; and the offset or systematic error introduced by failure to consider certain variables which affect the process output.

Rules for interpretation with respect to random error must be agreed upon by the parties concerned. These rules should reflect an assessment of the risk of accepting an unsatisfactory weight or rejecting a satisfactory weight, either with respect to a contractual requirement or an end use requirement. This aspect of the problem is not considered in the examples which follow.

The systematic error offsets the results of one measurement process from those obtained by other measurement processes. Failure to consider the factors contributing to systematic error can create a situation in which one facility, acting according to the mutually agreed upon rules mentioned above, would judge an item to be satisfactory, but a second facility would judge the same item to be unsatisfactory. The former would accept, but the latter would reject. The example presented is illustrative of this aspect of the problem.

Testing for compliance with adjustment tolerance can be conducted in different ways. The role of the balance, or weighing device, is to establish an observed mass difference between the standard and the test object under local environmental conditions. The observed difference is a number which varies as a function of the actual difference and the standard deviation of the process. The role of the known standard is to provide a point of departure for the test, or adjustment. The adjustment process is a continuation of the test. If, in the test, it is judged that the weight is "out of tolerance," material is added or removed by some appropriate procedure until, on retest, an "in tolerance" condition is obtained.

Both the test weight and the standard must be well characterized, that is, one must know the effective material density, and the coefficient of expansion. The characteristics of the local environment must also be known. The degree to which this information must be known depends upon the size of the tolerance band. Certainly one would want the uncertainty of the computed corrective terms to be some small fraction of the tolerance. In the limit, uncertainties in these terms smaller than about one standard deviation of the measurement process are not significant.

Given: A known platinum standard, density 21.5 g/cm³ at 20 °C, with the following assigned values:

Apparent Mass versus Brass	Mass	Coefficient of Cubical Expansion
1g−0.003 mg	1g−0.090 mg	0.000 026

[1] Adjustment tolerances for various types of weights were established in NBS Circular No. 3 [87]. Revised and expanded tables were given in NBS Circular 547 [87]. A specification for field standards, NBS Handbook 105-1 [89], was prepared in 1969. Current tables for adjustment tolerances are being prepared by Subcommittee E31 of the American Society for Testing Materials.

and a test weight of density 8.3909 g/cm^3 at 20 °C, coefficient of cubical expansion 0.000 054. Test, or adjust, the weight to comply with tolerance specification based on apparent mass versus brass values. (All current tolerance specifications for weights in the United States are based on apparent mass versus brass values.) The weighings are to be made in an environment of 1.02 mg/cm^3 density at a temperature of 23 °C.

Since the test material is the same as the material used to define apparent mass, from eq (9) in section 2.4, the mass value and the apparent mass value will be identical. The local observed difference between the platinum standard and a hypothetical test object of value exactly one, with characteristics the same as the real test object, can be computed from eq (8) of section 2.3:

$$S\left[1-\frac{\rho[1+\alpha_{\mathscr{S}}(t-t_0)]}{d_s^0}\right]-X\left[1-\frac{\rho[1+\alpha_{\mathscr{X}}(t-t_0)]}{d_x^0}\right]=K(O_1-O_2)$$

$$0.99991\left[1-\frac{0.00102[1+0.000\,026(23-20)]}{21.5}\right]-1\left[1-\frac{0.00102[1+0.000\,054(23-20)]}{8.3909}\right]=K(O_1-O_2)$$

or

$$K(O_1-O_2)\approx-0.000\,016\text{g}$$

This means that, in this specific weighing environment, the test weight, which appears to be heavier than the platinum standard by the amount 0.016 mg, would have a mass value, X, or an apparent mass value, X^*, of 1 gram. Having established this, there are several courses of action which could produce useful results. For example, the test weight should be heavier than the standard by 0.016 mg, so, with the standard on the balance, the balance could be set to read (1 g – 0.016 mg) or 0.999984 g. Under this condition, any similar test weight with a value, or adjusted to give a value, within a reading of 1 g plus or minus the tolerance limits would be acceptable.

In comparison, from the assigned apparent mass value of the platinum standard $S^*=1$ g – 0.003 mg, one might erroneously conclude that a test weight with value 0.003 heavier than the standard would be acceptable. The difference in result between the two approaches is 0.013 mg, an amount which is significant with a typical process precision of 0.002 mg. Weights with values near the tolerance limit would be accepted in one case and rejected in the other. However, if the process precision and the tolerance limits are large compared to the 0.016 mg systematic error, routinely applying a correction of this magnitude may not be significant.

If, in this example, the weight being tested, or adjusted, had been made from material of density 7.8 g/cm^3 at 20 °C, the coefficient of expansion being 0.000045, an additional calculation is required. The mass of test material which will satisfy the definition of apparent mass versus brass at the specified conditions can be computed from eq (14) of section 2.4:

$$X=\frac{X^*\left(1-\frac{0.0012}{8.3909}\right)}{\left(1-\frac{0.0012}{d_x^0}\right)}=\frac{X^*\left(1-\frac{0.0012}{8.3909}\right)}{\left(1-\frac{0.0012}{7.8}\right)}$$

This value must be substituted in eq (8) of section 2.3:

$$S\left[1-\frac{\rho[1+\alpha_{\mathscr{S}}(t-t_0)]}{d_s^0}\right]-\left[\frac{X^*\left(1-\frac{0.0012}{8.3909}\right)}{\left(1-\frac{0.0012}{7.8}\right)}\right]\left[1-\frac{\rho[1+\alpha_{\mathscr{X}}(t-t_0)]}{d_x^0}\right]=K(O_1-O_2)$$

$$0.999\,91\left[1-\frac{0.00102[1+0.000\,026(23-20)]}{21.5}\right]-\left[1\frac{\left(1-\frac{0.0012}{8.3909}\right)}{\left(1-\frac{0.0012}{7.8}\right)}\right]\left[1-\frac{0.00102[1+0.000\,045(23-20)]}{7.8}\right]=K(O_1-O_2)$$

or

$$K(O_1-O_2)=-.000\,017\text{ g}$$

Having determined $K(O_1-O_2)$, the interpretation is the same as in the previous example.

2. Establishing the mass of an object with a direct reading instrument.

As before, one is concerned with the order of magnitude of the systematic error introduced when the value as given by the balance is assumed to be the mass of the test object. If the magnitude of this systematic error is large with reference to the precision of the process, a detailed accounting may be necessary.

Given: A balance indication of 97.875465 g for a test object; find the mass of the test object. The weights internal to the balance are of material density 7.78 g/cm^3 at 20 °C. The density of the test object is 1.5 g/cm^3 at 20 °C. The weighing has been made in an air density of 1.15 mg/cm^3 at 20 °C.

Part of the success of the direct reading balance can be attributed to careful control of weight material and careful adjustment of the internal weights. The internal weights are adjusted on an apparent mass basis, mostly with reference to material of density 8.3909 at 20 °C, but sometimes with reference to material of density 8.0 g/cm^3 at 20 °C (see sec. 2.4). The internal weights are usually adjusted with respect to standards made from the same material, the standards having been assigned appropriate apparent mass values. Since the material of the standard and of the internal weight are the same, no systematic error, other than that associated with measurement and adjustment process, is introduced in the transfer of value. The mass equivalent of the indicated value can be computed directly from equation (14) of section 2.5, considering the total reading, that is, the dial settings plus the indicated scale reading, as S^*.

$$S=\frac{S^*\left(1-\frac{0.0012}{8.3909}\right)}{\left(1-\frac{0.0012}{7.78}\right)}$$

Equation (8) of section 2.3 can be used with the following reasoning. Referring to eqs (17) and (18) of section 2.5, if the test weight $\mathscr{X}$ in equation (18) is identical in all respects to $\Sigma\mathscr{B}$, the indication, O_2, would also be identically zero. In the problem under consideration, the $\mathscr{X}$ of eq (18), which is the dial readings plus the scale reading, since the scale reading is merely a subdivision of the smallest dial weight, becomes a well characterized standard S. Equation (8) can now be used with $K(O_1-O_2)=0$. Hence:

$$S\left[1-\frac{\rho[1+\alpha_{\mathscr{S}}(t-t_0)]}{d_S^0}\right]-X\left[1-\frac{\rho[1+\alpha_{\mathscr{X}}(t-t_0)]}{d_x^0}\right]=0$$

$$97.875465\left[\frac{1-\frac{0.0012}{8.3909}}{1-\frac{0.0012}{7.78}}\right]\left[1-\frac{0.00115}{7.78}\right]-X\left[1-\frac{0.00115}{1.5}\right]=0$$

so that $X=97.937182$ grams.

The systematic error introduced by using the number indicated by the balance as an estimate of the mass of the test object is:

$$(97.875465-97.937182)g=-0.061717g$$

This is a large error relative to the expected precision for a measurement process at the 100 g level. For example, such an error might be significant in the preparation of titrating solutions by gravimetric methods. The error, however, is a proportional one; thus in the dispensing of such a solution, as from a plastic "squeeze" bottle, the error amounts to only about 0.61 mg per gram of solution dispensed. Again, while the error may be large relative to the available measurement precision, it may not be significant relative to a particular end use requirement.

3. Checking the indication of a precise direct reading balance with a set of calibrated weights.

In general, this task cannot be done with a set of weights known only to comply with some specified adjustment tolerance. A well characterized set of standards is required with assigned values such as shown in table 1. The determination of the uncertainty of the summation of values for the group of standards used is not treated in this discussion. It must be emphasized that the results of such a check must be evaluated with reference to the uncertainty of the values of the standards, the precision of the process, and the limits stated by the manufacturer. In some instances, it may be desirable to establish precise values for particular dial settings of the instrument, or for the weights internal to the instrument. Such service may be available from the instrument manufacturer. Detailed procedures are not discussed in this paper.

Given: A direct reading balance with internal weights adjusted on the basis of apparent mass versus brass, determine if the balance indicates the appropriate value for a test weight of mass 53.512 g. The weights internal to the balance are made from material of density 7.8 g/cm^3 at 20 °C and a coefficient of expansion 0.000045. The balance is located in an environment of air density 1.17 mg/cm^3 at a temperature of 22.5 °C.

TABLE 1

Mass (g)	Vol. at 20 °C (cm^3)	Coefficient of expansion
50.000 395	6.35329	0.000 045
3.000 016	0.38120	.000 045
0.499 951	.03012	.000 020
.010 004	.00371	.000 069
.002 007	.00074	.000 069

As in the previous example, a balance indication of 53.512 g would be observed for a test weight made from the same material as the internal weights of the instrument, and adjusted to have an apparent mass versus brass of 53.512 g. One can compute S as before:

$$S=\frac{53.512\left(1-\frac{0.0012}{8.3909}\right)}{\left(1-\frac{0.0012}{7.8}\right)}$$

Equations (7) and (8) of section 2.3 can be combined in part to obtain the following:

$$53.512\left[\frac{1-\frac{0.0012}{8.3909}}{1-\frac{0.0012}{7.8}}\right]\left[1-\frac{0.00117[1+0.000045(22.5-20)]}{7.8}\right]$$

$$-[53.000411-(0.00117)(6.73449)[1+0.000045$$

$$(22.5-20)]]-[.499951-(0.00117)(.03012)$$

$$[1+0.000020(22.5-20)]]-[.012011$$

$$-(0.00117)(.00445)[1+0.000069(22.5-20)]]$$

$$=K(O_1-O_2)\approx 0.000100\text{ g}$$

This result can be used in the following way. With the balance "zeroed" at some particular number, and with the appropriate dial settings for 53.512 g, the summation of the test weights should give an indication 0.100 mg less than the "zero" settings. If this is the case, and subject to a consideration of the factors mentioned above, one would conclude the balance indication is appropriate.

In the above three examples it is presumed that the controllable sources of systematic error have been minimized. These include temperature differentials, operator techniques, and instrument malfunction, as well as the constancy of the standards and built-in weights. Surveillance tests should be used to monitor the constancy of the standards. Details for such tests are available on request. Balance tests should be performed in accordance with ASTM E319-68 [88].

FORM NBS-114A (1-71)

U.S. DEPT. OF COMM. BIBLIOGRAPHIC DATA SHEET	1. PUBLICATION OR REPORT NO. NBS MN-133	2. Gov't Accession No.	3. Recipient's Accession No.

4. TITLE AND SUBTITLE

MASS AND MASS VALUES

5. Publication Date: January 1974

6. Performing Organization Code

7. AUTHOR(S) Paul E. Pontius

8. Performing Organization

9. PERFORMING ORGANIZATION NAME AND ADDRESS

NATIONAL BUREAU OF STANDARDS
DEPARTMENT OF COMMERCE
WASHINGTON, D.C. 20234

10. Project/Task/Work Unit No.

11. Contract/Grant No.

12. Sponsoring Organization Name and Address

Same as No. 9.

13. Type of Report & Period Covered

Final

14. Sponsoring Agency Code

15. SUPPLEMENTARY NOTES

16. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)

There are several bases for assigning mass values to weights to be used as mass standards. As a consequence a given weight may have several assigned mass values depending on the basis used. In many cases, the differences between these assigned values, although easily detectable with precise weighing equipment, are of no practical concern. However, in some instances these differences may be crucial. The first part of this paper is a historical summary of weighing, standards, and the assignment of value; and the interfacing of mass measurements with civilization. The second part of this paper discusses in detail the methods of assigning mass values. Ways to convert from values on one basis to values on another basis are discussed. Sample problems relating to the buoyant effect of the air are presented in the appendices.

17. KEY WORDS (Alphabetical order, separated by semicolons)

Apparent mass; buoyancy corrections; mass comparison; mass value; true mass; weighing.

18. AVAILABILITY STATEMENT

[X] UNLIMITED.

[ ] FOR OFFICIAL DISTRIBUTION. DO NOT RELEASE TO NTIS.

19. SECURITY CLASS (THIS REPORT): UNCLASSIFIED

20. SECURITY CLASS (THIS PAGE): UNCLASSIFIED

21. NO. OF PAGES: 39

22. Price: 70 cents

U. S. GOVERNMENT PRINTING OFFICE : 1974 O - 512- 546

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