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Platinum Resistance Thermometry

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Platinum Resistance Thermometry

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$$M(t') = 0.045\left(\frac{t'}{100\ °\mathrm{C}}\right)\left(\frac{t'}{100\ °\mathrm{C}}-1\right)\left(\frac{t'}{419.58\ °\mathrm{C}}-1\right)\left(\frac{t'}{630.74\ °\mathrm{C}}-1\right)\ °\mathrm{C}$$

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Platinum Resistance Thermometry

John L. Riddle, George T. Furukawa, and Harmon H. Plumb

The monograph describes the methods and equipment employed at the National Bureau of Standards for calibrating standard platinum resistance thermometers (SPRT) on the International Practical Temperature Scale of 1968 (IPTS−68). The text of the scale is clarified and characteristics of the scale are described. Several designs of SPRT's are shown and discussed in the light of the requirements and recommendations of the text of the IPTS−68. Possible sources of error such as those due to the internal and external self-heating effects and the immersion characteristics of SPRT's are described in detail. Precautions and limitations for the mechanical and thermal treatment and for the shipment of SPRT's are indicated, and a guide is given for those desiring the thermometer calibration services of NBS. The description of equipment employed at the National Bureau of Standards for maintaining the IPTS−68 includes the triple point of water cell, tin point cell, zinc point cell, oxygen normal boiling point comparison cryostat, the 13 to 90 K comparison cryostat, and the reference SPRT's upon which the NBS−IPTS−68 in the region 13 to 90 K is based. Methods are given for calculating temperatures from the calibration data and observed resistances; the propagation of calibration errors is discussed. Supplemental information given in the Appendices includes the authorized English version of the text of the IPTS−68, tabular values of the reference function used below 0 °C, tabular values of the differences between IPTS−68 and IPTS−48, analysis of the first derivatives at 0 °C of the IPTS−68 formulations, methods for calibrating potentiometers and Mueller bridges, and the derivation of the coefficients used in the analysis of error propagation.

Key words: Calibration; calibration errors; cryostat; fixed points; freezing points; International Practical Temperature Scale; platinum resistance thermometer; thermodynamic temperature scale; thermometry; triple point.

1. Introduction

The National Bureau of Standards (NBS) has the responsibility to establish, maintain, and develop the standards for physical measurements necessary for the nation's industrial and scientific progress and, in cooperation with other national laboratories, to establish international uniformity of the basic physical quantitites. Dissemination of these standards and the methodology of measurements is accomplished by appropriate publications, consultation, and calibration services. One of the important activities is the realization, reproduction, and transmission of the International Practical Temperature Scale of 1968 (IPTS−68), the present basis for assigning uniform values of temperature. Standard platinum resistance thermometers are the prescribed interpolation instruments for realizing the IPTS−68 from 13.8 to 904 K; they also serve as reference standards for calibrating many other types of thermometers, e.g., liquid-in-glass thermometers. Thus standard platinum resistance thermometers are the base for an extremely large fraction of the temperature measurements made in science and technology.

The information presented in this monograph is intended for those who use standard platinum resistance thermometers, who wish to submit such thermometers to the NBS for calibration, or who desire information that will aid in establishing local calibration facilities. Good practice in the use of these thermometers is emphasized to assist in the realization of accuracy that is commensurate with their capability, as well as to point out their inherent limitations. This monograph describes the calibration of standard platinum resistance thermometers at the NBS including the equipment, techniques, and procedures. The discussion of the IPTS−68, the defining fixed points, the interpolation formulae, the design of standard platinum resistance thermometers, and pertinent procedures which lead to "state of the art" measurements are presented in considerable detail.

2. Background and Basic Concepts

The development of the platinum resistance thermometer resulted in an internationally acceptable practical temperature scale defined to give values of temperatures close to those on the thermodynamic scale. The birth of platinum resistance thermometers as useful precision instruments occurred in 1887 when H. L. Callendar [16][1] reported that platinum resistance thermometers exhibited the prerequisite stability and reproducibility if they were properly constructed and treated with sufficient care. In the next four decades the platinum resistance thermometers gained such wide acceptance that their use was proposed and adopted by the Comité International des Poids et Measures in 1927 in defining values on a practical scale of temperature, the first International Temperature Scale [15]. Since that time improvements in the purity of platinum and other materials of construction, in thermometer design, and in calibration techniques have yielded improvements in the precision, accuracy, and range of temperatures that can be measured with platinum resistance thermometers. The international scale has been redefined both to take advantage of these improvements and to bring the scale more nearly into agreement with the thermodynamic scale.

2.1. Thermodynamic Temperature Scale

Temperature scales based on functions that can be derived from the first and second laws of thermodynamics, and hence that give values of temperature consistent with the entire system of logic ("thermodynamics") derivable from these laws, are said to be thermodynamic scales. The present thermodynamic scale was established in 1954 by assigning the value 273.16 kelvins to the triple point of water. [17] On the Kelvin thermodynamic scale, the values of temperature and the designation of the appropriate units and symbols have been adopted [51] as follows: The unit of measurement on this thermodynamic scale is the kelvin, indicated by the symbol K. The same name and symbol are also used to denote an interval of temperature. The designation of an interval of temperature by "degrees Celsius" is also permitted. The symbol of the value of temperature on the Kelvin thermodynamic scale is T. The Celsius thermodynamic scale is defined by the equation: $t=T-273.15$ K. The zero of the Celsius scale is 0.01 K below the triple point of water. (Although the value of the temperature of the ice point is close to and can be taken for all practical purposes to be 0 °C, the value of the ice point is no longer defined as 0 °C.) The unit of a Celsius thermodynamic temperature is the degree Celsius, symbol °C, which is equal to the unit of temperature on the Kelvin scale. The symbols of the values of temperature, t and T, contain the unit of temperature °C and K, respectively.

2.2. Practical Temperature Scales

Accurate measurements of values of temperature on the thermodynamic scale are fraught with experimental difficulties, yet all values of temperature should be referable to the Kelvin thermodynamic scale. "Practical" temperature scales are intended to give values of temperature that are comparatively easily reproduced and, therefore, utilitarian, but most are based on functions that are not related to the first and second laws of thermodynamics. In contrast, vapor pressure scales, e.g., that of ^{4}He [13], are based on the laws of thermodynamics as is also the radiation pyrometry scale. Because many physical laws are based on thermodynamic temperatures, the values of temperature on any practical scale should be as close to the thermodynamic scale as is possible or the differences between a practical scale and the thermodynamic scale should be appropriately documented so that conversions from the practical scale to the thermodynamic temperature scale are possible.

Many practical scales have been used in the last fifty years, but the only scales employing platinum resistance thermometers that have had widespread use in the United States were those defined and sanctioned by either the International Committee of Weights and Measures or the National Bureau of Standards. Successive scales that were defined by the International Committee of Weights and Measures are the International Temperature Scale of 1927 (ITS–27) [15], the International Temperature Scale of 1948 (ITS–48) [46], and the International Practical Temperature Scale of 1948 with the text revision of 1960 (IPTS–48) [48]. The differences in the definition of these scales primarily arose from steps to improve the reproducibility of the scale and the values of temperature between −182.97 and 630.5 °C remained essentially unchanged—the changes in the values of temperature were less than the uncertainty of the values on the scale at the time. These differences are summarized in Appendix B.

Because the ITS–27 [15] extended only 7 °C below the normal boiling point of oxygen (−182.962 °C), the National Bureau of Standards was motivated to develop a practical scale from the oxygen point down to approximately 11 K [28]. This scale has been variously referred to as the NBS provisional scale, the Hoge and Brickwedde scale, and most recently as the NBS–39 scale; directly related to this scale is the NBS−55 scale ($T_{\mathrm{NBS-55}}=T_{\mathrm{NBS-39}}-0.01$ K) [50]. These NBS scales have had widespread use but, together with the previously mentioned international scales, have now been supplanted by the International Practical Temperature Scale of 1968 (IPTS–68) [51] (see Appendix A for the complete text of IPTS–68 and Appendix C for the difference in values of temperature).

The text of the IPTS–68 introduced the first major changes in the international practical scale

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[1] Figures in brackets indicate the literature references on page 50.

since 1927. Changes were made to extend the range of the scale down to 13.81 K and to enhance its reproducibility as well as to improve its agreement with the thermodynamic scale. The changes in the values of temperature from the IPTS–48 to the IPTS–68 are tabulated in Appendix C; included in this appendix is a tabulation of the differences between the IPTS–68 and the NBS–55 scale which it replaced in the United States of America. Temperatures on the IPTS–68 may be expressed in either kelvins or degrees Celsius. The symbols and units of the international practical scale are like those described earlier in this section for use with the thermodynamic scale. The symbol for the value of temperature and for the unit of temperature on the International Practical Kelvin Scale are T_{68} and K (Int. 1968), respectively. For the International Practical Celsius Scale they are correspondingly, t_{68} and °C (Int. 1968). The relation between T_{68} and t_{68} is

$$t_{68} = T_{68} - 273.15 \text{ K}. \tag{2.1}$$

The subscripts or parenthetical parts of the designation need not be used if it is certain that no confusion will result from their omission.

In the text of the IPTS–1968, platinum resistance thermometers are defined as standard interpolation instruments for realizing the scale from −259.34 to 630.74 °C (or from 13.81 to 903.89 K). The resistance values (or ratios of resistance values, $R(t)/R(0)$, where $R(t)$ is the resistance at temperature t and $R(0)$ the resistance at 0 °C) of any particular standard platinum resistance thermometer are related to the values of temperature with specified formulae. The constants of the formulae are determined from the resistance values of the thermometer at stated defining fixed points and, usually, from the derivatives of the formula specified for the temperature range immediately above. [Hereafter, platinum resistance thermometers that meet the following specifications, based primarily on the text of the IPTS–68, will be referred to as standard platinum resistance thermometers and abbreviated as SPRT: The platinum resistor shall be very pure annealed platinum supported in such a manner that the resistor remains as strain-free as practical; the value of $R(100)/R(0)$ shall not be less than 1.3925; the stability of the thermometer shall be such that, when the thermometer is subjected to thermal cycling similar to that encountered in the normal process of calibrating it, the value of $R(0)$ does not change by more than $4 \times 10^{-6} R(0)$; the platinum resistor shall be constructed as a four-lead element and both the resistor and its leads shall be insulated in such a manner that the measured resistance of the platinum resistor is not affected by the insulator more than about $4 \times 10^{-7} R(0)$ at the temperatures of calibration or use; and the four-lead resistor shall be hermetically sealed in a protective sheath. (A SPRT that has a $R(0)$ value of about 25.5 Ω will be referred to as a 25-Ω SPRT.)] The text of the scale assigns an exact numerical value of temperature to each of the defining fixed points. The defining points and their assigned values are listed in table 1. The realization of the fixed points

TABLE 1. *Defining fixed points of the IPTS–68*[a]

Equilibrium state	Assigned value of International Practical Temperature		W^*
	T_{68} (K)	t_{68} (°C)	
Equilibrium between the solid, liquid and vapor phases of equilibrium hydrogen (triple point of equilibrium hydrogen)	13.81	−259.34	[d]0.0014 1207
Equilibrium between the liquid and vapor phases of equilibrium hydrogen at a pressure of 33 330.6 N/m² (25/76 standard atmosphere)	17.042	−256.108	[d]0.0025 3445
Equilibrium between the liquid and vapor phases of equilibrium hydrogen (boiling point of equilibrium hydrogen)	20.28	−252.87	0.0044 8517
Equilibrium between the liquid and vapor phases of neon (boiling point of neon)	27.102	−246.048	0.0122 1272
Equilibrium between the solid, liquid and vapor phases of oxygen (triple point of oxygen)	54.361	−218.789	[d]0.0919 7253
Equilibrium between the liquid and vapor phases of oxygen (boiling point of oxygen)	90.188	−182.962	[d]0.2437 9911
Equilibrium between the solid, liquid and vapor phases of water (triple point of water)[c]	273.16	0.01	
Equilibrium between the liquid and vapor phases of water (boiling point of water)[b,c]	373.15	100	1.3925 9668
Equilibrium between the solid and liquid phases of zinc (freezing point of zinc)	692.73	419.58	2.5684 8557

[a] Except for the triple points and one equilibrium hydrogen point (17.042 K) the assigned values of temperature are for equilibrium states at a pressure $p_0 = 1$ standard atmosphere (101 325 N/m²).

[b] The equilibrium state between the solid and liquid phases of tin (freezing point of tin) has the assigned value of $t_{68} = 231.9681$ °C and may be used as an alternative to the boiling point of water. At this value of temperature $W^* = 1.8925\,7109$.[d] The value of $t' = 231.929163$ °C.

[c] The water used should have the isotopic composition of ocean water (see sec. III.4, Appendix A).

[d] This value is slightly different from that given in the text of the IPTS–68 (see Appendix A).

and the use of interpolation formula are discussed in detail in sections 7 and 6, respectively.

The IPTS–68 is defined by the text of the scale. However, even if perfect experimental work were possible, the values of temperature would not be unique because of a lack of uniqueness in the definition of the scale itself. For example, the text of the scale specifies the minimum quality of the platinum to be used in the SPRT; yet different samples of suitable platinum may not give exactly the same value of a temperature between defining fixed points. The different values of temperature obtained would, however, be in accordance with IPTS–68 because the samples of platinum are within the definition of the scale. Another source of difference is the variation in the realization of the fixed points where small differences in the purity of the fixed-point samples could easily occur. This spread of temperatures (the "legal" spread of the scale) is the result of the looseness of the definition of the scale; it is small compared to the usual errors of experimental measurement, but does exist and is a practical necessity if the physical properties of real materials are to be used in realizing the scale.

3. Platinum Resistance Thermometer Construction

This discussion is primarily directed toward thermometers that are suitable as defining standards on the IPTS–68; however, many of the techniques described are applicable to any resistance thermometer. To be suitable as a SPRT as described in the text of the scale (Appendix A) the resistor must be made of platinum of sufficient purity that the finished thermometer will have a value of $R(100)/R(0)$ not less than 1.3925 or α, defined as $(R(100)-R(0))/100R(0)$, not less than 0.003925. This requirement provides a scale that is more closely bounded and is not unreasonable since platinum wire of sufficient purity to yield the above ratio is now produced in several countries. The typical SPRT has an ice-point resistance of about 25.5 Ω and its resistor is wound from about 61 cm of 0.075 mm wire. The wire is obtained "hard drawn," as it is somewhat easier to handle in this condition, but it is annealed after the resistor is formed. Although wires between 0.013 mm and 0.13 mm diameter are commonly used in industrial platinum thermometers, experience shows that finer wires tend to have lower values of $R(100)/R(0)$ (implying the presence of more impurities or strains).

The insulation material that supports the resistor and leads must not contaminate the platinum during the annealing of the assembled thermometer nor when subjected for extended periods of time to temperatures to which thermometer is normally exposed. The insulation resistance between the leads must be greater than 5×10^9 Ω at 500 °C if the error introduced by insulation leakage in the leads of a 25-Ω thermometer is to be less than the equivalent of 1 μΩ. For SPRT's the most commonly used insulation is mica. The primary difficulties in the use of mica are the evolution of water vapor at high temperatures and the presence of the iron oxide impurity which, if reduced, leaves free iron that will contaminate the platinum. The most usual mica is muscovite ($H_2KAl_3(SiO_4)_3$), sometimes called India or ruby mica; it contains about 5 percent water of crystallization. The dehydration starts at approximately 540 °C [41] and not only causes mechanical deterioration of the mica but supplies free water vapor that reduces the insulation resistance. Phlogopite mica ($H_2KMg_3Al(SiO_4)_3$) or "amber" mica is used for thermometers that are expected to function up to 630 °C. Although the room temperature resistivity of phlogopite mica is slightly lower than that of muscovite, it does not begin to dehydrate until well above 700 °C [41].

Some designs employ very high-purity alumina or synthetic sapphire insulation (see fig. 1) [1, 18, 23, 52]. Fused silica has also been used [18]. Although the properties of these materials are superior to mica at high temperatures, the materials are more difficult to fabricate and, additionally, they must be

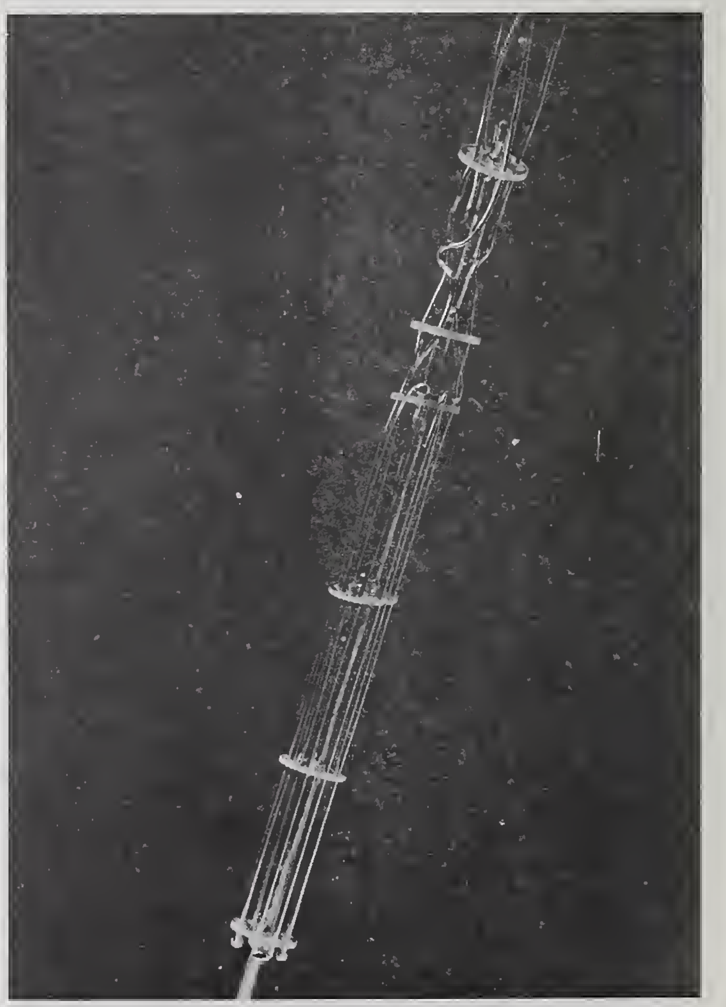

Figure 1. *Synthetic sapphire form for thermometer.*

Synthetic sapphire disks are employed to keep the platinum wires separated in a "bird-cage" type SPRT. A centrally located, heavier gage platinum wire maintains the spacings between the disks [23].

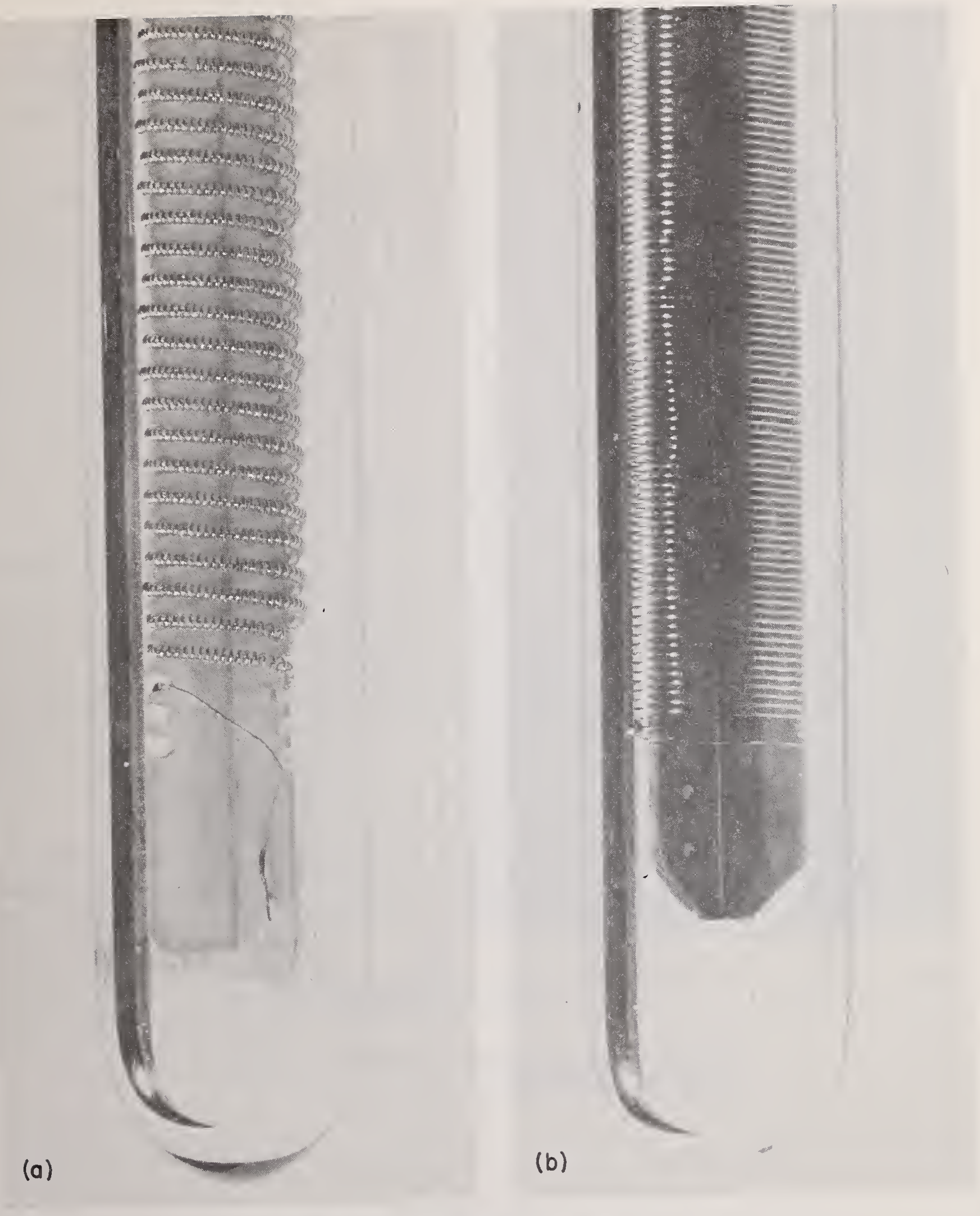

FIGURE 2. *Construction of the coils of some thermometers.*
(a) Coiled filament on mica cross [34].
(b) Single layer filament on mica cross [47].

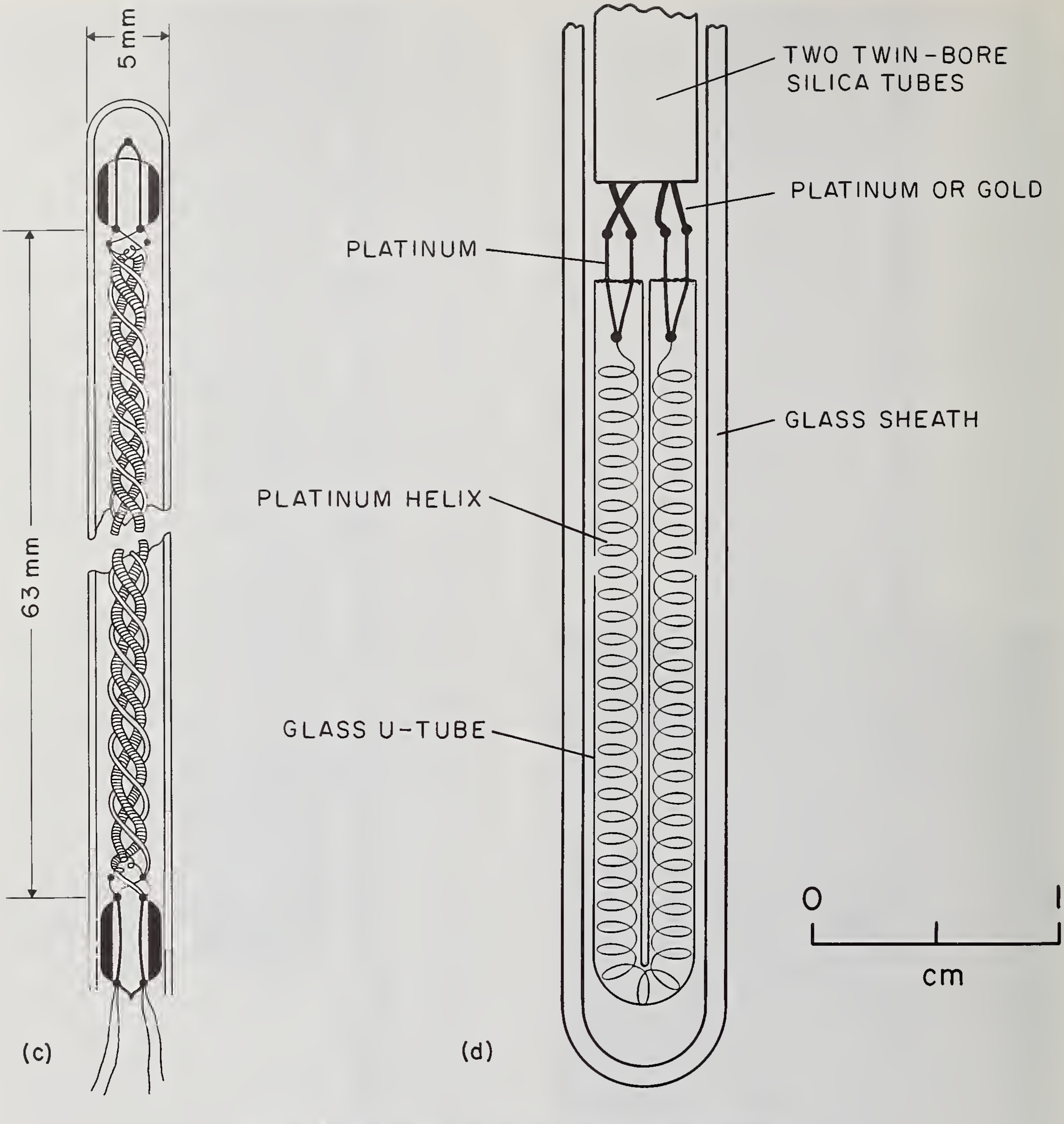

Figure 2—Continued. *Construction of the coils of some thermometers.*
(c) Coiled filament on twisted glass ribbon [49].
(d) Coiled filament in glass U-tube [2].

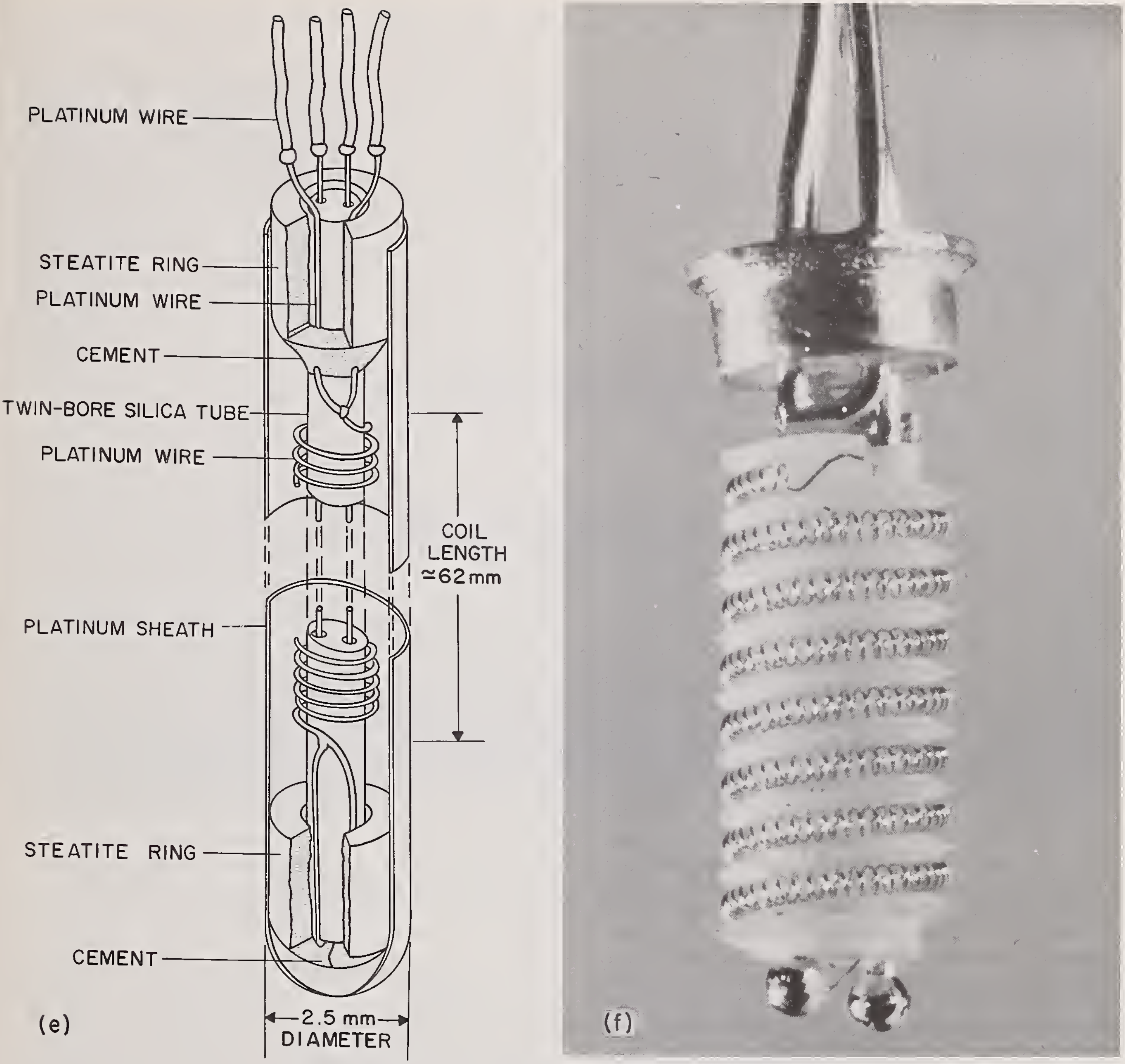

FIGURE 2—Continued. *Construction of the coils of some thermometers.*
(e) Single layer filament around twin-bore silica tubing [3].
(f) Coiled filament on helically grooved ceramic. (Courtesy the Minco Products, Inc.)

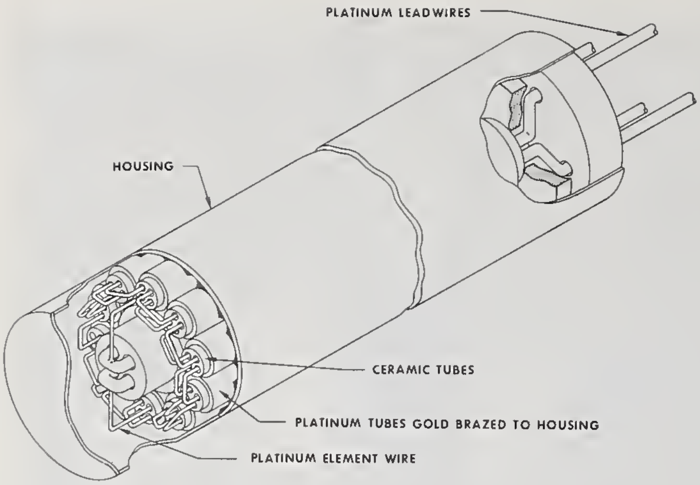

FIGURE 2—Continued. *Construction of the coils of some thermometers.*
(g) Filament threaded through four-hole ceramic tubes. (Courtesy the Rosemount Engineering Company.)

of very high purity because migration of impurities and the consequent contamination occurs much more easily at high temperatures. In cutting either mica or alumina great care must be taken to avoid or eliminate traces of metal that originate from either the cutting tool or from the metal clamps that are used to support the insulation during machining. A "carbide tipped" tool should be used for the cutting process. Unfortunately, attempts to remove the metal chemically from the mica result in contaminating the mica. Synthetic sapphire, however, can be chemically cleaned after machining.

The configuration of the resistor is inevitably the result of compromise between conflicting requirements. The resistor must be free to expand and contract without constraint from its support. This characteristic is the so called "strain-free" construction. If the platinum were not free to expand, the resistance of the platinum would not only be a function of temperature but would also relate to the strain that results from the differential expansion of the platinum and its support. Seven methods of approach toward achieving "strain-free" construction are illustrated in figure 2 [2, 3, 4, 5, 34, 47]. Because of the lack of adequate mechanical support, the wire in each of these designs may be strained by acceleration, e.g., shock or vibration. The thermal contact of the resistor with the protecting envelope or sheath is primarily through gas which, even if the gas is mostly helium, is obviously poor compared to the thermal contact that is possible through many solid materials. This poor thermal contact increases the self-heating effect and the response time of the thermometer. The designs shown in figures 2a, 2b, and 2g suffer less in these respects than the others. On the other hand, for calorimetric work the instrument of the lowest heat capacity is often preferred.

The sensing elements of all SPRT's have four leads (see fig. 2). The four leads define the resistor precisely by permitting measurements that eliminate the effect of the resistance of the leads. The resistor winding is usually "noninductive," often bifilar, but occasionally other configurations that tend to minimize inductance are used. This serves to reduce the pick-up of stray fields and usually improves the performance of the thermometer in a-c circuits. (If the resistor is to be measured using a-c, the electrical time constant, i.e., reactive component should be minimized.)

Because the junction of the leads is electrically a part of the measured resistor, the leads extending immediately from the resistor must also be of high-purity platinum; the lengths of these leads are often as short as 8 mm. Either gold or platinum wire is employed in continuing these leads within the thermometer. Gold does not seem to contaminate the platinum and is easily worked. Measurement of the resistor may be facilitated if the four leads are made of the same material with the length and diameter the same so that the leads have about equal resistances at any temperature within the temperature range of the thermometer. This statement is also applicable to the leads that are external to the protecting envelope. Figure 3 shows the arrangement of thermometer leads near the head of three SPRT's.

The hermetic seal through the soft glass envelope at the thermometer head is frequently made using short lengths of tungsten wire, to the ends of which platinum lead wires are welded. The external platinum leads are soft soldered to copper leads that are mechanically secured to the head. For d-c measurements, satisfactory external copper leads can be made from commercially available cable. The cable consists of twelve wires (No. 26 AWG solid bare copper wire) each insulated with double silk windings; additionally, another double silk wrapping encloses each group of three insulated wires. A double silk braid and a tight, waterproof, polyvinyl coating cover the entire four groups. After the appropriate end insulation is removed, the ends of the three wires in each group are twisted together to form a lead to which a lug is soldered. Stranded leads which do not have individually insulated strands should not be used, as the breakage of a single strand may cause "noise" in the resistance thermometer circuits which is difficult to locate and eliminate. The leads external to small capsule type thermometers are usually solid copper wires, although wires of other materials such as manganin are sometimes used; the leads to the thermometer must be placed in good thermal contact with the system to be measured by the capsule thermometer.

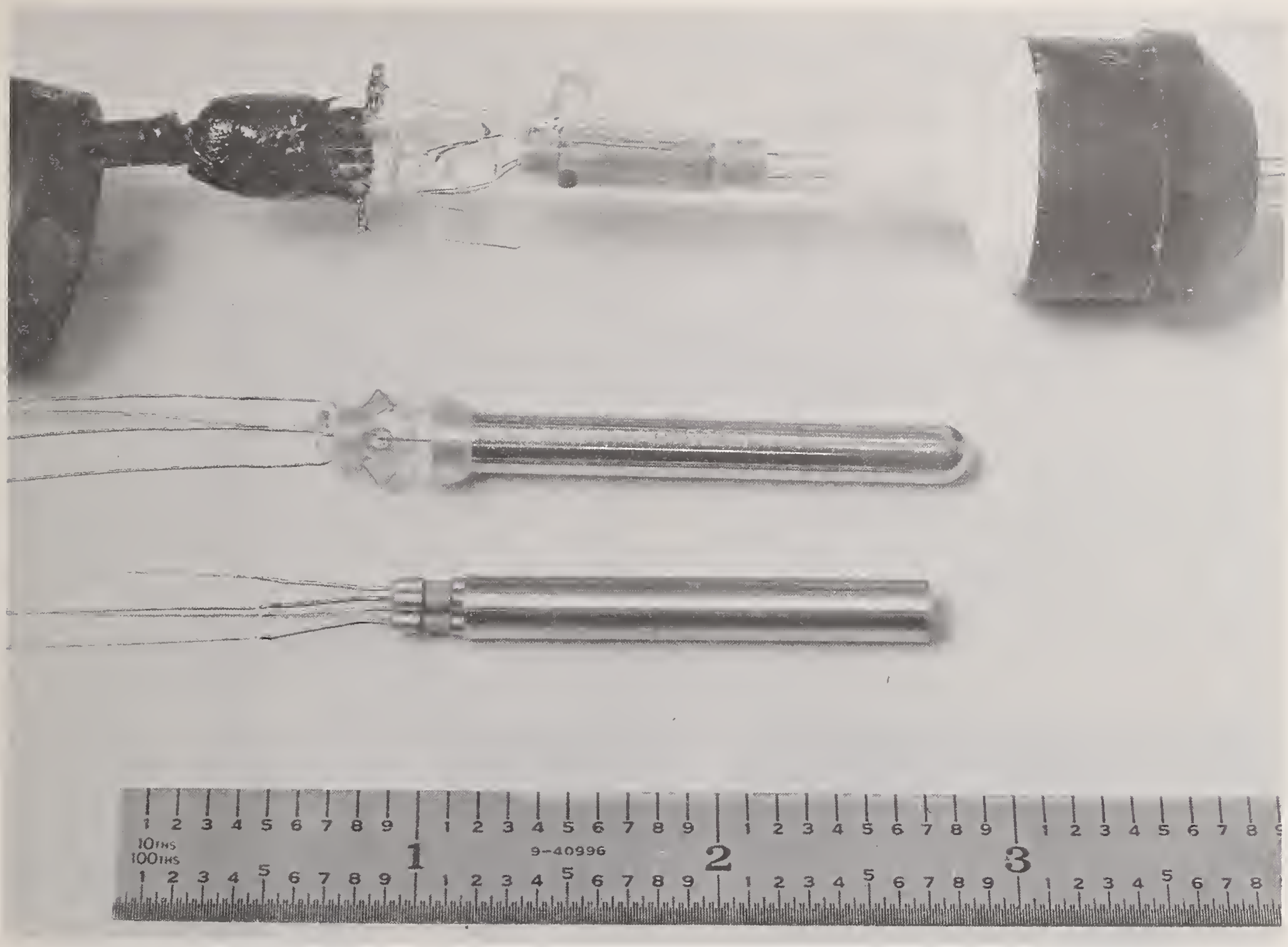

FIGURE 3. *"Heads" of three SPRT's.*

The upper head is that of a long stem thermometer. The internal leads are brought out through hermetic seals and are connected to external copper leads at the left.
In the center is a capsule type SPRT with leads brought out through glass seals. Capsule body is platinum.
The bottom capsule type SPRT shows the thermometer leads brought out through individual metal-ceramic-metal type seals. Capsule body is stainless steel.

4. Using the Thermometer

4.1. Mechanical Treatment of SPRT

A SPRT is a mechanically delicate instrument. As discussed in the section on thermometer construction, the platinum wire cannot be rigidly supported and at the same time be free to expand and contract with temperature changes. Shock, vibration, or any other form of acceleration may cause the wire to bend between and around its supports, thus producing strains that change its temperature-resistance characteristics. Strains in the platinum resistor normally will increase the resistance and decrease the value of α. If a tap of a thermometer with glass sheath is hard enough to be audible, but still not cause breakage, the action will typically increase the triple-point resistance of a 25 Ω SPRT by an amount between 1 and 100 $\mu\Omega$. (A change of 100 $\mu\Omega$ is equivalent to 0.001 °C in temperature.) Thermometers that have received repeated shocks of this kind through rough handling over a one year period have increased in resistance at the triple point of water by an amount equivalent to 0.1 °C. Similar changes may be caused by using the thermometer in a bath that transmits vibrations to the thermometer or by shipping the thermometer not suitably packed.

It is preferable to "hand carry" the thermometer to maintain the integrity of its calibration. If the thermometer must be shipped, it should first be placed in a rigid and moderately massive container that has been lined with material which softly conforms to the thermometer and protects it from mechanical shocks or vibrations. This container should then be packed in an appreciably larger box with generous room on all sides for soft packing material that will absorb or attenuate the shocks that might occur during shipment. Two reasonable package configurations are shown in figures 4 and 5.

In arranging storage for the thermometer in the laboratory, a container should be used that minimizes or eliminates the possibility of "bumping" the thermometer. This is a most reasonable precaution when one considers the amount of handling that many SPRT's receive during the course of many measurements. This precaution may be even more pertinent for SPRT's that are routinely used as a standard for calibrating other thermometers. Figure 6 shows a storage arrangement employed at NBS.

Care should also be exercised in protecting the thermometer from cumulative shocks that might be received during insertion into apparatus. Figure 19 (sec. 7) shows the simple provisions made at NBS for reducing mechanical shock when thermometers are inserted into triple-point of water cells. The polyethylene plastic tube at the top of the cell guides the SPRT into the thermometer well without "bumping", and the tapered entrance of the aluminum sleeve near the bottom of the well then guides the SPRT into the sleeve and onto the soft polyurethane sponge at the bottom.

4.2. Thermal Treatment of SPRT

With exception of specific instances, SPRT's, as they are generally used, are not greatly susceptible to damage from thermal shocks. In the case of capsule type thermometers, the metal to glass seals may be broken by rapid cooling, e.g., when a capsule thermometer that has been at ambient temperature is quickly immersed in liquid nitrogen. Sudden exposure of capsule thermometers to temperatures that are much higher than ambient seldom occurs since they are rarely used above 100 °C. Another specific instance of thermal shock is subjecting long-stem SPRT's to temperatures that are well above 600 °C and subsequently quenching them. This treatment can mechanically damage the thermometer but, even if no visible damage has been done, the calibration of the thermometer may have been altered by the freezing in of defects that occurred during the quenching [24].

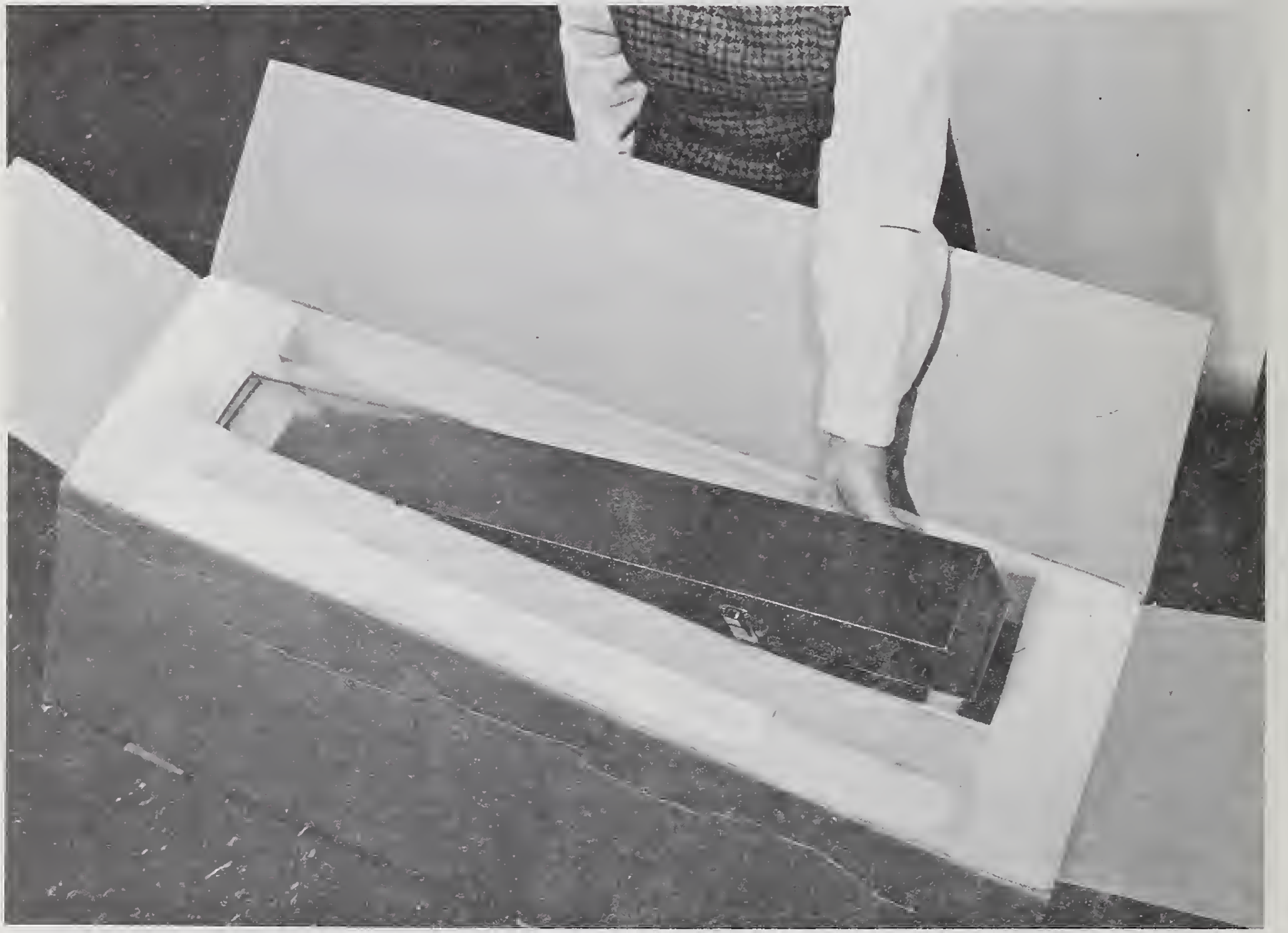

FIGURE 4. *A method for packaging a SPRT for shipment.*
The metal case contains a SPRT snugly nested in polyurethane foam. The metal case in turn is protected during shipment by a tightly fitted polyurethane foam lined box.

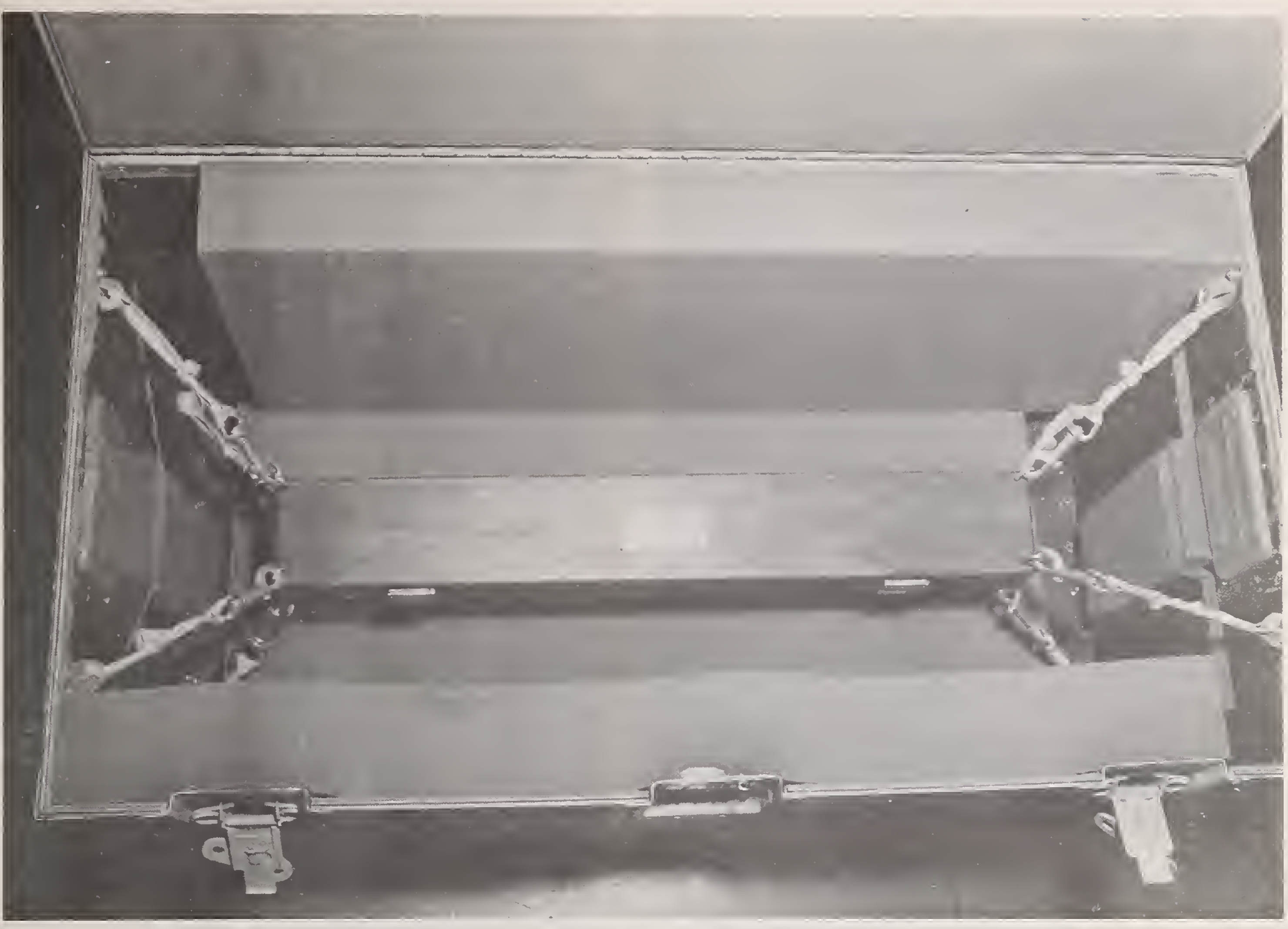

FIGURE 5. *Special container for SPRT shipment.*

The wooden case contains a SPRT snugly nested in polyurethane foam. Each of the eight corners of this wooden case is attached elastically to the large metal container. Blocks of polyurethane foam give additional protection in the event of "overtravel."

(courtesy the Space Division, North American Rockwell Corporation)

The upper temperature limit of a SPRT is restricted by the softening point of the material of the protecting sheath, the temperature at which the thermometer was annealed before calibration, the evolution of water and other contaminants from the sheath and insulators, and grain growth in the platinum wire. The concomitant changes are a function of both time and temperature and most are predictable in only a qualitative way.

Thermometer sheaths of borosilicate glass soften noticeably at temperatures above 500 °C, e.g., their use at 515 °C (1000 °F) is limited to a few hours unless they are specially supported to prevent deformation. Fused silica sheaths should be used for measurements above 500 °C. Platinum grain growth has been observed in SPRT's that have been used over a period of several hundred hours at 420 °C. Figure 7, made with an electron microscope, shows in detail a short section of platinum wire from a thermometer that was subjected to such a treatment. Grains which are one to two wire diameters long and as large as the wire in diameter may be clearly seen. The growth of large grains results in a thermometer that is more susceptible to calibration changes caused by mechanical shock; consequently, the thermometer may be considered unstable. The evolution of water within the thermometer sheath at temperatures below 500 °C has been observed in a few thermometers but seems to be avoidable if sufficient baking and evacuation are furnished during the fabrication of the thermometer. The presence of water may become more conspicuous when the thermometer is cooled, e.g., in a triple point cell. The presence of water, either within the thermometer or on the external insulated leads, will be evidenced during electrical measurements by the kick of the galvanometer when the current through the thermometer is reversed. The kick stems from the polarization of water on the insulation. The decay time (the time for repolarization of water on the insulation) of the galvanometer pulse is usually greater than a minute when moisture exists in the thermometer but is only of a few seconds' duration with moisture on silk insulated leads, indicating that the quantity and physical state of moisture and the effect of moisture on the insulation inside the

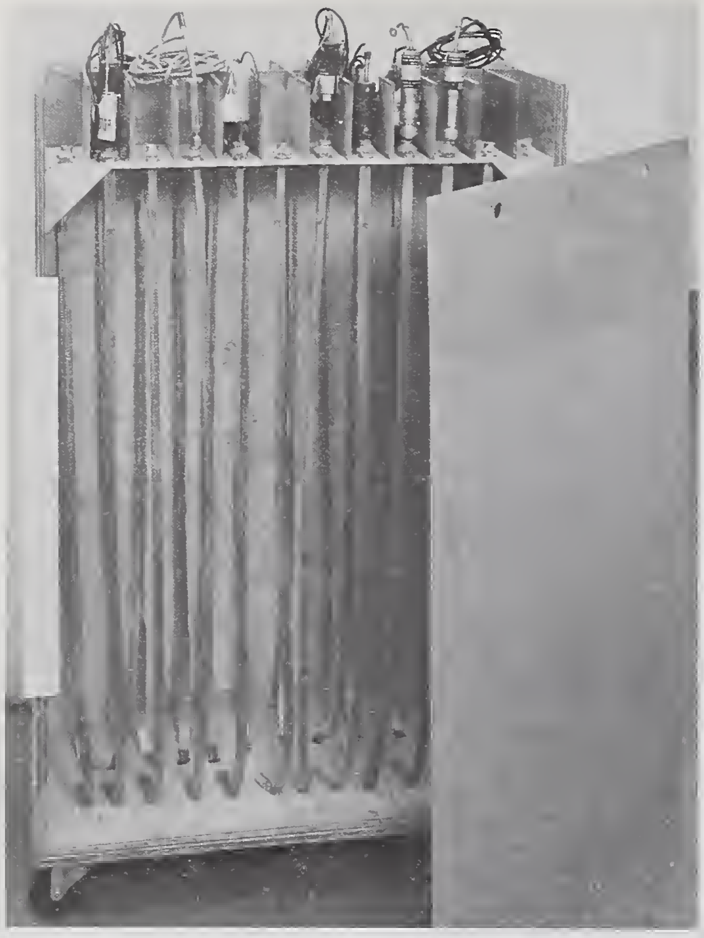

Figure 6. *A storage for SPRT's.*

The thermometers are stored vertically by slipping into the wire braid sleeves. The cover to the box has been removed only to show the arrangement of the wire braid sleeves.

thermometer are different from those on the external silk insulation.

Annealing may occur if the platinum wire was not properly annealed during the manufacturing process or (more likely) if it has been strained by mechanical shock since it was last well annealed. If the strains are sufficiently severe, noticeable annealing will occur at temperatures as low as 100 °C. The annealing process does have somewhat beneficial aspects because the thermometer tends to return toward the metallurgical state that it presumably was in during its previous calibration. (At NBS all long-stem SPRT's are annealed between 470 and 480 °C for 4 hours, removed from the furnace while still hot, and allowed to cool in air at ambient temperature prior to their calibration). The comparison of the resistance of the thermometer at a fixed point before and after annealing will show a shift downward in resistance and, usually, a shift upward in $R(t)/R(0)$ (see sec. 6 on the computation of temperature).

In the measurements at temperatures above 500 °C the difficulties discussed above are more likely to occur or be accelerated. More care must be taken in baking and evacuating the thermometer after assembly. For work near the upper temperature limit (630 °C), the SPRT's should have phlogopite mica or fused silica coil forms and a fused silica sheath. Also, care should be exercised to avoid the previously mentioned effect of freezing in of high temperature defects when cooling the thermometer.

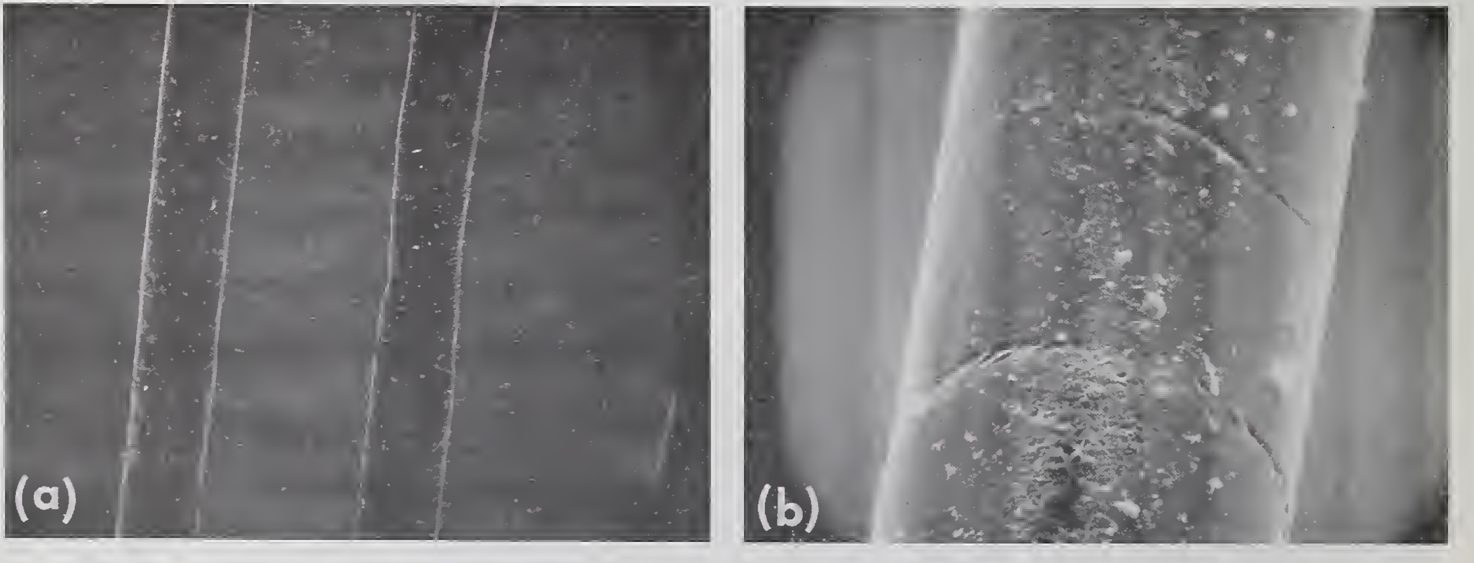

Figure 7. *Electron microphotograph of platinum thermometer wire subjected to a total of several hundred hours at 420 °C.*
Magnification: photograph (a) 172× (b) 850×. The photographs show: (1) grain growth; (2) migration. No attempt has been made to date to identify the white "flecks."

4.3. Thermometer Immersion

A thermometer is sufficiently immersed in the bath when there is no heat flow between the sensor and its environment through the thermometer leads or sheath that extend from the region of sensor. Regrettably, considering the ease with which the adequacy of immersion can be checked, insufficient immersion of a thermometer is still a very common error. The test simply involves a equilibrium temperature measurements at two depths of immersion, while the bath is maintained at very nearly constant temperature. If, after taking into account any change in the temperature of the bath, the temperatures at the different immersions do not agree, an immersion problem exists. Experimentalists are frequently led astray by the erroneous idea that the required immersion is strongly temperature dependent. Figure 8 shows the difference between the bath temperature and the temperature indicated by the thermometer as a function of immersion for two different thermometers. The immersion characteristics are more clearly seen in figure 9 where the same data are given in a semilog plot. A linear relationship between the immersion and the logarithm of the temperature difference is to be expected in simple cases and in practice this is a very useful approximation in the usual region of interest, namely, where the temperature difference is small. Figure 9 shows that for thermometer G in an ice bath, the temperature difference is reduced by a factor of 10 for each 3.3 cm of immersion. For thermometer M, the temperature difference is reduced by a factor of 10 for each 1.4 cm of immersion. If the error due to immersion is to be less than 0.025 mK, the temperature difference must be reduced by six orders of magnitude and, therefore, both thermometers must be immersed in the ice bath by six times the respective amounts cited above, or a total of 19.8 cm and 8.4 cm, respectively. Even if the temperature difference between the ambient and the bath were only 2.5 degrees the thermometer immersion could be decreased only slightly because, for the same accuracy, a reduction of the temperature difference by five factors of ten is still needed.

The similarity of the radial conductance of heat to or from the thermometer in the bath is important. Figure 10 shows the immersion characteristics of thermometers G and M in a triple-point of water cell rather than an ice bath. The immersion characteristics have changed for both thermometers, but particularly for thermometer M. This is caused primarily by the increase in thermal resistance radially from the thermometer to the ice-water interface of the triple point cell: the sheath of water that surrounds the thermometer within the triple-point cell well and the glass wall of the well contribute to the increased thermal resistance (see also sec. 4.4). The immersion problem would be even worse if the space between the thermometer well and the thermometer were filled with air instead of water. Typical immersion data for thermometers in tin and zinc freezing point cells are given in the discussion of that apparatus (see sec. 7).

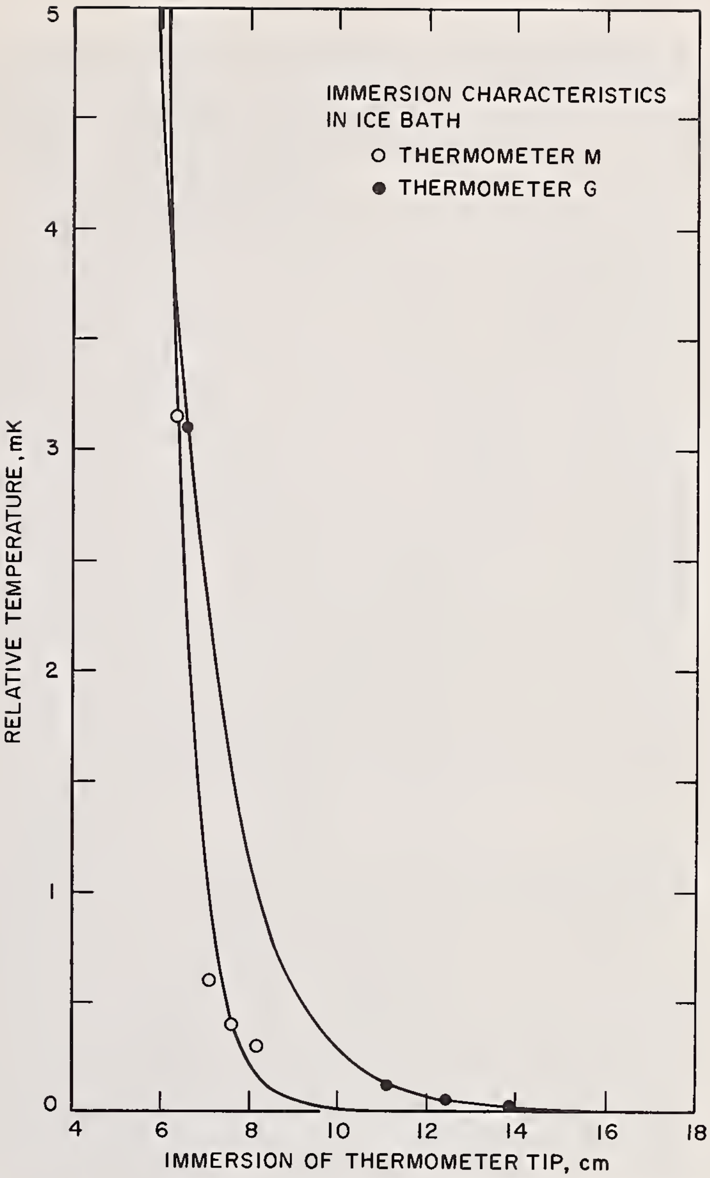

FIGURE 8. *Immersion characteristics in an ice bath of two long stem SPRT's with different sheath materials and internal construction.*

The plot shows the relative temperature as a function of the depth of thermometer immersion.

4.4. Heating Effects in SPRT

The measurement of resistance necessarily involves passing a current through the resistor. The resultant heating that occurs in the resistor and its leads raises their temperature above that of their surroundings until the resistor element attains a temperature sufficiently higher than the surroundings to dissipate the power developed. A typical steady-state profile of the radial temperature distribution caused by a current of 1 mA flowing in a 25 Ω SPRT is shown in figures 11 and 12. In figure

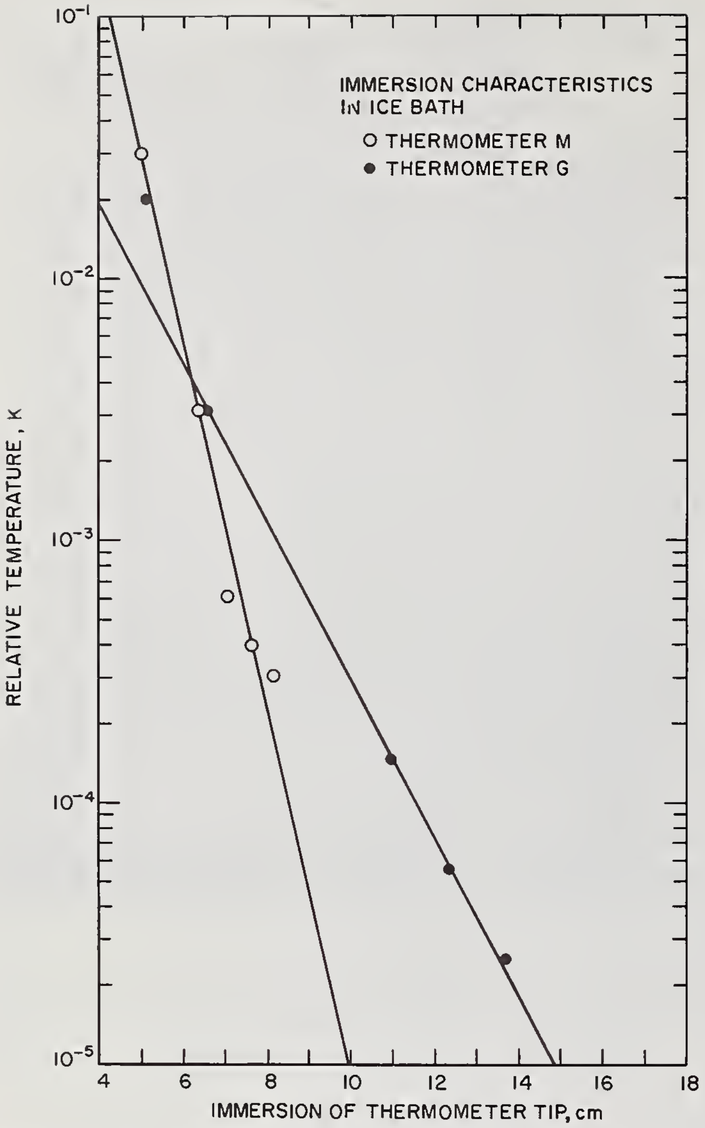

FIGURE 9. *Immersion characteristics in an ice bath of two long stem SPRT's with different sheath materials and internal construction.*

The data of figure 8 have been replotted to show the linear relationship between the logarithm of the relative temperature and the depth of thermometer immersion.

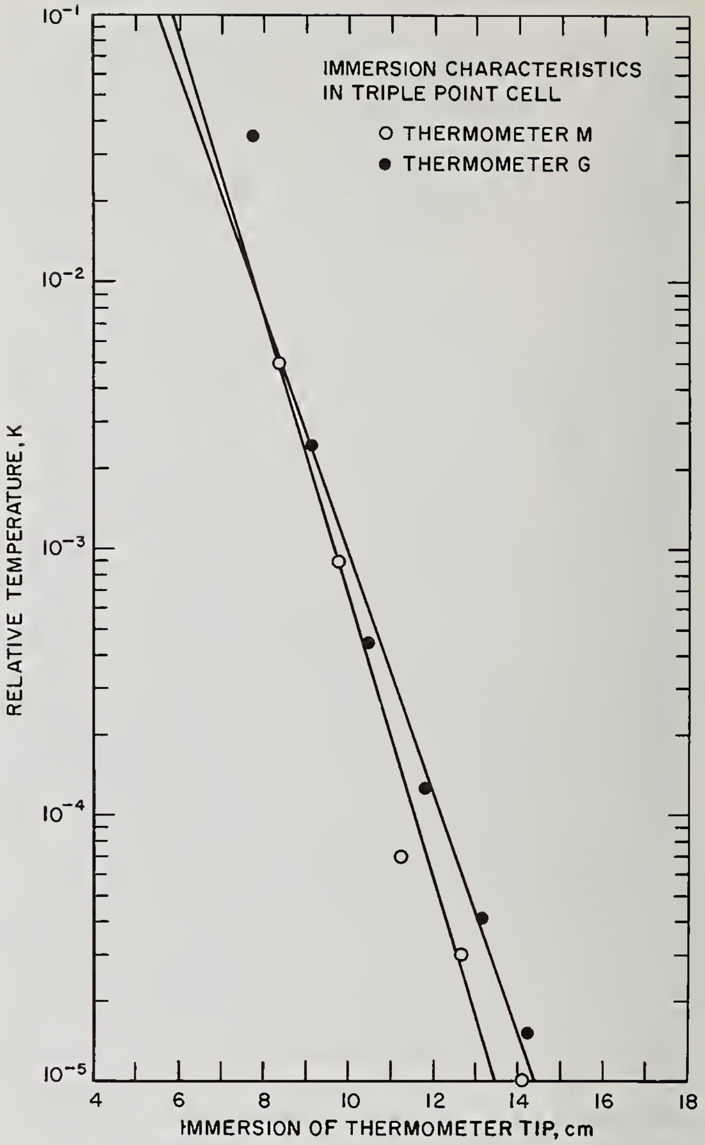

FIGURE 10. *Immersion characteristics in the triple point of water cell of two long stem SPRT's with different sheath materials and internal construction.*

The plot shows the relationship between the logarithm of relative temperature and the depth of thermometer immersion. Comparison with figure 9 shows that the immersion characteristics of the thermometers tend to be poorer in the triple-point cell than those in the ice bath. This change in the observed immersion characteristics is caused primarily by the higher resistance to radial heat flow in the triple-point cell.

11, the *internal* heating effect of the thermometer, i.e., between the platinum resistor and the outside wall of the protecting sheath, at a given environmental temperature is a function only of the thermometer construction and the current and is, therefore, the same during both calibration and use. This assumes that the thermometer resistor does not move within its protecting sheath. At the ice point, the internal heating effect may be measured by direct immersion in the ice bath and is typically between 0.3 mK/(mA)^2 and 1.2 mK/(mA)^2 for a 25 Ω SPRT. If the thermometer is used with the same current that was used during its calibration, the same internal heating effect occurs and no error is introduced in the measurement.

As figure 12 indicates, there is also an *external* heating effect, an extension of the heating effect beyond the outside of the thermometer sheath because the generated joule heat must flow to some external heat sink. The total heating effect, i.e., the combined internal and external heating effects, can easily be determined by calculating the resistance that would be measured at zero current; this can be performed either algebraically or graphically as shown in figure 13. When it is desirable to determine the internal heating effect only, the experimental conditions must be such that the external heating effect is negligible. This condition can be closely approximated by directly immersing the thermometer into an ice bath wherein the solid ice particles are in contact with the thermometer sheath, or into a metal (tin or zinc) freezing point apparatus in which the metal freezes directly on the thermometer. (The metal must be remelted before complete solidification or the thermometer will be crushed.)

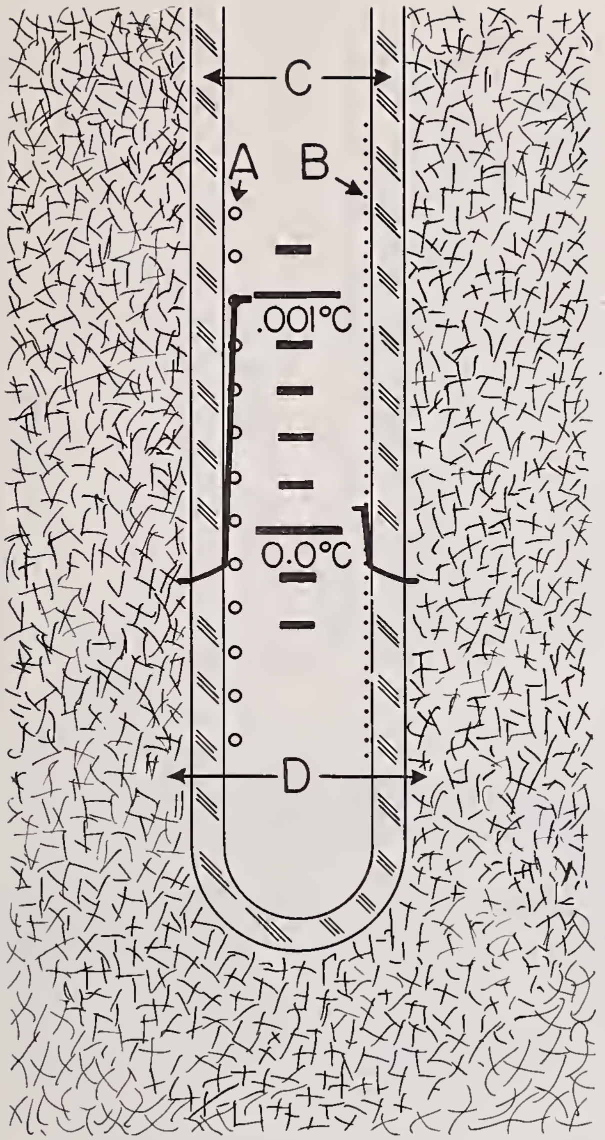

FIGURE 11. *Platinum resistance thermometers at 33 cm immersion in an ice bath.*

Temperature profile from the middle of thermometer coil out to the ice bath with 1 mA current.

A. Platinum coils of coiled filament thermometer; only coils of one side are indicated.
B. Platinum coils of single layer helix thermometer; only turns of one side are indicated.
C. Borosilicate glass thermometer envelope.
D. Finely divided ice and water.

The recommended measurement and calculation procedures are identical to those previously described for determining the total heating effect (see fig. 13). For the most precise work, all resistance measurements should be made at two currents and use made of the value of resistance calculated for zero current. Reiterating what was said earlier, an error due to the heating effect is introduced if the thermometer is not calibrated and used at the same current or is not in good thermal contact with its surroundings.

The external heating effect may be reduced by making the thermometer well a relatively close fit to the thermometer and placing a material of high thermal conductance in the annular space between the thermometer and the well. The material used to fill the space must not undergo an exothermic (or endothermic) reaction at the temperatures involved because such a reaction would additionally change the temperature of the thermometer from that of the surroundings. Examples of this difficulty that have been experienced in the NBS calibration laboratory are the slow decomposition of light mineral oil (at 122 °C) and oxidation of a steel bushing (at 444 °C). Difficulties associated with this type of reaction can be detected by comparing the derived values of resistance at zero current that have been obtained from measurements in which (i) the questionable material was used and (ii) the material was not present, or a better substitute was employed.

Another source of heat flux to and away from the platinum coil, and consequently a possible source of error, is radiation. If the sensor can "see" a surface that is appreciably hotter or colder, the power gained or lost by the resistor will result in its temperature being changed. In the triple-point of water measurements, radiation from lights in the room which is incident upon the top of the ice bath or triple point cell can easily produce an error of 0.0001 K (see sec. 7 on triple-point cell). The water-triple-point cell should, therefore, be immersed in an ice bath in which no extraneous radiation from sources above room temperature can reach the sensor of the SPRT. At a higher temperature (630 °C) an error as large as 0.01 °C can occur if a clear fused silica thermometer sheath is employed because "light piping" takes place. In this process radiation is conducted towards ambient temperatures within the wall of the thermometer sheath, being confined there by total internal reflections. Any technique which would eliminate these reflections would of course, eliminate this source of error. The error can be substantially reduced by coating the exterior wall with graphite paint as is depicted in figure 14 or by roughening (by sand-blasting) the external wall.

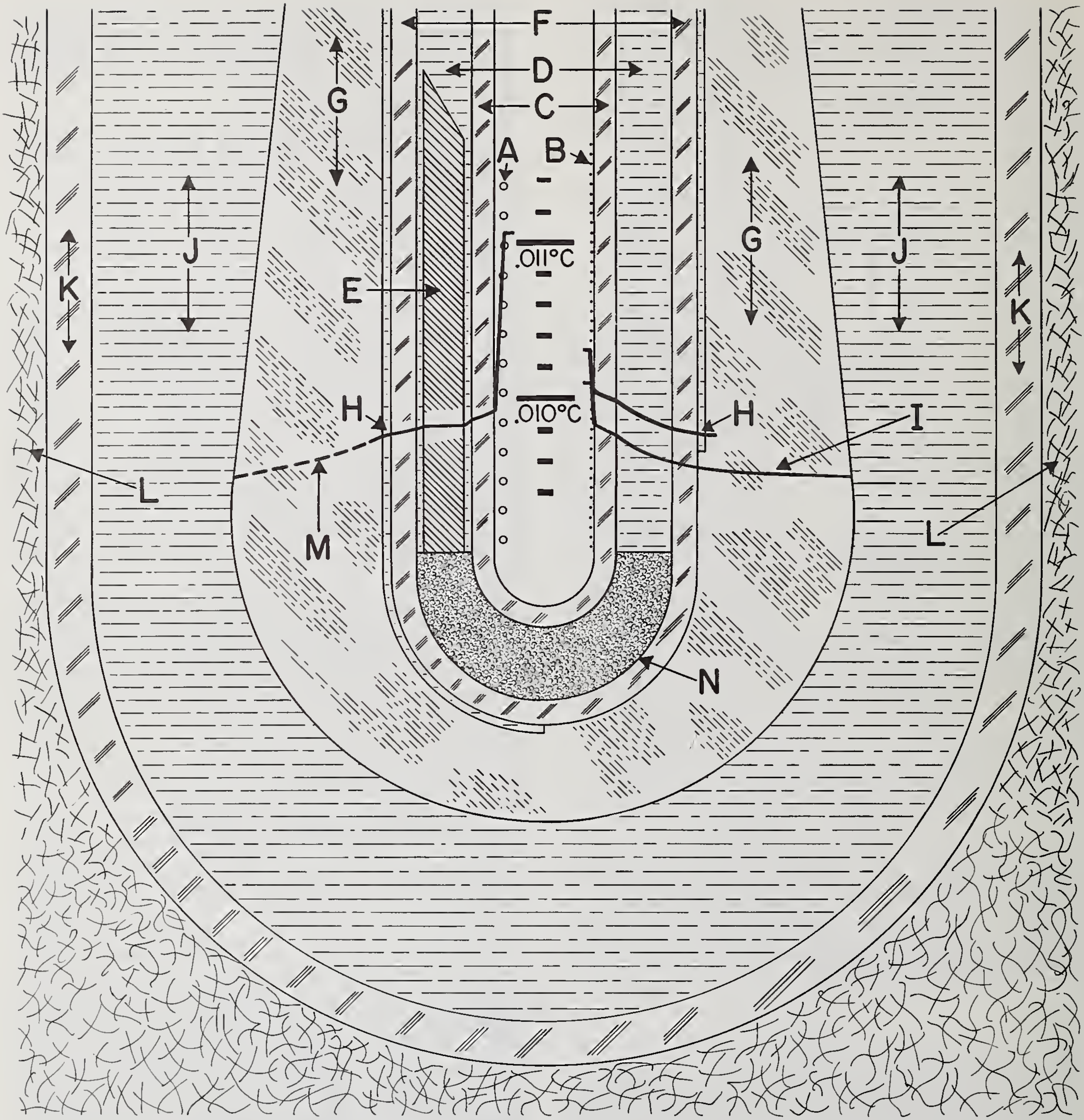

FIGURE 12. *Platinum resistance thermometers at 33 cm immersion in a water triple point cell.*

Temperature profile from the middle of thermometer coil out to the ice-water interface with 1 mA current.

A. Platinum coils of coiled filament thermometer; only coils of one side are indicated.
B. Platinum coils of single layer helix thermometer; only turns of one side are indicated.
C. Borosilicate glass thermometer envelope.
D. Water from ice bath.
E. Aluminum bushing (length not to scale).
F. Borosilicate glass thermometer well.
G. Ice mantle.
H. Inner melt.
I. No inner melting; temperature profile relative to the temperature of outer ice-water interface.
J. Water in cell.
K. Cell well (borosilicate glass).
L. Outside ice-water bath.
M. Temperature profile of the ice mantle.
N. Polyurethane sponge.

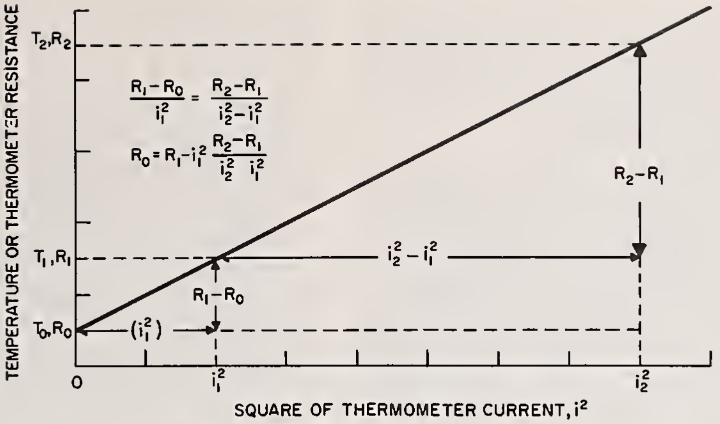

FIGURE 13. *Plot of SPRT resistance (temperature) versus the square of the thermometer current.*

The plot illustrates how the value of "zero current" thermometer resistance may be obtained graphically or algebraically from measurements at two currents.

R_0 = resistance of SPRT at "zero current."
R_1 and R_2 = resistances determined at currents i_1 and i_2, respectively.

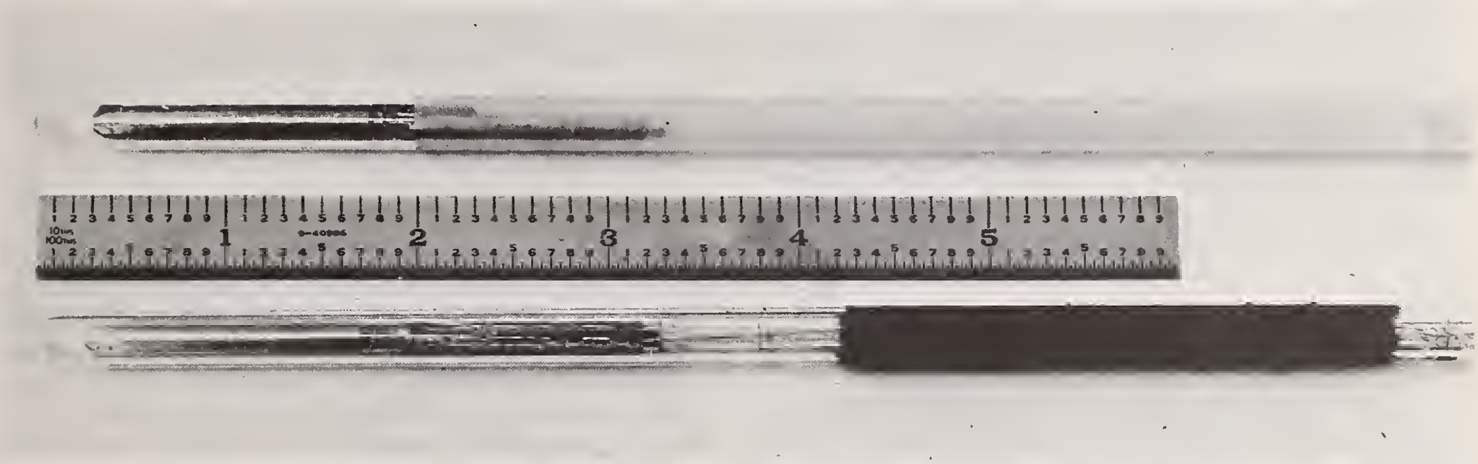

FIGURE 14. *Two methods for reducing radiation "piping" in fused silica thermometer sheaths.*
The sheath of the thermometer at the top was given a matte finish by "sand blasting." "Aqua dag" is shown painted on the sheath of the SPRT at the bottom.

5. Resistance Measurements

In this section, the salient features of instrumentation used to measure the resistance of SPRT's in the NBS calibration laboratory will be described so that the laboratory's general electrical measurement procedures may be understood. The discussion will neither give details on other instruments that are available for measuring electrical resistance nor special design features of particular instruments, nor a comparison of different measurement techniques. Some of the instruments will be mentioned by name and literature references will be cited to direct an interested reader to more detailed information.

The most suitable methods for and the limitations on achieving both accurate and reproducible resistance measurements depend on several factors. Johnson noise [31], which is inherent in any resistor (caused by the random movement of electrons within a conductor), is given by:

$$e = 7.43 \times 10^{-12} \sqrt{TR(\Delta f)}, \tag{5.1}$$

where e is the thermal agitation voltage in volts, Δf is the effective bandwidth in hertz, R is the resistance of the conductor in ohms, and T is the temperature of the resistor in kelvins. The constant 7.43×10^{-12} comes from $2\sqrt{k}$, where k is the Boltzmann constant. However, this frequently mentioned limitation is seldom the predominant source of uncertainty in resistance measurements with SPRT's. For example, using a 25-Ω thermometer at room temperature with an observation (averaging) time of 1 second, this noise is of the order of 0.6 nV—slightly less than the signal (1.0 nV) that results from a 1-μΩ unbalance when 1 mA is flowing through the thermometer. Spurious emf's, variations in contact resistance, mechanical or electrical disturbance of the detector system, and variations in thermometer lead unbalance, are sources of "noise," one or more of which contribute much more to the electrical measurement uncertainty than the Johnson noise in the vast majority of measurement systems.

The signal level at the null detector caused by the resistance unbalance can be raised by simply increasing the current through the thermometer. This may at times be a satisfactory solution; limitations are encountered, however, because increasing the current rapidly increases the uncertainty due to the self-heating of SPRT (see sec. 4) and may even introduce significant error due to the self heating of reference resistors in the bridge or potentiometer. The increased power dissipated in the thermometer may produce additional difficulties if the thermometer is used in a calorimeter system with a small heat capacity.

In addition to the uncertainty of null detection in a reasonable period of time, there is also the problem of referencing the unknown resistor to a single resistance standard. A large number of arrangements have been proposed and used for this purpose. They may be broadly classified as either bridge or potentiometer circuits that employ either alternating or direct currents with either resistors or inductive dividers to establish the ratio of the unknown resistor to a standard resistor. (A standard resistor is defined, hereafter, as a stable resistor or a combination of stable resistors of known value.)

5.1. Mueller Bridge

At the National Bureau of Standards the traditional instrument used with SPRT's is the Mueller bridge [37, 38]. There have been several modifications [21, 22, 26] of the bridge since its first appearance, but this discussion will only cover its principal features. A simple form of the bridge is shown schematically in figure 15. It is basically the classical equal-ratio-arm Wheatstone bridge with provisions for interchanging (commutating) the leads of a four-lead resistor in such a way that the average of two balances is independent of the resistances of the leads. Referring to figure 15 (NORMAL), when no current flows through the null detector, the voltages e_1 and e_2 are equal, and the bridge is said to be balanced. The equation of balance is:

$$R_{D_1}+R_C=R_X+R_T, \tag{5.2}$$

where R_{D_1} is the resistance of the variable decade balancing resistor,

R_C is the resistance of a lead from the bridge to the thermometer sensing element,

R_X is the resistance of the thermometer sensing element, and

R_T is a lead resistance similar to R_C.

After commutating the leads (marked C, T, c, and t) to the positions shown in figure 15 (REVERSE) the equation of balance is

$$R_{D_2}+R_T=R_X+R_C, \tag{5.3}$$

where

R_{D_2} is the resistance of the adjustable-decade balancing resistor that is required for the second balance of the bridge.

The addition of eqs (5.2) and (5.3) yields the value of R_X in terms of R_{D_1} and R_{D_2}:

$$R_{D_1}+R_{D_2}=2R_X \tag{5.4}$$

or

$$R_X=\frac{R_{D_1}+D_{D_2}}{2}. \tag{5.5}$$

An assumption made in the above equations is that R_T and R_C are constant during the time required for making the two balances.

Actually R_T and R_C need not be constant. If eqs (5.2) and (5.3) are rewritten as:

$$R_{D_1}+R_{C_1}=R_X+R_{T_1} \tag{5.6}$$

and

$$R_{D_2}+R_{T_2}=R_X+R_{C_2}\,, \tag{5.7}$$

and then added, there results

$$R_{D_1}+R_{D_2}+(R_{C_1}-R_{T_1})-(R_{C_2}-R_{T_2})=2R_X \tag{5.8}$$

or

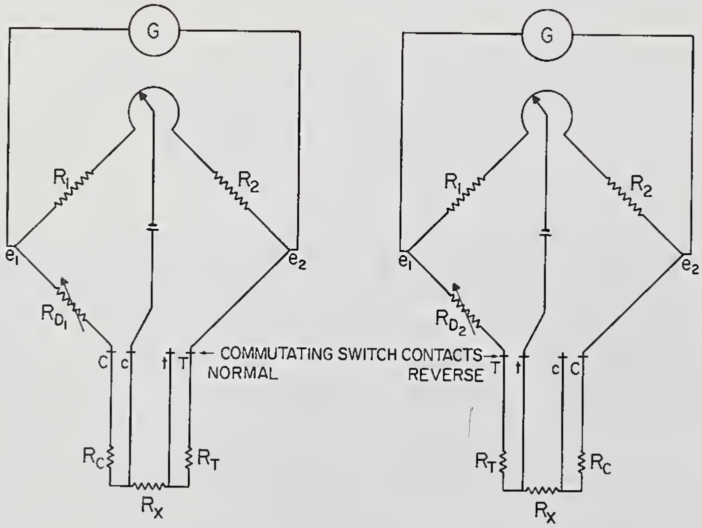

FIGURE 15. *Schematic of a Mueller bridge circuit in NORMAL and REVERSE thermometer connections.*

G is the null detector (galvanometer).
R_1 and R_2 represent the ratio arm resistors.
R_D represents the adjustable resistor.
R_C and R_T are the resistances of the "potential" leads of the SPRT.
R_X is the resistance of the SPRT.

$$R_X = \frac{R_{D_1} + R_{D_2}}{2} + \frac{(R_{C_1} - R_{T_1}) - (R_{C_2} - R_{T_2})}{2}. \qquad (5.9)$$

Thus, a sufficient condition for the measured resistance to be independent of the lead resistances is that the *difference* in the resistance of the two potential leads is constant during the period of observation. Equation (5.9) demonstrates that experimental emphasis could be placed on insuring that the leads be of equal length and cross section and that the temperature gradients between the leads be constant or only changing slowly, rather than the more difficult option of maintaining the temperature of the leads constant.

Successful operation of the Mueller bridge is dependent upon the reproducibility and self consistency (linearity) of the adjustable resistor indicated in figure 15 as R_D. The methods of accomplishing this include thermostating the resistors and employing special circuitry. (See fig. 1 of Appendix H. A procedure for calibrating the Mueller bridge is also described in Appendix H.) The circuitry has been designed to reduce the uncertainties that are associated with the variations in the contact resistances of the "dry" switches. For the 1-Ω and 0.1-Ω step decade resistors, the switch contact resistances are placed in series with the bridge ratio arms (R_1 and R_2), which are usually from 500 to 3000 Ω, so that the effect of the possible variations in the contact resistances (about 0.0005 Ω) can usually be neglected. For the measurement of the higher thermometer resistances this arrangement introduces uncertainties which may be significant. For a 25-Ω SPRT the uncertainty of 0.0005 Ω in 3000 Ω or 1 part in 6×10^6 in the ratio arm corresponds to only 1 μΩ or 0.01 mK when measuring 6 Ω (near −183 °C) but increases to 10 μΩ or 0.1 mK when measuring 60 Ω (near 350 °C). The decades with steps of 0.01 Ω or less are the Waidner-Wolff shunted decades [38] which reduce the effect of contact resistance in the switch by a factor of 250 or more. The switches for the 10-Ω step decade and the commutator are directly in series with the resistors in the adjustable arm of the bridge and the thermometer resistor; they have, therefore, mercury-wetted contacts. The mercury-wetted switches in the Mueller bridges employed at the National Bureau of Standards have an uncertainty of less than 2 μΩ when well maintained. All of the mercury contacts are normally cleaned every day before use. The mercury is removed by vacuum using a small polyethylene tip at the end of a vacuum line with a mercury trap. Fresh mercury is placed on each amalgamated contact. If the entire surface of the contact is not wetted by the new mercury, the surface is scrubbed (without removing the mercury) with the flat end of a solid copper rod about ¼ inch in diameter until the entire surface becomes wetted. The mercury is again removed and replaced with clean mercury. The switch is then reassembled and operated several times after which it is reopened to remove any mercury which has splashed onto the surrounding surfaces. The switch is then finally assembled for use. Switches with sliding contacts are exercised every day before use by revolving them 10 or 20 times; this is particularly important for the 1, 0.1, and 0.01 Ω decade switches. The sliding switches are cleaned occasionally with a lint-free cloth, either dry or moistened with benzene or "varsol". (Carbon tetrachloride is not recommended as it frequently contains impurities which will result in corrosion.) After cleaning, the contacts are lubricated with a light coating of pure petrolatum.

5.1.1. Bridge Ratio Arms

If the two ratio resistors, R_1 and R_2 in figure 15, change so as to become unequal, this may be compensated by adjusting the "tap" on the slide-wire resistor joining R_1 and R_2. Because this tap is in the battery arm of the bridge, the variations of its contact resistance are unimportant to the determination of thermometer resistances. The ratio arm resistors are adjusted to be equal by varying this tap on the slide wire between R_1 and R_2 until the interchanging of the ratio arms does not change the bridge balance. The accuracy of the ratio is limited by the uniformity of the resistance of the copper leads and switch contacts which are in series with R_1 and R_2 (those connecting the 1-Ω and 0.1-Ω decades) and, of course, by the sensitivity of the null detector.

Some versions of the bridge have incorporated into the commutator additional switch contacts which reverse the ratio arm resistors simultaneously with each commutation of the thermometer leads; the switch contacts of the 1-Ω and 0.1-Ω decades and their leads to the commutator switch are, however, not reversed. The effect of their variations must still be considered in the determination of the bridge resistance. Referring again to figure 15, but now with the ratio arms R_1 and R_2 interchanged in the reverse bridge connection, the equations of bridge balance with the normal and reverse connections are, respectively:

$$\frac{R_{D_1} + R_C}{R_T + R_X} = \frac{R_1}{R_2} = \frac{1+\epsilon}{1} \qquad (5.10)$$

and

$$\frac{R_{D_2} + R_T}{R_C + R_X} = \frac{R_2}{R_1} = \frac{1}{1+\epsilon}, \qquad (5.11)$$

where $\epsilon = (R_1 - R_2)/R_2$. After combining eqs (5.10) and (5.11) and eliminating R_T and R_C,

$$R_X = \frac{R_{D_1} + R_{D_2}(1+\epsilon)}{2+\epsilon}. \qquad (5.12)$$

By adding and subtracting $(R_{D_1} + R_{D_2})/2$ from the right-hand side of eq (5.12) and combining, the relation

$$R_x = \frac{(R_{D_1} + R_{D_2})}{2} + \frac{(R_{D_1} - R_{D_2})\epsilon}{2(2+\epsilon)} \quad (5.13)$$

is obtained. Equation (5.13) shows that if the bridge is operated to yield $R_{D_1}=R_{D_2}$, then $R_X=R_{D_1}=R_{D_2}$, without regard to the lack of equality of the ratio-arm resistors. In practice, because the thermometer leads (T and C) are usually made nearly equal, R_{D_1} and R_{D_2} will be only slightly different, typically much less than 0.002 Ω. Also, the equality of the ratio-arm resistors can be adjusted to better than 1 ppm with ease. Thus, the second term on the right of eq (5.13) is completely negligible in normally conducted Mueller bridge measurements which utilize simultaneous commutation of the SPRT leads and the ratio-arm resistors.

5.1.2. Bridge Current Reversal

The indicated balance of any *d-c* bridge is dependent upon the *iR* voltages across the elements of the circuit and the spurious emf's. The equations of bridge balance involve only resistance values; therefore, the effects of spurious emf's must be eliminated. This could be done by first observing the "indication" of the null detector with no bridge current, then balancing the bridge with current to the same indication. The indication of the null detector with no current includes the effect of any spurious emf's; when the bridge is balanced to the same indication with current in the bridge, the *iR* voltages are balanced if the effect of the spurious emf's remains unchanged. However, one should recognize that a change in the magnitude of the thermometer current will change the thermometer temperature (due to self heating) and that enough time must elapse before reading to allow the thermometer to attain a steady thermal state. During this time the effect of spurious emf's may change significantly. This problem can be simply surmounted by reversing the battery current with a snap action toggle switch, so that an essentially continuous heating power is retained in the thermometer (hence, a uniform self heating effect). In the process, the galvanometer or null detector sensitivity is effectively doubled. The rate at which the current reversals must be made is dependent on the rate of change of the spurious emf's.

5.2. A-C Bridge

If the current is reversed sufficiently rapidly, it is usually said that the bridge is an "a-c" bridge. Bridges operating at 400 hertz have been built at the NBS based on a design by Cutkosky [19]. These bridges were designed for use with SPRT's and include special provisions for use with thermometers having values of $R(0)$ as low as 0.025 Ω. The bridge utilizes an inductive ratio divider that eliminates the necessity of calibrating the bridge because the initial uncertainty of the divider is about 2 parts in 10^8 and appears to be stable. Additionally, the bridge requires only one manual resistance balance, the phase angle balance being automatic, and incorporates a built-in phase-sensitive null detector with which 1 μΩ in 25 Ω can be easily resolved. Small deviations from balance can be recorded continuously, the accuracy of recording these deviations being limited primarily by the resolution and linearity of the recorder. A small (usually less than 10 μΩ) error may be introduced in measuring a 25-Ω SPRT unless coaxial leads are used between the bridge and the thermometer head. (The heads of SPRT's have been modified to contain two BNC coaxial connectors. The two leads from one end of the SPRT coil were connected to the center "female" contracts of the BNC receptacles and the two leads from the other end of the SPRT were connected to the outer shells or the shield contacts.) For precision measurements the length of the pair of coaxial leads should not be greater than 15 meters to limit the dielectric losses of the shunt capacitance. Preliminary measurements on 25-Ω SPRT's indicate that, if the leads do not affect the measurements, the accuracy of the measured value in ohms of a thermometer element is limited by the accuracy to which the reference standard resistor is known. However, in the accurate determination of the resistance ratio, $R(t)/R(0)$, the stability, rather than the accuracy, of the reference standard is the important requirement. Further work is in progress at NBS to determine the dc-ac transfer characteristics of SPRT's.

5.3. Potentiometric Methods

The Mueller bridge and the Smith bridge methods [6, 7, 27] are suitable for a four-terminal determination of thermometer resistance if the lead resistances are relatively stable. When the lead resistances are variable, e.g., in measurements at low temperatures where the cryostat temperature is varying and the thermometer resistance is small, potentiometric methods become better suited to the measurements.

The potentiometric method of resistance measurement depends in principle upon the determination of the ratio of *iR* voltages developed across the SPRT and across a resistor of known resistance that is connected in series. The ratio is determined by comparisons with *iR* voltages that are developed in a separate resistance network and usually a separate current supply.

One of the major drawbacks of the potentiometric method has been the requirement of exceptionally high stability of current in both the potentiometer and in the SPRT circuits during the measurement period. However, the recent availability of highly stable current supplies and their continued improvement have made the potentiometric methods more popular.

Three circuits for the more commonly used potentiometric methods are illustrated in figure 16. To eliminate the effect of spurious emf's in the measurements employing circuits (a) and (b) the currents i_1 and i_2 are reversed. Four balances are necessary in each method, two with the current in one direction and two with the current reversed. Four balances are also necessary employing circuit (c) by reversal of the current i_1 and at the connection between the voltages to be compared. The readings for the normal and reverse connections are averaged. By using high quality reversing switches along with current sources of high resistance (to make the relative effect of small changes in resistance associated with the switching negligible compared to the total resistance of the circuit), the current in the circuit can be made more stable and the switch in series with the detector need not be disturbed during the current reversal process. Nevertheless, a shunt is needed across the

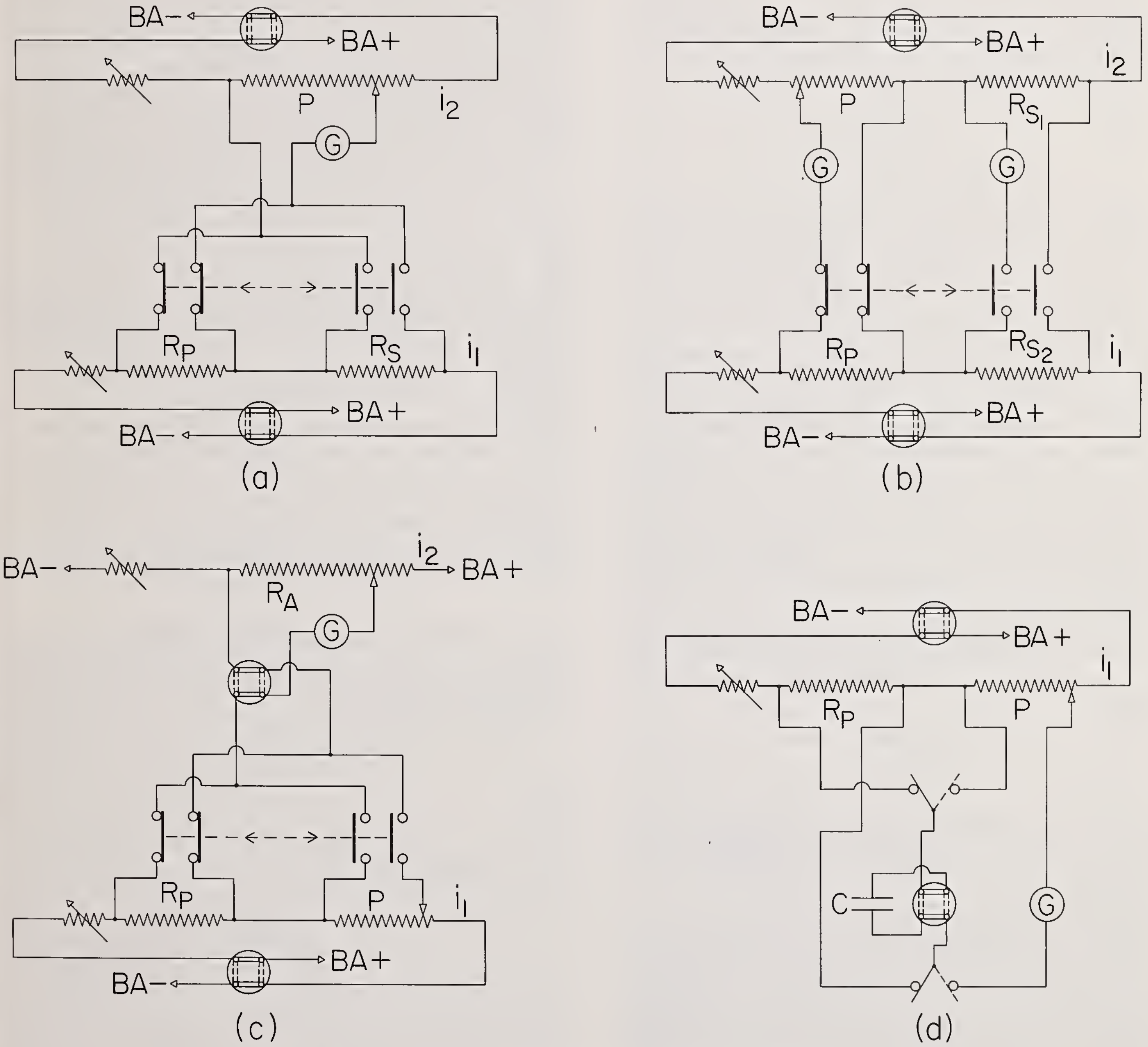

FIGURE 16. *Potentiometric circuits used to determine resistance values.*

R_p, the resistance to be determined.
R_s, standard resistor of precisely known value.
P, potentiometer or precisely known adjustable resistor.
R_A, adjustable resistor.
C, stable capacitor of low dielectric absorption, low absorption time constant, and high leakage resistance (polystyrene).
G, null detector
i_1, independent stable current in the unknown resistance circuit.
i_2, independent stable current in the potentiometer or in the voltage transfer instrument circuit.

detector to protect it because of asynchronous contact operation during the reversal. The discussion of the potentiometric methods to follow assumes that the required balances are made with current reversal.

Using circuit (a) of figure 16, the potentiometer P is successively balanced against the voltages i_1R_p and i_1R_s in terms of i_2P_p and i_2P_s, respectively, where R_p and R_s are the resistances of SPRT and standard resistor, respectively, and P_p and P_s are in the potentiometer resistance units. (P_p indicates the potentiometer resistance when its iR voltage is balanced against the SPRT; P_s indicates the potentiometer resistance when balanced against a standard resistor.) Thus, with $i_1R_p=i_2P_p$ and $i_1R_s=i_2P_s$, the SPRT resistance is given by

$$R_p=\frac{P_p}{P_s}R_s.$$

There is a wide variety of potentiometers that employ essentially the circuit shown in 16(a). The potentiometer iR voltages in some designs [43, 54, 55, 56] are developed with different currents in certain decades; in others, the iR voltages are adjusted by varying, by means of the potentiometer switches, the current through a fixed resistor. These designs keep the resistance of the detector circuit constant. In the "double potentiometer," which is designed to make two consecutive voltage balances more conveniently, there are duplicate sets of switches but only a single resistance network to develop iR voltages. A requirement of the potentiometer, whatever the design, is that the "resistance units" or the "iR voltage units" that are developed in the instrument for voltage balance be linear.

Two standard resistors are used in circuit (b). Current i_2 is adjusted until $i_2R_{s_1}=i_1R_{s_2}$; then P is adjusted until $i_2P_p=i_1R_p$. The SPRT resistance is given by:

$$R_p=\frac{R_{s_2}}{R_{s_1}}P_p. \tag{5.14}$$

In circuit (c), either current i_2 or resistance R_A is adjusted until $i_2R_A=i_1R_p$; then P is adjusted until $i_2R_A=i_1P_p$. The thermometer resistance is given directly by $R_p=P_p$.

The "isolating potential comparator" method described by Dauphinee [20] is an adaptation of circuit (c) where the voltage i_1R_p is set up as i_2R_A and measured as i_1P_p. The method is shown in circuit (d). The voltage i_1R_p appears across a high quality capacitor C and is compared with the voltage i_1P_p, P being adjusted until $i_1R_p=i_1P_p$. The break-before-make, double-pole chopper switches the capacitor alternately across R_p and then across P between 20 to 80 times a second. Extraneous voltages are canceled by reversing the current and the capacitor connections and averaging the second reading with the first.

The linearity of the potentiometer can be calibrated by comparing successive steps of one decade against the total (X) of the next lower decade. See Appendix I for a method of calibrating potentiometers.

6. Calculation of Temperatures from the Calibration Data and Observed Resistances

The SPRT is the standard interpolation instrument between the defining fixed points in the range 13.81 K to 630.74 °C. (See table 1 and Appendix A.) The "constants" of the interpolation formulae that relate the resistance of a particular SPRT to the value of its temperature on the IPTS–68 are obtained by resistance measurements at the defining fixed points. At NBS the "long stem" SPRT's are usually calibrated for application above 90 K; the capsule-type SPRT's are calibrated for use between 13 K and 250 °C or occasionally up to 400 °C. The equipment and procedures employed at NBS to achieve the fixed points are described in the next section. The resistance measuring instruments employed at NBS and other instruments that can be used for resistance measurements of SPRT have been described earlier in section 5. This section deals with the methods in use at NBS to obtain the constants of the interpolation formulae from the calibration measurements of an SPRT at the fixed points. Methods of calculating temperatures from the observed resistances, when the constants of the interpolation formulae are known are also described. At NBS all evaluations of equations and calculations pertaining to the SPRT are performed on a high-speed electronic digital computer (UNIVAC 1108).

6.1. Temperatures from 0 to 630.74 °C

From 0 to 630.74 °C the temperatures on the IPTS–68 are defined by

$$t_{68}=t'+M(t'), \tag{6.1}$$

where

$$M(t')=0.045\left(\frac{t'}{100}\right)\left(\frac{t'}{100}-1\right)\left(\frac{t'}{419.58}-1\right)\left(\frac{t'}{630.74}-1\right) \tag{6.2}$$

and

$$t'=\frac{1}{\alpha}\left(\frac{R(t')}{R(0)}-1\right)+\delta\left(\frac{t'}{100}\right)\left(\frac{t'}{100}-1\right). \tag{6.3}$$

In the official English version of the text of the scale (see Appendix A) the unit °C appears in the defining equations, e.g.,

$$t_{68} = t' + 0.045 \left(\frac{t'}{100\ ^\circ\mathrm{C}}\right)\left(\frac{t'}{100\ ^\circ\mathrm{C}} - 1\right)\left(\frac{t'}{419.58\ ^\circ\mathrm{C}} - 1\right)\left(\frac{t'}{630.74\ ^\circ\mathrm{C}} - 1\right)\ ^\circ\mathrm{C}. \qquad (6.4)$$

In this discussion the equations will be simplified by omitting the units. Similarly $R(t^\circ\mathrm{C})$ and $R(0^\circ\mathrm{C})$ will be simplified to $R(t)$ and $R(0)$. In addition, t_{68} will, henceforth, be abbreviated to t. The constants $R(0)$, α, and δ are determined from calibration resistance measurements of the SPRT at the triple point of water (TP), the steam point or the tin point, and the zinc point. (The tin point is now employed at NBS.) The constants may be more conveniently obtained from (6.1) and from the relation, equivalent to equation (6.3), given by:

$$W(t') = R(t')/R(0) = 1 + At' + Bt'^2 \qquad (6.5)$$

where $R(t')$ is the observed resistance at the temperature t' and $R(0)$ is the resistance at 0 °C. The constants A and B are related to α and δ by

$$A = \alpha(1 + \delta/100), \qquad (6.6)$$

$$B = -10^{-4}\alpha\delta; \qquad (6.7)$$

also,

$$\alpha = A + 100B, \qquad (6.8)$$

$$\delta = -10^4 B/(A + 100B). \qquad (6.9)$$

Equations (6.3) and (6.5), appear to be of the same form as the earlier formulations of the International Temperature Scale [15, 46, 48]. But, the value of temperature, t, on the IPTS–68 is not equal to t', and the value of t' is not the value of temperature on the IPTS–48 because the definitions of the two scales are different (see Appendix B).

The value t', obtained from (6.3) or (6.5), may be considered to be a first approximation to t, the value of temperature on the IPTS–68. Equation (6.2) gives the adjustment to be made to the value t' to yield t. This adjustment was included in the definition of the IPTS–68 with the intention of bringing the scale into closer agreement with the thermodynamic scale. Therefore, at any given hotness (except certain fixed points) the values of t' and t are different, but *they both represent the same hotness*; accordingly, the resistances $R(t')$ and $R(t)$ are equal. The observed SPRT resistance will, hereafter, be indicated by $R(t)$. See Appendix D.

At the NBS, the SPRT that is received for calibration is first annealed between 470 and 480 °C for about four hours and then allowed to cool in air at the ambient conditions. The calibration measurements at the fixed points are made in the following sequence: TP, zinc point, TP, tin point, and TP. If any observed resistance is questioned, the measurement is repeated. Any additional tin or zinc-point measurements will usually be bracketed before and after by TP measurements in order to check any change in the TP resistance that may occur. Whenever the TP resistance of a SPRT changes by more than 75 $\mu\Omega$ (depending upon the design) in the course of calibration the complete sequence of calibration, including the annealing, is repeated.

A measuring current of 1 mA through the resistor of the thermometer is normally used. The sequence of measurements with the Mueller bridge at each of these fixed points is, unless additional measurements are found necessary, NRRN, where N = normal and R = reverse connections of the bridge. The readings are corrected for the "bridge calibration" and "bridge zero" and averaged (see sec. 5 on Mueller bridge resistance measurements).

For the tin and zinc-point calibration measurements, the "equilibrium temperatures" at the external surface of the SPRT near the resistance coil are computed and assigned to the observed resistance values. The experimental equilibrium temperature value is slightly different from the IPTS–68 value of the fixed-point temperature; the value of the equilibrium temperature at the outer surface of the thermometer sheath is obtained by correcting the IPTS–68 fixed-point temperature value for the hydrostatic head of the liquid metal column (also, for any significant deviation from 1 atm gas pressure over the fixed-point cell) and for the *external* heating effect of the SPRT in the particular cell. In the case of the measurements in a triple point of water cell, the assigned value of the temperature is adjusted for the temperature effect of the hydrostatic head of the water column and external heating effect of the SPRT.

The value of $R(\mathrm{TP})$ that is measured after the $R(\mathrm{tin})$ measurement is employed to obtain $W(\mathrm{tin})$; in a similar manner, the value of $R(\mathrm{TP})$ measured after the $R(\mathrm{zinc})$ measurement is employed to obtain $W(\mathrm{zinc})$. The value of $R(0)$ is calculated from $R(\mathrm{TP})$ according to eq (6.5) by assigning $B = 0$ for the small temperature interval of the evaluation. The value of A employed is $3.98485 \times 10^{-3}\ ^\circ\mathrm{C}^{-1}$, which is an average value for SPRT's calibrated at NBS in the past few years. The uncertainty in the adjustment of the value from $R(\mathrm{TP})$ to $R(0)$ in this manner is about $\pm 1 \times 10^{-8}\ R(0)$ or $\pm 0.25\ \mu\Omega$ for a 25 Ω SPRT and is, therefore, negligible. The thermometer constants A and B are obtained by simultaneous solution of eq (6.5) from the values of $W(\mathrm{tin})$, $t'(\mathrm{tin})$, and $W(\mathrm{zinc})$, $t'(\mathrm{zinc})$. The constants α and δ are obtained from A and B according to eqs (6.8) and (6.9). The printed results of the computer analysis of calibration data include, in addition to tables of $W(t)$ or $R(t)$ versus t, the observed resistances (adjusted for bridge calibration and bridge zero) at the three fixed points, and the values of $R(0)$, α, δ, A, and B.

Several possibilities are available to the user for computing temperatures and for checking the calibration of his SPRT. First, in eq (6.3) $R(0)$, α, δ may be taken as invariant and t' calculated from the measured $R(t)$. Second, in eq (6.3) only α, and δ may be taken as invariant, requiring then the determination of $R(t)/R(0)$. The first method is equivalent to using a table of R versus t; the second method is equivalent to using a table of $R(t)/\mathrm{R}(0)$ versus t. Equation (6.3) may be rewritten as

$$t'=100\,\frac{R(t)-R(0)}{R(100)-R(0)}+\delta\left(\frac{t'}{100}\right)\left(\frac{t'}{100}-1\right). \qquad (6.10)$$

In eq (6.10), as a third method, $R(100)-R(0)$, α and δ may be taken as constant, requiring the determination of $R(t)-R(0)$ to calculate t'. The fourth possibility seen with equation (6.10) is to take only δ as constant which requires the determination of $\dfrac{R(t)-R(0)}{R(100)-R(0)}$. The ultimate extension of this procedure of reducing the SPRT constants is not to assume that anything has remained constant and recalibrate the thermometer every time it is used. In 80 years of platinum thermometry each of the above assumptions or its equivalent has been made by investigators.

From the standpoint of accuracy the worst of these assumptions is the first, which requires only the measurement of $R(t)$. This choice may be dictated, however, by convenience or economy. If values of $R(0)$ can be obtained, the choice is not as clear between using $R(t)/R(0)$ or using $R(t)-R(0)$ to calculate the value of temperature. Experience with a particular thermometer may help one make the choice. Lacking this, one may be guided by the experience of the NBS and the National Research Council of Canada which indicates that below 500 °C the second assumption, namely, that α and δ are constant and using the value of $R(t)/R(0)$ to calculate a value of temperature, is the most satisfactory. This assumption has the distinctive practical advantage of being dimensionless (i.e., being independent of the size of the resistance unit). If request is made at the time the SPRT is submitted to NBS for calibration, tables based on any one of the assumptions discussed in this paragraph, in a form suitable for linear interpolation, can be provided from the calibration data.

When many observations of $R(t)$, such as in specific-heat measurements, are to be converted to t and a high speed electronic computer is available, eqs (6.1), (6.2), and (6.3) or (6.10) may be more convenient than tables. Iteration methods are best employed in the solution of either eq (6.3) or (6.10). For example, an approximate t' can first be obtained neglecting the term containing δ. Improved approximations are obtained by substituting successive solutions into the complete equation, stopping when t' does not change more than desired limits.

If it is desired to generate a table of values of $W(t)$ versus values of t, entries for integral values of t at one-degree intervals seem quite useful. The error contributed by using simple linear interpolation within the one-degree intervals is less than 10^{-4} °C in the range from 0 to 630 °C. The calculation of the table is most easily performed employing eqs (6.1) and (6.5). The only difficulty is that (6.5) gives W as a function of t' rather than t, hence if W is to be tabulated as a function of t the value of t' corresponding to the value of t must be first obtained and used in (6.5). By rearranging (6.1),

$$t'=t-M(t'). \qquad (6.11)$$

Below 472 °C the error introduced by replacing (6.11) with $t'=t-M(t)$ is less than 1.2×10^{-4} °C. The error in the value of t' to be used may be reduced below 10^{-5} °C by a single iteration of eq (6.11). A very efficient equation for calculating the nth entry for tables with intervals of less than 10 degrees is

$$t'_n=t_n-M(t_n-M(t_n-M(t'_{n-1})), \qquad (6.12)$$

Equation (6.12) does not require iteration and it may be shown that it introduces an error of only $[dM(t')/dt']^2$ times the tabular interval.

6.2. Temperatures Below 0 °C

From 0 °C down to −259.34 °C, the temperatures on the IPTS−68 are defined by

$$W(t)=W^*(t)+\Delta W(t), \qquad (6.13)$$

where $W(t)=R(t)/R(0)$ is the ratio of the observed resistance of the SPRT at temperature t to that at 0 °C and $W^*(t)$ is a reference value of the resistance ratio given by the function

$$t=\sum_{i=1}^{20}A_i\,(\ln W^*(t))^i. \qquad (6.14)$$

In this monograph the symbol $W^*(t)$ will replace the symbol $W_{\mathrm{CCT-68}}(T_{68})$ used in the official text of the IPTS−68 to represent the reference (standard) values of W. The values of $W^*(t)$ represent a fictitious but not "ideal" platinum resistance thermometer. The constants A_i of the reference function are listed in Appendix A. The deviations, $\Delta W(t)$ of eq (6.13), are expressed by simple polynomial equations of specified form; the constants of these equations are determined from measurements of $W(t)$ at the appropriate defining fixed points and the corresponding values of $W^*(t)$. The intermediate temperatures are determined from the corresponding measured values of $W(t)$, the appropriate polynomial equation for $\Delta W(t)$, and eqs (6.13) and

(6.14). (See Appendix A which lists the four temperature ranges, the associated polynomial equations, and the fixed points defined by the IPTS−68 in the range −259.34 to 0 °C.) The text of the scale, given in Appendix A, states the deviation equations for use below the oxygen point as functions of T (in kelvins). This form of these equations will also be used in this monograph because of their greater simplicity. It follows that eq (6.13) becomes

$$W(T)=W^*(T)+\Delta W(T), \qquad (6.13a)$$

and the reference function (6.14) becomes

$$T=273.15+\sum_{i=1}^{20} A_i\,(\ln W^*(T))^i. \qquad (6.14a)$$

Procedures for obtaining the calibration constants for the polynomial equations will be described later in this section.

The calculation of $W^*(t)$ must be carried out with a large number of digits. At NBS approximately 17 decimal digits are used. A table of $W^*(t)$ at closely spaced values of t is given in Appendix E together with a short Fortran subroutine which will produce this table efficiently (no more than two iterations per entry).

The following procedure may be employed to convert the measured values of $W(t)$ to t. From eq (6.13) is obtained

$$W^*(t)=W(t)-\Delta W(t). \qquad (6.13b)$$

When eqs (6.14) and (6.13b) are combined,

$$t=\sum_{i=1}^{20} A_i \ln[W(t)-\Delta W(t)]^i. \qquad (6.15)$$

To solve for t, an approximate value of t is first obtained by neglecting $\Delta W(t)$ and evaluating eq (6.15) with the measured $W(t)$ only. (A closely spaced table of $W^*(t)$ could be employed for this evaluation.) The approximate value of t is used with the appropriate polynomial equation to obtain an approximate value of $\Delta W(t)$ which is then employed with $W(t)$ in eq (6.15) (or table of $W^*(t)$) to obtain t. The process is repeated until the value of t no longer changes more than the desired limits.

Considering the complexity of the reference function (eq (6.14)), a high speed electronic computer or a table compiled by such a computer at small enough temperature intervals (0.1 to 1 °C, depending upon the value of t) to permit linear interpolation is a necessity.

Although a table of $W^*(t)$ could be employed to obtain t from the measured value of $W(t)$ in accordance with the procedure outlined above, a direct interpolation in a table of t versus $W(t)$ (measured) is obviously more convenient. Such a table can be generated from the relation given by eq (6.13). Values of $\Delta W(t)$ can be calculated at appropriate values of t, using the polynomial equations obtained from calibration, and combined with the corresponding values of $W^*(t)$ to obtain $W(t)$. (For tables furnished by NBS, see Appendix J.)

6.2.1. −182.962 °C (90.188 K) to 0 °C (273.15 K)

The polynomial deviation function specified by the text of the IPTS−68 between −182.962 °C (90.188 K) and 0 °C (273.15 K) is:

$$\Delta W(t)=W(t)-W^*(t)=A_4t+C_4t^3(t-100). \qquad (6.16)$$

The constants A_4 and C_4 in eq (6.16) are determined by calibration measurements at the steam point, the triple point of water, and the boiling point of oxygen, or alternatively, at the zinc point, the tin point, the triple point of water, and the boiling point of oxygen. Calibration measurements at the oxygen point and again at the TP are added to the sequence of measurements described earlier (sec. 6.1) for the range 0 to 630.74 °C. The value of $W(t)$ at the oxygen point is obtained from R(oxygen) and $R(0)$. The value of α (see eq 6.3)) is required from the measurements above 0 °C in order to evaluate the constant A_4 of eq (6.16). From the definition of α,

$$\alpha^*=(W^*(100)-1)/100 \qquad (6.17)$$

and

$$\alpha=(W(100)-1)/100; \qquad (6.18)$$

hence,

$$\Delta W(100)=W(100)-W^*(100)=100(\alpha-\alpha^*). \qquad (6.19)$$

(The $\alpha^*(=0.0039259668\ ^\circ\mathrm{C}^{-1})$ is the value used in forming the $W^*(t)$ table [51].) Substituting eq (6.19) in eq (6.16) with $t=100$ °C, there is obtained

$$A_4=\alpha-\alpha^*=\Delta W(100)/100. \qquad (6.20)$$

The value of C_4 is determined by measurement of $W(t)$ at the boiling point of oxygen. The deviation $\Delta W(t)=W(t)-W^*(t)$ at the oxygen point and the value of A_4 from (6.19) gives the constant C_4

$$C_4=\frac{\Delta W(t)-A_4t}{t^3(t-100)}, \qquad (6.21)$$

where t is the oxygen normal boiling point temperature in °C.

The first and second derivative of the IPTS—68 reference function (eq(6.14)) have the same value at 0°C as those of the function $W(t)$ obtained from eqs (6.1), (6.2), and (6.3) with the thermometer constants $\alpha = 3.9259668 \times 10^{-3}\ ^\circ\mathrm{C}^{-1}$ and $\delta = 1.496334$ °C. On the other hand, these derivatives of the temperature-resistance function for real SPRT's are not in general continuous through 0 °C. The discontinuities are, however, very small and may be neglected. See Appendix F for detailed discussion of this point.

6.2.2. 13.81 to 90.188 K

From 13.81 to 90.188 K, the SPRT's that are received at the NBS are not calibrated at the defining fixed points established by phase equilibrium in accordance with the text of the IPTS–68, but they are calibrated by a procedure that is equivalent to the IPTS–68. The SPRT's are calibrated at NBS by intercomparison with SPRT standards that maintain the NBS–IPTS–68 (see sec. 7 for details). The temperatures of the intercomparison calibration measurements have been selected so that the measurements, which are made at temperatures very close to those of the defining fixed points, are supplemented by measurements suitably placed between these points. Usually sixteen measurements are made between 12 and 90 K. The data are analyzed by a method of least squares and values of resistances corresponding to the defining fixed-point temperatures are evaluated. The values of $W(T)$ are then computed for the defining fixed-point temperatures: 13.81, 17.042, 20.28, 27.102, 54.361, and 90.188 K, and applied to eq (6.13) or (6.13a) to obtain the deviation $\Delta W(T)$ at each of these temperatures. The discussion to follow describes procedures for obtaining the constants of the specified polynomial deviation functions.

Below the oxygen point the deviation, $\Delta W(T)$, of the value of $W(T)$ from the reference function, $W^*(T)$, is defined with a succession of polynomial equations, each covering a relatively short range. Each of these deviation equations is described by a three or four term power series in T (where T is the value of temperature on the International Practical Kelvin Scale: $T=t+273.15$ K) and is a smooth downward extension of the deviation curve found for the temperature range immediately above.

6.2.2.1. 54.361 to 90.188 K

In the range from the oxygen normal boiling point (90.188 K) down to the oxygen triple point (54.361 K) the deviation function is

$$\Delta W(T)=A_3+B_3T+C_3T^2. \qquad (6.22)$$

The three constants A_3, B_3, and C_3 are determined from the values of $\Delta W(T)$ at the two oxygen fixed points and the value of the first derivative of the deviation equation (6.16) specified for above the oxygen point,

$$\left(\frac{d\Delta W(t)}{dt}\right)_{O_2}=A_4-300C_4t^2+4C_4t^3, \qquad (6.23)$$

where the t in eq (6.23) is the value of the oxygen normal boiling point temperature (−182.962 °C). At the oxygen point the first derivative of the deviation function (eq (6.22)) extending below the oxygen boiling point is set equal to the first derivative of the deviation function (see eqs (6.16) and (6.23)) above the oxygen normal boiling point; i.e.,

$$\frac{d\Delta W(T)}{dT}=B_3+2C_3T=\left(\frac{d\Delta W(t)}{dt}\right)_{O_2}, \qquad (6.24)$$

at $t=-182.962$ °C and $T=90.188$ K. Thus, the deviation functions and their first derivatives are forced to be equal at the point of joining.

6.2.2.2. 20.28 to 54.361 K

The deviation function defined by eq (6.22) is joined smoothly at the oxygen triple point ($T=54.361$ K) in a completely analogous manner as described in section 6.2.2.1, with the deviation function between 20.28 and 54.361 K given by

$$\Delta W(T)=A_2+B_2T+C_2T^2+D_2T^3. \qquad (6.25)$$

The constants are determined from the values of $\Delta W(T)$ at the normal boiling points of equilibrium hydrogen (20.28 K) and neon (27.102 K) and the triple point of oxygen (54.361 K) and by equating its derivative and the derivative of eq (6.22) at the point of joining (54.361 K).

6.2.2.3. 13.81 to 20.28 K

A fourth deviation function

$$\Delta W(T)=A_1+B_1T+C_1T^2+D_1T^3 \qquad (6.26)$$

covers the range 13.81 to 20.28 K. The constants are determined from the values of $\Delta W(T)$ at the triple point (13.81 K) and the normal boiling point (20.28 K) of equilibrium hydrogen and at the temperature of 17.042 K (boiling point of equilibrium hydrogen at 25/76 atm) and by equating the derivative of eq (6.26) and the derivative of eq (6.25) at the point of joining (20.28 K).

6.2.3. Calibration Tables Between 13.81 and 273.15 K (0 °C)

At NBS, the coefficients of the deviation functions obtained through the procedure outlined above allow the values of $\Delta W(T)$ to be calculated over the temperature range specified for each of the deviation functions. Values of $W(T)$ for the thermometer being calibrated are then given by

$$W(T)=W^*(T)+\Delta W(T). \qquad (6.13a)$$

Values are calculated to provide tables of $W(T)$ or $R(T)$ versus T at equal temperature intervals sufficiently small to permit linear interpolation. In the region 13 to 90 K the tabulations are usually given at 0.1 K intervals and from 90 to 273 K at 1 K intervals.

6.3. Errors of Temperature Determinations

The usefulness of a measured value of temperature depends strongly upon both the amount of its uncertainty and the knowledge of the amount of its uncertainty. Reduction of the errors contributing to the uncertainty is primarily limited by the ingenuity of the experimentalist; there is, however, an inherent source of uncertainty or spread of temperature values on any practical scale which is predicated upon the use of real materials, even if "perfectly" calibrated thermometers are used and no error is introduced in the measurements. Unfortunately only scattered data are available from which one could infer the degree of this ambiguity or possible spread of values of the defined IPTS−68. McLaren [33] did not find a distinctive difference among the measured temperature values at 321 and 231 °C of eleven thermometers with α ranging from 0.003921 $°C^{-1}$ to 0.003926 $°C^{-1}$. (Five of the eleven thermometers met the requirements of IPTS−68, i.e., α equal to or greater than 0.0039250 $°C^{-1}$.) The sensitivity of McLaren's test was about ±0.5 mK; his results indicate a spread of temperature values which are an order of magnitude smaller than the spread reported earlier for seven thermometers of much less pure platinum (α from 0.003909 $°C^{-1}$ to 0.003925 $°C^{-1}$) by Hoge and Brickwedde [29]. These latter thermometers, while acceptable on the ITS−27 which was in use at the time of their work, were not made of sufficiently pure platinum to meet the requirement of the IPTS−68 that α equal or exceed 0.0039250 $°C^{-1}$. The thermometers that were investigated by Hoge and Brickwedde showed a spread of 7 mK between −190 and 0 °C, and spread of 1.3 mK between 0 and 100 °C. The six thermometers measured between 100 and 444 °C had a maximum spread of 12 mK. Data which would indicate the spread of interpolated values of temperature on the IPTS–68 below 0 °C due to the variations in presently acceptable platinum wire is sparse and inadequate [11, 36, 44]. On the basis of existing data, Bedford and Ma [9] estimated that the IPTS–68 is reproducible to ± 3 mK between 14 and 20 K, ± 1 mK between 20 and 54 K, ± 2 mK between 54 and 90 K, and ± 5 mK between 90 and 273 K. It should be pointed out that, in addition to unaccounted for variations in the platinum wire, a spread in the temperature values may also arise from incomplete definitions of the materials that define fixed points (e.g., their isotopic composition).

The remainder of this section will deal with errors that arise both from errors of measurement by the user and from errors of calibration of the thermometer.

Many of the same kinds of error sources plague both the calibrator and the user. The errors may be divided into two categories; first, a temperature error, i.e., a difference between the actual thermometer temperature and the temperature of the point of interest or reference temperature, and second, a resistance error, i.e., a difference between the measured resistance and the resistance of the sensor. For example, temperature error occurs because of an unknown difference between the temperature of the thermometer and the equilibrium temperature realized in a fixed-point apparatus used for calibration. (The calibrator is also plagued, of course, by any unknown difference between the equilibrium temperature realized in his fixed-point apparatus and the equilibrium temperature defined by IPTS−68.) Temperature error also occurs because of unknown temperature differences between the thermometer and the object of interest in the user's apparatus. The experimenter must decide where to place the thermometer to minimize the magnitude of this error. Several possible sources of temperature error are discussed in section 4. In considering the errors of thermometer resistance measurements one must not only examine the reproducibility and calibration of his measurement equipment but also his measurement technique. Sections 5, 7, and 4 indicate possible sources of resistance or resistance ratio error.

The following analysis describes the error in the calculated value of temperature that results from errors made in calibrating the SPRT at the fixed-points. Any additional imprecision and inaccuracy introduced by the user are assumed to be best known by him.

At any value of temperature, t, the total differential of t for the SPRT is,

$$dt = \sum_i \left(\frac{\partial t}{\partial t_i}\right) dt_i + \sum_i \left(\frac{\partial t}{\partial W_i}\right) dW_i, \qquad (6.27)$$

where t_i is the temperature attributed to the calibration fixed point and W_i is the value of the resistance ratio, $R(t_i)/R(0)$, attributed to that temperature. The analysis may be simplified by converting any error in the value of temperature attributed to the calibrating fixed point to an equivalent error in W at that fixed-point; then, dt_i of eq (6.27) vanishes and eq (6.27) reduces to

$$dt = \sum_i \left(\frac{\partial t}{\partial W_i}\right) dW_i, \qquad (6.28)$$

where dW_i now includes the contribution from errors in the values of temperature as well as the errors in the values of $R(t)/R(0)$ at the calibration points. By utilizing eq (6.28) the error in the value of temperature t on the IPTS−68 caused by an error in W_i corresponding to a positive unit error in t_i was evaluated as a function of temperature. Figure 17 shows the errors in the values of temperature that would result from the calibration errors at each of the fixed points. For each curve the calibration error is taken to have occurred at only one fixed point with no calibration error at the other fixed points.

The measurement at the triple-point of water should be a part of platinum resistance thermometry work and $W(t)$, i.e., $R(t)/R(0)$, should be used in calculating temperatures (see sec. 6.1). The value

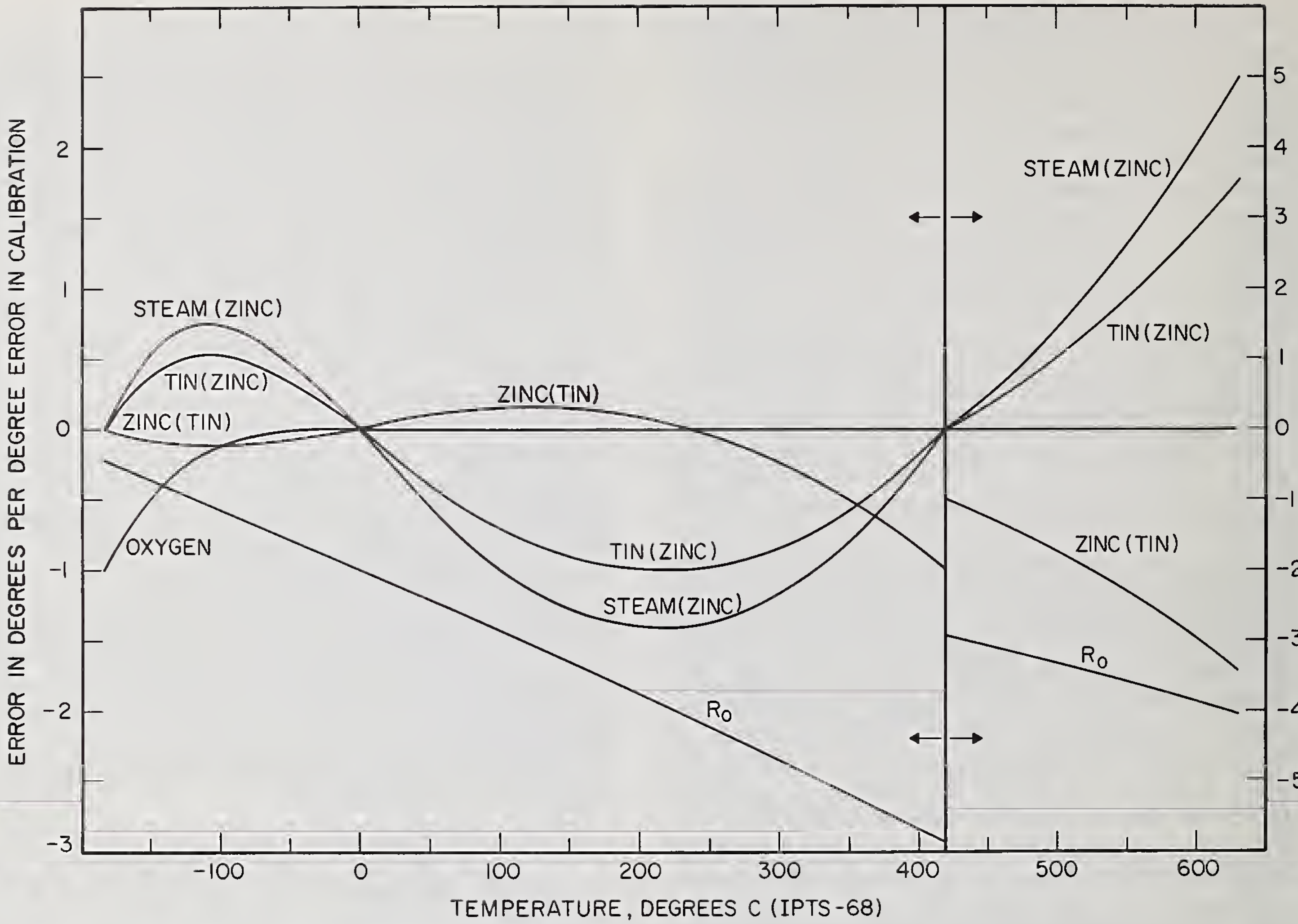

FIGURE 17. *The error at various temperatures propagated from errors made in the calibration of a platinum resistance thermometer.*

The curves show the error (i.e., departure from the IPTS–68) in the values of temperatures caused by a unit positive (hotter) error in realizing the temperature (hotness) of one given calibration point. The calibration at the triple point of water is assumed to have been made without error. The fixed point at which the error was made is indicated on the curve. The error curve depends not only upon the particular fixed point at which the error occurred, but also upon which other fixed points above 0 °C were employed in the calibration. The calibration at the other fixed point above 0 °C, indicated in parenthesis, is assumed to have been performed without error. A calibration error at the oxygen normal boiling point does not introduce an error in the measured values of temperature above 0 °C. The first derivatives of the ZINC(TIN) curve and the TIN(ZINC) curve are not continuous through 0 °C.

The curve marked R_0 shows the error that would be introduced if the experimenter makes a unit positive error in realizing the temperature (hotness) of 0 °C and then calculates the value of a temperature from the value of $R(t)/R(0\ °C)$.

of $W(t)$ is particularly sensitive to the errors of $R(0)$ in applications at high temperatures where any error of $R(0)$ becomes amplified. Figure 17 shows the error curve resulting from an error in $R(0)$ corresponding to a positive unit error in temperature.

The estimated uncertainties of the calibration measurements obtained at NBS are 0.002 or 0.003 K at the oxygen normal boiling point, 0.0002 K at the triple point of water, 0.001 K at the tin point, and 0.001 K at the zinc point. These uncertainties are estimates; hopefully, in the future, all of the desired documentation of these estimates will become available. The experimenter could estimate his overall error by statistically summing his measurement error with the calibration errors that can be obtained from the curves on figure 17 and the above estimate of calibration uncertainties. (For further details see Appendix G.)

7. Calibration

The calibration of a SPRT on the IPTS–68 involves the measurement of the resistance when the thermometer is at the temperature of the prescribed defining fixed points. This section deals with the equipment, the preparation of fixed-point cells, and the procedures employed at NBS to realize the temperatures of the prescribed defining fixed points for calibrating SPRT's.

In practice, thermometer calibrations are not made at exactly the temperatures of prescribed equilibrium conditions for the fixed points. For small known departures from the prescribed conditions a temperature correction can satisfactorily be made, e.g., such a departure may exist because of the hydrostatic head at the level of the thermometer resistor in calibrations that employ triple-point or freezing-point cells. (The effect of pressure deviations is given in the text of IPTS–68, Appendix A.) Gross departures from the prescribed conditions usually require extensive efforts to establish that the adjusted temperature and resulting calibration are sufficiently close to the IPTS–68.

The calibration apparatus at the NBS has been designed to be used with the great majority of SPRT's. Capsule type SPRT's, however, are mounted in special stainless-steel holders (fig. 18) before they are calibrated in fixed-point equipment with deep thermometer wells.

7.1. Triple Point of Water

The triple point of water (0.01 °C) is the most useful and important of the defining fixed points for calibrating SPRT's. The virtues of regular thermometer measurements at the triple point of water are so great that all but the most casual measurements of temperature with a SPRT should include a reference measurement at the triple point. The triple point is realized in a sealed glass cell (fig. 19) containing ice, water, and water vapor. When the cell is in use it may be placed in an ordinary crushed ice-water bath. Figure 19 shows the cell and ice-bath system used at NBS.

The cell is first immersed in the ice bath with the mouth of the re-entrant thermometer-well above the surrounding ice-water level. The well is thoroughly dried, then filled with crushed Dry Ice[2] and maintained full for about 20 min by replacing the sublimated Dry Ice. The initial freezing of water within the triple-point cell will occur several degrees below the triple-point temperature because water easily supercools. When the water does start to freeze fine needles of ice crystals (dendrites) are initially formed and protrude from the wall of the well into the liquid. The fine needles quickly cover the well but soon disappear to form a clear coating of ice (on the well) that will grow and become a 4 to 8 mm thick mantle in about 20 min. It is important to keep the well completely full of Dry Ice during this period. (If the Dry Ice level in the well is allowed to drop several inches and then the well is refilled, the ice mantle is very apt to crack. The desired triple-point temperature may not be achieved if a crack

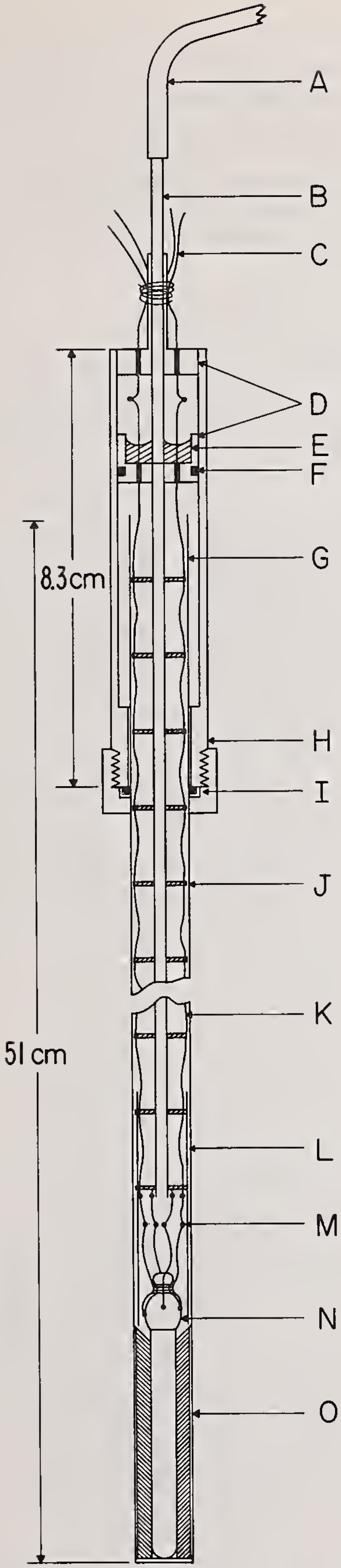

FIGURE 18. *Holder for capsule-type platinum resistance thermometers.*

Calibration measurements are performed in the holder at the triple point of water, tin point, and the normal boiling point of oxygen.

A. Elastomer tubing to helium gas source.
B. Thin (0.005") wall stainless steel tubing for purging the holder with helium gas before the vacuum tubing connector (*I*) is sealed.
C. Leads to measurement equipment (Mueller bridge).
D. Sections of mechanical tie-down (brass), soldered to the stainless purge tube, for guiding and fastening the incoming thermometer leads.
E. Hard wax for holding and sealing the 0.005" gold leads that extend down to the thermometer.
F. "O" ring vacuum seal to brass tube (*H*).
G. Thin wall stainless steel tube (7/16" o.d.) closed at the bottom.
H. Brass tube with lead seal at top and vacuum tubing connector (*I*) at the bottom.
I. Vacuum tubing connector for sealing the tube (*G*).
J. Polytetrafluoroethylene plastic lead spacers.
K. Insulated gold leads (4) passing through holes close to the outer diameter of the spacers (*J*) to attain good tempering. The lead insulation (not shown) is polytetrafluoroethylene tubing cut into a helix and held in tension to eliminate buckling. The four gold leads are welded at the bottom end to short sections of platinum leads.
L. Thin polytetrafluoroethylene sheet rolled into a cylinder to insulate the exposed leads (near the connections to the capsule thermometer) against the stainless steel tubing.
M. Connections between the short sections of platinum leads of the holder and the platinum leads of the capsule thermometer.
N. Capsule thermometer.
O. Aluminum sleeve to fit the thermometer and the stainless steel tube. The sleeve reduces the external heating effect of the thermometer.

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[2] Certain commercial materials are identified in this paper in order to adequately specify the experimental procedure. Such identification does not imply recommendation or endorsement by the National Bureau of Standards.

in the ice mantle extends from the well surface into the surrounding liquid water.) In the process of introducing Dry Ice into the well some of the Dry Ice may be deposited around the top of the cell, causing the water within the cell to freeze solidly across the top. The ice at the top of the cell should be melted immediately to avoid the possible breaking of the cell glass. Whenever ice is frozen solidly across the top surface of the cell water and a strong bond is formed between the thermometer well and the outer cell wall, any subsequent freezing of water below the surface ice can result in sufficient pressure to rupture the cell. The surface ice can be melted by raising the cell slightly and warming the top of the cell with the hands briefly while gently shaking the top of the cell sideways to "wash" the region with the cell water which facilitates the melting of the layer of ice. After 20 min no additional Dry Ice should be added and the remaining Dry Ice in the well should be allowed to sublime completely. Finally, when the Dry Ice in the well is completely gone, the cell is lowered deeper into the ice bath and the well allowed to fill with water. If the cell is raised high enough to see the mantle during the freezing process the magnification of the cylinder of water will give the impression that the cell is or is about to be frozen solid with ice although the coating of ice on the well may still be as little as 1 or 2 mm thick. If the cell is inverted, the true thickness of the ice may be seen. (The cell should not be inverted after the "inner-melt," described in the next paragraph, has been made.) An immersion type cooler may be used instead of Dry Ice for freezing the ice mantle [25]. However, care must be taken to avoid admitting the auxiliary heat-transfer liquid, e.g., alcohol, into the surrounding ice bath.

A second ice-water interface is formed by melting the ice immediately adjacent to the well surface. This is referred to as the "inner melt." The inner melt is made by inserting a glass tube at ambient temperature into the well for a few seconds. A test for the existence of the ice-water interface over the entire interior surface of the mantle is to give the cell a small rotational impulse and determine whether or not the ice mantle rotates freely around the axis of the thermometer well.

Because of some evidence [12] that the temperature of the triple-point cell is sometimes slightly low (the order of 2×10^{-4} °C) immediately after freezing, the cell should be prepared at least one day prior to its use. The reason for this low initial temperature and the subsequent gradual increase during one or two days to a steady value is not clearly established but is believed to be connected with structural strains that are produced when the ice is first frozen; presumably the strains are relieved with time as the ice anneals. The magnitude of the lower initial temperature and the rate of increase to a steady temperature value is dependent upon the specific technique that is employed in freezing the cell.

Figure 19 shows a triple-point cell immersed in an ice bath. In using the cell, a small soft plastic sponge (*J*) is first placed at the bottom of the well to reduce the mechanical shock that the thermometer might otherwise experience when it is lowered. Also, a closely fitting aluminum bushing (*I*) about 5 cm long is placed above the sponge (at the bottom of the well) to reduce the external self heating of

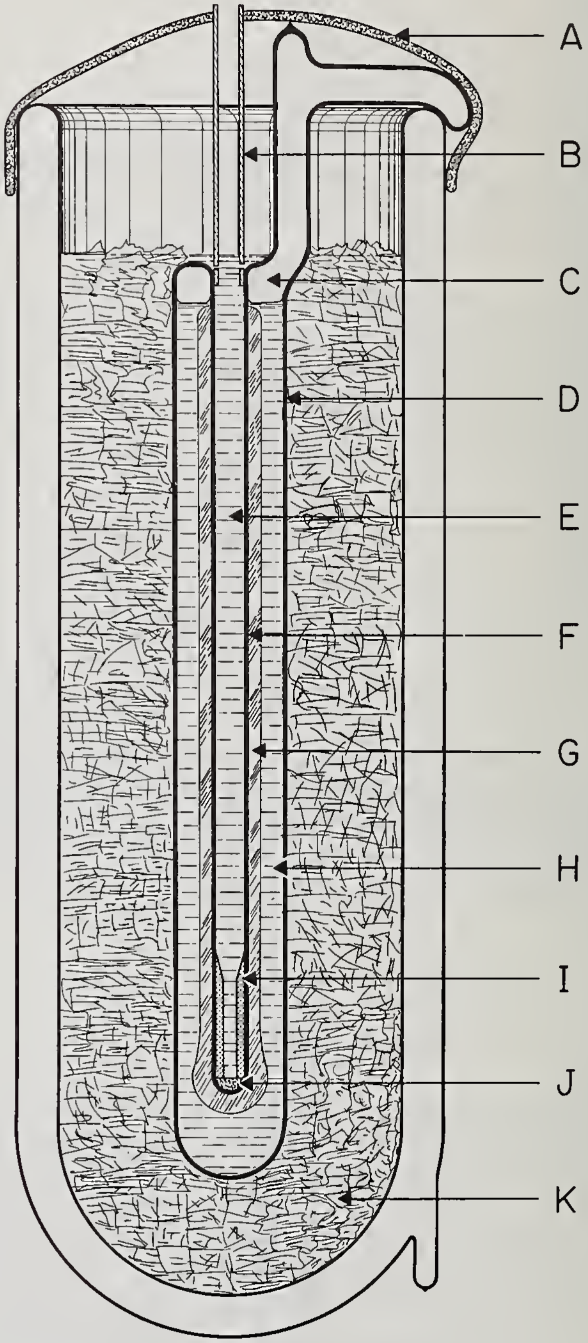

FIGURE 19. *Water triple point cell.*

A. Heavy black felt shield against ambient radiation.
B. Polyethylene tube for guiding the SPRT into the thermometer well.
C. Water vapor.
D. Borosilicate glass cell.
E. Water from ice bath.
F. Thermometer well (precision bore).
G. Ice mantle.
H. Air-free water.
I. Aluminum bushing with internal taper at upper end to guide the SPRT into the close-fitting inner bore.
J. Polyurethane sponge.
K. Finely divided ice and water.

the thermometer. In a half-inch well, filled only with water for thermal contact between the SPRT and the well, the external self heating of a typical 25-Ω SPRT (7.5 mm O.D.) is about 0.2 mK/(mA)^2. The bushing reduces this heating by a factor of five or more depending upon its fit with the well and the SPRT. (See sec. 4 for the discussion on the self heating of SPRT.) To eliminate the ambient room radiation (from ceiling lights in particular) a heavy black felt cloth (A) covers the top of the cell except for a hole through which the thermometer may be inserted. The thermometer is precooled in the ice bath that surrounds the cell before it is inserted into the cell. A polyethylene plastic tube (B) from the hole in the felt cloth to the re-entrant well provides a guide for inserting the SPRT. Before measurements are made a minimum of five minutes is allowed to elapse (with the thermometer current on and the bridge nearly balanced) for the thermometer to attain thermal equilibrium. If an ice particle is present in the well near the SPRT resistor, an error will occur in the calibration. The water in the well must be free of ice before the SPRT is inserted. Routine measurements made in calibrating thermometers in triple-point cells have an estimated standard deviation of less than 0.14 mK. Very careful work using two currents and extrapolating to the resistance value for zero current has yielded an estimated standard deviation of less than 0.04 mK.

7.2. Metal Freezing Points

Freezing points are advantageous because the effect of a change in pressure on the temperature, the value of dt/dp, is much smaller than that of boiling points. In a metal freezing-point cell, the temperature at the solid-liquid interface depends upon the concentration and kind of impurities; also, strains in the solid and grain size effect the temperature. The concentrations of impurities (solute) existing at the interface depend upon the amount and kinds of impurities in the sample, the amount of sample frozen, and the rate of freezing. To achieve a true temperature and phase equilibrium the net rate of freezing (or melting) must approach zero. This equilibrium condition may be illustrated by the binary composition versus temperature phase diagram shown in figure 20 in which the two constituents are completely miscible in both the liquid and solid phases, e.g., the system Ag-Au or Bi-Sb. The phase diagram of figure 20 is presented for its simplicity and, since metal samples employed in freezing-point cells are prepared by the zone refining process, the remaining impurities (combined as a single component in the illustration) are expected to form solid solution with the major component. (Most binary metal systems on which data exist do not form a continuous series of solid solutions; various degrees of immiscibility are usually found.) The figure shows the region of the

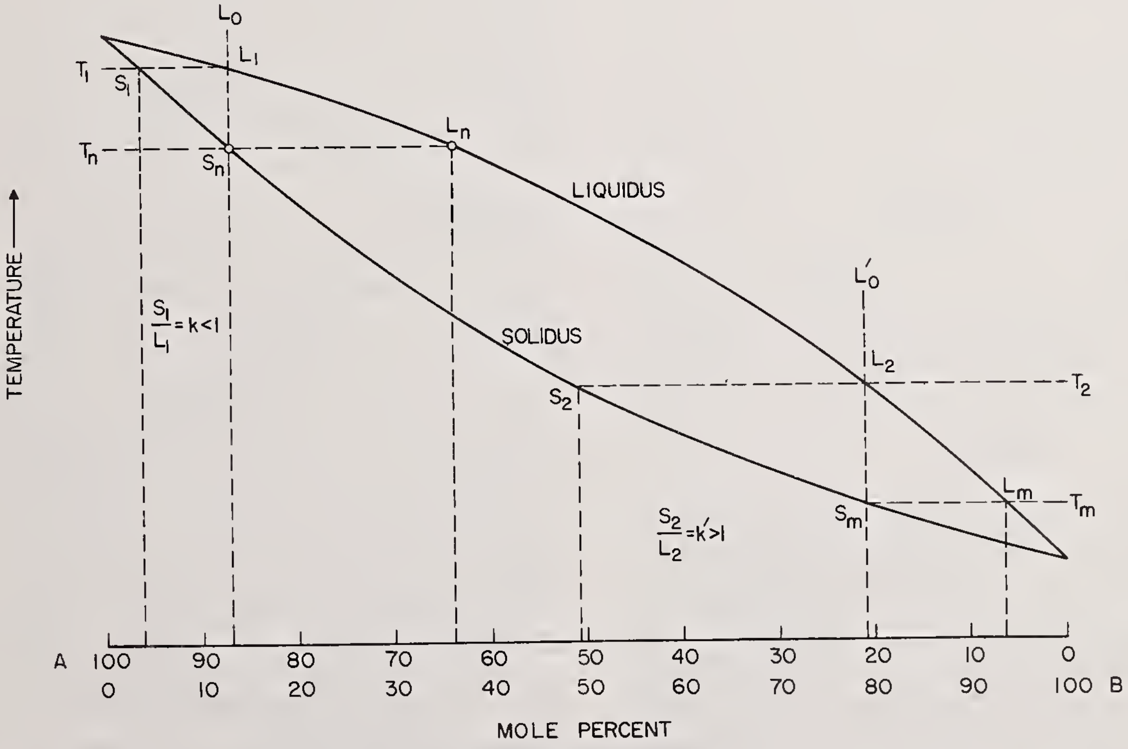

Figure 20 *Binary phase diagram of a system that is completely miscible in both solid and liquid phases.*

The solidus and liquidus curves represent compositions, respectively, of the solid and liquid phases that can coexist in equilibrium. The left side of the diagram shows that the solute concentration in the solid phase is less than that in the liquid phase, i.e., $k < 1$. The right side of the diagram, on the other hand, shows that the solute concentration in the solid phase is greater than that in the liquid phase, i.e., $k > 1$.

composition and temperature where the solid and liquid phases can coexist. The solidus curve indicates the temperature at which a solid solution of any given composition would begin to melt; the liquidus curve indicates the temperature at which a liquid solution of any given composition would begin to freeze (assuming no supercooling). Alternatively, when both solid and liquid phases are present in equilibrium, the composition of each phase is given by the intersections of the temperature line with the solidus and liquidus curves, respectively. An enlarged section of this phase diagram, showing the effect of a small amount of *B* in nearly pure *A*, is shown in figure 21a. If a completely melted sample of composition L_0 is allowed to cool under equilibrium conditions, no change in composition of the liquid occurs until the temperature reaches the liquidus curve (L_1 at temperature T_1) at which point solid (assuming no supercooling) of the composition S_1, given by the solidus curve at the same temperature, is formed. In the case shown here, the first solid contains a smaller proportion of *B* than the liquid from which it was formed; as more solid is formed the concentration of *B* in the liquid increases. When freezing under equilibrium conditions the composition of the solid phase moves from S_1 to S_n; the composition of the liquid phase moves from L_1 to L_n. The last of the liquid solidifies at T_n. Obviously the amounts of *A* and *B* in the completely solidified sample must be the same as in the original liquid. S_n must, therefore, equal L_0 and L_1 and no segregation occurs. The temperature range of the "freeze" is from T_1 to T_n. Although in most systems the solutes lower the freezing point, there are systems in which solutes raise the freezing point; antimony in tin is an example of the latter [30]. Such a system is illustrated by the extreme right portion of the equilibrium phase diagram where *B* is the major constituent as in figure 20. An enlarged section is shown in figure 21b. The analysis of the phase diagram is similar to that given previously. The most notable difference is that, as the diagram shows, the concentration of the solute is greater in the solid than in the liquid. The ratio (S/L) of solute concentration in the solid (S) to that in the liquid (L) is called the solute distribution coefficient (k) and, as given in figures 20 and 21 is less than unity for solutes that depress the freezing point and is greater than unity for solutes that elevate the freezing point. Except for very dilute solutions, depicted in figure 21, the solute distribution coefficient depends on concentration.

Phase diagrams are idealized conceptions of systems at equilibrium (often based on relatively little data). For equilibrium freezing the crystallization process must proceed at such a negligible rate that there is sufficient time for the diffusion of impurities within the solid matrix to achieve uniform distribution throughout the solid, i.e., there is

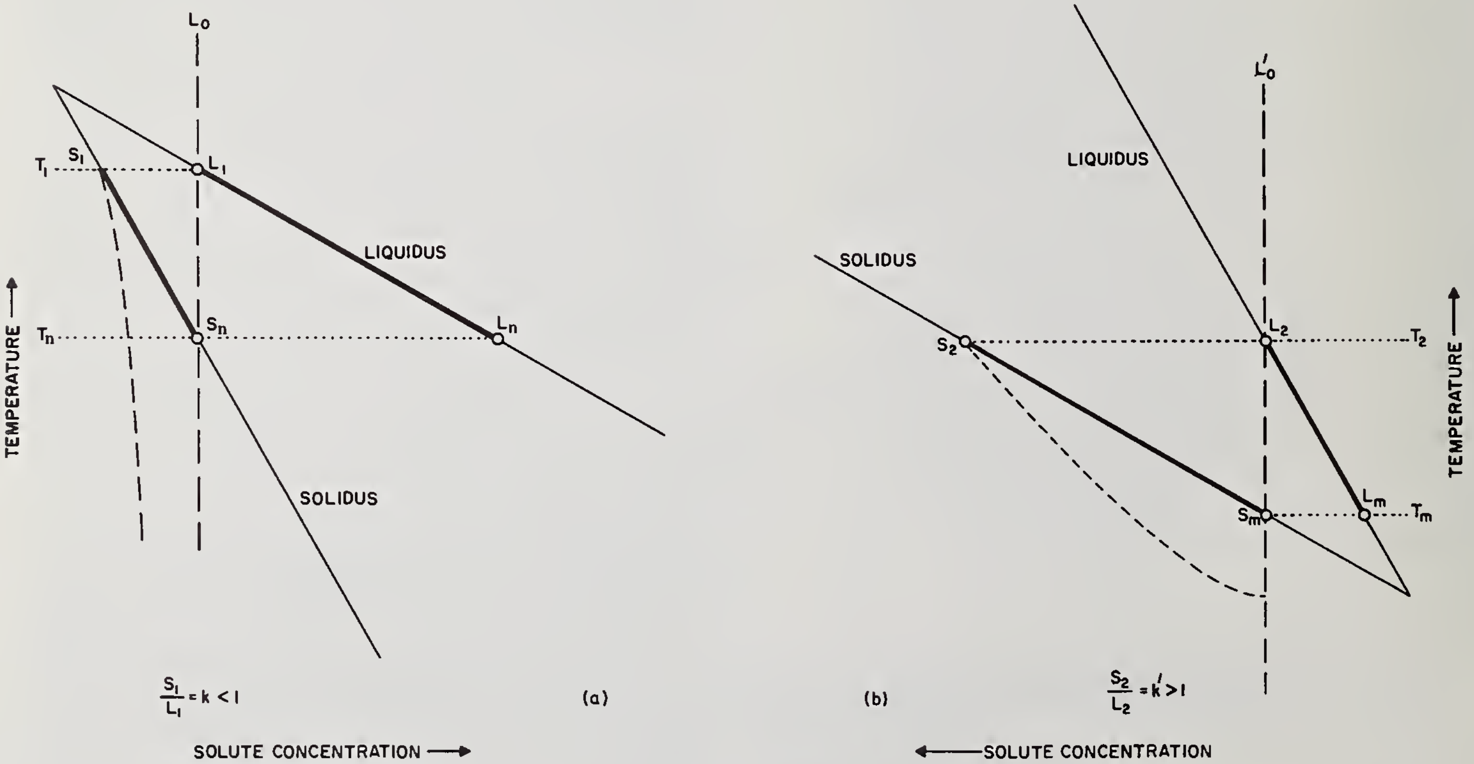

FIGURE 21. *Enlargement of a binary phase diagram in the region of high purity of each component.*

The solute distribution coefficient k is taken to be constant. Figure (a) represents a phase diagram when $k < 1$ and figure (b) when $k > 1$.

The heavy lines on the liquidus and solidus curves represent compositions of the liquid and solid phase during the equilibrium freezing process. The dashed line beginning at S_1 in (a) or S_2 in (b) and extending downward represents the average composition of the solid phase during a semiequilibrium freezing process. The composition of the solid at the freezing interface is represented by the solidus curve and that of the liquid phase in equilibrium with the freezing interface is represented by the liquidus curve. (The solute distribution coefficient may not be constant over the entire range of fraction frozen for semiequilibrium freeze; therefore, eq (7.1) may not be valid when g approaches unity.)

no concentration gradient in the solid. Experimentally this rate of freezing is never realized; however, freezing rates can be achieved which, while large compared to the diffusion rates in the solid matrix, are small compared to the diffusion rate in the liquid. This condition leads to a maximum segregation in the solid but homogeneity in the liquid; this condition will be referred to in this monograph as semiequilibrium freezing. The results of semiequilibrium freezing are shown in the phase diagram of figure 21 and on the cooling curve of figure 22. The phase diagram still represents the compositions at the solid-liquid interface but the solidus curve no longer is the average composition of the solid phase; the average composition of the solid is given by the dotted line beginning at S_1 in figure 21a. Compared to an equilibrium freeze, the freezing temperature range for the entire sample is increased due to the increased concentration of impurities at the interface near the end of the freeze. (This assumes no eutectic is formed in the equilibrium freeze.) For the case of semiequilibrium freezing Pfann [40][3] gives an expression equivalent to

$$S/L_0=k(1-g)^{k-1}, \tag{7.1}$$

where L_0 is the overall solute concentration and S is the solute concentration of the freezing interface after the fraction g of the original mass of liquid has frozen. (The equation is not applicable for the entire range of g. The derivation assumes the distribution coefficient k to be constant and the solute diffusion rate in the solid to be zero.) Figure 23 gives curves of relative solute composition of the freezing interface resulting from this expression for various values of k. Figure 22 compares the curve for $k=0.4$, replotted on a linear scale, with the curve that would be obtained for a corresponding equilibrium freeze. The semiequilibrium freeze is shown to have a broader freezing range.

Sufficiently rapid freezing causes a departure from the semiequilibrium freeze. As the velocity of the advancing solid-liquid interface increases, the rate of solute rejection into the liquid at the interface increases. When the impurity can not be uniformly distributed throughout the liquid by either diffusion or convective mixing, the concentration of the solute builds up at the interface as shown in figure 24b. When this occurs, the effective segregation of the solute decreases (i.e., the effective value of k approaches 1). Because of the increase in the solute concentrations at the interface, the temperature of the interface becomes lower. Figure 24c shows qualitatively the result of a freeze which is very rapid compared to the impurity diffusion rate in the liquid (and for which there is no other method of homogenization, e.g., stirring).

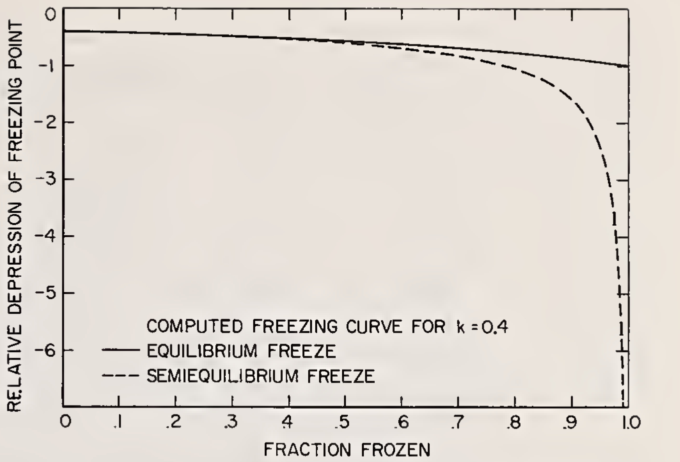

FIGURE 22. *Comparison of computed freezing curves for equilibrium and semiequilibrium freezing processes of a completely miscible (in solid and liquid phases) binary system with constant solute distribution coefficient* (k=*0.4*).

The solid line represents the equilibrium freezing process and the dashed line the semiequilibrium freezing process. In the equilibrium freezing process, the temperature, when the last trace of liquid freezes, is shown to be depressed on*e unit* for the "sample"; the temperature, when the first solid freezes, is shown to be depressed *0.4 units*. The freezing curve for the semiequilibrium freezing process is shown relative to that of the equilibrium freezing process. (The solute distribution coefficient may not be constant over the entire range of fraction frozen for semiequilibrium freeze; therefore, equation (7.1) may not be valid when g approaches unity.)

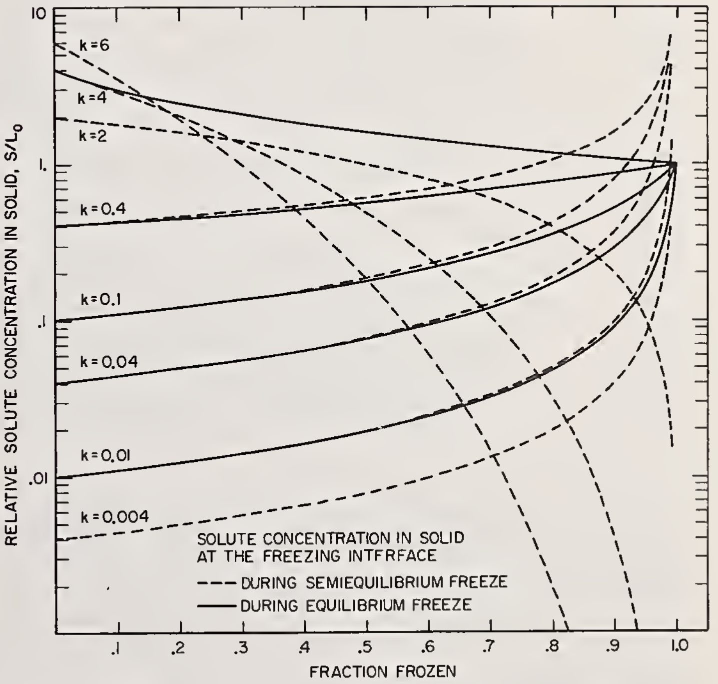

FIGURE 23. *Comparison of relative solute concentrations of the solid phase at the freezing interface during equilibrium and semiequilibrium freezing processes of a completely miscible (in solid and liquid) binary system with constant solute distribution coefficient k.*

Curves are given for various values of solute distribution coefficient. The solid curve represents an equilibrium freezing process and the dashed curve represents a semiequilibrium freezing process. (The solute distribution coefficient may not be constant over the entire range of fraction frozen for semiequilibrium freeze; therefore, eq (7.1) may not be valid when g approaches unity.)

A notable example of nonequilibrium freezing in metal freezing-point cells is the rapid freezing that occurs after a supercool. When the completely melted sample is cooled through the temperature at which the phase diagram indicates that the

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[3] Pfann [40] refers to *semiequilibrium freezing* as *normal freezing*.

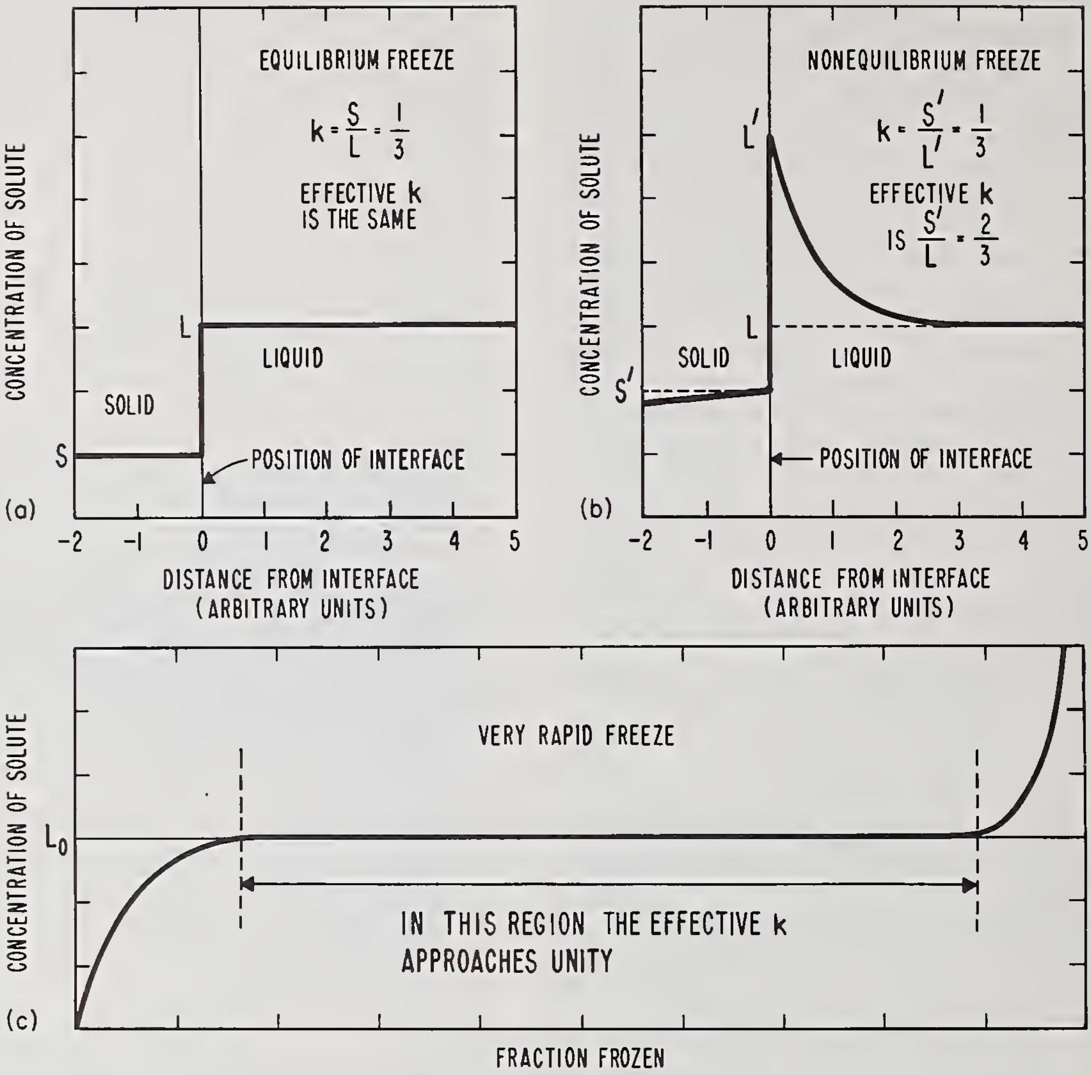


FIGURE 24. *Dependence of solute concentration upon the freezing rate.*

(a) The concentration of solute in the region near the freezing interface during an equilibrium freeze with the solid-liquid interface advancing at a negligible rate. There are no concentration gradients in neither liquid or solid phases.

(b) The solute concentration near the interface during a nonequilibrium freeze with the interface advancing rapidly relative to the rate of diffusion of the solute in the liquid. The solute concentration in the liquid at the interface increases until at steady state the flow of solute from the freezing liquid to the liquid at the interface equals the flow away from the liquid at the interface to the bulk of the liquid. The solute concentration of the frozen solid is shown to be increasing with the advancing interface.

(c) The solute distribution in the frozen solid approached by a very rapid freeze. The curve shows the initial transient rise of solute concentration of the solid to that corresponding to the solute distribution coefficient of unity.

solid should first appear (i.e., crosses the liquidus curve) no solid appears. The first solid appears at a somewhat lower temperature (typically 0.02 or 0.06 °C lower in zinc, 1 to 25 °C lower in tin, water, or antimony). The solid tends to grow most rapidly into the cooler parts of the liquid until the released heat of fusion raises the temperature of the liquid to the equilibrium temperature of the composition at the solid-liquid interface. The temperature to which a liquid supercools is not very reproducible even with the same sample. The amount of supercooling seems to depend on the purity of the sample, the thermal history of the melt, the occurrence of mechanical shock or vibration, and other effects not known or understood. The system may be far from equilibrium during the recovery from the supercool; the solid-liquid interface advances very rapidly and there is very little time for diffusion or convection to homogenize the liquid. As a result, the impurities rejected from the freezing liquid for the case where $k < 1$ become relatively concentrated in the liquid in the region of the interface. When the recovery from the supercool is nominally complete, the high concentration of impurities in the liquid at the interface will be reduced by diffusion and the temperature will rise until the flow of impurities from the freezing liquid to the liquid at the interface is equal to the flow away from the interface through the remaining "bulk" liquid.

Rapid freezing implies a rapid transfer of heat. Many authors [42, 45] have suggested that it may be necessary for the temperature at the solid-liquid interface to be significantly below the equilibrium temperature if freezing is to take place rapidly; i.e., the net rate of freezing is zero at the equilibrium temperature and increases only with decreasing solid-liquid interface temperatures. The metal in the two phases at the interface thus constitutes a "proportional" controller, supplying heat on demand and having a temperature "offset" from the equilibrium value if demand exists. This picture is intuitively appealing and is in qualitative agreement with the theory of reaction kinetics. But, the crucial questions would seem to be what is the rate of energy released per unit of temperature departure from equilibrium and how is the measurement of this separated from other events briefly outlined above that occur in nonequilibrium freezing? The difficulty of this measurement has prevented firm establishment of this model.

7.3. Metal Freezing Point Furnaces

The metal freezing-point cells are used in furnaces of the design shown in figure 25. At NBS, two furnaces are employed, one for the tin point and the other for the zinc point. Except for the metal sleeves employed at the top section of the furnace core, the two furnaces are the same. The core of the furnace is a stack of three cylindrical coaxial blocks of aluminum (top (G), center (L), and bottom (T)) that are thermally insulated from each other by Fiberfrax paper and surrounded by a nichrome wire heater (main heater, O) which extends the full length of the three blocks. The heater wire fits closely within each of the two holes of the 51 cm lengths of two-hole oval alumina tubing. These alumina tubings are very closely spaced around the outer surface of the core (parallel to its axis) and are held tightly against the outside of the blocks with three Inconel "garters." (See the periphery of the furnace core section drawings 1, 2, 3, 4, and 5.) The garters were made of Inconel wire bent into a form resembling a continuing sine-wave about one centimeter in amplitude and wavelength, then rolled to slightly reduce the thickness, and welded into a ring. The top and bottom blocks each have an additional heater (F and U) consisting of nichrome wire that passes through 98 mm lengths of two-hole, round alumina tubings which were selected to fit closely in the 12 holes of each block. To minimize the thermal time lags, the heater assemblies have small clearances between the heaters and the aluminum core blocks. (The leads extending through the core to the heater of the top core block are gold. The leads to the other heaters are heavy nichrome wire.) Extending down through holes that run the length of the three assembled blocks are six thin-wall (0.13 mm) stainless steel tubes (B) nominally 3.2 mm in diameter. The tubes pass through 3.22 mm holes in the end blocks (see a and b of figure 25). To permit the holes for these tubes to be accurately positioned through the 30.5 cm long center block, the center block was made in two cylindrical pieces. The end plate (Q) was attached later to the center core block. Grooves that closely fit the stainless steel tubes were milled on the outside of the inner cylinder which was then fitted tightly inside the second outer cylinder. The two cylinders were "shrunk together", a process that involved precooling the inner cylinder in liquid nitrogen and heating the outer cylinder. The tubes are wells into which thermocouples are placed for controlling the furnace temperature; they also serve as wells for small exploratory resistance thermometers to determine the temperature distribution within the furnace core. The entire core of the furnace, including all three blocks and the heaters, slips into a stainless steel tube (A) 11.4 cm in diameter with a 0.76 mm wall. Sheets of mica are wrapped around the main heater near each end to center the core within the tube, to enhance thermal contact between the tube and the main heater near the ends, and to reduce undesirable heat convection currents. The centering leaves a small annular air space between the tube and the alumina insulators of the main heater in the region of the center block; the thermal contact of the end blocks with the surrounding stainless tube serves as "thermal end guards" and reduces the thermal gradients in the middle section of the tube opposite the center block.

The weight of the core rests on a 0.51 mm wall stainless steel tube (W) 11.4 cm long and 2.5 cm in diameter. The space beneath the core is filled with loose Fiberfrax insulation. The outside of the furnace is a 35.6 cm diameter brass tube with end plates and is filled as indicated in figure 25 with Fiberfrax insulation. A rather widely spaced helical coil of 9.5 mm copper tubing is soldered to the outside brass tube (K) to permit water cooling; a comparable provision is made for the top end plate. The water cooling is not used when the furnace is at the tin point but is helpful at the zinc point in reducing convection currents around the furnace that cause gradients in the head of the thermometer which, in turn, may result in thermal emf's. The thermal insulation is sufficient so that, even at the zinc point with no water cooling, the outside of the furnace is not "hot to the touch." To achieve further thermal insulation, the furnace core is recessed from the top of the furnace as shown in figure 25. A major path of heat loss from the top of the core is along the 11.4 cm diameter stainless steel tube that contains the core; to reduce the heat loss the core was recessed and the tube was made relatively longer.

The top core block (G) of the tin furnace was designed to receive an aluminum sleeve (H) 10.2 cm long that is bored to fit as closely as possible the glass cell that holds the tin sample. By using removable metal sleeves a reasonable range of glass cell diameters can be accommodated while still achieving good thermal contact with the cell above the crucible where the heat shunts are located. The glass cell is of sufficiently uniform diameter to achieve a suitably close fit. In the design, allowance has been made for the differential expansion of the aluminum and the glass cell. (The NBS glass cells are generally ground on the

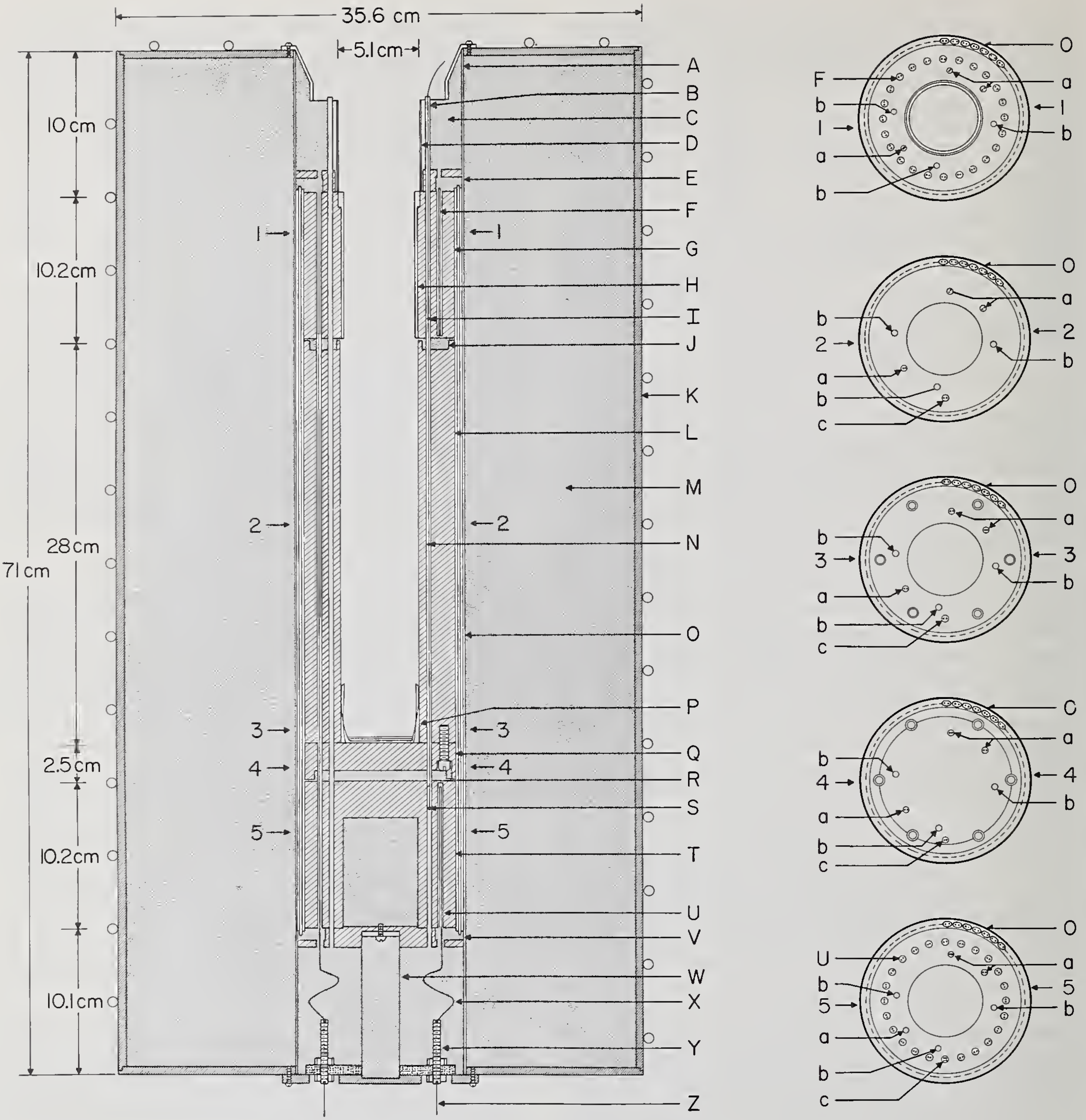


FIGURE 25. *Schematic of furnace body and core.*

A. Stainless steel tube. 11.4 cm o.d. × 0.76 mm wall.
B. Stainless steel tubes (6); 3.20 mm o.d. × 0.013 mm wall.
C. Insulation, Fiberfrax mats.
D. Stainless steel tube.
E. Insulation, Fiberfrax mats and mica sheets.
F. Control heaters for top core block.
G. Top core block.
H. Sleeve (aluminum or Inconel) for metal freezing point cell.
I. Elevation of thermocouple junction (Chromel-P/Alumel), top core block. (See *a*.)
J. Insulation, Fiberfrax sheets.
K. Brass shell, 4.8 mm thick.
L. Center core block.
M. Insulation, bulk Fiberfrax.
N. Elevation of thermocouple junction (Chromel-P/Alumel), center core block. (See *a*.)
O. Main heaters, held on with Inconel "garters."
P. "Spider" for centering the freezing point cell.
Q. Center core block end plate.
R. Insulation, Fiberfrax sheets.
S. Elevation of thermocouple junction (Chromel-P/Alumel) bottom core block. (See *a*.)
T. Bottom core block.
U. Control heaters for bottom core block.
V. Insulation, Fiberfrax mats and mica sheets.
W. Stainless steel tube support, 2.5 cm o.d. × 0.51 mm wall × 11.4 cm long.
X. Heater leads.
Y. Posts for heater leads.
Z. To electric power.
a. Wells (B) for control thermocouples (see I, N, and S).
b. Wells (B) for testing the temperature profile of furnace core.
c. Leads from top core block heaters.

outside to a uniform diameter.) Because aluminum sleeves bond to the aluminum top core block at the zinc point, split inconel sleeves are employed in the zinc point furnace. Also, because of the close tolerances needed for good thermal contact, the zinc-point cells are not permitted to cool to room temperature in the furnace, otherwise the borosilicate glass tube would be crushed owing to the differential contraction between the aluminum top end block and the glass.

The center core block (L) was bored to provide approximately 1.0 mm clearance for the most common (and largest) size of glass tubes, specifically 51 mm o.d.; this 1.0 mm annular space provides the thermal insulation that reduces the heat transfer to the crucible and metal sample during freezing or melting experiments. The design also reduces the need for close control or knowledge of the furnace temperature in many operations.

Three Chromel-P/Alumel thermocouples (TC) enclosed in alumina sheaths control the furnace temperature. Two of the thermocouples (differential) are referenced to the temperature at the middle of the center core block; the measuring junction of one is located approximately 1.2 cm up into the top core block and that of the second is similarly located down into the bottom core block. The third TC is referenced outside the furnace and the measuring junction is located in the middle of the center block.

Each differential TC is connected in series with a stable voltage source (powered by mercury cells) which is adjustable between ± 75 μV (± 1.5 °C). The center block TC connects to a reference junction, which is self-compensating for changes in room temperature, and to a voltage source that has (i) a range of 15 mV, (ii) a stability of 0.01 percent, and (iii) a reproducibility of setting of better than 2 μV. The combined output of each TC and voltage source is amplified by a chopper type DC amplifier whose output operates a "three-mode" controller. The zero stability of the amplifier is better than 0.5 μV. The output of each controller is an adjustable linear (proportional) function of the input, its rate of change, and its time integral. The signal from the controller operates a gate drive for a full-wave silicon-controlled rectifier which, in turn, controls the power supplied to the heater. The rectifiers do not cause detectable interference with any other equipment (low level null detectors, a-c and d-c bridges, etc.) in the room. Each aluminum block of the core is grounded by means of a heavy gold wire.

The control of the furnace was made very flexible in that several combinations of manual and automatic controls may be selected for operating the furnace, including the provision for independently setting the temperature offset between the center and top and between the center and the bottom core blocks. A freeze is usually conducted with each of the three heaters under the automatic control of their corresponding TC's. A plot of thermometer resistances (relative to that at the center of the center block) as a function of depth in the core of the furnace, while the furnace and a tin-point cell was at 235 °C, is shown in figure 26. These measurements were made with a small four-lead platinum resistance thermometer, 2.8 mm diameter and 2 cm long, having a nominal ice-point resistance of 50 Ω.

This furnace, which was designed and built at the NBS, is more sophisticated than necessary for calibrating thermometers at the tin and zinc fixed points with reasonable (better than 0.002 °C) accuracy. It was designed for special studies of the freezing and melting phenomena of tin and zinc. Tin and zinc freezes for calibrating SPRT's have been performed very successfully in many laboratories with furnaces that contain a single copper block and employed a manually controlled heater. Such calibrations have been performed in furnaces that exhibit core temperature gradients of 1.5 °C over the length of the crucible containing the sample [53] and relatively long response times. Realization of the tin and zinc freezing points with reasonable accuracy is principally dependent upon the high purity of freezing-point samples and the use of proper freezing and measurement techniques.

7.4. Tin-point Cell

The realization of either the tin-point or the steam-point is necessary for the calibration of SPRT's in accordance with the specifications set forth in the text of IPTS–68. The tin-point (231.9681 °C) has two distinctive advantages; first, the temperature is much closer than the steam point to the midpoint between the triple point of water and the zinc point and, therefore, tends to produce less average error in the calibration of the thermometer (see fig. 17), second, and of greater practical importance, the solid-liquid equilibrium temperature of tin is 8600 times less sensitive to pressure changes than the liquid-vapor equilibrium temperature of water at 1 atm. For the freezing point of tin, $dt/dp = +4.3 \times 10^{-6}$ °C/torr or $+2.2 \times 10^{-5}$ °C/(cm column of liquid tin). Therefore, knowledge of the pressure within ± 1 torr is adequate for determining the temperature of a tin-point cell.

The tin sample for the freezing-point cell must be of high purity (nominally $>99.999\%$) and the freezing apparatus must be designed and operated to interpose a solid-liquid interface of tin completely around the resistance element and lower part of the thermometer stem. The immersion of the resistor must be sufficient to prevent sensible heat flow from the resistor up the stem of the thermometer. In addition, the sample holder must be chemically inert and not introduce any impurities that would affect the freezing temperature of tin. At NBS the tin sample is contained in a closed crucible of high-purity graphite. A re-entrant well,

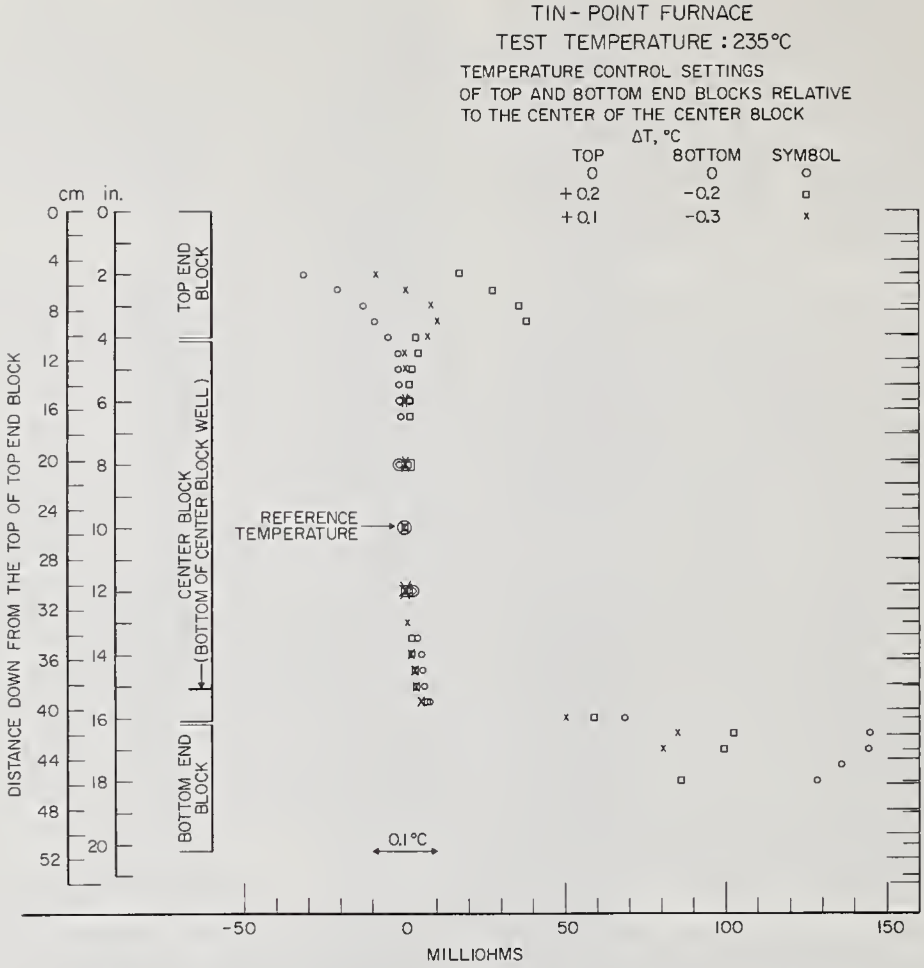


FIGURE 26. *Vertical temperature profile of tin-point furnace at 235 °C.*
The vertical distances are relative to the top of the top core block; the thermometer resistances are relative to that observed at 25.4 cm from the top of the top core block (close to the center of the center core block). The resistance of the thermometer at the reference temperature was about 95 Ω. The resistance thermometer (2.8 mm diameter × 2.0 cm long) sensed the average temperature of a section of well about 2 cm long.

also of high-purity graphite, is screwed into the lid. The assembly is shown schematically in figure 27. The cell assembly permits sealing the stem of the thermometer, as well as the crucible with its charge of tin, in helium gas which provides an inert atmosphere for both the crucible and the tin and additionally improves the thermal conductance to the thermometer.

Because of the relatively high thermal impedance of gas, even helium, considerable effort was made to obtain small clearances at the heat shunts to keep the thermal impedance as low as possible. This is practical with components of borosilicate glass and graphite because their thermal coefficients of expansion are similar. The outer borosilicate glass cell was formed from precision bore tubing and its external diameter was ground to fit the furnace sleeve (see (H) fig. 25). The two graphite heat shunts (G), shown in figure 27, were fitted closely to both the thermometer guide tube (F) and the glass cell (H). The shunts primarily serve to conduct heat to the inner glass tube of the tin-point cell, thereby improving the "immersion characteristics" of the thermometer. (See the end of this section for results of tests on the immersion characteristics of a SPRT in the tin-point cell.) The space between the heat shunts is loosely filled with high purity Fiberfrax (washed) insulation to eliminate convection currents and radiation losses from the top of the crucible and the heat shunts; it also vertically positions the heat shunts.

The equipment arrangement for filling the crucibles is shown in figure 28. The bell jar (which contains the metal sample and its funnel and crucible) is evacuated to about 0.01 torr by a mechanical pump that is "trapped" with a molecular sieve and the temperature of the enclosed assembly is then raised to a few degrees below the tin point. After 1 hour, the induction heater power is increased so that the tin sample will melt and flow into the crucible within about 15 min. Next, the sample is allowed to cool nearly to room temperature, the crucible is removed from the bell jar, and the crucible is slipped into the special borosilicate glass cell, figure 29. The glass cell is then purged with helium, lowered into the furnace that is used for realizing the freezing point, and the tin reheated to about 250 °C. Subsequently, the glass cell is

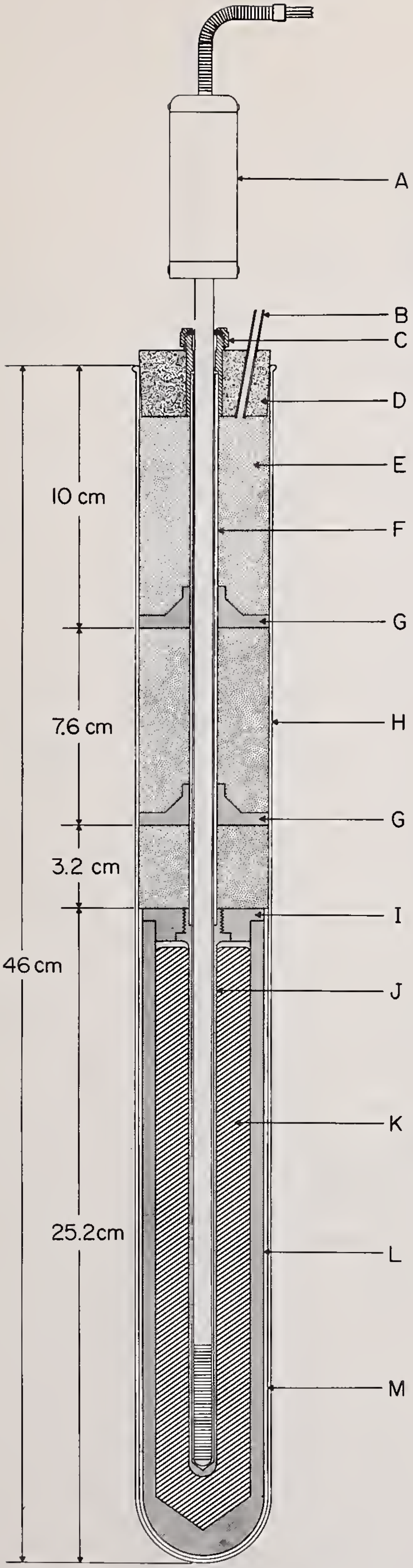


FIGURE 27. *Metal freezing-point cell.*

A. Platinum resistance thermometer.
B. To helium gas supply and pressure gauge.
C. Thermometer stem seal with silicone rubber.
D. Silicone rubber cap.
E. Insulation, washed Fiberfrax.
F. Thermometer guide tube, borosilicate glass.
G. Heat shunt, graphite.
H. Borosilicate glass cell.
I. Graphite cap (lid).
J. Graphite thermometer well.
K. Metal sample.
L. Graphite crucible.
M. Insulation, Fiberfrax paper.

raised out of the furnace and the graphite lid and well assembly are smoothly and quickly pushed into place. To perform the assembly easily the outside diameters of the crucible and lid were made equal and the inside dimensions of the glass tube only slightly (perhaps 0.01 mm) larger. The glass tube served to guide and correctly position the lid to cover the crucible. The lid is closely fitted to the crucible to minimize the possibility of later contamination. Finally, the crucible is again allowed to cool, removed from the special glass cell, and assembled as a freezing-point cell such as is shown in figure 27.

In the preparation for tin freezing point measurements, the tin-point cell is placed in the furnace (held about 10 °C above the tin point) until complete melting occurs. During the heating process the cell temperature, which is monitored with a SPRT, rises until melting begins, becomes nearly constant until the melting is completed, and then rises again. There is no reason to raise the temperature of the tin more than a few tenths of a degree above the melting point. In a furnace with large temperature gradients precautions should be taken to make certain that the metal is completely melted. There is no evidence to indicate that heating the tin to several degrees above its melting point in an inert atmosphere is harmful to the sample.

After the melting is completed, the furnace is allowed to cool to the tin point. A thermometer is inserted in the cell well and instrumented so that a temperature range of 25 °C below the tin point can be visually monitored within about 0.5 °C by a person, standing beside the furnace, holding a tin cell ready to be reinserted. A galvanometer is used for this purpose. When the cell has cooled to the tin point it is removed from the furnace. The cell temperature, after a few seconds, will suddenly decrease very rapidly, perhaps as much as 20 or 25 °C. As soon as its temperature *ceases* to drop, the cell is immediately lowered into the furnace again; subsequently, the cell temperature will increase even more rapidly than it previously decreased. If the cell temperature is examined in more detail, the final approach to the tin point (the last few ten-thousandths of a degree) will be seen to take a few minutes (see fig. 30). When the temperature plateau is reached, the SPRT is measured. It may then be removed and a second SPRT inserted. The second thermometer is preheated above the tin point so that its temperature, during insertion into the tin point cell, slightly exceeds that of the cell. Figure 31 shows the results of successive insertions of the same SPRT in a single tin freeze.

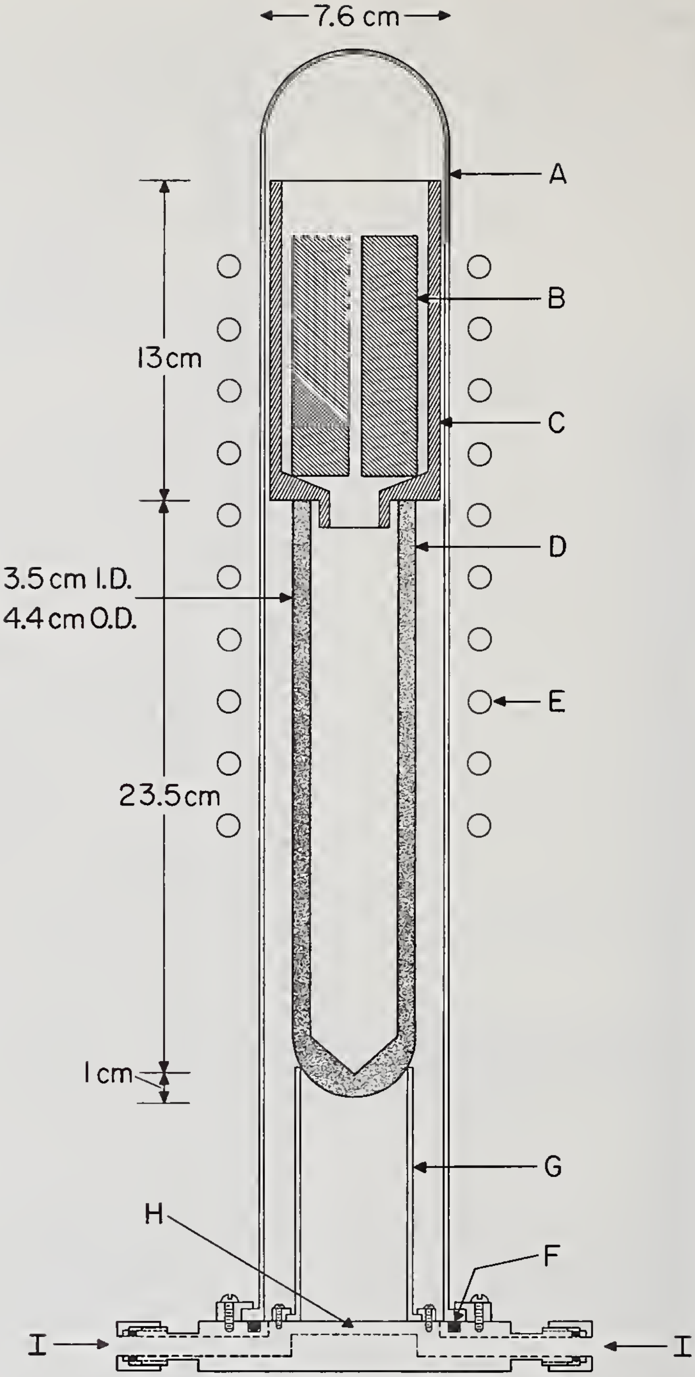
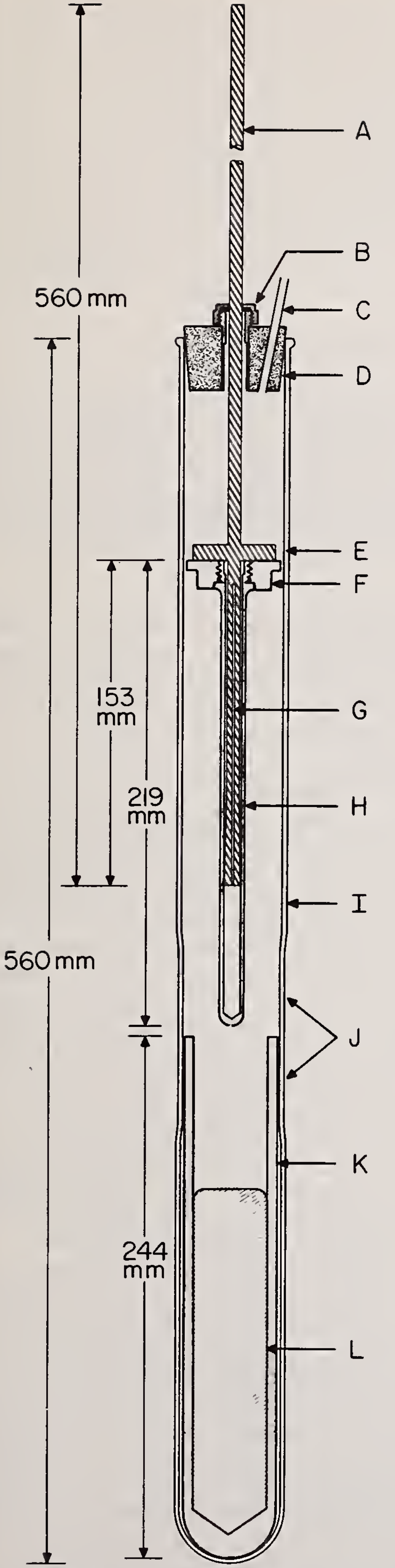


FIGURE 28. *Arrangement for filling the freezing-point cell with metal sample by induction heating.*

A. Borosilicate glass envelope.
B. Metal sample.
C. Graphite sample holder and funnel.
D. Graphite crucible.
E. Induction heater coils.
F. "O" ring groove.
G. Borosilicate glass stand.
H. Slot for pumping out and gas purging.
I. Connection to vacuum and purified argon supply.



- A. Stainless steel pusher rod.
- B. Gas seal with silicone rubber. Permits linear motion of the pusher rod (*A*).
- C. Inlet for purified helium gas that is used in purging and maintaining positive pressure of the gas during assembly.
- D. Silicone rubber cap.
- E. Stainless steel flange attached to the pusher rod for pressing against the graphite lid during assembly.
- F. Graphite lid for the metal sample cell.
- G. Slit on the pusher rod. The two halves are sprung out to hold up the graphite thermometer well and lid up while melting the metal sample.
- H. Graphite thermometer well.
- I. Borosilicate glass tube.
- J. Section of glass tube shrunk to fit the crucible and lid so that lid can be easily guided onto the crucible.
- K. Graphite crucible.
- L. Molten metal sample.

FIGURE 29. *Apparatus and method for installing graphite thermometer well and lid in the graphite crucible containing the molten, metal freezing-point sample.*

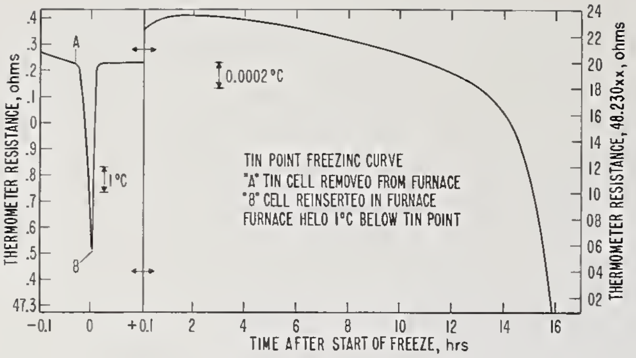


FIGURE 30. *The freezing curve of tin obtained using an a-c bridge with the head of the SPRT adapted for coaxial connectors.*

The furnace was controlled to be 0.9 K below the tin point. The mass of tin sample was 1300 g.

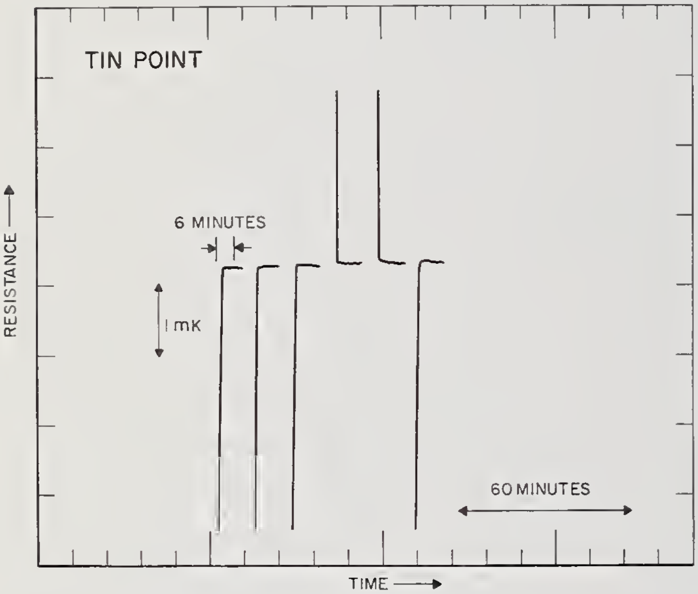


FIGURE 31. *Consecutive measurements of the resistance of a SPRT in the same tin freeze.*

(An a-c bridge was employed in the measurements.)

Following each equilibrium resistance observation, the SPRT was completely withdrawn from the tin-point cell, preheated close to the tin point in an auxiliary furnace, and reinserted into the cell. The data shows that the preheated SPRT comes to equilibrium in a very short time. In two of the cases shown, the temperature of the SPRT when inserted into the cell was slightly above the tin point. (For comparison when the SPRT was not preheated close to the cell temperature, see fig. 35.)

An experimental procedure similar to that described above for observing the supercooling of the tin may be used to check the preheating of the SPRT before inserting it into the tin-point cell. The thermometer should be preheated and held in the air over the tin cell while monitoring the decreasing thermometer temperature. As a first approximation, the thermometer can be inserted into the tin-point cell when its indicated temperature is 30 °C above the tin point. If subsequent close monitoring indicates that the thermometer temperature is rising, then the thermometer was too cold when it was inserted. Several thermometers may be consecutively calibrated during one freeze if each is properly preheated (see fig. 31). A simple check on the temperature constancy during a freeze is afforded by employing the same thermometer in both the first and last measurement in a series of SPRT calibrations. At NBS a single SPRT is set aside and only used for this purpose as an experimental control.

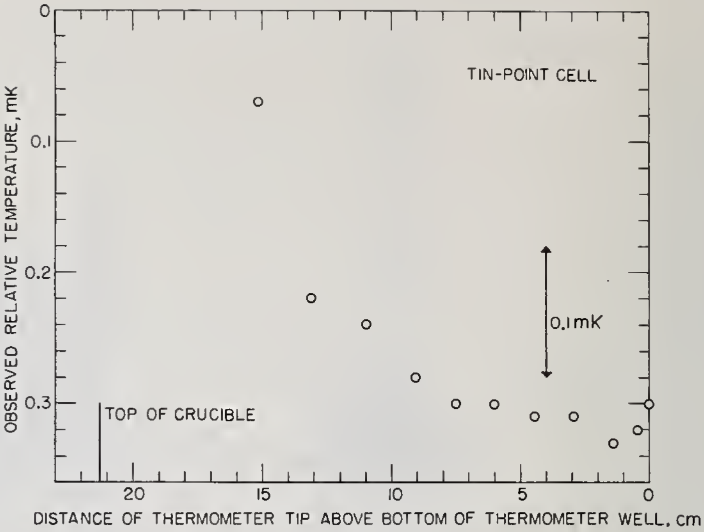


FIGURE 32. *Immersion characteristic of a SPRT in a tin-point cell.*

The data show that the indications of the SPRT are very nearly the same between about 8 or 10 cm above the bottom and the bottom of the thermometer well.

The immersion characteristics of a SPRT were tested in one of the tin-point cells that were described earlier. The observed resistances are shown in figure 32 as a function of the distance from the bottom of the crucible thermometer well. The results show that the immersion of the SPRT in the cell is more than adequate. This figure does not, however, show the temperature gradient expected from the change in pressure with depth.[4]

In the reduction of the calibration data the temperature at the point of immersion of the SPRT in the tin-point cell is employed. The value of temperature assigned in the text of the IPTS–68 is adjusted for the departure from the equilibrium conditions specified for the fixed point; i.e., an adjustment of temperature is made for the effect of the hydrostatic head of liquid tin and for any departure from 1 standard atm of the gas pressure in the cell.

The high reproducibility of the tin point may be seen in the results of tests (made at the NBS) that compare the freezing points of a number of tin samples. The average standard deviation of measurements on a given cell is ±0.05 mK from freeze to freeze (see fig. 33). All of the measurements are plotted relative to the mean of samples 6C and 6E. The individual measurements (one for each freeze) are shown as short horizontal bars; four freezes are represented for each cell with the exception of 6K

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[4] Recent work done at NBS indicates that the temperature gradient expected from the change in pressure with depth is distorted or obscured when observations are made from within the graphite thermometer well if the solid liquid interface is not very close to and surrounding the well. Further work in this area is in progress.

on which only two freezes were performed. The values are based on the determinations of $R(t)/R(0)$ that were obtained from extrapolations to zero thermometer current of observations of $R(t)$'s and $R(0)$'s at 1 and 2 mA. The $R(t)$'s were measured 1.5 hours after the initiation of a freeze. The $R(0)$'s were measured immediately after every determination of $R(t)$'s. The samples identified as 5 were reported to be 99.999 percent pure [35]; the samples identified as 6 were reported to be 99.9999 percent pure [35]. The sample identified 6M was accidentally overheated to a temperature above 500 °C.

7.5. Zinc-Point Cell

The equipment and procedures used at NBS for filling graphite crucibles with zinc are identical to those employed for tin, except that after evacuation at a temperature slightly below the tin point, an atmosphere of purified dry argon is admitted into the bell jar. Because of the relatively high vapor pressure of zinc at the melting point (over 0.1 torr), filling the graphite crucible by melting the sample in vacuum is less practical. The power of the induction heater is raised sufficiently to melt the zinc sample (approximately 1280 g) in about 30 min. In spite of the presence of argon gas, enough zinc vapor diffuses and is deposited on the surface of the bell jar to cause difficulty in observing when the zinc is completely melted. The sample is allowed to cool and the assembly of the zinc-point cell is carried out in the same manner as the tin sample (see sec. 7.4).

To realize the freezing point of zinc with an accuracy of about ±0.002 °C the furnace need not be very complex. However, a zinc sample of high purity and the use of proper freezing-point techniques are important. The experimental procedure for realizing the freezing-point of zinc differs appreciably from that employed for tin. The zinc is melted in the furnace while being monitored with a SPRT. The zinc freeze is initiated in the furnace, unlike the tin freeze where the cell is removed from the furnace to initiate the freeze. The furnace temperature is reduced so that the

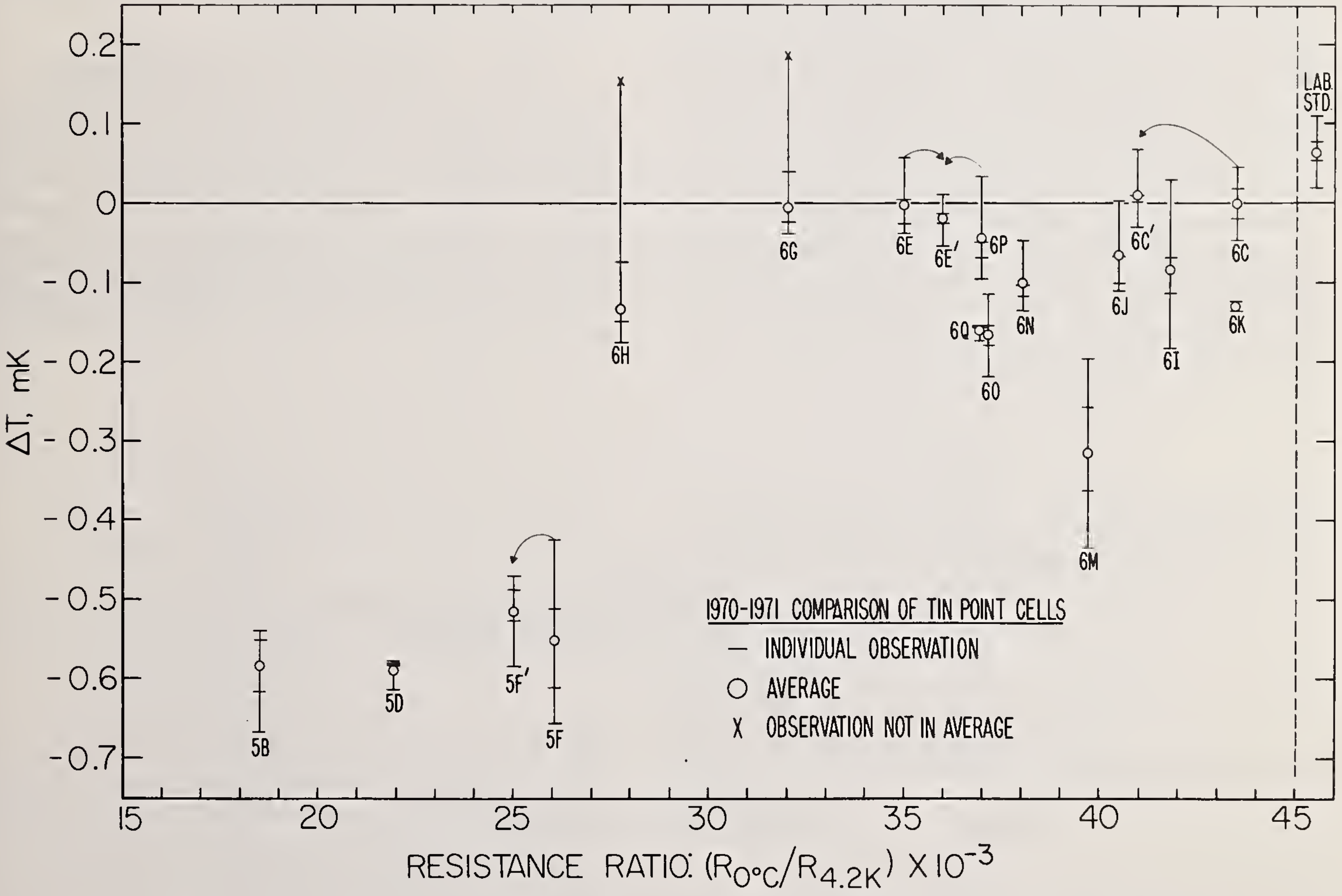

FIGURE 33. *Comparison of tin-point freezing plateau temperatures with resistance ratios:* R(*0 °C*)/R(*4.2 K*).

The prefixes 5 and 6 indicate samples nominally 99.999 percent pure (Lot No. 6637) and samples nominally 99.999 percent pure (Lot No. 6779), respectively. The alphabetic character identifies the cell. The prime (′) identifies a second set of measurements on the same cell. LAB. STD. is a tin-point cell that has been used for over four years at the NBS in the calibration of SPRT's. Observations from four separate freezes are shown for each cell. The plateau temperature selected was the reading 1.5 hours after the initiation of the freeze; the temperature corresponded to about 25 percent frozen. The temperatures of the SPRT during the previous hour differed typically 0.00002 or 0.00003 K but never more than 0.0001 K from the selected temperatures of any measurements. The observed plateau temperatures are shown relative to the mean value obtained for the tin-point cells 6C and 6E. Cell 6M was accidentally overheated above 500 °C before the measurements. Only two measurements were obtained for cell 6K. (Resistance ratio measurements were made by R. L. Powell at the Boulder Laboratories of the National Bureau of Standards.)

sample temperature, as measured within the well, reaches the zinc-point temperature at a cooling rate of 0.1 to 0.2 °C per minute. The zinc supercools below the zinc point by 0.02 to 0.06 °C (as measured by the SPRT) before the recalescence occurs. The subsequent rise toward the plateau temperature, while rapid at first, becomes very slow after several minutes (see fig. 34). To hasten this process the induced-freeze suggested by McLaren [32] is employed to form a second solid-liquid interface immediately adjacent to the well. (The first solid-liquid interface occurs at the crucible wall.) After the zinc cell has completed the initial rapid freeze following the supercool described above, two cold thermometers are inserted into the well to freeze the zinc around the well; a solid-liquid interface that completely surrounds the well is necessary and a single thermometer may not freeze a complete coating of zinc on the well. The resistance of the second thermometer will rise to a steady value (within 10^{-4} °C) within a few minutes (typically 10 to 15 min) after insertion and will remain nearly constant for a period of time that is determined by the freezing rate of the outer solid zinc layer and the purity of the zinc. (The rate of freezing, of course, depends principally upon the heat loss to the furnace, i.e., the temperature setting of the furnace). Freezes lasting as long as three days have been observed at NBS, but 12 to 16 hour freezes are more typical. As with tin, SPRT's may be consecutively calibrated in a single zinc freeze if the thermometers are preheated (see fig. 35). The preheating of the thermometer serves to minimize the growth of the inner freeze on the well and thereby extend the useful life of the freeze. Care must be exercised to avoid melting a hole in the inner freeze or loosening the mantle on the well so that it slides to the bottom of the crucible. To ensure that no melting and no excessive freezing occurs, the thermometer is preheated to a temperature slightly below the zinc point before inserting it into the cell. As with tin, a SPRT is set aside at NBS for checking the constancy of the freeze, and it is used for both the first and last measurements in each freeze. The freezing-point measurements are normally not conducted at exactly the specified pressure of 1 standard atmosphere and a correction is made for this departure. The pressure at the interface surrounding the thermometer resistor includes both the gas pressure over the sample and the pressure due to the head of liquid zinc. For zinc at the melting point the value of dt/dp is $+5.7\times10^{-6}$ °C/torr or $+2.74\times10^{-5}$ °C/cm column of liquid zinc [32].

The immersion characteristics of a SPRT were tested in one of the zinc-point cells that were described. The observed resistances are shown in figure 36 as a function of the distance from the bottom of the crucible thermometer well. The results show that the immersion of the SPRT in the cell is more than adequate.

7.6. Oxygen Normal Boiling Point

The oxygen normal boiling point calibration is realized at the NBS by reference to the NBS−55 scale adjusted to correspond to the IPTS−68 (see sec. 7.7). The temperature (hotness) assigned to the oxygen point, now maintained by SPRT standards, is 0.0019 K lower [8] than that maintained previously on the NBS−1955 scale [50]. This change resulted from efforts made to achieve uniformity

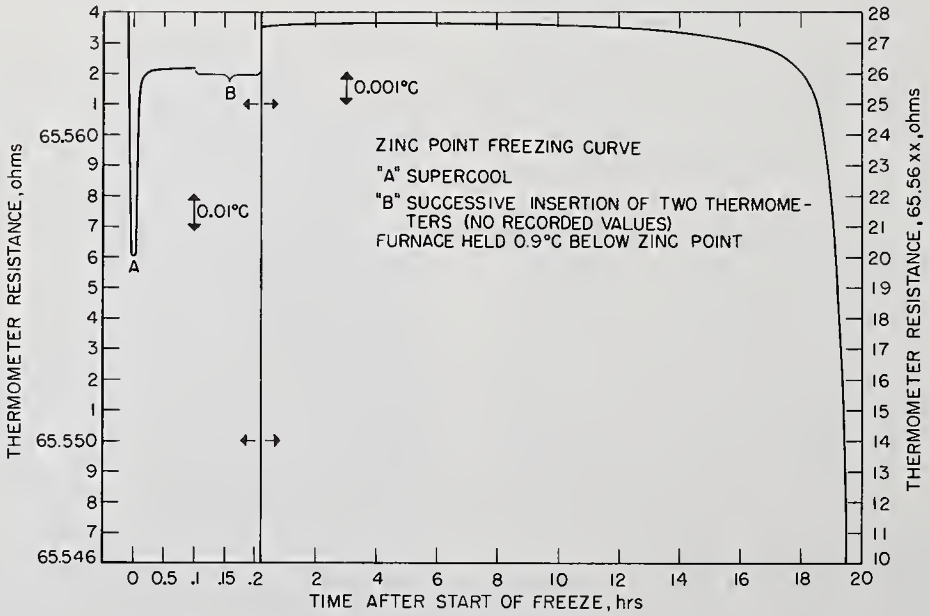

FIGURE 34. *The freezing curve of zinc obtained using an a-c bridge.*

(The head of the SPRT had been adapted for coaxial connectors.)

The furnace was controlled at 0.9 K below the zinc point. The mass of zinc sample was 1280 g. Two cold SPRT's were inserted in the cell at "B" to freeze a coating of zinc around the graphite thermometer well.

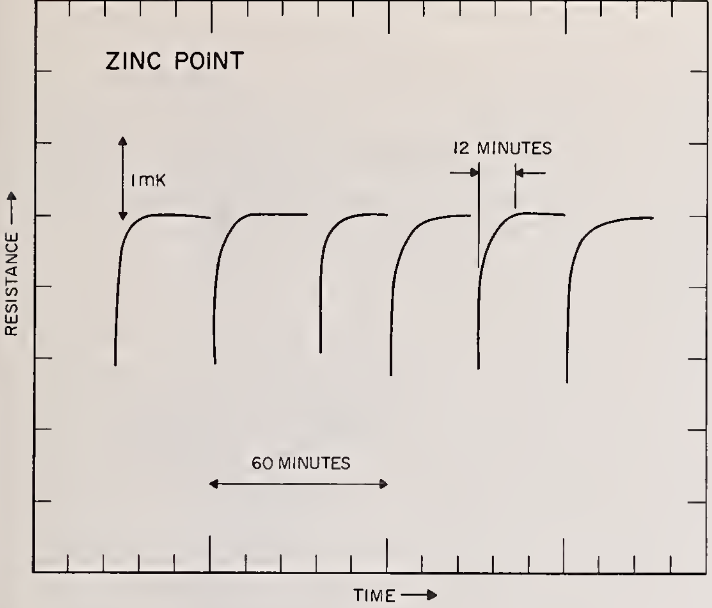

FIGURE 35. *Consecutive measurements of the resistance of a SPRT in the same zinc freeze.*

(An a-c bridge was employed in the measurements.)

Following each equilibrium resistance observation, the SPRT was completely withdrawn from the zinc-point cell, exposed to the ambient temperature for one minute, and reinserted into the cell. In the actual procedure employed in calibration the SPRT is preheated close to the zinc point (see text). The data show that the SPRT comes rapidly to equilibrium even when inserted into the cell relatively cold. (For comparison when the SPRT was preheated, see fig. 31.)

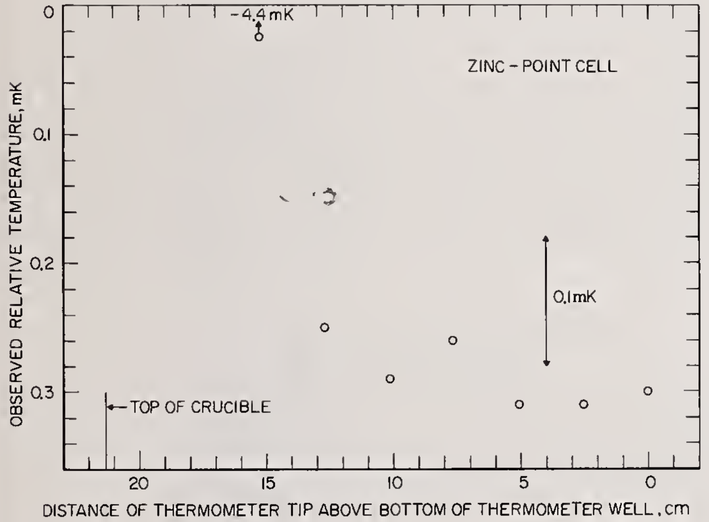

FIGURE 36. *Immersion characteristics of a SPRT in a zinc-point cell.*

The data show that the indications of the SPRT are very nearly the same between about 8 or 10 cm above the bottom and the bottom of the thermometer well.

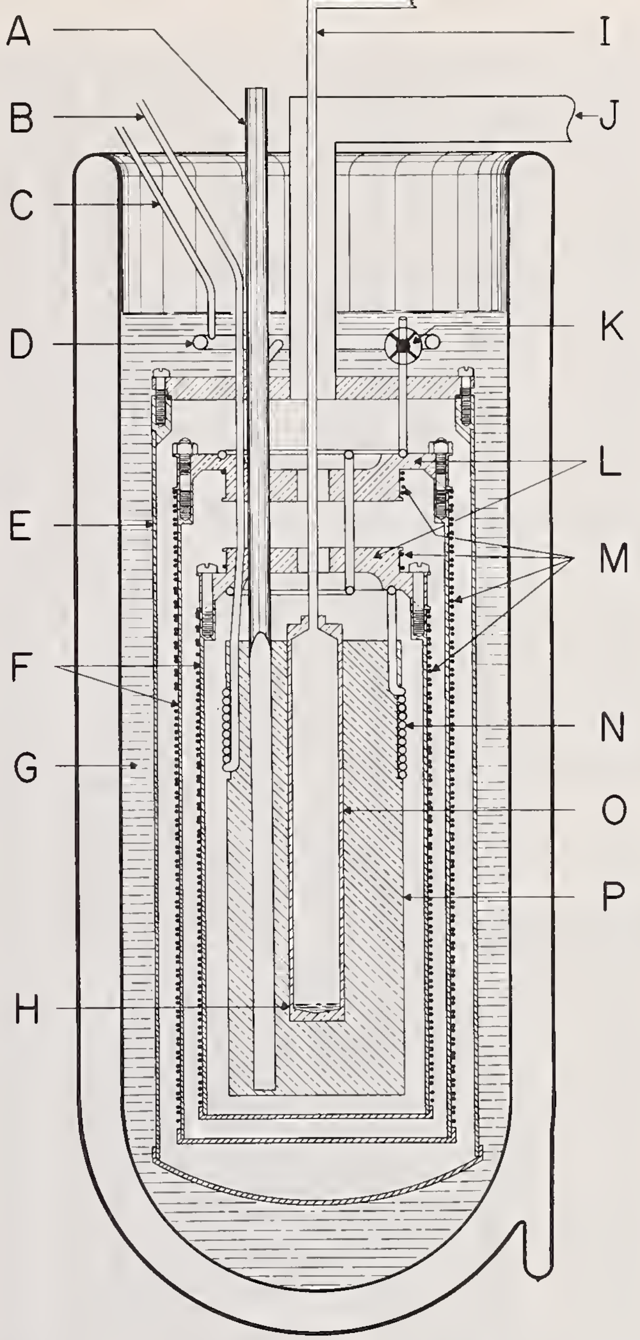

FIGURE 37. *Cryostat for comparison calibration of thermometers with standards at the oxygen normal boiling point.*

A. Thermometer well; eight wells are located around the copper block. The SPRT's are sealed at the top with molded silicone rubber.
B. Tube to vacuum pump to draw liquid nitrogen through cooling tubes.
C. Tube to helium gas supply.
D. Manifold for distributing helium gas to the thermometer wells.
E. Envelope (brass).
F. Radiation shields (copper).
G. Liquid nitrogen.
H. Liquid oxygen. (Not employed in comparison calibrations.)
I. Vapor-pressure tube, to differential pressure diaphragm and manometer.
J. High vacuum line.
K. Valve to control the liquid nitrogen input for cooling.
L. Top heat shields, to control thermometer well and thermometer stem temperatures.
M. Heaters.
N. Cooling tubes (thin-wall Monel).
O. Oxygen bulb.
P. Copper block.

in several national temperature scales; for details see reference [8]. Calibrations near the oxygen point are performed by inserting the SPRT's to be calibrated and an SPRT standard in the apparatus shown in figure 37. In the preparation for calibration the apparatus is evacuated and immersed in liquid nitrogen to the level indicated. There are eight thin-walled Monel wells (A) extending into the copper block (P). Two of these wells are one-half inch in diameter to accommodate capsule type thermometers in holders. Thermometers are sealed into the wells at the top by a molded band of very soft silicone rubber. The thermometer wells are filled with helium gas to a pressure that is slightly above atmospheric. The helium enhances the thermal contact between the wall of the well and

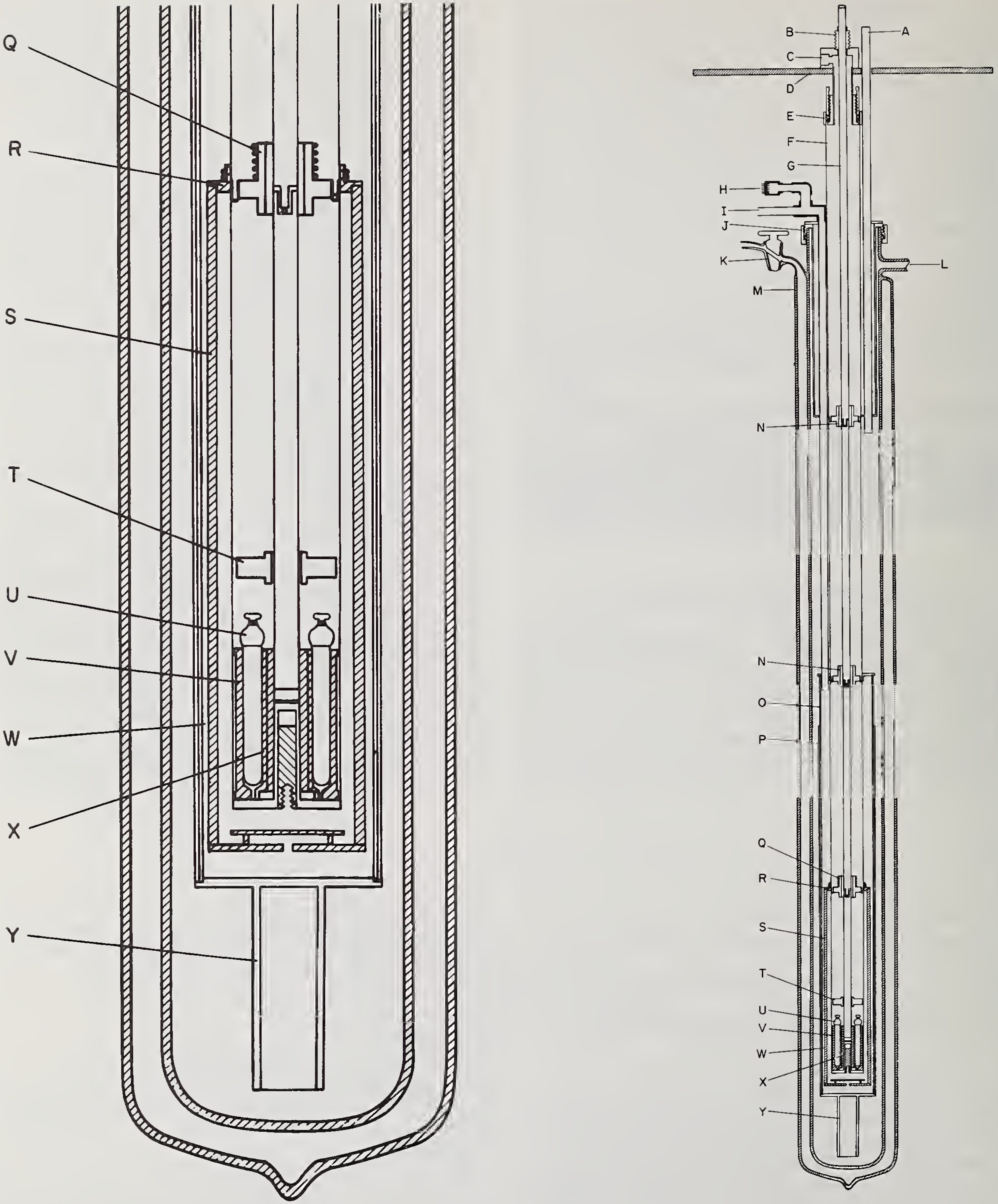

FIGURE 38. *Cryostat for the intercomparison of capsule type SPRT's.*

the thermometer and prevents condensible gases from entering the wells. The copper block (P) and shields (F and L) are maintained under vacuum. They are initially cooled by admitting nitrogen gas and/or liquid through the valve (K) shown just below the surface of the liquid nitrogen. The cool vapor flows through coils (N) on the top shields and coils on the massive center block, finally passing through (B) to a large-capacity vacuum pump. After cooling the block to a temperature near the oxygen point, the valve is closed and the nitrogen gas in the cooling coils is removed; the temperature of the block is brought within 1 K of the oxygen point, and the outer shields are controlled at the temperature of the block. Experience shows that the temperature of the block can be maintained more nearly constant by allowing the inner shields to "float" without heating at a temperature near that of the block. [Heaters (M) on the copper block and the inner shields are used, when desired, to attain higher temperatures]. The outer shields are controlled to be at the temperature of the block by means of one differential thermopile between the top shield and the block, and a second thermopile between the outer top shield and the outer side shield. The signals from these thermopiles go to commercial low level chopper type *d-c* amplifiers and thence to three-mode controllers; the output signals from the controllers are then raised to power levels that are sufficient to supply the shield heater (less than 1 *W*). The measurements on the thermometer to be calibrated and the standard thermometer are made simultaneously using two Mueller bridges. The standard deviation of the intercomparison is less than the equivalent of 0.1 mK.

7.7. Comparison Calibration Between 13.81 and 90.188 K

To determine temperatures below the oxygen normal boiling point, the text of IPTS–68 (Appendix A) assigns values to defining fixed points and prescribes the form of interpolation formulae. At the time the IPTS–68 was adopted no national laboratory could satisfactorily realize all the fixed points. The four temperature scales (e.g., NBS–55) upon which the extension of the IPTS–68 below the oxygen point was principally based are each highly reproducible. The text of the IPTS–68, therefore, formally recognized that the use of these national scales, adjusted by the published differences [8], will give a close approximation to the IPTS–68 below 90.188 K. At NBS a reference group of capsule type SPRT's maintains the "national scale" (NBS–55). The NBS version of IPTS–68 (NBS–IPTS–68) is achieved by making reference to the NBS–55 scale and utilizing the published differences between the NBS–55 scale and the IPTS–68 referred to above. Thermometers are calibrated by intercomparison in a nearly isothermal copper block with one of the thermometers of this reference group. A second thermometer from the reference group is also included in the comparison to serve as a check on both the reference thermometer being used and the comparison techniques. These two thermometers will be referred to, henceforth, as the first and second standards, respectively.

The comparison of capsule type SPRT's is performed in a cryostat that has been built and operated rather like an adiabatic calorimeter. The apparatus is shown in figure 38. The copper comparison block (V), shown enlarged in figure 39, has wells for six thermometers, two standards and four thermometers under test. The schematic in figure 40 shows that the first standard is wired independently of the other five thermometers. The wiring arrangement employed for the five thermometers requires only $2n+2$ leads for n thermometers, in this case, 12 leads, thus minimizing the number of leads and, therefore, the heat transfer through the leads with the surroundings. The first standard is electrically isolated from the others. This permits the use of two bridges to make simultaneous measurements on the first standard and any one of the thermometers to be calibrated or the second standard. The resistance measurements are made with two Mueller bridges.

← A. Guide for directing transfer tube into liquid helium Dewar.
B. Metal bellows to permit differential expansion between the central tube and the supporting thermometer block well.
C. Exit for the vacuum line and electrical leads to the thermometers, the thermocouples, and the heater on the copper thermometer block.
D. Supporting shelf.
E. Demountable "O" ring seal to the well around the thermometer block.
F. Well around the thermometer block.
G. Central tube for supporting the thermometer block.
H. Seal for the electrical leads from the vacuum can that surrounds the shield and lower portion of the thermometer block well.
I. Line from vacuum can "O".
J. Demountable "O" ring seal to the liquid helium Dewar.
K. Glass stopcock to permit reevacuation of liquid helium Dewar.
L. Line for pumping the space within the liquid helium Dewar.
M. Liquid helium Dewar.
N. Thermal tie-down for lead wires.
O. Vacuum can that surrounds the lower portion of the thermometer block well and the thermal shield.
P. Copper sleeve on vacuum can to maintain uniform temperature when liquid helium level is low.
Q. Thermal tie-down for leads similar to N with a heater and five-junction Chromel-P/constantan thermocouple for temperature control.
R. Copper ring on the thermometer block well with temperature control components similar to Q.
S. Heavy copper shield with temperature control components similar to Q.
T. Thermometer lead terminal block of anodized aluminum.
U. Capsule type platinum resistance thermometers thermally attached to copper block with vacuum grease.
V. Copper thermometer block with holes for six thermometers.
W. Location of thermocouple junctions placed on shield.
X. Reentrant "thumb" in the bottom of well. "Thumb" contains reference junctions for thermocouples on R and S.
Y. Heavy copper tail on vacuum can to reach liquid helium at a low level

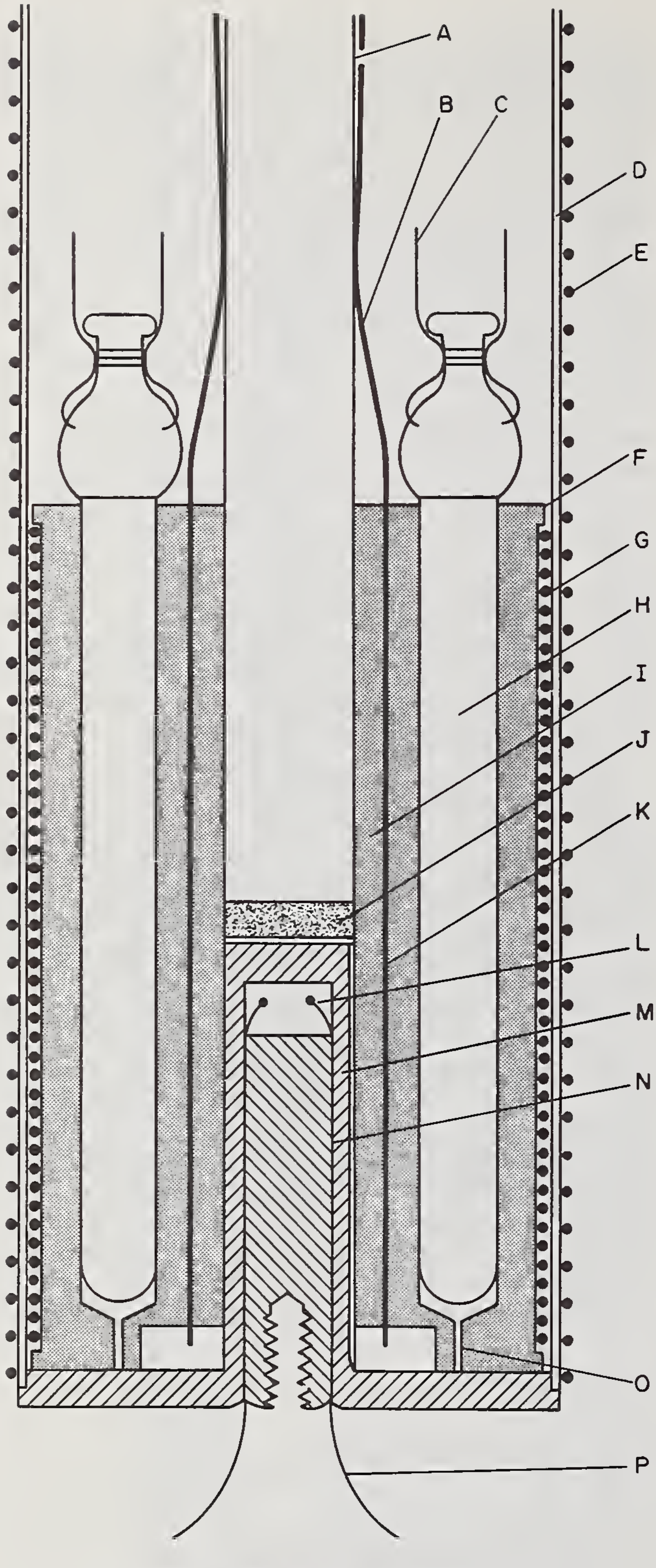

FIGURE 39. *Thermometer comparison block.*

A. Stainless steel tube to support thermometer comparison block.
B. Manganin leads for thermometers and copper leads for block heater.
C. Platinum leads of resistance thermometer.
D. Stainless steel well for thermometer block.
E. Heater distributed along well "D".
F. Copper thermometer block with wells for six thermometers.
G. Heater for thermometer block.
H. Platinum resistance thermometer.
I. Copper sleeve (attached to central supporting tube) with longitudinal grooves to serve as thermal tie-downs for manganin leads and thermocouples.
J. Brass plug to seal end of central support tube.
K. Location of grooves in sleeve "I."
L. Reference junctions for thermocouples on shield (at "W" in fig. 38) and thermometer block well ring heater (at "R" in fig. 38).
M. Closely fitted reentrant copper "thumb" (with a groove for venting gas).

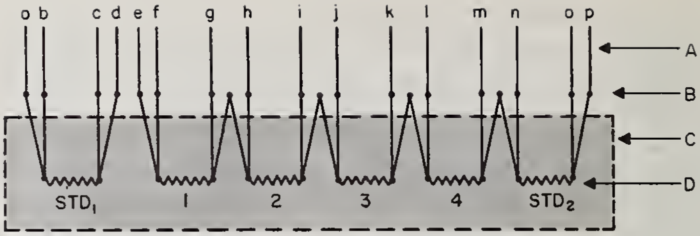

FIGURE 40. *Schematic of wiring capsule-type platinum resistance thermometers in the copper comparison block of the cryostat for calibration.*

To avoid excessive heating of the copper block, the bridge connections to the thermometers, wired in series, are made as follows:

SPRT	Leads used
1	efgh
2	ghij
3	ijkl
4	klmn
STD_2	mnop

A. Cryostat leads.
B. Thermometer connections made among them and with the cryostat leads.
C. Schematic of the copper comparison block.
D. Platinum resistance thermometers. STD_1 and STD_2 are reference standards; others are to be calibrated.

To minimize the possible change in the difference of resistance in the potential leads, the leads from the thermometers that pass up through the cryostat are of No. 26 AWG (0.404 mm diameter) manganin wire insulated with a heavy coating of Formvar. The choice represents a compromise among the requirements of low heat conductivity, the resistance seen by the galvanometer, and lead-resistance stability. The leads are "tempered" by being placed in good thermal contact with the series of "thermal tie downs" (N) shown in figure 41. The manganin leads and copper leads for thermocouples and heaters exit through a hard wax seal at (C) the top of the cryostat and then join to heavier copper leads. The external copper leads are thermally insulated to minimize both the temperature change of the leads and any possible gradient change between leads. The resistances of the copper sections of the leads and those of the manganin sections of the leads were separately adjusted for common equality to maintain the lead balance with changing room temperature. The thermometer leads are connected to the bridge through selector switches that employ either mercury-wetted contacts or multiple all-silver contacts in parallel.

The cryostat is constructed so that the central core, including the copper comparison block, is stationary while the remainder of the cryostat assembly may be lowered to expose the block for thermometer installation or removal. The only seals required for sealing the cryostat are at room temperature and consist of simple Viton "O" rings (E).

N. Copper plug with longitudinal grooves to serve as thermal tie-downs for thermocouples.
O. Vent.
P. Thermocouple leads to shield and thermometer block well ring heater.

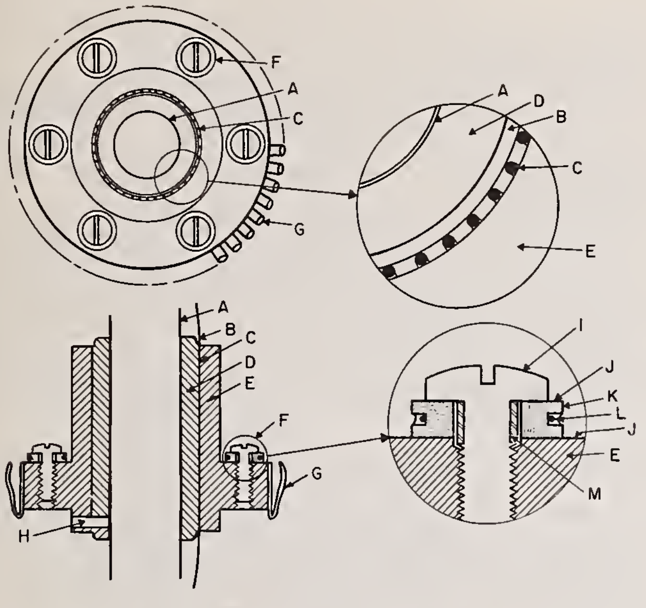

FIGURE 41. *Thermal tie-downs.*

A. Central support tube (stainless steel) for thermometer block.
B. Rounded corner, tangent to the bottom of the grooves for wire.
C. Round bottom grooves in cylinder "D" with wire cemented in place.
D. Copper cylinder soldered (tin-lead eutectic) to the central support tube.
E. Copper cylinder fitted to "D" and held in place by pin "H."
F. Thermocouple tie-down.
G. Spring fingers of beryllium copper heavily silver plated and gold "flashed."
H. 1.5 mm steel pin to hold cylinder "E" in position.
I. Pan head 2–56 brass screw to clamp thermocouple tie-down assembly to "E."
J. Insulating washer of 0.005 mm Mylar coated with vacuum grease.
K. Copper washer with pre-tinned groove for thermocouple wire.
L. Polyimide insulated thermocouple wire, placed at the bottom of the pretinned groove and "potted" in eutectic tin-lead solder.
M. Epoxy insulation on screw.

The cryostat proper (fig. 38) is surrounded with the usual arrangement—an inner Dewar flask (M), which contains liquid helium, guarded by an outer Dewar flask (not shown) that contains liquid nitrogen. During the initial cool down, the inner Dewar is filled with liquid nitrogen and the liquid nitrogen vapor pressure is reduced to about ⅓ atm. The vacuum can (*O*) of the cryostat is filled with ^{3}He gas to a pressure of about 0.1 torr to facilitate the cooling; under such conditions, the thermometer block cools to about 65 K in approximately 12 hours (overnight). The liquid nitrogen remaining in the inner Dewar is removed by applying a small over-pressure of ^{4}He gas which forces the nitrogen out through a 3.2 mm thin-wall stainless tube (not shown in fig. 38) that extends from the Dewar bottom to the room. (Precooling the cryostat from about 80 K to about 65 K by pumping on liquid nitrogen reduces the liquid helium required for cooling by about 2 liters.) Six to seven liters of liquid helium are usually required to cool the apparatus from 65 to 4.2 K and fill the inner helium Dewar.

As a part of the calibration procedure the thermometer resistance readings are first made at about 4 K, without the use of temperature controls. For the calibrations above 4 K, the ^{3}He gas in the vacuum can is removed to reduce the heat transfer and the temperature of the comparison block is controlled by the use of three sets of five-junction Chromel-P/Constantan differential thermopiles that sense temperature differences between the comparison block and the thermal shields. Appropriate automatic controls are employed to power three separate heaters to reduce the indicated temperature differences to zero. When the space in the vacuum can is evacuated to a pressure in the 10^{-6} torr range, most of the heat transfer to the comparison block from the surroundings is (i) down through the wall of the well (F) and (ii) within the well through the central thin-wall stainless tube and electrical leads. Consequently, two sets of thermopiles and heaters are affixed, one at (R) on the wall of the well and the other at (Q) within the well. These two locations are about 15 cm above the comparison block. The third thermopile and heater are mounted on the heavy copper shield which surrounds the lower end of the well where the comparison block rests. The thermopiles are all referenced to the temperature of the comparison block. If adequate temperature control of a system is to be achieved, the thermal lags of both the heaters and the temperature sensors (in this case thermopiles) must be small. In addition, the "immersion error" that is caused by heat conducted along the leads of the sensor should be small. These characteristics were achieved in the cryostat by employing the "thermal tie downs" shown in figures 38 and 41. (The selection of Chromel-P/Constantan thermocouples was influenced by their low thermal conductivity. There are other materials that possess better thermoelectric characteristics but their relatively high thermal conductivities were considered to be disadvantageous.) The spurious emf of each thermopile is balanced out with a simple emf injector that is adjustable between ±5 μV. The adjustment for the spurious emf is made at 4.2 K after equilibrium has been attained with ^{3}He gas still in the can.

The calibration resistance measurements are made at 16 temperatures; approximately 12.20, 13.81, 15.426, 17.042, 18.661, 20.28, 23.691, 27.102, 33.917, 40.732, 47.547, 54.361, 66.303, 78.245, 84.216, and 90. 188 K. The values of $W(T)$ at the IPTS–68 defining fixed-point temperatures are obtained by least square analysis of the comparison data (see sec. 6).

To change from one calibration temperature to the next higher temperature, current is supplied to the heater that is wound directly on the comparison block. Although the control of the shields is adequate to track the temperature of the block when it is rapidly heated at the lower temperatures (below about 40 K), the shields lag at higher temperatures when the block is being rapidly heated because of the limitations of the power amplifiers which drive their heaters. The time required for the system to attain thermal equilibrium after the heating of the block is stopped increases with temperature. At 13.8 K the thermometer drift will decrease to less than 10 $\mu\Omega$/min in a minute, while at 90 K more than 30 min is required to achieve this drift rate. Measurements are considered unsatisfactory if the indicated drift rate is more than 5 $\mu\Omega$/min at temperatures below 20 K or more than 10 $\mu\Omega$/min at temperatures above 20 K.

In order to achieve these low drift rates rather quickly, particularly at temperatures above 30 K, it was found necessary to uniformly heat the lower 20 cm section of the well (F) when the comparison block was being heated to the next higher calibration temperature. This was accomplished by powering a heater wound around the well (*E*) as shown in figure 39. An adjustable fraction (depending upon the temperature) of the voltage applied to the block heater is applied to the well heater to raise its temperature at the same time and rate as the block temperature.

In the calibration of the capsule type SPRT's the resistance measurements are carried out with a continuous current of 1 mA through the thermometer and with a commutator switching order of NRRNNRRN with the Mueller bridge. This order of measurements is not the optimum one (which is NRRNRNNR), but when a large number of measurements must be made, such as in the calibration of thermometers over this temperature range, the pattern used is less subject to gross errors. At approximately 90.2 K, measurements are made at two currents, 1 mA and 2 mA, to determine the self heating effect of the SPRT. Although infrequent, the glass to metal seal of a thermometer may leak; this causes a loss of helium and consequent increase in the self-heating effect of the thermometer. A thermometer having this defect may be unreliable. The treatment of the data resulting from these measurements differs markedly from those obtained above 90.188 K (see sec. 6).

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9. Appendixes

APPENDIX A.

The International Practical Temperature Scale of 1968*

Adopted by the Comité International des Poids et Mesures

Received February 17, 1969

Foreword

The Comité International des Poids et Mesures (CIPM) at its meeting in October 1968 agreed to adopt the International Practical Temperature Scale of 1968 (IPTS-68) in accordance with the decision of the 13^e Conférence Générale des Poids et Mesures, Resolution 8, of October 1967. This resolution gave the CIPM permission to introduce a new Scale to replace the existing IPTS-48 as soon as the Comité Consultatif de Thermométrie (CCT) made its recommendations.

There were two important reasons for revising the IPTS-48. The first of these was the need to extend the Scale to lower temperatures to achieve a unification of the existing national scales in the region 10 K—90 K. Secondly the Scale had not been revised significantly over the range from −183 °C to 1063 °C since its inception in 1927, and modern gas thermometer measurements showed that the Scale gave values considerably different from thermodynamic temperatures.

The CCT agreed on the basic data and methods of interpolation for a new definition of the IPTS at its 8th meeting held in Washington and Ottawa in September 1967. Provision was made to extend the Scale down to 13.81 K, the triple point of equilibrium hydrogen, and for the use of the best known values of thermodynamic temperatures. The value of c_2 used in the Planck equation was also revised as a result of a better knowledge of the values of fundamental physical constants. The CCT appointed a small sub-committee to prepare a text of the definition of the IPTS-68 in accordance with its recommendations, with instructions to resolve some points of detail. The final draft of the English text, which is reproduced below, received the approval of all members of the CCT before it was submitted to the CIPM. The official text of the IPTS-68 is the French text which was prepared by the Bureau International des Poids et Mesures from the English text, and is published as an annexe to Comptes rendus des séances de la Treizième Conférence Générale des Poids et Mesures.

C. R. BARBER

I. Introduction

The basic temperature is the thermodynamic temperature, symbol T, the unit of which is the kelvin, symbol K. The kelvin is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water[1].

The Celsius temperature, symbol t, is defined by

$$t = T - T_0 \tag{1}$$

where $T_0 = 273.15$ K. The unit employed to express a Celsius temperature is the degree Celsius, symbol °C, which is equal to the kelvin. A difference of temperature is expressed in kelvins; it may also be expressed in degrees Celsius.

The International Practical Temperature Scale of 1968 (IPTS-68) has been chosen in such a way that the temperature measured on it closely approximates the thermodynamic temperature; the difference is within the limits of the present accuracy of measurement.

The International Practical Temperature Scale of 1968 distinguishes between the International Practical Kelvin Temperature with the symbol T_{68} and the International Practical Celsius Temperature with the symbol t_{68}; the relation between T_{68} and t_{68} is

$$t_{68} = T_{68} - 273.15 \text{ K} . \tag{2}$$

The units of T_{68} and t_{68} are the kelvin, symbol K, and degree Celsius, symbol °C, as in the case of the thermodynamic temperature T and the Celsius temperature t.

The International Practical Temperature Scale of 1968 was adopted by the International Committee of Weights and Measures at its meeting in 1968 according to the power given to it by Resolution 8 of the 13th General Conference of Weights and Measures. It replaces the International Practical Temperature Scale of 1948 (amended edition of 1960).

II. Definition of the International Practical Temperature Scale of 1968[2] (IPTS-68)

1. Principle of the IPTS-68 and Defining Fixed Points

The IPTS-68 is based on the assigned values of the temperatures of a number of reproducible equilibrium states (defining fixed points) and on standard instruments calibrated at those temperatures. Interpolation between the fixed point temperatures is provided by formulae used to establish the relation between indications of the standard instruments and values of International Practical Temperature.

The defining fixed points are established by realizing specified equilibrium states between phases of pure substances. These equilibrium states and the values of the International Practical Temperature assigned to them are given in Table 1.

The standard instrument used from 13.81 K to 630.74 °C is the platinum resistance thermometer. The thermometer resistor must be strain-free, annealed pure platinum. The resistance ratio $W(T_{68})$, defined by

$$W(T_{68}) = R(T_{68})/R(273.15 \text{ K}) \tag{3}$$

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* The text in French of this Scale is published in Comptes rendus de la Treizième Conférence Générale des Poids et Mesures, 1967—1968, Annexe 2, and Comité Consultatif de Thermométrie, 8^e session, 1967, Annexe 18.

[1] 13th General Conference of Weights and Measures (1967), Resolutions 3 and 4.

[2] In this document Kelvin temperatures are used, in general, below 0 °C and Celsius temperatures are used above 0 °C. This avoids the use of negative values and conforms with general usage.

Table 1. *Defining fixed points of the IPTS-68*[a]

Equilibrium state	Assigned value of International Practical Temperature T_{68} (K)	t_{68} (°C)
Equilibrium between the solid, liquid and vapour phases of equilibrium hydrogen (triple point of equilibrium hydrogen)	13.81	−259.34
Equilibrium between the liquid and vapour phases of equilibrium hydrogen at a pressure of 33 330.6 N/m² (25/76 standard atmosphere)	17.042	−256.108
Equilibrium between the liquid and vapour phases of equilibrium hydrogen (boiling point of equilibrium hydrogen)	20.28	−252.87
Equilibrium between the liquid and vapour phases of neon (boiling point of neon)	27.102	−246.048
Equilibrium between the solid, liquid and vapour phases of oxygen (triple point of oxygen)	54.361	−218.789
Equilibrium between the liquid and vapour phases of oxygen (boiling point of oxygen)	90.188	−182.962
Equilibrium between the solid, liquid and vapour phases of water (triple point of water)[c]	273.16	0.01
Equilibrium between the liquid and vapour phases of water (boiling point of water)[b,c]	373.15	100
Equilibrium between the solid and liquid phases of zinc (freezing point of zinc)	692.73	419.58
Equilibrium between the solid and liquid phases of silver (freezing point of silver)	1235.08	961.93
Equilibrium between the solid and liquid phases of gold (freezing point of gold)	1337.58	1064.43

[a] Except for the triple points and one equilibrium hydrogen point (17.042 K) the assigned values of temperature are for equilibrium states at a pressure $p_0 = 1$ standard atmosphere (101 325 N/m²). In the realization of the fixed points small departures from the assigned temperatures will occur as a result of the differing immersion depths of thermometers or the failure to realize the required pressure exactly. If due allowance is made for these small temperature differences, they will not affect the accuracy of realization of the Scale. The magnitudes of these differences are given in section III.

[b] The equilibrium state between the solid and liquid phases of tin (freezing point of tin) has the assigned value of $t_{68} = 231.9681$ °C and may be used as an alternative to the boiling point of water.

[c] The water used should have the isotopic composition of ocean water, see section III, 4.

where R is the resistance, must not be less than 1.392 50 at $T_{68} = 373.15$ K. Below 0 °C the resistance-temperature relation of the thermometer is found from a reference function and specified deviation equations. From 0 °C to 630.74 °C two polynomial equations provide the resistance-temperature relation.

The standard instrument used from 630.74 °C to 1064.43 °C is the platinum-10% rhodium/platinum thermocouple, the electromotive force-temperature relation of which is represented by a quadratic equation.

Above 1337.58 K (1064.43 °C) the International Practical Temperature of 1968 is defined by the Planck law of radiation with 1337.58 K as the reference temperature and a value of 0.014 388 metre kelvin for c_2.

2. *Definition of the International Practical Temperature of 1968 in Different Temperature Ranges*

a) The range from 13.81 K to 273.15 K

From 13.81 K to 273.15 K the temperature T_{68} is defined by the relation

$$W(T_{68}) = W_{\mathrm{CCT\text{-}68}}(T_{68}) + \Delta W(T_{68}) \qquad (5)$$

where $W(T_{68})$ is the resistance ratio of the platinum resistance thermometer and $W_{\mathrm{CCT\text{-}68}}(T_{68})$ is the resistance ratio as given by the reference function set out in Table 2[3]. The deviations $\Delta W(T_{68})$ at the temperatures of the defining fixed points are obtained from the measured values of $W(T_{68})$ and the corresponding values of $W_{\mathrm{CCT\text{-}68}}(T_{68})$, see Table 4. To find $\Delta W(T_{68})$ at intermediate temperatures interpolation formulae are used. The range between 13.81 K and 273.15 K is divided into four parts in each of which $\Delta W(T_{68})$ is defined by a polynomial in T_{68}. The constants in the polynomials are determined from the values of $\Delta W(T_{68})$ at the fixed points and the condition that there should be no discontinuity in $\mathrm{d}\Delta W(T_{68})/\mathrm{d}T_{68}$ at the junctions of the temperature ranges.

From 13.81 K to 20.28 K the deviation function is

$$\Delta W(T_{68}) = A_1 + B_1 T_{68} + C_1 T_{68}^2 + D_1 T_{68}^3 \qquad (5)$$

where the constants are determined by the measured deviations at the triple point of equilibrium hydrogen, the temperature of 17.042 K and the boiling point of equilibrium hydrogen and by the derivative of the deviation function at the boiling point of equilibrium hydrogen as derived from Eq. (6).

From 20.28 K to 54.361 K the deviation function is

$$\Delta W(T_{68}) = A_2 + B_2 T_{68} + C_2 T_{68}^2 + D_2 T_{68}^3 \qquad (6)$$

where the constants are determined by the measured deviations at the boiling point of equilibrium hydrogen, the boiling point of neon and the triple point of oxygen and by the derivative of the deviation function at the triple point of oxygen as derived from Eq. (7).

From 54.361 K to 90.188 K the deviation function is

$$\Delta W(T_{68}) = A_3 + B_3 T_{68} + C_3 T_{68}^2 \qquad (7)$$

where the constants are determined by the measured deviations at the triple point and the boiling point of oxygen and by the derivative of the deviation function at the boiling point of oxygen as derived from Eq. (8).

From 90.188 K to 273.15 K the deviation function is

$$\Delta W(T_{68}) = A_4 t_{68} + C_4 t_{68}^3 \, (t_{68} - 100\ ^\circ\mathrm{C}) \qquad (8)$$

where $t_{68} = T_{68} - 273.15$ K and the constants are determined by the measured deviations at the boiling point of oxygen and the boiling point of water[1].

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[3] For the relation between the IPTS-68 and the "national scales" from which it was in part derived see Appendix I.

[4] If the freezing point of tin (see Note [b], Table 1) is used as a fixed point instead of the boiling point of water, $W(100\ ^\circ\mathrm{C})$ for the platinum thermometer should be calculated from Eqs. (9) and (10).

Table 2. *The reference function* $W_{\mathrm{CCT\text{-}68}}(T_{68})$ *for platinum resistance thermometers for the range from 13.81* K *to 273.15* K[a]

$$T_{68} = \left\{ A_0 + \sum_{i=1}^{20} A_i [\ln W_{\mathrm{CCT\text{-}68}}(T_{68})]^i \right\} \mathrm{K} \tag{22}$$

Coefficients A_i:

i	A_i	i	A_i
0	$0.273\,15 \times 10^3$	11	$0.767\,976\,358\,170\,845\,8 \times 10$
1	$0.250\,846\,209\,678\,803\,3 \times 10^3$	12	$0.213\,689\,459\,382\,850\,0 \times 10$
2	$0.135\,099\,869\,964\,999\,7 \times 10^3$	13	$0.459\,843\,348\,928\,069\,3$
3	$0.527\,856\,759\,008\,517\,2 \times 10^2$	14	$0.763\,614\,629\,231\,648\,0 \times 10^{-1}$
4	$0.276\,768\,548\,854\,105\,2 \times 10^2$	15	$0.969\,328\,620\,373\,121\,3 \times 10^{-2}$
5	$0.391\,053\,205\,376\,683\,7 \times 10^2$	16	$0.923\,069\,154\,007\,007\,5 \times 10^{-3}$
6	$0.655\,613\,230\,578\,069\,3 \times 10^2$	17	$0.638\,116\,590\,952\,653\,8 \times 10^{-4}$
7	$0.808\,035\,868\,559\,866\,7 \times 10^2$	18	$0.302\,293\,237\,874\,619\,2 \times 10^{-5}$
8	$0.705\,242\,118\,234\,052\,0 \times 10^2$	19	$0.877\,551\,391\,303\,760\,2 \times 10^{-7}$
9	$0.447\,847\,589\,638\,965\,7 \times 10^2$	20	$0.117\,702\,613\,125\,477\,4 \times 10^{-8}$
10	$0.212\,525\,653\,556\,057\,8 \times 10^2$		

The reference function $W_{\mathrm{CCT\text{-}68}}(T_{68})$ is continuous at $T_{68} = 273.15$ K in its first and second derivatives with the function $W(t_{68})$ given by Eqs. (9) and (10) for $\alpha = 3.925\,966\,8 \times 10^{-3}\ {}^\circ\mathrm{C}^{-1}$ and $\delta = 1.496\,334\ {}^\circ\mathrm{C}$.

[a] A tabulation of this reference function, sufficiently detailed to allow interpolation to an accuracy of 0.0001 K, is available from the Bureau International des Poids et Mesures, 92-Sèvres, France. A skeleton tabulation appears in this text as Table 3.

b) The range from 0 °C (273.15 K) to 630.74 °C

From 0 °C to 630.74 °C t_{68} is defined by

$$t_{68} = t' + 0.045 \left(\frac{t'}{100\ {}^\circ\mathrm{C}}\right) \left(\frac{t'}{100\ {}^\circ\mathrm{C}} - 1\right) \cdot \left(\frac{t'}{419.58\ {}^\circ\mathrm{C}} - 1\right) \left(\frac{t'}{630.74\ {}^\circ\mathrm{C}} - 1\right) {}^\circ\mathrm{C} \tag{9}$$

where t' is defined by the equation:

$$t' = \frac{1}{\alpha}\,[W(t') - 1] + \delta \left(\frac{t'}{100\ {}^\circ\mathrm{C}}\right) \left(\frac{t'}{100\ {}^\circ\mathrm{C}} - 1\right) \tag{10a}$$

where $W(t') = \dfrac{R(t')}{R(0\ {}^\circ\mathrm{C})}$. The constants $R(0\ {}^\circ\mathrm{C})$, α and δ are determined by measurement of the resistance at the triple point of water, the boiling point of water (or the freezing point of tin, see Note [b], Table 1) and the freezing point of zinc.

Eq. (10a) is equivalent to the equation

$$W(t') = 1 + A\,t' + B\,t'^2 \tag{10b}$$

where $A = \alpha(1 + \delta/100\ {}^\circ\mathrm{C})$ and $B = -\ 10^{-4}\,\alpha\,\delta\ {}^\circ\mathrm{C}^{-2}$.

c) The range from 630.74 °C to 1064.43 °C

From 630.74 °C to 1064.43 °C t_{68} is defined by the equation

$$E(t_{68}) = a + b t_{68} + c t_{68}^2 \tag{11}$$

where $E(t_{68})$ is the electromotive force of a standard thermocouple of rhodium-platinum alloy and platinum, when one junction is at the temperature $t_{68} = 0$ °C and the other is at temperature t_{68}. The constants a, b and c are calculated from the values of E at 630.74 °C ± 0.2 °C, as determined by a platinum resistance thermometer, and at the freezing points of silver and gold.

The wires of the standard thermocouple shall be annealed and the purity of the platinum wire shall be such that the ratio $W(100\ {}^\circ\mathrm{C})$ is not less than 1.3920. The rhodium-platinum wire shall contain nominally 10% rhodium and 90% platinum by weight. The thermocouple shall be such that the electromotive forces $E(630.74\ {}^\circ\mathrm{C})$, $E(\mathrm{Ag})$ and $E(\mathrm{Au})$ shall satisfy the following relations:

$$E(\mathrm{Au}) = 10\,300\ \mu\mathrm{V} \pm 50\ \mu\mathrm{V} \tag{12}$$

$$E(\mathrm{Au}) - E(\mathrm{Ag}) = 1183\ \mu\mathrm{V} + 0.158\ [E(\mathrm{Au}) - 10\,300\ \mu\mathrm{V}] \pm 4\ \mu\mathrm{V} \tag{13}$$

$$E(\mathrm{Au}) - E(630.74\ {}^\circ\mathrm{C}) = 4766\ \mu\mathrm{V} + 0.631\ [E(\mathrm{Au}) - 10\,300\ \mu\mathrm{V}] \pm 8\ \mu\mathrm{V}\,. \tag{14}$$

d) The range above 1337.58 K (1064.43 °C)

Above 1337.58 K (1064.43 °C) the temperature T_{68} is defined by the equation

$$\frac{L_\lambda(T_{68})}{L_\lambda(T_{68}(\mathrm{Au}))} = \frac{\exp\left[\dfrac{c_2}{\lambda T_{68}(\mathrm{Au})}\right] - 1}{\exp\left[\dfrac{c_2}{\lambda T_{68}}\right] - 1} \tag{15}$$

in which $L_\lambda(T_{68})$ and $L_\lambda(T_{68}(\mathrm{Au}))$ are the spectral concentrations at temperature T_{68} and at the freezing point of gold, $T_{68}(\mathrm{Au})$, of the radiance of a blackbody at the wavelength λ[5]; $c_2 = 0.014\,388$ metre kelvin.

III. Supplementary Information

The apparatus, methods and procedures described in this section represent good practice at the present time.

1. *Standard Resistance Thermometer*

A standard platinum resistance thermometer should be so designed and constructed that the four-terminal resistance element is as free as possible from strain and will remain so in use. Satisfactory resistors have been made with platinum wires of uniform diameter between 0.05 and 0.5 mm and with at least a short portion of each lead adjacent to the resistor also made of platinum. A commonly used value of $R(0\ {}^\circ\mathrm{C})$ is ≈ 25 ohms and the measuring current for such a thermometer is normally 1 or 2 milliamperes. All thermometer components in close proximity to the resistor must be clean and non-reactive with platinum.

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[5] Since $T_{68}(\mathrm{Au})$ is close to the thermodynamic temperature of the freezing point of gold and c_2 is close to the second radiation constant of the Planck equation, it is not necessary to specify the value of the wavelength to be employed in the measurements [see Metrologia 3, 28 (1967)].

Table 3. *Values of $W_{\text{CCT-88}}(T_{88})$, according to the equation given in Table 2, at integral values of T_{68}*

T_{68} K	$W_{\text{CCT-68}}(T_{88})$	T_{68} K	$W_{\text{CCT-68}}(T_{88})$	T_{88} K	$W_{\text{CCT-68}}(T_{88})$	T_{68} K	$W_{\text{CCT-68}}(T_{68})$	T_{68} K	$W_{\text{CCT-68}}(T_{68})$	T_{68} K	$W_{\text{CCT-68}}(T_{68})$
		50	0.075 377 56	100	0.286 302 01	150	0.498 611 35	200	0.704 966 94	250	0.907 383 09
		51	0.079 101 23	101	0.290 621 54	151	0.502 787 07	201	0.709 050 04	251	0.911 397 53
		52	0.082 875 95	102	0.294 938 41	152	0.506 960 58	202	0.713 131 61	252	0.915 410 74
		53	0.086 698 59	103	0.299 252 45	153	0.511 131 72	203	0.717 211 74	253	0.919 422 74
		54	0.090 566 00	104	0.303 563 59	154	0.515 300 65	204	0.721 290 26	254	0.923 433 43
		55	0.094 475 15	105	0.307 871 83	155	0.519 467 37	205	0.725 367 33	255	0.927 442 83
		56	0.098 423 36	106	0.312 177 10	156	0.523 631 80	206	0.729 442 88	256	0.931 451 01
		57	0.102 407 74	107	0.316 479 39	157	0.527 794 09	207	0.733 516 90	257	0.935 458 05
		58	0.106 425 83	108	0.320 778 56	158	0.531 954 17	208	0.737 589 47	258	0.939 463 71
		59	0.110 475 06	109	0.325 074 67	159	0.536 112 11	209	0.741 660 59	259	0.943 468 22
		60	0.114 553 12	110	0.329 367 65	160	0.540 267 92	210	0.745 730 26	260	0.947 471 52
		61	0.118 657 89	111	0.333 657 51	161	0.544 421 67	211	0.749 798 41	261	0.951 473 52
		62	0.122 787 22	112	0.337 944 16	162	0.548 573 36	212	0.753 865 18	262	0.955 474 30
13	0.001 230 61	63	0.126 939 14	113	0.342 227 68	163	0.552 722 91	213	0.757 930 43	263	0.959 473 85
14	0.001 459 73	64	0.131 111 89	114	0.346 508 00	164	0.556 870 48	214	0.761 994 30	264	0.963 472 19
15	0.001 745 41	65	0.135 303 63	115	0.350 785 19	165	0.561 016 06	215	0.766 056 72	265	0.967 469 31
16	0.002 094 74	66	0.139 512 84	116	0.355 059 10	166	0.565 159 58	216	0.770 117 70	266	0.971 465 13
17	0.002 515 12	67	0.143 738 00	117	0.359 329 89	167	0.569 301 12	217	0.774 177 30	267	0.975 459 80
18	0.003 014 28	68	0.147 977 73	118	0.363 597 54	168	0.573 440 76	218	0.778 235 45	268	0.979 453 25
19	0.003 599 62	69	0.152 230 58	119	0.367 861 99	169	0.577 578 48	219	0.782 292 23	269	0.983 445 41
20	0.004 277 80	70	0.156 495 41	120	0.372 123 31	170	0.581 714 23	220	0.786 347 56	270	0.987 436 42
21	0.005 054 95	71	0.160 771 08	121	0.376 381 51	171	0.585 848 06	221	0.790 401 51	271	0.991 426 14
22	0.005 936 68	72	0.165 056 43	122	0.380 636 57	172	0.589 979 99	222	0.794 454 09	272	0.995 414 71
23	0.006 928 04	73	0.169 350 49	123	0.384 888 51	173	0.594 110 08	223	0.798 505 23	273	0.999 401 99
24	0.008 033 16	74	0.173 652 40	124	0.389 137 32	174	0.598 238 35	224	0.802 555 06		
25	0.009 255 04	75	0.177 961 17	125	0.393 383 16	175	0.602 364 78	225	0.806 603 52		
26	0.010 595 85	76	0.182 276 05	126	0.397 625 94	176	0.606 489 31	226	0.810 650 54		
27	0.012 056 90	77	0.186 596 28	127	0.401 865 67	177	0.610 612 08	227	0.814 696 25		
28	0.013 639 01	78	0.190 921 07	128	0.406 102 42	178	0.614 733 10	228	0.818 740 59		
29	0.015 342 61	79	0.195 249 92	129	0.410 336 28	179	0.618 852 29	229	0.822 783 64		
30	0.017 167 68	80	0.199 582 12	130	0.414 567 09	180	0.622 969 72	230	0.826 825 31		
31	0.019 113 63	81	0.203 917 14	131	0.418 795 07	181	0.627 085 40	231	0.830 865 61		
32	0.021 179 44	82	0.208 254 45	132	0.423 020 15	182	0.631 199 39	232	0.834 904 61		
33	0.023 363 43	83	0.212 593 44	133	0.427 242 33	183	0.635 311 64	233	0.838 942 24		
34	0.025 663 35	84	0.216 933 88	134	0.431 461 69	184	0.639 422 13	234	0.842 978 57		
35	0.028 076 45	85	0.221 275 23	135	0.435 678 31	185	0.643 530 94	235	0.847 013 53		
36	0.030 599 53	86	0.225 617 12	136	0.439 892 10	186	0.647 638 07	236	0.851 047 26		
37	0.033 229 16	87	0.229 959 16	137	0.444 103 22	187	0.651 743 52	237	0.855 079 63		
38	0.035 961 55	88	0.234 301 05	138	0.448 311 59	188	0.655 847 30	238	0.859 110 69		
39	0.038 793 05	89	0.238 642 48	139	0.452 517 30	189	0.659 949 47	239	0.863 140 46		
40	0.041 719 68	90	0.242 983 15	140	0.456 720 33	190	0.664 049 96	240	0.867 168 94		
41	0.044 737 60	91	0.247 322 90	141	0.460 920 77	191	0.668 148 86	241	0.871 196 11		
42	0.047 842 92	92	0.251 661 28	142	0.465 118 61	192	0.672 246 07	242	0.875 221 99		
43	0.051 031 78	93	0.255 998 36	143	0.469 313 87	193	0.676 341 76	243	0.879 246 57		
44	0.054 300 36	94	0.260 333 69	144	0.473 506 60	194	0.680 435 77	244	0.883 269 94		
45	0.057 644 86	95	0.264 667 18	145	0.477 696 82	195	0.684 528 25	245	0.887 292 00		
46	0.061 061 61	96	0.268 998 70	146	0.481 884 59	196	0.688 619 13	246	0.891 312 69		
47	0.064 546 79	97	0.273 328 07	147	0.486 069 85	197	0.692 708 41	247	0.895 332 24		
48	0.068 096 90	98	0.277 655 16	148	0.490 252 74	198	0.696 796 17	248	0.899 350 49		
49	0.071 708 35	99	0.281 979 88	149	0.494 433 19	199	0.700 882 32	249	0.903 367 44		
50	0.075 377 56	100	0.286 302 01	150	0.498 611 35	200	0.704 966 94	250	0.907 383 09		

During fabrication it is recommended that the thermometer be evacuated while at about 450 °C and then filled with dry gas and hermetically sealed. It is desirable to have oxygen present in the gas filling to ensure that trace impurities in the platinum will remain in an oxidised state. After completion, the resistance element should be stabilized by heating at a temperature higher than its intended maximum operating temperature and in any case not lower than 450 °C.

Table 4. *Values of $W_{\text{CCT-88}}(T_{68})$, according to the data given in Table 2, at the fixed-point temperatures*

Fixed point	T_{68} (K)	t_{68} (°C)	$W_{\text{CCT-68}}$
e-H_2 triple	13.81	−259.34	0.001 412 06
e-H_2 17.042	17.042	−256.108	0.002 534 44
e-H_2 boiling	20.28	−252.87	0.004 485 17
Ne boiling	27.102	−246.048	0.012 212 72
O_2 triple	54.361	−218.789	0.091 972 52
O_2 boiling	90.188	−182.962	0.243 799 09
	273.15	0	1
H_2O boiling	373.15	100	1.392 596 68
Sn freezing	505.1181	231.9681	1.892 570 86

The insulation resistance of the components supporting the resistance element and leads must be high enough to avoid significant shunting of the element. For example, care must be taken to avoid condensation of water vapour between the leads at low temperatures, and intrinsic leakage in the insulators themselves at high temperatures. The insulators are usually fabricated from mica, silica or alumina, and these materials normally give adequate intrinsic insulation up to 500 °C. However, as the temperature approaches 630 °C, the problem becomes more critical and errors of 1 mK or greater may easily occur. In the case of mica, there is the additional difficulty that significant amounts of water may be released during its exposure to temperatures above 450 °C, and unless this moisture is removed by periodic pumping or by a desiccant the insulation resistance will deteriorate rapidly.

To ensure adequate stability in the resistance and the temperature coefficients of resistivity, the resistor of a standard platinum resistance thermometer should be maintained, as far as possible, in an annealed state. Added resistivity may arise both from the accidental cold working that results from normal thermometer handling and also as a result of rapid cooling when a thermometer is transferred rapidly from an environment above 500 °C to room temperature. This latter increase in resistance is due to quenched-in, non-equilibrium concentrations of vacancy defects and is retained as long as the thermometer remains below 200 °C. Much of the cold work and all of the quenched-in resistance may be removed by annealing at 500 °C for 30 min.

Significant errors can be caused by radiation loss from the thermometer by total reflection in the constructional components, particularly if these are of silica. Such loss in the sheath, but not in the internal components, can be suppressed by blackening the outer surface of the sheath (e.g., with a colloidal graphite suspension) or by sand-blasting the surface to produce a matt finish.

Table 5

Effect of pressure on the freezing-point temperatures of metals

Metal	Freezing point at 1 standard atmosphere (°C)	Pressure coefficient	
		kelvins per atmosphere	kelvins per centimetre of liquid
Mercury	−38.862	+0.005 4	+0.000 071
Indium	156.634	+0.004 9	+0.000 033
Tin	231.9681	+0.003 3	+0.000 022
Bismuth	271.442	−0.003 5	−0.000 034
Cadmium	321.108	+0.006 2	+0.000 048
Lead	327.502	+0.008 0	+0.000 082
Zinc	419.58	+0.004 3	+0.000 027
Antimony	630.74	+0.000 85	+0.000 005

The completed thermometer should be tested to establish that the depth of immersion is sufficient to avoid heat conduction errors. An effective way of doing this is to confirm that the apparent temperature gradient in a metal freezing point is in agreement with that to be expected from hydrostatic effects (see Table 5).

For temperatures below 90 K it is usual to use a small platinum resistance thermometer, generally not larger than 5 mm in diameter and 60 mm in length, that can be totally immersed in a uniform temperature zone with heat conduction down the leads being suppressed by attaching them to a suitable guard ring. In order to achieve good thermal contact between the resistor and its surroundings the resistor is contained in a thin sheath, commonly of platinum about 0.25 mm thick, which is filled with helium.

A useful criterion by which the efficiency of the annealing and the reliability of the thermometer may by judged is the constancy of its resistance at a reference temperature. The temperatures of the triple point of water (273.16 K) and the boiling point of helium (4.215 K) are commonly used for this purpose. The first of these is convenient for most high temperature thermometers, while the second is not only often conveniently attained for thermometers built into cryogenic apparatus but has the additional advantage that the resistance is relatively insensitive to temperature variations. In practice it is found that variations of resistance at the triple point of water for commercially produced high temperature thermometers should not exceed $4 \times 10^{-6}\,R\,(0\,°\mathrm{C})$ (equivalent to ≈ 1 mK above 40 K), and will not exceed $5 \times 10^{-7}\,R\,(0\,°\mathrm{C})$ over a reasonable period of use for the very best thermometers when these are handled with extreme care. For resistance thermometers used only at temperatures of 100 °C or less, variations should not exceed $5 \times 10^{-7}\,R(0\,°\mathrm{C})$.

The small temperature rise of the thermometers caused by the measuring current may be determined by measurements at two currents.

2. *Standard Thermocouple*

Satisfactory standard thermocouples have been made of wires of a uniform diameter between 0.35 and 0.65 mm. The thermocouple wires must be thoroughly annealed in order to ensure constancy of e.m.f. in use. For this purpose it is necessary to heat the platinum wire to a temperature of at least 1100 °C and the platinum-rhodium wire to 1450 °C. If the annealing is done before the wires have been mounted in their insulators the completed thermocouple must be heated again to a temperature of at least 1100 °C until its electromotive force has been stabilized and local inhomogeneities caused by strain have been removed. When this has been satisfactorily accomplished the thermocouple e.m.f. should not be changed by changes in the temperature gradients along the wires; it should not change, for example, with increase of depth of immersion in an enclosure at a uniform temperature.

3. *Pressure*

In practice pressures are usually determined by means of a mercury column. The mean density of pure mercury at the temperature t_{68} in a barometric column supported by the pressure p being measured is given, with sufficient accuracy over the temperature range from 0°C to 40°C and for the pressures relevant to these measurements, by the relation

$$\varrho\left(t_{68}, \frac{p}{2}\right) = \frac{\varrho(20°\mathrm{C}.\,p_0)}{[1+A(t_{68}-20°\mathrm{C})+B(t_{68}-20°\mathrm{C})^2]\times\left[1-\chi\left(\frac{p}{2}-p_0\right)\right]} \tag{16}$$

where

$A = 18\,115 \times 10^{-8}\,°\mathrm{C}^{-1}$, $B = 0.8 \times 10^{-8}\,°\mathrm{C}^{-2}$,
$\chi = 4 \times 10^{-11}\ \mathrm{N}^{-1}\ \mathrm{m}^2$,
$\varrho\,(20°\mathrm{C}, p_0) = 13\,545.87\ \mathrm{kg/m}^3$ is the density of pure mercury at $t_{68} = 20°\mathrm{C}$ under a pressure $p_0 = 1$ standard atmosphere (101 325 N/m²).

A sufficiently accurate value of the local gravity may be obtained by using the Potsdam system and applying a correction of $-14 \times 10^{-5}\ \mathrm{m/s}^2$ (-14 milligals)[6].

Hydrostatic head pressures within the fixed points

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[6] By Resolution 1 (1968), the International Committee of Weights and Measures decided that for metrological purposes, the value of the acceleration due to gravity at Potsdam, the reference point of this System, should be taken as 9.812 60 m/s² instead of 9.812 74 m/s², the value adopted initially.

cells cause small but significant temperature effects; these are summarized in Table 5.

4. Triple Point of Water

The temperature of the triple point of water can be realized in sealed glass cells containing only water of high purity and of substantially the isotopic composition of ocean water. The cells have an axial well for the thermometers and the triple-point temperature is obtained wherever the ice is in equilibrium with a liquid-vapour surface. At a depth h below the liquid-vapour surface, the equilibrium temperature t_{68} between ice and liquid water is given by

$$t_{68} = A + Bh \tag{17}$$

where $A = 0.01\ °\mathrm{C}$ and $B = -7 \times 10^{-4}\ \mathrm{m}^{-1}\ °\mathrm{C}$. The method recommended for preparing a triple-point cell consists of forming a thick layer of ice around the thermometer well by cooling from within, then melting enough of this sheath, also from within, to produce a new water-ice interface adjacent to the well. During the first hours following the preparation of the cell the temperature measured in the thermometer well rises fairly rapidly by a few ten thousandths of a kelvin becoming stable after from 1 to 3 days. This initial change of temperature is probably caused by the growth of the ice crystals or by the slow disappearance of strain in the crystals. A cell prepared in this way and kept in an ice bath is capable of maintaining a temperature constant to about 0.0001 K for several months. Even with cells from various sources, when used in this way, the differences in the temperatures obtained should in any case not exceed 0.0002 K. A significant rise in temperature of the thermometer above the triple-point temperature may be caused by artificial light or sunlight falling on the ice-covered cell and it is therefore recommended that measurements should be made with the cell suitably shielded from radiation.

Variations in the isotopic content of naturally occurring water are such that they will result in detectable differences in the triple-point temperature. Ocean water contains about 0.016 moles of deuterium, ^{2}H, per 100 moles of hydrogen, ^{1}H, and 0.04 moles of ^{17}O and 0.2 moles of ^{18}O per 100 moles of ^{16}O. This proportion of heavy isotopes is substantially the highest to be found in naturally occurring water. Continental surface water normally contains about 0.015 moles of ^{2}H per 100 moles of ^{1}H; water coming from polar snow may occasionally contain as little as 0.01 moles of ^{2}H per 100 moles of ^{1}H.

The operation of purifying the water may slightly modify its isotopic composition and the isotopic composition at an ice-water interface is slightly dependent on the freezing technique.

An increase of 0.001 moles of ^{2}H per 100 moles of ^{1}H corresponds to an increase of temperature of the triple point of 0.000 04 K; this is the difference between the triple points for ocean water and the normally occurring continental surface water. The extreme difference in the triple-point temperatures of naturally occurring water is 0.000 25 K.

5. Triple Point, 17.042 K Point and Boiling Point of Equilibrium Hydrogen

Hydrogen has two molecular modifications, designated by the prefixes *ortho* and *para*, caused by different relative orientations of the two nuclear spins in the diatomic molecules. The equilibrium *ortho para* composition is temperature dependent and at room temperature is about 75% orthohydrogen and 25% parahydrogen (so-called "normal hydrogen"). On liquefaction the composition changes slowly with time and there are corresponding changes in the physical properties. At the boiling point the equilibrium composition is 0.21% ortho- and 99.79% parahydrogen and the temperature is lower than that of normal hydrogen by about 0.12 K. The name equilibrium hydrogen means in this document that the hydrogen has its equilibrium *ortho-para* composition at the relevant temperature. In order to avoid errors in the realization of these fixed points caused by indeterminate composition it is advisable to use equilibrium hydrogen converted by the use of a catalyst such as ferric hydroxide. Hydrogen of high chemical purity should be used as may be obtained by diffusion through palladium.

The temperature of equilibrium between solid, liquid and vapour phases of hydrogen can be realized by using a sufficient quantity of liquid hydrogen with some catalyst in a cavity in a copper block in which platinum resistance thermometers are immersed and which is surrounded by a vacuum space. The temperature of the block is reduced until the hydrogen is solidified and then the temperature is allowed to rise slowly and the transition at the triple point is observed. The flat portion of the time-temperature curve can be constant to 0.0001 K for 30 min or more.

The temperature of equilibrium between liquid and gaseous hydrogen is normally realized by the static method. In this method a cavity in a block of metal of high thermal conductivity is maintained at a temperature close to the boiling point by immersing it in liquid hydrogen. In order to avoid temperature gradients due to hydrostatic pressure the liquid hydrogen makes contact with the top of the block only, the lower part being shielded by a vacuum jacket. The cavity contains a small quantity of very pure liquid hydrogen together with some catalyst. The vapour pressure of this hydrogen is transmitted by a thin tube of low heat conduction connected to a manometer outside the enclosure. Precautions must be taken to avoid direct radiation down this tube into the cavity and to ensure that the tube at every point is at a higher temperature than the temperature at the surface of the liquid hydrogen in the cavity. Comparisons are made between the vapour-pressure thermometer thus formed and platinum resistance thermometers immersed in closely fitting holes in the metal block adjacent to the cavity.

The validity of the measurements may be checked by showing that the values obtained are independent of the ratio of the volume of the liquid hydrogen to the volume of the vapour in the cavity.

The temperature T_{68} as a function of the vapour pressure of equilibrium hydrogen is given to an accuracy of a few millikelvins for the range from 13.81 K to 23 K by the equation

$$\lg \frac{p}{p_0} = A + \frac{B}{T_{68}} + CT_{68} + DT_{68}^2 \tag{18}$$

where

$A = 1.711\,466,\qquad B = -44.010\,46\ \mathrm{K},$

$C = 0.023\,590\,9\ \mathrm{K}^{-1},\quad D = -0.000\,048\,017\ \mathrm{K}^{-2},$

$p_0 = 101\,325\ \mathrm{N/m^2}.$

6. Boiling Point of Neon

The boiling point of neon can be realized in a manner similar to that described for hydrogen. The normal isotopic composition of neon is 0.0026 moles of ^{21}Ne and 0.088 moles of ^{22}Ne per 0.909 moles of ^{20}Ne.

The temperature T_{68} as a function of the vapour pressure of neon is given to an accuracy of ±0.0002 K for the range from 27 K to 27.2 K by the equation

$$T_{68} = \left[27.102 + 3.3144\left(\frac{p}{p_0} - 1\right) - 1.24\left(\frac{p}{p_0} - 1\right)^2 + 0.74\left(\frac{p}{p_0} - 1\right)^3\right]\mathrm{K}. \qquad (19)$$

7. Triple Point and Boiling Point of Oxygen

The triple point and the boiling point of oxygen can be realized in a manner similar to that described for hydrogen. Particular care must be given to the purity of the oxygen in the vapour-pressure thermometer. The oxygen is sufficiently pure when the normal boiling point remains constant with the removal of successive fractions of vapour. The temperature T_{68} as a function of the vapour pressure of oxygen is given to an accuracy of ±0.0001 K for the range from 90.1 K to 90.3 K by the equation

$$T_{68} = \left[90.188 + 9.5648\left(\frac{p}{p_0} - 1\right) - 3.69\left(\frac{p}{p_0} - 1\right)^2 + 2.22\left(\frac{p}{p_0} - 1\right)^3\right]\mathrm{K}. \qquad (20)$$

8. Boiling Point of Water

The temperature of equilibrium between liquid water and its vapour is usually realized by the dynamic method with the thermometer in the saturated vapour. For precise calibration it is preferable to use closed systems in which the boiler and manometer are connected to a manostat filled with air or preferably helium.

The boiler must be constructed in such a way that contamination of the water is avoided. The thermometer must be protected from radiation emitted by bodies which are at temperatures different from the boiling-point temperature. If the equilibrium temperature has been obtained the observed temperature (reduced to a constant pressure) will be independent of the time elapsed, variations in the rate of heat supplied to the liquid, and the depth of immersion of the thermometer.

A change in the proportion of deuterium in the water produces a change in the boiling point of water in the same direction as in the triple point but to about one third the extent.

The temperature t_{68} as a function of the vapour pressure of water is given, to an accuracy of ±0.0001 K for the range from 99.9 °C to 100.1 °C by the equation

$$t_{68} = \left[100 + 28.0216\left(\frac{p}{p_0} - 1\right) - 11.642\left(\frac{p}{p_0} - 1\right)^2 + 7.1\left(\frac{p}{p_0} - 1\right)^3\right]{}^{\circ}\mathrm{C}. \qquad (21)$$

9. Freezing Points of Tin and Zinc

Very reproducible temperatures can be realized by observing the flat part of the temperature versus time curve obtained during the slow freezing of very pure metals.

The melting and freezing of tin and zinc may be carried out in a crucible of very pure artificial graphite (99.999% by weight) about 5 cm in diameter provided with an axial thermometer well. The depth of immersion of the thermometer in the metal must be sufficient to eliminate the influence of thermal conduction along the thermometer leads on the temperature of the sensitive element. It is convenient to contain the crucible and ingot of metal in a pyrex or silica tube under an inert atmosphere and to heat it in a metal-block furnace.

The aim of the cooling technique in the determination of the freezing point is to ensure that the thermometer sensor is as nearly as possible enclosed by, and is in thermal equilibrium with, a solid-liquid interface: shortly after nucleation there should be present either a completed solid shell nucleated on and thickening from the crucible wall or a completed solid mantle induced on the thermometer well.

The equilibrium temperature between solid and liquid metal varies slightly with pressure. The magnitudes of these variations are given in Table 5.

High purity tin (99.9999% by weight) supercools by 20 K to 30 K before solidification when cooled from the liquid state. The following technique has been successful for nucleating an ingot of tin while avoiding excessive undercooling of the freezing-point furnace. Starting with the temperature a few kelvins above the freezing point, the furnace is slowly cooled at about 0.1 kelvin per minute until the melt reaches its liquidus-point temperature; the sample holder containing the melt and a monitoring resistance thermometer is then either withdrawn into the throat of the furnace or removed from the furnace entirely. Under either of these conditions the sample cools rapidly; when the rapid rise in temperature that indicates general nucleation is detected the sample holder is quickly replaced in the furnace which is still cooling slowly. Thereafter, as solidification proceeds slowly, a characteristic cooling curve for a high purity metal is realized having a temperature plateau that is reproducible for a particular sample to ±0.0001 K for durations dependent on the rate of furnace cooling.

High purity zinc (99.9999% by weight) is treated in a somewhat different manner since it does not supercool excessively. A thin layer of solid metal is produced on the central thermometer well by removing the thermometer when the melt reaches the liquidus temperature, and either cooling the thermometer to room temperature and replacing it, or inserting a silica rod in its place for about 30 s before replacing the thermometer.

A criterion for checking that the purity of a sample of zinc or of tin is satisfactory is that its melting range is not more than 0.001 K.

10. Freezing Points of Silver and Gold

The equilibrium temperatures between the solid and liquid phases of silver and of gold may be realized in covered crucibles of either very pure artificial

Table 6. *Secondary reference points*

Equilibrium state	International Practical Temperature T_{68} (K)	 t_{68} (°C)
Equilibrium between the solid, liquid and vapour phases of normal hydrogen (triple point of normal hydrogen)	13.956	259.194
Equilibrium between the liquid and vapour phases of normal hydrogen (boiling point of normal hydrogen)	20.397	−252.753
$\lg \frac{p}{p_0} = A + \frac{B}{T_{68}} + CT_{68} + DT_{68}^2$ (23)		
$A = 1.734\,791$, $B = -44.623\,68$ K, $C = 0.023\,186\,9$ K^{-1}, $D = -0.000\,048\,017$ K^{-2}		
for the temperature range from 13.956 K to 30 K.		
Equilibrium between the solid, liquid and vapour phases of neon (triple point of neon)	24.555	−248.595
Equilibrium between the liquid and vapour phases of neon		
$\lg \frac{p}{p_0} = A + \frac{B}{T_{88}} + CT_{68} + DT_{68}^2$ (24)		
$A = 4.611\,52$, $B = -106.3851$ K, $C = -0.036\,833\,1$ K^{-1}, $D = 4.248\,92 \times 10^{-4}$ K^{-2}		
for the temperature range from 24.555 K to 40 K.		
Equilibrium between the solid, liquid and vapour phases of nitrogen (triple point of nitrogen)	63.148	−210.002
Equilibrium between the liquid and vapour phases of nitrogen (boiling point of nitrogen)	77.348	−195.802
$\lg \frac{p}{p_0} = A + \frac{B}{T_{68}} + C \lg \frac{T_{68}}{T_0} + DT_{68} + ET_{68}^2$ (25)		
$A = 5.893\,139$, $B = -404.131\,05$ K, $C = -2.3749$, $D = -0.014\,250\,5$ K^{-1}, $E = 72.5342 \times 10^{-8}$ K^{-2}		
for the temperature range from 63.148 K to 84 K.		
Equilibrium between the liquid and vapour phases of oxygen		
$\lg \frac{p}{p_0} = A + \frac{B}{T_{68}} + C \lg \frac{T_{68}}{T_0} + DT_{68} + ET_{68}^2$ (26)		
$A = 5.961\,546$, $B = -467.455\,76$ K, $C = -1.664\,512$, $D = -0.013\,213\,01$ K^{-1}, $E = 50.8041 \times 10^{-6}$ K^{-2}		
for the temperature range from 54.361 K to 94 K.		
Equilibrium between the solid and vapour phases of carbon dioxide (sublimation point of carbon dioxide)	194.674	− 78.476
$T_{68} = \left[194.674 + 12.264 \left(\frac{p}{p_0} - 1\right) - 9.15 \left(\frac{p}{p_0} - 1\right)^2\right]$ K (27)		
for the temperature range from 194 K to 195 K.		
Equilibrium between the solid and liquid phases of mercury (freezing point of mercury)[a]	234.288	− 38.862
Equilibrium between ice and air-saturated water (ice point)	273.15	0
Equilibrium between the solid, liquid and vapour phases of phenoxybenzene (diphenyl ether) (triple point of phenoxybenzene)	300.02	26.87
Equilibrium between the solid, liquid and vapour phases of benzoic acid (triple point of benzoic acid)	395.52	122.37
Equilibrium between the solid and liquid phases of indium (freezing point of indium)[a]	429.784	156.634
Equilibrium between the solid and liquid phases of bismuth (freezing point of bismuth)[a]	544.592	271.442
Equilibrium between the solid and liquid phases of cadmium (freezing point of cadmium)[a]	594.258	321.108
Equilibrium between the solid and liquid phases of lead (freezing point of lead)[a]	600.652	327.502
Equilibrium between the liquid and vapour phases of mercury (boiling point of mercury)	629.81	356.66
$t_{68} = \left[356.66 + 55.552 \left(\frac{p}{p_0} - 1\right) - 23.03 \left(\frac{p}{p_0} - 1\right)^2 + 14.0 \left(\frac{p}{p_0} - 1\right)^3\right]$ °C (28)		
for $p = 90 \times 10^3$ N/m^2 to 104×10^3 N/m^2.		
Equilibrium between the liquid and vapour phases of sulphur (boiling point of sulphur)	717.824	444.674
$t_{68} = \left[444.674 + 69.010 \left(\frac{p}{p_0} - 1\right) - 27.48 \left(\frac{p}{p_0} - 1\right)^2 + 19.14 \left(\frac{p}{p_0} - 1\right)^3\right]$ °C (29)		
for $p = 90 \times 10^3$ N/m^2 to 104×10^3 N/m^2.		
Equilibrium between the solid and liquid phases of the copper-aluminium eutectic	821.38	548.23
Equilibrium between the solid and liquid phases of antimony (freezing point of antimony)[a]	903.89	630.74
Equilibrium between the solid and liquid phases of aluminium (freezing point of aluminium)	933.52	660.37
Equilibrium between the solid and liquid phases of copper (freezing point of copper)	1357.6	1084.5
Equilibrium between the solid and liquid phases of nickel (freezing point of nickel)	1728	1455
Equilibrium between the solid and liquid phases of cobalt (freezing point of cobalt)	1767	1494
Equilibrium between the solid and liquid phases of palladium (freezing point of palladium)	1827	1554
Equilibrium between the solid and liquid phases of platinum (freezing point of platinum)	2045	1772
Equilibrium between the solid and liquid phases of rhodium (freezing point of rhodium)	2236	1963
Equilibrium between the solid and liquid phases of iridium (freezing point of iridium)	2720	2447
Equilibrium between the solid and liquid phases of tungsten (temperature of melting tungsten)	3660	3387

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[a] See Table 5 for the effect of pressure variations on these freezing points.

graphite, or ceramic material or vitreous silica. If graphite is used it is advisable to prevent the access of air to the crucible so that the graphite is not oxidised. Molten silver should be protected so as to prevent the solution of oxygen into it and the consequent depression of the freezing point.

The ingot of metal should be heated to a uniform temperature a few kelvins above the melting point of the metal and then cooled slowly. The thermocouple to be calibrated, mounted in a protecting tube of suitable refractory material, with refractory insulators separating the two wires, is immersed in the molten metal which is then allowed to freeze. The depth of immersion of the thermocouple in the metal must be sufficient to eliminate the influence of thermal conduction along the wires of the thermocouple.

That the equilibrium temperature is obtained can be checked by the following criteria: the electromotive force of the thermocouple should be independent of small variations in the depth of immersion in the ingot of metal during successive freezes, and should remain constant for at least 5 min during one freeze.

A blackbody at the freezing point of gold is required for the establishment of the reference temperature for radiation temperature measurements. For the realization of such a blackbody the crucible containing the gold must be modified to provide a uniform temperature enclosure immersed in the gold. A blackbody enclosure is more readily achieved if the material employed for its construction has already a high emissivity and therefore graphite is very suitable for this purpose.

11. Secondary Reference Points

As well as the defining fixed points of the IPTS-68 given in Table 1 other reference points are available. Some of these points and their temperatures on the IPTS-68 are given in Table 6. Except for the triple points and the vapour pressure-temperature equations each temperature is that for a system in equilibrium under the pressure of one standard atmosphere.

Appendix I.

History of the Development of the International Temperature Scales; Differences between the IPTS-68 and the IPTS-48

The International Temperature Scale was adopted in 1927 to overcome the practical difficulties of the direct realization of thermodynamic temperatures by gas thermometry and to unify the existing national temperature scales. It was introduced by the 7th General Conference of Weights and Measures with the intention of providing a practical scale of temperature which was easily and accurately reproducible and which gave as nearly as possible thermodynamic temperatures.

The International Temperature Scale was revised in 1948. The experimental procedures by which the Scale was realized remained substantially unchanged, but two amendments were made to the definition of the Scale resulting in appreciable changes in the numerical values assigned to measured temperatures. The change in the value of the temperature of the freezing point of silver from 960.5 °C to 960.8 °C changed temperatures measured with the standard thermocouple (range 630 °C to 1063 °C); the maximum difference was about 0.4 K near 800 °C. The adoption of the value of 0.014 38 metre kelvin instead of 0.014 32 metre kelvin for the radiation constant c_2 changed all temperatures above the freezing point of gold, while the use of the Planck radiation formula instead of the Wien formula affected the very high temperatures. Temperatures above the freezing point of gold were decreased, for example, by 2.2 K at 1500 °C and by 6 K at 2000 °C. Also at this revision, in order to secure international uniformity of nomenclature, the 9th General Conference of Weights and Measures decided to abandon the word "Centigrade" and its French equivalent "Centésimal" in favour of the name "Celsius". That is "°C" was now regarded as the abbreviation of "degree Celsius".

Table 7. *Estimated uncertainties of the assigned values of the defining fixed points in terms of thermodynamic temperatures*

Defining fixed point	Assigned value	Estimated uncertainty (K)
Triple point of equilibrium hydrogen	13.81 K	0.01
17.042 K point	17.042 K	0.01
Boiling point of equilibrium hydrogen	20.28 K	0.01
Boiling point of neon	27.102 K	0.01
Triple point of oxygen	54.361 K	0.01
Boiling point of oxygen	90.188 K	0.01
Triple point of water	273.16 K	Exact by definition
Boiling point of water	100 °C	0.005
Freezing point of tin	231.9681 °C	0.015
Freezing point of zinc	419.58 °C	0.03
Freezing point of silver	961.93 °C	0.2
Freezing point of gold	1064.43 °C	0.2

An amended edition of the 1948 Scale was adopted by the 11th General Conference of Weights and Measures under the new title "International Practical Temperature Scale of 1948 (amended edition of 1960)", the numerical values of temperature remaining the same as in 1948. The new edition incorporated the new definition of the kelvin given by defining the thermodynamic temperature of the triple point of water as exactly 273.16 K (10th General Conference of Weights and Measures 1954, Resolution 3). It was also recognized at this time that the IPTS no longer represented the thermodynamic temperature as closely as possible and the text included a section on the differences between them. The IPTS-68 has been devised to bring these differences within the limits of accuracy to which the thermodynamic temperatures are known (see Table 7) and to extend the Scale to lower temperatures.

The IPTS-68 differs from the IPTS-48 in the following ways. The lower limit of the Scale is now 13.81 K instead of 90.18 K. The values assigned to the defining fixed points are modified where necessary to conform as nearly as possible to the thermodynamic temperatures, the only points remaining unchanged being the triple point of water, which is permanently fixed by definition, and the boiling point of water. The interpolation instruments remain the same as before but the standard platinum resistance thermometer is

Table 8

Approximate differences ($t_{68} - t_{48}$), in kelvins, between the values of temperature given by the IPTS of 1968 and the IPTS of 1948

t_{68} °C	0	−10	−20	−30	−40	−50	−60	−70	−80	−90	−100
−100	0.022	0.013	0.003	−0.006	−0.013	−0.013	−0.005	0.007	0.012		
− 0	0.000	0.006	0.012	0.018	0.024	0.029	0.032	0.034	0.033	0.029	0.022
t_{68} °C	0	10	20	30	40	50	60	70	80	90	100
0	0.000	−0.004	−0.007	−0.009	−0.010	−0.010	−0.010	−0.008	−0.006	−0.003	0.000
100	0.000	0.004	0.007	0.012	0.016	0.020	0.025	0.029	0.034	0.038	0.043
200	0.043	0.047	0.051	0.054	0.058	0.061	0.064	0.067	0.069	0.071	0.073
300	0.073	0.074	0.075	0.076	0.077	0.077	0.077	0.077	0.077	0.076	0.076
400	0.076	0.075	0.075	0.075	0.074	0.074	0.074	0.075	0.076	0.077	0.079
500	0.079	0.082	0.085	0.089	0.094	0.100	0.108	0.116	0.126	0.137	0.150
600	0.150	0.165	0.182	0.200	0.23	0.25	0.28	0.31	0.34	0.36	0.39
700	0.39	0.42	0.45	0.47	0.50	0.53	0.56	0.58	0.61	0.64	0.67
800	0.67	0.70	0.72	0.75	0.78	0.81	0.84	0.87	0.89	0.92	0.95
900	0.95	0.98	1,01	1.04	1.07	1.10	1.12	1.15	1.18	1.21	1.24
1000	1.24	1.27	1.30	1.33	1.36	1.39	1.42	1.44			
t_{68} °C	0	100	200	300	400	500	600	700	800	900	1000
1000		1.5	1.7	1.8	2.0	2.2	2.4	2.6	2.8	3.0	3.2
2000	3.2	3.5	3.7	4.0	4.2	4.5	4.8	5.0	5.3	5.6	5.9
3000	5.9	6.2	6.5	6.9	7.2	7.5	7.9	8.2	8.6	9.0	9.3

now required to have $W(100\,°\mathrm{C})$ at least equal to 1.3925 instead of 1.3920. Over the temperature range from 90.188 K to 273.15 K the Callendar-Van Dusen equation is no longer used for interpolation, instead the reference function $W_{\mathrm{CCT\text{-}68}}\ (T_{68})$ is employed. Above 0 °C, the Callendar equation is modified so that interpolated values of temperature conform more nearly with thermodynamic-temperature values. Finally the latest value of c_2, namely 0.014 388 metre kelvin is introduced in the Planck equation for determining temperatures above the freezing point of gold. The effect of all these changes is summarised in Table 8 which gives the differences between the values of temperature derived from the IPTS-68 and the IPTS-48.

In the range from 13.81 K to 90.188 K the IPTS-68 is based on the average of four "national scales" and on chosen "best" temperatures for the defining fixed points. These national scales are each defined in terms of platinum resistance thermometers calibrated against a gas thermometer and are highly reproducible.

The differences between the IPTS-68 and the national scales are published in Metrologia, Vol. 5, p. 47, (1969). This allows the use of the national scales and these differences to give a close approximation to the IPTS-68.

The text of this definition of the IPTS-68 conforms with the decision of the 13th General Conference of Weights and Measures to denote the unit of thermodynamic temperature by the name "kelvin", symbol K, and to denote a temperature interval by the same unit and symbol or by "degree Celsius" or "°C".

Appendix II.

Practical Scales of Temperature for Use over the Range from 0.2 K to 5.2 K

Temperatures can be derived from measured vapour pressures of ^{4}He and ^{3}He. The upper limits for use are set by the critical points of the gases (5.2 K for ^{4}He and 3.3 K for ^{3}He) and the lower limit by the vapour pressures becoming too low for practical measurement. The "1958 ^{4}He scale" and the "1962 ^{3}He scale" are the recommended scales[7] in which temperatures are denoted by T_{58} and T_{62} respectively.

The "1958 ^{4}He scale", recommended in 1958 by the International Committee of Weights and Measures, is defined by a table of ^{4}He vapour pressures versus temperature (Comité Consultatif de Thermométrie, 5^e session 1958, p. T 192 and Procès-Verbaux CIPM, 26-A, 1958, p. T 192)[8].

The "1962 ^{3}He scale", recommended by the International Committee of Weights and Measures in 1962, is defined by an equation giving the vapour pressure of ^{3}He as a function of temperature (Comité Consultatif de Thermométrie, 6^e session 1962, p. 184)[9].

In the temperature range between 0.9 K and the critical temperature of ^{3}He the temperatures T_{58} and T_{62} are believed to be in agreement to within 0.3 mK.

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[7] Recent measurements by the acoustic thermometer give temperatures higher than those of the He vapour pressure scales; the difference at the boiling point of ^{4}He is about 0.008 K.

[8] An expanded form of this table together with auxiliary information is given in the J. Res. Nat. Bur. Standards 64A.1 (1960).

[9] A table of values and information on measuring vapour pressures are given in the J. Res. Nat. Bur. Standards 68 A. 547, 559, 567. 579 (1964).

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APPENDIX B. Comparison of Assigned Fixed Point Values and Interpolating Formulae in the Temperature Ranges Defined by the Platinum Resistance Thermometer for ITS–27, ITS–48, IPTS–48, and IPTS–68

(a) Comparison of fixed point values

Defining fixed points	ITS–27 °C	ITS–48 °C	IPTS–48 °C	IPTS–68	
				°C	K
TP e-H_2				−259.34	13.81
BP e-H_2, 25/76 atm				−256.108	17.042
NBP e-H_2				−252.87	20.28
NBP Ne				−246.048	27.102
TP O_2				−218.789	54.361
NBP O_2	−182.97	−182.970	−182.97	−182.962	90.188
Ice point	0.000	0			
TP H_2O			0.01	0.01	273.16
NBP H_2O	100.000	100	100	100	373.15
FP Sn				231.9681	505.1181
FP Zn			419.505	419.58	692.73
NBP S	444.60	444.600	444.6		
Some secondary fixed points					
SP CO_2	−78.5	−78.5	−78.5	−78.476	194.674
FP Hg	−38.87	−38.87	−38.87	−38.862	234.288
Ice point			0.000	0	273.15
TP H_2O		0.0100			
FP Sn	231.85	231.9	231.91		
FP Pb	327.3	327.3	327.3	327.502	600.652
FP Zn	419.45	419.5			
NBP S				444.674	717.824
FP Sb	630.5	630.5		630.74	903.89

TP = triple point, BP = boiling point, NBP = normal boiling point, FP = freezing point, SP = sublimation point.

APPENDIX B. Comparison of Assigned Fixed Point Values and Interpolating Formulae in the Temperature Ranges Defined by the Platinum Resistance Thermometer for ITS–27, ITS–48, IPTS–48, and IPTS–68—Continued

(b) Comparison of temperature ranges and interpolating instrument and formulae

	ITS–27	ITS–48	IPTS–48	IPTS–68
$R(100)/R(0)$ of Pt.	$\geqq 1.390$	> 1.3910	$\geqq 1.3920$	$\geqq 1.3925$
Temperature Range	-190 to 0 °C	-182.970 to 0 °C	-182.97 to 0 °C	13.81 to 273.15 K
Interpolating Formula	[a] quartic	[a] quartic	[a] quartic	[c] reference function plus deviation [d] polynomials
Calibration Temperatures	NBP O_2, ice point, NBP H_2O, & NBP S	NBP O_2, ice point, NBP H_2O, & NBP S	NBP O_2, TP H_2O, NBP H_2O, & NBP S or FP Zn	TP e-H_2, BP e-H_2 (25/76 atm), NBP e-H_2, NBP Ne, TP O_2, NBP O_2, TP H_2O, NBP H_2O or FP Sn, & FP Zn.
Temperature Range	0 to 660 °C	0 to 630.5 °C	0 to 630.5 °C	0 to 630.74 °C
Interpolating Formula	[b] quadratic	[b] quadratic	[b] quadratic	Quadratic plus [e] correction function
Calibration Temperatures	ice point, NBP H_2O, & NBP S	ice point, NBP H_2O, & NBP S	TP H_2O, NBP H_2O, & NBP S	TP H_2O, NBP H_2O or FP Sn, & FP Zn.

[a] Quartic interpolation formula: $R(t)=R(0)\,[1+At+Bt^2+C\,(t-100)\,t^3]$.

[b] Quadratic interpolation formula: $R(t)=R(0)\,(1+At+Bt^2)$.

[c] *Reference function*

$$T_{68}=\sum_{i=0}^{20} A_i[\ln W^*\,(T)]^i \qquad \text{(Values of } A_i \text{ are given in Appendix A.)}$$

[d] *Deviation polynomials:* $\Delta W(T)=W(T)-W^*(T)$

13.81 to 20.28 K: $\Delta W(T)=A_1+B_1T+C_1T^2+D_1T^3$

20.28 to 54.361 K: $\Delta W(T)=A_2+B_2T+C_2T^2+D_2T^3$

54.361 to 90.188 K: $\Delta W(T)=A_3+B_3T+C_3T^2$

-182.962 to 0 °C: $\Delta W(t)=A_4t+C_4t^3(t-100\text{ °C})$

where $T=T_{68}$, $t=t_{68}$, $W^*=W_{\text{CCT-68}}$.

[e] *Quadratic interpolation formula plus correction function:* $R(t')/R\,(0)=1+At'+Bt'^2$ plus

$$t_{68}=t'+0.045\left(\frac{t'}{100\text{ °C}}\right)\left(\frac{t'}{100\text{ °C}}-1\right)\left(\frac{t'}{419.58\text{ °C}}-1\right)\left(\frac{t'}{630.74\text{ °C}}-1\right).$$

APPENDIX C. Tables of Difference in the Values of Temperature Between the IPTS-48 and IPTS-68 and Between the NBS-1955 and IPTS-68 in the Temperature Range 13 K to 630 °C

The values given are the calculated differences between values of temperature on the IPTS−48 and on the IPTS−68, and, below the oxygen point, between values on the NBS−55 scale and on the NBS−IPTS−68 (see sec. 7.7). The calculations above the oxygen point are based upon the differences of the scales as defined in their respective texts (see Appendix A). The assumption is made that the value of a temperature on each scale is uniquely defined by its text. (In contrast, refer to the last paragraph of sec. 2.2.) The further assumption is made that, on the IPTS−48, the value 419.505 °C recommended by the text of the IPTS−48 for the freezing point of zinc is exactly correct. The coefficients employed were $\alpha = 0.0039259665$, $\delta = 1.491356$, and $\beta = 0.1109843$ on the IPTS−48. These values were derived from the reference function (W^*) and the corresponding values of α and δ given in the text of the IPTS−68 (see Appendix A).

The calculation of the differences between values on the NBS−55 scale and the IPTS−68 are based upon the estimated differences published by Bedford, Durieux, Muijwijk, and Barber (Metrologia **5**, 47 (1969)). The calibration of a thermometer on the NBS−55 scale was first adjusted by these published differences. The difference (ΔW) between each of these adjusted values and the corresponding value of the reference function (W^*) was then smoothed by a least squares method using the nearest five points. These smoothed values were added to the reference function values and the resulting table subtabulated at 0.1 K intervals by cubic interpolation. The differences between these results and the calibration on the NBS−55 scale are tabulated here as the estimated differences between the NBS−55 scale and the IPTS−68.

The two scales formerly used, i.e., the IPTS−48 and the NBS−55 scale, as realized and maintained at the National Bureau of Standards, were continuous through their point of joining, the normal boiling point of oxygen. Bedford, et al. (loc. cit.), however, estimated that the "oxygen point" upon which these scales were based was too hot by 0.0019 K. The correction to be applied to values on the NBS−55 scale to get values on the NBS−IPTS−68 scale is, as previously stated, based on the estimates of Bedford, et al. and, therefore, includes a correction for their estimated error in the hotness of the NBS "oxygen point." In contrast, the corrections to be used to adjust values on the IPTS−48 to values on the IPTS−68 are based only upon the differences in the definitions of the two scales. This tacitly assumes that all fixed points are perfectly realized. The result of using the different bases for the temperature differences in the two ranges (above and below the oxygen point) is a discontinuity in the estimated corrections at the oxygen point.

The Corrections to Values of Temperature on the NBS-55 Scale. (The correction ($T_{68}-T_{NBS-55}$) is given with the argument in T_{NBS-55}.)

T K	Corr. mK	*T* K	Corr. mK	*T* K	Corr. mK	*T* K	Corr. mK
		15.0	2.04	20.0	8.94	25.0	7.76
		15.1	2.36	20.1	8.96	25.1	7.71
		15.2	2.61	20.2	8.95	25.2	7.65
		15.3	2.90	20.3	8.92	25.3	7.61
		15.4	3.19	20.4	8.90	25.4	7.55
		15.5	3.47	20.5	8.88	25.5	7.50
		15.6	3.72	20.6	8.87	25.6	7.47
		15.7	4.02	20.7	8.84	25.7	7.42
		15.8	4.35	20.8	8.85	25.8	7.39
		15.9	4.69	20.9	8.89	25.9	7.36
		16.0	5.02	21.0	8.90	26.0	7.32
		16.1	5.31	21.1	8.92	26.1	7.28
		16.2	5.55	21.2	8.90	26.2	7.25
		16.3	5.73	21.3	8.88	26.3	7.23
		16.4	5.92	21.4	8.86	26.4	7.21
		16.5	6.10	21.5	8.87	26.5	7.20
		16.6	6.27	21.6	8.86	26.6	7.17
		16.7	6.48	21.7	8.87	26.7	7.14
		16.8	6.72	21.8	8.90	26.8	7.12
		16.9	6.97	21.9	8.89	26.9	7.10
		17.0	7.13	22.0	8.89	27.0	7.08
		17.1	7.27	22.1	8.89	27.1	7.06
		17.2	7.45	22.2	8.88	27.2	7.04
		17.3	7.67	22.3	8.87	27.3	7.05
		17.4	7.89	22.4	8.84	27.4	7.06
		17.5	8.04	22.5	8.83	27.5	7.07
		17.6	8.17	22.6	8.84	27.6	7.08
		17.7	8.27	22.7	8.82	27.7	7.09
		17.8	8.33	22.8	8.82	27.8	7.10
		17.9	8.41	22.9	8.81	27.9	7.11
13.0	−12.01	18.0	8.50	23.0	8.81	28.0	7.12
13.1	−10.63	18.1	8.59	23.1	8.80	28.1	7.13
13.2	−9.27	18.2	8.65	23.2	8.75	28.2	7.14
13.3	−8.13	18.3	8.69	23.3	8.69	28.3	7.14
13.4	−7.07	18.4	8.76	23.4	8.62	28.4	7.17
13.5	−6.11	18.5	8.84	23.5	8.57	28.5	7.20
13.6	−5.29	18.6	8.86	23.6	8.55	28.6	7.23
13.7	−4.45	18.7	8.87	23.7	8.50	28.7	7.25
13.8	−3.64	18.8	8.87	23.8	8.46	28.8	7.28
13.9	−2.88	18.9	8.90	23.9	8.41	28.9	7.30
14.0	−2.20	19.0	8.90	24.0	8.36	29.0	7.32
14.1	−1.61	19.1	8.92	24.1	8.31	29.1	7.34
14.2	−1.14	19.2	8.94	24.2	8.24	29.2	7.38
14.3	−.68	19.3	8.96	24.3	8.17	29.3	7.42
14.4	−.23	19.4	8.87	24.4	8.11	29.4	7.46
14.5	.17	19.5	8.95	24.5	8.04	29.5	7.50
14.6	.51	19.6	8.96	24.6	7.99	29.6	7.54
14.7	.88	19.7	8.99	24.7	7.95	29.7	7.57
14.8	1.28	19.8	8.98	24.8	7.89	29.8	7.61
14.9	1.68	19.9	8.96	24.9	7.83	29.9	7.65
15.0	2.04	20.0	8.94	25.0	7.76	30.0	7.69

The Corrections to Values of Temperature on the NBS-55 Scale. (The correction ($T_{68} - T_{\mathrm{NBS-55}}$) is given with the argument in $T_{\mathrm{NBS-55}}$.) — Continued

T K	Corr. mK	T K	Corr. mK	T K	Corr. mK	T K	Corr. mK
30.0	7.69	35.0	11.03	40.0	15.42	45.0	15.12
30.1	7.73	35.1	11.14	40.1	15.46	45.1	15.09
30.2	7.75	35.2	11.26	40.2	15.50	45.2	15.06
30.3	7.77	35.3	11.39	40.3	15.53	45.3	15.02
30.4	7.79	35.4	11.51	40.4	15.57	45.4	14.99
30.5	7.82	35.5	11.63	40.5	15.59	45.5	14.96
30.6	7.85	35.6	11.75	40.6	15.62	45.6	14.92
30.7	7.89	35.7	11.87	40.7	15.64	45.7	14.89
30.8	7.92	35.8	11.98	40.8	15.66	45.8	14.85
30.9	7.97	35.9	12.10	40.9	15.69	45.9	14.82
31.0	8.00	36.0	12.22	41.0	15.71	46.0	14.80
31.1	8.03	36.1	12.34	41.1	15.74	46.1	14.77
31.2	8.06	36.2	12.46	41.2	15.76	46.2	14.74
31.3	8.09	36.3	12.58	41.3	15.78	46.3	14.71
31.4	8.13	36.4	12.69	41.4	15.79	46.4	14.69
31.5	8.18	36.5	12.81	41.5	15.80	46.5	14.65
31.6	8.22	36.6	12.91	41.6	15.80	46.6	14.62
31.7	8.27	36.7	13.02	41.7	15.80	46.7	14.59
31.8	8.31	36.8	13.12	41.8	15.81	46.8	14.56
31.9	8.37	36.9	13.23	41.9	15.81	46.9	14.54
32.0	8.42	37.0	13.32	42.0	15.81	47.0	14.50
32.1	8.46	37.1	13.42	42.1	15.80	47.1	14.48
32.2	8.53	37.2	13.52	42.2	15.80	47.2	14.44
32.3	8.59	37.3	13.62	42.3	15.79	47.3	14.42
32.4	8.64	37.4	13.71	42.4	15.79	47.4	14.39
32.5	8.70	37.5	13.80	42.5	15.79	47.5	14.35
32.6	8.76	37.6	13.89	42.6	15.77	47.6	14.32
32.7	8.82	37.7	13.98	42.7	15.76	47.7	14.28
32.8	8.88	37.8	14.06	42.8	15.74	47.8	14.25
32.9	8.95	37.9	14.15	42.9	15.73	47.9	14.21
33.0	9.02	38.0	14.24	43.0	15.71	48.0	14.17
33.1	9.10	38.1	14.32	43.1	15.68	48.1	14.13
33.2	9.19	38.2	14.39	43.2	15.67	48.2	14.08
33.3	9.27	38.3	14.47	43.3	15.65	48.3	14.03
33.4	9.35	38.4	14.54	43.4	15.63	48.4	13.98
33.5	9.45	38.5	14.60	43.5	15.59	48.5	13.93
33.6	9.54	38.6	14.66	43.6	15.56	48.6	13.88
33.7	9.63	38.7	14.73	43.7	15.53	48.7	13.83
33.8	9.73	38.8	14.80	43.8	15.51	48.8	13.78
33.9	9.83	38.9	14.86	43.9	15.48	48.9	13.74
33.0	9.92	39.0	14.92	44.0	15.45	49.0	13.68
34.1	10.02	39.1	14.97	44.1	15.42	49.1	13.62
34.2	10.13	39.2	15.02	44.2	15.39	49.2	13.56
34.3	10.23	39.3	15.08	44.3	15.36	49.3	13.50
34.4	10.34	39.4	15.14	44.4	15.33	49.4	13.45
34.5	10.45	39.5	15.19	44.5	15.30	49.5	13.39
34.6	10.57	39.6	15.24	44.6	15.27	49.6	13.33
34.7	10.68	39.7	15.29	44.7	15.23	49.7	13.27
34.8	10.81	39.8	15.34	44.8	15.19	49.8	13.21
34.9	10.92	39.9	15.38	44.9	15.16	49.9	13.16
35.0	11.03	40.0	15.42	45.0	15.12	50.0	13.09

The Corrections to Values of Temperature on the NBS-55 Scale. (The correction ($T_{68} - T_{\mathrm{NBS-55}}$) is given with the argument in $T_{\mathrm{NBS-55}}$.)—Continued

T K	Corr. mK	*T* K	Corr. mK	*T* K	Corr. mK	*T* K	Corr. mK
50.0	13.09	55.0	7.07	60.0	−.80	65.0	−.09
50.1	13.02	55.1	6.88	60.1	−.88	65.1	−.04
50.2	12.96	55.2	6.70	60.2	−.95	65.2	.03
50.3	12.90	55.3	6.51	60.3	−1.03	65.3	.09
50.4	12.83	55.4	6.33	60.4	−1.10	65.4	.14
50.5	12.77	55.5	6.13	60.5	−1.16	65.5	.20
50.6	12.70	55.6	5.95	60.6	−1.22	65.6	.26
50.7	12.63	55.7	5.76	60.7	−1.26	65.7	.32
50.8	12.56	55.8	5.57	60.8	−1.31	65.8	.38
50.9	12.48	55.9	5.38	60.9	−1.36	65.9	.44
51.0	12.41	56.0	5.20	61.0	−1.39	66.0	.49
51.1	12.33	56.1	5.02	61.1	−1.42	66.1	.54
51.2	12.25	56.2	4.83	61.2	−1.45	66.2	.58
51.3	12.18	56.3	4.65	61.3	−1.47	66.3	.63
51.4	12.09	56.4	4.46	61.4	−1.49	66.4	.67
51.5	12.01	56.5	4.28	61.5	−1.50	66.5	.72
51.6	11.93	56.6	4.10	61.6	−1.51	66.6	.76
51.7	11.84	56.7	3.92	61.7	−1.51	66.7	.80
51.8	11.74	56.8	3.74	61.8	−1.51	66.8	.84
51.9	11.65	56.9	3.56	61.9	−1.51	66.9	.87
52.0	11.55	57.0	3.38	62.0	−1.50	67.0	.91
52.1	11.45	57.1	3.21	62.1	−1.48	67.1	.94
52.2	11.35	57.2	3.03	62.2	−1.47	67.2	.98
52.3	11.24	57.3	2.86	62.3	−1.45	67.3	1.01
52.4	11.13	57.4	2.70	62.4	−1.43	67.4	1.03
52.5	11.02	57.5	2.53	62.2	−1.40	67.5	1.05
52.6	10.90	57.6	2.36	62.6	−1.37	67.6	1.07
52.7	10.77	57.7	2.19	62.7	−1.34	67.7	1.08
52.8	10.65	57.8	2.02	62.8	−1.30	67.8	1.08
52.9	10.52	57.9	1.86	62.9	−1.26	67.9	1.09
53.0	10.38	58.0	1.70	63.0	−1.22	68.0	1.08
53.1	10.24	58.1	1.55	63.1	−1.17	68.1	1.08
53.2	10.11	58.2	1.39	63.2	−1.13	68.2	1.07
53.3	9.97	58.3	1.24	63.3	−1.08	68.3	1.06
53.4	9.82	58.4	1.10	63.4	−1.03	68.4	1.05
53.5	9.66	58.5	.95	63.5	−.98	68.5	1.03
53.6	9.51	58.6	.81	63.6	−.93	68.6	1.01
53.7	9.35	58.7	.68	63.7	−.88	68.7	.99
53.8	9.19	58.8	.54	63.8	−.82	68.8	.96
53.9	9.02	58.9	.41	63.9	−.76	68.9	.93
54.0	8.85	59.0	.28	64.0	−.70	69.0	.89
54.1	8.69	59.1	.15	64.1	−.64	69.1	.85
54.2	8.52	59.2	.02	64.2	−.58	69.2	.80
54.3	8.34	59.3	−.10	64.3	−.52	69.3	.75
54.4	8.16	59.4	−.21	64.4	−.45	69.4	.71
54.5	7.98	59.5	−.31	64.5	−.40	69.5	.65
54.6	7.80	59.6	−.42	64.6	−.33	69.6	.58
54.7	7.62	59.7	−.52	64.7	−.28	69.7	.52
54.8	7.43	59.8	−.62	64.8	−.22	69.8	.45
54.9	7.25	59.9	−.71	64.9	−.15	69.9	.38
55.0	7.07	60.0	−.80	65.0	−.09	70.0	.31

The Corrections to Values of Temperature on the NBS-55 Scale. (The correction ($T_{68}-T_{\mathrm{NBS-55}}$) is given with the argument in $T_{\mathrm{NBS-55}}$.)—Continued

T K	Corr. mK	T K	Corr. mK	T K	Corr. mK	T K	Corr. mK
70.0	.31	75.0	−5.59	80.0	−9.03	85.0	−2.91
70.1	.23	75.1	−5.71	80.1	−9.01	85.1	−2.69
70.2	.15	75.2	−5.84	80.2	−8.98	85.2	−2.47
70.3	.06	75.3	−5.96	80.3	−8.95	85.3	−2.24
70.4	−.02	75.4	−6.08	80.4	−8.91	85.4	−2.00
70.5	−.10	75.5	−6.20	80.5	−8.88	85.5	−1.76
70.6	−.20	75.6	−6.31	80.6	−8.82	85.6	−1.52
70.7	−.29	75.7	−6.43	80.7	−8.77	85.7	−1.27
70.8	−.39	75.8	−6.55	80.8	−8.71	85.8	−1.02
70.9	−.49	75.9	−6.67	80.9	−8.65	85.9	−.76
71.0	−.59	76.0	−6.78	81.0	−8.59	86.0	−.50
71.1	−.70	76.1	−6.89	81.1	−8.52	86.1	−.24
71.2	−.81	76.2	−7.00	81.2	−8.44	86.2	.02
71.3	−.92	76.3	−7.11	81.3	−8.36	86.3	.29
71.4	−1.02	76.4	−7.21	81.4	−8.28	86.4	.56
71.5	−1.13	76.5	−7.31	81.5	−8.19	86.5	.83
71.6	−1.25	76.6	−7.42	81.6	−8.10	86.6	1.11
71.7	−1.36	76.7	−7.51	81.7	−8.00	86.7	1.38
71.8	−1.48	76.8	−7.61	81.8	−7.90	86.8	1.65
71.9	−1.60	76.9	−7.71	81.9	−7.80	86.9	1.92
72.0	−1.72	77.0	−7.80	82.0	−7.70	87.0	2.20
72.1	−1.85	77.1	−7.89	82.1	−7.59	87.1	2.47
72.2	−1.96	77.2	−7.98	82.2	−7.48	87.2	2.75
72.3	−2.08	77.3	−8.06	82.3	−7.36	87.3	3.02
72.4	−2.20	77.4	−8.14	82.4	−7.24	87.4	3.30
72.5	−2.32	77.5	−8.22	82.5	−7.12	87.5	3.57
72.6	−2.44	77.6	−8.30	82.6	−6.99	87.6	3.84
72.7	−2.58	77.7	−8.37	82.7	−6.86	87.7	4.12
72.8	−2.71	77.8	−8.44	82.8	−6.73	87.8	4.39
72.9	−2.84	77.9	−8.51	82.9	−6.59	87.9	4.65
73.0	−2.97	78.0	−8.57	83.0	−6.46	88.0	4.92
73.1	−3.10	78.1	−8.63	83.1	−6.32	88.1	5.18
73.2	−3.23	78.2	−8.69	83.2	−6.17	88.2	5.45
73.3	−3.36	78.3	−8.74	83.3	−6.02	88.3	5.70
73.4	−3.50	78.4	−8.78	83.4	−5.88	88.4	5.96
73.5	−3.64	78.5	−8.83	83.5	−5.72	88.5	6.22
73.6	−3.78	78.6	−8.87	83.6	−5.56	88.6	6.47
73.7	−3.91	78.7	−8.91	83.7	−5.40	88.7	6.72
73.8	−4.04	78.8	−8.94	83.8	−5.24	88.8	6.96
73.9	−4.17	78.9	−8.97	83.9	−5.07	88.9	7.21
74.0	−4.30	79.0	−9.00	84.0	−4.90	89.0	7.45
74.1	−4.43	79.1	−9.03	84.1	−4.72	89.1	7.68
74.2	−4.56	79.2	−9.05	84.2	−4.54	89.2	7.92
74.3	−4.69	79.3	−9.07	84.3	−4.34	89.3	8.15
74.4	−4.82	79.4	−9.08	84.4	−4.16	89.4	8.37
74.5	−4.95	79.5	−9.08	84.5	−3.96	89.5	8.58
74.6	−5.08	79.6	−9.09	84.6	−3.76	89.6	8.79
74.7	−5.20	79.7	−9.08	84.7	−3.56	89.7	9.00
74.4	−5.34	79.8	−9.07	84.8	−3.35	89.8	9.20
74.9	−5.46	79.9	−9.06	84.9	−3.13	89.9	9.40
75.0	−5.59	80.0	−9.03	85.0	−2.91	90.0	9.59

Estimated Corrections to Values of Temperature on the IPTS of 1948. (The correction ($t_{68}-t_{48}$) is given with the argument in t_{48}.)

t °C	Corr. °C	t °C	Corr. °C	t °C	Corr. °C	t °C	Corr. °C
		−150	−.012630	−100	.022386	−50	.028618
		−149	−.012963	−99	.023166	−49	.028183
		−148	−.013216	−98	.023921	−48	.027734
		−147	−.013392	−97	.024650	−47	.027272
		−146	−.013491	−96	.025352	−46	.026798
		−145	−.013514	−95	.026027	−45	.026311
		−144	−.013464	−94	.026675	−44	.025814
		−143	−.013343	−93	.027295	−43	.025305
		−142	−.013152	−92	.027886	−42	.024785
		−141	−.012893	−91	.028449	−41	.024256
		−140	−.012571	−90	.028983	−40	.023717
		−139	−.012186	−89	.029488	−39	.023170
		−138	−.011741	−88	.029963	−38	.022613
		−137	−.011240	−87	.030410	−37	.022049
		−136	−.010685	−86	.030827	−36	.021478
		−135	−.010079	−85	.031215	−35	.020899
		−134	−.009425	−84	.031574	−34	.020314
−183	.007936	−133	−.008725	−83	.031904	−33	.019722
−182	.009822	−132	−.007984	−82	.032204	−32	.019125
−181	.011224	−131	−.007203	−81	.032476	−31	.018523
−180	.012195	−130	−.006386	−80	.032720	−30	.017917
−179	.012785	−129	−.005536	−79	.032934	−29	.017306
−178	.013040	−128	−.004656	−78	.033121	−28	.016691
−177	.013003	−127	−.003748	−77	.033280	−27	.016073
−176	.012712	−126	−.002815	−76	.033411	−26	.015453
−175	.012202	−125	−.001860	−75	.033515	−25	.014829
−174	.011507	−124	−.000887	−74	.033593	−24	.014205
−173	.010656	−123	.000104	−73	.033644	−23	.013578
−172	.009676	−122	.001108	−72	.033669	−22	.012951
−171	.008592	−121	.002123	−71	.033668	−21	.012323
−170	.007427	−120	.003148	−70	.033642	−20	.011696
−169	.006200	−119	.004179	−69	.033591	−19	.011069
−168	.004931	−118	.005214	−68	.033517	−18	.010443
−167	.003637	−117	.006250	−67	.033418	−17	.009819
−166	.002332	−116	.007287	−66	.033296	−16	.009196
−165	.001029	−115	.008321	−65	.033152	−15	.008577
−164	−.000258	−114	.009351	−64	.032985	−14	.007960
−163	−.001520	−113	.010375	−63	.032797	−13	.007347
−162	−.002747	−112	.011390	−62	.032587	−12	.006739
−161	−.003930	−111	.012396	−61	.032357	−11	.006136
−160	−.005063	−110	.013390	−60	.032106	−10	.005537
−159	−.006138	−109	.014370	−59	.031837	−9	.004945
−158	−.007152	−108	.015337	−58	.031548	−8	.004360
−157	−.008099	−107	.016287	−57	.031240	−7	.003782
−156	−.008975	−106	.017220	−56	.030915	−6	.003213
−155	−.009779	−105	.018134	−55	.030573	−5	.002652
−154	−.010506	−104	.019028	−54	.030213	−4	.002100
−153	−.011156	−103	.019901	−53	.029838	−3	.001559
−152	−.011727	−102	.020753	−52	.029446	−2	.001029
−151	−.012218	−101	.021581	−51	.029040	−1	.000511
−150	−.012630	−100	.022386	−50	.028618	0	.000005

Estimated Corrections to Values of Temperature on the IPTS of 1948. (The correction ($t_{68}-t_{48}$) is given with the argument in t_{48}.)—Continued

t °C	Corr. °C	*t* °C	Corr. °C	*t* °C	Corr. °C	*t* °C	Corr. °C
0	+.000005	50	−.010366	100	.000000	150	.020374
1	−.000487	51	−.010321	101	.000341	151	.020823
2	−.000966	52	−.010269	102	.000686	152	.021273
3	−.001432	53	−.010209	103	.001034	153	.021723
4	−.001884	54	−.010141	104	.001386	154	.022174
5	−.002324	55	−.010065	105	.001742	155	.022625
6	−.002750	56	−.009981	106	.002101	156	.023077
7	−.003164	57	−.009890	107	.002464	157	.023528
8	−.003565	58	−.009791	108	.002830	158	.023981
9	−.003953	59	−.009685	109	.003200	159	.024433
10	−.004329	60	−.009571	110	.003572	160	.024886
11	−.004692	61	−.009450	111	.003948	161	.025338
12	−.005043	62	−.009322	112	.004327	162	.025791
13	−.005382	63	−.009187	113	.004710	163	.026244
14	−.005709	64	−.009045	114	.005095	164	.026696
15	−.006024	65	−.008896	115	.005483	165	.027149
16	−.006328	66	−.008740	116	.005873	166	.027602
17	−.006619	67	−.008577	117	.006267	167	.028054
18	−.006899	68	−.008408	118	.006663	168	.028506
19	−.007167	69	−.008232	119	.007062	169	.028958
20	−.007424	70	−.008050	120	.007463	170	.029409
21	−.007670	71	−.007861	121	.007867	171	.029860
22	−.007905	72	−.007666	122	.008274	172	.030311
23	−.008128	73	−.007465	123	.008682	173	.030761
24	−.008341	74	−.007258	124	.009093	174	.031211
25	−.008543	75	−.007044	125	.009507	175	.031660
26	−.008734	76	−.006825	126	.009922	176	.032109
27	−.008914	77	−.006600	127	.010339	177	.032557
28	−.009084	78	−.006369	128	.010759	178	.033004
29	−.009243	79	−.006133	129	.011180	179	.033451
30	−.009392	80	−.005891	130	.011604	180	.033896
31	−.009531	81	−.005643	131	.012029	181	.034341
32	−.009660	82	−.005390	132	.012456	182	.034785
33	−.009779	83	−.005132	133	.012884	183	.035228
34	−.009888	84	−.004868	134	.013315	184	.035670
35	−.009987	85	−.004599	135	.013747	185	.036112
36	−.010076	86	−.004325	136	.014180	186	.036552
37	−.010156	87	−.004046	137	.014615	187	.036991
38	−.010226	88	−.003763	138	.015051	188	.037429
39	−.010287	89	−.003474	139	.015489	189	.037866
40	−.010339	90	−.003180	140	.015928	190	.038301
41	−.010381	91	−.002882	141	.016368	191	.038736
42	−.010414	92	−.002579	142	.016809	192	.039169
43	−.010439	93	−.002272	143	.017251	193	.039601
44	−.010454	94	−.001960	144	.017695	194	.040031
45	−.010461	95	−.001644	145	.018139	195	.040460
46	−.010459	96	−.001323	146	.018584	196	.040888
47	−.010448	97	−.000999	147	.019031	197	.041314
48	−.010429	98	−.000670	148	.019478	198	.041739
49	−.010401	99	−.000337	149	.019925	199	.042162
50	−.010366	100	.000000	150	.020374	200	.042584

Estimated Corrections to Values of Temperature on the IPTS of 1948. (The correction ($t_{68}-t_{48}$) is given with the argument in t_{48}.)—Continued

t °C	Corr. °C	t °C	Corr. °C	t °C	Corr. °C	t °C	Corr. °C
200	.042584	250	.061015	300	.072611	350	.076876
201	.043004	251	.061320	301	.072765	351	.076897
202	.043422	252	.061621	302	.072916	352	.076915
203	.043839	253	.061921	303	.073065	353	.076931
204	.044254	254	.062217	304	.073210	354	.076945
205	.044668	255	.062510	305	.073353	355	.076957
206	.045079	256	.062801	306	.073492	356	.076967
207	.045489	257	.063088	307	.073629	357	.076975
208	.045897	258	.063373	308	.073763	358	.076981
209	.046303	259	.063655	309	.073893	359	.076985
210	.046707	260	.063933	310	.074021	360	.076987
211	.047109	261	.064209	311	.074146	361	.076987
212	.047510	262	.064482	312	.074267	362	.076985
213	.047908	263	.064752	313	.074386	363	.076981
214	.048304	264	.065019	314	.074502	364	.076976
215	.048698	265	.065283	315	.074615	365	.076969
216	.049091	266	.065544	316	.074726	366	.076960
217	.049481	267	.065802	317	.074833	367	.076949
218	.049869	268	.066057	318	.074937	368	.076937
219	.050255	269	.066309	319	.075039	369	.076923
220	.050638	270	.066558	320	.075138	370	.076907
221	.051020	271	.066804	321	.075234	371	.076890
222	.051399	272	.067047	322	.075327	372	.076871
223	.051776	273	.067287	323	.075418	373	.076851
224	.052150	274	.067524	324	.075505	374	.076830
225	.052523	275	.067758	325	.075590	375	.076806
226	.052893	276	.067989	326	.075673	376	.076782
227	.053260	277	.068217	327	.075752	377	.076756
228	.053626	278	.068441	328	.075829	378	.076729
229	.053988	279	.068663	329	.075903	379	.076701
230	.054349	280	.068882	330	.075975	380	.076671
231	.054707	281	.069097	331	.076043	381	.076640
232	.055062	282	.069309	332	.076110	382	.076608
233	.055415	283	.069519	333	.076173	383	.076575
234	.055766	284	.069725	334	.076234	384	.076541
235	.056114	285	.069928	335	.076293	385	.076505
236	.056459	286	.070129	336	.076349	386	.076469
237	.056802	287	.070326	337	.076402	387	.076432
238	.057142	288	.070520	338	.076453	388	.076394
239	.057480	289	.070711	339	.076501	389	.076355
240	.057814	290	.070899	340	.076547	390	.076315
241	.058147	291	.071084	341	.076591	391	.076274
242	.058476	292	.071266	342	.076632	392	.076233
243	.058803	293	.071444	343	.076670	393	.076191
244	.059127	294	.071620	344	.076707	394	.076148
245	.059449	295	.071793	345	.076741	395	.076105
246	.059768	296	.071962	346	.076772	396	.076061
247	.060084	297	.072129	347	.076802	397	.076016
248	.060397	298	.072292	348	.076829	398	.075971
249	.060707	299	.072453	349	.076854	399	.075926
250	.061015	300	.072611	350	.076876	400	.075880

Estimated Corrections to Values of Temperature on the IPTS of 1948. (The correction ($t_{68}-t_{48}$) is given with the argument in t_{48}.)—Continued

t °C	Corr. °C	t °C	Corr. °C	t °C	Corr. °C	t °C	Corr. °C
400	.075880	450	.074257	500	.079211	550	.100515
401	.075834	451	.074264	501	.079440	551	.101189
402	.075788	452	.074273	502	.079675	552	.101874
403	.075741	453	.074284	503	.079917	553	.102570
404	.075694	454	.074298	504	.080166	554	.103278
405	.075648	455	.074316	505	.080422	555	.103998
406	.075600	456	.074336	506	.080684	556	.104730
407	.075553	457	.074359	507	.080954	557	.105474
408	.075506	458	.074385	508	.081231	558	.106231
409	.075459	459	.074414	509	.081514	559	.106999
410	.075412	460	.074446	510	.081805	560	.107780
411	.075366	461	.074482	511	.082104	561	.108573
412	.075319	462	.074521	512	.082410	562	.109379
413	.075273	463	.074563	513	.082723	563	.110198
414	.075227	464	.074609	514	.083044	564	.111030
415	.075181	465	.074658	515	.083373	565	.111874
416	.075136	466	.074711	516	.083710	566	.112732
417	.075091	467	.074767	517	.084054	567	.113603
418	.075047	468	.074828	518	.084407	568	.114487
419	.075004	469	.074892	519	.084768	569	.115385
420	.074961	470	.074960	520	.085137	570	.116296
421	.074918	471	.075032	521	.085514	571	.117220
422	.074877	472	.075108	522	.085899	572	.118159
423	.074836	473	.075188	523	.086293	573	.119111
424	.074796	474	.075273	524	.086696	574	.120078
425	.074757	475	.075361	525	.087108	575	.121058
426	.074719	476	.075455	526	.087528	576	.122053
427	.074682	477	.075552	527	.087957	577	.123062
428	.074647	478	.075654	528	.088395	578	.124086
429	.074612	479	.075761	529	.088842	579	.125125
430	.074578	480	.075872	530	.089299	580	.126178
431	.074546	481	.075988	531	.089764	581	.127245
432	.074515	482	.076109	532	.090239	582	.128328
433	.074486	483	.076235	533	.090724	583	.129426
434	.074458	484	.076366	534	.091218	584	.130539
435	.074431	485	.076502	535	.091722	585	.131668
436	.074406	486	.076644	536	.092236	586	.132812
437	.074383	487	.076790	537	.092759	587	.133972
438	.074361	488	.076942	538	.093293	588	.135147
439	.074341	489	.077099	539	.093837	589	.136338
440	.074323	490	.077262	540	.094391	590	.137545
441	.074307	491	.077430	541	.094955	591	.138768
442	.074293	492	.077604	542	.095530	592	.140008
443	.074281	493	.077784	543	.096115	593	.141264
444	.074271	494	.077969	544	.096711	594	.142536
445	.074263	495	.078161	545	.097317	595	.143825
446	.074257	496	.078359	546	.097935	596	.145130
447	.074254	497	.078562	547	.098563	597	.146453
448	.074252	498	.078772	548	.099203	598	.147792
449	.074254	499	.078988	549	.099853	599	.149148
450	.074257	500	.079211	550	.100515	600	.150522

Estimated Corrections to Values of Temperature on the IPTS of 1948. (The correction ($t_{68} - t_{48}$) is given with the argument in t_{48}.)—Continued

t °C	Corr. °C	t °C	Corr. °C	t °C	Corr. °C	t °C	Corr. °C
600	.150522	608	.162148	616	.174953	624	.189006
601	.151913	609	.163683	617	.176640	625	.190853
602	.153322	610	.165236	618	.178346	626	.192722
603	.154748	611	.166808	619	.180073	627	.194611
604	.156192	612	.168399	620	.181819	628	.196522
605	.157654	613	.170009	621	.183585	629	.198453
606	.159134	614	.171637	622	.185372	630	.200406
607	.160632	615	.173285	623	.187178	631	.202381

APPENDIX D. Table of the Function

$$M(t') = 0.045 \left(\frac{t'}{100\ ^\circ\mathrm{C}}\right)\left(\frac{t'}{100\ ^\circ\mathrm{C}} - 1\right)\left(\frac{t'}{419.58\ ^\circ\mathrm{C}} - 1\right)\left(\frac{t'}{630.74\ ^\circ\mathrm{C}} - 1\right)\ ^\circ\mathrm{C}$$

and its derivative* from $t' = 0$ °C to $t' = 630$ °C

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* $\frac{dM(t')}{dt'} = \frac{0.045}{s^2 za}[4t'^3 - 3(s+z+a)t'^2 + 2(sz+sa+za)t' - sza]$

$(s = 100\ ^\circ\mathrm{C},\ z = 419.58\ ^\circ\mathrm{C},\ a = 630.74\ ^\circ\mathrm{C})$

t' °C	$M(t')$ °C	$dM(t')/dt'$	t' °C	$M(t')$ °C	$dM(t')/dt'$
0	.000000	−.000450	50	−.009124	.000040
1	−.000444	−.000437	51	−.009080	.000048
2	−.000875	−.000425	52	−.009029	.000055
3	−.001294	−.000413	53	−.008971	.000062
4	−.001701	−.000401	54	−.008906	.000068
5	−.002095	−.000389	55	−.008834	.000075
6	−.002478	−.000377	56	−.008755	.000082
7	−.002849	−.000365	57	−.008670	.000089
8	−.003208	−.000353	58	−.008578	.000095
9	−.003555	−.000342	59	−.008480	.000101
10	−.003891	−.000330	60	−.008375	.000108
11	−.004215	−.000319	61	−.008264	.000114
12	−.004528	−.000307	62	−.008147	.000120
13	−.004830	−.000296	63	−.008024	.000126
14	−.005121	−.000285	64	−.007895	.000132
15	−.005401	−.000274	65	−.007760	.000138
16	−.005670	−.000264	66	−.007619	.000144
17	−.005928	−.000253	67	−.007473	.000149
18	−.006176	−.000242	68	−.007320	.000155
19	−.006413	−.000232	69	−.007163	.000160
20	−.006639	−.000221	70	−.007000	.000166
21	−.006856	−.000211	71	−.006831	.000171
22	−.007062	−.000201	72	−.006657	.000176
23	−.007258	−.000191	73	−.006478	.000182
24	−.007444	−.000181	74	−.006294	.000187
25	−.007620	−.000171	75	−.006105	.000192
26	−.007787	−.000162	76	−.005911	.000196
27	−.007944	−.000152	77	−.005713	.000201
28	−.008091	−.000142	78	−.005509	.000206
29	−.008229	−.000133	79	−.005301	.000210
30	−.008357	−.000124	80	−.005088	.000215
31	−.008476	−.000115	81	−.004871	.000219
32	−.008586	−.000106	82	−.004649	.000224
33	−.008687	−.000097	83	−.004423	.000228
34	−.008780	−.000088	84	−.004193	.000232
35	−.008863	−.000079	85	−.003959	.000236
36	−.008937	−.000070	86	−.003720	.000240
37	−.009003	−.000062	87	−.003478	.000244
38	−.009061	−.000053	88	−.003231	.000248
39	−.009110	−.000045	89	−.002981	.000252
40	−.009151	−.000037	90	−.002727	.000256
41	−.009183	−.000029	91	−.002470	.000259
42	−.009208	−.000020	92	−.002209	.000263
43	−.009224	−.000012	93	−.001944	.000266
44	−.009233	−.000005	94	−.001676	.000270
45	−.009234	.000003	95	−.001404	.000273
46	−.009227	.000011	96	−.001130	.000276
47	−.009212	.000018	97	−.000852	.000279
48	−.009190	.000026	98	−.000571	.000283
49	−.009161	.000033	99	−.000287	.000285
50	−.009124	.000040	100	.000000	.000288

$$M(t') = 0.045\left(\frac{t'}{100\ ℃}\right)\left(\frac{t'}{100\ ℃}-1\right)\left(\frac{t'}{419.58\ ℃}-1\right)\left(\frac{t'}{630.74\ ℃}-1\right)\ °\mathrm{C}$$

and its derivative* from $t' = 0$ ℃ to $t' = 630$ ℃

* $$\frac{dM(t')}{dt'} = \frac{0.045}{s^2 za}\left[4t'^3 - 3(s+z+a)t'^2 + 2(sz+sa+za)t' - sza\right]$$

$(s = 100$ ℃, $z = 419.58$ ℃, $a = 630.74$ ℃)

t' °C	$M(t')$ °C	$dM(t')/dt'$
100	−.000000	.000288
101	.000290	.000291
102	.000582	.000294
103	.000878	.000297
104	.001176	.000299
105	.001476	.000302
106	.001779	.000304
107	.002085	.000307
108	.002393	.000309
109	.002703	.000311
110	.003015	.000313
111	.003330	.000316
112	.003646	.000318
113	.003965	.000319
114	.004285	.000321
115	.004608	.000323
116	.004932	.000325
117	.005257	.000327
118	.005585	.000328
119	.005914	.000330
120	.006244	.000331
121	.006576	.000333
122	.006909	.000334
123	.007244	.000335
124	.007579	.000336
125	.007916	.000337
126	.008254	.000339
127	.008593	.000340
128	.008933	.000340
129	.009274	.000341
130	.009616	.000342
131	.009958	.000343
132	.010302	.000343
133	.010645	.000344
134	.010990	.000345
135	.011335	.000345
136	.011680	.000346
137	.012026	.000346
138	.012372	.000346
139	.012718	.000346
140	.013065	.000347
141	.013411	.000347
142	.013758	.000347
143	.014105	.000347
144	.014451	.000347
145	.014798	.000347
146	.015144	.000346
147	.015491	.000346
148	.015837	.000346
149	.016182	.000345
150	.016527	.000345
150	.016527	.000345
151	.016872	.000345
152	.017217	.000344
153	.017560	.000343
154	.017904	.000343
155	.018246	.000342
156	.018588	.000341
157	.018929	.000341
158	.019269	.000340
159	.019608	.000339
160	.019947	.000338
161	.020284	.000337
162	.020620	.000336
163	.020956	.000335
164	.021290	.000334
165	.021623	.000332
166	.021955	.000331
167	.022285	.000330
168	.022614	.000328
169	.022942	.000327
170	.023268	.000326
171	.023593	.000324
172	.023916	.000322
173	.024238	.000321
174	.024558	.000319
175	.024876	.000318
176	.025193	.000316
177	.025508	.000314
178	.025821	.000312
179	.026132	.000310
180	.026442	.000308
181	.026749	.000306
182	.027054	.000304
183	.027358	.000302
184	.027659	.000300
185	.027958	.000298
186	.028255	.000296
187	.028550	.000294
188	.028843	.000291
189	.029133	.000289
190	.029421	.000287
191	.029707	.000284
192	.029990	.000282
193	.030271	.000280
194	.030549	.000277
195	.030825	.000275
196	.031098	.000272
197	.031369	.000269
198	.031637	.000267
199	.031902	.000264
200	.032165	.000261

Appendix D. Table of the Function—Continued

$$M(t')=0.045\left(\frac{t'}{100\ ^\circ\mathrm{C}}\right)\left(\frac{t'}{100\ ^\circ\mathrm{C}}-1\right)\left(\frac{t'}{419.58\ ^\circ\mathrm{C}}-1\right)\left(\frac{t'}{630.74\ ^\circ\mathrm{C}}-1\right)\ ^\circ\mathrm{C}$$

and its derivative* from $t'=0$ °C to $t'=630$ °C

* $\frac{dM(t')}{dt'}=\frac{0.045}{s^2 za}[4t'^3-3(s+z+a)t'^2+2(sz+sa+za)t'-sza]$

($s=100$ °C, $z=419.58$ °C, $a=630.74$ °C)

t' °C	$M(t')$ °C	$dM(t')/dt'$	t' °C	$M(t')$ °C	$dM(t')/dt'$
200	.032165	.000261	250	.041170	.000088
201	.032425	.000259	251	.041256	.000084
202	.032682	.000256	252	.041339	.000080
203	.032937	.000253	253	.041417	.000076
204	.033188	.000250	254	.041491	.000072
205	.033437	.000247	255	.041561	.000068
206	.033682	.000244	256	.041627	.000064
207	.033925	.000241	257	.041689	.000060
208	.034165	.000238	258	.041746	.000056
209	.034402	.000235	259	.041800	.000052
210	.034636	.000232	260	.041849	.000047
211	.034866	.000229	261	.041895	.000043
212	.035094	.000226	262	.041936	.000039
213	.035318	.000223	263	.041973	.000035
214	.035540	.000220	264	.042006	.000031
215	.035758	.000216	265	.042034	.000027
216	.035972	.000213	266	.042059	.000022
217	.036184	.000210	267	.042079	.000018
218	.036392	.000207	268	.042095	.000014
219	.036597	.000203	269	.042107	.000010
220	.036799	.000200	270	.042114	.000005
221	.036997	.000197	271	.042117	.000001
222	.037192	.000193	272	.042116	−.000003
223	.037383	.000190	273	.042111	−.000007
224	.037571	.000186	274	.042102	−.000012
225	.037756	.000183	275	.042088	−.000016
226	.037937	.000179	276	.042070	−.000020
227	.038114	.000176	277	.042048	−.000024
228	.038288	.000172	278	.042021	−.000029
229	.038459	.000169	279	.041990	−.000033
230	.038625	.000165	280	.041955	−.000037
231	.038789	.000161	281	.041916	−.000042
232	.038948	.000158	282	.041872	−.000046
233	.039104	.000154	283	.041824	−.000050
234	.039256	.000150	284	.041772	−.000054
235	.039404	.000147	285	.041715	−.000059
236	.039549	.000143	286	.041654	−.000063
237	.039690	.000139	287	.041589	−.000067
238	.039827	.000135	288	.041520	−.000072
239	.039960	.000131	289	.041446	−.000076
240	.040090	.000128	290	.041368	−.000080
241	.040215	.000124	291	.041285	−.000085
242	.040337	.000120	292	.041198	−.000089
243	.040455	.000116	293	.041107	−.000093
244	.040569	.000112	294	.041012	−.000097
245	.040679	.000108	295	.040912	−.000102
246	.040785	.000104	296	.040809	−.000106
247	.040887	.000100	297	.040700	−.000110
248	.040986	.000096	298	.040588	−.000115
249	.041080	.000092	299	.040471	−.000119
250	.041170	.000088	300	.040350	−.000123

Appendix D. Table of the Function — Continued

$$M(t')=0.045\left(\frac{t'}{100\ ℃}\right)\left(\frac{t'}{100\ ℃}-1\right)\left(\frac{t'}{419.58\ ℃}-1\right)\left(\frac{t'}{630.74\ ℃}-1\right)\ ℃$$

and its derivative* from $t'=0$ ℃ to $t'=630$ ℃

$$^{*}\ \frac{dM(t')}{dt'}=\frac{0.045}{s^2\,za}\left[4t'^3-3(s+z+a)t'^2+2(sz+sa+za)t'-sza\right]$$

$(s=100\ ℃,\ z=419.58\ ℃,\ a=630.74\ ℃)$

t' ℃	$M(t')$ ℃	$dM(t')/dt'$	t' ℃	$M(t')$ ℃	$dM(t')/dt'$
300	.040350	−.000123	350	.029063	−.000322
301	.040225	−.000127	351	.028740	−.000325
302	.040095	−.000132	352	.028412	−.000329
303	.03996	−.000136	353	.028082	−.000332
304	.039823	−.000140	354	.027748	−.000336
305	.039681	−.000144	355	.027410	−.000339
306	.039534	−.000149	356	.027069	−.000343
307	.039383	−.000153	357	.026725	−.000346
308	.039228	−.000157	358	.026378	−.000349
309	.039069	−.000161	359	.026027	−.000352
310	.038906	−.000166	360	.025673	−.000356
311	.038738	−.000170	361	.025316	−.000359
312	.038566	−.000174	362	.024955	−.000362
313	.038390	−.000178	363	.024592	−.000365
314	.038210	−.000182	364	.024225	−.000368
315	.038025	−.000186	365	.023855	−.000371
316	.037837	−.000191	366	.023482	−.000375
317	.037644	−.000195	367	.023106	−.000378
318	.037447	−.000199	368	.022727	−.000381
319	.037247	−.000203	369	.022345	−.000384
320	.037042	−.000207	370	.021960	−.000386
321	.036832	−.000211	371	.021572	−.000389
322	.036619	−.000215	372	.021181	−.000392
323	.036402	−.000219	373	.020787	−.000395
324	.036181	−.000223	374	.020391	−.000398
325	.035955	−.000227	375	.019992	−.000401
326	.035726	−.000231	376	.019590	−.000403
327	.035493	−.000235	377	.019185	−.000406
328	.035255	−.000239	378	.018778	−.000409
329	.035014	−.000243	379	.018368	−.000411
330	.034769	−.000247	380	.017955	−.000414
331	.034520	−.000251	381	.017540	−.000416
332	.034267	−.000255	382	.017122	−.000419
333	.034010	−.000259	383	.016702	−.000421
334	.033749	−.000263	384	.016280	−.000424
335	.033484	−.000267	385	.015855	−.000426
336	.033215	−.000271	386	.015427	−.000429
337	.032943	−.000274	387	.014997	−.000431
338	.032667	−.000278	388	.014565	−.000433
339	.032387	−.000282	389	.014131	−.000435
340	.032103	−.000286	390	.013695	−.000438
341	.031816	−.000289	391	.013256	−.000440
342	.031524	−.000293	392	.012815	−.000442
343	.031229	−.000297	393	.012373	−.000444
344	.030931	−.000300	394	.011928	−.000446
345	.030629	−.000304	395	.011481	−.000448
346	.030323	−.000308	396	.011032	−.000450
347	.030013	−.000311	397	.010582	−.000452
348	.029700	−.000315	398	.010129	−.000453
349	.029383	−.000318	399	.009675	−.000455
350	.029063	−.000322	400	.009219	−.000457

Appendix D. Table of the Function—Continued

$$M(t') = 0.045\left(\frac{t'}{100\ ℃}\right)\left(\frac{t'}{100\ ℃}-1\right)\left(\frac{t'}{419.58\ ℃}-1\right)\left(\frac{t'}{630.74\ ℃}-1\right)\ ℃$$

and its derivative* from $t' = 0$ ℃ to $t' = 630$ ℃

$$^{*}\ \frac{dM(t')}{dt'} = \frac{0.045}{s^2 za}\left[4t'^3 - 3(s+z+a)t'^2 + 2(sz+sa+za)t' - sza\right]$$

$(s = 100\ ° \quad z = 419.58\ ℃,\ a = 630.74\ ℃)$

t' ℃	$M(t')$ ℃	$dM(t')/dt'$	t' ℃	$M(t')$ ℃	$dM(t')/dt'$
400	.009219	−.000457	450	−.014725	−.000477
401	.008761	−.000459	451	−.015201	−.000476
402	.008301	−.000460	452	−.015677	−.000475
403	.007840	−.000462	453	−.016151	−.000474
404	.007377	−.000464	454	−.016625	−.000473
405	.006913	−.000465	455	−.017096	−.000471
406	.006447	−.000466	456	−.017567	−.000470
407	.005980	−.000468	457	−.018036	−.000468
408	.005511	−.000469	458	−.018503	−.000467
409	.005041	−.000471	459	−.018969	−.000465
410	.004570	−.000472	460	−.019433	−.000463
411	.004098	−.000473	461	−.019895	−.000461
412	.003624	−.000474	462	−.020356	−.000460
413	.003149	−.000475	463	−.020814	−.000458
414	.002673	−.000476	464	−.021271	−.000456
415	.002196	−.000477	465	−.021725	−.000453
416	.001718	−.000478	466	−.022178	−.000451
417	.001239	−.000479	467	−.022628	−.000449
418	.000760	−.000480	468	−.023076	−.000447
419	.000279	−.000481	469	−.023522	−.000444
420	−.000202	−.000482	470	−.023965	−.000442
421	−.000684	−.000482	471	−.024406	−.000439
422	−.001167	−.000483	472	−.024844	−.000437
423	−.001651	−.000484	473	−.025279	−.000434
424	−.002135	−.000484	474	−.025712	−.000431
425	−.002619	−.000485	475	−.026142	−.000429
426	−.003104	−.000485	476	−.026569	−.000426
427	−.003589	−.000485	477	−.026993	−.000423
428	−.004075	−.000486	478	−.027415	−.000420
429	−.004561	−.000486	479	−.027833	−.000417
430	−.005047	−.000486	480	−.028248	−.000413
431	−.005533	−.000486	481	−.028659	−.000410
432	−.006020	−.000486	482	−.029068	−.000407
433	−.006506	−.000486	483	−.029473	−.000403
434	−.006993	−.000486	484	−.029874	−.000400
435	−.007479	−.000486	485	−.030272	−.000396
436	−.007965	−.000486	486	−.030666	−.000392
437	−.008451	−.000486	487	−.031057	−.000389
438	−.008937	−.000486	488	−.031443	−.000385
439	−.009423	−.000485	489	−.031826	−.000381
440	−.009908	−.000485	490	−.032205	−.000377
441	−.010392	−.000484	491	−.032580	−.000373
442	−.010877	−.000484	492	−.032950	−.000369
443	−.011360	−.000483	493	−.033317	−.000364
444	−.011843	−.000483	494	−.033679	−.000360
445	−.012326	−.000482	495	−.034036	−.000355
446	−.012807	−.000481	496	−.034390	−.000351
447	−.013288	−.000480	497	−.034738	−.000346
448	−.013768	−.000479	498	−.035082	−.000342
449	−.014247	−.000478	499	−.035422	−.000337
450	−.014725	−.000477	500	−.035756	−.000332

Appendix D. Table of the Function — Continued

$$M(t') = 0.045 \left(\frac{t'}{100\ ^\circ\mathrm{C}}\right)\left(\frac{t'}{100\ ^\circ\mathrm{C}} - 1\right)\left(\frac{t'}{419.58\ ^\circ\mathrm{C}} - 1\right)\left(\frac{t'}{630.74\ ^\circ\mathrm{C}} - 1\right)\ ^\circ\mathrm{C}$$

and its derivative* from $t' = 0$ °C to $t' = 630$ °C

* $\frac{dM(t')}{dt'} = \frac{0.045}{s^2 za}[4t'^3 - 3(s + z + a)t'^2 + 2(sz + sa + za)t' - sza]$

($s = 100$ °C, $z = 419.58$ °C, $a = 630.74$ °C)

t' °C	$M(t')$ °C	$dM(t')/dt'$
500	−.035756	−.000332
501	−.036086	−.000327
502	−.036410	−.000322
503	−.036730	−.000317
504	−.037044	−.000312
505	−.037353	−.000306
506	−.037657	−.000301
507	−.037955	−.000296
508	−.038248	−.000290
509	−.038535	−.000284
510	−.038817	−.000279
511	−.039092	−.000273
512	−.039362	−.000267
513	−.039626	−.000261
514	−.039884	−.000255
515	−.040135	−.000248
516	−.040381	−.000242
517	−.040620	−.000236
518	−.040852	−.000229
519	−.041078	−.000223
520	−.041298	−.000216
521	−.041510	−.000209
522	−.041716	−.000202
523	−.041915	−.000195
524	−.042107	−.000188
525	−.042292	−.000181
526	−.042470	−.000174
527	−.042640	−.000167
528	−.042803	−.000159
529	−.042959	−.000152
530	−.043106	−.000144
531	−.043247	−.000136
532	−.043379	−.000128
533	−.043504	−.000121
534	−.043620	−.000113
535	−.043729	−.000104
536	−.043829	−.000096
537	−.043921	−.000088
538	−.044004	−.000079
539	−.044080	−.000071
540	−.044146	−.000062
541	−.044204	−.000053
542	−.044253	−.000045
543	−.044293	−.000036
544	−.044324	−.000027
545	−.044346	−.000017
546	−.044359	−.000008
547	−.044362	.000001
548	−.044356	.000011
549	−.044341	.000020
550	−.044316	.000030
550	−.044316	.000030
551	−.044281	.000040
552	−.044236	.000050
553	−.044181	.000060
554	−.044116	.000070
555	−.044041	.000080
556	−.043956	.000090
557	−.043860	.000101
558	−.043754	.000111
559	−.043637	.000122
560	−.043510	.000133
561	−.043371	.000144
562	−.043222	.000155
563	−.043062	.000166
564	−.042890	.000177
565	−.042707	.000189
566	−.042513	.000200
567	−.042307	.000212
568	−.042090	.000223
569	−.041861	.000235
570	−.041620	.000247
571	−.041367	.000259
572	−.041102	.000271
573	−.040825	.000283
574	−.040535	.000296
575	−.040233	.000308
576	−.039918	.000321
577	−.039591	.000334
578	−.039251	.000346
579	−.038898	.000359
580	−.038532	.000373
581	−.038153	.000386
582	−.037761	.000399
583	−.037355	.000412
584	−.036936	.000426
585	−.036503	.000440
586	−.036056	.000454
587	−.035596	.000467
588	−.035121	.000482
589	−.034633	.000496
590	−.034130	.000510
591	−.033613	.000524
592	−.033081	.000539
593	−.032535	.000554
594	−.031974	.000568
595	−.031398	.000583
596	−.030807	.000598
597	−.030201	.000614
598	−.029580	.000629
599	−.028944	.000644
600	−.028292	.000660

Appendix D. Table of the Function—Continued

$$M(t') = 0.045 \left(\frac{t'}{100\ ℃}\right)\left(\frac{t'}{100\ ℃} - 1\right)\left(\frac{t'}{419.58\ ℃} - 1\right)\left(\frac{t'}{630.74\ ℃} - 1\right)\ °\mathrm{C}$$

and its derivative* from $t' = 0$ ℃ to $t' = 630$ ℃

$$^{*}\ \frac{dM(t')}{dt'} = \frac{0.045}{s^2 za}\left[4t'^3 - 3(s+z+a)t'^2 + 2(sz+sa+za)t' - sza\right]$$

$$(s = 100\ ℃,\ z = 419.58\ ℃,\ a = 630.74\ ℃)$$

t' °C	$M(t')$ °C	$dM(t')/dt'$	t' °C	$M(t')$ °C	$dM(t')/dt'$
600	−.028292	.000660	616	−.015648	.000926
601	−.027624	.000675	617	−.014713	.000944
602	−.026941	.000691	618	−.013760	.000962
603	−.026241	.000707	619	−.012789	.000980
604	−.025526	.000723	620	−.011800	.000998
605	−.024795	.000739	621	−.010793	.001016
606	−.024047	.000756	622	−.009767	.001035
607	−.023283	.000772	623	−.008723	.001053
608	−.022503	.000789	624	−.007660	.001072
609	−.021705	.000806	625	−.006579	.001091
610	−.020891	.000822	626	−.005478	.001110
611	−.020061	.000839	627	−.004359	.001129
612	−.019213	.000856	628	−.003220	.001148
613	−.018348	.000874	629	−.002062	.001168
614	−.017465	.000891	630	−.000884	.001187
615	−.016566	.000909	631	.000313	.001207

APPENDIX E. Tables of Values of the Reference Function and Its Derivative from $T=13$ K to $T=273.15$ K and over the Equivalent Range of Temperature With the Argument in Degrees Celsius

$W^*(T)$ and its derivative are defined by the expressions

$$T=\sum_{i=0}^{20} A_i[\ln W^*(T)]^i$$

and

$$\frac{dW^*(T)}{dT}=\frac{1}{\frac{1}{W^*(T)}\sum_{i=1}^{20} iA_i[\ln W^*(T)]^{i-1}}$$

where $T=T_{68}$, $W^*(T)=WCCT_{-68}(T_{68})$, and the values of A_i are:

i	A_i	i	A_i
0	$0.273\,15\times10^3$	11	$0.767\,976\,358\,170\,845\,8\times10$
1	$0.250\,846\,209\,678\,803\,3\times10^3$	12	$0.213\,689\,459\,382\,850\,0\times10$
2	$0.135\,099\,869\,964\,999\,7\times10^3$	13	$0.459\,843\,348\,928\,069\,3$
3	$0.527\,856\,759\,008\,517\,2\times10^2$	14	$0.763\,614\,629\,231\,648\,0\times10^{-1}$
4	$0.276\,768\,548\,854\,105\,2\times10^2$	15	$0.969\,328\,620\,373\,121\,3\times10^{-2}$
5	$0.391\,053\,205\,376\,683\,7\times10^2$	16	$0.923\,069\,154\,007\,007\,5\times10^{-3}$
6	$0.655\,613\,230\,578\,069\,3\times10^2$	17	$0.638\,116\,590\,952\,653\,8\times10^{-4}$
7	$0.808\,035\,868\,559\,866\,7\times10^2$	18	$0.302\,293\,237\,874\,619\,2\times10^{-5}$
8	$0.705\,242\,118\,234\,052\,0\times10^2$	19	$0.877\,551\,391\,303\,760\,2\times10^{-7}$
9	$0.447\,847\,589\,638\,965\,7\times10^2$	20	$0.117\,702\,613\,125\,477\,4\times10^{-8}$
10	$0.212\,525\,653\,556\,057\,8\times10^2$		

The function $W^*(T)$ and its derivative were evaluated and printed employing the following FORTRAN routine:

```
'I FOR WSTAR, WSTAR
      DIMENSION TX(1200)
      INTEGER SCALE, SCALED
      DOUBLE PRECISION W,WW(1200),DWDTT(1200),WLOG,A(20),
     1DTDW,DWDTB4,DWDTNX,CHORD,T,TC,STEP,SIZE,TO,TSTART
      READ(5,98) (I,A(I),N=1,20)
   98 FORMAT (I2,D24. 16)
    1 READ(5,99,END=70) (TEND,STEP,SCALE)
   99 FORMAT (F8.0,D4.0,A6)
      STEP=(-ABS(STEP))
      IF (SCALED.EQ.SCALED) GO TO 10
      SCALE=SCALE
      SIZE=1.
      IF(SCALE.EQ. ' DEC C') T0=90
      IF(SCALE.EQ.'KELVIN') T0=273.15D00
      IF(SCALE.NE.' DEG F') GO TO 5
      SIZE=.5555555555555555D00
      T0=32.
    5 TSTART=STEP*IDINT(T0/STEP)
      DWDT=1/A(1)
      W=1+(TSTART-T0)*SIZE
   10 IEND=(TEND-TSTART)/STEP+1.5
      IF (TEND.GT.T0) GO TO 69
      IF (IEND.LT.1) GO TO 5
      CHORD=DWDT
      DWDTNX=DWDT
      W=W-CHORD*STEP*SIZE
      DO 35 I=1, IEND
      T=TSTART+(1-1)*STEP
      TX(I)=T
      TC=(T-T0)*SIZE
      W=W+CHORD*STEP*SIZE
      DWDTB4=DWDT
   15 WLOG=DLOG(W)
      T=A(20)*WLOG
      DO 20 J=1,19
   20* T=WLOG* (A(20*-J)+T)
      DT=T-TC
      W=W-DT*DWDTNX
      IF(ABS(DT)-.000001)25,15,15
   25 WW(I)=W
      WLOG=DLOG(W)
      DTDW=20*A(20)
      DO 30 J=1,19
   30 DTDW=(20-J)*A(20-J)+WLOG*DTDW
      DTDW=DTDW/W
      DWDTT(I)=DWDT
      DWDTT (I)=DWDT
      CHORD=(3.*DWDT-DWDTB4)/2
      DWDTNX=2.*DWDT-DWDTB4
   35 CONTINUE
      JEND=IEND+50
      IPAGES=(IEND-2)/100+1
      IF (IPAGES.LT.3) GO TO 55
      IT100=MOD(IDINT(TEND/STEP),100)
      IF(IT100.EQ.0) GO TO 55
      ISPACE=MOD((100-IT100),50)
```

```
       LINES=MOD((100+IT100),100)
       IPAGES=IEND-LINES-2)/100+1
       IF (LINES-50) 40,40,52
   40  PRINT 101,SCALE
       IF(ABS(IT100)-50.EQ.0) GO TO 50
       DO 45 J=1,ISPACE
   45  PRINT 199,
   50  JEND=IEND+ISPACE-50
       PRINT 201, (TX(J), WW(J), DWDTT(J), J=IEND,JEND, -1)
       GO TO 55
   52  PRINT 102, SCALE, SCALE
       IEND=IEND+ISPACE
       JEND=IEND-ISPACE+1
       PRINT 201,(TX(J-50),WW(J-50),DWDTT(J-50), J=IEND,JEND,-1)
       IEND=IEND-ISPACE
       JEND=IEND+ISPACE-50
       PRINT202,(TX(J),WW(J),DWDTT(J),TX(J-50)(WW(J-50),DWDTT(J-50),J=
      1IEND,JEND,-1)
   55  IEND=JEND-50
       JEND=IEND-50
       DO 60 K=1,IPAGES
       PRINT102,SCALE,SCALE
       PRINT202,(TX(J),WW(J),DWDTT(J),TX(J-50), WW(J-50), DWDTT(J-50), J=
      1IEND,JEND,-1)
       IEND=JEND-50
       JEND=IEND-50
   60  CONTINUE
       TSTART=TEND
   65  GO TO 1
   69  PRINT 999,
   70  STOP
  199  FORMAT(1X)
  100  FORMAT(1H1/  2X'TEMP.'5X'WSTAR'5X'DERIVATIVE'5X / 1X,A6,4X
      1'FUNCTION'5X'X 1000'7X )
  102  FORMAT(1H1/2(2X'TEMP.'5X'WSTAR'5X'DERIVATIVE'5X)/2(1X,A6,4X
      1'FUNCTION'5X'X 1000'7X))
  101  FORMAT(1H1/ 38X'TEMP.'5X'WSTAR'5X'DERIVATIVE'5X / 38X,A6,4X
      1'FUNCTION'5X'X 1000'7X )
  200  FORMAT(  1X,0PF6.1,2X,F10.8,3X,3PF9.7,6X )
  201  FORMAT( 38X,0PF6.1,2X,F10.8,3PF9.7,6X )
  202  FORMAT(2(1X,0PF6.1,2X,F10.8,3X,3PF9.7,6X))
  999  FORMAT (1H,'*****CAN NOT COMPUTE TABLE AS REQUESTED. RUN STOPPED'
      1/1X,'HERE. CHECK TABLE DISCRIPTION CARD *****')
       END
  XQT WSTAR
 1  0.2508462096788033D003
 2  0.1350998699649997D003
 3  0.5278567590085172D002
 4  0.2767685488541052D002
 5  0.3910532053766837D002
 5  0.6556132305780693D002
 7  0.8080358685598667D002
 8  0.7052421182340520D002
 9  0.4478475896389657D002
10  0.2125256535560578D002
11  0.7679763581708458D001
12  0.2136894593828500D001
13  0.4598433489280693D000
14  0.7636146292316480D-01
15  0.9693286203731213D-02
16  0.9230691540070075D-03
17  0.6381165909526538D-04
18  0.3022932378746192D-05
19  0.8775513913037602D-07
20  0.1177026131254774D-08
```

IPTS-68 Reference Function and Its First Derivative With the Argument in Kelvin

T K	$W^*(T)$	$dW^*(T)/dT$ ×1000 K^{-1}	T K	$W^*(T)$	$dW^*(T)/dT$ ×1000 K^{-1}
			15.0	.00174542	.3163198
			15.1	.00177737	.3227158
			15.2	.00180996	.3291831
			15.3	.00184321	.3357224
			15.4	.00187711	.3423344
			15.5	.00191168	.3490197
			15.6	.00194692	.3557792
			15.7	.00198283	.3626135
			15.8	.00201944	.3695234
			15.9	.00205674	.3765094
			16.0	.00209474	.3835722
			16.1	.00213346	.3907123
			16.2	.00217289	.3979300
			16.3	.00221305	.4052258
			16.4	.00225394	.4125998
			16.5	.00229557	.4200522
			16.6	.00233795	.4275832
			16.7	.00238109	.4351925
			16.8	.00242499	.4428801
			16.9	.00246967	.4506458
			17.0	.00251512	.4584892
			17.1	.00256137	.4664099
			17.2	.00260841	.4744074
			17.3	.00265625	.4824812
			17.4	.00270491	.4906307
			17.5	.00275438	.4988551
			17.6	.00280468	.5071538
			17.7	.00285581	.5155260
			17.8	.00290779	.5239710
			17.9	.00296061	.5324879
13.0	.00123062	.2033212	18.0	.00301429	.5410760
13.1	.00125120	.2082522	18.1	.00306883	.5497344
13.2	.00127228	.2132663	18.2	.00312424	.5584624
13.3	.00129386	.2183622	18.3	.00318052	.5672591
13.4	.00131595	.2235382	18.4	.00323769	.5761238
13.5	.00133857	.2287929	18.5	.00329575	.5850558
13.6	.00136171	.2341247	18.6	.00335470	.5940542
13.7	.00138539	.2395323	18.7	.00341456	.6031185
13.8	.00140962	.2450143	18.8	.00347533	.6122479
13.9	.00143440	.2505696	18.9	.00353701	.6214418
14.0	.00145974	.2561970	19.0	.00359962	.6306996
14.1	.00148564	.2618957	19.1	.00366316	.6400207
14.2	.00151212	.2676649	19.2	.00372763	.6494046
14.3	.00153918	.2735041	19.3	.00379304	.6588507
14.4	.00156682	.2794127	19.4	.00385940	.6683587
14.5	.00159506	.2853906	19.5	.00392671	.6779280
14.6	.00162390	.2914375	19.6	.00399499	.6875581
14.7	.00165335	.2975537	19.7	.00406423	.6972487
14.8	.00168342	.3037392	19.8	.00413444	.7069993
14.9	.00171410	.3099944	19.9	.00420563	.7168096
15.0	.00174542	.3163198	20.0	.00427780	.7266791

IPTS-68 Reference Function and Its Derivative With the Argument in Kelvin—Continued

T K	$W^*(T)$	$dW^*(T)/dT$ × 1000 K^{-1}	T K	$W^*(T)$	$dW^*(T)/dT$ × 1000 K^{-1}
20.0	.00427780	.7266791	25.0	.00925506	1.2810697
20.1	.00435097	.7366074	25.1	.00938376	1.2929735
20.2	.00442513	.7465942	25.2	.00951365	1.3048932
20.3	.00450029	.7566391	25.3	.00964474	1.3168281
20.4	.00457646	.7667415	25.4	.00977702	1.3287776
20.5	.00465364	.7769012	25.5	.00991049	1.3407410
20.6	.00473184	.7871176	25.6	.01004517	1.3527177
20.7	.00481106	.7973902	25.7	.01018104	1.3647070
20.8	.00489132	.8077186	25.8	.01031811	1.3767085
20.9	.00497261	.8181023	25.9	.01045638	1.3887215
21.0	.00505494	.8285406	26.0	.01059585	1.4007456
21.1	.00513832	.8390330	26.1	.01073653	1.4127801
21.2	.00522275	.8495789	26.2	.01087841	1.4248246
21.3	.00530824	.8601776	26.3	.01102149	1.4368786
21.4	.00539479	.8708284	26.4	.01116578	1.4489416
21.5	.00548240	.8815306	26.5	.01131128	1.4610132
21.6	.00557109	.8922834	26.6	.01145799	1.4730930
21.7	.00566086	.9030861	26.7	.01160590	1.4851804
21.8	.00575171	.9139377	26.8	.01175502	1.4972751
21.9	.00584365	.9248375	26.9	.01190536	1.5093766
22.0	.00593668	.9357846	27.0	.01205690	1.5214845
22.1	.00603081	.9467780	27.1	.01220965	1.5335984
22.2	.00612604	.9578167	27.2	.01236362	1.5457180
22.3	.00622237	.9688998	27.3	.01251880	1.5578427
22.4	.00631982	.9800263	27.4	.01267519	1.5699723
22.5	.00641838	.9911951	27.5	.01283279	1.5821063
22.6	.00651806	1.0024053	27.6	.01299161	1.5942443
22.7	.00661886	1.0136557	27.7	.01315164	1.6063859
22.8	.00672079	1.0249453	27.8	.01331289	1.6185307
22.9	.00682385	1.0362729	27.9	.01347535	1.6306783
23.0	.00692805	1.0476376	28.0	.01363902	1.6428283
23.1	.00703338	1.0590382	28.1	.01380391	1.6549803
23.2	.00713986	1.0704735	28.2	.01397002	1.6671338
23.3	.00724748	1.0819427	28.3	.01413734	1.6792885
23.4	.00735625	1.0934444	28.4	.01430588	1.6914438
23.5	.00746617	1.1049777	28.5	.01447563	1.7035995
23.6	.00757724	1.1165414	28.6	.01464660	1.7157549
23.7	.00768948	1.1281345	28.7	.01481878	1.7279097
23.8	.00780287	1.1397560	28.8	.01499218	1.7400633
23.9	.00791743	1.1514048	28.9	.01516679	1.7522154
24.0	.00803315	1.1630800	29.0	.01534262	1.7643654
24.1	.00815005	1.1747804	29.1	.01551966	1.7765129
24.2	.00826811	1.1865051	29.2	.01569792	1.7886573
24.3	.00838735	1.1982532	29.3	.01587740	1.8007982
24.4	.00850776	1.2100238	29.4	.01605808	1.8129350
24.5	.00862935	1.2218159	29.5	.01623998	1.8250672
24.6	.00875213	1.2336287	29.6	.01642310	1.8371944
24.7	.00887608	1.2454613	29.7	.01660742	1.8493158
24.8	.00900122	1.2573128	29.8	.01679296	1.8614311
24.9	.00912754	1.2691826	29.9	.01697971	1.8735397
25.0	.00925506	1.2810697	30.0	.01716767	1.8856409

IPTS-68 Reference Function and Its Derivative With the Argument in Kelvin — Continued

T K	*W**(*T*)	*dW**(*T*)/*dT* × 1000 K⁻¹	*T* K	*W**(*T*)	*dW**(*T*)/*dT* × 1000 K⁻¹
30.0	.01716767	1.8856409	35.0	.02807645	2.4686391
30.1	.01735684	1.8977343	35.1	.02832386	2.4796248
30.2	.01754721	1.9098192	35.2	.02857237	2.4905766
30.3	.01773880	1.9218951	35.3	.02882197	2.5014945
30.4	.01793159	1.9339615	35.4	.02907267	2.5123779
30.5	.01812559	1.9460176	35.5	.02932445	2.5232267
30.6	.01832080	1.9580630	35.6	.02957731	2.5340406
30.7	.01851720	1.9700970	35.7	.02983125	2.5448194
30.8	.01871481	1.9821191	35.8	.03008627	2.5555628
30.9	.01891363	1.9941286	35.9	.03034237	2.5662706
31.0	.01911364	2.0061250	36.0	.03059953	2.5769426
31.1	.01931485	2.0181076	36.1	.03085775	2.5875786
31.2	.01951726	2.0300758	36.2	.03111704	2.5981783
31.3	.01972087	2.0420291	36.3	.03137739	2.6087417
31.4	.01992567	2.0539668	36.4	.03163879	2.6192684
31.5	.02013166	2.0658884	36.5	.03190124	2.6297583
31.6	.02033884	2.0777932	36.6	.03216474	2.6402113
31.7	.02054722	2.0896806	36.7	.03242928	2.6506273
31.8	.02075678	2.1015501	36.8	.03269486	2.6610059
31.9	.02096753	2.1134010	36.9	.03296148	2.6713472
32.0	.02117946	2.1252328	37.0	.03322913	2.6816510
32.1	.02139257	2.1370449	37.1	.03349781	2.6919171
32.2	.02160687	2.1488366	37.2	.03376751	2.7021455
32.3	.02182234	2.1606075	37.3	.03403824	2.7123360
32.4	.02203899	2.1723570	37.4	.03430998	2.7224886
32.5	.02225681	2.1840844	37.5	.03458273	2.7326031
32.6	.02247580	2.1957893	37.6	.03485650	2.7426794
32.7	.02269597	2.2074710	37.7	.03513127	2.7527175
32.8	.02291730	2.2191291	37.8	.03540704	2.7627173
32.9	.02313979	2.2307631	37.9	.03568381	2.7726787
33.0	.02336345	2.2423723	38.0	.03596158	2.7826016
33.1	.02358827	2.2539562	38.1	.03624033	2.7924860
33.2	.02381424	2.2655145	38.2	.03652007	2.8023319
33.3	.02404137	2.2770464	38.3	.03680080	2.8121391
33.4	.02426965	2.2885517	38.4	.03708250	2.8219077
33.5	.02449908	2.3000297	38.5	.03736518	2.8316376
33.6	.02472965	2.3114801	38.6	.03764882	2.8413287
33.7	.02496137	2.3229023	38.7	.03793344	2.8509810
33.8	.02519423	2.3342959	38.8	.03821902	2.8605946
33.9	.02542823	2.3456605	38.9	.03850556	2.8701693
34.0	.02566336	2.3569956	39.0	.03879305	2.8797052
34.1	.02589963	2.3683009	39.1	.03908150	2.8892022
34.2	.02613702	2.3795758	39.2	.03937089	2.8986603
34.3	.02637554	2.3908200	39.3	.03966123	2.9080795
34.4	.02661518	2.4020332	39.4	.03995251	2.9174598
34.5	.02685595	2.4132149	39.5	.04024472	2.9268012
34.6	.02709783	2.4243648	39.6	.04053786	2.9361038
34.7	.02734082	2.4354825	39.7	.04083194	2.9453674
34.8	.02758492	2.4465677	39.8	.04112694	2.9545921
34.9	.02783013	2.4576200	39.9	.04142286	2.9637779
35.0	.02807645	2.4686391	40.0	.04171969	2.9729248

IPTS-68 Reference Function and Its Derivative With the Argument in Kelvin – Continued

T K	$W^*(T)$	$dW^*(T)/dT$ × 1000 K^{-1}	T K	$W^*(T)$	$dW^*(T)/dT$ × 1000 K^{-1}
40.0	.04171969	2.9729248	45.0	.05764487	3.3812367
40.1	.04201744	2.9820329	45.1	.05798335	3.3884379
40.2	.04231610	2.9911020	45.2	.05832255	3.3956019
40.3	.04261566	3.0001324	45.3	.05866247	3.4027289
40.4	.04291612	3.0091238	45.4	.05900310	3.4098188
40.5	.04321748	3.0180765	45.5	.05934443	3.4168718
40.6	.04351974	3.0269903	45.6	.05968647	3.4238879
40.7	.04382288	3.0358654	45.7	.06002921	3.4308672
40.8	.04412691	3.0447017	45.8	.06037264	3.4378096
40.9	.04443182	3.0534993	45.9	.06071677	3.4447153
41.0	.04473761	3.0622582	46.0	.06106158	3.4515843
41.1	.04504427	3.0709783	46.1	.06140708	3.4584167
41.2	.04535180	3.0796598	46.2	.06175327	3.4652125
41.3	.04566020	3.0883026	46.3	.06210013	3.4719718
41.4	.04596946	3.0969069	46.4	.06244766	3.4786946
41.5	.04627958	3.1054726	46.5	.06279586	3.4853811
41.6	.04659055	3.1139997	46.6	.06314473	3.4920311
41.7	.04690238	3.1224883	46.7	.06349427	3.4986449
41.8	.04721505	3.1309384	46.8	.06384446	3.5052225
41.9	.04752856	3.1393500	46.9	.06419531	3.5117639
42.0	.04784292	3.1477233	47.0	.06454681	3.5182692
42.1	.04815811	3.1560581	47.1	.06489896	3.5247385
42.2	.04847413	3.1643546	47.2	.06525176	3.5311718
42.3	.04879098	3.1726127	47.3	.06560520	3.5375693
42.4	.04910865	3.1808326	47.4	.06595927	3.5439308
42.5	.04942714	3.1890142	47.5	.06631398	3.5502567
42.6	.04974645	3.1971576	47.6	.06666932	3.5565468
42.7	.05006657	3.2052628	47.7	.06702529	3.5628013
42.8	.05038750	3.2133299	47.8	.06738188	3.5690202
42.9	.05070924	3.2213589	47.9	.06773909	3.5752037
43.0	.05103177	3.2293498	48.0	.06809692	3.5813517
43.1	.05135511	3.2373027	48.1	.06845536	3.5874645
43.2	.05167923	3.2452176	48.2	.06881441	3.5935419
43.3	.05200415	3.2530945	48.3	.06917407	3.5995842
43.4	.05232985	3.2609335	48.4	.06953433	3.6055913
43.5	.05265633	3.2687347	48.5	.06989519	3.6115635
43.6	.05298359	3.2764980	48.6	.07025664	3.6175007
43.7	.05331163	3.2842236	48.7	.07061869	3.6234030
43.8	.05364044	3.2919114	48.8	.07098132	3.6292705
43.9	.05397001	3.2995615	48.9	.07134454	3.6351033
44.0	.05430035	3.3071739	49.0	.07170834	3.6409015
44.1	.05463145	3.3147488	49.1	.07207272	3.6466652
44.2	.05496330	3.3222860	49.2	.07243767	3.6523943
44.3	.05529590	3.3297857	49.3	.07280320	3.6580892
44.4	.05562925	3.3372479	49.4	.07316929	3.6637497
44.5	.05596335	3.3446727	49.5	.07353595	3.6693761
44.6	.05629819	3.3520602	49.6	.07390316	3.6749683
44.7	.05663376	3.3594102	49.7	.07427094	3.6805266
44.8	.05697007	3.3667230	49.8	.07463927	3.6860509
44.9	.05730710	3.3739985	49.9	.07500815	3.6915414
45.0	.05764487	3.3812367	50.0	.07537757	3.6969982

IPTS-68 Reference Function and Its Derivative With the Argument in Kelvin – Continued

T K	$W^*(T)$	$dW^*(T)/dT$ × 1000 K^{-1}	T K	$W^*(T)$	$dW^*(T)/dT$ × 1000 K^{-1}
50.0	.07537757	3.6969982	55.0	.09447519	3.9291588
50.1	.07574755	3.7024213	55.1	.09486830	3.9330396
50.2	.07611806	3.7078109	55.2	.09526179	3.9368927
50.3	.07648911	3.7131670	55.3	.09565568	3.9407182
50.4	.07686069	3.7184898	55.4	.09604994	3.9445161
50.5	.07723280	3.7237793	55.5	.09644458	3.9482867
50.6	.07760544	3.7290357	55.6	.09683959	3.9520301
50.7	.07797861	3.7342591	55.7	.09723498	3.9557464
50.8	.07835229	3.7394494	55.8	.09763074	3.9594356
50.9	.07872650	3.7446070	55.9	.09802687	3.9630981
51.0	.07910122	3.7497318	56.0	.09842336	3.9667339
51.1	.07947644	3.7548239	56.1	.09882022	3.9703430
51.2	.07985218	3.7598835	56.2	.09921743	3.9739257
51.3	.08022842	3.7649107	56.3	.09961500	3.9774821
51.4	.08060516	3.7699056	56.4	.10001292	3.9810123
51.5	.08098240	3.7748682	56.5	.10041120	3.9845165
51.6	.08136013	3.7797987	56.6	.10080983	3.9879947
51.7	.08173836	3.7846973	56.7	.10120880	3.9914472
51.8	.08211707	3.7895640	56.8	.10160812	3.9948740
51.9	.08249627	3.7943988	56.9	.10200777	3.9982753
52.0	.08287595	3.7992021	57.0	.10240777	4.0016511
52.1	.08325611	3.8039737	57.1	.10280810	4.0050017
52.2	.08363674	3.8087140	57.2	.10320877	4.0083273
52.3	.08401785	3.8134229	57.3	.10360977	4.0116278
52.4	.08439943	3.8181006	57.4	.10401109	4.0149033
52.5	.08478147	3.8227472	57.5	.10441275	4.0181542
52.6	.08516398	3.8273628	57.6	.10481472	4.0213805
52.7	.08554694	3.8319476	57.7	.10521702	4.0245823
52.8	.08593036	3.8365016	57.8	.10561964	4.0277598
52.9	.08631424	3.8410250	57.9	.10602257	4.0309130
53.0	.08669857	3.8455179	58.0	.10642582	4.0340422
53.1	.08708334	3.8499805	58.1	.10682938	4.0371474
53.2	.08746856	3.8544127	58.2	.10723325	4.0402288
53.3	.08785422	3.8588148	58.3	.10763743	4.0432865
53.4	.08824032	3.8631869	58.4	.10804191	4.0463206
53.5	.08862686	3.8675291	58.5	.10844669	4.0493314
53.6	.08901383	3.8718415	58.6	.10885177	4.0523188
53.7	.08940123	3.8761243	58.7	.10925715	4.0552830
53.8	.08978905	3.8803775	58.8	.10966283	4.0582243
53.9	.09017730	3.8846014	58.9	.11006880	4.0611426
54.0	.09056597	3.8887959	59.0	.11047506	4.0640381
54.1	.09095506	3.8929613	59.1	.11088160	4.0669110
54.2	.09134456	3.8970977	59.2	.11128844	4.0697614
54.3	.09173448	3.9012052	59.3	.11169555	4.0725893
54.4	.09212480	3.9052839	59.4	.11210295	4.0753950
54.5	.09251554	3.9093339	59.5	.11251063	4.0781786
54.6	.09290667	3.9133554	59.6	.11291859	4.0809402
54.7	.09329821	3.9173485	59.7	.11332682	4.0836798
54.8	.09369014	3.9213134	59.8	.11373532	4.0863978
54.9	.09408247	3.9252501	59.9	.11414410	4.0890941
55.0	.09447519	3.9291588	60.0	.11455314	4.0917688

IPTS-68 Reference Function and Its Derivative With the Argument in Kelvin – Continued

T K	$W^*(T)$	$dW^*(T)/dT$ × 1000 K^{-1}	T K	$W^*(T)$	$dW^*(T)/dT$ × 1000 K^{-1}
60.0	.11455314	4.0917688	65.0	.13530364	4.2007677
60.1	.11496245	4.0944222	65.1	.13572380	4.2025038
60.2	.11537203	4.0970544	65.2	.13614414	4.2042243
60.3	.11578186	4.0996654	65.3	.13656464	4.2059294
60.4	.11619196	4.1022555	65.4	.13698532	4.2076192
60.5	.11660231	4.1048246	65.5	.13740617	4.2092937
60.6	.11701292	4.1073730	65.6	.13782718	4.2109531
60.7	.11742379	4.1099008	65.7	.13824836	4.2125975
60.8	.11783490	4.1124081	65.8	.13866970	4.2142270
60.9	.11824627	4.1148950	65.9	.13909120	4.2158416
61.0	.11865788	4.1173616	66.0	.13951287	4.2174415
61.1	.11906974	4.1198081	66.1	.13993469	4.2190268
61.2	.11948184	4.1222345	66.2	.14035667	4.2205976
61.3	.11989419	4.1246411	66.3	.14077881	4.2221539
61.4	.12030677	4.1270279	66.4	.14120110	4.2236958
61.5	.12071959	4.1293951	66.5	.14162355	4.2252236
61.6	.12113265	4.1317428	66.6	.14204615	4.2267371
61.7	.12154594	4.1340709	66.7	.14246889	4.2282367
61.8	.12195946	4.1363799	66.8	.14289179	4.2297223
61.9	.12237321	4.1386696	66.9	.14331484	4.2311939
62.0	.12278719	4.1409403	67.0	.14373803	4.2326518
62.1	.12320140	4.1431921	67.1	.14416137	4.2340961
62.2	.12361583	4.1454250	67.2	.14458485	4.2355267
62.3	.12403049	4.1476393	67.3	.14500847	4.2369439
62.4	.12444536	4.1498349	67.4	.14543224	4.2383476
62.5	.12486045	4.1520121	67.5	.14585614	4.2397380
62.6	.12527576	4.1541709	67.6	.14628019	4.2411152
62.7	.12569129	4.1563115	67.7	.14670436	4.2424793
62.8	.12610702	4.1584339	67.8	.14712868	4.2438303
62.9	.12652297	4.1605384	67.9	.14755313	4.2451683
63.0	.12693913	4.1626249	68.0	.14797771	4.2464935
63.1	.12735550	4.1646937	68.1	.14840243	4.2478058
63.2	.12777207	4.1667448	68.2	.14882727	4.2491055
63.3	.12818884	4.1687783	68.3	.14925225	4.2503925
63.4	.12860582	4.1707944	68.4	.14967735	4.2516671
63.5	.12902300	4.1727931	68.5	.15010258	4.2529291
63.6	.12944038	4.1747746	68.6	.15052794	4.2541788
63.7	.12985796	4.1767390	68.7	.15095342	4.2554163
63.8	.13027573	4.1786864	68.8	.15137902	4.2566415
63.9	.13069369	4.1806169	68.9	.15180475	4.2578547
64.0	.13111185	4.1825306	69.0	.15223059	4.2590558
64.1	.13153020	4.1844276	69.1	.15265656	4.2602449
64.2	.13194874	4.1863081	69.2	.15308264	4.2614222
64.3	.13236746	4.1881721	69.3	.15350884	4.2625877
64.4	.13278637	4.1900197	69.4	.15393516	4.2637415
64.5	.13320546	4.1918511	69.5	.15436159	4.2648838
64.6	.13362474	4.1936663	69.6	.15478813	4.2660144
64.7	.13404420	4.1954655	69.7	.15521479	4.2671336
64.8	.13446383	4.1972487	69.8	.15564156	4.2682414
64.9	.13488365	4.1990161	69.9	.15606844	4.2693379
65.0	.13530364	4.2007677	70.0	.15649543	4.2704231

IPTS-68 Reference Function and Its Derivative With the Argument in Kelvin – Continued

T K	$W^*(T)$	$dW^*(T)/dT$ × 1000 K^{-1}	T K	$W^*(T)$	$dW^*(T)/dT$ × 1000 K^{-1}
70.0	.15649543	4.2704231	75.0	.17796116	4.3119570
70.1	.15692252	4.2714973	75.1	.17839239	4.3125623
70.2	.15734973	4.2725603	75.2	.17882367	4.3131598
70.3	.15777703	4.2736124	75.3	.17925502	4.3137496
70.4	.15820445	4.2746535	75.4	.17968642	4.3143316
70.5	.15863197	4.2756838	75.5	.18011789	4.3149061
70.6	.15905958	4.2767033	75.6	.18054940	4.3154730
70.7	.15948731	4.2777121	75.7	.18098098	4.3160323
70.8	.15991513	4.2787103	75.8	.18141261	4.3165842
70.9	.16034305	4.2796980	75.9	.18184430	4.3171287
71.0	.16077107	4.2806751	76.0	.18227604	4.3176658
71.1	.16119918	4.2816419	76.1	.18270783	4.3181957
71.2	.16162739	4.2825983	76.2	.18313968	4.3187183
71.3	.16205570	4.2835445	76.3	.18357157	4.3192336
71.4	.16248410	4.2844805	76.4	.18400352	4.3197419
71.5	.16291260	4.2854064	76.5	.18443552	4.3202431
71.6	.16334118	4.2863222	76.6	.18486757	4.3207372
71.7	.16376986	4.2872280	76.7	.18529967	4.3212243
71.8	.16419863	4.2881240	76.8	.18573181	4.3217045
71.9	.16462749	4.2890101	76.9	.18616401	4.3221778
72.0	.16505643	4.2898864	77.0	.18659625	4.3226442
72.1	.16548546	4.2907530	77.1	.18702854	4.3231039
72.2	.16591458	4.2916101	77.2	.18746087	4.3235568
72.3	.16634378	4.2924575	77.3	.18789325	4.3240030
72.4	.16677307	4.2932954	77.4	.18832567	4.3244426
72.5	.16720244	4.2941240	77.5	.18875814	4.3248755
72.6	.16763190	4.2949431	77.6	.18919065	4.3253020
72.7	.16806143	4.2957529	77.7	.18962320	4.3257219
72.8	.16849105	4.2965535	77.8	.19005579	4.3261354
72.9	.16892074	4.2973449	77.9	.19048842	4.3265424
73.0	.16935052	4.2981273	78.0	.19092110	4.3269431
73.1	.16978037	4.2989005	78.1	.19135381	4.3273375
73.2	.17021030	4.2996649	78.2	.19178657	4.3277257
73.3	.17064030	4.3004202	78.3	.19221936	4.3281076
73.4	.17107038	4.3011667	78.4	.19265219	4.3284832
73.5	.17150053	4.3019045	78.5	.19308505	4.3288529
73.6	.17193076	4.3026335	78.6	.19351796	4.3292164
73.7	.17236106	4.3033539	78.7	.19395090	4.3295738
73.8	.17279143	4.3040656	78.8	.19438387	4.3299253
73.9	.17322187	4.3047688	78.9	.19481688	4.3302707
74.0	.17365238	4.3054635	79.0	.19524993	4.3306104
74.1	.17408296	4.3061498	79.1	.19568300	4.3309441
74.2	.17451361	4.3068277	79.2	.19611611	4.3312720
74.3	.17494433	4.3074974	79.3	.19654926	4.3315941
74.4	.17537511	4.3081587	79.4	.19698243	4.3319105
74.5	.17580596	4.3088119	79.5	.19741564	4.3322212
74.6	.17623687	4.3094570	79.6	.19784888	4.3325262
74.7	.17666785	4.3100939	79.7	.19828214	4.3328256
74.8	.17709889	4.3107229	79.8	.19871544	4.3331194
74.9	.17753000	4.3113439	79.9	.19914877	4.3334077
75.0	.17796116	4.3119570	80.0	.19958212	4.3336906

IPTS-68 Reference Function and Its Derivative With the Argument in Kelvin – Continued

T K	$W^*(T)$	$dW^*(T)/dT$ × 1000 K^{-1}	T K	$W^*(T)$	$dW^*(T)/dT$ × 1000 K^{-1}
80.0	.19958212	4.3336906	85.0	.22127523	4.3416637
80.1	.20001551	4.3339679	85.1	.22170939	4.3417150
80.2	.20044892	4.3342399	85.2	.22214357	4.3417627
80.3	.20088235	4.3345065	85.3	.22257775	4.3418067
80.4	.20131582	4.3347678	85.4	.22301193	4.3418471
80.5	.20174931	4.3350238	85.5	.22344612	4.3418839
80.6	.20218282	4.3352745	85.6	.22388031	4.3419171
80.7	.20261636	4.3355200	85.7	.22431450	4.3419467
80.8	.20304993	4.3357603	85.8	.22474870	4.3419728
80.9	.20348351	4.3359955	85.9	.22518289	4.3419955
81.0	.20391713	4.3362257	86.0	.22561709	4.3420146
81.1	.20435076	4.3364507	86.1	.22605130	4.3420303
81.2	.20478442	4.3366708	86.2	.22648550	4.3420427
81.3	.20521809	4.3368858	86.3	.22691971	4.3420516
81.4	.20565179	4.3370959	86.4	.22735391	4.3420572
81.5	.20608551	4.3373012	86.5	.22778812	4.3420594
81.6	.20651925	4.3375015	86.6	.22822232	4.3420584
81.7	.20695301	4.3376970	86.7	.22865653	4.3420541
81.8	.20738679	4.3378877	86.8	.22909073	4.3420465
81.9	.20782059	4.3380737	86.9	.22952494	4.3420357
82.0	.20825441	4.3382549	87.0	.22995914	4.3420217
82.1	.20868824	4.3384315	87.1	.23039334	4.3420045
82.2	.20912209	4.3386034	87.2	.23082754	4.3419842
82.3	.20955596	4.3387706	87.3	.23126174	4.3419608
82.4	.20998985	4.3389333	87.4	.23169593	4.3419342
82.5	.21042375	4.3390914	87.5	.23213013	4.3419047
82.6	.21085766	4.3392450	87.6	.23256431	4.3418720
82.7	.21129160	4.3393942	87.7	.23299850	4.3418363
82.8	.21172554	4.3395389	87.8	.23343268	4.3417977
82.9	.21215950	4.3396791	87.9	.23386686	4.3417560
83.0	.21259348	4.3398151	88.0	.23430103	4.3417114
83.1	.21302747	4.3399466	88.1	.23473520	4.3416639
83.2	.21346147	4.3400739	88.2	.23516937	4.3416135
83.3	.21389548	4.3401968	88.3	.23560352	4.3415602
83.4	.21432951	4.3403155	88.4	.23603768	4.3415041
83.5	.21476354	4.3404301	88.5	.23647182	4.3414451
83.6	.21519759	4.3405404	88.6	.23690597	4.3413833
83.7	.21563165	4.3406466	88.7	.23734010	4.3413188
83.8	.21606572	4.3407487	88.8	.23777423	4.3412514
83.9	.21649980	4.3408466	88.9	.23820835	4.3411814
84.0	.21693389	4.3409406	89.0	.23864247	4.3411086
84.1	.21736799	4.3410304	89.1	.23907657	4.3410332
84.2	.21780210	4.3411163	89.2	.23951067	4.3409551
84.3	.21823621	4.3411983	89.3	.23994476	4.3408743
84.4	.21867034	4.3412764	89.4	.24037885	4.3407909
84.5	.21910447	4.3413505	89.5	.24081292	4.3407048
84.6	.21953861	4.3414207	89.6	.24124699	4.3406162
84.7	.21997275	4.3414871	89.7	.24168105	4.3405251
84.8	.22040690	4.3415498	89.8	.24211509	4.3404314
84.9	.22084106	4.3416086	89.9	.24254913	4.3403352
85.0	.22127523	4.3416637	90.0	.24298316	4.3402365

IPTS-68 Reference Function and Its Derivative With the Argument in Kelvin – Continued

T K	$W^*(T)$	$dW^*(T)/dT$ × 1000 K^{-1}	T K	$W^*(T)$	$dW^*(T)/dT$ × 1000 K^{-1}
90.0	.24298316	4.3402365	95.0	.26466721	4.3325507
90.1	.24341718	4.3401353	95.1	.26510045	4.3323498
90.2	.24385119	4.3400317	95.2	.26553368	4.3321474
90.3	.24428519	4.3399257	95.3	.26596688	4.3319435
90.4	.24471917	4.3398172	95.4	.26640007	4.3317379
90.5	.24515315	4.3397063	95.5	.26683323	4.3315309
90.6	.24558711	4.3395931	95.6	.26726637	4.3313224
90.7	.24602107	4.3394775	95.7	.26769950	4.3311124
90.8	.24645501	4.3393596	95.8	.26813260	4.3309009
90.9	.24688894	4.3392394	95.9	.26856568	4.3306879
91.0	.24732286	4.3391169	96.0	.26899873	4.3304735
91.1	.24775676	4.3389921	96.1	.26943177	4.3302577
91.2	.24819065	4.3388651	96.2	.26986479	4.3300404
91.3	.24862454	4.3387358	96.3	.27029778	4.3298217
91.4	.24905840	4.3386043	96.4	.27073075	4.3296017
91.5	.24949226	4.3384706	96.5	.27116370	4.3293803
91.6	.24992610	4.3383348	96.6	.27159663	4.3291575
91.7	.25035992	4.3381968	96.7	.27202953	4.3289333
91.8	.25079374	4.3380566	96.8	.27246241	4.3287078
91.9	.25122753	4.3379143	96.9	.27289527	4.3284810
92.0	.25166132	4.3377700	97.0	.27332811	4.3282529
92.1	.25209509	4.3376236	97.1	.27376092	4.3280234
92.2	.25252884	4.3374751	97.2	.27419371	4.3277927
92.3	.25296258	4.3373245	97.3	.27462648	4.3275607
92.4	.25339631	4.3371720	97.4	.27505922	4.3273274
92.5	.25383002	4.3370174	97.5	.27549195	4.3270929
92.6	.25426371	4.3368608	97.6	.27592464	4.3268572
92.7	.25469739	4.3367023	97.7	.27635732	4.3266203
92.8	.25513105	4.3365418	97.8	.27678997	4.3263821
92.9	.25556470	4.3363794	97.9	.27722259	4.3261427
93.0	.25599833	4.3362150	98.0	.27765520	4.3259021
93.1	.25643194	4.3360487	98.1	.27808777	4.3256604
93.2	.25686554	4.3358806	98.2	.27852033	4.3254176
93.3	.25729912	4.3357106	98.3	.27895286	4.3251735
93.4	.25773268	4.3355388	98.4	.27938536	4.3249283
93.5	.25816622	4.3353651	98.5	.27981784	4.3246821
93.6	.25859975	4.3351896	98.6	.28025030	4.3244347
93.7	.25903326	4.3350122	98.7	.28068273	4.3241862
93.8	.25946675	4.3348332	98.8	.28111514	4.3239366
93.9	.25990023	4.3346523	98.9	.28154752	4.3236859
94.0	.26033368	4.3344697	99.0	.28197987	4.3234341
94.1	.26076712	4.3342853	99.1	.28241220	4.3231813
94.2	.26120054	4.3340992	99.2	.28284451	4.3229276
94.3	.26163394	4.3339115	99.3	.28327679	4.3226727
94.4	.26206732	4.3337220	99.4	.28370904	4.3224168
94.5	.26250069	4.3335309	99.5	.28414127	4.3221599
94.6	.26293403	4.3333381	99.6	.28457348	4.3219020
94.7	.26336735	4.3331436	99.7	.28500565	4.3216432
94.8	.26380066	4.3329476	99.8	.28543780	4.3213833
94.9	.26423394	4.3327499	99.9	.28586993	4.3211225
95.0	.26466721	4.3325507	100.0	.28630203	4.3208607

IPTS-68 Reference Function and Its Derivative With the Argument in Kelvin—Continued

T K	$W^*(T)$	$dW^*(T)/dT$ × 1000 K^{-1}	T K	$W^*(T)$	$dW^*(T)/dT$ × 1000 K^{-1}
100.0	.28630203	4.3208607	150.0	.49861134	4.1769430
101.0	.29062156	4.3181928	151.0	.50278711	4.1746153
102.0	.29493839	4.3154407	152.0	.50696058	4.1723135
103.0	.29925242	4.3126140	153.0	.51113175	4.1700371
104.0	.30356359	4.3097216	154.0	.51530066	4.1677859
105.0	.30787184	4.3067718	155.0	.51946733	4.1655591
106.0	.31217712	4.3037723	156.0	.52363178	4.1633567
107.0	.31647937	4.3007301	157.0	.52779405	4.1611778
108.0	.32077857	4.2976521	158.0	.53195415	4.1590222
109.0	.32507467	4.2945442	159.0	.53611210	4.1568895
110.0	.32936765	4.2914122	160.0	.54026793	4.1547790
111.0	.33365749	4.2882612	161.0	.54442167	4.1526903
112.0	.33794417	4.2850962	162.0	.54857332	4.1506230
113.0	.34222768	4.2819217	163.0	.55272292	4.1485767
114.0	.34650801	4.2787417	164.0	.55687048	4.1465508
115.0	.35078516	4.2755600	165.0	.56101603	4.1445449
116.0	.35505913	4.2723801	166.0	.56515958	4.1425586
117.0	.35932992	4.2692052	167.0	.56930115	4.1405914
118.0	.36359754	4.2660381	168.0	.57344077	4.1386427
119.0	.36786200	4.2628816	169.0	.57757844	4.1367123
120.0	.37212331	4.2597380	170.0	.58171420	4.1347998
121.0	.37638148	4.2566095	171.0	.58584805	4.1329045
122.0	.38063653	4.2534980	172.0	.58998001	4.1310262
123.0	.38488848	4.2504055	173.0	.59411011	4.1291644
124.0	.38913735	4.2473335	174.0	.59823835	4.1273188
125.0	.39338316	4.2442834	175.0	.60236475	4.1254889
126.0	.39762593	4.2412563	176.0	.60648933	4.1236743
127.0	.40186568	4.2382537	177.0	.61061210	4.1218746
128.0	.40610244	4.2352763	178.0	.61473308	4.1200896
129.0	.41033624	4.2323250	179.0	.61885229	4.1183188
130.0	.41456710	4.2294006	180.0	.62296973	4.1165619
131.0	.41879505	4.2265037	181.0	.62708542	4.1148185
132.0	.42302012	4.2236348	182.0	.63119937	4.1130884
133.0	.42724233	4.2207945	183.0	.63531160	4.1113711
134.0	.43146172	4.2179829	184.0	.63942211	4.1096663
135.0	.43567831	4.2152003	185.0	.64353093	4.1079738
136.0	.43989213	4.2124470	186.0	.64763807	4.1062933
137.0	.44410321	4.2097231	187.0	.65174352	4.1046244
138.0	.44831158	4.2070286	188.0	.65584732	4.1029669
139.0	.45251728	4.2043635	189.0	.65994946	4.1013205
140.0	.45672032	4.2017278	190.0	.66404996	4.0996849
141.0	.46092074	4.1991213	191.0	.66814884	4.0980600
142.0	.46511857	4.1965439	192.0	.67224609	4.0964453
143.0	.46931384	4.1939955	193.0	.67634173	4.0948408
144.0	.47350657	4.1914757	194.0	.68043577	4.0932461
145.0	.47769680	4.1889844	195.0	.68452823	4.0916611
146.0	.48188455	4.1865212	196.0	.68861910	4.0900855
147.0	.48606985	4.1840858	197.0	.69270840	4.0885191
148.0	.49025273	4.1816779	198.0	.69679614	4.0869618
149.0	.49443322	4.1792971	199.0	.70088233	4.0854132
150.0	.49861134	4.1769430	200.0	.70496697	4.0838734

IPTS-68 Reference Function and Its Derivative With the Argument in Kelvin—Continued

T K	$W^*(T)$	$dW^*(T)/dT$ × 1000 K^{-1}
200.0	.70496697	4.0838734
201.0	.70905008	4.0823419
202.0	.71313166	4.0808187
203.0	.71721172	4.0793037
204.0	.72129027	4.0777966
205.0	.72536731	4.0762973
206.0	.72944286	4.0748056
207.0	.73351693	4.0733215
208.0	.73758951	4.0718447
209.0	.74166062	4.0703751
210.0	.74573026	4.0689126
211.0	.74979844	4.0674572
212.0	.75386518	4.0660085
213.0	.75793046	4.0645666
214.0	.76199431	4.0631313
215.0	.76605673	4.0617025
216.0	.77011772	4.0602801
217.0	.77417729	4.0588640
218.0	.77823545	4.0574541
219.0	.78229220	4.0560503
220.0	.78634755	4.0546525
221.0	.79040151	4.0532606
222.0	.79445408	4.0518746
223.0	.79850526	4.0504942
224.0	.80255507	4.0491195
225.0	.80660350	4.0477504
226.0	.81065057	4.0463867
227.0	.81469628	4.0450285
228.0	.81874063	4.0436755
229.0	.82278363	4.0423277
230.0	.82682528	4.0409851
231.0	.83086560	4.0396475
232.0	.83490458	4.0383149
233.0	.83894223	4.0369871
234.0	.84297856	4.0356641
235.0	.84701356	4.0343459
236.0	.85104725	4.0330323
237.0	.85507963	4.0317232
238.0	.85911070	4.0304186
239.0	.86314047	4.0291183
240.0	.86716894	4.0278223
241.0	.87119611	4.0265305
242.0	.87522200	4.0252428
243.0	.87924660	4.0239590
244.0	.88326992	4.0226792
245.0	.88729196	4.0214031
246.0	.89131273	4.0201307
247.0	.89533222	4.0188618
248.0	.89935045	4.0175964
249.0	.90336742	4.0163344
250.0	.90738312	4.0150756
250.0	.90738312	4.0150756
251.0	.91139757	4.0138199
252.0	.91541076	4.0125672
253.0	.91942270	4.0113173
254.0	.92343340	4.0100702
255.0	.92744285	4.0088257
256.0	.93145105	4.0075837
257.0	.93545801	4.0063439
258.0	.93946374	4.0051063
259.0	.94346823	4.0038708
260.0	.94747148	4.0026371
261.0	.95147350	4.0014051
262.0	.95547429	4.0001747
263.0	.95947385	3.9989456
264.0	.96347218	3.9977178
265.0	.96746929	3.9964910
266.0	.97146517	3.9952650
267.0	.97545982	3.9940397
268.0	.97945325	3.9928147
269.0	.98344545	3.9915902
270.0	.98743643	3.9903656
271.0	.99142618	3.9891409
272.0	.99541471	3.9879159
273.0	.99940201	3.9866902
273.15	1.00000000	3.9865063

IPTS-68 Reference Function and Its First Derivative With the Argument in Celsius

t °C	$W^*(t)$	$dW^*(t)/dT$ × 1000 °C^{-1}	t °C	$W^*(t)$	$dW^*(t)/dT$ × 1000 °C^{-1}
−260.0	.00126168	.2107490	−255.0	.00309642	.5540898
−259.9	.00128300	.2158042	−254.9	.00315227	.5628522
−259.8	.00130484	.2209403	−254.8	.00320900	.5716830
−259.7	.00132719	.2261558	−254.7	.00326661	.5805814
−259.6	.00135007	.2314492	−254.6	.00332511	.5895467
−259.5	.00137349	.2368191	−254.5	.00338452	.5985782
−259.4	.00139744	.2422640	−254.4	.00344483	.6076751
−259.3	.00142194	.2477828	−254.3	.00350606	.6168368
−259.2	.00144700	.2533743	−254.2	.00356820	.6260627
−259.1	.00147262	.2590375	−254.1	.00363127	.6353522
−259.0	.00149881	.2647716	−254.0	.00369527	.6447048
−258.9	.00152557	.2705758	−253.9	.00376022	.6541199
−258.8	.00155293	.2764497	−253.8	.00382610	.6635970
−258.7	.00158087	.2823930	−253.7	.00389294	.6731357
−258.6	.00160941	.2884054	−253.6	.00396073	.6827354
−258.5	.00163855	.2944869	−253.5	.00402949	.6923958
−258.4	.00166831	.3006377	−253.4	.00409921	.7021165
−258.3	.00169868	.3068581	−253.3	.00416991	.7118970
−258.2	.00172968	.3131483	−253.2	.00424159	.7217370
−258.1	.00176131	.3195089	−253.1	.00431426	.7316359
−258.0	.00179358	.3259405	−253.0	.00438792	.7415936
−257.9	.00182650	.3324437	−252.9	.00446258	.7516094
−257.8	.00186007	.3390193	−252.8	.00453825	.7616831
−257.7	.00189431	.3456678	−252.7	.00461492	.7718143
−257.6	.00192921	.3523901	−252.6	.00469261	.7820023
−257.5	.00196479	.3591870	−252.5	.00477132	.7922469
−257.4	.00200105	.3660590	−252.4	.00485106	.8025475
−257.3	.00203800	.3730068	−252.3	.00493183	.8129036
−257.2	.00207565	.3800312	−252.2	.00501364	.8233146
−257.1	.00211401	.3871326	−252.1	.00509650	.8337801
−257.0	.00215308	.3943114	−252.0	.00518040	.8442993
−256.9	.00219288	.4015681	−251.9	.00526536	.8548717
−256.8	.00223340	.4089030	−251.8	.00535138	.8654965
−256.7	.00227466	.4163162	−251.7	.00543846	.8761731
−256.6	.00231667	.4238079	−251.6	.00552661	.8869007
−256.5	.00235942	.4313780	−251.5	.00561584	.8976786
−256.4	.00240294	.4390265	−251.4	.00570615	.9085058
−256.3	.00244723	.4467532	−251.3	.00579755	.9193817
−256.2	.00249230	.4545578	−251.2	.00589003	.9303052
−256.1	.00253815	.4624399	−251.1	.00598361	.9412756
−256.0	.00258479	.4703991	−251.0	.00607829	.9522917
−255.9	.00263223	.4784348	−250.9	.00617407	.9633528
−255.8	.00268048	.4865465	−250.8	.00627096	.9744577
−255.7	.00272954	.4947336	−250.7	.00636896	.9856055
−255.6	.00277943	.5029952	−250.6	.00646808	.9967951
−255.5	.00283014	.5113308	−250.5	.00656832	1.0080255
−255.4	.00288170	.5197395	−250.4	.00666969	1.0192956
−255.3	.00293409	.5282205	−250.3	.00677218	1.0306044
−255.2	.00298734	.5367731	−250.2	.00687581	1.0419507
−255.1	.00304145	.5453965	−250.1	.00698057	1.0533335
−255.0	.00309642	.5540898	−250.0	.00708648	1.0647516

IPTS-68 Reference Function and Its First Derivative With the Argument in Celsius—Continued

t °C	$W^*(t)$	$dW^*(t)/dT$ × 1000 °C^{-1}	t °C	$W^*(t)$	$dW^*(t)/dT$ × 1000 °C^{-1}
−250.0	.00708648	1.0647516	−245.0	.01388681	1.6610569
−249.9	.00719352	1.0762039	−244.9	.01405353	1.6732110
−249.8	.00730172	1.0876895	−244.8	.01422146	1.6853661
−249.7	.00741106	1.0992071	−244.7	.01439060	1.6975216
−249.6	.00752156	1.1107558	−244.6	.01456096	1.7096772
−249.5	.00763322	1.1223343	−244.5	.01473254	1.7218324
−249.4	.00774603	1.1339418	−244.4	.01490533	1.7339866
−249.3	.00786000	1.1455771	−244.3	.01507933	1.7461396
−249.2	.00797515	1.1572392	−244.2	.01525455	1.7582907
−249.1	.00809145	1.1689271	−244.1	.01543099	1.7704395
−249.0	.00820893	1.1806398	−244.0	.01560864	1.7825855
−248.9	.00832758	1.1923763	−243.9	.01578751	1.7947282
−248.8	.00844741	1.2041358	−243.8	.01596759	1.8068671
−248.7	.00856841	1.2159172	−243.7	.01614888	1.8190017
−248.6	.00869059	1.2277198	−243.6	.01633139	1.8311315
−248.5	.00881395	1.2395426	−243.5	.01651511	1.8432558
−248.4	.00893850	1.2513847	−243.4	.01670004	1.8553743
−248.3	.00906423	1.2632455	−243.3	.01688618	1.8674863
−248.2	.00919115	1.2751240	−243.2	.01707354	1.8795912
−248.1	.00931926	1.2870195	−243.1	.01726210	1.8916886
−248.0	.00944856	1.2989314	−243.0	.01745187	1.9037778
−247.9	.00957904	1.3108588	−242.9	.01764286	1.9158583
−247.8	.00971073	1.3228011	−242.8	.01783504	1.9279295
−247.7	.00984361	1.3347576	−242.7	.01802844	1.9399908
−247.6	.00997768	1.3467277	−242.6	.01822304	1.9520417
−247.5	.01011295	1.3587108	−242.5	.01841885	1.9640815
−247.4	.01024942	1.3707063	−242.4	.01861586	1.9761096
−247.3	.01038709	1.3827136	−242.3	.01881407	1.9881255
−247.2	.01052597	1.3947322	−242.2	.01901348	2.0001285
−247.1	.01066604	1.4067616	−242.1	.01921410	2.0121180
−247.0	.01080732	1.4188011	−242.0	.01941591	2.0240935
−246.9	.01094980	1.4308504	−241.9	.01961891	2.0360544
−246.8	.01109349	1.4429090	−241.8	.01982312	2.0480000
−246.7	.01123838	1.4549764	−241.7	.02002851	2.0599297
−246.6	.01138448	1.4670521	−241.6	.02023510	2.0718429
−246.5	.01153179	1.4791358	−241.5	.02044288	2.0837391
−246.4	.01168031	1.4912269	−241.4	.02065185	2.0956176
−246.3	.01183004	1.5033250	−241.3	.02086200	2.1074779
−246.2	.01198098	1.5154297	−241.2	.02107334	2.1193193
−246.1	.01213312	1.5275407	−241.1	.02128587	2.1311413
−246.0	.01228648	1.5396575	−241.0	.02149957	2.1429433
−245.9	.01244106	1.5517797	−240.9	.02171446	2.1547247
−245.8	.01259684	1.5639069	−240.8	.02193052	2.1664849
−245.7	.01275384	1.5760388	−240.7	.02214775	2.1782235
−245.6	.01291205	1.5881748	−240.6	.02236616	2.1899397
−245.5	.01307147	1.6003146	−240.5	.02258574	2.2016331
−245.4	.01323211	1.6124579	−240.4	.02280649	2.2133031
−245.3	.01339396	1.6246042	−240.3	.02302840	2.2249492
−245.2	.01355703	1.6367530	−240.2	.02325147	2.2365708
−245.1	.01372132	1.6489041	−240.1	.02347571	2.2481674
−245.0	.01388681	1.6610569	−240.0	.02370111	2.2597386

IPTS-68 Reference Function and Its First Derivative With the Argument in Celsius—Continued

t °C	$W^*(t)$	$dW^*(t)/dT$ × 1000 °C^{-1}	t °C	$W^*(t)$	$dW^*(t)/dT$ × 1000 °C^{-1}
-240.0	.02370111	2.2597386	-235.0	.03638008	2.7974138
-239.9	.02392766	2.2712837	-234.9	.03666031	2.8072404
-239.8	.02415536	2.2828024	-234.8	.03694152	2.8170283
-239.7	.02438422	2.2942942	-234.7	.03722371	2.8267775
-239.6	.02461422	2.3057584	-234.6	.03750688	2.8364880
-239.5	.02484537	2.3171947	-234.5	.03779101	2.8461597
-239.4	.02507766	2.3286027	-234.4	.03807611	2.8557927
-239.3	.02531109	2.3399819	-234.3	.03836217	2.8653868
-239.2	.02554565	2.3513318	-234.2	.03864919	2.8749421
-239.1	.02578135	2.3626520	-234.1	.03893716	2.8844585
-239.0	.02601818	2.3739421	-234.0	.03922608	2.8939361
-238.9	.02625614	2.3852018	-233.9	.03951594	2.9033747
-238.8	.02649522	2.3964305	-233.8	.03980675	2.9127745
-238.7	.02673543	2.4076280	-233.7	.04009850	2.9221354
-238.6	.02697675	2.4187939	-233.6	.04039118	2.9314574
-238.5	.02721918	2.4299277	-233.5	.04068479	2.9407404
-238.4	.02746273	2.4410292	-233.4	.04097932	2.9499846
-238.3	.02770739	2.4520979	-233.3	.04127478	2.9591898
-238.2	.02795315	2.4631337	-233.2	.04157116	2.9683562
-238.1	.02820001	2.4741362	-233.1	.04186845	2.9774837
-238.0	.02844798	2.4851050	-233.0	.04216665	2.9865723
-237.9	.02869703	2.4960398	-232.9	.04246576	2.9956221
-237.8	.02894718	2.5069405	-232.8	.04276578	3.0046329
-237.7	.02919842	2.5178066	-232.7	.04306669	3.0136050
-237.6	.02945074	2.5286380	-232.6	.04336850	3.0225383
-237.5	.02970415	2.5394344	-232.5	.04367120	3.0314327
-237.4	.02995863	2.5501956	-232.4	.04397478	3.0402884
-237.3	.03021419	2.5609212	-232.3	.04427925	3.0491053
-237.2	.03047081	2.5716111	-232.2	.04458460	3.0578836
-237.1	.03072851	2.5822652	-232.1	.04489083	3.0666231
-237.0	.03098726	2.5928830	-232.0	.04519793	3.0753239
-236.9	.03124708	2.6034646	-231.9	.04550589	3.0839861
-236.8	.03150796	2.6140096	-231.8	.04581472	3.0926096
-236.7	.03176988	2.6245180	-231.7	.04612441	3.1011946
-236.6	.03203286	2.6349895	-231.6	.04643496	3.1097409
-236.5	.03229688	2.6454240	-231.5	.04674636	3.1182488
-236.4	.03256194	2.6558212	-231.4	.04705861	3.1267181
-236.3	.03282804	2.6661812	-231.3	.04737170	3.1351490
-236.2	.03309518	2.6765038	-231.2	.04768564	3.1435415
-236.1	.03336334	2.6867888	-231.1	.04800041	3.1518955
-236.0	.03363253	2.6970360	-231.0	.04831601	3.1602111
-235.9	.03390275	2.7072455	-230.9	.04863245	3.1684884
-235.8	.03417398	2.7174171	-230.8	.04894971	3.1767274
-235.7	.03444623	2.7275506	-230.7	.04926779	3.1849282
-235.6	.03471949	2.7376460	-230.6	.04958669	3.1930907
-235.5	.03499376	2.7477032	-230.5	.04990641	3.2012150
-235.4	.03526903	2.7577222	-230.4	.05022694	3.2093011
-235.3	.03554530	2.7677028	-230.3	.05054827	3.2173491
-235.2	.03582257	2.7776450	-230.2	.05087040	3.2253591
-235.1	.03610083	2.7875486	-230.1	.05119334	3.2333310
-235.0	.03638008	2.7974138	-230.0	.05151707	3.2412648

IPTS-68 Reference Function and Its First Derivative With the Argument in Celsius – Continued

t °C	$W^*(t)$	$dW^*(t)/dT$ × 1000 °C^{-1}	t °C	$W^*(t)$	$dW^*(t)/dT$ × 1000 °C^{-1}
−230.0	.05151707	3.2412648	−225.0	.06863481	3.5905076
−229.9	.05184159	3.2491608	−224.9	.06899417	3.5965674
−229.8	.05216690	3.2570187	−224.8	.06935412	3.6025922
−229.7	.05249299	3.2648389	−224.7	.06971468	3.6085818
−229.6	.05281987	3.2726211	−224.6	.07007584	3.6145364
−229.5	.05314752	3.2803655	−224.5	.07043759	3.6204562
−229.4	.05347594	3.2880722	−224.4	.07079993	3.6263411
−229.3	.05380513	3.2957411	−224.3	.07116286	3.6321912
−229.2	.05413509	3.3033724	−224.2	.07152637	3.6380067
−229.1	.05446580	3.3109660	−224.1	.07189046	3.6437876
−229.0	.05479728	3.3185221	−224.0	.07225512	3.6495340
−228.9	.05512951	3.3260405	−223.9	.07262036	3.6552461
−228.8	.05546248	3.3335215	−223.8	.07298617	3.6609237
−228.7	.05579621	3.3409650	−223.7	.07335255	3.6665672
−228.6	.05613068	3.3483711	−223.6	.07371948	3.6721765
−228.5	.05646588	3.3557398	−223.5	.07408698	3.6777517
−228.4	.05680182	3.3630712	−223.4	.07445503	3.6832930
−228.3	.05713850	3.3703654	−223.3	.07482364	3.6888004
−228.2	.05747589	3.3776222	−223.2	.07519279	3.6942740
−228.1	.05781402	3.3848420	−223.1	.07556249	3.6997139
−228.0	.05815286	3.3920245	−223.0	.07593273	3.7051203
−227.9	.05849242	3.3991700	−222.9	.07630351	3.7104931
−227.8	.05883269	3.4062785	−222.8	.07667483	3.7158326
−227.7	.05917368	3.4133499	−222.7	.07704668	3.7211387
−227.6	.05951536	3.4203845	−222.6	.07741906	3.7264117
−227.5	.05985775	3.4273822	−222.5	.07779196	3.7316515
−227.4	.06020084	3.4343430	−222.4	.07816539	3.7368584
−227.3	.06054462	3.4412671	−222.3	.07853933	3.7420323
−227.2	.06088909	3.4481544	−222.2	.07891379	3.7471734
−227.1	.06123425	3.4550051	−222.1	.07928877	3.7522819
−227.0	.06158009	3.4618192	−222.0	.07966425	3.7573578
−226.9	.06192661	3.4685967	−221.9	.08004024	3.7624011
−226.8	.06227381	3.4753378	−221.8	.08041673	3.7674122
−226.7	.06262168	3.4820424	−221.7	.08079372	3.7723909
−226.6	.06297022	3.4887106	−221.6	.08117120	3.7773375
−226.5	.06331942	3.4953426	−221.5	.08154918	3.7822520
−226.4	.06366928	3.5019382	−221.4	.08192765	3.7871346
−226.3	.06401981	3.5084977	−221.3	.08230661	3.7919854
−226.2	.06437098	3.5150211	−221.2	.08268605	3.7968044
−226.1	.06472281	3.5215084	−221.1	.08306597	3.8015918
−226.0	.06507528	3.5279597	−221.0	.08344637	3.8063478
−225.9	.06542840	3.5343750	−220.9	.08382724	3.8110723
−225.8	.06578216	3.5407545	−220.8	.08420858	3.8157656
−225.7	.06613655	3.5470982	−220.7	.08459039	3.8204278
−225.6	.06649157	3.5534062	−220.6	.08497266	3.8250588
−225.5	.06684723	3.5596785	−220.5	.08535540	3.8296591
−225.4	.06720351	3.5659152	−220.4	.08573860	3.8342284
−225.3	.06756041	3.5721164	−220.3	.08612225	3.8387671
−225.2	.06791793	3.5782821	−220.2	.08650635	3.8432753
−225.1	.06827607	3.5844125	−220.1	.08689090	3.8477530
−225.0	.06863481	3.5905076	−220.0	.08727590	3.8522003

IPTS-68 Reference Function and Its First Derivative With the Argument in Celsius—Continued

t °C	*W**(*t*)	*dW**(*t*)/*dT* × 1000 °C⁻¹	*t* °C	*W**(*t*)	*dW**(*t*)/*dT* × 1000 °C⁻¹
−220.0	.08727590	3.8522003	−215.0	.10703128	4.0386911
−219.9	.08766134	3.8566175	−214.9	.10743530	4.0417606
−219.8	.08804722	3.8610046	−214.8	.10783963	4.0448065
−219.7	.08843354	3.8653617	−214.7	.10824426	4.0478289
−219.6	.08882029	3.8696890	−214.6	.10864919	4.0508280
−219.5	.08920748	3.8739866	−214.5	.10905443	4.0538038
−219.4	.08959509	3.8782546	−214.4	.10945995	4.0567565
−219.3	.08998313	3.8824931	−214.3	.10986578	4.0596863
−219.2	.09037159	3.8867023	−214.2	.11027189	4.0625932
−219.1	.09076046	3.8908823	−214.1	.11067829	4.0654773
−219.0	.09114976	3.8950332	−214.0	.11108498	4.0683390
−218.9	.09153947	3.8991550	−213.9	.11149196	4.0711781
−218.8	.09192959	3.9032481	−213.8	.11189922	4.0739949
−218.7	.09232012	3.9073125	−213.7	.11230676	4.0767895
−218.6	.09271105	3.9113482	−213.6	.11271458	4.0795621
−218.5	.09310239	3.9153555	−213.5	.11312267	4.0823127
−218.4	.09349412	3.9193345	−213.4	.11353104	4.0850415
−218.3	.09388625	3.9232852	−213.3	.11393968	4.0877486
−218.2	.09427878	3.9272080	−213.2	.11434859	4.0904341
−218.1	.09467169	3.9311027	−213.1	.11475776	4.0930982
−218.0	.09506500	3.9349697	−213.0	.11516721	4.0957410
−217.9	.09545869	3 9388089	−212.9	.11557691	4.0983625
−217.8	.09585276	3.9426206	−212.8	.11598688	4.1009631
−217.7	.09624721	3.9464048	−212.7	.11639710	4.1035427
−217.6	.09664204	3.9501618	−212.6	.11680759	4.1061014
−217.5	.09703724	3.9538916	−212.5	.11721832	4.1086395
−217.4	.09743282	3.9575943	−212.4	.11762931	4.1111570
−217.3	.09782876	3.9612702	−212.3	.11804055	4.1136540
−217.2	.09822507	3.9649193	−212.2	.11845204	4.1161308
−217.1	.09862174	3.9685417	−212.1	.11886378	4.1185874
−217.0	.09901878	3.9721376	−212.0	.11927576	4.1210238
−216.9	.09941617	3.9757072	−211.9	.11968798	4.1234404
−216.8	.09981392	3.9792505	−211.8	.12010045	4.1258370
−216.7	.10021202	3.9827677	−211.7	.12051315	4.1282140
−216.6	.10061047	3.9862589	−211.6	.12092609	4.1305714
−216.5	.10100927	3.9897242	−211.5	.12133926	4.1329093
−216.4	.10140841	3.9931638	−211.4	.12175267	4.1352278
−216.3	.10180790	3.9965778	−211.3	.12216631	4.1375271
−216.2	.10220773	3.9999664	−211.2	.12258018	4.1398074
−216.1	.10260789	4.0033296	−211.1	.12299427	4.1420686
−216.0	.10300839	4.0066676	−211.0	.12340859	4.1443109
−215.9	.10340923	4.0099806	−210.9	.12382313	4.1465345
−215.8	.10381039	4.0132686	−210.8	.12423790	4.1487394
−215.7	.10421188	4.0165319	−210.7	.12465288	4.1509258
−215.6	.10461370	4.0197704	−210.6	.12506808	4.1530938
−215.5	.10501583	4.0229845	−210.5	.12548350	4.1552435
−215.4	.10541829	4.0261741	−210.4	.12589913	4.1573750
−215.3	.10582107	4.0293394	−210.3	.12631497	4.1594884
−215.2	.10622416	4.0324806	−210.2	.12673103	4.1615839
−215.1	.10662756	4.0355978	−210.1	.12714729	4.1636615
−215.0	.10703128	4.0386911	−210.0	.12756376	4.1657214

IPTS-68 Reference Function and Its First Derivative With the Argument in Celsius – Continued

t °C	$W^*(t)$	$dW^*(t)/dT$ × 1000 °C^{-1}	t °C	$W^*(t)$	$dW^*(t)/dT$ × 1000 °C^{-1}
−210.0	.12756376	4.1657214	−205.0	.14861484	4.2484572
−209.9	.12798043	4.1677637	−204.9	.14903975	4.2497506
−209.8	.12839731	4.1697885	−204.8	.14946479	4.2510313
−209.7	.12881439	4.1717959	−204.7	.14988995	4.2522996
−209.6	.12923167	4.1737860	−204.6	.15031524	4.2535555
−209.5	.12964914	4.1757590	−204.5	.15074066	4.2547990
−209.4	.13006682	4.1777149	−20 .4	.15116620	4.2560304
−209.3	.13048469	4.1796538	−204.3	.15159187	4.2572496
−209.2	.13090275	4.1815758	−204.2	.15201765	4.2584567
−209.1	.13132100	4.1834812	−204.1	.15244356	4.2596518
−209.0	.13173944	4.1853699	−204.0	.15286958	4.2608350
−208.9	.13215808	4.1872421	−203.9	.15329573	4.2620064
−208.8	.13257689	4.1890979	−203.8	.15372198	4.2631661
−208.7	.13299589	4.1909374	−203.7	.15414836	4.2643141
−208.6	.13341508	4.1927607	−203.6	.15457485	4.2654505
−208.5	.13383445	4.1945679	−203.5	.15500145	4.2665754
−208.4	.13425399	4.1963591	−203.4	.15542816	4.2676889
−208.3	.13467372	4.1981344	−203.3	.15585499	4.2687911
−208.2	.13509362	4.1998939	−203.2	.15628192	4.2698819
−208.1	.13551370	4.2016377	−203.1	.15670896	4.2709616
−208.0	.13593395	4.2033660	−203.0	.15713611	4.2720302
−207.9	.13635437	4.2050788	−202.9	.15756337	4.2730877
−207.8	.13677496	4.2067762	−202.8	.15799073	4.2741343
−207.7	.13719572	4.2084583	−202.7	.15841819	4.2751700
−207.6	.13761665	4.2101253	−202.6	.15884576	4.2761949
−207.5	.13803775	4.2117772	−202.5	.15927343	4.2772091
−207.4	.13845901	4.2134141	−202.4	.15970120	4.2782126
−207.3	.13888043	4.2150361	−202.3	.16012907	4.2792055
−207.2	.13930201	4.2166434	−202.2	.16055704	4.2801879
−207.1	.13972376	4.2182360	−202.1	.16098511	4.2811598
−207.0	.14014566	4.2198140	−202.0	.16141328	4.2821214
−206.9	.14056772	4.2213775	−201.9	.16184154	4.2830727
−206.8	.14098994	4.2229266	−201.8	.16226989	4.2840138
−206.7	.14141231	4.2244615	−201.7	.16269834	4.2849447
−206.6	.14183483	4.2259821	−201.6	.16312688	4.2858655
−206.5	.14225750	4.2274887	−201.5	.16355551	4.2867764
−206.4	.14268032	4.2289812	−201.4	.16398423	4.2876773
−206.3	.14310330	4.2304598	−201.3	.16441305	4.2885683
−206.2	.14352642	4.2319246	−201.2	.16484195	4.2894495
−206.1	.14394968	4.2333757	−201.1	.16527094	4.2903210
−206.0	.14437309	4.2348131	−201.0	.16570001	4.2911828
−205.9	.14479664	4.2362370	−200.9	.16612917	4.2920350
−205.8	.14522034	4.2376474	−200.8	.16655842	4.2928777
−205.7	.14564417	4.2390445	−200.7	.16698775	4.2937109
−205.6	.14606815	4.2404283	−200.6	.16741716	4.2945347
−205.5	.14649226	4.2417989	−200.5	.16784665	4.2953492
−205.4	.14691651	4.2431564	−200.4	.16827623	4.2961544
−205.3	.14734089	4.2445009	−200.3	.16870588	4.2969504
−205.2	.14776541	4.2458324	−200.2	.16913562	4.2977373
−205.1	.14819005	4.2471512	−200.1	.16956543	4.2985150
−205.0	.14861484	4.2484572	−200.0	.16999532	4.2992838

IPTS-68 Reference Function and Its First Derivative With the Argument in Celsius – Continued

t °C	$W^*(t)$	$dW^*(t)/dT$ × 1000 °C^{-1}	t °C	$W^*(t)$	$dW^*(t)/dT$ × 1000 °C^{-1}
200.0	.16999532	4.2992838	−195.0	.19157018	4.3275324
−199.9	.17042529	4.3000436	−194.9	.19200296	4.3279174
−199.8	.17085533	4.3007946	−194.8	.19243577	4.3282962
−199.7	.17128545	4.3015367	−194.7	.19286862	4.3286688
−199.6	.17171564	4.3022701	−194.6	.19330150	4.3290353
−199.5	.17214590	4.3029948	−194.5	.19373442	4.3293958
−199.4	.17257624	4.3037108	−194.4	.19416738	4.3297503
−199.3	.17300664	4.3044183	−194.3	.19460037	4.3300987
−199.2	.17343712	4.3051172	−194.2	.19503340	4.3304413
−199.1	.17386767	4.3058077	−194.1	.19546646	4.3307780
−199.0	.17429828	4.3064898	−194.0	.19589955	4.3311088
−198.9	.17472896	4.3071636	−193.9	.19633268	4.3314337
−198.8	.17515971	4.3078291	−193.8	.19676584	4.3317530
−198.7	.17559053	4.3084864	−193.7	.19719903	4.3320665
−198.6	.17602141	4.3091354	−193.6	.19763225	4.3323744
−198.5	.17645236	4.3097765	−193.5	.19806551	4.3326766
−198.4	.17688336	4.3104094	−193.4	.19849879	4.3329732
−198.3	.17731444	4.3110344	−193.3	.19893210	4.3332643
−198.2	.17774557	4.3116515	−193.2	.19936544	4.3335498
−198.1	.17817677	4.3122607	−193.1	.19979881	4.3338299
−198.0	.17860802	4.3128620	−193.0	.20023221	4.3341046
−197.9	.17903934	4.3134557	−192.9	.20066563	4.3343739
−197.8	.17947071	4.3140415	−192.8	.20109908	4.3346378
−197.7	.17990215	4.3146198	−192.7	.20153256	4.3348964
−197.6	.18033364	4.3151904	−192.6	.20196606	4.3351498
−197.5	.18076519	4.3157535	−192.5	.20239959	4.3353979
−197.4	.18119679	4.3163092	−192.4	.20283314	4.3356408
−197.3	.18162845	4.3168574	−192.3	.20326672	4.3358786
−197.2	.18206016	4.3173982	−192.2	.20370032	4.3361112
−197.1	.18249193	4.3179317	−192.1	.20413394	4.3363388
−197.0	.18292375	4.3184579	−192.0	.20456758	4.3365614
−196.9	.18335562	4.3189768	−191.9	.20500125	4.3367789
−196.8	.18378754	4.3194887	−191.8	.20543494	4.3369915
−196.7	.18421952	4.3199934	−191.7	.20586865	4.3371991
−196.6	.18465154	4.3204910	−191.6	.20630238	4.3374019
−196.5	.18508361	4.3209816	−191.5	.20673613	4.3375999
−196.4	.18551574	4.3214653	−191.4	.20716990	4.3377930
−196.3	.18594791	4.3219420	−191.3	.20760369	4.3379813
−196.2	.18638012	4.3224118	−191.2	.20803750	4.3381649
−196.1	.18681239	4.3228749	−191.1	.20847132	4.3383437
−196.0	.18724470	4.3233312	−191.0	.20890516	4.3385180
−195.9	.18767705	4.3237807	−190.9	.20933902	4.3386875
−195.8	.18810945	4.3242236	−190.8	.20977290	4.3388525
−195.7	.18854190	4.3246599	−190.7	.21020680	4.3390129
−195.6	.18897439	4.3250896	−190.6	.21064070	4.3391688
−195.5	.18940692	4.3255128	−190.5	.21107463	4.3393202
−195.4	.18983949	4.3259294	−190.4	.21150857	4.3394671
−195.3	.19027210	4.3263397	−190.3	.21194252	4.3396095
−195.2	.19070476	4.3267436	−190.2	.21237649	4.3397477
−195.1	.19113745	4.3271412	−190.1	.21281047	4.3398814
−195.0	.19157018	4.3275324	−190.0	.21324447	4.3400108

IPTS-68 Reference Function and Its First Derivative With the Argument in Celsius – Continued

t °C	$W^*(t)$	$dW^*(t)/dT$ × 1000 °C^{-1}	t °C	$W^*(t)$	$dW^*(t)/dT$ × 1000 °C^{-1}
−190.0	.21324447	4.3400108	−185.0	.23495228	4.3416391
−189.9	.21367847	4.3401359	−184.9	.23538645	4.3415872
−189.8	.21411249	4.3402567	−184.8	.23582060	4.3415325
−189.7	.21454652	4.3403733	−184.7	.23625475	4.3414750
−189.6	.21498057	4.3404857	−184.6	.23668890	4.3414145
−189.5	.21541462	4.3405940	−184.5	.23712303	4.3413514
−189.4	.21584869	4.3406981	−184.4	.23755717	4.3412854
−189.3	.21628276	4.3407981	−184.3	.23799129	4.3412168
−189.2	.21671685	4.3408941	−184.2	.23842541	4.3411453
−189.1	.21715094	4.3409860	−184.1	.23885952	4.3410712
−189.0	.21758504	4.3410739	−184.0	.23929362	4.3409944
−188.9	.21801915	4.3411579	−183.9	.23972772	4.3409150
−188.8	.21845327	4.3412378	−183.8	.24016181	4.3408329
−188.7	.21888740	4.3413139	−183.7	.24059589	4.3407482
−188.6	.21932154	4.3413861	−183.6	.24102996	4.3406609
−188.5	.21975568	4.3414544	−183.5	.24146402	4.3405710
−188.4	.22018983	4.3415189	−183.4	.24189807	4.3404786
−188.3	.22062398	4.3415796	−183.3	.24233211	4.3403836
−188.2	.22105814	4.3416366	−183.2	.24276615	4.3402862
−188.1	.22149231	4.3416898	−183.1	.24320017	4.3401862
−188.0	.22192648	4.3417393	−183.0	.24363418	4.3400839
−187.9	.22236066	4.3417852	−182.9	.24406819	4.3399790
−187.8	.22279484	4.3418274	−182.8	.24450218	4.3398718
−187.7	.22322902	4.3418659	−182.7	.24493616	4.3397620
−187.6	.22366321	4.3419009	−182.6	.24537013	4.3396500
−187.5	.22409740	4.3419323	−182.5	.24580409	4.3395356
−187.4	.22453160	4.3419602	−182.4	.24623804	4.3394188
−187.3	.22496579	4.3419846	−182.3	.24667198	4.3392998
−187.2	.22539999	4.3420055	−182.2	.24710590	4.3391784
−187.1	.22583420	4.3420229	−182.1	.24753981	4.3390548
−187.0	.22626840	4.3420369	−182.0	.24797371	4.3389288
−186.9	.22670260	4.3420476	−181.9	.24840760	4.3388007
−186.8	.22713681	4.3420548	−181.8	.24884147	4.3386703
−186.7	.22757101	4.3420587	−181.7	.24927533	4.3385378
−186.6	.22800522	4.3420593	−181.6	.24970918	4.3384029
−186.5	.22843943	4.3420566	−181.5	.25014301	4.3382660
−186.4	.22887363	4.3420507	−181.4	.25057683	4.3381270
−186.3	.22930784	4.3420415	−181.3	.25101064	4.3379858
−186.2	.22974204	4.3420291	−181.2	.25144443	4.3378425
−186.1	.23017624	4.3420135	−181.1	.25187820	4.3376971
−186.0	.23061044	4.3419948	−181.0	.25231197	4.3375496
−185.9	.23104464	4.3419729	−180.9	.25274571	4.3374001
−185.8	.23147884	4.3419479	−180.8	.25317945	4.3372485
−185.7	.23191303	4.3419198	−180.7	.25361316	4.3370949
−185.6	.23234722	4.3418887	−180.6	.25404687	4.3369394
−185.5	.23278141	4.3418545	−180.5	.25448055	4.3367818
−185.4	.23321559	4.3418173	−180.4	.25491422	4.3366223
−185.3	.23364977	4.3417772	−180.3	.25534788	4.3364608
−185.2	.23408395	4.3417341	−180.2	.25578151	4.3362975
−185.1	.23451812	4.3416881	−180.1	.25621514	4.3361321
−185.0	.23495228	4.3416391	−180.0	.25664874	4.3359649

IPTS-68 Reference Function and Its First Derivative With the Argument in Celsius—Continued

t °C	$W^*(t)$	$dW^*(t)/dT$ × 1000 °C^{-1}	t °C	$W^*(t)$	$dW^*(t)/dT$ × 1000 °C^{-1}
−180.0	.25664874	4.3359649	−175.0	.27830405	4.3255391
−179.9	.25708233	4.3357959	−174.9	.27873660	4.3252957
−179.8	.25751590	4.3356249	−174.8	.27916911	4.3250511
−179.7	.25794945	4.3354522	−174.7	.27960161	4.3248053
−179.6	.25838299	4.3352775	−174.6	.28003407	4.3245585
−179.5	.25881651	4.3351011	−174.5	.28046652	4.3243105
−179.4	.25925001	4.3349229	−174.4	.28089894	4.3240615
−179.3	.25968349	4.3347430	−174.3	.28133133	4.3238114
−179.2	.26011696	4.3345612	−174.2	.28176370	4.3235602
−179.1	.26055041	4.3343777	−174.1	.28219604	4.3233079
−179.0	.26098383	4.3341925	−174.0	.28262836	4.3230546
−178.9	.26141724	4.3340056	−173.9	.28306065	4.3228003
−178.8	.26185064	4.3338169	−173.8	.28349292	4.3225448
−178.7	.26228401	4.3336267	−173.7	.28392516	4.3222885
−178.6	.26271736	4.3334347	−173.6	.28435738	4.3220311
−178.5	.26315069	4.3332411	−173.5	.28478957	4.3217727
−178.4	.26358401	4.3330459	−173.4	.28522173	4.3215133
−178.3	.26401730	4.3328489	−173.3	.28565387	4.3212530
−178.2	.26445058	4.3326505	−173.2	.28608598	4.3209917
−178.1	.26488383	4.3324505	−173.1	.28651807	4.3207295
−178.0	.26531707	4.3322488	−173.0	.28695013	4.3204663
−177.9	.26575028	4.3320456	−172.9	.28738216	4.3202022
−177.8	.26618348	4.3318409	−172.8	.28781417	4.3199371
−177.7	.26661665	4.3316346	−172.7	.28824615	4.3196712
−177.6	.26704980	4.3314269	−172.6	.28867810	4.3194044
−177.5	.26748294	4.3312176	−172.5	.28911003	4.3191366
−177.4	.26791605	4.3310068	−172.4	.28954193	4.3188680
−177.3	.26834914	4.3307946	−172.3	.28997380	4.3185986
−177.2	.26878221	4.3305809	−172.2	.29040565	4.3183283
−177.1	.26921525	4.3303658	−172.1	.29083747	4.3180571
−177.0	.26964828	4.3301492	−172.0	.29126926	4.3177851
−176.9	.27008128	4.3299313	−171.9	.29170103	4.3175123
−176.8	.27051427	4.3297119	−171.8	.29213276	4.3172386
−176.7	.27094723	4.3294912	−171.7	.29256447	4.3169642
−176.6	.27138016	4.3292691	−171.6	.29299616	4.3166889
−176.5	.27181308	4.3290455	−171.5	.29342781	4.3164129
−176.4	.27224597	4.3288207	−171.4	.29385944	4.3161361
−176.3	.27267884	4.3285945	−171.3	.29429104	4.3158585
−176.2	.27311169	4.3283671	−171.2	.29472261	4.3155801
−176.1	.27354452	4.3281383	−171.1	.29515416	4.3153010
−176.0	.27397732	4.3279082	−171.0	.29558567	4.3150212
−175.9	.27441010	4.3276768	−170.9	.29601716	4.3147407
−175.8	.27484286	4.3274442	−170.8	.29644862	4.3144593
−175.7	.27527559	4.3272103	−170.7	.29688005	4.3141774
−175.6	.27570830	4.3269752	−170.6	.29731146	4.3138947
−175.5	.27614098	4.3267389	−170.5	.29774283	4.3136113
−175.4	.27657365	4.3265013	−170.4	.29817418	4.3133271
−175.3	.27700628	4.3262625	−170.3	.29860550	4.3130424
−175.2	.27743890	4.3260226	−170.2	.29903679	4.3127569
−175.1	.27787149	4.3257814	−170.1	.29946805	4.3124708
−175.0	.27830405	4.3255391	−170.0	.29989928	4.3121841

IPTS-68 Reference Function and Its First Derivative With the Argument in Celsius—Continued

t °C	$W^*(t)$	$dW^*(t)/dT$ × 1000 °C^{-1}	t °C	$W^*(t)$	$dW^*(t)/dT$ × 1000 °C^{-1}
−170.0	.29989928	4.3121841	−120.0	.51175723	4.1696979
−169.0	.30421002	4.3092826	−119.0	.51592580	4.1674503
−168.0	.30851783	4.3063249	−118.0	.52009214	4.1652272
−167.0	.31282265	4.3033185	−117.0	.52425626	4.1630283
−166.0	.31712445	4.3002706	−116.0	.52841820	4.1608530
−165.0	.32142318	4.2971877	−115.0	.53257798	4.1587009
−164.0	.32571882	4.2940758	−114.0	.53673561	4.1565715
−163.0	.33001133	4.2909406	−113.0	.54089113	4.1544643
−162.0	.33430069	4.2877872	−112.0	.54504455	4.1523789
−161.0	.33858690	4.2846206	−111.0	.54919589	4.1503148
−160.0	.34286993	4.2814449	−110.0	.55334518	4.1482715
−159.0	.34714978	4.2782645	−109.0	.55749244	4.1462487
−158.0	.35142646	4.2750828	−108.0	.56163769	4.1442458
−157.0	.35569995	4.2719035	−107.0	.56578094	4.1422623
−156.0	.35997027	4.2687295	−106.0	.56992222	4.1402979
−155.0	.36423741	4.2655639	−105.0	.57406154	4.1383521
−154.0	.36850140	4.2624092	−104.0	.57819893	4.1364243
−153.0	.37276224	4.2592676	−103.0	.58233440	4.1345144
−152.0	.37701994	4.2561416	−102.0	.58646796	4.1326217
−151.0	.38127452	4.2530330	−101.0	.59059965	4.1307459
−150.0	.38552601	4.2499434	−100.0	.59472946	4.1288866
−149.0	.38977442	4.2468745	−99.0	.59885742	4.1270433
−148.0	.39401977	4.2438278	−98.0	.60298355	4.1252157
−147.0	.39826208	4.2408044	−97.0	.60710786	4.1234034
−146.0	.40250138	4.2378054	−96.0	.61123036	4.1216060
−145.0	.40673770	4.2348319	−95.0	.61535108	4.1198231
−144.0	.41097106	4.2318846	−94.0	.61947002	4.1180544
−143.0	.41520148	4.2289643	−93.0	.62358719	4.1162996
−142.0	.41942899	4.2260716	−92.0	.62770262	4.1145582
−141.0	.42365363	4.2232070	−91.0	.63181631	4.1128299
−140.0	.42787542	4.2203709	−90.0	.63592828	4.1111145
−139.0	.43209438	4.2175636	−89.0	.64003855	4.1094117
−138.0	.43631056	4.2147855	−88.0	.64414711	4.1077210
−137.0	.44052396	4.2120366	−87.0	.64825399	4.1060422
−136.0	.44473464	4.2093170	−86.0	.65235920	4.1043750
−135.0	.44894261	4.2066270	−85.0	.65646275	4.1027192
−134.0	.45314790	4.2039663	−84.0	.66056464	4.1010745
−133.0	.45735055	4.2013350	−83.0	.66466490	4.0994405
−132.0	.46155058	4.1987329	−82.0	.66876353	4.0978171
−131.0	.46574803	4.1961599	−81.0	.67286054	4.0962040
−130.0	.46994291	4.1936158	−80.0	.67695594	4.0946010
−129.0	.47413527	4.1911002	−79.0	.68104974	4.0930078
−128.0	.47832512	4.1886132	−78.0	.68514196	4.0914242
−127.0	.48251250	4.1861542	−77.0	.68923259	4.0898500
−126.0	.48669744	4.1837229	−76.0	.69332166	4.0882850
−125.0	.49087996	4.1813191	−75.0	.69740917	4.0867290
−124.0	.49506009	4.1789423	−74.0	.70149512	4.0851817
−123.0	.49923785	4.1765922	−73.0	.70557953	4.0836431
−122.0	.50341328	4.1742684	−72.0	.70966241	4.0821129
−121.0	.50758640	4.1719704	−71.0	.71374376	4.0805910
−120.0	.51175723	4.1696 79	−70.0	.71782359	4.0790772

IPTS-68 Reference Function and Its First Derivative With the Argument in Celsius—Continued

t °C	$W^*(t)$	$dW^*(t)/dT$ × 1000 °C^{-1}	t °C	$W^*(t)$	$dW^*(t)/dT$ × 1000 °C^{-1}
−70.0	.71782359	4.0790772	−20.0	.92002439	4.0111301
−69.0	.72190192	4.0775712	−19.0	.92403489	4.0098834
−68.0	.72597874	4.0760731	−18.0	.92804416	4.0086393
−67.0	.73005407	4.0745825	−17.0	.93205217	4.0073976
−66.0	.73412791	4.0730995	−16.0	.93605895	4.0061582
−65.0	.73820027	4.0716238	−15.0	.94006449	4.0049209
−64.0	.74227116	4.0701553	−14.0	.94406879	4.0036856
−63.0	.74634058	4.0686939	−13.0	.94807186	4.0024522
−62.0	.75040855	4.0672394	−12.0	.95207370	4. 012205
−61.0	.75447506	4.0657918	−11.0	.95607430	3.9999902
−60.0	.75854013	4.0643509	−10.0	.96007368	3.9987614
−59.0	.76260377	4.0629166	−9.0	.96407183	3.9975337
−58.0	.76666597	4.0614887	−8.0	.96806875	3.9963070
−57.0	.77072675	4.0600673	−7.0	.97206444	3.9950811
−56.0	.77478611	4.0586521	−6.0	.97605891	3.9938559
−55.0	.77884405	4.0572432	−5.0	.98005215	3.9926310
−54.0	.78290059	4.0558403	−4.0	.98404417	3.9914065
−53.0	.78695573	4.0544434	−3.0	.98803497	3.9901819
−52.0	.79100948	4.0530523	−2.0	.99202454	3.9889571
−51.0	.79506184	4.0516672	−1.0	.99601288	3.9877321
−50.0	.79911282	4.0502877	.0	1.00000000	3.9865063
−49.0	.80316242	4.0489138			
−48.0	.80721065	4.0475455			
−47.0	.81125751	4.0461827			
−46.0	.81530302	4.0448252			
−45.0	.81934716	4.0434730			
−44.0	.82338996	4.0421259			
−43.0	.82743142	4.0407841			
−42.0	.83147153	4.0394472			
−41.0	.83551031	4.0381154			
−40.0	.83954777	4.0367884			
−39.0	.84358389	4.0354661			
−38.0	.84761870	4.0341486			
−37.0	.85165219	4.0328357			
−36.0	.85568437	4.0315273			
−35.0	.85971525	4.0302233			
−34.0	.86374482	4.0289237			
−33.0	.86777310	4.0276283			
−32.0	.87180008	4.0263371			
−31.0	.87582577	4.0250500			
−30.0	.87985018	4.0237668			
−29.0	.88387331	4.0224875			
−28.0	.88789516	4.0212120			
−27.0	.89191573	4.0199401			
−26.0	.89593504	4.0186718			
−25.0	.89995308	4.0174069			
−24.0	.90396985	4.0161454			
−23.0	.90798537	4.0148870			
−22.0	.91199963	4.0136318			
−21.0	.91601263	4.0123795			
−20.0	.92002439	4.0111301			

APPENDIX F. Analysis of the First Derivatives at 0 °C of IPTS–68 Platinum Resistance Thermometer Formulations Above and Below 0 °C

The four specified deviation functions of the IPTS–68 in the interval 13.81 K to 273.15 K are formulated to join smoothly by constraining the values of the first derivatives of the formulations above and below each point of joining to be identical at this point (see the IPTS–68 document). However, the values of the first derivatives of the formulations above and below 0 °C of real SPRT's are not constrained to be equal at 0 °C. This discussion will show that the magnitude of this difference is negligible for SPRT's that have suitable thermometer constants. All SPRT's received at the NBS for calibration in the past few years had suitable constants (see the last page of this Appendix).

The IPTS–68 reference function in Celsius

$$t = \sum_{i=1}^{20} A_i[\ln W^*(t)]^i \text{ °C} \tag{1}$$

was formulated so that its first and second derivatives would have the same value at 0 °C as obtained from

$$t = t' + 0.045\left(\frac{t'}{s}\right)\left(\frac{t'}{s}-1\right)\left(\frac{t'}{z}-1\right)\left(\frac{t'}{a}-1\right) \text{ °C} \tag{2}$$

and

$$t' = \frac{1}{\alpha}[W(t')-1] + \delta\left(\frac{t'}{s}\right)\left(\frac{t'}{s}-1\right) \text{ °C} \tag{3}$$

with thermometer constants

$$\alpha = 3.9259668 \times 10^{-3}/\text{ °C}$$

and

$$\delta = 1.496334 \text{ °C}.$$

(The above selected values of α and δ given in the text of the IPTS–68 will, henceforth, be referred to as α^* and δ^*, respectively. In eqs (1), (2), and (3), $W^*(t) = W_{\text{CCT-68}}(t_{68})$ and $t = t_{68}$ (for convenience), $s = 100$ °C, $z = 419.58$ °C, $a = 630.74$ °C, and $W(t') = W(t) = R(t)/R(0)$.) The second term of eq (2) represents a "correction" to be made to the temperature value t' to obtain a temperature value t that is a closer approximation to the thermodynamic scale. The values t' and t represent the same temperature (hotness). For real SPRT's the derivative of the formulation between −182.962 °C and 0 °C is obtained from the definition of the IPTS–68 in this temperature range

$$W(t) = W^*(t) + \Delta W(t) \tag{4}$$

and the deviation function

$$\Delta W(t) = A_4 t + C_4 t^3 (t-100). \tag{5}$$

The constants of eq (5) are determined from the deviations at the oxygen point and the steam point. When differentiated with respect to t, there results from eqs (4) and (5), respectively,

$$\frac{dW(t)}{dt} = \frac{dW^*(t)}{dt} + \frac{d\Delta W(t)}{dt} \tag{6}$$

and

$$\frac{d\Delta W(t)}{dt} = A_4 + 4C_4 t^2 (t-75). \tag{7}$$

At 0 °C, the reference function (eq (1)) on differentiation yields

$$\left(\frac{dW^*(t)}{dt}\right)_0 = \frac{1}{A_1} \tag{8}$$

and eq (7) yields

$$\left(\frac{d\Delta W(t)}{dt}\right)_0 = A_4. \tag{9}$$

When eqs (6), (8), and (9) are combined, there results

$$\left(\frac{dW(t)}{dt}\right)_{\text{OB}} = \frac{1}{A_1} + A_4, \tag{10}$$

where the subscript OB refers to the formulation below 0 °C. The constant A_4 is obtained from the value of $W(100\text{ °C})$, i.e., from

$$W(100\text{ °C}) = 100\,\alpha + 1 \tag{11}$$

and

$$W^*(100\text{ °C}) = 100\,\alpha^* + 1. \tag{12}$$

The value of α in eq (11) can be obtained from measurements at the triple point of water and the steam point or at the triple point of water, tin point, and zinc point $(\alpha = [R(100) - R\ 0)/100R(0))$. Combining equations (11) and (12) with eq (5) ($t = 100$ °C) there results

$$A_4 = \alpha - \alpha^*. \tag{13}$$

When eq (10) and (13) are combined,

$$\left(\frac{dW(t)}{dt}\right)_{\text{OB}} = \frac{1}{A_1} + \alpha - \alpha^*. \tag{14}$$

The derivative of the formulation between 0 and 630.74 °C is obtained by differentiating eq (2) and the equation

$$W(t')=1+At'+Bt'^2; \tag{15}$$

there results, respectively,

$$\frac{dt}{dt'}=1+\frac{0.045}{s^2 za}\,[4t'^3-3(s+z+a)t'^2 +2(sz+sa+za)t'-sza], \tag{16}$$

and

$$\frac{dW(t)}{dt'}=A+2Bt'. \tag{17}$$

Equation (15) is equivalent to eq (3) with the thermometer constants $A=\alpha(1+\delta/100\ ^\circ\mathrm{C})$ and $B=-10^{-4}\ \alpha\delta\ ^\circ\mathrm{C}^{-2}$. At $t=0\ ^\circ\mathrm{C}$, $t'=0\ ^\circ\mathrm{C}$, and eqs (16) and (17) become respectively,

$$\left(\frac{dt}{dt'}\right)_0=1-\frac{0.045}{100}=0.99955 \tag{18}$$

and

$$\left(\frac{dW(t)}{dt'}\right)_0=A. \tag{19}$$

Combining eqs (18) and (19),

$$\left(\frac{dW(t)}{dt}\right)_{0A}=\left(\frac{dW(t)}{dt'}\right)_0\left(\frac{dt'}{dt}\right)_0=\frac{A}{0.99955}, \tag{20}$$

where the subscript OA refers to the formulation above 0 °C.

By subtracting eq (20) from eq (14), the difference in the derivatives at 0 °C of the two formulations becomes

$$\left(\frac{dW(t)}{dt}\right)_{0B}-\left(\frac{dW(t)}{dt}\right)_{0A}=\Delta\left(\frac{dW(t)}{dt}\right)_0=\frac{1}{A_1} +\alpha-\alpha^*-\frac{A}{0.99955}. \tag{21}$$

From the relations

$$A=\alpha(1+\delta/100\,^\circ\mathrm{C}) \tag{22}$$

and

$$B=-10^{-4}\,\alpha\delta\,^\circ\mathrm{C}^{-2} \tag{23}$$

A^* and B^* are obtained by substituting α^* and δ^* for α and δ, respectively. By substituting A^* and B^* in eq (15), differentiating, and combining with eq (16), there results

$$\left(\frac{dW^*(t)}{dt}\right)_{0A}=\frac{A^*}{0.99955}, \tag{24}$$

an expression that is very similar to eq (20). When eq (24) is compared with eq (8) and because the two derivatives are equal,

$$\frac{1}{A_1}=\frac{A^*}{0.99955}. \tag{25}$$

By substituting eq (25) for $1/A_1$ in eq (21), there results

$$\Delta\left(\frac{dW(t)}{dt}\right)_0=\alpha-\alpha^*+\frac{A^*-A}{0.99955}. \tag{26}$$

Although $\Delta\left(\frac{dW(t)}{dt}\right)_0$ can be evaluated from eq (26), the conversion of A^* and A in terms of B^* and B results in an equation that is simpler to evaluate. The substitution of the expression (22) for A^* and A and then of the expression (23) for the $\alpha\delta$ product results in

$$\Delta\left(\frac{dW(t)}{dt}\right)_0=\frac{0.00045(\alpha^*-\alpha)-100(B^*-B)}{0.99955}. \tag{27}$$

The α values of SPRT's calibrated at NBS in the past few years range between 3.925 and about $3.927\times10^{-3}\ ^\circ\mathrm{C}^{-1}$ and the B between -5.872 and $-5.877\times10^{-7}\ ^\circ\mathrm{C}^{-2}$; both α^* and B^* are about the average of the above limits. Therefore, $\alpha^*-\alpha$ is about $\pm1\times10^{-6}\,^\circ\mathrm{C}^{-1}$ and $B-B^*$ is about $\pm2.5\times10^{-10}\,^\circ\mathrm{C}^{-2}$. (Note that "100" in eq (27) has the unit °C associated with it.) When these values are substituted in eq (27)

$$\Delta\left(\frac{dW(t)}{dt}\right)_0\sim3\times10^{-8}\,^\circ\mathrm{C}^{-1}. \tag{28}$$

This may be taken as the maximum expected discontinuity. Any variation of this discontinuity is mostly dependent on the constant B. Inasmuch as $(dW(t)/dt)_0$ is about $4\times10^{-3}\ ^\circ\mathrm{C}^{-1}$, the relative discontinuity in the first derivatives of the two formulations at 0 °C is expected to be less than 8×10^{-6}. The discontinuity is negligible for most purposes.

APPENDIX G. Derivation of Differential Coefficients for the Analysis of Errors in Platinum Resistance Thermometry

This appendix deals with the propagation of errors in the temperature determination that results from the errors of calibration of a SPRT. The differential coefficients (the rate of change of the values of temperature with respect to the change of the resistance ratios, $R(t)/R(0)$, measured for calibration) are derived for the calibration measurements at the steam point, tin point, zinc point, and oxygen normal boiling point (NBP).

The total error, in the value of temperature that is obtained from measurements employing a SPRT, is the sum of the error introduced by the calibration measurements and the error from the experimenter's own measurements. The experimenter must determine his own observational error from a careful evaluation of his measurement techniques. He must also know the errors that may be introduced in the values of temperature caused by possible errors in the thermometer calibration measurements at the NBS. He must also be aware of possible errors from an unknown change in the calibration.

The variation among the SPRT's is yet another source of uncertainty; i.e., even though the measurements with the SPRT's are made at the same temperature (hotness), they do not yield exactly the same values of temperature. A number of scattered data do exist [1, 2, 3, 4], but there are no systematic high-precision measurements on the intercomparison of SPRT's that employ modern SPRT's and measuring equipment.

The errors of calibration at NBS may be separated into two types: (1) errors of deviation from realizing the fixed points and (2) errors of resistance measurements. The total differential of t for the SPRT is given by

$$dt = \sum \left(\frac{\partial t}{\partial t_i}\right) dt_i + \left(\frac{\partial t}{\partial W_i}\right) dW_i, \tag{1}$$

where t_i = temperature of the defining fixed points, i.e., for this discussion the oxygen NBP, the triple point of water, the steam point or the tin point, and the zinc point. The W_i refers to the measured resistance ratios $R(t_i)/R(0)$ corresponding to the fixed points. The dt_i or δt_i refers to a variation in the fixed-point temperature (hotness) experimentally realized, e.g., the deviation of the temperature of a tin-point cell. The differential coefficient $\left(\frac{\partial t}{\partial t_i}\right)$ represents the rate of change in the value of temperature with respect to a change in the ith fixed-point temperature (hotness); the coefficients may be derived from the prescribed interpolation formulae using the values of defining fixed temperatures. The differential coefficients are temperature dependent, i.e., $\left(\frac{\partial t}{\partial t_i}\right) = f(t)$.

The dW_i or δW_i refers to a variation in the resistance ratio measurement, e.g., the error of the resistance ratio measurement process with the SPRT at NBS. The differential coefficient $\left(\frac{\partial t}{\partial W_i}\right)$ represents the rate of change in the value of temperature with respect to a change in the measured resistance ratio at the ith fixed point; the coefficients are derived from the prescribed formulae that relate the measured resistance ratio to the constants of the SPRT. Similar to $\left(\frac{\partial t}{\partial t_i}\right)$, $\left(\frac{\partial t}{\partial W_i}\right)$ is also temperature dependent.

Some preliminary and limited investigations at NBS on the realization of the tin point and the triple point of water indicate that these fixed points are reproducible within ±0.1 mK. This value, although obtained employing the same thermometer over a short interval of a few days to determine the error attributable to the variations in the temperatures of the fixed-point cells, includes both the variations in the temperature (δt_i) of many freezes in different cells and the variations in the measurement process (δW_i). Over a period of several months the reproducibility of measurements on different SPRT's employing the same cell is estimated to be about ±0.2 mK. This value likewise includes any variations in the temperature of the cell as well as the variations in the measurement process. Thus, the errors of the realization of the fixed-point temperatures and the errors of calibration resistance measurements are difficult to separate. Therefore, the analysis of error propagation has been simplified by combining the two sources of errors in calibration measurements. Equation (1) then simplifies to

$$dt = \sum \left(\frac{\partial t}{\partial W_i}\right) dW_i. \tag{2}$$

In the expression the change or error δt_i in the value of temperature has been replaced by a corresponding change or error δW_i. At present, the total uncertainty (which includes δt_i and δW_i of eq (1)) of the calibration measurements is estimated to be 2 or 3 mK at the oxygen NBP, and 1 mK at the tin point, and 1 mK at the zinc point.

The differential coefficients $\left(\frac{\partial t}{\partial W_i}\right)$ have been derived separately for two ranges of temperature defined by the SPRT, the range 0 to 630.74 °C and the range −182.962 (90.188 K) to 0 °C (273.15 K). In the first range (0 to 630.74 °C), the coefficients have been derived for two types of calibration measurements: (1) at the triple point of water, the steam point, and the zinc point and (2) at the triple point of water, the tin point, and the zinc point. In the second range, the differential coefficients have also been derived for two types of calibration measurements: (1) at the oxygen NBP, the triple point of water, and the steam point and (2) at the oxygen NBP, the triple point of water, the tin point, and the zinc point.

The error δt in the value of temperature, that would arise from an error δW_i corresponding to a positive unit temperature (hotness), was calculated as a function of the temperature employing the derived differential coefficients; i.e., the function

$$\delta t = \left(\frac{\partial t}{\partial W_i}\right)\delta W_i \qquad (3)$$

was evaluated as a function of the temperature. When the calibration error at a particular fixed point was considered, the calibration measurements at the other fixed points were taken to be correct. The following cases have been calculated and plotted in figure 17.

(1) *The range 0 °C to 630.74 °C*

(a) calibration measurements made at the triple point of water, the steam point, and the zinc point; calibration error occurring only at the steam point or only at the zinc point,

(b) calibration measurements made at the triple point of water, the tin point, and the zinc point; calibration error occurring only at the tin point or only at the zinc point.

(2) *The range −182.962 °C (90.188 K) to 0 °C (273.15 K)*

(a) calibration measurements made at the oxygen NBP, the triple point of water, and steam point; calibration error occurring only at the oxygen NBP or only at the steam point,

(b) calibration measurements made at the oxygen NBP, the triple point of water, the tin point, and the zinc point; calibration error occurring only at the oxygen NBP, or only at the tin point, or only at the zinc point.

In the range −182.962 to 0 °C, the calibration errors at the oxygen NBP for the cases 2a and 2b yield the same temperature error δt; therefore, the analysis of only one of these cases is given.

As a separate analysis, the case of an error in the realization of the triple point of water by the experimenter has been calculated; the calibration of the SPRT at NBS was considered perfect and an error of resistance measurement δR corresponding to a positive unit temperature at the triple point of water was considered to have been made by the experimenter. The error δR in the triple point of water measurements was taken to be propagated in the determination of the resistance ratio $R(t)/R(0)$. The results of the analysis are also plotted in figure 17.

The following sections deal with the details of the derivation of the differential coefficients that were employed to avaluate the temperature error function given by eq (3) for the cases cited in the previous paragraphs. In the discussion, the variations in the measurements at the fixed points will be described in terms of the resistance ratio, $W(t_i) = R(t_i)/R(0)$, the ratio of observed resistance at the fixed-point temperature t_i to that at 0 °C. The variations in $W(t_i)$ will be taken to be centered in $R(t_i)$. Obviously, any error in $R(0)$ will be reflected in $W(t_i)$; thus, the present discussion is also applicable to possible errors in the determination of $R(0)$.

1. *0 °C to 630.74 °C*

From 0 to 630.74 °C, the temperature t on the IPTS−68 is defined by

$$t = t' + 0.045\left(\frac{t'}{s}\right)\left(\frac{t'}{s}-1\right)\left(\frac{t'}{z}-1\right)\left(\frac{t'}{a}-1\right)\,^{\circ}\mathrm{C} \qquad (4)$$

and

$$t' = \frac{1}{\alpha}\left(W(t')-1\right) + \delta\left(\frac{t'}{s}\right)\left(\frac{t'}{s}-1\right)\,^{\circ}\mathrm{C}, \qquad (5)$$

where

$$W(t_i') = W(t_i) = R(t_i)/R(0) \qquad (6)$$

$s = 100\,^{\circ}\mathrm{C}$, $z = 419.58\,^{\circ}\mathrm{C}$, $a = 630.74\,^{\circ}\mathrm{C}$, and $t = t_{68}$ (for convenience). The second term on the right side of eq (4) will, henceforth, be defined as

$$M(t') = 0.045\left(\frac{t'}{s}\right)\left(\frac{t'}{s}-1\right)\left(\frac{t'}{z}-1\right)\left(\frac{t'}{a}-1\right)\,^{\circ}\mathrm{C}. \qquad (7)$$

The function $M(t')$ represents a correction to t' to obtain the temperature t that is a closer approximation to the thermodynamic scale. The values t_i' and t_i, therefore, represent the same temperature (hotness) and $R(t_i') = R(t_i)$. Also, eq (5) is equivalent to

$$W(t') = 1 + At' + Bt'^2, \qquad (8)$$

where

$$A = \alpha(1 + \delta/100\,^{\circ}\mathrm{C}) \qquad (9)$$

and

$$B = -10^{-4}\alpha\delta\,^{\circ}\mathrm{C}^{-2} \qquad (10)$$

The constants of the IPTS−68 eqs (5) and (8) are determined by measurement of the SPRT resistance at the triple point of water, the steam point, and the zinc point or the triple point of water, the tin point, and the zinc point. To simplify the symbols and terminology, the following definitions are made:

$$\omega(t') = W(t') - 1 = At' + Bt'^2, \tag{11}$$

i.e.,

$$\omega_S = W_S - 1 = [R(100)/R(0)] - 1 \tag{12}$$

$$\omega_T = W_T - 1 = [R(231.9292)/R(0)] - 1, \tag{13}$$

and

$$\omega_Z = W_Z - 1 = [R(419.58)/R(0)] - 1. \tag{14}$$

The differential coefficients will be derived first for the SPRT formulation with calibration measurements at the triple point of water, the tin point, and the zinc point. The differential coefficients sought are

$$\left(\frac{\partial t}{\partial \omega_T}\right) \text{ and } \left(\frac{\partial t}{\partial \omega_Z}\right).$$

Because ω is a function of t', the conversion

$$\frac{\partial t}{\partial \omega} = \left(\frac{\partial t}{\partial t'}\right)\left(\frac{\partial t'}{\partial \omega}\right) \tag{15}$$

must be made. From eq (4),

$$\frac{\partial t}{\partial t'} = 1 + \frac{0.045}{s^2 za}\,[4t'^3 - 3(s+z+a)t'^2 + 2(sz+sa+za)t' - sza]$$

$$= 1 + \frac{dM(t')}{dt'} \tag{16}$$

Equation (16) will be employed to convert $\partial t'/\partial\omega$ to $\partial t/\partial\omega$.

From eq (11),

$$\omega_T = At'_T + Bt'^2_T \tag{17}$$

and

$$\omega_Z = At'_Z + Bt'^2_Z, \tag{18}$$

where t'_T is the defined tin point temperature and t'_Z is the defined zinc point temperature. (According to the definition of the IPTS–68, $t'_T = t_{68_T} - 0.038937$ and $t'_Z = t_{68_Z}$; henceforth, the "prime" symbol will not be employed in the designation of a fixed-point temperature.) Solving for A and B, there results

$$A = (\omega_T t_Z^2 - \omega_Z t_T^2)/t_T t_Z(t_Z - t_T), \tag{19}$$

and

$$B = (\omega_Z t_T - \omega_T t_Z)/t_T t_Z(t_Z - t_T). \tag{20}$$

Substituting in eq (11) for A and B, there is obtained

$$\omega(t') = \left[\frac{(\omega_T t_Z^2 - \omega_Z t_T^2)}{t_T t_Z(t_Z - t_T)}\right] t' + \left[\frac{(\omega_Z t_T - \omega_T t_Z)}{t_T t_Z(t_Z - t_T)}\right] t'^2. \tag{21}$$

The differential coefficients $\left(\frac{\partial t'}{\partial \omega_T}\right)$ and $\left(\frac{\partial t'}{\partial \omega_Z}\right)$ are obtained by implicit differentiation of eq (21); for ω_T, eq (21) yields

$$\frac{t_Z^2 t' - t_Z t'^2}{t_T t_Z(t_Z - t_T)}\,d\omega_T + \frac{[(\omega_T t_Z^2 - \omega_Z t_T^2) + 2(\omega_Z t_T - \omega_T t_Z)t']}{t_T t_Z(t_Z - t_T)}\,dt' = 0 \tag{22}$$

After transferring, eq (22) becomes

$$\frac{\partial t'}{\partial \omega_T} = \frac{t_Z t'(t_Z - t')}{\omega_T t_Z(2t' - t_Z) - \omega_Z t_T(2t' - t_T)}. \tag{23}$$

Similarly to obtain $\frac{\partial t'}{\partial \omega_Z}$, the differentiation of eq (21) yields

$$\frac{t_T t'^2 - t_T^2 t'}{t_T t_Z(t_Z - t_T)}\,d\omega_Z + \frac{[(\omega_T t_Z^2 - \omega_Z t_T^2) + 2(\omega_Z t_T - \omega_T t_Z)t']}{t_T t_Z(t_Z - t_T)}\,dt' = 0. \tag{24}$$

After transferring, eq (24) becomes

$$\frac{\partial t'}{\partial \omega_Z} = \frac{t_T t'(t_T - t')}{\omega_Z t_T(2t' - t_T) - \omega_T t_Z(2t' - t_Z)}. \tag{25}$$

The desired differential coefficients $\frac{\partial t}{\partial \omega_T}$ and $\frac{\partial t}{\partial \omega_Z}$ are obtained by combining eqs (23) and (25), respectively, according to eq (15), with eq (16); there results, respectively,

$$\frac{\partial t}{\partial \omega_T} = \left[1 + \frac{dM(t')}{dt'}\right] \frac{t_Z t'(t_Z - t')}{[\omega_T t_Z(2t' - t_Z) - \omega_Z t_T(2t' - t_T)]}, \tag{26}$$

and

$$\frac{\partial t}{\partial \omega_Z} = \left[1 + \frac{dM(t')}{dt'}\right] \frac{t_T t'(t_T - t')}{[\omega_Z t_T(2t' - t_T) - \omega_T t_Z(2t' - t_Z)]}. \tag{27}$$

Equations (26) and (27) represent the rate of change of temperature t with respect to a change in ω_T and ω_Z, respectively, at the temperature t'. Although both equations (26) and (27) can be converted to terms of t by employing the relation given by eq (4), the formulations given are simpler and practical to use. After calculating $\partial t/\partial \omega_T$ or $\partial t/\partial \omega_Z$ at t', then t' can be converted to t according to eq (4).

The error in the temperature t that results from the error in the tin point calibration is

$$\delta t = \left(\frac{\partial t}{\partial \omega_T}\right)_{t'} \delta \omega_T; \tag{28}$$

the error from the zinc point is

$$\delta t = \left(\frac{\partial t}{\partial \omega_Z}\right)_{t'} \delta \omega_Z. \tag{29}$$

Figure 17 shows the error that the experimenter would encounter for errors of $\delta\omega_T$ or $\delta\omega_Z$ corresponding to a unit positive error of temperature (hotness). The temperature in t' has been converted to t in the figure.

The derivation of the differential coefficients for the SPRT formulation with calibration measurements at the triple point of water, steam point, and zinc point follow the same procedures outlined above. The final equations may be obtained directly by substituting ω_S for ω_T and t_S for t_T in eqs (26) and (27); the results are

$$\frac{\partial t}{\partial \omega_S} = \left[1 + \frac{dM(t')}{dt'}\right] \frac{t_Z t'(t_Z - t')}{[\omega_S t_Z(2t' - t_Z) - \omega_Z t_S(2t' - t_S)]} \tag{30}$$

and

$$\frac{\partial t}{\partial \omega_Z} = \left[1 + \frac{dM(t')}{dt'}\right] \frac{t_S t'(t_S - t')}{[\omega_Z t_S(2t' - t_S) - \omega_S t_Z(2t' - t_Z)]}. \tag{31}$$

The errors in the temperature t that result from the errors in the calibration at the steam point and the zinc point are given by

$$\delta t = \left(\frac{\partial t}{\partial \omega_S}\right)_{t'} \delta \omega_S \tag{32}$$

and

$$\delta t = \left(\frac{\partial t}{\partial \omega_Z}\right)_{t'} \delta \omega_Z. \tag{33}$$

Figure 17 shows the error that the experimenter would encounter for errors of $\delta\omega_S$ and $\delta\omega_Z$ corresponding to a unit positive error of temperature (hotness).

2. *90.188 K (−182.962 °C) to 273.15 K (0 °C)*

From −182.962 to 0 °C the temperature t on the IPTS−68 [5] is defined by the relation

$$W(t) = W^*(t) + \Delta W(t), \tag{34}$$

where

$$W(t) = R(t)/R(0), \tag{35}$$

and $W^*(t)$ is the resistance ratio given by the reference function in Celsius

$$t = \sum_{i=1}^{20} A_i [\ln W^*(t)]^i. \tag{36}$$

From eq (36)

$$\delta t = (\partial t/\partial W^*(t))\delta W^*(t), \tag{36a}$$

and from eq (34)

$$\delta W^*(t) = -\delta \Delta W(t). \tag{36b}$$

For the derivative $\partial t/\partial W^*(t)$, see Appendix E. The experimenter must determine his observational error δW t). The deviation $\Delta W(t)$ is represented (in this temperature range and at 100 °C) by the function

$$\Delta W(t) = A_4 t + C_4 t^3 (t - 100\ ^\circ\mathrm{C}), \tag{37}$$

and the constants of the deviation equation are determined from the calibration measurements at the oxygen NBP, triple point of water, and the steam point. The deviation at 100 °C, $\Delta W(100)$, may also be obtained from the calculated $W(100)$ based on the calibration of the SPRT at the triple point of water, the tin point, and the zinc point. The differential coefficients will be derived for both cases.

For convenience, the symbols and terminologies used in the text of the IPTS−68 have been abbreviated as follows:

$$t = t_{68}, \tag{38}$$

$$W^*(t) = W_{\mathrm{CCT\text{-}68}}\ (t_{68}), \tag{39}$$

$$W(t) = W(t_{68}), \tag{40}$$

and

$$\Delta W(t) = \Delta W(t_{68}). \tag{41}$$

In addition, the temperature of the fixed points will be indicated as:

t_0 = oxygen NBP,

t_S = steam point,

t_T = tin point ($t_T = t_{68_T} - 0.038937$ °C).

and

t_Z = zinc point

and the deviations $\Delta W(t)$ at the fixed points will be indicated as

$$\Delta W_0 = W_0 - W_0^* \text{ (deviation at the oxygen NBP)} \tag{42}$$

$$\Delta W_S = W_S - W_S^* \text{ (deviation at the steam point).} \tag{43}$$

When the deviations at the oxygen NBP and the steam point are applied to eq (37), there results

$$\Delta W_0 = A_4 t_0 + C_4 t_0^3 (t_0 - 100) \tag{44}$$

$$\Delta W_S = A_4 t_S + C_4 t_S^3 (t_S - 100). \tag{45}$$

(For simplification the °C unit after 100 and 75 will be deleted in some of the equations from 44 through 51.)

The expressions for A_4 and C_4 become

$$A_4 = \frac{\Delta W_S t_0^3 (t_0 - 100) - \Delta W_0 t_S^3 (t_S - 100)}{t_S t_0^3 (t_0 - 100) - t_0 t_S^3 (t_S - 100)} \tag{46}$$

and

$$C_4 = \frac{\Delta W_0 t_S - \Delta W_S t_0}{t_S t_0^3 (t_0 - 100) - t_0 t_S^3 (t_S - 100)}. \tag{47}$$

The differential coefficients $\left(\frac{\partial \Delta W(t)}{\partial \Delta W_0}\right)$ and $\left(\frac{\partial \Delta W(t)}{\partial \Delta W_S}\right)$ are obtained by differentiation after combining eqs (46) and (47) with eq (37); there results

$$\frac{\partial \Delta W(t)}{\partial \Delta W_0} = \frac{-t t_s^3 (t_s - 100) + t_s t^3 (t - 100)}{t_s t_0^3 (t_0 - 100) - t_0 t_s^3 (t_s - 100)}. \tag{48}$$

When $t_s = 100$ °C is substituted in expression (48), there is obtained

$$\frac{\partial \Delta W(t)}{\partial \Delta W_0} = \frac{t^3 (t - 100)}{t_0^3 (t_0 - 100)} \tag{49}$$

For $\partial \Delta W(t) / \partial \Delta W_S$,

$$\frac{\partial \Delta W(t)}{\partial \Delta W_s} = \frac{t t_0^3 (t_0 - 100) - t_0 t^3 (t - 100)}{t_s t_0^3 (t_0 - 100) - t_0 t_s^3 (t_s - 100)} \tag{50}$$

and, when $t_S = 100$ °C is substituted wherever simplification can be made,

$$\frac{\partial \Delta W(t)}{\partial \Delta W_s} = \frac{t [t_0^2 (t_0 - 100) - t^2 (t - 100)]}{t_s t_0^2 (t_0 - 100)} \tag{51}$$

Figure 17 shows the error the experimenter would encounter for calibration errors of $\delta \Delta W_0$ or $\delta \Delta W_S$ corresponding to a unit positive error of temperature.

If the tin and zinc point calibrations are used, eqs (46) and (47) must be modified. Expressing W_S and W_S^* of eq (43) in the form of eq (8), there is obtained

$$\Delta W_S = (A - A^*) t_S + (B - B^*) t_S^2, \tag{52}$$

where

$$A^* = \alpha^* (1 + \delta^* / 100\ ^\circ\mathrm{C}) \tag{53}$$

and

$$B^* = 10^{-4} \alpha^* \delta^*\ ^\circ\mathrm{C}^{-2}, \tag{54}$$

where α^* ($3.9259668 \times 10^{-3}/^\circ$C) and δ^* (1.496334 °C) are the constants of the IPTS–68 reference function above 0 °C. The constants A and B are determined from calibration measurements at the tin and zinc points. By introducing the expressions for A and B from eqs (19) and (20) in eq (52), there results

$$\Delta W_S = \left[\frac{\omega_T t_Z^2 - \omega_Z t_T^2}{t_T t_Z (t_Z - t_T)} - A^* \right] t_S + \left[\frac{\omega_Z t_T - \omega_T t_Z}{t_T t_Z (t_Z - t_T)} - B^* \right] t_S^2. \tag{55}$$

Equation (55) is then substituted for ΔW_S in eqs (46) and (47). The eq (49) for the differential coefficient of $\partial \Delta W(t) / \partial \Delta W_0$ remains unchanged. Because the true value of ΔW_S is independent of whether the calibration is at the steam point or at the tin and zinc points, the error function $(\partial \Delta W(t) / \partial \Delta W_0) \delta \Delta W_0$ for the case 2b will be the same as that for case 2a with the steam-point calibration.

Instead of substituting eq (55) in eqs (46) and (47) and performing a long algebraic process, the total derivative of ΔW_S can be obtained in terms of ω_T and ω_Z, i.e.,

$$d \Delta W_S = \left(\frac{\partial \Delta W_S}{\partial \omega_T} \right) d\omega_T + \left(\frac{\partial \Delta W_S}{\partial \omega_Z} \right) d\omega_Z. \tag{56}$$

The expression (56) is employed to obtain $\delta \Delta W_S$, i.e.,

$$\delta \Delta W_S = \left(\frac{\partial \Delta W_S}{\partial \omega_T} \right) \delta \omega_T + \left(\frac{\partial \Delta W_S}{\partial \omega_T} \right) \delta \omega_Z, \tag{57}$$

and the error function $\left(\frac{\partial \Delta W(t)}{\partial \Delta W_S} \right) \delta \Delta W_S$ is then evaluated as

$$\partial \Delta W(t) = \frac{\partial \Delta W(t)}{\partial \Delta W_S} \left[\left(\frac{\partial \Delta W_S}{\partial \omega_T} \right) \delta \omega_T + \left(\frac{\partial \Delta W_S}{\partial \omega_Z} \right) \delta \omega_Z \right]. \tag{58}$$

The coefficient $\frac{\partial \Delta W(t)}{\partial \Delta W_S}$ is given by eq (51) with eq (55) substituted for ΔW_S. The differential coefficients $\frac{\partial \Delta W_S}{\partial \omega_T}$ and $\frac{\partial \Delta W_S}{\partial \omega_Z}$ are obtained from eq (55). Thus,

$$\frac{\partial \Delta W_S}{\partial \omega_T} = \frac{t_Z^2 t_S}{t_T t_Z (t_Z - t_T)} - \frac{t_Z t_S^2}{t_T t_Z (t_Z - t_T)}, \qquad (59)$$

which simplifies to

$$\frac{\partial \Delta W_S}{\partial \omega_T} = \frac{t_S (t_Z - t_S)}{t_T (t_Z - t_T)}. \qquad (60)$$

For $\frac{\partial \Delta W_S}{\partial \omega_Z}$, eq (55) becomes

$$\frac{\partial \Delta W_S}{\partial \omega_Z} = \frac{-t_T^2 t_S}{t_T t_Z (t_Z - t_T)} + \frac{t_T t_S^2}{t_T t_Z (t_Z - t_T)}, \qquad (61)$$

which simplifies to

$$\frac{\partial \Delta W_S}{\partial \omega_Z} = \frac{t_S (t_S - t_T)}{t_Z (t_Z - t_T)}. \qquad (62)$$

Combining eqs (60) and (62) with eq (58), there is obtained

$$\partial \Delta W(t) = \frac{\partial \Delta W(t)}{\partial \Delta W_S} \left[\frac{t_S (t_Z - t_S)}{t_T (t_Z - t_T)} \delta \omega_T + \frac{t_S (t_S - t_T)}{t_Z (t_Z - t_T)} \delta \omega_Z \right]. \qquad (63)$$

Figure 17 shows the error the experimenter would encounter for errors of $\delta \omega_T$ or $\delta \omega_Z$ corresponding to a positive unit error of temperature.

References to Appendix G

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[4] Sharevskaya, D. I., Orlova, M. P., Belyansky, L. B., and Galoushkina, G. A., Investigation of the Resistance-Temperature Properties of Platinum of Resistance Thermometry over the Range From 14 K to 90 K, Metrologia **5**, No. 4, 103–107 (1969).

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APPENDIX H. Calibration of Mueller Bridges

1. The Mueller Bridge

The Mueller bridge is designed for measurement of temperatures in conjunction with four-terminal standard platinum resistance thermometers (SPRT). This bridge is basically the equal arm Wheatstone bridge but with important modifications. One of these modifications permits the SPRT connections to be commutated so that the lead resistances of the SPRT can be canceled by averaging the two bridge readings (see sec. 3.3). Other modifications include a provision for adjusting the ratio arms to equality and for reducing the uncertainties due to variations in contact resistance. In the process of determining the thermometer resistance the bridge current is reversed so that the effect of any steady spurious emf's are eliminated. The current reversal method also doubles the apparent null detector sensitivity.

The Mueller bridges employed in the Platinum Resistance Thermometer Laboratory of the NBS have additional features including the simultaneous interchange of the ratio arms when the SPRT connections are commutated. Any errors arising from a small inequality of the ratio arms are thus made negligible when the two bridge readings are averaged. The schematic circuit diagram of this form of Mueller bridge is shown in figure H–1. For this discussion the upper two bridge arms will be referred to as the ratio arms, the lower left arm of the bridge plus the 0.1 and 1-Ω decades as the rheostat arm, and the lower right arm as the thermometer or the SPRT arm of the bridge.

This bridge has seven twelve-position and one four-position measuring dial switches; the 0.00001, 0.0001, 0.001, 0.01, and 0.1-Ω dials have positions − 1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and X (10) and the 1 and 10-Ω dials have positions 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, X (10), and Y (11). The 100-Ω dial has the four positions 0, 1, 2, and 3. Thus, resistances between 0 and 422.1111 Ω can be measured with the bridge. The 10 and 100-Ω dial switches have mercury-wetted contacts; the 0.00001, 0.0001, 0.001, and 0.01-dial switches, which have "dry" wiping contacts, have Waidner-Wolff resistance elements to reduce the effect of variations in the switch contact resistance. The 0.1 and 1-Ω dial switches have dry wiping contacts which are in series with the relatively high resistance of the ratio-arm resistors to minimize the effect of variations in the contact resistance. The

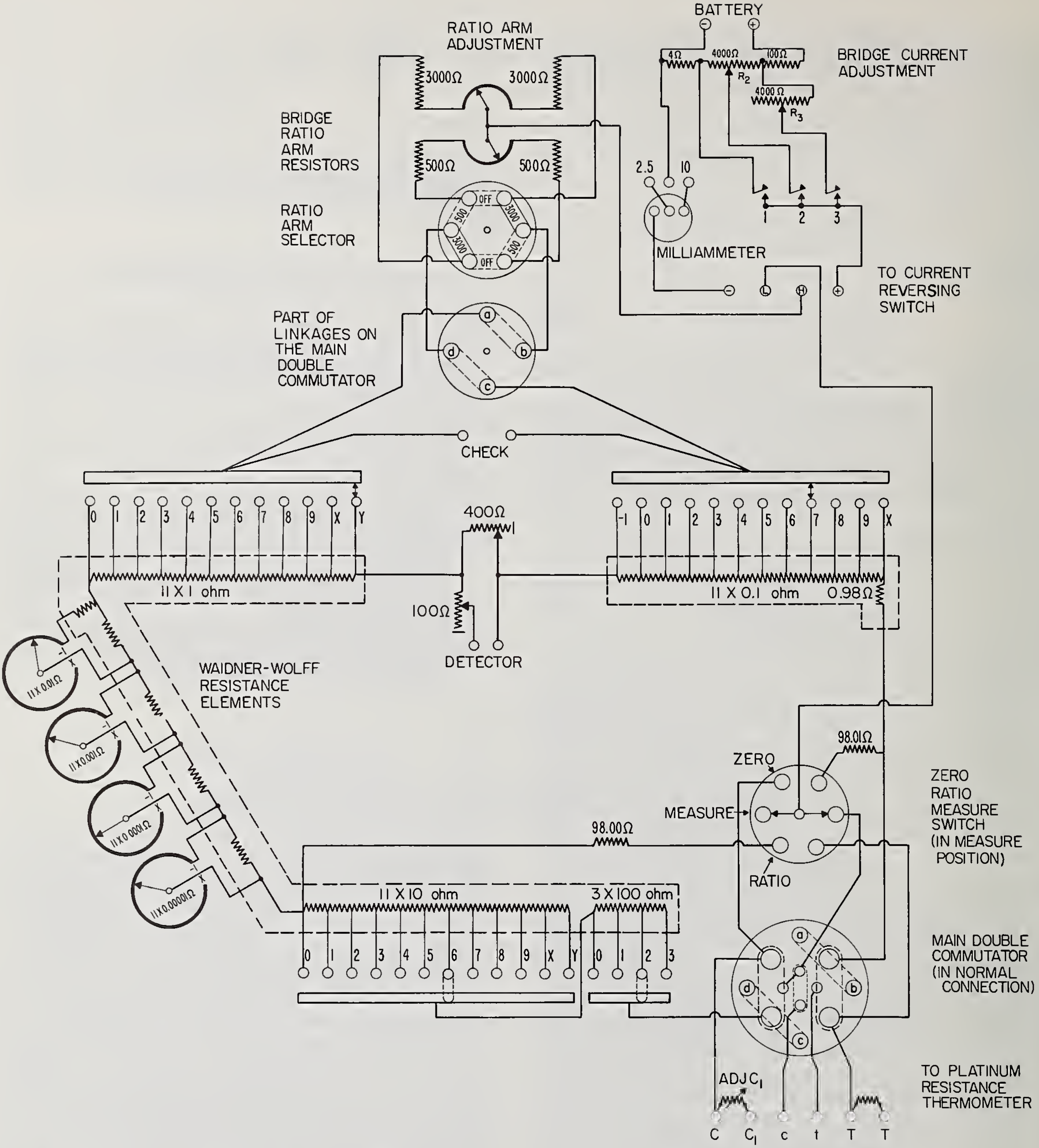

FIGURE H-1. *Schematic circuitry of Mueller bridge employed in the Platinum Resistance Thermometry Laboratory at the National Bureau of Standards.*

The resistors within the dashed lines are thermostated.

0.1-Ω dial switch operates "subtractively". A mercury-wetted, three-position switch permits the selection of 500 or 3000-Ω ratio arms or the "off" position. In the "off" position both ratio arm resistors are disconnected from the lower sections of the bridge. The operation of the bridge for adjustment of the ratio arms, for determination of the bridge zero, or for measurement of a SPRT resistance is selected by means of a three-position, ratio-zero-measure switch, which is a dry wiping contact switch for the older models of this Mueller bridge but a mercury-wetted switch for the newer models. A mercury-wetted switch (commutator switch) commutates the SPRT connection and simultaneously interchanges the ratio arms. The principal resistors of the bridge, shown enclosed in the shaded portion of figure H–1 are thermostated at about 35 °C.

Terminals are provided for batteries and for a current reversing switch. A milliammeter located in the battery circuit indicates the current through the SPRT (one-half of the bridge current). Variable resistors are incorporated in the bridge for adjusting the current. Terminals and variable resistors for damping are provided for the null detector. Special terminals and an adjustable resistor permit the equalization of the potential lead resistances of the SPRT.

2. *Auxiliary Equipment for Calibrating Mueller Bridges*

To calibrate the Mueller bridge, the following special equipment (other than null detector, current reversing switch, and batteries that are necessary in regular resistance measurements) is required:

(1) Adjustable decade resistance box that is sufficiently stable during the time required to intercompare two combinations of resistances (dial settings) of the Mueller bridge. At NBS a second Mueller bridge serves as this auxiliary adjustable resistor. (To avoid confusion the auxiliary Mueller bridge will, henceforth, be referred to as the auxiliary adjustable resistor.)

(2) Standard resistor of 10 Ω (preferably) or 100 Ω with a recent calibration to convert the bridge resistance into terms of the national resistance unit. Duplicate resistors will help reduce the calibration uncertainties.

(3) A thermometer and an insulated enclosure or thermostatted oil bath for the standard resistors.

3. *Preliminary Preparation*

3.1. *Bridge Thermostat*

The bridge resistors should have been thermostatted at a constant temperature at least one week prior to calibration. Safety provisions should be made to avoid any large deviation from the thermostatted temperature; e.g., auxiliary control and battery power should be provided to maintain the bridge temperature whenever the main power line is disrupted. The bridge should be recalibrated periodically and whenever the temperature control fails.

3.2. *Switches*

The old mercury should be removed from the mercury-wetted switches and the studs and links amalgamated with clean mercury. The studs of the dry wiping contact switches should be cleaned and lubricated with pure petroleum. The spaces between the wiping switch blades should be freed of accumulated dirt.

3.3. *Bridge Ratio Arms*

Either the ratio arm resistors of 500 or 3000 Ω may be selected; the equality of the ratio arms is adjusted separately by means of the slide-wire resistors which have the resistance range of about 0.02 percent of that of the ratio arm resistors. The "tap" (contact) on the slide wire resistor is in the battery circuit so that any variations in the contact resistance has no effect on the bridge balance. The choice of either 500- or 3000-Ω ratio arms is based upon the resistance of the SPRT to be determined, the voltage supply available, and the insulation resistance between the voltage supply and the bridge components. (One is particularly concerned with leakage between the voltage supply and the null detector or thermometer.) The contact and lead resistances associated with the 0.1- and 1-Ω dials is in series with the ratio arm resistors. For a given variation in the contact resistance of these switches, the uncertainty of a bridge reading due to these switches and their leads is greater for the 500-Ω ratio arm resistors than for the 3000 Ω resistors. Problems with insulation leakage, on the other hand, increase with both the value of the voltage supply and the resistance of the ratio arms.

To adjust the ratio arms to equality, the three-position, ratio-zero-measure dial switch is placed in the "ratio" position. In this position, two auxiliary resistors (each of about 98 Ω) are each connected to the rheostat and SPRT arms of the bridge; also, the 10- and 100-Ω resistors are "by-passed." The SPRT should be disconnected. The "tap" of the slide-wire resistor is initially placed near the midpoint. The bridge is balanced with the ratio arms in one position and balanced again with the ratio arms interchanged. The bridge dials are then set on the average of the two balance readings and the slide wire adjusted to balance the bridge. The setting on the slide-wire resistor should be checked by balancing the bridge again with the two positions of the ratio arms. The ratio arms have been adjusted to equality when the two bridge readings are the same. (All bridge balance readings, unless stated otherwise, are performed with current reversal.)

3.4. *Bridge Zero*

A residual resistance exists in the rheostat arm even with all measuring switches set on zero because of the resistances of the leads that connect the resistor net work and the resistances of the switch contacts. The Waidner-Wolff elements contribute also to the residual resistance. Therefore, in the bridge design a resistor (about 0.98 Ω) is placed in series with the 0.1-Ω decade resistors to balance the rheostat and SPRT arms of the bridge when all dials are set to zero. The remaining deviation from a perfect balance is the "bridge zero." The bridge zero must be subtracted algebraically from the dial readings of all measurements.

To determine the bridge zero, the bridge ratio arms are first equalized (see sec. 3.3) and the three-position, ratio-zero-measure switch is then placed on the "zero" position. The bridge zero is the average of the bridge balance readings obtained with the commutator switch in normal (N) and reverse (R) positions. If the contact resistance of the three-position selector switch varies between the N and R readings, the bridge zero will be in error. If the selector switch is considered unreliable, which it may be if a dry wiping contact switch is used for a selector switch, the bridge zero is determined by connecting a special shorting link made by soldering three copper wires (about AWG 18) into the form of an "H." The "H" should be about 5 cm high and 10 cm wide. One short vertical wire of the "H" is connected between the terminals C and T; the other short vertical wire of the "H" is connected between the terminals c and t. When using this special shorting link to determine the bridge zero the three-position selector switch should be in the "measure" position. The bridge zero is the average of the bridge balance readings obtained with the commutator in normal and reverse positions.

3.5. *Resistance of Leads Between the 1- and 0.1-Ω Decade Resistors and the Ratio Arm Switch*

The resistors of the 0.1- and 1-Ω step decades are enclosed, along with other principal resistors, in a thermostatted aluminum block. The resistances of the leads from these resistors to the dial switches must be sufficiently close so that their differences have negligible effect on the ratio of the ratio arms. The double commutator that interchanges the ratio arm resistors does not interchange the resistances of the leads from the resistors of the 0.1- and 1-Ω step decades, the switch contact resistances of these decades, nor the resistances of the leads from the dial switches of these decades to the ratio arm reversing switch (see bridge circuit diagram, fig. 4–1).

The design of the bridge permits the separate determination of the degree of equality of the leads from the 0.1- and 1-Ω decade resistors to their dial switches and of the leads from these switches to the ratio arm reversing switch. The "check" terminals are employed to test the degree of equality of the resistor lead plus the switch contact resistances; the resistances of the leads between the dial switches and the ratio arm reversing switch is determined by measurements using another bridge. By introducing branch points at locations a and c of the ratio-arm reversing switch, the degree of equality of the combined leads may be determined (see fig. H–2). The discussion to follow in this section (3.5) will assume that a pair of wires from points a and c are available at special terminals on the bridge. If the special wiring is not available, an equivalent circuit may be achieved by removing the double commutator and placing amalgamated heavy copper links from a to b and c to d. Wires are attached to the links close to the a and c studs of the switch and are then connected to a double-pole, double-throw (DPDT) all copper knife switch as shown schematically in figure H–2. Amalgamated links for the SPRT and battery are also placed on the double commutator studs corresponding to resistance measurement in the N connection (see fig. H–1). If the special wiring and terminals are available on the bridge, the double commutator is placed in the N position. The null detector is connected to the Mueller bridge through the DPDT switch so that the switch can be connected to the

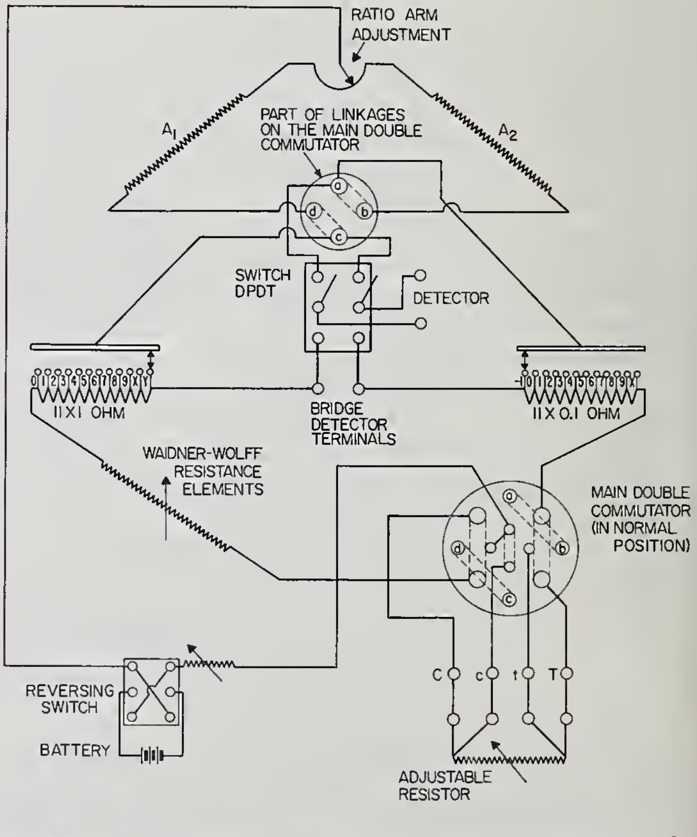

FIGURE H-2. *Schematic circuitry for testing the equality of the Mueller bridge leads between the ratio arm reversing switch (part of the main commutator switch) and 0.1-Ω and 1-Ω decades.*

The contact resistances of the decade switches are included as part of the bridge leads.

leads from positions a and c of the ratio-arm reversing switch or to the bridge galvanometer posts (see figs. H–1 and H–2). The ratio-zero-measure dial switch is placed on the "measure" position and the auxiliary adjustable resistor is connected as a four-terminal resistor to the SPRT posts. (At NBS the auxiliary Mueller bridge is prepared for this and subsequent bridge calibration applications by shorting the "check" terminals and by placing its commutator in the normal position, the galvanometer damping resistors in the open positions, the ratio-zero-measure switch in the "measure" position, and the ratio arm selector switch in the "off" position. Three heavy gauge copper wires are connected from the C and T posts and the shorted "check" terminals of the auxiliary bridge to the T, C, and c posts, respectively, of the bridge to be calibrated. In the above circuit, when the dials of the auxiliary Mueller bridge are set to zero, approximately equal resistances are introduced into the lower two arms of the Mueller bridge to be calibrated; the 0.1-Ω decade operates subtractively (see fig. H–1)).

Instead of testing all possible combinations of the switch positions of the 0.1- and 1-Ω decades, the maximum difference between the lead resistances is determined by comparing the 0 step of the 0.1-Ω decade with all steps of the 1-Ω decade and 0 step of the 1-Ω decade with all steps of the 0.1-Ω decade, totaling 22 comparisons. With 0.1- and 1-Ω decade dials on 0 and the DPDT switch thrown to the *a-c* position, the bridge is balanced (reading 1) employing the lower dials and the auxiliary adjustable resistor. The DPDT switch is then thrown to the galvanometer post position and the bridge is balanced (reading 2) using the lower dials only. The difference in the balance readings is, for all practical purposes, the difference in the lead resistance. The process is repeated with the 1-Ω decade on the 1 position. A sample of the complete test data is summarized in table 1. (The tabular values given are only for descriptive purposes and do not represent any actual data.) The maximum error in the bridge reading is, in proportional parts, $\frac{74}{500}\times 10^{-6}$ or $\frac{74}{3000}\times 10^{-6}$, depending upon the choice of ratio arm resistors. For this particular (fictitious) bridge the results demonstrate that in the determination of high resistances the 3000-Ω ratio arms should be employed, also that significantly large error may occur in the determination of small resistance differences in which the 0.1- and 1-Ω decade switches are moved.

After determining the differences in the lead resistances the auxiliary connections are removed and the Mueller bridge is placed in the normal operating condition.

TABLE 1. *Comparison and determination of the maximum difference in the lead resistance between the 0.1- and 1-ohm decades and the ratio arm reversing switch*

1-Ω decade with the 0.1-Ω decade on 0				0.1-Ω decade with the 1-Ω decade on 0			
Dial Position	Reading 1 μΩ	Reading 2 μΩ	Difference μΩ	Dial Position	Reading 1 μΩ	Reading 2 μΩ	Difference μΩ
0	142	136	+6	0	237	247	−10
1	521	508	+13	1	217	206	+11
2	506	500	+6	2	225	239	−14
3	517	506	+11	3	273	259	+24
4	511	501	+10	4	280	260	+20
5	496	502	−6	5	256	277	−21
6	499	507	−8	6	247	263	−16
7	501	498	−3	7	250	244	+15
8	500	489	−11	8	312	326	−16
9	497	511	−14	9	327	320	+7
X	504	488	−16	*X*	317	311	+6
Y	502	517	−15	−1	222	234	−12
1 ohm decade				0.1 ohm decade			
Maximum	+13			Maximum	+24		
Minimum	−16			Minimum	−21		
Difference	29			Difference	45		
Sum of difference=74							
Error=Bridge Reading×$\frac{74}{500}\times 10^{-6}$ Ω or Bridge Reading×$\frac{74}{3000}\times 10^{-6}$ Ω							

3.6. *Calibration of the Mueller Bridge in Terms of the National Unit of Resistance*

To calibrate the Mueller bridge in terms of the national unit of resistance the X position of the 10-Ω decade and/or the X position of the 1-Ω decade are compared with recently calibrated 100-Ω and/or 10-Ω standard resistors, respectively. The choice of the comparison depends upon the stability of the standard resistors and the bridge decades that are most used in the measurements. On the basis of absolute resistance, resistors of higher resistance are usually less stable. In the NBS Platinum Resistance Thermometry Laboratory the Mueller bridge is usually compared with 10-Ω standard resistors. (The bridge resistors may all be expressed in terms of, for example, the resistance of 0.1_X being exactly 1 bridge unit and, if the SPRT resistance ratio $R(t)/R(0)$ is employed in all temperature evaluations, the bridge unit is satisfactory. The resistance in terms of the national unit is, however, necessary for some comparisons, e.g., comparison of the measured R(0 °C) of the SPRT with that obtained at the National Bureau of Standards.)

The X position of the 1-Ω decade is compared with a 10-Ω standard resistor by measuring the standard, as a four-terminal resistor, directly with the Mueller bridge. The following table 2 gives an example of the results obtained. (One or two figures more than are experimentally realized are carried in the calculations for internal consistency.)

TABLE 2. *Comparison of X position of 1-Ω dial with a 10-Ω standard resistor*

(Standard resistor at 26.4 °C)

	Ohms
Standard at 25 °C	10.00026_{14}
Temperature correction to 26.4 °C	−0.0000047
Standard at 26.4 °C	10.0002567
Bridge reading, average of N and R readings	X.0010491
Bridge zero (subtract)	0.0001832
Bridge reading corrected for bridge zero	X.0008659
Standard resistor minus bridge reading corrected for bridge zero	−0.0006092
Resistance of 1_X	10−0.0006092 Ω

4. *Calibration of the Mueller Bridge Decade Steps*

The ratio arms of the Mueller bridge must be adjusted to equality before calibrating the 0.1 or 1-Ω decade steps. The double commutator is then placed in the "normal" position and the auxiliary adjustable resistor is connected as a four terminal resistor to the SPRT posts. As previously stated, all bridge balance readings are obtained with current reversal.

In principle the resistance steps of the Mueller bridge are calibrated internally by comparing each step of one decade with the X position of the next lower decade. The differences between the resistors can be obtained conveniently in terms of the lower decades. If upon analyzing the calibration data from the higher resistance decade down to the lower resistance decades the latter decades should show significant corrections, the differences expressed in terms of them could then be corrected and the calibration data reanalyzed (see step (18)).

The procedure used at the NBS Thermometry Laboratory for calibrating the Mueller bridge is as follows:

(1) Set to 1 on the 1-Ω decade of the bridge and to 0 on the 0.1-Ω decade and use the lower decades to balance an appropriate resistance in the auxiliary adjustable resistor. Galvanometer (detector) deflections are interpolated to obtain fractions of the 0.00001-Ω step.

(2) Without disturbing the external circuit, change the bridge setting to 0 on the 1-Ω decade and to X on the 0.1-Ω decade and again use the lower decades to balance the bridge. The dials of the auxiliary adjustable resistor should be adjusted so that the lower decades are near zero for the lesser of (1) or (2) bridge balances.

(3) Repeat (2) and then (1).

(4) Set to 2 on the 1-Ω decade of the bridge and to 0 on the 0.1-Ω decade and balance with a corresponding resistance in the auxiliary adjustable resistor, as in (1).

(5) Change the bridge setting to 1 on the 1-Ω decade and to X on the 0.1-Ω decade and proceed as in (2) and (3).

(6) Continue this process until each of the 1-Ω steps has been compared with the X position of the 0.1-Ω decade.

(7) The observations under (1), (2), and (3) yield the relation:

$$1_1+a_1=0.1_X+b_1, \tag{1}$$

where the Arabic numeral index indicates reference to the resistance change due to that individual step in the decade, the Roman numeral index X indicates reference to the total resistance change due to changing the decade switch from position 0 to position X (or 10), and where a_1 and b_1 are the corrected averages of the respective lower decade readings, including galvanometer interpolations, which were obtained in the balancings of the bridge. The relation (1), when transferred to

$$1_1-0.1_X=b_1-a_1=c_1, \tag{2}$$

indicates the difference in resistance between step 1 of 1-Ω decade and X of the 0.1-Ω decade.

Equations similar to (2) are obtained from (4), (5), and (6) leading to the remainder of the set:

$$\begin{aligned}
1_2 - 0.1_X &= c_2\\
1_3 - 0.1_X &= c_3\\
1_4 - 0.1_X &= c_4\\
1_5 - 0.1_X &= c_5\\
1_6 - 0.1_X &= c_6\\
1_7 - 0.1_X &= c_7\\
1_8 - 0.1_X &= c_8\\
1_9 - 0.1_X &= c_9\\
1_{10} - 0.1_X &= c_{10}.
\end{aligned} \tag{3}$$

Table 3 summarizes the results of such a comparison between the 1-Ω decade and 0.1_X. In the two-digit notation of the dial pairs that are given in column 1, the first digit indicates the dial position of the decade being compared and the second digit the dial position of the next lower decade. The values listed in the second column correspond to the resistance differences, the c's, of eqs (2) through (3).

(8) Summing eqs (2) and (3), there is obtained

$$\sum_{i=1}^{10} 1_i - 10 \times 0.1_X = \sum_{i=1}^{10} c_i, \tag{4}$$

but

$$\sum_{i=1}^{10} 1_i = 1_X. \tag{5}$$

The comparison of 1_X with a 10-Ω standard resistor showed that

$$1_X = 10 - 0.0006092 \ \Omega. \tag{6}$$

(See table 2, sec. 3.6). Combining eqs (4), (5), and (6),

$$0.1_X = \left(10 - 0.0006092 - \sum_{i=1}^{10} c_i\right)/10. \tag{7}$$

From table 3, $\sum_{i=1}^{10} c_i = -0.0003746 \ \Omega$ and, therefore, $0.1_X = 1 - 0.00002346 \ \Omega$. The relation (7) is shown evaluated at the bottom of table 3.

(9) Each step of column 2 (table 3) is corrected for the real value of 0.1_X according to eqs (2) and (3) in terms of the national resistance unit. Column 3 lists the corrected values.

(10) The stepwise cumulative corrections of the dial positions are listed in column 4 and rounded to the nearest 0.1 μΩ in column 5. The last entry of columns 4 and 5 must agree with the value of 1_X obtained by comparison with the standard resistor. This serves as a check on arithmetic operations.

TABLE 3. *Calibration of 1-Ω decade by comparison with 0.1_X*

Dial pairs compared	Difference 0.1 μΩ	Corrected 0.1 μΩ	Σ, 0.1 μΩ	Nearest 0.1 μΩ
10−0X	−359	−593.6	−593.6	−594
20−1X	−298	−532.6	−1126.2	−1126
30−2X	−408	−642.6	−1768.8	−1769
40−3X	−406	−640.6	−2409.4	−2409
50−4X	−385	−619.6	−3029.0	−3029
60−5X	−368	−602.6	−3631.6	−3632
70−6X	−376	−610.6	−4242.2	−4242
80−7X	−386	−620.6	−4862.8	−4863
90−8X	−414	−648.6	−5511.4	−5511
X0−9X	−346	−580.6	−6092.0	−6092

Σ differences = −3746

$0.1_X = (10 - 0.0006092 - (-0.0003746))/10$
$= 1 - 0.00002346 \ \Omega.$

(11) The 0.1-Ω decade steps are compared with 0.01_X of the next lower dial following the procedures given in (1), (2), (3), (4), (5), and (6). A set of relations given by (2) and (3) is then obtained. Table 4 shows the results of a comparison.

(12) A relation similar to (4) is given by

$$\sum_{i=1}^{10} 0.1_i - 10 \times 0.01_X = \sum_{i=1}^{10} c_i, \tag{8}$$

where

$$\sum_{i=1}^{10} c_i = -0.0000318 \ \Omega \text{ and}$$

$$\sum_{i=1}^{10} 0.1_i = 0.1_X = 1 - 0.00002346 \ \Omega$$

(see step (8) and table 3).

There results similar to relation (7)

$$\begin{aligned} 0.01_X &= (1 - 0.00002346 - (-0.0000318))/10\\ &= 0.1 + 0.00000083 \ \Omega. \end{aligned} \tag{9}$$

(13) Each dial step of column 2 (table 4) is corrected for the real value of 0.01_X in terms of the national resistance unit. Column 3 lists the corrected dial steps. Column 4 lists the stepwise cumulative corrections and column 5 lists values of column 4 rounded to the nearest 0.1 μΩ.

(14) The 0.01-Ω decade steps are compared with 0.001_X of the next lower decade following the same procedures given in (1), (2), (3), (4), (5) and (6). The measurements are analyzed according to (7), (8), (9), (10), (13) and (14). The results of such a comparison are summarized in table 5. The 0.001-Ω and 0.001-Ω decade steps are calibrated in the same manner; the results are given in tables 6 and 7, respectively.

Table 4. *Calibration of 0.1-Ω decade by comparison with 0.01_X*

Dial pairs compared	Difference 0.1 μΩ	Corrected 0.1 μΩ	Σ, 0.1 μΩ	Nearest 0.1 μΩ
10–0X	–51	–42.7	–42.7	–43
20–1X	–07	+1.3	–41.4	–41
30–2X	–17	–8.7	–50.1	–50
40–3X	–42	–33.7	–83.8	–84
50–4X	–59	–50.7	–134.5	–134
60–5X	–42	–33.7	–168.2	–168
70–6X	–28	–19.7	–187.9	–188
80–7X	–34	–25.7	–213.6	–214
90–8X	–28	–19.7	–233.3	–233
X0–9X	–10	–1.7	–235.0	–235

Σ differences = –318

$0.01_X = (1 - 0.00002346 - (-0.0000318))/10$
$= 0.1 + 0.00000083\ \Omega.$

Table 5. *Calibration of 0.01–Ω decade by comparison with 0.001_X*

Dial pairs compared	Difference 0.1 μΩ	Corrected 0.1 μΩ	Σ, 0.1 μΩ	Nearest 0.1 μΩ
10–0X	+7	+16.8	+16.8	+17
20–1X	–19	–9.2	+7.6	+8
30–2X	+13	+22.8	+30.4	+30
40–3X	–19	–9.2	+21.2	+21
50–4X	–30	–20.2	+1.0	+1
60–5X	–4	+5.8	+6.8	+7
70–6X	–5	+4.8	+11.6	+12
80–7X	+6	+15.8	+27.4	+27
90–8X	–27	–17.2	+10.2	+10
X0–9X	–12	–2.2	+8.0	+8

Σ differences = –90

$0.001_X = (0.1 + 0.00000083 - (-0.0000090))/10$
$= 0.01 + 0.00000098\ \Omega.$

Table 6. *Calibration of 0.001–Ω decade by comparison with 0.0001_X*

Dial pairs compared	Difference 0.1 μΩ	Corrected 0.1 μΩ	Σ, 0.1 μΩ	Nearest 0.1 μΩ
10–0X	+2	–0.4	–0.4	0
20–1X	+4	+1.6	+1.2	+1
30–2X	+2	–0.4	+0.8	+1
40–3X	+4	+1.6	+2.4	+2
50–4X	+1	–1.4	+1.0	+1
60–5X	+4	+1.6	+2.6	+3
70–6X	+3	+0.6	+3.2	+3
80–7X	+4	+1.6	+4.8	+5
90–8X	+4	+1.6	+6.4	+6
X0–9X	+6	+3.6	+10.0	+10

Σ differences = +34

$0.0001_X = (0.01 + 0.00000098 - 0.0000034)/10$
$= 0.001 - 0.00000024\ \Omega.$

Table 7. *Calibration of 0.0001-Ω decade by comparison with 0.00001_X*

Dial pairs compared	Difference 0.1 μΩ	Corrected 0.1 μΩ	Σ, 0.1 μΩ	Nearest 0.1 μΩ
10–0X	+1	+0.5	+0.5	+0
20–1X	+1	–1.5	–1.0	–1
30–2X	–1	–1.5	–2.5	–2
40–3X	0	–0.5	–3.0	–3
50–4X	+2	+1.5	–1.5	–2
60–5X	–1	–1.5	–3.0	–3
70–6X	+1	+0.5	–2.5	–2
80–7X	+1	+0.5	–2.0	–2
90–8X	0	–0.5	–2.5	–2
X0–9X	+1	+0.5	–2.0	–2

Σ differences = +3

$0.00001_X = (0.001 - 0.00000024 - 0.0000003)/10$
$= 0.0001 - 0.00000005\ \Omega.$

Table 8. *Calibration of 0.00001 Ω decade*

Dial pairs compared	Difference 0.1 μΩ	Corrected 0.1 μΩ	Σ, 0.1 μΩ	Nearest 0.1 μΩ
10–0X	+2	+1.6	+1.6	+2
20–1X	+1	+0.6	+2.2	+2
30–2X	0	–0.4	+1.8	+2
40–3X	0	–0.4	+1.4	+1
50–4X	0	–0.4	+1.0	+1
60–5X	–2	–2.4	–1.4	–1
70–6X	0	–0.4	–1.8	–2
80–7X	+2	+1.6	–0.2	0
90–8X	+1	+0.6	+0.4	0
X0–9X	0	–0.4	0	0

Σ differences = +4

$0.000001_X = (0.0001 - 0.00000005 - 0.0000004)/10$
$= 0.00001 - 0.00000004\ \Omega.$

(15) The 0.00001-Ω decade is calibrated by the method of galvanometer deflection. Set the 0.00001-Ω dial on zero with the bridge nearly balanced by appropriately setting the auxiliary adjustable resistor. Step to position X and record the galvanometer deflection. Assume 1/10 of the deflection to correspond to 0.000001_X which will be adjusted to the true value in the manner as the previous X's were adjusted. Set the 0.00001-Ω dial back to zero position and step to the 1 position. Record the deflection difference with respect to the imaginary 0.000001_X. Balance and repeat with other steps of the dial. The results of a sample data are summarized in table 8. The sum of the galvanometer deflections corresponds to 0.0001 – 0.00000005 Ω (see the bottom of table 7).

(16) The 10-Ω decade is calibrated by comparison with the X position of the 1-Ω decade following the procedures (1), (2), (3), (4), (5), (6), and (7). The results of the comparison are summarized in column 2 of table 9. The resistance of 1_X is 10-0.0006092 ohms (see table 2). Column 3 (table 9) lists the dial steps corrected to the national resistance units. Column 4 lists the stepwise cumulative corrections.

(17) The 100-Ω dial steps are calibrated by comparison with 10_x. Column 2 to table 10 summarizes the results. The resistance of $10_x=100\text{-}0.003717$ Ω (see table 9). The column 3 (table 10) lists the corrected dial steps and column 4 lists the stepwise cumulative corrections.

(18) All differences, i.e., c's of eq (2), in the dial readings should be examined to determine whether they are changed significantly when the dial readings are corrected according to the calibration. If the correction is significant, then a second analysis should be made employing the corrected differences.

(19) The decade steps of the sample data have been compared with the X of the next lower decade in terms of the lower decade steps (see column 2 of tables 3 to 10). The decades from 0.00001-Ω to 1-Ω are compared in terms of the 0.00001-Ω decade dial positions; the 10-Ω decade and the three 100-Ω steps are compared in terms of the 0.00001- and 0.0001-Ω steps. The 0.001-Ω decade dial was used at one occasion with the 100-Ω dials (see table 10). In as much as the lower dials have very small corrections, the bridge resistors including the auxiliary adjustable resistor should be adjusted so that the comparison measurements would involve changes in dial positions of the less significant dials wherever possible (see step (2)). By following this procedure the calibration correction to the differences (c's of eq (2)) can be made small or negligible and a second analysis (step (18)) of the calibration data based on the corrected values of c's may be avoided. In the sample calibration data the corrections to the lower dials are in the range of interpolated values of the least significant dial (0.00001-Ω). The corrections to the differences (c's) are negligible.

(20) The results of calibration given in tables 3 to 10 are summarized in table 11 for rapid reference in making bridge dial corrections. Corrections of less than one μΩ may be taken as zero without significantly increasing the uncertainty of the resistance measurements.

(21) For further information reference may be made to the following:

(a) Brooks, P. P. B., Calibration procedures for direct-current resistance apparatus, Nat. Bur. Stand. (U.S.), Monogr. 39, 53 pages (March 1, 1962).

(b) Brooks, P. P. B., Calibration of Mueller Thermometer bridges, 39 pages (October 1965), from the National Bureau of Standards List of Publications LP38 (August 1966), Calibration Procedures of the NBS, Electrical Reference Standards Section.

TABLE 9. *Calibration of 10-Ω decade by comparison with* 1_X

Dial pairs compared	Difference μΩ	Corrected μΩ	Σ, μΩ
10 − 0X	+289	−320	−320
20 − 1X	+227	−382	−702
30 − 2X	+287	−322	−1024
40 − 3X	+015	−594	−1618
50 − 4X	+167	−442	−2060
60 − 5X	−009	−618	−2678
70 − 6X	+272	−337	−3015
80 − 7X	+576	−033	−3048
90 − 8X	+267	−342	−3390
X0 − 9X	+282	−327	−3717

$1_X = 10 - 0.0006092$ Ω.

TABLE 10. *Calibration of 100-Ω dial steps by comparison with* 10_X

Dial pairs compared	Difference μΩ	Corrected μΩ	Σ, μΩ
10 − 0X	−945	−4662	−4662
20 − 1X	−1710	−5427	−10089
30 − 2X	+973	−2744	−12833

$10_X = 100 - 0.003717$ Ω.

TABLE 11. *Summary of Mueller bridge calibration*
Bridge corrections in micro-ohms

Dial	Bridge decades							
position	100	10	1	0.1	0.01	0.001	0.0001	0.00001
1	− 4662	− 320	− 59.4	− 4.3	+1.7	0	0	+0.2
2	−10089	− 702	−112.6	− 4.1	+0.8	+0.1	−0.1	+0.2
3	−12833	−1024	−176.9	− 5.0	+3.0	+0.1	−0.2	+0.2
4		−1618	−240.9	− 8.4	+2.1	+0.2	−0.3	+0.1
5		−2060	−302.9	−13.4	+0.1	+0.1	−0.2	+0.1
6		−2678	−363.2	−16.8	+0.7	+0.3	−0.3	−0.1
7		−3015	−424.2	−18.8	+1.2	+0.3	−0.2	−0.2
8		−3048	−486.3	−21.4	+2.7	+0.5	−0.2	0
9		−3390	−551.1	−23.3	+1.0	+0.6	−0.2	0
X		−3717	−609.2	−23.5	+0.8	+1.0	−0.2	0

APPENDIX I. Calibration of the Linearity of a Potentiometer

The equipment required for calibrating the linearity of a potentiometer is:

(a) A source of adjustable stable voltage, e.g., a second potentiometer or an adjustable well regulated electronic voltage supply.

(b) A detector of sufficient sensitivity. (For the resolution of 1 $\mu\Omega$ at 1 ma in the SPRT a sensitivity of one nanovolt is needed.)

The calibration procedure, analogous to that of the Mueller bridge (see Appendix H), is the comparison of successive voltage steps of one decade against the X position of the next lower decade. The sensitivity of the detector should be sufficient to permit interpolation of the last dial of the potentiometer. The current reversal procedure will double the detector sensitivity and cancel any steady extraneous emf's.

1. Calibration of the Decade Switches

Let ${}_j\Delta_i$ be the difference in nanovolts between the ith step of decade j and the X position of $j+1$ (next lower) decade, i.e.,

$$ {}_j\Delta_i = ({}_jV_i - {}_jV_{i-1}) - {}_{j+1}V_X, \tag{1}$$

where V is the voltage associated with the dial position. (The decade $j=1$ is the most significant decade.) The *average difference* between each of the steps of decade j and X of decade $j+1$ is:

$$ {}_j\bar{\Delta} = \frac{1}{10}\sum_{i=1}^{10} {}_j\Delta_i = ({}_jV_X - 10\,{}_{j+1}V_X)/10. \tag{2}$$

The difference between each observed ${}_j\Delta_i$ and ${}_j\bar{\Delta}$ is

$$ {}_j\Delta_i - {}_j\bar{\Delta} = {}_j\Delta_i - \frac{1}{10}\sum_{i=1}^{10} {}_j\Delta_i = {}_j\delta_i $$

$$ = ({}_jV_i - {}_jV_{i-1}) - {}_jV_X/10. \tag{3}$$

Equation (3) shows that the calibration process reduces to the comparison of each step of the decade with the average step of the decade. The deviations from linearity ${}_jD_n$ at the nth step of decade j is the algebraic sum of the deviations from the average difference of the 1st to nth steps inclusive, i.e.,

$$ {}_jD_n = \sum_{i=1}^{n} {}_j\delta_i \tag{4}$$

$$ = \sum_{i=1}^{n}\left[{}_j\Delta_i - \frac{1}{10}\sum_{i=1}^{10} {}_j\Delta_i \right] = {}_jV_n - n\,{}_jV_X/10. \tag{5}$$

In eqs (4) and (5), ${}_jD_{10} \equiv 0$. Table 1 shows an example of calibration data and calculations of linearity corrections on "decade 1," the most significant dial with steps nominally 0.01 V, of a six-dial potentiometer. The record of the differences between dial positions and X of the next lower decade is given in nanovolts, but the actual laboratory record is recorded as galvanometer deflections and the sensitivity.

The procedure for determining the linearity of the lower decades is the same as that for decade 1, i.e., the dial positions of decade 2 are compared with the X position of decade 3 and so forth. Tables of differences ${}_j\Delta_i$, differences ${}_j\delta_i$ from the mean ${}_j\bar{\Delta}$, and the deviations from linearity ${}_jD_n$ are constructed as shown in table 1 for decade 1.

The linearity of the least significant decade is determined from galvanometer deflections. The galvanometer deflection corresponding to each step is compared with the average deflection for the decade (see eq (3)). Since the least significant decade can tolerate relatively large deviations in the resistance elements, usually its linearity is found to be adequate for most measurements.

The above calibration data yield the deviation from linearity of a dial position in terms of the average step of the decade (see eq (5)). The overall linearity of the measuring decades is determined in terms of the linearity of the dial positions of a decade and the relation of its X position with the average step of the next higher decade (eq (2)), i.e., ${}_i\bar{\Delta}$ or 6.7 nV (see table 1 for the deviation of the average step of decade 1 from X position of decade 2). The X position of decade 2 is 6.7 nV *smaller* than the average step of decade 1; each step or the average step of decade 2 is, therefore, 6.7/10 nV smaller. The total deviation from linearity of the nth step of decade 2 is the sum of the deviations from linearity within the decade (as determined from comparison

TABLE 1. *Example of calibration of potentiometer dial 1 steps = nominally 0.01 V*

Position i dial 1	${}_1\Delta_i$ nanovolts	${}_1\delta_i$ nanovolts	${}_1D_n$ nanovolts
1	21	14.3	14
2	33	26.3	41
3	−47	−53.7	−13
4	−52	−58.7	−72
5	34	27.3	−44
6	43	36.3	−8
7	62	55.3	47
8	−11	−17.7	29
9	27	20.3	50
X	−43	−49.7	0

$\sum_{i=1}^{10} {}_1\Delta_i = 67$ nV.

${}_1\bar{\Delta} = 6.7$ nV.

of the 2d decade steps with the X position of the 3d decade) and the deviation between decades 1 and 2, i.e.,

$$ {}_2\epsilon_n = {}_2D_n - n\,{}_1\bar{\Delta}/10 \tag{6}$$

${}_2\epsilon_n$ is the total deviation from linearity of the nth step of decade 2.

The total deviation from linearity of the mth step of decade 3 is:

$$ {}_3\epsilon_m = {}_3D_m - m[{}_2\bar{\Delta}/10 + {}_1\bar{\Delta}/100]. \tag{7}$$

The total deviation from linearity of the kth step of decade 4 is:

$$ {}_4\epsilon_k = {}_4D_k - k[{}_3\bar{\Delta}/10 + {}_2\bar{\Delta}/100 + {}_1\bar{\Delta}/1000]. \tag{8}$$

For the pth step of decade 5 the total deviation from linearity is:

$$ {}_5\epsilon_p = {}_5D_p - p[{}_4\bar{\Delta}/10 + {}_3\bar{\Delta}/100 + {}_2\bar{\Delta}/1000 + {}_1\bar{\Delta}/10000]. \tag{9}$$

The term containing ${}_1\bar{\Delta}/10000$ or even ${}_2\bar{\Delta}/1000$ may be insignificant. The deviations from linearity of even lower decades are calculated as an extension of the above.

2. *Calibration of the Range Switches*

In contrast to the Mueller bridge, potentiometers have range switches with $x1$, $x0.1$, or even $x0.01$ factors. Any deviations from the nominal factors are determined by comparing the X position of decade 1 on the $x0.1$ range and 1 position of decade 1 on the $x1$ range *corrected for linearity*. The $x0.01$ range is compared in the same manner with $x0.1$ range.

2.1. To Check the x0.1 Range Factor

With the range switch set on $x1$ position, adjust the external voltage source until it is balanced against the step 1 of the first ($j=1$) decade. When a balance is achieved, change the range switch to $x0.1$ position and the first decade to step X. Since the step 1 of the first decade is 1 ppm too high (from the calibration given in table 1), the dial settings should be $X00001$ on the six-dial potentiometer being employed in the example given here. This gives an accurate 10 to 1 ratio for the dial settings. If the potentiometer current with the range switch set on $x0.1$ position is too high, then (using current reversals) the galvanometer deflections will have the same sense with the increase in the dial setting. If the current is too low, then the galvanometer deflection will have the opposite sense with the increase in the dial setting. If, e.g., galvanometer deflection is equivalent to $+10$ nanovolts, the range factor correction becomes $+1\times10^{-6}$.

2.2. To check the x0.01 Range Factor

The $x0.01$ range is compared with the $x0.1$ range in the same manner as above between $x0.1$ and $x1$ ranges. The range factor correction for the $x0.01$ range relative to the $x1$ range is then the combined correction factors (very nearly the sum of the correction factors) for the $x0.1$ range and the $x0.01$ range relative to the $x0.1$ range.

3. *Combined Decade Switch and Range Switch Corrections*

The potentiometer voltage equation with calibration correction is:

$$V=F(1+K)\ (E_1+E_2+E_3+E_4+E_5+E_6$$

where

$$+\epsilon_1+\epsilon_3+\epsilon_3+\epsilon_4+\epsilon_5+\epsilon_6)$$

$F=$ range factor $x1$, $x0.1$, or $x0.01$
$K=$ range factor correction for $x0.1$ or $x0.01$ ranges, zero for $x1$ range
E_1 through $E_6=$ nominal dial readings for the decades
ϵ_1 through $\epsilon_6=$ total deviations from linearity of the dial settings.

The above calibration procedure is suitable for a "simple" potentiometer with one working battery. When the battery current is branched (split-circuit), for example in the Wenner voltage-difference potentiometer [1], the ratio of the currents in the two branches must be precisely adjusted before comparing a decade in one branch with a decade in the other and checked periodically during this intercomparison.

Instruments with more than one potentiometer in tandem, for example the White potentiometer [4, 5, 6] and other more recent designs [3], require precise standardization of the currents when the decades between two potentiometers are compared.

For application of universal ratio sets to potentiometer calibration see NBS Monograph 39 by Brooks [2].

References to Appendix I

[1] Behr, L., The Wenner potentiometer, Rev. Sci. Instr. **3**, No. 3, 109–120 (1932).

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APPENDIX J. Guide for Obtaining the Calibration of Standard Platinum Resistance Thermometers at the National Bureau of Standards

The following guides are intended for users and those immediately connected with the application of SPRT's who are interested in obtaining calibration services at the NBS or who are planning to purchase SPRT's.

The SPRT calibration equipment and procedures employed by NBS are for four-terminal thermometers that meet the IPTS−68 specification of quality (see sec. 2) and that are constructed within certain external physical dimensions and immersion requirements. To obtain calibration services within the regular time schedule, the SPRT's must conform to these specifications. Any deviations from these specifications will result in additional cost, time, or rejection from calibration, e.g., the calibration of an over-sized SPRT is prohibitively expensive.

1. *Inquiries and Purchase Orders for the SPRT Calibration Service*

1.1. Inquiries and purchase orders shall be directed to the attention of:

Temperature Section, Heat Division
B04 Physics Building
National Bureau of Standards
Washington, D.C. 20234

1.2. Attention is called to paragraphs (d), (e), and (f) of Section 200.100 in NBS Special Publication 250, Calibration and Test Services of the National Bureau of Standards (1970 edition) which deal with shipping, insurance, and risk of loss or damage of instruments sent to NBS for calibration and test. These three paragraphs are quoted below for convenience of reference.

> (d) Shipping and insurance coverage instructions should be clearly and legibly shown on the purchase order for the calibration or test. The customer must pay shipping charges to and from NBS; shipments from NBS will be made collect. The method of return transportation should be stated and it is recommended that return shipments be insured, since NBS will not assume liability for their loss or damage. For long-distance shipping it is found that air express and air freight provide an advantage in reduction of time in transit. If return shipment by parcel post is requested or is a suitable mode of transportation, shipments will be prepaid by NBS but without covering insurance. When no shipping or insurance instructions are furnished, return shipment will be made by common carrier collect and uninsured.
>
> (e) NBS will not be responsible for the risk of loss or damage to any item during shipment to or from the Bureau. Any arrangements for insurance covering this risk must be made by the customer. Return shipment will be made by the Bureau as indicated in paragraph (d) of this Section. The purchase order should always show the value of the equipment and if transit insurance is carried by the customer this fact should be stated.
>
> (f) The risk of loss or damage in handling or testing of any item by NBS must be assumed by the customer, except when it is determined by the Bureau that such loss or damage was occasioned solely by the negligence of bureau personnel.

1.3. The user of the SPRT shall assume the responsibility to provide the following information either directly or through his procurement group. If any one item of the requested information is lacking, the calibration will be delayed. To minimize the "paper work," all necessary information should be forwarded in a single communication.

1.3.1. Manufacturer, model number, and serial number of the SPRT.

1.3.2. The temperature range in which the SPRT will be employed.

1.3.3. Specific calibration desired. (See under section 4 the list of test services that are available.)

1.3.4. Specify the type and quantity of thermometer calibration tables desired. (See under section 4.3 the list of tables that are available.)

1.3.5. If more than one type of calibration service, e.g., the calibration of standard resistors is also being purchased from NBS on the same purchase order, enough copies of purchase orders should be prepared so that one copy can accompany each type of instrument to be tested.

1.3.6. If insurance of the return shipment is desired it must be requested. Such insurance is recommended.

2. *Shipment of SPRT to NBS for the Calibration Service*

2.1. Direct shipments of SPRT by common carrier to the attention of:

Temperature Section, Heat Division
B04 Physics Building
National Bureau of Standards
Route 70S and Quince Orchard Road
Gaithersburg, Maryland 20760

2.2. Precautions should be exercised to protect the SPRT from mechanical shock that would alter its calibration. In shipment, the SPRT must be softly supported within a rigid case but must not be free to "rattle" within the case. The packing material must, therefore, be resilient and not become compacted. An outer case must be sufficiently rigid and strong to withstand the treatment usually given by shippers. *Styrofoam is not sufficiently rigid to be used as an outer case.* The SPRT's will not be returned in containers that are obviously unsuitable, such as those closed by nailing; a wooden box should be closed with screws. Suitable containers will be provided, for a fee, whenever the SPRT shipping container is not satisfactory for re-use. *The user shall assume the responsibility of instructing his shipping department concerning packaging and the desired carrier for shipping his SPRT.*

Two methods suitable for packaging the SPRT for shipment are illustrated in figures 4 and 5 of section 4.7.

3. *Requirements to Qualify for Regular Calibration Schedule*

3.1. *Quality:*

The SPRT must be a four-terminal resistor that conforms with the IPTS–68 specifications for construction *practice* and quality (see Appendix A).

3.2. *Dimensions:*

3.2.1. *Long stem type SPRT*

3.2.1.1. The maximum length of the resistance element shall be 5 cm. The entire element must be located near one end of the SPRT.

3.2.1.2. The maximum outside dimension of the SPRT sheath shall be 7.52 mm (0.296″) for a minimum length of 46 cm (18″).

3.2.1.3. The maximum overall length of glass-sheathed SPRT's shall be 1 meter. The maximum overall length of SPRT's that have sheaths of thin metal is limited by the height to the ceiling of the NBS calibration laboratory; SPRT's up to 4 meters can be accommodated.

3.2.1.4. The suitability of the SPRT for calibration at NBS also depends upon the immersion characteristics of the thermometer. Although the thermometer may meet the preceding requirements of dimension, the immersion characteristics of the thermometer may be too poor for calibrations that employ the existing equipment at NBS.

3.2.2. *Capsule type SPRT*

3.2.2.1. The maximum length of the resistance element shall be 5 cm. The element must be located at one end of the SPRT.

3.2.2.2. The maximum over-all length of the SPRT shall be 8 cm.

3.2.2.3. The sheath, at the end that contains the resistant element, shall be reasonably uniform and shall not exceed 5.72 mm (0.225″) outside diameter for a minimum length of 5 cm and maximum length of 6 cm.

3.2.2.4. The outside diameter of any part of the SPRT, including the head where the four leads emerge through a hermetic seal, shall not exceed 9.6 mm (0.38″); the length of the enlarged section shall not exceed 2 cm.

4. *Calibration Services*

4.1. *Long stem SPRT*

4.1.1. Calibration at the triple point of water, tin point, and zinc point. A table will be furnished with entries at 1-degree intervals between −50 and +500 °C.

4.1.2. Calibration at the triple point of water, tin point, zinc point, and oxygen normal boiling point. A table will be furnished with entries at 1-degree intervals between −183 and +500 °C.

4.1.3. Thermometers which will withstand temperatures above 500 °C, such as quartz sheathed thermometers, calibrated under 4.1.1 or 4.1.2 will be furnished tables to 631 °C.

4.2. *Capsule type SPRT*

The capsule type SPRT's are not generally calibrated above the tin-point temperature at the NBS. Because of the uncertain electrical insulation characteristics between the leads at the zinc point, any calibration performed with the capsule type SPRT's at the zinc point will not be of as high a quality as is obtained with the long stem SPRT's. Instead, the thermometer constant B is assumed on the basis of the values of B for other SPRT's calibrated at NBS during the past few years.

4.2.1. Calibration at the triple point of water and tin point. A table will be furnished with entries at 1-degree intervals between −50 and +250 °C.

4.2.2. Calibration at the triple point of water, tin point, and oxygen normal boiling point. A table will be furnished with entries at 1-degree intervals between −183 and +250 °C.

4.2.3. Calibration at the triple point of water, the tin point and a comparison calibration between 13 and 90 K. A table will be furnished with entries at 0.1-degree intervals between 13 and 90 K and at 1-degree intervals between 90 and 600 K.

4.3. *Tables*

The user has the choice, as a part of the calibration service, as to the form of table for his thermometer.

4.3.1. The argument (the temperature) can be °C, °F, or K.

4.3.2. The function can be the resistance ratio $R(t)/R(0)$, the resistance $R(t)$, or the resistance difference $R(t)-R(0)$. For most applications the resistance ration $R(t)/R(0)$ versus temperature is recommended.

4.3.3. If no specific request is made, the table furnished for the long stem SPRT will be the resistance ratio $R(t)/R(0)$ versus temperature in °C. The table furnished for the capsule type SPRT will be the resistance versus temperature in K whenever the comparison calibration between 13 and 90 K is requested; otherwise the table furnished will be the resistance ratio $R(t)/R(0)$ versus temperature in °C.

5. *Non-Standard Platinum Resistance Thermometers*

A very special configuration of platinum resistance thermometer may be desired for certain applications. For the calibration of such thermometers the user may contact the NBS (see 1.1.) concerning the possibility of a calibration at NBS or advice on the calibration of the thermometer in the user's laboratory employing a SPRT that has been calibrated at the NBS. The information on dimensions and wiring configuration should be furnished with the inquiry. If the calibration can be performed at NBS, the fee for the special calibration will be furnished.

Calorimetric type platinum resistance thermometers, which have the resistance element wound on a flat mica sheet and sandwiched between two mica sheets and then inserted into a flattened tube, do not meet the specifications of the IPTS–68. A calibration of these thermometers may be furnished by NBS through special arrangement.

Inquiries concerning the calibration of other resistance thermometers, such as germanium resistance thermometers, are welcomed. The NBS will attempt, whenever possible, to meet reasonable calibration requests of services; the cost of such services will be negotiated. Appropriate discussion should take place directly between the user and the NBS technical staff.

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16. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)

The monograph describes the methods and equipment employed at the National Bureau of Standards for calibrating standard platinum resistance thermometers (SPRT) on the International Practical Temperature Scale (IPTS-68). The official text of the scale is clarified and characteristics of the scale are described. Several designs of SPRT's are shown and discussed in the light of the requirements and recommendations on the text of the IPTS-68. Possible sources of error such as those due to the internal and external self-heating effects and the immersion characteristics of SPRT's are described in detail. Precautions and limitations for the mechanical and thermal treatment and for the shipment of SPRT's are indicated, and a guide is given for those desiring the thermometer calibration services of NBS. The description of equipment employed at the National Bureau of Standards for maintaining the IPTS-68 includes the triple point of water cell, tin point cell, zinc point cell, oxygen normal boiling point comparison cryostat, the 13 to 90 K comparison cryostat, and the reference SPRT's upon which the NBS-IPTS-68 in the region 13 to 90 K is based. Methods are given for calculating temperatures from the calibration data and observed resistances; the propagation of calibration errors is discussed. Supplemental information given in the Appendices includes the authorized English version of the text of the IPTS-68, tabular values of the "reference function" used below 0 °C, tublar values of the differences between IPTS-68 and IPTS-48, analysis of the first derivatives at 0 °C of the IPTS-68 formulations, methods for calibrating potentiometers and Mueller bridges, and the derivation of the coefficients used in the analysis of error propagation.

17. KEY WORDS (Alphabetical order, separated by semicolons) Calibration; calibration errors; cryostat; fixed points; freezing point; International Practical Temperature Scale; platinum resistance thermometer; Thermodynamic Temperature Scale; thermometry; triple point.

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