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Information Section
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THE STANDARDIZATION OF INDUCTORS AT RADIO FREQUENCIES

(This Circular is not complete without Letter Circular, "The Secondary Standardization of Radio Wavemeters," referred to herein as LC 75).

The purpose of this Circular is to describe in some detail the measurement of the apparent inductance at any radio frequency of a fixed or variable inductor, its pure inductance and effective capacity over any range of radio frequencies, and its resistance at any radio frequency. As the basis of these measurements, it is assumed that standards of frequency (wave length), capacity, and resistance are already available. The apparatus required is as follows: a variable condenser whose capacity at the frequencies employed is known for every scale reading and whose effective resistance at these frequencies and scale readings is either known or is negligible in comparison with the resistance of the coil; a wavemeter standardized in terms of frequency or of wave length; a thermoelement and wall galvanometer, current-square meter, or radio-frequency ammeter; several non-inductive resistors of known resistance; and a source of undamped, unmodulated waves. If the resistance of the coil is not to be measured, the resistance of the condenser need not be known, the resistors are not used, the current-measuring instrument may be replaced by any device capable of indicating resonance, and a source of damped or modulated waves may be used if it is constant in frequency and power.

The method of measurement is the same for a variable inductor as for a fixed inductor. Separate descriptions are hence unnecessary. When the terms "apparent inductance," "effective capacity," etc., are used it will be understood that in the case of a variable inductor "apparent inductance at the given scale reading" etc. are meant.

The observations to be made are the same whether the apparent inductance at various specified frequencies or the pure inductance and effective capacity are to be measured. They consist simply of the readings necessary for standardization in terms of frequency or of wave length of the circuit composed of the standardized condenser, the inductance to be measured, and connecting leads spaced well apart and as short

as is possible without bringing the coil to within about ten centimeters of the condenser. With this circuit is combined the device used to indicate resonance. The device must be combined in such a way as to cause no appreciable change in the capacity or inductance of the circuit.

If the inductor to be measured is not originally provided with a mounting of some kind, it should be supported during the measurement in such a way as to introduce as little imperfect dielectric as possible into its immediate field. In the case of a light coil it will often be practicable to suspend it from the leads joining it to the condenser.

Standardization as to frequency or wave length is effected by tuning the generator to resonance with the circuit which contains the inductor being measured (the condenser being set at any convenient reading) and measuring the frequency (wave length) of the generator by means of the standard wavemeter. The precautions to be taken in this procedure are described in LC 75.

The apparent inductance at any frequency, the pure inductance, and the effective capacity are determined by means of the following equations:

$$(1) \qquad L_a = \frac{25330 \times 10^6}{f^2 C} \qquad\qquad L_a = \frac{0.2818 \lambda^2}{C} \qquad (2)$$

$$(3) \qquad C_o = \frac{C_2 f_2{}^2 - C_1 f_1{}^2}{f_1{}^2 - f_2{}^2} \qquad\qquad C_o = \frac{C_2 \lambda_1{}^2 - C_1 \lambda_2{}^2}{\lambda_2{}^2 - \lambda_1{}^2} \qquad (4)$$

$$(5) \qquad L_p = \frac{25330 \times 10^6}{f^2 (C+C_o)} \qquad\qquad L_p = \frac{0.2818 \lambda^2}{(C+C_o)} \qquad (6)$$

Equations (2), (4), and (6) are equivalent respectively to equations (1), (3), and (5), the difference between the two sets being that one involves f, the frequency in kilocycles per second; the other λ, the wave length in meters. The other quantities appearing in the equations are as follows: L_a, the apparent inductance in microhenries; C, the condenser capacity in micromicrofarads which in series with the inductance L_a forms a circuit resonant to the frequency f or the wave length λ; C_1 and C_2, the capacities corresponding similarly to f_1 and f_2 or λ_1 and λ_2; C_o, the effective capacity in micromicrofarads; L_p, the pure inductance in microhenrics.

It is seen from equations (3), (4), (5), and (6) that two observations are necessary and sufficient for a determination of C_o and L_p. A determination made from only two observations can not, however, be depended on. Six or more observations should be made. These can be paired in any convenient way and made to yield three independent values of C_o. If the disagreement among these three values is not too great, their average is taken as the effective capacity of the coil. The substitution of this average and the values of capacity and frequency or wave length in equation (5) or (6) gives six partly interdependent values of L_p. If these agree nearly enough, their average is taken as the pure inductance of the coil.

A much easier and sometimes more accurate method of determining the effective capacity from the observations taken is a graphical solution based on the following considerations[1]

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[1] Circular 74 of the Bureau of Standards, "Radio Instruments and Measurements," p.138

Equations (5) and (6) are readily put in the forms:

$$L_p(C+C_o)=(25330 \times 10^6)\frac{1}{f^2} \quad (7) \qquad L_p(C+C_o)= 0.2818 \ \lambda^2 \quad (8)$$

It is evident from these equations (7) and (8) that C bears a linear relation to $\frac{1}{f^2}$ and to λ^2; that is, if C is plotted horizontally against $\frac{1}{f^2}$ or λ^2 plotted vertically, the points representing the different observations will all lie on the same straight line. Now if $\frac{1}{f^2}$ or λ^2 is to become zero (it is to be understood that the discussion here is simply mathematical; no physical significance is to be attached to zero wave length) the right hand member of each equation and consequently the left hand member must become zero. Hence C must equal $-C_o$. (Again no physical meaning is intended. The interpretation of the mathematical processes used is to be found in the next sentence). That is, observed values of C and $\frac{1}{f^2}$ or λ^2 are plotted as already indicated, a straight line is drawn through the plotted points, and the point in which this line intersects the horizontal axis indicates the effective capacity. The effective capacity is given, in terms of the scale to which C is plotted, by the distance at which this point lies to the left of the vertical axis.

Since actual observations will not be free from errors, the plotted points will not lie precisely on a straight line. The degree of inaccuracy of the observations will be indicated

by the dispersion of these points about a straight line drawn among them in such a way that the dispersion is as small as possible and about the same on the two sides of the line. Points far out of alignment with the rest should be disregarded in drawing the line and the corresponding readings should be discarded in calculating the pure inductance. The value of condenser capacity used should be chosen small enough that the observations can be plotted on a large coordinate sheet to a scale so open that the effective capacity will be readable to within the error of measurement. The capacity should, however, not be made small enough to bring the natural frequency of the circuit anywhere near the natural frequency of the coil, for in this region the equations are apt to break down, the $(C, \frac{1}{f^2})$ graph, or the (C, λ^2) graph ceasing to be a straight line. Except for errors in the calibration of the standard instruments, it may safely be assumed that the error of measurement is as small as the capacity represented by the greatest horizontal distance from the $(C, \frac{1}{f^2})$ or (C, λ^2) line of any of the points which determine it. And this dispersion will include errors in the calibration of the standards also, unless they are of special kinds. If two different condensers are used in turn in the circuit containing the coil under test, it may in general be safely assumed that the greatest horizontal distance from the $(C, \frac{1}{f^2})$, or (C, λ^2) line among not less than six points is greater than the possible error of the result.

If equation (3) or (4) is used, the choice of condenser capacity is the same as in the graphical method; that is, it should be as small as possible without bringing the frequency of the resonant circuit anywhere near the natural frequency of the coil.

The accuracy of the inductance measurement will be largely determined by the accuracy with which frequencies or wave lengths can be measured. Since L_p is linearly related to $\frac{1}{f^2}$ and λ^2 rather than to $\frac{1}{f}$ and λ, the percentage of error in measuring f or λ will be doubled in measuring L_p.

In standardizing a variable inductor, the required measurements are made at enough scale readings to make it possible to draw a calibration curve for the instrument. The points on the scale mentioned in LC 75 as suitable for the standardization of a wavemeter are also suitable for the standardization of an inductor.

The resistance of an inductor can be measured either by the resistance-variation method or by the reactance-variation method

as described in LC 75. The inductor is joined by short, well-spaced leads to a standard condenser having negligible or known resistance. A thermoelement and wall galvanometer, a current-square meter, or a radio-frequency ammeter is inserted in the circuit. From the measured resistance of the circuit at the required radio frequency is subtracted the direct-current resistance of the leads and current-measuring device together with the resistance of the condenser if that is appreciable. The remainder is taken as the resistance of the inductor.

It has thus far been assumed that it is possible to induce an alternating electromotive force directly in the coil under test. Some coils are, however, shielded, coupling thus being prevented, and others are so wound that it is not possible to couple them closely enough to a generator to induce an appreciable current in them. In testing such coils, the following method may sometimes be used to advantage.

A circuit is formed of a variable condenser, an inductor (not the one to be measured; it is introduced later), a current measuring instrument, and the necessary leads. This circuit is tuned to resonance with a generator of undamped, unmodulated waves of frequency $f = \frac{\omega}{2\pi}$. The resistance, R_1, of this circuit is measured by the resistance-variation method or the reactance-variation method and the capacity at resonance, C_1, of the condenser is noted. The coil to be measured is then connected across the terminals of the condenser. Care must be taken that no electromotive force is induced in this coil by the generator. The circuit is again tuned to resonance, the condenser capacity, C_2, is noted, and the resistance, R_2, of the circuit is measured. The characteristics of the coil are then given by the equations:

$$(9) \qquad R = \frac{D}{\left(\frac{d}{C_1}\right)^2 + (C_2 \omega D)^2}$$

$$(10) \qquad X \equiv \frac{1}{C_2 \omega} \left[1 + \frac{\frac{d}{C_1}}{\left(\frac{d}{C_1}\right)^2 + (C_2 \omega D)^2} \right]$$

In these equations

$d \equiv C_2 - C_1$ $\qquad$ $D \equiv R_2 - R_1$

R is the resistance of the coil under test and X its reactance at the frequency f. Hence

$$X = L_a \omega = L_p \omega - \frac{1}{C_o \omega}$$

R, R_1 and R_2 are in ohms, C_1, C_2, and C_o are in farads, L_a and L_p are in henries, and ω is 2π times the frequency in cycles per second.

The usefulness of this method is impaired by the fact that certain relations between the quantities involved are unfavorable to accurate measurement. For example, if C_1 and C_2 or R_1 and R_2 are near together, the error of measurement may be as large as or larger than d or D. In other cases it will be found that C_2 or R_2 is so large that serious difficulties are introduced in the measurement.

The characteristics of a coil with an alternating electromotive force applied to its terminals may be found to be appreciably different from the characteristics of the same coil with an electromotive force induced in it directly.

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