X-BAND ATMOSPHERIC ATTENUATION FOR AN EARTH TERMINAL MEASUREMENT SYSTEM

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The National Institute of Standards and Technology has developed an Earth Terminal Measurement System to be used by the Camp Parks Communications Annex in determining satellite effective isotropic radiated power and antenna gain. In determining these quantities the effects of atmospheric attenuation must be taken into account. This paper provides an overview of the methods used for determining atmospheric attenuation with emphasis on a tipping-curve method. An error analysis is also provided.

Key words: antenna gain; atmosphere; atmospheric attenuation; error analysis; satellite communications; satellite effective isotropic radiated power (EIRP).

1. Introduction

The Electromagnetic Fields Division of the National Institute of Standards and Technology (NIST) has developed an Earth Terminal Measurement System (ETMS) for the Camp Parks Communications Annex of the Department of Defense. The ETMS is designed to determine G/T for the earth terminal, satellite effective isotropic radiated power (EIRP), and the gain of the ETMS antenna at its output feed [1]. The ETMS is designed to operate in the X-band from 7 to 10 GHz. In determining these quantities the ETMS uses a calibrated radio star as a signal source. The determination of G/T, EIRP, and antenna gain requires that the measurement data be corrected for the effects of the atmosphere on the radio star flux and the satellite signal. The methods of correcting for the atmospheric attenuation effects and the errors associated with these methods are the subject of this report.
1.1 Attenuation Mechanisms

Two principal effects on a radio signal at frequencies of 7 to 10 GHz, traveling through the atmosphere, are gaseous attenuation and hydrometeor attenuation [2].

Gaseous attenuation is the process in which gas molecules absorb part of the radio signal and thereby reduce its amplitude. The main sources of gaseous absorption in the frequency band from 7 to 10 GHz are oxygen molecules (O$_2$) and water vapor (H$_2$O). Determining the level of absorption due to water vapor can be a challenge since the water vapor content of the atmosphere varies considerably with the atmospheric conditions. The level of absorption due to water vapor is especially sensitive to variations in temperature and relative humidity. However, it is relatively straightforward to determine the level of absorption due to oxygen, since the atmospheric content of oxygen changes very little.

Hydrometeor attenuation is a reduction in the radio signal amplitude due to scattering by hydrometeors. Hydrometeors are particles formed by the condensation of water vapor in the atmosphere and include rain, clouds, fog, snow, and ice [2]. Snow and ice are usually not important at frequencies below 30 GHz.

In addition to the effects of gaseous attenuation and hydrometeor attenuation there are scintillation effects. Scintillation is a fluctuation in the amplitude (and phase) caused by small atmospheric irregularities along the transmission path. Dry scintillations are associated with the passage of cumulus clouds [3] and cause mean signal variations of a few tenths of a decibel. Scintillation lasts longer and occurs more frequently in the presence of rain than in its absence. Scintillation in the presence of rain is called wet scintillation.
Scintillation can be difficult to correct for. For example, scintillation causes a significant change in the mean temperature along the transmission path especially for small sky brightness temperatures [3].

1.2 Theoretical Determination

Much effort has been expended in the study of atmospheric propagation [4]. Daywitt [5] derived theoretical equations to be used by the ETMS in estimating the effects of the atmosphere. Considerable work has been done since then to improve upon the values of the molecular absorption coefficients [6-10].

Shaw of the Institute of Telecommunications Sciences (ITS) developed a FORTRAN program in 1981 based on the work of Liebe [6] to calculate the effects of the atmosphere on the radio signal. The author has obtained this program from Liebe and updated it to include Liebe's more recent results [7-10]. This program will be used in estimating some of the errors.

Despite these improvements, the uncertainties in the atmospheric loss remain close to 50%, as found by Daywitt [5], primarily because there has been little reduction in the uncertainties of various atmospheric quantities such as the temperature, pressure, and water vapor profiles. The effects of water vapor, cloud droplet concentrations, and rain are the major problems.

A 50% uncertainty is not serious in the clear sky case because the zenith attenuation remains below 0.05 dB at 8 GHz. The uncertainty in the zenith attenuation is estimated to be less than 0.03 dB. However, when clouds are present the zenith attenuation may exceed 0.5 dB; in the presence of rain the
zenith attenuation can exceed 1 dB. The uncertainties associated with these conditions lead to unacceptably large errors in the determination of G/T, EIRP, and antenna gain. It is desirable, therefore, to determine the atmospheric attenuation by measurement.

1.3 Measurement

There are several methods for determining the atmospheric attenuation using measurement techniques. Direct attenuation measurements can be made by observing the extinction of an extraterrestrial source (such as the sun) as a function of zenith angle or elevation angle [11]. The attenuation can also be measured by measuring the sky brightness temperature as a function of elevation [11]. Since radiative equilibrium and local thermodynamic equilibrium hold for the troposphere, where the attenuation of X-band signals occurs, the atmosphere must reradiate whatever it absorbs and thus contribute to the sky brightness. Finally, a dual channel radiometer can be used to determine the amount of total water vapor along a ray path (Guiraud et al. [12]).

**Direct attenuation measurement**

In the direct attenuation measurement, which can take hours, the antenna is pointed at an extraterrestrial source and the antenna temperature measured as a function of the zenith angle. If the attenuation is measured in decibels then the antenna temperature in K is given by [11]

\[
T_a(\theta) = T_a' 10^{-0.1a_0 \csc \theta} + T_s(\theta),
\]  

(1)
where $T_a(\theta)$ is the temperature of the source at the antenna, $T'_a$ is the temperature of the source in the absence of the atmosphere, $T_s$ is the apparent sky temperature, $a_0$ is the zenith attenuation, and $\theta$ is the elevation angle. If $T_s$ is balanced out or is negligible then (1) can be written as

$$10 \log T_a(\theta) = -a_0 \csc\theta + 10 \log T'_a.$$  \hspace{1cm} (2)

Therefore if $10 \log T_a$ is plotted against $\csc\theta$ the slope is equal to the zenith attenuation.

**Sky brightness measurement**

The zenith attenuation can also be determined by measuring the brightness temperature of the sky and using values of the mean atmospheric temperature along the transmission path as derived from atmospheric models. For this case the zenith attenuation is given by Hogg et al. [13] as

$$a_0 = 10 \log [(T_m - T_c)/(T_m - T_z)],$$  \hspace{1cm} (3)
where $T_m$ is the mean temperature along the transmission path, $T_c$ is the cosmic background temperature, and $T_z$ is the brightness temperature in the zenith direction. $T_c$ and $T_z$ are obtained by measurement while $T_m$ is determined from atmospheric models and is weakly model dependent with a value of about 270 K.

Dual channel radiometer

Guiraud et al. [12] showed that the total integrated water vapor and liquid water along the transmission path can be determined by measuring the brightness at two frequencies of millimeter waves, 20.6 and 31.6 GHz. The dual channel radiometer is calibrated against another method of determining integrated water vapor and liquid water (usually radiosonde). This needs to be done for each geographical location. For example, for Denver CO, Guiraud et al. found that the total integrated water vapor, $V$ (in cm), along the transmission path is given by

$$V = -0.19 + 0.118T_{b1} - 0.0560T_{b2}, \quad (4)$$

and the total integrated liquid water, $L$ (in cm), is

$$L = -0.018 - 0.00114T_{b1} + 0.00284T_{b2}. \quad (5)$$
In equations (4) and (5) $T_{b1}$ is the brightness temperature of the sky at 20.6 GHz in the direction of the transmission path and $T_{b2}$ is the brightness temperature at 31.6 GHz. The attenuation can be determined by combining the values for $V$ and $L$ with theoretical absorption coefficients given in dB/cm. A limitation exists for this technique. At 20.6 and 31.6 GHz radiometers saturate (that is, $T_b$ approaches the ambient temperature) during conditions of medium to heavy rainfall. As a result reliable observations can only be guaranteed in the presence of clouds that do not produce rain [14, 15]. Hence, the dual frequency radiometer technique cannot be used to determine the attenuation during rain conditions.
2. Radiative Transfer in an Atmosphere

The derivative of the specific intensity, $I$, of an electromagnetic signal traveling through an atmosphere that emits, absorbs, and scatters is given by (Chandrasekhar [16])

\[
\frac{dI(s)}{ds} = -\alpha(s)I(s) + \alpha(s)J(s),
\]  

(6)

where $\alpha(s)$ is the absorption coefficient, $J$ is the source function describing the contribution due to emission and scattering, and $s$ is the path length. When local thermodynamic equilibrium (LTE) applies, as is the case for the troposphere (Westwater [17]) then the source is equal to the Planck function, $B(T)$. When the Rayleigh-Jeans approximation also applies (that is, when $h\nu \ll kT$) as is the case for microwave frequencies then

\[
B(T) = \frac{(2kT)}{\lambda^2},
\]  

(7)

where $k$ is Boltzmann's constant and $\lambda$ is the wavelength. The solution to equation (6) is $I = (2kT)/(\lambda^2)$, from which we get
\[ T_b = T'_b \exp \left[ - \int_0^{\infty} \alpha(s') ds' \right] + \int_0^{\infty} T(s) \alpha(s) \exp \left[ - \int_s^{\infty} \alpha(s') ds' \right] ds, \] (8)

where \( T_b \) is the brightness temperature of the extraterrestrial source at the antenna, \( T'_b \) is the brightness temperature of the extraterrestrial source (for example, the sun, the moon, the cosmic background) with no intervening atmosphere, and \( T(s) \) is the atmospheric kinetic temperature.

For a plane parallel atmosphere, which is a good approximation for the earth's atmosphere provided the elevation angle is greater than about 15° (the zenith angle is less than about 75°), \( s \) can be replaced by \( s = h \csc \theta \), where \( h \) is the height in the atmosphere. In addition, if we replace \( T(s) \) by a mean atmospheric temperature, \( T_m \), (see for example, Falcone et al. [11]), equation (8) becomes

\[ T_b(\theta) = T'_b \cdot 10^{-a_0 \csc \theta / 10} + T_m \left[ 1 - 10^{-a_0 \csc \theta / 10} \right], \] (9)

where \( a_0 \) is the attenuation in the zenith direction in decibels.
3. Sky Brightness Tipping-Curve Method

3.1 Equations

If we wish to look at the sky and determine the sky brightness, we want to look in directions that have only the cosmic background as an extraterrestrial radio source. For this case equation (9) becomes

\[ T_s(\theta) = T_c \ 10^{(-a_0 \csc \theta / 10)} + T_m \left[ 1 - 10^{(-a_0 \csc \theta / 10)} \right], \]  

Equation (10)

where \( T_s \) is the apparent sky brightness temperature. If we put all the terms involving \( a_0 \) on one side of the equation and then take 10 times the logarithm of both sides of the equation we get

\[ a_0 \csc \theta = 10 \log \left[ \frac{(T_m - T_c)}{(T_m - T_s(\theta))} \right]. \]  

Equation (11)

Equation (11) can be written as the equation of a line if we make the substitutions

\[ y = 10 \log \left[ \frac{(T_m - T_c)}{(T_m - T_s(\theta))} \right], \]
\[ x = \csc \theta. \]

Equation (11) then becomes

\[ y = a_0 x + b, \]

where \( b \) is nominally equal to 0 (that is, in the absence of measurement errors \( b = 0 \)).

Thus if we plot the value of \( (T_m - T_c)/(T_m - T_s(\theta)) \) in dB against the value of \( \csc \theta \) we should get a straight line. We can therefore make measurements at many values of the elevation angle and fit a least-squares line to the results; the resulting slope is the zenith attenuation. Once the zenith attenuation is known the attenuation in the direction of interest can be determined by \( a(\theta) = a_0 \csc \theta \) provided that the weather conditions are similar to those in the directions of the sky brightness data. This is known as the tipping-curve method. The advantage of the tipping-curve method is that for a least-squares fit, errors that are constant with \( \theta \) do not cause errors in \( a_0 \) since they affect the intercept and not the slope. Note from equation (11) that as \( T_s(\theta) \to T_m \), that is as the sky brightness temperature approaches the mean atmospheric temperature, we get saturation and it becomes impractical to determine \( a_0 \) using this method. For
microwave frequencies $T_m = 270 \text{ K}$ [12] so that saturation occurs for an elevation angle of $\theta = 15^\circ$ when $a_o \approx 3 \text{ dB}$. This occurs for X-band only if it is raining at a rate of greater than about 50 mm/h (about 2 in/h).

### 3.2 Error Analysis for Sky Brightness Tipping-Curve Method

To estimate the error associated with the tipping-curve method as applied to sky brightness measurements we can take the differential of equation (11) and then evaluate the individual terms. We then find after some rearrangement of terms that the uncertainty $\delta a$ in the attenuation is

$$
\delta a(\theta) = 4.343 \left[ \frac{T_m(T_s(\theta) - T_c)}{(T_m - T_c)(T_m - T_s(\theta))} \cdot \frac{\delta T_m}{T_m} + \frac{T_c}{(T_m - T_s(\theta))} \cdot \frac{\delta T_c}{T_c} +
\right.
$$

$$
\left. + \frac{T_s(\theta)}{(T_m - T_s(\theta))} \cdot \frac{\delta T_s}{T_s} \right] + a_0 \cot \theta \delta \theta.
$$

(12)

Due to the least-squares fit, errors which are independent of $\theta$ do not affect $a_0$. Therefore, since $T_c$ and its error are independent of $\theta$, $\delta T_c = 0$, and the second term in the brackets of equation (12) can be neglected. We will assume that the uncertainty $\delta \theta$ in the pointing is approximately 20% of the half-power beamwidth.
A value of $T_m = 270$ K is used by Guiraud et al. [12] for moderate humidity conditions at a frequency of 21 GHz. This is based on the US Standard Atmosphere. Changes in the ground temperature, the relative humidity, and non-uniform stratification effects would cause the value of $T_m$ to change. Falcone et al. [11] used a value of $T_m = 1.12 T_g - 50$, where $T_g$ is the ground temperature. For the US Standard Atmosphere conditions Slobin [18] finds that using a two component model for $T_m$ gives better results and the best temperatures are 265 K for the oxygen component and 280 K for the water vapor component. From this we would expect that for a US Standard Atmosphere and 0% humidity $T_m \approx 265$ K. Because the ratio of the water vapor absorption to the oxygen absorption coefficient at X-band is less than at K-band we expect that $T_m$ will be closer to the average temperature of oxygen at X-band than at 21 GHz. To estimate the value of $T_m$, Shaw's program was modified to calculate the value of $T_m$ along the transmission path. Our results are that $T_m \approx 265$ K would be a better value for the mean atmospheric temperature for X-band. There are several variables that contribute to the uncertainty in $T_m$. They are uncertainties in (1) the dependence of $T_m$ on ground temperature, (2) the dependence of $T_m$ on the relative humidity at ground level, (3) the atmospheric temperature and density profiles, and (4) non-uniform stratification. Only uncertainty (4) varies with the antenna pointing direction.

To estimate the uncertainties in $T_m$ due to nonuniform stratification we can calculate $T_m$ using our computer model for different conditions. The worst case occurs when most of the sky is clear with a thick cloud in one direction. Our computer model predicts a maximum error of about $\pm 10$ K in $T_m$ for this case.
A word of caution: in the presence of scintillation the value of $T_m$ can be considerably reduced even for rms variations of only 0.2 dB especially if the sky brightness is below 100 K [3]. In extreme cases the mean atmospheric temperature can be reduced below 200 K. In this error analysis we will assume measurements will be avoided at such times.

Determining the uncertainty in $T_s$ is a little more complicated. A simple radiometer system measures the output temperature for two conditions. In the first condition we consider the situation where the sum of the signals from the antenna and the reference source is measured. Then the output temperature, $T_{out}^{(1)}$, is

$$T_{out}^{(1)} = T_{ant}(\theta) \eta + T_{Amb}(1-\eta) + T_e + T_n.$$  \hfill (13a)

For the second condition we consider the situation where only the reference noise source is measured. Then the output temperature, $T_{out}^{(2)}$, is

$$T_{out}^{(2)} = T_e + T_n.$$  \hfill (13b)

where $T_{ant}$ is the antenna temperature at the antenna output feed, $\eta$ is the transmission efficiency from the antenna output feed to the receiver input, $T_{Amb}$
is the ambient temperature, \( T_e \) is the effective receiver noise temperature, and \( T_n \) is the reference noise temperature. The output \( y \) of a radiometer is then given by [3]

\[
y = H \frac{(T_{\text{out}}^{(1)} - T_{\text{out}}^{(2)})}{T_{\text{out}}^{(1)}},
\]

where \( H \) is the receiver gain factor which can be determined using a calibrated precision IF attenuator. We solve for \( T_{\text{ant}} \) in terms of \( y, \eta, H, T_n, T_e, \) and \( T_{\text{Amb}} \), we get

\[
T_{\text{ant}}(\theta) = \frac{y(T_n + T_e)}{H\eta} - \frac{(1-\eta)T_{\text{Amb}}}{\eta}.
\]

This indicates that \( T_{\text{ant}} \) is determined not only by the brightness temperature in the antenna pointing direction but also by contributions from other directions that occur in the side- and backlobes. Thus the sky temperature in the direction \( \theta \) is given by

\[
T_s(\theta) = \frac{T_{\text{ant}}(\theta)}{k(\theta)},
\]
where

\[ k(\theta) = \frac{\int T_b(\Omega) D(\Omega - \Omega_0(\theta)) \, d\Omega}{T_s(\theta) \int D(\Omega - \Omega_0(\theta)) \, d\Omega}, \]  

Equation (17) is approximately given by

\[ k(\theta) \approx \frac{T_s(\theta) \Omega_0 + 0.5(\Omega_a - \Omega_0)(\bar{T}_s + T_g)}{T_s(\theta) \Omega_a}, \]  

where \( \Omega_0 \) is the solid angle subtended by that part of the sky at \( T_s \) as described following equation (20), \( \Omega_a \) is the total solid angle subtended by the antenna, \( \bar{T}_s \) is the average sky brightness temperature, and \( T_g \) is the temperature of the ground. Ideally we would like the side- and backlobes to be low enough so that the contribution of the second term in the numerator of equation (18) will be small. Note that as \( \Omega_0 \rightarrow \Omega_a \), \( k \rightarrow 1 \) and \( T_{ant} \rightarrow T_s \). However, unless the capability to shape the beam exists, \( \Omega_0 \) does not normally approach \( \Omega_a \) so, unless \( T_s(\theta) \) is much greater than \( \bar{T}_s + T_g \), the second term in the numerator of equation (18) cannot be neglected. This condition on \( T_s(\theta) \) will not be met at any frequency.
Further, even at millimeter-wave frequencies \( \Omega_0 \) is unlikely to approach \( \Omega_a \) in value for the 18 m (60 ft) dish.

The peak antenna directivity is given by

\[
D_0 = \frac{4\pi (\text{peak power emitted per solid angle})}{\text{total radiated power}}.
\]  

If the peak power is \( P_0 \) then the total radiated power is \( P_0 \Omega_a \). Thus the peak directivity is given by

\[
D_0 = \frac{4\pi P_0}{P_0 \Omega_a} = \frac{4\pi}{\Omega_a}.
\]  

By assuming that the pattern for the antenna has the form

\[
\frac{(\sin(f\theta))^2}{(f\theta)^2},
\]

we can estimate the value of \( \Omega_0 \) and \( \Omega_a \) if we know the value of \( f \). In this equation \( \theta \) is the angle relative to the z-axis of the antenna. The value of \( f \) can be determined by knowing the value of the location of the first null. Then \( f \)
is given by \( f = \pi / \theta_{\text{null}} \). At 8 GHz the location of the first null for the Camp Parks 18 m (60 ft) dish is 0.16°. To obtain \( \Omega_0 \) we will integrate from 0 to 1.5°. We choose this range of \( \theta \) since \( T_s \) changes by about 10% over this range of \( \theta \) values. We find \( \Omega_0 \approx 1.15 \times 10^{-5} \) steradians, and \( \Omega_a \approx 2.18 \times 10^{-5} \). These numbers imply a peak directivity of 57.6 dB which is a little less than the value of the gain found for the Camp Parks 18 m (60 ft) dish. To get an improved value of \( k(\theta) \) detailed knowledge of the pattern is necessary. In addition, an initial guess for \( T_s \) is necessary. Using the above values for \( \Omega_0 \) and \( \Omega_a \) we find that \( k(\theta) \) is approximately

\[
k(\theta) \approx 0.54 + \left( \frac{0.23}{T_s(\theta)} \right) \frac{(T_s + T_e)}{H}. \tag{21}
\]

We will assume that the uncertainty in \( k \) is approximately one-third of the value of the second term in equation (21). \( k \) can be strongly dependent on \( \theta \) when the second term in equation (21) is large compared to the first. This is the case when measuring the sky brightness since \( T_g \) is greater than \( T_s \).

Combining equations (15) and (16) we get

\[
T_s(\theta) = \frac{1}{k(\theta)} \left[ \frac{y(T_n + T_e)}{H \eta} - \frac{(1-\eta)T_{\text{Amb}}}{\eta} \right]. \tag{22}
\]
If we take the differential of equation (22) to find $\delta T_s$ and simplify terms we get

$$
\frac{\delta T_s}{T_s} = \frac{1}{k(\theta)} \left[ \frac{1.5 \delta T_{\text{out}}}{\eta T_s} + \frac{(\eta k T_s + (1-\eta)T_{\text{Amb}})(\delta T_n + \delta T_e)}{\eta T_s (T_n + T_e)} - \frac{(1-\eta)\delta T_{\text{Amb}}}{\eta T_s} \right]
$$

$$
- \frac{\delta k}{k} + \frac{1}{k(\theta)} \left[ \frac{(T_{\text{Amb}} - \eta k T_s)}{\eta T_s} \right] \frac{\delta \eta}{\eta} + \left[ \frac{(1-\eta)T_A - \eta k T_s}{\eta T_s} \right] \frac{\delta H}{H}, \quad (23)
$$

where we have used

$$
\delta y \approx 1.5H \frac{\delta T_{\text{out}}}{(T_n + T_e)},
$$

and $\delta T_{\text{out}}$ is the measurement uncertainty in determining $T_{\text{out}}^{(1)}$ and $T_{\text{out}}^{(2)}$. Most of the terms in equation (23) are independent of $\theta$. Uncertainties in $T_n$, $T_e$, $T_A$, $\eta$, and $H$ are independent of $\theta$ and therefore do not cause uncertainties in $a_0$. If we drop the terms that are 0 in equation (12) then we get
\[ \delta a \sim 4.343 \left[ \frac{(T_s(\theta) - T_c) \delta T_m}{(T_m - T_c)(T_m - T_s(\theta))} \right. \left. + \frac{T_s(\theta)}{(T_m - T_s(\theta))} \cdot \delta k + \frac{1.5 T_s(\theta)}{\eta(T_m - T_s(\theta))} \cdot \frac{\delta T_{\text{out}}}{kT_s} \right] + a_0 \cot \theta \delta \theta, \tag{24} \]

where \( \delta T_m \) is the uncertainty in \( T_m \) due to nonuniform stratification. The error due to nonuniform stratification is determined by assuming a clear sky in all directions but one in which there is a thick cloud.

We will use the following values for various variables, some based on March and April 1988 Camp Parks measurements: \( \eta \sim 0.8, T_n \sim 800 \text{ K}, T_e \sim 300 \text{ K}, \) and \( T_{\text{out}} \sim 1100 \text{ K}, \) and \( \delta T_{\text{out}} \sim 5 \text{ K}. \) The variable \( \delta T_{\text{out}} \) represents random noise in the receiver.

We will evaluate the errors for four different weather conditions: (1) clear sky (zenith \( T_s \approx 6 \text{ K}, a_0 = 0.05 \text{ dB}), (2) medium clouds (zenith \( T_s \approx 14 \text{ K}, a_0 = 0.2 \text{ dB}), (3) heavy clouds (zenith \( T_s \approx 31 \text{ K}, a_0 = 0.5 \text{ dB}), and (4) moderate rain (zenith \( T_s \approx 56 \text{ K}, a_0 = 1 \text{ dB}). \) We will also assume that the errors at an elevation angle of \( 30^\circ \) are approximately equal to the average errors at other angles.

For clear sky conditions the largest contributions to the uncertainty in the attenuation is caused by the uncertainty in \( k. \) The uncertainty in \( k \) that we have
used in our calculations requires that \((\Omega_a - \Omega_0)\) be known to about 30%. This requires a detailed knowledge of the antenna pattern and a good initial estimate of \(T_s\). Improving on the uncertainty in \(k\) is difficult since the antenna backlobes are likely to be affected by the direction in which the antenna is pointing.

As we can see from table 1 the uncertainty in the attenuation for clear sky conditions is about 120% of the attenuation and exceeds the theoretical uncertainty for clear sky conditions. We conclude that for clear sky conditions it is better to determine atmospheric attenuation using theory. This agrees with the conclusions of Brussard [3].

The uncertainty for medium cloud conditions is found in table 2. The total uncertainty for medium cloud conditions is about equal to the attenuation. There is considerable uncertainty in the attenuation using the sky brightness tipping-curve method for medium cloud conditions but it can be used as a check and is perhaps as good as theory since estimates of cloud droplet densities are necessary for the theory.

The uncertainty for heavy cloud conditions is found in table 3. The total uncertainty for heavy cloud conditions is about 60% of the attenuation and equal to about 0.3 dB. This may be adequate for many measurements but may be larger than is acceptable for calibrations of gain, \(G/T\), or EIRP.

For moderate rain conditions of 10 mm/h (0.4 in/h) the total uncertainty in the attenuation is about 0.5 dB as indicated in table 4. This is about 50% of the attenuation. In rain conditions, however, scintillation often has a significant effect on the attenuation. If significant scintillation occurs it will be
virtually impossible to determine the attenuation at a given time since statistical methods must be used to determine a distribution of attenuations due to scintillation. Because of the probability of scintillation it is recommended that EIRP, G/T, and gain measurements not be done during rain conditions.

In examining the uncertainties of tables 1-4 we find that the greatest source of uncertainty under all conditions is the uncertainty in the value of \( k \). To get even a moderately good value of \( k \) the uncertainty in the antenna pattern must be no greater than 30% in the back- and sidelobes. The requirement on knowledge of the antenna pattern is difficult to meet for the backlobes since the backlobes usually depend on the orientation of the antenna relative to its mount. In addition, a good initial guess for \( T_s \) is necessary.
Table 1. Sky brightness tipping curve atmospheric attenuation uncertainties for clear sky conditions ($a_0 = 0.05$ dB)

<table>
<thead>
<tr>
<th>Error source</th>
<th>Value</th>
<th>Uncertainty</th>
<th>Contribution to the uncertainty in attenuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>30°</td>
<td>±0.03°</td>
<td>$&lt; 0.001$ dB</td>
</tr>
<tr>
<td>$T_m$ (nonuniform strat.)</td>
<td>265 K</td>
<td>±10 K</td>
<td>0.01 dB</td>
</tr>
<tr>
<td>$T_s$</td>
<td>k</td>
<td>8.4</td>
<td>0.05 dB</td>
</tr>
<tr>
<td>$T_s$</td>
<td>$T_{out}$</td>
<td>1100 K</td>
<td>0.02 dB</td>
</tr>
</tbody>
</table>

RSS attenuation error for $a_0 = 0.05$ dB
Linear sum attenuation error for $a_0 = 0.05$ dB

Table 2. Sky brightness tipping curve zenith attenuation uncertainties for medium cloud conditions ($a_0 \approx 0.2$ dB)

<table>
<thead>
<tr>
<th>Error source</th>
<th>Value</th>
<th>Uncertainty</th>
<th>Contribution to the uncertainty in attenuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>30°</td>
<td>±0.03°</td>
<td>$&lt; 0.001$ dB</td>
</tr>
<tr>
<td>$T_m$ (nonuniform strat.)</td>
<td>265 K</td>
<td>±10 K</td>
<td>0.02 dB</td>
</tr>
<tr>
<td>$T_s$</td>
<td>k</td>
<td>3.4</td>
<td>0.14 dB</td>
</tr>
<tr>
<td>$T_s$</td>
<td>$T_{out}$</td>
<td>1100 K</td>
<td>0.05 dB</td>
</tr>
</tbody>
</table>

RSS attenuation error for $a_0 = 0.2$ dB
Linear sum attenuation error for $a_0 = 0.2$ dB

23
Table 3. Sky brightness tipping curve zenith attenuation uncertainties for heavy cloud conditions \((a_0 \approx 0.5 \text{ dB})\)

<table>
<thead>
<tr>
<th>Error source (\theta)</th>
<th>Value</th>
<th>Uncertainty</th>
<th>Contribution to the uncertainty in attenuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\theta)</td>
<td>30°</td>
<td>±0.03°</td>
<td>&lt; 0.001 dB</td>
</tr>
<tr>
<td>(T_m) (nonuniform strat.)</td>
<td>265 K</td>
<td>±10 K</td>
<td>0.04 dB</td>
</tr>
<tr>
<td>(T_s)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(k)</td>
<td>2.0</td>
<td>±0.5</td>
<td>0.3 dB</td>
</tr>
<tr>
<td>(T_{out})</td>
<td>1100 K</td>
<td>±5 K</td>
<td>0.09 dB</td>
</tr>
</tbody>
</table>

RSS attenuation error for \(a_0 = 0.5 \text{ dB}\) 0.32 dB
Linear sum attenuation error for \(a_0 = 0.5 \text{ dB}\) 0.43 dB

Table 4. Sky brightness tipping curve zenith attenuation uncertainties for moderate rain conditions \((a_0 \approx 1.0 \text{ dB})\)

<table>
<thead>
<tr>
<th>Error source (\theta)</th>
<th>Value</th>
<th>Uncertainty</th>
<th>Contribution to the uncertainty in attenuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\theta)</td>
<td>30°</td>
<td>±0.03°</td>
<td>&lt; 0.001 dB</td>
</tr>
<tr>
<td>(T_m) (nonuniform strat.)</td>
<td>265 K</td>
<td>±10 K</td>
<td>0.1 dB</td>
</tr>
<tr>
<td>(T_s)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(k)</td>
<td>1.5</td>
<td>±0.3</td>
<td>0.5 dB</td>
</tr>
<tr>
<td>(T_{out})</td>
<td>1100 K</td>
<td>±5 K</td>
<td>0.16 dB</td>
</tr>
</tbody>
</table>

RSS attenuation error for \(a_0 = 1.0 \text{ dB}\) 0.53 dB
Linear sum attenuation error for \(a_0 = 1.0 \text{ dB}\) 0.76 dB
4. Attenuation From Radio Source Extinction Curve

4.1 Equations

If we measure the signal from a radio star then equation (9) becomes

\[
T_\star(\theta) = (T'_\star + T_c)10^{-a_0 \csc \theta / 10} + T_m \left[ 1 - 10^{-a_0 \csc \theta / 10} \right], \tag{25}
\]

where \(T_\star(\theta)\) is the brightness temperature of the radio star at the antenna, \(T'_\star\) is the brightness temperature of the star in the absence of the atmosphere. If we subtract the sky contribution (equation (10)) from equation (25) then

\[
a_0 \csc \theta = 10 \log \left[ \frac{T'_\star}{(T_\star(\theta) - T_S(\theta))} \right]. \tag{26}
\]

Just as in the case of measuring the sky brightness, if the log of the argument in equation (26) is plotted versus \(\csc \theta\) we should get a line. We can as before make a least-squares fit to the data and eliminate errors that are independent of \(\theta\) since they contribute only to the uncertainty in the intercept and not the slope, \(a_0\). To obtain a good extinction curve in a few hours the brightness temperature of the radio source should be measured about every 10 min. Once the zenith attenuation is determined the attenuation in other directions can be
determined by noting that \( a(\theta) = a_0 \csc \theta \) provided that the weather in that direction is similar to the weather in the direction of the extinction curve data.

If \( T^*_\star \) is much greater than both \( T_c \) and \( T_m \), as is the case in general only for the sun, then \( T^*_\star \) is much greater than \( T_s \). In this case equation (26) becomes

\[
a_0 \csc \theta = 10 \log \left[ \frac{T^*_\star}{T^*_\star(\theta)} \right],
\]

and the uncertainty in \( a_0 \) becomes

\[
\delta a \sim 4.343 \left( \frac{\delta k}{k} + \frac{1.5 \delta T_{\text{out}}}{\eta k T^*_\star(\theta)} \right) + a_0 \cot \theta \delta \theta.
\]

For the sun \( T^*_\star \sim 10^4 \) K for X-band [19]. Because the sun is so bright the contribution of the sky and the ground to the antenna temperature is small and \( k = 0.330 \pm 0.005 \) for all conditions. Also, \( k \) is very nearly independent of \( \theta \) so
that \( T_{\ast} \) can be replaced by \( T_{\text{ant}}/k \) in equation (27) with \( k \) a constant. Thus we can plot \( \log(T_{\text{ant}}) \) against \( \csc \theta \) and get a good value for \( a_0 \). We will assume that 

\[
\frac{\delta T_{\text{out}}}{T_{\text{out}}} \approx 0.01 \quad \text{so that} \quad \delta T_{\text{out}} \approx 40 \text{ K for the sun.}
\]

We will estimate the error due to nonuniform stratification by considering the case where the sun is mostly in a clear sky region but a light cloud passes in front of the sun at some time during the measurements. Other types of clouds must be considered to be a change in weather conditions.

When the sun is the source the attenuation uncertainty is the same for most conditions. This uncertainty is found in table 5. The results of table 5 show that the atmospheric attenuation can be determined to within an RSS uncertainty of about \( \pm 0.14 \) dB for most weather conditions.

Since this technique uses the sun as the source it is necessary that the weather conditions remain stable over several hours so the sun can cover a large enough range of elevation angles without the stratification changing significantly. This will permit a meaningful least squares fit to be made. When the weather conditions change a new fit of the antenna temperature versus \( \csc \theta \) must be made. In addition, at the time measurements are made the sun must not be producing radio bursts such as those that are associated with solar flares. Finally, this technique is no better at measuring attenuation due to scintillation than any other technique.
Table 5. Extinction curve zenith attenuation uncertainties using the sun as the source

<table>
<thead>
<tr>
<th>Error source</th>
<th>Value</th>
<th>Uncertainty</th>
<th>Contribution to the uncertainty in attenuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$ non-uniform stratification</td>
<td>30°</td>
<td>$\pm 0.03^\circ$</td>
<td>$&lt; 0.001$ dB $0.1$ dB</td>
</tr>
<tr>
<td>$T_*$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>k</td>
<td>0.330</td>
<td>$\pm 0.005$</td>
<td>$0.07$ dB</td>
</tr>
<tr>
<td>$T_{\text{out}}$</td>
<td>4000 K</td>
<td>$\pm 40$ K</td>
<td>$0.07$ dB</td>
</tr>
<tr>
<td>RSS attenuation error for $a_0 = 1.0$ dB</td>
<td></td>
<td></td>
<td>$0.14$ dB</td>
</tr>
<tr>
<td>Linear sum attenuation error for $a_0 = 1.0$ dB</td>
<td></td>
<td></td>
<td>$0.24$ dB</td>
</tr>
</tbody>
</table>
5. Dual Frequency Radiometer Technique

5.1 Theory

A dual frequency radiometer uses two frequencies such that one is more sensitive to water vapor and the other is more sensitive to liquid water [12]. It is usual for one frequency to be near, but less than, the frequency of the water vapor absorption line at 22.2 GHz and the other to be near 30 or 32 GHz. The first line is more sensitive to water vapor and the second to liquid water.

If the total atmospheric transmission at frequency \( \nu \) is \( \exp(-a_\nu/4.343) \) then \( a_\nu \) can be determined by [15]

\[
a_\nu = 10 \log \left( \frac{T_m - T_c}{T_m - T_{S\nu}} \right),
\]

(29)

where \( a_\nu \) is the attenuation in dB, \( T_m \) is the mean atmospheric temperature along the transmission path, \( T_c \) is the cosmic background, and \( T_{S\nu} \) is the sky brightness in the direction of the transmission path. This is equivalent to equation (11) with the subscript \( \nu \) added to explicitly show that the attenuation is a function of frequency. The attenuation can also be written as (Westwater [15])

\[
a_\nu = a_{\nu V} V + a_{\nu L} L + a_{d\nu},
\]

(30)
where $\alpha_{V\nu}$ and $\alpha_{L\nu}$ are path-averaged mass absorption coefficients for water vapor and liquid water respectively, $\alpha_{d\nu}$ is the dry air attenuation, and V and L are the integrated amounts of water vapor and liquid water.

In the usual circumstances $\alpha_{V\nu}$, $\alpha_{L\nu}$, and $\alpha_{d\nu}$ are not known since they depend on the unknown temperature, pressure, and humidity profiles (Hogg et al. [20]). It is necessary, therefore, to determine V and L using statistical methods. This is accomplished by taking many radiosondings and correlating the temperature, pressure, and humidity profiles with different ground conditions (Westwater [15], Guiraud et al. [12]). Such a process must be repeated for each location. In addition, since radiosondes do not measure liquid water, theoretical cloud models are necessary.

Equations (29) and (30) can be evaluated for both the upper ($\nu=2$) and lower ($\nu=1$) frequencies and we can then solve for V and L. We obtain

$$V = \frac{\alpha_{L2}f_1 - \alpha_{L1}f_2}{(\alpha_{V1}\alpha_{L2} - \alpha_{V2}\alpha_{L1})},\quad (31)$$

and

$$L = \frac{-\alpha_{V2}f_1 + \alpha_{V1}f_2}{(\alpha_{V1}\alpha_{L2} - \alpha_{V2}\alpha_{L1})},\quad (32)$$

where
\[ f_1 = a_1 - a_{d1}, \]

\[ f_2 = a_2 - a_{d2}. \]

\( a_1 \) and \( a_2 \) are determined using equation (29) and \( a_{d1} \) and \( a_{d2} \) are determined from theory plus average radiosonde data.

Also, \( V \) and \( L \) are usually measured in centimeters of precipitable water vapor. The \( \alpha \)'s can be determined from their value in dB/km and further noting that 1 g/m\(^3\) over a path length of 1 km is equal to 1 mm of precipitable water vapor.

5.2 Error Analysis for the Dual Channel Radiometer Technique

According to Westwater [15], \( V \) and \( L \) can be measured to an accuracy of about 15%. The molecular absorption constants can be determined theoretically to an accuracy of about 15% and can be path averaged to an accuracy of about 15%. As a result we get the uncertainties found in table 6.

<table>
<thead>
<tr>
<th>conditions</th>
<th>Zenith attenuation</th>
<th>RSS uncertainty</th>
<th>Linear sum uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>clear</td>
<td>0.05 dB</td>
<td>0.01 dB</td>
<td>0.02 dB</td>
</tr>
<tr>
<td>mod. cloud</td>
<td>0.2 dB</td>
<td>0.05 dB</td>
<td>0.09 dB</td>
</tr>
<tr>
<td>hvy. cloud</td>
<td>0.5 dB</td>
<td>0.12 dB</td>
<td>0.22 dB</td>
</tr>
</tbody>
</table>
The dual frequency radiometer gives results accurate to about 0.2 dB even for heavy cloud conditions. During rain conditions the dual frequency radiometer does not give accurate results because the radiometer becomes saturated.
6. Discussion

None of the methods that we have evaluated are ideal. Overall, however, the most accurate results can be obtained by using a dual frequency radiometer to measure the integrated water vapor and liquid along the transmission path.

The dual frequency radiometer cannot be used during rain since it becomes saturated. This is not a serious problem since it is also undesirable to make measurements during rainstorms because scintillation usually occurs. The dual frequency radiometer would require the acquisition of an additional system. Also, much effort would be needed to develop a large statistical data base so that atmospheric profiles can be correlated with ground conditions and thus accurate path averaged absorption coefficients determined. Also, the dual frequency radiometer can account for nonuniform stratification better than the other techniques especially at millimeter waves. Only the Camp Parks Communications Annex can decide if its needs warrant the time and expenditure required for this technique.

The extinction curve method with the sun as the radio source is the next most accurate technique to use. The most serious constraint for this technique is that it can be used only when the sun is above the horizon. Second, it cannot be used during violent solar events such as solar flares since the sun's brightness temperature changes rapidly with time; however, as long as there are no such transient events the sun's actual brightness temperature need not be known. Third, the weather must be stable for at least two hours so that the sun can cover a large enough portion of the sky for a least-squares line to be meaningful. Finally, the weather conditions in the direction of the sun must be
similar to those in the direction of the radio calibration source so that the zenith attenuation derived from the extinction curve can be used in determining the attenuation in the direction of the radio calibration source. The advantage of this technique is that it gives results accurate to about 0.15 dB under many weather conditions. It is necessary to find a separate extinction curve if the weather conditions change (for example, when the type of clouds are different). Also, this technique can be used at millimeter wave frequencies since the sun is a strong source at these frequencies.

The sky brightness tipping curve technique will be difficult to implement since it requires a detailed knowledge of the antenna pattern including the side and backlobes. Because the backlobes of an antenna are partly determined by the interaction of the antenna with its mount the backlobes will change with antenna orientation. This makes acquiring the necessary information on the antenna pattern extremely difficult if not impossible. A good first guess of the sky temperature, accurate to at least 30%, is also necessary to determine k. Since we are trying to measure $T_s$ in order to determine the attenuation this is not desirable. The sky brightness tipping curve technique could be used with a horn or small dish which is designed to direct a large portion of the energy into the mainbeam (about 90% or more at X-band). A smaller antenna also has the advantage that its entire pattern can be characterized on a near-field range.

The accuracy of the extinction curve method can be improved by combining it with a modified tipping-curve method. This modified technique would consist of measuring the sky temperature difference profile; that is, we measure $\Delta T = T_s(AZ(\text{sun}), \theta) - T_s(AZ(\text{radio star}), \theta)$. Then the attenuation in the direction of the radio star is given by
\[ a(\text{AZ(radio star)}, \theta) = a_0(\text{AZ(sun)}, \theta) + \Delta a(\theta), \quad (33) \]

where \( a_0 \) is determined from equation (27) using the extinction curve method and \( \Delta a \) is given by

\[ \Delta a \csc \theta = 10 \log \left( \frac{T_m - T_s}{T_m - T_s - \Delta T} \right). \quad (34) \]

In equation (34) the value of \( T_s \) is determined from equation (10) using the value of \( a_0 \) as determined from the extinction curve method.

For millimeter waves nonuniform stratification of the atmosphere will be a more serious problem for both the tipping-curve method and the extinction curve method. This is because clouds cause a larger change in the attenuation at millimeter waves than at X-band.

A theoretical determination of the attenuation can be very useful for clear sky conditions. For these conditions it is nearly as accurate as the dual frequency radiometer technique. The atmospheric attenuation computer program developed at ITS can also be used as a check on other techniques of attenuation determination since it can estimate attenuation due to clouds if they can be classified (for example, cirrus, cumulus). The value of the theoretical method for determining attenuation could be increased if radiosonde data is used to create a database that correlates atmospheric profiles with ground conditions. Profiles specific to weather conditions can be used in this program.
7. Conclusions and Recommendations

We recommend that the modified version of the computer program written by Shaw at ITS be implemented for the Camp Parks ETMS. This program gives accurate results for clear sky conditions and can also be used to estimate attenuation in the presence of clouds.

In addition, we recommend that the extinction curve method with the sun as the radio source be combined with the modified tipping-curve method to determine the atmospheric attenuation. Other than the dual frequency radiometer technique this method is the most accurate technique for determining the atmospheric attenuation.

Third, the Camp Parks Communications Annex should determine whether they need to be able to determine accurately the atmospheric attenuation during the night as well as the day and if increased accuracy for nonuniform stratification conditions is required. In this event they should consider implementing the dual frequency radiometer.

Fourth, the Camp Parks Communications Annex should consider the acquisition of a smaller antenna for millimeter-wave measurements. A smaller antenna with a diameter of less than about 3 m (10 ft) could have its entire pattern determined on a spherical near-field range. Since gains generally increase with increasing frequency, smaller antennas are possible at millimeter-wave frequencies. For example, a 50% efficient 3 m dish at 22 GHz would have a gain of about 54 dB.

A final caution: calibration measurements should not be made during rain conditions since scintillation is likely to be present. Scintillation may also
occur in the presence of cumulus clouds. Under no circumstances should calibrations of gain, G/T, or EIRP be made during scintillations since atmospheric attenuation cannot be accurately determined under these conditions.
8. Acknowledgments

The author wishes to thank Bill Daywitt for his suggestions and discussions regarding this topic. The author also thanks Hans Liebe of the Institute of Telecommunications Sciences for providing the author with several relevant references as well as for providing the atmospheric attenuation computer program developed at ITS. Finally, the author acknowledges the support of this work by the Camp Parks Communications Annex.
9. References


**X-BAND ATMOSPHERIC ATTENUATION FOR AN EARTH TERMINAL MEASUREMENT SYSTEM**

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**PERFORMING ORGANIZATION**
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**ABSTRACT**
The National Institute of Standards and Technology has developed an Earth Terminal Measurement System to be used by the Camp Parks Communications Annex in determining satellite effective isotropic radiated power and antenna gain. In determining these quantities the effect of atmospheric attenuation must be taken into account. This paper provides an overview of the methods used for determining atmospheric attenuation with emphasis on a tipping-curve method. An error analysis is also provided.

**KEY WORDS**
antenna gain; atmosphere; atmospheric attenuation; error analysis; satellite communications; satellite effective isotropic radiated power (EIRP)

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