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# Survey Sample Design for Microfilm Inspection at the National Archives 

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National Archives and Records Administration Seventh and Pennsylvania Avenue，NW
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#### Abstract

This report describes the statistical design of a sample survey to monitor the condition of microfilm in a large collection maintained by the National Archives. The design criterion developed for the survey ensures that the number of rolls of film inspected will be large enough to achieve a pre-chosen probability of detecting a specified amount of damaged film that might exist in the population. Tables and formulas are given to satisfy the design criterion under a range of conditions. Other practical aspects of survey design are discussed including the sampling frame, stratification, sample selection procedure, pilot testing, and the use of replicated sampling.


Key Words and Phrases: Sample Size Determination, Sampling Fraction, Sampling Frame, Sample Survey, Statistics, Stratification.

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Regular surveys of microfilm collections are necessary in order to monitor the condition of the film and to detect any serious problems of degradation or other damage to the records. This report deals specifically with the design of a survey for the large microfilm collection maintained by the National Archives and Records Administration (NARA) at National Underground Storage, Inc. (NUS) in Boyers, Pennsylvania

The practical survey procedure envisaged consists of two separate phases. The first phase has the purpose of detecting and identifying any parts of the microfilm collection where serious degradation has occurred. After this initial broad inspection is carried out, a follow-up investigation will be conducted as the second phase of the survey. The purpose of the second phase follow-up study is to evaluate the extent of any problems discovered in the first phase and to recommend or initiate appropriate corrective action. This report addresses the statistical aspects of designing the first phase survey of the microfilm.

As thus described, the purpose of this survey is somewhat different in character from many typical sample surveys. It is more common for a survey to be conducted for the purpose of estimating the percentage or number of items in a population that belong to a given category (e.g. damaged) than for the purpose of detecting or locating the presence of damaged items.

This report describes a method of statistically assessing the merits of a given sampling design in terms of the probability of detecting a group of damaged microfilm rolls in a population of rolls. It is found that a rule based on selecting a fixed proportion of the population, no matter how large or small the population, is a statistically defensible approach. Some of the statistical properties of this approach are characterized.

## 2. Characteristics of the NUS Population, Sample Units and Frame

The storage area at NUS consists of an underground vault in a tunnel of a limestone mine. The storage area covers about 5000 square feet and contains about 40 rows of shelving, with six to ten compartments per row.

A summary description of the shelving and its contents is given in Appendix 1. The summary is based on notes taken by Alan Calmes when he visited the site in August, 1985. A sketched map of the facility is included in the Appendix.

Based on the summary in Appendix 1, there are about 124,000 rolls of microfilm at NUS, of which about 50,000 are 16 mm format and 74,000 are 35 mm format.

The standard inspection procedure for microfilm, as described in McCamy (1964) in the context of inspection for aging blemishes, can be used for evaluating the condition of individual rolls of microfilm. Since that procedure is designed for evaluating the film on a 100 foot roll basis, the appropriate sampling unit for this survey is the 100 foot roll. This means that a workable procedure will be needed for inspecting a randomly chosen $100^{\prime}$ roll that is spliced together with nine other $100^{\prime}$ rolls on a $1000^{\prime}$ core.

A critical element in the successful execution of a survey of any population is the existence of an adequate frame. A frame is a literal or conceptual list that contains exactly one entry for each and every member of the population to be sampled. In the present context, an adequate frame would have to be capable of uniquely identifying every $100^{\prime}$ roll of film in the NUS storage facility. A number of finding aids and other forms of intellectual control related to the contents of the microfilms exist. Unfortunately, none of these forms a complete listing of the population of microfilm.

A computer listing of the microfilm by "Control Number" exists. This listing appears to be a useful starting point, but is not fully adequate as a sampling frame at present. One important deficiency of the Control Number list is that there is apparently no simple and accurate way to figure out how many $100^{\prime}$ rolls of film are represented by each entry on the list. In addition, it is not possible to manipulate the list (e.g. sort it, create statistical summaries of the data it contains) owing to the inflexibility of the computer hardware and/or software environment in which it resides.

An improved version of the computerized Control Number list would be a useful tool to aid in conducting periodic inspection surveys of the microfilm population. A well-designed and maintained computer index to the population could also be useful in other ways, including recording and monitoring the results of the periodic inspections themselves. An adequate computerized list should contain at least the following information:

- Identification number - ideally at the individual roll level
- Number of rolls represented by each entry, if more than 1
- Source of microfilm (NARA or Other)
- Film size ( 16 mm or 35 mm )
- Film type (camera negative, duplicate negative, duplicate positive, etc.)
- Storage location
- Year produced (at least grouped into the major categories defined by: before 1955, 1956-1965, and after 1965)

For the purposes of conducting the inspection survey, it is not necessary to include information on the intellectual content of the film rolls, but such information is essential for other purposes.

A very important use for the information contained in the frame will be in aiding the (second phase) follow-up investigations that will track down any significant problems or deterioration detected in the (first phase) survey. The identification of a problem in the survey sample would lead to follow-up checks of other rolls of film that belong to the same Control Number, or are housed in the same box, or were produced at the same time by the same supplier, or have some other salient characteristic in common with the sample roll(s). For convenience in what follows, microfilm rolls that can be logically linked together through sharing some such characteristic will be referred to as a "logical group" (or simply a "group" when the context is clear.) To enable efficient use of the logical group structure in the survey, it will be important that the computerized list, or sampling frame, contain the necessary information for locating all rolls in the population that belong to the same logical group as any sampled roll.

An important consideration in the planning of a statistical survey is the choice of the sample size, which in the present case amounts to the number of rolls of microfilm from the population to be inspected. Clearly, as the sample size increases, the precision of the inference from the sample data to the population increases (as long as the quality of the inspection procedure does not degrade with increasing sample size.) Therefore, the ideal of inspecting the whole population must be balanced against the limited resources available for conducting the inspection.

The data from a sample survey are typically put to many uses, all of which are affected by the sample size. In order to treat the choice of sample size mathematically, it is necessary to adopt a model for the population and sampling procedure and to study the intended primary use of the survey data in terms of that model.

Suppose that there exist in the population some number "A" of rolls of microfilm that are significantly deteriorated, or are at substantial risk of deterioration. In the statistical quality control literature related to this problem, these rolls correspond to defective items in a lot of manufactured goods. For brevity such rolls will be designated as "defective" rolls in this report. If the individual defective rolls are isolated and randomly distributed throughout the population, then a survey sample has a very small chance of solving the problem that they represent, for this would require that the sample identify all of the defective rolls in the population in order for appropriate action to be taken in each instance. Fortunately, the practical situation is more favorable, because a single defective roll identified in the survey will typically lead to the discovery of others like it in the follow-up process. It is fruitful to model this aspect of the survey.

As described in Section 2, the rolls of microfilm can be categorized in several ways and conceptually divided into "logical subgroups" that have certain properties in common. As a starting point, consider the case in which all of the defective rolls of microfilm belong to a single identifiable group. It is not necessary to assume that the group consists exclusively of defective rolls; the model and mathematical development to follow only depend on the number of defective rolls contained in the group. For this reason, and for simplicity, the following development will focus on only the defective rolls in a group, often speaking as though the group consists entirely of defective rolls.

Conceptually, if the sample inspection procedure identifies at least one defective roll of film, then the follow-up phase of the survey project can examine the entirety of the logical group to which the defective roll belongs and appropriately respond to the identified problem throughout that group. In a sense, the main job of the survey, then, is to discover any such problems that exist within the population. The sample size required to satisfy the needs of the survey can be determined by specifying the probability that a group containing at least a specified number of defective rolls will be discovered.

The following symbols are used to denote the quantities of interest:

```
    n = sample size (number of rolls inspected)
    N = population size (total number of rolls in storage at NUS)
    f = n/N, the "sampling fraction"
    A = number of defective rolls in the population
P
        group of defective rolls (i.e. the probability that one
        or more defective rolls is selected into the random
        sample)
```

Derivations of the formulas discussed in this section and in section 4 are given in Appendix 3.

Under the assumption of simple random sampling, the probability of detection is

$$
\begin{equation*}
P_{d} \simeq 1-(1-f)^{A} . \tag{3.1}
\end{equation*}
$$

In this equation a key role is played by the proportion of the population sampled, $f=n / N$, which is called the sampling fraction in statistical literature.

Figure 1 shows a plot of $P_{d}$ versus the sampling fraction for several values of $A$. The plot illustrates that the probability of detection increases as the sampling fraction increases and as the number of defective rolls increases.


Figure 1. Plot of probability of detection, $P_{d}$, vs. sampling fraction, $f$, for various numbers of defective rolls, A. For a fixed number of defectives, the probability of detection increases as the sampling fraction increases. For a given sampling fraction, $\mathrm{P}_{\mathrm{d}}$ decreases as the number of defectives in the population decreases

If the values of $P_{d}$ and $A$ are specified, it follows from equation (3.1) that the sampling fraction is given by

$$
\begin{equation*}
f \simeq 1-\left(1-P_{d}\right)^{1 / A} . \tag{3.2}
\end{equation*}
$$

The interpretation of equation (3.2) is that the appropriate sample size, $n$, depends on the value of the population size, $N$. In fact, it says that $n$ is directly proportional to N , the proportionality factor being determined by $\mathrm{P}_{\mathrm{d}}$ and A as specified by (3.2). Therefore, the values of $P_{d}$ and A determine the sampling fraction and so the required sample size, $n$, can be calculated as a fixed fraction of the population size, N.

Table 3.1 gives the values of the sampling fraction for a selection of values of $P_{d}$ and $A$.

Table 3.1.
Sampling Fraction, f, for Selected Values of $\mathrm{P}_{\mathrm{d}}$ and A , Under the
Assumption That All Defective Rolls of Microfilm Belong to a Single Identifiable Group

| No. of Defective <br> Rolls, | Probability of Detection, $P_{d}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $50 \%$ | $75 \%$ | $90 \%$ | $95 \%$ | $99 \%$ |
|  | .023 | .045 | .074 | .095 | .142 |
| 50 | .014 | .027 | .045 | .058 | .088 |
| 100 | .0069 | .014 | .023 | .030 | .045 |
| 200 | .0035 | .0069 | .011 | .015 | .023 |
| 300 | .0023 | .0046 | .0076 | .0099 | .015 |
| 600 | .0012 | .0023 | .0038 | .0050 | .0076 |
| 1000 | .00069 | .0014 | .0023 | .0030 | .0046 |
| 3000 | .00023 | .00046 | .00077 | .0010 | .0015 |

As an example, suppose we wish to insure that the probability of detection is at least $95 \%$ for a group containing 200 defective rolls. From Table 3.1, the appropriate value of the sampling fraction is $\mathrm{f}=.015$, or 1.5\%. Thus, if the population consists of a total of $N=100,000$ rolls of microfilm, the sample size should be $1.5 \%$ of 100,000 or $n=1500$. Similarly, for a population of $N=40,000$ rolls, the sample size should be $n=600$ (which is $1.5 \%$ of 40,000 .)

In practice, it is unlikely that the defective rolls of microfilm in the population will all belong to a single identifiable group. If the defective rolls are distributed across several groups, the follow-up phase of the inspection plan will locate all of them only if at least one member of each and every defective group is identified by the sample inspection. It seems intuitively clear that a larger sample size will be needed to achieve a reasonable probability of detecting all of the logical groups containing defectives in this case.

One simple way to extend the probability model to analyze the situation involving several groups is to assume that the groups are all equal in size. Specifically, let

$$
\begin{aligned}
k= & \text { number of groups containing defective rolls } \\
A / k= & \text { number of defective rolls per group } \\
P_{d}(k)= & \text { probability of detecting all of the defective groups } \\
& \text { (i.e., the probability that one or more members of each } \\
& \text { defective group are included in the sample) }
\end{aligned}
$$

In this case, it can be shown (section A3.2 of Appendix 3) that the probability of simultaneously detecting all of the defective groups is given (approximately) by

$$
\begin{equation*}
P_{d}(k) \simeq\left[1-(1-f)^{A / k}\right]^{k} \tag{4.1}
\end{equation*}
$$

Again the sampling fraction, $f=n / N$, plays a key role in determining the probability of detection. The plot of $P_{d}(k)$ versus $f$ in Figure 2 illustrates how, for a fixed number of defective rolls in the population ( $A=300$ is shown), the probability of simultaneous detection decreases as the number of groups increases. Putting it another way, the sampling fraction, $f$, must be substantially larger if a given number of defective rolls are distributed across several groups compared to the case of all defectives belonging to a single group.

As was true in the case of a single group, the equation for probability of detection (4.1) can be solved for the sampling fraction in terms of the other parameters. The solution is

$$
\begin{equation*}
\mathrm{f} \simeq 1-\left[1-P_{\mathrm{d}}(\mathrm{k})^{1 / \mathrm{k}}\right]^{\mathrm{k} / \mathrm{A}} \tag{4.2}
\end{equation*}
$$

Formula (4.2) can be used to determine the appropriate sampling fraction for specified values of $A, k$, and $P_{d}(k)$. From the sampling fraction and the population size, $N$, the sample size, $n$, can be found by multiplying the population size by the sampling fraction: $n=f \times N$.


Figure 2. Plot of probability of detection, $\mathrm{P}_{\mathrm{d}}(\mathrm{k})$, vs. sampling fraction, $f$, showing dependence on number of groups, $k$. The plot is constructed assuming that a total of 300 defective rolls are equally distributed among $k$ groups. The probability of simultaneously detecting all $k$ groups in a single survey decreases as the number of groups increases.

Current regulations relating to inspection of microform records (NARA, 1985, section 1230.22 ) derive from a rule that the sample shall constitute a $1 \%$ sample of the population. This rule, which was originally proposed on intuitive grounds, and without a rigorous statistical basis, is consistent with the model and theoretical development outlined in Sections 3 and 4 of this report. Additional detail on the nature of these regulations is contained in Appendix 2, where selected portions of a 1986 draft revision to the regulations have been reproduced.

The theoretical development in this report provides a statistical justification for a sample size rule based on setting a fixed sampling fraction that will be used with populations of varying sizes. Equations (3.1) and (4.1) provide a statistical interpretation of the merits of any sampling plan specified by a given sampling fraction. Candidate sampling plans can be evaluated in terms of the probability of detecting a set of $A$ defective items, where the number $A$ is chosen to be relevant to a particular situation. Table 5.1 presents the probabilities of detection for a range of sampling fractions and group sizes. The table contains separate entries corresponding to the assumptions that all defective items belong to one group [using equation (3.1)] or that the defective items are distributed across 5 or 10 groups [using (4.1)].

Table 5.1 shows, for example, that if there are $A=100$ defective rolls of film in a population, a $5 \%$ random sample has a $99.4 \%$ chance of detecting at least one of them. If those 100 defectives all belong to a single logical group, they could all be found (and presumably fixed) by an follow-up inspection of the $100 \%$ of group to which the sample defective(s) belong. If the 100 defective rolls are distributed in 5 distinct (but individually identifiable) groups (of 20 each), that same $5 \%$ random sample of the population has only a $10.9 \%$ chance of detecting all 5 groups. Further, if the 100 defectives are situated in 10 groups (of 10 each), then the $5 \%$ random sample has only a $0.01 \%$ chance of locating at least one member of each of the 10 groups.

Table 5.2 is an extension of Table 3.1 and gives values of the sampling fraction corresponding to various values of $A, k$ and $P_{d}(k)$. As an example of the use of Table 5.2 , suppose it is desired to insure that the probability of detection is at least $99 \%$ for simultaneous detection of 5 groups of defectives, each of which contains 600 defectives. For this example, we wish to find the sampling fraction, $f$, in Table 5.2 corresponding to $k=5$ groups, probability of detection $P_{d}(k)=99 \%$, and total number of defective rolls $A=5 \times 600=3000$. The tabulated value of f is .010 , which implies that a $1 \%$ sample of the population will be required. If the population contains 124,000 rolls of microfilm, for example, the required sample size would be $1 \%$ of 124,000 or 1,240 rolls to be inspected.

Table 5.1.
Probability of Detection, $P_{d}(k)$ in Percent, for Various Values of $A, f$, and $k$.

| No. of Groups k | Number of Defective | Sampling Fraction, $f=n / N$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |
|  | Rolls, A | . 001 | . 005 | . 01 | . 05 | . 10 |
| 1 | 30 | 3.0\% | 14.0\% | $26.0 \%$ | $78.5 \%$ | 95.8\% |
| 1 | 50 | 4.9 | 22.2 | 39.5 | 92.3 | 99.5 |
| 1 | 100 | 9.5 | 39.4 | 63.4 | 99.4 | $99.9+$ |
| 1 | 300 | 25.9 | 77.8 | 95.1 | 99.9+ | 99.9+ |
| 1 | 1000 | 63.2 | 99.3 | 99.9+ | 99.9+ | 99.9+ |
| 1 | 3000 | 95.0 | 99.9+ | 99.9+ | 99.9+ | 99.9+ |
| 5 | 30 | 0.0 | 0.0 | 0.0 | 0.1 | 2.3 |
| 5 | 50 | 0.0 | 0.0 | 0.0 | 1.0 | 11.7 |
| 5 | 100 | 0.0 | 0.0 | 0.02 | 10.9 | 52.3 |
| 5 | 300 | 0.0 | 0.1 | 1.9 | 79.0 | 99.1 |
| 5 | 1000 | 0.02 | 10.2 | 48.7 | 99.9+ | 99.9+ |
| 5 | 3000 | 1.9 | 77.6 | 98.8 | 99.9+ | 99.9+ |
| 10 | 30 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 10 | 50 | 0.0 | 0.0 | 0.0 | 0.0 | 0.01 |
| 10 | 100 | 0.0 | 0.0 | 0.0 | 0.01 | 1.4 |
| 10 | 300 | 0.0 | 0.0 | 0.0 | 8.9 | 64.8 |
| 10 | 1000 | 0.0 | 0.01 | 1.0 | 94.2 | $99.9+$ |
| 10 | 3000 | 0.0 | 8.1 | 60.5 | 99.9+ | $99.9+$ |

Table 5.2.
Sampling Fraction, $f=n / N$,
for Various Values of $A, k$, and $P_{d}(k)$.

| No. of Groups k | Number of Defective Rolls, A | Probability of Detection, $\mathrm{P}_{\mathrm{d}}(\mathrm{k})$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 50\% | 75\% | 90\% | 95\% | 99\% |
| 1 | 30 | . 023 | . 045 | . 074 | . 095 | . 142 |
| 1 | 50 | . 014 | . 027 | . 045 | . 058 | . 088 |
| 1 | 100 | . 0069 | . 014 | . 023 | . 030 | 045 |
| 1 | 200 | . 0035 | . 0069 | . 011 | . 015 | . 023 |
| 1 | 300 | . 0023 | . 0046 | . 0076 | . 0099 | . 015 |
| 1 | 600 | . 0012 | . 0023 | . 0038 | . 0050 | . 0076 |
| 1 | 1000 | . 00069 | . 0014 | . 0023 | . 0030 | . 0046 |
| 1 | 3000 | . 00023 | . 00046 | . 00077 | . 0010 | . 0015 |
| 5 | 30 | . 29 | . 38 | . 48 | . 53 | . 64 |
| 5 | 50 | . 18 | . 25 | . 32 | . 37 | . 46 |
| 5 | 100 | . 097 | . 13 | . 18 | . 20 | . 27 |
| 5 | 200 | . 050 | . 070 | . 092 | . 11 | . 14 |
| 5 | 300 | . 034 | . 047 | . 062 | . 074 | . 098 |
| 5 | 600 | . 017 | . 024 | . 032 | . 038 | . 050 |
| 5 | 1000 | . 010 | . 014 | . 019 | . 023 | . 031 |
| 5 | 3000 | . 0034 | . 0048 | . 0064 | . 0076 | . 010 |
| 10 | 30 | . 59 | . 70 | . 78 | . 83 | . 90 |
| 10 | 50 | . 42 | . 51 | . 60 | . 65 | . 75 |
| 10 | 100 | . 24 | . 30 | . 37 | . 41 | . 50 |
| 10 | 200 | . 13 | . 16 | . 20 | . 23 | . 29 |
| 10 | 300 | . 086 | . 11 | . 14 | . 16 | . 21 |
| 10 | 600 | . 044 | . 058 | . 073 | . 084 | . 11 |
| 10 | 1000 | . 027 | . 035 | . 045 | . 051 | . 067 |
| 10 | 3000 | . 0090 | . 012 | . 015 | . 017 | . 023 |

## 6. Stratification

Stratification refers to the process of dividing the population into nonoverlapping subpopulations, or strata, that will be treated separately in the sample survey. Since the stratification is done before sampling, the frame must contain enough information to unequivocally assign each sample unit to one and only one stratum before sampling.

There are four major reasons for stratification that might apply to this survey of microfilm: administrative convenience (including the need to prepare separate summary reports for each stratum), statistical efficiency, existence of strata having significantly different historical or monetary value, and allocation of resources for future surveys.

Administrative convenience. It is often useful to stratify a large population into smaller pieces so that survey work can be organized independently in each stratum. An additional advantage is that summary reports and any statistical analyses of the data from each stratum can be carried out separately.

Statistical efficiency. The theory outlined in sections 3 and 4 implies that the probability of detection depends on $A$, the total number of defective rolls in the subpopulation (or stratum), and on $k$, the number of identifiable groups to which the defective rolls belong. This implies that an efficient stratification will divide the population according to expected values of $A$ and $k$.

For example, one stratum might consist of rolls for which the sizes of the identifiable subgroups (possible values of A if the rolls turn out to be defective) are all relatively large compared to another stratum consisting of smaller group sizes. Organizing the population into strata in this way would have the advantage of making it possible to sample a larger fraction of the stratum consisting of smaller groups. Similarly, if a stratum could be formed in which it was expected that at most $k=1$ group could contain defective rolls, a relatively small sampling fraction would suffice, saving a heavier sampling effort for another stratum in which $k \geq 1$ is expected.

Strata of significantly different historical or monetary value. A stratum consisting of relatively more valuable microfilm can be sampled more intensively than a less valuable stratum. For example, only camera negative film was sampled in the 1984 survey conducted at the National Archives.

Planning for future surveys. If experience with the population suggests that one stratum is significantly more at risk than another, a stratified approach provides for the possibility of sampling the lower risk stratum less often or less intensively in a periodic inspection program. This approach was described in the proposed revision to the Federal Property Management Regulations on Micrographics Records Management that the author helped prepare in November, 1986. Relevant portions of that document are reproduced in Appendix 2.

In this section, it is assumed that a suitable list frame, as described in Section 2, is available for use in sample selection. The sampling procedure that will be described is a systematic sampling scheme which is practical to implement and which can be used for list frames having entries that represent more than one roll of microfilm. The details of the recommended sampling procedure will be described and illustrated by working through a hypothetical example.

Step 1. By referring to Table 5.1 and/or Table 5.2, choose the appropriate sampling fraction, $f$, to achieve the desired probability of detection, $\mathrm{P}_{\mathrm{d}}$, for relevant values of $A$, the assumed number of defectives in the population, and $k$, the assumed number of equal groups to which the $A$ defectives belong.

Example: Suppose we want to design a survey so that there will be a $90 \%$ chance of detecting the presence of each of 10 groups that contain at least 100 defectives, should such groups exist. In this example we want $P_{d}=.90$, for $k=10$ groups and $A=100 \times 10=1000$ total defectives. From Table 5.2, we find that a sampling fraction of $f=0.0446$, or $4.46 \%$ of the population, will be required to guarantee a $90 \%$ chance of detecting at least one defective roll from each of the assumed 10 groups of defectives.

Step 2. Calculate the sampling interval, $S$, by the formula $S=1 / f$ and round down to the nearest whole number. The sample for inspection will be chosen as "every Sth" unit in the population.

Example: Given that $f=0.0446$, we find $S=1 /(0.0446)=22.42$. Rounding down to the nearest whole number yields $S=22$. Thus the sample will consist of every 22 nd unit from the population.

Step 3. Choose a random starting number, R, between 1 and S. The first unit selected will be serial number $R$ and the rest of the sample will be chosen as every Sth unit after that.

Example: Consulting a table of random numbers, we locate a "random" starting point by placing a finger on the table without looking. Then, scanning down the located column (or across the row or moving diagonally - it doesn't matter since the table is completely random) the first (two-digit) number encountered between 01 and 22 (=S) is 09 , say. Thus the random starting number will be $R=9$. The sample rolls of microfilm will consist of serial numbers 9 , $31(=9+22), 53(=31+22), 75(=53+22)$, etc.

Step 4. Assign "serial numbers" to all the rolls of microfilm in the population and select the sample rolls.

Example: Table 7.1 below represents a hypothetical population which will be used to illustrate the sample selection procedure. The population consists of 68146 rolls of microfilm represented in a list frame with 5000 entries. The population is to be sampled using random start $\mathrm{R}=9$ and sampling interval $\mathrm{S}=22$.

Table 7.1
Hypothetical Population for Example

| List Entry Number | Number of Rolls Per Entry | Serial Numbers Assigned | Serial Numbers for Selected Sample Rolls |
| :---: | :---: | :---: | :---: |
| 1 | 23 | 1-23 | 9 |
| 2 | 11 | 24-34 | 31 |
| 3 | 7 | $35-41$ | - |
| 4 | 19 | 42-60 | 53 |
| 5 | 6 | $61-66$ | - |
| 6 | 12 | 67-79 | 75 |
| 7 | 14 | 80-94 | - |
| 8 | 28 | 95-123 | 97, 119 |
| 9 | 5 | 124-128 | - |
| 10 | 13 | 129-132 | - |
| - | - | . $\cdot$ | . |
| - | - | . . |  |
| 5000 | 12 | 68134-68146 | 68143 |
|  | ```Total = 68146 rolls in population``` |  | Total number of rolls selected for the sample would be 3097. |

Notice that two rolls (serial numbers 97 and 119) are to be selected from list entry number 8. This will happen occasionally for list entries that correspond to more than 22 rolls (or, more generally, $S$ rolls) of microfilm.

In assigning the serial numbers, it is not necessary to actually number every roll in the population. The serial numbers are used simply as a conceptual device to keep track of the varying numbers of rolls per list entry and, in effect, to insure that every roll in the population has a equal chance of being selected into the sample.

Step 5. Select the individual microfilm rolls tor be inspected by making random selections from the list entries identified in step 4.

Example: The procedure described in Table 7.1 results in selection of a set of list entries corresponding to the sample of microfilm rolls to be inspected. The final selection of the individual microfilm rolls from the selected list entries is accomplished by simple random sampling, as follows: First, physically locate all of the microfilm rolls that belong a chosen list entry and count them. It may be expected that the actual number of rolls for a list entry will not always be exactly the number predicted. In the hypothetical example, we may imagine that list entry \#1 is actually found to contain 26 rolls, rather than the 23 rolls that were assumed when Table 7.1 was constructed. The sample selection is completed by drawing one roll at random from the actual number of rolls found, using a table of random numbers or a computerized random number generator. In the example, a random number between 1 and 26 would be drawn and the corresponding roll selected for inspection. Continuing the example, note that two rolls would be drawn at random from the group corresponding to list entry \#8 because two serial numbers were identified as belonging to that group.

## 8. Use of a Geographic Frame for Sampling

The term "geographic frame" is used here to mean a conceptual frame that identifies each roll of microfilm in the population with its physical storage location. A geographic frame can be used to obtain reasonably complete coverage of the population, with the exception that if some group of film is heavily used, items from that group would tend to be under-represented in the sample because items in use at the time of the survey would be unavailable for inspection. This type of frame is less desirable than a list because it is less stable over time and does not lend itself to creating and maintaining records of data obtained from repeated surveys. However, if a list frame is not available and can not be constructed in a timely and cost-effective manner, useful results still can be obtained from a survey conducted by a geographic frame.

The procedure for sampling from a geographic frame follows essentially the same steps described in Section 7 of this report, with appropriate adaptations. In particular, the "list entries" are replaced by convenient physical storage units, such as storage shelves or drawers.

To begin, the sampling fraction, sampling interval, and random starting number are chosen following exactly the same methods described in steps 1,2 , and 3 in Section 7. The number of rolls of microfilm on each shelf (using shelf in place of list entry) must be determined and used to assign serial numbers to the rolls as was illustrated in Table 7.1. The shelves are selected systematically in the same way list entries were selected in Table 7.1. Similarly, the number of rolls that will be inspected on a given shelf is determined by the number of selected serial numbers that happen to belong to that shelf. Finally, the individual rolls are chosen at random from the rolls on selected shelves following the method described in step 5 of Section 7 .

## 9. Other Practical Considerations for Sampling

Pilot Test. Any survey that is planned will need a well-defined data collection procedure and some sort of form or questionnaire for recording the field data. These procedures should be tested by use in a realistic pilot survey of some small portion of the population. This test need not be large inspection of 10 or 20 rolls of microfilm should suffice - but it should be conducted by one or more of the inspectors that will be involved in the fullscale survey. A pilot test will almost always lead to improvements in the data recording form(s) and often uncovers serious deficiencies in the planned survey procedures.

Replication. The easiest, and often most convincing, way to evaluate the statistical uncertainty in a survey is to repeat it and compare results. In fact, replication can be built into a survey by simply dividing the workload into approximately equal pieces and conducting the survey in parallel and independently (e.g. different inspectors, equipment, data summarization) on each piece. This method of organizing survey work, using about 4 to 10 subsamples, has been used extensively and very effectively in practical work in many fields. Examples are described by Deming (1960, Chapter 6) and Sudman (1976, pp. 171-178).

As a concrete example, suppose that it is desired to take a $2 \%$ sample (i.e. 1 in 50 ) of a population. The survey could be divided into 4 subsamples by having each of 4 inspectors conduct a 1 in 200 sample (of $0.5 \%$ of the population.) To make the 4 subsamples as comparable as possible, it would be well to use so-called "interpenetrating" subsamples by having each of the 4 inspectors choose systematic samples with different random starts, but counting from the same point of origin in the population.

## 10. Acknowledgements

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Appendix 1.
SUMMARY OF PHYSICAL LAYOUT AT NUS IN BOYERS, PA

This appendix is based on notes taken by Alan Calmes during an August, 1985, visit to National Underground Storage, Inc. in Boyers, Pennsylvania. The raw summary data are reproduced in Table A1.1. A sketch of the physical layout of the facility is given in Figure 3.

## Observations on Physical Layout at NUS

- There are about
49,654 rolls of 16 mm film $\begin{array}{r}(119 \text { million frames) }) \\ +\quad 74,326 \text { rolls of } 35 \mathrm{~mm} \text { film } \\ \hline 123,980 \text { rolls }\end{array}$ ( 89 million frames $)$
208 million frames
in the entire storage area at NUS.
- These rolls take up only about 56 compartments ( 10 shelves per compartment.)
- The number of rolls per compartment is quite variable:
- many have 3000 rolls per compartment
- many have 1600 " "
- many have 2112 " "
- Max = 5416 " "
- Min = 100 " "
- Mean = 2200 " "
- Most compartments have either 16 mm or 35 mm film, but not both.

Table A1.1
Rough Count of Number of Rolls of Stored Microfilm at NUS (based on A. Calmes' notes from August, 1985, visit to Boyers, PA)

| Aisle | Compartment | No. of 100 foot rolls* |  |
| :---: | :---: | :---: | :---: |
|  |  | 16 mm | 35 mm |
| 1 | 2 | - | 3000 |
|  | 3 | - | 3000 |
|  | 4 | - | 3000 |
|  | 5 | - | 3000 |
|  | 6 | - | 3000 |
|  | 7 | - | 3000 |
|  | 8 | - | 3000 |
| 2 | 3 | - | 3000 |
|  | 4 | - | 1660 |
|  | 5 | - | 2954 |
|  | 6 | - | 3780 |
|  | 7 | - | 3780 |
|  | 8 | - | 3780 |
| 3 | 3 | - | 3600 |
|  | 4 | - | 3600 |
|  | 5 | - | 3600 |
|  | 6 | 1700 | 1040 |
|  | 7 | - | 1600 |
|  | 8 | - | 1600 |
| 4 | 2 | - | 1600 |
|  | 3 | - | 1600 |
|  | 4 | - | 1600 |
|  | 5 | - | 1600 |
|  | 6 | 680 | 2200 |
|  | 7 | 544 | 1280 |
|  | 8 | - | 1600 |
| 5 | 3 | - | 1600 |
|  | 4 | - | 1600 |
|  | 5 | 1520 | 800 |
|  | 6 | 3040 | - |
|  | 7 | 3040 | - |
|  | 8 | 2702 | - |
|  | 9 | 2112 | - |
|  | 10 | 2112 | - |

Table A1.1, Continued

| Aisle | Compartment | No. of 100 foot rolls* |  |
| :---: | :---: | :---: | :---: |
|  |  | 16 mm | 35 mm |
| 6 | 2 | 2112 | - |
|  | 3 | 2112 | - |
|  | 4 | 2112 | - |
|  | 5 | 2112 | - |
|  | 6 | 2112 | - |
|  | 7 | 2112 | - |
|  | 8 | 2112 | - |
|  | 9 | 5416 | - |
| 7 | 2 | 2396 | - |
|  | 3 | 2720 | - |
|  | 4 | 2176 | 264 |
|  | 5 | 304 | 1408 |
|  | 6 | 1976 | 480 |
|  | 7 | 2432 | 160 |
|  | 8 | - | 1440 |
|  | 9 | - | 100 |
|  | Subtotals | 49,654 | 74,326 |
|  | Grand Total |  |  |

$*$ counts each 1000 foot roll as $10 \times 100$ foot rolls.

$\left[\begin{array}{lll}\text { Aisle 1, Compartment 2 } & \text { through } \\ \text { Aisle 7, } & \text { Compartment } 9 & \text { are in use. }\end{array}\right]$
Figure 3. Sketch of the NARA underground storage facility at National Underground Storage, Inc., Boyers, PA. August, 1985.

SELECTED PORTIONS OF THE NOVEMBER, 1986, DRAFT REVISION TO 36 CFR Part 1230, Subpart B
Standards for the Storage, Use and Disposition of Microform Records.
Section 1230.22
(a) Permanent Records
(1) Unstratified samples.
(i) Master films of permanent and unscheduled records microfilmed to dispose of the original record shall be inspected on a 3-year cycle. At each cycle, the inspection shall be made on a randomly selected sample consisting of 1000 microform units, or $1 \%$ of the total number of microform units in the collection, whichever is smaller. The term "microform unit" refers to a single [100'] roll of microfilm, a microfiche, or similar appropriate unit for inspection.
(ii) To facilitate inspection, an inventory of microfilm must be maintained, listing each microform series/publication by production date, producer, processor, format, and results of previous inspections.
(iii) The elements of the inspection shall consist of (1) an inspection for aging blemishes following the guidelines in the AIIM/MS HB96 [AIIM Special Interest Publication \#34, Association for Information and Image Management, Silver Spring, Maryland]; (2) a rereading of resolution targets; (3) a remeasurement of density; and (4) a certification of the environmental conditions under which the microforms are stored, as specified in sec. 1230.20 (a).
(iv) An inspection log shall be maintained. Information to be contained in the log shall include (1) a complete description of all records tested (title; number or identifier for each unit of film; and inclusive dates, names, or other data identifying the records on the unit of film) ; (2) the date of inspection; (3) the elements of inspection; (4) the defects uncovered; and (5) the corrective action taken. In addition, the log shall contain the results of all archival film tests required by sec. 1230.14.
(v) The results of the inspection shall be reported to the Office of Records Administration, National Archives (NI), Washington, DC 20408, 30 days after the inspection is completed. Reports shall include (1) the quantity of microform records on hand, i.e., number of rolls, number of microfiche, etc.; (2) the quantity of microforms inspected; (3) the condition of the microforms; (4) any defects discovered; and (5) corrective action taken. A copy of the inspection report shall be stored with the microfilm and have the same retention period as the microfilm.
(2) Stratified samples
(i) When the records required by sec. 1230.22(b) are maintained, it may be possible to divide a microform collection into two strata, with one stratum having an appreciably lower risk of deterioration than the other. This is determined by analysis of the information derived from previous inspections. One stratum consists of microform series/publications which showed no signs of deterioration in past inspections. In such circumstances, it will be sufficient to inspect the latter, lower risk stratum only in alternate inspection cycles. i.e., every 6 years, while continuing to inspect the former, higher risk stratum every 3 years. When the population is stratified in this way, the sample size for inspection may, in some cases, be reduced in alternate inspection cycles, as follows: when the inspection schedule calls for inspection of only the higher risk stratum, the required sample size may be computed as the smaller of 1000 units or $1 \%$ of the high risk stratum only.
(ii) When stratification is used in the inspection program, the stratum definitions must be well documented, including the reasons used to determine which microform records would be placed in the respective strata. This information must be included in the inspection report submitted to the National Archives.
(iii) This stratification of the collection shall not be used for an inspection cycle until an inspection of the entire collection has been conducted at least twice. In any case, the inspection procedures shall follow the unstratified plan described in sec. 1230. 22 (a) above at least every 6 years.

## MATHEMATICAL DEVELOPMENT OF FORMULAS FOR $P_{d}$

## A3.1 Derivation of Formulas for Probability of Detection

The formulas for the probability of detection, $P_{d}$, are derived as follows.

Define the random variable $X$ to be the number of defective rolls of microfilm drawn into a simple random sample of size $n$. In summary, the essential quantities are denoted as:

$$
\begin{aligned}
\mathrm{N}= & \text { population size (total number of rolls) } \\
\mathrm{n}= & \text { sample size } \\
\mathrm{f}= & \mathrm{n} / \mathrm{N}, \text { the sampling fraction } \\
\mathrm{A}= & \text { number of defective rolls in the population } \\
\mathrm{X}= & \text { number of defective rolls drawn in a simple random sample } \\
& \text { (without placement) of size } n
\end{aligned}
$$

The probability law of the random variable $X$ is the Hypergeometric Distribution (Cochran, 1977, pp. 55-57). Thus $P_{d}$ can be calculated as

$$
\begin{equation*}
\mathrm{P}_{\mathrm{d}}=1-\frac{\binom{N-A}{n}}{\binom{N}{n}} \tag{A3.1}
\end{equation*}
$$

The practical range of values of A will typically be less than $10 \%$ of the population size, or else a survey sample would not be needed to locate defective rolls. In this case, $A / N$ is less than 0.1 and the probability in (A3.1) can be well-approximated by use of the binomial distribution (Schilling, 1982, pp. 64-65). Using the so-called f-binomial approximation, leads to the expression

$$
\begin{equation*}
P_{d} \simeq 1-(1-f)^{A} \tag{A3.2}
\end{equation*}
$$

Using equation (A3.2), one can solve for the sampling fraction, $f=n / N$, in terms of $P_{d}$ and $A$. The solution is

$$
\begin{equation*}
\mathrm{f} \simeq 1-\left(1-P_{d}\right)^{1 / A} \tag{A3.3}
\end{equation*}
$$

The theoretical model adopted here has been used previously by Schilling (1978) in the development of a lot sensitive sampling plan for compliance testing. As in the present application, Schilling's work deals with the problem of trying to detect defective items in an isolated lot (or single finite population) of items. Unfortunately, the tables provided in that work
are too specialized to be useful in the present application to microfilm sampling. In particular, Schilling's main table applies only to the case $\mathrm{P}_{\mathrm{d}}=$ 0.90 and the smallest value of $f$ given is 0.01 .

It can be shown (Schilling, 1978) that the approximation used in equation (A3.2) is conservative in that the exact value of $P_{d}$, from equation (A3.1), is actually greater than or equal to the value calculated by the approximate formula, (A3.2). This means that the simple formula (A3.3) for the sampling fraction, and Tables 3.1 and 5.2 which were calculated from it, tend to yield recommended sampling fractions that are slightly larger than would be found if an exact calculation based on (A3.1) were performed.

## A3.2 Effect of Distributing a Given Number of Defective Rolls Across Several Groups

The formulas in section A3.1 assume that all the defective microfilm rolls belong to the same identifiable group. If instead the A defective rolls are assumed to belong to several logical groups, it would be necessary to get at least one representative from each group in the sample in order to be able to locate all the defective rolls in the follow-up phase of the inspection program.

Formally, we consider the case in which the A defective rolls are distributed equally among $k$ groups, each of size $A / k$. The relevant probability of detection is the probability that at least one member of each group is drawn in the random sample. Let

$$
\begin{aligned}
& X_{i}= \text { number of members of group } i \text { in the sample } \\
&(i=1, \ldots, k) \text {, and } \\
& P_{d}(k)=\text { probability that } X_{i} \geq 1 \text { for all } i, 1 \leq i \leq k .
\end{aligned}
$$

Treating the $\mathrm{X}_{\mathrm{i}}$ as approximately independent random variables, it follows from (A3.2) that

$$
\begin{equation*}
P_{d}(k) \simeq\left[1-(1-f)^{A / k}\right]^{k} . \tag{A3.4}
\end{equation*}
$$

Formula (A3.4) was used in the construction of Figure 2.
As was the case for a single group, equation (A3.4) can easily be solved for the sampling fraction in terms of the other parameters. The solution is

$$
\begin{equation*}
\mathrm{f} \simeq 1-\left[1-P_{d}(k)^{1 / k}\right]^{k / A} . \tag{A3.5}
\end{equation*}
$$

## A3. 3 Relation to National and International Standards on Sampling

The author is aware that it would have been desirable to refer to national or international standards for rules on choosing appropriate sample sizes. The following discussion explains why this was not possible.

The sampling standards that are most relevant to the microfilm inspection problem are MIL-STD-105D (U.S. Department of Defense, 1963) and its international adaptation, ISO 2859 (ISO, 1974). Both are discussed in detail in Schilling (1982).

In parallel with this report, the tables and rules given in the standards are based on the hypergeometric probability model, and on binomial and Poisson approximations to that model (see, for example paragraph 11.1 of ISO 2859.) Further correspondences in notation and concepts between those standards and the present report are as follows:

This Work
$\begin{array}{ll}\mathrm{P}_{\mathrm{d}} & 1-\mathrm{P}_{\mathrm{a}},\left(\mathrm{P}_{\mathrm{a}}=\text { probability of acceptance }\right) \\ \mathrm{A} / \mathrm{N} & \text { LQ, Limiting Quality } \\ \mathrm{n} & \mathrm{n}, \text { Sample Size } \\ \mathrm{N} & \text { Lot or Batch Size }\end{array}$
MIL-STD-105D

ISO 2859 and

An important difference between this microfilm inspection problem and the problems addressed by MIL-STD-105D and ISO 2859 is that the sampling standards are designed to control the proportion of defective items in a long series of lots or batches. In contrast, there is only one "lot" of microfilm to be inspected in the problem considered here.

These standards do describe, as a secondary application, the use of their tables for sampling "isolated lots," a situation which more nearly matches the microfilm inspection problem. Unfortunately, the values of $\mathrm{P}_{\mathrm{a}}\left(=1-\mathrm{P}_{\mathrm{d}}\right)$ in the standard tables are only available for a very limited selection of values of $n$ and $N$. Hence the usefulness of the standards is similarly limited in the present context. In particular, the plots and tables given in sections 3 and 5 of this report could not have been derived from the sparse values given in the tables in the standards.

## A3.4. Comparison With Textbook Formulas for Sample Size

Equations (A3.3) and (A3.5) are significantly different from the formulas for sample size usually given in textbooks on sampling. Those sources use a different criterion based on specifying the desired length of a confidence interval for the estimated proportion of defective items in a population, rather than the criterion used in this report based on $P_{d}$. Specifically, the usual formula for sample size (Cochran, 1977, section 4.4) is:

$$
\begin{equation*}
n=\frac{n_{0}}{1+\left(n_{0}-1\right) / N} \tag{A3.6}
\end{equation*}
$$

where $\quad n_{0}=t^{2} P Q / d^{2}$
and $\quad \mathrm{N}=$ population size
$\mathrm{P}=$ proportion of defective items in the population ( $=A / N$ )
$\mathrm{Q}=1-\mathrm{P}$
$t=1.96$ for $95 \%$ confidence level (for example)
d $=$ desired bound on the error of estimation (half-width of a 95\% confidence interval)

For a large population ( $N$ large relative to $n$ ) the value of $n$ is wellapproximated by simply calculating $n_{0}$. The formula for $n_{0}$ is widely used for planning surveys (e.g. MSTC, 1981) to good effect when the goal of the survey is to estimate the proportion of defective items in the population.

The effect of the population size in formula (5.1) is significantly different in character, as well as detail, in comparison to the recommended formula (3.2). Specifically, the textbook formula (5.1) is only weakly dependent on population size whenever N is large compared to n (Cochran, 1977, page 76). In contrast, the recommended formula (3.2) shows that $n$ should be chosen to be directly proportional to N .

In summary, the reason the sample size formulas in this report are so different from the usual formulas is that a different criterion has been used, based on $\mathrm{P}_{\mathrm{d}}$, to determine the required sample size. The sample size formulas given is textbooks are derived by considering how large a sample is needed to estimate the proportion of defective rolls with a $95 \%$ confidence interval of a given length. Our concern here is not with estimation, but rather with detection of a group of defective rolls of microfilm.
4. TITLE AND SUBTITLE

SURVEY SAMPLE DESIGN FOR MICROFILM INSPECTION AT THE NATIONAL ARCHIVES

## 5. AUTHOR(S)

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$\square$ Document describes a computer program; SF-185, FIPS Software Summary, is attached.
11. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)

This report describes the statistical design of a sample survey to monitor the condition of microfilm in a large collection maintained by the National Archives. The design criterion developed for the survey ensures that the number of rolls of film inspected will be large enough to achieve a pre-chosen probability of detecting a specified amount damaged film that might exist in the population. Tables and formulas are given to satisfy the design criterion under a range of conditions. Other practical aspects of survey design are discussed including the sampling frame, stratification, sample selection procedure, pilot testing, and the use of replicated sampling.
12. KEY WORDS (Six to twelve entries: alphabetical order; capitolize only proper names; ond separate key words by semicolons) sample size determination; sample survey; sampling fraction; sampling frame; statistics; stratification
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