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Abstract

The ASME Performance Test Code, PTC 19.3, establishes the velocity ratings of thermowells in terms of a simplified resonant frequency estimate. The theoretical basis for this is examined and compared with discrete element methods. It is found that the code calculation fails to guarantee conservative velocity ratings for shorter thermowells. The implications of this finding and possible remedies are discussed.

Keywords: Thermowell design, velocity ratings, vortex shedding, critical frequency estimates.

Introduction

The traditional starting point for thermowell design and application is the ASME Performance Test Code, PTC 19.3 [1]. While it is the formal basis for the static pressure and velocity ratings of thermowells used in performance testing, it is at the same time, the de facto standard for thermowells used throughout industry. This rating is derived from a simplified analysis that describes the tendency for thermowell to experience flow-induced resonance when the vortex shedding rate coincides with the natural frequency of a transverse bending mode. In spite of the widespread acceptance of the code, there is a growing body of evidence that thermowells designed to its criteria are subject to increased risk of failure at considerably lower velocities. There are a number of reasons for this including use of an out-dated description of vortex shedding, limitations of the critical frequency estimate, and reliance on an incomplete stress analysis [3]. Similar conclusions are reached for thermowells designed to the code criteria although used in non-code applications [4]. A more complete discussion of flow-induced vibration as it relates to thermowells and intrusive pipe fittings generally may be found in the literature [5]-[9]. The present discussion is restricted to that aspect of the code dealing with resonant frequency calculations.

The code takes a tabular approach to the critical frequency estimates. This is no longer justified given the range of computational tools now available, so it is beneficial to compare the theory used to develop these tables with the results of discrete element calculations. While the code estimate is found to be adequate for the longer thermowells, it is found to over-estimate the critical frequency (and thus velocity rating) of short thermowells commonly recommended for high velocity applications. It also lacks sufficient accuracy for use in more rigorous designs or in forensic studies. These deficiencies can only be remedied through the use of modern analysis methods for both the critical frequency and stress estimates.

Murdock's Approximation

The PTC 19.3 critical frequency calculation [1,2] combines a single-degree-of-freedom (SDOF), discrete-mass model of the thermowell with a displacement variable taken as the tip deflection of a continuous beam. This highly unusual approach differs significantly from accepted methods for estimating the critical frequencies of vibrating beams or establishing the forced response, yet it appears to be conservative. The theoretical basis for this method is explained but not endorsed in the following.

The traditional form of Rayleigh's method is based on the fact that, in the absence of losses, the amplitudes of the kinetic and potential energies, in harmonically vibrating systems, are equal. Following the treatment in reference [10] for beams in which the shear and rotation of the neutral axis are ignored, the resonant frequency of the beam may be written as:

$$\omega_0^{\;2} = \frac{\int EI\left[\dfrac{d^2y}{dx^2}\right]^2 dx}{\int y^2 m\, dx}. \tag{1}$$

The transverse deflection $y(x)$, with x the spanwise coordinate for a beam, is characterized by an elastic modulus, E, a moment of inertia, I, and lineal mass density, m.

This relationship is exact where the deflection profile is a solution of the fourth-order Stürm-Liouville problem for the beam and forms the least upper bound for all estimates based on approximate mode shapes. One variation of this method uses the static deflection $y(x)$ of the thermowell (as a horizontal beam) sagging under its own weight. Such a solution does satisfy the boundary conditions, but given that it is a particular integral for a weight-loaded beam, rather than an homogeneous mode shape, it can only provide a crude estimate.

A discrete mass version of the latter results in a critical frequency estimate given by:

$$\omega_0^{\;2} = \frac{g \sum m_i y_i}{\sum m_i y_i^{\;2}}, \tag{2}$$

where y_i is the static displacement of the individual masses, m_i, and g is the acceleration of gravity. This approach suffers from the same limitations as the continuous model but is less accurate and convergent only to the extent that the displacements represent actual dynamic modes of the system.

The Rayleigh method by itself only produces a least upper bound for the resonant frequency. For design purposes, given the imprecise method used in the code, a lower bound is preferred to minimize the risk of exposing the thermowell to flow-induced resonance. Since the maximum deflection of a cantilever always occurs at its tip, a lower bound can be constructed by the expedient of replacing the SDOF displacement by the tip displacement of the continuous beam. The resonant frequency is now bounded by:

$$\sqrt{\frac{g}{y_{tip}}} \le \omega_0 \le \sqrt{\frac{g}{y_m}}\,. \tag{3}$$

The lower bound constitutes the Murdock estimate used in PTC 19.3 while the upper bound is the SDOF Rayleigh estimate. The analytical expression for the tip deflection, y_{tip}, as derived in reference [2] is only valid for linearly tapered thermowells in which shear can be ignored.

The Code Method

To simplify the calculations, PTC 19.3 relies on tabulated frequency factors with the resonant frequency of the thermowell expressed as:

$$f_0 = \frac{K_f}{L^2}\sqrt{\frac{E}{\gamma}} \tag{4}$$

where f_o ($=\omega_0/2\pi$) is the resonant frequency of the thermowell, K_f is a frequency factor according to the Murdock approximation, E is the modulus of elasticity, L is the unsupported length of the thermowell, and γ is the specific weight of the material.

Recasting the Murdock critical frequency estimate in the form of Eq. (4) results in an analytical expression for the frequency factors tabulated in the code. They may also be determined by direct numerical integration of Eq. (11) of reference [2]. No evidence of a reported ill-condition [2] is observed. The frequency factors developed from the Murdock formula [2] are shown in Fig. (1) for the Design Classes currently recommended in the code. The thermowell dimensions for these classes in both customary US units and in metric equivalents are summarized in Table I, below.

Table I: ASME PTC 19.3 Thermowell Dimensions (As adopted 1974). Metric equivalents, in parentheses, are approximate.

Size\Class	I	II	III	IV	V
Root	0.8125" (2.06 cm)	0.9375" (2.38 cm)	1.1250" (2.86 cm)	1.2500" (3.18 cm)	1.4375" (3.65 cm)
Tip	0.6250" (1.59 cm)	0.7500" (1.91 cm)	0.9375" (2.38 cm)	1.0625" (2.70 cm)	1.2500" (3.18 cm)
Bore (Nominal)	0.2500" (0.64 cm)	0.3750" (0.95 cm)	0.5625" (1.43 cm)	0.6875" (1.75 cm)	0.8750" (2.22 cm)

Note: Thermowell lengths used in the calculations and as suggested by the code are 2.5", 4.5", 7.5", 10.5", 16", and 24". A metric conversion of: 1”=25.4 mm, applies.

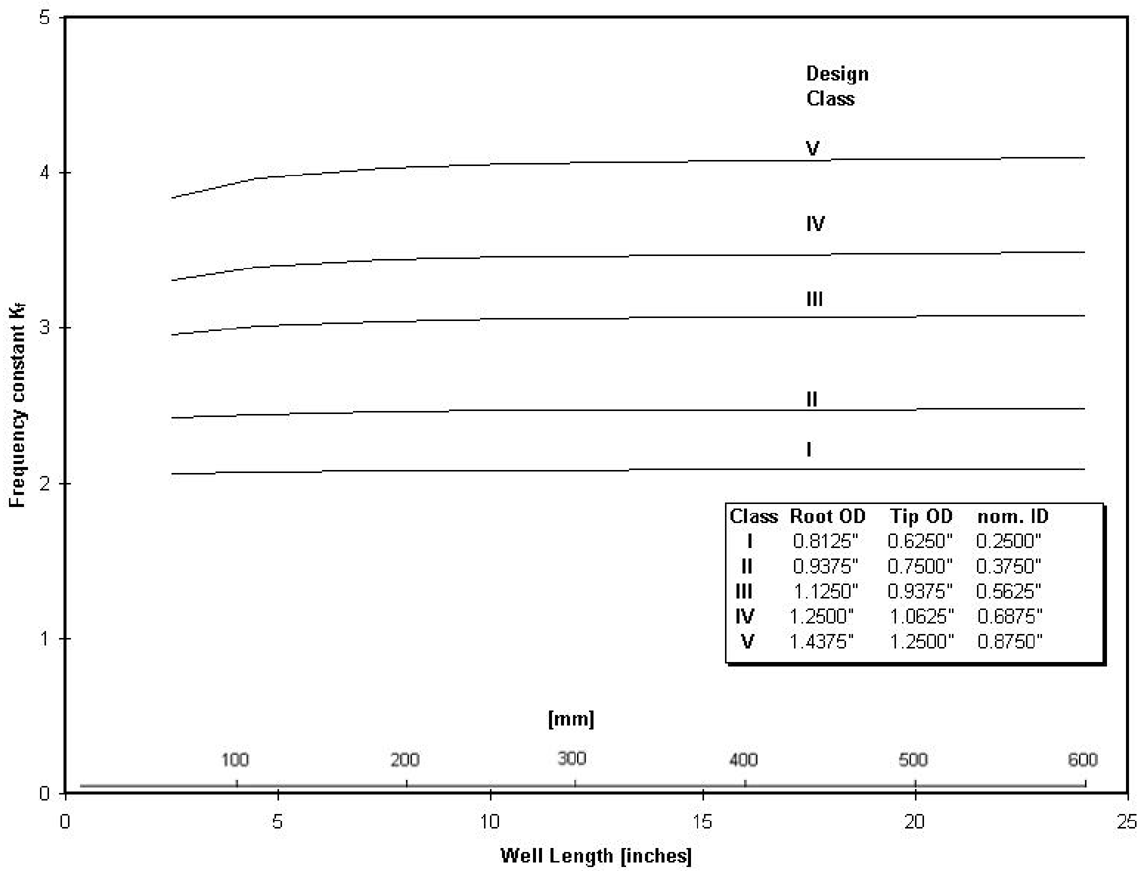

Fig. 1: PTC 19.3 frequency factors using the analytic expression in reference [2] for the design classes considered in the code (see Table I).

For completeness, the equivalent Murdock estimate for a hollow cylinder is given by:

$$K_f \cong \sqrt{\frac{g(D^2+d^2)}{8\pi^2}}\,, \tag{5}$$

where D is the outside diameter at the base (root) and d the nominal inside diameter (or bore) of the thermowell. This result is easily compared to the exact theory and suggests that in having ignored shear effects, the Murdock calculation under-estimates the actual resonant frequency of the uniform beam by some 25 %.

Discrete Beam Comparisons

The frequency factors for the PTC 19.3 designs can be inferred from discrete beam calculations in similar fashion. Two beam models are considered to illustrate the importance of shear as the thermowell aspect ratio approaches unity. The discrete beam calculations are based on the Euler-Bernoulli beam elements that allows only simple bending and the Timoshenko beam elements that includes both shear and rotation of the neutral axis. Transfer matrix methods are used to establish the mode shapes and critical frequencies [10,11], with numerical error of the method held to less than 0.2 % in comparison with independent calculations [12]. The resulting frequency factors for PTC 19.3 designs are presented in Fig. 2. Rayleigh estimates for these designs [13] are shown as heavy dashed lines.

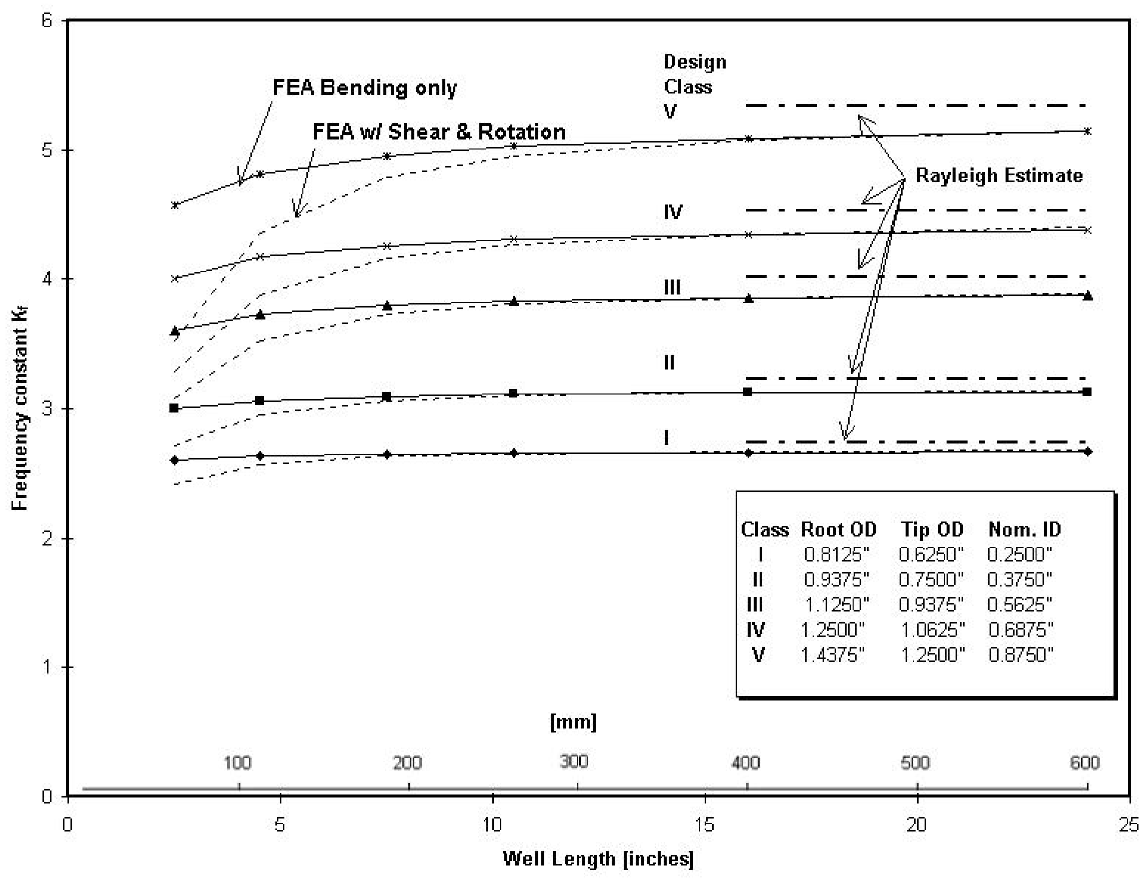

Fig. 2: Comparison of the frequency factors developed from two discrete beam calculations one that includes simple bending only and the other that includes shear and rotation of the neutral axis.

For aspect ratios L/D >5, the Murdock calculation under-estimates the critical frequency by some 15 % to 20 % and therefore is sufficient for design. This difference is also in line with that found for hollow cylinders. When shear and rotation of the neutral axis are included in the beam calculations, it is discovered that the code estimate is no longer conservative for small aspect ratios. While the lack of accuracy of the code method is a concern in forensic evaluations, arbitrary use of improved estimates should be approached cautiously to avoid a significant reduction in design margins [3]. In application critical, high velocity applications, a finite element based, dynamic analysis is recommended. It is noted that finite element methods that fail to include shear and rotation of the neutral axis, although an considerable improvement, suffer from the same defect as the Murdock calculation for small aspect ratios.

Conclusions

The analytical expression for critical frequency developed by Murdock duplicates the frequency factors contained in PTC 19.3, and is better suited for direct calculations. While it is generally conservative, discrete element calculations that include shear and rotation of the neutral axis are recommended where more accurate estimates are required or where severe service conditions are expected. Caution is advised, however, in simply combining these improved estimates within the frame work of the current code, without consideration of tensile failure in bending.

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