Web-Based 3D Visualization in a Digital Library of Mathematical Functions

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Abstract

The National Institute of Standards and Technology (NIST) is developing a digital library of mathematical functions to replace the widely used National Bureau of Standards Handbook of Mathematical Functions [Abramowitz and Stegun 1964]. The NIST Digital Library of Mathematical Functions (DLMF) will provide a wide range of information about high level functions for scientific, technical and educational users in the mathematical and physical sciences. Clear, concise 3D visualizations that allow users to examine poles, zeros, branch cuts and other key features of complicated functions will be an integral part of the DLMF. Specially designed controls will enable users to move a cutting plane through the function surface, select the surface color mapping, choose the axis style, or transform the surface plot into a density plot. To date, Virtual Reality Modeling Language and Extensible 3D (VRML/X3D) standards have been used to implement these capabilities in more than one hundred 3D visualizations for the DLMF. We discuss the development of these visualizations, focusing on the design and implementation of the VRML code, and show several examples.

Keywords: virtual reality modeling language, VRML, extensible 3D, X3D, digital library, 3D visualization, special functions

1. Introduction

Fundamentally unchanged since its initial publication in 1964, the National Bureau of Standards (NBS) Handbook of Mathematical Functions [Abramowitz and Stegun 1964] continues to be a popular resource in the mathematical and scientific community. Consequently, the National Institute of Standards and Technology (NIST), the successor organization to NBS, has decided to update the handbook, publishing it on the web as the Digital Library of Mathematical Functions (DLMF) [Lozier 2002]. The contents of the DLMF, also being published in hardcopy format, will include mathematical formulas, references, methods of computation, graphs, and links to software for more than 40 different functions. The web site will feature interactive navigation, a mathematical equation search, and dynamic interactive graphics. It is slated for completion in 2005, but a mockup version with limited capabilities can be found at http://dlmf.nist.gov .

High level, or special, mathematical functions such as the Bessel functions, gamma and beta functions, hypergeometric functions and others are important for solving many problems in the mathematical and physical sciences. The graphs of many of these functions exhibit zeros, poles, branch cuts and other singularities that can be better understood if the function surface can be manipulated to show these features more clearly. A digital library offers a unique opportunity to create cutting-edge 3D visualizations that not only help scientists and other technical users, but also make the library more accessible to educators, students and others interested in an introduction to the field of special functions.

After investigating web-based 3D graphics file formats, we selected the Virtual Reality Modeling Language (VRML) to create our visualizations. VRML is a standard 3D file format for which browsers and plug-ins are publicly available on the Internet, and its functionalities satisfy our web-based visualization requirements. The emerging XML based standard, X3D, is the successor to VRML and will support all VRML functions. We have successfully translated and tested several of our VRML files to confirm that conversion to X3D format is both feasible and practical.

The DLMF web site will be organized into chapters, and most chapters will have a section called "Graphs and Visualizations" where links to 2D and 3D figures can be found. Each 3D image will be associated with a link to a VRML/X3D file that the user can manipulate interactively.

Although we continue to address such issues as data accuracy, the clipping of surfaces falling outside a specified range, and the generation of computational meshes [Saunders and Wang 2000] to capture significant features of a function, this paper assumes these issues have been resolved and focuses primarily on the creation of the VRML files. In particular, we describe the development and implementation of MathViewer, a VRML prototype, or object-defining and extensibility routine, which controls the interactive manipulation of the 3D surfaces. In Section 2 we discuss the user capabilities provided by MathViewer. Section 3 describes the high and low-level VRML prototypes that make up MathViewer, Section 4 shows still images of visualizations created for the DLMF, Section 5 discusses some initial results using an X3D implementation of MathViewer, and finally, Section 6 offers some conclusions and suggestions for enhancements.

2. MathViewer

While customary VRML browser controls such as rotate, zoom, and pan permit a user to examine a 3D display from an arbitrary direction, we also wanted to provide options for more extensive manipulation of function surfaces. Discussions with DLMF editors and authors led to several suggestions concerning the types of interaction users might desire. We developed the VRML prototype MathViewer to implement several of these capabilities discussed below.

MathViewer generates a display consisting of a 3D function surface with coordinate axes, and a main control panel with five buttons. The first four buttons generate subordinate control displays that provide cutting plane control, axes and label style control, scaling control, and color map control. The fifth button, "none", deletes the subordinate display. Figure 1 shows the modulus of the gamma function in the CosmoPlayer VRML browser. The dashboard display is specific to CosmoPlayer.
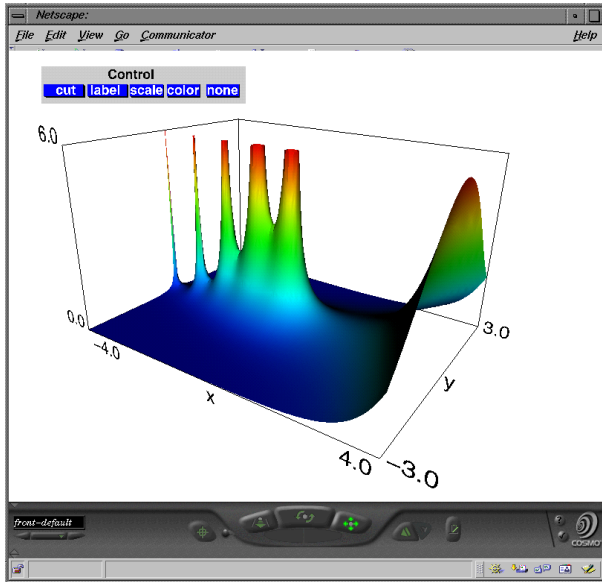

Figure 1 Gamma function display in CosmoPlayer.

(1) Cutting Plane Control

Cutting plane control allows the user to examine the intersection of a plane with a function surface as the plane moves through the surface. The plane can be parallel to coordinate axis x, y, or z. The intersection curves are projected onto opposite faces of the bounding box, and displayed in a separate pop up window. The user moves the cutting plane by selecting VCR-like buttons to play, stop, reverse, rewind, or fast-forward, or uses the slider bar to move the plane to a specific position and clicks "go" to display the intersection. The calculation of the intersection curves occurs in real time.

 Figure 2 shows a cutting plane moving in the z, or vertical axis, direction for the Jacobian elliptic function, dn(x, κ). The cutting plane control panel is below the function on the left, and on the bottom right is the pop up window showing the intersection curves. The axis labels, "x", "n", "z", displayed on the control panel and pop up window correspond to the axes shown in the surface figure. When the user moves the cutting plane with the "play" button, the location of the cutting plane is displayed with the pop up window showing the curves of intersection. When the cutting plane is moved with the slider bar, the location of the cutting plane appears under the slider bar in the control panel. When the "go" button is hit, the intersection curves and cutting plane location appear in the pop up window.
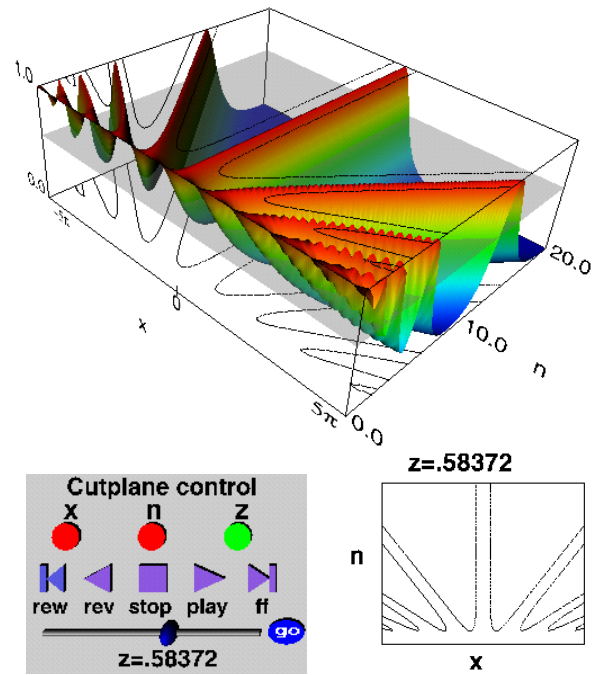

Figure 2 Jacobian elliptic function $z = $ dn(x, κ), for $\kappa = 1 - e^{-n}$, n=0 to 20 and real argument, x, $-5\pi \leq x \leq 5\pi$ with cutting plane at z=0.58372.

(2) Axis and Label Control

Using the axis and label control panel, a user can select the style preferred. This may be convenient if the user wishes to do a screen capture of the display for use in a publication, or elsewhere. Figure 3 shows the four axis styles that are available: front, back, triad, and no axes. Both the front and back styles allow the user to enclose the surface with a bounding box.
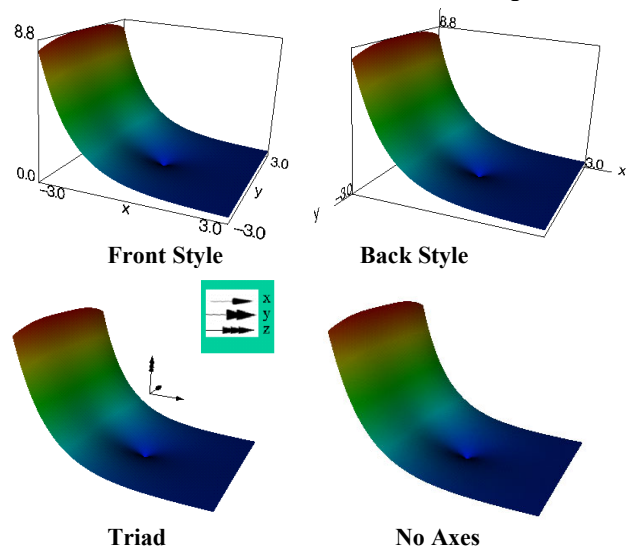

Figure 3 Axis and Labeling Styles

Expressions for mathematical functions often contain special symbols or Greek letters which are not supported by most VRML

browsers. When such characters are needed we generate image files using other software and imbed them into the VRML file. Examples of this can be seen in some of the axis labels for the surfaces shown in Figure 13.

(3) Color Map Control

Visualizations in the DLMF represent either real-valued or complex-valued functions of the form, $z=f(x, y)$. For real valued functions only a height based color map, where height = z, is provided. For complex-valued functions, the user has the option of using a height based color mapping where height = |z|, or a mapping based on the phase, or argument, of z. The height based mapping is determined by the following formula where Color is RGB color (red, green blue) and value is the height scaled to lie between 0 and 1, inclusive:

Color = (0, value/0.25, 1), if $0 \le$ value $< 0.25,$
Color = (0, 1, (0.5-value)/0.25), if $0.25 \le$ value $< 0.5,$
Color = ((value-0.5)/0.25), 1, 0), if $0.5 \le$ value $< 0.75,$
Color = (1, (1-value)/0.25, 0), if $0.75 \le$ value $\le 1.$

Currently the user is given two options for the phase based mapping: four color map or continuous spectrum. In the four color based method, the colors blue, green, red, and yellow are used to indicate whether the phase of z lies in the first, second, third, or fourth quadrant, respectively. The continuous spectrum based method maps the phase information to the color so that 0, $\pi/2$, π, and $3\pi/2$ correspond to red, yellow, cyan, and blue with a smooth transition between the colors. Figures 1-3 show height based color mappings. The surfaces in Figures 4 and 5 illustrate the four color and continuous spectrum based mappings.
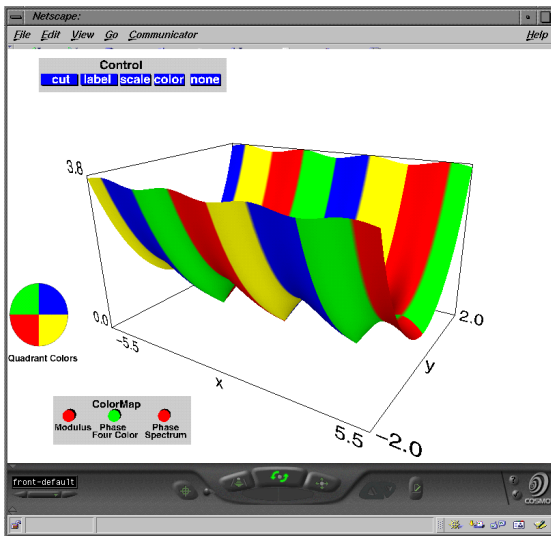

Figure 4 Modulus of complex sine function with four color map.