Thermodynamic Data to Support High-Temperature Syngas Quench Design: Vapor Liquid Equilibrium Calculations

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ABSTRACT

A procedure has been developed, based on a dilute-solution model of the liquid phase and the rigorous virial expansion for the vapor phase, for modeling the vapor-liquid equilibrium between water and a synthesis gas consisting primarily of H₂, CO, and CO₂. Calculations of the vapor-liquid equilibrium have been performed at temperatures and pressures typical for a quench step in an IGCC power plant. The uncertainty in the calculated vapor-phase mole fraction of water is less than 0.01, and is due primarily to the omission of higher-order terms in the virial expansion.

Other thermodynamic calculation methods were examined and compared to the more rigorous calculations. The ideal-gas assumption for the vapor phase seriously underestimates (by about 0.09 mole fraction at typical conditions) the water content of the equilibrium vapor. The Peng-Robinson equation performs much better, producing vapor-phase mole fractions of water that differ from the more rigorous results by amounts similar to the uncertainty in the virial approach.

This work was performed for the Program for Technology Innovation of the Electric Power Research Institute (EPRI). A version of this report with additional process-related comments from the sponsor perspective is being published separately as an EPRI Report.
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INTRODUCTION

Background

The purpose of this work is to provide data relevant to the design of Integrated Gasification Combined Cycle (IGCC) power plants. In some proposed designs, a hot synthesis gas, consisting mostly of H₂, CO, and CO₂, is quenched with water. The amount of water in the resulting gas is a significant factor in the economics of the cycle. Previous efforts to model this step, using a variety of thermodynamic models, have come up with significantly different results. This project was undertaken to perform calculations with a rigorous thermodynamic model in order to provide reliable data for the water vapor content, with reasonable estimates of uncertainties.

Problem Definition

The calculations performed consist of the computation of thermodynamic equilibrium between vapor and liquid (vapor-liquid equilibrium, or VLE). This equilibration takes place at a specified temperature T and absolute pressure p. Based on input from the sponsor, a “baseline” case was defined at a temperature of 495 K (approximately 431 °F) and a pressure of 4.5 MPa (approximately 650 psia).

The “baseline” composition for the dry gas was also taken from sponsor input and is shown in Table 1-1. Other, trace components in the gas phase may exist (such as H₂S, NH₃, and CH₄), but they are outside the scope of this project. While the main result of interest (mole fraction of water in equilibrium vapor) is not very sensitive to the total amount of water in the system, for consistency all calculations assumed five times as many moles of water as moles of dry gas.

| Table 1-1 |
| Baseline Composition of Synthesis Gas |
| Gas | Mole Fraction |
| Ar  | 0.01 |
| H₂  | 0.39 |
| N₂  | 0.01 |
| CO  | 0.41 |
| CO₂ | 0.18 |
INTRODUCTION

Approach

In order to solve the VLE problem, the fugacity of each component in the liquid phase is equated with its fugacity in the vapor phase under the constraint of fixed $T$ and $p$. In the terminology of chemical engineering, this is known as a “flash” calculation. The computational algorithm for solving the flash problem is described in Appendix A. This algorithm requires a model for the liquid-phase and vapor-phase fugacities of each component as a function of temperature, pressure, and composition.

For modeling the fugacities in the liquid phase at these moderate pressures, where the solubility of the gases in water is small, it is sufficient to invoke the dilute-solution approximation known as Henry’s Law. This primarily requires knowing the Henry’s constant for each gas in water as a function of temperature. Additional data on the partial molar volumes of gases in solution provide a small correction for the effect of pressure on the fugacity. The liquid-phase fugacity model used in this work is completely documented in Appendix B.

The most important factor in determining the water content of the equilibrium vapor is the vapor-phase fugacity model. At the conditions of interest, an ideal-gas assumption (common in many engineering calculations) would seriously overestimate the fugacity of water, leading to an underestimation of the equilibrium water mole fraction. The equation-of-state approaches commonly used in engineering thermodynamics are designed primarily for hydrocarbon mixtures, so they cannot be expected to be reliable for mixtures of water with nonpolar gases as encountered in this work.

Our approach here is to model the vapor-phase fugacities by the virial expansion, which is a rigorous series of corrections to the ideal-gas law. At the moderate pressures of interest here, the second virial coefficient (first correction to the ideal gas) provides sufficient accuracy. Therefore, if reliable values of the second virial coefficients among all components in the mixture can be obtained, accurate values of the vapor-phase fugacities can be obtained. Appendix C documents the second-virial-coefficient data used in this work, and their use in fugacity calculations.

For purposes of comparison, we also performed vapor-phase fugacity calculations with the ideal-gas model, and with a common engineering equation of state model (the Peng-Robinson equation, applied only to the vapor phase). These models are also described in Appendix C.
2
RESULTS

Baseline Case Results

Table 2-1 shows the results of VLE calculations for the baseline case as defined in Chapter 1 (495 K, 4.5 MPa), for all three of the vapor fugacity calculation methods considered in this work. While the quantity of direct interest in this project is the vapor-phase mole fraction of water, all compositions in both phases are shown for completeness.

The rigorous calculation from virial coefficients produces a vapor mole fraction of water of 0.5731, much larger than would be obtained from an ideal-gas calculation. Perhaps surprisingly, use of the Peng-Robinson equation for vapor fugacities produces results in fairly good agreement with the more rigorous calculation.

Table 2-1
Equilibrium Liquid (x) and Vapor (y) Mole Fractions Calculated for the Baseline Case, with Vapor Fugacities Computed by the Ideal-Gas (IG), Virial (Vir), and Peng-Robinson (PR) Methods

<table>
<thead>
<tr>
<th>i</th>
<th>x(IG)</th>
<th>x(Vir)</th>
<th>x(PR)</th>
<th>y(IG)</th>
<th>y(Vir)</th>
<th>y(PR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₂O</td>
<td>0.9986</td>
<td>0.9988</td>
<td>0.9989</td>
<td>0.4817</td>
<td>0.5731</td>
<td>0.5666</td>
</tr>
<tr>
<td>Ar</td>
<td>6.87×10⁻⁶</td>
<td>6.15×10⁻⁶</td>
<td>5.96×10⁻⁶</td>
<td>0.0052</td>
<td>0.0043</td>
<td>0.0043</td>
</tr>
<tr>
<td>H₂</td>
<td>2.98×10⁻⁴</td>
<td>2.72×10⁻⁴</td>
<td>2.68×10⁻⁴</td>
<td>0.2026</td>
<td>0.1668</td>
<td>0.1693</td>
</tr>
<tr>
<td>N₂</td>
<td>4.89×10⁻⁶</td>
<td>4.38×10⁻⁶</td>
<td>4.32×10⁻⁶</td>
<td>0.0052</td>
<td>0.0043</td>
<td>0.0043</td>
</tr>
<tr>
<td>CO</td>
<td>2.87×10⁻⁴</td>
<td>2.56×10⁻⁴</td>
<td>2.53×10⁻⁴</td>
<td>0.2131</td>
<td>0.1754</td>
<td>0.1780</td>
</tr>
<tr>
<td>CO₂</td>
<td>7.58×10⁻⁴</td>
<td>6.32×10⁻⁴</td>
<td>6.14×10⁻⁴</td>
<td>0.0922</td>
<td>0.0762</td>
<td>0.0774</td>
</tr>
</tbody>
</table>

Effects of Temperature and Pressure

To assess the effect of temperature and pressure on the results, calculations were performed both over a range of temperatures (480 K to 510 K, fixing the pressure at its baseline value of 4.5 MPa), and over a range of pressures (4.0 MPa to 5.0 MPa, fixing the temperature at its baseline value of 495 K). The results of these calculations are shown in Figures 2-1 and 2-2.
RESULTS

Figure 2-1
Effect of Temperature on Equilibrium Mole Fraction of Water in Vapor at 4.5 MPa

Figure 2-2
Effect of Pressure on Equilibrium Mole Fraction of Water in Vapor at 495 K
RESULTS

Figure 2-1 shows that, as would be expected (because of the increase of the vapor pressure of water with temperature), increasing the temperature at constant pressure results in more water in the vapor phase. The difference between the virial result and the other two vapor fugacity methods grows slowly with temperature, but over the entire range shown the ideal-gas results are seriously in error, while the Peng-Robinson results are reasonable.

Figure 2-2 (note that the scale is different from that in Figure 2-1) shows that an increase in pressure at constant temperature results in a reduced mole fraction of water in the vapor. This also makes physical sense; the partial pressure of water is determined primarily by its vapor pressure as a function of temperature, so any increase in pressure must come from increasing the amount of other components in the synthesis gas. Over this range of pressures at fixed temperature, the relative differences among the three vapor fugacity methods is nearly constant.

Estimation of Uncertainties

Three main sources of uncertainty in the calculated vapor-phase water content can be identified in our model. In the discussion below, we will use the term “uncertainty” to refer to an expanded uncertainty with coverage factor $k=2$, which approximately corresponds to a 95% confidence interval.

Uncertainty from Liquid Fugacity Model

The liquid-phase fugacity model introduces only negligible uncertainty. The fugacity of pure water, which is the main part of this model that determines the water content in the vapor, is determined mainly by the vapor pressure of water, which has an uncertainty of approximately 0.02% in this temperature range. [1] A small correction for pressure is on the order of 1%, and it is determined by the liquid molar volume, which is known to within 0.1%. [1]

The gas solubility in the liquid phase is determined primarily by the Henry's constants for the different gases in water, which have uncertainties of roughly 5% in this temperature range. [2] However, because the resulting solubilities are still quite small (see Table 2-1), the only significant effect of this uncertainty on the VLE results is on the mole fractions of the gases in the liquid phase; the water content in the vapor is not significantly affected.

Uncertainty from Second Virial Coefficients

The water-gas and water-water second virial coefficients all have some uncertainty associated with them (see references in Appendix C). We tested the impact of this uncertainty on the results by perturbing each of these virial coefficients (excluding those for the trace gas components argon and nitrogen) by its ($k=2$) uncertainty and repeating the calculations for the baseline case. The results are shown in Table 2-2. The results in the table show that this contribution to the uncertainty is also relatively small, contributing to the uncertainty of the water vapor mole fraction by about 0.002 for the second virial coefficient of pure water and by less than 0.001 for each of the water-gas coefficients.
RESULTS

Table 2-2
Effect on Water Content in Vapor of Perturbing Second Virial Coefficient Upward by its (κ=2) Uncertainty

<table>
<thead>
<tr>
<th>Second Virial Coefficient Perturbed</th>
<th>Water Mole Fraction in Vapor</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Baseline case)</td>
<td>0.5731</td>
</tr>
<tr>
<td>H$_2$O-H$_2$O</td>
<td>0.5708</td>
</tr>
<tr>
<td>H$_2$H$_2$O</td>
<td>0.5729</td>
</tr>
<tr>
<td>CO-H$_2$O</td>
<td>0.5724</td>
</tr>
<tr>
<td>CO$_2$H$_2$O</td>
<td>0.5728</td>
</tr>
</tbody>
</table>

There are also uncertainties in the second virial coefficients not involving water, but their influence on the water content of the vapor is relatively negligible.

Uncertainty from Higher Virial Coefficients

The truncation of the virial expansion after the first correction necessarily introduces some error due to the omission of higher-order terms. These errors will increase with increasing density. We can estimate the effect of higher-order terms by considering the binary system H$_2$O-CO$_2$. For this system, experimental values of third virial coefficients in the temperature range of interest are available for pure water, pure CO$_2$, and for the binary interactions. [1,3,4] These values have large uncertainties, but they serve the purpose of estimating the magnitude of the uncertainty due to virial coefficients beyond the second.

We establish a “baseline” case for the H$_2$O-CO$_2$ mixture at the same $T$ and $p$ as our main case. For this case, calculation with our second-virial model yields a water vapor mole fraction of 0.5788. Repeating the calculation with the inclusion of third virial coefficients produces a vapor mole fraction of 0.5857. Further analysis reveals that almost all of this change is due to the third virial coefficient of pure water.

We therefore conclude that the omission of higher virial coefficients causes the vapor mole fraction of water to be underestimated by an amount on the order of 0.005 to 0.01. This is the largest contribution to the uncertainty in our calculation. The fact that most of this deviation appears to be due to the third virial coefficient of pure water suggests that including only that coefficient could reduce this source of uncertainty. However, the improvement in uncertainty would be limited by the large uncertainty in water’s third virial coefficient; two sources for this coefficient differ from each other by about 25%. [1,5]

Effect of Gas Composition

Some calculations were performed to assess the effect of different gas compositions on the equilibrium water content of the vapor. This was done for each of the major components of the synthesis gas by adding 0.01 to its mole fraction, and reducing the other major components proportionately. The resulting effects were quite small. Increasing the H$_2$ mole fraction in the
synthesis gas by 0.01 changed the water equilibrium mole fraction by −0.00005. Similar
increments for CO and CO₂ produced changes of −0.00001 and +0.00009, respectively. So, we
can say that the water content in the vapor is enhanced if there is more CO₂ in the synthesis gas
and reduced if there is more H₂ in the synthesis gas, but these effects are quite small.

**Liquid-Phase Composition**

While the water content of the equilibrated vapor is the main quantity of interest in this study, it
is worthwhile to make a few observations about the content of the liquid phase. The calculated
values for the different gas solubilities at the baseline conditions are shown in Table 2-1.
Because CO₂ is more soluble in water than most gases, its mole fraction in the equilibrium liquid
is larger than those of the other gases combined, despite its being only 18 % of the synthesis gas.
This CO₂ solubility may be important if basic components such as NH₃ are present.

Because the same liquid-phase fugacity model was used in all calculations, the differing liquid
compositions shown in Table 2-1 are attributable entirely to the effect on the equilibrium of the
vapor fugacity models. In particular, the ideal-gas model overestimates the amount of gas in the
liquid phase, because it overestimates the equilibrium mole fractions of the synthesis gases in the
vapor phase.

Liquid-phase compositions may be much less accurate if a different liquid fugacity model is
used. A few sample calculations were performed where the Peng-Robinson equation was used
not only for the vapor fugacities, but also for the liquid fugacities. This option might be more
easily available in, for example, process simulation packages. While the vapor-phase
compositions from these calculations were still quite accurate, the mole fractions of the gases in
the liquid phase were significantly in error, by up to a factor of two in some cases. However, if
the liquid-phase mole fractions are of no interest, such an approach could be reasonable.
Main Results

Calculations were performed of the vapor-liquid equilibrium between water and a typical synthesis gas at conditions of temperature and pressure appropriate for a “quench” of the gas. For the liquid phase, these calculations used the best available data for Henry’s constants of gases in water and related quantities. For the vapor phase, they used the thermodynamically rigorous virial expansion truncated at the level of the second virial coefficient. When all uncertainties are considered, the equilibrium mole fraction of water in the vapor phase can be considered accurate to within 0.01, with the largest contribution to the uncertainty coming from the neglect of higher-order virial coefficients.

Other methods of computing the vapor-phase fugacity were examined. The ideal-gas assumption seriously underestimates the water content in the vapor. The Peng-Robinson equation produces only a slight underestimation, similar in size to the uncertainty of the more rigorous calculation. It therefore could be a reasonable option for engineering calculations for this system, at least if the form of the Peng-Robinson equation used is that documented in Appendix C (in particular, the water-specific \( a(T) \) term, Eq. (C25), is important). Preferably, the use of the Peng-Robinson equation in this context would only be for the vapor phase, with Henry’s law being used for the liquid phase. If the Peng-Robinson equation were used for both phases, the solubilities of gas components in the liquid would not be accurately represented.

The effects of temperature and pressure on the water content of the vapor are as expected; an increase of temperature increases the water content, while an increase of pressure reduces it. The effect of gas composition on the equilibrium water content is very small.

Note on “Adiabatic” Flash Calculations

While the scope of this project was limited to flash calculations at fixed temperature and pressure, a closer approximation to the real industrial situation would be what is commonly known as an “adiabatic” flash calculation, in which the inlet vapor and liquid streams are brought together at a fixed pressure and the outlet temperature and vapor and liquid compositions are determined under the constraint of constant enthalpy. While rigorous adiabatic flash calculations would be a major additional effort beyond the scope of this work, we can offer some qualitative guidance regarding how the results of this work would translate into such a situation.

Because the solubility of the synthesis gas in water is small, the enthalpy balance will be dominated by the vaporization of the water. More vaporization will result in more evaporative
cooling; this will be attenuated somewhat by the lower vapor pressure of the water as the temperature decreases. For a given ratio of quench water to synthesis gas, methods that underestimate the equilibrium water content of the vapor at a fixed temperature and pressure (such as the ideal-gas assumption) will overestimate the outlet temperature from the quench step in an adiabatic situation.

**Possible Enhancements**

Finally, we consider some possibilities for enhancement in the current calculations. These are considered in two categories. The first category involves improvements in the accuracy of the calculations described here. The second category involves enhancements in the scope of the calculations to encompass additional aspects not considered here.

In the first category, as mentioned in Chapter 2, the primary source of uncertainty is the omission of higher-order terms, particularly the third virial coefficient of pure water. Including these higher-order contributions could potentially reduce the uncertainty of the calculated vapor water content by approximately a factor of two. The most important task for such an enhancement would be a critical evaluation of the third virial coefficient of water, in order to estimate its best value and it uncertainty. Uncertainties could also be reduced to a lesser extent by improvements in the theoretical results for the water-gas second virial coefficients; in particular, the H₂O/CO and H₂O/CO₂ interactions are not known as accurately as one might hope.

In the second category, implementing “adiabatic” calculations as described in the previous section is feasible, but would entail significant additional effort to superimpose an enthalpy balance on the VLE calculations. The inclusion of additional trace components in the synthesis gas, beyond those listed in Table 1-1, could also be considered. Sufficient data exist to add O₂ and CH₄; parameters for these gases are listed in Appendices B and C. A significant trace component is H₂S. Sufficient data exist to incorporate H₂S in the liquid-phase fugacity model, but the H₂O/H₂S interaction for the vapor-phase virial model is not known. With regard to potential effects of these trace components, it can be noted that, as pointed out in Chapter 2, the water content of the equilibrium vapor is not very sensitive to small changes in the synthesis gas composition.

Finally, there is a significant complication introduced by the presence of NH₃ in the synthesis gas. NH₃ dissolved in the liquid phase would undergo acid-base reactions with dissolved CO₂ and H₂S; these reactions would significantly enhance the solubilities of these gases. As a result, the present approach would significantly underestimate the amount of NH₃ in the equilibrium liquid, and the solubility of the acid gases would also be somewhat underestimated. However, as long as the NH₃ content in the synthesis gas was small, the effect on the equilibrium water content of the vapor (the main quantity of interest in this work) would be small. In principle it would be possible to incorporate liquid-phase acid-base reactions into the model, but it would be a major effort.
VAPOUR-LIQUID FLASH CALCULATIONS

Definition of Problem

The calculation defined for the project is the classic "flash" problem, discussed in many textbooks on chemical engineering thermodynamics [6,7].

The known inputs are temperature $T$, pressure $p$, and overall composition (in this case, specified flows of water in one input stream and synthesis gas of specified composition in another input stream). The output variables are the mole fractions of each component at equilibrium in both the vapor and liquid phases, and the fraction of the total feed in each phase. The following equations must be solved under the constraint that both phases are at $T$ and $p$:

$$f_i^\text{vap} = f_i^\text{liq}$$ (for each component $i$) \hspace{1cm} (A1)

$$z_i = \alpha x_i + (1 - \alpha) y_i$$ (for each component $i$), \hspace{1cm} (A2)

where $f_i$ is the fugacity of component $i$ (which is a function of $T$, $p$, and composition), $z_i$, $y_i$, and $x_i$ are the mole fraction in the total feed, the equilibrium vapor, and the equilibrium liquid, and $\alpha$ is the fraction (by mole) of the total feed in the liquid phase at equilibrium. Equation (A1) is the condition of chemical equilibrium between the phases, while Eq. (A2) represents conservation of mass for each component.

For phase-equilibrium calculations, it can be more convenient to work with the fugacity coefficient, defined by $\phi_i = f_i/(x_i p)$. Equation (A1) then becomes

$$\frac{\phi_i^\text{liq}}{\phi_i^\text{vap}} = \frac{y_i}{x_i} = K_i$$ (for each component $i$), \hspace{1cm} (A3)

where the so-called "K-factor" $K_i$ is a key parameter in the flash algorithm.

Description of Algorithm

Many algorithms have been proposed for solving the "flash" problem; for systems such as this where the phases are very dissimilar, a simple algorithm can be used, consisting of the following steps:
VAPOR-LIQUID FLASH CALCULATIONS

1. Assume an equilibrium composition for each phase. In this case, we initially assume that the liquid phase is pure water, while the vapor phase is the composition of the synthesis gas.

2. With this assumed composition, compute the fugacity coefficients $\phi_i^{\text{liq}}$ and $\phi_i^{\text{vap}}$ for each component in each phase, yielding $K_i$ for each component (see Eq. A3).

3. Perform an “ideal flash,” solving Eqs. (A2) and (A3) with the values of $K_i$ fixed at their values determined in Step 2. This is a simple matter of finding a value for $\Phi$ that satisfies the material balance constraint (A2) and the constraints that $\sum x_i = 1$ and $\sum y_i = 1$. We use an ideal flash algorithm originally programmed by Topliss [8].

4. If the compositions computed in Step 3 have not changed (within some tolerance), the calculation has converged. Otherwise, store the new vapor and liquid compositions determined in Step 3, and go back to Step 2.

In the systems considered here, this algorithm typically converges within about five iterations. We note that, due to the model adopted here for liquid fugacities (see Appendix B), $\phi_i^{\text{liq}}$ is always independent of composition, so it only needs to be computed the first time in Step 2. For calculations where an ideal-gas model is assumed for the vapor, $\phi_i^{\text{vap}}$ is also independent of composition (because it is always 1 for all species), so the initial ideal flash produces the final answer.
LIQUID-PHASE FUGACITY MODEL

Fugacity of Gaseous Solutes

The fundamental thermodynamic quantity describing the solubility of a gas in a liquid is the Henry’s constant $k_H$, defined by

$$k_H = \lim_{x_i \to 0} \left( \frac{f_i}{x_i} \right),$$  \hspace{1cm} (B1)

where $f_i$ and $x_i$ are, respectively, the liquid-phase fugacity and mole fraction of the solute $i$. While this definition can be applied at any state of the solvent, in this work we restrict our attention to vapor-liquid equilibrium conditions in single solvents, so that $k_H$ is a function only of temperature along the vapor-liquid saturation curve of the solvent.

At small solute gas pressures, Eq. (B1), with the limit removed, would be sufficient to relate the solute fugacities to the mole fraction. However, there is a (small but not negligible) thermodynamic correction for higher pressures, giving the result: [6]

$$\ln \left( \frac{f_i}{x_i} \right) = \ln k_H + \frac{\bar{v}_i^* (p - p^s)}{RT},$$  \hspace{1cm} (B2)

where $\bar{v}_i^*$ is the infinite-dilution partial molar volume of solute $i$ in the solvent, $p$ is the pressure, $p^s$ is the saturation vapor pressure of the solvent, $R$ is the molar gas constant, and $T$ is the absolute temperature. If the pressure is equal to the solvent vapor pressure $p^s$, the correction term becomes zero and Eq. (B1) is recovered.

For phase-equilibrium calculations as implemented in this work, what is required is not the fugacity $f_i$ but the fugacity coefficient $\phi_i = f_i / (x_i p)$. Manipulation of Eq. (B2) produces the following expression for the solute fugacity coefficient $\phi_i$:

$$\phi_i = \frac{k_H \exp \left( \frac{\bar{v}_i^* (p - p^s)}{RT} \right)}{p}.$$  \hspace{1cm} (B3)
LIQUID-PHASE FUGACITY MODEL

The most important factor in Eq. (B3) is the Henry’s constant \( k_H \). We use values from the critical data evaluation of Fernández-Prini et al., who correlated their results for each solute as a function of temperature with a theory-based expression: [2]

\[
\ln\left(\frac{k_H}{p^p}\right) = A/T_R + \frac{B\tau^{0.35}}{T_R} + C(T_R)^{-0.41} \exp \tau,
\]  

(B4)

where \( \tau = 1 - T_R \), \( T_R = T / T_c \), and \( T_c \) is the critical temperature of the solvent (647.096 K for \( H_2O \)). Table B-1 contains the parameters \( A \), \( B \), and \( C \) for each solute in water, along with the minimum and maximum temperatures of correlation.

<table>
<thead>
<tr>
<th>Solute</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>( T_{\text{min}} ) (K)</th>
<th>( T_{\text{max}} ) (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ar</td>
<td>-8.40954</td>
<td>4.29587</td>
<td>10.52779</td>
<td>273.19</td>
<td>568.36</td>
</tr>
<tr>
<td>( H_2 )</td>
<td>-4.73284</td>
<td>6.08954</td>
<td>6.06066</td>
<td>273.15</td>
<td>636.09</td>
</tr>
<tr>
<td>( N_2 )</td>
<td>-9.67578</td>
<td>4.72162</td>
<td>11.70585</td>
<td>278.12</td>
<td>636.46</td>
</tr>
<tr>
<td>( O_2 )</td>
<td>-9.44833</td>
<td>4.43822</td>
<td>11.42005</td>
<td>274.15</td>
<td>616.52</td>
</tr>
<tr>
<td>CO</td>
<td>-10.52862</td>
<td>5.13259</td>
<td>12.01421</td>
<td>278.15</td>
<td>588.67</td>
</tr>
<tr>
<td>( CO_2 )</td>
<td>-8.55445</td>
<td>4.01195</td>
<td>9.52345</td>
<td>274.19</td>
<td>642.66</td>
</tr>
<tr>
<td>( CH_4 )</td>
<td>-10.44708</td>
<td>4.66491</td>
<td>12.12986</td>
<td>275.46</td>
<td>633.11</td>
</tr>
</tbody>
</table>

Both Eqs. (B3) and (B4) require \( p^p \), the vapor pressure of water at the temperature of interest. This is calculated from the international standard correlation of Wagner and Pruss, which has the following functional form:

\[
\ln\left(\frac{p^p}{p_c}\right) = (T_c / T)(a_1\tau + a_2\tau^{1.5} + a_3\tau^3 + a_4\tau^{3.5} + a_5\tau^4 + a_6\tau^{7.5}),
\]  

(B5)

where \( p_c \) is water’s critical pressure (22.064 MPa), and \( \tau \) and \( T_c \) are defined below Eq. (B3). The values of the coefficients are \( a_1 = -7.85951783 \), \( a_2 = 1.84408259 \), \( a_3 = -11.7866497 \), \( a_4 = 22.6807411 \), \( a_5 = -15.9618719 \), \( a_6 = 1.80122502 \). [9]

The final quantity required for computation of the solute fugacity coefficients by Eq. (B3) is the solute partial molar volume \( \bar{v}_i^\infty \) at infinite dilution in water. This quantity is calculated as described by Fernández-Prini and Japat based on a molecular perturbation theory. [10,11] Their final expression is

\[
\bar{v}_i^\infty = RT\chi_T \left\{ 1 + \frac{\eta}{1-\eta} \left[ 1 + \frac{r + r^2}{1-\eta} + \frac{9\eta r^2}{(1-\eta)^2} + \frac{(1+2\eta)^2}{(1-\eta)^3}r^3 \right] \right\},
\]  

(B6)

B-2
where $\eta$ is the “packing fraction” of the pure solvent (water), defined by $\eta = \pi N_A \rho_w d_w^3 / 6$, where $\rho_w$ is the molar density of water, $d_w$ is the characteristic diameter of a water molecule (taken as 0.27 nm in all cases here), and $N_A$ is Avogadro’s number. $r$ is the ratio $d_i/d_w$, where $d_i$ is a characteristic diameter for the solute molecule. $\bar{\eta}_T$ is the isothermal compressibility of pure water.

The pure-water properties $\rho_w$ and $\bar{\eta}_T$ are obtained from the NIST Standard Reference Database for water properties. [12] The only parameter required for the calculation of $\bar{\eta}_i^{\infty}$ is then $d_i$, which is correlated as a function of temperature by one of two functional forms: [11]

\[
d_i = d_i^c + b_i(T - 298K), \tag{B7a}
\]

\[
d_i = d_i^c + b_i' \ln(T / 298K). \tag{B7b}
\]

Table B-2 contains the parameters $d_i^c$ and $b_i$ or $b_i'$ for all the solutes considered in this work.

<table>
<thead>
<tr>
<th>Solute</th>
<th>$d_i^c$ (nm)</th>
<th>$b_i$ (nm/K) or $b_i'$ (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ar</td>
<td>0.241</td>
<td>$-2.01 \times 10^{-4}$</td>
</tr>
<tr>
<td>H$_2$</td>
<td>0.266</td>
<td>$-1.25 \times 10^{-4}$</td>
</tr>
<tr>
<td>N$_2$</td>
<td>0.339</td>
<td>$-1.70 \times 10^{-4}$</td>
</tr>
<tr>
<td>O$_2$</td>
<td>0.342</td>
<td>$-2.00 \times 10^{-4}$</td>
</tr>
<tr>
<td>CO</td>
<td>0.353</td>
<td>$-2.53 \times 10^{-4}$</td>
</tr>
<tr>
<td>CO$_2$</td>
<td>0.427</td>
<td>$-0.2246$</td>
</tr>
<tr>
<td>CH$_4$</td>
<td>0.369</td>
<td>$-3.05 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

We note that, at the pressures and temperatures of interest in this work, the correction to the solute fugacity coefficients provided by the $\bar{\eta}_i^{\infty}$ term is typically on the order of one or two percent. Its accuracy is therefore not crucial in the context of the present project.

**Fugacity of Liquid Water**

Under the conditions considered here (dilute solution of solute gases), the calculation of the fugacity of liquid water is straightforward, given merely by its fugacity in the pure state $f_w^{\text{pure}}$ at the temperature and pressure of interest (obtained from [12]) multiplied by its mole fraction (which is slightly less than 1 due to the solubility of the solute gases). The equation for the fugacity coefficient becomes
LIQUID-PHASE FUGACITY MODEL

\[ \phi_w = \frac{f_w^{\text{pure}} \left( 1 - \sum_i x_i \right)}{x_w p} = \frac{f_w^{\text{pure}}}{p}. \]  

(B8)

Impact on Calculations

It can be noted that the main quantity of interest in this project (the amount of water in the equilibrium vapor phase) is not very sensitive to the liquid-phase fugacity calculations (other than the calculation of \( f_w^{\text{pure}} \), which is well-known). The solute fugacity coefficients as calculated in Eq. (B3) primarily influence the solubility of the gases \( x_i \). While these solubilities may be of interest for some applications, they are small enough that their impact on the equilibrium fugacity of water is generally less than 1%.

We also note that, due to the structure of the model, both the solute fugacity coefficients \( \phi_i \) (Eq. B3) and the water fugacity coefficient \( \phi_w \) (Eq. B8) depend only on temperature and pressure, but not on the liquid-phase composition. This simplifies the vapor-liquid equilibrium calculations, as discussed in Appendix A.
VAPO PHASE FUGACITY MODEL

Calculation from Virial Coefficients

For gas-phase systems, a rigorous description of the thermodynamics is provided by the virial expansion, which is written as a series of corrections to the ideal-gas law:

\[
\frac{P}{\rho RT} = z = 1 + B\rho + C\rho^2 + \ldots, \quad (C1)
\]

where \( P \) is the pressure, \( \rho \) the molar density, \( R \) the molar gas constant, and \( T \) the absolute temperature. The quotient \( z \) (which is 1 for an ideal gas) is usually called the compressibility factor, \( B \) is called the second virial coefficient, \( C \) the third virial coefficient, and so forth. An alternative form of the expansion can be formulated as a power series in \( P \) rather than \( \rho \), but it has been shown that the pressure form is generally less accurate than the density form when truncated at the same level. [13]

For mixtures, the virial coefficients are rigorously given as mole-fraction-weighted sums over characteristic binary quantities (for \( B \)), ternary quantities (for \( C \)), etc. For \( B \), the expression is

\[
B_{\text{mix}}(T) = \sum_{i} \sum_{j} y_i y_j B_{ij}(T), \quad (C2)
\]

where \( y \) is the mole fraction of the species in the vapor mixture and the sums extend over all components. \( B_{ij} \) is completely determined by the interactions between one molecule of species \( i \) and one of species \( j \). For a given pair, it is a function of temperature only. If \( i = j \), \( B_{ii}(T) \) is just the pure-component value of \( B(T) \); these values are well known for the substances of interest here. Obtaining reliable values of \( B_{ij}(T) \) for unlike pairs is more difficult, and has been the main focus of this project.

At moderate pressures, it is adequate to truncate Eq. (C1) after the term involving the second virial coefficient. Standard thermodynamic manipulations then produce the following expression for the fugacity coefficient for each species: [6]

\[
\ln \phi_i = 2\rho \sum_j y_j B_{ij} - \ln z, \quad (C3)
\]

where the sum is over all species including \( i \). The molar density \( \rho \) (and therefore the compressibility factor \( z \)) is solved for iteratively from Eq. (C1) once all the virial coefficients are
known. Use of Eq. (C3) requires knowledge of all the second virial coefficients $B_{ij}(T)$ for all pairs of species in the system.

**B(T) Data for Pure Components**

$B(T)$ for pure water is taken from the correlation of Harvey and Lemmon, which has the form

$$B(T) / B^0 = a_1 (T^*)^{-0.5} + a_2 (T^*)^{-0.8} + a_3 (T^*)^{-3.35} + a_4 (T^*)^{-8.3}, \quad (C4)$$

where $B^0 = 1000 \text{ cm}^3/\text{mol}$, $T^* = T/100 \text{ K}$, $a_1 = 0.34404$, $a_2 = -0.75826$, $a_3 = -24.219$, and $a_4 = -3978.2$. [14]

$B(T)$ for pure oxygen is taken the work of from Wagner et al.:

$$B(T) / B^0 = b_1 + b_2 (T^*)^{-0.25} + b_3 (T^*)^{-3.5} + b_4 (T^*)^{-4.5} + b_5 (T^*)^{-5.5}, \quad (C5)$$

where $B^0 = 1000 \text{ cm}^3/\text{mol}$, $T^* = T/11 \text{ K}$, $b_1 = 0.143389$, $b_2 = -0.629863$, $b_3 = -0.577814 \times 10^7$, $b_4 = 0.695858 \times 10^9$, and $b_5 = -0.246023 \times 10^{11}$. [15]

$B(T)$ for pure hydrogen is taken from a correlation by Hodges et al.:

$$B(T) / B^0 = a_1 (T^*)^{-0.33} + a_2 (T^*)^{-1.4} + a_3 (T^*)^{-1.8} + a_4 (T^*)^{-2.2}, \quad (C6)$$

where $B^0 = 1 \text{ cm}^3/\text{mol}$, $T^* = T/100 \text{ K}$, $a_1 = 42.0803$, $a_2 = -143.982$, $a_3 = 146.918$, and $a_4 = -47.5601$. [16]

For other pure species, $B(T)$ is taken from the reference-quality equations of state implemented in the NIST Refprop database. [3] The expressions for $B(T)$ derived from these equations take the form:

$$B(T) / \rho_c = \sum_i a_i \tau_i^b_i, \quad (C7)$$

where $\rho_c$ is the critical density of the fluid, $\tau = T_c / T$, where $T_c$ is the critical temperature of the fluid, and the $a_i$ and $b_i$ are a series of parameters specific to the fluid. Table C-1 gives parameters $a_i$ and $b_i$ for use in Eq. (C7), and Table C-2 lists the values of the critical constants.
Table C-1
Parameters for Eq. (C4) for $B(T)$ of Ar, N$_2$, CO, CO$_2$, and CH$_4$

<table>
<thead>
<tr>
<th>Param.</th>
<th>Ar</th>
<th>N$_2$</th>
<th>CO</th>
<th>CO$_2$</th>
<th>CH$_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>0.08872230499</td>
<td>0.924803575275</td>
<td>0.90554</td>
<td>0.388568232032</td>
<td>0.04367901028</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.705148051673</td>
<td>-0.492448489428</td>
<td>-2.4515</td>
<td>2.93854759427</td>
<td>0.6709236199</td>
</tr>
<tr>
<td>$a_3$</td>
<td>-1.68201156541</td>
<td>0.564857472948</td>
<td>0.53149</td>
<td>-5.58671885349</td>
<td>-1.765577859</td>
</tr>
<tr>
<td>$a_4$</td>
<td>-0.149090144315</td>
<td>-1.61720005987</td>
<td>-0.12778</td>
<td>-0.767531995925</td>
<td>0.1830487909</td>
</tr>
<tr>
<td>$a_5$</td>
<td>-0.120248046009</td>
<td>-0.481395031883</td>
<td>(not used)</td>
<td>2.16589615432</td>
<td>0.1511883679</td>
</tr>
<tr>
<td>$a_6$</td>
<td>-0.321813917507</td>
<td>-0.043576236045</td>
<td>(not used)</td>
<td>-0.370556852701</td>
<td>-0.4289363877</td>
</tr>
<tr>
<td>$a_7$</td>
<td>0.332300176958</td>
<td>(not used)</td>
<td>(not used)</td>
<td>-0.0167758797004</td>
<td>-0.01932040831</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0</td>
<td>0.25</td>
<td>0.25</td>
<td>0</td>
<td>-0.5</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.25</td>
<td>0.875</td>
<td>1.125</td>
<td>0.75</td>
<td>0.5</td>
</tr>
<tr>
<td>$b_3$</td>
<td>1</td>
<td>0.5</td>
<td>1.5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$b_4$</td>
<td>2.75</td>
<td>0.75</td>
<td>3.625</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>$b_5$</td>
<td>4</td>
<td>2</td>
<td>(not used)</td>
<td>1.5</td>
<td>1</td>
</tr>
<tr>
<td>$b_6$</td>
<td>3</td>
<td>4</td>
<td>(not used)</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>$b_7$</td>
<td>3.5</td>
<td>(not used)</td>
<td>(not used)</td>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>

Table C-2
Critical Parameters and Acentric Factor for Substances Considered in this Work

<table>
<thead>
<tr>
<th>Substance</th>
<th>$T_c$ (K)</th>
<th>$p_c$ (kPa)</th>
<th>$\rho_c$ (mol/dm$^3$)</th>
<th>$\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>H$_2$O</td>
<td>647.096</td>
<td>22064.</td>
<td>17.873728</td>
<td>0.3443</td>
</tr>
<tr>
<td>Ar</td>
<td>150.687</td>
<td>4863.</td>
<td>13.4074</td>
<td>-0.00219</td>
</tr>
<tr>
<td>H$_2$</td>
<td>33.145</td>
<td>1296.4</td>
<td>15.508</td>
<td>-0.219</td>
</tr>
<tr>
<td>N$_2$</td>
<td>126.192</td>
<td>3395.8</td>
<td>11.1839</td>
<td>0.0372</td>
</tr>
<tr>
<td>O$_2$</td>
<td>154.581</td>
<td>5043.</td>
<td>13.63</td>
<td>0.0222</td>
</tr>
<tr>
<td>CO</td>
<td>132.86</td>
<td>3494.</td>
<td>10.85</td>
<td>0.0497</td>
</tr>
<tr>
<td>CO$_2$</td>
<td>304.1282</td>
<td>7377.3</td>
<td>10.6249</td>
<td>0.22394</td>
</tr>
<tr>
<td>CH$_4$</td>
<td>190.564</td>
<td>4599.2</td>
<td>10.139</td>
<td>0.01142</td>
</tr>
</tbody>
</table>

**B(T) Data for Aqueous Binaries**

The $B_{ij}(T)$ for pairs where water is one of the components are the most important factors determining the vapor-phase fugacity coefficient in Eq. (C3), and therefore are the most
VAPOR-PHASE FUGACITY MODEL

important in determining the equilibrium mole fraction of water in the vapor. For the most part, these are obtained from a series of recently published works making use of quantum chemistry. [16,17,18,19,20,21] The exception is the H₂O/CO₂ system, where the experimental data of Patel et al. cover the temperature range of interest and were fitted for this work to a simple functional form. [4]

The equations for \( B_{ij}(T) \) for most of these pairs take a form similar to that of Eq. (C4):

\[
B_{ij}(T) / B^0 = a_1 (T^*)^a + a_2 (T^*)^b + a_3 (T^*)^c + a_4 (T^*)^d ,
\]

(C8)

where \( B^0 = 1000 \text{ cm}^3/\text{mol} \) and \( T^* = T/100 \text{ K} \). Values of the \( a \) and \( b \) coefficients for each aqueous binary for use in Eq. (C8) are given in Table C-3. Information about the uncertainty of these correlations may be found in the original references.

Table C-3
Parameters for Eq. (C8) for \( B_{ij}(T) \) for H₂O with Various Gases

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 )</td>
<td>96.1941</td>
<td>33.047</td>
<td>67.595</td>
<td>124.605</td>
<td>485.062</td>
<td>−3308.1</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>−211.074</td>
<td>−250.41</td>
<td>−249.83</td>
<td>−214.421</td>
<td>−2054.24</td>
<td>(not used)</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>−96.4425</td>
<td>285.42</td>
<td>−204.38</td>
<td>−102.818</td>
<td>7422.08</td>
<td>(not used)</td>
</tr>
<tr>
<td>( a_4 )</td>
<td>−12.6006</td>
<td>−186.78</td>
<td>(not used)</td>
<td>−22.360</td>
<td>−5919.89</td>
<td>(not used)</td>
</tr>
<tr>
<td>( b_1 )</td>
<td>−0.31</td>
<td>−0.21</td>
<td>−0.24</td>
<td>−0.33</td>
<td>−0.18</td>
<td>−2.66</td>
</tr>
<tr>
<td>( b_2 )</td>
<td>−0.82</td>
<td>−1.50</td>
<td>−1.06</td>
<td>−0.73</td>
<td>−1.70</td>
<td>(not used)</td>
</tr>
<tr>
<td>( b_3 )</td>
<td>−2.24</td>
<td>−2.26</td>
<td>−3.22</td>
<td>−2.03</td>
<td>−2.10</td>
<td>(not used)</td>
</tr>
<tr>
<td>( b_4 )</td>
<td>−4.60</td>
<td>−3.21</td>
<td>(not used)</td>
<td>−4.07</td>
<td>−2.25</td>
<td>(not used)</td>
</tr>
</tbody>
</table>

The only aqueous binary for which \( B_{ij}(T) \) is not described by Eq. (C8) is H₂O/CH₄, for which the correlation is

\[
B_{ij}(T) / (1 \text{ cm}^3 / \text{mol}) = \sum_{i=1}^{6} a_i (T_0 / T)^i ,
\]

(C9)

where \( T_0 = 599.216 \text{ K} \), \( a_1 = 109.447 \), \( a_2 = −227.694 \), \( a_3 = 182.497 \), \( a_4 = −80.944 \), \( a_5 = 18.368 \), and \( a_6 = −1.673 \). [18]

**B(T) Data for Nonaqueous Binaries**

The \( B_{ij}(T) \) for pairs where neither component is water play a minor but not negligible role in the phase equilibrium. For these binaries, we make use of a corresponding-states correlation developed by Tsonopoulos. [22] The correlation takes the form
\[ \frac{B(T)p_c}{RT_c} = F^{(0)} \left( \frac{T}{T_c} \right) + \omega F^{(1)} \left( \frac{T}{T_c} \right), \]  \hspace{1cm} (C10) 

with 

\[ F^{(0)} \left( \frac{T}{T_c} \right) = 0.1455 - \frac{0.330}{T_R} - \frac{0.1385}{T_R^2} - \frac{0.0121}{T_R^3} - \frac{0.000607}{T_R^8}, \]  \hspace{1cm} (C11) 

\[ F^{(1)} \left( \frac{T}{T_c} \right) = 0.0637 + \frac{0.331}{T_R^2} - \frac{0.423}{T_R^3} - \frac{0.008}{T_R^8}. \]  \hspace{1cm} (C12) 

\( p_c \) and \( T_c \) are the critical pressure and temperature, respectively. \( \omega \) is the acentric factor, a semi-theoretical parameter related to the degree to which a substance deviates from a "simple" fluid; values for \( \omega \) are well-known for most common fluids. \( T_R \) is the reduced temperature, \( T_R = T/T_c \). Critical properties and acentric factors for all fluids used in this work are taken from the NIST Refprop database [3] and are listed in Table C-2.

For "quantum" gases (those such as H\(_2\) with such low molecular masses that their thermodynamic properties are affected by quantum mechanics), temperature-dependent "effective" critical constants are used, defined as follows:

\[ T_c = \frac{T_c^0}{1 + \frac{21.8}{MT}}, \]  \hspace{1cm} (C13) 

\[ p_c = \frac{p_c^0}{1 + \frac{44.2}{MT}}. \]  \hspace{1cm} (C14) 

\( T_c^0 \) and \( p_c^0 \) are hypothetical values that the critical temperature and pressure would have in the absence of quantum effects, and \( M \) is the relative molar mass. For H\(_2\), the only quantum fluid considered in this project, \( T_c^0 = 43.6 \) K, \( p_c^0 = 2.05 \) MPa, and \( M = 2.01594 \).

In order to use Eq. (C10) to derive \( B_{ij}(T) \) for unlike pairs, combining rules are required to obtain characteristic values of \( T_c, p_c, \omega, \) and \( M \) (if one of the pair is H\(_2\)) for use in the Tsonopoulos correlation. As suggested in the literature, the following combining rules are used: [6,22]

\[ T_{c_{ij}} = \left( T_{c_{i}} T_{c_{j}} \right)^{1/2} \left( 1 - k_{ij} \right), \]  \hspace{1cm} (C15) 

\[ p_{c_{ij}} = \frac{4T_{c_{ij}} (p_{c_{i}} v_{c_{i}}/T_{c_{i}} + p_{c_{j}} v_{c_{j}}/T_{c_{j}})}{\left( v_{c_{i}}^{1/3} + v_{c_{j}}^{1/3} \right)^3}, \]  \hspace{1cm} (C16)


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VAPOR-PHASE FUGACITY MODEL

\[ \omega_j = 0.5\left(\omega_i + \omega_j\right) \], \hspace{1cm} (C17)

\[ M_{ij}^{-1} = 0.5\left(M_i^{-1} + M_j^{-1}\right) \]. \hspace{1cm} (C18)

The binary parameter \( k_{ij} \) in Eq. (C15) is set to zero in the absence of data, but can be adjusted to fit experimental data if they are available. Table C-4 lists values of \( k_{ij} \) (which is the same as \( k_{ji} \)) and the corresponding sources of mixture data for a number of binaries in this work. For those not listed in the table, a value of 0 is used.

**Table C-4**

**Binary Parameters \( k_{ij} \) for Eq. (C15) for Pairs of Nonaqueous Gases**

<table>
<thead>
<tr>
<th>Component ( i )</th>
<th>Component ( j )</th>
<th>( k_{ij} )</th>
<th>Data Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ar</td>
<td>H(_2)</td>
<td>-0.12</td>
<td>[23]</td>
</tr>
<tr>
<td>Ar</td>
<td>N(_2)</td>
<td>-0.01</td>
<td>[23]</td>
</tr>
<tr>
<td>Ar</td>
<td>O(_2)</td>
<td>0.00</td>
<td>[23]</td>
</tr>
<tr>
<td>Ar</td>
<td>CO</td>
<td>-0.01</td>
<td>[24]</td>
</tr>
<tr>
<td>Ar</td>
<td>CO(_2)</td>
<td>0.05</td>
<td>[23]</td>
</tr>
<tr>
<td>Ar</td>
<td>CH(_4)</td>
<td>0.02</td>
<td>[23]</td>
</tr>
<tr>
<td>H(_2)</td>
<td>N(_2)</td>
<td>-0.04</td>
<td>[24]</td>
</tr>
<tr>
<td>H(_2)</td>
<td>CO</td>
<td>-0.04</td>
<td>[24]</td>
</tr>
<tr>
<td>H(_2)</td>
<td>CO(_2)</td>
<td>-0.04</td>
<td>[25]</td>
</tr>
<tr>
<td>H(_2)</td>
<td>CH(_4)</td>
<td>-0.05</td>
<td>[24]</td>
</tr>
<tr>
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<td>O(_2)</td>
<td>-0.01</td>
<td>[23]</td>
</tr>
<tr>
<td>N(_2)</td>
<td>CO</td>
<td>-0.02</td>
<td>[26]</td>
</tr>
<tr>
<td>N(_2)</td>
<td>CO(_2)</td>
<td>-0.01</td>
<td>[23,27]</td>
</tr>
<tr>
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<td>CH(_4)</td>
<td>0.03</td>
<td>[23,28]</td>
</tr>
<tr>
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<td>CO(_2)</td>
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<td>[23]</td>
</tr>
<tr>
<td>O(_2)</td>
<td>CH(_4)</td>
<td>0.04</td>
<td>[23]</td>
</tr>
<tr>
<td>CO</td>
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<td>-0.04</td>
<td>[29]</td>
</tr>
<tr>
<td>CO</td>
<td>CH(_4)</td>
<td>0.01</td>
<td>[24]</td>
</tr>
<tr>
<td>CO(_2)</td>
<td>CH(_4)</td>
<td>0.05</td>
<td>[23,27]</td>
</tr>
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</table>
Supplementary Calculation with Peng-Robinson Equation of State

For comparison purposes, some calculations for this project were performed with the semiempirical Peng-Robinson equation of state. [30,31] The Peng-Robinson equation has the following form:

\[ p = \frac{RT}{v - b} \frac{a(T)}{v(v + b) + b(v - b)}, \]  

(C19)

where \( p \) is the pressure, \( T \) is the absolute temperature, \( v \) is the molar volume, and \( R \) is the molar gas constant. \( a \) and \( b \) are substance-specific parameters; \( b \) is a constant depending only on the critical properties, while \( a \) is a function of temperature in a manner that depends on the acentric factor \( \omega \):

\[ b = 0.778 \frac{RT_c}{p_c}, \]  

(C20)

\[ a(T) = a(T_c) \alpha(T), \]  

(C21)

\[ a(T_c) = 0.45724 \frac{(RT_c)^2}{p_c}, \]  

(C22)

\[ \alpha(T) = \left[ 1 + \beta \left( 1 - \sqrt{T/T_c} \right) \right]^2, \]  

(C23)

\[ \beta = 0.37464 + 1.54226\omega - 0.26992\omega^2. \]  

(C24)

We make use of a special parameterization introduced by Peng and Robinson for \( a(T) \) for water: [31]

\[ a_w(T) = \left[ 1.0085677 + 0.82154 \left( 1 - \sqrt{T/T_c} \right) \right]^2, \text{ for } \sqrt{T/T_c} < 0.85. \]  

(C25)

When \( \sqrt{T/T_c} \geq 0.85 \), Eqs. (C23-C24) are used as with other fluids.

Use of the Peng-Robinson equation for mixtures requires mixing rules for the parameters \( a \) and \( b \) based on their pure component values:

\[ a = \sum_i \sum_j y_i y_j a_{ij}, \]  

(C26)

\[ b = \sum_i y_i b_i, \]  

(C27)
where $y_i$ is the mole fraction of component $i$. For unlike pairs, a combining rule is needed for $a_{ij}$ based on pure-component values $a_i$ and $a_j$:

$$a_{ij} = (a_i a_j)^{1/2}.$$  \hfill (C28)

It is possible to introduce an adjustable binary parameter into Eq. (C28) (similar to that in Eq. (C15)) in order to attempt to fit experimental data more closely, but that enhancement was not explored in this project.

From the Peng-Robinson equation, the equation for the fugacity coefficient $\phi_i$ of a component $i$ is: [6]

$$
\ln \phi_i = \frac{b_i}{b} \left( \frac{pv}{RT} - 1 \right) - \ln \left( \frac{p(v-b)}{RT} \right) - \frac{a}{2\sqrt{2}bRT} \left[ 2 \sum_j y_j a_{ji} \right] \ln \frac{v + \left(1 + \sqrt{2}\right)b}{v + \left(1 - \sqrt{2}\right)b}.
$$  \hfill (C29)

The accuracy of our implementation of the Peng-Robinson equation was tested by verifying that calculated results for densities and fugacity coefficients matched those from an independently programmed implementation available in a NIST database. [3]
REFERENCES


REFERENCES


