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U.S. DEPARTMENT OF COMMERCE
Technology Administration
National Institute of Standards and Technology

Office of Applied Economics
Building and Fire Research Laboratory
Gaithersburg, Maryland 20899

The Optimal Pricing of Publicly Supplied Private Goods: A Case Study of NIST Standard Reference Materials

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Benefits and Costs of Research: A Case Study of Cybernetic Building Systems

Robert E. Chapman

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ABSTRACT

This study provides a framework for determining optimal prices and production plans for a welfare-maximizing public enterprise that produces multiple goods, faces a budget constraint, and is obligated to meet all demand. Public enterprises often operate under conditions of decreasing marginal cost where first-best, profit-maximizing rules lead to deficits. In order to cover costs, prices therefore need to exceed marginal cost. A public-sector pricing model in the Boiteux tradition computes price and output combinations that minimize the loss from charging prices that do not equal marginal cost.

The report describes the Boiteux model and its extensions. The Ramsey version of the model is applied to the pricing problem of the Standard Reference Materials Program (SRMP) at the National Institute of Standards and Technology (NIST). The NIST SRMP supplies samples of materials whose physical or chemical properties are precisely characterized; they are used as intermediate goods by firms and laboratories to calibrate manufacturing equipment or scientific apparatus for quality control. The SRMP is faced with the problem of how to calculate prices that will cover the cost of the program and will result in quantities that just meet demand at those prices.

The model was applied to a group of 11 SRMs. After estimating their demand and cost functions and combining them with the theoretical principles of the model, optimal prices and production plans were calculated for the group of 11 SRMs for the years 1978 to 1992. The results fulfilled the optimality requirements of the Ramsey-Boiteux model: Deviations of price from marginal cost were inversely proportionate to the goods' price elasticities of demand, and the corresponding optimal quantities of SRMs maintained the same proportions as the quantities that would have been demanded at prices equal to marginal cost. In every year of the study period there would have been a welfare gain if Ramsey prices had been charged rather than average-cost prices, and unit sales and revenues would have been higher than they were under the actual pricing policy of the SRMP in the years from 1978 to 1992.

The analysis shows that in the case of NIST SRMs the Ramsey-Boiteux model can provide concrete and relatively simple pricing rules that yield welfare-optimizing prices and quantities.

KEYWORDS

average-cost pricing; Boiteux model; budget constraint; cost recovery; decreasing-cost production; demand analysis; economic analysis; inverse-elasticity rule; marginal-cost pricing; multi-product public enterprises; optimal pricing; Ramsey prices; second-best pricing; standard reference materials; user fees; welfare maximization.

PREFACE

This study is based in part on a doctoral dissertation in Economics, submitted to The George Washington University. The task of setting prices and determining saleable quantities is more complex in the public sector than in the private sector where profit-maximization offers the appropriate signals. Government agencies therefore welcome information that can help them formulate their pricing policies. NIST's Standard Reference Materials Program (SRMP) has been faced with reductions in tax appropriations and has been under pressure, as have been many other agencies, to recover the cost of its program from user fees. This study applies a theoretical public-sector pricing model to a group of standard reference materials to see whether the model can help the SRMP to develop price and production strategies that would cover costs without losing customers.

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EXECUTIVE SUMMARY

The purpose of the study was to determine whether a public-sector pricing model in the Boiteux tradition can provide a pricing rule that would lead to welfare-maximizing prices and optimal output levels for Standard Reference Materials (SRMs). SRMs are samples of materials whose physical and chemical properties are precisely characterized and certified by NIST. Manufacturing firms and science laboratories use these materials for quality control.

Since the early 1980's, NIST's Standard Reference Materials Program (SRMP), like other government agencies, has been increasingly under pressure to cover the cost of its goods and services from user fees rather than from tax appropriations. In the wake of an across-the-board price increase of 25 percent in 1982, which was intended to increase revenues from user fees, sales of SRMs declined by about 16 percent and revenues increased by only about four percent. The SRMP had little information on and no method to include in its pricing formula the effect of price increases on demand. So it was not clear to what extent each individual SRM was responsible for the decline in sales. In addition, other factors may also have influenced demand. The Ramsey-Boiteux model explicitly takes into account the response of users to price changes and calculates prices that cover costs and meet demand.

Background

SRMs are produced under conditions of decreasing marginal costs, as is the case for many publicly supplied goods and services. The cost of basic research and development that is embodied in an SRM is very high, but once the prototype exists, the cost of producing one more SRM remains constant or decreases. In the private sector, an enterprise under these conditions would evolve into a natural monopoly and in general would exploit its monopoly power beyond the level needed to cover costs. But at monopoly prices, fewer SRMs would be sold than would be optimal to maintain the nation's technological infrastructure. To encourage an efficient level of supply, the monopoly can be regulated or taken over by the government. In the case of SRMs, the argument in favor of government production is that the SRMP is associated with NIST and can benefit from research that NIST conducts in many areas of science and technology. Private-sector firms would likely underinvest in the basic research needed to develop SRMs. A second argument for government production of SRMs is that NIST provides a non-partisan, non-proprietary, and therefore objective source of standard reference materials. This property, which reduces transaction costs for firms that use SRMs for quality control, carries through to "secondary" standard reference materials that are produced from and are traceable to NIST's "primary" SRMs.

The first-best pricing rule for a profit-maximizing enterprise dictates that price be set equal to marginal cost. However, when marginal cost is constant or declining, the resulting prices do not cover total costs. In the private sector, a profit-maximizing enterprise, in order to stay in business, would reduce output and charge monopoly prices. Public enterprises—who are expected to maximize welfare rather than profits—usually resort to average-cost pricing if their deficits cannot be covered from tax appropriations. Average-cost pricing is a reasonable pricing rule in the sense that its prices cover costs and those who use the good or service pay for it. But it does not produce prices that result in optimal production plans. For prices and production plans to be optimal, demand-side information has to be included in the pricing formula in addition to the cost information that determines average-cost prices.

The Ramsey-Boiteux Model

The Ramsey-Boiteux model is especially useful for public enterprises that produce multiple goods, as is the case for the SRMP. Ramsey (1927), Boiteux (1956), and others addressed the problem in the context of raising optimal taxes to cover the deficits of national railroads and utilities. A number of economists since then have extended the Ramsey-Boiteux model to make it applicable to other pricing problems. Its main feature, as it applies to SRMs, is that it takes into account not only production and other costs on the supply side but also systematically incorporates the response of customers to price changes. Combined with the budget constraint, it uses this information to calculate optimal prices that cover costs and result in saleable quantities. In doing so, the formula calculates simultaneously the optimal deviations from marginal cost for all goods in the program. These deviations vary among SRMs depending on whether the demand response to a percentage increase in price results in a revenue increase (inelastic demand) or a revenue decrease (elastic demand). The prices of SRMs with inelastic demand can be increased relatively more than the prices of SRMs with elastic demand. The result is that combined sales are prevented from falling to a level where total revenue would decline.

It is relatively uncomplicated to implement the model because the following simplifications apply to SRMs:

(1) SRMs are intermediate goods. They are not sold to final consumers but to manufacturing firms and laboratories, and they usually constitute a small percentage of the total inputs into the final product. No direct information is therefore needed about how SRM price increases affect the wealth of the nation from the point of view of consumers of final products. This also means that we need not be concerned about how SRM prices might be used as a means of income redistribution.

(2) SRMs are not substitutes for each other; that is, customers will not buy more cement SRMs, for example, because the price of rubber SRMs has increased.

Because of these simplifications, the only information needed to implement the model are cost data for SRMs and the price elasticities of demand that express the response of customers to changes in SRM prices. The pricing formula reduces to the Ramsey (inverse-elasticity) version of the public-sector model, where the deviation from marginal cost for each SRM is inversely proportional to its own price elasticity of demand. The price estimation is thus based on a system of $n + 1$ equations—one equation for each of the n

SRM k and one for the revenue-cost constraint—to be solved for the prices of all n SRMs and the proportionality factor λ. The formula is as follows:

$$\frac{(P_k - MC_k)}{P_k} = \frac{-\lambda}{\eta_{kk}}; \qquad k = 1,...,n \qquad \text{(ES-1)}$$

subject to

$$Revenue - Fixed\ Costs - Variable\ Costs = 0 \qquad \text{(ES-2)}$$

where

$(P_k - MC_k)/P_k$	=	percentage deviation of price from marginal cost for each SRM k,
γ	=	proportionality factor, measure of welfare effect of deviation from MC.
η_{kk}	=	price elasticity of demand (percentage change in quantity divided by percentage change in price) of each SRM

and where the budget constraint is expressed as a breakeven constraint.

Demand and Cost Functions for SRMs

The demand for SRMs:
To obtain the price elasticities of demand that are needed to apply the Ramsey pricing formula, it is necessary to analyze the demand for individual SRMs. The quantity demanded may be more responsive to price changes for some SRMs than for others because of differences among demanders, differences in the availability or usefulness of alternative means of quality control, and because of the general business conditions prevailing in the production sectors that use SRMs. In general it is true that the less price elastic a good, the less danger there is of losing customers. If, on the other hand, the price is comparatively price elastic, customers will leave the market or purchase less if the price is increased, and total revenue will decrease. Therefore the seller refrains from large price increases for goods that are very price elastic.

In light of these considerations, a demand function for each of the 24 SRMs in the test group was developed by regression analysis. We looked at the demand for an SRM as a derived demand, since SRMs are intermediate goods. In the demand model we postulated that firms will buy SRMs if the expected increase in profits is higher from using SRMs than from using other means of quality control or no quality control. The demand for measures of quality control in the form of NIST SRMs thus depends on the price of the

SRM, the price of private-sector substitutes, and other factors that might influence the quantity demanded.

We estimated demand functions for 24 SRMs, which were selected by the SRMP. Price and sales data were available for 15 years from 1978 to 1992, when the demand analysis was performed. We were able to get statistically significant price elasticities of demand for the 11 SRMs listed below (see the appendix for a detailed set of statistics on the elasticity estimates for each SRM):

SRM	Price Elasticity
1261 A LA Steel AISI 4340	1.699
187B&C Borax pH	0.781
1625 SO_2 Permeation Tube, 10 cm	2.096
40H Sodium Oxalate Redox	0.533
27F Iron Ore, Sibley Powder	1.251
39I Benzoic Acid Combustion	0.833
120B&C Phosphate Rock	0.876
1361A CU & CR Coatings	1.287
1635 Trace Elements in Coal	2.668
122H Cast Iron Car Wheel	2.978
53E Bearing Metal, Lead Base	1.902

The price elasticities indicate that the impact of price changes on the quantity demanded varies among SRMs. If the coefficient is between 0 and 1 (in absolute terms), we say that the demand is inelastic, meaning that a one percent increase in price will lead to a less than one percent decrease in the quantity demanded for this SRM. If the coefficient is between 1 and ∞, we say that the demand is elastic, meaning that a one percent increase in price will lead to a more than one percent decrease in the quantity demanded. A coefficient of 1 indicates unitary elasticity, that is, a one percent change in price leads to a one percent change in the quantity demanded. The implications for raising revenue are as follows:

- if the demand is inelastic, an increase in price leads to an increase in total revenue, and a decrease in price leads to a fall in total revenue;
- if the demand is elastic, an increase in price leads to a decrease in total revenue, and a decrease in price leads to an increase in total revenue;
- if the demand has unitary elasticity, total revenue is not affected by changes in price.

Cost functions for SRMs:
The empirical cost functions were constructed from the fixed and variable costs allocated to each of the 11 SRMs according to the SRMP accounting conventions. The SRMP

categorizes as recoverable fixed costs of the program all costs that are not production costs, except for the portion of operating costs that is associated directly with the sale and distribution of individual SRMs. Fixed costs consist of development costs to produce prototypes of SRMs, most of the operating costs that have to do with project management, an overhead charge, and miscellaneous other expenses paid to other agencies and NIST divisions for technical support. Variable costs include production costs and the part of operations costs that are sales-related costs for advertising, shipping, and billing. Production costs comprise the labor, raw materials, inventory and obsolescence, and special equipment for processing a prototype into a certified SRM.

The combination of cost and demand functions with the theoretical Ramsey formula allows us to estimate the cost-covering prices and quantities for the group of 11 SRMs.

Optimal Prices for NIST SRMs

The two-SRM case:
We first tested the application of the model with two SRMs, calculating their optimal prices for the year 1982. We used marginal costs, demand functions, price elasticities, and a revenue-cost constraint for SRMs 1261A LA Steel and 187 B&C Borax pH.

Table ES-1 shows the Ramsey prices and quantities that resulted from the calculation. They are compared with the average-cost prices charged and quantities sold by the SRMP in 1982, and also with the marginal-cost prices and the corresponding quantities that could be sold if there were no budget constraint and the deficit could be covered from tax appropriations.

As expected, the Ramsey formula calculated prices higher than marginal-cost prices. They have to be higher if they are to cover the deficit that would result from marginal-cost pricing. Note too that the Ramsey price is lower than the actual, average-cost, price charged by the SRMP in 1982 for the price-elastic SRM 1261A and higher for the price-inelastic SRM 187B&C. Quantities are closer than with average-cost pricing to the welfare-maximizing quantities that would be sold if marginal-cost pricing were applied. Total revenue, an indication of the size of the program, is higher under Ramsey pricing than under average-cost pricing.

The optimal deviation from marginal cost needed to cover costs was calculated to be 29 percent for SRM 1261A and 62 percent for SRM 187B&C, the inverses of the price elasticities. In the two-product case the optimal deviation is just simply the inverse of the price elasticity of demand, whereas in the multiproduct case it is *proportionate* to the inverse of the price elasticity.

In order to determine whether the loss of welfare from second-best pricing would be smaller under Ramsey pricing than under average-cost pricing, we followed the conventional treatment of calculating the change in consumer and producer surplus due to a price increase and comparing the resulting deadweight losses for the two pricing rules.

For the two SRMs under discussion the deadweight loss is 16 percent of total revenue under Ramsey pricing and 23 percent under average-cost pricing, a clear indication, that Ramsey pricing in this example would have reduced the loss of welfare by switching from average-cost pricing to Ramsey pricing.

Table ES-1. Comparison of Prices and Quantities under Three Different Pricing Formulas[1]

SRM	Pricing Formula	Quantity	Revenue	Average Cost	Total Cost	Deficit
	Ramsey Price					
SRM 1261A SRM 187B&C Total	$81 $77	305 76	$24,794 $5,846 **$30,640**	$86 $58	$26,247 $ 4,393 **$30,640**	-$1,453 +$1,453 **0**
	Marginal-Cost Price					
SRM 1261A SRM 187B&C Total	$58 $29	541 163	$31,378 $4,727 **$36,105**	$74 $47	$39,946 $ 7,517 **$47,463**	-$8,568 -$2,790 **-$11,358**
	Average-Cost Price					
SRM 1261A SRM 187B&C Total	$94 $59	238 93	$22,372 $5,487 **$27,859**	$94 $59	$22,372 $5,487 **$27,859**	0 0 **0**

Multi-SRM case:

The results of the two-SRM case generalize to any number of SRMs. The model was solved simultaneously for the 11 SRM, first for the year 1982, and then for all the years of the study period from 1978 to 1992. The results show that the required optimality conditions for second-best pricing under a breakeven constraint are met. The calculated optimal deviations from marginal cost are inversely proportionate to their price elasticities for all SRMs in the group. As required, the formula produced unequal deviations in which prices of SRMs with inelastic demands diverge from their marginal costs by a relatively wider margin than prices of SRMs with elastic demand. The deviations from marginal cost of the actual, average-cost prices show no such relationship to the SRMs' price elasticities.

[1]Throughout, the stated empirical estimates were calculated by computer and rounded, and so the summed entries in the tables do not precisely match the shown totals.

As far as the corresponding Ramsey quantities are concerned, the calculations show that saleable quantities differ proportionately from the levels that would be observed if prices were set at marginal cost. If one thinks of the consequence of a deviation of price from marginal cost as a distortion of relative demand patterns, then the loss of welfare is minimized if the relative quantities of the SRMs sold are kept unchanged from their marginal-cost proportions. This is true for Ramsey quantities but not for the quantities that correspond to the actual, average-cost prices.

Figure ES-1 compares revenues for the group of 11 SRMs under Ramsey pricing, marginal-cost pricing, and average-cost pricing from 1978 to 1992. Figure ES-2 compares the number of saleable units of SRMs under the three pricing rules. As can be seen from the graphical representation, Ramsey pricing would have resulted in higher revenues and a greater number of combined units sold than did average-cost pricing. The corresponding deadweight-loss calculations also showed that Ramsey pricing would have reduced the loss of welfare that comes from having to deviate from marginal-cost pricing.

Conclusions and Implications for SRMP Pricing Policy

The analysis shows that in the case of NIST SRMs, the Ramsey-Boiteux model can provide concrete and relatively simple pricing rules that yield welfare-optimizing prices and quantities. As the model predicts, the deviations from marginal cost needed to cover costs are relatively greater for the price-inelastic SRMs and relatively smaller for the price-elastic SRMs. Because the price increases are optimal, the inefficiency due to having to charge prices higher than marginal-cost is minimized, and there is no distortion of relative demand patterns. Combined unit sales and revenues would have been higher under Ramsey pricing than under the average-pricing policy of the SRMP in the years from 1978 to 1992.

The Ramsey model tested in this study could assist the SRMP in several ways in its formulation of a successful pricing and production policy:

(1) The model explicitly takes into account the price elasticities of demand in its derivation of cost-covering prices. This means that the price increases that are necessary to cover the cost of the program as a whole can be distributed among SRMs in a way that recovers a proportionately greater portion of the costs from those SRMs whose demand is price inelastic without decreasing revenue. The prices of SRMs whose demand is relatively more price elastic can be increased by less in order to keep customers.

(2) Since prices are estimated on the basis of the demand functions, knowing the prices is tantamount to knowing the corresponding quantities. These quantity estimates could be used as benchmarks for planning renewal batches of SRMs.

(3) The budget constraint, which is an integral part of the Ramsey model, can be formulated to meet the SRMP's objective. For example, the desired size of the program could be one of the exogenous variables. To cover the cost of the program, it may not be necessary to expand sales by lowering prices, but to increase prices of SRMs for which no

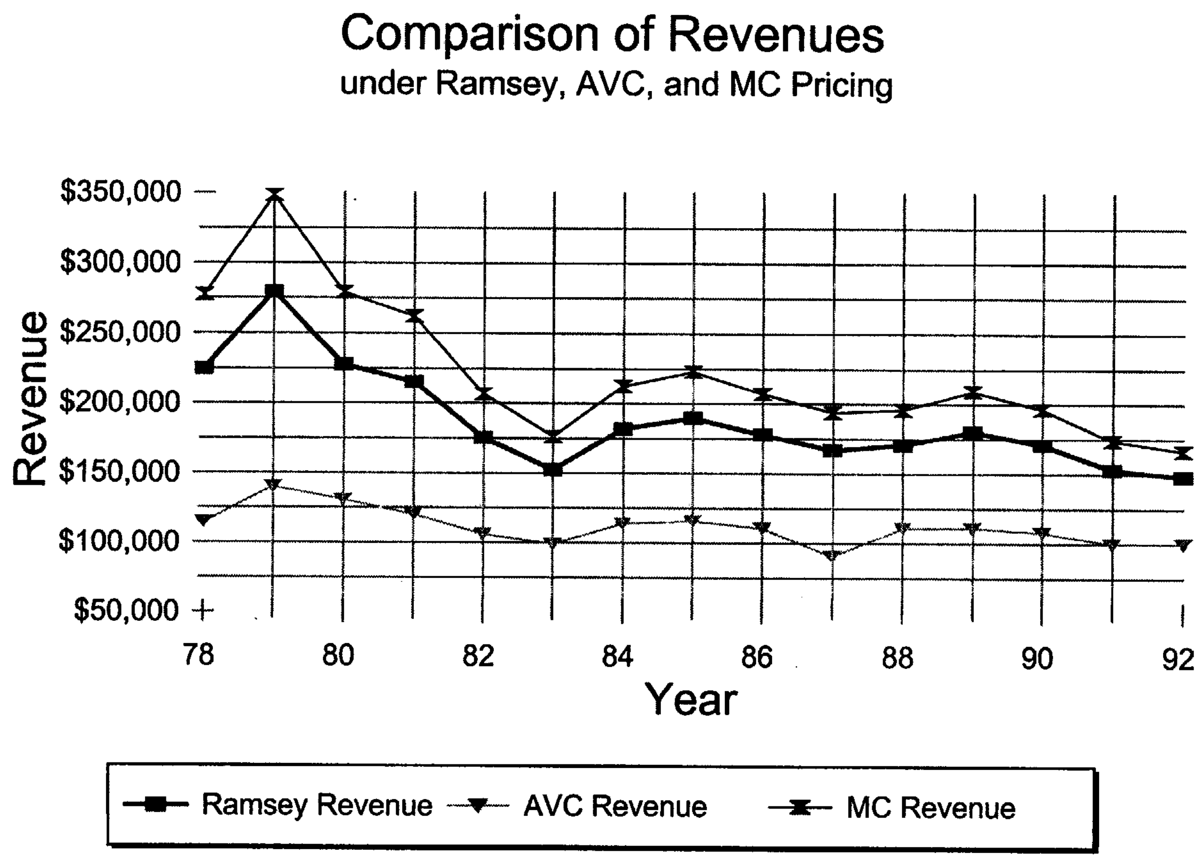

Figure ES-1. Comparison of Ramsey, AVC, and MC Revenues for the Years 1978 to 1992.

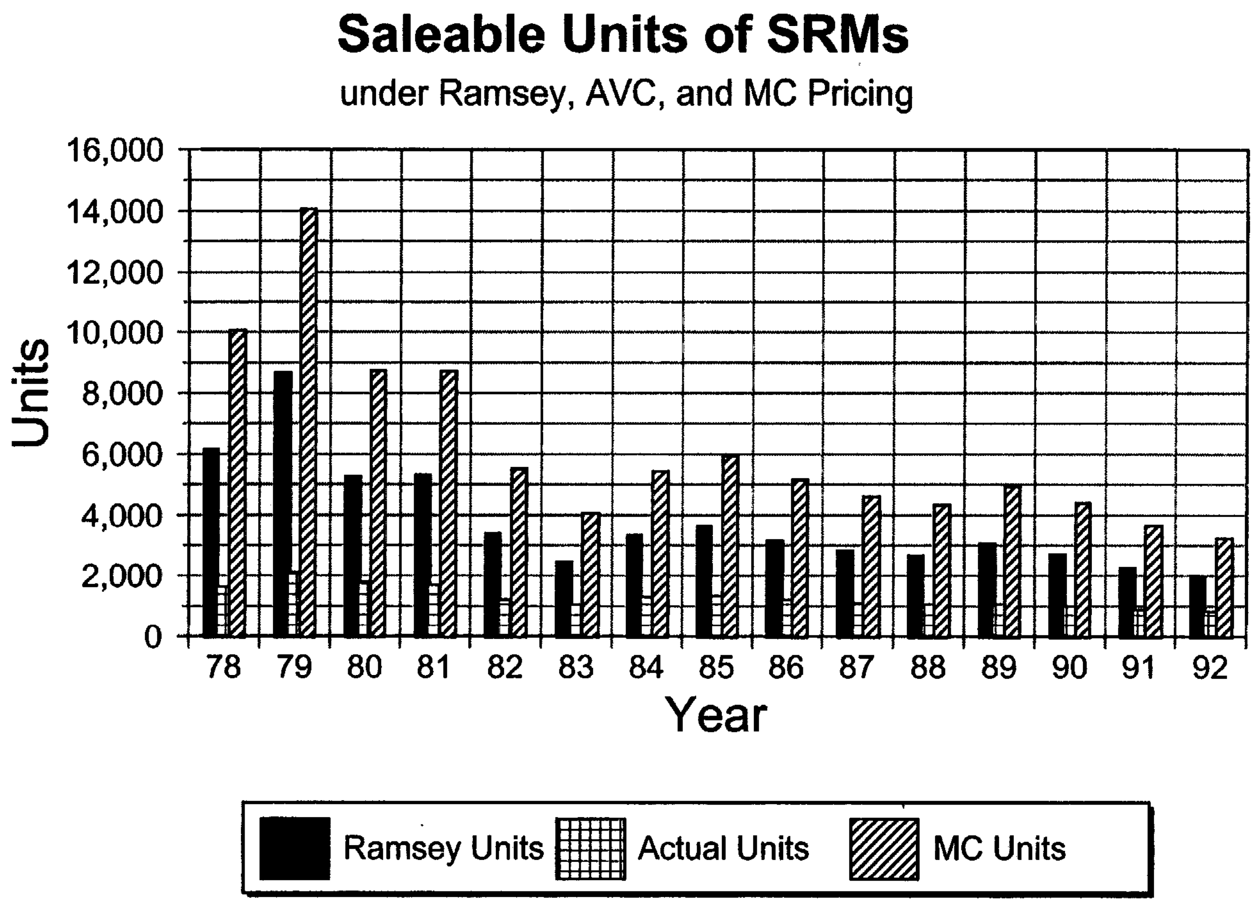

Figure ES-2. Saleable Units of SRMs under Ramsey, AVC, and MC Pricing for the Years 1978 to 1992

satisfactory secondary standards are available. Hence the SRMP might decide to focus on the supply of SRMs whose demand is price inelastic. Then to determine the prices that correctly reflect the true value of the standard to the user will require the optimal combination of demand and cost information that is provided by the model.

(4) The ability of the SRMP to base pricing policies on economic theory rather than on arbitrary rules of thumb or on accounting conventions improves the reliability of its decisions and strengthens its position in budget negotiations with the NIST budget office and with the Congress.

(5) The Government Performance and Results Act (GPRA) of 1993 is intended to improve federal program effectiveness and public accountability. The application of the Ramsey model to pricing decisions at NIST would go a long way in providing information to managers and in helping to articulate program goals. Since welfare maximization implies efficient allocation of resources, pricing policies based on this concept are in agreement with the intent of the GPRA.

1. INTRODUCTION

1.1 Background, Scope, and Results

Public discussion of government enterprises is now filled with suggestions that they could be run more efficiently in the private sector. Whether this proposition has merit as far as any one government agency is concerned is a complicated issue to resolve and is the subject of a body of economic theory all of its own. However, as a corollary, internal subsidization of public enterprises has increasingly fallen out of favor by politicians, economists, and voters since the early 1980's. As a consequence, managers of public enterprises have had to pay closer attention to covering their deficits from user fees rather than from tax appropriations as in the past. Since this usually means that the public enterprises have to charge higher prices for their services, renewed attention is being paid to the theory of public pricing in an attempt to find welfare-maximizing prices that cover costs and meet demand.

Solutions to the public-sector pricing problem come not from welfare economics but from the theory of taxation that goes back to Ramsey (1927) and Boiteux (1956). The pricing problem can be described as that of a government enterprise that is dedicated to maximizing social welfare; it charges marginal-cost prices for its service to guarantee the efficient allocation of resources. Any shortcomings are financed out of tax revenues. However, taxes, in order to be optimal, have to be lump-sum. Since lump sum taxation is in general not feasible, any excise taxes or income taxes will distort prices somewhere in the economy and move society to a point below its utility-possibility frontier. Once we recognize that a first-best optimum is not possible and that we are in the realm of the second-best, we need to consider other possible solutions to the problem of having to cover the cost of providing the service. To charge distortionary user fees may be less detrimental than imposing distortionary taxes. For one thing, charging user fees may satisfy the intent of the benefits-received principle of pricing and taxation more adequately than subsidization from tax revenues. Since non-users can be distinguished from users, and intensive users from non-intensive users, they can be charged accordingly, and so the entire cost of the service will be paid for by its users.

The purpose of this study is to examine whether a public-sector pricing model in the Boiteux tradition can provide a pricing rule for calculating welfare-maximizing prices for a multi-product public enterprise that produces under conditions of decreasing marginal cost, faces a budget constraint, and is obligated to meet all demand. These are the conditions that are generally relevant for many government agencies. In our investigation we focus on the National Institute of Standards and Technology (NIST) of the Department of Commerce, whose Standard Reference Materials (SRM) Program produces about 2000 standard reference materials, each with its own demand function, but with shared production costs. SRMs are intermediate goods that are sold as inputs to firms in industry and to science and research laboratories in the private and public sectors. They are samples of materials whose chemical and physical properties are characterized to within a very narrow margin of uncertainty. Firms and laboratories use them to calibrate manufacturing equipment or scientific apparatus. SRMs are publicly provided but they are private goods in the sense that they can be sold in different quantities to users that can be identified and charged a price.

SRMs are produced under conditions of decreasing, or at least constant, marginal cost. Until the early 1980's the resulting deficits were covered by tax appropriations. When, during the Reagan administration, Congressional subsidies were phased out, the SRM Program (henceforth called SRMP) was forced to increase prices in order to recover its deficits through increased revenues. A 25 percent increase was implemented in 1982. It was imposed on all SRMs, partly as lump-sum surcharges and partly as *ad valorem* increases. Since the demand functions of SRMs are not known, the increases were based solely on cost considerations. Unit sales decreased by 16 percent and revenues increased by only about 4 percent. Since then, the SRMP has been experimenting to devise a pricing scheme that would cover costs and maintain demand. Their formula distributes the total costs of the program among the units of SRMs it expects to sell, based on the previous year's sales. There is little information as to what effect these prices have on demand or whether or not they maximize welfare. Besides, if fewer SRMs are sold than predicted, costs are still not covered.

The public-sector pricing model of the Ramsey-Boiteux type seems well suited to solve the pricing problem of the SRMP. After analyzing the Boiteux model and its extensions we found that the following circumstances that characterize the supply of SRMs and the assumptions that can be made allow us to apply the model in its Ramsey version:

- SRMs are intermediate goods that are not sold to consumers so that SRM prices do not directly enter the utility functions of consumers and thus do not affect the social welfare function;
- they constitute a small portion of the inputs into the production of final goods, so that indirect effects from price increases that are passed on by the private-sector firm to consumers are insignificant and can be ignored;
- income effects are negligible for the same reason, so that we can deal with compensated demand;
- because the SRMP is small with respect to the private sector, its prices do not significantly affect the price-cost margins of private monopolistic firms and we can assume perfect competition among the firms that are users of SRMs; and
- SRMs are not substitutes for each other, so that the cross elasticities of demand are zero.

Thus many of the restrictions of the extended Boiteux model do not apply. This means that the informational and computational requirements that usually make the implement-ation of the Boiteux model "a devastatingly complex task,"[2] are considerably reduced.

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[2]See Tresch (1981), ch. 15, p. 127.

This, of course, is true only in the abstract. When it came to the actual implementation of the model, there were many details regarding data availability that were devastating enough. For example, one of the requirements that public enterprises now face—and which is just beginning to be formally recognized by the SRMP—is that if their goods are sold at the market they have to take into account demand-side information when determining cost-covering prices and saleable quantities. When setting prices, the SRMP at present considers only costs and does not take into account demand-related information, at least not in any systematic way. Since price elasticities of demand are an integral part of the inverse-elasticity feature of the Ramsey model, it was necessary to develop demand functions for the selected SRMs before being able to implement the model.

SRMs have been sold by NIST since the beginning of the century, and so we expected a wealth of data to be available. However, because the SRMP in the past was always more concerned about the scientific side of supplying SRMs than the economic side of it, it did not keep the sparse sales-related data it had collected over the years. Most unfortunately, historical data of more recent years, which were stored on tapes, burnt in a fire at a central government storage facility in 1978. We were left with data for the years from 1978 to 1992 which we used to derive demand functions for 24 SRMs. Because of the dearth of data, only eleven of the price elasticities of demand were statistically significant, and we used only this group of 11 SRMs to implement the model.

The implementation of the Ramsey version of the model resulted in cost-covering prices and quantities for the group of 11 SRMs. They fulfilled the conditions for optimality required by the theory of second-best. We compared these quasi-optimal prices and quantities with marginal-cost prices and quantities and with the actual prices charged by the SRMP and the corresponding quantities. We found that for all years from 1978 to 1992 the quasi-optimal prices and quantities were closer to the marginal-cost price and quantities than the actual, average-cost, prices and quantities. As a consequence the deadweight losses from the unavoidable deviations from marginal cost would have been smaller under Ramsey pricing than under the average-cost pricing practiced by the SRMP. We also found that under Ramsey pricing more SRMs would have been sold and total revenue would have been higher than under average-cost pricing. These results imply that the SRMP can cover costs and achieve a gain in welfare by charging relatively higher prices for those SRMs whose demand is price-inelastic and relatively lower prices for those SRMs whose demand is price-elastic.

1.2 Organization of Dissertation

The remainder of this introductory chapter explains in the context of SRMs the objectives and constraints of public-sector enterprises that make second-best pricing necessary. We also look at the public-good properties of SRMs. Chapter 2 describes the past history of prices, production, and costs of the NIST Standard Reference Materials Program. In chapter 3 we describe the Boiteux model and the extensions by various economists (Rees (1968), Feldstein (1972a, b, c), Hagen (1979), and others) that make it more applicable to real-world applications. The derivation of the demand functions for the SRMs is the subject of chapter 4. Chapter 5 combines the theoretical model with the pricing and production conditions of the SRMP and implements the Ramsey version of the model to derive the empirical results. In chapter 5 we also take a closer look at the cost functions for SRMs. Chapter 6 summarizes the results and conclusions and interprets them with respect to their implications for the pricing policies of the SRMP and of public enterprises in general. Chapter 6 also suggests topics for further research.

1.3 Objectives and Constraints of the Model as they Apply to SRMs

1.3.1 Objectives

The task of covering the costs of government services is a difficult one for several reasons: First, government agencies generally are expected to maximize social welfare rather than profits and so may not be able to apply the private-sector profit-maximizing prices that cover the cost of production and efficiently ration access. The social welfare function is the formal expression of some ethical standards generally accepted by society. These standards are the basis for a "normative" theory of public pricing. Even though, in practice, these normative policy prescriptions are often difficult to implement they are still very important because they offer benchmark models useful for critically evaluating the public enterprise's day-to-day operations. In this study we limit ourselves to investigating the pricing of SRMs in relation to the normative theory of public pricing without regard to the political or bureaucratic objectives the SRMP might have, such as output maximization or revenue maximization. We deal with the "positive" theory of public pricing only to the extent that we compute the deadweight loss that is incurred by the SRMP's pricing rule in order to compare it with the deadweight loss from Ramsey pricing.

1.3.2 Market Failure and Public-Good Components of SRMs

A look at the production and cost structure of SRMs shows that they are produced under decreasing average and marginal costs, as is the case for many publicly supplied private goods.[3] A large amount of basic research is embodied in an SRM even before it gets to the stage of a prototype. Because of the high cost of the basic research and development, the production of SRMs evolves into a natural monopoly. A profit-maximizing monopoly in the private sector would exploit its monopoly power and raise prices and cut output beyond the levels needed to cover costs. Fewer SRMs would be developed and sold than are optimal to meet national macroeconomic objectives.

Decreasing-cost production in itself, however, is not a sufficient justification for public production. Private-sector firms could supply these goods and services as regulated monopolies. There is an ongoing discussion as to whether or not government production is justified in the case of SRMs since they are private intermediate goods that are sold to

[3] A thorough theoretical treatment of decreasing-cost industries can be found in Tresch (1981), chapter 9.

identifiable users—mostly manufacturing and research firms—who are able to pay a fee. The arguments usually advanced in support of government production of SRMs are twofold:

(1) The number of SRMs that would be developed in the private sector might be insufficient for maintaining the country's desired technological and scientific infrastructure. The SRMP has the advantage of being associated with the National Institute of Standards and Technology who conducts publicly funded basic research in many and very diverse areas of scientific investigation. The characterization of physical and chemical properties that lead to certified SRMs are often byproducts of that research. These basic research results are a positive externality the cost of which cannot easily be assigned to particular users. Private sector firms would underinvest in basic research if they were unable to capture its benefits and so some SRMs would not be developed at all.

(2) The second public-good property is the "traceability" of secondary SRMs to the "primary" SRMs that NIST produces. Because NIST SRMs are certified and supplied by a government agency, they have the benefit of originating from a non-partisan, non proprietary, and therefore presumably objective source. This benefit extends to secondary standards that are produced by private-sector U.S. (and foreign) suppliers who use NIST's primary reference materials as measurement standards.

Whether or not these arguments are valid in the case of the SRMP has not been demonstrated empirically.[4] The benefits of externalities are usually difficult to measure or to express in dollars. The dollar value of the merit-good component of SRMs and the extent to which they should therefore be subsidized from tax appropriations is not known. We assume that to some extent this public-good aspect is taken into account by the fact that the costs of basic research that is embodied in SRMs are not, or only partially, charged to SRMs' development and production but are paid for instead through other projects. For example, the SRMs for air and water pollution are byproducts of research undertaken to investigate pollution problems from energy-generating systems. Also, some $500,000 in tax revenue are still appropriated annually to the SRMP for developing new SRMs.

With respect to the second public-good property, the traceability of secondary standards to NIST standards, it is likely that most of the benefits of, for example, lower transaction costs, would either be appropriated by producers through higher profits or by consumers of the final product through lower prices. Consequently, in neither of these cases would there be any need for the general taxpayer to compensate the SRMP for benefits received but not paid for. This leaves us with having to solve the problem of how to price goods that are produced under conditions of decreasing or constant marginal costs.

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[4]See Link (1996) who suggests a methodology for evaluating public-sector research and development and uses NIST projects and programs as case studies, albeit not the SRMP.

1.3.3 The Budget Constraint

Determining that an R&D program is appropriate to the public sector still leaves the issue of public accountability, that is, whether the public monopoly performs well enough to maximize welfare and use its resources efficiently. Public enterprises are generally not expected to fully exploit their monopoly power, so when it comes to the determination of public pricing policies, budget constraints are explicitly formulated to achieve a certain deficit or surplus. Deficits are sometimes allowed if there are reasons for subsidizing the production of a good, surpluses may be used to finance other public expenditures. Usually government enterprises are mandated to cover costs rather than to achieve a deficit or a surplus, so that a breakeven constraint is relevant. A breakeven constraint is taken as an indicator of both efficiency and public-spiritedness.

In the case of the SRMP, the mandate has always been to cover the costs of the program. In our application, therefore, we assume a breakeven constraint, even though the Ramsey model can accommodate any type of profit constraint.

1.3.4 Production

A further constraint concerns the production side of the model. The technology of private firms is not explicitly modeled. This means that we take the private sector as exogenous and assume that the public enterprise adapts to its behavior. We assume further that the public enterprise produces efficiently along the production possibility frontier $g(z) = 0$. Without any further restrictions, this allows for decreasing, constant, or increasing returns to scale. By doing so we avoid having to deal with the subject of X-inefficiency of public firms and with arguments that claim that allocative efficiency, which depends on price ratios, cannot be improved by changing relative prices.[5] We also assume that the usual requirements of convexity and monotonicity of the production function are fulfilled so that the existence and optimality of the solution to the optimization problem is guaranteed.

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[5]Bös (1994), in chapter 2, discusses in detail several theories on efficient production in the public and private sectors.

2. HISTORY OF STANDARD REFERENCE MATERIALS

2.1 Organizational Background

2.1.1 What are Standard Reference Materials?

SRMs are produced by NIST, an agency of the U.S. Department of Commerce. The broad mission of NIST is to provide support for the scientific and technological infrastructure of the United States. It does this by carrying out research that provides measurement methods, standards, and data. The results are disseminated through two types of services. The first type of services, which is non-reimbursable, contributes to the broad base of scientific and technological knowledge in the public domain. It consists of publications, lectures, memberships in standards committees and scientific societies, informal advice to government agencies and industries, and, more recently, administration of quality awards and grants programs. The second type comprises all those services that benefit identifiable customers with well-defined transaction points where user fees can be charged. These include, measurement assurance programs, calibration services, laboratory accreditation, and specifically the supply of SRMs with which this paper is concerned.

In its literature, NIST defines SRMs and their uses in the following way:

> *"SRMs are well-characterized, homogeneous, stable materials with specific properties measured and certified by NIST. They are used widely throughout the United States and the world to help develop test methods of proven accuracy, to calibrate instruments and measurement systems used to maintain quality control, to help assure equity in buyer-seller transactions, and to assure the long-term reliability and integrity of the measurement process."*

SRMs are sold as small quantities of materials that have been analyzed and whose dimensions, composition, or properties are certified to be the "true" values within a very narrow range. For example, a bottle of simulated rainwater contains exactly 2.69 milligrams of sulfate per liter, 0.205 milligrams of potassium, and a number of known quantities of other substances. The sample is intended to be used as a benchmark. Knowing its precise composition, a user can calibrate laboratory equipment for measuring acid rain.

The history of SRMs for steel illustrates their use for quality control. The very first SRM was developed in 1906 at the request of railroad equipment manufacturers. Ingredients such as carbon, silicon, chromium, and nickel are required in exact proportions to assure that wheels of railroad carriages do not break. Since then over 90 SRMs for steel have been developed, and 90 percent of the U.S. steel production is now quality-controlled by SRMs.

2.1.2 Users of SRMs

Some level of measurement service is required by virtually any institution in the private or public sectors. Figure 2-1 shows the number of SRMs sold by NIST and the sales revenues collected in each of the years from 1978 to 1992. In 1992 NIST sold over 48,000 units of SRMs, producing over $7 million (in 1982 dollars)[6] in sales revenues.

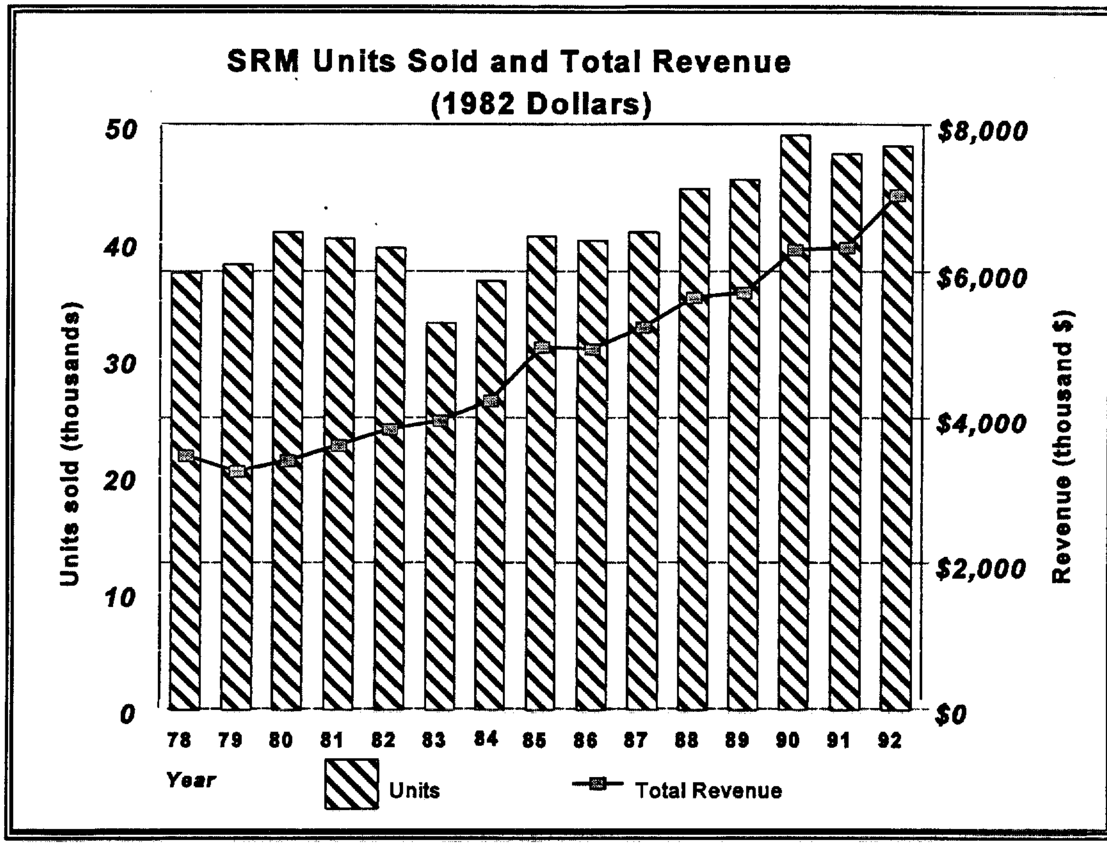

Figure 2-1. Sales and Revenues of Standard Reference Materials

The pie chart in figure 2-2 shows the distribution of SRMs among user groups. Over 70 percent of all customers are industrial firms who buy SRMs of chemical, ferrous and non-ferrous metals, and of engineering, and nuclear materials. Virtually all of the top manufacturing companies in the U.S. buy SRMs.

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[6]The Producer Price Index for Intermediate Materials less Food and Energy (1982=100) (henceforth referred to as the PPI) is used throughout to convert current dollars to 1982 dollars. To ensure comparability, the PPI has been calculated on a fiscal-year instead of a calendar-year basis.

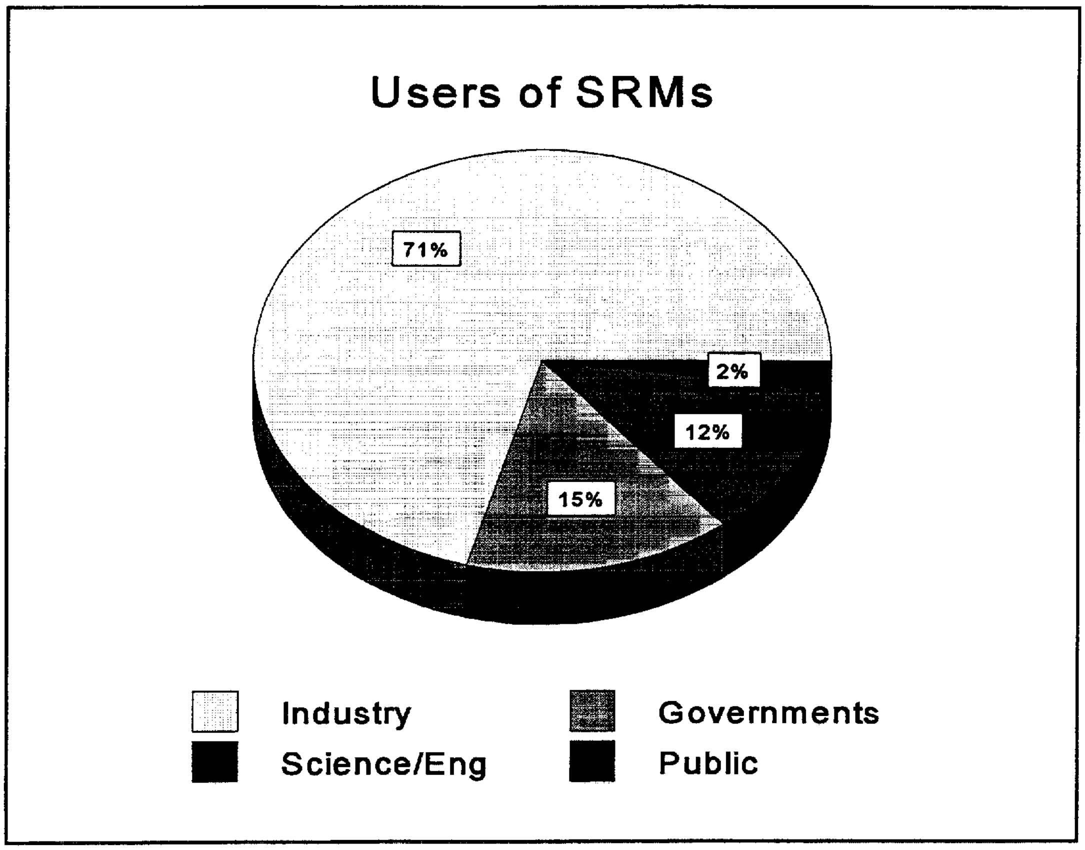

Figure 2-2. Distribution of SRM Users

Fifteen percent of SRM purchases are made by other federal agencies and state and local governments who need measurement-related technology in order to carry out their responsibilities in national defense and energy matters, public health and safety, natural resource management, environment, education, and technology transfer.

About 12 percent of SRM users belong to the scientific and engineering research community made up of industrial research laboratories, universities and hospitals. Its members, interested in advancing the state of knowledge in their fields, rely on NIST for making available the best and latest measurement techniques. The remaining two percent of SRMs are sold to the general public.

Among the SRM buyers just described, an estimated 20 percent of NIST's customers are intermediate laboratories that buy "primary" standards from NIST and then produce "secondary" standards for those users whose day-to-day production activities do not require the top quality represented by NIST-produced standard reference materials. The secondary standards are, however, "traceable" to NIST, i.e., users have direct access to NIST to verify their measurements if the need arises.

About one quarter of all SRMs are sold to users outside the U.S., either because corresponding foreign SRMs are not available or because the SRMs produced in the U.S. are less expensive.

2.1.3 Why is NIST Selling SRMs?

SRMs can be characterized as publicly supplied private goods. A good is characterized as a private good if it is bought in different quantities and if people who do not pay can be excluded from buying it. In this respect SRMs are similar to many other private goods that are either supplied or regulated by the government. Examples are public utilities (energy, communication, transportation); basic goods industries (nuclear energy, coal, oil, steel); finance (savings banks, insurance); education (schools, universities); and health (public health programs, hospitals).

The original Congressional mandate for NIST's production of SRMs was intended to assure that standards were available to U.S. industry in socially desirable quantities at socially optimal prices. The reason why SRMs have been produced publicly rather than privately is that the cost of basic research and the cost of developing SRMs are high enough to preclude the existence of more than one firm to satisfy the demand for primary SRMs. The production of SRMs exhibits economies of scale and thus decreasing, or at least constant, marginal costs. Under these conditions, the market creates a "natural" monopoly, which in the private sector would lead to lower output and higher prices than is socially optimal.

2.1.4 The Structure of SRM Production

Figure 2-3 lists the steps that lead to a finished SRM and the organizational categories into which NIST divides these steps. The fundamental properties of substances and materials that are to be certified as standards are byproducts of basic research that is being conducted at NIST independently of the SRMP. For example, the SRMs for air and water pollution emanated from research undertaken to investigate pollution problems stemming from energy-generating systems.

The SRMP enters at the development stage. It coordinates user requests, suggestions by NIST scientists for new SRMs, funding, timing of development, production of prototypes, production of SRM batches, and marketing and distribution activities.

2.1.5 SRM Program Goals and Constraints

The present SRMP Office was created in 1965 to centralize the SRM activities within NIST (then the National Bureau of Standards) and to extend the program beyond its traditional areas of physical, engineering, and chemical measurements into new areas

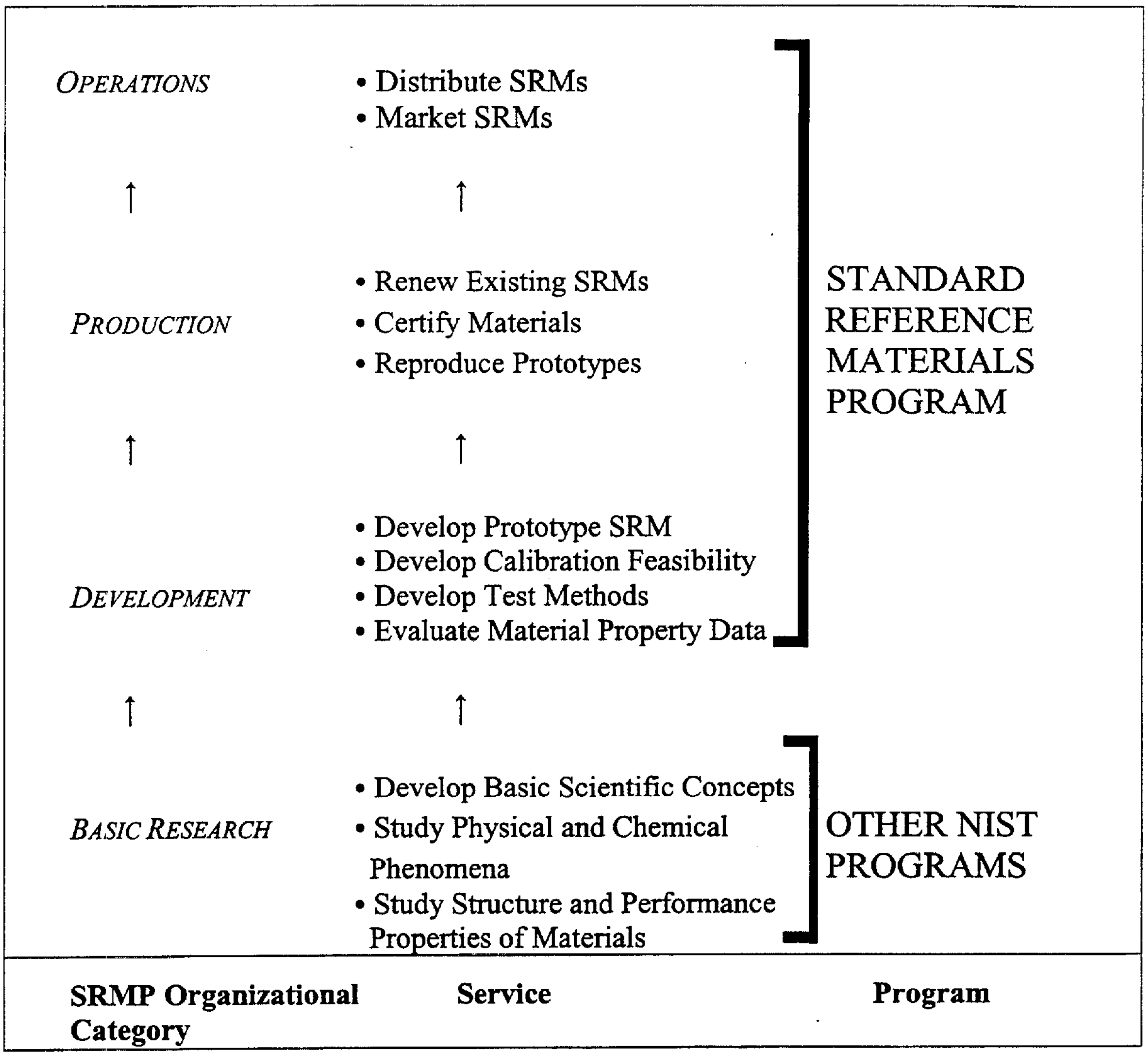

Figure 2-3. Hierarchy of SRM Production

such as computer technology, nuclear physics, health, and environment. At that time the SRMP Office stated its goals as follows:

1. *Maintain the high degree of quality expected from NBS, while expanding the output of SRMs.*
2. *Produce a more balanced output by moving into areas not covered heretofore. Stress particularly areas of national concern, especially health, pollution control, and technological advances.*
3. *Produce more SRMs which characterize the frontiers of scientific measurements.*
4. *Keep old customers happy by producing renewal SRMs as required."*

Given the nature of the demand for SRMs, the cost structure, and the constraints on pricing and funding of the program, these goals may not all be simultaneously attainable. Since the early 1980's, the Congress has been phasing out the subsidies from tax revenues that were usually relied on in the past tot cover deficits. The SRMP management subsequently had to increase prices in order to produce sufficient sales revenue to cover the cost of the program. This has made it necessary, more so than previously, to take into account demand-side information when setting prices in order to prevent losing customers.

2.2 Economic Background

2.2.1 Demand

The demand for SRMs is a derived demand. SRMs are intermediate goods that are used as inputs in a firm's production of final goods. The firm will buy SRMs as long as the value of the increase in productivity in the production of the final good is greater or equal to the price the firm has to pay for SRMs. If this condition is not met, the firm will substitute other means of quality control for SRMs. Whether or not a firm will buy SRMs, therefore, depends mainly on its price, but also on the availability and the prices of substitutes. In some cases, there are no direct substitutes for the primary SRMs that NIST produces, and consequently the price elasticity of demand for these SRMs is expected to be relatively low. Likewise, NIST faces an inelastic demand curve for certain SRMs whose use is mandated by government regulation, as in pollution control, for example, or for SRMs that are designated in contract specifications. But, according to NIST experience, if good secondary standards are available, many firms switch to secondary standards or use them intermittently with NIST's primary standards when prices of primary standards increase. These SRMs have a relatively higher price elasticity of demand.

2.2.2 Cost Structure

NIST's accounting practices attribute SRM costs to the four organizational categories listed in figure 2-3, namely Basic Research, Development, Production, and Operations.

1) Basic research costs are covered by general research funds through congressional appropriations to NIST or through contracts with other agencies or private-sector firms. The SRMP excludes basic research costs from any of its cost calculations and accounts only for development, production, and operations costs.

2) Development costs constitute the largest proportion of the costs of an SRM. They are mainly equipment and labor costs incurred in generating a prototype of a specific SRM from the basic properties and characteristics that were determined for the material through basic research. The cost of development for one SRM amounts from between $250,000 to over $1 million. These costs are independent of how many units of the SRM are eventually prepared and sold.

3) Production costs cover the preparation of individual units and their certification. These are costs of labor, raw materials, and equipment for processing a prototype into

certified units. Production costs also include inventory and obsolescence costs. All of production costs are variable costs.

4) Operations costs include the fixed costs of overhead, costs for buildings and computer facilities, and the cost of managerial staff. All sales-related costs of operations, such as advertising, shipping, and billing are variable costs. Roughly 75 percent of operations costs are fixed costs and 25 percent are variable costs.

Since in terms of a single SRM, fixed costs constitute a much larger portion of total costs than variable costs, it is reasonable to assume that average cost per unit of SRM is declining and marginal cost is decreasing, or at least constant, over the relevant range of demand. This feature of decreasing or constant marginal costs has important implications for the pricing of SRMs because the first-best pricing formula where price equals marginal cost will not allow the SRMP to cover the cost of its program.

2.2.3 Funding and Pricing in the SRM Program

U.S. Code 275 of 1906, mandates that "the Secretary of Commerce shall charge for services performed." This general policy is detailed in the U.S. Dept of Commerce Administrative Order No. 203-5, issued in 1961, which states:

> *"The objectives of the program are to provide fair and equitable charges for services rendered by the Department, thereby reducing the burden of cost on the general taxpayer; reduce pressures for special services; and provide a yardstick to evaluate future legislative and program requirements."*

The interpretation of these directives has differed over the years depending on each Administration's degree of conservatism with respect to government spending. In the earlier years of the SRMP, it was considered acceptable in general to request additional Congressional appropriations when deficits accrued. But since the early 1980's the SRMP has been under increasing pressure to set its prices so as to generate sufficient revenue to cover costs.

2.2.3.1 Funding

NIST's SRM budget is financed from a Working Capital Fund (WCF) of about $15 million, which was made available by Congress in 1965 to operate the SRMP. Any capital "borrowed" from the fund to meet annual expenditures has to be repaid to the fund out of sales revenues.

The direct appropriations that are still available can be used only for developing new SRMs. The rationale for financing some of the research at the development stage through tax revenues is the same as that for financing some of the basic research, namely that there is a fundamental or generic component to SRM development, the benefit of which cannot be imputed to any particular industry or user. In addition, the use of direct, non-reimbursable appropriations is also considered justified in some cases for developing SRMs that are technologically desirable but whose sales potential is uncertain.

At present, direct appropriations for development amount to about $0.5 million (in current dollars) annually. This amount covered about 40 percent of the development expenditures in 1992, for example. The remainder has to be recovered from user fees. Before 1983, $1.1 million annually was appropriated for development from tax revenues. In 1982 Congress instructed the SRMP to recover, over the coming years, progressively larger portions of development costs through user fees, up to the point where direct appropriations would be reduced to the present 0.5 million per year.

In addition to the reimbursable part of development costs, the WCF has to be reimbursed from sales revenues for all production and operations costs. In practice, however, the SRMP has not been successful in covering these costs and, until 1990, additional appropriations have been needed in most years to offset the deficit in the WCF. Figure 2-4 compares sales revenues with expenditures, in constant 1982 dollars. The distance between the two curves that are superimposed on the bar graph shows the deficit in the WCF from 1978 to 1992. It has been declining steadily since 1983 and was eliminated in1990.

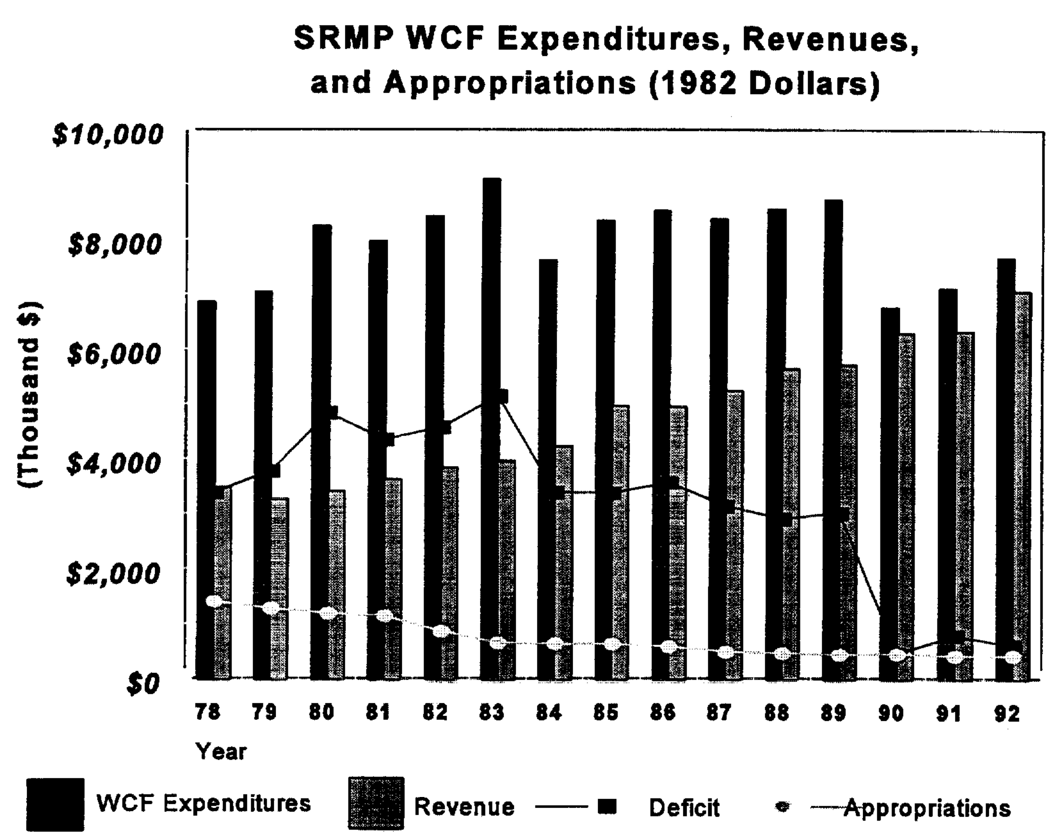

Figure 2-4 Deficit of SRM Program

However, the deficit may have been eliminated not because of sufficiently increased sales revenues but rather at the expense of the development of new SRMs and of renewals of existing SRMs. Figure 2-5 shows that since 1983 the number of completed new SRMs and renewals of existing SRMs has declined. In 1983, for example, there were a total of 193 new SRMs and renewals. By 1992 there were 102. It appears that during the years when direct appropriations were high, SRM development and renewals expanded vigorously. The decline is taking place at a time when advances in technology and a renewed emphasis on improving the competitiveness of American manufacturing in world markets make it imperative to develop more new SRMs rather than fewer.

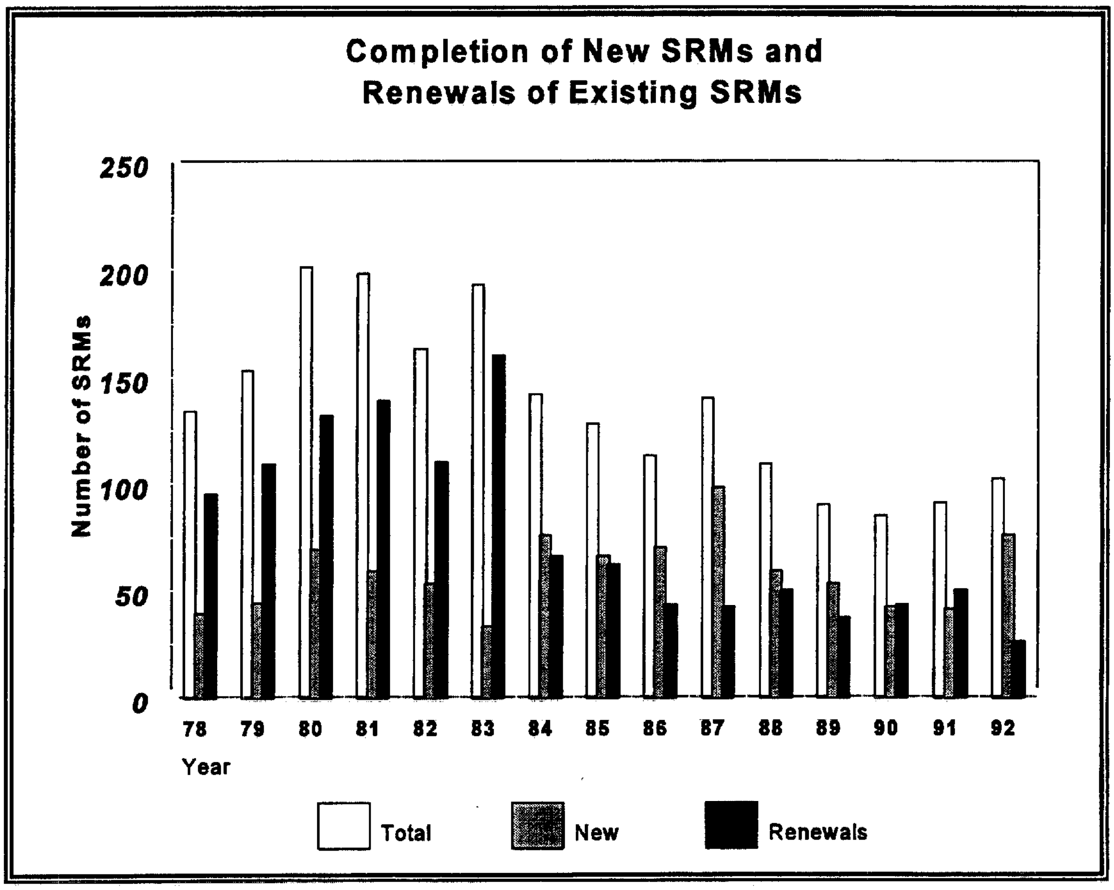

Figure 2-5. New SRMs and Renewals

2.2.3.2 Pricing

In an attempt to eliminate deficits through increased sales revenues rather than direct appropriations, the SRMP has in recent years made several adjustments to its pricing formula. The basis of the pricing formula is the Unit Production Cost (UPC) to which various surcharges are added, some *ad valorem* and some lump-sum surcharges. The UPC represents the actual cost of labor and materials used in the production of an SRM from its prototype. Since the UPC is arrived at by averaging total production costs over the number of units produced, the price for an individual SRM (produced under conditions of decreasing or, at least, constant marginal cost) becomes lower, the higher the quantity produced. For this reason—and because there is little information available on SRM demand functions—SRMP managers have tended to overestimate the number of units to be produced annually. Before 1972 there was a tendency to routinely produce too large a supply of SRMs. The average inventory holding period was 10 to 12 years, and many of the SRMs deteriorated or became technologically obsolete and could no longer be sold. Because production costs funded by the WCF are amortized over 10 years, and unsaleable SRMs were not accounted for as an annual cost of production, they did not appear as losses. The U.S. General Accounting Office (GAO) demanded a change in accounting practices in 1972, and subsequently an *obsolescence surcharge* was added to the pricing formula as a means to cover the cost of unsaleable SRMs.[7] By 1977 the average inventory holding period had fallen to its present level of 5.2 years. The obsolescence surcharge is now 10 percent of UPC.

Another surcharge to UPC is the *development surcharge* introduced in 1982 in order to compensate for the reduction in direct appropriations. Initially, the SRMP attempted to recover immediately the loss in appropriations for development through a surcharge that increased the SRM prices by 25 percent on the average. In the wake of this price increase, unit sales overall dropped by 16 percent in 1983, and sales revenues increased by less than 5 percent. Table 2-1 shows the change in units sold and the percentage changes per category between 1982 and 1983. It is clear from this table that the price elasticities of demand vary, even though the figures refer only to *categories* of SRMs and only to one year and do not result from a systematic demand analysis of *individual* SRMs.

Figure 2-6 shows that the rate of the 1982/83 price change is much higher than the rates of change in other years. Because of the unexpected drop in sales in 1983, the development surcharge was reduced to 5 percent of UPC in the following year with the intention to increase it more gradually to 40 percent of UPC by 1992.

Figure 2-6 also shows that because of the attempt to recover immediately the full amount of the loss in appropriations, the average price per SRM unit at NIST grew at an annual percentage rate considerably higher than that of the annual change in the PPI. During the years between 1980 and 1984, the average rate of increase in NIST prices was 15.9 percent, whereas the average growth in producer prices was 6.6 percent. As figure 2-6

[7]U.S. General Accounting Office, *Substantial Losses Incurred Under the Standard Reference Materials Program,* B-114821, dated 11/10/72. A contemporaneous NIST report (*Economic Analysis of Production and Pricing of Standard Reference Materials,* by H. E. Marshall and R. T. Ruegg, unpublished) came to similar conclusions and pointed out the need for developing demand functions for SRMs in order to substantiate conclusions suggested by economic theory.

shows, the difference in average growth rates in the years from 1984 to 1992 was much smaller, at least in absolute values (4.5 percent compared with 2.1 percent).

Table 2-1. Change in SRM Unit Sales per Category from 1982 to 1983

CATEGORY	1982	1983	% CHANGE
Engineering	8,088	5,734	-29
Chemicals	4,810	3,813	-21
Metals	12,527	9,994	-20
Nonmetals	4,563	3,886	-15
Health	3,669	3,639	-1
Radioactivity	1,704	1,729	+1
Environmental	4,162	4,313	+4
TOTALS	39,523	33,108	-16

The third surcharge to the UPC is an *operations surcharge.* It is arrived at by dividing the total operation-related costs of the program by the number of SRMs expected to be sold in a particular year. In 1992, the average operations cost was $64 per unit. Another fixed amount of one dollar is added to the price of each SRM for overhead costs paid to the Department of Commerce

In summary, the pricing formula for SRMs is as follows:

Price = Unit Production Cost
+ development surcharge (35% in 1991)
+ obsolescence surcharge (10%)
+ operations surcharge (multiples of $64)
+ overhead ($1)

Because some of the additions to UPC are lump-sum, the percentage increases are different for each SRM. But the differences are arbitrary and not based on the differences in price elasticities of demand. Some attempt at systematic price differentiation was made during the late 1960s. The SRMP "assigned" to lower-cost SRMs part of the production costs of SRMs that were considered "too expensive." The GAO, however, reprimanded the SRMP Office and recommended that it "establish sales prices of SRMs on the basis of their actual production costs." This reprimand is interpreted by the SRMP management as meaning that it is limited to strictly charging an average-cost price (defined according to its own accounting convention), even though in reality it does not know the true average cost of an individual SRM.

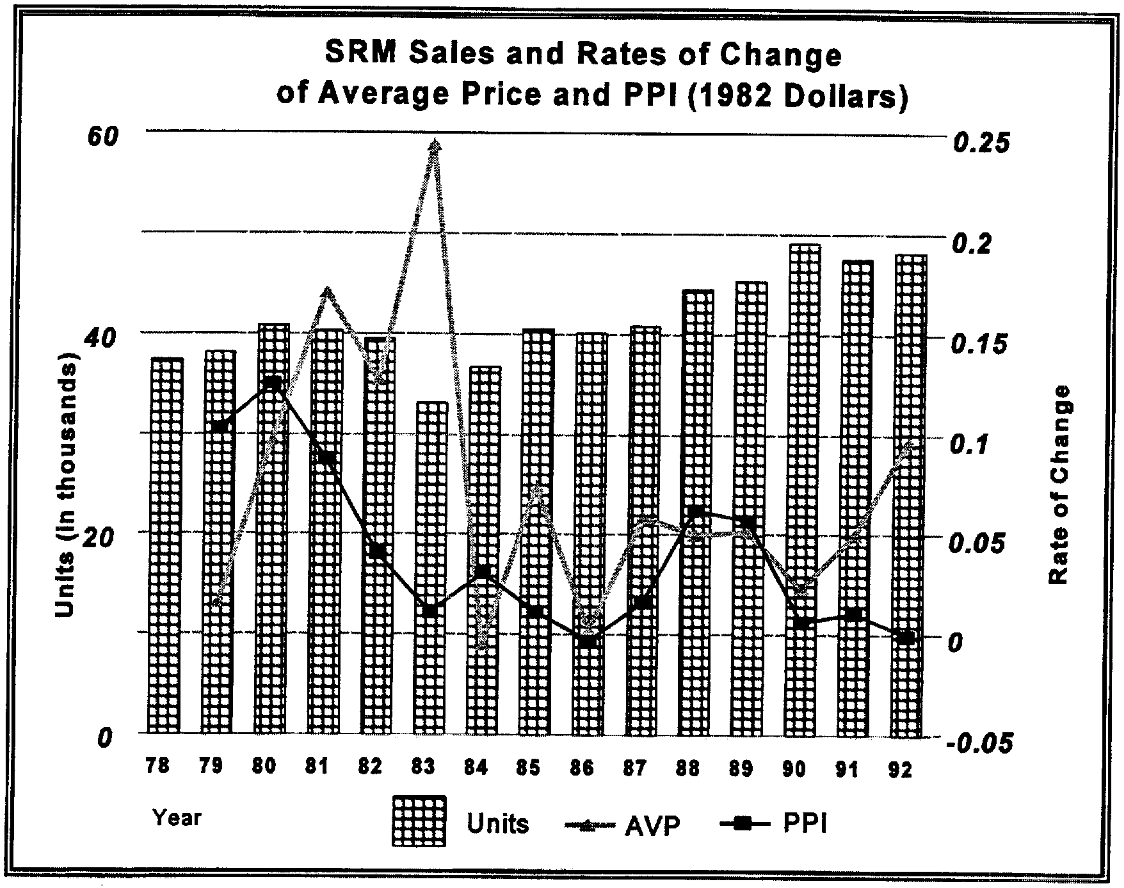

Figure 2-6. **SRM Sales and Rates of Change of Average Price and PPI from 1978 to 1992**

2.3 Conclusions

The present trial-and-error method of increasing and decreasing surcharges has not succeeded in covering costs and ensuring a saleable supply of SRMs. Congress, the GAO, and the NIST Budget Office have urged the SRMP management to use an approach based on economic theory and to collect more reliable information on the sensitivity of SRM sales to price changes. The 1972 GAO report recommended, for example, that NIST "adopt policies and procedures for determining the sales potential of proposed SRMs and the quantities to be produced, and criteria for establishing sales prices for SRMs that will provide for operating the SRMP on a self-sustaining basis to the extent practicable."

In our study we address the second part of this admonition, that is the determination of prices and production plans for *existing* SRMs. Determining the sales potential of *proposed* SRMs requires a different set of variables that need to take into account technological considerations and the implications to industry of not developing an SRM at all or of developing it later rather than sooner. These latter types of decisions are also an important part of managing the SRMP and need to be a topic of future research.

3. THE PUBLIC-SECTOR PRICING MODEL IN THE BOITEUX TRADITION

3.1 Introduction

The theory of pricing publicly supplied goods arises not from welfare economics but rather from the theory of taxation and public utility regulation. The early literature (Ramsey, 1927; Pigou, 1928; Hotelling, 1938; Manne 1952; Boiteux, 1956) addressed the problem in terms of optimal taxation to be raised by the government to cover the deficits of nationalized railroads and utilities. These are the types of industries that are subject to decreasing-cost production, where marginal-cost pricing will not cover costs. Boiteux's model became the intellectual precursor to much of the more recent theoretical and empirical work on the pricing problem of public enterprises. He was manager of the nationalized French electricity industry when he published his paper on the management of public monopolies subject to budgetary constraints. Boiteux, building on Ramsey's work, introduced a general equilibrium approach to the problem and thus took into account the interdependence between public-sector outputs and between these and those of the rest of the economy, a consideration that had been missing in the literature on the theory of taxation and utility pricing of the earlier models.

Since Boiteux's analysis is the basis for much of the public-sector pricing theory developed since then, the chapter focuses on his model but follows an analysis by Bös (1994) who incorporates into the Boiteux model subsequent extensions that deal with various restrictions and make it more generally applicable. As originally presented, the Boiteux model (extensively reviewed by Drèze (1964) and Rees (1968)) analyzes the public-sector pricing problem in the context of a many-person, N goods and N factors model in which all other markets are perfectly competitive and the government has the ability to redistribute endowment income lump-sum to satisfy interpersonal equity. Rees (1968) and Hagen (1979) examine the implications of the welfare maximization criterion when the unregulated private sector is not perfectly competitive. Feldstein (1972a, b, c) develops a distributional equity factor and shows how the Boiteux efficiency prices should be modified to reflect the principle of distributional equity, making unnecessary the assumption in Boiteux's model of a given distribution of lump-sum incomes. Feldstein (1972c) also investigates optimal differential pricing rules for public firms that sell intermediate goods as inputs to producers rather than as final goods to consumers. To deal with markets in disequilibrium, Drèze (1984) develops pricing rules for the public sector subject to a budget constraint, in an economy where the private sector experiences excess supply of labor and of commodities.

We first present the general extended Boiteux model and derive the marginal conditions that are needed to compute the optimal prices and production plans. The solution equation has five terms which represent the efficiency and equity effects of 'second-best' government pricing decisions. In interpreting their economic relationships, we present a short summary of how the extensions to the basic Boiteux model have contributed to the development of public-sector economic theory in more recent years. The economic interpretation shows how the model provides a number of different pricing rules, depending on the budget constraint, distributional objectives, monopolistic private markets, or intermediate-goods properties.

It will become clear that implementing the full extended Boiteux model presents a challenge with respect to information and computation. There may be applications where all the conditions of the extended Boiteux model are relevant, but often many of its restrictions do not apply in specific cases so that the informational and computational requirements become less severe. For example, some simplifying assumptions can be made in the case of pricing SRMs because they are intermediate goods whose prices do not in any significant way directly influence consumer welfare. This and some other assumptions, which we will describe later, allow us to apply the model as an 'intermediate-goods' case. In section 3.5 of this chapter we will adapt the basic model accordingly. The description in chapter 5 of its implementation will be based exclusively on the intermediate-goods version of the model.

3.2 General Assumptions of the Public-Sector Pricing Model

The basic principles of pricing publicly supplied private goods can best be understood in a theoretical full-information approach to price setting.[8] The basic principles can also be shown most clearly if we concentrate on a normative approach. In doing so, we consider a welfare-maximizing firm that does not pursue other political or bureaucratic objectives, such as winning votes or maximizing output.

The basic pricing model consists of the following components:

- maximization of the welfare function

subject to

- market-clearing conditions
- the enterprise's technology, and
- a revenue-cost constraint.

3.2.1 The Objective Function

The main actor in the model is the management of the public enterprise. We assume that it is interested in maximizing a Paretian welfare function of the form

$$W(v^1,...,v^H); \quad \partial W/\partial v^h > 0; \qquad h = 1,...,H \tag{3.1}$$

[8]Bös (1994) deals extensively with problems of incentive-compatible price regulation in case of imperfect information.

where welfare depends on individual consumers' utilities. In order to facilitate the economic interpretation of the marginal conditions on which the analysis centers, the welfare function is defined over the 'budget space' rather than the 'commodity space,' that is, the welfare function is optimized with respect to prices rather than quantities. So we deal in terms of individual indirect utility functions $v^h(p, r^h)$ and Marshallian demand functions $x_i^h(p, r^h)$. The consumer's optimum utility depends on a vector of prices p $(p_o,...,p_n)$, and on lump sum incomes r^h. Labor is chosen as the numeraire, with $p_o = 1$, so that prices $p_1,...,p_n$ are expressed in terms of labor hours at which the goods $x^h = (x_i^h,...,x_n^h)$ are consumed or supplied (positive quantities are net demand, negative quantities net supply; x_0 = labor supply). r^h are exogenously given, non-labor, lump-sum incomes.

3.2.2 Constraint I: Market-clearing Conditions

The market clearing conditions link the public enterprise to the general market economy. By taking into account the existence of many private firms, the agency management takes into account the demand for its products by consumers and producers. The model assumes that there are J private unregulated enterprises, $j = 1, ..., J$, and one public enterprise. The production plans of the private enterprises are $y^j = (y_o^j, ..., y_n^j)$, for the public enterprise they are $z = (z_o, ..., z_n)$. Positive quantities are net outputs, negative quantities are net inputs. This 'netput' concept can accommodate goods produced by the agency that are sold both as intermediate goods to producers and as final goods to consumers.

The basic model assumes that supply and demand are in equilibrium according to the market clearing conditions

$$\sum_h x_i^h(p, r^h) - z_i - \sum_j y_i^j(p) = 0; \qquad i = o, ..., n \tag{3.2}$$

where positive x_i^h is private net demand, positive y_i^j is net supply by a private enterprise and positive z_i is net supply by the public enterprise in question. The negative quantities y_o^j and z_o are the respective labor inputs for private-sector firms and the public enterprise. Any firm's output is used either for consumption or as an input for its own or other firms' production. Consumers buy goods from and supply labor to private firms and the public enterprise. The net profits of private firms and the public sector are equal to total lump sum payments.

3.2.3 Constraint II: Production

The public enterprise is assumed to produce efficiently according to a production function

$$g(z) = 0 \tag{3.3}$$

Defining technology by $g(z) = 0$ implies that the public enterprise produces at points along its production-possibilities frontier rather than at any point below it. Such points are characterized by allocational efficiency in production (meaning that none of the factor inputs needs to be adjusted so as to increase the production of some good without reducing the production of any other good) and by the different price ratios by which they are supported. The implications of this assumption with regard to X-inefficiency, allocative inefficiency, cost minimization, and long- and short-run production are discussed in detail in Bös (1994, pp. 47-66). By not further restricting the production function $g(z) = 0$, we allow for decreasing, constant, or increasing returns to scale.

The technology of private firms is given exogenously and the public enterprise is expected to adjust to it, as is usual in second-best analyses that focus on public-sector production. In this model, the private sector is exogenous, but we assume that private supply functions $y_i^j(p)$ exist and are known to the public enterprise.

3.2.4 Constraint III: Profits and Deficits

We assume that the government, as the superior authority, has decided that some goods, z_i, should be produced in the public sector and that the public enterprise's management has the right to set prices of particular goods (labeled $k \in K \subset I$). In addition, in order to limit profits or deficits, the government has imposed a revenue-cost constraint of the type

$$\sum_{i=o}^{n} p_i z_i = \Pi^o \qquad (3.4)$$

$\Pi = 0$ implies break-even pricing, $\Pi = 0$ determines a deficit, and $\Pi > 0$ a profit. In its most general formulation, $\Pi = \Pi(p,z,\rho)$, the revenue-cost constraint depends on prices p and netputs z, and on an exogenously fixed factor ρ, which may be determined by a variety of ideological or economic motives regarding, for example, the desired size of the public sector, fears of losing votes because of high public-utility deficits, or unfavorable comparisons of resource uses in the public and private sectors.

This definition can accommodate many variations of revenue-cost constraints depending on the type of restriction intended for the public enterprise. For example, an enterprise may be instructed to cover a percentage of its costs by selling its products and receive financing of its deficit from elsewhere, either through matching grants or tax appropriations. Or an enterprise may be constrained to cover its production costs, where total costs may be distributed among the different goods produced by the enterprise. A third example is provided by 'mark-up on cost' regulation where an enterprise sets its price to cover all its costs plus what it considers a normal profit.

The revenue-cost constraint can thus include factors exogenously given by the government or endogenously determined by the firm. This is usually the case with regulated public enterprises, where the government assigns, for example, the rate of return and the public enterprise decides on the capital input. In a full-information model all factors could be determined endogenously by all the variables and functions of the model. The resulting pricing formulas will look alike whether the factors of the revenue constraint are exogenously given or wether they are endogenously determined. As is usual in Boiteux-type models, we will apply an exogenously fixed budget constraint, $\Pi = \Pi^o$, assumed to be given by the government. This could be a break-even constraint, for example, or a constraint based on the previous year's budget and determined before prices are set. A lower value of Π^o is typically associated with a lower price level and consequently higher demand for the good in question.

Directly incorporating the market clearing constraint and the government budget constraint leads to a realistic second-best model in which the management of the public enterprise accepts the constraints as given and uses the instruments available to it to maximize its welfare function. The instruments that management has available and can control are prices and production plans.

The controlled prices $\{p_k, k \in K \subset I\}$ are a subset of all prices. Prices of goods that are supplied or demanded by only the public enterprise will be controlled in any case. There may also be non-regulated prices of publicly supplied or demanded goods where the public enterprise has to accept prices that are fixed by private enterprises or by government agencies outside the model. That is, the uncontrolled prices $\{p_i, i \notin K\}$ are exogenously given in this model where the public sector adjusts to the unregulated private economy. The wage rate p_0, which serves as the numeraire, is not regulated.

The production plans under the control of the public enterprise $\{z_i, i \in I\}$, are a subset of all net production plans of the economy, both of the public sector and the private sector $\{z_i, y_i, i = o,...,n; j = o,...,J\}$. The production plans whose prices are controlled by the public enterprise are a subset of z_i, denoted z_k, $k \in K \subset I$.

If, in addition to efficient allocation of resources, the public enterprise has distributional equity as one of its objectives, then individual lump-sum incomes (r^h, $h = 1,...,H$) are a third instrument under management's control.

3.3 Solving the Model

Before deriving the marginal conditions for optimal prices and quantities, we want to discuss briefly some questions that have been raised in the public-sector literature. These questions challenge (1) the existence of marginal-cost equilibria under economies of scale, even in a first-best pricing situation and (2) the optimality of marginal-cost pricing under economies of scale. The Boiteux framework does not handle questions concerning the existence of equilibria, such as whether marginal-cost firms go bankrupt because of losses; or whether consumers go bankrupt if they are liable as shareholders of the public enterprise. It assumes that deficits are financed by lump-sum taxes. Likewise, if an equilibrium does exist, it is assumed that the second-order conditions are fulfilled and a unique optimum exists.

Very restrictive assumptions are necessary to answer these challenges. Beato (1982) discusses special distributions and endowments to assure positive individual incomes with respect to the *existence* of an equilibrium when deficits have to be financed by taxes. Bös (1994, p. 122) suggests that further research should investigate whether a marginal-cost equilibrium is assured when deficits are financed by taxes and goods and factors are inelastic in supply. He also suggests two-part tariffs where the fixed part, aggregated over all customers, covers the difference between total costs and the revenue that would result from marginal-cost pricing.

The *optimality* of marginal-cost and second-best equilibria is discussed by Guesnerie (1975), Dierker (1986) and others. Dierker (1991) deals with the possibility that second-best optimality cannot be taken for granted when prices obey the Ramsey rule. First Dierker argues that lump-sum redistribution cannot be taken as a valid assumption in second-best pricing. This means that the public enterprise would always have to include income distribution as one of its instruments when setting its prices. Secondly, Dierker develops the following three assumptions that are needed in addition to the usual assumptions of the Boiteux-Ramsey model:

(1) The assumption of monotonicity requires that consumers buy more output if prices decline but not so much more that no room is left for an increase in the consumption of the numeraire good, for instance leisure. This assumption is made to ensure that in a Boiteux equilibrium not all prices can be lowered simultaneously and still lead to a feasible allocation.

(2) The convexity assumption requires that if an initial output bundle is compatible with the revenue-cost constraint and a terminal bundle is also, then compatibility must hold for any linear combination of the two bundles.

(3) Compensated demand must be less elastic than supply. This condition is fulfilled with certainty if demand is inelastic and returns to scale are not so strongly increasing that more output is sold for less money at marginal-cost prices.

These assumptions are fairly restrictive. Dierker (1991, section 4), however, also demonstrates that the problems of existence and optimality are less severe if the technology of the public enterprise is convex up to some well-defined fixed cost. Bös (1994, p. 75) conveniently argues that in any empirical case, the restrictive assumptions are either fulfilled or not and so their investigation is appropriate then, but since they cannot be justified by the usual microeconomic theory, it is not sensible to treat them in general theoretical analyses. We will thus ignore Dierker's restrictions in the following discussion of the theoretical model. Since from the past pricing history there is no evidence that the above three conditions are not met in the case of SRMs, and since the effects of SRM price changes on income distribution are insignificant, we assume the existence and optimality of the second-best equilibrium arrived at in the empirical part of the analysis.

3.3.1 Optimal Prices and Quantities

In the discussion below we follow a stepwise procedure by first looking at the marginal conditions for prices and quantities and then extending it to include the effect of public-sector price changes on lump-sum incomes (section 3.3.2).

The following Lagrangean function $\mathcal{L}$ combines the social welfare function, the market-clearing conditions, the enterprise's technology, and the budget constraint. The solution to this equation gives the prices and quantities that maximize welfare in a second-best environment.

$$\mathcal{L} = W(v^1,...,v^H) - \sum_{i=o}^{n} \alpha_i \left[\sum_h x_i^h (p,r^h) - z_i - \sum_j y_i^j(p)\right] - \beta g(z) - \bar{\gamma}\left[\Pi^o - \sum_{i=o}^{n} p_i z_i\right] \tag{3.5}$$

Differentiating with respect to prices, quantities, and the three Lagrangean multipliers results in a system of five equations in five unknowns. The marginal conditions for optimal prices and quantities are given in (3.6) and (3.7)

$$\frac{\partial \mathcal{L}}{\partial p_k}: \sum_h \frac{\partial W}{\partial v^h}\frac{\partial v^h}{\partial p_k} - \sum_i \alpha_i \left(\sum_h \frac{\partial x_i^h}{\partial p_k} - \sum_j \frac{\partial y_i^j}{\partial p_k}\right) + \bar{\gamma} z_k = 0; \quad k \in K \tag{3.6}$$

$$\frac{\partial \mathcal{L}}{\partial z_i}: \alpha_i - \beta \frac{\partial g}{\partial z_i} + \bar{\gamma} p_i = 0; \quad i = o,...,n \tag{3.7}$$

If we assume that the public enterprise also controls the distribution of lump-sum incomes, r^h, then the solution of the Lagrangean function $\mathcal{L}$ yields a third marginal condition for computing optimal lump sum incomes.

$$\frac{\partial \mathcal{L}}{\partial r^h}: \frac{\partial W}{\partial v^h}\frac{\partial v^h}{\partial r^h} - \sum_i \alpha_i \frac{\partial x_i^h}{\partial r^h} = 0 \tag{3.8}$$

From equations (3.6), (3.7), (3.8), together with the constraints, the optimal prices, lump sum incomes, quantities, and the Lagrangean multipliers can be computed.

Concentrating on prices and quantities alone (and neglecting lump-sum incomes) we substitute (3.7) into (3.6) and obtain

$$\sum_h \frac{\partial W}{\partial v^h}\frac{\partial v^h}{\partial p_k} - \sum_i \left(\beta\frac{\partial g}{\partial z_i} - \bar{\gamma}p_i\right)\left[\sum_h \frac{\partial x_i^h}{\partial p_k} - \sum_j \frac{\partial y_i^j}{\partial p_k}\right] + \bar{\gamma}z_k = 0. \qquad (3.9)$$

We divide these equations by $\beta_0 := \beta(\partial g/\partial z_0) > 0$[9], where β_0 is the initial endowment of labor. We furthermore define $\lambda := \partial W/\partial v_h/\beta_0)$; $\gamma := \bar{\gamma}/\beta_0$; $c_i := (\partial g/\partial z_i)/(\partial g/\partial z_0)$.

- $\lambda^h \geq 0$ is the normalized marginal social welfare of individual utility. The welfare function is chosen so that λ^h increases with decreasing individual utility.
- γ is a 'normalized' measure of the welfare effects of the size of the public enterprise's deficit. If the revenue-cost constraint Π^o exceeds the unconstrained welfare-optimal profit, then $0 < \gamma < 1$.
- c_i is a shadow price that measures the marginal labor costs of publicly producing good i (for $z_i > 0$; otherwise it is a partial marginal rate of transformation). In this presentation of the model c_i is used as marginal costs.[10]

Using these new symbols, the marginal conditions (3.10) can be rewritten as follows:

$$\sum_h \lambda^h \frac{\partial v^h}{\partial p_k} - \sum_i (c_i - \gamma p_i)\left[\sum_h \frac{\partial x_i^h}{\partial p_k} - \sum_j \frac{\partial y_i^j}{\partial p_k}\right] + \gamma z_k = 0 \qquad (3.10)$$

Since the term for controlling the distribution of lump-sum incomes is not included in equation (3.10), the assumption is that optimal lump-sum incomes are exogenously given. For our purposes it is sufficient to assume that the public enterprise's management controls prices and production plans but accepts individual lump-sum incomes as given and optimally distributed at the outset.

Equation (3.10) expresses a combination of distinct equity and efficiency effects of the particular government pricing and production decisions and second-best conditions considered in a Boiteux-type model. In order to make the economic relationships clearer and to be able to interpret the terms of the equation in relation to the price-cost difference $(p_i - c_i)$ instead of $(\gamma p_i - c_i)$, Bös restates (3.10) as follows, after adding $(1 - \gamma)\Sigma_i p_i$ $[\Sigma_h(\partial x_i^h/\partial p_k) - \Sigma_j(\partial y_i^j/\partial p_k)]$ to both sides of the equation:

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[9]Differentiate the Lagrangean function $\mathcal{L}$ with respect to initial endowments of labor z_0. Then $\alpha_0>0$ and $\beta_0>0$ follow from economic plausibility. See Drèze and Marchand (1976: 67).

[10]This is valid if the public enterprise operates at minimum cost, which implies that marginal rates of input substitution equal input prices. See Bös (1994: 62-66) for a derivation of cost functions for enterprises that do not minimize costs.

$$\sum_h \lambda^h \frac{\partial v^h}{\partial p_k} - (1-\gamma)\sum_h \sum_j p_i \frac{\partial x_i^h}{\partial p_k} - \sum_i (c_i - p_i)\left[\sum_h \frac{\partial x_i^h}{\partial p_k} - \sum_j \frac{\partial y_i^j}{\partial p_k}\right] = $$
$$= -\gamma z_k - (1-\gamma)\sum_i \sum_j p_i \frac{\partial y_i^j}{\partial p_k}; \qquad k \in K. \tag{3.11}$$

We now explain the economic relationships of the five terms in equation (3.11) from left to right with respect to their equity and efficiency effects.

3.3.1.1 Distributional objectives

The first two terms of equation (3.11) represent the distributional objectives of the model. The first term, $\Sigma_h \lambda^h(\partial v^h/\partial p_k)$, represents the social valuation of the effect of the change in price p_k. It refers to the price *structure*, differentiating between necessities and luxuries. Its absolute value is high for necessities and low for luxuries. Bös makes this clearer by applying Roy's identity:

$$\sum_h \lambda^h \frac{\partial v^h}{\partial p_k} = -\sum_h \lambda^h x_k^h \cdot \frac{\partial v^h}{\partial r^h}; \quad k \in K. \tag{3.12}$$

He also defines a 'distributional characteristic,' F_k, of any good $k \in K$ as a distributionally weighted sum of individual consumption shares: [11]

$$F_k := \sum_h \lambda^h \frac{\partial v^h}{\partial r^h} \cdot \frac{x_k^h}{x_k}; \quad k \in K. \tag{3.13}$$

Since the social valuation of changes in the individual lump-sum incomes, λ^h $(\partial v^h/\partial r^h)$, is a decreasing function of individual incomes, it brings about the distributional weighting in (3.13). The term λ^h $(\partial v^h/\partial r^h)$ is a combination of the social and individual valuations of incomes and utility. λ^h is the social valuation of individual utility, and $\partial v^h/\partial r^h$ is the individual marginal utility of lump-sum incomes. λ^h decreases with increasing income, $\partial v^h/\partial r^h$ increases with decreasing income. Society chooses the welfare function in such a way that at the optimum λ^h $(\partial v^h/\partial r^h)$ is positive but decreasing with income.

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[11] Feldstein (1972a, b, c) defines this distributional characteristic as the weighted average of the marginal utilities, i.e., each household's marginal utility weighted by that household's consumption of good *i*.

The second distributional term in equation (3.11) refers to the price *level*; its absolute value is larger the smaller γ. A smaller γ typically results from a lower revenue-cost constraint Π^o. A lower revenue-cost constraint is associated with a lower level of prices. Lower prices, in turn, imply higher demand and thus reinforce the distributional objectives. For example, the level of prices of a welfare-maximizing deficit enterprise will be lower than that of an unregulated monopolist, for whom γ would be equal to 1.

Applying the Slutsky equation to the second term in (3.11) to separate the substitution and income effects gives us the following equation:

$$(1 - \gamma)\sum_h \sum_i p_i \frac{\partial x_i^h}{\partial p_k} = (1 - \gamma)\left[\sum_h \sum_i p_i \frac{\partial \hat{x}_i^h}{\partial p_k} - \sum_h \sum_i p_i x_k^h \frac{\partial x_i^h}{\partial r^h}\right]$$

$$= -(1 - \gamma)\sum_h x_k^h \tag{3.14}$$

where $\hat{x}_i^h$ denotes compensated demand. Since the compensated expenditures for any individual h do not react to price changes, we have

$$\sum_i p_i(\partial \hat{x}_i^h / \partial p_k) = 0 \tag{3.15a}$$

Moreover, differentiating the individual budget constraint $((p_i x_i^h) = r^h)$ yields

$$\sum_i p_i(\partial x_i^h / \partial r^h) = 1. \tag{3.15b}$$

The first two terms in (3.11) can therefore be written as

$$-F_k x_k + (1 - \gamma)x_k, \tag{3.16}$$

3.3.1.2 Allocation in the public sector

The third and fourth terms of equation (3.11) reflect the allocation in the public sector, the core of the Boiteux model. The expression $(p_i - c_i)$ in the third term represents, in a second-best pricing situation, by how much prices should deviate from marginal cost. Since the public-sector allocation not only depends on the supply side but also on the price sensitivity of demand for publicly supplied goods, the consumer demand for public supply is represented as

$$\frac{\partial z_i^D}{\partial p_k} := \sum_h \frac{\partial x_i^h}{\partial p_k} - \sum_j \frac{\partial y_i^j}{\partial p_k}, \tag{3.17}$$

where z_i^D is a Marshallian demand function.

With its emphasis on second-best pricing, the Boiteux model also stresses the importance of the revenue-cost constraint. In our version of the model, this constraint is represented by the fourth term $-\gamma z_k$.

3.3.1.3 The interaction between the public enterprise and the private sector

The fifth term of equation (3.11) reflects the adjustment of the public sector to 'monopolistic pricing' in the private unregulated economy. Some consumer prices may deviate from marginal costs because of monopoly power of entrepreneurs, rule-of-thumb pricing, or commodity taxation. In a second-best approach, welfare is better maximized if unavoidable distortions, reflecting the relative scarcity of goods, are compensated by systematic distortions throughout the economy.[12] In Boiteux's model there are two sets of firms: those in the private sector are perfect competitors, and those in the public sector are subject to the rules for quasi-optimal pricing. Subsequent extensions to the model, by Lancaster-Lipsey (1956-57) and Green (1962) for example, stress the interaction between the private and the public sectors.

If the private sector is perfectly competitive, this fifth term in equation (3.11) vanishes since for price-taking competitive firms, producing at the profit-maximizing level,

$$\sum_i p_i(\partial y_i^j/\partial p_k) = 0. \tag{3.18}$$

The term does not vanish in case of monopolistic pricing. The deviation from marginal cost can be introduced into the analysis as a price-cost margin. For monopolists, who change their levels of production from the efficient ones, marginal costs c_i^j (= $-dy_o^j/dy_i^j$ *for* $y_i^j > 0$) can be interpreted as 'producer prices.' In the case of efficient production,

$$\sum_i c_i^j \frac{\partial y_i^j}{\partial p_k} = 0; \qquad j = 1, ..., J. \tag{3.19}$$

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[12]See Lipsey and Lancaster (1956-57), Green (1962), and Schmalensee (1981), who discuss how the chosen public-sector pricing structure can counteract distortions in the private sector from price-cost deviations.

Hence the fifth term in equation (3.11) can be expanded (Hagen, 1979) to reflect the influence of price-cost margins on public prices:

$$(1 - \gamma)\sum_i \sum_j p_i \frac{\partial y_i^j}{\partial p_k} = (1 - \gamma)\sum_i \sum_j (p_i - c_i^j)\frac{\partial y_i^j}{\partial p_k}; \qquad k \in K. \tag{3.20}$$

Taking into account the above reformulations, the five terms of equation (3.11) can be rewritten as

$$\begin{aligned} F_k x_k - (1 - \gamma)x_k + \sum_i (c_i - p_i)\frac{\partial z_i^D}{\partial p_k} &= \\ = \gamma z_k - (1 - \gamma)\sum_i \sum_j (c_i^j - p_i)\frac{\partial y_i^j}{\partial p_k}; \qquad k \in K. \end{aligned} \tag{3.11a}$$

Equation (3.11a) contains the basic marginal conditions for optimal prices and quantities. It represents the general framework for a public enterprise's policy since it contains the interaction between public and private supply, includes distributional welfare judgments, and uses the usual, non-compensated demand for public supply.

3.3.2 Compensating for Income Effects

Boiteux' original model used compensated demand, that is, it assumed that incomes are redistributed optimally by some sort of compensating lump-sum payments. Normally, in a consumer-surplus approach, when compensated demand is assumed, this redistribution procedure is not made explicit and thus hides the implied value judgments. Bös (1994, p. 81) points out this weakness by making explicit the redistribution required to obtain compensated demand functions in the Boiteux model. To arrive at the basic conditions for the case of compensated demand, the derivation of equation (3.11a) has to include equation (3.8) in addition to equations (3.6) and (3.7). Again substituting Roy's identity for equation (3.8), we obtain

$$\frac{\partial W}{\partial v^h}\frac{\partial v^h}{\partial p_k} = -\sum_i \alpha_i x_k^h \frac{\partial x_i^h}{\partial r^h}; \qquad h = i,\dots,H; \qquad k \in K. \tag{3.8a}$$

At the optimum, the distributional valuations of the government and all income effects cancel out. This is so because incomes are redistributed in such a way that for each consumer the weighted sum of all income effects that result from changing price p_k is equal to the government's valuation of the individual's utility change because of the change in the price p_k. Distributional considerations are no longer needed so that the pricing structure now only concerns allocation. At the same time all income effects are eliminated, leaving only the substitution effect and thus compensated demand.

Using transformations analogous to those in the above 'non-compensated' case, we obtain the basic marginal conditions for the case of compensated demand:

$$\sum_i (c_i - p_i)\frac{\partial \hat{z}_i}{\partial p_k} = \gamma z_k - (1 - \gamma)\sum_i \sum_j (c_i^j - p_i)\frac{\partial y_i^j}{\partial p_k}; \qquad k \in K. \qquad (3.21)$$

This equation corresponds to the equation for Marshallian demand but omits the first two (distributional) terms in equation (3.11a) since we are now dealing with compensated demand $\hat{z}_i$.

3.4 Economic Interpretation of the Extended Boiteux Model

The marginal conditions of equation (3.21) represent a general normative theory of public pricing. Bös (1994) gives an economic interpretation to these general results by examining special cases of the marginal conditions. Imposing restrictive assumptions on the Boiteux model, he examines the pricing rules of marginal-cost pricing, Ramsey pricing, Feldstein pricing, and the adjustment to private monopolistic pricing with respect to allocation, distribution, and stabilization policies. Since these conditions often are not realized in practice, the specific models can be looked at as benchmark models that draw attention to the factors that are important when formulating public pricing policy, namely the dependence on marginal costs, price elasticities of demand, distributional value judgments, and monopolistic pricing in the private sector.

We summarize Bös' treatment of these special cases, listing the restrictions that lead to the different pricing rules and their economic meaning. Their derivations are described in detail in Bös (1994).

3.4.1 Marginal-cost Pricing in the Public Sector

The assumptions that allow marginal-cost pricing in a first-best environment are the following:

(1) only prices of publicly produced goods are controlled; all other prices in the public sector are equal to marginal cost c_i;
(2) the private sector is perfectly competitive;
(3) the distribution of lump-sum incomes is optimal; so we deal with compensated demand
(4) there is no revenue-cost constraint on the public sector;

In this case, equation (3.19) reduces to

$$\sum_{i \in K} (p_i - c_i) \frac{\partial \hat{z}_i}{\partial p_k} = 0; \qquad k \in K. \tag{3.22}$$

Assuming that $\partial \hat{z}_i / \partial p_k$ is a regular matrix, we get the marginal-cost pricing rule $p_i = c_i(z)$, $i \in K$.

The enterprise sets all controlled prices equal to marginal costs c_i. The marginal-cost pricing rule is valid for natural monopolies as well as for competitive enterprises in the public sector. In the case of decreasing-cost industries, the marginal-cost pricing rule leads to 'welfare-optimal' deficits which normatively have to be financed by lump-sum taxation. These deficits are not indicative of mismanagement. The optimization model that produces the marginal-cost pricing rule also results in instructions for optimal quantities of outputs and inputs and thereby prescribes cost minimization.

3.4.1.1 Effects of marginal-cost pricing on allocation, distribution, stabilization

Under the usual assumptions of the Boiteux-type model, the marginal-cost pricing rule, derived from the unconstrained maximization of welfare, results in a first-best allocation of resources. It arrives at a Pareto-optimal allocation of goods among consumers and leads to the first-best utilization of capacity in the private enterprise's production. If both public and private enterprises follow marginal-cost pricing, the allocation between publicly and privately produced goods is also a first-best allocation. In the decreasing-cost case marginal-cost pricing leads to an extension of the public sector, because prices will be lower than cost-covering prices, and the demand for publicly supplied goods will therefore be greater.

No general conclusions can be drawn with respect to the effects of marginal-cost pricing on income redistribution. Marginal-cost pricing does not have income redistribution as its main objective, but it may have some distributional consequences. For example, if in the decreasing-cost case, the comparatively lower prices are for goods that are consumed mainly by lower-income consumers, the distributive effect may be positive. On the other hand, the positive effect may be offset by the fact that in this model the deficits have to be financed by (possibly regressive) lump-sum taxation. Similarly, in the case of two-part tariffs, such as peak-load pricing, there may be an unfavorable distributional effect if those consumers who cannot shift to off-peak demand belong to lower-income groups.

No general statements on the effect of changes in marginal-cost prices on stabilization can be made. The effect depends on whether or not there are scale economies, whether we are looking at the short run or long run, how the government spends the revenue it receives at higher prices, and on whether we have a competitive or monopolistic market structure.

3.4.2 Ramsey Pricing

By restricting the public enterprise to meet a revenue-cost constraint, we are required to find the second-best set of prices. The analysis dates back to Ramsey (1927) who, in the context of optimal taxation, derived a formal mathematical solution to the optimal pricing problem in industries in which marginal-cost prices do not cover total costs. To explain the optimal Ramsey policy, we consider the following case:

(1) only prices of publicly produced goods are controlled; all other prices in the public sector are equal to marginal cost c_i;
(2) the private sector is perfectly competitive;
(3) the distribution of lump-sum incomes is optimal; so we deal with compensated demand;
(4) the public enterprise is restricted by an exogenously fixed deficit or profit Π^o.

In this case the marginal conditions of equation (3.19) reduce to

$$\sum_{i \in K} (p_i - c_i) \frac{\partial \hat{z}_i}{\partial p_k} = -\gamma z_k; \qquad k \in K \tag{3.23}$$

where $\gamma < 1 < 0$ when, as in the most relevant cases, Π^o exceeds the unconstrained welfare-optimal profit.

Ramsey pricing is characterized by a tradeoff between the level of prices and the structure of prices. A low Π^o implies a low price level and a larger government sector. The structure of prices is determined by the price elasticities of demand: prices of goods that are price-inelastic can be increased by a higher percentage to meet Π^o than prices of goods that are price-elastic. The tradeoff exists because a low Π^o will favor low-income consumers, but a high γ, which meets the revenue-cost constraint by increasing prices of price-inelastic goods relatively more, will have the opposite effect on income distribution if these goods are bought mainly by low-income consumers. If $\gamma = 1$, Ramsey pricing converges to monopoly pricing for compensated demand. An agency that chooses Ramsey pricing behaves as if it were an unconstrained monopolist inflating all compensated price elasticities by a factor of $1/\gamma$. When $0 < \gamma < 1$, this leads to a lower level of prices than would be the case for a profit-maximizing monopolist.

The formula in equation (3.23) corresponds to one of the most widely recognized forms of the Ramsey theorem, saying that the optimal deviation between price and marginal cost will be proportionate to the deviation between the marginal revenue and marginal cost of that good.

The Ramsey pricing condition can also be expressed in terms of elasticities. By dividing all terms by p_i and rearranging them, we get

$$\sum_{i \in K} \frac{(p_i - c_i)}{p_i} \frac{\partial \hat{z}_k}{\partial p_i} \frac{p_i}{\hat{z}_i} = -\gamma; \qquad k \in K, \tag{3.24}$$

where $(p_i - c_i)/p_i$ is the Lerner Index, L_i, and $(\partial \hat{z}_k/\partial p_i)(p_i/\hat{z}_i)$ is the compensated price elasticity of demand, η_{ki}.[13]

The economic interpretation becomes quite complicated when more than two goods are considered. In the many-goods case, prices must be chosen so that the weighted sum of the compensated cross-price elasticities is the same for all goods. For this case, Bös solves for the Lerner indices according to Cramer's rule and arrives at $L = \Re$ the Ramsey index, where η becomes an M times M-dimensional matrix of compensated demand elasticities and $-\gamma$ a column vector of M elements. See Bös (1994), section 8.1.2.

3.4.2.1 The inverse-elasticity rule

In the case of two publicly supplied goods, $k \in K$, whose outputs are subject to a budget constraint and all *cross elasticities of demand are zero*, the Ramsey Index is equal to the Lerner Index:

$$\Re_k = L_k = -\frac{\gamma}{\eta_{kk}}; \qquad k \in K. \tag{3.25}$$

where η_{kk} is the own compensated price elasticity of demand. This form of the Ramsey theorem is known as the *inverse elasticity rule.* It asserts that the Lerner Index, that is, the optimal *percentage* deviation of the price of any good from its marginal cost, $(p_i - c_i)/p_i$, should be inversely proportional to its own price elasticity of demand, $(\partial \hat{z}_k/\partial p_i)(p_i/\hat{z}_i)$. It also implies that the optimal percentage deviation of price from marginal cost will be larger, the smaller the absolute value of the good's price elasticity. This means that if prices do have to deviate from marginal costs, but in a way that produces relatively little distorting effects on demand, the bulk of the price rise should fall on goods whose demands are comparatively price-inelastic. A further implication of the inverse elasticity rule is that if all price elasticities in question are equal, prices should be set proportional to marginal costs.

The Lerner Index is positive for prices that exceed marginal costs ($\gamma > 0$) and negative for prices that are below marginal cost ($\gamma < 0$). A breakeven constraint imposed on an enterprise producing under decreasing cost conditions would require positive Lerner Indices, that is, prices exceeding marginal cost. Negative Lerner Indices would apply if an enterprise is directed to follow a low pricing policy. The economic effects are different

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[13] Because of assumption (2) in the first paragraph of this section and because substitution effects are symmetric, $\partial \hat{z}_i/\partial p_k = \partial \hat{z}_k/\partial p_i$.

for positive and negative Lerner Indices; in the case of positive indices, prices are relatively higher for price-inelastic goods, whereas for negative Lerner Indices, the reverse is true.

When *cross-price elasticities are not zero*, the elasticity term in the Lerner Indices consists of so-called 'superelasticities' H_{kk}, which take into account the interplay of direct and cross elasticities. For good 1, this superelasticity is

$$H_{11} = \left[\frac{\eta_{22} - \eta_{12}}{\eta_{11}\eta_{22} - \eta_{12}\eta_{21}}\right]^{-1} = \eta_{11}\left[\frac{1 - \dfrac{\eta_{12}\eta_{21}}{\eta_{11}\eta_{22}}}{1 - \dfrac{\eta_{21}}{\eta_{22}}\dfrac{p_2 z_2}{p_1 z_1}}\right] \tag{3.26}$$

For good 2, H_{22} is analogous, and hence, when cross elasticities are not zero, the Lerner Index in the two-good case is

$$L_k = -\frac{\gamma}{H_{kk}}; \qquad k = 1, 2. \tag{3.27}$$

In this formulation the percentage deviation is greater if the direct-price elasticity is low and if the two goods are substitutes. In the case where the two goods are substitutes, the cross-price elasticity is greater than zero and a price increase of one good will increase the revenue from the other good.

A convenient formulation of the inverse elasticity rule, which eliminates the Lagrangean parameter γ, states that the optimal percentage deviation of the price of any good from its marginal cost will vary inversely with the elasticity of the demand for that good. In the two-goods case, when cross elasticities are zero, the formula becomes

$$\frac{L_1}{L_2} = \frac{\eta_{22}}{\eta_{11}}. \tag{3.28}$$

When non-zero cross elasticities have to be taken into account, we get, in the two-goods case,

$$\frac{L_1}{L_2} = \frac{H_{22}}{H_{11}} = \frac{\eta_{22} - \eta_{12}}{\eta_{11} - \eta_{21}}. \tag{3.29}$$

3.4.2.2 Effect of Ramsey pricing on allocation, distribution, and stabilization

The effect of Ramsey pricing on allocation, distribution, and stabilization depends on the value of the revenue-cost constraint Π^0. When there is a breakeven constraint, $\Pi^0 = 0$, in the decreasing-cost case, at least one Ramsey price will exceed marginal cost; if cross-elasticities of demand are zero, all Ramsey prices will be above marginal-cost prices. The effect on allocation in this case will be that the public sector will be smaller than it would be under marginal-cost pricing without a revenue-cost constraint.

If, in order to meet some distributional objective, the *level* of prices is kept low so that $\Pi^0 < 0$, low-income consumers will be subsidized. The *structure* of prices will be influenced by the characteristics of the inverse elasticity rule which requires relatively higher price increases for price-inelastic goods. If lower-income consumers buy a larger proportion of these price-inelastic goods, this will counteract the subsidization of this group of consumers.

With respect to stabilization, no general statement can be made. The stabilizing effect of Ramsey prices will depend on the elasticities of demand during business cycles. A Ramsey policy will have anticyclical effects if, when $\Pi^0 \neq 0$, prices and quantities are optimally determined.

3.4.3 Feldstein Pricing

Ramsey and Boiteux did not explicitly incorporate distributional equity into their analyses. Ramsey's derivation of optimal excise taxes was based on a single individual with optimal lump sum redistribution of income by the government. Boiteux assumed optimal lump sum redistribution of income for his model. It is more realistic in a second-best framework to deny the existence of optimal lump sum redistribution and to assume explicitly that social marginal utilities of income are unequal. Feldstein (1972a) extended the Boiteux model to include distributional equity in the derivation of the optimal structure of public prices. Even though, for practical and political reasons, public pricing is not considered first and foremost an instrument of distribution policy, it is plausible, at least in the normative context, that by using pricing policies to improve income distribution, a higher level of social welfare can be achieved than by relying on an income tax as the only policy instrument.

To derive the Feldstein pricing formula, Bös (1994) considers the following special case of the Boiteux model:

(1) only prices of publicly produced goods are controlled;
(2) the private sector is perfectly competitive;
(3) all lump-sum incomes are *exogenously given*, and we are dealing with non-compensated demand;
(4) there is a revenue-cost constraint Π^0.

The fact that the optimal prices for government production decisions now explicitly include adjustments for equity, makes the assumption of compensated demand functions no longer necessary; we can deal with Marshallian demand functions. However, the revenue-cost constraint is still needed to ascertain that the public enterprise at least covers the cost of production. Because the internal subsidization of the poor may increase the demand for lower-priced goods at the expense of the demand for higher-priced goods, revenues may decrease, implying a tendency toward a deficit.

Revenues would also decrease if private competitors were to enter the public enterprise's market to supply the good that serves as the subsidizer. So Bös introduces the additional assumption that goods with publicly controlled prices are neither supplied nor demanded by private firms. In this case $z_k = x_k$, and equation (3.11a) simplifies to

$$\sum_{i=0}^{n} (p_i - c_i)\frac{\partial z_i^D}{\partial p_k} = -(1 - F_k)z_k; \qquad k \in K, \tag{3.30}$$

where z_i^D represents non-compensated demand and F_k the distributional characteristics of equation (3.13), that is,

$$F_k := \sum_h \lambda^h \frac{\partial v^h}{\partial r^h} \frac{x_k^h}{x_k}; \qquad k \in K. \tag{3.13}$$

The distributional characteristic F_k will be higher for necessities than luxuries, first because the weight λ^h will be higher if x_k^h is a necessity, and second, because the individual marginal utility of lump-sum income $\partial v^h/\partial r^h$ will be higher for lower income. A higher F_k implies lower prices for necessities.[14]

An agency that chooses Feldstein pricing behaves as if it were an unconstrained monopolist inflating all price elasticities by a factor of $1/(1 - F_k)$. The greater F_k, the greater the value of $1/(1 - F_k)$, and the greater the social valuation of individual consumption. This implies that the elasticities of necessities will be overestimated to a greater degree than those of luxuries.

If regulated goods are supplied and demanded also by private firms and $x_k \neq z_k$, then equation (3.30) needs to be extended to include private net supply x_k.

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[14]Feldstein's distributional coefficient also says that if preferences are identical and homothetic then changes in the distribution of income will have no effect on aggregate goods demands and factor supplies. With income elasticities constant and equal for all people, the percentage increases in demand (factor supplies) by the gainers in an income redistribution will be exactly offset by equal percentage decreases in demand (factor supplies) by the losers, no matter how much income is redistributed.

$$\sum_{i=0}^{n} (p_i - c_i)\frac{\partial z_i^D}{\partial p_k} = -(1 - F_k)x_k - \gamma(z_k - x_k); \qquad k \in K. \tag{3.31}$$

In this case the price will be reduced for distributional reasons and also because of competition by private firms. The higher F_k, and the greater the consumption from private supply x_k, the greater the tendency toward a lower price for the regulated good.

3.4.3.1 Comparison of allocational and distributional effects of Ramsey and Feldstein pricing

The Ramsey model is usually looked at as resulting in a purely allocational pricing rule, and the Feldstein model in a pricing rule where income distribution also matters. In a comparison of Ramsey and Feldstein prices, Bös shows that this distinction may not hold in all cases[15]. In certain perverse cases—depending on the relationship between compensated (Ramsey) and noncompensated (Feldstein) elasticities—the Feldstein optimum may imply a tendency towards an increase in the price of necessities. Further, if low-income earners are allowed to buy more than 50 percent of a luxury and high-income earners more than 50 percent of a necessity, a reversal of the distributional characteristics F_k could also cause the Feldstein price for a necessity to be higher than the Ramsey price. But these distinctions can clearly be made only if the regulated goods are not substitutes or complements. If the cross-price elasticities are non-zero, then a comparison between Feldstein and Ramsey prices becomes almost impossible.

3.4.4 Adjustment to Private Monopolistic Pricing

The assumption of perfect competition in the private sector, which has been maintained in the description of the extended Boiteux model so far, is of course not very realistic. There are always some prices in the private sector that deviate from marginal costs because of monopoly power, the use of rule of thumb pricing, or taxation. Second-best theory says that if price-cost margins are distorted in some markets, then first-best competitive efficiency rules are in general no longer optimal for other markets (Lipsey and Lancaster, 1956-57; Diamond and Mirrlees, 1971; Schmalensee, 1981). The remaining prices then should also deviate from marginal-cost prices to reflect the relative scarcity of goods. To compensate for unavoidable distortions, public prices for substitutes have to be higher than under marginal-cost pricing but lower than in the private sector. If, for example, the deviation of price from marginal cost for a substitute good in the public sector is less than in the private sector, the demand for the private good will decrease and the private sector will adjust its pricing toward marginal cost. For publicly supplied complements, prices would have to be lower than marginal cost to bring the combined price structure for both

[15] Bös (1994) p.146.

complements closer to what it would have been under marginal-cost pricing. The welfare-increasing effect of the public-sector adjustment to monopolistic pricing will be the more significant, the larger is the public sector in relation to the private sector.

In the extended Boiteux model, the marginal conditions of equation (3.21) include the interaction between the public and private sectors. To focus on this aspect of the model, we make the following assumptions:

(1) only prices of publicly produced goods are controlled;
(2) the unregulated private sector is *not* perfectly competitive;
(3) the distribution of lump-sum incomes is optimally chosen, and so we deal with compensated demand;
(4) there exists a revenue-cost constraint Π^0.

Equation (3.21), adjusted to take into account the price-cost margins in the private sector, becomes

$$\sum_i (p_i - c_i)\frac{\partial \hat{z}_i}{\partial p_k} = -\gamma z_k - (1 - \gamma)\sum_i \sum_j (p_i - c_i^j)\frac{\partial y_i^j}{\partial p_k}; \quad k \in K. \tag{3.32}$$

Bös (1994, p. 153) rewrites the equation to solve for the price of a single good k as follows:

$$\begin{aligned} p_k = c_k - \frac{\gamma z_k}{\partial \hat{z}_k / \partial p_k} - \sum_{i \neq k} (p_i - c_i)\frac{\partial \hat{z}_i / \partial p_k}{\partial \hat{z}_k / \partial p_k} - \\ -(1 - \gamma)\sum_i \sum_j (p_i - c_i^j)\frac{\partial y_i^j / \partial p_k}{\partial \hat{z}_k / \partial p_k}; \quad k \in K. \end{aligned} \tag{3.33}$$

The effect of the revenue-cost constraint (assuming $\gamma > 0$) is for the price p_k to exceed marginal cost since $\partial \hat{z}_k / \partial p_k < 0$. The remaining two terms on the right-hand side of the equation describe the reallocation between public and private sectors. If the public good is a net substitute and if all prices exceed the respective marginal costs, then p_k will be higher than c_k. If good k is a complement to some of the other goods, then no clear answer is possible.

As Bös points out, these results are valid only if we assume that, in maximizing profits, the private-sector monopolist disregards his relation with the public-sector enterprise. If the private and public economic agents act as oligopolists, then their particular reaction patterns on price and output determination have to be analyzed and an optimal pattern has to be specified. Bös (1994) treats this problem in a framework of regulation through competition in a 'mixed' market environment. Examples of firms where interactions of this kind may take place are telecommunications or electricity distribution, where firms may be obliged to connect other licensed firms to their public network or grid. Bös suggests that many more case studies will be necessary to understand strategic behavior in regulated oligopolistic markets.

3.5 Optimal Pricing of Publicly Supplied Intermediate Goods

Bös' adaptation of the basic Boiteux model to the public supply of intermediate goods is especially interesting in our case because we want to apply the model to Standard Reference Materials (SRMs), which are intermediate goods.

3.5.1 Modification of the Basic Model

The extensions to the Boiteux model that deal with intermediate goods go back to Rees (1968) and Feldstein (1972c). Feldstein, in addition, included considerations of income distribution in his treatment of intermediate goods, as discussed above in section 3.4.3. Intermediate goods are bought by firms rather than by final consumers and serve as inputs into the production of final goods. In Bös' analysis, electricity and transportation services, for example, are considered publicly supplied goods and services that can be used either by firms as intermediate goods or by consumers as final goods. A business telephone call and a private telephone call are two different goods that can be priced differently.

The pricing of intermediate goods is fully included in the model described above. Any price p_k can be the price of a good sold as an intermediate good only to firms and not to consumers. However, since the prices of intermediate goods do not appear as arguments in the individual utility functions, they do not directly influence welfare. However, these prices may have an indirect effect on welfare for two reasons:

(1) private monopolistic firms may increase the prices of the final consumption goods produced by them if they have to pay more for the intermediate goods they buy as inputs from the public enterprise. To include this effect, the model provides a link between the prices of privately supplied final consumption goods and the prices of the inputs bought by the private firm from the public enterprise.

(2) private monopolistic firms may shift part of the burden of price increases back to the public enterprise, if the public enterprise buys goods from private firms that themselves buy intermediate goods from the public enterprise. To include this effect, the model provides a link between prices of public intermediate goods and prices of private intermediate goods.

To show these links in the model, Bös partitions the various goods into five subsets:

(i) Publicly provided intermediate goods, z_k, are sold by the public enterprise and bought by private firms, but not by private consumers. The public enterprise controls prices and quantities, so that p_k and z_k are instruments of the public enterprise. These conditions imply that
$z_k > 0$, $(k \in K \subset I)$, *i.e.*, K is a subset of all goods.
$y_k^j < 0$, i.e., the goods are inputs of the private firms,
$x_k^h = 0$, i.e., the goods are not purchased by consumers;
$\Sigma_j y_k^j + z_k = 0$ is the market-clearing condition.

(ii) Publicly provided consumption goods z_b $(b \in B \subset I)$, sold by the public enterprise and bought by consumers but not by private firms. Prices p_b and quantities z_b are instruments of the public enterprise. These conditions (analogous to those under (1)) imply that
$z_b > 0$,
$x_b^h > 0$,
$y_b^j = 0$, and the market clearing condition is
$\Sigma_h x_b^h = z_b$.

(iii) Privately provided intermediate goods y_a^j $(a \in A \subset I)$, sold by private firms to the public enterprise but not to consumers (or other private firms). Any single price p_a is functionally dependent $(p_a(\cdot))$ on public prices p_k and on the wage p_o. These conditions imply that
$z_a < 0$,
$y_a^j > 0$,
$x_a = 0$, and the market clearing condition is
$\Sigma_j y_a^j + z_a = 0$.

(iv) Privately provided consumption goods y_m^j $(m \in M \subset I)$, sold by private firms to consumers but not to other private firms or to the public enterprise. Any single price p_m is functionally dependent $(p_m(\cdot))$ on public prices p_k and the wage rate p_o. These conditions imply that
$z_m = 0$,
$x_m^h > 0$,
$y_m^j > 0$, and the market clearing condition is
$\Sigma_h y_m^j = \Sigma_h x_m^h$.

(v) Labor is supplied by consumers to private firms and the public enterprise. Labor is the numeraire and the wage rate is p_o. The market equilibrium condition is $x_o = y_o + z_o$.

Consumer utility depends directly on the prices of consumer goods and the wage rate and indirectly on the regulated prices p_k through the reaction functions $p_m(\cdot)$, if they buy consumption goods produced by private firms with the use of publicly supplied intermediate goods.

3.5.2 Conditions for Optimal Prices and Quantities

As far as prices for publicly provided *consumption* goods (p_b, $b \in B \subset I$) are concerned, the conditions of the extended Boiteux model apply directly. The prices of these consumption goods are determined according to the principles of the basic model, as described and interpreted in the previous sections of this chapter. Assuming uncompensated demand (see Feldstein pricing, equation (3.31)) and monopolistic private markets (see equation (3.32)), the equation determining prices for publicly supplied consumption goods is as follows:

$$\sum_i (p_i - c_i)\frac{\partial z_i^D}{\partial p_b} = -(1 - F_b)z_b - (1 - \gamma)\sum_i \sum_j (p_i - c_i^j)\frac{dy_i^j}{dp_b}; \quad \forall b, \qquad (3.34)$$

The pricing structure for publicly provided *intermediate* goods (p_k, $k \in K \subset I$) takes into account the effect of changes in p_k on the prices p_m of the consumption goods supplied by the private firm to consumers. The equation for pricing publicly provided intermediate goods is thus as follows:

$$\sum_i (p_i - c_i)\frac{\partial z_i^D}{\partial p_k} = -(1 - INT_k)z_k - (1 - \gamma)\sum_i \sum_j (p_i - c_i^j)\frac{dy_i^j}{dp_k}; \quad \forall k, \qquad (3.35)$$

where the responses of private-sector prices of consumption goods and private-sector prices of intermediate goods to public-sector price changes of intermediate goods are incorporated in the term

$$\frac{dy_i^j}{dp_k} := \frac{\partial y_i^j}{\partial p_k} + \sum_m \frac{\partial y_i^j}{\partial p_m}\frac{\partial p_m}{\partial p_k} + \sum_a \frac{\partial y_i^j}{\partial p_a}\frac{\partial p_a}{\partial p_k}. \qquad (3.36)$$

and where INT_k equals

$$INT_k = \frac{1}{z_k} \cdot \left((1 - \gamma)\left[z_k - \sum_m x_m \frac{\partial p_m}{\partial p_k} \right] + \sum_m x_m F_m \frac{\partial p_m}{\partial p_k} \right). \qquad (3.37)$$

In an interpretation of the term $(1 - INT_k)$ analogous to that of the term $(1 - F_k)$ in equation (3.30), we can say that for intermediate goods a profit-maximizing monopolist would adjust the (non-compensated) price elasticities of demand by a factor of $1/(1 - INT_k)$. The meaning of the term INT_k is more easily understood if we look at some special cases of the basic pricing model for intermediate goods.

3.5.3 Special Cases

3.5.3.1 Distributional consequences of public-sector intermediate goods prices are insignificant

Equation (3.35) exhibits the same formal structure as the Feldstein-pricing formula for uncompensated demand in the basic model. Feldstein (1972c) emphasizes in his model for intermediate goods the distributional role of public-sector prices. Bös (1994), in his treatment of intermediate goods pricing, maintains that only a very weak case can be made for including distributional effects from public-sector pricing of intermediate goods. He argues that even in the pricing of publicly supplied consumer goods, the distributional role of public pricing is only indirect compared with that of income taxation or subsidization. In the case of intermediate goods the distributional consequences are even more diffuse since the influence of intermediate goods prices on the distributional position of consumers is even further removed. If the distributional effects are ignored, the weighted average of distributional characteristics vanishes from the model, leaving the general Ramsey structure, which focuses on the allocational effect of public-sector prices.

In the following subsection we focus on the allocational characteristics of the intermediate-goods version of the model.

3.5.3.2 Prices of private firms do not respond to public intermediate goods prices

In practical applications of the model it is entirely possible that private monopolistic prices p_m do not respond to price changes of publicly provided intermediate goods ($\partial p_m/\partial p_k = 0$). This could be the case, for example, if the intermediate good constitutes a small percentage of the inputs in the production of the private good. Since price changes then do not carry through to the private goods bought by the consumer ($\partial p_d/\partial p_k = 0$), there is no effect on the individual consumer's utilities. There is consequently no effect on the welfare function either. Hence the term *INT* in equation (3.36) reduces to $-\gamma z_k$. The pricing formula for intermediate goods then becomes

$$\sum_i (p_i - c_i)\frac{\partial z_i^D}{\partial p_k} = -\gamma z_k - (1 - \gamma)\sum_i \sum_j (p_i - c_i^j)\frac{\partial y_i^j}{\partial p_k}; \quad k \in K. \tag{3.38}$$

Equation (3.38) has the same structure as equation (3.32) for publicly supplied goods in general. Analogously, prices depend only on the price elasticities of demand of the private firms and on the interaction between the public firm supplying intermediate goods and the private sector firms. If the private sector were perfectly competitive, and if it is assumed that individual lump-sum incomes are optimally distributed to compensate the consumer for all income effects, the pricing formula for the public enterprise's intermediate goods would be the general Ramsey formula of equation (3.23), where

$\partial z_i^D / \partial p_k$ becomes $\partial \hat{z}_i / \partial p_k$, i.e.,

$$\sum_{i \in K} (p_i - c_i) \frac{\partial \hat{z}_i}{\partial p_k} = -\gamma z_k; \qquad k \in K \tag{3.23}$$

3.6 Conclusions

We have presented a public-sector pricing model in the Boiteux tradition, starting with a general equilibrium framework, discussing its extensions, and presenting particular cases that rely on simplifications that make the model more operational but are acceptable only in special problems. It is clear that second-best pricing rules, even in their simplest form, are more complex than the simple marginal-cost rule of the first-best optimality model. In addition to production costs, second-best rules need other elements such as demand elasticities. Further, the pricing and production decision rules that emerge from the model are sensitive to policy objectives and behavioral assumptions and differ accordingly in their informational requirements. It is therefore important to understand the logic of their derivation and to be aware that effects throughout the economy depend on price and cost changes but may also depend on simultaneous changes in other policy objectives. Fortunately, in practical applications, many of the difficulties are actually insignificant in specific contexts. In many cases there exist only a few prices associated with a few commodities that may be sold to only a specific group of producers or consumers. In these cases a piecemeal approach with relatively modest informational requirements may be sufficient to derive prices that are superior to average-cost or rule-of-thumb prices.

4. THE DEMAND FOR STANDARD REFERENCE MATERIALS

4.1 Introduction

This chapter focuses on the demand for Standard Reference Materials (SRMs) in an attempt to shed light on how price increases affect the sales, and thus the revenues, of SRMs. The quantity demanded may be more responsive to price changes for some SRMs than for others because of differences among demanders, differences in the availability or usefulness of substitutes, and because of the general business conditions prevailing in the production sectors that use SRMs. Information on the responsiveness of sales to price changes is useful in light of the fact that the Standard Reference Materials Program (SRMP) has to observe a revenue-cost constraint and is aiming at meeting all demand for SRMs.

In economic terms this price responsiveness is measured by the "price elasticity of demand." It is defined as the proportionate change in the quantity demanded resulting from a (small) proportionate change in price. In general it is true that the less price elastic a good, the more easily its price can be increased in order to cover costs because the sellers need not be afraid of losing customers. If, on the other hand, the price is comparatively price elastic, customers will leave the market (or purchase less) if the price is increased, and revenue will decrease. Therefore the seller refrains from large price increases for goods that are very price elastic.

In the case of the SRMP, information on the price elasticities of demand for SRMs contributes to a meaningful assessment of its pricing policy in several ways: (1) Given that the SRMP is producing SRMs under conditions of decreasing, or at least constant, marginal cost, knowledge of price elasticities enables it to determine cost-covering prices and production plans that maximize welfare. (2) Since the SRMP allocates its fixed costs among all the SRMs it expects to sell, it does so more efficiently when prices are differentiated on the basis of demand elasticities. (3) The SRMP can use information on price elasticities of demand to formulate production policies that will focus on SRMs with relatively inelastic demand rather than on SRMs with relatively elastic demand for which (apparently) substitutes can be found more easily.

In light of these considerations, SRMs have to be looked at as different goods and a separate demand function estimated for each SRM. The price elasticities of demand resulting from the estimated demand functions can then be used to shed light on how the price for each SRM might deviate optimally from its marginal cost. The theory of pricing for public enterprises who are subject to a revenue-cost constraint is well developed; it supplies pricing formulas that can be tested if the price elasticities of demand for the products in question are available. Since the choice of price is tantamount to the choice of (saleable) output levels, the two steps of (1) obtaining price elasticities of demand, and (2) applying them to the theoretically appropriate pricing formula, represent a complete tool for evaluating pricing and production decisions.

In section 4.2 we first present a model of the derived demand for SRMs, explaining how firms who use SRMs as inputs might base their demand on the expected increase in profits from using SRMs. From this model we derive the demand functions for NIST SRMs in section 4.3. We estimate by linear and logarithmic regression the coefficients of the independent variables that affect the sales of SRMs. Section 4.4 discusses the results, which show clearly, as expected, that the price coefficients are negative and that the degree of sensitivity of SRM sales to price changes varies among SRMs.

4.2 The Derivation of the Demand for NIST SRMs

4.2.1 The Factor Demand of the Firm

The demand for SRMs is a derived demand. SRMs are intermediate goods that are purchased as inputs by producers. Firms typically use them for quality control, to facilitate seller-customer transactions, or as measurement tools in research. As quality control measures, SRMs contribute to more efficient use of man-hours and materials, reduce the number of rejects among outputs, and improve the quality of manufactured goods. In commercial transactions, the use of SRMs in production reduces the uncertainty about specific characteristics of a good and thus reduces negotiating time. In research, SRMs play a role in the development of new methods, processes, and products. In all of these cases, firms expect that the use of SRMs will increase their profits.

Because NIST is a government agency that does not seek to derive profits from the sale of SRMs, it is assumed to be non-partisan and objective. The "primary" SRMs that NIST produces are therefore considered to be of uncontested reliability and objectivity and therefore generally preferred to either commercially produced "secondary" SRMs— even if traceable to NIST's primary SRMs—or to other competing commercial U.S. or foreign SRMs. Whether a producer, for example, will purchase the primary NIST SRM or not depends on a number of factors that affect the firm's costs and revenues. It depends on the price of SRMs, the price of secondary or other SRMs or of substitute means of quality control, the cost in terms of inefficiency of doing without SRMs, the share that SRM costs represent in the firms's total expenditures for inputs, and other variables that affect profits, such as business conditions in the manufacturing sector in which SRMs are used or the general state of the economy.

The following model expresses the profit function of firms that use SRMs and other means of quality control as inputs into their production of final goods.

$$\max E(\pi) = P_s r_s + (1 - P_s) r_m - wS - mM \tag{4.1a}$$

where

$E(\pi)$	=	the expected increase in profits from either using SRMs as quality control measures, or using alternative or no measures of quality control;
P_s	=	the probability of using SRMs;
$(1-P_s)$	=	the probability of not using SRMs;
r_s	=	the increase in revenue produced by using SRMs;
r_m	=	the increase in revenue produced by using other or no means of quality control;
w	=	the price of SRMs;
S	=	the quantity of SRMs used;
m	=	the price of alternative measures of quality control;
M	=	the quantity of alternative means of quality control.

Firms will buy SRMs if the expected increase in profits is higher from using SRMs than from using other means of quality control. They may use both at the same time. To measure the contribution of SRMs to the increase in profits, we take the expected increase in profits due to SRMs less the expected increase in profits due to other means, or less the expected cost of not using quality control at all.

Rearranging the terms of equation (4.1a) gives us the following equation:

$$\max E(\pi) = P_s(r_s - r_m) + r_m - wS - mM \tag{4.1b}$$

Differentiating equation (4.1b) with respect to S and M, we obtain the necessary optimum conditions as follows:

$$\frac{\partial E(\pi)}{\partial S} = \frac{R(S,M) \cdot \partial P(S,M)}{\partial S} - w = 0 \tag{4.2a}$$

$$\frac{\partial E(\pi)}{\partial M} = \frac{R(S,M) \cdot \partial P(S,M)}{\partial M} - m = 0 \tag{4.2b}$$

where

$R(S,M)$	=	$(r_s - r_m)$, the difference in revenue increase between using SRMs and alternative means of quality control, expressed as a function of the number of units used.
$P(S,M)$	=	the probability of an increase in revenue produced by the use of SRMs, taking into account the probability of an increase in revenue produced by the use of alternative measures of quality control. We assume the following definition of P(S,M):

$P(S,M) = (1-\beta e^{-\tau S - \varsigma M})$,

where

β = a constant representing some effect on revenue even if no quality control is implemented;

τ = a measure of the effectiveness of using SRMs;

ς = a measure of the effectiveness of using other measures of quality control.

The test conditions for this model are the particular values of the prices *w* and *m*. We want to observe the changes in the level of *S* and *M* as their prices change. A profit-maximizing firm will employ *S* and *M* up to the point where the marginal contribution of each factor to the expected increase in profits is equal to the cost of acquiring an additional unit of *S* or *M*.

To assure that equations (4.1a) and (4.1b), i.e., the resulting quantities of *S* and *M*, pertain to a maximum expected profit $E(\pi)$, second-order conditions are needed:

$$R_{11} < 0,\ R_{22} < 0,\ R_{11}R_{22} - R_{12} > 0.$$

The purpose of the analysis is to formulate a refutable hypothesis as to how firms react to changes in the parameters they face, in this case to changes in *w, m*, and *R*. Equations (4.2a and 4.2b) are two implicit relations in five unknowns, *S, M, w, m*, and *R*. It is possible to solve for *M* and *S* implicitly in terms of m, w, and R.

$$\begin{aligned} S &= S^*(m,w,R) \\ M &= M^*(m,w,R) \end{aligned} \qquad (4.3)$$

Equations (4.3) represent the factor demand curves for *S* and *M*.[16] These relations indicate the amount of each, SRMs and other measures, that will be used as inputs as a function of their prices and the increase in revenue they produce. In order to solve for S^* and M^*, the functions have to be defined. Once we have solved for equations (4.3), we get the comparative statics of the profit-maximizing model as the following six partial derivatives:

$$\frac{\partial S^*}{\partial w}, \frac{\partial S^*}{\partial m}, \frac{\partial S^*}{\partial R}, \frac{\partial M^*}{\partial w}, \frac{\partial M^*}{\partial m}, \frac{\partial M^*}{\partial R}$$

[16]These factor demand curves are not the marginal product curves. The marginal product functions are expressed in terms of M and S, while factor demand curves are expressed in terms of prices and increased revenue and are dependent on the behavioral assertions of the model.

The partials indicate the marginal changes in the use of S and M due to changes in their prices and a change in the increase in revenue earned by the firm. The factor demand functions are those levels of S^* and M^* that the entrepreneur employs to keep the value of the marginal products of S and M equal to their prices for any change in revenue. The sufficient conditions for the maximization of profits imply that the factor demand curves must be downward sloping in their respective factor prices. A change in the price of S or M will result in a change in the usage of that factor in the opposite direction. This constitutes the refutable hypothesis of the model.

No refutable hypothesis emerges from the cross effects $\partial S^*/\partial m$ and $\partial M^*/\partial w$. The cross effects are equal but their signs can be either positive or negative, depending on whether S and M are complements or substitutes. So all events relating to the change in SRM use when the cost of other methods increases are possible.

In the expressions relating to the effects of changes in SRM use due to a change in revenue, $\partial S^*/\partial R$ and $\partial M^*/\partial R$ also depend on whether S and M are substitutes or complements. We cannot say whether the use of S increases or not with a change in the price of M; it can only be said that an increase in revenue cannot lead to a decrease in both S^* and M^*.

4.2.2 Estimating the Demand for NIST SRMs

In our treatment of the determination of a firm's factor demand we focus on SRMs. Analogous to equations (4.3) a simple representation of the production function of a firm's measurement technology solely in terms of SRMs can be expressed as

$$m = m(P,S,X) \qquad (4.4)$$

where

- m = the demand for measurement technology in the form of NIST SRMs
- P = the price of SRMs
- S = the price of private-sector substitutes for SRMs
- X = other factors that might influence the quantity demanded of SRMs and thus the change in revenue.

The additional variable X, which might influence the quantity demanded of NIST SRMs, represents factors such as the level of sales of the firm's output, government legislation requiring the use of certain standards, business conditions in the manufacturing sector or economy-wide, or technological changes.

The economic model of derived input demand implies that the quantity demanded of a NIST SRM, m, is negatively related to its price, P, holding constant, or controlling for, the effects on the quantity demanded of changes in the price of private-sector substitutes for the NIST SRM, S, as well as for the effects of other demand factors, X. The quantity demanded of a NIST SRM is expected to be positively related to the price of substitutes. Other demand factors can have either a positive or negative influence on quantity demanded. This hypothetical relationship is depicted in figure 4-1, where m is a function of the change in P, and where S and X are held constant.

Demand for SRMs

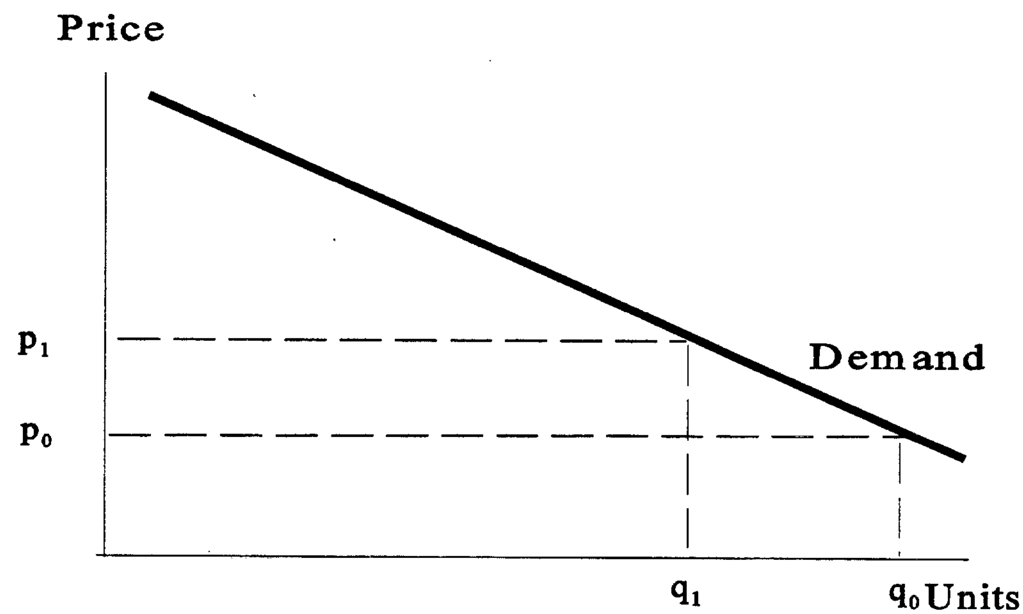

Figure 4-1. The Quantity Demanded of SRMs as a Declining Function of Price

4.2.3 Specifying the SRM Demand Equation

The purpose of the empirical estimation is to determine whether there is indeed a negative relationship between the unit sales of SRMs and their prices, and if so, whether it is quantitatively significant. We assume that the demand for an individual SRM at time t, m_t, consists of a systematic component—which depends on the price charged by NIST relative to the price for private-sector substitutes, p_t ($=P_t/S_t$), as well as on other, non-price factors, X, affecting m_t—and a stochastic component, ε_t, representing a surrogate for all variables that cannot be separately included in the model but which collectively affect m_t.

We assume the systematic relationship between the quantity demanded of SRMs and the variables to be linear and express it as a standard linear model as follows:

$$m_t = \beta_1 + \beta_2 p_t + \Sigma\beta_j X_{jt}; \quad t = 1,...,15; \; j = 1,...,n. \tag{4.5}$$

where the variables are defined as above, and X_{jt} represents factors other than relative price that affect the quantity demanded of NIST SRMs.

In the context of the empirical demand estimation, the model may be expressed as follows:

$$Units_t = \beta_1 + \beta_2 RPrice_t + \beta_3 BCon_t + \beta_4 Dummy + \epsilon_t \tag{4.6}$$

where the quantity demanded, m_t, is expressed as *UNITS* of SRMs, p_t is expressed as *RPrice*, the price of an SRM relative to prices of similar inputs, and X_{jt} is expressed as either *BCons*, the "business conditions" prevailing in the manufacturing sector that uses SRMs, or as *GPDI*, a proxy for cyclical business fluctuations economy-wide. Dummy variables are used as additional non-price variables to account for specific conditions relating to an SRM.

As discussed above, β_2 is expected to have a negative sign, meaning that if the SRM price (*RPrice*) rises relative to prices for similar inputs, the number of units sold by NIST (*UNITS*) will decrease. We also hypothesize that SRM sales are positively related to an increase in business activity (*BCon*) in the particular manufacturing, service, or utilities sector that uses the SRM, so that β_3 is expected to have a positive sign; β_4 is expected to be either positive or negative, depending on whether the dummy variable is specified as detrimental or conducive to SRM sales. To the error term ε_t are relegated all minor and random influences not captured by the specified variables.

We also estimate a second, often used, functional form of the demand equation. It is non-linear in its variables and describes a constant-elasticity demand curve of the form

$$m_t = \beta_1 \, p_t^{\beta_2} \, \Sigma X_{jt}^{\beta_j} \tag{4.7}$$

It is linear in the logarithms of the variables (except for the dummy variables, *D*) and can thus be estimated by linear regression. Analogous to equation (4.6), it is of the form

$$lnUnits_t = \ln\beta_1 + \beta_2 \ln RPrice_t + \beta_3 \ln BCon_t + \beta_4 D + \epsilon_t \tag{4.8}$$

An attractive feature of the log model is that the estimated slope coefficients directly measure the price elasticities of demand.

4.2.4 Data

4.2.4.1 List of SRMs analyzed

We use the same basic demand model, in its linear and logarithmic versions, to estimate demand functions separately for each of the 24 SRMs listed below:

Metals:		
1.	122 H	Cast Iron Car Wheel
2.	1261 A	LA Steel AISI 4340
3.	53E	Bearing Metal, Lead Base
4.	527	Zinc, Base C
Nonmetals:		
5.	635	Portland Cement, Blue
6.	637	Portland Cement, Pink
7.	120	B&C Phosphate Rock
8.	27 F	Iron Ore, Sibley Powder
Chemicals/Rubber/Plastics:		
9.	370 E	Zinc Oxide Rubber Composite
10.	37 G&H	Sulfur Rubber Composite
11.	40 H	Sodium Oxalate Redox
Engineering:		
12.	185 E,F&G	Potassium Hydrogen Phthalate pH
13.	187 B&C	Borax pH
14.	189 A	Potassium Tetroxalate pH
15.	39 I	Benzoic Acid Combustion
Environmental:		
16.	1575	Pine Needles
17.	1620, &A,B	Sulfur in Fuel Oil, 5%
18.	1622,A,B&C	Sulfur in Fuel Oil, 2%
19.	1625	Permeation Tube, 10 cm
20.	1635	Trace Elements in Coal
Health:		
21.	911 A&B	Cholesterol
22.	1577 &A&B	Bovine Liver
Science/Metrology:		
23.	935 A	Potassium Dichromate-UV
24.	1361	Cu & CR Coating on Steel

4.2.4.2 Description of variables

Tables A-1a to A-1g in appendix A summarize for each SRM the description of the dependent and independent variables and list their mean values.

Dependent variables:

For each SRM, the dependent variable *UNITS* is specified as *UNIDOM* (*LUNIDOM* for the logarithmic model), that is, as the number of units sold in a given fiscal year (FY) by the SRMP in the domestic market. It corresponds to the measure of the quantity demanded, m_t, in equation (4.5). The regressions are also estimated with dependent variables *UNITOT* and *LUNITOT*, representing the total number of units sold, including exports. However, we consider an estimate of the impact of price changes on domestic sales (*UNIDOM*), that is, sales excluding exports, to be a more accurate measure because the independent non-price variables which serve as proxies for cyclical economic conditions apply to SRM-using sectors of U.S. markets only and do therefore not reflect business activity related to the exported portion of SRM sales. The results of the estimates using total sales (*UNITOT*, including exports) are included in some of the tables and appendixes as supportive and comparative information only.

Independent variables:

The variable *RPrice* measures the effect of NIST price changes relative to price changes in markets for inputs similar to SRMs. We divide the NIST price by the Producer Price Index, adjusted to the government fiscal year, (PPI FY), for Intermediate Materials (less food and energy). We use this PPI as a proxy for prices of substitutes and thus measure the impact of SRM prices compared to prices of similar intermediate goods. This is done because time-series data for prices of substitutes for individual SRMs are not available.

The effects on the quantity demanded of cyclical decreases or increases in economic activity are captured by the variable *BCon*. This variable represents for each SRM the value of product shipments (in the manufacturing sector) or similar measures of business activity, such as receipts (in the service sector) or value of energy sold to end users (for utilities), depending on the economic sector using the SRM being analyzed. If an SRM is used in more than one manufacturing sector, *BCon* is specified individually for each sector as *BCon1*, *BCon2*, etc. The data used to construct the *BCon* variables comes from Census data (1977, 1982, 1987), converted to time series data by using Census Bureau bridge tables at the 5- and 7-digit (product class and product) SIC levels. First each SRM is mapped to the particular SIC product class. Then the dollar values of product shipments (or receipts or sales when appropriate) for all product classes pertaining to each SRM is summed for each *BCon* variable and each year and listed under the appropriate industry name. The names of the industries associated with each *BCon* variable are given in tables A-1a to A-1g in appendix A.

The value of product shipments, as represented by the *BCon* variables, relates SRM sales to the output produced by SRM-using firms in manufacturing, service, or utilities sectors. It is one obvious way of accounting for non-price economic influences on SRM sales. A different non-price variable that could also be used as a proxy for economic conditions is investment expenditures. We also tested the model with the non-price variable *GPDI*, the U.S. Gross Private Domestic Investment. The *GPDI* variable represents business conditions economy-wide rather than by sector as do the *BCon* variables. The estimation that maps SRM sales to the exact economic sectors in which they are used may result in more accurate estimates, but having to prepare a multitude of time-series data sets from Census data is extremely time-consuming and error-prone and is likely to prevent the model from being used by the SRMP. We therefore wanted to compare the results obtained using the *BCon* variables with those obtained using the *GPDI*. Estimation of demand functions by the SRMP will be much simpler if GPDI can be used to measure the influence of changes in the economy on SRM sales. In addition, using just one economic variable other than *RPrice*, instead of several *BCon* variables for each SRM, has the advantage of saving degrees of freedom (df), which is an important consideration, given the limited number of observations available.[17]

Dummy variables:

The two dummy variables used in the equations for some SRMs are *BKOR* and *LoSt*. *BKOR* controls for surges in the number of SRM units sold, which were explained by SRMP staff to be due to the fact that after a period of insufficient NIST inventory there were back-orders to be filled in the year in which the particular SRMs was renewed. The dummy variable *LoSt* accounts for low stock during years prior to an SRM renewal that prevented all orders from being filled.

An additional dummy variable *STRUC* was initially added to the list of variables, in an attempt to account for structural or legislative changes, such as the decline of the steel industry, or for new legislation mandating the use of certain SRMs, but the coefficients for *STRUC* were not significant and did not contribute to improving the fit of the model. Because the available information describing these structural changes is largely anecdotal and based on hunches by the SRMP staff, this dummy variable was omitted from the final analysis. More research needs to be done to properly match SRMs, time periods, and types of possible structural changes. In an attempt to take these structural effects into account in some way, we use instead for some SRMs (122H Cast Iron Car Wheel and 53E Bearing Metal, Lead Base) specific producer price indexes, (e.g., PPI for iron and steel, PPI for nonferrous metals) rather than the general PPI for intermediate goods.

[17] df = number of observations minus parameters to be estimated; the statistical "goodness of fit" is calculated from the remaining degrees of freedom.

4.3 Model Versions

Eight different versions of the simple derived demand function for NIST SRMs are estimated, using annual sales and price data, combined with non-price data representing economic conditions economy-wide or in the manufacturing or service sectors utilizing SRMs.

The most plausible version of the basic linear regression model uses as the dependent variable the number of units sold in the U.S. (*UNIDOM*), and as non-price explanatory variables the *BCon* variables. In another set of regression runs, the *BCon* variables are replaced by *GPDI* as the non-price explanatory variable. Further, because of the data limitations with respect to SRM sales and price figures, we also estimate the regressions with the total number of units sold (*UNITOT*), including exports, as the dependent variable, to gain additional information. We also tested the basic linear model in its logarithmic, that is, constant-elasticity, version.

To summarize, the following versions of the basic demand function are estimated:

Version	**No.**
Linear regression on the number of units sold in the U.S. market (*UNIDOM*), with *BCons* as non-price explanatory variables	V1
Logarithmic version of the above regression.	V2
Linear regression on the number of units sold in the U.S. market (*UNIDOM*), with *GPDI* as the only non-price explanatory variable (except for dummy variables in some cases)	V3
Logarithmic version of the above regression.	V4
Linear regression on the total number of units sold (including exports) (*UNITOT*), with *BCons* as non-price explanatory variables	V5
Logarithmic version of the above regression.	V6
Linear regression on the total number of units sold (including exports) (*UNITOT*), with *GPDI* as the only non-price explanatory variable (except for dummy variables in some cases)	V7
Logarithmic version of the above regression.	V8

All versions were also tested with a *TIME* variable to check whether SRM sales show a trend—an average increase or decrease over time—not explained by the other independent variables of the model. The *TIME* coefficients turned out to be not statistically significant, and so we have omitted the model versions including the trend variable from the discussion of the regression results.

4.4 Discussion of Regression Results

4.4.1 The Impact of Price Changes on the Demand for SRMs

Table 4-1 lists the price coefficients of demand for each of the 24 SRM equations, for all versions of the basic regression model. (See table A-2 in the appendix for detailed statistics.) After comparing the estimates obtained with the model versions using the *BCon* variables with those using the *GPDI* variable as the non-price variable measuring the influence of changes in the economy on SRM sales, we conclude that the results are similar enough to allow us to choose the *GPDI* version as the operational model. This implies that for the purposes of our model, economy-wide fluctuations in gross private domestic investment can be considered as good an indicator of non-price economic impacts on the demand for SRMs as the SIC-specific industry output represented by the *BCon* variables. The variable *UNITOT* for total SRM sales (including exports) showed the expected correspondence with domestic sales but are of no further interest for our analysis. Our discussion focuses therefore on versions V3 and V4, which use domestic sales (*UNIDOM*, *LUNIDOM*) as the independent variable and *GPDI* (and *lnGPDI*) as the proxies for economic conditions.

Table 4-1 shows that most of the relative price coefficients are negative. There are some positive price coefficients which are, however, statistically not significant (except those for 637 Portland Cement).[18] For 11 of the 24 SRMs, the negative price coefficients are statistically significant at either the 0.05 or 0.10 (two-tail test) level. In spite of the limited number of observations, which makes the results somewhat tentative, the analysis indicates that the quantity demanded of each SRM is indeed negatively related to a change in its own price.

4.4.2 The Price Elasticity of Demand for SRMs

Because we want to evaluate the effect on revenue of a change in price, we need to look at the price elasticity of demand. We would like to know just how sensitive the quantity demanded of any one of those SRMs is to a change in its price. It is the differences in the

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[18] The coefficients with positive signs were mostly in the environmental and health categories. Another variable may be needed to account for legislative changes that mandate the use of these SRMs. 635 and 637 Portland Cements also showed positive coefficients. Since the SRMP has phased in new cement SRMs in recent years, the quantity demanded of cement SRMs may not be adequately represented by SRMs 635 and 637.

Table 4-1. Coefficients of Relative Price (RPrice, LnRPrice) Estimated by Linear and Logarithmic Regression

	DEPENDENT VARIABLES	U.S DEMAND (UNIDOM)				TOTAL DEMAND (UNITOT)			
	INDEPENDENT VARIABLES	RPrice BCon	LnRPriceL nBcon	RPrice GPDI	LnRPrice LnGPDI	Price BCon	LnRPrice LnBCon	RPrice GPDI	LnPrice LnGPDI
	MODEL VERSIONS	V1	V2	V3	V4	V5	V6	V7	V8
METALS	122 H Cast Iron Car Wheel	-3.87	-1.04	-11.90**	-2.98**	-8.93	-1.83	-19.23**	-2.73**
	1261 A LA Steel AISI 4340	-4.68**	-2.37**	-3.87**	-1.70**	-5.82**	-2.36**	-4.92**	-1.72**
	53 E Bearing Metal, Lead Base	-1.47**	-5.16**	-1.40**	-1.90**	-1.78**	-4.09**	-1.57**	-3.47**
	627 Zinc Base C	-0.25	-0.78	-0.05	-0.22	-0.69	-1.43	0.02	-0.27
NonMET	635 Portland Cement, Blue´	0.83	0.30	1.31	0.40	1.01	0.23	1.29	0.25
	637 Portland Cement, Pink	2.16**	0.80**	1.75*	0.58	3.21**	0.78**	2.68**	0.64*
	120 BC Phosphate Rock	-0.74**	-1.17**	-0.57*	-0.88*	-1.14**	-1.10**	-0.94**	-0.98**
	27 F Iron Ore, Sibley Powder	0.39	1.00	-0.61**	-1.25**	1.60	1.58	-1.37**	-1.45**
Ch	370 E Zinc Oxide Rubber Composite	-1.11*	-1.32*	-0.96	-1.20	-4.08**	-2.82**	-2.67	-2.10
Rb	371 GH Sulfur Rubber Composite	0.20	0.11	-0.27	-0.20	-0.58	-0.42	-0.39	-0.32
Pl	40 H Sodium Oxalate Redox	-1.27*	-0.67	-1.02**	-0.55**	-1.25*	-0.58	-1.04**	-0.50**
ENGIN	185 EFG Pot. Hydrogen Phthalate pH	-0.01	-0.13	-0.44	-0.30	-0.98	-0.44	-0.68	-0.31
	187 BC Borax pH	-1.50	-0.79	-1.21*	-0.78*	-3.11	-1.25	-1.90*	-0.87
	189 A Potassium Tetroxalate pH	0.77	1.02	-0.26	-0.25	0.97	1.10	-0.30	-0.25
	39 I Benzoic Acid Combustion	-6.42**	-1.78**	-3.28**	-0.83*	-9.04**	-1.55**	-4.55	-0.72
									continued

	Table 4-1 continued	V1	V2	V3	V4	V5	V6	V7	V8
ENVIRON	1575 Pine Needles	-0.58	-1.13	0.23	0.20	-0.45	-0.30	0.58	0.53
	1620 &AB Sulfur in Fuel Oil, 5%	-0.18	-0.38	0.04	-0.09	-0.06	-0.13	0.21	0.10
	1622 ABC Sulfur in Fuel Oil, 2%	0.81	0.15	0.55	0.21	0.97	0.21	0.82	0.29
	1625 Permeation Tube, 10 cm	-0.36	-0.96	-0.67**	-2.10**	-0.41	-0.82	-0.76**	-2.01**
	1635 Trace Elements in Coal	-0.29	-0.08	-0.67*	-2.67*	-0.68	-0.92	-1.37**	-2.99**
HLTH	911 AB Cholesterol	-0.75	-0.61	-0.31	-0.40	-0.39	-0.10	-0.22	-0.13
	1577 &AB Bovine Liver	-0.87	-0.27	0.49	0.73	1.58	1.43	1.50	1.40
SC Mt	935 A Potassium Dichromate-UV	0.15	-0.23	0.23	0.16	0.30	-0.52	0.22	0.13
	1361A CU & CR Coating on Steel	-0.58**	-1.06**	-0.69*	-1.29**	-0.61**	-1.02**	-0.70*	-1.17**

Coefficients stated are for the independent variables RPrice (Relative Price) and LnRPrice (natural log of Relative Price).
The SRM groups are: Metals, Nonmetals, Chemicals/Rubber/Plastics, Engineering, Environmental, Health, Science/Metrology.
Bcons (LnBCons): Value of product shipments (in million $) as proxies for business conditions in SRM user sector.
GPDI (LnGPDI): U.S. Gross Private Domestic Investment (in million $).
** statistically significant at the 0.05 level (two-tail test)
* statistically significant at the 0.10 level (two-tail test).

price elasticities that will allow the SRMP to "trade off" relatively higher and lower price increases among SRMs. In taking advantage of these differences in elasticities, the public-sector pricing rule determines optimally differentiated prices that cover the cost of the program as a whole while minimizing the deadweight losses that result from deviating from marginal-cost pricing.

The price elasticity of demand, E_p, that is, the percentage change in units of SRMs demanded, m, for a given (small) percentage change in price, p, is defined as follows:

$$E_p = \frac{\frac{\Delta m}{m}}{\frac{\Delta p}{p}} = \frac{\Delta m}{\Delta p}\frac{p}{m} \tag{4.9}$$

In the logarithmic models the coefficient of the relative price variable measures the price elasticity of demand directly. In linear models the price elasticity can been calculated by multiplying the price coefficient by the ratio of the mean relative price to the mean number of units sold.

The estimated price elasticity, E_p, is expected to be always negative because of the inverse relationship between quantity and price implied by the "law of demand," and represented by the downward sloping demand curve. However, traditionally the negative sign is omitted and the price elasticity is expressed as an absolute value. The price elasticities of demand are to be interpreted as follows:

If	E_p	=	0, the demand is perfectly inelastic,
	E_p	=	1, the demand has unitary elasticity,
	E_p	=	∞, the demand is perfectly elastic,
	$0<E_p<1$,		we say the demand is inelastic,
	$1<E_p<\infty$,		we say the demand is elastic.

The implication for policy decisions meant to raise revenue is that

(1) if the demand is inelastic ($E_p < 1$), an increase in price leads to an increase in total revenue, and a decrease in price leads to a fall in total revenue;

(2) if the demand is elastic ($E_p > 1$), an increase in price leads to a decrease in total revenue, and a decrease in price leads to an increase in total revenue;

(3) if the demand has unitary elasticity ($E_p = 1$), total revenue is not affected by changes in price.

The price elasticities for the quantity demanded of all 24 SRMs, for both the linear and logarithmic versions of the models, are listed in table A-2 of appendix A.

As expected, model V4, the logarithmic version using *GPDI,* resulted in the "best" fit. It represents constant-elasticity demand curves. For the implementation of the public-sector pricing model it is very useful to be able to assume constant price elasticities because it greatly facilitates the computation of optimal prices and quantities (as will be described in chapter 5).

In the implementation of the pricing model we calculate optimal prices and quantities for only the group of SRMs whose relative price coefficients are statistically significant at either the 0.05 or 0.10 level (two-tail test). Table 4-2 lists the demand equations for these 11 SRMs.[19] According to these estimates, SRMs 120 B&C, 40 H, 187 B&C and 39 I, for which $E_p < 1$, have price-inelastic demand, whereas SRMs 122 H, 1261 A, 53 E, 27 F, 1625, 1635, and 1361, for which $E_p > 1$, have price-elastic demand.

4.4.3 Effects of Other Factors

For most of the SRMs the coefficients of the *BCon* variables and the *GPDI* variable have the expected positive signs, that is, SRM sales and business activity move in the same direction. An increase in economic activity (as expressed, for example, as a higher value of product shipments or private investment) tends to increase SRM sales; a decrease has the opposite effect.

The dummy variables to adjust for backorders (*BKOR*) and low stock (*LoST*) had the expected signs in every case and were statistically significant.

All versions of the demand model were tested for autocorrelation of residuals. The test showed that there is no reason to believe that the disturbance term relating to one observation is influenced by the disturbance relating to any other observation. In other words, the test for autocorrelation showed that if sales were lower in a certain year, there was no reason to expect them to be lower in the following year.

4.5 Implications of Regression Results for the SRM Program

The estimated demand functions indicate that changes in the prices of SRMs do influence the quantity demanded for SRMs 120 B&C, 40 H, 187 B&C and 39 I, for which $E_p < 1$, a price increase would increase revenues while reducing sales by a relatively smaller percentage. For SRMs 122 H, 1261 A, 53 E, 27 F, 1625, 1635, and 1361, for which $E_p > 1$, a price decrease would raise revenues by a greater percentage than the percentage decrease in price.

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[19]The results and diagnostic statistics of the regression estimates for these 11 SRMs are listed in tables A-4a to A-4k in Appendix A.

Table 4-2. Estimated Demand for Selected SRMs

1261 A LA Steel AISI 4340	$\ln D_1 = 0.894 - 1.699 \ln P_2 + 0.941 \ln GPDI$
187 B&C Borax pH	$\ln D_2 = -2.277 - 0.781 \ln P_7 + 0.765 \ln GPDI$
1625 SO_2 Permeation Tube, 10 cm	$\ln D_3 = 37.772 - 2.096 \ln P_9 - 1.734 \ln GPDI$
40H Sodium Oxalate Redox	$\ln D_4 = 8.376 - 0.553 \ln P_6 - 0.086 \ln GPDI$
27 F Iron Ore, Sibley Powder	$\ln D_5 = 4.913 - 1.251 \ln P_5 + 0.309 \ln GPDI$
39I Benzoic Acid Combustion	$\ln D_6 = -2.793 - 0.833 \ln P_8 + 0.888 \ln GPDI$
120 B&C Phosphate Rock	$\ln D_7 = 9.902 - 0.876 \ln P_4 - 0.151 \ln GPDI - 1.144\ LoSt$
1361A CU & CR Coatings	$\ln D_8 = 0.578 - 1.287 \ln P_{11} + 0.784 \ln GPDI - 2.940\ LoSt$
1635 Trace Elements in Coal	$\ln D_9 = 52.978 - 2.668 \ln P_{10} - 2.799 \ln GPDI + 0.816\ BKOR$
122 H Cast Iron Car Wheel	$\ln D_{10} = 5.751 - 2.978 \ln P_1 + 0.938 \ln GPDI$
53 E Bearing Metal, Lead Base	$\ln D_{11} = 28.384 - 1.902 \ln P_3 - 1.311 \ln GPDI$

**Relative price coefficients significant at 0.05 level (two-tail test)
* Relative price coefficients significant at 0.10 level (two-tail test)

An analogous interpretation is that the less price elastic a good, the easier it is to increase its price in order to meet a prescribed budgetary goal. On the other hand, if the demand is comparatively price-elastic, the customer will leave the market and switch to secondary SRMs or alternative methods of quality control if the NIST SRM price is increased. Therefore, the SRMP should refrain from price increases for those SRMs that are very price-elastic, if the goal is to keep those customers. If, on the other hand, higher price elasticities are taken as a sign that other means of quality control or secondary standards can replace NIST's primary SRMs relatively more easily than the SRMs that show inelastic demand, then NIST may want to focus its technological efforts on primarily recertifying and renewing those latter, inelastic, SRMs.

The evidence that the impact of price changes on the quantity demanded varies among SRMs can be used advantageously when determining a pricing formula for the SRMP as a whole. As we have seen from the discussion in chapter 3, Boiteux-type pricing models for publicly supplied private goods can make use of the differences in price elasticities of demand to calculate optimal deviations from each SRM's marginal cost to produce a pricing formula that covers the cost of the program.

5. OPTIMAL PRICES FOR NIST STANDARD REFERENCE MATERIALS

5.1 Introduction

The historical background of the pricing policy and the rationale for decreasing-cost production of the NIST Standard Reference Materials Program (SRMP) have been presented in chapter 2. Because an analysis of the pricing policy of a public enterprise of the importance of the National Institute of Standards and Technology requires a careful examination of the empirical framework, we have spent a great deal of time and effort estimating demand functions for a number of SRMs. The demand analyses are described in chapter 4. We will take a closer look at the empirical cost functions later in this chapter. The theoretical principles that apply to the solution of the NIST SRMP pricing problem have been described in detail in chapter 3. By combining the theoretical and empirical findings we want to test the following two propositions:

(1) If prices have to deviate from marginal costs to avoid a deficit, there is a welfare gain to be obtained from charging Ramsey prices rather than average-cost prices.

(2) The quasi-optimal prices[20] provided by the Ramsey pricing rule lead to higher, more nearly optimal output levels when compared with the actual, average-cost pricing policy practiced by the SRMP now.

The issues raised by these two hypotheses are important ones for public-sector enterprises. Public enterprises ought to meet demand and maximize welfare. These goals are not always clearly defined in the policy statements that managers of public enterprises have to follow. Managers often prefer more tangible indicators such as maximization of output or of revenues. Also, since welfare maximization usually implies lower prices, they are often faced with the problem of avoiding deficits. The application of the public-sector pricing model in this chapter is intended to show that, at least in some cases, the Ramsey pricing rule may represent a fairly practical way of determining prices and output levels that cover costs and maximize social welfare. The Ramsey results could be valid even if one-shot policy changes aimed at welfare maximization for the whole agency are impractical. Stepwise welfare improvements could be achieved by using the price elasticities and the Ramsey rule to determine optimal price-cost margins in a piecemeal fashion.

In the implementation of the public-sector pricing model we intend to show how, in order to be optimal in the second-best sense, SRM prices need to differ from marginal-cost prices or from the average-cost prices charged by the SRMP now. We demonstrate the

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[20]In this chapter we use the terms "Ramsey prices" and "quasi-optimal prices" interchangeably when we mean prices that are optimal when the public enterprise is faced with an exogenously given revenue-cost constraint. We will refer to "optimal" prices when we mean marginal-cost prices that are optimal in the "first-best" sense.

empirical estimation by first calculating the quasi-optimal prices and their deviations from marginal cost for two SRMs for the year 1982. Subsequently, we extend the estimation procedure to a group of 11 SRMs for which the demand functions in chapter 4 showed statistically significant price elasticities of demand. We compute the quasi-optimal prices for these SRMs for the years from 1978 to 1992. In each case, we also compute the deadweight losses associated with the deviations from marginal cost to examine whether a welfare gain can be obtained by shifting from average-cost pricing to Ramsey pricing. To get an idea whether in the limit the pricing formula behaves as expected, we also calculate, for the year 1982, Ramsey prices with the price elasticities varying by plus or minus one standard deviation. Further, we examine the implications for prices, production plans, and deadweight loss, if the SRMP focused on supplying only SRMs that are price-inelastic. The results of the empirical model with respect to their policy implications will be discussed in chapter 6.

5.1.1 Special Assumptions Regarding SRMs as Intermediate Goods

Since we are interested in testing the model by determining prices for SRMs, which are publicly supplied intermediate goods, we look at optima of a general Boiteux model that have the properties discussed in section 3.5 of chapter 3. We summarize these properties as follows:

The public agency produces outputs $z_k > 0$, $k \in K$, where K is a subset of all goods. The public firm sets the prices p_k of its outputs. All goods k are intermediate goods, hence we have $y_k < 0$ and $x_k = 0$. That is, the goods are inputs of the private firms and are not purchased by consumers. We do not deal with corner solutions where some good switches from being an intermediate good to being also a consumption good. Hence we only consider optima of our general model where $\partial x_k^h/\partial p_k = 0$. We also have to bear in mind that the consumer budget constraints and utility functions are independent of p_k and of x_k^h. This means that the price p_k does not influence consumer decisions, at least not directly. So we also have $\partial x_i^h/\partial p_k = 0$ for $i \neq k$. More generally, indirect consumer utility and also welfare do not directly depend on prices p_k. At the optimum, of course, prices p_k will have an influence on utility and welfare because they influence the production plans and hence the consumption possibilities of all other goods.

Some special assumptions are also necessary with respect to labor inputs. We assume that the public firm pays higher wages than the private firms. This has been shown in many empirical studies. Moreover, it seems plausible in our model where the public firm maximizes welfare and where the wage rate p_0 is the only instrument of the public firm that directly influences welfare. If the public wage is higher than the private wage, everyone would like to work in the public firm. Hence we assume that the public firm chooses the optimal employment z_0 and rations demand so that $z_0 = x_0$. The remaining labor supply goes to the private sector where an equilibrium of $y_1 = x_1$ is achieved. We are not interested in modelling rationing schemes,[21] so we assume that the employees of the public firm are picked at random from total labor supply. It is further assumed that private and public labor supply share the available total amount of labor L, so that $x_0 + x_1 = L$ and $z_0 = L - y_1$.

The above assumptions and the following circumstances that characterize the public supply of SRMs eliminate some of the restrictions of the extended Boiteux model so that SRM prices depend only on the price elasticities of demand and on the budget constraint as required by the Ramsey rule.

(1) NIST SRMs are intermediate goods that are sold only as inputs to firms and not as final goods to consumers. We therefore do not need to deal with changes in consumer demand directly due to changes in SRM prices ($\partial x_k^h/\partial p_k = 0$).

(2) SRMs are intermediate goods and, in most cases, represent a small proportion of inputs into final goods. For this reason, the effects of SRM price changes on income redistribution are assumed to be insignificant. The Feldstein distributional term in equation (3.34) thus vanishes from the model.

(3) We assume that the firms buying SRMs operate in general under conditions of perfect competition, or, at least, that the extent of the price-cost margins ($p_i > c_i$) for the output of private monopolistic firms (of both consumer goods and intermediate goods) is not influenced by the pricing of SRMs. This assumption eliminates the last term in equation (3.35) which adjusts public prices for monopolistic pricing in the private sector.

(4) Because SRMs do constitute a small percentage of the inputs into the production of private goods, private monopolistic firms do not in any significant way pass on price changes to consumers ($\partial x_i^h/\partial p_k = 0$ for $i \neq k$). Hence the indirect effect of a change in the price of an SRM on consumer welfare can be ignored and the term INT_k in equation (3.35) reduces to $-\gamma z_k$.

(5) The SRMP is so small that its prices do not influence the labor demand of the private sector. So we need not include in the calculation of SRM prices the marginal conditions that model the response of private labor demand to SRM prices.

(6) Finally, in the case of SRMs, a further simplifying assumption can be made. NIST SRMs are not substitutes for each other, that is, a customer will not substitute, for example, a rubber SRM for a cement SRM because the price of the cement SRM has increased. Thus the cross elasticities of demand are zero.

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[21]For modelling labor rationing schemes see Drèze (1984), Bös (1994).

Taking into account assumptions (1) to (6), we are left with the inverse-elasticity rule of equation (3.24):

$$\sum_{i \in k} \frac{(p_i - c_i)}{p_i} \frac{\partial \hat{z}_k}{\partial p_i} \frac{p_i}{\hat{z}_k} = -\gamma; \qquad k \in K, \tag{3.24}$$

where $(p_i - c_i)/p_i$ is the percentage deviation from marginal cost for SRM i, $(\partial \hat{z}_k/\partial p_i)(p_i/\hat{z}_k)$[22] is the compensated price elasticity of demand for the intermediate good, and γ is a measure of the welfare effect of the size of the deviation from marginal cost needed to meet the budget constraint.

Each estimation is thus based on a system of equations—one equation for each of n SRMs and for the exogenously given revenue-cost constraint—to be solved for the prices of the n SRMs and γ, the Lagrangean parameter.

$$\sum_{k \in K} \frac{(p_k - c_k)}{p_k} \eta_{kk} = -\gamma; \qquad k = 1,\ldots,n \tag{5.1}$$

$$\sum_{k=1}^{n} p_k \hat{z}_k = \Pi^0 \tag{5.2}$$

where

p_k	=	the price of SRM, ($k = 1,\ldots,n$),[23]
c_k	=	the marginal cost of SRM k,
η_{kk}	=	the own price elasticity of demand of SRM k,
$\hat{z}_k$	=	the estimated quantity demanded of SRM k,
Π^0	=	the value of the revenue-cost constraint, expressed as the sum of the differences between revenue and cost for each good, and
γ	=	a measure of the welfare effect of the size of the deviation from marginal cost needed to meet the budget constraint.

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[22]Because of the assumption of perfect competition and because substitution effects are symmetric, $\partial \hat{z}_k/\partial p_i = \partial \hat{z}_i/\partial p_k$.

[23]Since the prices of all SRMs under investigation are controlled by the SRMP, the subscript i is no longer needed.

The way the budget constraint is formulated influences the size of the program. If the budget constraint allows a deficit ($\Pi^0 < 0$), more SRMs are sold at lower prices. If the budget constraint requires a profit ($\Pi^0 > 0$), prices need to be higher and sales necessarily decrease. Usually government agencies are confronted with a breakeven constraint ($\Pi^0 = 0$). The SRMP, for example, is not required to make a profit but rather to cover the cost of the program. In this case the size of the program is conditioned upon the size of the deviation from marginal cost, determined by prices and outputs, and perhaps other, exogenously given, factors.

The proportionality factor γ, calculated from the Lagrangean parameter of the budget constraint, is a measure of the welfare effect of the size of the deviation from marginal cost. It depends on all the variables and functions of the model. $\gamma = 0$ if marginal-cost pricing is applied (since marginal-cost pricing is welfare-maximizing); $\gamma = 1$, if the entire monopoly profit is exploited. γ is between zero and one when the budget constraint is a breakeven constraint and prices have to be higher than marginal-cost prices, as is the case for decreasing-cost production. The factor γ is calculated by the model to inflate the price elasticities of demand of all goods by $1/\gamma$, the proportion required to cover costs. In other words, prices necessarily exceed marginal cost but only to the extent needed to cover the deficit, thus minimizing the loss of welfare that results from these unavoidable deviations.

5.1.2 The Demand Functions

The derivation of the demand functions for 24 SRMs is described in detail in chapter 4. The natural log version of the constant-elasticity demand function best represents the demand for SRMs over the 15-year time period. The natural log version has the added advantage that the estimated price coefficients can be used directly as the price elasticities of demand. Further, because the price elasticities are constant, we can assume that the quasi-optimal quantities calculated by the model follow the same demand functions. We also assume that the functions go through the point p_0, q_0 (in figure 4-1), representing the actual SRMP prices and quantities, so that the SRMP prices and quantities which we compare with the estimated Ramsey prices and quantities, are always on the estimated demand curve.[24]

In the empirical implementation of the Ramsey pricing rule, we calculate Ramsey prices for the 11 SRMs for which the price elasticities of demand turned out to be statistically significant at either the 5 percent or 10 percent level of confidence (two-tailed). The demand equations for these 11 SRMs are listed in table 5-1 (copied from table 4-2 in chapter 4). It is the fact that some of the SRMs have price-elastic and some have

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[24]We are hereby assuming that the SRMP knows the demand functions for the SRMs and can therefore accurately determine their average cost prices. In actuality, this is not the case. The SRMP uses the number of units sold in the previous year to approximate average cost prices for the coming year.

Table 5-1. Estimated Demand for Selected SRMs[25]

1261 A LA Steel AISI 4340	$\ln D_1 = 0.894 - 1.699 \ln P_2 + 0.941 \ln GPDI$
187 B&C Borax pH	$\ln D_2 = -2.277 - 0.781 \ln P_7 + 0.765 \ln GPDI$
1625 SO_2 Permeation Tube, 10 cm	$\ln D_3 = 37.772 - 2.096 \ln P_9 - 1.734 \ln GPDI$
40 H Sodium Oxalate Redox	$\ln D_4 = 8.376 - 0.553 \ln P_6 - 0.086 \ln GPDI$
27 F Iron Ore, Sibley Powder	$\ln D_5 = 4.913 - 1.251 \ln P_5 + 0.309 \ln GPDI$
39I Benzoic Acid Combustion	$\ln D_6 = -2.793 - 0.833 \ln P_8 + 0.888 \ln GPDI$
120 B&C Phosphate Rock	$\ln D_7 = 9.902 - 0.876 \ln P_4 - 0.151 \ln GPDI - 1.144\ LoST$
1361 A CU & CR Coatings	$\ln D_8 = 0.578 - 1.287 \ln P_{11} + 0.784 \ln GPDI - 2.940\ LoSt$
1635 Trace Elements in Coal	$\ln D_9 = 52.978 - 2.668 \ln P_{10} - 2.799 \ln GPDI + 0.816\ BKOR$
122 H Cast Iron Car Wheel	$\ln D_{10} = 5.751 - 2.978 \ln P_1 + 0.938 \ln GPDI$
53 E Bearing Metal, Lead Base	$\ln D_{11} = 28.384 - 1.902 \ln P_3 - 1.311 \ln GPDI$

price-inelastic demand that offers an opportunity to "trade off" relatively higher and lower price increases in a way that will cover costs and minimize the reduction in the quantity demanded by users of SRMs.

Since we have defined the problem in terms of setting quasi-optimal prices rather than quasi-optimal outputs, we will use these demand functions to estimate the quantities demanded that correspond to those quasi-optimal prices.

5.1.3 The Empirical Cost Functions

The derivation of cost functions for the SRMs selected for the analysis was not a straightforward matter. The SRMP produces nearly 2000 SRMs whose production and sale necessarily involve common fixed costs. In theory the Ramsey formula could calculate the quasi-optimal prices for all SRMs in the program, subject to a breakeven constraint covering the entire program. Fixed costs in this case would not have to be assigned to individual SRMs but could simply be taken from the account of total expenditures of the SRMP as listed in table 5-2. In our illustration of the model, however, we are calculating quasi-optimal prices for a subgroup of SRMs and so we need to apportion to the budget constraint each SRM's fixed-cost contribution.

[25]The statistics for each of these equations are listed in tables A-3a to A-3k in the appendix.

Table 5-2. Recoverable SRMP Expenditures in 1990

Types of Costs	Costs in Thousands of 1990-$	Costs in Thousands of Constant 1982-$
Fixed Costs		
Development	$741	$615
Operations: Project Mgt.	$2,173 (75% of Operations)	$1,804
Overhead	$2	$1.66
Misc. Other	$163	$135
Total Fixed Costs	**$3,604**	**$2,991**
Variable Costs		
Production	$3, 892	$$3,230
Operations: Sales & Distr.	$724 (25% of Operations)	$601
Total Variable Costs	**$4,616**	**$3,831**
Total Costs	**$8,220**	**$6,823**

Baumol (1978) discusses the difficulty, for a multi-product, decreasing-cost firm, of dividing total costs into average fixed and variable costs and imputing them in a unique manner to one or the other of the products. The reasons relevant to the SRM situation are the following:

(1) In a multi-product firm it is generally not possible to define the average cost corresponding either to the firm's output as a whole or to any of the individual products because products are not measured in common units.

(2) It is generally not possible to calculate separate average costs for each product because some of the costs are attributable directly to particular goods and some are incurred in common for many or all outputs simultaneously.

(3) Prices cannot be used to determine costs because they themselves are endogenous variables of the problem, declining monotonically with increasing output levels.

Baumol concludes that only by using arbitrary accounting conventions is it possible to assign average costs to individual products, and this is exactly what the SRMP does. We have adopted the SRMP accounting conventions, combined with some simplifying assumptions, to construct cost functions for the selected SRMs.

The SRMP categorizes as (recoverable) fixed costs of the program all costs that are not production costs, except for the portion of operating costs that is associated directly with the sale and distribution of individual SRMs.[26] Fixed costs consist of development costs to produce prototypes of SRMs, most of the operating costs that have to do with project management, an overhead charge, and miscellaneous other expenses paid to other agencies and NIST divisions for technical support. It needs to be pointed out that prototype development costs are assigned annually to each SRM as a uniform surcharge. In other words, these fixed costs are not charged to an SRM as a lump sum at the time they are incurred but rather divided evenly over time and units. Thus these "shares" are included in each year's budget as recoverable amounts.

Variable costs include production costs and the part of operations costs that are sales-related costs for advertising, shipping, and billing. Production costs comprise the cost of labor, raw materials, inventory and obsolescence, and special equipment for processing a prototype into a certified SRM.

To determine the total average costs (= price) of an SRM, the SRMP divides total fixed costs by the number of SRMs it expects to sell during the coming fiscal year. It can do so because all SRMs are measured in "units." (The fixed costs to be recovered are thus assigned to each SRM equally, irrespectively of the actual fixed cost incurred for the particular SRM). To this average fixed cost is added the variable cost of the SRM. The variable cost is calculated by dividing the total production cost of a batch of SRMs by the number of units in the batch. The size of the batch is determined by technical considerations rather than by estimated demand, but once a batch is produced, the variable cost remains constant until the batch is used up (which for most SRMs takes several years). In any given year it is these per unit fixed costs and variable costs multiplied by the number of units sold from a particular batch that constitute the cost to be recovered for each SRM.

For the SRMs we selected for our analysis we calculated the fixed costs by estimating the demand for a specific year using the demand equations listed in table 5-1. To this estimated number of units we then assigned the corresponding fixed cost according to the formula used by the SRMP. The resulting fixed cost for each SRM, summed over the SRMs in the group, then constitutes the total fixed cost to be recovered in a particular year for the group of SRMs selected for analysis. In the computation of the Ramsey prices and quantities that follow we consider these costs fixed in each year and thus independent of the number of units calculated by the Ramsey formula.

On the basis of the SRMP accounting conventions and the assumptions just described, the cost functions of our analysis can thus be expressed as linear functions, additively separable between the 11 SRMs:

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[26] After conversations with several staff members of the SRMP, we used an estimate of 25 percent for the variable cost portion of operating costs.

$$C(\hat{z}_k) = C \bullet z_k + \frac{\partial C}{\partial z_k} \bullet \hat{z}_k \quad k=1,...,11, \tag{5.3}$$

where

$\hat{z}_k$	=	the quasi-optimal quantity estimated
z_k	=	the estimate of the actual quantity, used to compute fixed costs, and
$\partial C/\partial z_k$	=	constant marginal costs, varying from year to year.

The process of combining cost and demand functions to estimate Ramsey prices will be illustrated with two SRMs in the next section.[27]

5.2 Estimation of Ramsey Prices for two SRMs

5.2.1 System of Equations

We first test the model by calculating welfare-maximizing prices and quantities for two SRMs for the year 1982. 1982 is the last year in which SRMP costs for research and development of prototypes were covered by tax appropriations and also the year before a 25-percent across-the board price increase demonstrated that price increases that disregard price elasticities of demand may not produce the desired cost-covering revenue.

For two SRMs the system of equations, including a breakeven budget constraint, to be solved for p_1, p_2, and γ, is as follows:

$$\sum_{k=1}^{2} \frac{(p_k - c_k)}{p_k} \eta_{kk} = -\gamma \tag{5.4}$$

$$\Pi^0 = \sum_{k=1}^{2} p_k \hat{z}_k - \sum_{k=1}^{2} (FC_k + VC_k)) = 0 \tag{5.5}$$

[27] Since the technology of the SRMs is clearly convex up to well-defined fixed costs, we do not further explore the restrictions imposed by Dierker (see chapter 3, section 3.3) on Ramsey pricing with respect to the existence and optimality of equilibria.

where

c_k	=	marginal cost of SRM k, where $k = 1, ...2$,
η_{kk}	=	the own price elasticity of demand of SRM k,
$p_k\hat{z}_k$	=	the revenue produced by SRM k,
FC_k	=	fixed costs associated with the supply of SRM k,
VC_k	=	$c_k\hat{z}_k$ = variable costs associated with the supply of SRM k,
$\hat{z}_k$	=	$\beta_1 . p_k^{\beta_2} . GPDI^{\beta_3}$, according to the demand function described for each of the SRMs in chapter 4, and
γ	=	measure of the welfare effect of the size of the deficit to be covered.

5.2.2 Fixed and Variable Costs

5.2.2.1 SRM 1261A LA Steel AISI 4340:

One of the SRMs that the SRMP selected for analysis is SRM 1261A Low Alloy Steel AISI. This standard reference material provides analytical values for elements. It is furnished in solid 150-gram units or as chips. The principal consideration in issuing this SRM, as all SRMs, is to provide a homogeneous material so that an investigator in one laboratory can be assured that the material used is the same as that being investigated in another laboratory.

Sales of this SRM have decreased from between 300 and 400 units in the late seventies to less than 100 units in the early nineties. A benchmark price that takes into account the price elasticity of demand would help the SRMP to set a price that minimizes the decrease in quantity demanded and the ensuing loss of revenue.

Fixed and Variable Costs of SRM 1261A: We estimate the fixed cost of supplying SRM 1261A on the basis of the predicted quantity demanded for 1982. Applying the 1982 SRMP price and FY GPDI to the natural log version of the demand equation in chapter 4, and listed in table 5-1, we get the following estimate of quantity demanded, z_1, for 1982:

$$\ln z_1 = 0.89 - 1.70 \ln 94.00 + 0.94 \ln 474{,}571$$
$$z_1 = 238 \text{ units}$$

where subscript 1 refers to SRM 1261A.

We then use this estimate of quantity demanded to derive the fixed costs of SRM 1261 for the year 1982 according to the 1990 SRMP accounting formula (adjusted by the 1982 PPI for Intermediate Goods). The 1982 fixed costs for operations, overhead, and miscellaneous other costs amount to \$36 per unit of SRM 1261A and hence to \$8,568 for 238 units.

The variable cost for SRM 1261A was estimated to be $58 per unit based on the production cost of processing a prototype into a certified SRM and on the costs of selling and distributing it. The variable cost is used as an approximation to marginal cost in the computation of the Ramsey prices.

The resulting revenue-cost equation for SRM 1261A for the year 1982 is thus

$$R - C = p_1 \cdot \hat{z}_1 - 8568 - 58 \cdot \hat{z}_1,$$

where $\hat{z}_1$ is the quasi-optimal number of units of SRM 1261A corresponding to quasi-optimal price p_1, the Ramsey price.

5.2.2.2 SRM 187B&C Borax pH (Ion Activity)

This SRM is used to prepare solutions of known hydrogen ion concentrations to calibrate commercial pH instruments. Sales of SRM 187B&C have remained fairly stable, between about 90 and 100 units from 1978 to 1992. It is possible that in this case, prices could be increased by relatively more without losing revenue.

The revenue-cost equation for SRM 187B&C was derived in the same way as that for SRM 1261A, based on the estimated demand for 1982 and the SRMP accounting formula.

Using 1982 prices and GPDI, the quantity demanded of SRM 187B&C is estimated as

$$\ln z_2 = -2.28 - 0.78 \ln 59.00 + 0.76 \ln 474{,}571$$
$$z_2 = 93 \text{ units},$$

where the subscript 2 refers to SRM 187B&C.

Fixed and Variable Costs of SRM 187B&C: The fixed cost for SRM 187B&C in 1982 amounts to $2,790 and the marginal cost to $29. The resulting revenue-cost equation for 1982 is thus

$$R - C = p_2 \cdot \hat{z}_2 - 2790 - 29 \cdot \hat{z}_2.$$

5.2.3 Computation of Ramsey Prices for SRM 1261A and SRM 187B&C

The Ramsey formula simultaneously takes into account the price elasticities of all SRMs and calculates quasi-optimal prices and quantities, given the budget constraint. After rearranging the terms of equations (5.3) and (5.4) we get the following system to be solved for prices and for the Lagrangean parameter γ:

$$p_1 = \frac{c_1 \eta_{11}}{\eta_{11} + \gamma} \tag{5.6}$$

$$p_2 = \frac{c_2 \eta_{22}}{\eta_{22} + \gamma} \tag{5.7}$$

$$\Pi^0 = p_1 \hat{z}_1 + p_2 \hat{z}_2 - FC_1 - FC_2 - c_1 \hat{z}_1 - c_2 \hat{z}_2 = 0 \tag{5.8}$$

Substituting into this system of equations the values for η_{kk}, c_k, FC_k, and the regression equation for estimating $\hat{z}_k$, allows us to solve simultaneously for the cost-covering prices and quantities for SRMs 1261A and 187B&C for 1982:

$$p_1 = \frac{58(-1.70)}{-1.70 + \gamma} \tag{5.9}$$

$$p_2 = \frac{29(-0.78)}{-0.78 + \gamma} \tag{5.10}$$

subject to the breakeven budget constraint:

$$p_1 (0.89 - 1.70 \ln p_1 + 0.94 \ln 474571) + p_2 (-2.28 - 0.78 \ln p_2 + 0.76 \ln 474571)$$

$$- 8568 - 58(0.89 - 1.70 \ln p_1 + 0.94 \ln 474571)$$

$$- 2790 - 29(-2.28 - 0.78 \ln p_2 + 0.76 \ln 474571) = 0 \tag{5.11}$$

The Ramsey prices and quantities of SRMs 1261A and 187B&C derived from the solution of this system of equations are presented in table 5-3 together with the corresponding revenues and costs. Also listed are the quantities that would have been demanded if marginal-cost prices had been charged. The table further lists the average-cost prices actually charged in 1982 by the SRMP and the corresponding quantities of SRMs 1261 and 187B&C.

Table 5-3. Comparison of Prices and Quantities under Different Pricing Formulas[28]

SRM	Pricing Formula	Quantity	Revenue	Average Cost	Total Cost	Deficit
	Ramsey Price					
SRM 1261A SRM 187B&C Total	$81 $77	305 76	$24,794 $5,846 **$30,640**	$86 $58	$26,247 $ 4,393 **$30,640**	-$1,453 +$1,453 **0**
	Marginal-Cost Price					
SRM 1261A SRM 187B&C Total	$58 $29	541 163	$31,378 $4,727 **$36,105**	$74 $47	$39,946 $ 7,517 **$47,463**	-$8,568 -$2,790 **-$11,358**
	Average-Cost Price					
SRM 1261A SRM 187B&C Total	$94 $59	238 93	$22,372 $5,487 **$27,859**	$94 $59	$22,372 $5,487 **$27,859**	0 0 **0**

The Ramsey formula calculated quasi-optimal prices higher than marginal-cost prices, as expected. The Ramsey price is lower than the actual price charged by the SRMP in 1982 for the price-elastic SRM 1261A and higher than the actual price for the price-inelastic SRM 187B&C. The saleable quantities differ inversely. Total revenue under Ramsey pricing is higher than under average-cost pricing. Under marginal-cost pricing, as economic theory predicts, the size of the program would be larger since more SRMs could be sold at the lower price, but the deficit would have to be covered from tax appropriations. The value of γ in this example was calculated as 0.49.

[28]Throughout, the stated empirical estimates were calculated by computer and rounded, and so the summed entries in the tables do not precisely match the shown totals.

5.2.4 The Optimal Deviation from Marginal Cost

Both Ramsey prices and average-cost prices deviate from marginal costs and so are second-best in the Pareto sense. However, given that a breakeven constraint has to be met, the deviations from marginal cost of Ramsey prices are optimal because the loss of welfare is minimized. This is not the case when average-cost pricing is applied.

Returning to equation (5.4) and rearranging terms, we can now calculate the Lerner index, that is, the percentage deviation from marginal cost of the Ramsey and the average-cost prices:

$$\frac{(p_k - c_k)}{p_k} = \frac{-\gamma}{\eta_{kk}} \tag{5.4}$$

For SRM 1261A the welfare-maximizing deviation from marginal cost is 29 percent, as follows:

$$\frac{(81.34 - 58)}{81.34} = \frac{-0.49}{-1.70} = 0.29. \tag{5.12}$$

For SRM 187B&C it is 62 percent:

$$\frac{(77.18 - 29)}{77.18} = \frac{-0.49}{-0.78} = 0.62 \tag{5.13}$$

As expected, the percentage deviation from marginal cost is relatively higher for the price-inelastic SRM and relatively lower for the price-elastic SRM. In the two-product case, the economic interpretation of the ratio is simply that the optimal deviations from marginal cost are equal to the inverse of their price elasticities (0.62/0.29 is approximately equal to 1.70/0.78 = 2.13). In the multi-product case (where γ does not factor out) the price-cost margin for each SRM is required to be *proportionate* to the inverse of its price elasticity, as we shall see later in this chapter.

The deviations from marginal cost for the average-cost prices actually charged by the SRMP in 1982 are 38 percent for SRM 1261A and 51 percent for SRM 187B&C. The ratio (0.51/0.38 = 1.34) of these deviations does not correspond to the inverse elasticity ratio; the increase over marginal cost for the price-elastic SRM should have been less and for the price-inelastic one should have been greater than the one that actually resulted from the average-cost prices charged by the SRMP.

5.2.5 Evaluating the Welfare Effect of the Deviation from Marginal Cost

5.2.5.1 γ as a measure of the welfare effect of the deviation from marginal cost

In the two-goods case analyzed here, the welfare effect of the size of the needed deviation from marginal-cost is measured by the shadow price, γ. γ is the Lagrangean parameter associated with the budget constraint. In the case of Ramsey pricing, the positive price-cost margin needed to meet the breakeven constraint lies somewhere between the marginal-cost and the profit-maximizing monopoly price. It was calculated to be 0.49 in our example of the two-product analysis. Ramsey pricing implies that the public enterprise acts as a profit-maximizing monopolist who inflates all price elasticities of demand by a factor of $1/\gamma$, where γ determines the proportionate deviation from marginal cost just sufficient to cover the deficit. The profit-maximizing choice of a monopolist would be to reduce his output to where marginal revenue equals marginal cost and charge the corresponding monopoly price. Since $\gamma = 1$ when monopoly prices are charged and $\gamma = 0$ when marginal-cost prices are charged, the factor of 0.49 indicates that in our example the welfare loss from Ramsey pricing is about half of what it would be under full monopoly pricing.

5.2.5.2 Difference in deadweight loss as a measure of welfare gain

Once we have solved for the Ramsey prices and quantities, it is reasonably straightforward to calculate the difference in deadweight loss as a measure of welfare gain due to a shift from average-cost prices to Ramsey prices. Based on the conventional treatment[29] of a reduction in consumer surplus due to a price increase, it can in general be said that a given price rise will cause a greater loss in consumer surplus, not offset by a gain in producer surplus, the more elastic the demand curve. That is the reason why welfare maximization requires a smaller deviation of price from marginal cost for any good whose demand is relatively elastic.

Following the concept of consumer and producer surplus, the welfare-maximizing pricing rule in the presence of a budget constraint can also be expressed as a maximization of consumer surplus, *S*, plus profits, *R-C*, (Bös and Eichstaedt, 1984), subject to the exogenously given budget constraint:

$$\max\ S(p) + R(z(p),p) - C(z(p)) \tag{5.14}$$

subject to

$$R(z(p),\ p) - C(z(p)) = \Pi^0 \tag{5.15}$$

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[29]See, for example, Hicks (1947), Harberger (1954) for a discussion of consumer surplus and deadweight loss measures.

The solution of this constrained objective function leads to the same welfare-maximizing conditions as the Ramsey rule and the same second-best quasi-optimal prices. This is the area *S* under the demand curve in figure 5-1. The analysis reduces to comparing points *A* and *B* in figure 5-1.[30] Point *A* represents the Ramsey equilibrium, point *B* the marginal-cost equilibrium. Any increase in price reduces the number of units SRM users are willing to buy at that price and thus reduces welfare as defined in equation (5.14) by the deadweight loss (triangle *ABC*).

In the case of individual SRMs, where $R \neq C$, the reduction in welfare due to an increase in price under the Ramsey rule would have to include both the deadweight loss and the difference between revenue and cost. For an SRM whose average cost curve lies above the demand curve, as is the case with SRM 1261A, the reduction in surplus would consist of triangles *ABC* and *AFE* in figure 5-1. However, since the Ramsey rule calculates prices for all SRMs simultaneously in a way that exactly offsets negative and positive differences between revenue and cost, the loss of welfare due to the deviation from marginal cost can be measured by the sum of the deadweight losses over all SRMs. This is illustrated for the two-SRM case in table 5-4, column 2. Thus, for the program as a whole, if it is subject to a breakeven constraint, it is simply the consumer surplus that needs to be maximized.

To see whether in the case of SRMs, there is a welfare gain from a shift to Ramsey pricing from the actual, average-cost pricing, we need to compare the difference in size of 'consumer surplus plus profit' for the two pricing rules, that is, we need to examine which one of the pricing rules results in a smaller deadweight loss from the deviation from marginal cost.

If the demand function between *A* and *B* is assumed to be linear and if income effects are assumed to be zero, the deadweight loss can be estimated by the following equation:

$$DWL = \frac{dp \bullet dz}{2} \tag{5.16}$$

We use this equation and the Ramsey and average-cost prices and quantities in table 5-3 to approximate the deadweight losses for SRMs 1261A and 187B&C in 1982. Table 5-4 shows that, as expected, the deadweight loss for 1982 for the two SRMs combined is

[30]This assumes that the good is efficiently produced along the technology frontier g(z) = 0 and marginal cost is constant.

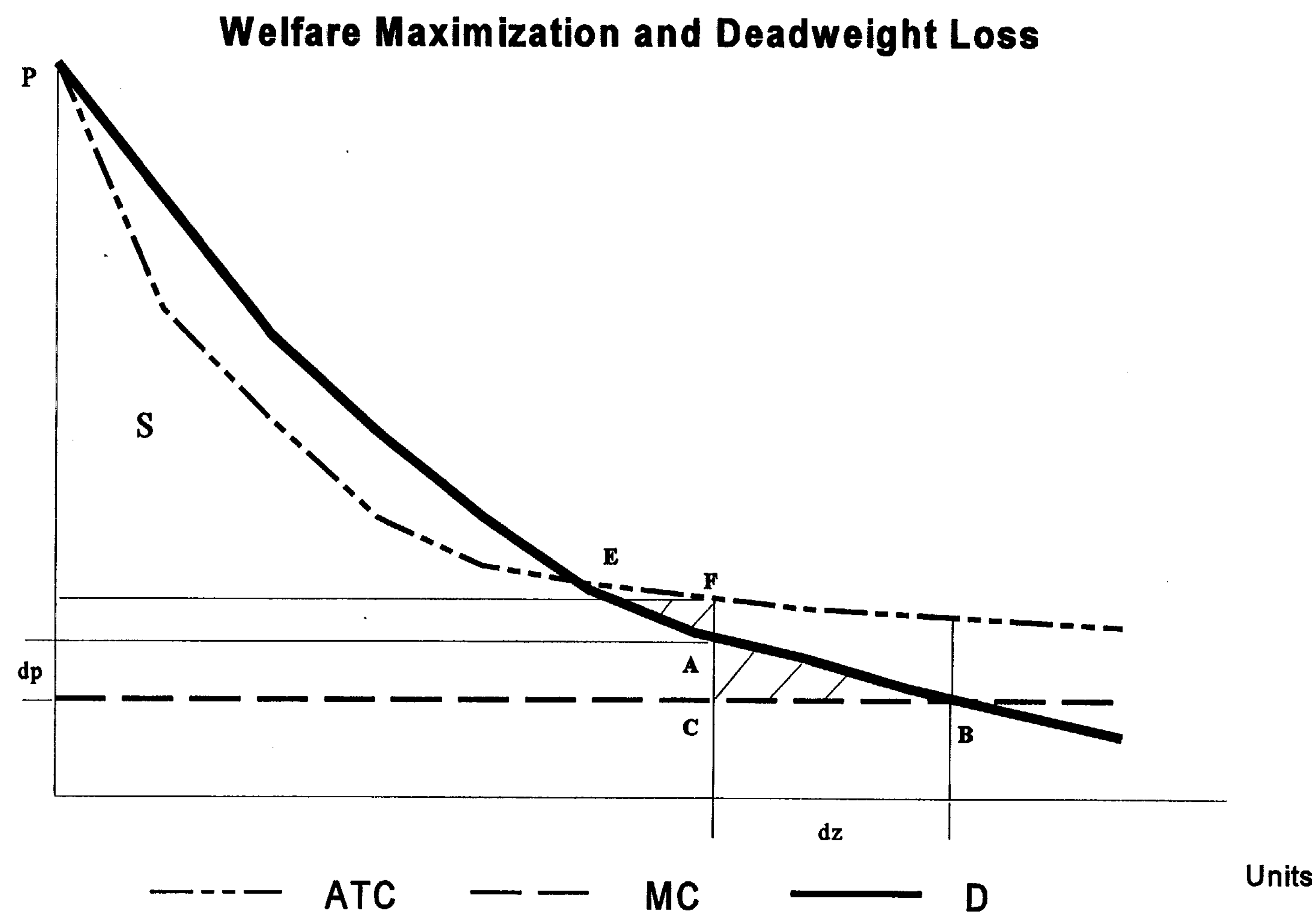

Figure 5-1. Deadweight Loss Expressed as Consumer Surplus plus Revenue minus Cost

smaller as a dollar amount and as a percentage of total revenue under the welfare-maximizing Ramsey pricing rule than under the average-cost pricing rule of the SRMP.[31]

Table 5-4. Deadweight Losses with Second-best Pricing of SRMs 1261A and 187B&C - 1982

SRM	DWL + Deficit with Ramsey Pricing	DWL with Average-cost Pricing	Ratios of DWL to Revenue	
			w/Ramsey Price	w/AVC Price
SRM 1261A	\$2,754 + \$1,453	\$5,454	0.17	0.24
SRM 187B&C	\$2,096 - \$1,453	\$1,050	0.11	0.19
Total	**\$4,850**	**\$6,504**	**0.16**	**0.23**

The deadweight loss is greater in absolute values for the relatively elastic SRM and smaller for the relatively inelastic SRM, corresponding to their price elasticities of demand. The welfare loss as a ratio of DWL to revenue (from table 5-3) is also lower under Ramsey pricing than under average-cost pricing for the SRMs individually and for the total values.

It can be concluded that a welfare gain of approximately \$1,654 (= \$6,504-\$4,850) could have been obtained in 1982 if the SRMP had charged a lower-than-average-cost price for the price-elastic SRM 1261A and a higher-than-average-cost price for the price-inelastic SRM 187B&C.

5.3 Estimation of Ramsey Prices and Quantities for More than two SRMs

5.3.1 Ramsey Prices for 11 SRMs

The results of the two-SRM case generalize to any number of SRMs. Equations analogous to equations (5.6) to (5.8) were solved simultaneously for the 11 SRMs under discussion. Table 5-5 compares 1982 Ramsey prices and quantities for these SRMs with actual and marginal-cost prices and quantities, and the corresponding revenues, costs, and deviations from marginal costs. For all 11 SRMs combined, Ramsey prices would have

[31]Figure 5-1 depicts the usual representation of deadweight loss due to an increase in the price of a good whose demand curve is linear. Because in our case all deviations from marginal cost are calculated simultaneously, the total deadweight loss is not a simple sum of the individual triangles. In our calculations, the quantity base of these triangles represents the total "general equilibrium" change in each SRM quantity in response to the ***entire*** set of price distortions. This also takes into account an additional source of loss, the loss from the deviation from marginal costs of SRM 2 due to the fact that SRM 1 is no longer priced at marginal cost. For the derivation of the algebraic expression (in the context of second-best theory of taxation) for the sum of the deadweight triangles plus this additional source of deadweight loss, see Tresch (1981), ch. 15, p. 316.

Table 5-5. Comparison of SRM Prices and Quantities under Ramsey, Average-cost, and Marginal-Cost Pricing Rules - 1982

		Ramsey Pricing					Average-cost Pricing				Marginal-cost Pricing				
SRM	η_{kk}	Price	$(p_k-c_k)/p_k$	Q	Revenue	Cost	Price	$(p_k-c_k)/p_k$	Q	Revenue (= Cost)	P=MC	Q	Revenue	Cost	Deficit
1261A	-1.70	$77.12	0.25	334	$25,735	$27,922	$94	0.38	238	$22,372	$58	541	$31,407	$39,975	$8,568
187B&C	-0.78	62.96	0.54	89	5,591	5,365	59	0.51	93	5,487	29	163	4,718	7,508	2,790
1625SO_2	-2.10	147.68	0.20	103	15,272	16,346	174	0.32	74	12,876	118	166	19,529	23,673	4,144
40H	-0.55	146.92	0.76	89	13,127	9,177	85	0.59	121	10,285	35	198	6,913	12,963	6,050
27F	-1.25	51.26	0.34	56	2,874	3,436	85	0.60	30	2,550	34	94	3,186	4,716	1,530
39I	-0.83	62.72	0.51	214	13,419	13,563	64	0.52	210	13.440	31	385	11,930	18,860	6,930
120BC	-0.88	75.13	0.48	63	4,741	4,817	77	0.49	62	4,774	39	112	4,371	6,727	2,356
1361A	-1.29	144.20	0.33	84	12,068	11,280	131	0.26	93	12,183	97	139	13,522	16,684	3,162
1635	-2.70	67.69	0.16	172	11,663	12,397	103	0.45	56	5,768	57	273	15,534	18,110	2,576
122H	-2.98	32.61	0.14	2,066	67,375	67,229	71	0.62	203	14.413	28	3,254	91,100	100,438	9,338
53E	-1.90	29.54	0.22	123	3,623	3,954	65	0.65	27	1,755	23	197	4,541	9,072	4,531
Totals				**3,393**	**175,487**	**175,487**			**1,207**	**105,903**		**5,521**	**206,750**	**258,725**	**51,975**

led to higher sales (3393 units), without resulting in a deficit, than did actual prices (1207 units). With a revenue of $175.5 thousand compared with $105.9 thousand, the size of the program for the 11 SRMs would have been greater under Ramsey pricing than it was under average-cost pricing.

A further interesting interpretation of the results shows that the required optimality rules for second-best pricing under a breakeven constraint are met (Baumol and Bradford, 1970). Table 5-5 shows that for all SRMs the calculated optimal deviations from marginal cost are inversely proportionate to their price elasticities, as required. The proportionality factor, γ, is found to be 0.42. For example, for SRM 1361A, whose price elasticity is 1.29, the Lerner Index is 0.33, which is equal to γ/η_{kk}:

$$\frac{\$144.20-\$97}{\$144.20} \approx \frac{-0.42}{-1.29} \approx 0.3273 \tag{5.17}$$

For SRM 40H, whose price elasticity is -0.55, the Lerner Index is 0.76. This relationship holds for the remaining SRMs. Also, the figures in table 5-5 show that, as required for welfare maximization, the formula produced unequal deviations in which prices of SRMs with inelastic demands diverge from their marginal costs by a relatively wider margin than prices of SRMs with elastic demand. By contrast, the deviations from marginal cost of the actual, average-cost prices, which are listed in table 5-5 for comparison, show no such relationship to the SRMs' price elasticities.

5.3.2 Ramsey Quantities for 11 SRMs

Another requirement for the second-best optimality of Ramsey prices is that (after compensation for income effects) purchases change proportionately from the levels that would be observed if prices were set at marginal costs. The rule asserts that the divergence between price and marginal cost must be such that the estimated percentage changes in quantity demanded resulting from an increase in all prices from the given marginal-cost levels are approximately the same for all goods. In general, if $\Delta p_i = p_i - c_i$, the relationship can be expressed as

$$\frac{dz_i}{dp_i} \Delta p_i = -\gamma z_i. \tag{5.18}$$

The left-hand side of equation (5.18) can be approximated to Δz_i, the change in the ith good that would result from a shift to second-best prices from their marginal-cost prices:

$$\Delta z_i = \kappa z_i, \quad \textit{where } \kappa = -\gamma. \tag{5.19}$$

As can be seen from table 5-6, the proportions[32] among SRM quantities are approximately the same under Ramsey pricing as under marginal-cost pricing. The relationship of equation (5.19) holds approximately for all SRMs. For example, the change in the quantity demanded of SRM 1361A is

$$-55 \text{ units} = (84 - 139 \text{ units}) \approx -0.42 \cdot 139 \text{ units}.$$

Table 5-6. Proportional Relationships of Output Quantities under Ramsey Pricing, Marginal-Cost Pricing, and Average-Cost Pricing

SRM	Elasticity	Quantity Proportions		
	η_{kk}	w/Ramsey Price	w/MC Price	w/ AVC Price
1261A	-1.70	1.00	1.00	1.00
187B&C	-0.78	3.76	3.33	2.56
1625 SO2	-2.10	3.23	3.27	3.12
40H	-0.55	3.73	2.74	1.97
27F	-1.25	5.95	5.78	7..93
39I	-0.83	1.56	1.41	1.13
120BC	-0.88	5.29	4.83	3.84
1361A	-1.29	3.99	3.88	2.56
1635	-2.67	1.94	1.99	4.25
122H	-2.98	0.16	0.17	1.17
53	-1.90	2.72	2.74	1.21

If one thinks of the consequence of a deviation of price from marginal cost as a distortion of relative demand patterns, then the loss of welfare is minimized if the *relative* quantities of the SRMs sold are kept unchanged from their marginal-cost pricing proportions. In terms of elasticities, this means that if we need to reduce quantities by an equal percentage, the price of an SRM with elastic demand needs to be raised less than the price of an SRM with inelastic demand. These relationships hold for the Ramsey quantities we calculated but not for the quantities that correspond to the actual, average-cost, prices (shown in column 5 of table 5-6).

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[32]The quantity ratios are calculated for SRM $k = 1$ over SRMs $k = 1,...11$.

5.3.3 Comparison of Deadweight Losses for 11 SRMs

Table 5-7 lists the deadweight losses and the deficits for each of the 11 SRMs as well as the combined deadweight losses, both under Ramsey pricing and actual pricing. As in the two-goods case, there would have been a welfare gain if the Ramsey pricing rule had been used to determine prices and production plans. For the year 1982, the deadweight loss of $18,970 under Ramsey pricing is approximately one-fifth of the $91,379 resulting from average-cost pricing. As a percentage of total revenue, the total deadweight loss for 1982 amounts to about 11 percent of total revenue with Ramsey pricing, whereas it amounts to about 86 percent with average-cost pricing. Because output levels vary from their marginal-cost levels according to the differences in the price elasticities rather than in an arbitrary fashion unrelated to the demand functions of the individual SRMs, the lower deadweight loss confirms that the use of the Ramsey formula would result in prices and quantities closer to their welfare-maximizing levels than those called forth by the average-cost pricing policy that the SRMP follows at present.

Table 5-7. Deadweight Losses with Second-best Pricing of 11 SRMs - 1982

SRM	DWL with Ramsey Pricing	DWL with Average-cost Pricing	Ratios of DWL to Total Revenue	
			w/Ramsey Price	w/AVC Price
1261A	$1,987	$5,463	0.08	0.24
187B&C	1,255	1,045	0.22	0.19
1625 SO2	921	2,562	0.06	0.20
40H	6,053	1,913	0.46	0.19
27F	325	1,625	0.11	0.64
39I	2,710	2,885	0.20	0.21
120B&C	885	951	0.19	0.20
1361A	1,315	789	0.11	0.06
1635	536	4,980	0.05	0.86
122H	2,740	65,587	0.04	4.55
53E	235	3,579	0.07	2.04
Total	**$18,970**	**$91,379**	**0.11**	**0.86**

5.4 Results of Empirical Estimates for the Years 1978 to 1992

The same calculations as for the year 1982 were carried out for the other years in the study period, the years 1978 to 1992. Table 5-8 presents a summary of the totals for each year of the number of units sold, the corresponding revenues, and the deadweight losses under Ramsey pricing, average-cost pricing, and marginal-cost pricing. Figure 5-2 shows a graphical representation of the revenues and figure 5-3 of the units sold. In each year, Ramsey pricing would have resulted in a greater number of units sold and higher revenues than average-cost pricing.

Table 5-8. Summary of Total Units, Revenues, and DWL for Ramsey, Average-cost, and Marginal-cost Pricing

Year	γ	Ramsey Units	AVC Units	Ramsey Rev	AVC Rev	MC Units	MC Rev	MC Deficit	Ramsey DWL	AVC DWL	Ramsey DWL/Rev	AVC DWL/Rev
78	0.44	6,138	1,625	224,822	114,313	10,056	277,357	56,147	22,571	145,095	0.10	1.27
79	0.43	8,649	2,124	279,641	139,867	14,058	348,300	66,388	25,265	183,692	0.09	1.31
80	0.44	5,262	1,794	227,517	130,621	8,738	278,959	59,477	24,313	126,524	0.11	0.97
81	0.43	5,305	1,696	215,161	119,986	8,717	262,130	55,587	21,833	126,531	0.10	1.06
82	0.42	3,383	1,207	175,161	105,903	5,521	206,730	51,975	19,225	91,375	0.11	0.86
83	0.42	2,464	1,044	152,373	99,018	4,051	176,058	44,936	18,020	71,731	0.12	0.72
84	0.42	3,322	1,310	181,840	113,696	5,438	212,439	52,084	20,416	93,513	0.11	0.82
85	0.42	3,639	1,344	189,534	115,473	5,936	222,821	53,170	20,715	100,786	0.11	0.87
86	0.42	3,145	1,200	177,633	110,319	5,143	206,914	50,828	20,093	92,253	0.11	0.84
87	0.42	2,824	1,076	166,781	90,588	4,604	193,655	47,996	18,692	69,917	0.11	0.77
88	0.42	2,654	1,066	170,545	110,666	4,340	195,672	50,980	19,924	84,608	0.12	0.77
89	0.42	3,048	1,055	180,180	111,111	4,942	208,800	51,928	19,998	95,390	0.11	0.86
90	0.42	2,707	993	170,726	108,017	4,397	196,219	50,322	19,453	87,316	0.11	0.81
91	0.41	2,241	883	153,017	99,833	3,648	173,684	46,459	18,026	74,652	0.12	0.75
92	0.41	1,978	831	148,349	99,888	3,226	166,454	46,373	18,008	69,321	0.12	0.69
Totals		**56,759**	**19,248**	**2,813,280**	**1,669,299**	**92,815**	**3,326,192**	**784,650**	**306,552**	**1,512,704**	**0.11**	**0.91**
1982 Present Values				**2,435,046**	**1,402,528**				**258,654**	**1,361,048**	**0.11**	**0.97**

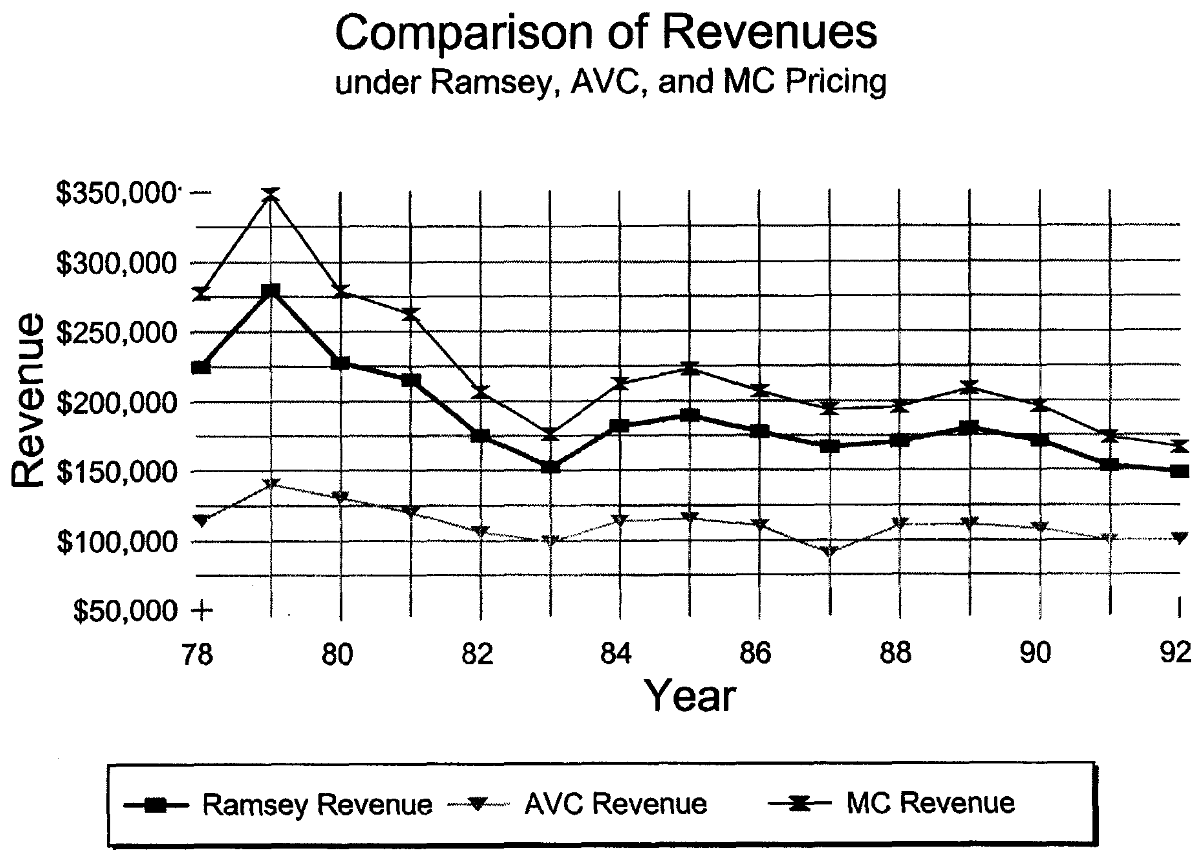

Figure 5-2. Comparison of Ramsey, AVC, and MC Revenues for the Years 1978 to 1992

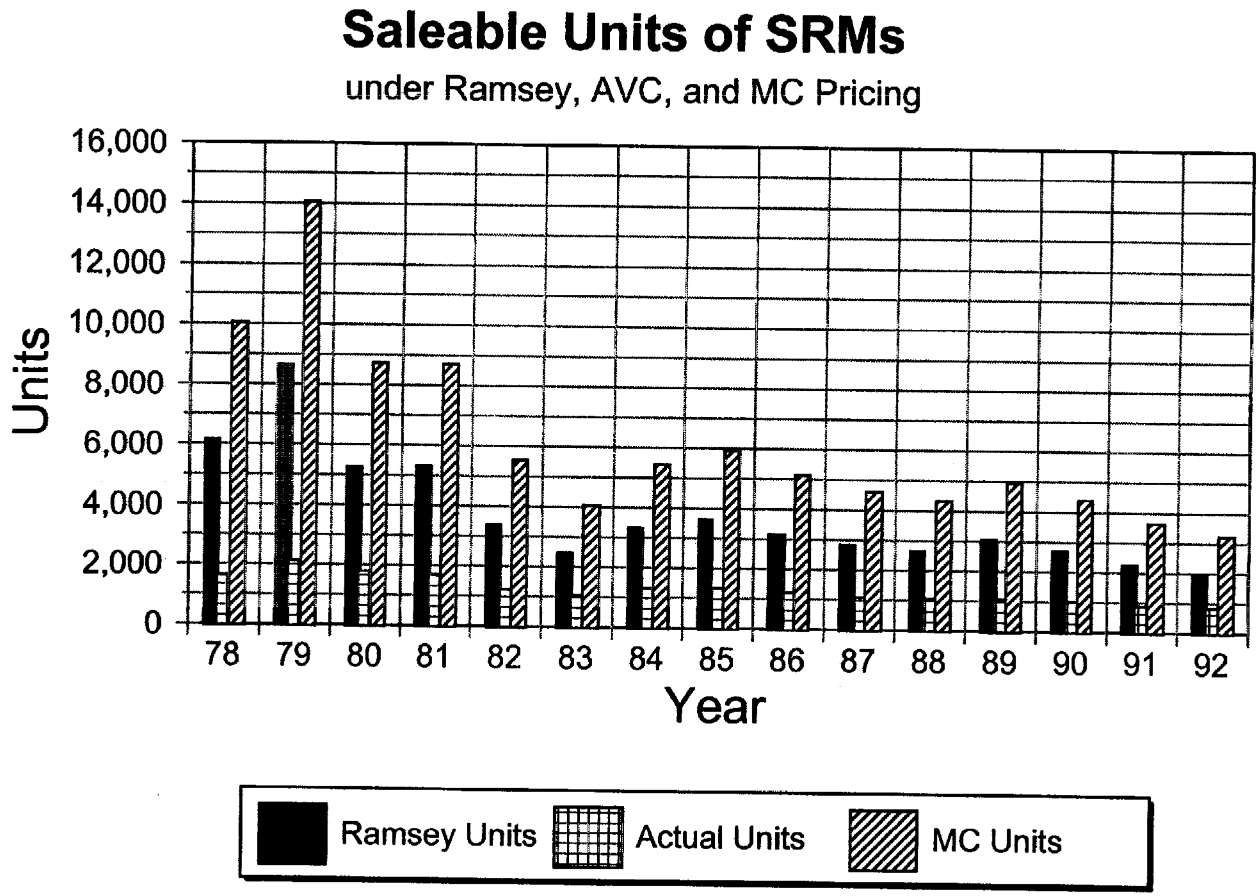

Figure 5-3. Saleable Units of SRMs under Ramsey, AVC, and MC Pricing for the Years 1978 to 1992

The estimated annual deadweight losses for the years 1978 to 1992, listed in table 5-8 and plotted in figure 5-4, are lower in each year for Ramsey pricing than for average-cost pricing in absolute values and as ratios of deadweight loss to revenue. The deadweight loss in dollars under average-cost pricing is almost five times the deadweight loss under Ramsey pricing. The ratio of total deadweight loss to total revenue for the study period was 0.91 under average-cost pricing and 0.11 under Ramsey pricing. This result makes intuitive sense; if all prices are set at their optimal levels, deadweight loss is likely to be lower than if prices are set on the basis of some rule of thumb.

Another way of looking at these deadweight losses is to discount them to present values as of some base year. In our case we choose 1982 as the base year and discount all annual amounts to that year before summing them.[33] The calculations show that in absolute values the amounts are of course lower in present-value dollars, but the relationships of total average-cost deadweight loss to Ramsey deadweight loss and of deadweight losses to revenues remains essentially unchanged, as is evident from the amounts in the last row of table 5-8. Under average-cost pricing, the deadweight loss is still about five times higher than under Ramsey pricing, and the ratio of average-cost deadweight loss to average-cost revenue is even slightly higher at 0.97. The total discounted welfare gain that could have been obtained if Ramsey pricing had been applied to this group of 11 SRMs alone during the 15-year period amounts to about $1.1 million (=$1,361,048 - $258,654). The increase in revenue from Ramsey pricing would have been about $1 million (from $1,402,528 to $2,435,046). The increase in units sold would have been 37,511 (=56,759 - 19,248).

5.5 Sensitivity Analysis

Because the price elasticities were estimated with only 15 observations, they are somewhat tentative. To see how prices and quantities would change with a change in elasticities, we performed a simple sensitivity analysis for the year 1982 by changing the estimated price elasticities of all 11 SRMs by ± one standard error. We also calculated Ramsey prices for only those SRMs that have inelastic demand.

5.5.1 Price Elasticities Lower than Estimated Coefficients

By decreasing the estimated price elasticities by one standard error we assumed that all 11 SRMs had lower elasticities than those estimated by regression analysis. The resulting Ramsey prices and quantities, which are listed in table 5-9, approach marginal-cost prices and quantities, with a γ of close to zero. The Lerner Indices range from 0.01 to 0.02. These results indicate that if all SRMs had less price-elastic demands, a smaller deviation

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[33] A real discount rate of 10 percent was used. This is the discount rate that was in effect in 1982 for evaluating time-distributed costs and benefits of projects in the Federal Government. See *Circular A-94 (1972)* and revisions.

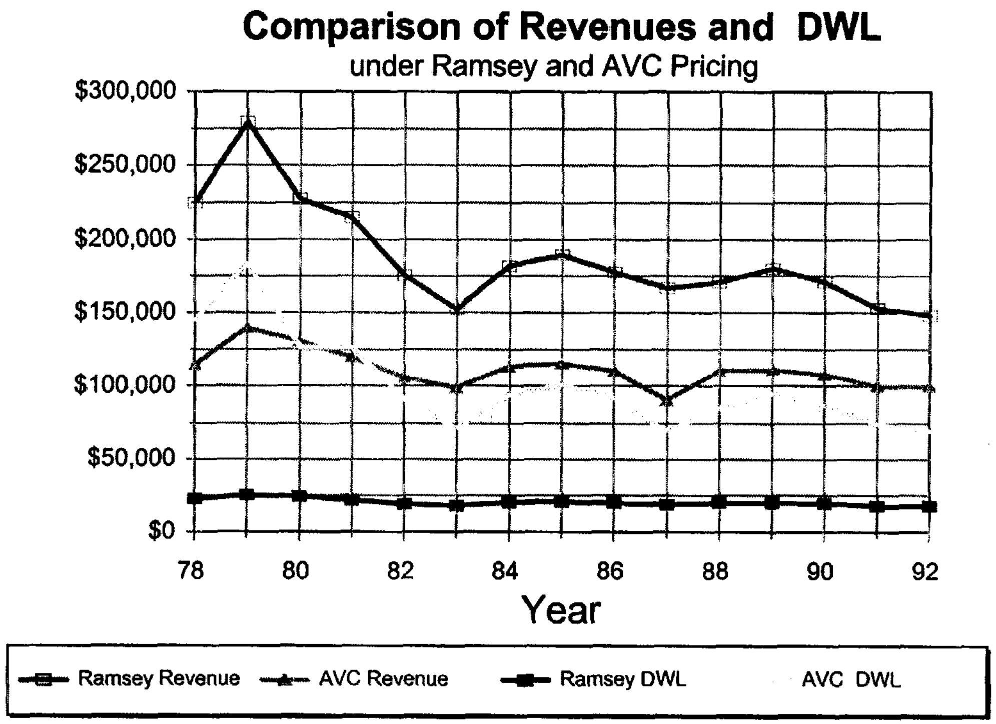

Figure 5-4. Comparison of Revenues and DWL with Ramsey and AVC Pricing for the Years 1978 to 1992

from marginal cost would be needed to cover the deficit. The deadweight loss would be smaller and thus the welfare-effect of the deviation from marginal cost would be close to zero. The marginal-cost prices and the corresponding quantities that were calculated with the same low elasticities still resulted in a deficit equal to the fixed costs.

5.5.2 Price Elasticities Higher Than Estimated Coefficients

Calculating Ramsey prices with price elasticities higher by one standard error for all 11 SRMs significantly increases prices and reduces quantities (see table 5-9), to levels approaching monopoly prices and quantities. In the case of SRM 1635, the calculations showed that the quantity corresponding to the Ramsey price is reduced to zero (precisely 0.24) units. It appears from these results that in the limit, when all SRMs have elastic demand, prices would have to be raised to a level where some SRMs would no longer be produced at all.

Table 5-9. Results of Sensitivity Analysis - 1982

Estimates for Lower and Upper Bounds of Price Elasticity of Demand		
Estimated Measure	**η_{kk} - 1 se (less elastic)**	**η_{kk} + 1 se (more elastic)**
	Ramsey Prices	
γ	0.01	1.0
No. of Units	163,561	205
Revenue	$7,813,648	$12,778
Deficit	0	$36,684
DWL	$206	$12,403
DWL/Revenue	≈0	0.97
	Marginal-Cost Prices	
γ	0	0
No. of Units	164,897	854
Revenue	$7,827,788	$29,208
Deficit	$51,975	$51,975
DWL	0	0
DWL/Revenue	0	0
	Average-cost Prices	
γ	N/A	N/A
No. of Units	39,444	119
Revenue	$2,819,218	$9,530
Deficit	0	0
DWL	$2,821,680	$13,4513
DWL/Revenue	≈1.0	1.08

In this scenario, the Ramsey formula did not succeed in calculating cost-covering prices. Even with γ set to one—meaning that the enterprise would exploit all of its monopoly power—costs are higher than revenues. The public enterprise would not be able to cover its average costs and would not choose to stay in business.

The two scenarios are highly unlikely events since not all of the SRMs' price elasticities would simultaneously deviate in the same direction from their estimated average values. Hence the result of the above exercise is without any practical usefulness. In a real-life situation, some elasticities might be higher and some lower than their estimated averages, thus partially offsetting the effect of a divergence from the estimated values. A different set of Ramsey prices and quantities would result, but one not likely to match either the monopoly or marginal-cost level.

5.5.3 Application of Ramsey Rule to Price-inelastic SRMs Only

SRMP management, in their discussions of pricing and production plans, sometimes advance the argument that perhaps the SRMP should focus on producing only those SRMs whose price elasticities are low. The assumption is that buyers must find it easier to use secondary standard reference materials or to find alternative ways of quality control for SRMs with relatively higher price elasticities. We have therefore applied the Ramsey rule to only those SRMs in the group of 11 that have price-inelastic demand.

Optimal prices in this case can be relatively lower, and thus quantities higher, because a relatively smaller deviation from marginal cost is needed to cover the deficit if fewer price-elastic SRMs in the group have to be "subsidized." Consequently the deadweight loss is relatively lower if the program focuses on these inelastic SRMs. Table 5-10 illustrates this result for the year 1982. It compares optimal prices, quantities and deadweight losses of a subset of four inelastic SRMs with those of the same SRMs when calculated as part of the larger group of eleven (as listed in table 5-5).

A corollary to the above result is that less of the public enterprise's monopoly power has to be exploited to meet the budget constraint. In this illustration, γ, the measure of the welfare effect of the deviation from marginal cost, is 0.38 for pricing the inelastic SRMs by themselves, compared with 0.42 for pricing them as part of the group that also contains elastic SRMs.

Table 5-10. **Deadweight Losses for SRMs with Price-Inelastic Demand - 1982**

SRM	η_{kk}	**Values calculated for inelastic SRMs alone**					**Values calculated for inelastic SRMs as part of group of 11 SRMs**				
		γ	**Ramsey Price**	**Quantity**	**Deviation from MC**	**DWL**	γ	**Ramsey Price**	**Quantity**	**Deviation from MC**	**DWL**
187B&C	-0.781	0.37	$55.64	93	0.48	$935	0.42	$63.15	84	0.54	$1,348
40H	-0.553		$108.09	106	0.68	$3,349		$148.17	89	0.76	$6,144
39I	-0.833		$56.25	234	0.45	$1,901		$62.89	213	0.51	$2,732
120B&C	-0.876		$68.05	69	0.43	$628		$75.31	63	0.48	$892
Totals						**$6,813**					**$11,116**

5.6 Conclusions

The preceding implementation of the second-best pricing model attempts to show that welfare optimal pricing can be achieved by relatively simple rules if certain conditions are fulfilled. By applying the Ramsey formula to cost and demand functions for a group of 11 NIST SRMs, we were able to calculate quasi-optimal prices that satisfy the two propositions stated at the beginning of this chapter.

(1) Estimates of deadweight losses showed that a welfare gain can be achieved from using Ramsey pricing rather than the average-cost pricing formula that the SRMP uses at present.

(2) The quasi-optimal prices provided by the Ramsey pricing rule lead to higher, more nearly optimal output levels when compared with the actual, average-cost pricing policy practiced by the SRMP now.

The cost-covering prices calculated satisfied the optimality requirements of the Ramsey theorem in the following way: The prices deviate from marginal costs in a systematic manner as required for welfare maximization in the presence of a budget constraint. For all the SRMs under investigation, the calculated percentage deviations of price from marginal cost are inversely proportionate to the SRMs' own price elasticities of demand. Consequently, the prices correctly show unequal deviations from marginal cost in which SRMs with elastic demands have prices close to their marginal costs and SRMs with inelastic demand have prices that diverge by a relatively wider margin.

Since we know the relevant demand functions, the choice of price is tantamount to the choice of saleable output levels. The calculated Ramsey prices yielded outputs that were reduced by the same proportion from the quantities that would be demanded at prices equal to the corresponding marginal costs.

6. SUMMARY AND POLICY IMPLICATIONS

6.1 Summary

The driving force behind the argument in favor of special pricing rules for public enterprises is the assertion that governments ought to maximize welfare rather than profits. Further, public enterprises often operate under conditions of constant or decreasing marginal cost where first-best profit-maximizing rules fail to cover costs. If the deficits that then arise cannot be paid for through lump-sum taxation, one has to deal with a problem in the area of the second best. Resource allocation now is to be optimal under the added constraint that the government enterprise raise enough revenue to cover the cost of its program.

A public-sector pricing model in the Boiteux tradition provides optimal pricing rules for multi-product public enterprises that have a mandated budget constraint. The price and output combinations that it computes minimize the deadweight loss due to the unavoidable deviations of price from marginal cost. Since this pricing rule takes into account price elasticities of demand, it is superior to the average-cost pricing that most U.S. government enterprises have adopted. Average-cost pricing does not explicitly include demand-side information.

We have first described the Boiteux model and its extensions. To apply it we focused on the Ramsey version of the model and applied it to the pricing problem of the Standards Reference Materials Program at the National Institute of Standards and Technology. It supplies samples of materials whose physical or chemical properties are precisely characterized; they are used as intermediate goods by firms who use them to calibrate scientific apparatus or manufacturing equipment. The pricing history of the SRM Program is described in chapter 2.

Since the price elasticities of demand are an important ingredient of the Ramsey model, we first estimated, by regression analysis, the derived demand for 24 SRMs, using a constant-elasticity demand function. For the analysis we used only the 11 SRMs whose price elasticities were significant at the 5 or 10 percent levels of confidence. The cost functions were derived according to the accounting conventions of the SRM Program. Combining the theoretical principles of the model, the price elasticities from the demand analysis, and the fixed and variable costs of the empirical cost functions, we calculated quasi-optimal prices and production plans for the group of 11 SRMs for the years 1978 to 1992. We compared these with the prices and quantities that would have been derived if marginal-cost pricing were applied, and with the actual, average-cost prices and quantities in those years, which are based on the pricing policies followed by the SRM Program now.

The resulting price and quantity deviations from the first-best optimum fulfilled the required optimality requirements of the Ramsey model. Prices were inversely proportionate to their price elasticities of demand and quasi-optimal quantities deviated by the same proportions from the quantities that would have been demanded at prices equal to their corresponding marginal costs.

By comparing deadweight losses, we found that in every year of the study period there would have been a welfare gain if Ramsey prices had been charged rather than average-cost prices, and unit sales and revenues would have been higher than they were under the actual pricing policy of the SRM Program in the years from 1978 to 1992.

The analysis shows that in the case of NIST SRMs the Ramsey-Boiteux model can provide concrete and relatively simple pricing rules that yield welfare-optimizing prices and quantities.

6.2 Policy Implications

6.2.1 Policy Implications for NIST

When, in the early 1980's, the Congressional subsidies covering development costs of SRMs were phased out, the SRM Program was forced to increase prices in order to recover through increased revenues the no-longer-available appropriations. A 25 percent overall price increase was implemented in 1982. It was imposed across all SRMs, partly as lump sum increases and partly as *ad valorem* increases. The increases were based on cost considerations since the demand functions of the SRMs were not known. The result was that there was on average a 16 percent decrease in sales, and revenue increased by less than 5 percent. Since that time, the SRM Program has been struggling to devise a pricing scheme that would cover costs and maintain demand. Their formula tries to distribute the total costs of the program among the units of SRMs it expects to sell, based on the previous year's sales. If fewer SRMs are sold than predicted, deficits are incurred.

The Ramsey model tested in this study could assist the SRMP in several ways in their formulation of a successful pricing policy:

(1) The model explicitly takes into account the price elasticities of demand in its derivation of cost-covering prices. This means that the price increases that are necessary to cover the cost of the program as a whole can be distributed among SRMs in a way that recovers a proportionately greater portion of the costs from those SRMs whose demand is price inelastic without decreasing revenue. The prices of SRMs whose demand is relatively more price elastic can be increased by less in order to keep customers.

(2) Since prices are estimated on the basis of the demand functions, knowing the prices is tantamount to knowing the corresponding quantities. These quantity estimates could be used as benchmarks for planning renewal batches of SRMs.

(3) The budget constraint, which is an integral part of the Ramsey model, can be formulated to meet the SRM Program's objectives, since it implies a mixture of endogenous and exogenous determination of the deviation from marginal cost. Prices and outputs are endogenous but exogenous variables could be included, such as the desired size of the program. To cover the cost of the program, it may not be necessary to expand sales by lowering prices, but to increase prices of SRMs for which no satisfactory secondary standards are available. In this context, the SRM Program might decide to focus on the supply of SRMs whose demand is price inelastic. Then to determine the prices that correctly reflect the true value of the standard to the user will require the optimal combination of demand and cost information that is provided by the model.

(4) Second-best pricing formulas allow many combinations of prices and outputs, none of which may be optimal from the welfare-maximization perspective. The Ramsey model provides the theoretical and practical basis for calculating prices that minimize the loss of welfare incurred by unavoidable deviations from marginal-cost prices. The ability of the SRM Program to base pricing policies on economic theory rather than on arbitrary rules of thumb or on accounting conventions improves the reliability of its decisions and strengthens its arguments in budget negotiations with the NIST budget office and with the Congress.

(5) Many of the restrictions of the general extended Boiteux model can be relaxed in the case of SRMs. For example, NIST SRMs are intermediate goods that are not sold to consumers but to firms, they constitute a small percentage of total input costs of the final product, income effects can be ignored, and their cross elasticities are zero. Therefore, as we have shown, it is possible to reduce the informational and computational requirements and make the model operational in practice.

It is also possible to apply the model in a piecemeal fashion to only certain groups of SRMs. In this case Ramsey pricing would not maximize welfare for the program as a whole but would produce a welfare *improvement* when compared with average-cost pricing, all other things equal. At the very least, the calculated prices and quantities could serve as benchmarks leading to price differentiation that is more in line with actual demand than the differentiation that occurs in an arbitrary fashion through uneven across-the-board price increases based on average cost.

(6) The Government Performance and Results Act (GPRA) of 1993, Public Law 103-62, is intended to improve federal program effectiveness and public accountability. It is explicitly noted in the GPRA that "Federal managers are severely disadvantaged in their effort to improve program efficiency and effectiveness because of insufficient articulation of program goals and inadequate information on program performance." The application of the Ramsey model to pricing decisions at NIST would go a long way in providing

information to managers and in helping to articulate program goals. Since welfare maximization implies efficient allocation of resources, pricing policies based on this concept are in agreement with the intent of the GPRA.

6.2.2 Policy Implications for Government Enterprises in General

We have shown that the Ramsey-Boiteux model provides concrete and relatively simple pricing rules that yield policy directives under specific assumptions. There are many examples where welfare-maximizing pricing has actually been applied.[34] But in many cases policy prescriptions of the welfare kind fail in practice because their goals are not clearly articulated, the various actors engaged in public-sector pricing and investment have differing objectives and insufficient information. The model in its full-information form may be too complex to be implemented. Rees (1968), for example, suggests that the cost of obtaining information and of estimating the values of the parameters, and the costs of introducing and maintaining the optimal pricing system may outweigh the welfare gain arising from it. However, since his discussion of this problem in 1968, econometric methods and data collection methods have vastly improved. At the same time, the willingness of legislatures to finance program deficits through tax appropriations has considerably declined so that it has become more urgent to find efficient ways to cover costs while maintaining services. We rather agree with Baumol and Bradford (1970), that "there exists a highly sophisticated and well-developed body of literature indicating what should be done in such circumstances." And if—as is the case with most practical applications—many of the restrictions of the full-information Boiteux model do not apply, the estimation of the needed parameters is straightforward, as has been shown by the implementation of the model in this study. The theory is too elegant and the results too convenient not to make use of them for critically evaluating and improving the day-to-day operations of public enterprises.

6.3 Suggestions for Further Research

The findings of this study were arrived at by testing whether prices and quantities of SRMs change in a way predicted by the public-sector pricing model when certain assumptions regarding the variables are changed. In doing so we assumed that the SRMs existed, that their costs were well defined and that the results of one pricing rule could be compared with another. We think we answered the questions that we asked and gained additional insight into how SRMs should be priced. The attempt to answer one question, however, usually results in the formulation of others that are worthy of additional research efforts.

The issue that deserves primary consideration in the case of SRMs is a logical extension of this study. It has to do with the problem of how to decide whether or not to develop an SRM in the first place and how quickly to develop it. The question of how the SRMP

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[34]Bös (1985) describes the development of optimal pricing rules for bus and underground services of London Transport. They were used as benchmarks for evaluating actual pricing policies. Their planned implementation was prevented by a change in administration. See also Quoilin (1976) who describes the use of an optimal pricing model at the Electricité de France.

decides on the size of its investment in new SRMs should be related to the cost to society of not having these SRMs available or having them available later than would be optimal. These costs are not known, and any related investment cost is not included in the cost accounting of the SRM Program. The decline in recent years in the number of new SRMs developed and the decline in renewals of SRMs, as evidenced by the graph in figure 2-5, may mean that even though the SRM Program is covering its costs, the true cost of not making SRMs available is not included in their cost accounting. At present the development costs are determined as an arbitrarily fixed surcharge on the existing SRMs expected to be sold. A dynamic formulation of the model may be needed to include the aspect of time in determining these costs so that they can be included in the budget constraint and thus reflected in the resulting quasi-optimal prices.

Another possible topic of investigation concerns the fundamental issue whether SRMs should be supplied by the government at all or whether they could be supplied more efficiently in the private sector, even as a regulated natural monopoly. The questions to be answered are whether the availability of NIST laboratories and the traceability of SRMs to a government agency are sufficient reasons to warrant the public supply of SRMs. With the wave of privatization of government enterprises in many countries around the world came a renewed interest in the economic theory of denationalization that could provide insights on this issue, even if only to confirm the legitimacy of SRM production by NIST.

APPENDIX

Regression Statistics

on

Estimation of SRM Demand Functions

Table A-1a. Description of Demand Variables for SRMs in Metals Category

METALS	Description	Means			
DEPENDENT VARIABLES		122H	1261A	53E	627
UNIDOM	Number of units sold by NIST annually in the U.S. FY 1978-92	259	283	25	26
LUNIDOM	Natural log of the above variable	2.33	2.41	1.32	1.40
UNITOT	Total number of units sold by NIST (including exports) FY 1978-92	415	374	34	43
LUNITOT	Natural log of the above variable	2.55	2.53	1.48	1.61
INDEPENDENT VARIABLES					
RPrice	Average price per unit divided by the PPI(FY) FY 1978-92	74.43	105.52	67.04	86.47
LnRPrice	Natural log of the above variable	1.87	2.01	1.82	1.94
BCon: Primary Metal Industries[1]	Value of product shipments in millions of 1982 dollars FY 1978-92	65,447.71	51,380.65	9,684.41	
LnBCon:	Natural log of the above variable	74.81	4.70	3.98	
BCon: Industrial Machinery and Equipment	Value of product shipments in millions of 1982 dollars FY 1978-92	45,953.55		186,708.19	
LnBCon:	Natural log of the above variable	4.65		5.27	
BCon: Fabricated Metal Products	Value of product shipments in millions of 1982 dollars FY 1978-92		130,181.15		
LnBCon:	Natural log of the above variable		5.11		
BCon: Non-ferrous Metals, Die-Castings, Foundries	Value of product shipments in millions of 1982 dollars FY 1978-92				9,684.41
LnBCon:	Natural log of the above variable				3.98

Table A-1a continued	Description of Independent Variables	122H	1261A	53E	627
BCon: Hardware	Value of product shipments in millions of 1982 dollars FY 1978-92				7,071.26
LnBCon:	Natural log of the above variable				3.85
GPDI	Gross Private Domestic Investment in millions of 1982 dollars FY 1978-92	608,292.47	608,292.47	608,292.47	608,292.47
LnGPDI	Natural log of the above variable	5.78	5.78	5.78	5.78

[1]The BCon variables are composites of those SIC product classes that pertain to the particular SRM; thus the numerical values may not be identical for the same BCon name. For example, *Primary Metal Industries* for 122H, Cast Iron, comprises the relevant SIC product classes for ferrous metals, whereas for 53E, Bearing Metal–Lead, *Primary Metals Industries* comprises the relevant SIC product classes for nonferrous metals.

Table A-1b. Description of Demand Variables for SRMs in Non-Metals Category

NON-METALS	Description	Means			
DEPENDENT VARIABLES		635[a]	637[a]	120BC	27F
UNIDOM	Number of units sold by NIST annually in the U.S. FY 1978-89 FY 1978-92	160	165	55	35
LUNIDOM	Natural log of the above variable FY 1978-89 FY 1978-92	2.21	2.21	1.71	1.53
UNITOT	Total number of units sold by NIST (including exports) FY 1978-89 FY 1978-92	248	250	96	65
LUNITOT	Natural log of the above variable FY 1978-89 FY 1978-92	2.39	2.39	1.96	1.79
INDEPENDENT VARIABLES					
RPrice	Average price per unit divided by the PPI(FY) FY 1978-89 FY 1978-92	63.30	63.30	82.69	81.51
LnRPrice	Natural log of the above variable FY 1978-89 FY 1978-92	1.79	1.79	1.91	1.91
BCon: Cement and Concrete Products	Value of product shipments in millions of 1982 dollars FY 1978-89	21,812.84	21,812.84		
LnBCon:	Natural log of the above variable FY 1978-89	4.34	4.34		
BCon: Chemicals and Allied Products	Value of product shipments in millions of 1982 dollars FY 1978-92			14,119.94	
LnBCon:	Natural log of the above variable FY 1978-92			4.15	

Table A-1b continued	Description of Independent Variables	635[a]	637[a]	120BC	27F
BCon: Stone, Clay, and Glass Products	Value of product shipments in millions of 1982 dollars FY 1978-92			17,621.68	
LnBCon:	Natural log of the above variable FY 1978-92			4.24	
BCon: Iron Ores and Ferroalloy Ores	Value of shipments in millions of 1982 dollars FY 1978-92				2,989.45
LnBCon:	Natural log of the above variable FY 1978-92				3.46
BCon: Blast Furnaces and Steel Mills	Gross Private Domestic Investment in millions of 1982 dollars FY 1978-92				43,632.08
LnBCon:	Natural log of the above variable FY 1978-92				4.63
GPDI	Gross Private Domestic Investment in millions of 1982 dollars; FY 1978-89 FY 1978-92	594,930.2	594,930.2	608,292.5	608,292.5
LnGPDI	Nat. log of above variable FY 1978-89 FY 1978-92	5.77	5.77	5.78	5.78
LoST (dummy)	Low Stock in 1987; 1987= 1, otherwise = 0			0.07	

See footnote in table A-1a.
[a]Only the years 1978 to 89 were used for SRMs 635 and 637 because new cement SRMs were phased in after 1989.

Table A-1c. Description of Demand Variables for SRMs in Chemicals/Rubber/Plastics Category

CHEMICALS/RUBBER/PLASTICS	Description	Means		
DEPENDENT VARIABLES		370E	371GH	40H
UNIDOM	Number of units sold by NIST annually in the U.S. FY 1978-92	87	55	125
LUNIDOM	Natural log of the above variable	1.93	1.73	2.09
UNITOT	Total number of units sold by NIST (including exports) FY 1978-92	147	83	144
LUNITOT	Natural log of the above variable	2.14	1.91	2.15
INDEPENDENT VARIABLES				
RPrice	Average price per unit divided by the PPI(FY) FY 1978-92	92.38	74.41	80.62
LnRPrice	Natural log of the above variable	1.96	1.87	1.90
BCon: Rubber and Plastics Products	Value of product shipments in millions of 1982 dollars FY 1978-92	20,027.78	20,027.78	
LnBCon:	Natural log of the above variable	4.30	4.30	
BCon: Chemicals and Allied Products	Value of product shipments in millions of 1982 dollars FY 1978-92			26,896.20
LnBCon:	Natural log of the above variable			4.42
BCon: Medical and Dental Laboratories	Receipts in millions of 1982 dollarsFY 1978-92			5,823.41
LnBCon:	Natural log of the above variable			3.74
GPDI	Gross Private Domestic Investment in millions of 1982 dollars FY 1978-92	608,292.5	608,292.5	608,292.5
LnGPDI	Natural log of above varaiable	5.78	5.78	5.78

See footnote in table A-1a.

Table A-1d. Description of Demand Variables for SRMs in Engineering Category

ENGINEERING	Description	Means			
DEPENDENT VARIABLES		185EFG	187BC	189A	39I
UNIDOM	Number of units sold by NIST annually in the U.S. FY 1978-92	168	102	51	252
LUNIDOM	Natural log of the above variable	2.22	2.00	1.66	2.39
UNITOT	Total number of units sold by NIST (including exports) FY 1978-92	206	137	58	423
LUNITOT	Natural log of the above variable	2.31	2.13	1.72	2.61
INDEPENDENT VARIABLES					
	Average price per unit divided by the PPI(FY) FY 1978-92	73.95	69.20	75.76	71.32
LnRPrice	Natural log of the above variable	1.86	1.83	1.87	1.84
BCon: Alkalies, Industrial Gases & Chemicals, Inorganic Pigments	Value of product shipments in millions of 1982 dollars FY 1978-92	17,484.46	17,484.46	17,484.4 6	
LnBCon:	Natural log of the above variable	4.24	4.24	4.24	
BCon: Commercial Physical and Biological Research, Testing Labs.	Receipts in millions of 1982 dollars FY 1978-92	4,863.89	4,863.89	4,863.89	
LnBCon:	Natural log of the above variable	3.66	3.66	3.66	
BCon: Explosives	Value of product shipments in millions of 1982 dollars FY 1978-92				680.15
LnBCon:	Natural log of the above variable				2.83

Table A-1d continued	Description of Independent Variables	185EFG	187BC	189A	391
BCon: Petroleum Refining	Value of product shipments in millions of 1982 dollars FY 1978-92				145,898.59
LnBCon:	Natural log of the above variable				5.15
BCon: Laboratory Apparatus, Analytical Instruments, Measuring Devices	Value of product shipments in millions of 1982 dollars FY 1978-92				7,091.35
LnBCon:	Natural log of the above variable				3.84
	Value of electricity in kWh sold by electric utilities, in millions of 1982 dollars; FY 78-92				134,666.97
LnBCon:	Natural log of the above variable				5.13
GPDI	Gross Private Domestic Investment in millions of 1982 dollars FY 1978-92	608,292.5	608,292.5	608,292.5	608,292.5
LnGPDI	Natural log of the above variable	5.78	5.78	5.78	5.78
LOST (dummy)	Low stock in 1978, 79, 80; 1878, 79, 80 = 1; otherwise = 0	0.20		0.20	

See footnote in table A-1a.

Table A-1e. Description of Demand Variables for SRM in Environmental Category

ENVIRONMENTAL	Description	Means				
DEPENDENT VARIABLES		1575	1620&AB[1]	1622ABC	1625	1635
UNIDOM	Number of units sold by NIST annually in the U.S. FY 1978-92 FY 1980-92	67	91	198	60	37
LUNIDOM	Nat. log of the above variable FY 1978-92 FY 1980-92	1.81	1.94	2.28	1.66	1.44
UNITOT	Total number of units sold by NIST (including exports) FY 1978-92 FY 1980-92	126	114	236	69	73
	Nat. log of the above variable FY 1978-92 FY 1980-92	2.09	2.05	2.36	1.72	1.73
INDEPENDENT VARIABLES						
RPrice	Average price per unit divided by the PPI(FY) FY 1978-92 FY 1980-92	102.17	97.61	99.53	185.48	107.20
LnRPrice	Nat. log of the above variable FY 1978-92 FY 1980-92	2.01	1.98	1.99	2.25	2.03
BCon: Food and Kindred Products	Value of product shipments in millions of 1982 dollars FY 1978-92	279,193.2 8				
LnBCon:	Nat. log of the above variable FY 1978-92	5.45				
BCon: Commercial Physical & Biological Research, Testing Labs.	Receipts in millions of 1982 dollars FY 1978-92	4,863.89			4,863.89	
LnBCon:	Nat. log of the above variable FY 1978-92	3.66			3.66	

Table A-1e continued	Description of Independent Variables	1575	1620&AB	1622ABC	1625	1635
BCon: Crude Oil Production	Value of shipments in millions of 1982 dollarsFY 1980-92		63,119.06	63,119.06		
LnBCon:	Nat. log of the above variable FY 1980-92		4.78	4.78		
BCon: Petroleum refining	Value of product shipments in millions of 1982 dollars FY 1980-92		145,322.73	145,322.73		
LnBCon:	Nat. log of the above variable FY 1980-92		5.15	5.15		
BCon: Electricity Sales to End Users	Value in kWh sold by electric utilities, in millions of 1982 dollars FY 1978-92 FY 1980-92		136,584.34	136,584.34	134,666.97	
LnBCon:	Nat. log of the above variable FY 1978-92 FY 1980-92		5.13	5.13	5.13	
BCon: Bituminous, Subbituminous Coal, Lignite & Anthracite Production	Value of production in millions of 1982 dollars FY 1978-92					21,723.67
LnBCon:	Nat. log of the above variable FY 1978-92					4.34
BCon: Carbon Black (Channel and Furnace Process)	Value of product shipments in millions of 1982 dollars FY 1978-92					663.76
LnBCon:	Nat. og of the above variable FY 1978-92					2.82
BCon: Asphalt and Paving Mixtures & Blocks	Value of product shipments in millions of 1982 dollars FY 1978-92					3,196.17
LnBCon:	Nat. log of the above variable FY 1978-92					3.50
GPDI	Gross Private Domestic Investment in 1982 dollars FY 1978-92 FY 1980-92	608,292.5	743,516.1	743,516.1	608,292.5	608,292.5
LnGPDI	Nat. Log of the above variable FY 1978-92 FY 1980-92	5.78	5.82	5.82	5.78	5.78
BKOR:	Backorders filled 1980; 1980=1, otherw. = 0			0.08		0.07

For SRMs 1620 &1620AB and 1622ABC price and sales data were available for FY1980-92 only. See footnote in table A-1a.

Table A-1f. Description of Demand Variables for SRMs in Health Category

HEALTH	Description	Means	
DEPENDENT VARIABLES		911 A&B	1577&AB
UNIDOM	Number of units sold by NIST annually in the U.S. FY 1978-92	103	153
LUNIDOM	Natural log of the above variable FY 1978-92	2.00	2.17
UNITOT	Total number of units sold by NIST (including exports) FY 1978-92	133	295
LUNITOT	Natural log of the above variable FY 1978-92	2.12	2.45
INDEPENDENT VARIABLES			
RPrice	Average price per unit divided by the PPI(FY) FY 1978-92	93.32	118.06
LnRPrice	Natural log of the above variable FY 1978-92	1.94	2.07
BCon: Food, incl. Fresh and Frozen Meat	Value of product shipments in millions of 1982 dollars FY 1978-92	174,350.56	
LnBCon:	Natural log of the above variable FY 1978-92	5.24	
BCon: Medicinals and Botanicals	Value of product shipments in millions of 1982 dollars FY 1978-92	29,808.06	
LnBCon:	Natural log of the above variable FY 1978-92	4.46	
BCon: Medical Laboratories	Receipts in millions of 1982 dollars FY 1978-92	4,417.79	
LnBCon:	Natural log of the above variable FY 1978-92	3.60	

Table A-1f continued	Description of Independent Variables	911 A&B	1577&AB
BCon: Food and Kindred Products	Value of product shipments in millions of 1982dollars FY 1978-92		83,994.85
LnBCon:	Natural log of the above variable FY 1978-92		4.92
BCon: Commercial Physical & Biological Research; Testing Labs.	Receipts in millions of 1982 dollars FY 1978-92		4,83.89
LnBCon:	Natural log of the above variable FY 1978-92		3.66

See footnote in table A-1a.
Means of GPDI and LnGPDI as in table A-1a.

Table A-1g. Description of Demand Variables for SRMs in Science/Metrology Category

SCIENCE/METROLOGY	Description	Means	
DEPENDENT VARIABLES		935&A	1361A
UNIDOM	Number of units sold by NIST annually in the U.S. FY 1978-92	55	92
	Natural log of the above variable FY 1978-92	1.71	1.88
UNITOT	Total number of units sold by NIST (including exports) FY 1978-92	72	99
LUNITOT	Natural log of the above variable FY 1978-92	1.84	1.93
INDEPENDENT VARIABLES			
RPrice	Average price per unit divided by the PPI(FY) FY 1978-92	83.37	157.29
LnRPrice	Natural log of the above variable FY 1978-92	1.90	2.072.19
BCon: Pharmaceuticals; Soaps; Industrial Chemicals; Agricultural Chemicals	Value of product shipments in millions of 1982 dollars FY 1978-92	116,746.23	
LnBCon:	Natural log of the above variable FY 1978-92	5.06	
BCon: Laboratory Apparatus; Analytical Instruments; Measuring Devices	Value of product shipments in millions of 1982 dollars FY 1978-92	7,091.35	
LnBCon:	Natural log of the above variable FY 1978-92	3.84	

Table A-1g continued	Description of Independent Variables	935&A	1361A
BCon: Metal Cans; Sheeting, Plating and Coating	Value of product shipments in millions of 1982 dollars FY 1978-92		34,603.08
LnBCon:	Natural log of the above variable FY 1978-92		4.54
LoST (dummy)	Low stock in 1978; 1978 = 1; otherwise = 0		0.07

See footnote in table A-1a.
Means of GPDI and LnGPDI as in table A-1a.

Table A-2. Elasticities of Demand for SRMs With Respect to Relative Price of SRMs[1]

	Independent Variables	U.S. DEMAND (UNIDOM)			
	Dependent Variables Model Versions	RPrice BCon V1	LnRPrice LnBCon V2	RPrice GPDI V3	LnRPrice LnGPDI V4
METALS	122 H Cast Iron Car Wheel	-1.11 (-5.91 to 3.69)	-1.04 (-4.76 to 2.68)	-3.49** (-9.93 to 2.46)	-2.90** (-4.2 to -1.75)
	1261 A LA Steel AISI 4340	-1.74* (-2.67 to -0.82)	-2.37* (-3.82 to -0.92)	-1.44** (-1.44 to -2.27)	-1.70** (-2.83 to -0.57)
	53 E Bearing Metal, Lead Base	-3.99** (-6.63 to -1.36)	-5.16** (-8.35 to -1.97)	-3.75** (-4.85 to -2.65)	-1.90** (-2.12 to -1.69)
	627 Zinc Base C	-0.83 (-2.52 to 0.85)	-0.78 (-2.14 to 0.58)	-0.17 (-2.06 to 1.72)	-0.22 (-2.23 to 1.79)
NonMETALS	635 Portland Cement, Blue	0.33 (-0.14 to 0.79)	0.30 (-0.14 to 0.74)	0.52 (-0.21 to 1.24)	0.40 (-0.23 to 1.03)
	637 Portland Cement, Pink	0.83 (0.34 to 1.31)	0.80 (0.30 to 1.30)	0.67** (-0.08 to 1.42)	0.58* (-0.14 to 1.32)
	120 BC Phosphate Rock	-1.11* (-2.24 to 0.01)	-1.17* (-2.24 to -0.10)	-0.86* (-1.82 to -0.10)	-0.88** (-1.85 to 0.09)
	27 F Iron Ore, Sibley Powder	0.90 (-1.48 to 3.27)	1.00 (-1.66 to 3.66)	-1.42** (-2.48 to 0.37)	-1.25** (-2.27 to -0.27)
Ch Rb Pl	370 E Zinc Oxide Rubber Composite	-1.18* (-5.51 to 3.15)	-1.32* (-2.89 to 0.25)	-1.02 (-2.43 to 0.39)	-1.20 (-2.79 to 0.39)
	371 GH Sulfur Rubber Composite	0.27 (-1.25 to 1.79)	0.11 (-1.70 to 1.92)	-0.36 (-1.80 to 1.07)	0.20 (-1.52 to 1.92)
	40 H Sodium Oxalate Redox	-0.82* (-1.68 to 0.05)	-0.67 (-1.51 to 0.17)	-0.66** (-1.17 to -0.14)	-0.55** (-1.05 to 0.06)
ENGINEERING	185 EFG Pot. Hydrogen Phthalate pH	-0.00 (-1.40 to 1.39)	-0.13 (-1.56 to 1.30)	-0.19 (-0.93 to 0.54)	-0.31 (-1.05 to 0.43)
	187 BC Borax pH	-1.02 (-3.04 to 1.01)	-0.79 (-2.81 to 1.23)	-0.82* (-1.75 to 0.11)	-0.78* (-2.58 to 0.16)
	189 A Potassium Tetroxalate pH	1.14 (-2.67 to 4.96)	1.02 (-1.93 to 3.97)	-0.39 (-1.84 to 1.07)	-0.25 (-1.51 to 1.01)
	39 I Benzoic Acid Combustion	-1.82** (-2.86 to -0.78)	-1.78** (-2.77 to -0.79)	-0.93** (-1.86 to 0.00)	-0.83* (-1.75 to 0.09)

	TableA-2 continued	A1	A2	A3	A4
ENVIRONMENTAL	1575 Pine Needles	-0.88 (-3.22 to 1.47)	-1.13 (-3.55 to 1.29)	-0.35 (-1.37 to 2.06)	0.20 (-1.61 to 2.01)
	1620 &AB Sulfur in Fuel Oil, 5%	-0.19 (-1.20 to 0.82)	-0.38 (-1.37 to 0.61)	-0.04 (-0.73 to 0.81)	-0.09 (-0.79 to 0.61)
	1622 ABC Sulfur in Fuel Oil, 2%	0.41 (-1.48 to 2.29)	0.15 (-1.48 to 1.78)	0.28 (-0.78 to 1.34)	0.21 (-0.78 to 1.20)
	1625 Permeation Tube, 10 cm	-1.11 (-3.13 to 0.91)	-0.96 (-3.40 to 1.48)	-2.07** (-3.08 to -1.06)	-2.10** (-3.26 to -0.94)
	1635 Trace Elements in Coal	-0.85 (-2.91 to 1.20)	-0.08 (-2.72 to 2.56)	-1.97* (-4.10 to 0.16)	-2.67* (-5.86 to 0.52)
HEALTH	911 AB Cholesterol	-0.68 (-2.70 to 1.34)	-0.61 (-2.64 to 1.42)	-0.38 (-0.93 to 0.37)	-0.40 (-1.10 to 0.30)
	1577 &AB Bovine Liver	-0.67 (-3.09 to 1.75)	-0.28 (-2.92 to 2.36)	0.38 (-1.15 to 1.91)	0.73 (-0.95 to 2.41)
SC MT	935 A Potassium Dichromate-UV	0.23 (-1.37 to 1.82)	-0.23 (-2.04 to 1.58)	0.35 (-0.35 to 1.05)	0.16 (-0.65 to 0.97)
	1361 CU & CR Coating on Steel	-0.99** (-1.92 to -0.06)	-1.06* (-2.00 to -0.12)	-1.18* (-2.39 to 0.02)	-1.29* (-2.46 to -0.12)

[1]Estimated ranges in parantheses are calculated using as upper and lower bounds the upper and lower bounds of the 95% confidence intervals around the estimated coefficients of the price variables.

****significant at 0.05 level (two-tail test).**
*** significant at 0.10 level (two-tail test).**

Table A-3a. SRM 122H Cast Iron Car Wheel - Group 101 Ferrous Metals

SIC Industries: Metal Industries; Industrial Machinery and Equipment.

Regression Results for Estimation of U.S. Demand by OLS	
DEPENDENT VARIABLE	**LUNIDOM** **Nat. Log of Domestic Sales in Units (150 g)**
INDEPENDENT VARIABLES	**Regression Coefficients**
Constant	**5.7513** **(9.1373)** **(t = 0.63)**
RPrice (LnRPrice) **Relative Price**	**-2.9779**** **(0.5646)** **(t = -5.27)**
GPDI (LnGPDI) **Gross Private Domestic** **Investment (million $)**	**0.9381** **(0.7577)** **(t = 1.24)**
No. of Observations: **R^2:** **Adjusted R^2:** **Durbin-Watson:**	**15** **0.72** **0.69** **2.5215**

Standard error and t-statistic in parentheses
**** statistically significant at the 0.05 level (two-tail test)**

Table A-3b. SRM 1261A LA Steel AISI 4340 - Group 101 Ferrous Metals

SIC Industries: Primary Metal Industries; Fabricated Metal Products.

Regression Results for Estimation of U.S. Demand by OLS	
DEPENDENT VARIABLE	**LUNIDOM** **Nat. Log of U.D. Demand in Units (disk)**
INDEPENDENT VARIABLES	**Regression Coefficients**
Constant	**0.8944** **(10.6770)** **(t = 0.08)**
RPrice (LnRPrice) **Relative Price**	**-1.6987**** **(0.5182)** **(t = -3.28)**
GPDI (LnGPDI) **Gross Private Domestic Investment (million $)**	**-0.9410** **(0.9027)** **(t = -1.04)**
No. of Observations: **R^2:** **Adjusted R^2:** **Durbin-Watson:**	**15** **0.5090** **0.4271** **0.6880**

Standard error and t-statistic in parentheses
**** statistically significant at the 0.05 level (two-tail test)**

Table A-3c. SRM 53E Bearing Metal, Lead Base - Group 102 Nonferrous Metals

SIC Industries: Primary Metal Industries; Industrial Machinery and Equipment.

Regression Results for Estimation of U.S. Demand by OLS	
DEPENDENT VARIABLE	LUNIDOM Nat. Log of Domestic Sales in Units (150 g)
INDEPENDENT VARIABLES	Regression Coefficients
Constant	28.3847 (14.3933) (t = 1.97)
RPrice (LnRPrice) Relative Price	-1.9017* (0.9881) (t = -1.92)
GPDI (LnGPDI) Gross Private Domestic Investment (million $)	-1.3113 (1.1533) (t = -1.14)
No. of Observations: R^2: Adjusted R^2: Durbin-Watson:	15 0.53 0.45 1.1251

Standard error and t-statistic in parentheses
*** statistically significant at the 0.10 level (two-tail test)**

Table A3-d. SRM 120 B&C Phosphate Rock - Group 111 Geological Materials and Ores

SIC Industries: Chemicals and Allied Products; Stone, Clay, and Glass Products.

Regression Results for Estimation of U.S. Demand by OLS	
DEPENDENT VARIABLE	LUNIDOM Nat. Log of Domestic Sales in Units (100 ml)
INDEPENDENT VARIABLES	Regression Coefficients
Constant	9.9018
RPrice (LnRPrice) Relative Price	-0.8757* (0.4394) (t = -1.99)
GPDI (LnGPDI) Gross Private Domestic Investment (million $)	-0.1512 (0.1670) (t = -0.91)
LOST (Dummy) Low stock in 1987	-1.1437** (0.2426) (t = -4.72)
No. of Observations: R^2: Adjusted R^2: Durbin-Watson:	15 0.7268 0.6523 2.2190

Standard error and t-statistic in parentheses
*** statistically significant at the 0.10 level (two-tail test)**

Table A-3e. SRM 27F Iron Ore - Group 111 Geological Materials and Ores

SIC Industries: Iron Ores and Ferroalloy Ores; Blast Furnaces and Steel Mills.

Regression Results for Estimation of U.S. Demand by OLS	
DEPENDENT VARIABLE	**LUNIDOM** **Nat. Log of Domestic Sales in Units (150 g)**
INDEPENDENT VARIABLES	**Regression Coefficients**
Constant	**4.9131** **(6.1942)** **(t = 0.79)**
RPrice (LnRPrice) **Relative Price**	**-1.2507**** **(0.4560)** **(t = -2.74)**
GPDI (LnGPDI) **Gross Private Domestic Investment (million $)**	**0.3087** **(0.5076)** **(t = 0.61)**
No. of Observations: **R^2:** **Adjusted R^2:** **Durbin-Watson:**	**15** **0.3961** **0.2955** **2.7793**

Standard error and t-statistic in parentheses
**** statistically significant at the 0.05 level (two-tail test)**

Table A-3f. SRM 40H Sodium Oxalate Redox - Group 104 High Purity Materials

SIC Industries: Chemicals and Allied Products; Medical and Dental Laboratories.

Regression Results for Estimation of U.S. Demand by OLS	
DEPENDENT VARIABLE	LUNIDOM Nat. Log of Domestic Sales in Units (150 g)
INDEPENDENT VARIABLES	Regression Coefficients
Constant	8.3756 (4.8114) (t = 1.74)
RPrice (LnRPrice) Relative Price	-0.5533** (0.2266) (t = -2.44)
GPDI (LnGPDI) Gross Private Domestic Investment (million $)	-0.0864 (0.3886) (t = -0.22)
No. of Observations: R^2: Adjusted R^2: Durbin-Watson:	15 0.4043 0.3051 2.6970

Standard error and t-statistic in parentheses
**** statistically significant at the 0.05 level (two-tail test)**

Table A-3g. SRM 187 B&C Borax pH - Group 201 Ion Activity

SIC Industries: Alkalies, Industrial Gases and Chemicals, Inorganic Pigments; Commercial Pysical and Biological Research; Testing Laboratories.

Regression Results for Estimation of U.S. Demand by OLS	
DEPENDENT VARIABLE	LUNIDOM Nat. Log of Domestic Sales in Units (150 g)
INDEPENDENT VARIABLES	Regression Coefficients
Constant	-2.2767 (6.2262) (t = -0.37)
RPrice (LnRPrice) Relative Price	-0.7809* (0.4432) (t = -1.76)
GPDI (LnGPDI) Gross Private Domestic Investment (million $)	0.7650 (0.5722) (t = 1.34)
No. of Observations: R^2: Adjusted R^2: Durbin-Watson:	15 0.2063 0.0740 2.0962

Standard error and t-statistic in parentheses
* statistically significant at the 0.10 level (two-tail test)

Table A-3h. SRM 39I Benzoic Acid Combustion - Group 203 Thermodynamic Properties

SIC Industries: Explosives; Petroleum Refining; Laboratory Apparatus, Analytical Instruments, Measuring Devices; Electricity.

Regression Results for Estimation of U.S. Demand by OLS	
DEPENDENT VARIABLE	LUNIDOM Nat. Log of Domestic Sales in Units (150 g)
INDEPENDENT VARIABLES	Regression Coefficients
Constant	-2.7934 (6.8760) (t = -0.41)
RPrice (LnRPrice) Relative Price	-0.8327* (0.4208) (t = -1.98)
GPDI (LnGPDI) Gross Private Domestic Investment (million $)	0.8884 (0.5884) (t = 1.51)
No. of Observations: R^2: Adjusted R^2: Durbin-Watson:	15 0.2524 0.1278 0.9123

Standard error and t-statistic in parentheses
* statistically significant at the 0.10 level (two-tail test)

Table A-3i. SRM 1625 SO_2 Permeation Tube - Group 107 Analyzed Gases

SIC Industries: Electiricty; Commerical Physical and Biological Research; Testing Laboratories.

Regression Results for Estimation of U.S. Demand by OLS	
DEPENDENT VARIABLE.	LUNIDOM Nat. Log of Domestic Sales in Units (150 g)
INDEPENDENT VARIABLES	Regression Coefficients
Constant	37.7722 (12.5455) (t = 3.01)
RPrice (LnRPrice) Relative Price	-2.0958** (0.5310) (t = -3.95)
GPDI (LnGPDI) Gross Private Domestic Investment (million $)	-1.7341 (1.0375) (t = 1.67)
No. of Observations: R^2: Adjusted R^2: Durbin-Watson:	15 0.7494 0.7076 2.4445

Standard error and t-statistic in parentheses
** statistically significant at the 0.05 level (two-tail test)

Table A-3j. SRM 1635 Trace Elements in Coal - Group 108 Fossil Fuels

SIC Industries: Bituminous Coal, Lignite, and Anthracite; Carbon Black Channel and Furnace Processes; Asphalt and Paving Mixtures, and Blocks.

Regression Results for Estimation of U.S. Demand by OLS	
DEPENDENT VARIABLE	**LUNIDOM** **Nat. Log of Domestic Sales in Units (150 g)**
INDEPENDENT VARIABLES	**Regression Coefficients**
Constant	52.9784 (12.2346) (t = 4.33)
RPrice (LnRPrice) **Relative Price**	-2.6676* (1.4503) (t = -1.84)
GPDI (LnGPDI) **Gross Private Domestic Investment (million $)**	-2.7991** (0.9833) (t = 2.85)
BKOR (dummy) **Backorders filled in 1980**	0.8157 (0.5181) (t = 1.57)
No. of Observations: **R^2:** **Adjusted R^2:** **Durbin-Watson:**	15 0.7357 0.6637 1.9345

Standard error and t-statistic in parentheses
**** statistically significant at the 0.05 level (two-tail test)**
*** statistically significant at the 0.10 level (two-tail test)**

Table A-3k. SRM 1361A Cu and Cr Coating on Steel - Group 207 Metrology

SIC Industries: Metal Cans; Sheeting, Plating, Coating.

Regression Results for Estimation of U.S. Demand by OLS	
DEPENDENT VARIABLE	**LUNIDOM** **Nat. Log of Domestic Sales in Units (150 g)**
INDEPENDENT VARIABLES	**Regression Coefficients**
Constant	**0.5785**
RPrice (LnRPrice) **Relative Price**	**-1.2873**** **(0.5274)** **(t = -2.44)**
GPDI (LnGPDI) **Gross Private Domestic Investment (million $)**	**0.7836** **(0.8440)** **(t = 0.93)**
LOST (Dummy) **Low Stock in 1978**	**-1.2940*** **(0.3651)** **(t = -8.05)**
No. of Observations: **R^2:** **Adjusted R^2:** **Durbin-Watson:**	**15** **0.8568** **0.8178** **1.8432**

Standard error and t-statistic in parentheses
**** statistically significant at the 0.05 level (two-tail test)**
*** statistically significant at the 0.10 level (two-tail test)**

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