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A MODIFIED OPTIMAL ALGORITHM FOR ACTIVE STRUCTURAL CONTROL

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ABSTRACT

This study presents a modification to two linear optimal control algorithms, namely classical and instantaneous, to achieve a greater reduction in structural displacements and control forces. The modification consists of building a library of gain matrices and selecting the gain matrix that would result in the maximum control force without exceeding the control system capacity. The modification was used to compute the response of several single-degree-of-freedom (SDOF) systems, a multi-degree-of-freedom (MDOF) system, and a base isolated structure. Based on the examples considered, the modification results in a reduction of up to 45 % in the peak control forces and structural displacements as compared to existing algorithms.

The study shows that the external excitation influences the selection of the control system parameters such as controller capacity and gain matrices. These parameters, therefore, should be determined according to the seismic excitation intensity expected at the site.

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1. INTRODUCTION

Active control may be used to reduce the response of structures to earthquakes and wind. When an active control system is used, the external loads are partially resisted by control forces, the magnitudes of which are computed by control algorithms.

Many control algorithms have been introduced and tested numerically and experimentally to prove their reliability and efficiency. Among them are classical optimal control, instantaneous optimal control, pole assignment, independent modal space control, predictive control, and many others. References to these algorithms can be found in [7]. Of these, two algorithms, classical and instantaneous, have been used extensively to control the response of structures to seismic loads. Both algorithms are effective in reducing the response. This study briefly reviews these two algorithms and proposes a modification to improve their performance in reducing the control forces and the structural displacements. The modified algorithms, which use varying weighting matrices to compute the control forces, are examined for several single and multi-degree-of freedom structures under different seismic excitations to demonstrate their effectiveness.

2. CLASSICAL AND INSTANTANEOUS OPTIMAL CONTROL ALGORITHMS

The governing differential equation of motion for an n-degree of freedom structure with mass matrix M, damping matrix C, and stiffness matrix K with m controllers is given by:

$$\mathrm{M}\ \ddot{x}(t) + \mathrm{C}\ \dot{x}(t) + \mathrm{K}\ x(t) = \mathrm{D}\ u(t) + \mathrm{E}\ f(t) \tag{2.1}$$

where the n-dimensional vector $x(t)$ represents the displacement, the r-dimensional vector $f(t)$ the external excitation, and the m-dimensional vector $u(t)$ the control force. The matrix D (size m x n) and matrix E (size n x n) define the locations of the controllers and excitations, respectively.

Using the state-space representation, equation (2.1) takes the form:

$$\dot{z}(t) = \mathrm{A}\ z(t) + \mathrm{B}\ u(t) + \mathrm{H}\ f(t) \tag{2.2}$$

where $z(t)=\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}$ is a 2n-dimensional state vector. The system matrix A, and the matrices B and H are given in [7].

2.1 Classical Optimal Control

In the classical linear optimal control theory, the control force vector $u(t)$ is selected by minimizing a cost function over the response duration. The function to be minimized is a linear quadratic expression which takes the form:

$$\mathrm{J} = \int_0^{t_f} [z^T(t)\ \mathrm{Q}\ z(t) +\ u^T(t)\ \mathrm{R}\ u(t)]\ \mathrm{dt} \tag{2.3}$$

where t_f is the duration of excitation, Q (size 2n x 2n) is a positive semi-definite matrix, and R (size m x m) is a positive definite matrix. The matrices Q and R are referred to as weighting matrices which are assumed constant during the excitation. If the elements of Q are larger than those of R, the reduction of $z(t)$ is given priority over the reduction of $u(t)$. For a closed-loop control configuration, minimizing equation (2.3) would result in a control force vector $u(t)$ regulated only by the state vector $z(t)$ such that:

$$u(t) = \mathrm{G}(t)\ z(t) \tag{2.4}$$

and

$$\mathrm{G}(t) = -\frac{1}{2}\ \mathrm{R}^{-1}\ \mathrm{B}^{\mathrm{T}}\ P(t) \tag{2.5}$$

where the matrix $\mathrm{G}(t)$ (size m x 2n) represents the gain matrix, and the matrix $P(t)$ (size 2n x 2n) is the solution of the classical Riccati equation which after ignoring the excitation term reduces to:

$$\dot{P}(t) + P(t)\ \mathrm{A} - \frac{1}{2}\ P(t)\ \mathrm{B}\ \mathrm{R}^{-1}\ \mathrm{B}^{\mathrm{T}}\ P(t) + \mathrm{A}^{\mathrm{T}}\ P(t) + 2\ \mathrm{Q} = 0, \qquad P(t_f) = 0 \tag{2.6}$$

Equation (2.6) must be solved backward in time since $P(t)$ is specified at t_f. In structural applications, the elements of $P(t)$ remain constant during the response and drop to zero at t_f [7]. Setting $P(t)$ to a constant, equation (2.6) takes the form :

$$P\,A - \frac{1}{2}\,P\,B\,R^{-1}\,B^{T}\,P + A^{T}\,P + 2\,Q = 0 \tag{2.7}$$

The gain matrix $G(t)$ which also remains constant during the excitation is equal to:

$$G = -\frac{1}{2}\,R^{-1}\,B^{T}\,P \tag{2.8}$$

2.2 Instantaneous Optimal Control

Yang et al [9] argued that the above procedure is not truly optimal because the excitation term in equation (2.6) is ignored. In addition, the method is not feasible for open-loop and closed-open-loop systems where the excitation term cannot be ignored and the equation must be solved backward in time with *a priori* knowledge of the excitation term. Consequently, they proposed the instantaneous optimal control algorithm which is based on minimizing the cost function at each instant of time rather than over the response duration. The cost function to be minimized is the integrand in equation (2.3), i.e.

$$J(t) = z^{T}(t)\,Q\,z(t) + u^{T}(t)\,R\,u(t) \tag{2.9}$$

After some mathematical manipulation, the control force vector $u(t)$ for a closed-loop system may be computed from equation (2.4) with a constant gain matrix in the form:

$$G = -\frac{1}{2}\,\Delta t\,R^{-1}\,B^{T}\,Q \tag{2.10}$$

It is apparent that the formulation of the gain matrix is arbitrary and depends on the choice of time interval Δt and weighting matrices R and Q. In another study, Yang et al [12] indicated that the selection of Δt and Q should satisfy Lyapunov stability and energy criteria. They suggested several methods for selecting Q and used them to solve different cases.

It should be noted that for a given control force, the classical and instantaneous optimal control algorithms with constant weighting matrices result in a reduction in response. By using variable Q or R matrices, the same reduction in response can be achieved under a smaller control force, or put another way, the same control force would result in a larger reduction in displacements than those attained with constant Q and R matrices. Moreover, one of the features of an active control system is that the structural characteristics may be adjusted to account for changes in the excitation. This may be accomplished by replacing the system matrix A by [A+BG] which would result in changing the damping and stiffness coefficients during the response. For fixed gain matrices, the system matrix [A+BG] will remain the same during the response with no changes in the modified stiffness and damping coefficients. One may need, however, to change these parameters during the response by varying the gain matrices according to the excitation, structural properties, and controller capacity. For these reasons, varying gain matrices can improve the performance of a control system compared to constant gain matrices.

3. MODIFIED OPTIMAL CONTROL ALGORITHMS

It is known that the larger the elements of the Q matrix, the larger is the control force vector and consequently, the smaller is the displacement vector. If one is interested in minimizing vibrations, the elements of Q should be kept large to achieve a large control force vector. Since the capacity of a control system is limited, it is not always possible to achieve the required control force vector to keep the vibrations within the desired limits. A modification to the algorithms discussed in the previous section is presented herein. It consists of varying the weighting matrix Q during the response to minimize the displacement and control force vectors. The modification makes the optimum use of a control system with a given capacity by varying the weighting matrix Q to achieve the maximum reduction in the response without exceeding the capacity. The necessary steps for the analysis are described below.

1- Select different Q matrices. The selection of Q matrices depends on the range of control forces to be used in the analysis. This range should include the control system capacity and allow the optimum use of control force in the analysis. A series of ten Q matrices were used for this study. Minimum and maximum Q matrices were selected to correspond to control forces of one-half and twice the system capacity, respectively. Between these two limits, eight equally spaced Q matrices were chosen.

2- Establish a library of gain matrices corresponding to the Q matrices using equations (2.7 and 2.8) for the classical algorithm and equation (2.10) for the instantaneous algorithm. Rank the gain matrices in decreasing order in the library as shown in Figure 3.1. The gain matrices are computed and stored off-line; hence, no additional on-line computations are encountered during the response.

3- Specify a reference control force U_{ref}. In this study, U_{ref} is assumed as 95 % of the controller capacity U^{*}_{b}. The reason for this assumption will be discussed in the next step, thus:

$$U_{ref} = 0.95\ U_b \tag{3.1}$$

4- At time t, estimate the response at the next time step $t + \Delta t$ using the first three terms of the Taylor series expansion:

$$z_{est}(t + \Delta t) = \frac{5}{2}\ z(t)\ \text{-}\ 2\ z(t - \Delta t) + \frac{1}{2}\ z(t - 2\Delta t) \tag{3.2}$$

5- Starting with the largest gain matrix in the library, estimate the control force vector $u(t)$ from equation (2.4). If this force exceeds U_{ref}, select the next largest gain matrix and repeat the process. If the force does not exceed U_{ref}, use the gain matrix to compute the control force vector for the next time step. It should be noted that the computed and estimated (see step 4) control force vectors will not be identical. The difference between the two, however, was less than 5 % for the examples considered. This is the reason that U_{ref} was selected as 95 % of U_b in equation (3.1).

The above procedure has a negligible effect on the CPU time needed to perform the on-line computations. For all the examples considered in this study, the CPU time required to sweep the gain matrices (usually not all the ten gain matrices are swept) at a given time step was extremely small, consequently, the modification has minor or no effect on the time delay of the control system.

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* The selection of the controller capacity is based on the trade-off between the cost of the control system and the desired reduction in response.

The above procedure guarantees that the gain matrix used for the next time step will result in the optimum use of the control system without exceeding the capacity. A diagram for the modification to be added to the current optimal control algorithms is shown in Figure 3.1.

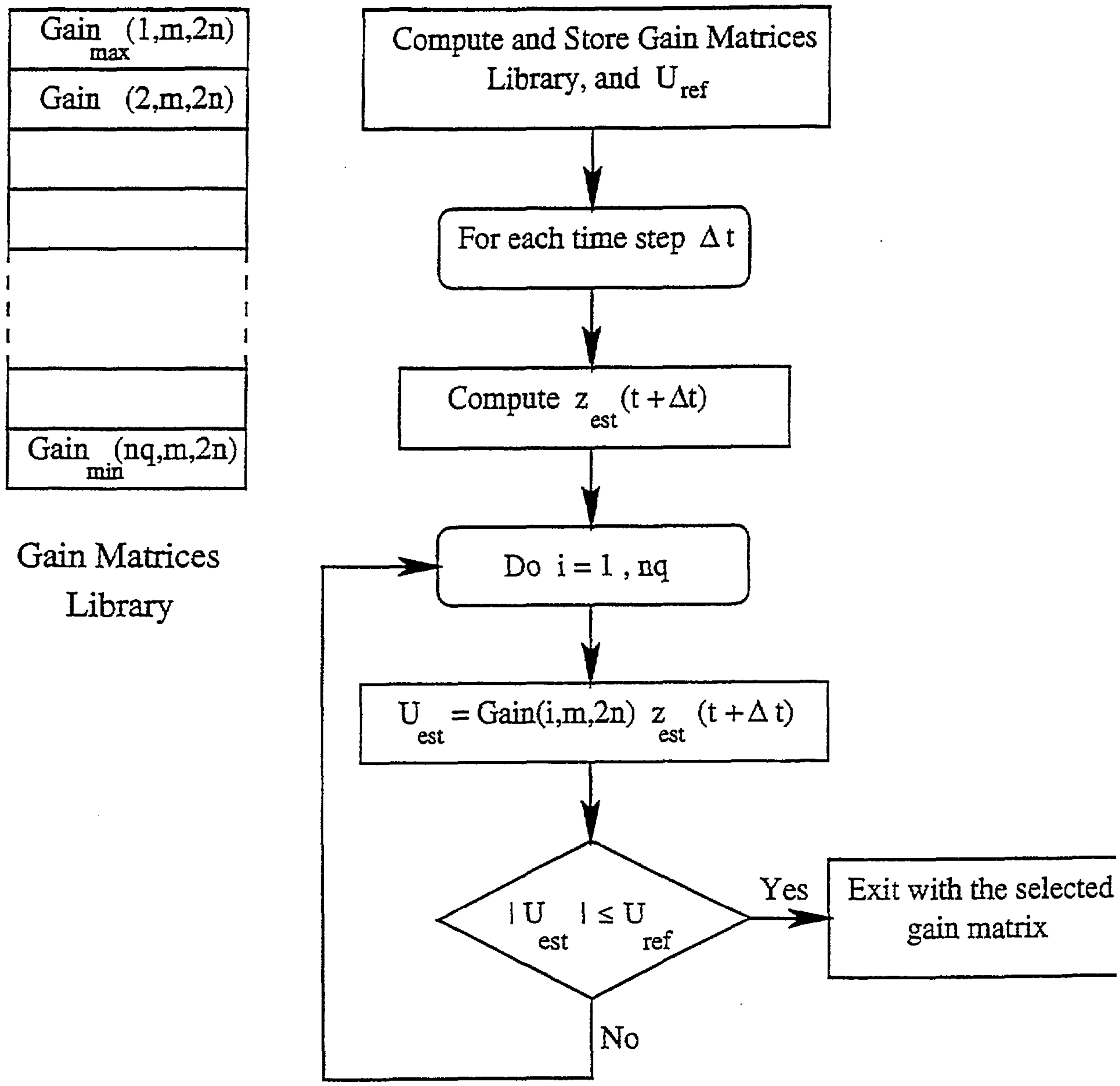

Figure 3.1 Block Diagram for the Modification to the Optimal Control Algorithms.

4. NUMERICAL EXAMPLES

To illustrate the effectiveness of the modification in reducing displacements and control forces, several examples -- a series of single-degree of freedom structures, a multi degree of freedom structure, and a base isolated structure -- are considered. For each example, comparisons between the modified algorithm (variable Q matrices) and the classical or instantaneous algorithms (constant Q matrices) are carried out in two ways. The first compares the responses from the classical or instantaneous algorithm with the responses from the modified algorithm for the same displacements, to show the reduction in the control force. The second compares the responses from the classical or instantaneous algorithm with the responses from the modified for the same control forces, to illustrate the reduction in the displacement.

4.1 SDOF Structures Using Modified Classical Algorithm

Four linear SDOF structures with periods of 0.2 s, 0.4 s, 1.0 s, and 2.0 s, a mass of 16 x 10^3 kg, and a damping ratio of 5 %, with a single controller are used to demonstrate the advantages of the modified algorithm over the classical. The Q and R matrices are selected as:

$$Q = q\, I_{(2n \times 2n)} \qquad\qquad R = I_{(m \times m)} \tag{4.1}$$

where I is the identity matrix. The structures were subjected to the S00E component of the El Centro record of the Imperial Valley earthquake of May 18, 1940, with a peak ground acceleration of 0.348g. The structures were first analyzed using the classical algorithm with a constant Q matrix. Several values of q ranging from zero (no control force) to 10^6 (an arbitrary large number to provide a substantial control force) were used in the analysis. Figure 4.1(a) shows the variation of the maximum controlled displacement as a ratio of the uncontrolled displacement and Figure 4.1(b) shows the control force as a ratio of the structure's mass for different q values. As noted from the figures, the larger the q, the larger the control force, and as expected, the smaller the displacement.

The four SDOF structures were analyzed for different U_{ref} using the modified algorithm with a library of ten gain matrices for each. The Q matrices were selected as discussed in step 1 of section 3. The results are presented in Table 4.1. Shown in the table are the maximum relative displacement x_{max} and absolute acceleration a_{max} for each structure with no control and the maximum displacement x_{max}, acceleration a_{max}, and control force U_{max} computed by the modified algorithm along with the corresponding U_{ref}, q_{max}, and q_{min}. As expected, the computed control force U_{max} may be somewhat different from the reference force U_{ref}, but the difference between the two remains within 5 % of U_{ref}.

In order to demonstrate the efficiency of the proposed modification, the same four structures were analyzed using the classical algorithm with a constant Q matrix. Each structure was analyzed twice; first with a Q matrix which resulted in the same displacement as that of the modified algorithm, and second with a Q matrix which resulted in the same maximum control force. The results together with the root mean square of the control force RMS(U) are presented in Tables 4.2(a) and 4.2(b) where a significant reduction in the maximum displacement or the maximum control force is observed. For the same displacement, Table 4.2(a), the modified algorithm gives a reduction in the control force of up to 45 %, and for the same control force, Table 4.2(b), a reduction in the displacement of up to 48 % is achieved. Comparison of the RMS values in Table 4.2(b) shows that the RMS values from the modified algorithm are greater than those from the classical indicating that the former results in a more efficient use of the controller in reducing the response. Figure 4.2 shows the relative displacement, absolute acceleration, and control force computed by the modified algorithm for the first ten seconds of the response for the structure with the period T=0.4 s. Also

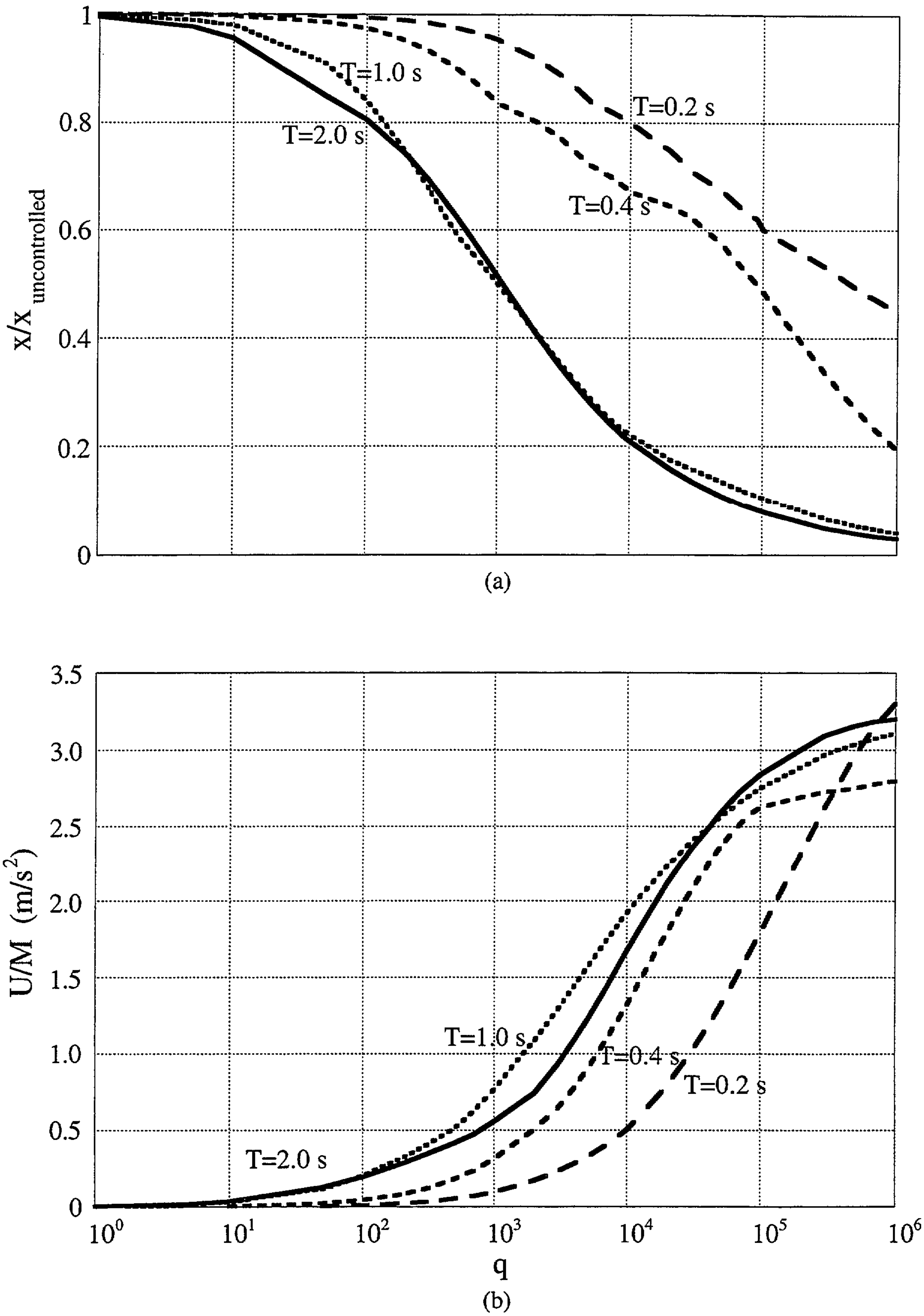

Figure 4.1 Traditional Classical Optimal Control for SDOF Structures.
(a) Variation of x_{max} with q (b) Variation of U/M with q

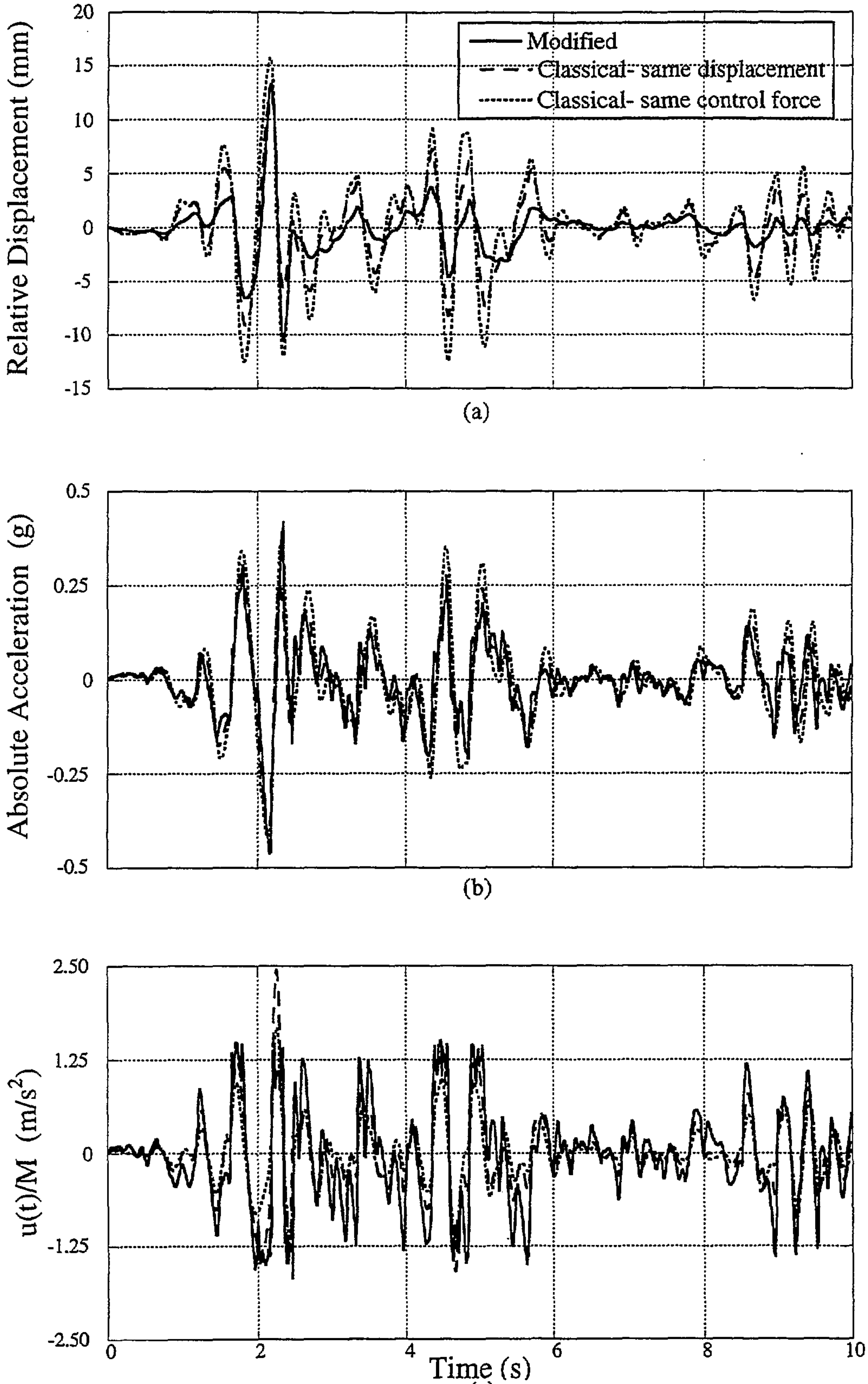

Figure 4.2 Response of the 0.4 s SDOF Structure Using Modified and Traditional Analyses.
(a) Relative Displacement (b) Absolute Acceleration (c) Control Force

Table 4.1. Summary of the Response of SDOF Structures Using the Modified Algorithm.

T	No Control		U_{ref}	q_{max}	q_{min}	x_{max}	a_{max}	U_{max}
s	x_{max} mm	a_{max} g	kN	x 10^5	x 10^5	mm	g	kN
0.2	6.6	0.65	2790	10	0.30	3.0	0.45	2791
0.4	24.4	0.62	2540	10	0.07	13.5	0.46	2664
1.0	128.0	0.52	2290	0.7	0.01	28.7	0.25	2301
2.0	176.5	0.18	2040	0.7	0.01	24.9	0.15	2045

Table 4.2(a). Comparison of Classical and Modified Algorithms for Same Displacement.

T	Classical				Modified				%
s	x_{max} mm	a_{max} g	U_{max} kN	RMS(U) kN	x_{max} mm	a_{max} g	U_{max} kN	RMS(U) kN	Reduction U_{max}
0.2	3.0	0.37	4890	731	3.0	0.45	2791	785	43
0.4	13.5	0.40	3978	739	13.5	0.46	2664	986	32
1.0	28.7	0.26	3094	665	28.7	0.25	2301	869	26
2.0	24.9	0.25	3726	810	24.9	0.15	2045	843	45

Table 4.2(b). Comparison of Classical and Modified Algorithms for Same Control Force.

T	Classical				Modified				%
s	x_{max} mm	a_{max} g	U_{max} kN	RMS(U) kN	x_{max} mm	a_{max} g	U_{max} kN	RMS(U) kN	Reduction U_{max}
0.2	3.6	0.42	2791	470	3.0	0.45	2791	785	13
0.4	16.0	0.42	2664	528	13.5	0.46	2664	986	16
1.0	41.4	0.22	2301	526	28.7	0.25	2301	869	30
2.0	48.5	0.15	2045	541	24.9	0.15	2045	843	48

RMS values are computed for the first 15 seconds of the response.

shown are the responses computed by the classical algorithm using a) the same displacement and b) the same control force as those of the modified algorithm. The figure demonstrates the advantages of the modified algorithm.

In order to test the modified algorithm for different excitations, the structure with the period of 0.4 *s* was analyzed using the S16E component of the accelerogram for the Pacoima Dam record of the San Fernando earthquake of Feb. 9, 1971, with a peak ground acceleration of 1.17 g. The U_{ref} and the library of gain matrices used in the previous example were also used in this analysis. The modified algorithm did not result in a reduction in the control force or displacement for this excitation. When, however, the ground acceleration was scaled down to 30 % (approximately the same peak ground acceleration as the S00E component of El Centro, 1940) reductions in control force of about 31 % and in displacement of about 38 % were noted. This illustrates that a control system designed for a given excitation may not be effective for another excitation, and emphasizes the fact that the control system should be considered as an integral part of the structural system and designed based on the expected seismic excitation. Since the ground motion at any location varies for different earthquakes, a statistical analysis of a set of records scaled to the expected peak ground acceleration may be used to obtain the controller capacity and the corresponding library of gain matrices when designing a control system.

4.2 MDOF Structure Using Modified Classical Algorithm

A six-story frame was used to test the effectiveness of the modified algorithm in reducing the response and the control force in a MDOF structure. The column stiffnesses are k_i = 3000 kN/m, floor masses $m_i = 1.0 \times 10^3$ kg, and the damping ratio is assumed to be 5 %. A controller is placed on the top floor and U_{ref} is assumed to be 760 kN. The excitation is the S00E component of the El Centro record of the Imperial Valley earthquake of May 18, 1940. The Q and R matrices are the same as in equation (4.1).

A procedure similar to that used in the previous example was also used to obtain the displacements and control forces for this example. The structure was first analyzed using the classical algorithm where the relationship between Q and the maximum control force was found. Ten Q matrices were determined as before and the structure was analyzed with the modified algorithm. Again, the classical analysis was carried out twice: once for the top floor displacement the same as that of the modified algorithm and then for the same maximum control force. The results for the three analyses are shown in Table 4.3 where for the same control force (765 kN), a reduction in the top floor displacement of approximately 21 % (26.1 mm compared to 33.0 mm) and for the same maximum top floor displacement (26.1 mm), a reduction in the control force of 25 % (765 kN compared to 1019 kN) are achieved with the modified algorithm. Observations similar to those for a SDOF system regarding the RMS of the control force can also be made for MDOF structures. Figure 4.3 shows the top floor displacement, acceleration, and the control force using the modified and classical algorithms for the first ten seconds of the response. The figure demonstrates the advantages of the modified algorithm.

4.3 Base-Isolated Structure Using Modified Instantaneous Algorithm

The six story frame of the previous example supported at its base by an isolator with a linear stiffness k_b of 90 kN/m, a mass m_b of 1.4×10^3 kg, and a damping ratio of 20 % is considered in this example. A controller is placed at the base to control the displacement of the isolator. The fundamental periods are 0.48 s for the unisolated and 1.85 s for the isolated structures. For the instantaneous optimal control, Yang et al [12] indicated that for the Q matrix to satisfy the stability

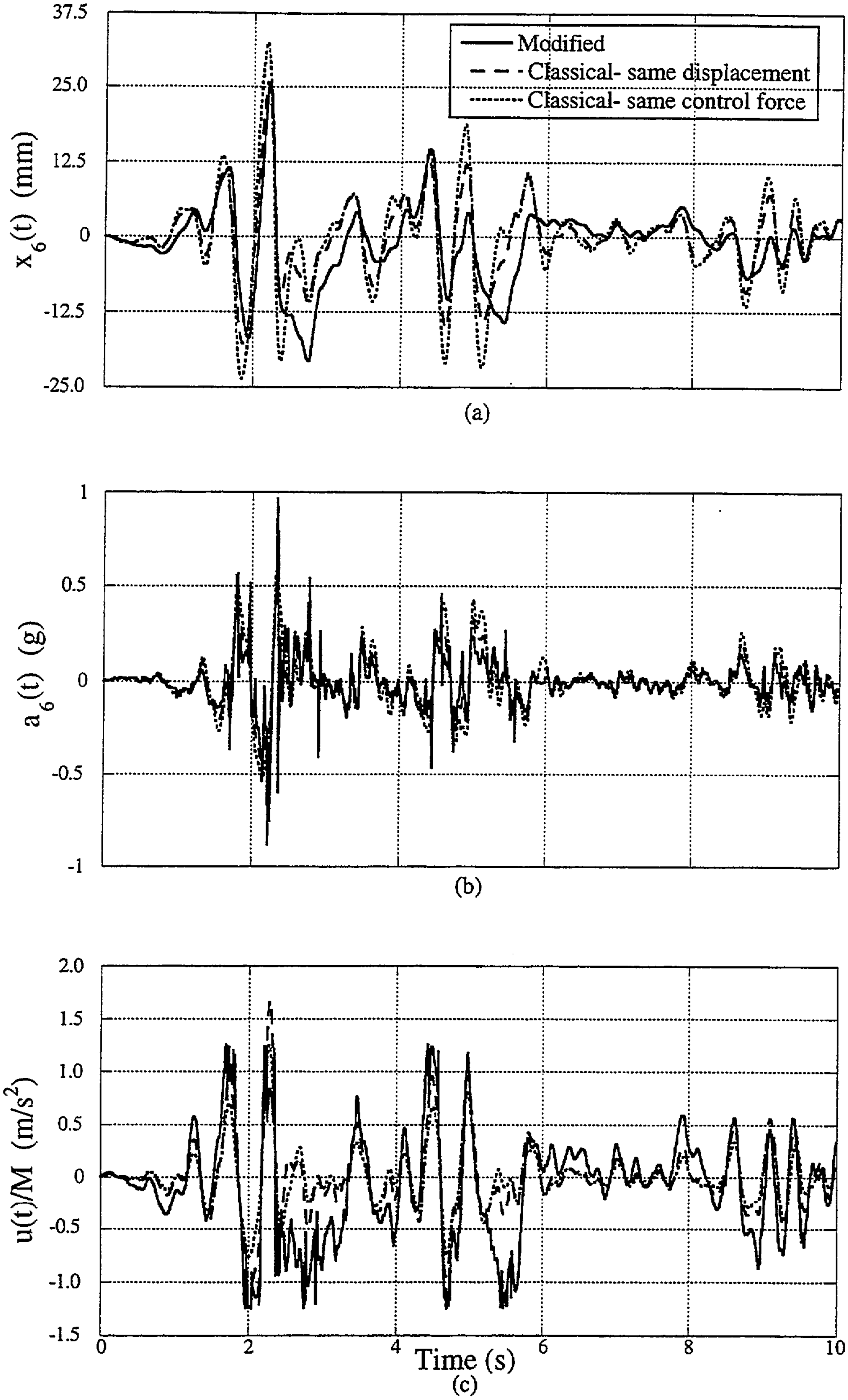

Figure 4.3 Response of the Six-Story Building Using Modified and Traditional Analyses.
(a) Relative Displacement (b) Absolute Acceleration (c) Control Force

Table 4.3. Summary of the Response of the Six-Story Building Using Modified and Classical Algorithms.

Level	No Control		Modified Algorithm U_{max}=765 *kN*		Classical Algorithm U_{max}= 1019 *kN*		Classical Algorithm U_{max}=765 *kN*	
	x max mm	a max g	x max mm	a max *g*	x max mm	a max g	x max mm	a max g
Top	57.2	1.09	26.1	0.95	26.1	0.44	33.0	0.59
5	53.6	0.99	22.9	0.54	24.5	0.46	31.5	0.54
4	47.2	0.88	20.6	0.53	22.4	0.47	27.9	0.53
3	38.6	0.81	17.0	0.55	18.8	0.46	23.1	0.51
2	27.4	0.65	12.4	0.49	13.7	0.43	16.8	0.48
1	14.5	0.51	6.6	0.51	7.4	0.40	8.9	0.42
		RMS(U)	285 kN		191 kN		142 kN	

Table 4.4. Summary of the Response of the Six-Story Base-Isolated Building Using Modified and Instantaneous Algorithms.

Level	No Control		Modified Algorithm U_{max}=343 *kN*		Classical Algorithm U_{max}=605 *kN*		Classical Algorithm U_{max}=343 *kN*	
	x max mm	a max g	x max mm	a max *g*	x max mm	a max g	x max mm	a max g
Top	91.5	0.14	51.0	0.23	51.8	0.28	64.5	0.18
5	90.9	0.14	50.8	0.22	51.3	0.21	64.3	0.16
4	90.2	0.14	50.3	0.17	50.8	0.19	63.8	0.15
3	89.2	0.13	49.8	0.15	50.0	0.17	63.0	0.15
2	87.6	0.13	49.3	0.16	49.5	0.17	62.2	0.15
1	86.1	0.12	48.5	0.16	48.8	0.18	61.0	0.14
Base	83.8	0.12	47.7	0.18	47.7	0.24	59.7	0.17
		RMS(U)	155 kN		160 kN		104 kN	

RMS values are computed for the first 15 seconds of the response.

condition of the algorithm, one choice is the Riccati matrix where $Q\Delta t$ in equation (2.10) is set equal to P. Another choice, used in this example, is to satisfy the continuos Lyapunov equation:

$$A^T Q + Q A = - I_o \tag{4.2}$$

In the above equation, I_o is any symmetric positive semi-definite matrix which is selected to have the form:

$$I_o = \phi \, 1_{(2n \times 2n)} \tag{4.3}$$

where 1 is a matrix with all elements equal to unity. By varying ϕ in equation (4.3), different Q matrices are determined from equation (4.2) and the corresponding gain matrices G are computed from equation (2.10). To select the gain matrices, a procedure similar to that in example 1 was used in this example. The response of the structure to the S00E component of El Centro using the modified algorithm and the instantaneous algorithm --one with the same control force and the other with the same base displacement-- are presented in Table 4.4. The results show that for the same base displacement (47.7 mm), a reduction in the control force of approximately 43 % (343 kN compared to 605 kN) and for the same control force (343 kN), a reduction in the base displacement of about 20 % (47.7 mm compared to 59.7 mm) is achieved with the modified algorithm. The base displacement, acceleration, and the control force for the first ten seconds of the response are shown in Figure 4.4.

4.4 Discussions

Based on the methodology and the examples presented, the advantages of using the modified algorithm are:

1- The proposed modification with variable gain matrices results in a greater reduction in response than the traditional algorithms with constant Q matrices. The main reason for this reduction is a better distribution and a more efficient use of the control force.

2- Using variable gain matrices, the system matrix [A+BG] and consequently, the stiffness and damping characteristics will also vary during the response. Hence, the structure responds more efficiently to the external loads by better utilizing the structural properties and the control system capacity.

3- The analysis indicates that control systems should be considered as part of the structural system. Similar to designing structural elements, the control system capacity and gain matrices should be selected to resist the expected seismic loads. A statistical analysis of earthquake records scaled to the expected acceleration may be used to determine the control system capacity and gain matrices.

4- Finally, when using control algorithms with constant gain matrices, the control force may exceed the controller capacity under unexpected external loading conditions. It has been suggested [8] that in such cases the control force be set equal to the controller capacity. While this suggestion insures that the control force does not exceed the capacity, it does not maintain the optimality of the algorithm. The proposed modification, however, guarantees that the computed control force does not exceed the capacity and that it maintains the optimality of the algorithm.

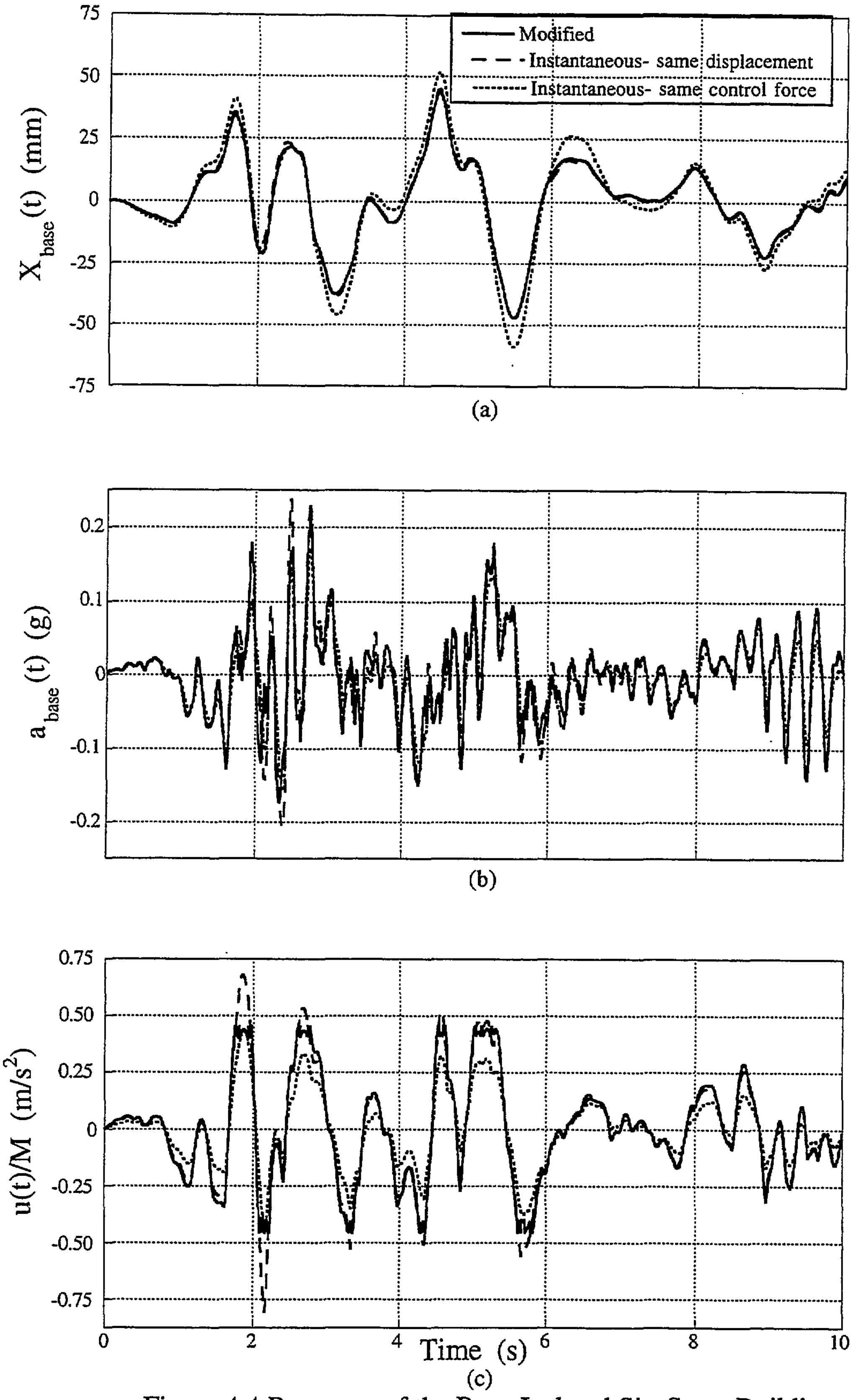

Figure 4.4 Response of the Base-Isolated Six-Story Building Using Modified and Traditional Analyses.
(a) Relative Displacement (b) Absolute Acceleration (c) Control Force

5. CONCLUSIONS

A modification to two optimal control algorithms which results in a greater reduction in structural displacements and control forces is introduced. The modification consists of building a library of gain matrices and selecting the gain matrix that makes the optimal use of the control force without exceeding the controller capacity. The modification was applied to several SDOF systems, a MDOF system, and a base isolated structure. Based on the examples used, the modification results in a reduction of up to 45 % in control forces or in displacements as compared to the existing algorithms.

The study indicates that the control system should be considered as part of the structural system, where the external loads are resisted by structural rigidity as well as control forces. Similar to the structural elements, the selection of control system parameters (controller capacity and gain matrices) should be based on the expected seismic excitation.

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APPENDIX A. SYMBOLS AND NOTATION

1	unity matrix
A	system matrix
a_{max}	maximum absolute acceleration response
B	control force location matrix in state-space
C	damping matrix
D	control force location matrix
E	excitation location matrix
f	excitation vector
G	gain matrix
g	gravity acceleration
H	excitation location matrix in state-space
I	identity matrix
I_o	any symmetric positive semi-definite matrix
J	performance index
K	Stiffness matrix
M	mass matrix
n	number of degrees of freedom
P	Riccati matrix
q	operator or multiplier
Q	weighting matrix
R	weighting matrix
t	time
u	control force vector
U_b	control force capacity
U_{ref}	reference control force
x	displacement vector
x_{max}	maximum relative displacement
z	state vector
Δt	time increment
ϕ	multiplier or operator