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Abstract

The lifting force of a fluid upon an immersed body was analyzed and reported by Archimedes in the third century B.C.. In present times many people employed in metrology find it difficult to grasp the concept of buoyancy as it applies to mass measurement. Antiquated techniques in use today propagate ignorance of the concept. The modern electronic balance makes possible a simple educational two balance experiment that provides the observer a clear picture of the buoyant force and Newton's third law. The experiment, when performed carefully, yields an excellent volume determination of the demonstration mass and highlights the use of the SI units of mass, length and temperature and the derived units of force, pressure and density.

<u>Introduction</u>

Beginning with Archimedes in the third century B.C., Archimedes' principle has been in the literature and is well known. It is often succinctly expressed by saying "that solids will be lighter in fluid by the weight of the fluid displaced." What has been described is the principle of buoyancy. The Harper Encyclopedia of Science [] has a concise explanation of buoyancy :

> The principle of buoyancy has its origin in the law of fluid pressure, which says that pressure varies directly with depth. Thus the upward pressure on the bottom of a submerged solid (assumed rectangular for the sake of simplicity) is greater than the downward pressure on the upper face. The net upward or buoyant, force is equal to the difference in weight between two fluid columns whose bases are the upper and lower faces of the solid. Hence the buoyant force is equal to the weight of the portion of fluid displaced by the solid. For a floating body, the buoyant force also equals the weight of the floating body itself. If a body is denser than the fluid in which it is submerged, buoyancy proves insufficient to support the body which thereupon sinks to the bottom.

From this description and the simple relationship,

$$\text{Density} = \text{Mass/Volume},$$

it can be shown that the buoyant force is the product of the fluid density , the volume of the mass displacing the fluid and the earth's gravity.

In the practice of classical mass metrology we must account for the buoyant force on a mass that is denser than fluid in which it is immersed. The following thought problem was constructed to help those still struggling with Archimedes' buoyancy 23 centuries after publication of his work. Perhaps the invention and use of the electronic calculator will eventually sink the use of "apparent mass" which does more to perpetuate ignorance of buoyancy than any other contrivance, i.e. mass is mass and nothing more.

THE PIGGYBACK THOUGHT BALANCE EXPERIMENT

Consider a test object O of mass M_O suspended by a massless fiber as shown in figure 1. From Newton's second law, we know that the downward gravitational force, exerted on O and transmitted through the fiber, is given by:

$$F_N = M_O\, g$$

where M_O is the mass of the object and g is the acceleration due to gravity. From Archimedes' principle as just described, we know that the object also experiences an upward buoyant force whose magnitude is given by

$$F_B = \rho V_O g$$

where ρ is the density of the fluid surrounding the object and V_O is the volume of the object.

The tension in the fiber is equal to the difference of the magnitudes of the two opposing forces. Taking the downward force to be positive, the tension may be written as

$$T = F_N - F_B$$

In the International System of Units (SI) mass is expressed in kilograms (kg), length in meters (m) and time in seconds (s). In SI units, volume is expressed in cubic meters (m^3), density in kilograms per cubic meter (kg/m^3), and the unit of force derived from Newtons second law, is equal to one kilogram-meter per second per second ($kg\text{-}m/sec^2$) is called the newton.

Now we consider the experimental setup shown in figure 2. A fiber connects test object O to an electronic balance above. Immediately below O is a second electronic balance whose pan contains a beaker of water and a supporting block stowed atop the beaker. For this experiment we will ignore possibilities such as evaporation of water and chipping of the block, and will assume that the tare force experiment. With the apparatus set up in an ordinary laboratory, the force seen by the upper balance will be given by

$$T_1 = M_0 g - \rho_a V_0 g$$

$$= (M_0 - \rho_a V_0)g$$

where ρ_a is the density of the surrounding air.

The second stage of the experiment is shown in figure 3. Now the beaker has been placed atop the block such that test object O is submerged in water (with luck no air bubbles cling to it), and does not touch the beaker. Object O is now buoyed up by the water instead of air, we must now compute a new value for the force seen by the upper balance

$$T_2 = (M_0 - \rho_w)V_0 g$$

where ρ_w is the density of water. Since the density of water is 800 times the density of air, we know that T_2 will be less than T_1, and that the difference

$$T_2 - T_1 = (M_0 - \rho_w V_0)g - (M - \rho_a)V_0 g$$

$$= (\rho_a - \rho_w)V_0 g$$

will be a negative number. We have assumed that the mass of the test object is unaffected by its immersion. Newton's third law tells us that the change in force seen by the upper balance must be matched by an equal and opposite change in force seen on the lower balance. This means that the new force given on the lower balance is given by

$$-(T_2 - T_1) = -(\rho_a - \rho_w)V_0 g$$

THE LABORATORY EXPERIMENT

Let I_{U1} be the number indicated by the upper balance prior to immersion of the test object.

I_{L1} be the number indicated by the lower balance prior to immersion of the test object.

I_{U2} be the number indicated by the upper balance with the test object submerged.

I_{L2} be the number indicated by the lower balance with the test object submerged.

The various values of I are proportional to the respective values of force exerted on the balances. The value of k is the same for both balances by virtue of the calibration described below. For this experiment, we can write

$$kI_{U1} = (M - \rho_a V_O)g$$

$$kI_{L1} = F_T$$

$$kI_{U2} = (M - \rho_w V_O)g$$

$$kI_{L2} = F_T + (\rho_w - \rho_a)V_O g$$

from the results of the previous discussion. With simple algebra we find that

$$I_{U1} - I_{U2} = -(I_{L1} - I_{L2})$$

This expression is useful because it allows all the readings to be cross-checked. It was derived using nothing more than Archimedes' principle and Newton's second and third laws.

Prior to beginning the experiment the two electronic balances are placed side by side and calibrated with a standard mass S. After calibration both balances give the same indication when S is placed on their respective pans and the air density is unchanged. We can predict the change in both balance indications in the experiment in kilograms if the terms are expressed in S.I. units.

$$I_{U1} - I_{U2} = (\rho_w - \rho_a)V_{mo}$$

$$-(I_{L1} - I_{L2}) = -(\rho_w - \rho_a)V_{mo}$$

The density of water [4] can be calculated from the water temperature measurement and likewise the air density [5] from air temperature, barometric pressure and relative humidity.

The experiment was conducted using an 85-g silicon crystal of known volume [6]. (One could use a precision sphere and measure its diameter and calculate its volume). The silicon crystal volume was 37.01596 cm^3. The experiment was performed immediately after the balances were calibrated in situ with the standard mass. First the water temperature was measured and then the balances were adjusted to indicate zero just prior to loading. The crystal was attached to a hook on the upper balance for weighing below the pan and the beaker of water and blocks on the lower balance pan. All of the remaining instrument indications were then recorded. The blocks and beaker of water were then arranged to submerge the crystal and after reaching stabilty the balance indications were again recorded. We can now calculate for each balance the difference between indications and compare them to the observed difference. The calculated and observed differences are tabulated in table 1.

TABLE 1

BALANCE RESPONSE			
Upper	Lower	Upper	Lower
Calculated kg		Observed kg	
0.0369082	-0.0369082	0.0369079	-0.0369070
0.0369083	-0.0369083	0.0369105	-0.0369084
0.0368886	-0.0368886	0.0368881	-0.0368881
0.0368886	-0.0368886	0.0368887	-0.0368884

Within this set of data, the balances do indicate the equal and opposite responses in kilograms within 60 ppm.

An ancient and useful method of volume determination was used to obtain the crystal volume from the observations made on either balance. Electronic balances are usually calibrated [2,3] by adjusting the balance to indicate zero when the pan is empty and indicate the nominal value, I_C, of the calibration mass S when it is loaded on the mechanism. I_C and S are close to each other in value and S has a density ρ_S of approximately 8 g/cm³. We can express the force imposed on the balance by an

$$\left(\frac{S\left(1-\frac{\rho_a}{\rho_S}\right)I_x}{I_c}\right)g = (X-\rho_a V_x)g$$

object of unknown mass , X, and the corresponding balance indication, I_X, as follows:

$$\left(\frac{S\left(1-\frac{\rho_a}{\rho_s}\right)l_x}{l_c}\right)g = (X - \rho_a V_x)g \qquad \text{eq(1)}$$

Equation (1) is rearranged to obtain the volume V_X. We substitute M_0 and V_0 for X and V_X respectively in equation (2). The expression for the crystal volume (could be any object) is

$$V_m = \frac{S\left(1-\frac{\rho_a}{\rho_s}\right)(l_1-l_3)}{l_c(\rho_w-\rho_a)} \qquad \text{eq(2)}$$

The upper balance data was used to calculate the silicon crystal volume. The crystal volume determined from the four experiments is 37.0239 cm_3 and the standard deviation is 0.003 cm^3. The difference between the measured volume and the known volume is -0.008 cm^3 and is statistically significant. The difference is most likely caused from gas bubbles adhering to the submerged crystal. However, the uncertainty of the measured volume is adequate for use in ordinary weighing.

We note that mass is not in the expression for volume above. Furthermore, when equation (2) is solved for the mass X, g is not present, the same is true of equation (1). Although sufficient precision was not achieved in the experiment to observe the effect from separating, the balances one would expect to see 0.0000003 kg per meter per kg due to the gradient in the earth's gravitational field.

CONCLUSION

The piggyback balance experiment is easy to perform and useful in teaching students about the opposition of the gravitational and buoyant forces. This can be especially useful for anyone engaged in high-accuracy gravimetric measurements. There is the additional advantage of teaching the importance of the ancillary measurements to achieve accurate measurements. This is especially true of the volume determination.

References:

[1] The Harper Encyclopedia of Science, James R. Newman, Editor (Harper and Row Evanston and Sigma; New York, Washington 1967) p.223

[2] Schoonover, R.M.,"A Look at the Analytical Balance," Anal. Chem. 973A-980A (1982)

[3] Schoonover,R.M., et.al."The Eclectronic Balance and Some Gravimetric Applications", Proceeding of the 39th International Instrumentation Symposium, Instrument Society of America, May 1993.

[4] Kell, G.S.;"Density, Thermal Expansivity, and Compresibilty of Liquid Water from to 150 °C: Correlations and Tables for Atmospheric Pressure and Saturation Reviewed and Expressed on 1968 Temperature Scale" Journal of Chemical Engineering Data, Vol 20, No. 1 (1975).

[5] Davis, R.S.; "Equation for the Determination of Density of Moist Air (1981/91)" Metrologia (1992), 29.

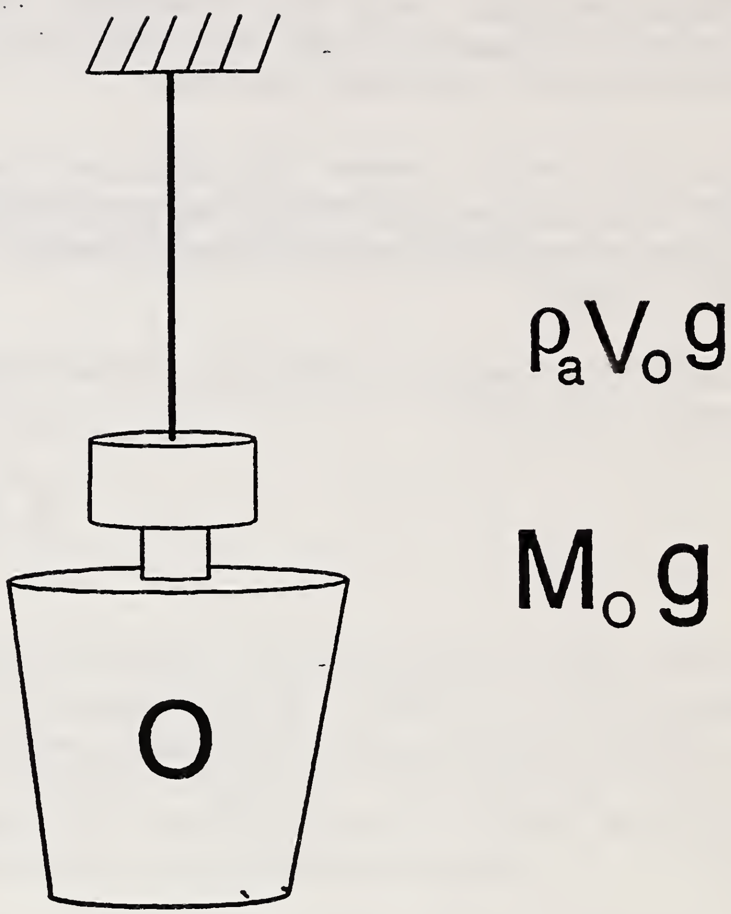

Figure 1. Free body diagram showing forces on object O.

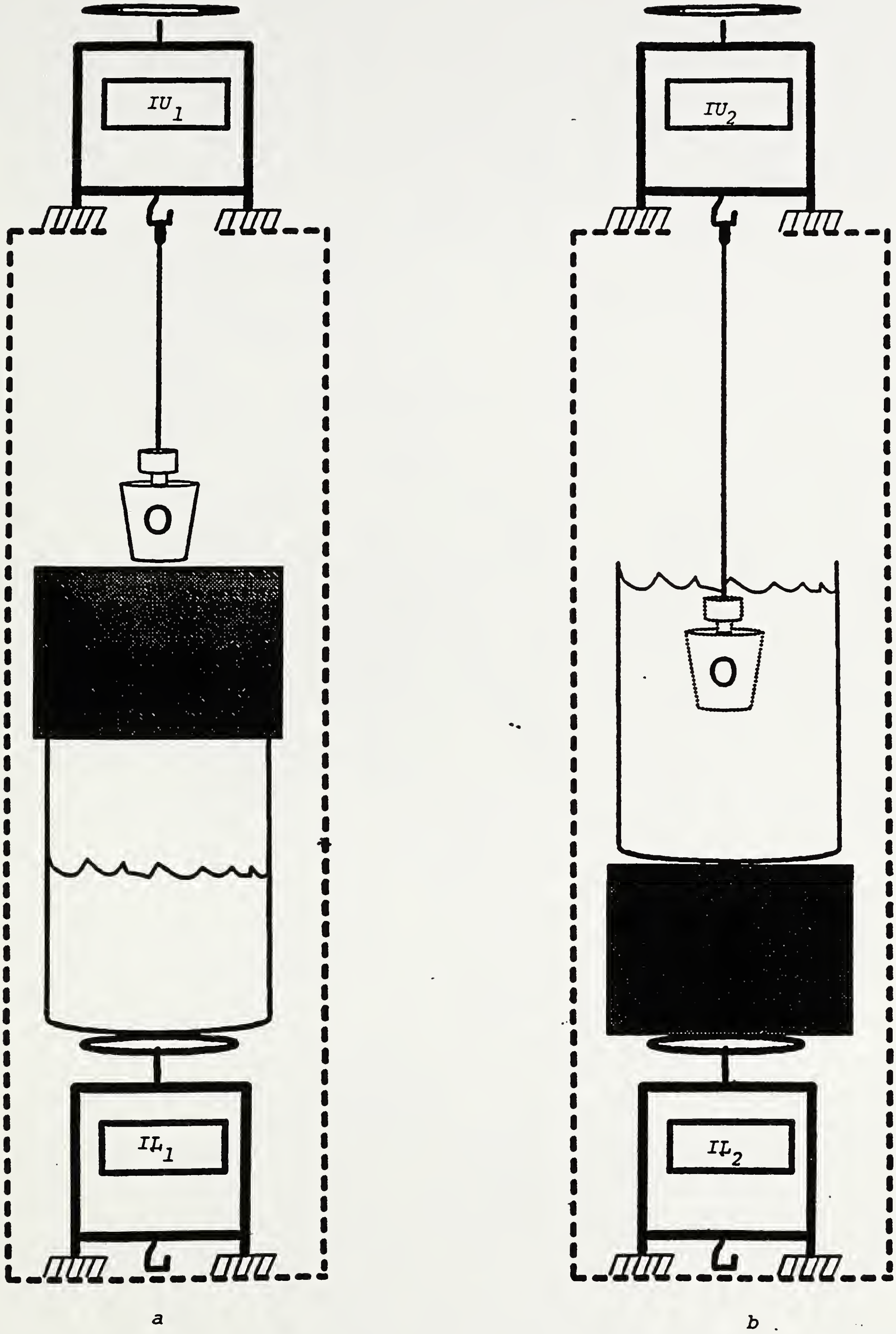

Figure 2. Experimental setup.