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Prepared under the
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**U.S. DEPARTMENT of COMMERCE**
Barbara Hackman Franklin, *Secretary*
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# ABSTRACT

Regression analysis was performed on published data from nondestructive and cylinder compressive strength testing of concrete. The nondestructive tests investigated were: rebound hammer, probe penetration, pulse velocity, pullout, and break-off. Regression analysis accounted for the error in both the nondestructive and the compressive strength data and their constant coefficient of variation. Data for each nondestructive test were grouped by coarse aggregate type and aggregate mass fraction. The results of the regression analysis are given, along with the parameters required to estimate compressive strength from subsequent nondestructive tests. A common format for the analysis and reporting of nondestructive-destructive regression experiments is suggested.

**Keywords**: break-off, nondestructive testing, probe penetration, pullout, pulse velocity, rebound hammer, regression analysis, strength estimation

# Contents

# List of Figures



# List of Tables

# 1 INTRODUCTION

The following report was written to facilitate nondestructive evaluation of the in-place compressive strength of concrete using limited destructive testing. Typically, a small number of destructive and nondestructive tests are performed in tandem at noncritical locations in the structure to develop a regression relationship between the two tests. Nondestructive tests are then performed at critical portions of the structure and the regression relationship is used to predict the in-place compressive strength.

For structures in which there exist very few, if any, non-critical locations for destructive testing, the prediction of the in-place strength using nondestructive techniques must be performed with little or no destructive test data. If destructive tests are not permitted in the structure, the in-place strength must be estimated from either laboratory tests using the same mix design or from nondestructive results from structures with a similar mix design. In cases where no destructive evaluation can be performed and laboratory tests are neither feasible nor possible, the assessment of the in-place strength must be based solely upon published results.

This report is a compilation of existing nondestructive evaluation (NDE) data that have been published. The intent was to develop regression relationships between NDE test results and cylinder compressive strength for typical concrete mix designs. In the absence of destructive tests, the relationships developed here can be used as guidelines in the assessment of the in-place strength of concrete based solely upon NDE tests. The five NDE tests considered are: rebound hammer, probe penetration, ultrasonic pulse velocity, break-off, and pullout. These five tests comprise an overwhelming majority of the NDE tests performed.

There are aspects of each test that warrant attention when assessing the in-place strength of concrete. Each test has limitations in what it can and can not measure. An in-depth review of each test can be found in the CRC Handbook on Nondestructive Testing of Concrete[1].

# 2 LITERATURE SEARCH

Published data were obtained from a literature search of both the COMDEX and NTIS systems. The search was based upon keywords that were unique to a specific NDE test and general to the subject of NDE testing and strength evaluation.

The literature search yielded 544 abstracts. Based upon the information available in the abstracts, approximately 70 of the publications were requested. These publications were then separated by test method. For each test method, there were less than ten publications containing relevant data; the remainder typically discussed the test itself. Of the few reports that contained data, only a about half, or less, included sufficient information for classification by aggregate type and content. The result is a database of information for five nondestructive tests methods that is based upon ten reports, which are referenced at the end of Appendix A.

# 3 DATA INTEGRITY

While some of the data used in this report were results from careful experimentation, the remainder of the data were garnered from experiments with poor experimental designs (*e.g.* few if any replications) or the regression of the data was performed incorrectly. In order to compare the quality of the different reports used in this survey, nine criteria were selected and each report was given a rating for each criterion. The nine criteria that were used in the ranking are described in detail in Appendix B and are variations of the eleven criteria of Oak Ridge National Laboratory report ORNL/NRC/LTR–90/22 [3]. The ORNL report describes a materials database and the use of rankings for the published data used in the database. The eleven criteria of the Oak Ridge report were altered slightly to reflect the requirements for NDE tests. The criteria are numbered one to nine in Appendix A and are ranked from left to right in the data label tables. A rating of a "*" for a single criterion means that criterion does not apply to that reference. For example, references that did not perform any least squares analysis (criterion nine) have a "*" in the last column. Due to the small number of experiments available, all of the raw data with sufficient information was used, regardless of the quality of the written report.

One problem encountered when evaluating the data was the lack of replications of the destructive tests: nondestructive tests would be repeated some number of times and would be compared to a single cylinder compression test. The lack of compression test replications made it difficult to establish a variability for the test results.

Another common mistake made in some of the reports was the application of ordinary least squares analysis (OLS) to establish regression relationships. Ordinary least squares analysis has two requirements of the data: the variance in the data must be constant and the independent variable must be known without error; although in practice, OLS analysis is applied to problems in which the error in the independent variable is much smaller than the error in the dependent variable. Relationships between compressive strength measurements and NDE measurements violate both of these assumptions. First, it has been demonstrated that the coefficient of variation of destructive and nondestructive tests is constant[2]; the standard deviation of the averaged values grows linearly with the estimated mean value. Second, the coefficients of variation of destructive and nondestructive tests are all nonzero, and have similar magnitudes. Therefore, proper regression analysis must address both the changing variability and the non-negligible variability in the independent variable.

It should be noted that only a small number of compressive strength data from existing structures was available from the published data. Since most of the published works reported NDE test results obtained from laboratory experiments, only limited NDE data were obtained from tests performed on existing structures. The impact is two-fold: laboratory experiments are often, but not always, correlated to cast cylinder specimens, rather than cored specimens; evaluation of an existing structure would exhibit greater variability due to variations occurring in the field during normal production and placement of concrete.

# 4 SUBDIVISION OF RESULTS

For each NDE test, data were subdivided by coarse aggregate type and by aggregate content by mass. Water:cement ratios ($w/c$) are also given for comparison purposes. The data for ultrasonic pulse velocity, rebound hammer, and probe penetration are subdivided by coarse aggregate content ($CA$), the data for pullout and break-off are subdivided by total aggregate content ($A_T$).

The decision to use aggregate content by mass, rather than by volume, was governed by practicality. The researchers that did publish their mix designs generally omitted the density of their aggregate. Therefore, the specific gravity of the aggregate would have to be estimated using typical densities. Since very few authors published their aggregate densities, the vast majority of volume fractions would be estimated values.

The range of $CA$ values for which data were grouped was established subjectively. Individual experiments were arranged into a table according to $CA$ and ($w/c$). The data were generally divided into thirds: a middle range that spanned 5%, data below that range, and data above that range. This separation usually divided the data into 'reasonably' sized groups. A middle range of 5% was chosen because it represented a moderate range of accuracy achievable in typical mix proportions. However, often there existed insufficient data to subdivide the results beyond a single group.

The data are subdivided by aggregate type (limestone or granite) and either coarse aggregate or total aggregate content because of their influence upon NDE results. The rebound hammer and probe penetration tests give a measure of hardness and are influenced by surface effects. Given two concrete specimens with the same compressive strength, the specimen with the harder (Moh's hardness) aggregate, or more aggregate, will "appear" to have a greater strength based upon rebound hammer or probe penetration results.

Results indicate that the pullout and break-off test may depend upon both the aggregate type and maximum aggregate size[1]. Unfortunately, there was insufficient data to further subdivide the respective test results by maximum aggregate size. Also, since the maximum aggregate sizes, for all of the NDE tests typically ranged from 10–20mm, there would have been insufficient data to develop relationships over a wide range of aggregate sizes.

# 5 STATISTICAL PROCEDURE

In developing relationships between NDE and compressive strength data, ordinary least squares analysis will not work for two reasons: the variability of the data is not constant and both variables have coefficients of variation of similar magnitude. However, this does not preclude regression analysis. Since the coefficient of variation of NDE results remains relatively constant, the data can be transformed into an acceptable form through a change of variables. Additionally, regression of data when there exists error in both variables has been addressed by Mandel[4].

Mandel's method is used for univariate linear regression. This method was preferred because it can be implemented with existing least-squares, linear regression methods. The problem of nonconstant error in both variables is alleviated by a change of coordinates by taking the logarithm of the initial quantities[5]. The results of the regression analysis are then transformed back into the original coordinate system.

Using correct regression analysis is important when establishing predictions. Typically, when using the same functional form for the regression, ordinary least squares and a technique that considers the variability in the independent variable will both give regression coefficients that are nearly equal. Hence, both methods yield nearly identical estimates of the true mean in-place compressive strength. The difference between the two techniques is the estimated prediction interval. Ordinary least squares will yields a noticeably smaller, and, therefore, unrepresentative prediction interval than a method that considers the error in the independent variable[6][7][8].

Obtaining an accurate estimate for the prediction interval is very important in applications concerning estimates of strength. The strength of a structure depends upon the lowest values of strengths that exist in the structure. This is somewhat analogous to the fact that a chain is only as strong as its weakest link. An accurate estimate of the lower bound of strengths is paramount to establishing the integrity of the structure.

## 5.1 Logarithmic Approximation

The analysis in the following section requires the mean value of the logarithm of both the NDE and compressive strength data. This analysis is complicated by the fact that very few of the publications included the individual measurements. Often only the mean value of the measured quantity is published. Therefore, an estimate of the mean value of the logarithms must be estimated from the mean measured value published.

Let $W_i$ represent individual observations of either NDE or compressive strength data and let $X_i = \ln(W_i)$. For $n$ replicates, the desired quantity is

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} \ln\left(W_i\right)$$

Individual $W_i$ can be defined by the mean, $\overline{W}$, and a residual, $\epsilon_i$:

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} \ln\left(\overline{W} + \epsilon_i\right)$$

Since the coefficient of variation of NDE tests and the compressive tests is significantly less than one ($\approx$0.10), the quantity $\epsilon_i$ is small compared with $\overline{W}$. Therefore, a second-order Taylor expansion should serve as an accurate estimate:

$$\overline{X} \approx \frac{1}{n}\sum_{i=1}^{n} \left[\ln\left(\overline{W}\right) + \frac{\epsilon_i}{\overline{W}} - \frac{1}{2}\frac{\epsilon_i^2}{\overline{W}^2}\right]$$

The summation of the second term is, by definition, zero and the third term can be related to the measured variance $s_W^2$:

$$\overline{X} \approx \ln\left(\overline{W}\right) - \frac{1}{2}\frac{(n-1)}{n}\frac{s_W^2}{\overline{W}^2} \tag{1}$$

since

$$s_W^2 = \frac{1}{n-1}\sum_{i=1}^{n} \epsilon_i^2$$

The third term can also be related to the measured coefficient of variation $\xi_W$:

$$\overline{X} \approx \ln\left(\overline{W}\right) - \frac{1}{2}\frac{(n-1)}{n}\xi_W^2 \tag{2}$$

from the definition of coefficient of variation.

This approximation will be used in the following section to calculate the approximate value of the mean logarithm based upon the reported mean values of NDE and compressive strength results.

## 5.2 Transformation of Variables

The following development is taken from Stone and Reeve[8], which references Mandel's method. The notation is the same as Stone and Reeve with only minor changes.

The directly measured quantity is either an NDE measurement or a compressive strength measurement. The observable quantities, $W$ and $C$, represent the NDE and compressive strength measurements, respectively. The observables are assumed to have constant coefficients of variation; *i.e.* the standard deviations vary linearly with estimated mean value.

The model for the true relationship between compressive strength and an NDE results is a power law,

$$c_i = \alpha_0 w_i^{\alpha_1} \tag{3}$$

with a multiplicative error model,

$$C_i = c_i(1+\eta_i) \qquad\qquad W_i = w_i(1+\theta_i) \tag{4}$$

Substituting Eqn. 4 into Eqn. 3 and taking the natural logarithm of both sides yields the following relationship:

$$\ln(C_i) = \ln(\alpha_0) + \alpha_1 \ln(W_i) + [\ln(1+\eta_i) - \alpha_1 \ln(1+\theta_i)] \tag{5}$$

This is analogous to a linear model

$$y_i = \beta_0 + \beta_1 x_i \tag{6}$$

after the following substitutions,

$$Y_i = \ln(C_i) \qquad X_i = \ln(W_i) \qquad \beta_0 = \ln(\alpha_0) \qquad \beta_1 = \alpha_1 \tag{7}$$

The following error model,

$$Y_i = y_i + \epsilon_i \qquad\qquad X_i = x_i + \delta_i \tag{8}$$

is correct if $\eta_i$ and $\theta_i$ are distributed lognormally, rather than normally. Stone and Reeve argue that the assumption of a lognormal distribution is sound over typical mean and variance values used in NDE testing of concrete. Additionally, there exists a direct relation between $s_X^2$ and $s_W^2$:

$$s_X^2 = \left(\frac{\partial X}{\partial W}\right)^2 s_W^2 = \frac{s_W^2}{W^2} = \xi_W^2 \tag{9}$$

There is an analogous relationship between $s_Y^2$ and $s_C^2$.

The remaining development is based upon the following model:

$$Y_i = \beta_0 + \beta_1 X_i + (\epsilon_i - \beta_1 \delta_i) \tag{10}$$

where $\epsilon_i$ and $\delta_i$ follow normal distributions: $N(0, \sigma_\epsilon^2)$, and $N(0, \sigma_\delta^2)$.

To facilitate further development, let the following quantities be defined, as in [8]:

$n_R$ = the number of different reports used in developing a single regression relationship,

$n_{A_i}$ = the number of different concrete ages or mixes tested in a given report,

$n = \sum_{i=1}^{n_R} n_{A_i}$ –the number of observables

$n_{X_i}$ = the number of NDE replicates for $i$th age or mix, and

$n_{Y_i}$ = the number of compressive strength replicates for $i$th age or mix.

Estimates of the true means $x_i$ and $y_i$ are calculated from the individual measurements:

$$\overline{X}_i = \frac{1}{n_{X_i}} \sum_{j=1}^{n_{X_i}} X_{ij} \qquad\qquad \overline{Y}_i = \frac{1}{n_{Y_i}} \sum_{j=1}^{n_{Y_i}} Y_{ij} \tag{11}$$

However, values of $X_{ij}$ and $Y_{ij}$ were not available. Therefore, the approximation from Eqn. 2 is used:

$$\overline{X}_i \approx \ln\left(\overline{W}_i\right) - \frac{1}{2}\frac{(n_{X_i}-1)}{n_{X_i}}\xi_{W_i}^2 \qquad\qquad \overline{Y}_i \approx \ln\left(\overline{C}_i\right) - \frac{1}{2}\frac{(n_{Y_i}-1)}{n_{Y_i}}\xi_{C_i}^2 \tag{12}$$

The pooled variances are the sum of weighted variances:

$$s_X^2 = \frac{1}{\sum(n_{X_i}-1)} \sum_{i=1}^{n} (n_{X_i}-1) s_{X_i}^2 \tag{13}$$

$$= \frac{1}{\sum(n_{X_i}-1)} \sum_{i=1}^{n} (n_{X_i}-1)\xi_{W_i}^2 \quad (14)$$

$$s_Y^2 = \frac{1}{\sum(n_{Y_i}-1)} \sum_{i=1}^{n} (n_{Y_i}-1)s_{Y_i}^2 \quad (15)$$

$$= \frac{1}{\sum(n_{Y_i}-1)} \sum_{i=1}^{n} (n_{Y_i}-1)\xi_{C_i}^2 \quad (16)$$

using the relationship from Eqn. 9 above.

In some cases, the average value was reported without an associated variance. To account for this, typical values were used from the following table [2]:

| Test Method | Coefficient of Variation (%) |
|---|---|
| Cylinder Compression (ASTM C 39) | 4 |
| Core Compression (ASTM C 42) | 5 |
| Pullout (ASTM C 900) | 8 |
| Probe Penetration (ASTM C 803) | 5 |
| Rebound Hammer (ASTM C 805) | 12 |
| Pulse Velocity (ASTM C 597) | 2 |

Table 1: Assumed coefficient of variation for data with none given.

Let $\overline{n}_X$ and $\overline{n}_Y$ be the mean number of replications:

$$\overline{n}_X = \frac{1}{n}\sum_{i=1}^{n} n_{X_i} \qquad \overline{n}_Y = \frac{1}{n}\sum_{i=1}^{n} n_{Y_i} \quad (17)$$

The degrees of freedom associated with $s_X^2$ and $s_Y^2$ are:

$$\nu_X = n(\overline{n}_X - 1) \qquad \nu_Y = n(\overline{n}_Y - 1) \quad (18)$$

The values of $\sigma_\delta^2$ and $\sigma_\epsilon^2$ are estimated from the pooled variance of mean values:

$$s_\delta^2 = \frac{s_X^2}{\overline{n}_X} \qquad s_\epsilon^2 = \frac{s_Y^2}{\overline{n}_Y} \quad (19)$$

The relative values of these two quantities indicates whether ordinary least squares analysis is applicable. Let $\lambda$ represent the ratio

$$\lambda = \frac{s_\epsilon^2}{s_\delta^2} \quad (20)$$

which is a constant. The ratio indicates the relative size of the variance of the dependent variable ($s_\epsilon$) with respect to the variance of the independent variable ($s_\delta$). For large values

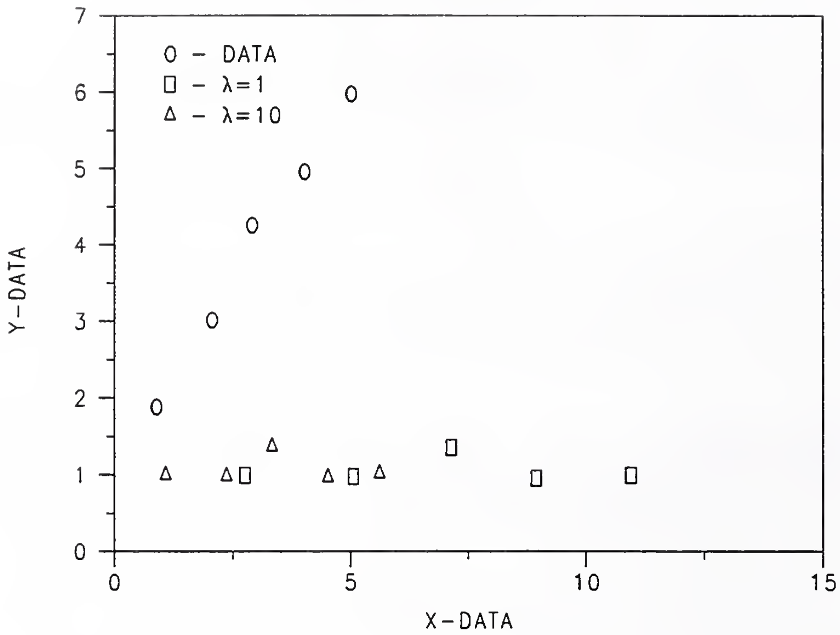


Figure 1: Rotation and scaling of data using Mandel's method.

of $\lambda$, the application of ordinary least square analysis is valid. For very small values of $\lambda$, ordinary least squares can be applied to the data, with the variability entirely attributed to the independent variable. However, for values of $\lambda$ near one, a modified regression technique, like the one proposed by Mandel[4], must be used.

The quantity $\lambda$ is an parameter to the method developed by Mandel. It is based upon measurements or experience.

## 5.3 Mandel's Method

With the variables now transformed to ones with constant variance, regression analysis based upon the method proposed by Mandel [4] can be performed. Simply put, Mandel's method consists of a rotation and a scaling of the original data. The data are rotated about their y-axis intercept until they fall upon a horizontal line, as in Fig. 1, which shows arbitrary data, $(\overline{X}, \overline{Y})$, denoted by circles, having constant variance in both the $x$ and $y$ directions. Two arbitrary values of $\lambda$ were chosen and the resulting transformed data in $(u, v)$ coordinates for both values of $\lambda$ are denoted by squares and triangles. Both values of $\lambda$ rotated the data to approximately the same $y$ value. However, smaller values of $\lambda$ scale the data a greater amount in the $x$ direction.

Returning to the NDE data, the linearized data $(\overline{X}, \overline{Y})$ are transformed into $(u, v)$ coordinates:

$$v_i = \overline{Y}_i - b\overline{X}_i \qquad u_i = \overline{X}_i + k\overline{Y}_i \tag{21}$$

The values of $k$ and $b$ are calculated from the following equations which assume $\overline{X}_i$ and $\overline{Y}_i$ are independent:

$$b = \frac{S_{XY} + kS_{YY}}{S_{XX} + kS_{XY}} \qquad k = \frac{b}{\lambda} \tag{22}$$

These two equations can be solved either through a quadratic relationship or by iterating the two equations, assuming an initial value of zero for $k$.

The following are the definitions of $S_{XX}$, $S_{YY}$,

$$S_{XX} = \sum_{i=1}^{n} \left(\overline{X}_i - \overline{\overline{X}}\right)^2 \qquad S_{YY} = \sum_{i=1}^{n} \left(\overline{Y}_i - \overline{\overline{Y}}\right)^2 \tag{23}$$

and $S_{XY}$:

$$S_{XY} = \sum_{i=1}^{n} \left(\overline{X}_i - \overline{\overline{X}}\right)\left(\overline{Y}_i - \overline{\overline{Y}}\right) \tag{24}$$

where

$$\overline{\overline{X}} = \frac{1}{n}\sum_{i=1}^{n} \overline{X}_i \qquad \overline{\overline{Y}} = \frac{1}{n}\sum_{i=1}^{n} \overline{Y}_i \tag{25}$$

With the data transformed into $(u, v)$ coordinates, ordinary least squares is performed to calculate values for $b$ and $k$. These values are then used to estimate the values of $\beta_0$ and $\beta_1$ in Eqn. 6.

Consider the effects of extreme values of $\lambda$. For large values of $\lambda$ (large error in $Y$ compared with $X$), the value of $k$ approaches zero and value of $b$ is the slope calculated from ordinary least squares analysis. For small values of $\lambda$, the value of $k$ approaches infinity and $b$ again approaches the value of the slope of ordinary least squares analysis. More correctly, after exchanging the $(x, y)$ pairs, $b$ approaches the reciprocal of the slope.

Once the data are transformed into $(u, v)$ coordinates, the following quantities are calculated:

$$\overline{u} = \frac{1}{n}\sum_{i=1}^{n} u_i \qquad \overline{v} = \frac{1}{n}\sum_{i=1}^{n} v_i \tag{26}$$

$$S_{uu} = \sum_{i=1}^{n} (u_i - \overline{u})^2 \qquad S_{vv} = \sum_{i=1}^{n} (v_i - \overline{v})^2 \tag{27}$$

and the "residual" standard deviation,

$$s_e = \sqrt{S_{vv}^2 / (n-2)} \tag{28}$$

which is a measure of scatter of the transformed data about its regression line.

Estimates of $\beta_0$ and $\beta_1$ can be calculated:

$$\hat{\beta}_1 = b \qquad \hat{\beta}_0 = \overline{\overline{Y}} - b\overline{\overline{X}} \tag{29}$$

along with their estimated variance:

$$s^2_{\hat{\beta}_1} = \frac{(1+kb)^2}{S_{uu}} s^2_e \qquad s^2_{\hat{\beta}_0} = \frac{s^2_e}{n} + \overline{\overline{X}} s^2_{\hat{\beta}_1} \tag{30}$$

With the linear regression accomplished, a test of the model can be performed using an F-test proposed by Stone and Reeve [8]. If the linear model is correct, $\sigma^2_e = \sigma^2_\epsilon + \beta^2_1 \sigma^2_\delta$ and the statistic

$$F = \frac{s^2_e}{s^2_\epsilon + \hat{\beta}^2_1 s^2_\delta} \tag{31}$$

follows an F-distribution with $n-2$ and $n^*$ degrees of freedom, where $n^*$ is an approximation of the effective degrees of freedom:

$$n^* = \frac{(s^2_\epsilon + \hat{\beta}^2_1 s^2_\delta)^2}{\left[\frac{s^4_\epsilon}{n\overline{n}_Y} + \frac{(\hat{\beta}_1 s_\delta)^4}{n\overline{n}_X}\right]} \tag{32}$$

A significance level of less than about 1% suggests considerable model error.

This statistic is measure of the relative difference between the estimated residual error due to the model and the error based upon measurement. If the value of $F$ is near one, the distances between the data and the regression line are roughly equal to the errors in the data. Values of $F$ far greater than one suggest that the data are much farther away from the regression line than the error in the data suggest they should be.

## 5.4 Future Predictions of In-place Strength

The previous section addressed how regression analysis was performed to obtain the best-fit regression equation for the transformed NDE and compressive strength data. This section describes a method to predict the in-place compressive strength based upon future NDE test measurements. The approach used to estimate in-place strength based upon future measurements is taken from Stone and Reeve[8] and Mandel [4]. Future tests are denoted with the superscript ($'$).

The use of any previously developed correlation requires that the following three assumptions are true:

1. The functional relationship remains the same from the previous measurements to the new measurements. (*e.g.* the functional relationship does not change due to additional processes such as degradation.)

2. The logarithms of compressive strengths are normally distributed.

3. The ratio of variances is given by:

$$\sigma^2_{Y'}/\sigma^2_{X'} = \sigma^2_Y/\sigma^2_X \tag{33}$$

Let

$$\begin{aligned}
m &= \text{the number of replication of the in-place test,} \\
W'_j &= \text{the observed } j\text{th replicate of the in-place test,} \\
X'_j &= \ln(W'_j), \\
\overline{X}' &= \frac{1}{m}\sum_{j=1}^{m} X'_j, \\
s^2_{X'} &= \frac{1}{\nu_{X'}}\sum_{j=1}^{m}\left(X'_j - \overline{X'}\right)^2, \\
\nu_{X'} &= m-1 \;\; \text{– the degrees of freedom in estimator } s^2_{X'}, \text{ and} \\
s^2_{\overline{X}'} &= s^2_{X'}/m
\end{aligned}$$

The estimate of the true logarithm of in-place concrete strength, $y'$, is

$$\overline{Y}' = \overline{v} + b\overline{X}' = \hat{\beta}_0 + \hat{\beta}_1\overline{X}' \tag{34}$$

The estimated variance of $\overline{Y}'$ is

$$s^2_{\overline{Y}'} = U + V \tag{35}$$

The quantities $U$ and $V$ are defined as:

$$U = \left[\frac{1}{n} + (1+kb)^2 \frac{\left(\overline{X}' - \overline{\overline{X}}\right)^2}{S_{uu}}\right] s^2_e \tag{36}$$

$$V = b^2 s^2_{\overline{X}'} \tag{37}$$

The degrees of freedom associated with $s^2_{\overline{Y'}}$ are:

$$\nu_{Y'} = \frac{(U+V)^2}{\left(U^2/(n-1) + V^2/(m-1)\right)} \tag{38}$$

From this, an estimate of the true mean compressive strength, $c'$ can be determined, along with an estimate of its standard deviation $\sigma_{c'}$:

$$\overline{C}' = e^{\hat{\beta}_0}\overline{W}'^{\hat{\beta}_1} \qquad\qquad s_{\overline{C}'} = \overline{C}' s_{\overline{Y}'} \tag{39}$$

However, what is typically sought is an estimate of the variance of the individual observation, $\sigma_{Y'}$. From this, estimates of percentile-strengths can be determined. Let the transformed characteristic strength, $TCS = \ln(CS)$, be defined as the $100\theta$ percentile of the distribution of transformed compressive strengths in the sample under investigation; Stone and Reeve suggest $\theta = 0.10$ as a common value. Since the logarithms of compressive strength are normally distributed, the transformed characteristic strength is $TCS = y' + \Phi^{-1}(\theta)\,\sigma_{Y'}$, where $\Phi$ is the cumulative distribution function of the standard normal distribution.

The variance of the future predicted value $Y'$, $\sigma^2_{Y'}$, can be estimated from Eqn. 33:

$$s_{Y'} = s_{X'}\frac{s_Y}{s_X} \tag{40}$$

The distribution of $s_{Y'}$ must be derived to establish a best estimator for $TCS$. Stone and Reeve derived the relations:

$$s^2_{Y'}/\sigma^2_{Y'} \sim \frac{[\chi^2(\nu_{X'})/\nu_{X'}]\,[\chi^2(\nu_Y)/\nu_Y]}{[\chi^2(\nu_X)/\nu_X]} \sim C^2(\nu_{X'},\nu_Y,\nu_X) \tag{41}$$

$$s_{Y'}/\sigma_{Y'} \sim C(\nu_{X'},\nu_Y,\nu_X) \tag{42}$$

using the $\chi^2$ distribution, and the $C^2$ and $C$ distributions defined in Appendix A of Stone and Reeve [8]. The mean and variance of the $C$ and the $C^2$ distributions are

$$\mu_C = \left(\frac{2\nu_X}{\nu_{X'}\nu_Y}\right)^{1/2}\frac{\Gamma[(\nu_{X'}+1)/2]}{\Gamma[\nu_{X'}/2]}\frac{\Gamma[(\nu_Y+1)/2]}{\Gamma[\nu_Y/2]}\frac{\Gamma[(\nu_X-1)/2]}{\Gamma[\nu_X/2]} \tag{43}$$

$$\sigma^2_C = \frac{\nu_X}{(\nu_X-2)} - \mu^2_C \tag{44}$$

$$\mu_{C^2} = \frac{\nu_X}{(\nu_X-2)} \tag{45}$$

$$\sigma^2_{C^2} = \left(\frac{\nu_{X'}+2}{\nu_{X'}}\right)\left(\frac{\nu_Y+2}{\nu_Y}\right)\left(\frac{\nu^2_X}{(\nu_X-2)(\nu_X-4)}\right) - \mu^2_{C^2} \tag{46}$$

From this, an unbiased estimator of $TCS$ is derived:

$$\widehat{TCS} = \overline{Y'} + \Phi^{-1}(\theta)s_{Y'}/\mu_C \tag{47}$$

with a variance

$$s^2_{\widehat{TCS}} = s^2_{\overline{Y'}} + \left(\Phi^{-1}(\theta)\right)^2\frac{s^2_{Y'}\sigma^2_C}{\mu^2_C\mu_{C^2}} \tag{48}$$

Stone and Reeve found that the statistic

$$T = \frac{\widehat{TCS} - TCS}{s_{\widehat{TCS}}} \tag{49}$$

can be approximated by the Student's $t$-distribution. Therefore, ordinary $t$-distribution analysis is applicable. For example, for some minimum required strength, $RS$, there is a transformed required strength $TRS = \ln(RS)$. The probability that the characteristic strength is greater than the required strength is given by:

$$P\{CS > RS\} = F_{\nu_T}\left[\frac{(\widehat{TCS} - TRS)}{s_{\widehat{TCS}}}\right] \tag{50}$$

where $F_{\nu_T}[]$ is the cumulative $t$-distribution function with $\nu_T$ degrees of freedom. The quantity $\nu_T$ is given by:

$$\nu_T = \frac{(M+N)^2}{(M^2/\nu' + N^2/\nu'')} \tag{51}$$

where:

$$M = s_{\overline{Y}'}^2 \tag{52}$$

$$N = \frac{(\Phi^{-1}(\theta))^2 s_{\overline{Y}'}^2 \sigma_C^2}{\mu_C^2 \mu_{C^2}} \tag{53}$$

$$\nu' = \nu_{Y'} \tag{54}$$

$$\nu'' = 2\frac{\mu_{C^2}^2}{\sigma_{C^2}^2} \tag{55}$$

# 6 METHOD MODIFICATION

## 6.1 Failed Model Test

When the F-test from Eqn. 31 was performed as a test of the linear model, eleven of the fourteen regression relations in Appendix A failed with $F$-probabilities of 1.000, suggesting the data lie farther from the regression line than the extent of the errors in the data. There were two possibilities for these failures: the assumed linear model for the logarithms is incorrect, or during the combining of similar mix data, additional sources of error are not accounted for, giving an uncharacteristically small estimate of the errors in the data. Since there has already been extensive research performed establishing regression relations between NDE and compressive strength test, the validity of the linear relationship appears to be beyond question. A possible explanation for unaccounted error may be due to the combining of similar mix data used in establishing the regression relationships.

## 6.2 Unknown Nuisance Factors

Typically, a regression relationship is established for a single mix of concrete. In this report, results from similar mixes have been combined in order to make predictions for future mixes that fall within that range of mixes. Since results from a single mix cannot be expected to exhibit sufficient variability to account for combining results from other mixes [1], the

[1] References to the variability between similar mixes assumes additional variability due to production and placement of the concrete

errors reported for a single mix are unrepresentative of the variability due to combining results from similar mixes. Therefore, the $F$-test failures could be partially attributed to the missing variability between mixes, which is an unknowable nuisance factor.

One possible solution would be to evaluate each mix separately. An $F$-test could be performed for each mix, determining the validity of the data. The problem with this approach is all of the results in Appendix A are from approximately 100 different mixes, many of which have only a small number of data points. A problem would still arise when attempting to use the results from the 100 separate regressions since any future mix would have a relatively small probability of matching one of the mixes exactly. The results from one or more regression relations would have to be combined to estimate the results for the new specimen.

## 6.3 Adjusted Variability

The basic problem is that other deterministic factors have an effect upon compressive strength, but are not included in the model because they are unknown and/or unknowable. In a sense, this is problem without a direct solution. A solution is to modify the existing data to probabilistically circumvent this problem.

A solution used herein was to artificially adjust the pooled variances to account for the variability between mixes. In this report, the pooled variances $s_\epsilon^2$ and $s_\delta^2$ were artificially adjusted using a self-validation procedure. Data were randomly separated into two nearly equal sized groups, group $A$ and group $B$. Regression analysis using Mandel's method was performed upon the data in group $A$ only. The pooled variances $s_\epsilon^2$ and $s_\delta^2$ of group $A$ were both multiplied by an artificial factor $\mathcal{F}_A$, keeping $\lambda$ constant. The value of $\mathcal{F}_A$ was varied until the fraction of the data in group $B$ within $(\overline{Y}' \pm s_{\overline{Y}'}\, t_{\nu,\frac{\alpha}{2}})$ was 50% for $\alpha = 0.50$. With the value of $\mathcal{F}_A$ now fixed, the fraction of data in group $B$ within the limits of $\alpha = 0.10$ was recorded and compared to 90%. Using the same $\mathcal{F}_A$ and regression coefficients, the fraction of the data in group $A$ within the same limits was also recorded. Keeping the same groups $A$ and $B$, the process was repeated by establishing regression coefficients from the data in group $B$. The process of splitting the data into two groups was repeated three times and the final value of $\mathcal{F}$ was calculated from the average of the six values.

To simplify the procedure, $s_{\overline{X}'}^2$ was replaced with $s_\delta^2$ in Eqn. 37. Since groups $A$ and $B$ were chosen randomly, the value of $s_\delta^2$ for both groups were nearly equal.

The validation tests were performed both as a method to achieve a value for $\mathcal{F}$ and as a check of the distribution of the data. Each fraction of data within the aforementioned ranges is reported for inspection. The validation tests are considered successful if the reported results are near 50% and 90%. If a test gives numbers which are close to 50% and 90%, the data are normally distributed. If numbers for both groups are near 50% and 90%, the data in each group predict the behavior of the other group well.

The results from the validation test are given only for comparison purposes. The intention is to give the reader a qualitative feel for the quality of the data.

## 6.4 Lack of Replication

A few of the data sets contained compressive strength values from a single observation, even over multiple authors. Therefore, the pooled variance for the complete set of data was $\frac{0}{0}$. A rigorous approach would require a bootstrapping method which begins with an estimate of $s_\epsilon^2$ and then consecutively updates $s_\epsilon^2$ from $(s_\epsilon^2)^{i+1} = (s_e^2)^i - (\hat{\beta}_1{}^2 s_\delta^2)^i$, where the superscript denotes iteration number, by iterating Mandel's method. This method should work for a single data set. However, the data in this report originate from slightly different regression relations, hence the need for $\mathcal{F}$ described in the previous section. Without $\mathcal{F}$, the calculated value of $s_e^2$ would be uncharacteristically large. The remaining difficulty is that there are two unknown independent parameters, $\mathcal{F}$ and $s_\epsilon^2$, and only one minimizing procedure, $(s_\epsilon^2)^{i+1} - (s_\epsilon^2)^i = 0$.

Since values of $\mathcal{F}$ are typically on the order of 5–10, the exact value of $s_\epsilon^2$ may not be critical. Instead, a value for $s_\epsilon^2$ could be estimated from engineering knowledge and $\mathcal{F}$ could be calculated for this estimate. If the value of $\mathcal{F}$ approached the value of one, then the results were sensitive to the value of $s_\epsilon^2$ and the bootstrap method was used. For data sets which had large values of $\mathcal{F}$, the value of $s_Y^2$ was estimated from the values in Table 1.

In two cases, **PV_L2** and **SH_G2** [2], the value of $s_\epsilon^2$ had to be established by a bootstrap method because of limited replications of compressive strength measurements and the estimated value of $\mathcal{F}$ was close to one. In both cases, $s_Y^2$ was varied while iterating $s_\epsilon^2 = s_e^2 - \hat{\beta}_1^2 s_\delta^2$. The value of $s_Y^2$ was varied, rather than $s_\delta^2$, because **SH_G2** had only two data points with replications, and due to the small number of data points, eight, it was easier to use this approach and let the factor $\overline{n}_Y$ 'smear' the effects of the two data points with replications over the other data points.

# 7 RESULTS

The results of the regression calculations on available data appear at the end of this report in Appendix A. Results are given in a common format for comparison.

Each data point plotted on the graph represents the average of some number of replications of both the nondestructive test and the cylinder compression test. The replications were conducted on similar specimens, all at the same age. However, not all of the data points represent samples at the same age. A single published reference may have data points for samples having ages ranging from one day to 100 days, or older. All of the data points for a single mix, regardless of age, are used in the regression analysis because a single mix at different ages follows the same regression relationship.

The results of the regression analysis are superimposed upon the data. The solid line represents the regression relationship between the compressive strength and the NDE test. It is the best estimate of the mean compressive strength for a given NDE test result. The dashed lines represent the limits of a 95% confidence interval for the estimated mean strength.

[2] Bold letters indicate the name of the data group in Appendix A

The limits are calculated from

$$\overline{Y}' \pm t_{0.975,n} s^2_{\overline{Y}'} \tag{56}$$

and $s^2_\delta$ was substituted for $s^2_{\overline{X}'}$. The reader should note that the characteristic strength will lie outside the range since it is based upon the population of strengths.

## 7.1 Format of Results

The regression results appear in Appendix A. The results are first separated by NDE test, then by aggregate type, granite and limestone. For a specific aggregate type, the results are finally subdivided by coarse aggregate content, $CA$, or by total aggregate content $A_T$. The results for a single regression analysis appear on two pages. The first page shows plots of the data along with $(w/c)$ and $CA$. On the second page are the self-validation results and the regression results.

There are two plots on the first page of each regression analysis. The left plot shows the transformed data; the data are plotted in '*log*' space. The plot on the right shows the original data with the regression relationship superposed. These plots allow visual inspection of the data, giving qualitative information about the validity of the model and the quality of the data; the reader could consider the range of the data and consider the effect of possible outliers. The right-hand plot of the raw data has each data point labelled by author, referenced at the end of Appendix A. Below both plots is a table of author index, $(w/c)$, $CA$, and rating. The rating for each of the nine criteria are given so the reader may consider each criterion individually when comparing different data sets.

The second page displays two tables. The top table contains the results of the self-validation test. A validation of $A \to B$ means that regression analysis was performed upon the data in group $A$ and the results compared with the data in group $B$. The columns 'self50' and 'self90' show the fraction of the data from the group on the left side of the '$\to$' within the limits defined by $\alpha = .50$ and $\alpha = .90$ respectively. The columns 'cap50' and 'cap90' are the fraction of the data from the group on the right side of the '$\to$' within the limits defined by the same values of $\alpha$; it's the fraction of data in the second group 'captured' by the regression relation based upon the first group. The value of $\mathcal{F}$ on the bottom row of the table is the average of the six values above it. The averaged value of $\mathcal{F}$ and the regression values at the bottom of the page were applied to all of the data and the fraction of the data within the limits defined by $\alpha = .50$ and $\alpha = .90$ are given in the columns 'self50' and 'self90'.

The table at the bottom of the second page gives the results of the regression analysis. These values are required for making predictions of future values and their associated variances.

## 7.2 Pullout Test Considerations

There exists a number of variations in pullout test methods. Two considerations are the nature of the test (cast in place or drilled) and apex angle ($A_x$). The pullout results reported in Appendix A include data from three configurations: cast in place ($A_x \approx 70^o$), cast in place LOK-test ($A_x \approx 62^o$), drilled CAPO-test ($A_x \approx 62^o$). Results from the LOK-test and CAPO-test are noted at the top of the respective pages. The apex angles are noted in the data reference table given below each plot of the data. Information about the CAPO-test can be found in a report be Petersen [9].

## 7.3 Example

Consider ultrasonic pulse velocity tests performed upon a concrete structure with gravel coarse aggregate with $CA = 50\%$. The corresponding regression data appears on page 24 labeled **PV_G2**. Five UPV readings were tabulated along with their logarithms, $X_i' = \ln(W_i')$:

| | | | | | |
|---|---|---|---|---|---|
| $W'$: | 3.930 | 4.061 | 3.945 | 4.034 | 4.025 |
| $X'$: | 1.369 | 1.401 | 1.372 | 1.395 | 1.393 |

The following statistics are calculated from the logarithms:

$$\overline{X}' = 1.386 \qquad s^2_{X'} = 0.0002100 \qquad \nu_{X'} = 4 \qquad s^2_{\overline{X}'} = 0.00004200$$

Statistics of the estimated mean logarithm of compressive strength are calculated:

$$\overline{Y}' = 2.785 \qquad U = .003837 \qquad V = .003783 \qquad s^2_{\overline{Y}'} = 0.007621 \qquad \nu_{Y'} = 15.19$$

using Eqns. 34, 36, 37, 35, and 38 respectively. Note that $\hat{\beta}_0 = -10.37$.

The predicted mean strength (MPa) is

$$\overline{C} = \exp(2.785) = 16.2$$

The following quantities are required for comparisons between the characteristic and required strengths:

$$\mu_C = 0.9405 \qquad \sigma^2_C = 0.1208 \qquad \mu_{C^2} = 1.005406 \qquad \sigma^2_{C^2} = 0.5245$$

Assume the transformed characteristic strength is defined as the lowest 10% of the transformed strength population, $\theta = .10$, and $\Phi^{-1}(\theta) = -1.282$. The transformed characteristic strength and its variance are:

$$\begin{aligned} \widehat{TCS} &= 2.785 + (-1.282)\frac{0.8828}{0.9405} = 2.665 \\ s^2_{\widehat{TCS}} &= 0.007621 + (-1.282)^2\frac{.007793 \times .1208}{.8845 \times 1.005} = .009362 \end{aligned}$$

The quantity $s^2_{\overline{Y}'}$ is calculated from Eqn. 40. The quantities $s^2_{\overline{X}}$ and $s^2_{\overline{Y}}$ are calculated from Eqn. 19.

Suppose one wants to test whether there is a greater than 95% probability that the transformed characteristic strength is greater than a transformed required strength of 10.00MPa, $TRS = \ln(10.00)$. The statistic and associated degrees of freedom are:

$$T = \frac{2.665 - \ln(10.000)}{0.09676} = 3.746 \qquad \nu_T = 19.01$$

From Eqn. 50, the probability that $TCS > TRS$ is 99%. Therefore, the sample tested does satisfy this criteria. The strength at the 95% probability is 12.1MPa.

# 8 FUTURE MODIFICATIONS

The intent of this work was to establish reference data with which NDE measurements could be used to predict the in-place strength of concrete without the need for companion destructive tests. This report recognizes that the mixes represented in the regression results section represents a limited number of possible mixes. To perpetuate the usefulness of this report, the data to date have been compiled in such a manner as to facilitate the incorporation of additional data into the existing database.

It is also important to note that the mere existence of additional data is not sufficient to prove useful to the established database. As noted previously in this report, typical methods of regression analysis had to be modified to accommodate data from experiments that were conducted poorly, from a statistical perspective. Therefore, the usefulness of future data requires that it also be from experiments which incorporate sound statistical principles. These considerations would include, but are not limited to, the following: replications of the data, randomization of the experiment, the publication of all raw data, etc.

# 9 STANDARD PROCEDURES

To facilitate future regression analysis of nondestructive testing data, the concrete industry should consider the adoption of two standards: a standard format for reporting destructive and nondestructive test results, and a standard method for the regression analysis of nondestructive tests. Two advantages of this are uniformity and correctness. Results from a regression analysis would be familiar to other researchers who may wish to incorporate the results into a composite problem. Also, the raw data would be in a format suitable for incorporation into databases; if these standards had been in place previously, there would have been a large quantity of additional data that could have been included into this report, but were not included due to a lack of required information. Finally, the standards would insure that sound statistical methods would be used, insuring that the results are both accurate and meaningful.

# 10 SUMMARY

This report has presented a statistical method for establishing a regression relationship between non-destructive and destructive cylinder compressive strength measurements when data from similar mixes must be combined to form a single regression relationship. This method addresses both the constant coefficient of variation in the data and the error on both variables. The method also attempts to account for unknown nuisance factors by artificially increasing the pooled variances of the data, using a self validation technique as a constraint.

The methods developed herein can be used to estimate both the true mean, and point estimates. Although estimates of the true mean are reported more often, the variance of point estimates yield information about the distribution of the strength population. Information about the strength population is required for calculating the characteristic strengths needed to asses structural integrity.

Difficulties experienced while compiling this information suggest that standards, or simple guidelines, should be required for the reporting of nondestructive and destructive test data. If guidelines had been in place before this report was written, a large number of data from additional sources could have been included, but were not included because critical information was missing.

# ACKNOWLEDEMENT

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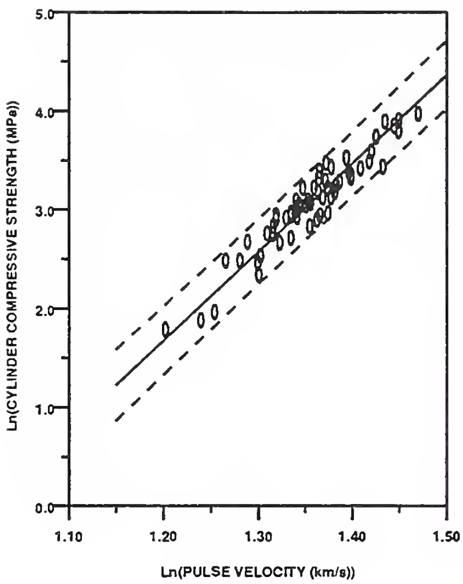
[9] C.G. Petersen, "LOK-test and CAPO-test Development and Their Application", *Proc. Instn. Civ. Engrs*, Part 1, **76**, 539–549, May 1984.

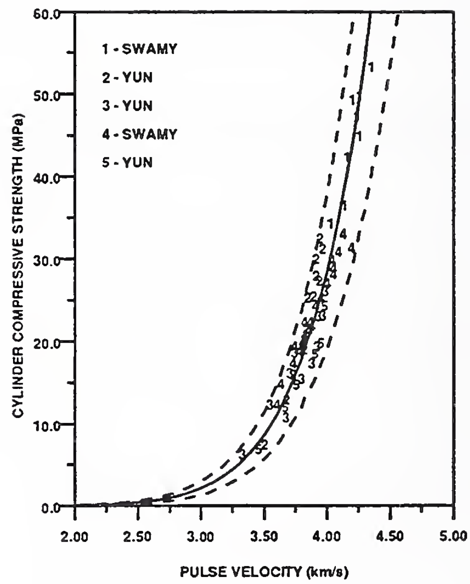
# A REGRESSION RESULTS

The results appear on consecutive pages. The first page displays the data, both the transformed and actual forms. Displaying the data lets the analyst get a visual "feel" for the data. The second page gives the results for the validation tests and the regression analysis. The regression results are used to make future predictions. The validation results give an indication of the distribution of the data and the data's ability to predict future observations.

PULSE VELOCITY: **PV_G1** - Gravel

| W/C | CA |
|---|---|
| .37–.72 | .44–.49 |




| # | EXPERIMENTER | w/c | CA | RATING |
|---|---|---|---|---|
| 1 | SWAMY1 | .44 | .44 | AACBDBBBB |
| 2 | YUN | .37–.44 | .45–.47 | AABBBABBD |
| 3 | YUN | .46–.53 | .47–.49 | AABBBABBD |
| 4 | SWAMY1 | .60–.72 | .46 | AACBDBBBB |
| 5 | YUN | .56 | .49 | AABBBABBD |

PULSE VELOCITY: **PV_G1** - Gravel

| W/C | CA |
|---|---|
| .37–.72 | .44–.49 |

| Validation | $\mathcal{F}$ | self50 | cap50 | self90 | cap90 |
|---|---|---|---|---|---|
| A→B | 12.0 | .52 | .52 | .97 | .97 |
| B→A | 12.8 | .58 | .52 | 1.00 | .97 |
| | | | | | |
| A→B | 10.3 | .41 | .50 | .97 | .97 |
| B→A | 15.6 | .67 | .50 | 1.00 | .97 |
| | | | | | |
| A→B | 12.5 | .53 | .50 | .90 | .91 |
| B→A | 11.7 | .53 | .50 | .97 | 1.00 |
| | | | | | |
| All Data | 12.5 | .50 | | .98 | |

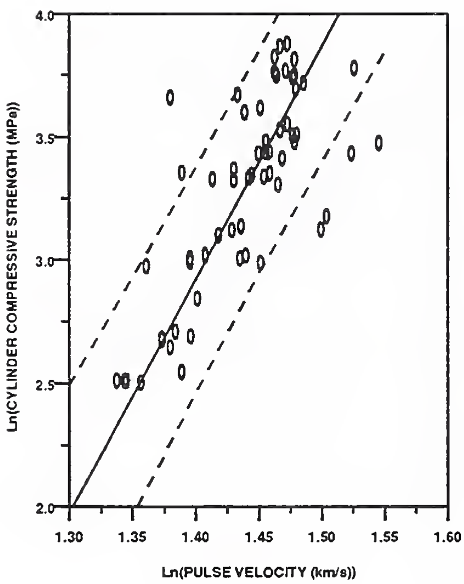
| | | | |
|---|---|---|---|
| $\hat{\alpha}_0$: | $\exp(-9.079)$ | $S_{uu}$: | 4.160 |
| $\hat{\alpha}_1$: | 8.959 | $s_e$: | 0.1423 |
| $s_{\hat{\alpha}_0}$: | $0.4833\hat{\alpha}_0$ | $\overline{n}_X$: | 10.06 |
| $s_{\hat{\alpha}_1}$: | 0.3558 | $\overline{n}_Y$: | 2.452 |
| $\overline{\overline{X}}$: | 1.357 | $s_\delta^2$: | 0.0003311 |
| $\overline{\overline{Y}}$: | 3.082 | $s_\epsilon^2$: | 0.006483 |
| $k$: | 0.4575 | $n$: | 62 |
| $b$: | 8.959 | F-prob: | 0.009275 |

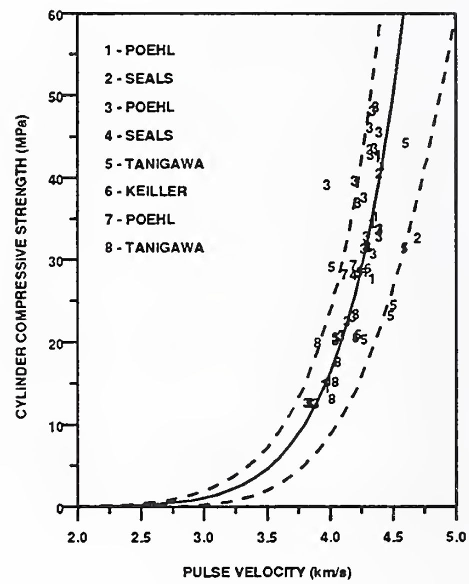
NOTE -

F-prob = cumulative distribution function (CDF) value of F-statistic. If F-prob is greater than 0.99, conclude the linear model is inadequate.

PULSE VELOCITY: **PV_G2** - Gravel

| W/C | CA |
|---|---|
| .33–.70 | .50–.55 |




| # | EXPERIMENTER | w/c | CA | RATING |
|---|---|---|---|---|
| 1 | POEHL | .33 | .53 | CABABBBB* |
| 2 | SEALS | .40 | .52–.55 | AABAABCBD |
| 3 | POEHL | .48 | .54 | CABABBBB* |
| 4 | SEALS | .55 | .51–.53 | AABAABCBD |
| 5 | TANIGAWA | .45–.55 | .50 | AABBBABBB |
| 6 | KEILLER | .66 | .55 | CABAABBBD |
| 7 | POEHL | .64 | .55 | CABABBBB* |
| 8 | TANIGAWA | .70 | .50 | AABBBABBB |

PULSE VELOCITY: **PV_G2** - Gravel

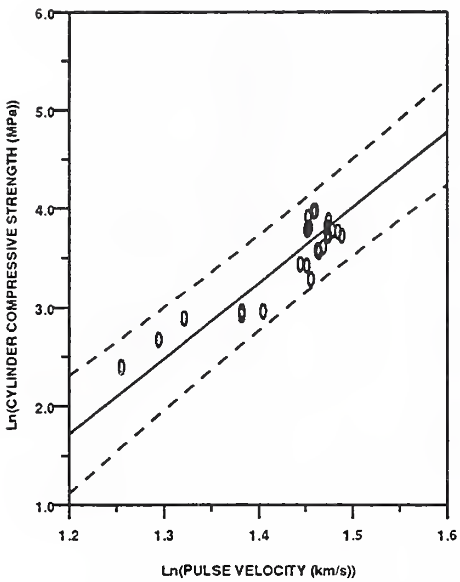
| W/C | CA |
|---|---|
| .33–.70 | .50–.55 |

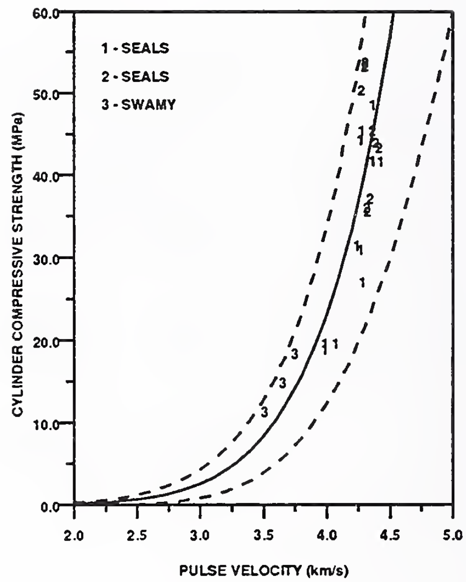
| Validation | $\mathcal{F}$ | self50 | cap50 | self90 | cap90 |
|---|---|---|---|---|---|
| A→B | 7.9 | .53 | .52 | .90 | .81 |
| B→A | 20.6 | .52 | .50 | .77 | .87 |
| | | | | | |
| A→B | 11.6 | .40 | .52 | .70 | .81 |
| B→A | 18.6 | .68 | .50 | 1.00 | .80 |
| | | | | | |
| A→B | 22.0 | .57 | .52 | .83 | .90 |
| B→A | 10.0 | .52 | .50 | .90 | .83 |
| | | | | | |
| All Data | 15.1 | .54 | | .85 | |

| | | | |
|---|---|---|---|
| $\hat{\alpha}_0$: | exp(−10.37) | $S_{uu}$: | 1.206 |
| $\hat{\alpha}_1$: | 9.491 | $s_e$: | 0.2997 |
| $s_{\hat{\alpha}_0}$: | 1.346$\hat{\alpha}_0$ | $\overline{n}_X$: | 7.098 |
| $s_{\hat{\alpha}_1}$: | 0.9354 | $\overline{n}_Y$: | 5.623 |
| $\overline{\overline{X}}$: | 1.438 | $s_\delta^2$: | 0.0005505 |
| $\overline{\overline{Y}}$: | 3.281 | $s_\epsilon^2$: | 0.02043 |
| $k$: | 0.2557 | $n$: | 61 |
| $b$: | 9.491 | F-prob: | 0.9191 |

PULSE VELOCITY: **PV_G3** - Gravel

| W/C | CA |
|---|---|
| .40–.82 | .56–.57 |




| # | EXPERIMENTER | w/c | CA | RATING |
|---|---|---|---|---|
| 1 | SEALS | .40 | .57 | AABAABCBD |
| 2 | SEALS | .55 | .56 | AABAABCBD |
| 3 | SWAMY1 | .82 | .56 | AACBDBBBB |

PULSE VELOCITY: **PV_G3** - Gravel

| W/C | CA |
|---|---|
| .40–.82 | .56–.57 |

| Validation | $\mathcal{F}$ | self50 | cap50 | self90 | cap90 |
|---|---|---|---|---|---|
| A→B | 7.4 | .50 | .50 | 1.00 | 1.00 |
| B→A | 11.4 | .50 | .50 | 1.00 | .92 |
| | | | | | |
| A→B | 10.3 | .67 | .50 | 1.00 | 1.00 |
| B→A | 4.0 | .21 | .50 | .93 | .92 |
| | | | | | |
| A→B | 9.3 | .54 | .54 | 1.00 | 1.00 |
| B→A | 11.2 | .62 | .54 | 1.00 | 1.00 |
| | | | | | |
| All Data | 8.9 | .50 | | 1.00 | |

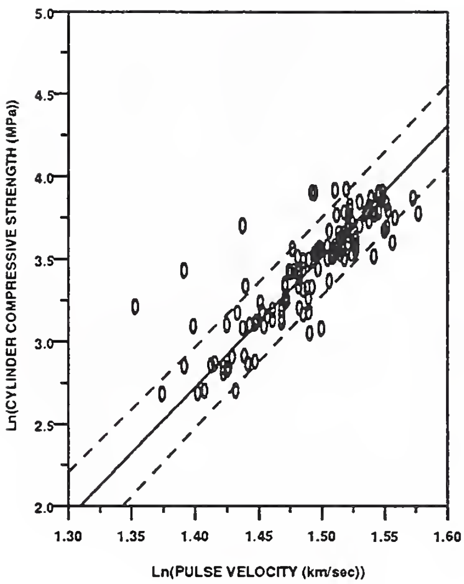
| | | | |
|---|---|---|---|
| $\hat{\alpha}_0$: | $\exp(-7.468)$ | $S_{uu}$: | 0.9002 |
| $\hat{\alpha}_1$: | 7.649 | $s_e$: | 0.2113 |
| $s_{\hat{\alpha}_0}$: | $1.062\hat{\alpha}_0$ | $\overline{n}_X$: | 4.038 |
| $s_{\hat{\alpha}_1}$: | 0.7390 | $\overline{n}_Y$: | 1.000 |
| $\overline{\overline{X}}$: | 1.435 | $s_\delta^2$: | 0.0008815 |
| $\overline{\overline{Y}}$: | 3.511 | $s_\epsilon^2$: | 0.02225 |
| $k$: | 0.3030 | $n$: | 26 |
| $b$: | 7.649 | F-prob: | 0.7688 |

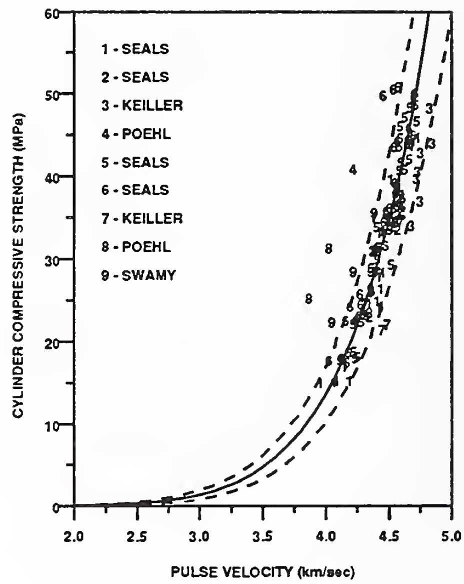
NOTE -

No replications for compressive strength data, $\overline{n}_Y = 1.0$.

PULSE VELOCITY: **PV_L1** - Limestone

| W/C | CA |
|---|---|
| .40–.78 | .46–.55 |




| # | EXPERIMENTER | w/c | CA | RATING |
|---|---|---|---|---|
| 1<br>2 | SEALS(FA:Sand)<br>SEALS(FA:Limestone) | .40 | .52–.55 | AABAABCBD |
| 3 | KEILLER | .47 | .47–.51 | CABAABBBD |
| 4 | POEHL | .45 | .51 | CABABBBB* |
| 5<br>6 | SEALS(FA:Sand)<br>SEALS(FA:Limestone) | .55 | .51–53 | AABAABCBD |
| 7 | KEILLER | .66 | .50–.54 | CABAABBBD |
| 8 | POEHL | .61–.78 | .54 | CABABBBB* |
| 9 | SWAMY1 | .60 | .46 | AACBDBBBB |

PULSE VELOCITY: **PV_L1** - Limestone

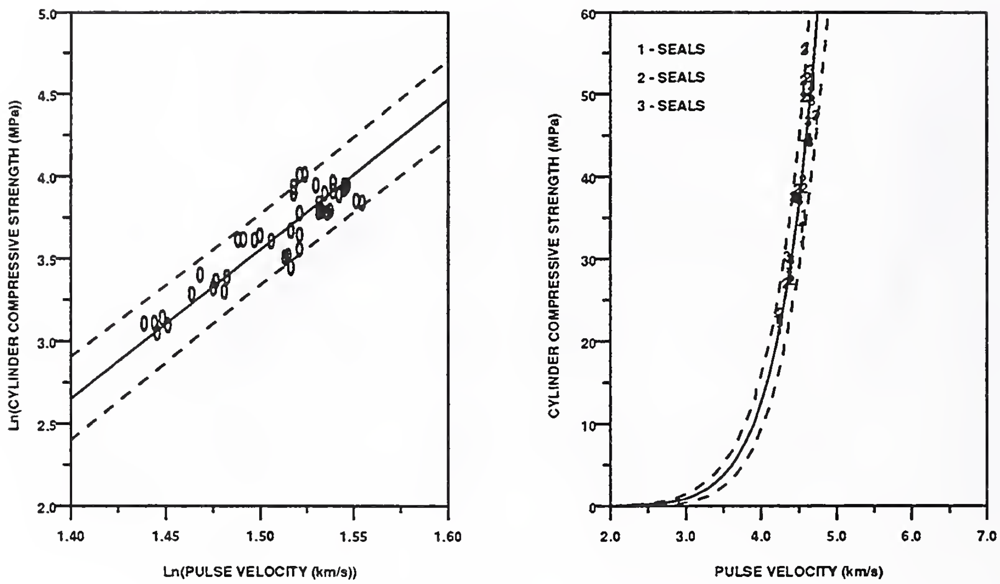
| W/C | CA |
| --- | --- |
| .40–.78 | .46–.55 |

| Validation | $\mathcal{F}$ | self50 | cap50 | self90 | cap90 |
| --- | --- | --- | --- | --- | --- |
| A→B | 1.5 | .36 | .52 | .81 | .78 |
| B→A | 2.3 | .55 | .51 | .84 | .84 |
| A→B | 2.5 | .55 | .51 | .88 | .85 |
| B→A | 1.17 | .34 | .51 | .76 | .75 |
| A→B | 1.5 | .48 | .51 | .80 | .81 |
| B→A | 2.8 | .61 | .50 | .88 | .82 |
| All Data | 2.0 | .50 | | .83 | |

| | | | |
| --- | --- | --- | --- |
| $\hat{\alpha}_0$: | $\exp(-8.407)$ | $S_{uu}$: | 4.567 |
| $\hat{\alpha}_1$: | 7.945 | $s_e$: | 0.1898 |
| $s_{\hat{\alpha}_0}$: | $0.6039\hat{\alpha}_0$ | $\overline{n}_X$: | 3.286 |
| $s_{\hat{\alpha}_1}$: | 0.4048 | $\overline{n}_Y$: | 1.323 |
| $\overline{\overline{X}}$: | 1.491 | $s_\delta^2$: | 0.0002202 |
| $\overline{\overline{Y}}$: | 3.442 | $s_\epsilon^2$: | 0.003909 |
| $k$: | 0.4477 | $n$: | 133 |
| $b$: | 7.945 | F-prob: | 1.000 |

PULSE VELOCITY: **PV_L2** - Limestone

| W/C | CA |
|---|---|
| .40–.55 | .56–.57 |




| # | EXPERIMENTER | w/c | CA | RATING |
|---|---|---|---|---|
| 1 | SEALS(FA:Sand) | .40 | .57 | AABAABCBD |
| 2 | SEALS(FA:Limestone) | | | |
| 3 | SEALS | .55 | .56 | AABAABCBD |

PULSE VELOCITY: **PV_L2** - Limestone

| W/C | CA |
|---|---|
| .40–.55 | .56–.57 |

| Validation | $\mathcal{F}$ | self50 | cap50 | self90 | cap90 |
|---|---|---|---|---|---|
| A→B | 1.6 | .67 | .52 | 1.00 | .91 |
| B→A | 1.1 | .43 | .52 | .91 | .90 |
| A→B | 0.94 | .50 | .50 | .73 | .95 |
| B→A | 1.4 | .55 | .50 | .95 | .86 |
| A→B | 0.81 | .38 | .52 | .76 | .96 |
| B→A | 1.5 | .65 | .52 | 1.00 | .81 |
| All Data | 1.0 | .50 | | .86 | |

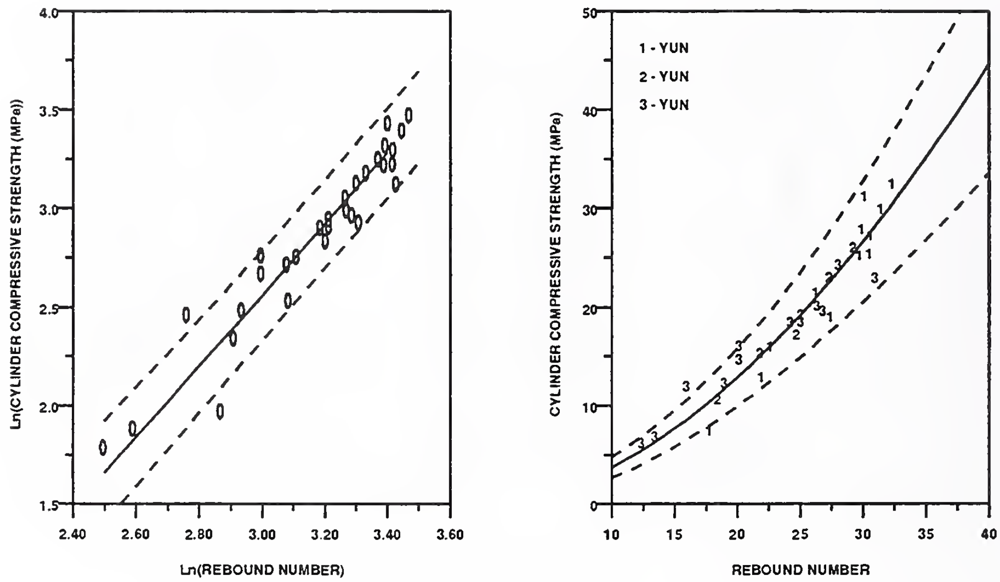
| | | | |
|---|---|---|---|
| $\hat{\alpha}_0$: | exp(−10.06) | $S_{uu}$: | 0.6489 |
| $\hat{\alpha}_1$: | 9.080 | $s_e$: | 0.1220 |
| $s_{\hat{\alpha}_0}$: | $0.9041\hat{\alpha}_0$ | $\overline{n}_X$: | 3.000 |
| $s_{\hat{\alpha}_1}$: | 0.5988 | $\overline{n}_Y$: | 1.000 |
| $\overline{\overline{X}}$: | 1.510 | $s_\delta^2$: | 0.0001333 |
| $\overline{\overline{Y}}$: | 3.644 | $s_\epsilon^2$: | 0.003721 |
| $k$: | 0.3253 | $n$: | 44 |
| $b$: | 9.080 | F-prob: | 0.5385 |

NOTE -

No replications of compressive strength data, $\overline{n}_Y = 1.0$. Since $\mathcal{F} \approx 1$, bootstrap was used: $\mathcal{F} \equiv 1.0$, and $s_Y^2 = (0.061)^2$.

REBOUND HAMMER: **RH_G1** - Gravel

| W/C | CA |
|---|---|
| .37–.56 | .45–.49 |



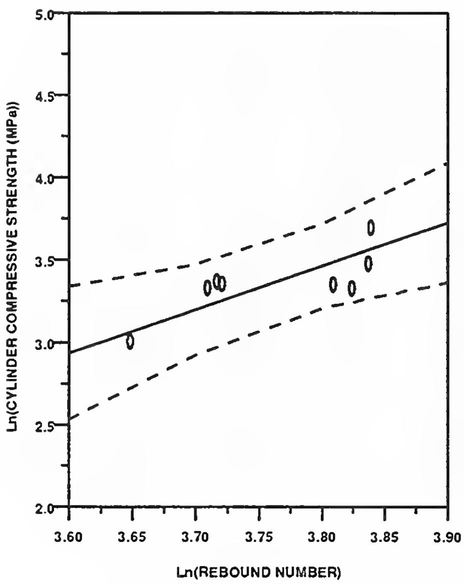

| # | EXPERIMENTER | w/c | CA | RATING |
|---|---|---|---|---|
| 1 | YUN | .37–.44 | .45–.47 | AABBBABBD |
| 2 | YUN | .46 | .49 | AABBBABBD |
| 3 | YUN | .53–.56 | .47–.49 | AABBBABBD |

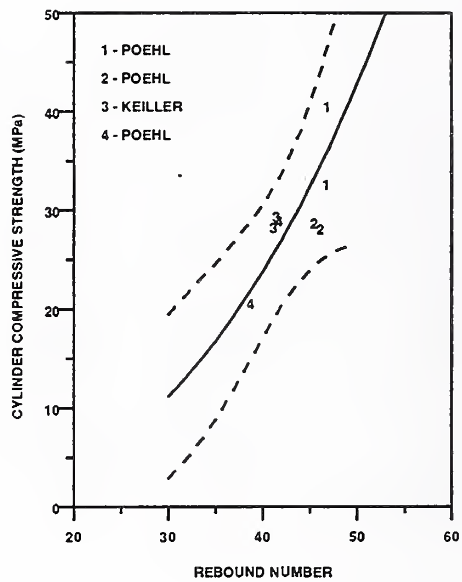
| W/C | CA |
|---|---|
| .37–.56 | .45–.49 |

| Validation | $\mathcal{F}$ | self50 | cap50 | self90 | cap90 |
|---|---|---|---|---|---|
| A→B | 3.5 | .50 | .50 | .86 | .81 |
| B→A | 4.6 | .63 | .50 | .94 | .79 |
| A→B | 3.5 | .57 | .50 | .79 | .88 |
| B→A | 4.2 | .63 | .50 | .81 | .86 |
| A→B | 1.6 | .25 | .50 | .69 | .79 |
| B→A | 10.1 | .79 | .50 | 1.00 | .88 |
| All Data | 4.6 | .60 | | .87 | |

| | | | |
|---|---|---|---|
| $\hat{\alpha}_0$: | $\exp(-2.843)$ | $S_{uu}$: | 138.3 |
| $\hat{\alpha}_1$: | 1.800 | $s_e$: | 0.1324 |
| $s_{\hat{\alpha}_0}$: | $0.3155\hat{\alpha}_0$ | $\overline{n}_X$: | 15.00 |
| $s_{\hat{\alpha}_1}$: | 0.09926 | $\overline{n}_Y$: | 4.000 |
| $\overline{\overline{X}}$: | 3.170 | $s_\delta^2$: | 0.003527 |
| $\overline{\overline{Y}}$: | 2.863 | $s_\epsilon^2$: | 0.001462 |
| $k$: | 4.342 | $n$: | 30 |
| $b$: | 1.800 | F-prob: | 0.8953 |

| W/C | CA |
|---|---|
| .33–.66 | .53–.55 |




| # | EXPERIMENTER | w/c | CA | RATING |
|---|---|---|---|---|
| 1 | POEHL | .33 | .53 | CABABBBB* |
| 2 | POEHL | .48 | .54 | CABABBBB* |
| 3 | KEILLER | .66 | .55 | CABAABBBD |
| 4 | POEHL | .64 | .55 | CABABBBB* |

REBOUND HAMMER: **RH_G2** - Gravel

| W/C | CA |
|---|---|
| .33–.66 | .53–.55 |

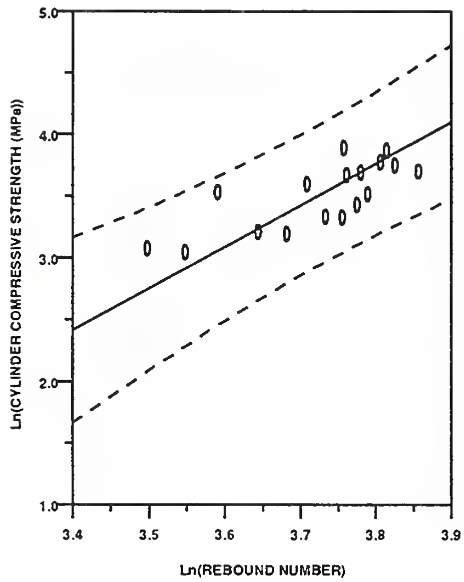
| Validation | $\mathcal{F}$ | self50 | cap50 | self90 | cap90 |
|---|---|---|---|---|---|
| A→B | 6.1 | 1.00 | .50 | 1.00 | 1.00 |
| B→A | 1.0 | .25 | .50 | 1.00 | .75 |
| | | | | | |
| A→B | 1.6 | .75 | .50 | 1.00 | 1.00 |
| B→A | 21. | .75 | .50 | 1.00 | .75 |
| | | | | | |
| A→B | 17. | 1.00 | .50 | 1.00 | .75 |
| B→A | 6.2 | 1.00 | .50 | 1.00 | 1.00 |
| | | | | | |
| All Data | 1.0 | .25 | | 1.00 | |

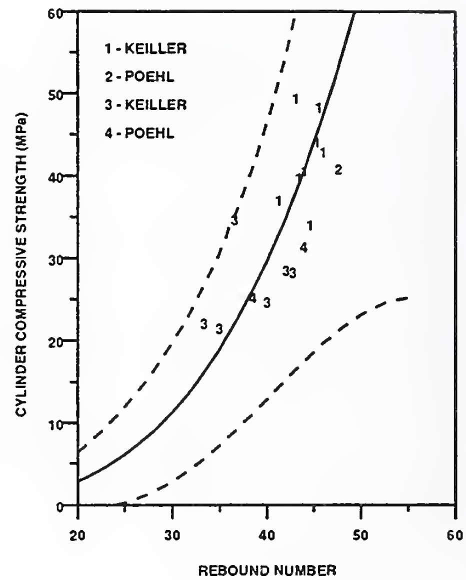
| | | | |
|---|---|---|---|
| $\hat{\alpha}_0$: | $\exp(-6.555)$ | $S_{uu}$: | .1401 |
| $\hat{\alpha}_1$: | 2.635 | $s_e$: | 0.1432 |
| $s_{\hat{\alpha}_0}$: | $2.998\hat{\alpha}_0$ | $\overline{n}_X$: | 14.25 |
| $s_{\hat{\alpha}_1}$: | 0.7967 | $\overline{n}_Y$: | 1.750 |
| $\overline{\overline{X}}$: | 3.763 | $s_\delta^2$: | 0.001077 |
| $\overline{\overline{Y}}$: | 3.362 | $s_\epsilon^2$: | 0.006914 |
| $k$: | 0.4106 | $n$: | 8 |
| $b$: | 2.635 | F-prob: | 0.7776 |

NOTE -

Only two data points had contained replications of the compressive strength measurements. To simplify the problem, the data were treated as if no replications existed. Since $\mathcal{F} \approx 1$, bootstrap was used: $\mathcal{F} \equiv 1.0$, and $s_Y^2 = (0.11)^2$.

| W/C | CA |
|---|---|
| .45–.78 | .47–.54 |




| # | EXPERIMENTER | w/c | CA | RATING |
|---|---|---|---|---|
| 1 | KEILLER | .47 | .47–.51 | CABAABBBD |
| 2 | POEHL | .45 | .51 | CABABBBB* |
| 3 | KEILLER | .66 | .50–.54 | CABAABBBD |
| 4 | POEHL | .61–.78 | .53 | CABABBBB* |

REBOUND HAMMER: **RH_L1** - Limestone

| W/C | CA |
|---|---|
| .45–.78 | .47–.54 |

| Validation | $\mathcal{F}$ | self50 | cap50 | self90 | cap90 |
|---|---|---|---|---|---|
| A→B | 1.1 | .30 | .57 | .90 | .86 |
| B→A | 3.0 | .71 | .50 | 1.00 | 1.00 |
| A→B | 12.5 | 1.00 | .56 | 1.00 | 1.00 |
| B→A | 10.1 | .89 | .50 | 1.00 | 1.00 |
| A→B | 2.9 | .67 | .50 | 1.00 | .88 |
| B→A | 3.0 | .50 | .56 | 1.00 | 1.00 |
| A→B | 3.6 | .56 | .50 | 1.00 | 1.00 |
| B→A | 5.7 | 1.00 | .56 | 1.00 | .89 |
| All Data | 5.2 | .53 | | 1.00 | |

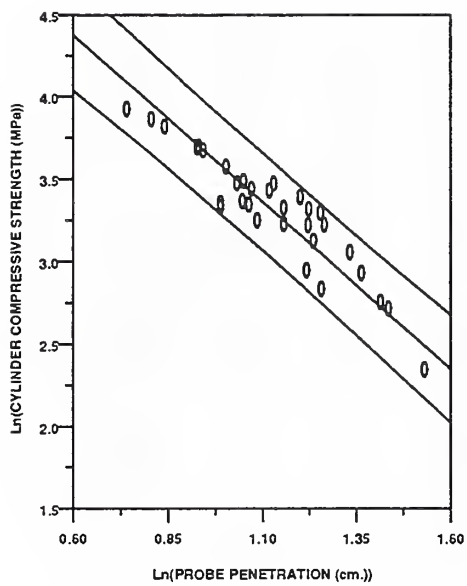
| | | | |
|---|---|---|---|
| $\hat{\alpha}_0$: | $\exp(-9.054)$ | $S_{uu}$: | 33.90 |
| $\hat{\alpha}_1$: | 3.372 | $s_e$: | 0.2291 |
| $s_{\hat{\alpha}_0}$: | $2.664\hat{\alpha}_0$ | $\overline{n}_X$: | 12.53 |
| $s_{\hat{\alpha}_1}$: | 0.7149 | $\overline{n}_Y$: | 3.471 |
| $\overline{\overline{X}}$: | 3.725 | $s_{\delta}^2$: | 0.005904 |
| $\overline{\overline{Y}}$: | 3.507 | $s_{\epsilon}^2$: | 0.003910 |
| $k$: | 5.092 | $n$: | 17 |
| $b$: | 3.372 | F-prob: | 0.2560 |

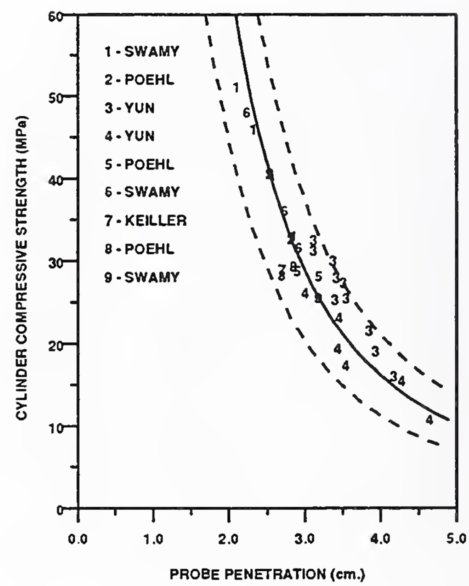
NOTE -

A fourth verification set was performed to average over the apparent anomalous second set.

PROBE PENETRATION: **PP_G1** - Gravel

| W/C | CA |
|---|---|
| .33–.82 | .44–.56 |




| # | EXPERIMENTER | w/c | CA | RATING |
|---|---|---|---|---|
| 1 | SWAMY2 | .44 | .44 | AACBDBBBB |
| 2 | POEHL | .33 | .53 | CABABBBB* |
| 3 | YUN | .37–.44 | .45–.47 | AABBBABBD |
| 4 | YUN | .46 | .49 | AABBBABBD |
| 5 | POEHL | .48 | .54 | CABABBBB* |
| 6 | SWAMY2 | .60 | .46 | AACBDBBBB |
| 7 | KEILLER | .66 | .55 | CABAABBBD |
| 8 | POEHL | .64 | .55 | CABABBBB* |
| 9 | SWAMY2 | .82 | .56 | AACBDBBBB |

PROBE PENETRATION: **PP_G1** - Gravel

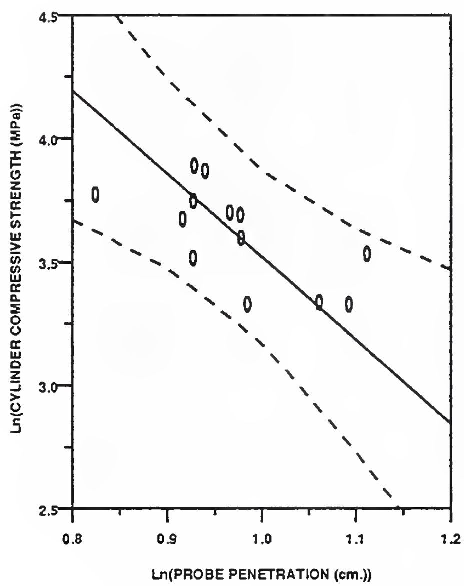
| W/C | CA |
|---|---|
| .33–.82 | .44–.56 |

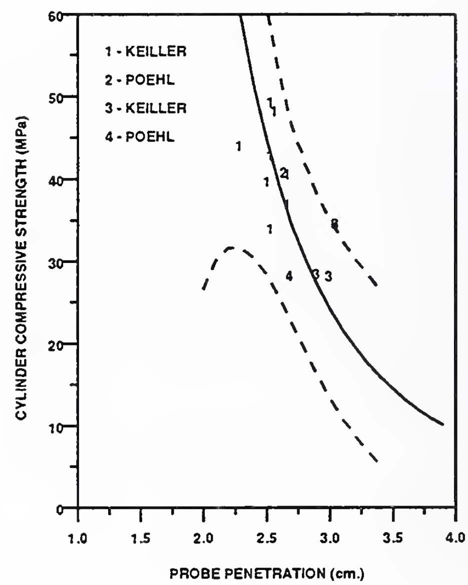
| Validation | $\mathcal{F}$ | self50 | cap50 | self90 | cap90 |
|---|---|---|---|---|---|
| A→B | 1.1 | .53 | .53 | 1.00 | 1.00 |
| B→A | 1.1 | .41 | .53 | 1.00 | .93 |
| | | | | | |
| A→B | 0.84 | .38 | .50 | .88 | .88 |
| B→A | 1.27 | .63 | .50 | 1.00 | 1.00 |
| | | | | | |
| A→B | 2.8 | .80 | .53 | 1.00 | 1.00 |
| B→A | 0.1 | .18 | .53 | .53 | .80 |
| | | | | | |
| All Data | 1.2 | .50 | | .97 | |

| | | | |
|---|---|---|---|
| $\hat{\alpha}_0$: | exp(5.594) | $S_{uu}$: | 1759. |
| $\hat{\alpha}_1$: | -2.029 | $s_e$: | 0.1407 |
| $s_{\hat{\alpha}_0}$: | $0.1664\hat{\alpha}_0$ | $\overline{n}_X$: | 3.500 |
| $s_{\hat{\alpha}_1}$: | 0.1462 | $\overline{n}_Y$: | 3.063 |
| $\overline{\overline{X}}$: | 1.126 | $s_\delta^2$: | 0.004906 |
| $\overline{\overline{Y}}$: | 3.309 | $s_\epsilon^2$: | 0.0004745 |
| $k$: | -20.98 | $n$: | 32 |
| $b$: | -2.029 | F-prob: | 0.4631 |

PROBE PENETRATION: **PP_L1** - Limestone

| W/C | CA |
|---|---|
| .45–.78 | .47–.54 |




| # | EXPERIMENTER | w/c | CA | RATING |
|---|---|---|---|---|
| 1 | KEILLER | .47 | .47–.51 | CABAABBBD |
| 2 | POEHL | .45 | .51 | CABABBBB* |
| 3 | KEILLER | .66 | .50–.54 | CABAABBBD |
| 4 | POEHL | .61–.78 | .54 | CABABBBB* |

PROBE PENETRATION: **PP_L1** - Limestone

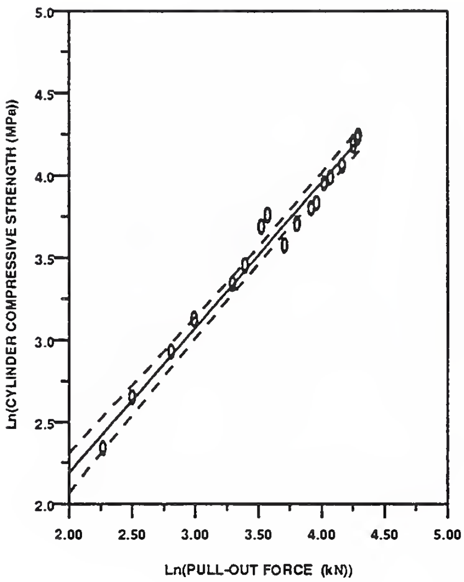
| W/C | CA |
|---|---|
| .45–.78 | .47–.54 |

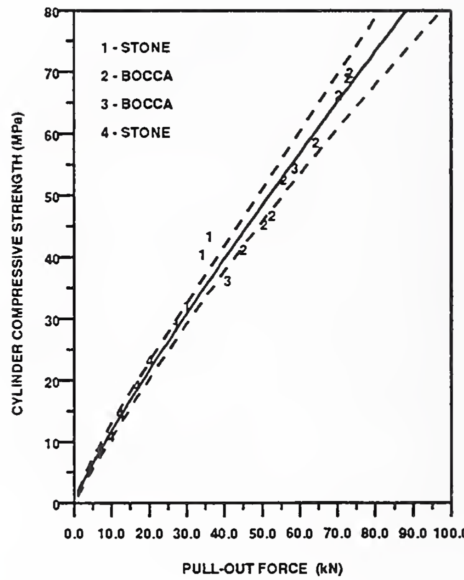
| Validation | $\mathcal{F}$ | self50 | cap50 | self90 | cap90 |
|---|---|---|---|---|---|
| A→B | 4.2 | .43 | .50 | 1.00 | 1.00 |
| B→A | 0.0 | .67 | .57 | 1.00 | 1.00 |
| A→B | 6.3 | .83 | .57 | 1.00 | 1.00 |
| B→A | 0.0 | .43 | .67 | 1.00 | 1.00 |
| A→B | 2.1 | .67 | .57 | 1.00 | .86 |
| B→A | 5.0 | .57 | .50 | 1.00 | 1.00 |
| All Data | 2.9 | .46 | | .92 | |

| | | | |
|---|---|---|---|
| $\hat{\alpha}_0$: | exp(6.889) | $S_{uu}$: | 4.493 |
| $\hat{\alpha}_1$: | -3.368 | $s_e$: | 0.2067 |
| $s_{\hat{\alpha}_0}$: | $1.003\hat{\alpha}_0$ | $\overline{n}_X$: | 3.000 |
| $s_{\hat{\alpha}_1}$: | 1.031 | $\overline{n}_Y$: | 3.538 |
| $\overline{\overline{\overline{X}}}$: | 0.9719 | $s_\delta^2$: | 0.001882 |
| $\overline{\overline{\overline{Y}}}$: | 3.615 | $s_\epsilon^2$: | 0.002232 |
| $k$: | -2.840 | $n$: | 13 |
| $b$: | -3.368 | F-prob: | 0.9214 |

PULLOUT: **PO_G1** - Gravel

| W/C | $A_T$ |
|---|---|
| .38–.66 | .70–.79 |




| # | EXPERIMENTER | w/c | $A_T$ | Apex Angle ($^o$) | RATING |
|---|---|---|---|---|---|
| 1 | STONE | .38 | .70 | 70 | AABAACAAA |
| 2 | BOCCA | .38–.40 | .74–.78 | 67 | CABABCACD |
| 3 | BOCCA | .38–.40 | .74–.78 | 67 | CABABCACD |
| 4 | STONE | .66 | .79 | 70 | AABAACAAA |

PULLOUT: **PO_G1** - Gravel

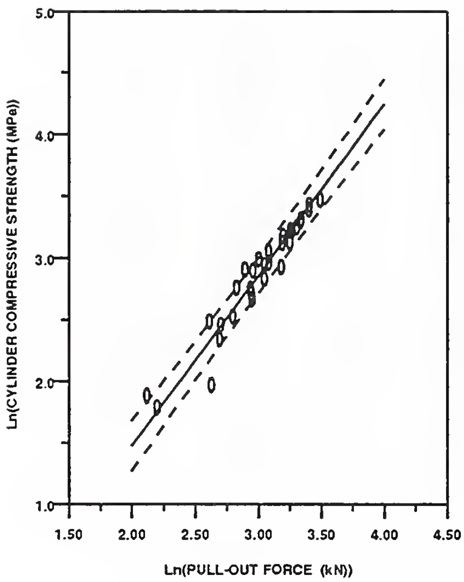
| W/C | $A_T$ |
|---|---|
| .38–.66 | .70–.79 |

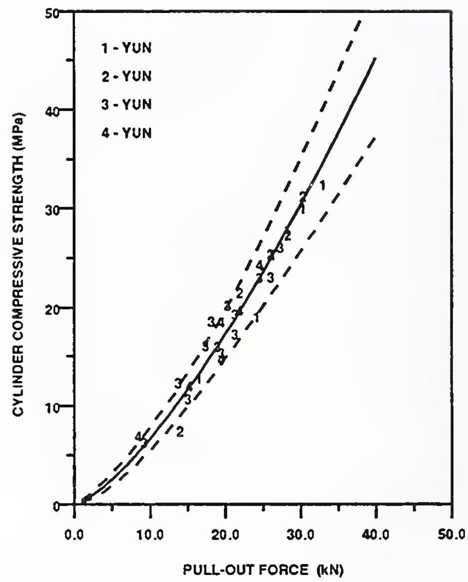
| Validation | $\mathcal{F}$ | self50 | cap50 | self90 | cap90 |
|---|---|---|---|---|---|
| A→B | 1.33 | 1.00 | .56 | 1.00 | 1.00 |
| B→A | 0.0 | .78 | .78 | 1.00 | 1.00 |
| | | | | | |
| A→B | 0.0 | .30 | .63 | .70 | .88 |
| B→A | .336 | .88 | .50 | 1.00 | 1.00 |
| | | | | | |
| A→B | 0.0 | .60 | .63 | 1.00 | 1.00 |
| B→A | .841 | .88 | .50 | 1.00 | .90 |
| | | | | | |
| All Data | 1.0 | .84 | | 1.00 | |

| | | | |
|---|---|---|---|
| $\hat{\alpha}_0$: | exp(+0.4196) | $S_{uu}$: | 114.4 |
| $\hat{\alpha}_1$: | 0.8839 | $s_e$: | 0.08230 |
| $s_{\hat{\alpha}_0}$: | 0.1186$\hat{\alpha}_0$ | $\overline{n}_X$: | 17.11 |
| $s_{\hat{\alpha}_1}$: | 0.03249 | $\overline{n}_Y$: | 15.56 |
| $\overline{\overline{X}}$: | 3.602 | $s_\delta^2$: | 0.0002434 |
| $\overline{\overline{Y}}$: | 3.603 | $s_\epsilon^2$: | 0.00005904 |
| $k$: | 3.645 | $n$: | 18 |
| $b$: | 0.8839 | F-prob: | 1.000 |

PULLOUT: **PO_G2** - Gravel LOK-test

| W/C | $A_T$ |
|---|---|
| .37–.56 | .73–.80 |




| # | EXPERIMENTER | w/c | $A_T$ | Apex Angle ($^o$) | RATING |
|---|---|---|---|---|---|
| 1 | YUN | .37 | .73 | 62 | AABBBABBD |
| 2 | YUN | .44 | .76 | 62 | AABBBABBD |
| 3 | YUN | .46–.53 | .78–.79 | 62 | AABBBABBD |
| 4 | YUN | .56 | .80 | 62 | AABBBABBD |

PULLOUT: **PO_G2** - Gravel LOK-test

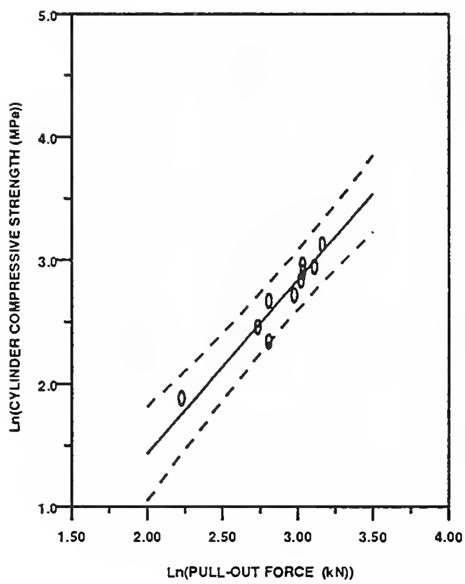
| W/C | $A_T$ |
|---|---|
| .37–.56 | .73–.80 |

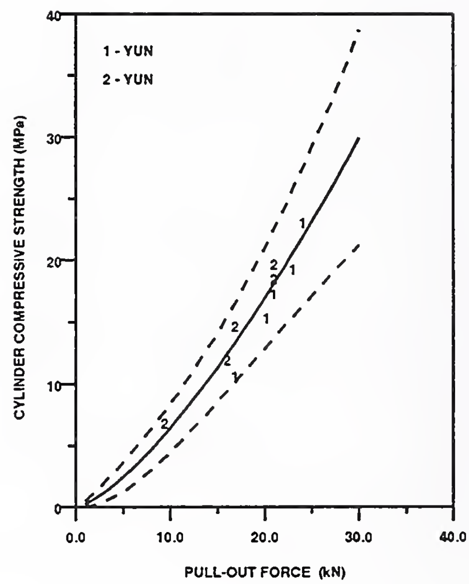
| Validation | $\mathcal{F}$ | self50 | cap50 | self90 | cap90 |
|---|---|---|---|---|---|
| A→B | .015 | .56 | .50 | 1.00 | 1.00 |
| B→A | .100 | .57 | .50 | 1.00 | .94 |
| | | | | | |
| A→B | .09 | .69 | .50 | 1.00 | 1.00 |
| B→A | .09 | .64 | .50 | 1.00 | 1.00 |
| | | | | | |
| A→B | .037 | .75 | .50 | 1.00 | 1.00 |
| B→A | .05 | .57 | .50 | 1.00 | .75 |
| | | | | | |
| All Data | 1.00 | .97 | | 1.00 | |

| | | | |
|---|---|---|---|
| $\hat{\alpha}_0$: | exp(−1.303) | $S_{uu}$: | 673.7 |
| $\hat{\alpha}_1$: | 1.386 | $s_e$: | 0.1211 |
| $s_{\hat{\alpha}_0}$: | $0.2112\hat{\alpha}_0$ | $\overline{n}_X$: | 8.000 |
| $s_{\hat{\alpha}_1}$: | 0.06989 | $\overline{n}_Y$: | 4.000 |
| $\overline{\overline{X}}$: | 3.005 | $s^2_\delta$: | 0.002312 |
| $\overline{\overline{Y}}$: | 2.863 | $s^2_\epsilon$: | 0.0003179 |
| $k$: | 10.08 | $n$: | 30 |
| $b$: | 1.386 | F-prob: | 1.0000 |

PULLOUT: **PO_G3** - Gravel CAPO-test

| W/C | $A_T$ |
|---|---|
| .46–.56 | .78–.80 |




| # | EXPERIMENTER | w/c | $A_T$ | Apex Angle ($^o$) | RATING |
|---|---|---|---|---|---|
| 1 | YUN | .46 | .78 | 62 | AABBBABBD |
| 2 | YUN | .56 | .80 | 62 | AABBBABBD |

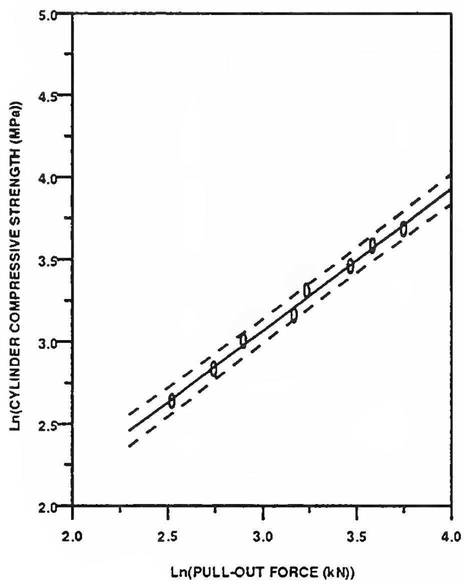
| W/C | $A_T$ |
|---|---|
| .46–.56 | .78–.80 |

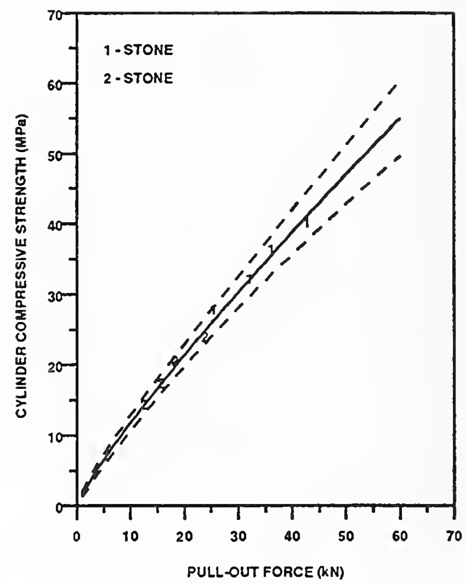
| Validation | $\mathcal{F}$ | self50 | cap50 | self90 | cap90 |
|---|---|---|---|---|---|
| A→B | 0.0 | 1.00 | 1.00 | 1.00 | 1.00 |
| B→A | 0.0 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | | | | |
| A→B | 0.0 | 1.00 | 1.00 | 1.00 | 1.00 |
| B→A | 0.03 | 1.00 | .50 | 1.00 | 1.00 |
| | | | | | |
| A→B | 0.0 | 1.00 | 1.00 | 1.00 | 1.00 |
| B→A | 0.0 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | | | | |
| All Data | 1.00 | 1.00 | | 1.00 | |

| | | | |
|---|---|---|---|
| $\hat{\alpha}_0$: | exp(−1.380) | $S_{uu}$: | 497.9 |
| $\hat{\alpha}_1$: | 1.405 | $s_e$: | 0.1122 |
| $s_{\hat{\alpha}_0}$: | $0.4140\hat{\alpha}_0$ | $\overline{n}_X$: | 8.000 |
| $s_{\hat{\alpha}_1}$: | 0.1426 | $\overline{n}_Y$: | 4.000 |
| $\overline{\overline{X}}$: | 2.892 | $s_\delta^2$: | 0.04807 |
| $\overline{\overline{Y}}$: | 2.683 | $s_\epsilon^2$: | 0.0003468 |
| $k$: | 19.47 | $n$: | 10 |
| $b$: | 1.405 | F-prob: | 0.7359 |

PULLOUT: **PO_L1** - Limestone

| W/C | $A_T$ |
|---|---|
| .38–.66 | .70–.79 |




| # | EXPERIMENTER | w/c | $A_T$ | Apex Angle ($^o$) | RATING |
|---|---|---|---|---|---|
| 1 | STONE | .38 | .70 | 70 | AABAACAAA |
| 2 | STONE | .66 | .79 | 70 | AABAACAAA |

PULLOUT: **PO_L1** - Limestone

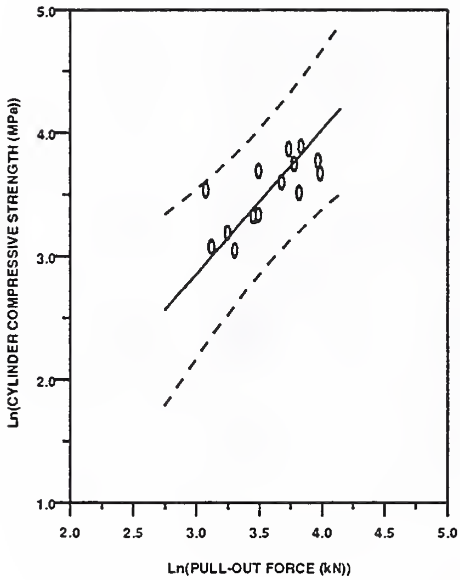
| W/C | $A_T$ |
|---|---|
| .38–.66 | .70–.79 |

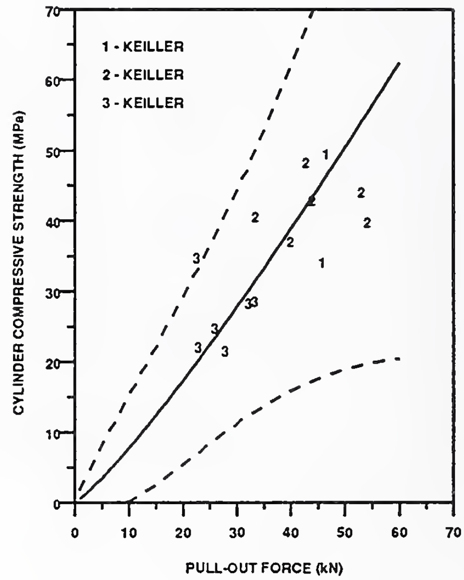
| Validation | $\mathcal{F}$ | self50 | cap50 | self90 | cap90 |
|---|---|---|---|---|---|
| A→B | 1.2 | .75 | .50 | 1.00 | 1.00 |
| B→A | 0.10 | .50 | .50 | 1.00 | 1.00 |
| A→B | 0.03 | .50 | .50 | 1.00 | 1.00 |
| B→A | 1.1 | .75 | .50 | 1.00 | 1.00 |
| A→B | 2.4 | 1.00 | .50 | 1.00 | 1.00 |
| B→A | 2.7 | 1.00 | .50 | 1.00 | 1.00 |
| All Data | 1.0 | .50 | | 1.00 | |

| | | | |
|---|---|---|---|
| $\hat{\alpha}_0$: | exp(0.4701) | $S_{uu}$: | 10.40 |
| $\hat{\alpha}_1$: | 0.8637 | $s_e$: | 0.03225 |
| $s_{\hat{\alpha}_0}$: | $0.09147\hat{\alpha}_0$ | $\overline{n}_X$: | 8.375 |
| $s_{\hat{\alpha}_1}$: | 0.02862 | $\overline{n}_Y$: | 5.000 |
| $\overline{\overline{X}}$: | 3.171 | $s_\delta^2$: | 0.001023 |
| $\overline{\overline{Y}}$: | 3.209 | $s_\epsilon^2$: | 0.0004101 |
| $k$: | 2.155 | $n$: | 8 |
| $b$: | 0.8637 | F-prob: | 0.4922 |

PULLOUT: **PO_L2** - Limestone LOK-test

| W/C | $A_T$ |
|---|---|
| .47–.66 | .72–.82 |




| # | EXPERIMENTER | w/c | $A_T$ | Apex Angle (°) | RATING |
|---|---|---|---|---|---|
| 1 | KEILLER | .47 | .72 | 62 | CABAABBBD |
| 2 | KEILLER | .47 | .79 | 62 | CABAABBBD |
| 3 | KEILLER | .66 | .77–.82 | 62 | CABAABBBD |

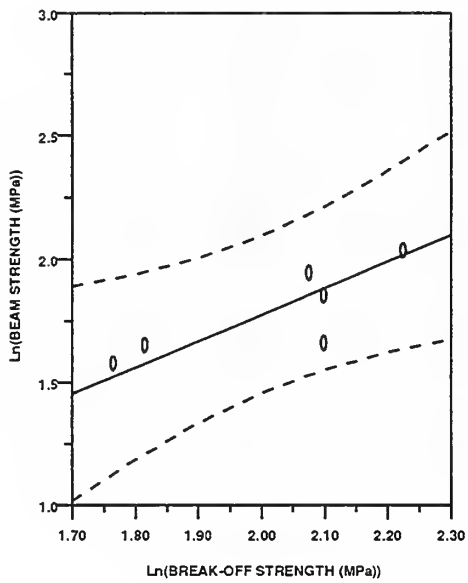
| W/C | $A_T$ |
|---|---|
| .47–.66 | .72–.82 |

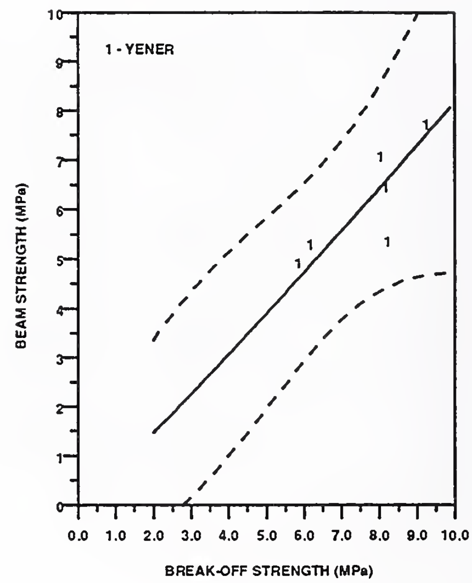
| Validation | $\mathcal{F}$ | self50 | cap50 | self90 | cap90 |
|---|---|---|---|---|---|
| A→B | 4.7 | .43 | .57 | 1.00 | .86 |
| B→A | 13.1 | .86 | .57 | 1.00 | 1.00 |
| A→B | 1.9 | .43 | .57 | 1.00 | .86 |
| B→A | 18.0 | .57 | .57 | 1.00 | 1.00 |
| A→B | 0.0 | .71 | .57 | 1.00 | 1.00 |
| B→A | 31. | .86 | .57 | 1.00 | 1.00 |
| All Data | 11.5 | .64 | | .93 | |

| | | | |
|---|---|---|---|
| $\hat{\alpha}_0$: | exp(−0.6444) | $S_{uu}$: | 78.01 |
| $\hat{\alpha}_1$: | 1.167 | $s_e$: | 0.2478 |
| $s_{\hat{\alpha}_0}$: | 1.019$\hat{\alpha}_0$ | $\overline{n}_X$: | 6.000 |
| $s_{\hat{\alpha}_1}$: | 0.2849 | $\overline{n}_Y$: | 4.000 |
| $\overline{\overline{X}}$: | 3.570 | $s_\delta^2$: | 0.05026 |
| $\overline{\overline{Y}}$: | 3.520 | $s_\epsilon^2$: | 0.007470 |
| $k$: | 7.850 | $n$: | 14 |
| $b$: | 1.167 | F-prob: | 0.3597 |

BREAK-OFF: **BO_G1** - Gravel

| W/C | $A_T$ |
|---|---|
| .35–.53 | .38–.46 |




| # | EXPERIMENTER | w/c | $A_T$ | RATING |
|---|---|---|---|---|
| 1 | YENER | .35–.53 | .38–.46 | CAAABCCBD |

BREAK-OFF: **BO_G1** - Gravel

| W/C | $A_T$ |
|---|---|
| .35–.53 | .38–.46 |

| Validation | $\mathcal{F}$ | self50 | cap50 | self90 | cap90 |
|---|---|---|---|---|---|
| A→B | 22.7 | 1.00 | .67 | 1.00 | 1.00 |
| B→A | 0.0 | 1.00 | 1.00 | 1.00 | 1.00 |
| | | | | | |
| A→B | 0.0 | .67 | .67 | 1.00 | 1.00 |
| B→A | 0.0 | .67 | .67 | 1.00 | 1.00 |
| | | | | | |
| A→B | 3.7 | 1.00 | .67 | 1.00 | 1.00 |
| B→A | 11.7 | 1.00 | .67 | 1.00 | 1.00 |
| | | | | | |
| All Data | 6.3 | .67 | | 1.00 | |

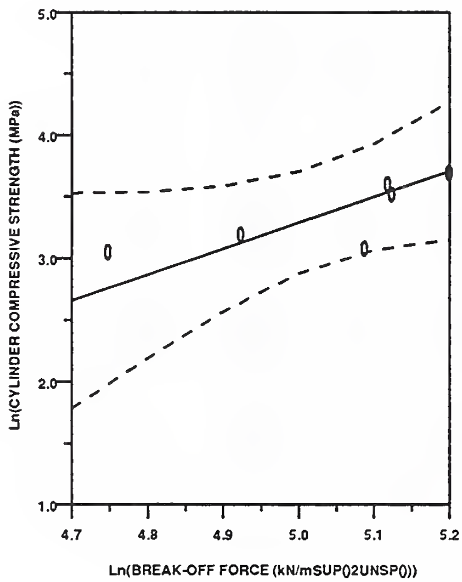
| | | | |
|---|---|---|---|
| $\hat{\alpha}_0$: | exp(−0.3660) | $S_{uu}$: | 1.111 |
| $\hat{\alpha}_1$: | 1.070 | $s_e$: | 0.1287 |
| $s_{\hat{\alpha}_0}$: | $0.6939\hat{\alpha}_0$ | $\overline{n}_X$: | 7.000 |
| $s_{\hat{\alpha}_1}$: | 0.3438 | $\overline{n}_Y$: | 7.000 |
| $\overline{\overline{X}}$: | 2.012 | $s_\delta^2$: | 0.009095 |
| $\overline{\overline{Y}}$: | 1.788 | $s_\epsilon^2$: | 0.005734 |
| $k$: | 1.698 | $n$: | 6 |
| $b$: | 1.070 | F-prob: | 0.6003 |

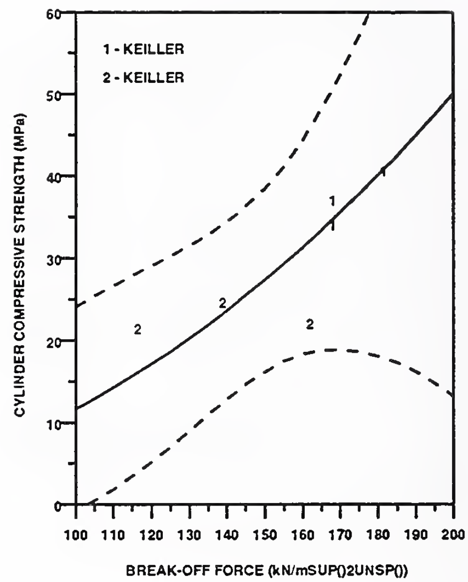
NOTE -

The apparently poor performance of the self-verification should be attributed to the small size of the data set.

BREAK-OFF: **BO_L1** - Limestone

| W/C | $A_T$ |
|---|---|
| .47–.66 | .47–.54 |




| # | EXPERIMENTER | w/c | $A_T$ | RATING |
|---|---|---|---|---|
| 1 | KEILLER | .47 | .47–.51 | CABAABBBD |
| 2 | KEILLER | .66 | .50–.54 | CABAABBBD |

BREAK-OFF: **BO_L1** - Limestone

| W/C | $A_T$ |
|---|---|
| .47–.66 | .47–.54 |

| Validation | $\mathcal{F}$ | self50 | cap50 | self90 | cap90 |
|---|---|---|---|---|---|
| A→B | 0.0 | .67 | 1.00 | 1.00 | 1.00 |
| B→A | 1.7 | 1.00 | .67 | 1.00 | 1.00 |
| A→B | 3.2 | 1.00 | .67 | 1.00 | 1.00 |
| B→A | 0.0 | .67 | 1.00 | 1.00 | 1.00 |
| A→B | 0.0 | 1.00 | 1.00 | 1.00 | 1.00 |
| B→A | 2.5 | 1.00 | .67 | 1.00 | 1.00 |
| All Data | 1.2 | .67 | | .83 | |

| | | | |
|---|---|---|---|
| $\hat{\alpha}_0$: | $\exp(-7.242)$ | $S_{uu}$: | 23.15 |
| $\hat{\alpha}_1$: | 2.105 | $s_e$: | 0.2503 |
| $s_{\hat{\alpha}_0}$: | $4.205\hat{\alpha}_0$ | $\overline{n}_X$: | 6.000 |
| $s_{\hat{\alpha}_1}$: | 0.8353 | $\overline{n}_Y$: | 4.000 |
| $\overline{\overline{X}}$: | 5.033 | $s_\delta^2$: | 0.002454 |
| $\overline{\overline{Y}}$: | 3.354 | $s_\epsilon^2$: | 0.0007224 |
| $k$: | 7.153 | $n$: | 6 |
| $b$: | 2.105 | F-prob: | 0.9986 |

NOTE -

The apparently poor performance of the self-verification should be attributed to the small size of the data set.

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# B QUALITY RATINGS

Each of the data sets used in determining the regression coefficients had different aspects that made them either more or less desirable than others in respect to the quality of the data published. An attempt was made to quantify different aspects of reports and rate them for different criteria. The nine criteria chosen were based upon the eleven data quality criteria of the Oak Ridge National Laboratory report ORNL/NRC/LTR-90/22. Since the Oak Ridge report had a slightly different intent for their ratings, only those criteria that could be related to this report were kept and the definitions were altered to address specific aspects of nondestructive testing and the reporting of the results.

Nine criteria were chosen and defined on the following page. Each criterion can receive a rating from A to D, with A being the best, or most desirable. In some cases, the absence of information was deemed more desirable than the publication of incorrect information (see #4, #6, and #8). The Table 2 lists the justification for each rating awarded.

## B.1 Nine Criteria

#1 Completeness of Material Description -

Rating of the thoroughness with which the mix designs and material properties of the mix constituents were reported. This would include consideration of the water:cement ratio, large aggregate-fine aggregate ratios, type of coarse aggregate, type of fine aggregate, type of cement, etc.

#2 Type of Input -

Describes the type of data used in the investigation reported. The data may be from either experimental work performed by the author or from referenced work.

#3 Completeness of Data -

What fraction of the data performed in the experiment are published. Considers whether all of the data are given or only the average values from a number of replications, and whether the number of replications was included.

#4 Completeness of Resources -

Rates experimental proficiency. Evaluates whether the experimental apparatus was used properly based upon the authors account of the procedure.

#5 Quality of Resources -

Evaluates the condition and appropriateness of equipment used.

#6 Consistency of Results -

Considers whether separately measured but related responses from a single experiment are correlated.

#7 Precision and Scatter -

A rating of the degree to which the data scatter. This rating is relative to all of the other publications.

#8 Uncertainty and Bias -

Describes any notable bias in the results by considering the description of the experiment.

#9 Statistical Methods -

Describes how the experimenters analyzed their data. It considers whether data were properly transformed to account for constant coefficient of variation. Also determines whether error in both observables was accounted for in the regression analysis.

| Criterion | A | B | C | D |
|---|---|---|---|---|
| 1 | Given: w/c, agg/cement, FA/CA, and material properties | Given: w/c, agg/cement, FA,CA, but no aggregate type | Given: w/c, total aggregate content | Given: No w/c nor aggregate type |
| 2 | Only experimental data is used | Mostly experimental, some referenced data used | Some experimental, mostly referenced data used | Only referenced data used |
| 3 | All data published and used | All averages and variablities reported | No variability given for averaged values | No data given |
| 4 | Excellent description of NDE tests | Fair description of NDE tests | No tests explained | Incorrect explanation of NDE test |
| 5 | Specified equipment and brand name | Used equipment specified by appropriate test standard | Outdated equipment used | No description of equipment |
| 6 | Strong consistencies evident | Good consistencies | Does not apply | Inconsistencies in test results |
| 7 | Relatively excellent precision | Good precision | Fair precision | Poor precision |
| 8 | Experiment explained well and no bias evident | No bias expected from explanation given | Cannot be sure whether bias exists | Strong bias evident |
| 9 | Transformed data properly, did address errors in $x$ and $y$ data | Transformed data properly, did not address errors in $x$ and $y$ data | Did not transform data properly, did address errors in $x$ and $y$ data | Did not transform data properly, did not address errors in $x$ and $y$ data |

Table 2: Justification for data rating.

| NIST-114A (REV. 3-90) | U.S. DEPARTMENT OF COMMERCE<br>NATIONAL INSTITUTE OF STANDARDS AND TECHNOLOGY | 1. PUBLICATION OR REPORT NUMBER<br>NISTIR 4874 |
|---|---|---|
| | | 2. PERFORMING ORGANIZATION REPORT NUMBER |
| | **BIBLIOGRAPHIC DATA SHEET** | 3. PUBLICATION DATE<br>JULY 1992 |

**4. TITLE AND SUBTITLE**

Nondestructive Evaluation of the In-Place Compressive Strength of Concrete Based Upon Limited Destructive Testing

**5. AUTHOR(S)**

Kenneth Snyder; Nicholas Carino; James Clifton

**6. PERFORMING ORGANIZATION (IF JOINT OR OTHER THAN NIST, SEE INSTRUCTIONS)**

U.S. DEPARTMENT OF COMMERCE
NATIONAL INSTITUTE OF STANDARDS AND TECHNOLOGY
GAITHERSBURG, MD 20899

**7. CONTRACT/GRANT NUMBER**

**8. TYPE OF REPORT AND PERIOD COVERED**

**9. SPONSORING ORGANIZATION NAME AND COMPLETE ADDRESS (STREET, CITY, STATE, ZIP)**

**10. SUPPLEMENTARY NOTES**

**11. ABSTRACT (A 200-WORD OR LESS FACTUAL SUMMARY OF MOST SIGNIFICANT INFORMATION. IF DOCUMENT INCLUDES A SIGNIFICANT BIBLIOGRAPHY OR LITERATURE SURVEY, MENTION IT HERE.)**

Regression analysis was performed on published data from nondestructive and cylinder compressive strength testing of concrete. The nondestructive tests investigated were: rebound hammer, probe penetration, pulse velocity, pullout, and break-off. Regression analysis accounted for the error in both the nondestructive and the compressive strength data and their constant coefficient of variation. Data for each nondestructive test were grouped by coarse aggregate type and aggregate mass fraction. The results of the regression analysis are given, along with the parameters required to estimate compressive strength from subsequent nondestructive tests. A common format for the analysis and reporting of nondestructive-destructive regression experiments is suggested.

**12. KEY WORDS (6 TO 12 ENTRIES; ALPHABETICAL ORDER; CAPITALIZE ONLY PROPER NAMES; AND SEPARATE KEY WORDS BY SEMICOLONS)**

break-off; nondestructive testing; probe penetration; pull-out; pulse velocity; rebound hammer; regression analysis; strength estimation

**13. AVAILABILITY**

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