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Model for the Non-Perturbative QCD Vacuum

Michael Danos

U.S. DEPARTMENT OF COMMERCE
National Institute of Standards and Technology
Gaithersburg, MD 20899

Daniel Gogny
Daniel Iracane

Centre d'Etudes de Bruyeres-le-Chatel
91680 Bruyeres-le-Chatel, France

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Abstract

By treating the high-momentum gluon and the quark sector as an in principle calculable effective Lagrangian we obtain a non-perturbative vacuum state for QCD as an infrared quark-gluon condensate. This vacuum is removed from the perturbative vacuum by an energy gap and supports a Meissner- Ochsenfeld effect. It is unstable below a minimum size and it also suggests the existence of a universal hadronization time. This vacuum thus exhibits all the properties required for color confinement.

1. Introduction

By now it is widely believed that the confinement in QCD, in analogy with superconductivity, results from the existence of a physical vacuum which is removed from the remainder of the spectrum by an energy density gap and which exhibits a Meissner-Ochsenfeld effect [1]. More particularly, it is believed that these characteristics of the physical vacuum result from the infrared properties of QCD. With this in mind we have constructed in a recent paper [2] a simple model for QCD which concentrates on the low-energy part and parametrizes in a precisely defined manner the high-energy part of the theory, and we have shown by means of a Bogoliubov transformation that this simple model has a non-perturbative solution for the vacuum arising in the low-energy part which indeed possesses the desired characteristics. In that paper the fields were formulated in a translationally non-invariant manner which is needed to describe the bag-vacuum interface. In the present paper where we concentrate on the vacuum state we use instead the translationally invariant Fourier expansion. This formulation is not only directly well suited for the description of the vacuum state, but it also is easier to use to obtain explicit solutions for the "wave function of the vacuum." For a full treatment of the structure of the hadrons one, of course, will have to revert to the formulation of reference 2. At any rate, in the present paper we will demonstrate in detail how to construct the parameters of the theory which arise from the high-energy part of QCD and evaluate some of them in lowest order. Owing to the asymptotic freedom of QCD this is a perfectly legitimate procedure, of course subject to the limitations in the accuracy of the results inherent in any perturbation expansion. We also will derive explicit expressions for the quasi-particle spectrum which were left unspecified in Ref. 2. We should here re-emphasize that the theory does not contain any ingredients extraneous to QCD. Therefore, in principle it can be enlarged in a straightforward manner to yield approximations approaching the actual solution of the QCD vacuum problem.

The most extensive recent treatments of the confinment problem are based on the evaluation of Wilson loops on a four-dimensional lattice [3,4] which, of course, precludes translational or Lorentz covariance. This method avoids the explicit introduction of the structure of the vacuum. The properties of the vacuum itself are discussed in several papers which show that it can have the character of a superconductor, i.e., that a superconductive vacuum is compatible with QCD [5]. A quite general discussion of the application of the Bogoliubov transformation to QCD recently has been given by Brise et al [6] and by Schütte [7].

There exists also a series of papers concerning the QCD vacuum utilizing the concepts of the bag model [8]. Even though the obtained results may be correct, it could be argued that an investigation of the color confinement in terms of a model which includes color confinement as one of its basic assumptions involves a circular argument. It is precisely to avoid this possible pitfall that we have been careful to use only pure QCD concepts in our treatment.

The present paper is organized as follows. In section 2 we introduce the method by which we isolate the infrared part of the theory. This method defines a framework in which one can perform a perturbation expansion of the high-energy part in full anology to the perturbation expansion of QED (see the discussion preceding eq (A.9) Appendix A): owing to the well-known asympotic freedom of QCD one presumably can have at least as much confidence in the results of this treatment as for the case of QED. The perturbatively treated high-energy part then yields the parameters which determine the dynamics of the infrared part, to be treated non-perturbatively. The degree of approximation with which one chooses to compute the high-energy part defines the "model;" the inaccuracies of the "model" reside in the residual terms of the expansion of the high-energy part, exactly as in QED for the solutions. By continuing the expansion the accuracy of the "model" can be improved, again exactly as in QED for the solutions. In other words, in this section we show how our "model" is related to, and is derived from, the QCD Lagrangian. We perform the analysis in the non-covariant Hamiltonian formalism in the Schrödinger picture, discussed in detail in references 9-11, and as for the gauge which must be fixed before writing the Hamiltonian, we chose to work in the Coulomb gauge. The formal drawback of not being manifestly Lorentz covariant, which it shares with the formulations of the theory involving a lattice, is outweighed by the fact that this treatment lends itself immediately to achieving non-perturbative solutions. Still, in contrast to the lattice theories, our treatment is translationally and ratotionally, i.e., Galileo, invariant and hence does not suffer from the difficulties associated with the center-of-mass motion, known in nuclear physics as the appearance of spurious translational and rotational states. We then write down the non-perturbative vacuum state in terms of the Bogoliubov transformation and diagonalize the "model" Hamiltonian in that space. We discuss the conditions for the appearance of a superconducting state of the vacuum, and obtain as a side-result that the Bose-Einstein condensation is a singular limit of the BCS condensation. We conclude this section by showing explicitly how in principle to achieve the exact solution.

Even though an exhaustive description of the Meissner-Ochsenfeld effect requires a correct treatment of the vacuum-bag interface, i.e., the breaking of translational invariance, we show in section 3 that our present simple model already points toward the existence of that effect. In particular we show that a color field is damped exponentially when penetrating the physical vacuum in that its quanta acquire an effective mass in the physical vacuum, of course without the intervention of a Higgs field. In that section we also discuss the consequences of our non-manifestly-covariant treatment, and in particular the question concerning the properties of the boost for our solutions. We further sketch the manner in which the physical vacuum condenses from the perturbative vacuum, using as an example the decay of a π^0 into two photons. We find indications for the existence of a universal time constant for this process, or, equivalently, for a universal hadronization time, similar for all hadrons. The existence of such a condensation time may play an important role in the development of the early Universe. Finally, we give a summary of our results.

Some technical points are developed in the appendicies. In Appendix A, we demonstrate the treatment of the high-energy part by evaluating explicitly a selection of low-order contributions. Since we work in a non-covariant framework we demonstrate in Appendix B how to carry out the ultraviolet renormalization in this framework. Further technical details are contained in the Doctoral Thesis of D. Iracane [9].

2. The Gap

In order for the results of the model theory to have any bearing on QCD, the model should contain as many as possible of the essential ingredients of QCD. We now discuss our reasoning.

We shall use only the gauge sector of QCD,

$$L_G = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} \tag{2.1}$$

$$F^a_{\mu\nu} = \partial_\mu A_\nu{}^a - \partial_\nu A_\mu{}^a - g f^{abc}(A_\mu{}^b A_\nu{}^c) \tag{2.2}$$

which by itself contains no scale. As is generally done, in the renormalization procedure one must chose a renormalization point which then indeed provides a scale (see Appendix B), which shows up for example in the running coupling constant. At the present time one customarily fixes the scale by comparison with experimental data. We here will follow the same procedure.

As explained, e.g., in references 10-12, one may derive in the Schrödinger picture of the fields a field-theoretic Schrödinger equation from the field- theoric Lagrangian, which for stationary states reduces to

$$H|S\rangle = E|S\rangle \tag{2.3}$$

where $|S\rangle$ is the state vector describing an eigenstate of the system. It is here that the formal covariance of the treatment is lost: the Hamiltonian is defined in a given Lorentz frame, say, the lab system. It is worth while to recall that in QCD the construction of the Hamiltonian is non-trivial [13,14] owing to the non-Abelian nature of the fields. However, since the effects arising from this difficulty become important only at the two-loop level [14] or are associated with the Coulomb term, they do not enter our picture at this point.

We now proceed to isolate the infrared part of the theory. To that end we break the Hilbert space into two parts and rewrite eq (2.3) as

$$H\binom{F}{G} = \begin{pmatrix} X & Y \\ Y^+ & Z \end{pmatrix} \binom{F}{G} = E\binom{F}{G} \tag{2.4a}$$

from which one may eliminate G to obtain

$$XF + Y \frac{1}{E-Z} Y^+F \equiv H_{eff} F = EF \tag{2.4b}$$

which, of course, is still exact. Nonetheless, H_{eff} is now an effective Hamiltonian. Our aim is to achieve such a division of the Hilbert space that the infrared problem be limited to the space F. In that case the nonperturbative treatment can be limited to a small part of the Hilbert space. This then allows the application of a number of well-known powerful techniques which, for technical reasons, could not be employed in the full Hilbert space. For example, the ultraviolet renormalization can be taken care of in the space G, say, by perturbative methods (see Appendix B), totally independently from the treatment of the infrared sector, which will be assigned to the space F.

In other words, the space F can and must be treated by nonperturbative methods; this is not the case for the space G. The precise way in which the space G is to be treated by perturbative methods is discussed in Appendix A, before eq (A.9).

We now return to the discussion of eq (4b). Since we are dealing here with an effective Hamiltonian, besides the above mentioned absence of manifest covariance H_{eff} will not necessarily be manifestly gauge invariant. Again, this is a well-known feature of the effective interactions, evident, for example, in the Breit potential for QED. Of course, it does not invalidate the obtained results. The form of the operators X,Y,Z depends on the division of the Hilbert space, and in QCD also on the choice of the gauge. We shall take the space F to consist of that part of the Hilbert space which contains only low momentum transverse gluons, while G contains the rest of the Hilbert space, i.e., quarks and high momentum gluons. The precise specification of F will be given below. Finally, once F and G have been specified one can introduce projection operators P and Q = 1 - P which project respectively on the parts F and G on the Hilbert space:

$$P\,F = F \quad , \quad P\,G = 0 \quad , \quad P^2 = P \tag{2.5a}$$

Herewith one can specify the operators of (4):

$$\begin{aligned} X &= P\,H\,P \quad , \quad Z = Q\,H\,Q \\ Y &= P\,H\,Q \quad , \quad Y^{+} = Q\,H\,P \quad . \end{aligned} \tag{2.5b}$$

Before discussing the specifics of our model we specify our choice of the representation of the transversal vector field operators [10-12]. To wit, we use a plane wave expansion for the (Schrödinger picture) field operators which are defined so as to obey

$$[a_{\underset{\sim}{k}ia},\ a^{+}_{\underset{\sim}{k}'i'a'}] = \delta_{\underset{\sim}{k}\underset{\sim}{k}'}\ \delta_{aa'}\ (\delta_{ii'} - \frac{k_i k_{i'}}{k^2}) \quad . \tag{2.6}$$

Here $\underset{\sim}{k}$ specifies the momentum, i the polarization, and a the color. Herewith the fields are written as

$$A = \frac{1}{(2\pi)^{3/2}} \sum\!\!\int \frac{d^3k}{\sqrt{2\omega_k}} [a_{\underset{\sim}{k}ia}\ \varepsilon_i \ell^{\,i\underset{\sim}{k}\cdot\underset{\sim}{r}} + a^{+}_{\underset{\sim}{k}ia}\ \varepsilon_i \ell^{\,-i\underset{\sim}{k}\cdot\underset{\sim}{r}}] \quad . \tag{2.7}$$

where ε_i are the real Cartesian components of the polarization vectors. We now specify the space F as containing all states which contain arbitrary numbers of transversal gluons with $|k| < M$. The states in G then contain at least one quark (in our case $q\bar{q}$ pair), or at least one gluon with $|k| > M$, in addition to an arbitrary number of soft gluons.

We now are ready to specify the model. We shall test whether employing a non-perturbative solution suffices for developing an energy gap for the vacuum. To that end we make the simplest possible choice for the vacuum state, and we consider the particular coherent pairing state,

$$|V'\rangle = e^{\frac{1}{2}\int_0^M d^3k\, \theta_k (a^+_{\underset{\sim}{k}} \cdot a^+_{\underset{\sim}{\bar{k}}})} |0\rangle \tag{2.8}$$

where the dot indicates formation of a singlet in both Minkowski and color spaces, and $\underset{\sim}{\bar{k}} = - \underset{\sim}{k}$. Note that θ_k depends only on $k = |\underset{\sim}{k}|$. In eq (2.8) we have omitted the time dependence; it will have to be added to achieve the complete Schrödinger picture state vector. Also, the state $|V'\rangle$ is not normalized. In equation (2.8) the separation between the spaces F and G is given as the upper limit of the integral over the momenta, denoted by M. To recall, this separation supposedly reflects the mass scales of the complete theory; since the space F contains no further scale this separation energy is the only new scale parameter of the theory as far as the treatment of the space F is concerned. The complete theory of course is independent of the value chosen for M.

It may be worth while to point out that our method in essence is the momentum space analog of Wigner's R-matrix treatment of short-range strongly interacting systems which splits position space into two parts, i.e., the internal and the external regions; the non-perturbative treatment can be confined to the internal region. The complete theory in both cases is independent of the choice of the separation radius — in momentum or in position space. Of course, upon solution up to a given accuracy this independence is lost; this is true both for our treatment and for the R-matrix treatment. However, in both cases the approximations are made in a well-defined manner; the "model" is thus not *ad hoc*, but is in fact a well-defined approximation to the exact solution, and, in principle the "model" calculation can be rendered as accurate as one desires (by computing higher order Feynman type graphs for our case, and by retaining more terms in the series defining the R-matrix in Wigner's case).

The state (2.8) contains an unspecified number of pairs. It cannot be achieved in a perturbative treatment as it is connected with the perturbative vacuum $|0\rangle$ only by an infinite number of applications of the Hamiltonian. The parameter θ_k is a variational parameter which will be used to find a (local) minimum of the Hamiltonian. For technical reasons it is advantageous to replace eq (2.8) by

$$|V\rangle = e^{\frac{1}{2}\int d^3k\, \theta_k B_{\underset{\sim}{k}}} |0\rangle \quad , \tag{2.9}$$

$$B_{\underset{\sim}{k}} = (a^+_{\underset{\sim}{k}} \cdot a^+_{\underset{\sim}{\bar{k}}} - a_{\underset{\sim}{k}} \cdot a_{\underset{\sim}{\bar{k}}}) \quad . \tag{2.10}$$

This state, the operator (2.10) being anti-Hermitian, respects the normalization of $|0\rangle$.

Note that it would be misleading to call the state of the system described by $|V\rangle$ a "gluon condensate." Namely, even though only the gluons are treated explicitly in (2.9), (2.10), the actual state contains also an unspecified number of $q\bar{q}$-pairs. However, being in the part G of the Hilbert space they are hidden in the effective force, which we now discuss. To that end we rewrite (2.4) as a variational problem

$$\langle F|X|F\rangle + \langle F|Y \frac{1}{E-Z} Y^{+}|F\rangle = E \tag{2.11}$$

$$\delta\{\langle F|X|F\rangle + \langle F|Y \frac{1}{E-Z} Y^{+}|F\rangle\} = 0 \ . \tag{2.12}$$

The first term of (2.11) is simply,

$$H_0 = \int^{M} d^3k \ \langle|k|\rangle \ a^{+}_{\underset{\sim}{k}} \cdot a_{\underset{\sim}{k}} \equiv \int^{M} d^3k \ \omega_k \ a^{+}_{\underset{\sim}{k}} \cdot a_{\underset{\sim}{k}} \ , \tag{2.13}$$

It is the second term of (2.11), i.e., in the effective force, where the essential model assumptions have to be made since an exact inversion of the operator (E-Z) is not possible, even though it concerns the space G and the inversion could be treated perturbatively. To begin with, the form (2.9) of F requires that the number of Fock space operators be even. The simplest possibility is to allow four operators, as shown in figure 1a. In principle, the effect of very complicated high-order graphs can be contained within this interaction term.

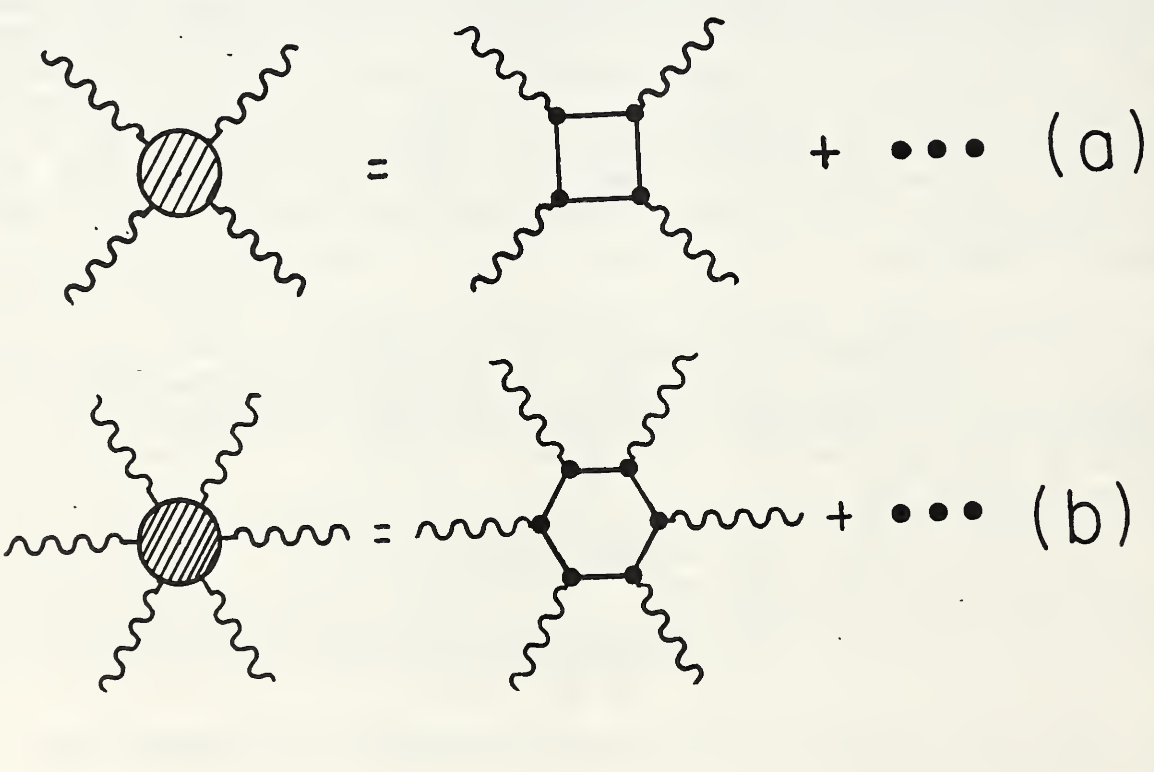

Fig. 1. (a) Graphs represented by (2.15),
(b) Graphs represented by (2.17).

Of course, the introduction of an effective interaction to describe the effects of the full Hilbert space on a limited subspace is not new. Recall, for example, the well-known case of the effective interaction in QED [15,16]. There the effective Euler-Heisenberg Lagrangian for the vacuum polarization is

$$\int d^3x \, L_{eff,QED} = -\frac{2e^4}{15\pi^2 m^4} \int d^3x \, (E^4 - 2E^2B^2 + B^4 + 7(E \cdot B)^2) \qquad (2.14)$$

where m is the electron mass. This form was derived as a local approximation to the non-local higher order corrections, and hence is valid for low momentum transfers, i.e., in the long wavelength limit. It allows a reasonably accurate estimate of the QED vacuum polarization effects. Since it is a Lagrangian it is not yet directly applicable in our case which requires an interaction Hamiltonian. In any case, in terms of the QED canonical fields, ϕ and π, the Lagrangian (2.14) shows the presence of terms of the form ϕ^4, $\phi^2\pi^2$, and π^4, with a preponderance of the term $\phi^2\pi^2$, i.e., $(B \cdot E)^2$. We shall write our effective interaction Hamiltonian in terms of similar local products of the gluon fields. As far as QED is concerned, we note here that (2.14) does not lead to a gap since its form factor tends to zero in the long wavelength limit.

Taking a hint from QED we will write the effective interaction as a series in powers of the field operators of the space F, i.e., in essence an expansion in powers of the gluon density. Taking over the results of Appendix A, eqs (A.9) and (A.10) we have for the lowest order term

$$X + \frac{1}{E-Z} Y^+ \rightarrow H_I' = \alpha_0 \int d^3x \, :(A \cdot A)^2: \quad . \qquad (2.15)$$

We here have introduced the dimensionless strength parameter α_0, which depends implicitly on the energy M which separates the spaces F and G. In higher order it also contains the QCD running coupling constant g, which again contains a scale, Λ, which arises as a renormalization constant in perturbative QCD; as shown in Appendix A the coupling constant can be written, as usual,

$$g^2 = \frac{g_0^2}{\log \frac{k^2}{\Lambda^2}} \quad , \qquad (2.16)$$

where k^2 is the momentum transfer of the particular matrix element of the operator Y. The form (2.15) is taken to encompass also the effect of the 4-field term of order g^2 contained in eqs (2.1), (2.2).

From the results of Appendix A where a number of lowest order graphs are computed one may conclude that in QCD the quantity α_0 seems to be negative, which is the sign needed for the possibility of the existence of a condensate. However, at the same time the resulting Hamiltonian is not positive definite, i.e., the Hamiltonian $H_0 + H_I$ is not bounded from below; the particle density would tend to infinity. Therefore, with this Hamiltonian one cannot expect the vacuum state to be stable. This, of course, is also true if one takes for

H_I an attractive QED-type Coulomb force [17]. (See below the discussion of the effect of the vertex Π^2A^2.) Since the original Hamiltonian representing a renormalizable theory presumably is bounded from below the effective Hamiltonian (2.15) certainly is an insufficient representation of the effective force H_{eff} of (2.5). To achieve a bounded Hamiltonian one has to continue the expansion begun in (2.15). To that end we add the sixth-order term which we write as

$$H'_{II} = \frac{\beta_0}{M^2} \int d^3x \, :(A\cdot A)^3: \tag{2.17}$$

where we have extracted explicitly the scale M^2 in order to define the dimensionless interaction strength parameter β_0. Analogously to the case of eq. (2.15), the numerical value of β_0 depends on the choice of the separation energy M, and it contains the running QCD coupling constant g. It can be computed perturbatively in the space G; from Appendix A we see that to the same level of assurance of the validity of the results as for α_0, the constant β_0 seems to be positive. Herewith we have reproduced the essential characteristics of the effective interaction: attraction at low density, stabilized at higher density. In the discussion of the solutions below we will generalize the form of the operators (2.15,2.17).

In order to solve our model we introduce the Boson analogue of the Bogoliubov quasi-particle transformation [18,19] which we write as

$$\begin{pmatrix} b^+_{\underset{\sim}{k}ia} \\ b_{-\underset{\sim}{k}ia} \end{pmatrix} = \begin{pmatrix} u_k & -v_k \\ -v_k & u_k \end{pmatrix} \begin{pmatrix} a^+_{\underset{\sim}{k}ia} \\ a_{-\underset{\sim}{k}ia} \end{pmatrix} \tag{2.18}$$

with

$$u_k^2 - v_k^2 = 1 \tag{2.19}$$

for each $k = |\underset{\sim}{k}|$. Hence the quasi-particle operators b and b^+ also obey the commutation relations (2.6). Furthermore we demand

$$b|V\rangle = 0 \tag{2.20}$$

where $|V\rangle$ is taken to be of the form (2.9). In order to fulfill (2.19) identically we introduce the Bogoliubov angle θ_k and write

$$u_k = \cosh\theta_k \tag{2.21a}$$

$$v_k = \sinh\theta_k \quad . \tag{2.21b}$$

We now look for the best solution achievable with the form (2.9) by searching for the minimum energy. To that end we need the expression for the energy. We shall use the complete fourth-order interaction which is, as shown in Appendix A,

$$H_I = \alpha_0 \int d^3x:(A\cdot A)^2: + \frac{\gamma_0}{M^2} \int d^3x:(A\cdot A)(\Pi\cdot\Pi): + \frac{\delta_0}{M^4} \int d^3x:(\Pi\cdot\Pi)^2 \quad . \tag{2.15a}$$

In that case, in order to stabilize the system, one also must augment (2.17) as follows

$$H_{II} = \frac{\beta_0}{M^2} \int d^3x{:}(A\cdot A)^3{:} + \frac{\eta_0}{M^8} \int d^3x{:}(\Pi\cdot\Pi)^3 \quad . \tag{2.17a}$$

We re-emphasize that all the strength parameters of the effective Hamiltonian is principle can be computed perturbatively in the space G. Using (13) we find, forming the expectation value of (15a) together with (17a) in the physical vacuum $|V\rangle$,

$$\frac{E}{\Omega} \equiv \zeta = \int_0^M d^3k \, \frac{(x_k-1)^2}{4x_k} \, \omega_k + \frac{\alpha}{8} Y^2 + \frac{1}{M^2} \frac{\beta}{12} Y^3 + \frac{\gamma}{4M^2} YZ + \frac{\delta}{8M^4} Z^2 + \frac{\eta}{12M^8} Z^3 \tag{2.22}$$

$$\Omega = 2(N^2 - 1)\tilde{\Omega} \tag{2.22a}$$

In this expression $(2\pi)^3 \tilde{\Omega}$ is the volume of the quantization box and $2(N^2-1) = 16$ for SU(3). Also, the powers of M have been extracted which according to Appendix A are needed to make all strength constants dimensionless. Further,

$$x_k = e^{2\theta_k} = (u_k + v_k)^2 \tag{2.23}$$

$$\alpha = 8 \, \frac{\alpha_0}{4(2\pi)^3} \left(2(N^2-1) + \frac{4}{3}\right) \tag{2.24a}$$

$$\beta = 12 \, \frac{\beta_0}{8(2\pi)^6} \left[2(N^2-1) + \frac{4}{3}\right]\left[2(N^2-1) + \frac{8}{3}\right] \tag{2.24b}$$

$$\gamma = 4 \, \frac{\gamma_0}{4(2\pi)^3} \, 2(N^2-1) \tag{2.24c}$$

$$\delta = 8 \, \frac{\delta_0}{4(2\pi)^3} \left[2(N^2-1) + \frac{4}{3}\right] \tag{2.24d}$$

$$\eta = 12 \, \frac{\eta_0}{8(2\pi)^6} \left[2(N^2-1) + \frac{4}{3}\right]\left[2(N^2-1) + \frac{8}{3}\right] \tag{2.24e}$$

$$Y = \int_0^M d^3k \, \frac{x_k-1}{\omega_k} \tag{2.25a}$$

$$Z = \int_0^M d^3k \, \left(\frac{1}{x_k} - 1\right) \omega_k \quad . \tag{2.25b}$$

Except for the factor $2(N^2-1)/(2\pi)^3$ the expression in (2.22) thus is directly the energy density. The quantities (2.24) are simply numerical constants, while the quantities (2.25) are also numerical constants, which, however, being functionals, depend on the actual form of the solution.

To obtain the minimum of (2.22) by varying θ_k it is most convenient to perform the variation independently with respect to u_k and v_k. To that end we add the condition (2.19) to the minimization, and search for

$$\begin{pmatrix} \frac{\partial}{\partial u_k} \\ \\ \frac{\partial}{\partial v_k} \end{pmatrix} \left(\zeta - \frac{1}{2} \int d^3k \; \varepsilon_k \left(u_k^2 - v_k^2 - 1\right)\right) = 0 \tag{2.26}$$

where ε_k is the Lagrange parameter. This leads to the following condition

$$\varepsilon_k \begin{pmatrix} u_k \\ \\ -v_k \end{pmatrix} = \begin{pmatrix} \omega_k + \Gamma_k & \Delta_k \\ \\ \Delta_k & \omega_k + \Gamma_k \end{pmatrix} \begin{pmatrix} u_k \\ \\ v_k \end{pmatrix} , \tag{2.27}$$

where

$$\Gamma_k = \frac{1}{2} \frac{G}{\omega_k} + \frac{1}{2} F\omega_k \tag{2.28a}$$

$$\Delta_k = \frac{1}{2} \frac{G}{\omega_k} - \frac{1}{2} F\omega_k \tag{2.28b}$$

$$G = \alpha Y + \frac{\beta}{M^2} Y^2 + \frac{\gamma}{M^2} Z \tag{2.28c}$$

$$F = \frac{\gamma}{M^2} Y + \frac{\delta}{M^4} Z + \frac{\eta}{M^8} Z^2 \quad . \tag{2.28d}$$

The solution of (2.27) is

$$\varepsilon_k = \sqrt{\omega_k^2 + G} \quad \sqrt{1 + F} \quad . \tag{2.29}$$

As seen from the eqs. (2.23) through (2.25), the equations are too involved to allow an analytical treatment. The actual solution must be found by numerical methods. We will show the results of such a calculation below in figure 2.

With the above quantities we can rewrite (2.23) in the form

$$x_k = \frac{\omega_k\sqrt{1 + F}}{\sqrt{\omega_k^2 + G}} \quad . \tag{2.23a}$$

This form will be convenient in our discussion below.

In order to proceed with the discussion of the spectrum we need the expression of the Hamiltonian in terms of the quasi-particle operators, The Wick decomposition of eq. (2.26) yields

$$\frac{H}{\Omega} = \zeta + \frac{1}{2}\int d^3k(v_k,u_k)\begin{pmatrix}\omega_k + \Gamma_k & \Delta_k \\ \Delta_k & \omega_k + \Gamma_k\end{pmatrix}\begin{pmatrix}u_k \\ v_k\end{pmatrix}(b^+_{\underset{\sim}{k}}\cdot b^+_{\underset{\sim}{-k}} - b_{\underset{\sim}{k}}\cdot b_{\underset{\sim}{-k}})$$

$$+ \int d^3k(u_k,v_k)\begin{pmatrix}\omega_k + \Gamma_k & \Delta_k \\ \Delta_k & \omega_k + \Gamma_k\end{pmatrix}\begin{pmatrix}u_k \\ v_k\end{pmatrix}(b^+_{\underset{\sim}{k}}\cdot b_{\underset{\sim}{k}}) + v_{residual} \quad . \tag{2.30}$$

Here $v_{residual}$ represents also the higher order terms of the perturbation expansion of H_{eff} beyond H_I and H_{II}. Upon insertion of the solution, i.e., using the results (2.28)-(2.29), eq. (2.30) acquires the form

$$H = E + \int d^3k\ \varepsilon_k\ b^+_{\underset{\sim}{k}}\cdot b_{\underset{\sim}{k}} + V_{residual} \quad . \tag{2.31}$$

This shows that for the case where $V_{residual}$ can be neglected, ε_k indeed is the quasi-particle energy and b^+ creates quasi-particles. From (2.29) we see immediately that the existence of a gap requires $G > 0$. Namely, for $\alpha = \beta = \gamma = 0$ we have $\varepsilon_k = \omega_k\ \sqrt{1 + \delta Z}$ which does not exhibit a gap, only a dilatation of the spectrum. On the other hand, if $G > 0$ then indeed $\sqrt{G}$ is the gap energy, i.e., the quasi-particle mass; the spectrum again is modified by the dilatation factor.

We now are in the position to discuss the character of the solutions. In view of the fact that we have not performed a full calculation of H_{eff} we shall do this freely, i.e., without prejudicing the sign and magnitude of the interaction constants of (2.22). That means, we will study the behavior of the solutions for a selection of cases when only some of the constants at a time are not zero. This way we will learn all the different forms the solutions can have; in the general case the characteristics of the solutions will fall into one or another of these types depending on the values of the parameters. Thus there actually is no need to study the general case, which, owing to the complexity of the equations (2.23) through (2.29) at any rate is too involved to allow an analytic solution; the solution must and can be obtained numerically.

We begin with the case $\alpha, \beta \neq 0$, all other constants = 0. In that case $F = 0$ from (2.28d) and the dilatation factor = 1. Thus, if $G > 0$ one may obtain a gap. Note that Γ_k and Δ_k, i.e., the mean field and the pairing strength, are singular for $k \to 0$, as a result of the relativistic measure contained in (2.7). At the same time the Bogoliubov angle, $\theta = \frac{1}{2} \log (k/\sqrt{k^2 + G})$ has there a singularity — confirming that the condensation is an infrared phenomenon. The behavior of the solutions is most easily seen for the limit of high densities, i.e., for $|\theta_k| >> 1$. In that case the energy density can be written as a power series in the particle density $\rho_k = v_k^2 \sim a^+a$. We find

$$\theta >> 1 \quad , \quad G < 0 \quad , \quad \zeta \sim \omega\rho + \alpha\rho^2 + \beta\rho^3 \quad , \tag{2.32a}$$

$$\theta << 1 \quad , \quad G > 0 \quad , \quad \zeta \sim \omega\rho + \alpha \left(\frac{1}{4\rho} - 1\right)^2 + \beta \left(\frac{1}{4\rho} - 1\right)^3 \quad . \tag{2.32b}$$

In the case (2.32a), for $\alpha < 0$, $\beta > 0$ is required to stabilize the system. We now discuss the two types of solutions (2.32a) and (2.32b), which both are accessible given a set of parameters α, β, owing to the form (2.25a) which can yield either sign for G. We thus investigate ζ as function of G. The extrema of this curve give the possible states of the system.

The branch $G > 0$ poses no difficulty. Depending on the magnitude of the parameters the curve has one minimum. The branch $G < 0$ requires a more careful analysis. To wit: ε_k, eq (2.29), has a branchpoint at $k^2 = |G|$ and is imaginary for $k^2 < |G|$. Since the Hamiltonian is hermitean this indicates that the space of the variational functions is inadequate. That this is indeed the case one sees by investigating the following Ansatz for the Bogoliubov angle:

$$\theta_k = 0 \quad \text{for} \quad 0 \leq k < m \tag{2.33a}$$

$$\theta_k = \phi_m \, \Delta_m(k) + \tilde{\theta}_k \tag{2.33b}$$

where the distribution $\Delta_m(k)$ is defined by

$$\Delta_m(k) = \int \frac{d^2\hat{k}}{4\pi \, \tilde{\Omega}} \, \delta^3(k - m) \tag{2.33c}$$

and where $\tilde{\theta}_k$ has support in the interval (m,M]. Here the function $\tilde{\theta}_k$ and the constants ϕ_m and m are the variational parameters. The distribution Δ is built to be idempotent so that the decomposition (2.33b) holds for all analytical functions of θ_k. Consequently (2.25a) is replaced by

$$\Upsilon = \int_m^M d^3k \, \frac{\tilde{x}_k - 1}{\omega_k} + \frac{x_m - 1}{\omega_m \tilde{\Omega}} \tag{2.34}$$

where

$$\tilde{x}_k = e^{2\tilde{\theta}_k} \quad ; \quad x_m = e^{2\phi_m} \quad . \tag{2.35}$$

Herewith the energy density becomes

$$\zeta = \int_m^M d^3k\ \omega_k \frac{(x_k - 1)^2}{4x_k} + \omega_k \frac{(x_m - 1)^2}{4x_m\tilde{\Omega}} + \frac{\alpha}{8}\gamma^2 + \frac{\beta}{12M^2}\gamma^3 \quad . \qquad (2.36)$$

Now we find that the minimum of the branch (2.32a) arrises at the solution,

$$m = 0 \quad , \quad \tilde{\theta}_k = 0 \quad , \quad G = 0 \quad , \quad \zeta_{min} = \frac{\alpha^3}{24\beta^2} M^4 \qquad (2.37a)$$

with

$$x_m = -\frac{\alpha M^2}{\beta} m\, M^2\, \tilde{\Omega} \quad . \qquad (2.37b)$$

For a quantization box of size L^3 this gives for $m \to 0$,

$$x_m \to -\frac{\alpha}{\beta}\frac{1}{8\pi^2} M^2 L^2 \quad . \qquad (2.37c)$$

In fact, this singular case of the Bogoliubov transformation turns out to be precisely the Bose-Einstein condensation with a spectrum $\varepsilon_k = k$ and with a population density ϕ_0 at $k = 0$. This way we have the important result:

> If the minimum at G = 0 of (2.36) is lower than the minimum of (2.32b), no gap ensues and we have Bose-Einstein condensation with a vacuum state $e^{\frac{1}{2}\theta_0 a_0^+ a_0^+}|0\rangle$; only if the minimum of (2.32b) is lower, we have a superconducting vacuum $|V\rangle$ with a non-vanishing gap.

This is shown in figure 2, for two values of the ratio α/β. The dashed parts show the energy for the variational functions without the distribution (2.33), the full lines show the complete result.

We next consider the case $\delta < 0$, $\eta > 0$, $\alpha = \beta = \gamma = 0$. Here then $G = 0$, eq (2.28c), and the spectrum (2.29) shows no gap, only a dilatation. Minimization leads to $x_p = \sqrt{1+F}$ and one again has two minima. As η increases the minimum at $F < 0$ becomes shallower. Again $\eta > 0$ is required to stabilize the solutions.

Combining the two cases one has the spectrum (2.29). As one "switches on" the terms Π^2, Π^4, one finds that the effects tend to go in opposite directions. Thus, e.g., for $\theta_k < 0$, the factor G increases while $\sqrt{1+F}$ decreases. Which of these trends wins out depends on the relative magnitudes of all the coefficients; the analysis here must be performed numerically on a case by case basis.

The last case concerns the vertex $A^2\Pi^2$, i.e., $\gamma \neq 0$. For stability it requires the presence of a repulsive term, i.e., either $\alpha > 0$, or $\beta > 0$ if one has $\alpha < 0$. We shall take $\alpha > 0$ and all other coefficients = 0. We also shall take $\gamma > 0$, which is the case for the Coulomb term, even though γ could have either sign. According to (2.28c), (2.28d) here both F and $G \neq 0$. However, for $\alpha > 0$ the case $G = 0$ is possible if $Y < 0$ which is the case for $\theta_k < 0$. We find the following solutions.

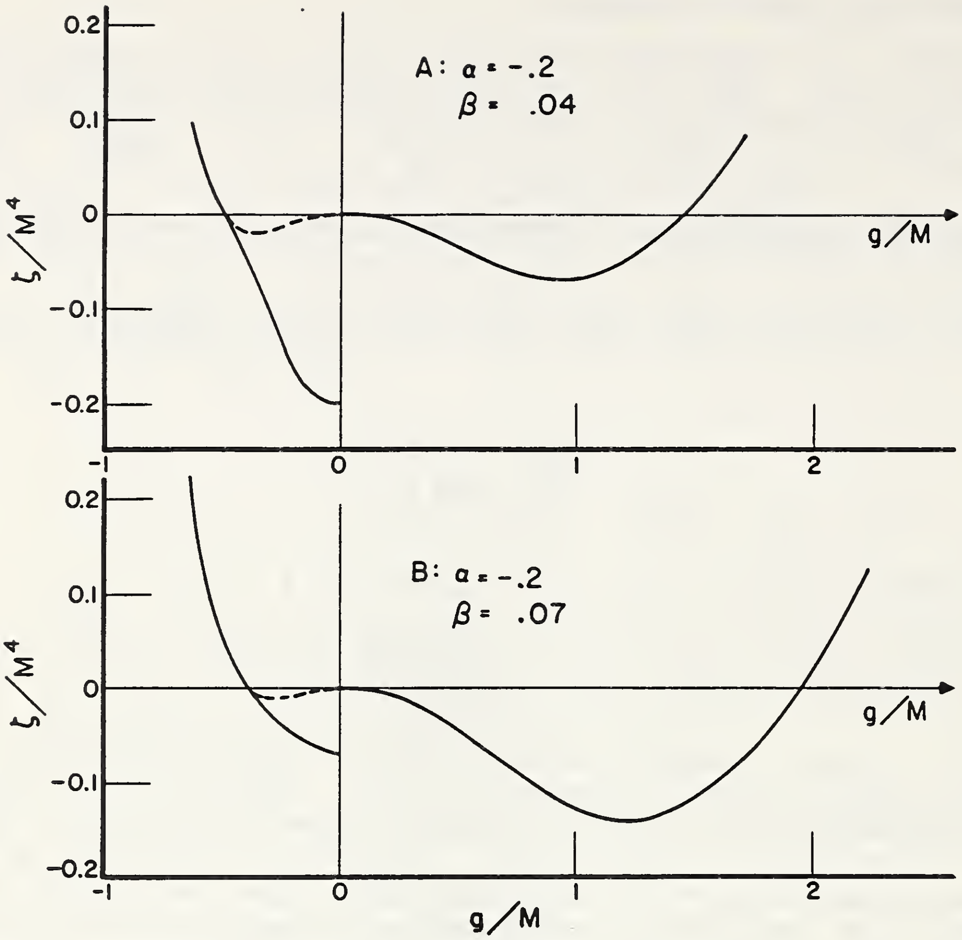

Fig. 2 Energy density ζ as function of effective gap energy g, defined as $g = +\sqrt{G}$ for $G > 0$, $g = -\sqrt{|G|}$ for $G < 0$. Full curve: solution of (36), i.e., the variational space includes the distribution (33); dashed curve: without the distribution (33). Case A: Bose-Einstein condensation; Case B: pairing condensation exhibiting gap.

For G = 0 there exists a Bose-Einstein type condensation with (see eq (2.35))

$$\tilde{x}_k = \sqrt{1 + F} \quad , \quad k > 0 \tag{2.37a}$$

where F is the non-trivial solution of the cubic equation

$$F = -\frac{\gamma^2}{\alpha} \pi \left(\frac{1}{\sqrt{1 + F}} - 1\right) \quad . \tag{2.37b}$$

This solution disappears for $\alpha > \alpha_c = 3\pi\gamma^2/4$. If $\alpha \to 0$, then $F \to \infty$, and

$$\zeta \to -\tfrac{1}{2}\pi^2 \, M^4 \, \frac{\gamma^2}{\alpha} \quad . \tag{2.38}$$

The other solution which exhibits a gap exists for $G > 0$; $-1 < F < 0$. In that case the Π^2 of H_0 and the Π^2 of the vertex play against each other; they have the same dependence on the density. We find for this solution that the energy of the condensate is finite only if $\gamma < \gamma_c = \frac{1}{2}\pi$. In this regime saturation is provided by the term Π^2 in H_0. Then we find for $\alpha = 0$

$$\lim_{\gamma \to \gamma_c} \zeta = -\frac{\pi}{36} M^4 \quad ,$$

$$\lim_{\substack{\gamma \to \gamma_c \\ k \to 0}} \varepsilon_k = \frac{2}{3} M \quad , \tag{2.39}$$

while at the same time $F \to -1$ and $G \to +\infty$. It therefore is possible to have a gap arise from $A^2\Pi^2$, however, then $\gamma < \gamma_c$ is necessary. This is not the case for a form factor $1/\Delta$, i.e., for the QED form of the Coulomb term. In that case H is not bounded from below, which, by the way, shows that the <u>replacement of the QCD Coulomb term by the QED form is not justifiable</u> for the infrared.

Finally, we return to the question of the relation of our results to the "exact" solution. There are two ingredients. First we note that owing to the presence of $V_{residual}$, $|V\rangle$ is not an eigenstate of the Hamiltonian since H, eq (2.30), connects states which differ in the number of quasi-particle pairs, i.e.,

$$\langle V_n | H | V_{n\pm\nu} \rangle \neq 0 \quad , \quad \nu = 1, 2, 3 \tag{2.40}$$

where (symbolically, integration over k implied)

$$|V_n\rangle = (b^+ b^+)^n \; |V\rangle \tag{2.41}$$

By diagonalyzing H in the space of the functions $|V_n\rangle$ one can obtain an improved form for the physical vacuum. In view of the forms (2.9), (2.10) for $|V\rangle$ it might be advantageous in the diagonalization of H to use instead of (2.33) the form

$$|\bar{V}_n\rangle = L_n^{(1/2)}(\theta B) \; e^{-(\theta/2)B} \; |0\rangle \tag{2.42}$$

which may be a better approximation to an n-quasi-pair state than (2.41). In eq (2.42) $L_n^{(1/2)}(x)$ is the (normalized) Laguerre polynomial. This improved vacuum would have a lower energy than our simple Bogoliubov vacuum $|V\rangle$, eq. (2.9), i.e., it would lead to a larger gap. However, this only would represent an improvement in the numerical accuracy of the results and has no bearing on the qualitative features. At any rate, we recognize at this point that in fact the space defined by (2.33) or (2.34) with $0 \leqslant n \leqslant N$ for $N \to \infty$ is complete for the space F, eq. (2.4), thus allowing for the "exact" solution. The other ingredient, of course, is the construction of the full effective Hamiltonian, i.e., the continuation of the expansion eqs. (2.15), (2.17), to higher order terms.

3. Discussion of the Solutions

Upon solving eqs (2.22), (2.27) with (2.15a), (2.17a) one achieves an approximation to the solution of eqs (2.11) and (2.12). Being only an approximation the solution inevitably will have inaccuracies, some of them numerical, some of them of principle. Nevertheless, as we now discuss, our solution encompasses all essential features expected from the physical QCD vacuum. The most prominent of these is the color Meissner-Ochsenfeld effect, needed for confinement. To wit, it must follow from the properties of $|V\rangle$ that in the bulk a presence of a color field is incompatible with the existence of $|V\rangle$. However, the understanding of the confinement problem demands an understanding of the transition region $|V\rangle \leftrightarrow |0\rangle$, i.e., of the structure of the vacuum- "bag" interface. Since a full description of this interface requires the presence of quarks which are needed to stabilize the "bag," we reserve the complete treatment of this problem to a forthcoming paper. At this juncture we shall give only a qualitative description, without addressing the question of the structure of the interface.

Consider a "weak" field, A_e, in the region of the perturbative vacuum, say region W. In that case the higher order terms are unimportant, and the system in region W is well described by

$$H_W = \frac{1}{2} \int_W d^3x \,:(\dot{A}\cdot\dot{A} - A\cdot\nabla^2 A): \quad . \tag{3.1}$$

In the physical vacuum, say region Ω, however, the presence of the condensate does not allow the neglect of the higher terms, and hence

$$H_\Omega = \frac{1}{2} \int_\Omega d^3x \,:(\dot{A}\cdot\dot{A} - A\cdot\nabla^2 A): + \alpha\int_\Omega d^3x \,:(A\cdot A)^2: + \beta\int_\Omega d^3x \,:(A\cdot A)^3: \quad . \tag{3.2}$$

Denoting the condensate field by A_c and keeping in (3.2) only the terms quadratic in A_e, we have, in the mean-field approximation,

$$H = H_W + H_\Omega = \frac{1}{2} \int_\Omega d^3x \,:(\dot{A}_e\cdot\dot{A}_e - A_e\cdot\nabla^2\Delta A_e): + m \int_\Omega d^3x \,:(A_e\cdot A_e): \tag{3.3}$$

where

$$m = \langle V| \,:\alpha(A_c\cdot A_c) + \beta(A_c\cdot A_c)^2:\, |V\rangle \quad . \tag{3.4}$$

Herewith we have for the equations of motion for $A_e|0\rangle$

$$\ddot{A}_e = \nabla^2 A_e - mA_e \qquad x \text{ in } \Omega \tag{3.5}$$

$$\ddot{A}_e = \nabla^2 A_e \qquad x \text{ in } W \quad . \tag{3.6}$$

Recalling (2.28c) we see from (3.4), (3.5) that in the physical vacuum $\sqrt{G}$ plays the role of a gluon mass. Note that this mass arises dynamically directly within the framework of QCD. In particular, no Higgs field had to be introduced, and no equivalent to a Higgs field has emerged in the form of any new quanta. To continue, as long as $m > 0$, in the limit of small energy of A_e, for $\nu^2 < m$ we have from eq (3.5)

$$q^2 = \nu^2 - m < 0 \tag{3.7}$$

i.e., an exponential damping of A_e in Ω, i.e., in the physical vacuum [20] while according to eq (3.6) the field can freely propagate in the perturbative vacuum with $k^2 = \nu^2$. Note that we here consider the case $m > 0$ in agreement with the discussion in the previous section. Also recall that the magnitude of m is related to the gap size, i.e., the value of H, eq (2.22) at the minimum. That indicates that a high-energy gluon, i.e., for $\nu^2 > m$, which could penetrate the physical vacuum, will melt that vacuum, by emitting quasi-particles and in the process lose energy until $\nu^2 < m$, and thus in the end will be turned around. In other words, both low and high energy gluons are totally reflected at the $|0\rangle - |V\rangle$ interface.

Next, consider the energy-momentum character of $|V\rangle$. Namely, since our treatment is not manifestly covariant a boost of the solution must be carried out in detail. However, since our treatment does not break translational invariance, we at least, in contrast to the bag model, have no difficulties associated with the center-of-mass problem. We begin by discussing the energy of the solution $|V(t)\rangle$, which here we write in full, i.e., (2.9) augmented by its time dependence. The expectation value (2.22),

$$\langle V(t)|\ H\ |V(t)\rangle = E_V < 0 \tag{3.8}$$

cannot actually be the physical eigenvalue of the vacuum; in the utilized quantized form all energies must be non-negative. The result (3.8) simply implies the need for a kind of gauge transformation. It is equivalent to the case of classical electrodynamics where one can shift the energy scale arbitrarily up or down by the addition of a constant scalar potential, i.e., by a global gauge transformation. The same can be done here by a re-definition of the phase of the state vector $|V\rangle$:

$$|V_0\rangle = e^{+iE_V t}\ |V(t)\rangle\ ; \tag{3.9}$$

with this phase the new state vector obeys

$$\frac{\partial}{\partial t}\ |V_0\rangle = 0 \quad . \tag{3.10}$$

Remembering that we work in the Schrödinger picture this then yields for the vacuum energy

$$E_0 = 0 \quad . \tag{3.11}$$

This way the vacuum $|V_0\rangle$ does not contribute to the cosmological term of gravity. On the other hand, now the perturbative vacuum acquires the energy $|E_V|$, which is the energy needed to replace $|V_0\rangle$ by $|0\rangle$. Since in our model system the state $|V_0\rangle$ occupies a volume $\tilde{\Omega}$ in position space, the number $|E_V|$ actually represents an energy density

$$\tilde{\zeta} = \frac{|E_V|}{\tilde{\Omega}} \quad . \tag{3.12}$$

With this normalization the state $|V_0>$ has the momentum four-vector

$$P = (E,\vec{k}) = (0,0) \qquad (3.13)$$

which indeed remains the same in all frames. However, a formal boost of $|V>$ would be wrong since the separation F - G of the Hilbert space in eqs (2.4), (2.5) is not boost-invariant. Therefore, in order to describe the vacuum in a boosted system one must perform the complete calculation in that boosted system. Then, of course, the form of $|V>$ in that system is exactly the same as in the original system. Of course, this formal boost non-invariance does not invalidate the accuracy of the solutions for a given frame of reference, say, the lab system.

The final point concerns the dynamics of the condensation. Such a process must take place for example in the annihilation of a qq pair, e.g., a π^0, into photons. In this process the emitted photons carry away both the energy of the $q\bar{q}$-system in the perturbative vacuum, and the latent heat of the perturbative vacuum of the "bag" volume. This process involves a non-perturbative change in the structure of the system: The vacuum state $|V>$ is connected with the perturbative vacuum $|0>$ in a non-perturbative manner. Hence the process cannot happen instantaneously. Because of the non-perturbative nature of this process the usual procedure of the graph expansion of the time-dependent perturbation theory is not feasible. A direct treatment is therefore called for. A full treatment again requires the inclusion of the quark degrees of freedom. Therefore we here only sketch the procedure, concentrating on the time development of the vacuum, $|V(t)>$, from $|0>$ at $t = 0$ to $|V>$ at large t.

Quite generally, the vacuum state vector is given by the expansion into quasi-pair states (2.41)

$$|V(t)> = \sum_n C_n(t)\ |\tilde{V}_n(t)> \qquad (3.14)$$

where the parameter θ contained in (2.9), (2.10), may be taken to be either time- dependent, or time-independent owing to the completeness of the set (41) with fixed θ. The time-dependence of $|\tilde{V}_n(t)>$ is given by its expectation value as in (3.8), renormalized with (3.9). At any rate, the time-dependence of $|V(t)>$ is governed by

$$\frac{\partial}{\partial t}\ |V(t)> = -i\ H\ |V(t)> \quad . \qquad (3.15)$$

This Schrödinger equation can be solved by a time-dependent Bogoliubov transformation, i.e., by the transformation (2.18) where the coefficients u, v, are taken to be c-number functions of t. However, without actually solving (3.57) the character of the solution can be inferred from the sudden approximation where it is assumed that the annihilation photons are emitted at $t = 0$ without a change in the structure of the "bag." In that approximation we have in terms of the asymptotic states (where θ is given by (2.21)),

$$|V(0)> = |0> \approx \sum C_n\ |\tilde{V}_n(0)> \ , \qquad (3.58)$$

with time-independent coefficients C_n. Since the individual components of $|V(t)\rangle$, eq (3.14) have a time-dependence given by their respective energies the higher terms will interfere away in a time given by the quasi-particle pair excitation spectrum. An exponential decay into the physical vacuum, i.e., the damping of the quasi-periodic beats between the components of (3.16), arises here in view of the continuum character of the final state (of the emitted γ-rays in our example) in the familiar manner upon integration over the energy.

Note that this time development is associated only with the quasi-particle spectrum, i.e., with the characteristics of the vacuum state. Hence the condensation time constant should be essentially the same for all processes. In other words, the condensation time is a characteristic of the vacuum; it may play an important role in the development of the early Universe in that it may lead to a perhaps substantial undercooling of the system.

In summary, we have demonstrated that in our model, which we believe reflects with sufficient accuracy the infrared aspects of QCD, a non-perturbative vacuum exists which is a quark-gluon condensate, and which exhibits an energy gap and repells color fields. Thus this vacuum has all the characteristics required for color confinement.

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APPENDIX A

We here give a justification for the form of the effective interaction eqs. (2.15), (2.17). We will limit the discussion to the lowest order graphs. The following terms with four external fields are possible:

$$I = :(A\cdot A)(A\cdot A): C_1 \qquad (A.1)$$

$$II = :(A\cdot A)(\Pi\cdot\Pi): C_2 \qquad (A.2)$$

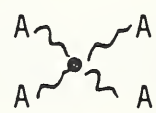
$$III = :(\Pi\cdot\Pi)(\Pi\cdot\Pi): C_3 \qquad (A.3)$$

Equations (A.1), (A.2), (A.3) are written in an obvious symbolic fashion. In terms of the gluon operators, the operators A and Π are

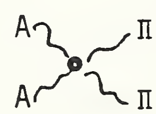
$$A_{ki\beta} = a^{+}_{ki\beta} + a_{\bar{k}i\beta} \qquad (A.4)$$

$$\Pi_{ki\beta} = a^{+}_{ki\beta} - a_{\bar{k}i\beta} \quad . \qquad (A.5)$$

Here the indices denote the momentum, the polarization and the color respectively. The three operators I, II, III do not have the same weight since the non-linear terms of the QCD Lagrangian are not symmetric in A and Π. Only the following elementary four-field vertices exist:

$V_4 =$ 

(A.6a)

$V_c =$ 

. (A.7a)

Hence the operator Y, eq (4), which connects our spaces F and G can be written using as in (56) as

$$Y = P\,(V_4 + V_c)\,Q \quad . \qquad (A.8)$$

Thus in second order, the operator I can be achieved by iterating either (A.6a) or (A.7a); the operator II by either iterating (A.7a) or by a product of (A.6a) and (A.6b); and the operator III by iterating (A.7a). Evidently the operator I has the greatest statistical weight. (Recall here, that the effective interaction is achieved by summing over the intermediate states of space G, eqs. (4a),(4b): the elementary vertices make up only a small part of the interaction.)

We now sketch the evaluation in second order of the operator I. This involves the contraction of the internal lines of the loop (see eq. (4b) for the effective Hamiltonian), i.e., the integration in the space G. We have, explicitly

$$V_4 = \frac{1}{4}\, g^2 \int d^3x\, A^{\alpha}_{i} A^{\beta}_{j} A^{\gamma}_{i} A^{\delta}_{j}\, f^{a\alpha\beta}\, f^{a\gamma\delta}$$

$$= \frac{1}{2}\,\frac{g^2}{(4\pi)^3} \int dp\,dq\,dk\,d\ell\; A^{\alpha}_{pi}\, A^{\beta}_{qj}\, A^{\gamma}_{ki}\, A^{\delta}_{\ell j}\; f^{a\alpha\beta}\, f^{a\gamma\delta}\; \delta^3(p+q+k+\ell)/\sqrt{\omega_p\omega_q\omega_k\omega_\ell} \qquad (A.6b)$$

$$V_c = -\frac{1}{2} g^2 \int d^3x d^3y \, A^{\alpha}_{xi} \Pi^{\beta}_{xi} \frac{1}{\Delta} (x-y) \, A^{\gamma}_{yj} \Pi^{\delta}_{yj} \, f^{a\alpha\beta} f^{a\gamma\delta}$$

$$= -\frac{g^2}{(4\pi)^3} \int dpdqdkd\ell \, A^{\alpha}_{pi} \, \Pi^{\beta}_{qj} \, \frac{4}{|p+q|^2} \, A^{\beta}_{kj} \, \Pi^{\delta}_{\ell j} \, f^{a\alpha\beta} f^{a\gamma\delta} \, \delta^3(p+q+k+\ell) \quad . \qquad (A.7b)$$

Note that V_c involves the Coulomb propagator. In the loop the integration will be over high momenta; hence the replacement of the covariant derivative by the simple derivative, i.e., the QCD Coulomb propagator by the QED propagator in (A.7b). Next consider the propagator $(E - Z)^{-1}$ of (4b). Since E is the desired eigenvalue, and we are interested in the lowest eigenstate, E is either zero, if no condensation takes place, or negative and equal the condensate energy, in case of condensation. Still, E drops out also in that case. Namely, since the space G contains also low momentum gluons, Z is very complex and looks like H_{eff}. Indeed, Z can be written as $Z = E + \int d^3k \; \omega_k \; a^+_k \cdot a_k + Z_{residual}$, and $(E-Z)^{-1}$ can be expanded in powers of $Z_{residual}$, since for space G, $\omega_k \geqslant M$. In principle, $Z_{residual}$ could be evaluated in an iterative manner; to the approximation of a small $Z_{residual}$, the propagator becomes simply the reciprocal of the sum of the one-particle energies ω_k.

We now have for the interaction Hamiltonian, omitting for the time being the color couplings,

$$g^4 \int \frac{\delta^3(p+q+p'+q')}{\sqrt{\omega_p \omega_q \omega_{p'} \omega_{q'}}} \; \frac{\delta^3(k+\ell+k'+\ell')}{\sqrt{\omega_k \omega_\ell \omega_{k'} \omega_{\ell'}}} \; A_p A_q \Big[A_{p'} A_{q'} \frac{-1}{H_0} A_{k'} A_{\ell'}$$

$$+ \frac{\omega_{p'} \omega_{q'}}{|p'+p|^2} \; \Pi_{p'} \Pi_{q'} \frac{-1}{H_0} \Pi_{k'} \Pi_{\ell'} \; \frac{\omega_{k'} \omega_{\ell'}}{|k'+k|^2} \Big] A_k A_\ell \qquad (A.9)$$

The form factor in this expression is

$$-g^4 \int_M^U d^3p' \left[\frac{1}{\omega_{p'} \omega_{p'+p+q}} + \frac{\omega_{p'} \omega_{p'+p+q}}{|p'+p|^2} \right] \frac{1}{\omega_{p'} + \omega_{p'+p+q}} \qquad (A.10a)$$

which for small external momenta becomes

$$\sim -g^4 \int dpdqdkd\ell \, \frac{\delta^3(p+q+k+\ell)}{\sqrt{\omega_p \omega_q \omega_k \omega_\ell}} \, (A_p \cdot A_q)(A_k \cdot A_\ell) \int_M^U d^3r/\omega_r^3 \quad . \qquad (A.10b)$$

Here U is an upper limit associated with the ultraviolet renormalization, which we will discuss below; and M is the mass introduced in (8) which specifies the boundary between the spaces F and G. Note that V_4 and V_C yield the same dependence of the effective interaction on M. Adding the contribution of the elementary vertex (A.6) we have, in position space,

$$H_{eff}(A^4) \sim \left(g^2 - g^4 \log \frac{U}{M}\right) \int d^3x \, (A_x \cdot A_x)^2 \tag{A.11}$$

For the force (A.2) we have

$$g^4 \int \frac{\delta^3(p+q+p'+q')}{\sqrt{\omega_p \omega_q \omega_{p'} \omega_{q'}}} \, \frac{\delta^3(k+\ell+k'+\ell')}{\sqrt{\omega_{k'} \omega_{\ell'}}} \, A_p A_q \, A_{p'} A_{q'} \, \frac{-1}{H_0} \, A_{k'} A_{\ell'} \, \Pi_k \Pi_\ell \, \frac{\sqrt{\omega_k \omega_\ell}}{|\ell+\ell'|^2} \tag{A.12}$$

with the form factor

$$g^4 \int d^3p' \, \frac{1}{\omega_{p'} \omega_{p'+p+q}} \, \frac{1}{|p'+\ell|^2} \, \frac{1}{\omega_{p'} + \omega_{p'+q+\ell}} \tag{A.13a}$$

which has the small momentum limit

$$\sim g^4 \int d^3p d^3q d^3k d^3\ell \; \delta^3(p+q+k+\ell) \, \frac{\sqrt{\omega_k \omega_\ell}}{\sqrt{\omega_p \omega_q}} \, (A_p \cdot \Pi_k) \, (A_q \cdot \Pi_\ell) \int_M^U d^3r/\omega_r^5 \; . \tag{A.13b}$$

This interaction requires no ultraviolet renormalization. Thus we find

$$H_{eff}(A^2\Pi^2) \sim g^4/M^2 \int d^3x \, (A_x \cdot \Pi_x)^2 \tag{A.14}$$

to which, of course, must be added the Coulomb term (A.7b).

The operator (A.3) arises from

$$g^4 \int \frac{\delta^3(p+q+p'+q')}{|p+p'|^2} \, \frac{\sqrt{\omega_p \omega_q}}{\sqrt{\omega_{p'} \omega_{q'}}} \, \Pi_p \Pi_q \, A_{p'} A_{q'} \, \frac{-1}{H_0} \, A_{k'} A_{\ell'} \, \Pi_k \Pi_\ell \, \frac{\sqrt{\omega_k \omega_\ell}}{\sqrt{\omega_{k'} \omega_{\ell'}}} \, \frac{\delta^3(k+\ell+k'+\ell')}{|k+k'|^2} \; . \tag{A.15}$$

As before we note the form factor

$$\sim -g^4 \int d^3p' \, \frac{1}{\omega_{p'} \omega_{p'+p+q}} \, \frac{1}{|p'+p|^2} \, \frac{1}{|p'+k|^2} \, \frac{1}{\omega_{p'} + \omega_{p'+p+q}} \; , \tag{A.16a}$$

its low momentum limit

$$\sim -g^4 \int d^3p d^3q d^3k d^3\ell \; \sqrt{\omega_p \omega_q \omega_k \omega_\ell} \; \Pi_q \Pi_p \Pi_k \Pi_\ell \; \delta^3(p+q+k+\ell) \int_M^U d^3r/\omega_r^7 \; , \tag{A.16b}$$

and the resulting position space form

$$H_{eff}(\Pi^4) \sim -g^4/M^2 \int d^3x \, (\Pi_x \cdot \Pi_x)^2 \quad . \tag{A.17}$$

Again no renormalization is required.

We now proceed to the higher terms which are supposed to stabilize the Hamiltonian. They result from continuing the graph expansion of the effective interaction (4b). The lowest order graphs arise by replacement of the two-point loop by a three-point loop, i.e., a triangle. Thus for the term in (A.6) we have

$$g^6 \int \frac{\delta^3(p+q+p'+q')}{\sqrt{\omega_p \omega_q \omega_{p'} \omega_{q'}}}$$

$$\times A_p A_q A_{p'} A_{q'} \frac{-1}{H_0} \frac{\delta^3(k+\ell+k'+\ell')}{\sqrt{\omega_k \omega_\ell \omega_{k'} \omega_{\ell'}}} A_k A_\ell A_{k'} A_{\ell'} \frac{-1}{H_0} \frac{\delta^3(r+s+r'+s')}{\sqrt{\omega_r \omega_s \omega_{r'} \omega_{s'}}} A_r A_s A_{r'} A_{s'} \tag{A.18}$$

As previously, the form factor

$$\sim g^6 \int_M^U d^3p' \, \frac{1}{\omega_{p'} \omega_{p'+p+q} \omega_{p'-r-s}} \, \frac{1}{\omega_{p'}+\omega_{p'+p+q}} \, \frac{}{\omega_{p'}+\omega_{p'-r-s}} \tag{A.19a}$$

for small external momenta is

$$\sim g^6 \int d^3p d^3q d^3k d^3\ell d^3r d^3s \, \frac{\delta^3(p+q+k+\ell+r+s)}{\sqrt{\omega_p \omega_q \omega_k \omega_\ell \omega_r \omega_s}} \, A_p A_q A_k A_\ell A_r A_s \int_M^U d^3p'/\omega_{p'}^5 \quad . \tag{A.19b}$$

No renormalization is required and we find

$$H_{eff}(A^6) \sim g^6/M^2 \int d^3x \, (A_x \cdot A_x)^3 \tag{A.20}$$

Indeed, they have the opposite sign of the fourth-power interaction owing to the presence of two (negative) energy denominators. Finally, using the Coulomb vertex (A.7a) we have

$$g^6 \int \frac{\delta^3(p+q+p'+q')}{|p+p'|^2} \, \frac{\delta^3(k+\ell+k'+\ell')}{|k+k'|^2} \, \frac{\delta^3(r+s+r'+s')}{|r+r'|^2} \, \frac{\sqrt{\omega_p \omega_q \omega_k \omega_\ell \omega_r \omega_s}}{\sqrt{\omega_{p'} \omega_{q'} \omega_{k'} \omega_{\ell'} \omega_{r'} \omega_{s'}}}$$

$$\times \Pi_p \Pi_q A_{p'} A_{q'} \frac{-1}{H_0} \Pi_k \Pi_\ell A_{k'} A_{\ell'} \frac{-1}{H_0} \Pi_r \Pi_s A_{r'} A_{s'} \tag{A.21}$$

$$\sim g^6 \int_M^U d^3p' \frac{1}{\omega_{p'}\omega_{p'+p+q}\omega_{p'-r-s}} \frac{1}{\omega_{p'}+\omega_{p'+p+q}} \frac{1}{\omega_{p'}+\omega_{p'-r-s}}$$

$$\times \frac{1}{|p+p'|^2} \frac{1}{|p+q+k+p'|^2} \frac{1}{|s-p'|^2} \qquad \text{(A.22a)}$$

$$\sim g^6 \int d^3p d^3q d^3k d^3\ell d^3r d^3s \; \Pi_p\Pi_q\Pi_k\Pi_\ell\Pi_r\Pi_s \; \delta^3(p+q+k+\ell+r+s) \sqrt{\omega_p\omega_q\omega_k\omega_\ell\omega_r\omega_s}$$

$$\times \int_M^U d^3t/\omega_t^{11} \qquad \text{(A.22b)}$$

and the position space form

$$H_{eff}(\Pi^6) \sim g^6/M^8 \int d^3x \, (\Pi_x \cdot \Pi_x)^3 \quad . \qquad \text{(A.23)}$$

We now consider the renormalization which is required for the term $\sim A^4$, (A.11) (see also Appendix B). Since QCD is a renormalizable theory the renormalization can be performed by using counterterms in the Lagrangian. Developing the coupling constant up to second order in the Goldstone series yields

$$g = g_0(1 + \tfrac{1}{2} c \, g_0{}^2 \log U/\mu) \qquad \text{(A.24)}$$

where μ defines the renormalization point. (Of course, the form (A.24) corresponds to that found from the renormalization group equations.) At the same time, summation of all graphs up to second order which describe soft gluon scattering has the form factor

$$g^2 - g^4 c \log U/t \qquad \text{(A.25)}$$

when t is the momentum transfer. Using (A.24) the ultra-violet momentum U drops out and (A.25) becomes instead

$$g_0{}^2 - g_0{}^4 c \log M/t \quad . \qquad \text{(A.26)}$$

We now return to our computation of the effective force. For each graph of the Goldstone series there exists an equivalent graph for the coupling constant. The difference between the two graphs resides in the difference between the energy denominators: The propagators for the effective force contain the perturbed energies, as the Goldstone series is a Brilloin-Wigner expansion. The difference between the two series thus vanishes in the ultraviolet limit, and hence we find for the effecive interaction

$$H_{eff}(A^4) \sim (g^2 - g^4 c \log \frac{U}{M}) = \alpha_s = g_0{}^2(1 - c g_0{}^2 \log \frac{\mu}{M}) \qquad \text{(A.27)}$$

Finally, comparing with the form (16) we find

$$\mu = e^{1/c g_0{}^2} \Lambda >> \Lambda \qquad \text{(A.28)}$$

for our case.

APPENDIX B

The familiar renormalization procedure, say in QED, is carried out in the framework of the covariant perturbation expansion. Here we instead have to employ the perhaps less familiar non-covariant Goldstone expansion. Of course, the differences between the two treatments are only formal. In either case, renormalization is achieved by introducing in the interaction Lagrangian suitable counter-terms. The counter-terms are employed to cancel the ultraviolet divergence, i.e., they belong to the perturbative domain. In the present context we are interested in the renormalization of the coupling constant. The procedure then consists in calculating the corrections to the coupling canstant to the same order as the physical effect - precisely as in the covariant treatment. And, quite importantly, since in our case the renormalization of concern is associated with the ultraviolet divergence the procedure is carried out in the space G. Hence no infrared singularities will be encountered.

Now to the details. Since our calculation of the matrix elements, Appendix A, is carried to second order we need the evaluation of the coupling constant to first order. Beginning with the QCD Hamiltonian

$$H_{QCD} = H_0 + V \quad , \quad V = V_3 + V_4 \tag{B.1}$$

$$V_3 = g\, A^3 \tag{B.2a}$$

$$V_4 = g^2\, A^4 + g^2\, A\Pi\, A\Pi \quad , \tag{B.2b}$$

we have for the two-gluon state

$$|\psi\rangle = a^+_{p',i',\alpha'}\; a^+_{q',j',\beta'}\; |0\rangle/\sqrt{2} \tag{B.3}$$

the "unperturbed" energy η_0 given by

$$\eta_0\; |\phi\rangle = H_0\; |\psi\rangle \quad . \tag{B.4}$$

The "perturbed" energy then is given in the Goldstone expansion as

$$\eta = \sum_c \langle\psi|\; V\left(\frac{1}{\eta_0 - H_0}\, V\right)^n\; |\psi\rangle \tag{B.5}$$

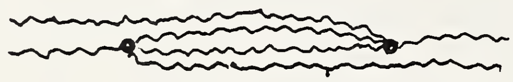
where the sum is over the connected graphs. To achieve a finite η one writes the coupling constant g of V as a power series in g_0. The coefficients of that series in a renormalizable theory then can be so determined by the definition of a small number of elementary divergences that all infinities cancel in each order of g_0 in the expansion (B.5) when inserting g as a function of g_0.

We now consider the scattering of two soft gluons, i.e., $\eta_0 \approx 0$. For the treatment up to fourth order we thus must calculate

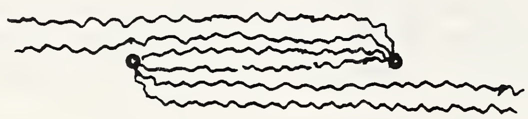
$$V_4 \frac{1}{-H_0} V_4 + V_4 \frac{1}{-H_0} V_3 \frac{1}{-H_0} V_3 + V_3 \frac{1}{-H_0} V_4 \frac{1}{-H_0} V_3$$
$$+ V_3 \frac{1}{-H_0} V_3 \frac{1}{-H_0} V_4 + V_3 \frac{1}{-H_0} V_3 \frac{1}{-H_0} V_3 \frac{1}{-H_0} V_3 \quad . \tag{B.6}$$

We here sketch the evaluation of the first term of (B.6). This term comprises the three Goldstone graphs

(B.7a)



(B.7b)



(B.7c)

The loop contains two hard gluons, (B.3) with $|p'|$, $|q'| > M$. Therefore the loop propagator is $|\psi\rangle \frac{1}{-\omega_{p'} - \omega_{q'}} \langle\psi|$. Applying V_4 on $|\psi\rangle$ yields

$$V_4|\psi\rangle = \frac{g^2}{(4\pi)^3} \int \frac{d^3p\, d^3q}{\sqrt{\omega_p \omega_q \omega_{p'} \omega_{q'}}} d^3(p+q-p'-q')\, A_{pi\alpha} A_{qj\beta} [f^{a\alpha'\beta'} f^{a\alpha\beta} \delta^t_{ii'}(p') \delta^t_{jj'}(q')$$

$$+ f^{a\alpha\alpha'} f^{a\beta\beta'} (\delta_{ij} \delta^t_{i'n}(p') \delta^t_{nj'}(q') - \delta^t_{ji'}(p') \delta^t_{j'i}(q') - \frac{\omega_{p'}\omega_{q'}}{|p-p'|^2}$$

$$\times \delta^t_{i'i}(p') \delta^t_{j'j}(q'))] \qquad \text{(B.8a)}$$

$$\approx \frac{g^2}{(4\pi)^3} \int d^3p d^3q \frac{d^3(p+q-p'-q')}{\sqrt{\omega_p \omega_q}} \frac{1}{\omega_{p'}} A_{pi\alpha} A_{qj\beta} [f^{a\alpha'\beta'} f^{a\alpha\beta} \delta^t_{ii'}(p') \delta^t_{jj'}(q')$$

$$+ f^{a\alpha\alpha'} f^{a\beta\beta'} (\delta_{ij} \delta^t_{i'j'}(p') - \delta^t_{ij'}(p') \delta^t_{i'j}(p') - \delta^t_{ii'}(p') \delta^t_{jj'}(p'))] \qquad \text{(B.8b)}$$

In the form (B.8b) we have neglected the external momenta p and q with respect to p' and q'. The first graph (B.7a) thus gives

$$V_4 \frac{1}{-H_0} V_4 = -\frac{g^2}{(4\pi)^3} \int d^3p d^3q d^3k d^3\ell \frac{d^3(p+q+k+\ell)}{\sqrt{\omega_p \omega_q \omega_k \omega_\ell}} A_{pi\alpha} A_{qj\beta} A_{kn\gamma} A_{\ell m\delta}$$

$$\times \int \frac{d^3r}{r^3} [3 f^{a\alpha\beta} f^{a\gamma\delta}\ \delta^t_{in}(t) \delta^t_{jm}(t) + 2 f^{a\alpha c} f^{a\beta d} f^{b\gamma c} f^{b\delta d}$$

$$\times (\delta^\ell_{ij} \delta^\ell_{nm} - \delta_{ij} \delta^t_{nm} - \delta_{nm} \delta^t_{ij} + \delta^t_{in} \delta^t_{jm} + \delta^t_{im} \delta^t_{jn})] \qquad \text{(B.9a)}$$

$$= - \frac{g^2}{4\pi)^3} \int d^3p d^3q d^3k d^3\ell \frac{d^3(p+q+k+\ell)}{\sqrt{\omega_p \omega_q \omega_k \omega_\ell}} A_{pi\alpha} A_{qj\beta} A_{kn\gamma} A_{\ell m\delta} \int \frac{d^3r}{\omega_r^3}$$

$$\times [f^{a\alpha\beta} f^{a\gamma\delta} \delta_{in}\delta_{jm} + 2f^{a\alpha c} f^{a\gamma d} f^{b\delta d} f^{b\gamma c}$$

$$\times (- \frac{1}{3} \delta_{ij}\delta_{nm} + \frac{4}{5} \delta_{in}\delta_{jm} + \frac{4}{5} \delta_{jn}\delta_{im})] \qquad \text{(B.9b)}$$

In (B.9b) the angular integrations have been carried out. Similar expressions arise for the graphs (B.7b) and (B.7c). Since the external momenta are small, all three graphs have propagators $1/2\omega_{p'}$, i.e., the energy of the hard loop gluons. The radial integration then yields the factor log(U/t) where t is the (small) momentum transfer of the graph. We note the appearance of two distinct color tensors:

$$N^{\gamma\delta}_{\alpha\beta} = f^{a\alpha\beta} f^{a\gamma\delta} \qquad \text{(B.10a)}$$

$$M^{\gamma\delta}_{\alpha\beta} = f^{a\alpha c} f^{a\beta d} f^{b\gamma c} f^{b\delta d} \qquad \text{(B.10b)}$$

Thus, for example, $N^{\gamma\delta}_{\alpha\alpha} = N^{\gamma\gamma}_{\alpha\beta} = 0$, while $M^{\gamma\delta}_{\alpha\gamma} = M^{\alpha\delta}_{\alpha\gamma} = 2M^{\gamma\alpha}_{\alpha\delta} = N^2\delta_{\gamma\delta} \neq 0$. It is the part containing the tensor $N^{\gamma\delta}_{\alpha\beta}$ which is employed in the renormalization discussed in Appendix A. The tensor $M^{\gamma\delta}_{\alpha\beta}$ must be cancelled when calculating all terms of (B.6).

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By treating the high-momentum gluon and the quark sector as an in principle calculable effective Lagrangian we obtain a non-perturbative vacuum state for QCD as an infrared quark-gluon condensate. This vacuum is removed from the perturbative vacuum by an energy gap and supports a Meissner- Ochsenfeld effect. It is unstable below a minimum size and it also suggests the existence of a universal hadronization time. This vacuum thus exhibits all the properties required for color confinement.

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