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ASSEMBLY CODE TO COMPUTE SINE AND COSINE USING THE CORDIC ALGORITHM

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Assembly Code to Compute Sine and Cosine Using the CORDIC Algorithm

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Abstract

The CORDIC algorithm is commonly used to approximate certain elementary functions. Many microprocessor and microcontroller chips without the availability of math coprocessor chips could benefit from the efficient implementation of this algorithm. The focus of this work is to report on a specific implementation in assembly code (for an 8051 microcontroller) that computes the sine and cosine to eleven bits of accuracy.

1. Introduction

From the early 1970's and into the 1980's, the CORDIC (COordinate Rotation DIgital Computer) algorithm (first used by Volder [4]) has been selected for use in many hand-held calculators offering the multiply, divide, square root, sine, cosine, tangent, arctangent, sinh, cosh, tanh, arctanh, ln, and exp functions [1]. The CORDIC algorithm's usefulness for these calculators can be seen in that all of these functions can be approximated using the same set of iterative equations (in binary form) [2]

$$x_{k+1} = x_k - m\delta_k y_k 2^{-\kappa}$$

$$y_{k+1} = y_k + \delta_k x_k 2^{-\kappa}$$

$$z_{k+1} = z_k - \delta_k \varepsilon_k$$

$$\delta_k = \pm 1, \quad \text{for } k = 0, 1, \dots, n,$$
(1)

where m = 1, 0, or -1, is a mode indicator and ε_k are constants stored prior to the execution of the algorithm and depend on m. Appropriate selection of initial values, x_0 , y_0 , z_0 , and the sign of each δ_k will generate approximations of each of the elementary functions mentioned.

Many modern microprocessors and microcontrollers do not have high speed hardware multipliers on-chip making function approximation by polynomial methods relatively slow. This explains the utility and popularity of math coprocessor chips in many computers. If, in addition, there is some reason that a math coprocessor chip is not feasible, one might consider using the CORDIC equations in software to compute elementary functions on the microprocessor or microcontroller. It would make sense to write this code in assembly language to maximize the speed of execution.

The two-fold task of this report is to include as much of the theory behind the CORDIC iterations (1) as is necessary and to give an example of the CORDIC algorithm in assembly code written for the Intel Corp. 8051 microcontroller. The 8051 does have an on-chip

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multiplier. However, since the 8051 has only an eight bit multiplier (requiring multiple precision multiplication), the use of polynomial approximation algorithms to approximate the elementary functions may not be faster than the CORDIC iterations.

Since we merely intend to demonstrate the effectiveness of the CORDIC algorithm, only sine and cosine functions will be considered.

This work was sponsored by the US Bureau of Mines (BOM) in support of their efforts in computer-assisted underground coal mining.

2. Instructions for Use of the 8051 Code to Compute Sine and Cosine

The theory behind the CORDIC algorithm is elegantly presented in [2] and will not be repeated except to mention that on page 322 line 3, x_0 should equal K not 1/K.

Equations 2 specify fourteen iterations of the CORDIC algorithm with constants and initial values defined for the computation of sine and cosine only. After completion of the fourteenth iteration, x_{14} and y_{14} will give the approximations to cosine and sine, respectively. This will give the sine and cosine of any angle, θ , between 0 and $\pi/2$. This result will be accurate to approximately $\pm 2^{-11} \approx \pm 0.000488$. Angles between $\pi/2$ and 2π can be handled by appropriate domain reduction.

$$x_{k+1} = x_k - m\delta_k y_k 2^{-k}$$

$$y_{k+1} = y_k + \delta_k x_k 2^{-k}$$

$$z_{k+1} = z_k - \delta_k \varepsilon_k$$

$$\varepsilon_k = \tan^{-1} 2^{-k}$$

$$\delta_k = \begin{cases} -1, & \text{if } z_k < 0 \\ 1, & \text{if } z_k \ge 0 \end{cases}$$

$$K = \prod_{k=0}^{13} \cos \varepsilon_k$$

$$x_0 = K, y_0 = 0, \text{ and } z_0 = \theta$$

$$(2)$$

A negative aspect of the CORDIC algorithm is that even if the user wants only the sine and not the cosine (or vice versa), the algorithm must compute the undesired quantity as well as the desired one. Note as well that, if one wanted to make the result more accurate (or less accurate), a simple increase (or decrease) in the number of iterations is not sufficient. One must also change the value of K as well as the number of ε_k 's stored in memory.

The assembly language program (called CORDIC and listed in the Appendix) declares the following three variables as two-byte (one-word) public variables: ?Angle_16?byte, ?Sine_16?byte, and ?Cosine_16?byte.

Here is the typical way CORDIC can be used: The calling program desires to compute the Sine or Cosine of a 16-bit (one-word) quantity in radians called θ . The calling program stores θ in the two bytes of ?Angle_16?byte, storing the least significant byte at ?Angle_16?byte and the most significant byte at ?Angle_16?byte+1. The CORDIC program requires θ to be a positive number in radians between 0 and 2π . Since the largest possible value of θ , 2π , has three bits to the left of the decimal point, the calling program must send θ with the decimal point assumed to be between bit location 13 and bit location 12 for the 16-bit θ (with numbering of locations from 0 to 15). In other words, the input, θ , has a fixed decimal point location assumed by CORDIC.

3. Two Examples of How θ , the Input to CORDIC, Must Be Represented

Example 1: $\theta = 2\pi$

 2π in binary form is 110.0100100010000₂. So, if one wanted the sine of θ when $\theta = 2\pi$, the calling program would put 00010000 at ?Angle_16?byte and 11001001 at ?Angle_16?byte+1. Then CORDIC would be executed after which the sine and cosine would be found as 16-bit public variables in locations ?Sine_16?byte, and ?Cosine_16?byte.

Example 2: $\theta = 0.2984$ radians

Since $0.2984_{10} = 0.01001100011001_2$, the calling program would put 10001100 (8C₁₆) at ?Angle_16?byte and 00001001 (09₁₆) at ?Angle_16?byte+1.

4. An Example of a Comparison of the Approximation for Sine and Cosine Using CORDIC to the "True" Values

As a simple example of the operation of the CORDIC algorithm, assume that θ = 0.2984 radians as in section 3, example 2. Computing the sine and cosine using the CORDIC algorithm we get that x_{14} = 0.11110100101100010101_2 and $y_{14} = 0.010010110011111001001_2$. These are approximations for the "exact" values, cos0.2984 = 0.11110100101011111101_2 and $\sin 0.2984 = 0.0100101101000011_2$. A comparison of the above two sets of binary numbers shows that the CORDIC algorithm is accurate only to about the eleventh significant binary digit as claimed in section 2. This is because we iterated only fourteen times. One can chose to iterate any number of times up to and including sixteen for varying degrees of accuracy (as long as the appropriate changes in the constants of equations 2 are made). NIST chose a level of accuracy for the algorithm to be that which seems as sufficient for calculations involving the positioning of underground coal mining machines. If it is too accurate or too slow in execution, one can always sacrifice accuracy for speed.

5. Conclusion

The general operation of the CORDIC algorithm has been given with the focus on a specific implementation in 8051 assembly code to compute the sine and cosine to eleven bits of accuracy.

This work can assuredly be expanded. It would be interesting to use a form of the CORDIC algorithm that allows for multiplication [3], making use of the 8051's on chip multiplier. Also useful would be to compare the performance of CORDIC with that of polynomial methods of approximating elementary functions.

The source code listed in the appendix is in the public domain and will be made available to all who request it from the author.

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6. Acknowledgements

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8. Appendix

NAME CORDIC PUBLIC ?Angle_16?byte,?Cosine 16?byte,?Sine 16?byte CORDIC CODE SEGMENT CODE CORDIC DATA SEGMENT DATA RSEG CORDIC DATA ?Angle_16?byte: DS 2 ?Cosine 16?byte: DS 2 ?Sine_16?byte: DS 2 K: DS 1 XTMP_0: DS 1 XTMP 1: DS 1 YTMP 0: DS 1 YTMP_1: DS 1 X 0: DS 1 X_1: DS 1 Y_0: DS 1

Y_1: DS	1	
Z_0: DS	1	
Z_1: DS	1	
E_0: DS	1	
E_1: DS	1	
RSEG	CORDIC_CODE	
E_00:	DB 22H,19H,0D6H,0	DEH,0D7H,07H,0FBH,03H
DB	0FFH,01H,00H,01H,80	0H,00H,40H,00H
DB	20H,00H,10H,00H,08H	H,00H,04H,00H
DB	02H,00H,01H,00H	
Angle_16:		
MOV	X_0,#6FH	;INITIALIZE X[0]
MOV	X_1,#13H	
MOV	Y_0,#00H	;INITIALIZE Y[0]
MOV	Y_1,#00H	
MOV	R2,#0	;INITIALIZE SIGN INDICATOR
		REGISTER AS POSITIVE FOR
		;BOTH SINE AND COSINE.
CLR BIT.	С	;CLEAR THE BORROW (CARRY)
MOV	A,?Angle_16?byte	;PLACE LOWER BYTE OF ANGLE IN
SUBB	А #44Н	SUBTRACT LOWER BYTE BY PI/2
MOV	7. 0 A	PLACE RESULT IN LOWER BYTE
1110 V	2_0,11	OF Z[0]
MOV	A,?Angle_16?byte+1	;PLACE UPPER BYTE OF ANGLE IN ACCUM
SUBB	A,#32H	SUBT (WITH BORROW) UPPER
		BYTE OF PI.
MOV	Z_1,A	;PLACE RESULT IN UPPER BYTE OF Z[0]
JC	Add_PiDiv2	;IF BORROW SET, THE ANGLE ;WAS [0,PI/2].
NOW CH	ECK IF THE ANGLE IS	IN [PI/2.PI]. IF NOT CONTINUE
MOV	R2,#2	INITIALIZE SIGN INDICATOR
		REGISTER POSITIVE FOR SINE
		;NEGATIVE FOR COSINE.
MOV	A,Z_0	;PLACE LOWER BYTE OF ANGLE IN
		;ACCUMULATOR

SUBB	A,#44H	;SUBTRACT LOWER BYTE BY PI/2
MOV	Z_0,A	;PLACE RESULT IN LOWER BYTE
		;OF Z[0]
MOV	A,Z_1	;PLACE UPPER BYTE OF ANGLE IN
		;ACCUM.
SUBB	A,#32H	SUBT WITH BORROW UPPER
		;BYTE BY PI/2
MOV	Z_1,A	;PLACE RESULT IN UPPER BYTE
		;OF Z[0]
JC	Twos	;IF BORROW SET, ANGLE WAS IN
		;[PI/2,PI)
NOW CI; CONTIN	HECK IF THE ANGLE IS	S BETWEEN PI AND 3PI/2, IF NOT
MOV	R2,#3	;INITIALIZE SIGN INDICATOR
		;REGISTER NEGATIVE FOR BOTH
		;SINE AND COSINE
MOV	A,Z_0	;PLACE LOWER BYTE OF ANGLE IN
		;ACCUM.
SUBB	A,#44H	;SUBTRACT LOWER BYTE BY PI/2
MOV	Z_0,A	PLACE RESULT IN LOWER BYTE
		;OF Z[0]
MOV	A,Z_1	;PLACE UPPER BYTE OF ANGLE IN
		;ACCUM
SUBB	A,#32H	;SUBT (WITH BORROW) UPPER
		;BYTE OF PI
MOV	Z_1,A	;PLACE RESULT IN UPPER BYTE
		;OF Z[0]
JC	Add_PiDiv2	;IF BORROW SET, ANGLE WAS IN
		;[PI,3PI/2)
IF WE G	ET THIS FAR, THE ANG	LE IS BETWEEN 3PI/2 AND 2PI.
MOV	R2,#1	;INITIALIZE SIGN INDICATOR
		REGISTER POSITIVE FOR
		;COSINE AND NEGATIVE FOR
		;SINE
MOV	A,Z_0	;PLACE LOWER BYTE OF ANGLE IN
		;ACCUM
SUBB	A,#44H	;SUBTRACT LOWER BYTE BY PI/2
MOV	Z_0,A	;PLACE RESULT IN LOWER BYTE

			;OF Z[0]
	MOV	A,Z_1	;PLACE UPPER BYTE OF ANGLE IN :ACCUM.
	SUBB	A,#32H	;SUBT WITH BORROW UPPER :BYTE BY PI/2
	MOV	Z_1,A	;PLACE RESULT IN UPPER BYTE :OF Z[0]
Τv	vos:		
	MOV	A,Z_0	;FORM THE TWOS COMPLEMENT ;OF Z[0]
	CPL	A	
	ADD	A.#1	
	MOV	Z 0,A	
	MOV	A,Z_1	
	CPL	A	
	ADDC A,	#O	
	MOV	Z_1,A	
	AJMP	Cordic_Algo	
Ad	ld_PiDiv2	2:	
	MOV	A,Z_0	
	ADD	A,#44H	;ADD BACK PI/2
	MOV	Z_0,A	
	MOV	A,Z_1	
	ADDC	A,#32H	
	MOV	Z_1,A	
•	IT IS AT	THIS POINT THAT THE	E Cordic Algorithm BEGINS
Сс	ordic_Alg	0:	
	MOV	DPTR,#E_00	;INIT DATA POINTER AT CORDIC
	CONSTA	ANTS	
	MOV	R1,#0	;INIT THE LOOP COUNTERS
	MOV	K,#0	
,	BELOW IS	S THE CORDIC LOOP	
Сс	ordic_Loo	p:	
	MOV	R0,K	;Temporarily store K for Shift_XY
	MOV	XTMP_0,X_0	;Temporarily Store X[K]
	MOV	XTMP_1,X_1	
	MOV	YTMP_0,Y_0	;Temporarily Store Y[K]
	MOV	YTMP 1 Y 1	

MOV	A,#0	;Temporarily Store E[K]
MOVC	A,@A+DPTR	
MOV	E_0,A	
MOV	A,#1	
MOVC	A,@A+DPTR	
MOV	E_1,A	
INC	DPTR	
INC	DPTR	
SET UP 1	THE CONTROL REGISTE	ER, R3, THAT WILL CONTAIN INFO
;ON THE	NEGATIVITY	
;OF X[K], Y	r[K], AND Z[K]	
MOV	R3,#0	
MOV	A,X_1	
ANL	A,#80H	
RL	А	
ORL	A,R3	
MOV	R3,A	
MOV	A,Y_1	
ANL	A,#80H	
RL	А	
RL	А	
ORL	A,R3	
MOV	R3,A	
MOV	A,Z_1	
ANL	A,#80H	
RL	А	
RL	А	
RL	А	
ORL	A,R3	
MOV	R3,A	
INC	R3	THIS STEP REQUIRED FOR
		LATER DJNZ INSTRUCTIONS
;COMPUT	TE Z[K+1]	
MOV	A,#80H	
ANL	A,Z_1	TEST FOR Z NEGATIVE
JNZ	Add_Z	
MOV	A,E_0	FORM TWOS COMPLEMENT OF
		;E[K] IF Z[K] IS POSITIVE.
		,

;SINCE THEN A SUBTRACTION ;IS REQUIRED

CPL	Α
ADD	A,#1
MOV	E_0,A
MOV	A,E_1
CPL	А
ADDC A	,#O
MOV	E_1,A
Add_Z:	
MOV	A,E_0
ADD	A,Z_0
MOV	Z_0,A
MOV	A,E_1
ADDC	A,Z_1
MOV	Z_1,A
;COMPUI	E X[K+1] AND Y[K+1]
CASE1:	
DJNZ	R3,CASE2
ACALL	Shift_XY
ACALL	Twos_Y_Shfted
AJMP	Add_XY
CASE2:	
DJNZ	R3,CASE3
ACALL	Abs_X
ACALL	Shift_XY
ACALL	Twos_X_Shfted
ACALL	Twos_Y_Shfted
AJMP	Add_XY
CASE3:	
DJNZ	R3,CASE4
ACALL	Abs_Y
ACALL	Shift_XY
AJMP	Add_XY
CASE4:	
DJNZ	R3,CASE5
ACALL	Abs_X
ACALL	Abs Y

ACALL	Shift_XY
ACALL	Twos_X_Shfted
AJMP	Add_XY
CASE5:	
DJNZ	R3,CASE6
ACALL	Shift_XY
ACALL	Twos_X_Shfted
AJMP	Add_XY
CASE6:	
DJNZ	R3,CASE7
ACALL	Abs_X
ACALL	Shift_XY
AJMP	Add_XY
CASE7:	
DJNZ	R3,CASE8
ACALL	Abs_Y
ACALL	Shift_XY
ACALL	Twos_X_Shfted
ACALL	Twos_Y_Shfted
AJMP	Add_XY
CASE8:	
ACALL	Abs_X
ACALL	Abs_Y
ACALL	Shift_XY
ACALL	Twos_Y_Shfted
Add_XY:	
;FORM X[K+1]
MOV	A,YTMP_0
ADD	A,X_0
MOV	X_0,A
MOV	A,YTMP_1
ADDC	A,X_1
MOV	X_1,A
;FORM Y[H	K+1]
MOV	A,XTMP_0
ADD	A,Y_0
MOV	Y_0,A
MOV	A,XTMP 1

.

ADDC	A,Y_1	
MOV	Y_1,A	
;INCREI	MENT K AND TES	T IF WE'VE LOOPED 14 TIMES YET
INC	K	
INC	R1	
CJNE	R1,#0EH,Long	_Jump
AJMP	Cordic_End	
Long_Jun	np:	
LJMP	Cordic_Loop	
Cordic_E	nd:	
IF THI; ANSW;	E COMPUTED AI TER,	NSWER IS THE NEGATIVE OF THE TRUE
;TEST ;SIGN.	IF ANSWERS AR	E NEGATIVE OR POSITIVE AND CHANGE
MOV	A,#3	;LEAVE SIGN OF
		ANSWERS POSITIVE IF
		;THE ANGLE IS $[0,PI/2)$ OR R2 = 0
ANL	A,R2	
JZ	The_End	
0 9		
MOV	A,#2	SKIP NEGATION OF COSINE
		;IF ANGLE IS IN
		[3PI/2,2PI] OR R2 = 1
ANL	A,R2	
JZ	Twos_Y	
Twos_X:		
MOV	A,X_0	;FORM THE TWOS COMPLEMENT
		;OF THE COSINE
CPL	А	;FOR ANGLES IN [PI/2,3PI/2)
ADD	A,#1	;OR R2 = 2 OR 3.
MOV	X_0,A	
MOV	A,X_1	
CPL	А	
ADDC	A,#0	
MOV	X_1,A	
Twos_Y:		
MOV	A,#1	;SKIP NEGATION OF SINE IF THE :ANGLE IS IN (PI/2.PI)

	ANL	A,R2	
	JZ	The_End	
	MOV	A,Y_0	FORM THE TWOS COMPLEMENT
			;OF THE SINE
	CPL	А	;FOR ANGLES IN [PI,2PI) OR
	ADD	A,#1	;EQUIVALENTLY, WHEN $R2 = 1$:OR 3.
	MOV	Y 0.A	
	MOV	A.Y 1	
	CPL	A	
	ADDC A	.#0	
	MOV	Y 1.A	
Tł	ne End:	- <u></u> - ,	
	AJMP	The Real End	
Ał	os X:		
	CLR	С	
	MOV	A.XTMP 0	
	SUBB	A.#1	
	MOV	XTMP 0.A	
	MOV	A,XTMP 1	
	SUBB	A,#0	
	MOV	XTMP 1.A	
	RET		
Ał	os_Y:		
	CLR	С	
	MOV	A,YTMP_0	
	SUBB	A,#1	
	MOV	YTMP_0,A	
	MOV	A,YTMP_1	
	SUBB	A,#0	
	MOV	YTMP_1,A	
	RET		
SI	hift_XY:		
	MOV	A,R0	
	JZ	End_Shift_XY	
	DEC	RO	
	CLR	С	
	MOV	A,XTMP 1	

RRC Α XTMP_1,A MOV MOV A,XTMP_0 RRC Α MOV XTMP_0,A CLR С MOV A, YTMP_1 RRC Α MOV YTMP_1,A MOV A,YTMP_0 RRC Α YTMP_0,A MOV AJMP Shift_XY End Shift XY: RET Twos_X_Shfted: MOV A,XTMP_0 CPL Α ADD A,#1 MOV XTMP_0,A MOV A,XTMP_1 CPL Α ADDC A,#0 MOV XTMP_1,A RET Twos_Y_Shfted: A,YTMP_0 MOV CPL Α ADD A,#1 YTMP_0,A MOV A,YTMP_1 MOV CPL Α ADDC A,#0 YTMP_1,A MOV RET The_Real_End: ?Cosine_16?byte,X_0 MOV ?Cosine_16?byte,X_1 MOV

MOV ?Sine_16?byte,Y_0 MOV ?Sine_16?byte,Y_1 END

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