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THERMAL ANALYSIS OF DIRECTLY BURIED CONDUIT HEAT DISTRIBUTION SYSTEMS

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Prepared for:
Tri-Service Building Materials Committee

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ABSTRACT

The calculations of heat losses and temperature fields for directly buried conduit heat distribution systems were performed using finite element method. The finite element analysis solved two-dimensional, steady-state heat transfer problems involving two insulated parallel pipes which were encased in the same conduit casing and in separate casings, and the surrounding earth. Descriptions of the theoretical basis, computational scheme, and the data input and outputs of the developed computer programs are presented. Numerical calculations were carried out for predicting the temperature distributions within the existing high temperature hot water distribution system with two insulated pipes covered in the same metallic conduit and the surrounding soil. The predicted results generally agree with the experimental data obtained at the test site. The deviations between the predicted and measured values are found to range from 0 to 17 percent with an average of 6 percent. The rates of heat loss from two insulated pipes encased in separate conduits were calculated for different pipe sizes, fluid temperatures and insulation thicknesses. The results were compared with the predictions from a steady-state, one-dimensional radial heat conduction model. The discrepancies between finite element and radial conduction models in pipe heat loss values are discussed.

Key words: Computer program, direct burial, district heating and cooling, finite element method, heat loss, heat transfer, underground heat distribution system.

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1. Introduction

District heating and cooling systems can deliver hot water and chilled water economically and efficiently, from the central plant to the end users through insulated piping networks. A centralized system is often more economical to produce heating and cooling energy since it cost less to use local energy sources or to burn low-cost fuels such as coal and municipal solid waste. In addition, a more efficient delivery of energy can be obtained with district heating and cooling systems due to reduced operating and maintenance costs and improved pipeline design and construction methods compared to individual local building plants.

Military institutions and facilities maintain approximately 6000 miles of heat distribution systems. With the majority of these systems installed more than 25 years ago, an extensive repair or replacement can be anticipated due to the deteriorated pipe insulation and the severely corroded carrier pipes and conduit casings. Nonmetallic piping materials can overcome the corrosion problems experienced with steel piping. However, most nonmetallic materials have fairly low operating temperature limits and become softer or decomposition at elevated temperatures. In direct burial conduit heat distribution systems, hot water or steam are conveyed through a pair of parallel buried pipes, which are covered by thermal insulation and encased in separate exterior conduits. The transmission heat losses from insulated piping to the surrounding earth account for the major portion of the operating costs.

Various experimental techniques, such as field testing for measuring the heat loss from the underground pipes, are expensive and time consuming. Mathematical modeling is an alternative approach, which provides a relatively inexpensive and rapid means for predicting the performance of heat distribution system. It can also be used for assessing the effects of various system variables such as pipe size, geometrical configuration, insulation thickness and operating fluid temperatures on the system performance. In the design of a new underground system or the improvement of the existing one, mathematical modeling is a valuable tool as any modifications to the design can be implemented in the model and tested easily. The predicted results from mathematical modeling, for example, the surface temperature of outer conduit casing, can be used as a guide for selecting proper conduit and coating materials for the underground systems. The prediction of the heat loss from the buried pipes to the surrounding earth is information needed in pipeline design. This is due to heat transfer to or from the pipes which has a significant affect to both the pumping requirements of the fluid carried through the pipes and the selection of optimum pipe insulation thicknesses based on life-cycle cost analysis. The determination of heat losses and surface temperatures of heat distribution systems can provide a basis for refinement of guide specification for improved guidance on design, construction, and installation procedures of efficient and reliable underground systems.

This report presents the computational procedures to predict the heat losses and temperature distributions within and around two insulated pipes encased in the same conduit casing and in separate casings. The report also describes the theoretical basis of computer models which are developed based on the finite element technique to solve a two-dimensional steady-state heat transfer between the surfaces of two parallel buried pipes and the ground surface. A comparison of the predicted results from the computer models with the field data obtained from the directly buried conduit heat distribution system installed at Fort Jackson, South Carolina, by the U. S. Army Cold Regions Research and Engineering Laboratory (CRREL) is given in this report.

2. Theoretical Basis

The heat transfer to the surrounding soil from two insulated pipes installed horizontally and encased in a conduit casing, which is buried at a finite depth into the earth, involves complex geometrical configurations, composite materials and three modes of energy transport including conduction, convection and radiation. With this nonlinear heat transfer problem, it is not possible to obtain closed-form analytical solutions for the rates of heat flow in the vicinity of a directly buried conduit system. To obtain approximate solutions to the differential equations governing heat flows, the finite element method is used because of its adaptability to irregularly shaped geometries and flexibility for different element sizes.

2.1 Governing Equation and Boundary Conditions

With reference to the Cartesian coordinate system, the differential equation describing two-dimensional heat conduction within an isotropic solid under steady-state conditions with no internal heat generation is given by:

$$\frac{\partial}{\partial x}\left[k_x \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial y}\left[k_y \frac{\partial T}{\partial y} \right] = 0 \qquad (1)$$

with the constant temperature specified on a portion of the boundary:

$$T = Tb \quad : \quad \text{on } S_1 \qquad (2)$$

and the convection boundary conditions specified on a part of the boundary:

$$k_x \frac{\partial T}{\partial x} l_x + k_y \frac{\partial T}{\partial y} l_y + h\,(T - T_a) = 0 \; ; \; \text{on } S_2 \qquad (3)$$

where T is temperature, k_j is thermal conductivity in the j-direction, x and y are Cartesian coordinates, T_b is the prescribed temperature for the boundary segment S_1, S_2 is the boundary segment subject to convective heat transfer, lx and ly are direction cosines of a vector perpendicular to S_2, h is the convective heat transfer coefficient, and T_a is the temperature of the external environment.

2.2 Method of Solution

Three-node triangular elements are used along with a linear variation of temperatures within each element as the shape function to discretize the domain of interest. Since the temperature distribution is a function of space, the local temperature within each triangular element can be expressed of the form:

$$T(x,y) = [N(x,y)] \{T_i\} \qquad (4)$$

where N(x,y) and T_i are the shape functions and temperature at node i, respectively.

Using the approximation for the unknown temperatures as given by equation 4 and the Galerkin weighted residual method [1-4], the heat conduction equation can be changed into a system of simultaneous equations, which can be written in a matrix form as:

$$[K] \{T\} = \{F\} \qquad (5)$$

where [K] is the conductance matrix, {T} is the column vector of unknown nodal temperatures, and {F} is the column matrix consisting of the boundary conditions.

The typical elements of the matrix and vectors in equation 5 can be expressed as:

$$K_{ij} = \int_V \left(k_x \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + k_y \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y}\right) dV + \int_{S_2} h\, N_i\, N_j\, d\,S \qquad (6)$$

$$Fi = \int_{s_2} h\, Ta\, Ni\, ds \qquad (7)$$

Assemblage of these element equations followed by modification of the assembled equations to account for constant temperature and convective heat transfer boundary conditions gives the global system of equations. This system of linear equations is solved for the unknown temperature vector by the LU decomposition method [5]. In this method, the conductance matrix is factored into the product of a lower-triangular matrix and an upper-triangular matrix. The resulting set of equations is then solved using forward substitution followed by backward substitution.

The monthly average earth temperature for a given depth used as the prescribed temperature along the outermost perimeter of the earth region is a function of the site location, month of year, and the depth below the ground surface. It can be estimated from the following equation consisting of a harmonic function [6]:

$$T = T_a + T_b \exp\left(-y\sqrt{w/2\alpha}\right) \sin\left[2\pi(t-3)/12 - y\sqrt{w/2\alpha}\right] \qquad (8)$$

where T = the monthly average earth temperature, ^{o}C (^{o}F)

T_a = the annual average earth temperature of the site, ^{o}C (^{o}F)

T_b = the annual amplitude of the monthly average temperature cycle, ^{o}C (F)

y = the depth from the ground surface, m (ft)

w = angular frequency of the annual cycle, rad/h.

α = thermal diffusitivity of the soil, m^2/h (ft^2/h)

t = the elapsed time from January, month

Heat transfer by free convection in the airspace and radiative exchange between the surfaces bounded by the insulated pipe and its horizontal conduit casing is treated as a fictitious non-transparent material having equivalent combined convective and radiative conductances in energy transport process. The rate of convective transfer can be expressed in terms of an equivalent heat conduction of the form [7,8]:

$$q = K_e \quad (T_h - T_c)/L \tag{9}$$

where q = the average heat flow rate, W/m^2 ($Btu/h \cdot ft^2$)

K_e = the effective thermal conductivity of the enclosed air layer, $W/m.K$ ($Btu/h \cdot ft. ^oF$)

T_h = the temperature of the hot surface, $^oC(^oF)$

T_c = the temperature of the cold surface, $_oC(_oF)$

L = the characteristic thickness of air layer, m(ft).

An effective thermal conductivity, which is defined as the thermal conductivity of stationary air in an enclosed space necessary to transfer the same amount of heat as the moving air, is used to account for free convection in air confined between the heated insulated pipe and the cooler conduit casing. The ratio of the effective to actual thermal conductivity of the enclosed airspace is a complex function of geometrical configuration of wall surfaces, the thermal properties and motion of confined air, the characteristic dimension or the thickness of the air layer, and the temperature difference between the hot and cold surfaces [8].

Free convection heat transfer in the annular space between two long horizontal concentric cylinders was studied experimentally using optical methods and theoretically using numerical techniques by Kuehn and Goldstein [9]. Their experimental results using air and water as the working media were correlated in terms of the effective thermal conductivity. The average effective to actual thermal conductivity for air can be expressed as below:

$$K_e/K_f = 0.159\ R_a^{\ 0.272} \tag{10}$$

where K_f = thermal conductivity of air at T_f

R_a = Rayleigh number, PrGr

Pr = Prandtl number, υ/α

υ = Kinematic viscosity of air

α = thermal diffusivity of air

Gr = Grashof number, $g\beta(T_i-T_o)\ L^3/\upsilon^2$

g = acceleration of gravity

β = thermal expansion coefficient at T_f

L = gap width, $L = R_o-R_i$

R_i, R_o = radius of the inner and outer cylinder, respectively

T_i, T_o = temperature of the inner and outer cylinder, respectively

For the case of two long, horizontal pipes encased in the same conduit casing, no specific experimental data or correlation results on free convection in confined space are available. In the absence of experimental correlation, the heat transfer by free convection in airspaces between two small eccentric horizontal cylinders and a large cylinder may be treated similarly as natural convection around a horizontal cylinder. It can be estimated by an expression of the form [10].

$$Nu = 0.53\ R_{aD}^{\ 0.25} \tag{11}$$

where Nu = Nusselt number, $hL/K_f = Ke/K_f$

h = average heat transfer coefficient

L = hypothetical gap width, $L = (D_o - D)/2$

D = effective diameter of the two small cylinders, which is defined as the diameter of a cylinder having the perimeter equal to the total perimeter of the small cylinders.

D_o = the diameter of the large cylinder

R_{aD} = Rayleigh number, Pr GrD

Gr_D = Grashof number, $g\beta(T_i-T_o)\ D^3/\nu^2$

The radiant exchange between the outer surfaces of the insulated pipes and the inner surface of the conduit casing is treated as a multiple radiative interchange between two infinitely long concentric cylinders with diffuse - gray surfaces of different emissivities and temperatures. The equivalent thermal conductance due to radiative transfer to be added to the convective component to yield the overall conductance can be determined from the following expression:

$$K_r = h_r L \tag{12}$$

where $h_r = \sigma\ (T_i + T_o)\ (T_i^2 + T_o^2)/\ [1/\epsilon_i + (Di/Do)(D_o)(1/\epsilon_o - 1)]$

σ = the Stefan-Boltzmann constant

ϵ_i, ϵ_o = the total emissivity of the exposed surface of the inner and outer cylinder, respectively

D_i, D_o = the outside diameter of the inner cylinder and the inside diameter of the outer cylinder, respectively

L = the gap width, $(D_o-D_i)/2$

For an insulated piping system, the surface film resistance between the hot fluid and the steel pipe and the thermal resistance of pipe wall can generally be neglected compared to the thermal resistance of the insulation. The rate of heat loss from an insulated pipe can be calculated from the following equation along with the calculated value of average temperature drop across the pipe insulation layer:

$$q = 2\pi k_i(T_1 - T_2)/\ln(r_2/r_1) \tag{13}$$

where q = the heat loss rate per unit length of the insulated pipe, W/m (Btu/h.ft)

k_i = thermal conductivity of insulation material, W/m.K (Btu.in/h.ft.^{2} ^{o}F)

r_1,r_2 = the inside and outside radius of the insulation layer, respectively, m(ft)

T_1 = the surface temperature of the insulation layer at inner radius Y_1, which is assumed to be the same as the working fluid temperature, ^{o}C (^{o}F)

T_2 = the surface temperature of the insulation layer at outer radius, ^{o}C (^{o}F)

The thermal conductivity of pipe insulation is generally a function of its mean temperature. For calcium silicate insulation, the thermal conductivity function is based on a look-up table and stored in a subprogram to provide the temperature dependent thermal conductivity value. The thermal conductivity of mineral wool insulation is obtained using a least-squares regression analysis of the test data from the pipe insulation used for the underground systems installed at Fort Jackson, South Carolina. It can be expressed by

$$K_i = 0.242 + 1.5501 \times 10^{-4} T_m + 7.500 \times 10^{-7} Tm^2 \quad (14)$$

where K_i = thermal conductivity of the insulation, Btu-in/h.ft^2 ^oF

T_m = the mean temperature of the insulation, ^oF

The thermal conductivity, kinematic viscosity and Prandtl number of air are a function of its mean temperature and can respectively be approximated as

$$\begin{aligned} K_a &= 0.01319 + 2.50 \times 10^{-5} T_a \\ \nu_a &= 1.2624 \times 10^{-4} + 5.40 \times 10^{-7} T_a \\ Pr &= 0.7185 - 1.275 \times 10^{-4} T_a \end{aligned} \quad (15)$$

where

K_a = thermal conductivity of air, Btu/h.ft.^oF

Ta = the bulk air temperature within the airspace, ^oF

ν_a = kinematic viscosity of air, ft^2/s

The bulk air temperature can be approximated by the average of the outer surface temperature of the insulation and the inner surface temperature of the conduit casing. For the case of two pipes encased in the same conduit,

an effective insulation outer surface temperature is defined to be equal to the weighted average of the external surface temperatures of two insulated pipes.

3. Description of Computer Programs

Two finite element computer programs called DIRECT1 and DIRECT2 have been developed for numerical modeling of two-dimensional, steady-state heat conduction. This involves directly buried conduit distribution systems with two parallel pipes encased in the same conduit casing and in separate casings, respectively. Each computer program consists of a main program and nine subroutines and the computer codes are written in FORTRAN language. In addition to reading in portions of input data, the main program calls pertinent subroutines, calculates the conductance matrix and excitation vector modified for the given convection and constant temperature boundary conditions, and prints out the calculated nodal temperatures.

Subroutine TGO calculates the average undisturbed earth temperatures at various depths for the month of interest. Subroutine PIPEO is used to read in the dimensions of the insulated pipes and the inner and outer regions of the surrounding earth, echos the data to allow the checking of input data, and generates the rectangular coordinates for each nodal point of the two-pipe system. Subroutines INSULK and SOILK provide the thermal conductivity of pipe insulation including calcium silicate and mineral wool, and soil thermal conductivity, respectively by linear interpolation of sets of thermal conductivity versus mean temperature data. Subroutines TGXX furnish the boundary temperatures of the outer earth region and TWOPIP calculates the rate of heat loss from the two insulated pipes to the soil surrounding the heat distribution system. Subprograms EQUIKO determines the equivalent thermal conductivity of airspace within the pipe and SOLVE is used to solve system of simultaneous equations by LU decomposition method [5]. Subroutine PIPEHL calculates the average temperature drops across the pipe insulation layers, the rates of heat loss from both underground pipes, and prints out the results of these calculations.

A computer program called DBJACKS similar to the computer code DIRECT1 in program structures and flow control, has been developed to simulate the conditions of thermal measurements performed by the U.S. Army Cold Regions Research and Engineering Laboratory for the high temperature hot water distribution system installed at Fort Jackson, South Carolina. A listing of the source codes of these computer programs is given in Appendix B.

In using the computer codes, the underground systems to be modeled numerically are first divided into regions of interest. These include pipe insulation covering the carrier pipes, airspace within the pipes, the conduit casing, back-fill region, and the inner and outer earth regions in the vicinity of the distribution system. These regions are then discretized into triangular elements. To increase the degree of calculation accuracy, the smaller element sizes are used in the areas of anticipated higher temperature gradients. The larger elements are employed for the areas having smaller temperature gradients. In labeling of the nodal points, the nodes along the inner and outer boundaries with the specified temperatures and convective heat fluxes are numbered. This is done first to obtain a reduced number of simultaneous equations, which will be solved for the unknown nodal temperatures.

The finite element meshes for the direct burial conduit systems for distributions of steam and hot water are shown in Figures 1 through 3. These figures are divided into three sets: the "a" figures illustrate the underground system consisting of both pipes encased in separate conduits the "b" figures show the distribution system involving two pipes covered by the same outer conduit, and the "c" figures illustrate the underground system installed at Fort Jackson and having both pipes encased in the same conduit. Figures 1.a to 1.c show the finite element design for the inner and outer earth regions surrounding the distribution pipes. The finite element grids for back-fill region adjacent to the underground system are shown in Figures 2.a through 2.c. Figures 3.a through 3.c illustrate the finite element meshes for pipe insulation, the conduit casing, and airspace between the insulated pipes and the casing. The finite element meshes contained in the computer programs consist of 248 triangular elements and 142 nodal points for program DIRECT2, 212 elements and 124 nodes for DIRECT1, and 224 elements and 131 nodes for DBJACKS, respectively. Table 1 presents a summary of finite element meshes representing regions of the major components of underground systems where both pipes are encased in separate conduits and in the same conduit.

4. Comparison of Calculated Results with Experimental Data

The direct burial conduit heat distribution system installed at Fort Jackson, South Carolina and modeled numerically, consists of two 5-inch (127 mm) high temperature hot water pipes with the heat supply pipe placed directly above the heat return pipe. Each pipe is insulated by a 1.5 inches (38 mm) thick layer of mineral wool. The hot water supply and return lines are encased in a 20 inches (508 mm) outside diameter steel conduit having a wall thickness of 1/8 inch (3.2 mm). Both pipes are separated by a distance of 9.5 inches (235 mm) between pipe centers. The conduit casing is located at 3 ft 8 inches (1.12 m) beneath the ground surface. In numerical simulation, the piping system is assumed to be surrounded by earth having a thermal conductivity of 7.0 Btu.in/h.ft^2 ^{o}F and an average temperature of 50 ^{o}F. The ground surface is exposed to ambient air of seasonally varying temperatures. All experimental data obtained by the CRREL from the test site were recorded at 6 - hour intervals. Numerical calculations were made based on the average values over a two-day period of the measured heat supply and return pipe temperatures, and the experimentally determined undisturbed earth temperatures at various depths.

Two input data files SDTAIFJ and SDTA2FJ are created prior to execution of computer program DBJACKS for numerical simulations of the thermal performance of the directly buried conduit heat distribution system installed at Fort Jackson. These input data files along with the outputs from the computer program are presented in Appendix A. The SDTALFJ shown in Appendix A.1 refers to the main program and subroutines PIPEJ and TGO for data input. It contains the title of the computer run, total numbers of nodal points and triangular elements. The first element number of pipe insulation, the first node numbers of pipe insulation for the heat supply and return pipes, and the month of interest are also included. In addition data for run control parameters, thermal conductivity and dimensions of the inner earth region, estimated average air temperature, and temperature drop across airspace are presented. Also shown are thermal conductivities of the conduit casing and the back-fill soil, the total emissivities and average temperatures of outer surface of pipe insulation and inner surface of the conduit casing, the pipe fluid temperatures, the material type, thermal conductivity and thicknesses of pipe insulation, the pipe sizes and locations of the pipe centers, the size and depth of the outer casing, their thermal properties and dimensions of the outer earth region, and the annual average earth temperature and amplitude of the monthly temperature cycle for the site involved. As shown in Appendix A.2, the element data file SDTA2FJ consists of the element number, the node numbers for its three vertices and the material type for each triangular element, total number of elements subject to convection boundary, the surface convection coefficients and ambient temperatures for three sides of each element that experiences convection loss.

The boundary conditions for numerical simulations are the constant temperatures of working fluids flowing inside the carrier pipes, the specified undisturbed earth temperatures along the outermost perimeter of the earth region, and rate of convective heat transfer between the ground surface in the vicinity of the underground system and the ambient air. The undisturbed earth temperatures at various depths for a specific month were calculated from equation 8 in which the two parameters: the average earth temperature of the site and the best-fit of the measured earth temperature data to the equation through the method of least squares. The system of matrix equations is solved for the unknown nodal temperatures using the LU decomposition technique. With adjustment of insulation and soil thermal conductivities to account for temperature effect, an iterative procedure is used until the heat loss from the underground pipes reaches a steady-state condition.

Computer calculations of pipe heat loss and temperature distributions in the vicinity of the underground system were performed on the mainframe Cyber 855 at NIST. These calculations required approximately 5 to 8 seconds of execution time. The outputs from the computer program included: the rates of heat loss from carrier pipes, the resultant temperature at each nodal point, average temperature drops across pipe insulation layers, equivalent thermal conductivities of airspace due to free convection and radiative transfer, and cartesian coordinates of all nodal points. A listing of output file SOUTFJ from program DIRECFJ for the sample case is given in Appendix A.3.

Numerical predictions of the thermal performance of this high temperature hot water distribution system were carried out to simulate the tests conducted during the January 1-10, 1987 period. A comparison of the predicted temperature distributions within and around the two insulated pipes encased in the same metallic conduit with the corresponding measured values obtained by the CRREL at the test site is tabulated in Table 2. The average value and the range of the calculated surface temperatures at 8 nodal points distributed evenly around the outer surface of pipe insulation, or the inner surface of the conduit casing are also presented in Table 2 for comparison. In general, the predicted results are in reasonably good agreement with the experimental data. The difference between the calculated and measured temperatures for all measuring locations with the exception of the outermost boundary is found to range from 0 to 23 percent with an average of 7 percent. A large discrepancy between the predicted and experimental temperatures on the outer surface of pipe insulation for the heat supply line can be explained by the mean equivalent thermal conductivity used in calculations. It is too large compared to the actual local equivalent conductivity which would result in relatively low insulation surface temperature. Kuehn and Goldstein [9] reported that the local equivalent conductivity or local heat transfer coefficient of horizontal concentric cylinders varied with angular position. The highest value occurred at the bottom of the inner cylinder with the smallest value at the top. With the absence of an expression for the local heat transfer coefficient, it is relevant to use the average value of the predicted temperatures on a cylindrical surface in correspondence to the mean heat transfer coefficient employed in numerical calculations.

As illustrated in Table 2, the deviations of the calculated results based on average surface temperatures from the measured values are found to vary from 0 to 17 percent with a mean of 6 percent. Figure 4 shows the temperature data predicted with the finite element models for temperatures at selected locations within and around the directly buried conduit system at Fort Jackson 4 against the temperature values determined by thermocouples at the test site. A similar plot for comparing the temperature results from thermocouple measurements with the predicted values including the average values of eight local surface temperatures on the outer surface of the insulated pipe and on the inner surface of the conduit casing is shown in Figure 5. These plots show that a reasonably good correlation exists between the temperature values obtained from the model predictions and the field measurements because all the data points lie close to the line of perfect agreement.

5. Comparisons of Finite Element and Thermal Analysis Programs with Guide Specification Table

A thermal analysis computer program called DIRECTP, was developed based on steady-state, one-dimensional heat conduction for an insulated pipe with an airspace conduit buried horizontally beneath the ground surface. The theoretical treatments considered the condition that hot fluid flows inside a steel pipe and heat is transferred radially by conduction and convection through a series of cylindrical shells. These consist of steel pipe, thermal insulation, airspace and conduit casing, and the soil layer around the underground pipe. The calculation procedure is essentially similar to that used to derive the pipe insulation tables for the maximum permissible heat loss values listed in Guide Specification CEGS-15705 [11]. A listing of the source code of thermal analysis computer program is given in Appendix B. The outputs from thermal analysis program include the heat loss from a single pipe of directly buried conduit system, the mean insulation thermal conductivity, and the inner and outer surface temperatures of both the pipe insulation and the conduit casing.

Some calculations using the finite element computer simulation program, DIRECT2, and the thermal analysis program were carried out on typical direct burial systems with nominal 6-in. (152 mm) and 3-in. (76 mm) pipes and different working fluid temperatures ranging from 200 ^{o}F to 400 ^{o}F. The results on pipe heat loss were compared with the corresponding maximum permissible heat loss values given in Table 5 of Guide Specification CEGS-15705. In these calculations, the heat supply and return pipes were located side by side at 4.0 ft. (1.22m) below the ground surface, and separated by a distance of 1.5 ft. (0.46m) between pipe centers. The thicknesses of calcium silicate insulation used for the pipes were varied from 1.5 inches (38mm) to 3.0 inches (76 mm). An average ground temperature of 55 ^{o}F (12.8C) and the thermal conductivity of soil equal to 5 Btu.in/h.ft^{2}.^{o}F (0.72 W/m.C) were used in calculations. The thicknesses of outer casing and airspace between the insulated pipe and the casing were 1/8 inch (3mm) and about 1 inch (25mm), respectively.

The pipe heat losses predicted by the finite element and thermal analysis computer programs for directly buried systems for hot water and steam distributions are shown in Table 3. In general, for the heat supply pipes, the calculated heat loss values from the finite element model are approximately 5 percent smaller compared to the values listed in the table of the guide specification. For the heat return pipes having the identical pipe size as the heat supply pipes, the finite element model gives about 22 percent smaller heat loss values in comparison to the guide specification table. For the case of steam distribution as illustrated in the table, this difference in heat loss values for the heat return pipe become greater with the increased size of the heat supply pipe. The rate of heat loss from a single pipe, especially for the heat return pipe, is considerably influenced by the pipe size and working fluid temperature of its neighboring pipe, which serves as a heat source or heat sink to continuously add or remove a portion of heat from the thermal field involved. The heat loss results calculated by thermal analysis program are consistently about 3 percent greater than the maximum allowable heat loss values tabulated in the guide specification table.

6. Conclusions

The temperature distributions in underground heat distribution systems, which consist of a pair of insulated pipes encased in the same conduit and in separate conduits within the surrounding soil, and the heat loss rates from the buried pipes were calculated using computer simulation programs utilizing the finite-element method.

General formulation of the relevant equations governing heat flow and boundary conditions for steady-state, two-dimensional heat conduction problems are presented. The computational scheme, the input data required for executing the simulation programs, and the outputs from the computer runs for sample cases are described. The spatial distributions of temperatures predicted by the finite element models for a high temperature hot water distribution system installed at Fort Jackson involving two insulated pipes encased in the same conduit are generally consistent with the experimental data obtained by the CRREL at the test site. The difference between the predicted and measured temperatures is found to range from 0 to 17 percent with an average of approximately 6 percent.

A comparison was made of the pipe heat losses predicted by the finite element and the thermal analysis models for a range of pipe working fluid temperatures and insulation thicknesses with the maximum permissible heat loss values listed in Table 5 of Guide Specification CEGS-15705. The finite element model gives approximately 5 percent smaller heat loss value for the heat supply pipe and 22 to 27 percent less in heat loss for the heat return pipe compared to the guide specification table, depending upon the pipe size, separation distance and working fluid temperature of the neighboring pipe. It is anticipated that the pipe located in the vicinity serves as a heat source or heat sink to interact with the thermal field involved. The pipe heat loss values predicted by the thermal analysis model derived based on steady-state, one-dimensional radial heat conduction through a single insulated pipe are found to agree within 3 percent of the values tabulated in the guide specification. Experimental data and correlation equations of local heat transfer coefficient are needed for a more accurate prediction of heat flows in an airspace between the insulated pipe and the inner surface of the conduit casing.

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Table 1 A Summary of Finite Element Meshes for Underground Systems with Two Pipes Encased in Separate Conduits and in the Same Conduit

	Pipes in Separate Conduits	Pipes in Same Conduits	Simulation Hot Water System
1. Inner Earth Region	Elements 1 through 32 (Fig. 1.a)	Elements 1 through 32 (Fig. 1.b)	Elements 1 through 32 (Fig. 1.c)
2. Outer Earth Region	Elements 33 through 90 (Fig. 1.a)	Elements 33 through 90 (Fig. 1.b)	Elements 33 through 96 (Fig. 1.c)
3. Pipe Insulation	Elements 91 through 122 (Fig. 3.a)	Elements 91 through 122 (Fig. 3.c)	Elements 97 through 128 (Fig. 3.c)
4. Airspace Between Insulated Pipe and Conduit Casing	Elements 123 through 154 (Fig. 3.a)	Elements 123 through 148 (Fig. 3.b)	Elements 129 through 160 (Fig. 3.c)
5. Conduit Casing	Elements 155 through 186 (Fig. 3.a)	Elements 149 through 164 (Fig. 3.b)	Elements 161 through 176 (Fig. 3.c)
6. Backfill Region	Elements 187 through 248 (Fig. 2.a)	Elements 165 through 212 (Fig. 2.b)	Elements 177 through 224 (Fig. 2.c)
Total Number of Nodal Points:	142	124	131
Total Number of Elements:	248	212	224

TABLE 2. COMPARISON OF CALCULATED RESULTS WITH EXPERIMENTAL DATA

DATE	VALUE	OUTDOOR AIR TEMP. (F)	SURFACE TEMP. (DEG. F) TOP PIPE	TOP INSUL.	BOTTOM PIPE	BOTTOM INSUL.	CONDUIT (INSIDE)	AIRSPACE TEMP. (F) TOP	MID	BOT
1/1-1/2/87	Meas.	41	325	156	222	127	121	140	146	116
	Calc.			122(131) [122-148]		134(127) [118-147]	118(117) [115-118]	120	148	117
	Diff. (%)			21.8(16.0)		5.5(0.0)	2.5(3.3)	14.3	1.4	0.9
1/3-1/4/87	Meas.	39	323	153	206	122	119	140	142	113
	Calc.			118(127) [118-143]		129(123) [114-142]	115(113) [111-115]	117	143	113
	Diff. (%)			22.9(17.0)		5.7(0.8)	3.4(5.0)	16.4	0.7	0.0
1/5-1/6/87	Meas.	42	322	152	213	122	118	138	142	113
	Calc.			120(129) [120-145]		131(125) [116-144]	115(113) [111-115]	119	145	115
	Diff. (%)			21.1(15.1)		7.4(2.5)	2.5(4.2)	13.8	2.1	1.8
1/7-1/8/87	Meas.	46	323	154	220	124	118	139	144	114
	Calc.			123(131) [123-148]		134(127) [118-147]	119(117) [115-119]	121	148	118
	Diff. (%)			20.1(14.9)		8.1(2.4)	0.8(0.8)	12.9	2.8	3.5
1/9-1/10/87	Meas.	48	321	153	225	125	118	139	144	114
	Calc.			124(133) [124-150]		135 [120-149]	120(119) [117-120]	122	149	119
	Diff. (%)			19.0(13.1)		8.0(3.2)	1.7(0.8)	12.2	3.5	4.4

Note: The numerical values inside the parentheses and the brackets are the average value, and the range of the calculated surface temperatures at 8 nodal points distributed evenly around the cylindrical surface, respectively.

DATE	VALUE	EARTH TEMP. (F) 2-in. DEEP	EARTH TEMP. (F) 9-in. DEEP	UNDISTURBED EARTH TEMP. (F) 3-in. DEEP	UNDISTURBED EARTH TEMP. (F) 26-in. DEEP	UNDISTURBED EARTH TEMP. (F) 56-in. DEEP	SYSTEM HEAT LOSS (Btu/h-ft)
1/1-	Meas.	56	66	46	50	51	
1/2/	Calc.	57	61	47	49	51	110
87	Diff. (%)	1.8	7.5	2.1	2.0	0.0	
1/3-	Meas.	56	66	45	49	50	
1/4/	Calc.	54	59	46	48	50	106
87	Diff. (%)	3.6	10.6	2.2	2.0	0.0	
1/5-	Meas.	56	66	44	48	49	
1/6/	Calc.	57	61	45	47	49	107
87	Diff. (%)	1.8	7.6	2.3	2.1	0.0	
1/7-	Meas.	56	66	45	49	49	
1/8/	Calc.	60	65	46	47	49	108
87	Diff. (%)	7.1	1.5	2.2	4.1	0.0	
1/9-	Meas.	56	66	47	50	50	
1/10	Calc.	62	66	48	49	51	108
/87	Diff. (%)	10.7	0.0	2.1	2.0	2.0	

Table 3. Comparison of Pipe Heat Loss Values (Btu/h.ft) Predicted by Finite Element Computer Program with Calculated Values Obtained from Thermal Analysis Program and Guide Specification Table

Calculated by	Pipe Working Fluid Temperature (Degree F)			
	200	250	350	400
1. 3-in. Pipe:				
FECP	29.8 (25.5)a (22.9)e	36.2 (33.1)b (28.8)f	61.7a	60.7b
TACP	33.8	39.9	62.5	61.7
CEGS	32.7	38.7	60.6	59.5
Insulation Thickness(in.):	1.5	2.0	2.0	3.0
2. 6-in. Pipe:				
FECP	42.1 (34.3)c	57.4 (49.3)d	80.0c 81.8e	93.9d 97.5f
TACP	48.8	67.8	81.5	97.2
CEGS	47.8	65.2	80.3	95.8
Insulation Thickness(in.):	1.5	1.5	2.5	2.5

Notes:

1. FECP and TACP represent finite element and thermal analysis computer programs, respectively. CEGS denotes Guide Specification CEGS-15705, which contains the heat loss values listed in Table 5.

2. The heat loss values in parentheses are calculated based on the pipe used as the heat return pipe.

3. Superscripts a through f denote pairs of a two-pipe system in calculations.

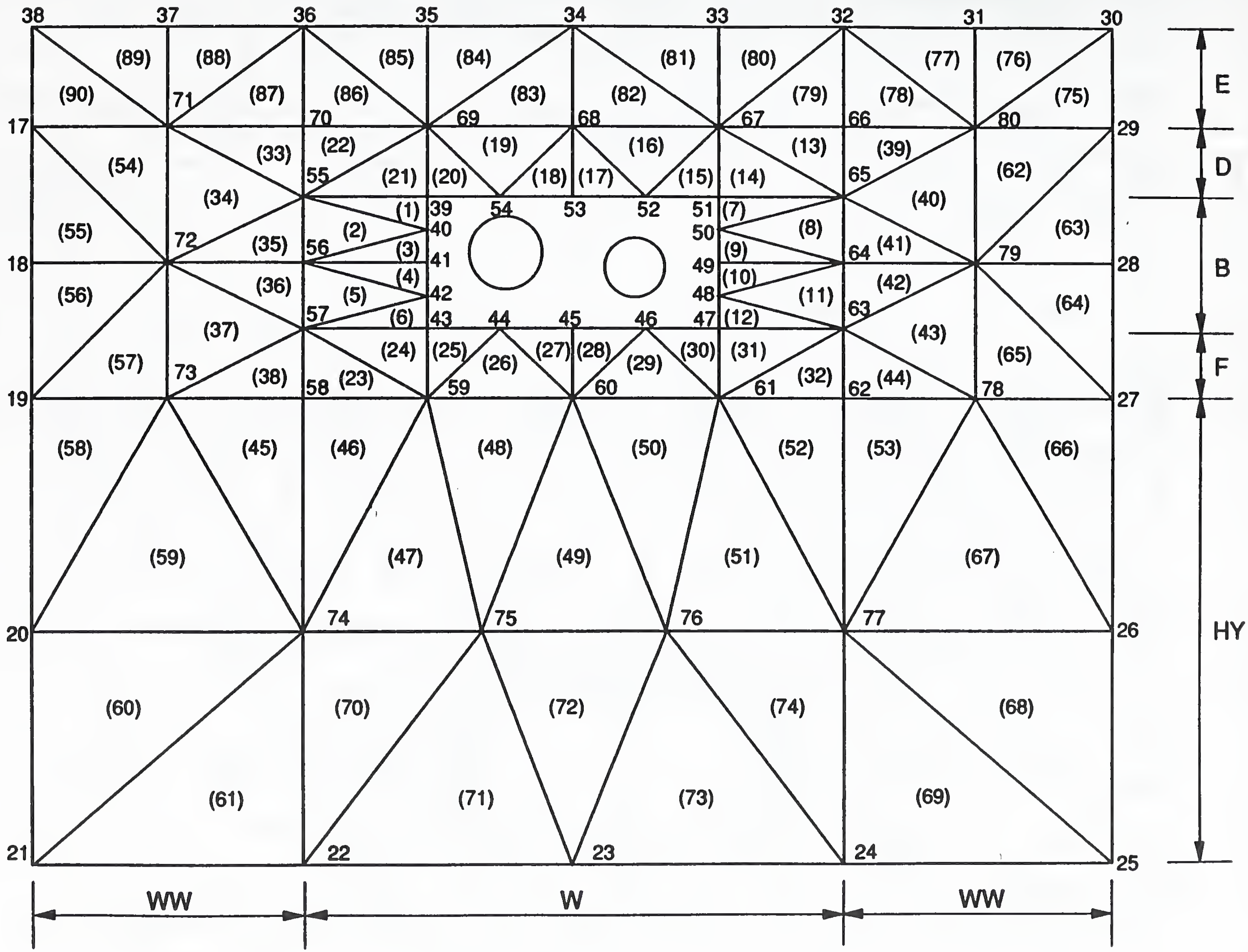

Figure 1.a Finite Element Mesh for Inner and Outer Earth Regions with Both Pipes in Separate Conduits

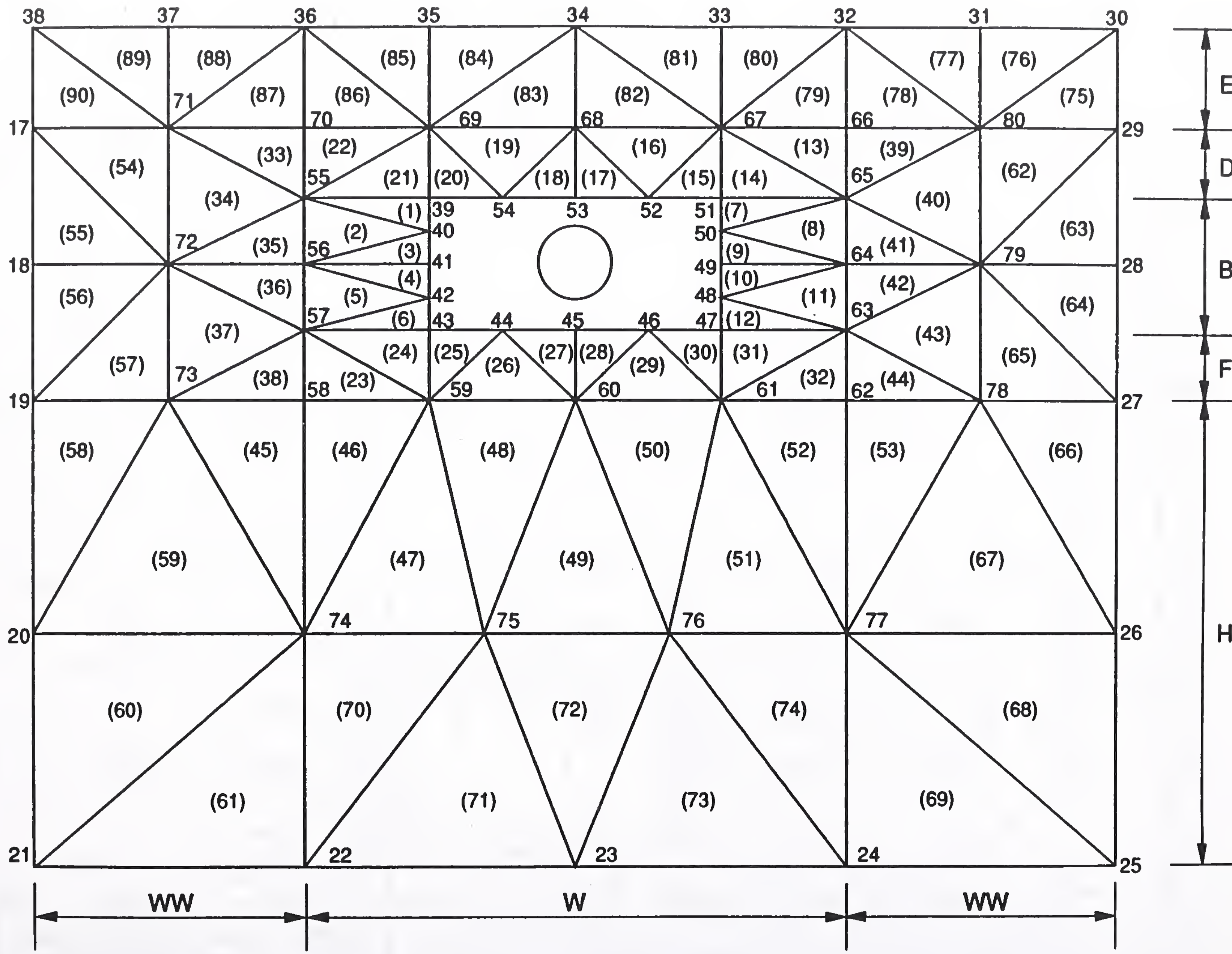

Figure 1.b Finite Element Mesh for Inner and Outer Earth Regions with Both Pipes in the Same Conduit

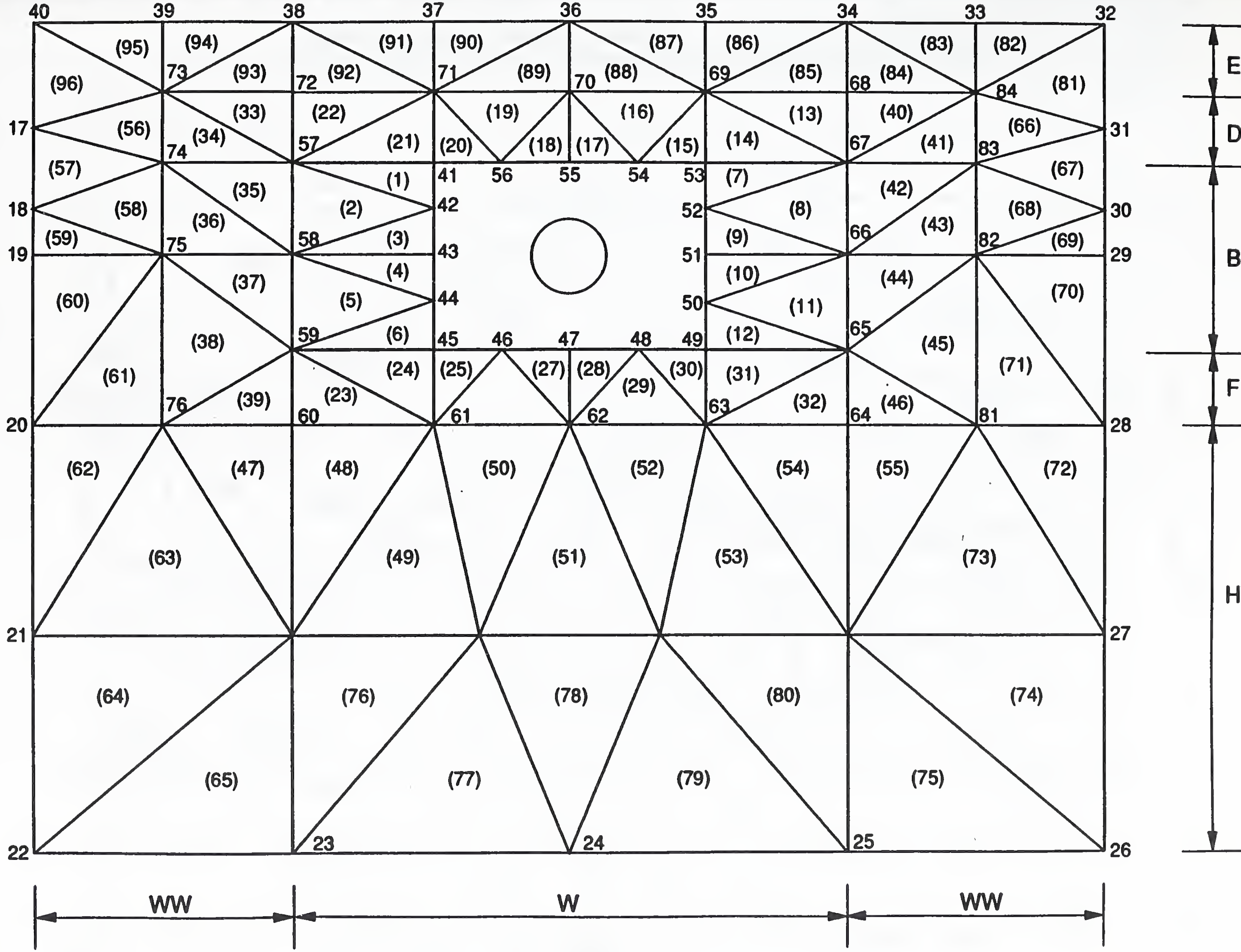

Figure 1.C Finite Element Mesh for Inner and Outer Earth Regions for Underground System Installed at Fort Jackson

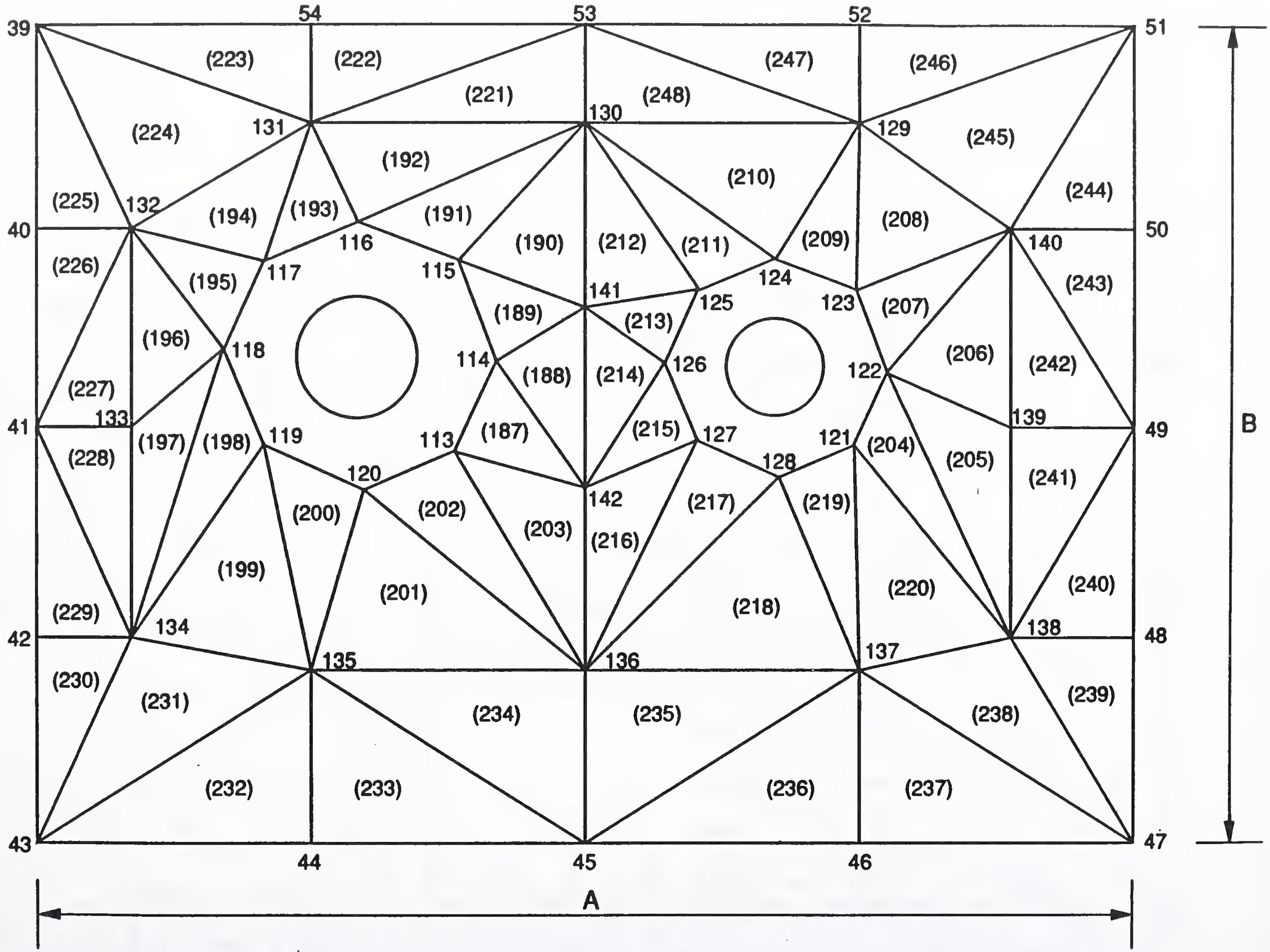

Figure 2.a Finite Element Mesh for Back-fill Region Surrounding Insulated Pipes Encased in Separate Conduits

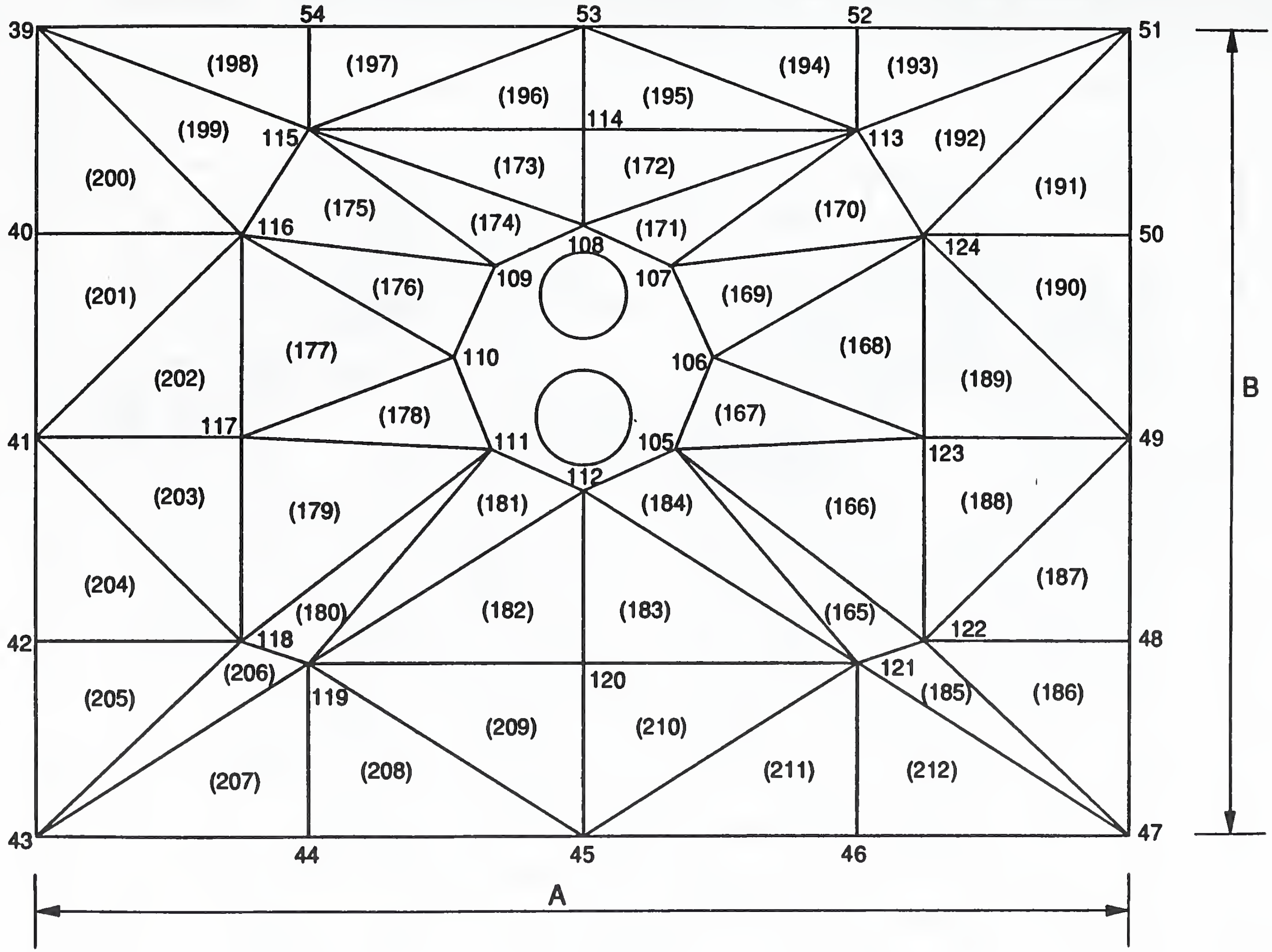

Figure 2.b Finite Element Mesh for Back-fill Region Surrounding Insulated Pipes Encased in the Same Conduit

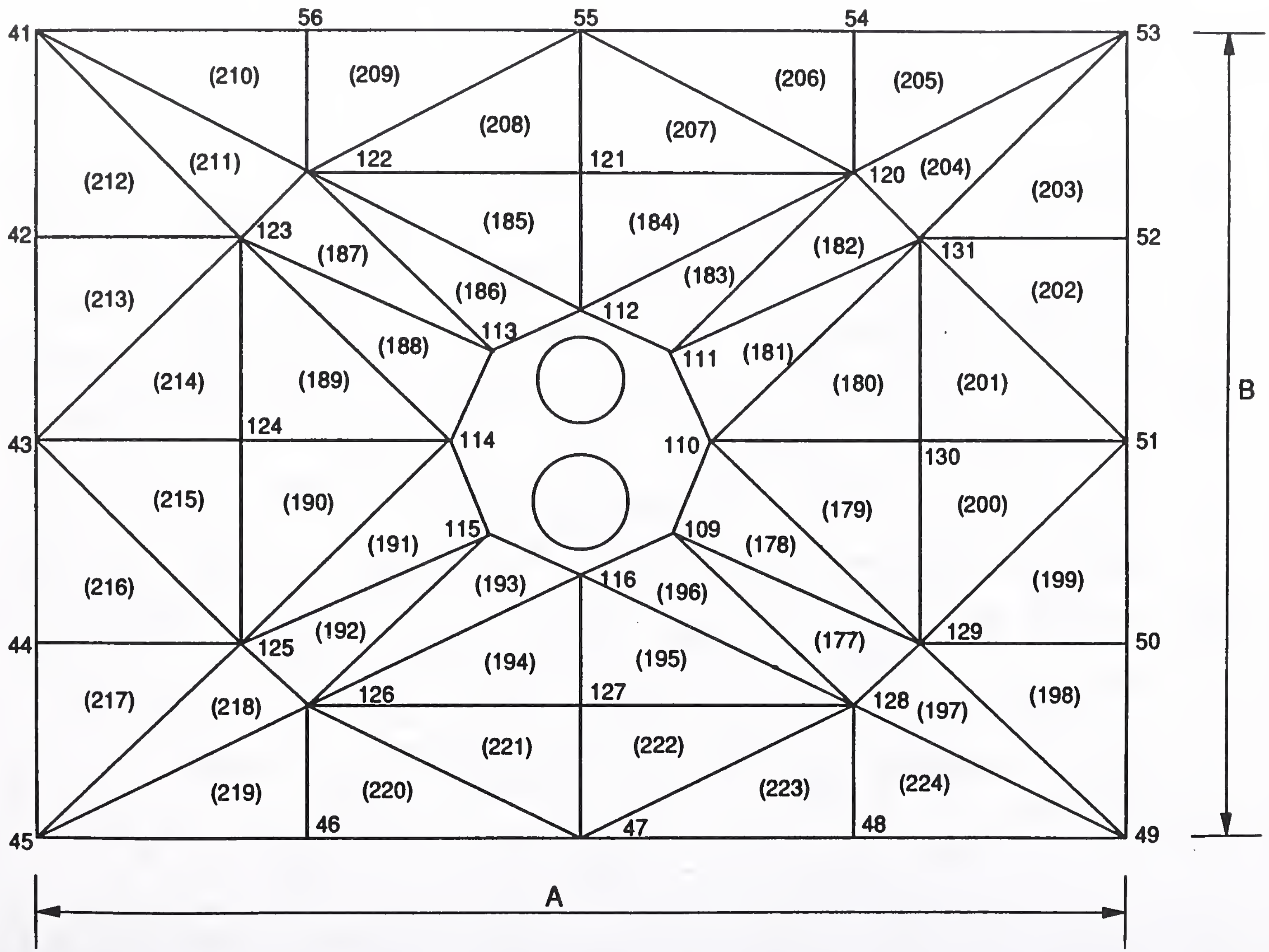

Figure 2.c Finite Element Mesh for Back-fill Region Around Underground System Installed at Fort Jackson

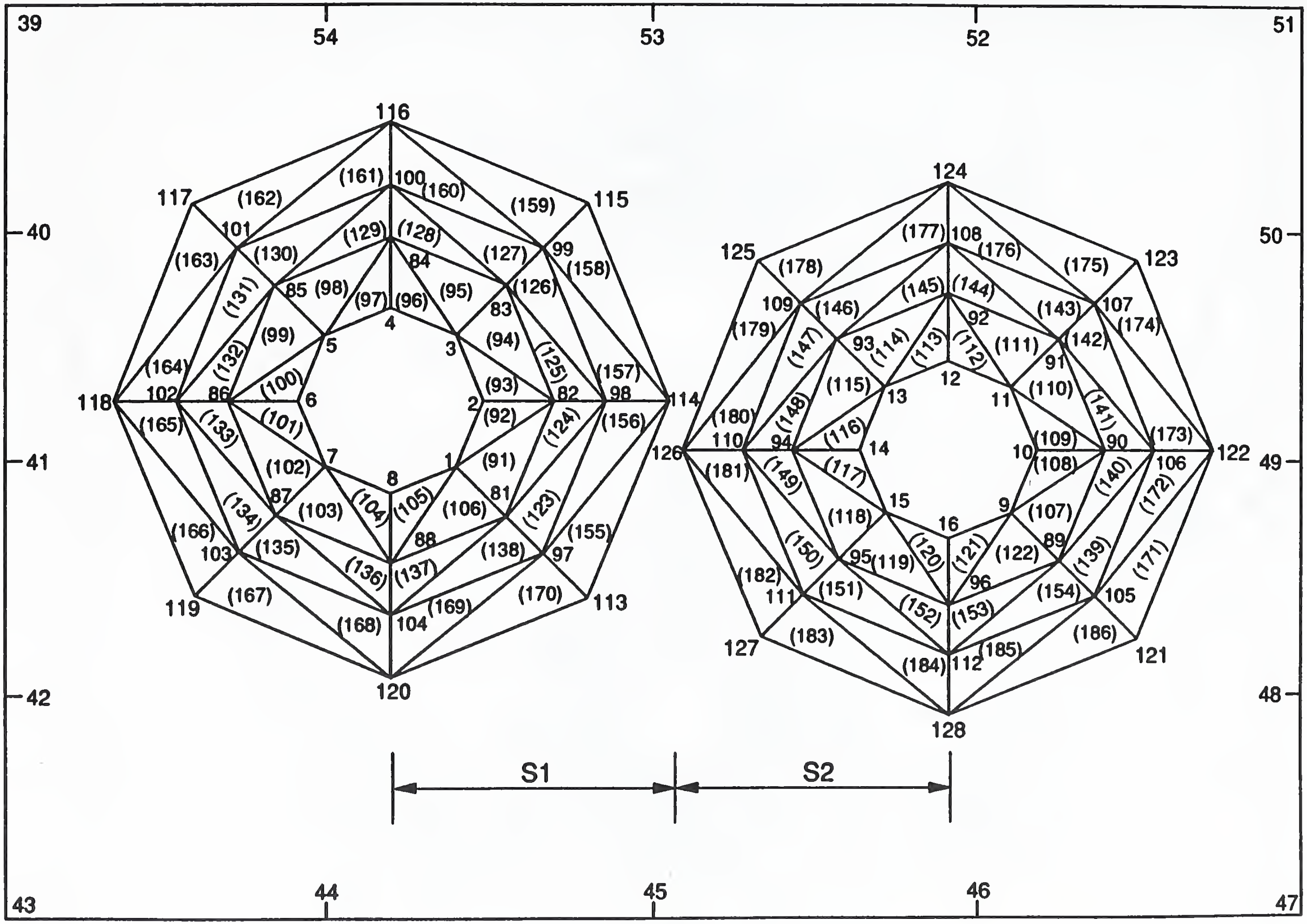

Figure 3.a Finite Element Design for Pipe Insulation, Airspace Layer and Conduit Casing for Two Pipes in Separate Conduits

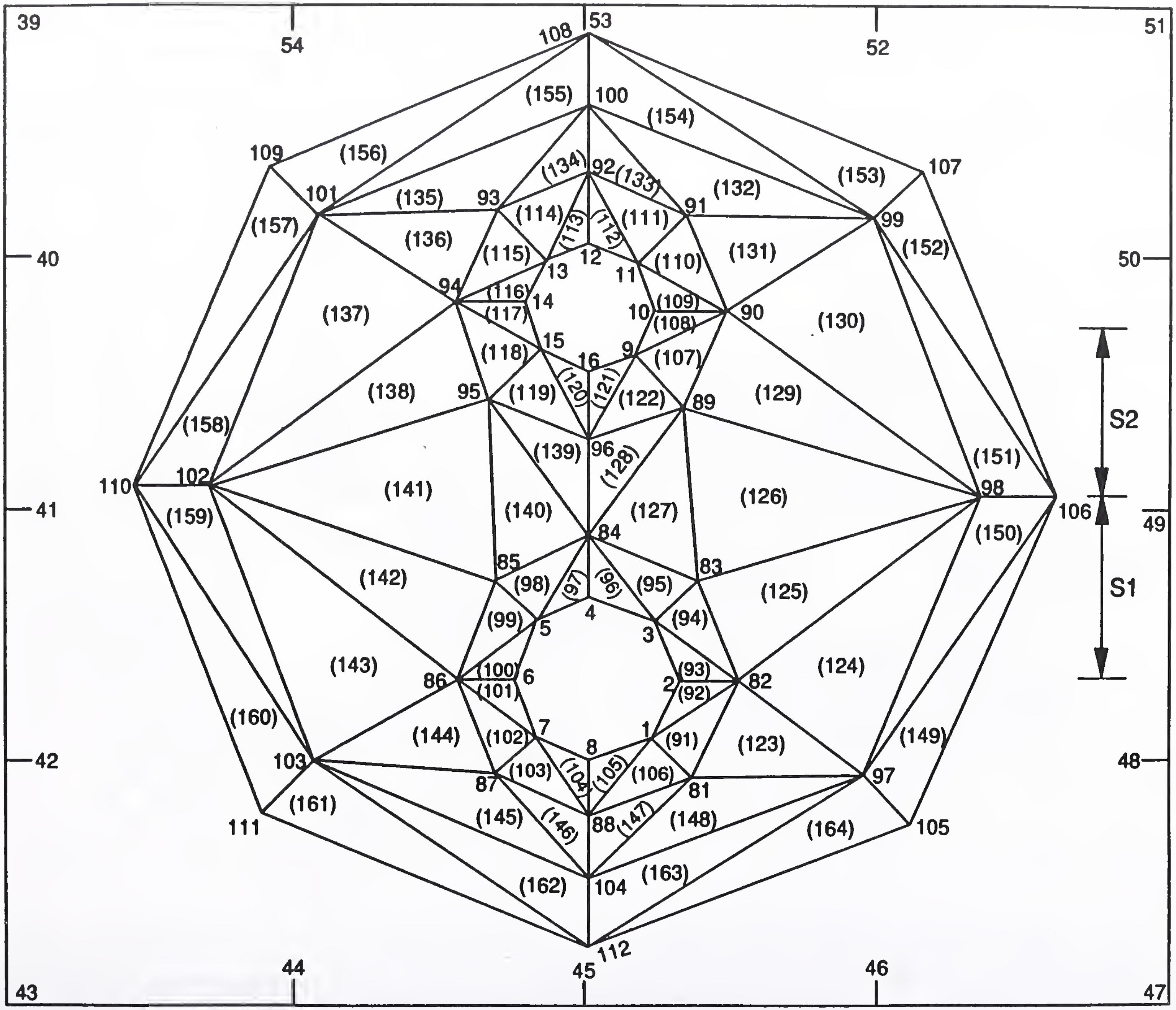

Figure 3.b Finite Element Design for Pipe Insulation, Airspace Layer and Conduit Casing for Two Pipes in the Same Conduit

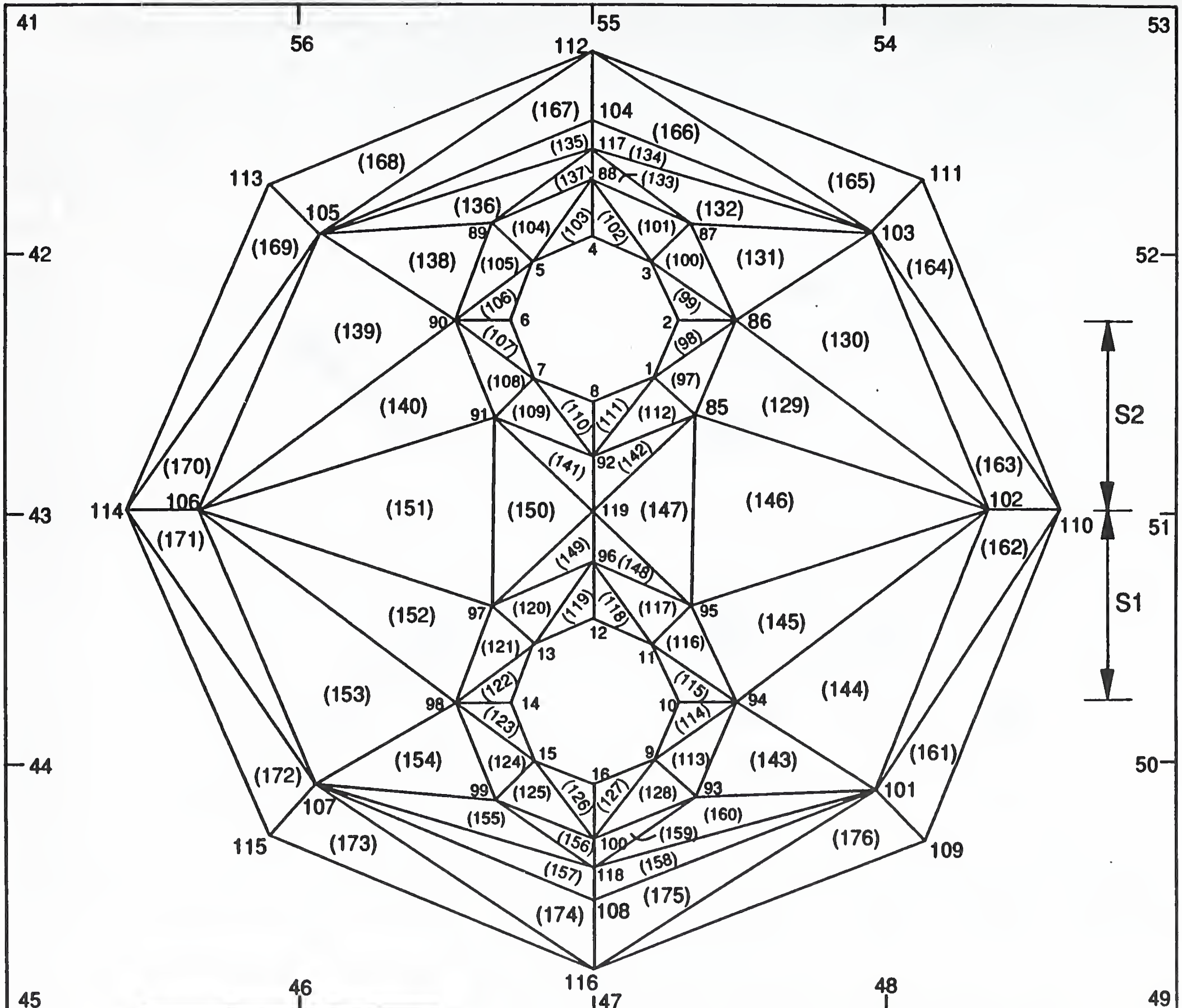

Figure 3.c Finite Element Design for Pipe Insulation, Airspace Layer and Conduit Casing for Underground System Installed at Fort Jackson

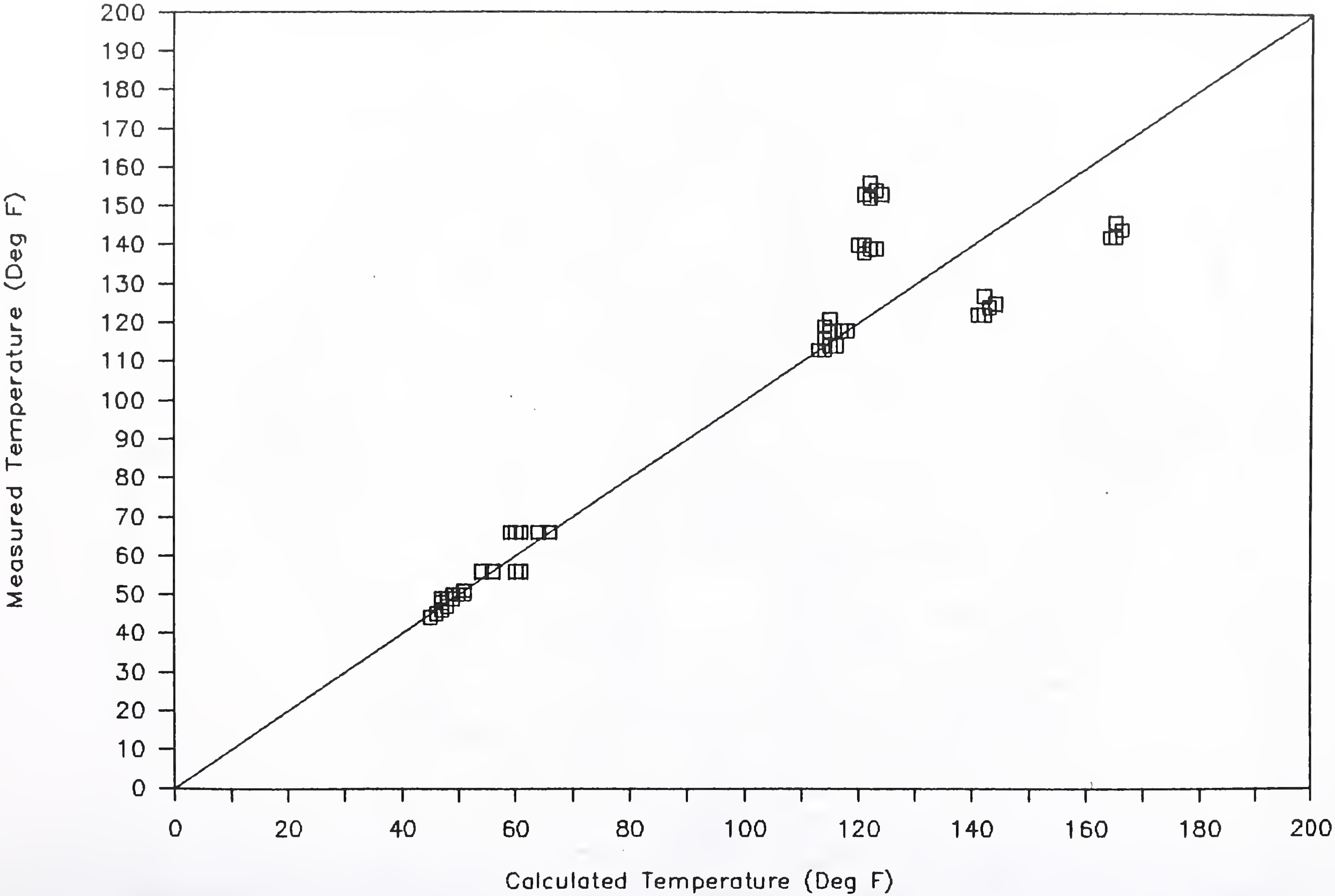

Figure 4. A Comparison Between the Temperature Values Predicted with Finite Element Models and the Values Obtained from Thermocouple Measurements

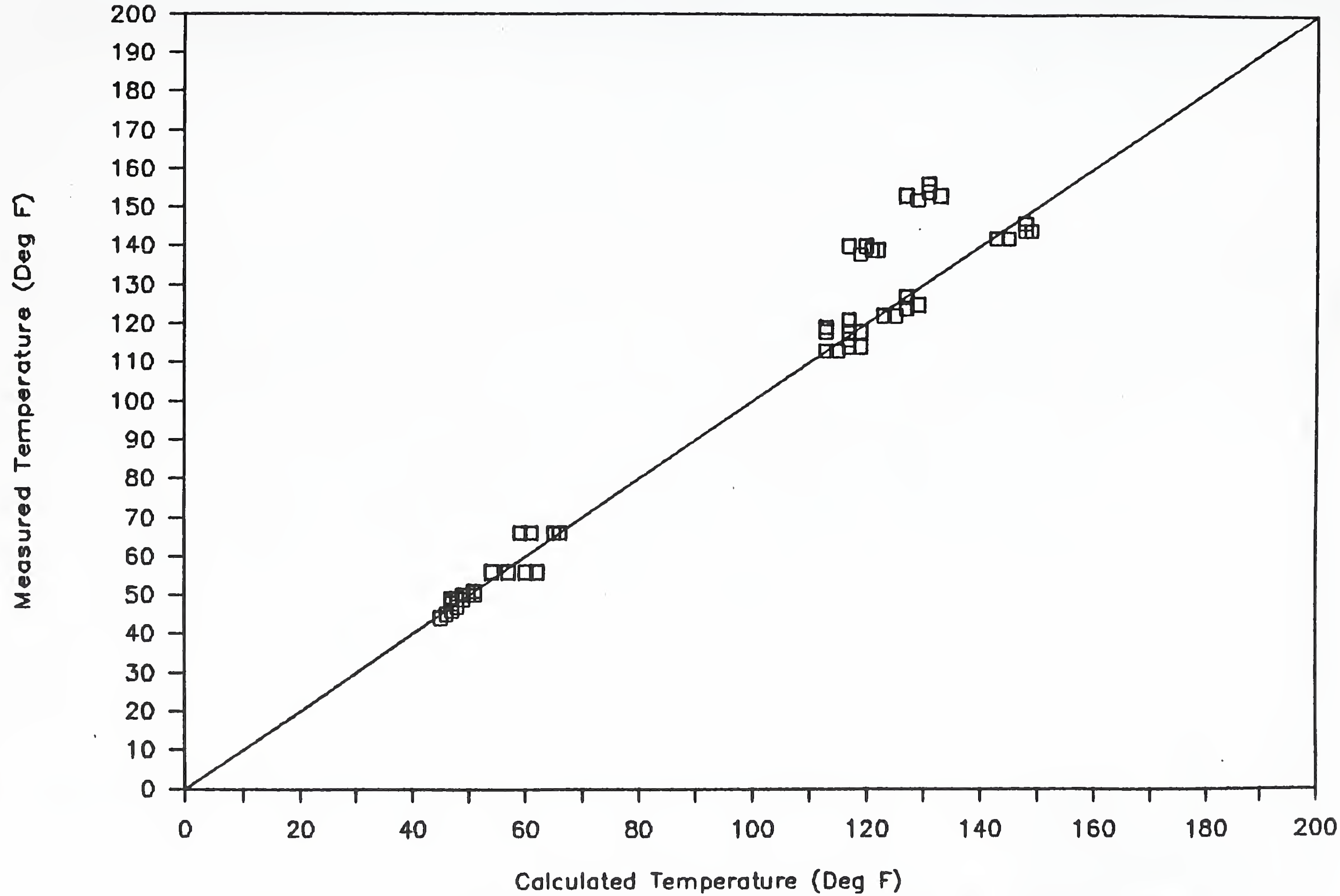

Figure 5. A Comparison of the Calculated Temperatures Including the Average Values for Cylindrical Surfaces with the Measured Temperatures

APPENDIX A. The Input Data Files and the Outputs from the Computer Programs

A.1 A Listing of SDTAIFJ Input File for Program DBJACKS

```
HOT WATER SYSTEM(PIPES IN ONE CASING)AT FORT JACKSON,1/7/87
131,224,31,97,85,93,101
1,1
7.,7.,0.583,0.583
140.0,18.0
372.0,7.0
0.8,0.9,140.0,115.0
323.0,220.0
0.34,7.0,1,2
5.563,5.563
1.47,1.47
19.75,0.125,54.0
0.385,0.385,55.0
0.167,10.0,10.0
3.833,7.50
55.0,11.10,0.019
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A.2 A Listing of SDTA2FJ Input File for Program DBJACKS

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91,0.50,0.0,0.0,46.0,46.0,46.0
94,0.50,0.0,0.0,46.0,46.0,46.0
95,0.50,0.0,0.0,46.0,46.0,46.0
```

A.3 Output File SOUTFJ from Program DBJACKS

```
HOT WATER SYSTEM(PIPES IN ONE CASING)AT FORT JACKSON,1/7/87
   TP1     TP2      KI      KG      D1      D2
323.00  220.00     .34    7.00    5.56    5.56
  THI1    THI2    DIAC    THKC    DEPC      S1      S2      TG
  1.47    1.47   19.75     .13   54.00     .39     .39   55.00
     E      WW      HY   MONTH
   .17   10.00   10.00       1
     W       H       D       F       A       B      WW      HY
  5.00    8.83     .58     .58    3.83    7.50   10.00   10.00
   XC1     YC1     XC2     YC2      B1      B2
 2.500   4.115   2.500   4.885   4.115   4.885
   DI1     DI2      S1      S2    THI1     KII     KIG     TP1     TP2
  5.56    5.56     .39     .39    1.47     .34    7.00    323.    220.
    Q1      Q2      QT      KP
 76.12   36.82  112.94    .522

 PRANTL=      .7006  GRASOF=    .6752E+08
 CONKA=     8.8033   RADKA=     1.4159  (BTU-IN/H-FT**2-DEF F)
 X(M),M=1,NN
  2.66    2.73    2.66    2.50    2.34    2.27    2.34    2.50    2.66    2.73
  2.66    2.50    2.34    2.27    2.34    2.50  -10.00  -10.00  -10.00  -10.00
-10.00  -10.00     .00    2.50    5.00   15.00   15.00   15.00   15.00   15.00
 15.00   15.00   10.00    5.00    4.42    2.50     .58     .00   -5.00  -10.00
   .58     .58     .58     .58     .58    1.54    2.50    3.46    4.42    4.42
  4.42    4.42    4.42    3.46    2.50    1.54     .00     .00     .00     .00
   .58    2.50    4.42    5.00    5.00    5.00    5.00    5.00    4.42    2.50
   .58     .00   -5.00   -5.00   -5.00   -5.00     .00    1.67    3.33    5.00
 10.00   10.00   10.00   10.00    2.75    2.85    2.75    2.50    2.25    2.15
  2.25    2.50    2.75    2.85    2.75    2.50    2.25    2.15    2.25    2.50
  3.08    3.32    3.08    2.50    1.92    1.68    1.92    2.50    3.09    3.33
  3.09    2.50    1.91    1.67    1.91    2.50    2.50    2.50    2.50    3.46
  2.50    1.54    1.12    1.12    1.12    1.54    2.50    3.46    3.87    3.87
  3.87
 Y(M),M=1,NN
  4.28    4.12    3.95    3.88    3.95    4.12    4.28    4.35    5.05    4.89
  4.72    4.65    4.72    4.89    5.05    5.12     .25    2.63    4.50    8.83
 13.83   18.83   18.83   18.83   18.83   18.83   13.83    8.83    4.50    2.63
   .25     .00     .00     .00     .00     .00     .00     .00     .00     .00
   .75    2.63    4.50    6.38    8.25    8.25    8.25    8.25    8.25    6.38
  4.50    2.63     .75     .75     .75     .75     .75    4.50    8.25    8.83
  8.83    8.83    8.83    8.83    8.25    4.50     .75     .17     .17     .17
   .17     .17     .17     .75    4.50    8.83   13.83   13.83   13.83   13.83
  8.83    4.50     .75     .17    4.37    4.12    3.86    3.76    3.86    4.12
  4.37    4.47    5.14    4.89    4.63    4.53    4.63    4.89    5.14    5.24
  5.08    4.50    3.92    3.68    3.92    4.50    5.08    5.32    5.09    4.50
  3.91    3.67    3.91    4.50    5.09    5.33     .04     .04    4.50    2.21
  2.21    2.21    2.63    4.50    6.38    6.79    6.79    6.79    6.38    4.50
  2.63
     QQ     ARRAY
 .00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
 .00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
 .00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
 .00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
 .00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
 .00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
 .00000E+00  .32296E+03  .11500E+03  .64208E+02  .28748E+02
 .44080E+02  .28748E+02  .64208E+02  .11500E+03  .32296E+03
 .00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
 .00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
 .00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
 .00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
 .00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
 .00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
 .00000E+00  .00000E+00  .17716E+01  .11762E+02  .22941E+02
 .44507E+02  .45673E+02  .10771E+02  .10771E+02  .45673E+02
 .44507E+02  .22941E+02  .11762E+02  .17716E+01  .18136E+02
 .18136E+02  .18136E+02  .18136E+02  .18136E+02  .18136E+02
 .18136E+02  .18136E+02  .12353E+02  .12353E+02  .12353E+02
 .12353E+02  .12353E+02  .12353E+02  .12353E+02  .12353E+02
 .00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
 .00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
 .00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
 .00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
```

```
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00
PRANTL=     .7021  GRASOF=   .4822E+08
CONKA=    7.9538   RADKA=    1.4421  (BTU-IN/H-FT**2-DEF F)
KASP=    9.3960  (BTU-IN./H-FT**2-DEG F)

AVERAGE VALUES OF PIPE INSULATION THERMAL CONDUCTIVITY :
 KI1 =        .340   KI2 =        .340   BTU-IN/H-FT**2-DEG F

AVERAGE TEMPERATURE DROPS ACROSS INSULATION :
 T1=     187.05   T2=      87.73  DEG F

HEAT LOSSES FROM UNDERGROUND PIPES :
 Q1=     78.49   Q2=      36.81   QT=     115.30   BTU/H-FT
    QQ     ARRAY
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .32296E+03  .11500E+03  .64208E+02  .28748E+02
.44080E+02  .28748E+02  .64208E+02  .11500E+03  .32296E+03
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .17716E+01  .11762E+02  .22941E+02
.44507E+02  .45673E+02  .10771E+02  .10771E+02  .45673E+02
.44507E+02  .22941E+02  .11762E+02  .17716E+01  .16226E+02
.17420E+02  .16019E+02  .17350E+02  .16019E+02  .17420E+02
.16226E+02  .17598E+02  .10343E+02  .10751E+02  .10484E+02
.10873E+02  .10484E+02  .10751E+02  .10343E+02  .10711E+02
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00
PRANTL=     .7028  GRASOF=   .5110E+08
CONKA=    8.0130   RADKA=    1.4244  (BTU-IN/H-FT**2-DEF F)
KASP=    9.4375  (BTU-IN./H-FT**2-DEG F)

AVERAGE VALUES OF PIPE INSULATION THERMAL CONDUCTIVITY :
 KI1 =        .318   KI2 =        .293   BTU-IN/H-FT**2-DEG F

AVERAGE TEMPERATURE DROPS ACROSS INSULATION :
 T1=     191.57   T2=      92.47  DEG F

HEAT LOSSES FROM UNDERGROUND PIPES :
 Q1=     75.17   Q2=      33.42   QT=     108.59   BTU/H-FT
    QQ     ARRAY
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .32296E+03  .11500E+03  .64208E+02  .28748E+02
.44080E+02  .28748E+02  .64208E+02  .11500E+03  .32296E+03
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .17716E+01  .11762E+02  .22941E+02
.44507E+02  .45673E+02  .10771E+02  .10771E+02  .45673E+02
.44507E+02  .22941E+02  .11762E+02  .17716E+01  .16156E+02
```

```
.17371E+02  .15945E+02  .17298E+02  .15945E+02  .17371E+02
.16156E+02  .17554E+02  .10296E+02  .10721E+02  .10440E+02
.10848E+02  .10440E+02  .10721E+02  .10296E+02  .10680E+02
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00  .00000E+00  .00000E+00  .00000E+00  .00000E+00
.00000E+00
PRANTL=      .7028   GRASOF=    .5086E+08
CONKA=    8.0017    RADKA=     1.4239  (BTU-IN/H-FT**2-DEF F)
KASP=    9.4257  (BTU-IN./H-FT**2-DEG F)

AVERAGE VALUES OF PIPE INSULATION THERMAL CONDUCTIVITY :
 KI1 =        .317    KI2 =        .292    BTU-IN/H-FT**2-DEG F

AVERAGE TEMPERATURE DROPS ACROSS INSULATION :
 T1=      191.80    T2=       92.67   DEG F

HEAT LOSSES FROM UNDERGROUND PIPES :
 Q1=      75.00    Q2=       33.38    QT=     108.38   BTU/H-FT
```

M	I	J	K	MAT.	C
1	41	57	42	1	.5833
2	42	57	58	1	.5833
3	42	58	43	1	.5833
4	43	58	44	1	.5833
5	44	58	59	1	.5833
6	44	59	45	1	.5833
7	52	67	53	1	.5833
8	52	66	67	1	.5833
9	51	66	52	1	.5833
10	50	66	51	1	.5833
11	50	65	66	1	.5833
12	49	65	50	1	.5833
13	67	68	69	1	.5833
14	67	69	53	1	.5833
15	53	69	54	1	.5833
16	54	69	70	1	.5833
17	54	70	55	1	.5833
18	55	70	56	1	.5833
19	56	70	71	1	.5833
20	56	71	41	1	.5833
21	41	71	57	1	.5833
22	57	71	72	1	.5833
23	59	60	61	1	.5833
24	45	59	61	1	.5833
25	45	61	46	1	.5833
26	46	61	62	1	.5833
27	46	62	47	1	.5833
28	47	62	48	1	.5833
29	48	62	63	1	.5833
30	48	63	49	1	.5833
31	49	63	65	1	.5833
32	63	64	65	1	.5833
33	57	72	73	6	.5833
34	57	73	74	6	.5833
35	57	74	58	6	.5833
36	58	74	75	6	.5833
37	58	75	59	6	.5833
38	59	75	76	6	.5833
39	59	76	60	6	.5833
40	67	84	68	6	.5833
41	67	83	84	6	.5833
42	66	83	67	6	.5833
43	66	82	83	6	.5833
44	65	82	66	6	.5833
45	65	81	82	6	.5833
46	64	81	65	6	.5833
47	60	76	77	6	.5833
48	60	77	61	6	.5833
49	61	77	78	6	.5833
50	61	78	62	6	.5833

51	62	78	79	6	.5833
52	62	79	63	6	.5833
53	63	79	80	6	.5833
54	63	80	64	6	.5833
55	64	80	81	6	.5833
56	17	74	73	6	.5833
57	17	18	74	6	.5833
58	18	75	74	6	.5833
59	18	19	75	6	.5833
60	19	20	75	6	.5833
61	20	76	75	6	.5833
62	20	21	76	6	.5833
63	21	77	76	6	.5833
64	21	22	77	6	.5833
65	22	23	77	6	.5833
66	31	84	83	6	.5833
67	31	83	30	6	.5833
68	30	83	82	6	.5833
69	29	30	82	6	.5833
70	28	29	82	6	.5833
71	28	82	81	6	.5833
72	27	28	81	6	.5833
73	27	81	80	6	.5833
74	26	27	80	6	.5833
75	25	26	80	6	.5833
76	23	78	77	6	.5833
77	23	24	78	6	.5833
78	24	79	78	6	.5833
79	24	25	79	6	.5833
80	25	80	79	6	.5833
81	32	84	31	6	.5833
82	32	33	84	6	.5833
83	33	34	84	6	.5833
84	34	68	84	6	.5833
85	34	69	68	6	.5833
86	34	35	69	6	.5833
87	35	36	69	6	.5833
88	36	70	69	6	.5833
89	36	71	70	6	.5833
90	36	37	71	6	.5833
91	37	38	71	6	.5833
92	38	72	71	6	.5833
93	38	73	72	6	.5833
94	38	39	73	6	.5833
95	39	40	73	6	.5833
96	40	17	73	6	.5833
97	1	85	86	2	.0251
98	1	86	2	2	.0277
99	2	86	3	2	.0277
100	3	86	87	2	.0249
101	3	87	88	2	.0249
102	4	3	88	2	.0276
103	5	4	88	2	.0276
104	5	88	89	2	.0249
105	5	89	90	2	.0249
106	5	90	6	2	.0277
107	6	90	7	2	.0277
108	7	90	91	2	.0251
109	7	91	92	2	.0254
110	7	92	8	2	.0280
111	8	92	1	2	.0280
112	1	92	85	2	.0254
113	9	93	94	2	.0237
114	10	9	94	2	.0248
115	10	94	11	2	.0248
116	11	94	95	2	.0238
117	11	95	96	2	.0241
118	11	96	12	2	.0251
119	12	96	13	2	.0251
120	13	96	97	2	.0241
121	13	97	98	2	.0238
122	13	98	14	2	.0248
123	14	98	15	2	.0248

124	15	98	99	2	.0237
125	15	99	100	2	.0236
126	16	15	100	2	.0247
127	16	100	9	2	.0247
128	9	100	93	2	.0236
129	85	102	86	3	.7865
130	86	102	103	3	.7865
131	86	103	87	3	.7865
132	87	103	117	3	.7865
133	87	117	88	3	.7865
134	103	104	117	3	.7865
135	104	105	117	3	.7865
136	89	117	105	3	.7865
137	88	117	89	3	.7865
138	89	105	90	3	.7865
139	90	105	106	3	.7865
140	90	106	91	3	.7865
141	91	119	92	3	.7865
142	92	119	85	3	.7865
143	93	101	94	3	.7865
144	94	101	102	3	.7865
145	95	94	102	3	.7865
146	95	102	85	3	.7865
147	95	85	119	3	.7865
148	96	95	119	3	.7865
149	96	119	97	3	.7865
150	97	119	91	3	.7865
151	97	91	106	3	.7865
152	97	106	98	3	.7865
153	98	106	107	3	.7865
154	98	107	99	3	.7865
155	99	107	118	3	.7865
156	99	118	100	3	.7865
157	107	108	118	3	.7865
158	108	101	118	3	.7865
159	100	118	93	3	.7865
160	93	118	101	3	.7865
161	101	109	110	4	31.0000
162	101	110	102	4	31.0000
163	102	110	103	4	31.0000
164	110	111	103	4	31.0000
165	103	111	112	4	31.0000
166	103	112	104	4	31.0000
167	104	112	105	4	31.0000
168	112	113	105	4	31.0000
169	105	113	114	4	31.0000
170	105	114	106	4	31.0000
171	106	114	107	4	31.0000
172	107	114	115	4	31.0000
173	107	115	116	4	31.0000
174	107	116	108	4	31.0000
175	108	116	101	4	31.0000
176	101	116	109	4	31.0000
177	109	128	129	5	.5833
178	109	129	110	5	.5833
179	110	129	130	5	.5833
180	110	130	131	5	.5833
181	111	110	131	5	.5833
182	111	131	120	5	.5833
183	111	120	112	5	.5833
184	112	120	121	5	.5833
185	112	121	122	5	.5833
186	112	122	113	5	.5833
187	113	122	123	5	.5833
188	113	123	114	5	.5833
189	114	123	124	5	.5833
190	114	124	125	5	.5833
191	114	125	115	5	.5833
192	115	125	126	5	.5833
193	115	126	116	5	.5833
194	116	126	127	5	.5833
195	116	127	128	5	.5833
196	116	128	109	5	.5833

```
 197    49   129   128     5      .5833
 198    49    50   129     5      .5833
 199    50    51   129     5      .5833
 200    51   130   129     5      .5833
 201    51   131   130     5      .5833
 202    51    52   131     5      .5833
 203    52    53   131     5      .5833
 204    53   120   131     5      .5833
 205    53    54   120     5      .5833
 206    54    55   120     5      .5833
 207    55   121   120     5      .5833
 208    55   122   121     5      .5833
 209    55    56   122     5      .5833
 210    56    41   122     5      .5833
 211    41   123   122     5      .5833
 212    41    42   123     5      .5833
 213    42    43   123     5      .5833
 214    43   124   123     5      .5833
 215    43   125   124     5      .5833
 216    43    44   125     5      .5833
 217    44    45   125     5      .5833
 218    45   126   125     5      .5833
 219    45    46   126     5      .5833
 220    46    47   126     5      .5833
 221    47   127   126     5      .5833
 222    47   128   127     5      .5833
 223    47    48   128     5      .5833
 224    48    49   128     5      .5833
TEMPERATURE ARRAY : T(I), I=1,NN
.32300E+03  .32300E+03  .32300E+03  .32300E+03  .32300E+03
.32300E+03  .32300E+03  .32300E+03  .22000E+03  .22000E+03
.22000E+03  .22000E+03  .22000E+03  .22000E+03  .22000E+03
.22000E+03  .45533E+02  .47363E+02  .49045E+02  .52456E+02
.54680E+02  .55395E+02  .55395E+02  .55395E+02  .55395E+02
.55395E+02  .54680E+02  .52456E+02  .49045E+02  .47363E+02
.45533E+02  .45423E+02  .48811E+02  .57715E+02  .57478E+02
.58719E+02  .57478E+02  .57715E+02  .48811E+02  .45423E+02
.64637E+02  .80731E+02  .95188E+02  .87191E+02  .79263E+02
.80951E+02  .81831E+02  .80951E+02  .79263E+02  .87191E+02
.95188E+02  .80731E+02  .64637E+02  .66091E+02  .66905E+02
.66091E+02  .63715E+02  .86835E+02  .77950E+02  .75833E+02
.76850E+02  .78578E+02  .76850E+02  .75833E+02  .77950E+02
.86835E+02  .63715E+02  .59022E+02  .59157E+02  .60490E+02
.59157E+02  .59022E+02  .49342E+02  .51146E+02  .60832E+02
.62056E+02  .62748E+02  .63459E+02  .63459E+02  .62748E+02
.62056E+02  .60832E+02  .51146E+02  .49342E+02  .13652E+03
.12995E+03  .12256E+03  .12338E+03  .12256E+03  .12995E+03
.13652E+03  .14814E+03  .11803E+03  .12476E+03  .13369E+03
.14728E+03  .13369E+03  .12476E+03  .11803E+03  .11836E+03
.11648E+03  .11728E+03  .11898E+03  .11719E+03  .11898E+03
.11728E+03  .11648E+03  .11523E+03  .11647E+03  .11728E+03
.11898E+03  .11717E+03  .11898E+03  .11728E+03  .11647E+03
.11521E+03  .12109E+03  .11750E+03  .14756E+03  .82806E+02
.85590E+02  .82806E+02  .85938E+02  .10489E+03  .91640E+02
.90360E+02  .92821E+02  .90360E+02  .91640E+02  .10489E+03
.85938E+02

AVERAGE VALUES OF PIPE INSULATION THERMAL CONDUCTIVITY :
 KI1 =        .317   KI2 =        .292   BTU-IN/H-FT**2-DEG F

AVERAGE TEMPERATURE DROPS ACROSS INSULATION :
 T1=     191.80   T2=      92.67  DEG F

HEAT LOSSES FROM UNDERGROUND PIPES :
 Q1=     75.00   Q2=      33.38   QT=    108.38  BTU/H-FT
```

APPENDIX B. A Listing of Computer Programs

```
      PROGRAM DIRECT1
C THIS IS A MAIN PROGRAM FOR HEAT LOSS ANALYSIS OF DIRECTLY BURIED
C CONDUIT UNDERGROUND HEAT DISTRIBUTION SYSTEMS WITH BOTH INSULATED
C PIPES IN THE SAME CASING. BASED ON THE FINITE ELEMENT METHOD USING
C THREE - NODE LINEAR TRIANGULAR ELEMENTS.
C SUBROUTINES CALLED: PIPEO,TGO,SOILK,INSULK,TGXX,SOLVLE,PIPEHL,TWOPIP,
C                     EQUIKO.
C     INPUT DATA FILES: SDATA1 AND SDATA2
C     OUTPUT FILE: SOUTPUT
C     X(I): THE X-COORDINATE OF NODAL POINT I,IN FT
C     Y(I): THE Y-COORDINATE OF NODAL POINT I,IN FT
C     (NODE(M,I),I=1,3): THREE NODAL POINTS OF ELEMENT M
C     M     ELEMENT INDEX
C     NE    TOTAL NUMBER OF ELEMENTS
C     NN    TOTAL NUMBER OF NODAL POINTS
C     MZ    TOTAL NUMBER OF KNOWN NODAL TEMPERATURES
C     C     THERMAL CONDUCTIVITY, BTU-IN/HR/FT**2/DEG F
C     L     THICKNESS OF THE ELEMENT, FT
C     T(I): THE TEMPERATURE OF NODAL POINT I, IN DEG F
      REAL L,KK,KI,KG,KIX1,KIX2,KTCT,KASP,KCAS,KBF
      CHARACTER*4 TITLE(15)
      DIMENSION Q(150),T(150),X(150),Y(150),KK(150,150)
      DIMENSION AS(250),B2IZ(250),B3IZ(250),B2JZ(250),B2KZ(250),
     & B3JZ(250),B3KZ(250)
      DIMENSION CC(250),TGX(12,6),QQ(150),NODE(250,3),MAT(250)
      DIMENSION HIJ(250),HJK(250),HKI(250),TIJ(250),TJK(250),
     & TKI(250),HHIJ(250),HHJK(250),HHKI(250),IXCB(250)
      DIMENSION CK(150,150),DQ(150),XT(150),INDX(150),VV(150)
      COMMON/PP/TP1,TP2,KI,KG,D1,D2,TH1,TH2,DP1,DP2,S1,S2,TG,
     & WW,HY,MONTH
      COMMON /EKC/D1P,D2P,DIC,THK1,THK2
      COMMON /ST/AO,BO,DIFF
      PI=4.*ATAN(1.)
      OPEN (8,FILE='SDATA1')
      OPEN (7,FILE='SOUTPUT',STATUS='NEW',FORM='FORMATTED')
      OPEN (9,FILE='SDATA2')
C READ IN THE TITLE OF THE PROBLEM TO BE ANALYZED
      READ (8,2,ERR=2000) TITLE
2     FORMAT(15A4)
      WRITE (7,3) TITLE
3     FORMAT(1X,15A4)
C READ TOTAL NUMBER OF NODAL POINTS, TOTAL NUMBER OF TRIANGULAR
C ELEMENTS, TOTAL NUMBER OF KNOWN NODAL TEMPERATURES, AND THE
C FIRST ELEMENT INDEX OF PIPE INSULATION
      READ (8,*) NN,NE,MZ,MINS
C SET THE UNIT NUMBER OF THE PRINTER
      MO=7
C READ MONTH OF INTEREST AND THE INDEX FOR FINITE ELEMENT GRID DATA
C TO BE PRINTED OUT : ICALB = 1  PRINT OUT NODAL COORDINATES
C                           = 0  NO PRINT OUT
      READ (8,*) MONTH,ICALB
C  READ THE THERMAL CONDUCTIVITY (IN BTU-IN./H-FT**2 - DEG F),
C  THICKNESS (IN INCHES) OF THE SIDE, THE DEPTH (IN FT.) OF EARTH
C  COVER (IN FT.), AND THE THICKNESS (IN FT.) OF BOTTOM BED OF
C  THE INNER EARTH REGION.
      READ (8,*) KTCT,TRTK,D,F
C  READ IN THE ESTIMATED AVERAGE TEMPERATURE OF AIR INSIDE THE
C  AIRSPACE BETWEEN INSULATED PIPES AND OUTER CASING, IN DEG F,
C  AND THE TEMPERATURE DIFFERENCE BETWEEN THE INSULATED PIPE
C  SURFACE TEMPERATURE AND THE INNER SURFACE TEMPERATURE OF THE
C  CASING, IN DEG F.
      READ (8,*) TAS,TDEL
C READ IN THE THERMAL CONDUCTIVITY (IN BTU-IN/H-FT**2-DEG F) OF
C THE PIPE CASING AND THE CONDUCTIVITY OF POURED-IN INSULATION
C MATERIAL OR BACK-FILL SOIL SURROUNDING THE PIPES IN THE
C INNERMOST REGION.
      READ (8,*) KCAS,KBF
C READING IN INPUT DATA FOR CALCULATIONS OF PIPE HEAT LOSS AND
C GENERATION OF THE COORDINATES OF NODAL POINTS
      CALL PIPEO(X,Y,TRTK,D,F,INXK)
      CALL TWOPIP(1)
      CALL EQUIKO(TAS,TDEL,KASP)
      IF(ICALB.EQ.1) THEN
```

```
      WRITE(7,5)
5     FORMAT('  X(M),M=1,NN')
      WRITE(7,7) (X(I),I=1,NN)
7     FORMAT(10F7.2)
      WRITE(7,10)
10    FORMAT('  Y(M),M=1,NN')
      WRITE(7,7) (Y(I),I=1,NN)
      END IF
C  CALCULATIONS OF UNDISTURBED EARTH TEMPERATURES AT VARIOUS DEPTHS
      CALL TGO(TGX,PI,Y)
C  INITIALIZATION OF THE INDEX OF CONVECTION BOUNDARY FOR ELEMENT N
      DO 12 N=1,NE
12    IXCB(N)=0
C  PERFORM ITERATIONS TO ACCOUNT FOR THE TEMPERATURE EFFECTS ON SOIL
C  AND INSULATION THERMAL CONDUCTIVITIES
         DO 24 I=1,NN
24          T(I)=TG
       DO 26 I=1,NE
          HIJ(I)=0.
          HJK(I)=0.
          HKI(I)=0.
          TIJ(I)=0.
          TJK(I)=0.
          TKI(I)=0.
          HHIJ(I)=0.
          HHJK(I)=0.
          HHKI(I)=0.
26      CONTINUE
C     READING IN THE ELEMENT NUMBER AND ITS NODAL POINTS AND THE
C     MATERIAL TYPE, WHICH INCLUDES
C        MAT(J) = 1  SOIL IN INNER EARTH REGION
C               = 2  PIPE INSULATION
C               = 3  AIR SPACE SURROUNDING THE PIPE IN CASING
C               = 4  OUTER CASING OF THE INSULATED PIPE
C               = 5  POURED-IN INSULATION MATERIAL IN BACK-FILL
C                    REGION
C               = 6  NATIVE SOIL IN THE OUTER EARTH REGION
      DO 30 I=1,NE
         READ(9,*) J,(NODE(J,K),K=1,3),MAT(J)
         IF (MAT(J).EQ.1) CC(J)=KTCT/12.
         IF (MAT(J).EQ.2) CC(J)=KI/12.
         IF (MAT(J).EQ.3) CC(J)=KASP/12.
         IF (MAT(J).EQ.4) CC(J)=KCAS/12.
         IF (MAT(J).EQ.5) CC(J)=KBF/12.
         IF (MAT(J).EQ.6) CC(J)=KG/12.
30    CONTINUE
C   READ IN TOTAL NUMBER OF ELEMENTS HAVING BOUNDARY SEGMENTS SUBJECT
C   TO CONVECTIVE HEAT TRANSFER
      READ (9,*) NECB
C   READ IN ELEMENT NUMBER, CONVECTIVE HEAT TRANSFER COEFFICIENTS,
C   AND AMBIENT TEMPERATURES FOR THREE BOUNDARY SEGMENTS
      DO 35 I=1,NECB
        READ (9,*) M,HIJ(M),HJK(M),HKI(M),TIJ(M),TJK(M),TKI(M)
        IXCB(M)=1
35    CONTINUE
      ITER=1
38    DO 40 I=1,NN
      DO 40 J=1,NN
          Q(I)=0.
          KK(I,J)=0.
          QQ(I)=0.
          CK(I,J)=0.
          DQ(I)=0.
          VV(I)=1.0
          INDX(I)=1
40    CONTINUE
      L=1.
      DO 180 M=1,NE
        I=NODE(M,1)
        J=NODE(M,2)
        K=NODE(M,3)
        IF(MAT(M).EQ.3) CC(M)=KASP/12.
        C=CC(M)
```

```
      IF ((INXK.EQ.0).OR.(ITER.EQ.1)) GO TO 60
C  DETERMINE SOIL AND INSULATION THERMAL CONDUCTIVITIES BASED ON THE
C  MEAN TEMPERATURES
      TM=(T(I)+T(J)+T(K))/3.
      IF(MAT(M).EQ.2)  CALL INSULK(TM,C)
      IF(MAT(M).EQ.6)  CALL SOILK(TM,KG,C)
      IF(MAT(M).EQ.1)  CALL SOILK(TM,KG,C)
      CC(M)=C
60    XI=X(I)
      XJ=X(J)
      XK=X(K)
      YI=Y(I)
      YJ=Y(J)
      YK=Y(K)
      CXX=C
      CXY=0.
      CYX=0.
      CYY=C
      B2I=YJ-YK
      B3I=XK-XJ
      B2J=YK-YI
      B3J=XI-XK
      B2K=YI-YJ
      B3K=XJ-XI
C   CALCULATE THE ELEMENT AREA
      SA=0.5*(XJ*B2J+XI*B2I+XK*B2K)
      SA=ABS(SA)
      A2=SA*2.
      AS(M)=A2
      B2I=B2I/A2
      B3I=B3I/A2
      B2J=B2J/A2
      B3J=B3J/A2
      B2K=B2K/A2
      B3K=B3K/A2
      B2IZ(M)=B2I
      B3IZ(M)=B3I
      B2JZ(M)=B2J
      B3JZ(M)=B3J
      B2KZ(M)=B2K
      B3KZ(M)=B3K
      BII=SA*L*(B2I*B2I*CXX+B2I*B3I*CXY+B3I*B2I*CYX+B3I*B3I*CYY)
      BIJ=SA*L*(B2I*B2J*CXX+B2I*B3J*CXY+B3I*B2J*CYX+B3I*B3J*CYY)
      BIK=SA*L*(B2I*B2K*CXX+B2I*B3K*CXY+B3I*B2K*CYX+B3I*B3K*CYY)
      BJI=SA*L*(B2J*B2I*CXX+B2J*B3I*CXY+B3J*B2I*CYX+B3J*B3I*CYY)
      BJJ=SA*L*(B2J*B2J*CXX+B2J*B3J*CXY+B3J*B2J*CYX+B3J*B3J*CYY)
      BJK=SA*L*(B2J*B2K*CXX+B2J*B3K*CXY+B3J*B2K*CYX+B3J*B3K*CYY)
      BKI=SA*L*(B2K*B2I*CXX+B2K*B3I*CXY+B3K*B2I*CYX+B3K*B3I*CYY)
      BKJ=SA*L*(B2K*B2J*CXX+B2K*B3J*CXY+B3K*B2J*CYX+B3K*B3J*CYY)
      BKK=SA*L*(B2K*B2K*CXX+B2K*B3K*CXY+B3K*B2K*CYX+B3K*B3K*CYY)
      KK(I,I)=KK(I,I)+BII
      KK(I,J)=KK(I,J)+BIJ
      KK(I,K)=KK(I,K)+BIK
      KK(J,I)=KK(J,I)+BJI
      KK(J,J)=KK(J,J)+BJJ
      KK(J,K)=KK(J,K)+BJK
      KK(K,I)=KK(K,I)+BKI
      KK(K,J)=KK(K,J)+BKJ
      KK(K,K)=KK(K,K)+BKK
      IF(IXCB(M).EQ.0) GO TO 130
C   ADDITION OF CONVECTION TERMS TO THE ELEMENT MATRIX TO ACCOUNT
C   FOR CONVECTION ON BOUNDARY
C   READING IN CONVECTIVE HEAT TRANSFER COEFFICIENTS AND AMBIENT
C   TEMPERATURES FOR THREE BOUNDARY SEGMENTS
      HHIJ(M)=HIJ(M)*L*SQRT((X(I)-X(J))**2+(Y(I)-Y(J))**2)/6.
      HHJK(M)=HJK(M)*L*SQRT((X(J)-X(K))**2+(Y(J)-Y(K))**2)/6.
      HHKI(M)=HKI(M)*L*SQRT((X(K)-X(I))**2+(Y(K)-Y(I))**2)/6.
      KK(I,I)=HHIJ(M)*2.+HHKI(M)*2.+KK(I,I)
      KK(I,J)=HHIJ(M)+KK(I,J)
      KK(I,K)=HHKI(M)+KK(I,K)
      KK(J,I)=HHIJ(M)+KK(J,I)
      KK(J,J)=HHIJ(M)*2+HHJK(M)*2.+KK(J,J)
      KK(J,K)=HHJK(M)+KK(J,K)
```

```
      KK(K,I)=HHKI(M)+KK(K,I)
      KK(K,J)=HHJK(M)+KK(K,J)
      KK(K,K)=HHJK(M)*2.+HHKI(M)*2.+KK(K,K)
      HHIJ(M)=TIJ(M)*3.*HHIJ(M)
      HHJK(M)=TJK(M)*3.*HHJK(M)
      HHKI(M)=TKI(M)*3.*HHKI(M)
130   Q(I)=Q(I)+HHIJ(M)+HHKI(M)
      Q(J)=Q(J)+HHIJ(M)+HHJK(M)
      Q(K)=Q(K)+HHJK(M)+HHKI(M)
180   CONTINUE
185   FORMAT('       M      I      J      K     MAT.     C')
187   FORMAT(1X,5I6,F10.4)
C   DETERMINE OUTER SURFACE TEMPERATURES OF UNDERGROUND PIPES
      DO 200 I=1,8
        T(I)=TP1
        II=I+8
        T(II)=TP2
200   CONTINUE
C   DETERMINE OUTER BOUNDARY TEMPERATURES OF EARTH REGION
      CALL TGXX(T,TGX,MONTH)
      MZ1=MZ+1
      DO 260 I=MZ1,NN
        SUM=0.
        DO 250 J=1,MZ
250       SUM=SUM+KK(I,J)*T(J)
        QQ(I)=Q(I)-SUM
260   CONTINUE
      IF(ICALB.EQ.1) THEN
         WRITE(7,280)
280      FORMAT(6X,'QQ    ARRAY')
         WRITE(7,285) (QQ(I),I=1,NN)
285      FORMAT (5E12.5)
      END IF
C  RENAMING OF MATRICES
      MN=NN-MZ
      DO 300 I=1,MN
         K=MZ+I
        DO 290 J=1,MN
            KL=MZ+J
290         CK(I,J)=KK(K,KL)
            XT(I)=T(K)
            DQ(I)=QQ(K)
300      CONTINUE
C  SOLUTION OF SIMULTANEOUS EQUATIONS
C  SET PHYSICAL DIMENSION OF MATRIX A
       NP=150
      CALL SOLVLE(CK,MN,NP,INDX,VV,DQ)
      DO 310 I=1,MN
           K=MZ+I
           T(K)=DQ(I)
310    CONTINUE
C   CALCULATIONS OF EQUIVALENT THERMAL CONDUCTIVITIES FOR AIR
C   SPACE INSIDE THE OUTER CASING
C
C   CALCULATE AVERAGE SURFACE TEMPERATURE OF INSULATED PIPES
      SU1=0.0
      SU2=0.0
      DO 312 I=1,8
        L1=I+80
        L2=I+88
        SU1=SU1+T(L1)
        SU2=SU2+T(L2)
312   CONTINUE
      TSM1=SU1/8.
      TSM2=SU2/8.
C  DETERMINE THE EFFECTIVE SURFACE TEMPERATURE OF INSULATED PIPES,
C  IN DEG F.
      DIP1=D1P+2.*THK1
      DIP2=D2P+2.*THK2
      TEFPS=(DIP1*TSM1+DIP2*TSM2)/(DIP1+DIP2)
C   CALCULATE THE INNER SURFACE TEMPERATURE OF THE CASING
      SU3=0.0
      DO 314 I=1,8
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         L3=I+96
 314     SU3=SU3+T(L3)
        TSIC=SU3/8.
C    DETERMINE THE TEMPERATURE DIFFERENCE BETWEEN THE EFFECTIVE SURFACE
C    TEMPERATURE OF INSULATED PIPES AND THE INNER SURFACE OF THE CASING,
C    IN DEG F
        TDEL=ABS(TEFPS-TSIC)
        TAS=(TEFPS+TSIC)/2.
        CALL EQUIKO(TAS,TDEL,KASP)
        WRITE(7,320) KASP
 320    FORMAT('  KASP=',F10.4,2X,'(BTU-IN./H-FT**2-DEG F)')
 330    FORMAT('  TEMPERATURE ARRAY : T(I), I=1,NN ')
C   CALCULATE THE MEAN VALUES OF INSULATION THERMAL CONDUCTIVITY FOR
C   PIPES 1 AND 2
 350    SKI1=0.
        SKI2=0.
        DO 400 LN=1,16
           LM=MINS+LN-1
           LL=LM+16
           SKI1=SKI1+CC(LM)
           SKI2=SKI2+CC(LL)
 400    CONTINUE
        KIX1=SKI1/16.
        KIX2=SKI2/16.
        R1=D1/24.
        R2=D2/24.
        TH1X=TH1/12.
        TH2X=TH2/12.
        IF(ICALB .EQ. 0) MO=11
C   CALCULATIONS OF THE HEAT LOSSES FROM THE UNDERGROUND PIPES
        CALL  PIPEHL(T,R1,R2,TH1X,TH2X,KIX1,KIX2,MO,QTX)
        HLOSS=QTX
        IF(ITER.EQ.1) HLOSX=0.
C    DETERMINE IF PIPE HEAT LOSS VALUE HAS CONVERGED, OR CONTINUE
C    ITERATIONS IF REQUIRED
        DELQT=ABS(HLOSS-HLOSX)/HLOSS
        IF(DELQT.LE. 0.010) GO TO 2010
        ITER=ITER+1
        HLOSX=HLOSS
        GO TO 38
 2000   WRITE (7,2005)
 2005   FORMAT (1X,'THERE ARE SOME ERRORS IN INPUT DATA')
 2010   IF(ICALB .EQ. 1) THEN
        WRITE (7,185)
        DO 2020 I=1,NE
          WRITE (7,187) I,(NODE(I,J),J=1,3),MAT(I),CC(I)
 2020   CONTINUE
        END IF
        WRITE (7,330)
        WRITE (7,285) (T(I),I=1,NN)
        CALL PIPEHL(T,R1,R2,TH1X,TH2X,KIX1,KIX2,7,QTX)
        STOP
        END

        SUBROUTINE TGO(TGX,PI,Y)
C    THIS SUBROUTINE CALCULATES THE UNDISTURBED EARTH TEMPERATURES
C    AT VARIOUS DEPTHS
        DIMENSION TGX(12,6),Y(150)
C       READING IN THE ANUAL AVERAGE TEMPERATURE AND AMPLITUDE OF THE
C       MONTHLY NORMAL TEMPERATURE CYCLE OF THE SITE, IN DEG F, AND
C       THERMAL DIFFUSIVITY OF SOIL, IN FT**2/H.
        READ (8,*) AO,BO,DIFF
        W=2.*PI/12.
        WZ=2.*PI/(8760*DIFF*2)
        ZZ=SQRT(WZ)
        DO 1 I=1,12
        DO 1 J=1,6
        Z=ZZ*Y(31-J)
 1      TGX(I,J)=AO+BO*EXP(-Z)*SIN(W*(I-3)-Z)
        RETURN
        END

        SUBROUTINE TGXX(T,TGX,MONTH)
```

```
C THIS SUBROUTINE PROVIDES OUTER BOUNDARY TEMPERATURES OF EARTH REGION
      DIMENSION T(150),TGX(12,6)
      T(30)=TGX(MONTH,1)
      DO 1 I=1,8
         II=I+30
1        T(II)=T(30)
      DO 5 I=2,6
         I15=I+15
         JI=31-I
         T(I15)=TGX(MONTH,I)
         T(JI)=TGX(MONTH,I)
5     CONTINUE
      DO 10 I=1,3
         I21=I+21
10     T(I21)=TGX(MONTH,6)
      RETURN
      END

      SUBROUTINE INSULK(TM,C)
C  THIS SUBROUTINE DETERMINES THE THERMAL CONDUCTIVITY OF PIPE
C  INSULATION (CALCIUM SILICATE) AS A FUNCTION OF THE MEAN
C  TEMPERATURE.
      REAL KN(16),KINS
      DIMENSION TN(16)
      DATA KN /0.375,0.40,0.42,0.45,0.48,0.50,0.53,0.555,0.58,0.61,
     & 0.63,0.66,0.68,0.74,0.82,0.90/
      DO 5 J=1,16
      IF(J .LE. 13) THEN
         TN(J)=100.+(J-1)*50.
      ELSE
         TN(J)=700.+(J-13)*100.
      END IF
5     CONTINUE
      IF(TM .GT. TN(1)) GO TO 10
      KINS=KN(1)
      GO TO 100
10    IF(TM .LT. TN(16)) GO TO 20
      KINS=KN(16)
      GO TO 100
20    DO 50 I=1,15
      T1=TM-TN(I)
      IF(T1 .NE. 0.) GO TO 30
      KINS=KN(I)
      GO TO 100
30    T2=TN(I+1)-TM
      IF(T2 .NE. 0.) GO TO 40
      KINS=KN(I+1)
      GO TO 100
40    P=T1*T2
      IF(P .LT. 0.) GO TO 50
      KINS=KN(I)+T1*(KN(I+1)-KN(I))/(TN(I+1)-TN(I))
      GO TO 100
50    CONTINUE
100   C=KINS/12.
      RETURN
      END

      SUBROUTINE  SOILK(TM,KG,C)
C THIS ROUTINE DETERMINES THE THERMAL CONDUCTIVITY OF SOIL AS A
C FUNCTION OF MEAN TEMPERATURES.
      REAL K(14),KG
      DIMENSION TX(14)
      DATA K/1.1,1.1,1.1,1.0,0.4,0.31,0.25,0.19,0.15,0.11,0.09,0.07,
     & 0.05,0.05/
      DO 1 I=1,14
1        TX(I)=50.+(I-1)*25.
      IF(TM.GT.TX(1)) GO TO 5
      ZK=1.1
      GO TO 50
5     IF(TM.LT.TX(14)) GO TO 10
      ZK=0.05
      GO TO 50
10    DO 25 I=1,13
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```
      T1=TM-TX(I)
      IF(T1.NE.0) GO TO 15
      ZK=K(I)
      GO TO 50
15    CONTINUE
      T2=TM-TX(I+1)
      IF(T2.NE.0.) GO TO 20
      ZK=K(I+1)
      GO TO 50
20    CONTINUE
      P=T1*T2
      IF(P.GT.0) GO TO 25
      ZK=K(I+1)+T2*(K(I+1)-K(I))/25.
      GO TO 50
25    CONTINUE
50    C=ZK*KG/(1.1*12.)
      RETURN
      END

      SUBROUTINE PIPEHL(T,R1,R2,TH1,TH2,ZKS1,ZKS2,MO,QT)
C  THIS SUBROUTINE CALCULATES THE AVERAGE TEMPERATURE DROPS ACROSS THE
C  PIPE INSULATIONS AND THE RATES OF HEAT LOSS FROM THE UNDERGROUND
C  PIPES IN DIRECTLY BURIED CONDUIT SYSTEM
      DIMENSION T(150)
      PI=4.*ATAN(1.)
      SUM1=0.
      SUM2=0.
      N1=8
      DO 1 I=1,N1
         K1=I
         K2=I+8
         K3=I+80
         K4=I+88
         SUM1=SUM1+T(K1)-T(K3)
         SUM2=SUM2+T(K2)-T(K4)
1     CONTINUE
      T1=SUM1/N1
      T2=SUM2/N1
      ZKIS1=ZKS1*12.
      ZKIS2=ZKS2*12.
      Q1=ZKS1*2.*PI*T1/LOG((R1+TH1)/R1)
      Q2=ZKS2*2.*PI*T2/LOG((R2+TH2)/R2)
      QT=Q1+Q2
      IF(MO .EQ. 11) GO TO 50
      WRITE(MO,5) ZKIS1,ZKIS2
5     FORMAT(/'  AVERAGE VALUES OF PIPE INSULATION THERMAL',
     &' CONDUCTIVITY :',/,'   KI1 = ',F10.3,'   KI2 = ',F10.3,
     & '   BTU-IN/H-FT**2-DEG F ')
      WRITE(MO,10) T1,T2
10    FORMAT(/'  AVERAGE TEMPERATURE DROPS ACROSS INSULATION : ',/,
     & '   T1= ',F10.2,'   T2= ',F10.2,'  DEG F')
      WRITE(MO,20) Q1,Q2,QT
20    FORMAT(/,2X,'HEAT LOSSES FROM UNDERGROUND PIPES :'/'   Q1=',
     & F10.2, '   Q2=',F10.2,'   QT=',F10.2,'  BTU/H-FT')
50    RETURN
      END

      SUBROUTINE PIPEO(X,Y,TRTK,D,F,INXK)
C THIS SUBROUTINE READS IN THE INPUT DATA TO BE USED FOR CALCULATIONS
C OF THE HEAT LOSSES FROM THE UNDERGROUND PIPES AND GENERATES X AND Y-
C COORDINATES OF NODAL POINTS FOR THE TWO PIPE SYSTEM.
      REAL KII,KIG,KI,KG
      DIMENSION X(150),Y(150)
      COMMON /PP/TP1,TP2,KII,KIG,DI1,DI2,THI1,THI2,B1,B2,S1,S2,TG,
     & WW,HY,MONTH
      COMMON /EKC/D1P,D2P,DIC,THK1,THK2
C READ TEMPERATURE OF PIPE NUMBERS 1 AND 2, IN DEG F
      READ (8,*) TP1,TP2
C READ THERMAL CONDUCTIVITY OF THERMAL INSULATION AND SOIL,
C RESPECTIVELY, IN BTU-IN./H-FT**2 - DEG F, AND INDEX OF THERMAL
C CONDUCTIVITY : INXK = 0  CONSTANT THERMAL CONDUCTIVITY
C                     = 1  TEMPERATURE DEPENDENT THERMAL CONDUCTIVITY
      READ (8,*) KII,KIG,INXK
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C READING IN THE OUTSIDE DIAMETERS OF STEEL PIPES 1 AND 2, IN INCHES
      READ (8,*) DI1,DI2
C READING IN THE THICKNESS OF THERMAL INSULATION USED FOR PIPES 1
C AND 2, RESPECTIVELY, IN INCHES
      READ (8,*) THI1,THI2
C READ IN THE INSIDE DIAMETER AND THICKNESS OF OUTER CASING, AND THE
C DEPTH OF ITS CENTER BELOW GROUND SURFACE, IN INCHES.
      READ (8,*) DIAC,THKC,DEPC
C   READING IN THE VERTICAL DISTANCES (IN FT.) FROM HORIZONTAL
C   CENTERLINE OF THE INNER EARTH REGION TO CENTERS OF PIPES 1 AND
C   2, RESPECTIVELY, AND THE AVERAGE EARTH TEMPERATURE, IN DEG F.
      READ (8,*) S1,S2,TG
C   READ IN THE THICKNESS OF EARTH COVER,WIDTH AND DEPTH OF OUTER
C   EARTH REGION SURROUNDING THE UNDERGROUND SYSTEM, IN FT.
      READ (8,*) E,WW,HY
      WRITE(7,10) TP1,TP2,KII,KIG,DI1,DI2
10    FORMAT('   TP1    TP2     KI     KG     D1     D2'/6F7.2)
      WRITE(7,20) THI1,THI2,DIAC,THKC,DEPC,S1,S2,TG
20    FORMAT('   THI1   THI2   DIAC   THKC   DEPC     S1     S2     TG',
     & /8F7.2)
      WRITE(7,30) E,WW,HY,MONTH
30    FORMAT('      E      WW     HY   MONTH '/3F7.2,I7)
C   READ IN THE INSIDE WIDTH AND HEIGHT OF THE INNER EARTH REGION,
C   FOR LOOSE-FILL INSULATION SYSTEM, IN FT.
      READ (8,*) A,B
C   CHANGE TO ENGINEERING UNITS
      D1=DI1/12.
      R1=D1*0.5
      D2=DI2/12.
      R2=D2*0.5
      D1P=DI1/12.
      D2P=DI2/12.
      DEP=DEPC/12.
      DIC=DIAC/12.
      KI=KII/12.
      KG=KIG/12.
      W=A+2*TRTK/12
      H=B+E+D+F
      WRITE(7,40) W,H,D,F,A,B,WW,HY
40    FORMAT('      W      H      D      F      A      B     WW     HY'
     &,/,8F7.2)
      PI=4.*ATAN(1.)
      TH1=THI1/12.
      TH2=THI2/12.
      THCA=THKC/12.
      THK1=THI1/12.
      THK2=THI2/12.
C   DETERMINE THE X AND Y-COORDINATES OF EARTH COVER, SIDES AND BOTTOM
C   BED OF THE INNER EARTH REGION (NODAL POINTS 39 TO 70)
      DO 50 I=1,5,2
         I65=I+65
         X(I65)=W-(I-1)*W/4.
50       Y(I65)=E
      DO 60 I=1,3,2
         I66=I+66
         X(I66)=(W+A)*0.5 - A*(I-1)*0.5
60       Y(I66)=E
      DO 65 I=1,4
         I38=I+38
         I42=I+42
         I46=I+46
         I50=I+50
         I1=I-1
         X(I38)=(W-A)*0.5
         Y(I38)=(D+E)+I1*B/4.
         X(I42)=0.5*(W-A)+I1*A/4.
         Y(I42)=D+E+B
         X(I46)=(W+A)*0.5
         Y(I46)=(D+E+B)-I1*B/4.
         X(I50)=0.5*(W+A)-I1*A/4.
         Y(I50)=D+E
65    CONTINUE
      DO 70 I=1,3
```

```
         I54=I+54
         I58=I+58
         I62=I+62
         X(I54)=0.0
         Y(I54)=(D+E)+B*(I-1)/2.
         X(I58)=(W-A)*0.5+(I-1)*A*0.5
         Y(I58)=H
         X(I62)=W
         Y(I62)=(D+E)+(3-I)*B/2.
70    CONTINUE
         X(58)=0.0
         Y(58)=H
         X(62)=W
         Y(62)=H
C     THE X AND Y-COORDINATES OF OUTER BOUNDARY EARTH SURROUNDING THE
C     DIRECTLY BURIED CONDUITS (NODAL POINTS 17 TO 38,AND 71 TO 80)
      DO 72 I=1,2
         I16=I+16
         I27=I+27
         X(I16)=-WW
         Y(I16)=E+(H-E)*(I-1)*0.5
         X(I27)=W+WW
         Y(I27)=E+(H-E)*(2-I)*0.5
72    CONTINUE
      DO 75 I=1,3
         I18=I+18
         I21=I+21
         I24=I+24
         X(I18)=-WW
         Y(I18)=H+HY*(I-1)*0.5
         X(I21)=W*(I-1)*0.5
         Y(I21)=H+HY
         X(I24)=W+WW
         Y(I24)=H+HY*(3-I)*0.5
75    CONTINUE
      DO 77 I=1,5,2
         I31=I+31
         X(I31)=W-(I-1)*W/4.
77       Y(I31)=0.0
      DO 78 I=1,3,2
         I32=I+32
         X(I32)=(W+A)*0.5-A*(I-1)*0.5
78       Y(I32)=0.0
      DO 80 I=1,2
         I29=I+29
         I36=I+36
         X(I29)=W+WW*(3-I)*0.5
         Y(I29)=0.
         X(I36)=-WW*0.5*I
         Y(I36)=0.
80    CONTINUE
      DO 82 I=1,3
         I70=I+70
         I77=I+77
         X(I70)=-WW*0.5
         Y(I70)=E+(H-E)*(I-1)*0.5
         X(I77)=W+WW*0.5
         Y(I77)=E+(H-E)*(3-I)*0.5
82    CONTINUE
      DO 85 I=1,4
         I73=I+73
         X(I73)=W*(I-1)/3.
85       Y(I73)=H+HY*0.5
C     X AND Y-COORDINATES OF THE CENTERS OF THE PIPES
      XC1=W*0.5
      B1=DEP+S1
      YC1=B1
      XC2=W*0.5
      B2=DEP-S2
      YC2=B2
      WRITE(7,90) XC1,YC1,XC2,YC2,B1,B2
90    FORMAT('    XC1    YC1    XC2    YC2     B1     B2'/6F7.3)
C     THE X AND Y-COORDINATES OF NODAL POINTS AT THE INNER AND OUTER
```

```
C    SURFACES OF PIPE INSULATION (NODAL POINTS 1 TO 16, AND 81 TO
C    96)
       DO 95 I=1,8
          THETA=2.*PI*I/8.
          I8=I+8
          I80=I+80
          I88=I+88
          X(I)=XC1+0.5*D1*SIN(THETA)
          Y(I)=YC1+0.5*D1*COS(THETA)
          X(I8)=XC2+0.5*D2*SIN(THETA)
          Y(I8)=YC2+0.5*D2*COS(THETA)
          X(I80)=XC1+(TH1+R1)*SIN(THETA)
          Y(I80)=YC1+(TH1+R1)*COS(THETA)
          X(I88)=XC2+(TH2+R2)*SIN(THETA)
          Y(I88)=YC2+(TH2+R2)*COS(THETA)
95     CONTINUE
C   THE X AND Y-COORDINATES OF NODAL POINTS AT THE INNER AND OUTER
C   SURFACES OF PIPE CASINGS (NODAL POINTS 97 TO 112)
       DO 97 I=1,8
          THETA=2.*PI*I/8.
          I96=I+96
          I104=I+104
          X(I96)=XC1+DIC*0.5*SIN(THETA)
          Y(I96)=DEP+DIC*0.5*COS(THETA)
          X(I104)=XC1+(THCA+DIC*0.5)*SIN(THETA)
          Y(I104)=DEP+(THCA+DIC*0.5)*COS(THETA)
97     CONTINUE
C    THE X AND Y-COORDINATES OF NODAL POINTS IN BACK-FILL SOIL, OR
C    POURED-IN INSULATION SURROUNDING THE PIPES (NODAL POINTS 113 TO
C    124)
         YUP=0.5*(Y(108)-Y(53))
         XLT=0.5*(X(110)-X(41))
         YLO=0.5*(Y(45)-Y(112))
         XRT=0.5*(X(49)-X(106))
       DO 100 I=1,3
          I112=I+112
          I115=I+115
          I118=I+118
          I121=I+121
          X(I112)=0.5*(W+A)-0.25*A*I
          Y(I112)=(D+E)+YUP
          X(I115)=0.5*(W-A)+XLT
          Y(I115)=(D+E)+0.25*B*I
          X(I118)=0.5*(W-A)+0.25*A*I
          Y(I118)=(D+E+B)-YLO
          X(I121)=0.5*(W+A)-XRT
          Y(I121)=(D+E+B)-0.25*B*I
100    CONTINUE
       RETURN
       END

       SUBROUTINE TWOPIP(IREPT)
C   THIS SUBROUTINE DETERMINES THE HEAT LOSSES FROM TWO PIPES TO THE
C   UNDERGROUND SURROUNDING THE HEAT DISTRIBUTION SYSTEM.
       REAL KII,KIG
       COMMON /PP/T1,T2,KII,KIG,DI1,DI2,THI1,THI2,B1,B2,S1,S2,TG,
      & WW,HY,MONTH
       PI=4.*ATAN(1.)
       X1=2.*PI
       R1=DI1/24.
       R2=DI2/24.
       TH1X=THI1/12.
       TH2=THI2/12.
       ZK1=KII/12.
       ZK2=ZK1
       D1=B1
       D2=B2
       ZKS=KIG/12.
       DO 10 I=1,IREPT
2      TH1=TH1X+0.1*(I-1)
       S=S1+S2
       A=R1+R2+TH1+TH2+0.05
       THI1=TH1*12.
```

```
      IF(A .LT. S) A=S
      C1=X1*ZK1/LOG((R1+TH1)/R1)
      C2=X1*ZK2/LOG((R2+TH2)/R2)
      P11=1.+C1/(X1*ZKS)*LOG((2*D1)/(R1+TH1))
      P12=C2/(X1*ZKS)*LOG((A*A+(D1+D2)**2)/(A*A+(D1-D2)**2))*0.5
      P21=C1/(X1*ZKS)*LOG((A*A+(D1+D2)**2)/(A*A+(D1-D2)**2))*0.5
      P22=1.+C2/(X1*ZKS)*LOG((2*D2)/(R2+TH2))
      DEL=P12*P21-P11*P22
      ZKP1=C1*(P12-P22)/DEL
      ZKP2=C2*(P21-P11)/DEL
      TP1=(P12*T2-P22*T1)/(P12-P22)
      TP2=(P21*T1-P11*T2)/(P21-P11)
      Q1=ZKP1*(TP1-TG)
      Q2=ZKP2*(TP2-TG)
      QT=Q1+Q2
      TAVG=(T1+T2)*0.5
      ZK=QT/(TAVG-TG)
      WRITE(7,6) DI1,DI2,S1,S2,THI1,KII,KIG,T1,T2
6     FORMAT('   DI1   DI2    S1     S2  THI1    KII    KIG    TP1    TP2',
     &/,7F6.2,1X,2F6.0)
      WRITE(7,8) Q1,Q2,QT,ZK
8     FORMAT('    Q1     Q2     QT        KP'/,3F7.2,2X,F6.3/)
10    CONTINUE
      RETURN
      END

      SUBROUTINE EQUIKO(TAS,TDEL,KASP)
C   THIS ROUTINE CALCULATES EQUIVALENT THERMAL CONDUCTIVITY OF AIR
C   SPACE SURROUNDING INSULATED PIPES INSIDE THE OUTER CASING.
      REAL KASP
      COMMON /EKC/D1P,D2P,DIC,THK1,THK2
      PI=4.*ATAN(1.)
C   CALCULATE THERMAL CONDUCTIVITY, IN BTU-FT/H-FT**2-DEG F, AND
C   KINEMATIC VISCOSITY, IN FT**2/S, OF AIR
      THKAIR=0.01319 + TAS*2.5E -5
      VAIR=1.2624E -4 + TAS*5.4E -7
C   CALCULATE THE EFFECTIVE DIAMETER OF THE INSULATED PIPES,
C   IN FT, AND THE CHARACTERISTIC LENGTH OF AIR SPACE
      DEFPIP=(D1P+2.*THK1)+(D2P+2.*THK2)
      CL=DIC-DEFPIP
C   CALCULATE THE PRANDTL NUMBER OF AIR , AND GRASHOF NUMBER AND
C   EQUIVALENT THERMAL CONDUCTIVITY, IN BTU-IN/H-FT**2-DEG F, OF
C   AIR SPACE
      PRANTL=0.71849 - TAS * 1.275E -4
      GRASOF=32.2 * TDEL *(CL**3.)/((VAIR**2.)*(TAS+459.7))
      KASP=12. *THKAIR*0.42*(PRANTL*GRASOF)**0.219
      RETURN
      END

      SUBROUTINE SOLVLE(A,N,NP,INDX,VV,B)
C   GIVEN AN NXN MATRIX A, WITH PHYSICAL DIMENSION NP, THIS ROUTINE
C   REPLACE IT BY THE LU DECOMPOSITION OF A ROWWISE PERMUTATION OF
C   ITSELF. INDX IS AN OUTPUT VECTOR WHICH RECORD THE ROW PERMUTATION
C   EFFECTED BY THE PARTIAL PIVOTING; VV IS VECTOR OF SCALING FACTORS.
C
C   THIS ROUTINE IS USED TO SOLVE THE LINEAR SET OF EQUATIONS :
C   [A][X]=[B]
C
      DIMENSION A(NP,NP),INDX(N),VV(N),B(N)
C
C   FORM IMPLICIT SCALING VECTOR VV
C
      DO 12 I=1,N
        AAMAX = 0.0
        DO 11 J=1,N
         IF(ABS(A(I,J)).GT.AAMAX) AAMAX=ABS(A(I,J))
   11   CONTINUE
        IF(AAMAX.EQ.0.) THEN
          WRITE(7,100) I
100     FORMAT(1X,'ERROR:SINGULAR MATRIX - ZERO ROW : ROW',I5)
          RETURN
        END IF
       VV(I) = 1.0/AAMAX
```

```
   12 CONTINUE
C
C   CROUT METHOD: LOOP OVER COLUMNS
C
      DO 19 J=1,N
        DO 14 I=1,J-1
          SUM = A(I,J)
          DO 13 K=1,I-1
            SUM = SUM - A(I,K)*A(K,J)
   13     CONTINUE
        A(I,J) = SUM
   14   CONTINUE
C
C   PIVOT IMPLEMENTATION
C
        AAMAX = 0.0D0
        DO 16 I=J,N
          SUM = A(I,J)
          DO 15 K=1,J-1
            SUM = SUM - A(I,K)*A(K,J)
   15     CONTINUE
          A(I,J) = SUM
          DUM = VV(I)*ABS(SUM)
          IF(DUM.GE.AAMAX) THEN
            IMAX = I
            AAMAX = DUM
          ENDIF
   16   CONTINUE

        IF(J.NE.IMAX) THEN
          DO 17 K=1,N
            DUM = A(IMAX,K)
            A(IMAX,K) = A(J,K)
            A(J,K) = DUM
   17     CONTINUE
        VV(IMAX) = VV(J)
      ENDIF
      INDX(J) = IMAX
      IF(A(J,J).EQ.0.0) THEN
        WRITE(7,110) J
110    FORMAT(1X,'ERROR: SINGULAR MATRIX - ZERO " DIAG " : ROW',I5)
        RETURN
      END IF
      IF(J.NE.N) THEN
        DUM = 1.0/A(J,J)
        DO 18 I=J+1,N
          A(I,J) = A(I,J)*DUM
   18   CONTINUE
      END IF
   19 CONTINUE
C
C   FORWARD SUBSTITUTION
C
      II = 0
      DO 22 I=1,N
        LL = INDX(I)
        SUM = B(LL)
        B(LL) = B(I)
        IF(II.NE.0) THEN
        DO 21 J=II,I-1
          SUM = SUM - A(I,J)*B(J)
   21     CONTINUE
        ELSE IF(SUM.NE.0.0) THEN
         II = I
        END IF
        B(I) = SUM
   22 CONTINUE
C
C   BACKWARD SUBSTITUTION
C
      DO 24 I=N,1,-1
        SUM = B(I)
        IF(I.LT.N) THEN
```

```
          DO 23 J=I+1,N
          SUM = SUM - A(I,J)*B(J)
23        CONTINUE
        END IF
        B(I) = SUM/A(I,I)
24    CONTINUE
      RETURN
      END
```

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      PROGRAM DIRECT2
C THIS IS A MAIN PROGRAM FOR HEAT LOSS ANALYSIS OF DIRECTLY BURIED
C CONDUIT UNDERGROUND HEAT DISTRIBUTION SYSTEMS WITH INSULATED PIPES
C IN SEPARATE CASINGS, BASED ON THE FINITE ELEMENT METHOD USING
C THREE - NODE LINEAR TRIANGULAR ELEMENTS.
C SUBROUTINES CALLED: PIPES,TGO,SOILK,INSULK,TGXX,SOLVLE,PIPEHL,TWOPIP,
C                     EQUIKC.
C     INPUT DATA FILES: CDATA1 AND CDATA2
C     OUTPUT FILE: COUTPUT
C     X(I): THE X-COORDINATE OF NODAL POINT I,IN FT
C     Y(I): THE Y-COORDINATE OF NODAL POINT I,IN FT
C     (NODE(M,I),I=1,3): THREE NODAL POINTS OF ELEMENT M
C     M      ELEMENT INDEX
C     NE     TOTAL NUMBER OF ELEMENTS
C     NN     TOTAL NUMBER OF NODAL POINTS
C     MZ     TOTAL NUMBER OF KNOWN NODAL TEMPERATURES
C     C      THERMAL CONDUCTIVITY, BTU-IN/HR/FT**2/DEG F
C     L      THICKNESS OF THE ELEMENT, FT
C     T(I): THE TEMPERATURE OF NODAL POINT I, IN DEG F
      REAL L,KK,KI,KG,KIX1,KIX2,KTCT,KASP,KCAS,KBF
      CHARACTER*4 TITLE(15)
      DIMENSION Q(150),T(150),X(150),Y(150),KK(150,150)
      DIMENSION AS(250),B2IZ(250),B3IZ(250),B2JZ(250),B2KZ(250),
     & B3JZ(250),B3KZ(250)
      DIMENSION CC(250),TGX(12,6),QQ(150),NODE(250,3),MAT(250)
      DIMENSION HIJ(250),HJK(250),HKI(250),TIJ(250),TJK(250),
     & TKI(250),HHIJ(250),HHJK(250),HHKI(250),IXCB(250)
      DIMENSION CK(150,150),DQ(150),XT(150),INDX(150),VV(150)
      COMMON/PP/TP1,TP2,KI,KG,D1,D2,TH1,TH2,DP1,DP2,S1,S2,TG,
     & WW,HY,MONTH
      DIMENSION TAS(2),TDEL(2),KASP(2),THGP(2),DIC(2),TEPS(2),
     & TSIC(2)
      COMMON /EKC/D1P,D2P,THK1,THK2,EINS,ECAS
      COMMON /ST/AO,BO,DIFF
      PI=4.*ATAN(1.)
      OPEN (8,FILE='CDATA1')
      OPEN (7,FILE='COUTPUT',STATUS='NEW',FORM='FORMATTED')
      OPEN (9,FILE='CDATA2')
C  READ IN THE TITLE OF THE PROBLEM TO BE ANALYZED
      READ (8,2,ERR=2000) TITLE
2     FORMAT(15A4)
      WRITE (7,3) TITLE
3     FORMAT(1X,15A4)
C  READ TOTAL NUMBER OF NODAL POINTS, TOTAL NUMBER OF TRIANGULAR
C  ELEMENTS, TOTAL NUMBER OF KNOWN NODAL TEMPERATURES, AND THE
C  FIRST ELEMENT INDEX OF PIPE INSULATION
      READ (8,*) NN,NE,MZ,MINS
C  READ IN THE NUMBER OF ITERATIONS TO ACCOUNT FOR THE TEMPERATURE
C  EFFECT ON INSULATION AND SOIL THERMAL CONDUCTIVITIES
      READ (8,*) MREPT
C  SET THE UNIT NUMBER OF THE PRINTER
      MO=7
C  READ MONTH OF INTEREST AND THE INDEX FOR FINITE ELEMENT GRID DATA
C  TO BE PRINTED OUT : ICALB = 1  PRINT OUT NODAL COORDINATES
C                            = 0  NO PRINT OUT
      READ (8,*) MONTH,ICALB
C   READ THE THERMAL CONDUCTIVITY (IN BTU-IN./H-FT**2 - DEG F),
C   THICKNESS (IN INCHES) OF THE SIDE, THE DEPTH (IN FT.) OF EARTH
C   COVER (IN FT.), AND THE THICKNESS (IN FT.) OF BOTTOM BED OF
C   THE INNER EARTH REGION.
      READ (8,*) KTCT,TRTK,D,F
C   READ IN THE ESTIMATED AVERAGE TEMPERATURE OF AIR INSIDE THE
C   ANNULUS BETWEEN INSULATED PIPE AND OUTER CASING, IN DEG F,
C   AND THE TEMPERATURE DIFFERENCE BETWEEN THE INSULATED PIPE
C   SURFACE TEMPERATURE AND THE INNER SURFACE TEMPERATURE OF THE
C   CASING, IN DEG F, FOR PIPE NO. 1 AND 2, RESPECTIVELY.
      READ (8,*) (TAS(I),TDEL(I),I=1,2)
C  READ IN THE THERMAL CONDUCTIVITY (IN BTU-IN/H-FT**2-DEG F) OF
C  THE PIPE CASING AND THE CONDUCTIVITY OF POURED-IN INSULATION
C  MATERIAL OR BACK-FILL SOIL SURROUNDING THE PIPES IN THE
C   INNERMOST REGION.
      READ (8,*) KCAS,KBF
C  READ IN TOTAL EMMISSIVITY OF OUTER SURFACE OF PIPE INSULATION
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C   AND INNER SURFACE OF CONDUIT CASING, RESPECTIVELY,AND THE
C   ESTIMATED AVERAGE VALUE OF OUTER SURFACE TEMPERATURES FOR
C   PIPES NO. 1 AND 2, RESPECTIVELY, AND THE ESTIMATED INNER
C   SURFACE TEMPERATURE OF THE CONDUIT CASING FOR PIPES NO. 1
C   AND 2, RESPECTIVELY
      READ(8,*) EINS,ECAS,(TEPS(I),I=1,2),(TSIC(I),I=1,2)
C   READING IN INPUT DATA FOR CALCULATIONS OF PIPE HEAT LOSS AND
C   GENERATION OF THE COORDINATES OF NODAL POINTS
      CALL PIPES(X,Y,TRTK,D,F,INXK,THGP,DIC,ITYPE)
      CALL TWOPIP(1)
      CALL EQUIKC(TAS,TDEL,THGP,DIC,TEPS,TSIC,KASP)
      IF(ICALB.EQ.1) THEN
      WRITE(7,5)
5     FORMAT('  X(M),M=1,NN')
      WRITE(7,7) (X(I),I=1,NN)
7     FORMAT(10F7.2)
      WRITE(7,10)
10    FORMAT('  Y(M),M=1,NN')
      WRITE(7,7) (Y(I),I=1,NN)
      END IF
C   CALCULATIONS OF UNDISTURBED EARTH TEMPERATURES AT VARIOUS DEPTHS
      CALL TGO(TGX,PI,Y)
C   INITIALIZATION OF THE INDEX OF CONVECTION BOUNDARY FOR ELEMENT N
      DO 12 N=1,NE
12    IXCB(N)=0
C   PERFORM ITERATIONS TO ACCOUNT FOR THE TEMPERATURE EFFECTS ON SOIL
C   AND INSULATION THERMAL CONDUCTIVITIES
         DO 24 I=1,NN
24          T(I)=TG
       DO 26 I=1,NE
          HIJ(I)=0.
          HJK(I)=0.
          HKI(I)=0.
          TIJ(I)=0.
          TJK(I)=0.
          TKI(I)=0.
          HHIJ(I)=0.
          HHJK(I)=0.
          HHKI(I)=0.
26      CONTINUE
C     READING IN THE ELEMENT NUMBER AND ITS NODAL POINTS AND THE
C     MATERIAL TYPE, WHICH INCLUDES
C        MAT(J) = 1  SOIL IN INNER EARTH REGION
C               = 2  PIPE INSULATION
C               = 3  AIR SPACE SURROUNDING THE PIPE IN CASING
C               = 4  OUTER CASING OF THE INSULATED PIPE
C               = 5  POURED-IN INSULATION MATERIAL IN BACK-FILL
C                    REGION
C               = 6  NATIVE SOIL IN THE OUTER EARTH REGION
      DO 30 I=1,NE
         READ(9,*) J,(NODE(J,K),K=1,3),MAT(J)
         IF (MAT(J).EQ.1) CC(J)=KTCT/12.
         IF (MAT(J).EQ.2) CC(J)=KI/12.
         IF (MAT(J).EQ.3) THEN
            IF ((J.GT.122).AND.(J.LT.139)) CC(J)=KASP(1)/12.
            IF ((J.GT.138).AND.(J.LT.155)) CC(J)=KASP(2)/12.
         END IF
         IF (MAT(J).EQ.4) CC(J)=KCAS/12.
         IF (MAT(J).EQ.5) CC(J)=KBF/12.
         IF (MAT(J).EQ.6) CC(J)=KG/12.
30    CONTINUE
C   READ IN TOTAL NUMBER OF ELEMENTS HAVING BOUNDARY SEGMENTS SUBJECT
C   TO CONVECTIVE HEAT TRANSFER
      READ (9,*) NECB
C   READ IN ELEMENT NUMBER, CONVECTIVE HEAT TRANSFER COEFFICIENTS,
C   AND AMBIENT TEMPERATURES FOR THREE BOUNDARY SEGMENTS
      DO 35 I=1,NECB
        READ (9,*) M,HIJ(M),HJK(M),HKI(M),TIJ(M),TJK(M),TKI(M)
        IXCB(M)=1
35    CONTINUE
      ITER=1
38    DO 40 I=1,NN
      DO 40 J=1,NN
```

```
            Q(I)=0.
            KK(I,J)=0.
            QQ(I)=0.
            CK(I,J)=0.
            DQ(I)=0.
            VV(I)=1.0
            INDX(I)=1
40     CONTINUE
       L=1.
       DO 180 M=1,NE
         I=NODE(M,1)
         J=NODE(M,2)
         K=NODE(M,3)
         IF(MAT(M).EQ.3) THEN
            IF((M.GT.122).AND.(M.LT.139)) CC(M)=KASP(1)/12.
            IF((M.GT.138).AND.(M.LT.155)) CC(M)=KASP(2)/12.
         END IF
         C=CC(M)
         IF ((INXK.EQ.0).OR.(ITER.EQ.1)) GO TO 60
C  DETERMINE SOIL AND INSULATION THERMAL CONDUCTIVITIES BASED ON THE
C  MEAN TEMPERATURES
         TM=(T(I)+T(J)+T(K))/3.
         IF(MAT(M).EQ.2)  CALL INSULK(TM,C,ITYPE)
         IF(MAT(M).EQ.6)  CALL SOILK(TM,KG,C)
         IF(MAT(M).EQ.1)  CALL SOILK(TM,KG,C)
         CC(M)=C
60     XI=X(I)
       XJ=X(J)
       XK=X(K)
       YI=Y(I)
       YJ=Y(J)
       YK=Y(K)
       CXX=C
       CXY=0.
       CYX=0.
       CYY=C
       B2I=YJ-YK
       B3I=XK-XJ
       B2J=YK-YI
       B3J=XI-XK
       B2K=YI-YJ
       B3K=XJ-XI
C   CALCULATE THE ELEMENT AREA
       SA=0.5*(XJ*B2J+XI*B2I+XK*B2K)
       SA=ABS(SA)
       A2=SA*2.
       AS(M)=A2
       B2I=B2I/A2
       B3I=B3I/A2
       B2J=B2J/A2
       B3J=B3J/A2
       B2K=B2K/A2
       B3K=B3K/A2
       B2IZ(M)=B2I
       B3IZ(M)=B3I
       B2JZ(M)=B2J
       B3JZ(M)=B3J
       B2KZ(M)=B2K
       B3KZ(M)=B3K
       BII=SA*L*(B2I*B2I*CXX+B2I*B3I*CXY+B3I*B2I*CYX+B3I*B3I*CYY)
       BIJ=SA*L*(B2I*B2J*CXX+B2I*B3J*CXY+B3I*B2J*CYX+B3I*B3J*CYY)
       BIK=SA*L*(B2I*B2K*CXX+B2I*B3K*CXY+B3I*B2K*CYX+B3I*B3K*CYY)
       BJI=SA*L*(B2J*B2I*CXX+B2J*B3I*CXY+B3J*B2I*CYX+B3J*B3I*CYY)
       BJJ=SA*L*(B2J*B2J*CXX+B2J*B3J*CXY+B3J*B2J*CYX+B3J*B3J*CYY)
       BJK=SA*L*(B2J*B2K*CXX+B2J*B3K*CXY+B3J*B2K*CYX+B3J*B3K*CYY)
       BKI=SA*L*(B2K*B2I*CXX+B2K*B3I*CXY+B3K*B2I*CYX+B3K*B3I*CYY)
       BKJ=SA*L*(B2K*B2J*CXX+B2K*B3J*CXY+B3K*B2J*CYX+B3K*B3J*CYY)
       BKK=SA*L*(B2K*B2K*CXX+B2K*B3K*CXY+B3K*B2K*CYX+B3K*B3K*CYY)
       KK(I,I)=KK(I,I)+BII
       KK(I,J)=KK(I,J)+BIJ
       KK(I,K)=KK(I,K)+BIK
       KK(J,I)=KK(J,I)+BJI
       KK(J,J)=KK(J,J)+BJJ
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```
      KK(J,K)=KK(J,K)+BJK
      KK(K,I)=KK(K,I)+BKI
      KK(K,J)=KK(K,J)+BKJ
      KK(K,K)=KK(K,K)+BKK
      IF(IXCB(M).EQ.0) GO TO 130
C    ADDITION OF CONVECTION TERMS TO THE ELEMENT MATRIX TO ACCOUNT
C    FOR CONVECTION ON BOUNDARY
C    READING IN CONVECTIVE HEAT TRANSFER COEFFICIENTS AND AMBIENT
C    TEMPERATURES FOR THREE BOUNDARY SEGMENTS
      HHIJ(M)=HIJ(M)*L*SQRT((X(I)-X(J))**2+(Y(I)-Y(J))**2)/6.
      HHJK(M)=HJK(M)*L*SQRT((X(J)-X(K))**2+(Y(J)-Y(K))**2)/6.
      HHKI(M)=HKI(M)*L*SQRT((X(K)-X(I))**2+(Y(K)-Y(I))**2)/6.
      KK(I,I)=HHIJ(M)*2.+HHKI(M)*2.+KK(I,I)
      KK(I,J)=HHIJ(M)+KK(I,J)
      KK(I,K)=HHKI(M)+KK(I,K)
      KK(J,I)=HHIJ(M)+KK(J,I)
      KK(J,J)=HHIJ(M)*2+HHJK(M)*2.+KK(J,J)
      KK(J,K)=HHJK(M)+KK(J,K)
      KK(K,I)=HHKI(M)+KK(K,I)
      KK(K,J)=HHJK(M)+KK(K,J)
      KK(K,K)=HHJK(M)*2.+HHKI(M)*2.+KK(K,K)
      HHIJ(M)=TIJ(M)*3.*HHIJ(M)
      HHJK(M)=TJK(M)*3.*HHJK(M)
      HHKI(M)=TKI(M)*3.*HHKI(M)
130   Q(I)=Q(I)+HHIJ(M)+HHKI(M)
      Q(J)=Q(J)+HHIJ(M)+HHJK(M)
      Q(K)=Q(K)+HHJK(M)+HHKI(M)
180   CONTINUE
C    DETERMINE IF FINITE ELEMENT GRID DATA ARE TO BE PRINTED OUT
      IF ((ICALB.EQ.1).AND.(IREPT.EQ.MREPT)) THEN
      WRITE(7,185)
185   FORMAT('      M     I     J     K    MAT.    C')
      DO 190 I=1,NE
      WRITE(7,187) I,(NODE(I,J),J=1,3),MAT(I),CC(I)
187   FORMAT(1X,5I6,F10.4)
190   CONTINUE
      END IF
C    DETERMINE OUTER SURFACE TEMPERATURES OF UNDERGROUND PIPES
      DO 200 I=1,8
        T(I)=TP1
        II=I+8
        T(II)=TP2
200   CONTINUE
C    DETERMINE OUTER BOUNDARY TEMPERATURES OF EARTH REGION
      CALL TGXX(T,TGX,MONTH)
      MZ1=MZ+1
      DO 260 I=MZ1,NN
        SUM=0.
        DO 250 J=1,MZ
250       SUM=SUM+KK(I,J)*T(J)
        QQ(I)=Q(I)-SUM
260   CONTINUE
      IF(ICALB.EQ.1) THEN
         WRITE(7,280)
280      FORMAT(6X,'QQ    ARRAY')
         WRITE(7,285) (QQ(I),I=1,NN)
285      FORMAT (5E14.5)
      END IF
C  RENAMING OF MATRICES
      MN=NN-MZ
      DO 300 I=1,MN
         K=MZ+I
        DO 290 J=1,MN
            KL=MZ+J
290         CK(I,J)=KK(K,KL)
            XT(I)=T(K)
            DQ(I)=QQ(K)
300      CONTINUE
C  SOLUTION OF SIMULTANEOUS EQUATIONS
C  SET PHYSICAL DIMENSION OF MATRIX A
       NP=150
      CALL SOLVLE(CK,MN,NP,INDX,VV,DQ)
      DO 310 I=1,MN
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            K=MZ+I
            T(K)=DQ(I)
310     CONTINUE
C    CALCULATIONS OF EQUIVALENT THERMAL CONDUCTIVITIES FOR AIR
C    SPACES IN INSULATED PIPES
C
C    CALCULATE AVERAGE SURFACE TEMPERATURE OF INSULATED PIPES
      SU1=0.0
      SU2=0.0
      DO 312 I=1,8
        L1=I+80
        L2=I+88
        SU1=SU1+T(L1)
        SU2=SU2+T(L2)
312   CONTINUE
      TEPS(1)=SU1/8.
      TEPS(2)=SU2/8.
C    CALCULATE THE INNER SURFACE TEMPERATURE OF THE CASING
      SU3=0.0
      SU4=0.0
      DO 314 I=1,8
        L3=I+96
        L4=I+104
        SU3=SU3+T(L3)
314     SU4=SU4+T(L4)
      TSIC(1)=SU3/8.
      TSIC(2)=SU4/8.
C    DETERMINE THE TEMPERATURE DIFFERENCE BETWEEN THE INSULATED PIPE
C    AND THE INNER SURFACE OF THE CASING FOR PIPES 1 AND 2, RESPECTIVELY.
      DO 316 I=1,2
      TAS(I)=(TEPS(I)+TSIC(I))/2.
      TDEL(I)=ABS(TEPS(I)-TSIC(I))
316   CONTINUE
      CALL EQUIKC(TAS,TDEL,THGP,DIC,TEPS,TSIC,KASP)
      WRITE(7,320) (I,KASP(I),I=1,2)
320   FORMAT('  KASP(',I1,')=',F10.4,2X,'(BTU-IN./H-FT**2-DEG F)'/,
     &       '  KASP(',I1,')=',F10.4,2X,'(BTU-IN./H-FT**2-DEG F)')
330   FORMAT('  TEMPERATURE ARRAY : T(I), I=1,NN ')
C  CALCULATE THE MEAN VALUES OF INSULATION THERMAL CONDUCTIVITY FOR
C  PIPES 1 AND 2
350   SKI1=0.
      SKI2=0.
      DO 400 LN=1,16
         LM=MINS+LN-1
         LL=LM+16
         SKI1=SKI1+CC(LM)
         SKI2=SKI2+CC(LL)
400   CONTINUE
      KIX1=SKI1/16.
      KIX2=SKI2/16.
      R1=D1/24.
      R2=D2/24.
      TH1X=TH1/12.
      TH2X=TH2/12.
      IF(ICALB .EQ. 0) THEN
        MO=11
        IF(IREPT .EQ. MREPT) MO=7
      END IF
C  CALCULATIONS OF THE HEAT LOSSES FROM THE UNDERGROUND PIPES
      CALL  PIPEHL(T,R1,R2,TH1X,TH2X,KIX1,KIX2,MO,QTX)
      HLOSS=QTX
      IF(ITER.EQ.1) HLOSX=0.
C    DETERMINE IF PIPE HEAT LOSS VALUE HAS CONVERGED, OR CONTINUE
C    ITERATIONS IF REQUIRED
      DELQT=ABS(HLOSS-HLOSX)/HLOSS
      IF(DELQT.LE. 0.010) GO TO 2010
      ITER=ITER+1
      HLOSX=HLOSS
      GO TO 38
2000  WRITE (7,2005)
2005  FORMAT (1X,'THERE ARE SOME ERRORS IN INPUT DATA')
2010  IF(ICALB .EQ. 1) THEN
      WRITE (7,330)
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      WRITE (7,285) (T(I),I=1,NN)
      END IF
      STOP
      END

      SUBROUTINE TGO(TGX,PI,Y)
C   THIS SUBROUTINE CALCULATES THE UNDISTURBED EARTH TEMPERATURES
C   AT VARIOUS DEPTHS
      DIMENSION TGX(12,6),Y(150)
C     READING IN THE ANUAL AVERAGE TEMPERATURE AND AMPLITUDE OF THE
C     MONTHLY NORMAL TEMPERATURE CYCLE OF THE SITE, IN DEG F, AND
C     THERMAL DIFFUSIVITY OF SOIL, IN FT**2/H.
      READ (8,*) AO,BO,DIFF
      W=2.*PI/12.
      WZ=2.*PI/(8760*DIFF*2)
      ZZ=SQRT(WZ)
      DO 1 I=1,12
      DO 1 J=1,6
      Z=ZZ*Y(31-J)
1     TGX(I,J)=AO+BO*EXP(-Z)*SIN(W*(I-3)-Z)
      RETURN
      END

      SUBROUTINE TGXX(T,TGX,MONTH)
C THIS SUBROUTINE PROVIDES OUTER BOUNDARY TEMPERATURES OF EARTH REGION
      DIMENSION T(150),TGX(12,6)
      T(30)=TGX(MONTH,1)
      DO 1 I=1,8
         II=I+30
1        T(II)=T(30)
      DO 5 I=2,6
         I15=I+15
         JI=31-I
         T(I15)=TGX(MONTH,I)
         T(JI)=TGX(MONTH,I)
5     CONTINUE
      DO 10 I=1,3
         I21=I+21
10     T(I21)=TGX(MONTH,6)
      RETURN
      END

      SUBROUTINE INSULK(TM,C,ITYPE)
C  THIS SUBROUTINE DETERMINES THE THERMAL CONDUCTIVITY OF PIPE
C  INSULATIONS (CALCIUM SILICATE AND MINERAL WOOL) AS A FUNCTION
C  OF THE MEAN TEMPERATURE.
      REAL KN(16),KINS
      DIMENSION TN(16)
      DATA KN /0.375,0.40,0.42,0.45,0.48,0.50,0.53,0.555,0.58,0.61,
     & 0.63,0.66,0.68,0.74,0.82,0.90/
C   DETERMINE THE THERMAL CONDUCTIVITY OF CALCIUM SILICATE
      IF (ITYPE .EQ. 1) THEN
      DO 5 J=1,16
      IF(J .LE. 13) THEN
         TN(J)=100.+(J-1)*50.
      ELSE
         TN(J)=700.+(J-13)*100.
      END IF
5     CONTINUE
      IF(TM .GT. TN(1)) GO TO 10
      KINS=KN(1)
      GO TO 100
10    IF(TM .LT. TN(16)) GO TO 20
      KINS=KN(16)
      GO TO 100
20    DO 50 I=1,15
      T1=TM-TN(I)
      IF(T1 .NE. 0.) GO TO 30
      KINS=KN(I)
      GO TO 100
30    T2=TN(I+1)-TM
      IF(T2 .NE. 0.) GO TO 40
      KINS=KN(I+1)
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      GO TO 100
40    P=T1*T2
      IF(P .LT. 0.) GO TO 50
      KINS=KN(I)+T1*(KN(I+1)-KN(I))/(TN(I+1)-TN(I))
      GO TO 100
50    CONTINUE
100   C=KINS/12.
C   DETERMINE THE THERMAL CONDUCTIVITY OF MINERAL WOOL
      ELSE IF (ITYPE .EQ. 2) THEN
      KINS=0.2420 + TM*1.5501E -4 + TM*TM*7.5000E -7
      C=KINS/12.
      ELSE
       RETURN
      END IF
      RETURN
      END

      SUBROUTINE  SOILK(TM,KG,C)
C THIS ROUTINE DETERMINES THE THERMAL CONDUCTIVITY OF SOIL AS A
C FUNCTION OF MEAN TEMPERATURES.
      REAL K(14),KG
      DIMENSION TX(14)
      DATA K/1.1,1.1,1.1,1.0,0.4,0.31,0.25,0.19,0.15,0.11,0.09,0.07,
     & 0.05,0.05/
      DO 1 I=1,14
1        TX(I)=50.+(I-1)*25.
      IF(TM.GT.TX(1)) GO TO 5
      ZK=1.1
      GO TO 50
5     IF(TM.LT.TX(14)) GO TO 10
      ZK=0.05
      GO TO 50
10    DO 25 I=1,13
      T1=TM-TX(I)
      IF(T1.NE.0) GO TO 15
      ZK=K(I)
      GO TO 50
15    CONTINUE
      T2=TM-TX(I+1)
      IF(T2.NE.0.) GO TO 20
      ZK=K(I+1)
      GO TO 50
20    CONTINUE
      P=T1*T2
      IF(P.GT.0) GO TO 25
      ZK=K(I+1)+T2*(K(I+1)-K(I))/25.
      GO TO 50
25    CONTINUE
50    C=ZK*KG/(1.1*12.)
      RETURN
      END

      SUBROUTINE PIPEHL(T,R1,R2,TH1,TH2,ZKS1,ZKS2,MO,QT)
C   THIS SUBROUTINE CALCULATES THE AVERAGE TEMPERATURE DROPS ACROSS THE
C   PIPE INSULATIONS AND THE RATES OF HEAT LOSS FROM THE UNDERGROUND
C   PIPES IN DIRECTLY BURIED CONDUIT SYSTEM
      DIMENSION T(150)
      PI=4.*ATAN(1.)
      SUM1=0.
      SUM2=0.
      N1=8
      DO 1 I=1,N1
         K1=I
         K2=I+8
         K3=I+80
         K4=I+88
         SUM1=SUM1+T(K1)-T(K3)
         SUM2=SUM2+T(K2)-T(K4)
1     CONTINUE
      T1=SUM1/N1
      T2=SUM2/N1
      ZKIS1=ZKS1*12.
      ZKIS2=ZKS2*12.
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      Q1=ZKS1*2.*PI*T1/LOG((R1+TH1)/R1)
      Q2=ZKS2*2.*PI*T2/LOG((R2+TH2)/R2)
      QT=Q1+Q2
      IF(MO .EQ. 11) GO TO 50
      WRITE(MO,5) ZKIS1,ZKIS2
5     FORMAT(/'  AVERAGE VALUES OF PIPE INSULATION THERMAL',
     &' CONDUCTIVITY :',/,'   KI1 = ',F10.3,'   KI2 = ',F10.3,
     & '   BTU-IN/H-FT**2-DEG F ')
      WRITE(MO,10) T1,T2
10    FORMAT(/'  AVERAGE TEMPERATURE DROPS ACROSS INSULATION : ',/,
     & '   T1= ',F10.2,'   T2= ',F10.2,'  DEG F')
      WRITE(MO,20) Q1,Q2,QT
20    FORMAT(/,2X,'HEAT LOSSES FROM UNDERGROUND PIPES :'/'   Q1=',
     & F10.2, '   Q2=',F10.2,'   QT=',F10.2,'  BTU/H-FT')
50    RETURN
      END

      SUBROUTINE PIPES(X,Y,TRTK,D,F,INXK,THGP,DIC,ITYPE)
C THIS SUBROUTINE READS IN THE INPUT DATA TO BE USED FOR CALCULATIONS
C OF THE HEAT LOSSES FROM THE UNDERGROUND PIPES AND GENERATES X AND Y-
C COORDINATES OF NODAL POINTS FOR THE TWO PIPE SYSTEM.
      REAL KII,KIG,KI,KG
      DIMENSION X(150),Y(150),THGP(2),DIC(2)
      COMMON /PP/TP1,TP2,KII,KIG,DI1,DI2,THI1,THI2,B1,B2,S1,S2,TG,
     & WW,HY,MONTH
      COMMON /EKC/D1P,D2P,THK1,THK2,EINS,ECAS
C READ TEMPERATURE OF PIPE NUMBERS 1 AND 2, IN DEG F
      READ (8,*) TP1,TP2
C READ THERMAL CONDUCTIVITY OF THERMAL INSULATION AND SOIL,
C RESPECTIVELY, IN BTU-IN./H-FT**2 - DEG F, AND INDEX OF THERMAL
C CONDUCTIVITY : INXK = 0  CONSTANT THERMAL CONDUCTIVITY
C                     = 1  TEMPERATURE DEPENDENT THERMAL CONDUCTIVITY
C AND THE MATERIAL TYPE OF PIPE INSULATION, WHICH INCLUDES
C                 ITYPE = 1  CALCIUM SILICATE
C                       = 2  MINERAL WOOL
      READ (8,*) KII,KIG,INXK,ITYPE
C READING IN THE OUTSIDE DIAMETERS OF STEEL PIPES 1 AND 2, IN INCHES
      READ (8,*) DI1,DI2
C READING IN THE THICKNESS OF THERMAL INSULATION USED FOR PIPES 1
C AND 2, RESPECTIVELY, IN INCHES
      READ (8,*) THI1,THI2
C READ IN THE THICKNESSES OF AIR GAP AND OUTER CASING USED FOR
C PIPES 1 AND 2, RESPECTIVELY, IN INCHES.
      READ (8,*) (THGP(I),I=1,2),THCA1,THCA2
C READ THE DEPTHS FROM GROUND SURFACE TO THE CENTERS OF PIPES 1 AND
C 2, RESPECTIVELY, IN FT.
      READ (8,*) B1,B2
C   READING IN THE HORIZONTAL DISTANCES (IN FT.) FROM VERTICAL
C   CENTERLINE OF THE INNER EARTH REGION TO CENTERS OF PIPES 1 AND
C   2, RESPECTIVELY, AND THE AVERAGE EARTH TEMPERATURE, IN DEG F.
      READ (8,*) S1,S2,TG
C   READ IN THE THICKNESS OF EARTH COVER,WIDTH AND DEPTH OF OUTER
C   EARTH REGION SURROUNDING THE UNDERGROUND SYSTEM, IN FT.
      READ (8,*) E,WW,HY
      WRITE(7,10) TP1,TP2,KII,KIG,DI1,DI2
10    FORMAT('   TP1    TP2     KI     KG     D1      D2'/6F7.2)
      WRITE(7,20) THI1,THI2,B1,B2,S1,S2,TG
20    FORMAT('   THI1   THI2    DP1    DP2     S1     S2     TG',
     & /7F7.2)
      WRITE(7,25) (THGP(I),I=1,2),THCA1,THCA2
25    FORMAT('  THGP1  THGP2  THCA1  THCA2 '/4F7.2)
      WRITE(7,30) E,WW,HY,MONTH
30    FORMAT('      E      WW      HY   MONTH '/3F7.2,I7)
C   READ IN THE INSIDE WIDTH AND HEIGHT OF THE INNER EARTH REGION,
C   FOR LOOSE-FILL INSULATION SYSTEM, IN FT.
      READ (8,*) A,B
C   CHANGE TO ENGINEERING UNITS
      D1=DI1/12.
      R1=D1*0.5
      D2=DI2/12.
      R2=D2*0.5
      D1P=DI1/12.
      D2P=DI2/12.
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      DIC(1)=(DI1+2.*THI1+2.*THGP(1))/12.
      DIC(2)=(DI2+2.*THI2+2.*THGP(2))/12.
      KI=KII/12.
      KG=KIG/12.
      W=A+2*TRTK/12
      H=B+E+D+F
      WRITE(7,40) W,H,D,F,A,B,WW,HY
40    FORMAT('      W       H       D       F       A       B      WW      HY'
     &,/,8F7.2)
      PI=4.*ATAN(1.)
      TH1=THI1/12.
      TH2=THI2/12.
      THG1=THGP(1)/12.
      THG2=THGP(2)/12.
      THC1=THCA1/12.
      THC2=THCA2/12.
      THK1=THI1/12.
      THK2=THI2/12.
C     DETERMINE THE X AND Y-COORDINATES OF EARTH COVER, SIDES AND BOTTOM
C     BED OF THE INNER EARTH REGION (NODAL POINTS 39 TO 70)
      DO 50 I=1,5,2
         I65=I+65
         X(I65)=W-(I-1)*W/4.
50       Y(I65)=E
      DO 60 I=1,3,2
         I66=I+66
         X(I66)=(W+A)*0.5 - A*(I-1)*0.5
60       Y(I66)=E
      DO 65 I=1,4
         I38=I+38
         I42=I+42
         I46=I+46
         I50=I+50
         I1=I-1
         X(I38)=(W-A)*0.5
         Y(I38)=(D+E)+I1*B/4.
         X(I42)=0.5*(W-A)+I1*A/4.
         Y(I42)=D+E+B
         X(I46)=(W+A)*0.5
         Y(I46)=(D+E+B)-I1*B/4.
         X(I50)=0.5*(W+A)-I1*A/4.
         Y(I50)=D+E
65    CONTINUE
      DO 70 I=1,3
         I54=I+54
         I58=I+58
         I62=I+62
         X(I54)=0.0
         Y(I54)=(D+E)+B*(I-1)/2.
         X(I58)=(W-A)*0.5+(I-1)*A*0.5
         Y(I58)=H
         X(I62)=W
         Y(I62)=(D+E)+(3-I)*B/2.
70    CONTINUE
         X(58)=0.0
         Y(58)=H
         X(62)=W
         Y(62)=H
C     THE X AND Y-COORDINATES OF OUTER BOUNDARY EARTH SURROUNDING THE
C     DIRECTLY BURIED CONDUITS (NODAL POINTS 17 TO 38,AND 71 TO 80)
      DO 72 I=1,2
         I16=I+16
         I27=I+27
         X(I16)=-WW
         Y(I16)=E+(H-E)*(I-1)*0.5
         X(I27)=W+WW
         Y(I27)=E+(H-E)*(2-I)*0.5
72    CONTINUE
      DO 75 I=1,3
         I18=I+18
         I21=I+21
         I24=I+24
         X(I18)=-WW
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          Y(I18)=H+HY*(I-1)*0.5
          X(I21)=W*(I-1)*0.5
          Y(I21)=H+HY
          X(I24)=W+WW
          Y(I24)=H+HY*(3-I)*0.5
75     CONTINUE
       DO 77 I=1,5,2
          I31=I+31
          X(I31)=W-(I-1)*W/4.
77        Y(I31)=0.0
       DO 78 I=1,3,2
          I32=I+32
          X(I32)=(W+A)*0.5-A*(I-1)*0.5
78        Y(I32)=0.0
       DO 80 I=1,2
          I29=I+29
          I36=I+36
          X(I29)=W+WW*(3-I)*0.5
          Y(I29)=0.
          X(I36)=-WW*0.5*I
          Y(I36)=0.
80     CONTINUE
       DO 82 I=1,3
          I70=I+70
          I77=I+77
          X(I70)=-WW*0.5
          Y(I70)=E+(H-E)*(I-1)*0.5
          X(I77)=W+WW*0.5
          Y(I77)=E+(H-E)*(3-I)*0.5
82     CONTINUE
       DO 85 I=1,4
          I73=I+73
          X(I73)=W*(I-1)/3.
85        Y(I73)=H+HY*0.5
C    X AND Y-COORDINATES OF THE CENTERS OF THE PIPES
       XC1=W*0.5 - S1
       YC1=B1
       XC2=W*0.5 + S2
       YC2=B2
       WRITE(7,90) XC1,YC1,XC2,YC2
90     FORMAT('    XC1    YC1    XC2    YC2'/4F7.3)
C    THE X AND Y-COORDINATES OF NODAL POINTS AT THE INNER AND OUTER
C    SURFACES OF PIPE INSULATION (NODAL POINTS 1 TO 16, AND 81 TO
C    96)
       DO 95 I=1,8
          THETA=2.*PI*I/8.
          I8=I+8
          I80=I+80
          I88=I+88
          X(I)=XC1+0.5*D1*SIN(THETA)
          Y(I)=YC1+0.5*D1*COS(THETA)
          X(I8)=XC2+0.5*D2*SIN(THETA)
          Y(I8)=YC2+0.5*D2*COS(THETA)
          X(I80)=XC1+(TH1+R1)*SIN(THETA)
          Y(I80)=YC1+(TH1+R1)*COS(THETA)
          X(I88)=XC2+(TH2+R2)*SIN(THETA)
          Y(I88)=YC2+(TH2+R2)*COS(THETA)
95     CONTINUE
C   THE X AND Y-COORDINATES OF NODAL POINTS AT THE INNER AND OUTER
C   SURFACES OF PIPE CASINGS (NODAL POINTS 97 TO 128)
       DO 97 I=1,8
          THETA=2.*PI*I/8.
          I96=I+96
          I104=I+104
          I112=I+112
          I120=I+120
          X(I96)=XC1+(THG1+TH1+R1)*SIN(THETA)
          Y(I96)=YC1+(THG1+TH1+R1)*COS(THETA)
          X(I104)=XC2+(THG2+TH2+R2)*SIN(THETA)
          Y(I104)=YC2+(THG2+TH2+R2)*COS(THETA)
          X(I112)=XC1+(THC1+THG1+TH1+R1)*SIN(THETA)
          Y(I112)=YC1+(THC1+THG1+TH1+R1)*COS(THETA)
          X(I120)=XC2+(THC2+THG2+TH2+R2)*SIN(THETA)
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          Y(I120)=YC2+(THC2+THG2+TH2+R2)*COS(THETA)
97     CONTINUE
C    THE X AND Y-COORDINATES OF NODAL POINTS IN BACK-FILL SOIL, OR
C    POURED-IN INSULATION SURROUNDING THE PIPES (NODAL POINTS 129 TO
C    142)
         YUP=0.5*(Y(116)-Y(53))
         YUPR=0.5*(Y(124)-Y(53))
         IF (YUPR.LT.YUP) YUP=YUPR
         XLT=0.5*(X(118)-X(41))
         YLO=0.5*(Y(45)-Y(120))
         YLOW=0.5*(Y(45)-Y(128))
         IF (YLOW.LT.YLO) YLO=YLOW
         XRT=0.5*(X(49)-X(122))
      DO 100 I=1,3
          I128=I+128
          I131=I+131
          I134=I+134
          I137=I+137
          X(I128)=0.5*(W+A)-0.25*A*I
          Y(I128)=(D+E)+YUP
          X(I131)=0.5*(W-A)+XLT
          Y(I131)=(D+E)+0.25*B*I
          X(I134)=0.5*(W-A)+0.25*A*I
          Y(I134)=(D+E+B)-YLO
          X(I137)=0.5*(W+A)-XRT
          Y(I137)=(D+E+B)-0.25*B*I
100   CONTINUE
      DO 120 I=1,2
          I140=I+140
          X(I140)=X(53)
          Y(I140)=(D+E+YUP)+(B-YUP-YLO)*I/3.
120   CONTINUE
      RETURN
      END

      SUBROUTINE TWOPIP(IREPT)
C  THIS SUBROUTINE DETERMINES THE HEAT LOSSES FROM TWO PIPES TO THE
C  UNDERGROUND SURROUNDING THE HEAT DISTRIBUTION SYSTEM.
      REAL KII,KIG
      COMMON /PP/T1,T2,KII,KIG,DI1,DI2,THI1,THI2,B1,B2,S1,S2,TG,
     & WW,HY,MONTH
      PI=4.*ATAN(1.)
      X1=2.*PI
      R1=DI1/24.
      R2=DI2/24.
      TH1X=THI1/12.
      TH2=THI2/12.
      ZK1=KII/12.
      ZK2=ZK1
      D1=B1
      D2=B2
      ZKS=KIG/12.
      DO 10 I=1,IREPT
2     TH1=TH1X+0.1*(I-1)
      TH2=TH1
      S=S1+S2
      A=R1+R2+TH1+TH2+0.05
      THI1=TH1*12.
      IF(A .LT. S) A=S
      C1=X1*ZK1/LOG((R1+TH1)/R1)
      C2=X1*ZK2/LOG((R2+TH2)/R2)
      P11=1.+C1/(X1*ZKS)*LOG((2*D1)/(R1+TH1))
      P12=C2/(X1*ZKS)*LOG((A*A+(D1+D2)**2)/(A*A+(D1-D2)**2))*0.5
      P21=C1/(X1*ZKS)*LOG((A*A+(D1+D2)**2)/(A*A+(D1-D2)**2))*0.5
      P22=1.+C2/(X1*ZKS)*LOG((2*D2)/(R2+TH2))
      DEL=P12*P21-P11*P22
      ZKP1=C1*(P12-P22)/DEL
      ZKP2=C2*(P21-P11)/DEL
      TP1=(P12*T2-P22*T1)/(P12-P22)
      TP2=(P21*T1-P11*T2)/(P21-P11)
      Q1=ZKP1*(TP1-TG)
      Q2=ZKP2*(TP2-TG)
      QT=Q1+Q2
```

```
      TAVG=(T1+T2)*0.5
      ZK=QT/(TAVG-TG)
      WRITE(7,6) DI1,DI2,S1,S2,THI1,KII,KIG,T1,T2
6     FORMAT('   DI1    DI2     S1     S2   THI1    KII     KIG    TP1    TP2',
     &/,7F6.2,1X,2F6.0)
      WRITE(7,8) Q1,Q2,QT,ZK
8     FORMAT('    Q1     Q2      QT        KP'/,3F7.2,2X,F6.3/)
10    CONTINUE
      RETURN
      END

      SUBROUTINE EQUIKC(TAS,TDEL,THGP,DIC,TEPS,TSIC,KASP)
C   THIS ROUTINE CALCULATES EQUIVALENT THERMAL CONDUCTIVITY OF AIR
C   SPACE IN THE ANNULUS BETWEEN INSULATED PIPE AND THE CASING FOR
C   HORIZONTAL CONCENTRIC PIPES.
      REAL KASP
      DIMENSION TAS(2),TDEL(2),KASP(2),THGP(2),DIC(2),TEPS(2),
     & TSIC(2),CONKA(2),RADKA(2),DOP(2)
      COMMON /EKC/D1P,D2P,THK1,THK2,EINS,ECAS
      PI=4.*ATAN(1.)
      DOP(1)=D1P+2.*THK1
      DOP(2)=D2P+2.*THK2
C   CALCULATE THERMAL CONDUCTIVITY, IN BTU-FT/H-FT**2-DEG F, AND
C   KINEMATIC VISCOSITY, IN FT**2/S, OF AIR
      DO 30 I=1,2
      THKAIR=0.01319 + TAS(I)*2.5E -5
      VAIR=1.2624E -4 + TAS(I)*5.4E -7
      CL=THGP(I)/12.
C   CALCULATE THE PRANDTL NUMBER OF AIR , AND GRASHOF NUMBER AND
C   EQUIVALENT THERMAL CONDUCTIVITY, IN BTU-IN/H-FT**2-DEG F, OF
C   AIR SPACE
      PRANTL=0.71849 - TAS(I)* 1.275E -4
      GRASOF=32.2 * TDEL(I)*(CL**3.)/((VAIR**2.)*(TAS(I)+459.7))
      CONKA(I)=12.*THKAIR*0.159*(PRANTL*GRASOF)**0.272
C   CALCULATE THE EMMISIVITY TERM AND EQUIVALENT THERMAL
C   CONDUCTIVITY OF AIRSPACE DUE TO RADIATIVE TRANSFER, IN
C   BTU-IN/H-FT**2-DEG F
      ETOT=1./EINS + (DOP(I)/DIC(I))*(1./ECAS - 1.)
      TREP=(TEPS(I)+459.7)/100.
      TRSI=(TSIC(I)+459.7)/100.
      HRAD=0.1714/100.*(TREP+TRSI)*(TREP*TREP+TRSI*TRSI)/ETOT
      RADKA(I)=12. * HRAD * CL
30    CONTINUE
C   CALCULATE EQUIVALENT THERMAL CONDUCTIVITY OF AIRSPACE, IN
C   BTU-IN/H-FT**2-DEG F
      DO 50 I=1,2
      KASP(I)=CONKA(I) + RADKA(I)
      WRITE (7,45) I,CONKA(I),RADKA(I)
45    FORMAT('  I=',I5,2X,' CONKA=',F10.4,2X,' RADKA=',F10.4,
     & 2X,'(BTU-IN/H-FT**2-DEG F)')
50    CONTINUE
      RETURN
      END

      SUBROUTINE SOLVLE(A,N,NP,INDX,VV,B)
C   GIVEN AN NXN MATRIX A, WITH PHYSICAL DIMENSION NP, THIS ROUTINE
C   REPLACE IT BY THE LU DECOMPOSITION OF A ROWWISE PERMUTATION OF
C   ITSELF. INDX IS AN OUTPUT VECTOR WHICH RECORD THE ROW PERMUTATION
C   EFFECTED BY THE PARTIAL PIVOTING; VV IS VECTOR OF SCALING FACTORS.
C
C   THIS ROUTINE IS USED TO SOLVE THE LINEAR SET OF EQUATIONS :
C   [A][X]=[B]
C
      DIMENSION A(NP,NP),INDX(N),VV(N),B(N)
C
C   FORM IMPLICIT SCALING VECTOR VV
C
      DO 12 I=1,N
        AAMAX = 0.0
        DO 11 J=1,N
         IF(ABS(A(I,J)).GT.AAMAX) AAMAX=ABS(A(I,J))
   11   CONTINUE
        IF(AAMAX.EQ.0.) THEN
```

```
          WRITE(7,100) I
 100      FORMAT(1X,'ERROR:SINGULAR MATRIX - ZERO ROW : ROW',I5)
          RETURN
        END IF
       VV(I) = 1.0/AAMAX
   12 CONTINUE
C
C   CROUT METHOD: LOOP OVER COLUMNS
C
      DO 19 J=1,N
        DO 14 I=1,J-1
          SUM = A(I,J)
          DO 13 K=1,I-1
            SUM = SUM - A(I,K)*A(K,J)
   13     CONTINUE
        A(I,J) = SUM
   14   CONTINUE
C
C   PIVOT IMPLEMENTATION
C
        AAMAX = 0.0D0
        DO 16 I=J,N
          SUM = A(I,J)
          DO 15 K=1,J-1
            SUM = SUM - A(I,K)*A(K,J)
   15     CONTINUE
          A(I,J) = SUM
          DUM = VV(I)*ABS(SUM)
          IF(DUM.GE.AAMAX) THEN
            IMAX = I
            AAMAX = DUM
          ENDIF
   16   CONTINUE

        IF(J.NE.IMAX) THEN
          DO 17 K=1,N
            DUM = A(IMAX,K)
            A(IMAX,K) = A(J,K)
            A(J,K) = DUM
   17     CONTINUE
        VV(IMAX) = VV(J)
      ENDIF
      INDX(J) = IMAX
      IF(A(J,J).EQ.0.0) THEN
        WRITE(7,110) J
 110    FORMAT(1X,'ERROR: SINGULAR MATRIX - ZERO " DIAG " : ROW',I5)
        RETURN
      END IF
      IF(J.NE.N) THEN
        DUM = 1.0/A(J,J)
        DO 18 I=J+1,N
          A(I,J) = A(I,J)*DUM
   18   CONTINUE
      END IF
   19 CONTINUE
C
C   FORWARD SUBSTITUTION
C
      II = 0
      DO 22 I=1,N
        LL = INDX(I)
        SUM = B(LL)
        B(LL) = B(I)
        IF(II.NE.0) THEN
        DO 21 J=II,I-1
          SUM = SUM - A(I,J)*B(J)
   21     CONTINUE
        ELSE IF(SUM.NE.0.0) THEN
         II = I
        END IF
        B(I) = SUM
   22 CONTINUE
C
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C    BACKWARD SUBSTITUTION
C
        DO 24 I=N,1,-1
          SUM = B(I)
          IF(I.LT.N) THEN
            DO 23 J=I+1,N
            SUM = SUM - A(I,J)*B(J)
   23       CONTINUE
          END IF
          B(I) = SUM/A(I,I)
   24 CONTINUE
      RETURN
      END
```

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      PROGRAM DBJACKS
C THIS IS A MAIN PROGRAM FOR HEAT LOSS ANALYSIS OF DIRECTLY BURIED
C CONDUIT UNDERGROUND HEAT DISTRIBUTION SYSTEMS INSTALLED AT FORT
C JACKSON, SC, BASE ON THE FINITE ELEMENT METHOD USING THREE-NODE
C LINEAR TRIANGULAR ELEMENTS.  THE UNDERGROUND SYSTEMS CONSIST OF TWO
C INSULATED PIPES ENCASED IN THE SAME CONDUIT CASING.
C SUBROUTINES CALLED: PIPEJ,TGO,SOILK,INSULK,TGXX,SOLVLE,PIPEHL,TWOPIP,
C                     EQUIKO.
C     INPUT DATA FILES: SDTA1FJ AND SDTA2FJ
C     OUTPUT FILE: SOUTFJ
C     X(I): THE X-COORDINATE OF NODAL POINT I,IN FT
C     Y(I): THE Y-COORDINATE OF NODAL POINT I,IN FT
C     (NODE(M,I),I=1,3): THREE NODAL POINTS OF ELEMENT M
C     M      ELEMENT INDEX
C     NE     TOTAL NUMBER OF ELEMENTS
C     NN     TOTAL NUMBER OF NODAL POINTS
C     MZ     TOTAL NUMBER OF KNOWN NODAL TEMPERATURES
C     C      THERMAL CONDUCTIVITY, BTU-IN/HR/FT**2/DEG F
C     L      THICKNESS OF THE ELEMENT, FT
C     T(I): THE TEMPERATURE OF NODAL POINT I, IN DEG F
      REAL L,KK,KI,KG,KIX1,KIX2,KTCT,KASP,KCAS,KBF
      CHARACTER*4 TITLE(15)
      DIMENSION Q(150),T(150),X(150),Y(150),KK(150,150)
      DIMENSION AS(250),B2IZ(250),B3IZ(250),B2JZ(250),B2KZ(250),
     & B3JZ(250),B3KZ(250)
      DIMENSION CC(250),TGX(12,7),QQ(150),NODE(250,3),MAT(250)
      DIMENSION HIJ(250),HJK(250),HKI(250),TIJ(250),TJK(250),
     & TKI(250),HHIJ(250),HHJK(250),HHKI(250),IXCB(250)
      DIMENSION CK(150,150),DQ(150),XT(150),INDX(150),VV(150)
      COMMON/PP/TP1,TP2,KI,KG,D1,D2,TH1,TH2,DP1,DP2,S1,S2,TG,
     & WW,HY,MONTH
      COMMON /EKC/D1P,D2P,DIC,THK1,THK2
      COMMON /ST/AO,BO,DIFF
      PI=4.*ATAN(1.)
      NP=150
      OPEN (8,FILE='SDTA1FJ')
      OPEN (7,FILE='SOUTFJ',STATUS='NEW',FORM='FORMATTED')
      OPEN (9,FILE='SDTA2FJ')
C  READ IN THE TITLE OF THE PROBLEM TO BE ANALYZED
      READ (8,2,ERR=2000) TITLE
2     FORMAT(15A4)
      WRITE (7,3) TITLE
3     FORMAT(1X,15A4)
C  READ TOTAL NUMBER OF NODAL POINTS, TOTAL NUMBER OF TRIANGULAR
C  ELEMENTS, TOTAL NUMBER OF KNOWN NODAL TEMPERATURES, THE FIRST
C  ELEMENT INDEX OF PIPE INSULATION, THE FIRST NODE INDEXES OF
C  PIPE INSULATION FOR PIPES NO.1 AND NO.2, RESPECTIVELY, AND THE
C  FIRST NODE INDEX OF THE INNER SURFACE OF THE CONDUIT CASING.
      READ (8,*) NN,NE,MZ,MINS,NINS1,NINS2,NSIC
C  SET THE UNIT NUMBER OF THE PRINTER
      MO=7
C  READ MONTH OF INTEREST AND THE INDEX FOR FINITE ELEMENT GRID DATA
C  TO BE PRINTED OUT : ICALB = 1  PRINT OUT INTERMEDIATE RESULTS
C                                 DURING ITERATIONS
C                            = 0  NO PRINT OUT
      READ (8,*) MONTH,ICALB
C   READ THE THERMAL CONDUCTIVITY (IN BTU-IN./H-FT**2 - DEG F),
C   THICKNESS (IN INCHES) OF THE SIDE, THE DEPTH (IN FT.) OF EARTH
C   COVER (IN FT.), AND THE THICKNESS (IN FT.) OF BOTTOM BED OF
C   THE INNER EARTH REGION.
      READ (8,*) KTCT,TRTK,D,F
C   READ IN THE ESTIMATED AVERAGE TEMPERATURE OF AIR INSIDE THE
C   AIRSPACE BETWEEN INSULATED PIPES AND OUTER CASING, IN DEG F,
C   AND THE TEMPERATURE DIFFERENCE BETWEEN THE INSULATED PIPE
C   SURFACE TEMPERATURE AND THE INNER SURFACE TEMPERATURE OF THE
C   CASING, IN DEG F.
      READ (8,*) TAS,TDEL
C  READ IN THE THERMAL CONDUCTIVITY (IN BTU-IN/H-FT**2-DEG F) OF
C  THE PIPE CASING AND THE CONDUCTIVITY OF POURED-IN INSULATION
C  MATERIAL OR BACK-FILL SOIL SURROUNDING THE PIPES IN THE
C  INNERMOST REGION.
      READ (8,*) KCAS,KBF
C  READ IN TOTAL EMISSIVITIES OF OUTER SURFACE OF PIPE INSULATION
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C   AND INNER SURFACE OF CONDUIT CASING, RESPECTIVELY, AND THE ESTIMATED
C   AVERGE VALUES OF OUTER SURFACE TEMPERATURES OF PIPE INSULATION AND
C   THE INNER SURFACE TEMPERATURES OF THE OUTER CASING, IN DEG F.
        READ (8,*) EINS,ECAS,TEPS,TSIC
C   READING IN INPUT DATA FOR CALCULATIONS OF PIPE HEAT LOSS AND
C   GENERATION OF THE COORDINATES OF NODAL POINTS
        CALL PIPEJ(X,Y,TRTK,D,F,INXK,ITYPE)
        CALL TWOPIP(1)
        CALL EQUIKO(TAS,TDEL,EINS,ECAS,TEPS,TSIC,KASP)
        IF(ICALB.EQ.1) THEN
        WRITE(7,5)
5       FORMAT('  X(M),M=1,NN')
        WRITE(7,7) (X(I),I=1,NN)
7       FORMAT(10F7.2)
        WRITE(7,10)
10      FORMAT('  Y(M),M=1,NN')
        WRITE(7,7) (Y(I),I=1,NN)
        END IF
C   CALCULATIONS OF UNDISTURBED EARTH TEMPERATURES AT VARIOUS DEPTHS
        CALL TGO(TGX,PI,Y)
C   INITIALIZATION OF THE INDEX OF CONVECTION BOUNDARY FOR ELEMENT N
        DO 12 N=1,NE
12      IXCB(N)=O
C   PERFORM ITERATIONS TO ACCOUNT FOR THE TEMPERATURE EFFECTS ON SOIL
C   AND INSULATION THERMAL CONDUCTIVITIES
           DO 24 I=1,NN
24              T(I)=TG
         DO 26 I=1,NE
             HIJ(I)=0.
             HJK(I)=0.
             HKI(I)=0.
             TIJ(I)=0.
             TJK(I)=0.
             TKI(I)=0.
             HHIJ(I)=0.
             HHJK(I)=0.
             HHKI(I)=0.
26        CONTINUE
C       READING IN THE ELEMENT NUMBER AND ITS NODAL POINTS AND THE
C       MATERIAL TYPE, WHICH INCLUDES
C           MAT(J) = 1  SOIL IN INNER EARTH REGION
C                  = 2  PIPE INSULATION
C                  = 3  AIR SPACE SURROUNDING THE PIPE IN CASING
C                  = 4  OUTER CASING OF THE INSULATED PIPE
C                  = 5  POURED-IN INSULATION MATERIAL IN BACK-FILL
C                       REGION
C                  = 6  NATIVE SOIL IN THE OUTER EARTH REGION
        DO 30 I=1,NE
           READ(9,*) J,(NODE(J,K),K=1,3),MAT(J)
           IF (MAT(J).EQ.1) CC(J)=KTCT/12.
           IF (MAT(J).EQ.2) CC(J)=KI/12.
           IF (MAT(J).EQ.3) CC(J)=KASP/12.
           IF (MAT(J).EQ.4) CC(J)=KCAS/12.
           IF (MAT(J).EQ.5) CC(J)=KBF/12.
           IF (MAT(J).EQ.6) CC(J)=KG/12.
30      CONTINUE
C    READ IN TOTAL NUMBER OF ELEMENTS HAVING BOUNDARY SEGMENTS SUBJECT
C    TO CONVECTIVE HEAT TRANSFER
        READ (9,*) NECB
C    READ IN ELEMENT NUMBER, CONVECTIVE HEAT TRANSFER COEFFICIENTS,
C    AND AMBIENT TEMPERATURES FOR THREE BOUNDARY SEGMENTS
        DO 35 I=1,NECB
          READ (9,*) M,HIJ(M),HJK(M),HKI(M),TIJ(M),TJK(M),TKI(M)
          IXCB(M)=1
35      CONTINUE
        ITER=1
38      DO 40 I=1,NN
        DO 40 J=1,NN
             Q(I)=0.
             KK(I,J)=0.
             QQ(I)=0.
             CK(I,J)=0.
             DQ(I)=0.
```

```
          VV(I)=1.0
          INDX(I)=1
40    CONTINUE
      L=1.
      DO 180 M=1,NE
        I=NODE(M,1)
        J=NODE(M,2)
        K=NODE(M,3)
        IF(MAT(M).EQ.3) CC(M)=KASP/12.
        C=CC(M)
        IF ((INXK.EQ.0).OR.(ITER.EQ.1)) GO TO 60
C  DETERMINE SOIL AND INSULATION THERMAL CONDUCTIVITIES BASED ON THE
C  MEAN TEMPERATURES
        TM=(T(I)+T(J)+T(K))/3.
        IF(MAT(M).EQ.2)  CALL INSULK(TM,C,ITYPE)
        IF(MAT(M).EQ.6)  CALL SOILK(TM,KG,C)
        IF(MAT(M).EQ.1)  CALL SOILK(TM,KG,C)
        CC(M)=C
60    XI=X(I)
      XJ=X(J)
      XK=X(K)
      YI=Y(I)
      YJ=Y(J)
      YK=Y(K)
      CXX=C
      CXY=0.
      CYX=0.
      CYY=C
      B2I=YJ-YK
      B3I=XK-XJ
      B2J=YK-YI
      B3J=XI-XK
      B2K=YI-YJ
      B3K=XJ-XI
C   CALCULATE THE ELEMENT AREA
      SA=0.5*(XJ*B2J+XI*B2I+XK*B2K)
      SA=ABS(SA)
      A2=SA*2.
      AS(M)=A2
      B2I=B2I/A2
      B3I=B3I/A2
      B2J=B2J/A2
      B3J=B3J/A2
      B2K=B2K/A2
      B3K=B3K/A2
      B2IZ(M)=B2I
      B3IZ(M)=B3I
      B2JZ(M)=B2J
      B3JZ(M)=B3J
      B2KZ(M)=B2K
      B3KZ(M)=B3K
      BII=SA*L*(B2I*B2I*CXX+B2I*B3I*CXY+B3I*B2I*CYX+B3I*B3I*CYY)
      BIJ=SA*L*(B2I*B2J*CXX+B2I*B3J*CXY+B3I*B2J*CYX+B3I*B3J*CYY)
      BIK=SA*L*(B2I*B2K*CXX+B2I*B3K*CXY+B3I*B2K*CYX+B3I*B3K*CYY)
      BJI=SA*L*(B2J*B2I*CXX+B2J*B3I*CXY+B3J*B2I*CYX+B3J*B3I*CYY)
      BJJ=SA*L*(B2J*B2J*CXX+B2J*B3J*CXY+B3J*B2J*CYX+B3J*B3J*CYY)
      BJK=SA*L*(B2J*B2K*CXX+B2J*B3K*CXY+B3J*B2K*CYX+B3J*B3K*CYY)
      BKI=SA*L*(B2K*B2I*CXX+B2K*B3I*CXY+B3K*B2I*CYX+B3K*B3I*CYY)
      BKJ=SA*L*(B2K*B2J*CXX+B2K*B3J*CXY+B3K*B2J*CYX+B3K*B3J*CYY)
      BKK=SA*L*(B2K*B2K*CXX+B2K*B3K*CXY+B3K*B2K*CYX+B3K*B3K*CYY)
      KK(I,I)=KK(I,I)+BII
      KK(I,J)=KK(I,J)+BIJ
      KK(I,K)=KK(I,K)+BIK
      KK(J,I)=KK(J,I)+BJI
      KK(J,J)=KK(J,J)+BJJ
      KK(J,K)=KK(J,K)+BJK
      KK(K,I)=KK(K,I)+BKI
      KK(K,J)=KK(K,J)+BKJ
      KK(K,K)=KK(K,K)+BKK
      IF(IXCB(M).EQ.0) GO TO 130
C   ADDITION OF CONVECTION TERMS TO THE ELEMENT MATRIX TO ACCOUNT
C   FOR CONVECTION ON BOUNDARY
C   READING IN CONVECTIVE HEAT TRANSFER COEFFICIENTS AND AMBIENT
```

```
C    TEMPERATURES FOR THREE BOUNDARY SEGMENTS
        HHIJ(M)=HIJ(M)*L*SQRT((X(I)-X(J))**2+(Y(I)-Y(J))**2)/6.
        HHJK(M)=HJK(M)*L*SQRT((X(J)-X(K))**2+(Y(J)-Y(K))**2)/6.
        HHKI(M)=HKI(M)*L*SQRT((X(K)-X(I))**2+(Y(K)-Y(I))**2)/6.
        KK(I,I)=HHIJ(M)*2.+HHKI(M)*2.+KK(I,I)
        KK(I,J)=HHIJ(M)+KK(I,J)
        KK(I,K)=HHKI(M)+KK(I,K)
        KK(J,I)=HHIJ(M)+KK(J,I)
        KK(J,J)=HHIJ(M)*2+HHJK(M)*2.+KK(J,J)
        KK(J,K)=HHJK(M)+KK(J,K)
        KK(K,I)=HHKI(M)+KK(K,I)
        KK(K,J)=HHJK(M)+KK(K,J)
        KK(K,K)=HHJK(M)*2.+HHKI(M)*2.+KK(K,K)
        HHIJ(M)=TIJ(M)*3.*HHIJ(M)
        HHJK(M)=TJK(M)*3.*HHJK(M)
        HHKI(M)=TKI(M)*3.*HHKI(M)
130     Q(I)=Q(I)+HHIJ(M)+HHKI(M)
        Q(J)=Q(J)+HHIJ(M)+HHJK(M)
        Q(K)=Q(K)+HHJK(M)+HHKI(M)
180     CONTINUE
185     FORMAT('      M     I     J     K    MAT.    C')
187     FORMAT(1X,5I6,F10.4)
C    DETERMINE OUTER SURFACE TEMPERATURES OF UNDERGROUND PIPES
        DO 200 I=1,8
          T(I)=TP1
          II=I+8
          T(II)=TP2
200     CONTINUE
C    DETERMINE OUTER BOUNDARY TEMPERATURES OF EARTH REGION
        CALL TGXX(T,TGX,MONTH)
        MZ1=MZ+1
        DO 260 I=MZ1,NN
          SUM=0.
          DO 250 J=1,MZ
250         SUM=SUM+KK(I,J)*T(J)
          QQ(I)=Q(I)-SUM
260     CONTINUE
        IF(ICALB.EQ.1) THEN
           WRITE(7,280)
280        FORMAT(6X,'QQ    ARRAY')
           WRITE(7,285) (QQ(I),I=1,NN)
285        FORMAT (5E12.5)
        END IF
C   RENAMING OF MATRICES
        MN=NN-MZ
        DO 300 I=1,MN
           K=MZ+I
          DO 290 J=1,MN
              KL=MZ+J
290           CK(I,J)=KK(K,KL)
              XT(I)=T(K)
              DQ(I)=QQ(K)
300        CONTINUE
C   SOLUTION OF SIMULTANEOUS EQUATIONS
C   SET PHYSICAL DIMENSION OF MATRIX A
        CALL SOLVLE(CK,MN,NP,INDX,VV,DQ)
        DO 310 I=1,MN
             K=MZ+I
             T(K)=DQ(I)
310      CONTINUE
C    CALCULATIONS OF EQUIVALENT THERMAL CONDUCTIVITIES FOR AIR
C    SPACE INSIDE THE OUTER CASING
C
C    CALCULATE AVERAGE OUTER SURFACE TEMPERATURE OF INSULATED PIPES
        SU1=0.0
        SU2=0.0
        DO 312 I=1,8
          L1=I+NINS1-1
          L2=I+NINS2-1
          SU1=SU1+T(L1)
          SU2=SU2+T(L2)
312     CONTINUE
        TSM1=SU1/8.
```

```
      TSM2=SU2/8.
C  DETERMINE THE EFFECTIVE SURFACE TEMPERATURE OF INSULATED PIPES,
C  IN DEG F.
      DIP1=D1P+2.*THK1
      DIP2=D2P+2.*THK2
      TEFPS=(DIP1*TSM1+DIP2*TSM2)/(DIP1+DIP2)
C   CALCULATE THE INNER SURFACE TEMPERATURE OF THE CASING
      SU3=0.0
      DO 314 I=1,8
        L3=I+NSIC-1
314     SU3=SU3+T(L3)
      TSIC=SU3/8.
C   DETERMINE THE TEMPERATURE DIFFERENCE BETWEEN THE EFFECTIVE SURFACE
C   TEMPERATURE OF INSULATED PIPES AND THE INNER SURFACE OF THE CASING,
C   IN DEG F
      TDEL=ABS(TEFPS-TSIC)
      TAS=(TEFPS+TSIC)/2.
      CALL EQUIKO(TAS,TDEL,EINS,ECAS,TEPS,TSIC,KASP)
      WRITE(7,320) KASP
320   FORMAT('  KASP=',F10.4,2X,'(BTU-IN./H-FT**2-DEG F)')
330   FORMAT('  TEMPERATURE ARRAY : T(I), I=1,NN ')
C  CALCULATE THE MEAN VALUES OF INSULATION THERMAL CONDUCTIVITY FOR
C  PIPES 1 AND 2
350   SKI1=0.
      SKI2=0.
      DO 400 LN=1,16
         LM=MINS+LN-1
         LL=LM+16
         SKI1=SKI1+CC(LM)
         SKI2=SKI2+CC(LL)
400   CONTINUE
      KIX1=SKI1/16.
      KIX2=SKI2/16.
      R1=D1/24.
      R2=D2/24.
      TH1X=TH1/12.
      TH2X=TH2/12.
      IF(ICALB .EQ. 0) MO=11
C  CALCULATIONS OF THE HEAT LOSSES FROM THE UNDERGROUND PIPES
      CALL  PIPEHL(T,R1,R2,TH1X,TH2X,KIX1,KIX2,MO,QTX)
      HLOSS=QTX
      IF(ITER.EQ.1) HLOSX=0.
C   DETERMINE IF PIPE HEAT LOSS VALUE HAS CONVERGED, OR CONTINUE
C   ITERATIONS IF REQUIRED
      DELQT=ABS(HLOSS-HLOSX)/HLOSS
      IF(DELQT.LE. 0.010) GO TO 2010
      ITER=ITER+1
      HLOSX=HLOSS
      GO TO 38
2000  WRITE (7,2005)
2005  FORMAT (1X,'THERE ARE SOME ERRORS IN INPUT DATA')
2010  WRITE (7,185)
      DO 2020 I=1,NE
      WRITE (7,187) I,(NODE(I,J),J=1,3),MAT(I),CC(I)
2020  CONTINUE
      WRITE (7,330)
      WRITE (7,285) (T(I),I=1,NN)
      CALL PIPEHL(T,R1,R2,TH1X,TH2X,KIX1,KIX2,7,QTX)
      STOP
      END

      SUBROUTINE TGO(TGX,PI,Y)
C   THIS SUBROUTINE CALCULATES THE UNDISTURBED EARTH TEMPERATURES
C   AT VARIOUS DEPTHS
      DIMENSION TGX(12,7),Y(150)
C     READING IN THE ANUAL AVERAGE TEMPERATURE AND AMPLITUDE OF THE
C     MONTHLY NORMAL TEMPERATURE CYCLE OF THE SITE, IN DEG F, AND
C     THERMAL DIFFUSIVITY OF SOIL, IN FT**2/H.
      READ (8,*) AO,BO,DIFF
      W=2.*PI/12.
      WZ=2.*PI/(8760*DIFF*2)
      ZZ=SQRT(WZ)
      DO 1 I=1,12
```

```
      DO 1 J=1,7
      Z=ZZ*Y(33-J)
1     TGX(I,J)=AO+BO*EXP(-Z)*SIN(W*(I-3)-Z)
      RETURN
      END

      SUBROUTINE TGXX(T,TGX,MONTH)
C THIS SUBROUTINE PROVIDES OUTER BOUNDARY TEMPERATURES OF EARTH REGION
      DIMENSION T(150),TGX(12,7)
      T(32)=TGX(MONTH,1)
      DO 1 I=1,8
         II=I+32
1        T(II)=T(32)
      DO 5 I=2,7
         I15=I+15
         JI=33-I
         T(I15)=TGX(MONTH,I)
         T(JI)=TGX(MONTH,I)
5     CONTINUE
      DO 10 I=1,3
         I22=I+22
10     T(I22)=TGX(MONTH,7)
      RETURN
      END

      SUBROUTINE INSULK(TM,C,ITYPE)
C  THIS SUBROUTINE DETERMINES THE THERMAL CONDUCTIVITY OF PIPE
C  INSULATION (CALCIUM SILICATE AND MINERAL WOOL) AS A FUNCTION
C  OF THE MEAN TEMPERATURE.
      REAL KN(16),KINS
      DIMENSION TN(16)
      DATA KN /0.375,0.40,0.42,0.45,0.48,0.50,0.53,0.555,0.58,0.61,
     & 0.63,0.66,0.68,0.74,0.82,0.90/
C   DETERMINE THE THERMAL CONDUCTIVITY OF CALCIUM SILICATE
      IF (ITYPE .EQ. 1) THEN
      DO 5 J=1,16
      IF(J .LE. 13) THEN
         TN(J)=100.+(J-1)*50.
      ELSE
         TN(J)=700.+(J-13)*100.
      END IF
5     CONTINUE
      IF(TM .GT. TN(1)) GO TO 10
      KINS=KN(1)
      GO TO 100
10    IF(TM .LT. TN(16)) GO TO 20
      KINS=KN(16)
      GO TO 100
20    DO 50 I=1,15
      T1=TM-TN(I)
      IF(T1 .NE. 0.) GO TO 30
      KINS=KN(I)
      GO TO 100
30    T2=TN(I+1)-TM
      IF(T2 .NE. 0.) GO TO 40
      KINS=KN(I+1)
      GO TO 100
40    P=T1*T2
      IF(P .LT. 0.) GO TO 50
      KINS=KN(I)+T1*(KN(I+1)-KN(I))/(TN(I+1)-TN(I))
      GO TO 100
50    CONTINUE
100   C=KINS/12.
C   DETERMINE THE THERMAL CONDUCTIVITY OF MINERAL WOOL
      ELSE IF (ITYPE .EQ. 2) THEN
      KINS=0.2420 + TM*1.5501E -4 + TM*TM*7.5001E -7
      C=KINS/12.
      ELSE
      RETURN
      END IF
      RETURN
      END
```

```
      SUBROUTINE  SOILK(TM,KG,C)
C THIS ROUTINE DETERMINES THE THERMAL CONDUCTIVITY OF SOIL AS A
C FUNCTION OF MEAN TEMPERATURES.
      REAL K(14),KG
      DIMENSION TX(14)
      DATA K/1.1,1.1,1.1,1.0,0.4,0.31,0.25,0.19,0.15,0.11,0.09,0.07,
     & 0.05,0.05/
      DO 1 I=1,14
1        TX(I)=50.+(I-1)*25.
      IF(TM.GT.TX(1)) GO TO 5
      ZK=1.1
      GO TO 50
5     IF(TM.LT.TX(14)) GO TO 10
      ZK=0.05
      GO TO 50
10    DO 25 I=1,13
      T1=TM-TX(I)
      IF(T1.NE.0) GO TO 15
      ZK=K(I)
      GO TO 50
15    CONTINUE
      T2=TM-TX(I+1)
      IF(T2.NE.0.) GO TO 20
      ZK=K(I+1)
      GO TO 50
20    CONTINUE
      P=T1*T2
      IF(P.GT.0) GO TO 25
      ZK=K(I+1)+T2*(K(I+1)-K(I))/25.
      GO TO 50
25    CONTINUE
50    C=ZK*KG/(1.1*12.)
      RETURN
      END

      SUBROUTINE PIPEHL(T,R1,R2,TH1,TH2,ZKS1,ZKS2,MO,QT)
C  THIS SUBROUTINE CALCULATES THE AVERAGE TEMPERATURE DROPS ACROSS THE
C  PIPE INSULATIONS AND THE RATES OF HEAT LOSS FROM THE UNDERGROUND
C  PIPES IN DIRECTLY BURIED CONDUIT SYSTEM
      DIMENSION T(150)
      PI=4.*ATAN(1.)
      SUM1=0.
      SUM2=0.
      N1=8
      DO 1 I=1,N1
         K1=I
         K2=I+8
         K3=I+84
         K4=I+92
         SUM1=SUM1+T(K1)-T(K3)
         SUM2=SUM2+T(K2)-T(K4)
1     CONTINUE
      T1=SUM1/N1
      T2=SUM2/N1
      ZKIS1=ZKS1*12.
      ZKIS2=ZKS2*12.
      Q1=ZKS1*2.*PI*T1/LOG((R1+TH1)/R1)
      Q2=ZKS2*2.*PI*T2/LOG((R2+TH2)/R2)
      QT=Q1+Q2
      IF(MO .EQ. 11) GO TO 50
      WRITE(MO,5) ZKIS1,ZKIS2
5     FORMAT(/'  AVERAGE VALUES OF PIPE INSULATION THERMAL',
     &' CONDUCTIVITY :',/,'    KI1 = ',F10.3,'    KI2 = ',F10.3,
     & '   BTU-IN/H-FT**2-DEG F ')
      WRITE(MO,10) T1,T2
10    FORMAT(/'  AVERAGE TEMPERATURE DROPS ACROSS INSULATION : ',/,
     & '    T1= ',F10.2,'    T2= ',F10.2,'  DEG F')
      WRITE(MO,20) Q1,Q2,QT
20    FORMAT(/,2X,'HEAT LOSSES FROM UNDERGROUND PIPES :'/'    Q1=',
     & F10.2, '    Q2=',F10.2,'    QT=',F10.2,'  BTU/H-FT')
50    RETURN
      END
```

```
      SUBROUTINE PIPEJ(X,Y,TRTK,D,F,INXK,ITYPE)
C THIS SUBROUTINE READS IN THE INPUT DATA TO BE USED FOR CALCULATIONS
C OF THE HEAT LOSSES FROM THE UNDERGROUND PIPES AND GENERATES X AND Y-
C COORDINATES OF NODAL POINTS FOR THE TWO PIPE SYSTEM.
      REAL KII,KIG,KI,KG
      DIMENSION X(150),Y(150)
      COMMON /PP/TP1,TP2,KII,KIG,DI1,DI2,THI1,THI2,B1,B2,S1,S2,TG,
     & WW,HY,MONTH
      COMMON /EKC/D1P,D2P,DIC,THK1,THK2
C READ TEMPERATURE OF PIPE NUMBERS 1 AND 2, IN DEG F
      READ (8,*) TP1,TP2
C READ THERMAL CONDUCTIVITY OF THERMAL INSULATION AND SOIL,
C RESPECTIVELY, IN BTU-IN./H-FT**2 - DEG F, AND INDEX OF THERMAL
C CONDUCTIVITY : INXK = 0  CONSTANT THERMAL CONDUCTIVITY
C                     = 1  TEMPERATURE DEPENDENT THERMAL CONDUCTIVITY
C AND THE MATERIAL TYPE OF PIPE INSULATION, WHICH INCLUDES
C                ITYPE = 1  CALCIUM SILICATE
C                      = 2  MINERAL WOOL
      READ (8,*) KII,KIG,INXK,ITYPE
C READING IN THE OUTSIDE DIAMETERS OF STEEL PIPES 1 AND 2, IN INCHES
      READ (8,*) DI1,DI2
C READING IN THE THICKNESS OF THERMAL INSULATION USED FOR PIPES 1
C AND 2, RESPECTIVELY, IN INCHES
      READ (8,*) THI1,THI2
C READ IN THE INSIDE DIAMETER AND THICKNESS OF OUTER CASING, AND THE
C DEPTH OF ITS CENTER BELOW GROUND SURFACE, IN INCHES.
      READ (8,*) DIAC,THKC,DEPC
C   READING IN THE VERTICAL DISTANCES (IN FT.) FROM HORIZONTAL
C   CENTERLINE OF THE INNER EARTH REGION TO CENTERS OF PIPES 1 AND
C   2, RESPECTIVELY, AND THE AVERAGE EARTH TEMPERATURE, IN DEG F.
      READ (8,*) S1,S2,TG
C   READ IN THE THICKNESS OF EARTH COVER,WIDTH AND DEPTH OF OUTER
C   EARTH REGION SURROUNDING THE UNDERGROUND SYSTEM, IN FT.
      READ (8,*) E,WW,HY
      WRITE(7,10) TP1,TP2,KII,KIG,DI1,DI2
10    FORMAT('   TP1    TP2     KI     KG     D1     D2'/6F7.2)
      WRITE(7,20) THI1,THI2,DIAC,THKC,DEPC,S1,S2,TG
20    FORMAT('   THI1   THI2   DIAC   THKC   DEPC     S1     S2     TG',
     & /8F7.2)
      WRITE(7,30) E,WW,HY,MONTH
30    FORMAT('      E     WW     HY  MONTH '/3F7.2,I7)
C   READ IN THE INSIDE WIDTH AND HEIGHT OF THE INNER EARTH REGION,
C   FOR LOOSE-FILL INSULATION SYSTEM, IN FT.
      READ (8,*) A,B
C   CHANGE TO ENGINEERING UNITS
      D1=DI1/12.
      R1=D1*0.5
      D2=DI2/12.
      R2=D2*0.5
      D1P=DI1/12.
      D2P=DI2/12.
      DEP=DEPC/12.
      DIC=DIAC/12.
      KI=KII/12.
      KG=KIG/12.
      W=A+2*TRTK/12
      H=B+E+D+F
      WRITE(7,40) W,H,D,F,A,B,WW,HY
40    FORMAT('      W      H      D      F      A      B     WW     HY'
     &,/,8F7.2)
      PI=4.*ATAN(1.)
      TH1=THI1/12.
      TH2=THI2/12.
      THCA=THKC/12.
      THK1=THI1/12.
      THK2=THI2/12.
C   DETERMINE THE X AND Y-COORDINATES OF EARTH COVER, SIDES AND BOTTOM
C   BED OF THE INNER EARTH REGION (NODAL POINTS 41 TO 72)
      DO 50 I=1,5,2
        I67=I+67
        X(I67)=W-(I-1)*W/4.
50      Y(I67)=E
      DO 60 I=1,3,2
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```
          I68=I+68
          X(I68)=(W+A)*0.5 - A*(I-1)*0.5
 60       Y(I68)=E
       DO 65 I=1,4
          I40=I+40
          I44=I+44
          I48=I+48
          I52=I+52
          I1=I-1
          X(I40)=(W-A)*0.5
          Y(I40)=(D+E)+I1*B/4.
          X(I44)=0.5*(W-A)+I1*A/4.
          Y(I44)=D+E+B
          X(I48)=(W+A)*0.5
          Y(I48)=(D+E+B)-I1*B/4.
          X(I52)=0.5*(W+A)-I1*A/4.
          Y(I52)=D+E
 65    CONTINUE
       DO 70 I=1,3
          I56=I+56
          I60=I+60
          I64=I+64
          X(I56)=0.0
          Y(I56)=(D+E)+B*(I-1)/2.
          X(I60)=(W-A)*0.5+(I-1)*A*0.5
          Y(I60)=H
          X(I64)=W
          Y(I64)=(D+E)+(3-I)*B/2.
 70    CONTINUE
          X(60)=0.0
          Y(60)=H
          X(64)=W
          Y(64)=H
C   THE X AND Y-COORDINATES OF OUTER BOUNDARY EARTH SURROUNDING THE
C   DIRECTLY BURIED CONDUITS (NODAL POINTS 17 TO 40,AND 73 TO 84)
           X(17)=-WW
           Y(17)=E+0.083
           X(31)=W+WW
           Y(31)=E+0.083
       DO 72 I=1,2
          I17=I+17
          I28=I+28
          X(I17)=-WW
          Y(I17)=(D+E)+I*B*0.25
          X(I28)=W+WW
          Y(I28)=(D+E)+(3-I)*B*0.25
 72    CONTINUE
       DO 75 I=1,3
          I19=I+19
          I22=I+22
          I25=I+25
          X(I19)=-WW
          Y(I19)=H+HY*(I-1)*0.5
          X(I22)=W*(I-1)*0.5
          Y(I22)=H+HY
          X(I25)=W+WW
          Y(I25)=H+HY*(3-I)*0.5
 75    CONTINUE
       DO 77 I=1,5,2
          I33=I+33
          X(I33)=W-(I-1)*W/4.
 77       Y(I33)=0.0
       DO 78 I=1,3,2
          I34=I+34
          X(I34)=(W+A)*0.5-A*(I-1)*0.5
 78       Y(I34)=0.0
       DO 80 I=1,2
          I31=I+31
          I38=I+38
          X(I31)=W+WW*(3-I)*0.5
          Y(I31)=0.
          X(I38)=-WW*0.5*I
          Y(I38)=0.
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80     CONTINUE
       DO 82 I=1,2
          I72=I+72
          I74=I+74
          I80=I+80
          I82=I+82
          X(I72)=-WW*0.5
          Y(I72)=E+(I-1)*D
          X(I74)=-WW*0.5
          Y(I74)=(E+D+B*0.5)+(I-1)*(F+B*0.5)
          X(I80)=W+0.5*WW
          Y(I80)=(E+D+B+F)-(I-1)*(F+0.5*B)
          X(I82)=W+0.5*WW
          Y(I82)=(E+D)-(I-1)*D
82     CONTINUE
       DO 85 I=1,4
          I76=I+76
          X(I76)=W*(I-1)/3.
85        Y(I76)=H+HY*0.5
C    X AND Y-COORDINATES OF THE CENTERS OF THE PIPES
       XC1=W*0.5
       B1=DEP-S1
       YC1=B1
       XC2=W*0.5
       B2=DEP+S2
       YC2=B2
       WRITE(7,90) XC1,YC1,XC2,YC2,B1,B2
90     FORMAT('    XC1     YC1     XC2     YC2      B1      B2'/6F7.3)
C    THE X AND Y-COORDINATES OF NODAL POINTS AT THE INNER AND OUTER
C    SURFACES OF PIPE INSULATION (NODAL POINTS 1 TO 16, AND 85 TO
C    100)
       DO 95 I=1,8
          THETA=2.*PI*I/8.
          I8=I+8
          I84=I+84
          I92=I+92
          X(I)=XC1+0.5*D1*SIN(THETA)
          Y(I)=YC1+0.5*D1*COS(THETA)
          X(I8)=XC2+0.5*D2*SIN(THETA)
          Y(I8)=YC2+0.5*D2*COS(THETA)
          X(I84)=XC1+(TH1+R1)*SIN(THETA)
          Y(I84)=YC1+(TH1+R1)*COS(THETA)
          X(I92)=XC2+(TH2+R2)*SIN(THETA)
          Y(I92)=YC2+(TH2+R2)*COS(THETA)
95     CONTINUE
C   THE X AND Y-COORDINATES OF NODAL POINTS AT THE INNER AND OUTER
C   SURFACES OF PIPE CASINGS (NODAL POINTS 101 TO 116)
       DO 97 I=1,8
          THETA=2.*PI*I/8.
          I100=I+100
          I108=I+108
          X(I100)=XC1+DIC*0.5*SIN(THETA)
          Y(I100)=DEP+DIC*0.5*COS(THETA)
          X(I108)=XC1+(THCA+DIC*0.5)*SIN(THETA)
          Y(I108)=DEP+(THCA+DIC*0.5)*COS(THETA)
97     CONTINUE
C    THE X AND Y-COORDINATES OF NODAL POINTS AT THE CENTER OF
C    CONDUIT CASING,AND AT THE TOP AND THE BOTTOM OF AIRSPACE
C    (NODAL POINTS 117 TO 119).
          X(119)=W*0.5
          Y(119)=DEP
          X(117)=W*0.5
          Y(117)=0.5*(Y(88)-Y(104))
          X(118)=W*0.5
          Y(118)=0.5*(Y(108)-Y(100))
C    THE X AND Y-COORDINATES OF NODAL POINTS IN BACK-FILL SOIL, OR
C    POURED-IN INSULATION SURROUNDING THE PIPES (NODAL POINTS 120 TO
C    131)
         YUP=0.5*(Y(112)-Y(55))
         XLT=0.5*(X(114)-X(43))
         YLO=0.5*(Y(47)-Y(116))
         XRT=0.5*(X(51)-X(110))
       DO 100 I=1,3
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          I119=I+119
          I122=I+122
          I125=I+125
          I128=I+128
          X(I119)=0.5*(W+A)-0.25*A*I
          Y(I119)=(D+E)+YUP
          X(I122)=0.5*(W-A)+XLT
          Y(I122)=(D+E)+0.25*B*I
          X(I125)=0.5*(W-A)+0.25*A*I
          Y(I125)=(D+E+B)-YLO
          X(I128)=0.5*(W+A)-XRT
          Y(I128)=(D+E+B)-0.25*B*I
100     CONTINUE
        RETURN
        END

        SUBROUTINE TWOPIP(IREPT)
C   THIS SUBROUTINE DETERMINES THE HEAT LOSSES FROM TWO PIPES TO THE
C   UNDERGROUND SURROUNDING THE HEAT DISTRIBUTION SYSTEM.
        REAL KII,KIG
        COMMON /PP/T1,T2,KII,KIG,DI1,DI2,THI1,THI2,B1,B2,S1,S2,TG,
       & WW,HY,MONTH
        PI=4.*ATAN(1.)
        X1=2.*PI
        R1=DI1/24.
        R2=DI2/24.
        TH1X=THI1/12.
        TH2=THI2/12.
        ZK1=KII/12.
        ZK2=ZK1
        D1=B1
        D2=B2
        ZKS=KIG/12.
        DO 10 I=1,IREPT
2       TH1=TH1X+0.1*(I-1)
        S=S1+S2
        A=R1+R2+TH1+TH2+0.05
        THI1=TH1*12.
        IF(A .LT. S) A=S
        C1=X1*ZK1/LOG((R1+TH1)/R1)
        C2=X1*ZK2/LOG((R2+TH2)/R2)
        P11=1.+C1/(X1*ZKS)*LOG((2*D1)/(R1+TH1))
        P12=C2/(X1*ZKS)*LOG((A*A+(D1+D2)**2)/(A*A+(D1-D2)**2))*0.5
        P21=C1/(X1*ZKS)*LOG((A*A+(D1+D2)**2)/(A*A+(D1-D2)**2))*0.5
        P22=1.+C2/(X1*ZKS)*LOG((2*D2)/(R2+TH2))
        DEL=P12*P21-P11*P22
        ZKP1=C1*(P12-P22)/DEL
        ZKP2=C2*(P21-P11)/DEL
        TP1=(P12*T2-P22*T1)/(P12-P22)
        TP2=(P21*T1-P11*T2)/(P21-P11)
        Q1=ZKP1*(TP1-TG)
        Q2=ZKP2*(TP2-TG)
        QT=Q1+Q2
        TAVG=(T1+T2)*0.5
        ZK=QT/(TAVG-TG)
        WRITE(7,6) DI1,DI2,S1,S2,THI1,KII,KIG,T1,T2
6       FORMAT('   DI1   DI2    S1    S2  THI1    KII   KIG   TP1    TP2',
       &/,7F6.2,1X,2F6.0)
        WRITE(7,8) Q1,Q2,QT,ZK
8       FORMAT('    Q1     Q2     QT       KP'/,3F7.2,2X,F6.3/)
10      CONTINUE
        RETURN
        END

        SUBROUTINE EQUIKO(TAS,TDEL,EINS,ECAS,TEPS,TSIC,KASP)
C    THIS ROUTINE CALCULATES EQUIVALENT THERMAL CONDUCTIVITY OF AIR
C    SPACE SURROUNDING INSULATED PIPES INSIDE THE OUTER CASING.
        REAL KASP
        COMMON /EKC/D1P,D2P,DIC,THK1,THK2
        PI=4.*ATAN(1.)
C    CALCULATE THERMAL CONDUCTIVITY, IN BTU-FT/H-FT**2-DEG F, AND
C    KINEMATIC VISCOSITY, IN FT**2/S, OF AIR
        THKAIR=0.01319 + TAS*2.5E -5
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      VAIR=1.2624E -4 + TAS*5.4E -7
C   CALCULATE THE EFFECTIVE DIAMETER OF THE INSULATED PIPES,
C   IN FT, AND THE CHARACTERISTIC LENGTH OF AIR SPACE
      DEFPIP=(D1P+2.*THK1)+(D2P+2.*THK2)
      CL=(DIC - DEFPIP)*0.5
C   CALCULATE THE PRANDTL NUMBER OF AIR , AND GRASHOF NUMBER, AND
C   EQUIVALENT THERMAL CONDUCTIVITY OF AIRSPACE DUE TO FREE
C   CONVECTION, IN BTU-IN/H-FT**2-DEG F.
C
      CLC=DEFPIP
      PRANTL=0.71849 - TAS * 1.275E -4
      GRASOF=32.2 * TDEL *(CLC**3.)/((VAIR**2.)*(TAS+459.7))
      CONKA=12. *THKAIR*0.530* ((PRANTL*GRASOF)**0.250)
C    CALCULATE THE EMISSIVITY TERM AND EQUIVALENT THERMAL
C    CONDUCTIVITY OF AIRSPACE DUE TO RADIATIVE TRANSFER, IN
C    BTU-IN/H-FT**2-DEG F.
      ETOT=1./EINS + (DEFPIP/DIC)*(1./ECAS - 1.)
      TREP=(TEPS + 459.7)/100.
      TRSI=(TSIC + 459.7)/100.
      HRAD=0.1714/100. * (TREP+TRSI)*(TREP*TREP + TRSI*TRSI)/ETOT
      RADKA=12.0 * HRAD *CL
C    CALCULATE EQUIVALENT THERMAL CONDUCTIVITY OF AIRSPACE, IN
C    BTU-IN/H-FT**2-DEG F.
      KASP=CONKA + RADKA
      WRITE(7,50) PRANTL,GRASOF
50    FORMAT('  PRANTL=',F10.4,2X,'GRASOF=',E12.4)
      WRITE(7,60) CONKA,RADKA
60    FORMAT('  CONKA=',F10.4,2X,' RADKA=',F10.4,2X,
     &'(BTU-IN/H-FT**2-DEF F)')
      RETURN
      END

      SUBROUTINE SOLVLE(A,N,NP,INDX,VV,B)
C   GIVEN AN NXN MATRIX A, WITH PHYSICAL DIMENSION NP, THIS ROUTINE
C   REPLACE IT BY THE LU DECOMPOSITION OF A ROWWISE PERMUTATION OF
C   ITSELF. INDX IS AN OUTPUT VECTOR WHICH RECORD THE ROW PERMUTATION
C   EFFECTED BY THE PARTIAL PIVOTING; VV IS VECTOR OF SCALING FACTORS.
C
C   THIS ROUTINE IS USED TO SOLVE THE LINEAR SET OF EQUATIONS :
C   [A][X]=[B]
C
      DIMENSION A(NP,NP),INDX(N),VV(N),B(N)
C
C   FORM IMPLICIT SCALING VECTOR VV
C
      DO 12 I=1,N
        AAMAX = 0.0
        DO 11 J=1,N
         IF(ABS(A(I,J)).GT.AAMAX) AAMAX=ABS(A(I,J))
   11   CONTINUE
        IF(AAMAX.EQ.0.) THEN
          WRITE(7,100) I
100     FORMAT(1X,'ERROR:SINGULAR MATRIX - ZERO ROW : ROW',I5)
          RETURN
        END IF
       VV(I) = 1.0/AAMAX
   12 CONTINUE
C
C   CROUT METHOD: LOOP OVER COLUMNS
C
      DO 19 J=1,N
        DO 14 I=1,J-1
          SUM = A(I,J)
          DO 13 K=1,I-1
            SUM = SUM - A(I,K)*A(K,J)
   13     CONTINUE
        A(I,J) = SUM
   14   CONTINUE
C
C   PIVOT IMPLEMENTATION
C
        AAMAX = 0.0D0
        DO 16 I=J,N
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```
          SUM = A(I,J)
          DO 15 K=1,J-1
            SUM = SUM - A(I,K)*A(K,J)
   15     CONTINUE
          A(I,J) = SUM
          DUM = VV(I)*ABS(SUM)
          IF(DUM.GE.AAMAX) THEN
            IMAX = I
            AAMAX = DUM
          ENDIF
   16   CONTINUE

        IF(J.NE.IMAX) THEN
          DO 17 K=1,N
            DUM = A(IMAX,K)
            A(IMAX,K) = A(J,K)
            A(J,K) = DUM
   17     CONTINUE
        VV(IMAX) = VV(J)
      ENDIF
      INDX(J) = IMAX
      IF(A(J,J).EQ.0.0) THEN
        WRITE(7,110) J
110    FORMAT(1X,'ERROR: SINGULAR MATRIX - ZERO " DIAG " : ROW',I5)
        RETURN
      END IF
      IF(J.NE.N) THEN
        DUM = 1.0/A(J,J)
        DO 18 I=J+1,N
          A(I,J) = A(I,J)*DUM
   18   CONTINUE
      END IF
   19 CONTINUE
C
C    FORWARD SUBSTITUTION
C
      II = 0
      DO 22 I=1,N
        LL = INDX(I)
        SUM = B(LL)
        B(LL) = B(I)
        IF(II.NE.0) THEN
        DO 21 J=II,I-1
          SUM = SUM - A(I,J)*B(J)
   21     CONTINUE
        ELSE IF(SUM.NE.0.0) THEN
         II = I
        END IF
        B(I) = SUM
   22 CONTINUE
C
C    BACKWARD SUBSTITUTION
C
      DO 24 I=N,1,-1
        SUM = B(I)
        IF(I.LT.N) THEN
          DO 23 J=I+1,N
          SUM = SUM - A(I,J)*B(J)
   23     CONTINUE
        END IF
        B(I) = SUM/A(I,I)
   24 CONTINUE
      RETURN
      END
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11. ABSTRACT (A 200-WORD OR LESS FACTUAL SUMMARY OF MOST SIGNIFICANT INFORMATION. IF DOCUMENT INCLUDES A SIGNIFICANT BIBLIOGRAPHY OR LITERATURE SURVEY, MENTION IT HERE.)

The calculations of heat losses and temperature field for directly buried conduit heat distribution systems were performed using the finite element computer programs. The finite element analysis solved two-dimensional, steady-state heat transfer problems involving two insulated parallel pipes encased in the same conduit casing and in separate casings, and the surrounding earth. Descriptions of the theoretical basis, computational scheme, and the data input and outputs of the developed computer programs are presented. Numerical calculations were carried out for predicting the temperature distributions within the existing high temperature hot water distribution system with two insulated pipes covered in the same metallic conduit and the surrounding soil. The predicted results generally agree with the experimental data obtained at the test site. The deviations between the predicted and measured values are found to range from 0 to 17 percent with an average of 6 percent. The rates of heat loss from two insulated pipes encased in separate conduits were calculated for different pipe sizes, fluid temperatures and insulation thicknesses, and the results were compared with the predictions from steady-state, one-dimensional radial heat conduction model. The discrepancies between finite element and radial conduction models in pipe heat loss values are discussed.

12. KEY WORDS (6 TO 12 ENTRIES; ALPHABETICAL ORDER; CAPITALIZE ONLY PROPER NAMES; AND SEPARATE KEY WORDS BY SEMICOLONS)

Computer program, direct burial, district heating and cooling, finite element method, heat loss, heat transfer, underground heat distribution system

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