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A QUANTITATIVE APPROACH TO CAMERA FIXATION

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ABSTRACT

This paper deals with quantitative aspects of camera fixation for a static scene. In general, when the camera undergoes translation and rotation, there is an infinite number of points that produce equal optical flow for any instantaneous point in time. Using a camera-centered spherical coordinate system, it is shown how to find these points in space. For the case where the rotation axis of the camera is perpendicular to the instantaneous translation vector, these points lie on cylinders. If the elevation component of the optical flow is set to zero then these points form a circle (called the Equal Flow Circle or simply EFC) and a line, i.e., all points that lie on this circle or line are observed as having the same azimuthal optical flow. A special case of the EFCs is the Zero Flow Circle (ZFC) where both components of the optical flow are equal to zero. A fixation point is the intersection of all the ZFCs. Points inside and outside the ZFC can be quantitatively mapped using the EFCs.

In a set of experiments using simulated as well as real data, we show how the concept of the EFC and ZFC can be used to explain the optical flow produced by points near the fixation point, and to explicitly map the space while fixating. It is also shown experimentally that points near the fixation point may change the sign of their optical flow as the camera moves.

1. INTRODUCTION

Camera fixation is defined as actively controlling the camera motion so that a given visible point in 3-D space (a fixated point) is constantly imaged to the same point in the image plane. There are several advantages offered by fixation:

1. Determination of relative range. Because the imaged position of the 3-D fixation point remains constant, the point results in zero optical flow. However, the optical flow arising from static points in a 3-D neighborhood of the fixation point can be used to easily determine whether these points are in front of or behind the fixation point [4]. Also, these points will have relatively small optical flow values, allowing the use of gradient-based flow extraction methods [9],[12],[14].
2. Verifying range. If the range of a static object is hypothesized using some methods, then this range can be verified by pointing the camera optical axis at the object and then fixating on the object at the given range. If the optical flow of points on the object are near zero, then the range is verified.
3. Detailed analysis of objects. If a moving camera is fixated on an object of interest, then this object will be kept in the camera field of view for a long period of time, thus allowing detailed analysis of the object's properties. Also, multiple views of this object will be obtained, resulting in a more complete understanding of the object.
4. Increasing resolution. If a moving camera is fixated on a region of interest, then the camera field of view can be quite narrow. This results in high resolution imagery and allows detailed analysis of the region.
5. Motion compensation. If a camera with pan/tilt mechanism is mounted on a platform that is moving, then by fixating the camera at a very distant point, the camera will have no rotation in an inertial coordinate system. This is a way of maintaining camera orientation in an inertial frame without using an inertial navigation system.
6. Maintaining orientation. Imagine a camera that is free to rotate relative to a moving platform. If the platform undergoes mainly rotation, the camera may fixate for a short time on a point or a feature, then "jump" (saccade) by about 360 ° to fixate on the same point/feature again, followed by a second "jump", etc. By measuring the orientation of the camera relative to the platform, valuable information regarding the orientation of the vehicle in 3D is obtained.
7. Analysis of partially hidden surfaces. If an object of interest is partially occluded, it is possible to get a more complete view of the object by fixating the camera at a point on this surface. This can be done by ignoring points or objects that are closer to the camera (i.e., produce high values of optical flow) and integrate over time points that are close to the fixation point (i.e., produce low values of optical

flow).

The most general form of fixation arises from a six-degree-of-freedom camera motion. A specialized and simple form of fixation is where a camera undergoes translation and no rotation. In this case, the camera can be thought of as fixating on a point lying on the line through the camera pinhole point in the direction of the velocity vector.

In this paper we quantitatively analyze the case where a camera fixates on a point in 3-D space. The paper begins by describing the coordinate systems that are used, followed by deriving expressions for the optical flow in spherical coordinates for six-degree-of-freedom camera motion. These equations are solved to find sets of points in 3-D space that result in equal flow values (and in particular zero flow) for instantaneous camera motion. In the case where the rotation axis is perpendicular to the translation vector, the equal flow points form circles and lines at each instant of time. If the camera motion is further restricted to continuously fixate on a point, we show how the equal-flow circles can be used to quantitatively analyze the space and in particular the neighborhood of the fixation point. In a set of simulation as well as real-data experiments we show how the concept of the Equal Flow Circle (EFC) and Zero Flow Circle (ZFC) can be used to explain the optical flow produced by points near the fixation point, how the optical flow of points on a ZFC disappear at a time instant, and how EFCs and ZFCs can be used to explicitly map the space while fixating.

Previous work in the area of camera fixation has been mainly in the following categories: (1) fixation for qualitative depth estimation [4], (2) fixating on a moving target for tracking applications [2], and (3) stereo fixation for vergence control [4],[6]. Surprisingly, little previous work that leads to a quantitative analysis of single camera fixation has been done [1],[7]. Cutting [7] did an analysis of camera fixation. We use a different analysis approach than he used, and our results are an extension and elaboration of the results he obtained.

2. EQUATIONS OF MOTION AND OPTICAL FLOW

This section describes the equations that relate a point in 3-D space to the projection of that point in the image for general six-degree-of-freedom motion of the camera. Some of the equations can be found in many books, e.g., see [10].

In the following analysis, we assume a moving camera in a stationary environment. Suppose the coordinate system is fixed with respect to the camera as shown in Figure 1. Assume a pinhole camera model and that the pinhole point of the camera is at the origin of the coordinate system. We derive the optical flow components in the spherical coordinates ($R\theta\phi$). In this frame, angular velocities ($\dot{\theta}$ and $\dot{\phi}$) of any point in space, say P, are identical to the optical flow values at P' in the image domain. Figure 2 illustrates this concept: θ and ϕ of a point in space are the same as θ and ϕ of the projected point P' in the image domain, and therefore there is no need to convert angular velocities of points in 3D space to optical flow. In Figure 2 the image domain is a sphere. However, for practical purposes the surface of the image sphere can be mapped onto an image plane (or other surface). This choice is also good from an implementation point of view. The spherical coordinate system is a natural frame to work with when a camera is mounted on a two degree of freedom rotating frame (whose angles can be expressed as Euler angles).

We start with the derivation of the velocity of a 3-D point in the XYZ coordinates (Figure 1). Let the instantaneous coordinates of the point P be $\mathbf{R} = (X, Y, Z)^T$ (where the superscript T denotes transpose). If the instantaneous translational velocity of the camera is $\mathbf{t} = (U, V, W)^T$ and the instantaneous angular velocity is $\boldsymbol{\omega} = (A, B, C)^T$ then the velocity vector $\mathbf{V}$ of the point P with respect to the XYZ coordinate system is:

$$\mathbf{V} = -\,\mathbf{t} - \boldsymbol{\omega} \times \mathbf{R} \tag{1}$$

or:

$$V_X = -U - BZ + CY \tag{2}$$

$$V_Y = -V - CX + AZ \tag{3}$$

$$V_Z = -W - AY + BX \tag{4}$$

where V_X, V_Y, and V_Z are the components of the velocity vector $\mathbf{V}$ along the X, Y, and Z directions respectively.

To convert from $R\theta\phi$ to XYZ coordinates we use the relations:

$$X = R\ \cos\phi\ \cos\theta \tag{5}$$

$$Y = R\ \cos\phi\ \sin\theta \tag{6}$$

$$Z = R\ \sin\phi. \tag{7}$$

Similarly, to convert from XYZ to $R\theta\phi$ coordinates we use:

$$R = \sqrt{X^2+Y^2+Z^2} \tag{8}$$

$$\theta = \tan^{-1}\frac{Y}{X} \tag{9}$$

$$\phi = \sin^{-1}\frac{Z}{\sqrt{X^2+Y^2+Z^2}}. \tag{10}$$

In order to find the optical flow of a 3-D point in $R\theta\phi$ coordinates, we use the following relations and transformations (See [11] and Figure 1):

Let V_R, V_θ, and V_ϕ be the components of the vector $\mathbf{V}$ in spherical coordinates, and

$$V_{R\theta\phi} = \begin{bmatrix} V_R \\ V_\theta \\ V_\phi \end{bmatrix} \tag{11}$$

$$V_{XYZ} = \begin{bmatrix} V_X \\ V_Y \\ V_Z \end{bmatrix}. \tag{12}$$

Then:

$$V_{R\theta\phi} = [T_\phi][T_\theta]V_{XYZ} \tag{13}$$

where

$$[T_\theta] = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{X}{\sqrt{X^2+Y^2}} & \frac{Y}{\sqrt{X^2+Y^2}} & 0 \\ \frac{-Y}{\sqrt{X^2+Y^2}} & \frac{X}{\sqrt{X^2+Y^2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{14}$$

and

$$[T_\phi] = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{X^2+Y^2}}{\sqrt{X^2+Y^2+Z^2}} & 0 & \frac{Z}{\sqrt{X^2+Y^2+Z^2}} \\ 0 & 1 & 0 \\ \frac{-Z}{\sqrt{X^2+Y^2+Z^2}} & 0 & \frac{\sqrt{X^2+Y^2}}{\sqrt{X^2+Y^2+Z^2}} \end{bmatrix}. \tag{15}$$

Also (see [11]):

$$V_R = \dot{R} \tag{16}$$

$$V_\theta = R\,\dot{\theta}\cos\phi \tag{17}$$

$$V_\phi = R\,\dot{\phi} \tag{18}$$

where dot denotes first derivative with respect to time.

Using equations (2)-(18) yields the following expressions:

$$\begin{bmatrix} R\,\dot{\theta}\,\cos\phi \\ R\,\dot{\phi} \end{bmatrix} = \begin{bmatrix} -\sin\theta & \cos\theta & 0 \\ -\sin\phi\cos\theta & -\sin\phi\sin\theta & \cos\phi \end{bmatrix} \begin{bmatrix} -U-BZ+CY \\ -V-CX+AZ \\ -W-AY+BX \end{bmatrix} \tag{19}$$

or

$$\begin{bmatrix} \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \frac{-Y}{X^2+Y^2} & \frac{X}{X^2+Y^2} & 0 \\ \frac{-XZ}{\sqrt{X^2+Y^2}(X^2+Y^2+Z^2)} & \frac{-YZ}{\sqrt{X^2+Y^2}(X^2+Y^2+Z^2)} & \frac{\sqrt{X^2+Y^2}}{X^2+Y^2+Z^2} \end{bmatrix} \begin{bmatrix} -U-BZ+CY \\ -V-CX+AZ \\ -W-AY+BX \end{bmatrix}. \tag{20}$$

As mentioned earlier, $\dot{\theta}$ and $\dot{\phi}$ of a point in space (i.e., the angular velocities in the camera coordinate system) are the *same* as the optical flow components $\dot{\theta}$ and $\dot{\phi}$ (Figure 2).

Suppose that we want to determine the locus of points in 3D space that produce constant optical flow values $\dot{\theta}$ and constant optical flow values $\dot{\phi}$ in the image for a given arbitrary six-degree-of-freedom camera motion. To do so we simply set $\dot{\theta}$ and $\dot{\phi}$ in equation set (20) to the desired constants and solve for X, Y, and Z. All points in 3-D space that satisfy this solution are called Equal Flow Points. However, the solution to these two equations is not unique since there are three unknowns and two equations. In general, there is an infinite number of solutions.

4. A SPECIAL CASE

In this section we analyze a specific motion in the instantaneous XY (ϕ=0) plane of the camera coordinate system. Later we use this analysis for the case where the camera fixates on a single point in space.

Let the camera motion vectors $\mathbf{t}$ and ω be given as follows:

$$\mathbf{t} = (U, V, 0)^T \tag{21}$$

$$\omega = (0, 0, C)^T. \tag{22}$$

This means that the translation vector may lie anywhere in the instantanous XY plane while the rotation is about the Z axis. Substituting these motion vectors into equation set (20) yields:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \dfrac{-Y}{X^2+Y^2} & \dfrac{X}{X^2+Y^2} & 0 \\ \dfrac{-XZ}{\sqrt{X^2+Y^2}(X^2+Y^2+Z^2)} & \dfrac{-YZ}{\sqrt{X^2+Y^2}(X^2+Y^2+Z^2)} & \dfrac{\sqrt{X^2+Y^2}}{X^2+Y^2+Z^2} \end{bmatrix} \begin{bmatrix} -U+CY \\ -V-CX \\ 0 \end{bmatrix} \tag{23}$$

Setting $\dot{\theta}$ and $\dot{\phi}$ in equation set (23) to constants will result in a set of equal flow points for this specific motion.

4.1 EQUAL FLOW CIRCLES

For visualization purposes, we decided to examine the case where the optical flow value of $\dot{\theta}$ is constant and the optical flow value of $\dot{\phi}$ is zero. From equation set (23), the points in space that result from constant $\dot{\theta}$ (regardless of the value of $\dot{\phi}$) form a cylinder of infinite height whose equation is $\left[X+\dfrac{V}{2(C+\dot{\theta})}\right]^2+\left[Y-\dfrac{U}{2(C+\dot{\theta})}\right]^2=\left[\dfrac{V}{2(C+\dot{\theta})}\right]^2+\left[\dfrac{U}{2(C+\dot{\theta})}\right]^2$, as displayed in Figure 3a, and the points in space that result from $\dot{\phi}$=0 (regardless of the value of $\dot{\theta}$) are those that lie on (a) a plane whose equation is $Y=-\dfrac{U}{V}X$, or (b) a plane whose equation is Z=0 (i.e., the XY plane) as pictorially described in Figure 3b. The intersection of the cylinder with the planes is the desired solution (Figure 3c), i.e., the points in space that result in $\dot{\theta}$=*constant* and $\dot{\phi}$=0 optical flow values. Analytically, the following are the solutions (disallowing the case of X=0 and Y=0 which corresponds anomalous situation):

$$Z = 0 \text{ and } \left[X + \frac{V}{2(C+\dot{\theta})}\right]^2 + \left[Y - \frac{U}{2(C+\dot{\theta})}\right]^2 = \left[\frac{V}{2(C+\dot{\theta})}\right]^2 + \left[\frac{U}{2(C+\dot{\theta})}\right]^2. \tag{24}$$

$$X = -\frac{V}{C+\dot{\theta}} \text{ and } Y = \frac{U}{C+\dot{\theta}}. \tag{25}$$

These solutions are drawn in Figure 4. Solution (24) is an equation of a circle that lies in the XY plane. The radius of the circle is $\left[\left[\frac{V}{2(C+\dot{\theta})}\right]^2 + \left[\frac{U}{2(C+\dot{\theta})}\right]^2\right]^{\frac{1}{2}}$ and its center is at $\left[-\frac{V}{2(C+\dot{\theta})}, \frac{U}{2(C+\dot{\theta})}\right]$. The circle is tangent to the camera translation vector at the origin. This can be shown as follows: the slope $\frac{dY}{dX}$ of a tangent to the circle at any point is $\frac{dY}{dX} = \frac{-\left[X + \frac{V}{2(C+\dot{\theta})}\right]}{\left[Y - \frac{U}{2(C+\dot{\theta})}\right]}$. At the origin, this takes the value $\frac{V}{U}$, which is the same as the slope of the translation vector **t**.

Solution (25) is a straight line perpendicular to the XY plane and intersects this plane at the point $\left[-\frac{V}{C+\dot{\theta}}, \frac{U}{C+\dot{\theta}}, 0\right]$. This intersection point also lies on the circle defined in solution (24).

The meaning of these solutions is the following: all points in 3-D space that lie on the circle or the line described by solutions (24) and (25) and which are visible (i.e., unoccluded and in the field of view of the camera) produce the same instantaneous optical flow $\dot{\theta}$ and zero instantaneous optical flow $\dot{\phi}$. We call the circle on which equal flow points lie the Equal Flow Circle (EFC). Two sets of EFCs are illustrated in Figure 5. Figure 5a shows EFCs for the case where the camera undergoes instantaneous translation only. The label of each circle represents the optical flow $\dot{\theta}$ in the image that corresponds to points on this circle. Here, there is a straight line (a circle with an infinite radius) that corresponds to zero flow in the image. Figure 5b shows EFCs for the case where the camera undergoes instantaneous translation and rotation. Here, there is a circle with finite radius that produces zero flow ($\dot{\theta} = 0$ in the image domain).

4.2 ZERO FLOW CIRCLES

One of the EFCs corresponds to points in 3-D space that produce zero flow. We call this circle Zero Flow Circle (ZFC) [13]. The equation that describes the ZFC can be obtained by setting $\dot{\theta}$=0 in equation (24) i.e.,

$$Z = 0 \text{ and } \left[X + \frac{V}{2C}\right]^2 + \left[Y - \frac{U}{2C}\right]^2 = \left[\frac{V}{2C}\right]^2 + \left[\frac{U}{2C}\right]^2. \tag{26}$$

If the Z component of the camera rotation vector ω is positive (i.e., C>0), then visible points in the XY plane that are inside the ZFC produce positive optical flow ($\dot{\theta}>0$), while visible points outside the ZFC produce negative optical flow ($\dot{\theta}<0$) in the image (see Figure 6). This can be shown as follows. The points that are inside the ZFC satisfy

$$\left[X-\frac{V}{2C}\right]^2+\left[Y+\frac{U}{2C}\right]^2<\left[\frac{V}{2C}\right]^2+\left[\frac{U}{2C}\right]^2. \tag{27}$$

Given the constraint in (27), then for $C>0$, equation (24) is satisfied if and only if $\dot{\theta}>0$. A similar discussion holds for $C<0$.

4.3 THE EFCs and ZFCs AS A FUNCTION OF TIME

As the camera moves through 3-D space, the EFCs move with it. Figure 7 is an example of a camera path with some EFCs. At each instant of time, the radii of the EFCs are a function of the instantaneous motion parameters $\mathbf{t}$ and $\boldsymbol{\omega}$. The locations of the EFCs are such that they always contain the origin of the camera coordinate system (the same as the camera pinhole point), are tangent to the instantaneous translation vector $\mathbf{t}$, and are perpendicular to the instantaneous rotation vector $\boldsymbol{\omega}$. Each ZFC lies to the left or right of the translation vector depending on whether the instantanous rotation is positive or negative, respectively. A simpler case is detailed in Figure 8, where we assume motion along a straight line with constant speed v^*. Figure 8a shows a camera that undergoes translation only. The optical flow of the four points A, B, C, and D as a function of time is shown in Figure 8c. The vertical coordinate is the optical flow $\dot{\theta}$ and the horizontal coordinate is time. At time $t=t_2$, the points A,B,C, and D produce the same optical flow (optical flow graphs of all the four points in Figure 8c intersect) and thus appear on the same instantaneous EFC (as shown in Figure 8b).

4.4 EFCs, ZFCs, and FIXATION

If the point on which the camera fixates is visible in the image, then the corresponding image point will have zero optical flow during fixation. However, at a specific time instant, a fixation point is one out of many that may produce zero optical flow. For motion in an instantaneous XY plane, as described previously, these points lie on a circle and a line. Since points inside the circle produce flow values of opposite sign to those outside the circle, a point in 3-D space which is not the fixation point may produce different flow values at different instants of time.

By definition, a fixation point (if visible) produces zero optical flow at all instants of time during the motion. If the fixation point lies in the instantaneous XY plane, then the ZFC at each instant of time must contain the fixation point. (There are special cases in which more than one fixation point exists.)

The fixation point is the intersection of all the ZFCs.

In Figure 9 we analyze the case where a camera fixates while translating in a straight line. Assume that the translational velocity V^* is a positive constant. Let us consider the optical flow due to a point other than the fixation point. Figures 9a shows the fixation point F and an arbitrary point A, and their positions relative to the camera. Five instants of time have been chosen to explain the optical flow due to point A.

Let θ_F and θ_A be the angles of points F and A, respectively, in the camera coordinate system. Then:

- For $t < t_1$, point A lies outside the instantaneous ZFC (Figure 9b), $(\theta_A - \theta_F)$ is negative and a strictly decreasing function (Figure 9c), and the optical flow of that point $\dot{\theta}_A$ (which also equals $(\dot{\theta}_A - \dot{\theta}_F)$ because $\dot{\theta}_F = 0$) is negative (Figure 9d).

- At $t = t_1$, point A lies on the instantaneous ZFC (Figure 9b), $(\theta_A - \theta_F)$ is negative (Figure 9c), and the optical flow $\dot{\theta}_A$ is zero (Figure 9d).

- For $t_1 < t < t_2$, point A lies inside the instantaneous ZFC (Figure 9b), $(\theta_A - \theta_F)$ is negative (Figure 9c), and the optical flow $\dot{\theta}_A$ is positive (Figure 9d).

- At $t = t_2$, point A lies inside the instantaneous ZFC (Figure 9b), $(\theta_A - \theta_F)$ is zero, i.e., point A lies on the optical axis in front of the fixation point (Figure 9c), and the optical flow $\dot{\theta}_A$ is positive (Figure 9d).

- For $t_2 < t < t_4$, point A lies inside the instantaneous ZFC (Figure 9b), $(\theta_A - \theta_F)$ is positive (Figure 9c), and the optical flow $\dot{\theta}_A$ is positive and gets its maximum value at t_3 (Figure 9d).

- At $t = t_4$, point A lies on the instantaneous ZFC (Figure 9b), $(\theta_A - \theta_F)$ is positive (Figure 9c), and the optical flow $\dot{\theta}_A$ is zero (Figure 9d).

- For $t > t_4$, say $t = t_5$, point A lies outside the instantaneous ZFC (Figure 9b), $(\theta_A - \theta_F)$ is positive and a strictly decreasing function (Figure 9c), and the optical flow $\dot{\theta}_A$ is negative (Figure 9d). In fact, as $t \rightarrow \infty$, $\dot{\theta}_A$ approaches zero.

The point A is inside instantaneous ZFCs during the open time interval (t_1, t_4). In this period of time it seems to the viewer that it "moves to the left". During the intervals $(-\infty, t_1)$ and (t_4, ∞) the point seems to be "moving to the right". No relative motion is detected at t_1 or t_4.

Figure 10 shows EFCs during fixation at two different time instants. At time instant t_1 the angular velocity of the camera is $C = C_1$, and at time instant t_2 it is $C = C_2$. The location of an EFC at a particular time instant can be obtained analytically from equation (24). However, for fixated motion the relationship between the radius and the corresponding angular velocity of an EFC can be simply obtained by subtracting the instantaneous angular velocity of the camera from the angular velocity of EFCs of a "translating only" camera. For example, at time t_1 the angular velocity (or the $\dot{\theta}$) of the ZFC is $C_1 - C_1 = 0$, the $\dot{\theta}$ that corresponds to a circle whose radius is half the ZFC's radius is $2C_1 - C_1 = C_1$, the $\dot{\theta}$ that corresponds to a circle whose radius is one fourth the ZFC's radius is $4C_1 - C_1 = 3C_1$, etc. Similarly we can analyze circles at other time instants.

4.6 MAPPING THE SPACE WHILE FIXATING

The EFCs can be described in a more generalized form, using ratios between the $\dot{\theta}$ of each circle to the rotation parameter C, i.e., $\frac{\dot{\theta}}{C}$. For example (see Figure 10), a circle which is inside the ZFC and whose radius is half the ZFC's radius, has a normalized angular velocity (or optical flow $\dot{\theta}$) $\frac{\dot{\theta}}{C}$ of 1. Figure 11a shows EFCs with normalized values of $\frac{\dot{\theta}}{C}$. Obviously, the normalized value of the ZFC is 0. Figure 11b shows normalized EFCs during fixation.

Using the EFC (or the normalized EFC) concept it is possible to quantitatively map the space in such a way that any point on the XY plane can be located.

The method is based on relative mapping. Given (only) the *ratio* between the optical flow $\dot{\theta}$ of a point and the camera rotation parameter C, $\frac{\dot{\theta}}{C}$ (assuming $C \neq 0$), and the projection of the point in the image (i.e., θ and ϕ), then the location of the point relative to the fixation point (or relative to the camera) can be obtained. The instantaneous direction of motion of the camera should be known (can be obtained by searching in the image for a point that produces $\frac{\dot{\theta}}{C} = -1$ (Figure 11a), and $\dot{\phi}=0$). Refer to Figure 12: After locating the instantaneous direction of motion θ_t, the location of normalized EFCs in camera coordinates can be determined. $\frac{\dot{\theta}_P}{C}$ determines the location of the normalized EFC on which the point P is located *relative the ZFC*. The angle $(\theta_P - \theta_t)$ determines the exact location of the point on that EFC relative to the instantaneous direction of motion (Figure 12a).

Figure 12b is a suggestion for a more elegant mapping. This mapping consists of two orthogonal families of circles. One family is the EFCs. A point can be mapped relative to the fixation point by specifying two numbers that are derived from the optical flow (they specify two orthogonal circles) [15].

5. EXPERIMENTAL RESULTS

We performed three sets of experiments to test the theory of EFCs and ZFCs.

5.1. SYNTHETIC DATA

In this set of experiments we tested the concept of the EFC using a computer simulation of a camera that fixates on a point while translating in a straight line. The simulation program finds the optical flow $\dot{\theta}$ of points in the XY plane and assigns a color to each value of flow. Figure 13 shows the results. Each circle corresponds to a different value of $\dot{\theta}$. The Fixation point is always on the instantaneous ZFC. Other points (e.g., the center of the white square) may be inside (Figure 13a), on, or outside (Figure 13b) the instantaneous ZFC. This figure is the flow map for only two time instants. However, we produced an animation movie that shows these circles dynamically expanding and contracting during fixation.

5.2. REAL DATA

This set of experiments were used to test the theory of ZFCs with real data, and involved linear camera translation under fixation. We predicted qualitative values for optical flow based on ZFC analysis, and compared it with actual flow values obtained during experimentation. In all cases, the actual flow values are consistent with the predicted values.

We examined three fixation scenarios. All three experiments involved the same camera motion and the same objects in the scene. However, the configuration of the objects relative to the camera translation motion path was different in the three experiments. Three different objects were placed colinearly on a table. The camera motion involved a constant velocity translation with fixation on the middle object. The experimental scenarios and the results are shown in Figure 14 and are described in more detail below.

For each experiment, the imagery at a single scan line $\phi=0$ of the camera was recorded as a function of time [5[,[12]. This scanline is the projection of the *XY* plane. (The camera scanline was parallel to the table top at all times.) Each experiment thus resulted in a spatio-temporal image for the given scan line. The horizontal coordinate of such an image represents time for successive frames of the same scan line, and the vertical coordinate represents the pixels spatial positions on the scan line. These images are shown also in Figure 14, and are described in more detail below. In each case the fixated object is the darker horizontal line in each image, while the other two curves belong to the other objects. The optical flow is the time derivative of each curve, or the slope of the curves in time-space domain.

<u>Experiment 1:</u> (Figure 14a)

In this experiment the three objects (A1, A2, and the fixation point) lie on a line parallel to the direction of camera translation path. The points A1 and A2 produce optical flow that is changing as a function of time. There is one place for each point where there is a change in the optical flow sign. This is due to a change in relative location between the points and the ZFCs. First ($t=t_1$), the point A1 is inside the ZFC (producing positive $\dot{\theta}$ optical flow) and the point A2 is outside the ZFC (negative $\dot{\theta}$ flow). Then ($t=t_2$), the point A1 is on the ZFC (zero flow) and A2 is still outside the ZFC. When the camera is perpendicular to the direction of translation ($t=t_3$) A1 and A2 are outside the ZFC (negative $\dot{\theta}$ flow). Later ($t=t_4$), The point A1 is outside the ZFC (negative $\dot{\theta}$ flow) and the point A2 is on the ZFC (zero flow). Finally ($t=t_5$) A1 is outside the ZFC (negative $\dot{\theta}$ flow) and A2 is inside the ZFC (positive $\dot{\theta}$ flow).

Experiment 2: (Figure 14b)

In this experiment the three objects (A3, A4, and the fixated object) lie on a line perpendicular to the direction of camera translation. First at $t < t_2$ (say $t = t_1$), A4 is inside the ZFC (positive $\dot{\theta}$ flow) and A3 is outside it (negative $\dot{\theta}$ flow). Then ($t=t_2$), A4 is on the ZFC (zero flow) and A3 is outside the ZFC (negative $\dot{\theta}$ flow). For $t_2<t<t_3$ A3 and A4 are outside the ZFC (negative $\dot{\theta}$ flow). Then at ($t=t_3$) A3 is on the ZFC (zero flow) and A4 is outside the ZFC (negative $\dot{\theta}$ flow). Later ($t=t_4$), A3 stays inside the ZFC (positive $\dot{\theta}$ flow) while A4 is outside the ZFC (negative $\dot{\theta}$ flow). At $t = t_4$ the ZFC gets its minimum diameter (compared to other ZFCs during the same motion). Similarly, we can continue to analyze the time instants after t_4. Note that in this case each point changes the direction of its corresponding optical flow twice due to two passes through ZFCs. The Zero Flow Points are marked (two for A3, and two for A4).

Experiment 3: (Figure 14c)

In this case we examine a more general case. The points A5 and A6 lie on a line that is not parallel nor perpendicular to the direction of the camera direction. Here, as in experiment 2, for each point there are two changes in the sign of the optical flow that can be explained using the ZFCs. Due to experimental limitations only one change for each point is seen.

5.3 A ZFC EXPERIMENT

Refer to Figure 15. We examined the ZFC theory in one more way. For a camera undergoing linear translation while fixating on a point, we set up a circle of toothpicks on the table top. The circle was tangent to the line of camera motion. When a point on this circle was chosen as the fixation point, we verified that at the instant the camera passes the tangent point, the optical flow of all points on the circle become zero. Figure 15a shows a portion of this circle. The image at some time instant is shown in Figure 15b. Figure 15c shows one scan line of the image in Figure 15b as a spatio-temporal image. The horizontal line in this time-space image corresponds to the fixation point, since it does not change its vertical location as a function of time (i.e., produces zero optical flow). The optical flow of other points (change in their vertical location over time) changes sign. At some time instant (approximately the central horizontal line of the time-space image) they produce no optical flow. At this instant of time these points lie on a circle that includes the pinhole point of the camera. This circle is the ZFC. Using a program that extracts optical flow of points using gradient methods [12] we displayed the optical flow images at several time instants. Figures 15d, e, and f show the evolution of the optical flow of the points on the circle. In Figure 15d the camera started to move and there is optical flow from most points. In Figure 15e the points near the fixation point produce small values of optical flow. Figure

15f depicts the situation just before the camera arrives at the tangent point. When the camera is exactly at the tangent point, all optical flow values become zero.

A videotape (for presentation only) shows this experiment continuously.

6. DISCUSSION

In this paper we present a quantitative way for analyzing fixation. Using the concept of EFCs it is possible to locate points in space relative to the fixation point, and explain the behavior, e.g., direction of optical flow, of points near the fixation point as a function of time. The camera's instantaneous direction of translation and the fixation point determine the plane on which the EFCs can be found. We show that points on an EFC inside the ZFC produce optical flow that is opposite in sign to that produced by points outside the ZFC. (When a point in space crosses a ZFC it produces zero flow.)

For explanation purposes we analyzed a special case of motion. However, a similar approach (though it could be more difficult to visualize the results) can be taken for a more general motion of the camera. The analysis for the current motion can also be extended to find equal flow curves, i.e., curves that correspond to constant $\dot{\theta}$ and constant $\dot{\phi}$, or curves that correspond to constant $f(\dot{\theta},\dot{\phi})$ where $f(.)$ is a function of $\dot{\theta}$ and $\dot{\phi}$.

The coordinate system that we chose is a convenient one. However, the angular velocities of points in space are independent of their representation in the image. Other image coordinate systems may be chosen, in particular, (for practical purposes) the image domain may be planar. (Obviously, an appropriate conversion from the spherical coordinate system should be used).

This analysis complements the qualitative understanding of fixation: It shows that a point that is not the fixation point may change its optical flow sign, and is not restricted to "moving to the right (or to the left)" as has commonly been assumed.

In this paper we emphasized camera fixation without mentioning eye fixation. However, the theory that has been developed is independent of the visual sensing device, and thus it is suitable for eye fixation as well. The EFCs can be thought as properties of space rather than properties of points in the image domain.

7. FUTURE WORK

Currently, other cases of equal flow points are being investigated. Also, the mapping described in section 4.6 is being extended to 3D. We examine other cases of camera motion where the rotation vector is not perpendicular to the translation vector. We plan to exploit the EFCs (and their extensions) concept in a vision based navigation algorithm for a real-time robot system.

8. ACKNOWLEDGEMENTS

The author would like to thank Marty Herman, Don Orser and Karen Chaconas for fruitful discussions, and for reading and making important comments about this document. Jim Albus has continuously encouraged discovering and establishing vision related computational theories. Tsai Hong developed the time-space program, and Ramesh Rangachar helped in doing the time-space experiments. Richard Freemire helped to produce the animation movie of the EFCs. Hakan Yakali produced some high quality figures that appear in this paper.

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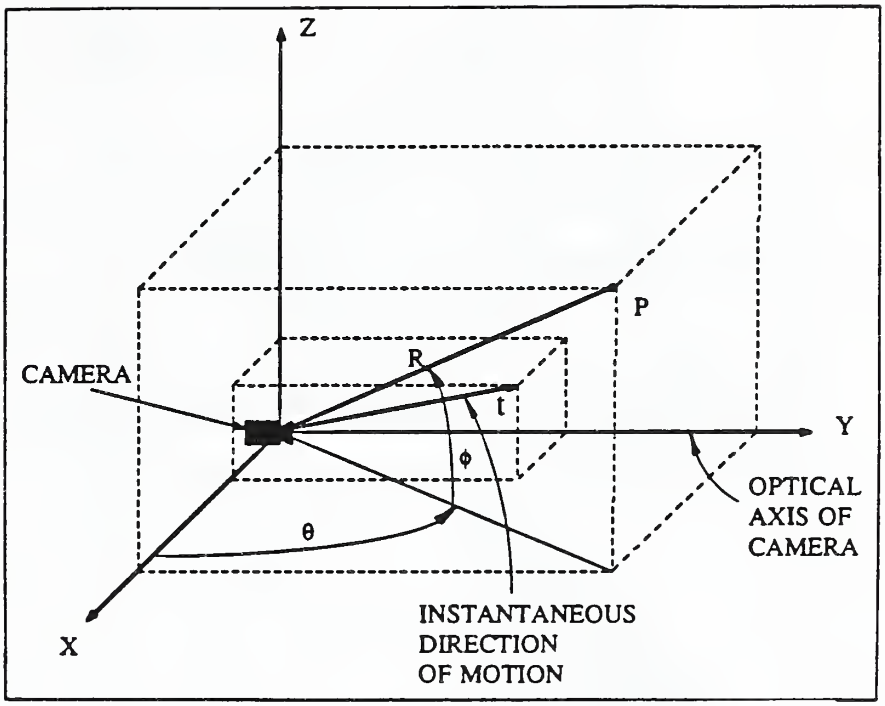


Figure 1: Coordinate system fixed to camera

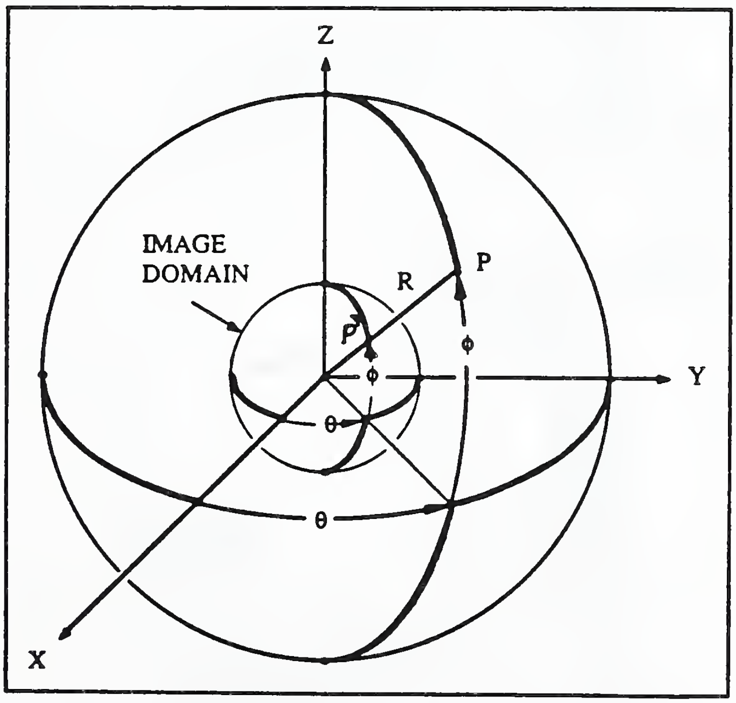


Figure 2: Image domain

(a)

(b)

(c)

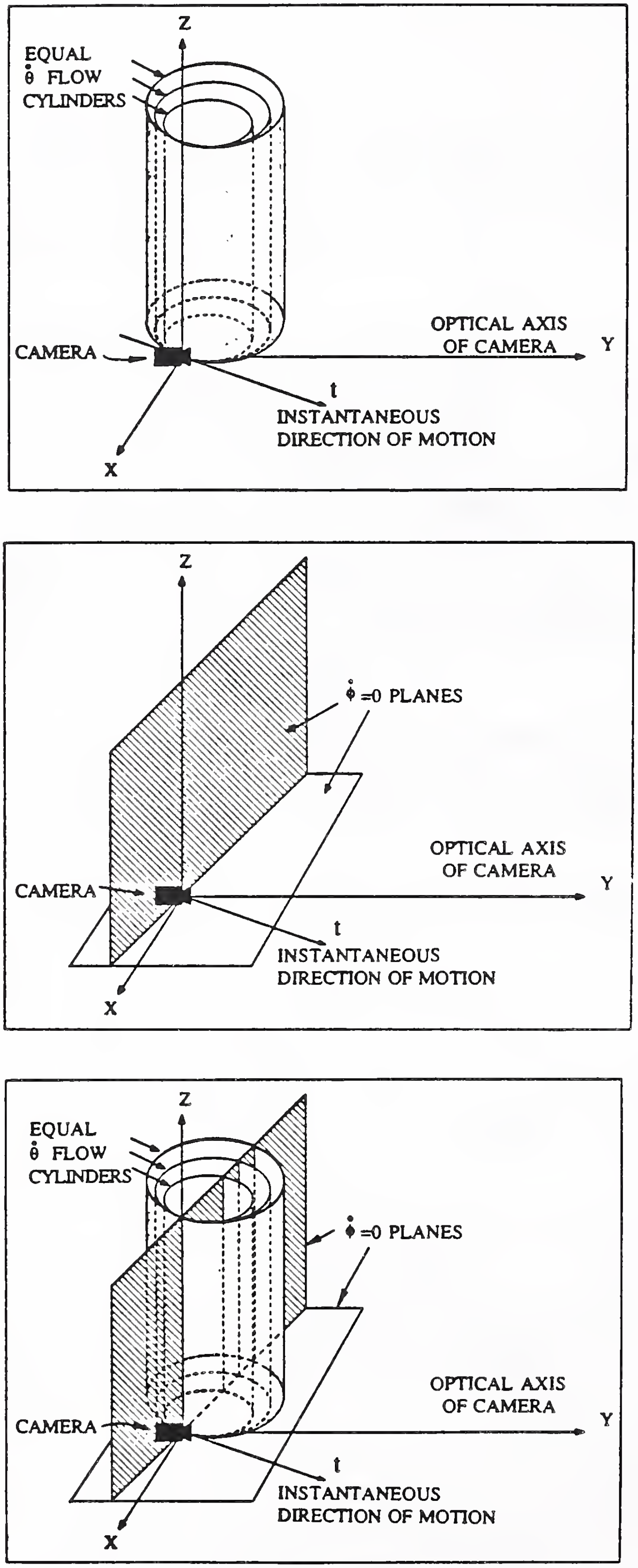


Figure 3: (a) Equal $\dot{\theta}$ cylinders
(b) $\dot{\phi}$=0 planes
(c) Intersections between $\dot{\theta}$ cylinders and $\dot{\phi}$=0 planes

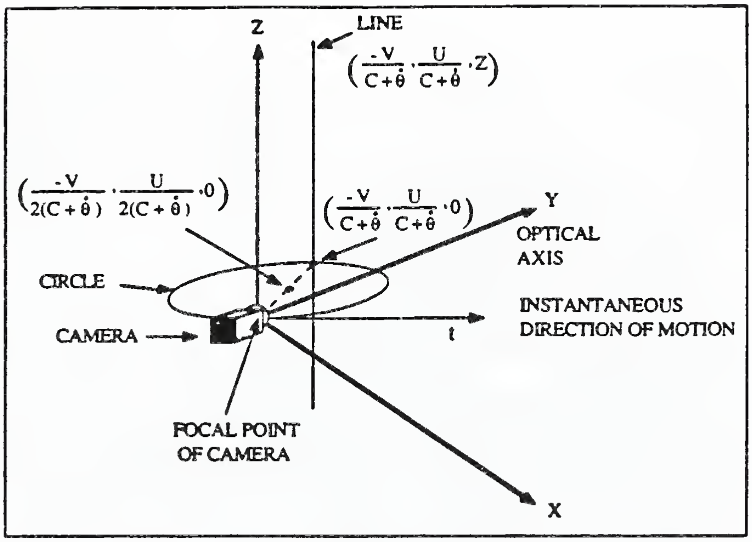
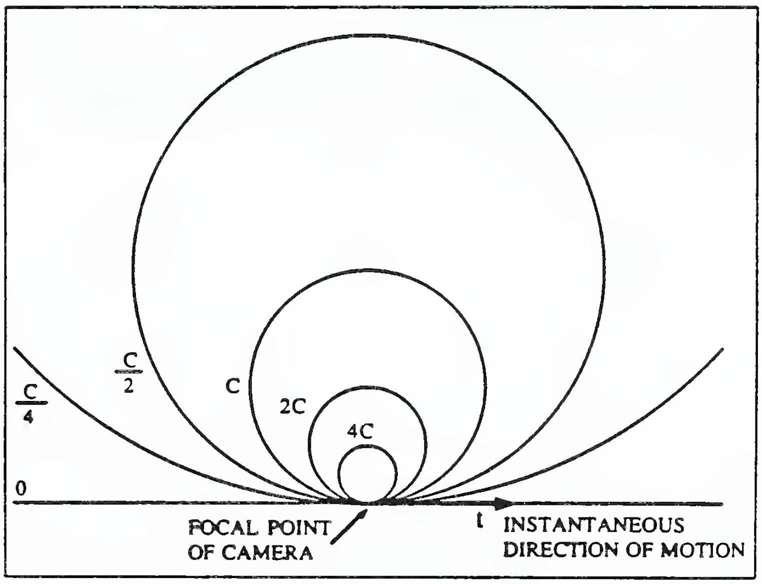


Figure 4: Pictorial description of solutions (24) and (25)

(a)



(b)

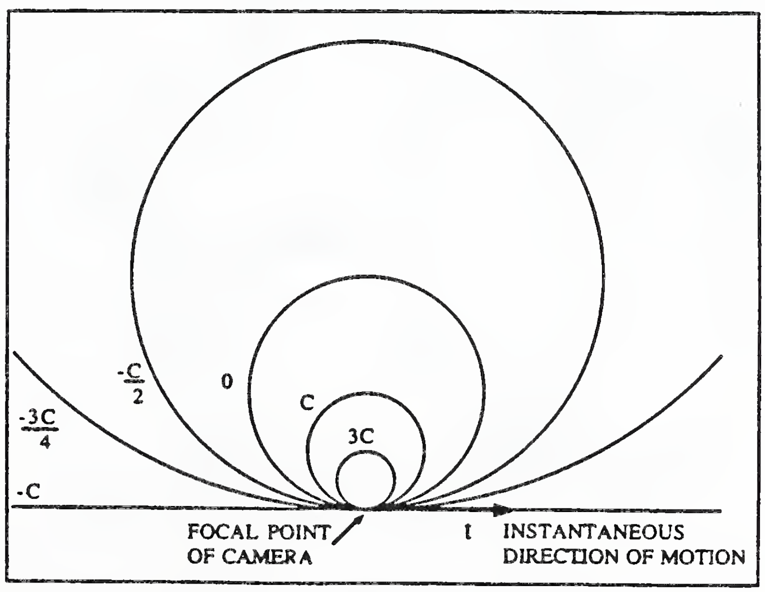


Figure 5: Optical flow values due to camera motion
(a) for translation only
(b) for translation and rotation

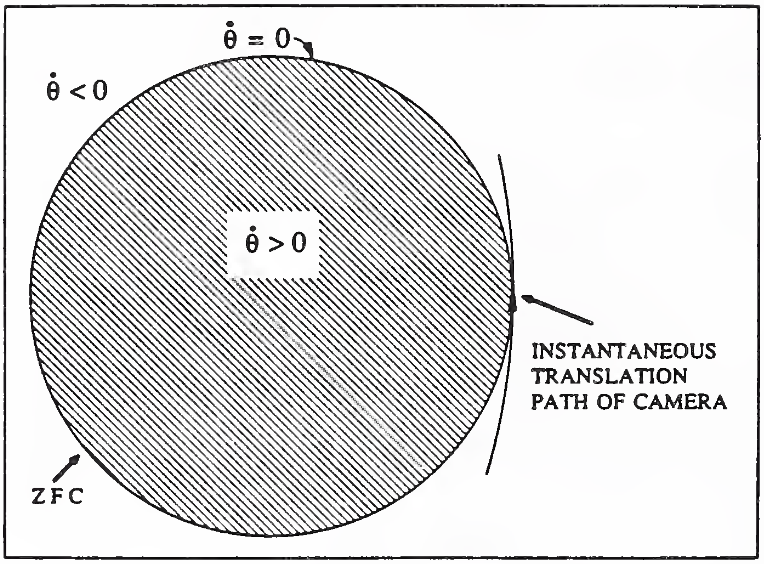


Figure 6: Optical flow signs inside and outside the ZFC

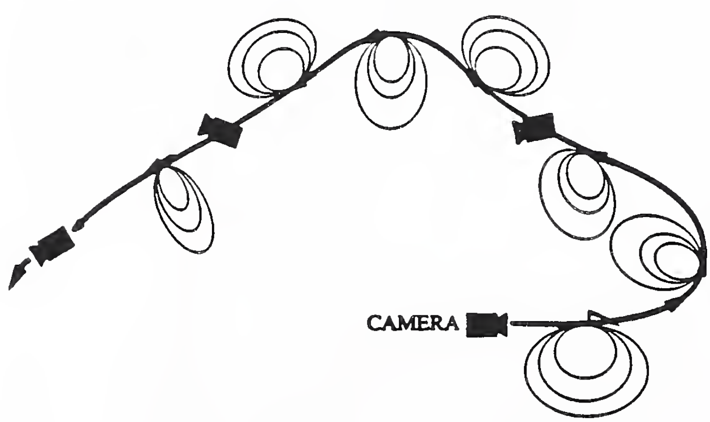


Figure 7: EFCs as a function of time

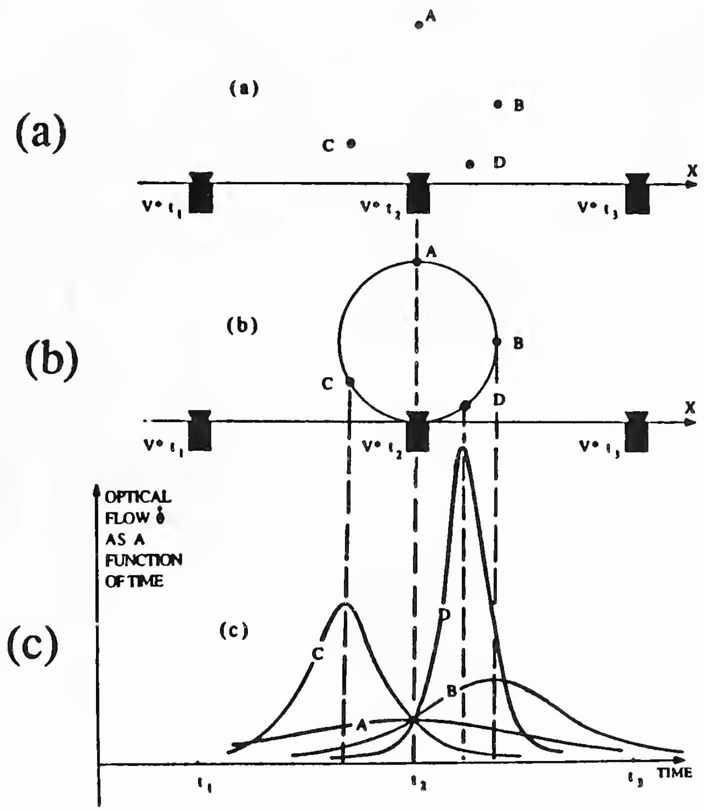


Figure 8: Optical flow and EFC
(a) translating camera
(b) an EFC
(c) optical flow as a function of time

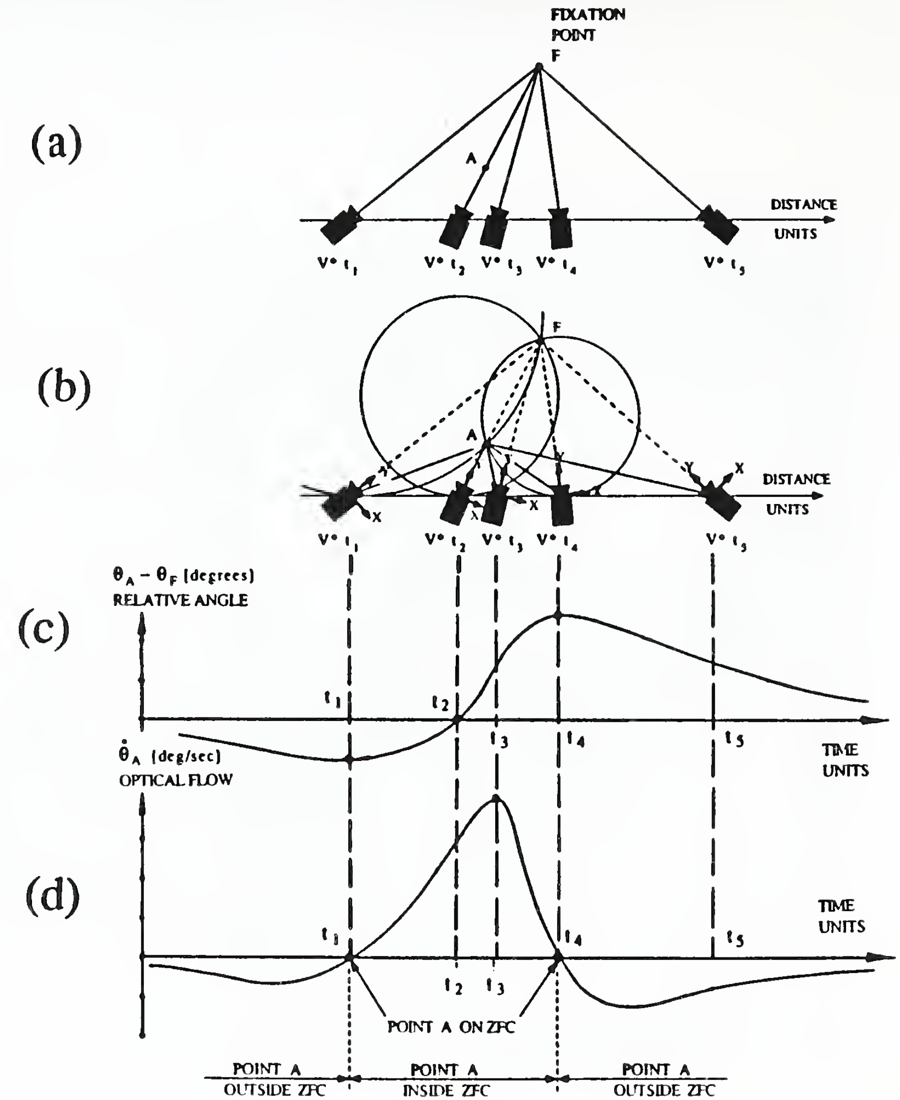


Figure 9: Fixation and the ZFCs
(a) fixating camera
(b) ZFCs during fixation
(c) location of point A relative to the fixation point
(d) optical flow due to point A

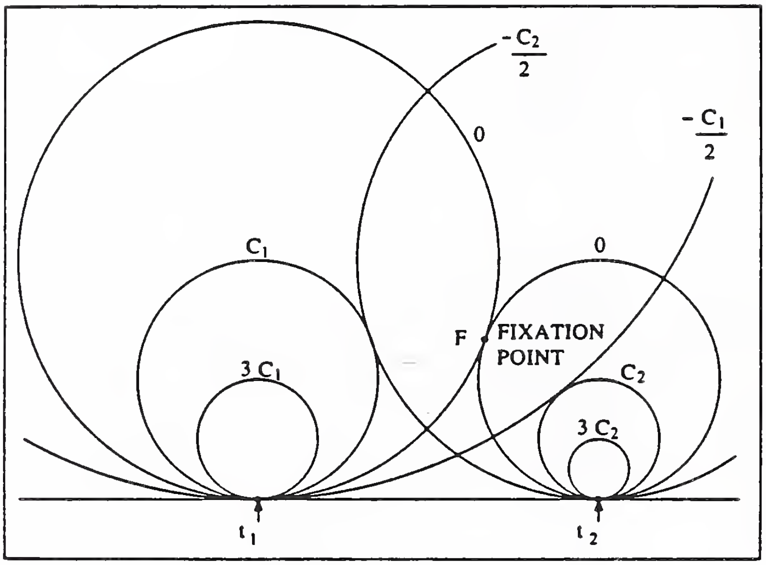
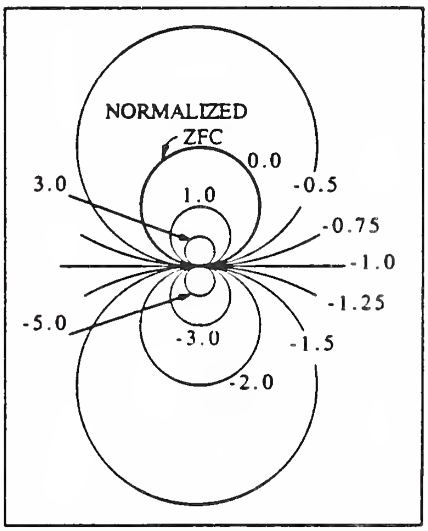


Figure 10: EFCs as a function of time

(a)



(b)

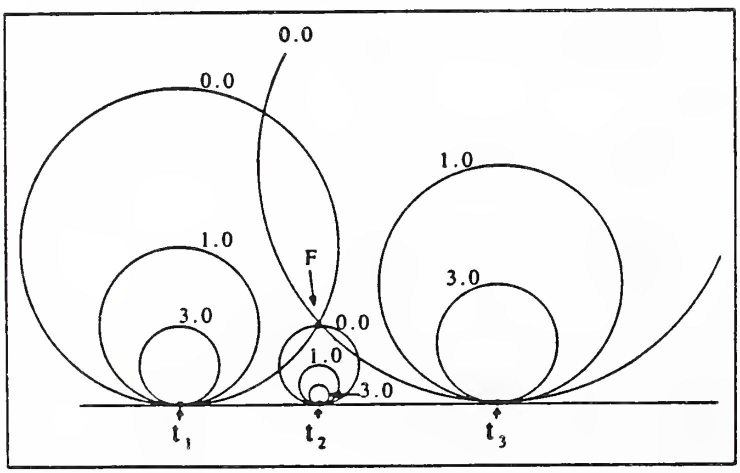
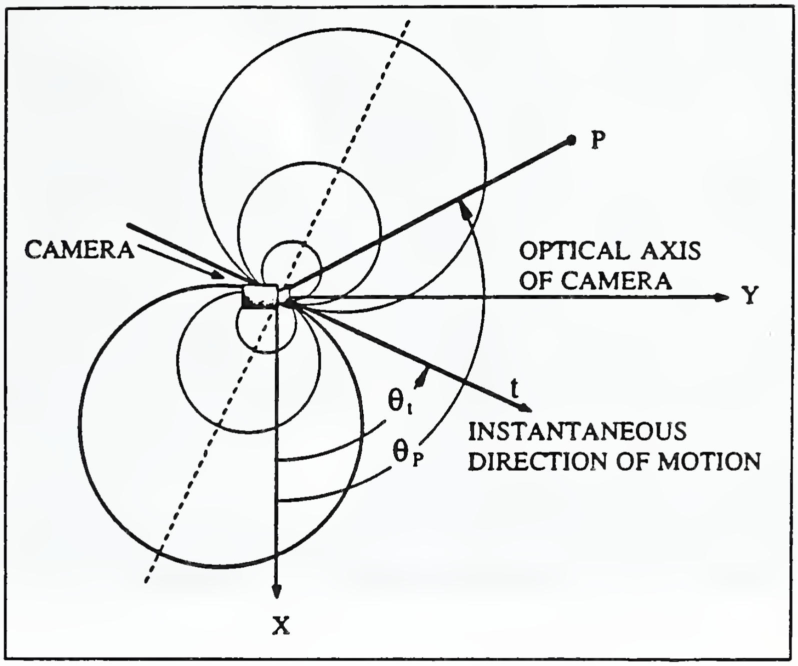


Figure 11: Normalized EFCs
(a) at a time instant
(b) during fixation

(a)



(b)

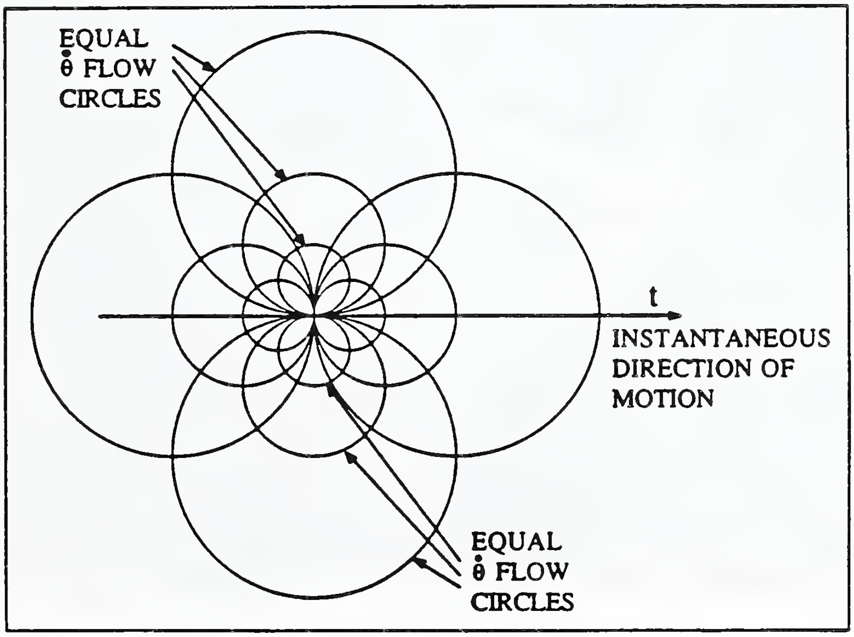
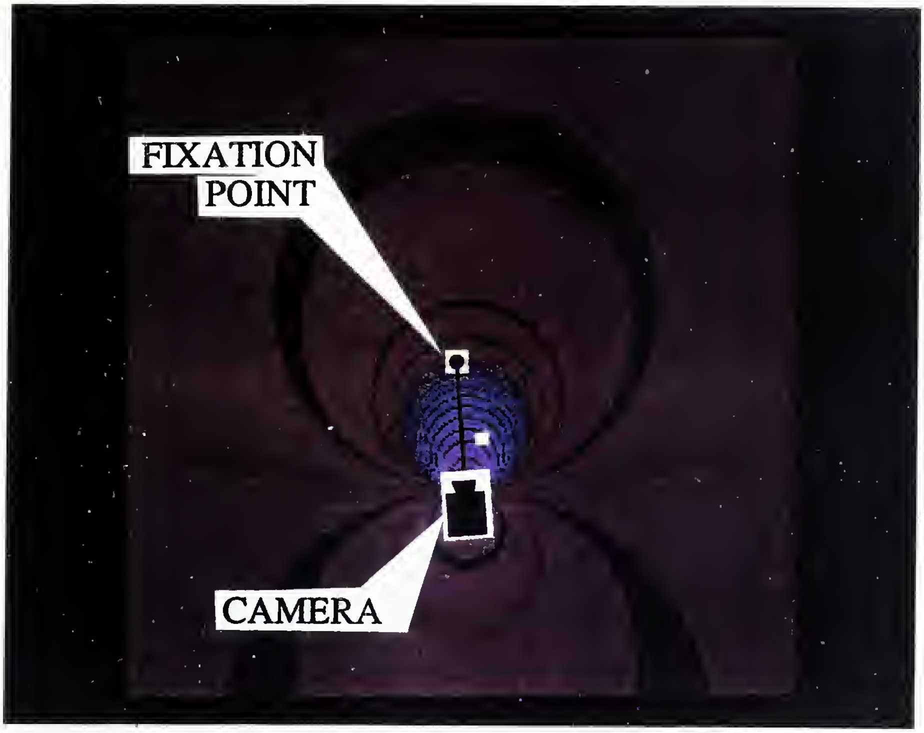


Figure 12: Mapping the *XY* plane using
(a) EFCs and angles
(b) EFCs and their orthogonal circles

(a)



(b)

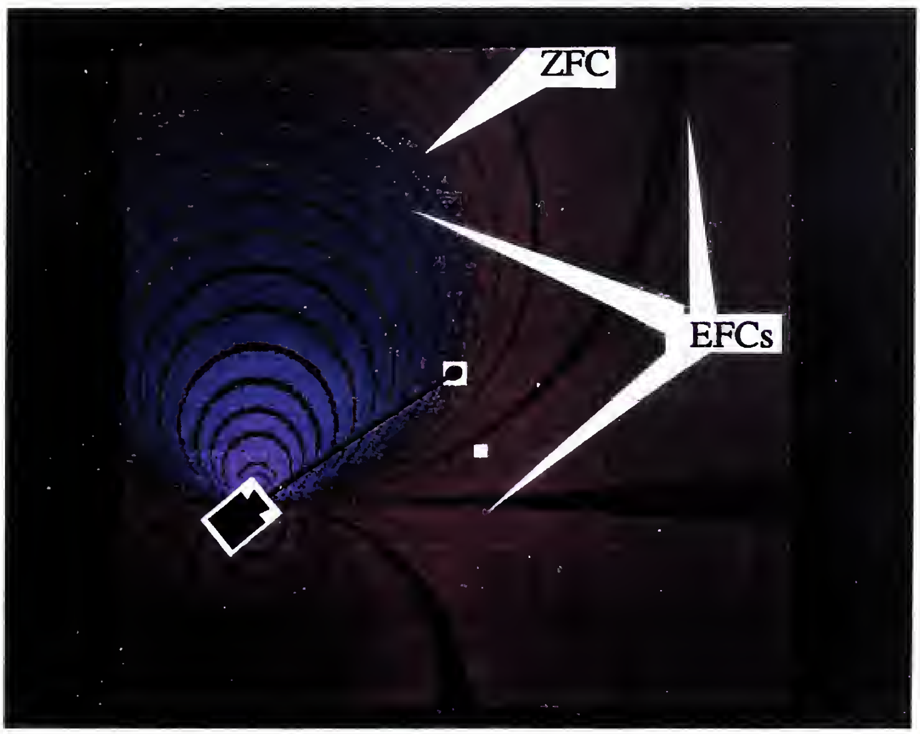


Figure 13: Results of simulated EFCs during fixation at two different locations of the camera

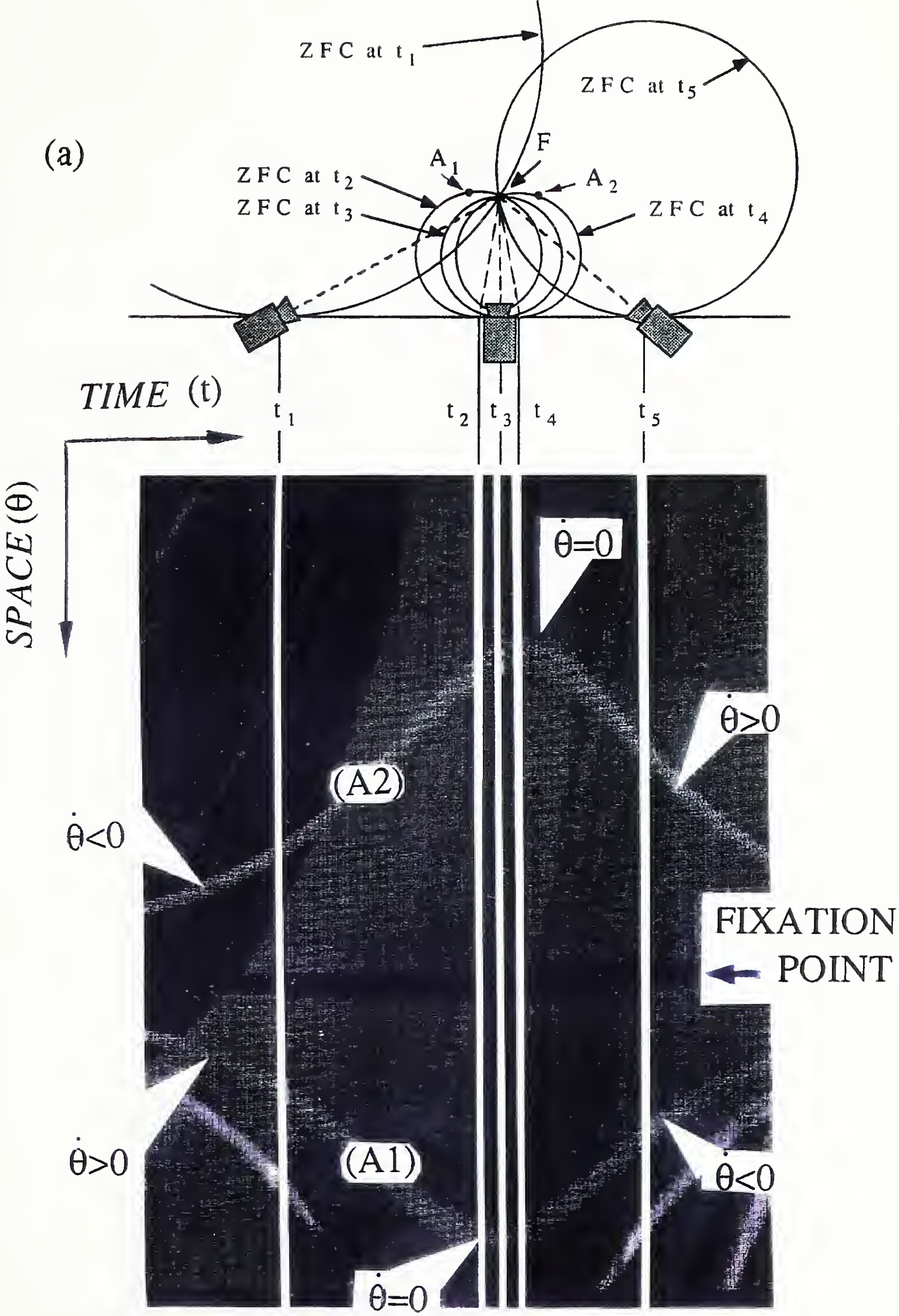


Figure 14a: Results in time-space domain for points A_1, A_2, and F

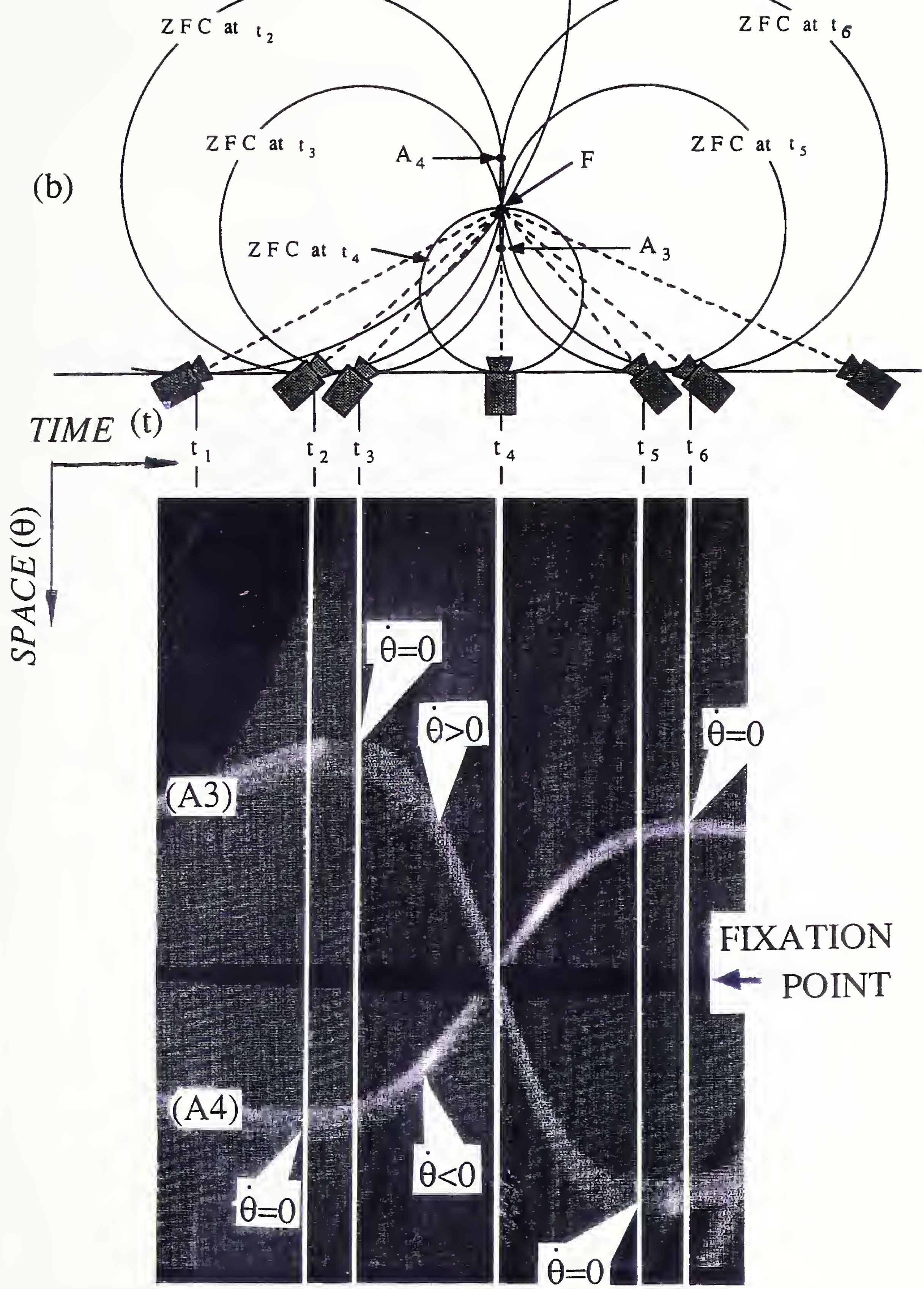


Figure 14b: Results in time-space domain for points A_3, A_4, and F

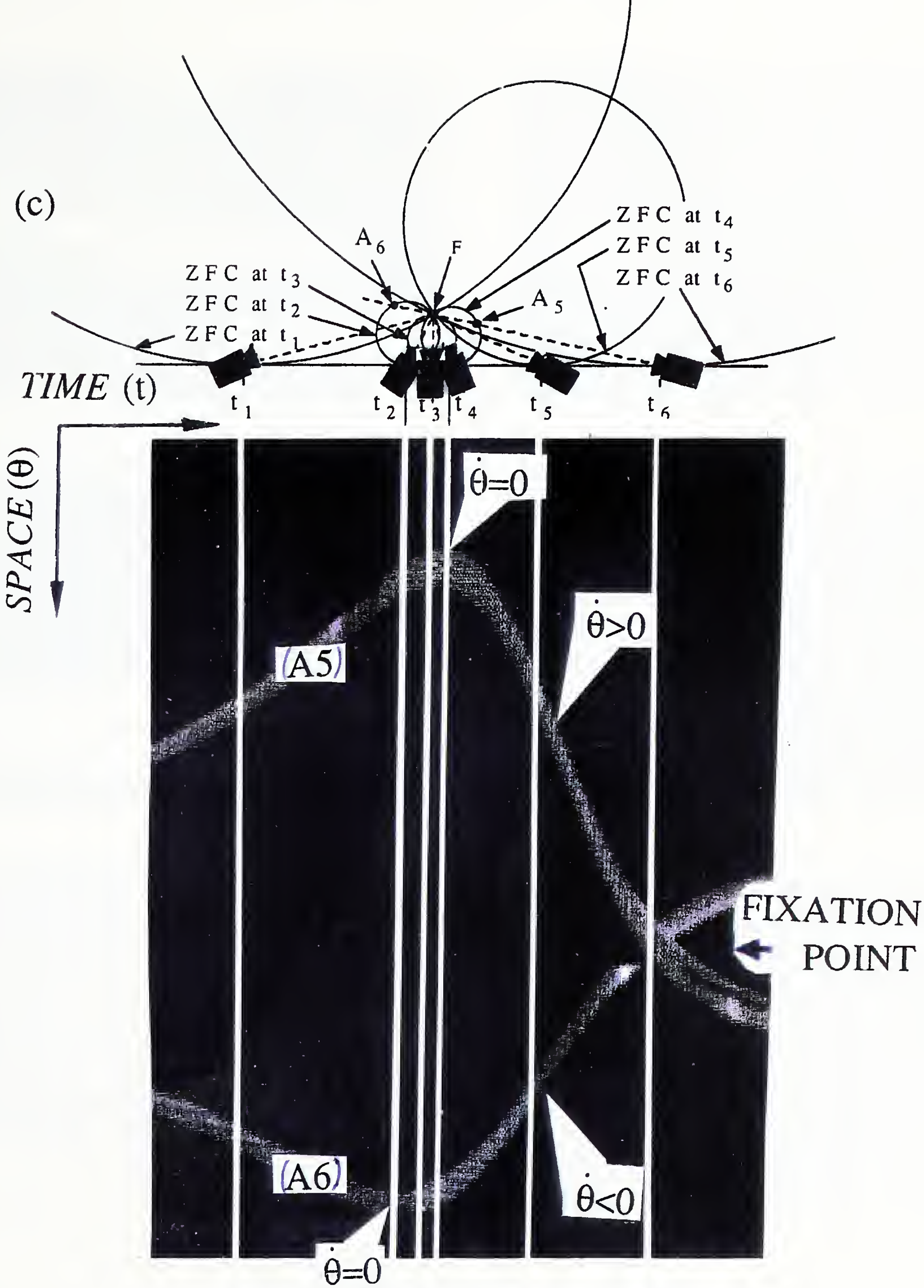


Figure 14c: Results in time-space domain for points A_5, A_6, and F

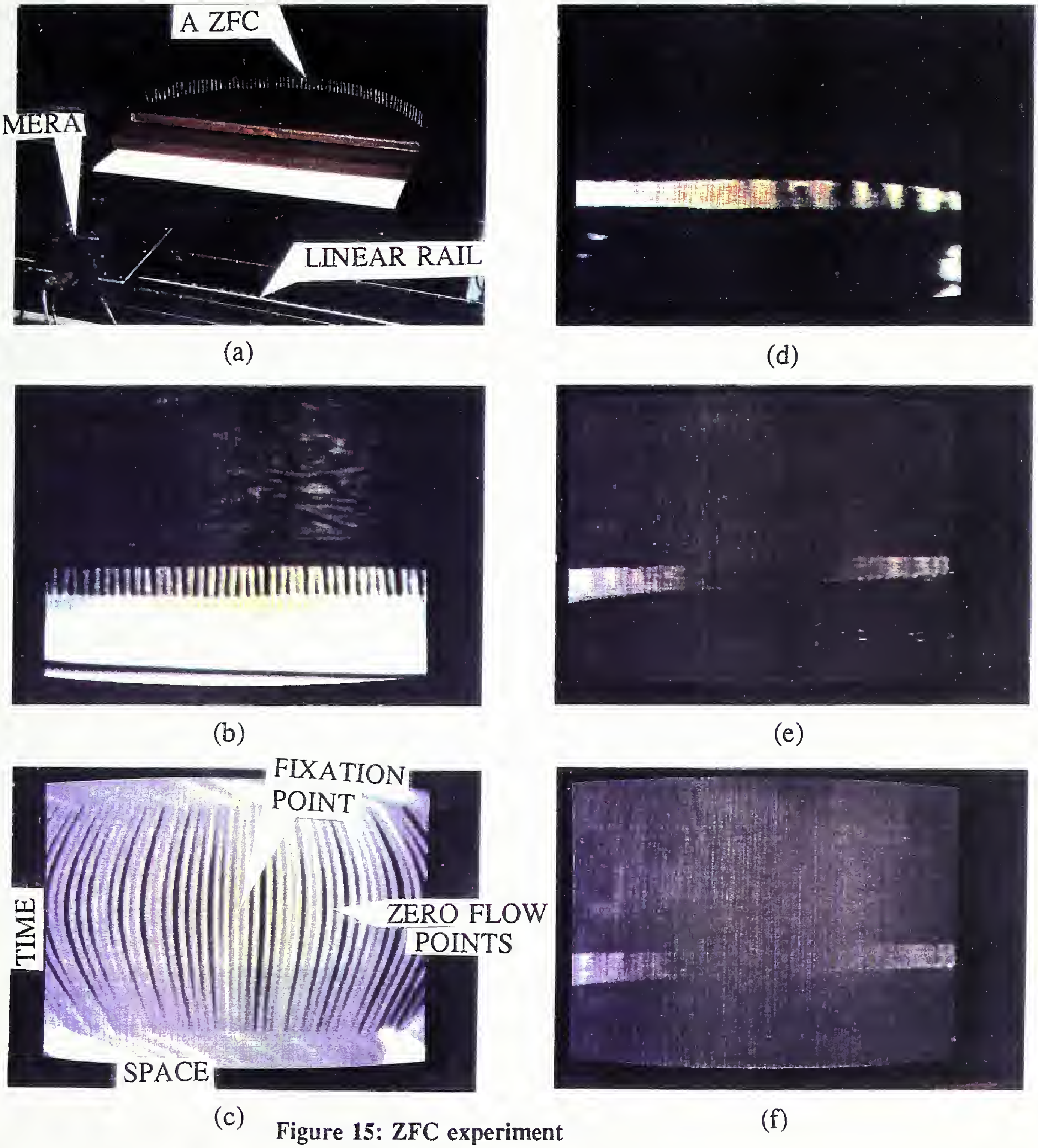


Figure 15: ZFC experiment
(a) setup
(b) camera's view at a time instant
(c) results in time-space domain
(d)-(f) optical flow at three time instants

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11. ABSTRACT (A 200-WORD OR LESS FACTUAL SUMMARY OF MOST SIGNIFICANT INFORMATION. IF DOCUMENT INCLUDES A SIGNIFICANT BIBLIOGRAPHY OR LITERATURE SURVEY, MENTION IT HERE.)

A fixation point is a point in 3-D space that projects to zero optical flow in an image over some period of time while the camera is moving. This paper deals with quantitative aspects of camera fixation for a static scene. In general, when the camera undergoes translation and rotation, there is an infinite number of points that produce equal optical flow for any instantaneous point in time. Using a camera-centered spherical coordinate system, it is shown how to find these points in space. For the case where the rotation axis of the camera is perpendicular to the instantaneous translation vector, these points lie on cylinders. If the elevation component of the optical flow is set to zero then these points form a circle (called the Equal Flow Circle or simply EFC) and a line, i.e., all points that lie on this circle or line are observed as having the same azimuthal optical flow. A special case of the EFCs is the Zero Flow Circle (ZFC) where both components of the optical flow are equal to zero. A fixation point is the intersection of all the ZFCs. Points inside and outside the ZFC can be quantitatively mapped using the EFCs.

In a set of experiments using simulated as well as real data, we show how the concept of the EFC and ZFC can be used to explain the optical flow produced by points near the fixation point, and to explicitly map the space while fixating. It is also shown experimentally that points near the fixation point may change the sign of their optical flow as the camera moves.

12. KEY WORDS (6 TO 12 ENTRIES; ALPHABETICAL ORDER; CAPITALIZE ONLY PROPER NAMES; AND SEPARATE KEY WORDS BY SEMICOLONS)

camera fixation, active vision, computer vision, visual motion, mobile robots, autonomous vehicles

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