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UNSTEADY LAMINAR FLOW IN A CIRCULAR TUBE: A TEST OF THE HERCOL (HERMITIAN COLLOCATION) COMPUTER CODE

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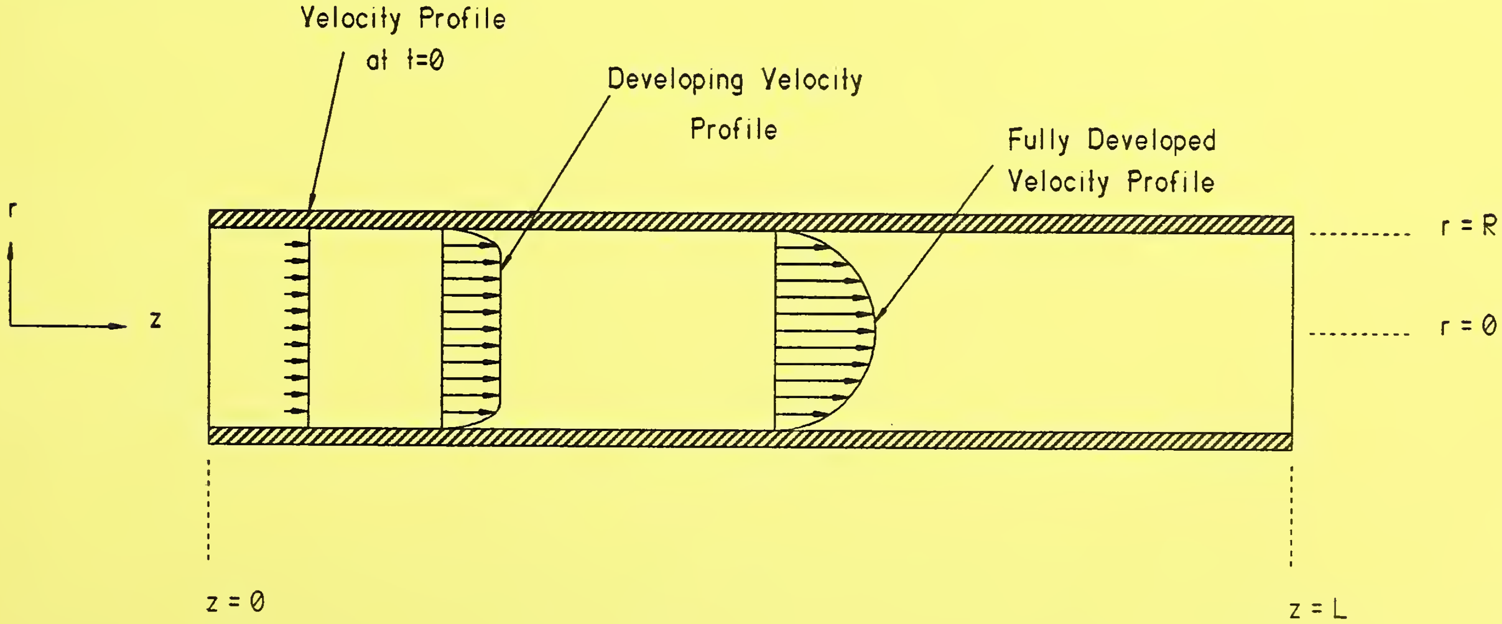

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A test of the HERCOL (Hermitian collocation) Computer Code

James F. Welch, James A. Hurley, Michael P. Glover,
Ryan D. Nassimbene, and Marilyn R. Yetzbacher

HERCOL, a computer code for the integration of second—order differential equations in one space dimension by Hermitian collocation, was used to calculate the unsteady velocity profiles for laminar flow in a circular tube. The code was tested for stability and accuracy on this problem for which an analytical solution exists prior to application to a like problem in which the initial and boundary conditions preclude the existence of analytical solutions.

The test problem is one in which a pressure gradient is imposed on a fluid initially at rest in a circular tube; the fluid accelerates, and, at steady state, has a parabolic velocity profile. A second example was constructed from the first; a pressure gradient equal but opposite in sign is imposed on the fluid with a fully developed parabolic velocity profile. At steady state, the velocity is again parabolic but in the opposite direction to that at the initial conditions.

Excellent agreement with the analytical solution was obtained in the first problem; in the second, the behaviour was as expected. This example is suitable for first—time users of the code.

Key words: numerical integration; partial differential equation; unsteady—state laminar flow;

INTRODUCTION

Velocity profiles for two examples of unsteady laminar flow of a fluid of constant density, ρ, and constant viscosity, μ, in a horizontal tube of length L and radius R (Figure 1) were calculated with the aid of HERCOL [1], a computer code for the integration of second–order differential operators in one space dimension. For the first example, the analytical solution [2,3] was compared with the results obtained from the code to assess the performance of HERCOL and to estimate its efficacy for the solution of similar problems for which no analytical solution exists. A simple modification of the first example to form a second was used to emphasize the speed and ease with which problems, with no readily available analytical solution may be solved with HERCOL.

In addition to serving as a test for HERCOL, the solution may be useful for estimating the time required for the flow in a sampling tube operated in a periodic fashion to achieve steady state.

UNSTEADY LAMINAR FLOW IN A CIRCULAR TUBE

A pressure gradient, $-\frac{\partial p}{\partial z}$, of constant magnitude $(p_0 - p_L)/L$ is imposed on a fluid contained in a tube shown in Figure 1. The fluid initially at rest is accelerated and at steady state has the Poiseuille velocity distribution given in Table 1

As shown in Table 2, the equations of continuity and motion are combined, and the result is transformed by a change of variables; the solution in the form of an infinite series is obtained by separation of variables [3].

The second example was constructed from the first; the fluid was assumed to have a Poiseuille velocity distribution initially, and the imposed pressure gradient had the same magnitude but opposite sign. The effect was to decelerate and reverse the flow. The equation of motion and related boundary conditions and initial conditions are listed in Table 3.

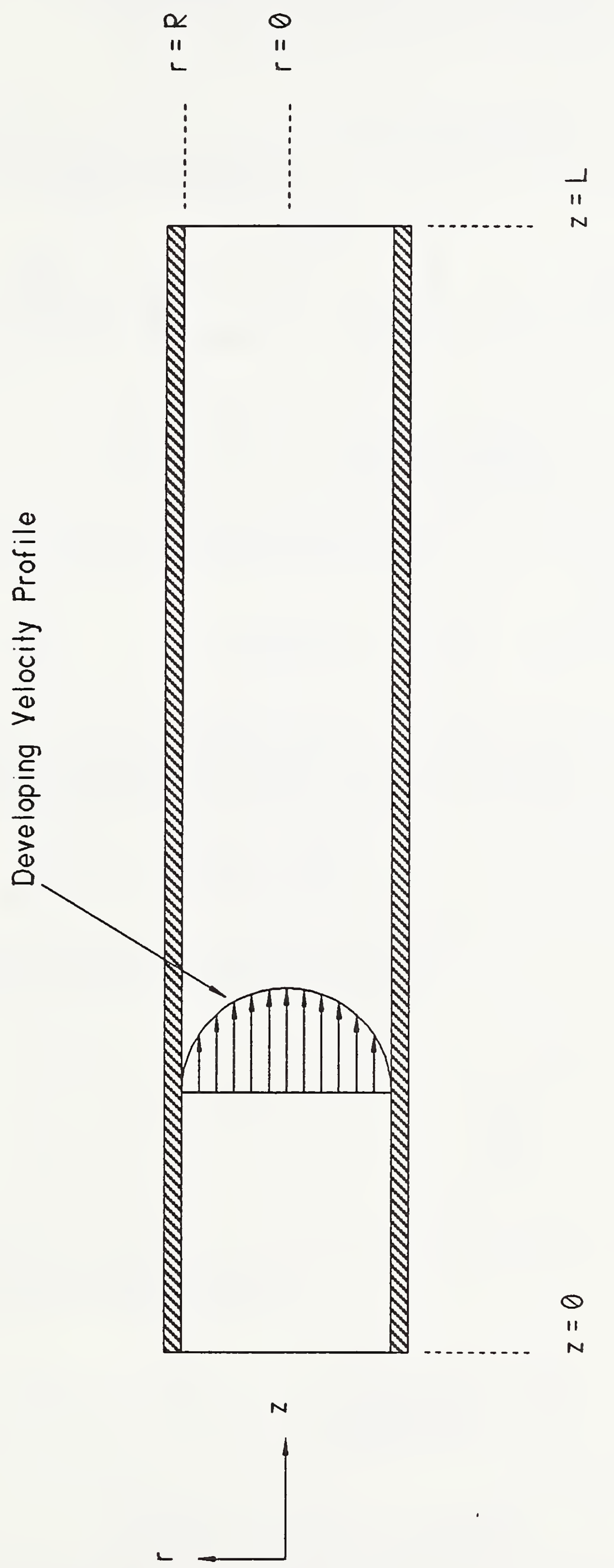

Figure 1. Schematic diagram of the cylindrical tube.

TABLE 1

POISEUILLE VELOCITY PROFILE

Dimensional Form	
Velocity Profile	$v_z = v_{max} [1 - (r/R)^2]$
Maximum Velocity	$v_{max} = (p_0 - p_L) \frac{R^2}{4\mu}$

Dimensionless Form	
Dimensionless Velocity	$\phi = v_z/v_{max}$
Dimensionless Position Variable	$\xi = r/R$
Dimensionless Velocity Profile	$\phi = 1 - \xi^2$

TABLE 2

EQUATIONS OF CONTINUITY AND MOTION
IN CYLINDRICAL COORDINATES (r,θ,z)
FOR A FLUID INITIALLY AT REST

EQUATION OF CONTINUITY ($v_r, v_\theta = 0$, ρ and μ constant)

$$\frac{\partial}{\partial z}(\rho v_z) = 0$$

EQUATION OF MOTION (v_r, v_θ, $\frac{\partial v_z}{\partial z} = 0$, ρ and μ constant)

$$\rho \frac{\partial v_z}{\partial t} = -\frac{\partial p}{\partial z} + \mu \left[\frac{1}{r}\frac{\partial}{\partial r}\left[r \frac{\partial v_z}{\partial r}\right]\right]$$

TRANSFORMATION OF VARIABLES

$\phi = \frac{v_z}{v_{max}}$ $\quad -\frac{\partial p}{\partial z} = (p_0 - p_L)/L$ $\quad \xi = r/R$ $\quad \tau = \frac{\mu t}{\rho R^2}$

TRANSFORMATION OF EQUATION OF MOTION

$$\frac{\partial \phi}{\partial \tau} = 4 + \frac{1}{\xi}\frac{\partial}{\partial \xi}\left[\xi \frac{\partial \phi}{\partial \xi}\right] \quad \text{or} \quad \frac{\partial \phi}{\partial \tau} = 4 + \left[\frac{1}{\xi}\frac{\partial \phi}{\partial \xi} + \frac{\partial^2 \phi}{\partial \xi^2}\right]$$

INITIAL BOUNDARY CONDITIONS

I.C.:	$\tau = 0$	$\phi = 0$ for $0 \leq \xi \leq 1$
B.C. 1:	$\xi = 0$	$\frac{\partial \phi}{\partial \xi} = 0$
B.C. 2:	$\xi = 1$	$\phi = 0$

ANALYTICAL SOLUTION [3]

$$\phi = (1-\xi^2) - 8 \sum_{n-1}^{\infty} \frac{J_0(\alpha_n \xi)}{\alpha_n^3 J_1(\alpha_n)} e^{-\alpha_n^2 \tau}$$

where α_n are the roots of $J_0(\alpha)$

TABLE 3

EQUATIONS OF CONTINUITY AND MOTION
IN CYLINDRICAL COORDINATES (r,θ,z)
FOR A FLUID INITIALLY IN MOTION

EQUATION OF CONTINUITY ($v_r, v_\theta = 0$, ρ and μ constant)

$$\frac{\partial}{\partial z}(\rho v_z) = 0$$

EQUATION OF MOTION (v_r, v_θ, $\frac{\partial v_z}{\partial z} = 0$, ρ and μ constant)

$$\rho \frac{\partial v_z}{\partial t} = -\frac{\partial p}{\partial z} + \mu \left[\frac{1}{r}\frac{\partial}{\partial r}\left[r \frac{\partial v_z}{\partial r}\right]\right]$$

TRANSFORMATION OF VARIABLES

$$\phi = \frac{v_z}{v_{max}} \qquad -\frac{\partial p}{\partial z} = -(p_0 - p_L)/L \qquad \xi = r/R \qquad \tau = \frac{\mu t}{\rho R^2}$$

TRANSFORMATION OF EQUATION OF MOTION

$$\frac{\partial \phi}{\partial \tau} = -4 + \frac{1}{\xi}\frac{\partial}{\partial \xi}\left[\xi \frac{\partial \phi}{\partial \xi}\right] \quad \text{or} \quad \frac{\partial \phi}{\partial \tau} = -4 + \left[\frac{1}{\xi}\frac{\partial \phi}{\partial \xi} + \frac{\partial^2 \phi}{\partial \xi^2}\right]$$

INITIAL BOUNDARY CONDITIONS

I.C.:	$\tau = 0$	$\phi = 0$ for $0 \leq \xi \leq 1$
B.C. 1:	$\xi = 0$	$\frac{\partial \phi}{\partial \xi} = 0$
B.C. 2:	$\xi = 1$	$\phi = 0$

COMPARISON OF NUMERICAL AND ANALYTICAL SOLUTIONS

Velocity profiles, Figure 2, for the example in which the fluid was initially at rest, were calculated on the interval 0 to 1 with steps of 0.1, [0 (0.1) 1.0], for the equation of motion written in the form

$$\xi \frac{\partial \phi}{\partial \tau} = 4\, \xi + \left[\frac{\partial \phi}{\partial \xi} + \xi \frac{\partial^2 \phi}{\partial \xi^2}\right]$$

to eliminate numerical difficulties at $\xi = 0$. Results from HERCOL are denoted by asterisks "*", analytical solutions are shown as dashed curves (— — —), and the steady—state solution is shown as a solid line (———). Tables 4 and 5 list values of ϕ for ξ on the interval [0, (0.1), 1.0] calculated from HERCOL, and from the analytical solution; for practical purposes, the results are identical.

Figure 3 is a plot of the velocity profiles for the second example; the fluid is initially in motion with a Poiseuille velocity distribution and a pressure gradient with the same magnitude as in the first example but with opposite sign as imposed on the flow at $\tau = 0$. At $\tau = 0.05$, the velocity profile has been distorted and at $\tau = 0.20$, the flow is in the opposite direction. At steady—state, the velocity profile is symmetric about $\phi = 0$ with respect to the initial velocity profile. A condition such as this may occur in the discharge lines of a surge—type gas compressor.

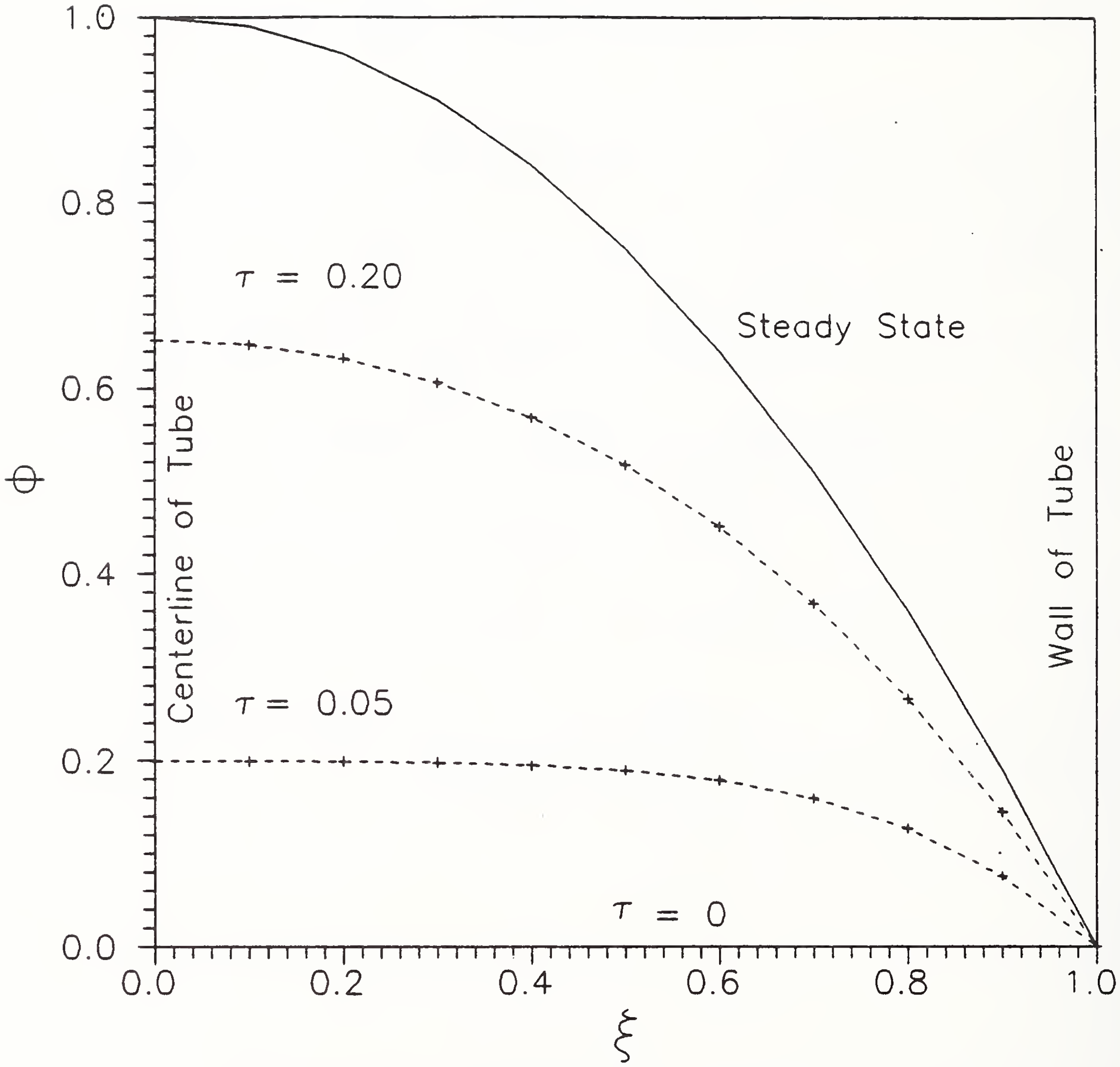

Figure 2. Velocity profiles for unsteady laminar flow in a circular tube, fluid at rest initially.

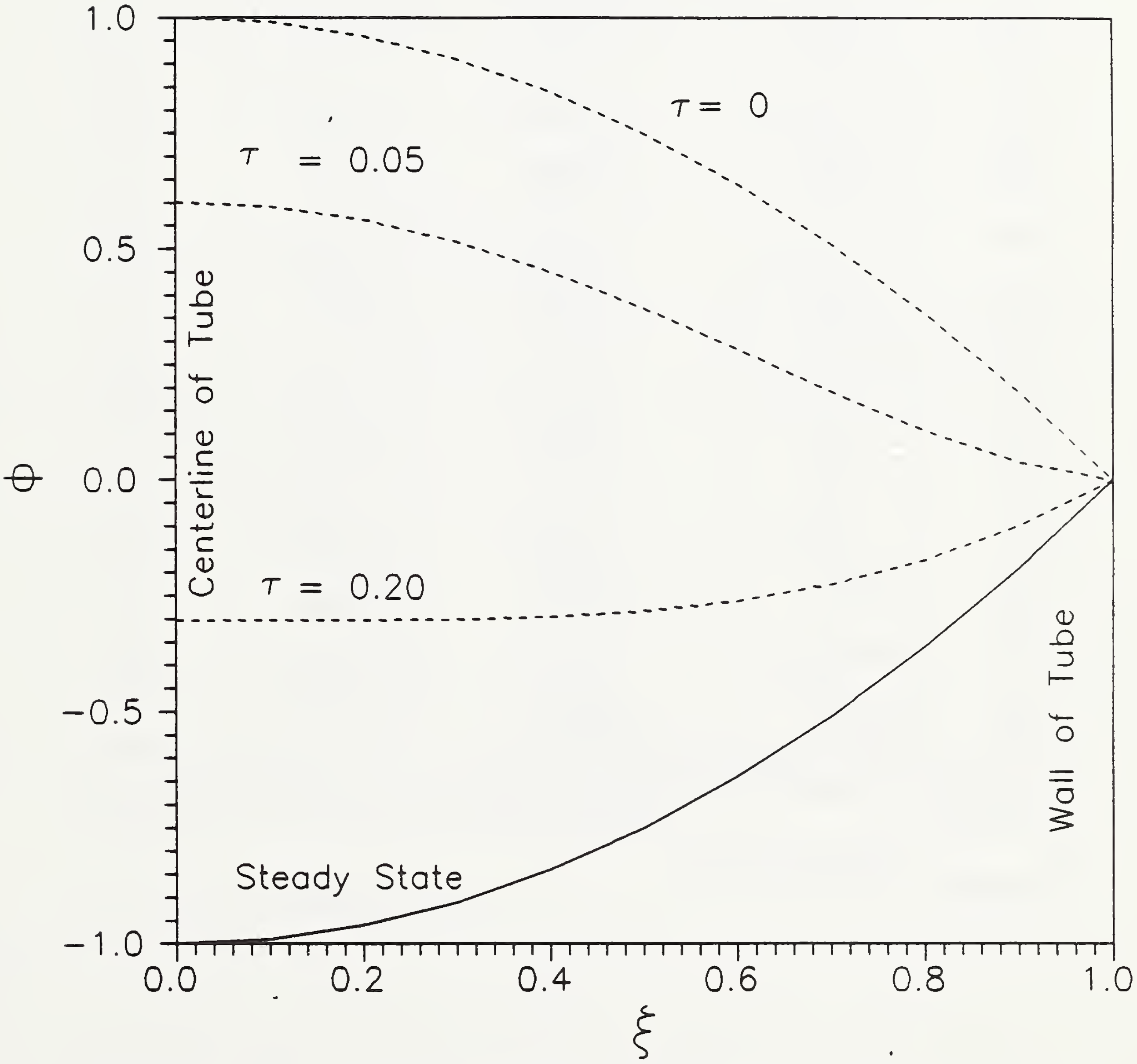

Figure 3. Velocity profiles for unsteady laminar flow in a cylindrical tube, fluid in motion initially.

TABLE 4

COMPARISON OF NUMERICAL AND ANALYTICAL SOLUTIONS OF

$$\frac{\partial \phi}{\partial \tau} = 4 + \left[\frac{1}{\xi}\frac{\partial \phi}{\partial \xi} + \frac{\partial^2 \phi}{\partial \xi^2}\right]$$

ON THE INTERVAL [0 (0.1) 1.0] AT $\tau = 0.05$

ξ	ϕ_{HERCOL}	$\phi_{\text{analytical}}$	% Difference
0.00	0.19962	0.19959	0.015
0.10	0.19948	0.19946	0.010
0.20	0.19897	0.19894	0.015
0.30	0.19772	0.19769	0.015
0.40	0.19497	0.19495	0.010
0.50	0.18935	0.18933	0.011
0.60	0.17861	0.17859	0.011
0.70	0.15936	0.15934	0.013
0.80	0.12701	0.12698	0.024
0.90	0.07594	0.07592	0.026
1.00	0.00000	0.00000	——

ϕ_{HERCOL} : numerical results from HERCOL

$\phi_{\text{analytical}}$: analytical solution

TABLE 5

COMPARISON OF NUMERICAL AND ANALYTICAL SOLUTIONS OF

$$\frac{\partial\phi}{\partial\tau} = 4 + \left[\frac{1}{\xi}\frac{\partial\phi}{\partial\xi} + \frac{\partial^2\phi}{\partial\xi^2}\right]$$

ON THE INTERVAL [0 (0.1) 1.0] AT $\tau = 0.20$

ξ	ϕ_{HERCOL}	$\phi_{\text{analytical}}$	% Difference
0.00	0.65181	0.65178	0.005
0.10	0.64680	0.64678	0.003
0.20	0.63158	0.63156	0.003
0.30	0.60551	0.60549	0.003
0.40	0.56759	0.56758	0.002
0.50	0.51646	0.51645	0.002
0.60	0.45049	0.45048	0.002
0.70	0.36782	0.36781	0.003
0.80	0.26650	0.26650	0
0.90	0.14454	0.14453	0.007
1.00	0.00000	0.00000	——

ϕ_{HERCOL} : numerical results from HERCOL

$\phi_{\text{analytical}}$: analytical solution

CONCLUSIONS

The performance of HERCOL was quite satisfactory for the solution of the equation of motion in cylindrical coordinates,

$$\frac{\partial \phi}{\partial \tau} = 4 + \left[\frac{1}{\xi}\frac{\partial \phi}{\partial \xi} + \frac{\partial^2 \phi}{\partial \xi^2}\right],$$

with combined Dirichlet, $\phi \Big|_{\xi}$, and Neumann, $\frac{\partial \phi}{\partial \xi}\Big|_{\xi}$, boundary conditions.

The code will be useful for generation of solutions for similar problems in which the initial and boundary conditions preclude analytical solutions, the equation of state is not a simple function, or the transport properties of the fluid are not constant.

ACKNOWLEDGMENTS

The comments and suggestions of J.M. Gary, the author of HERCOL, were quite helpful in this effort.

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[1] Gary, John M., "HERCOL Computer Program for the Solution of Initial Boundary—Value Problems with Hermitian Collocation," Applied and Computational Mathematics Division 719, National Institute of Standards and Technology, 325 Broadway, Boulder, CO 80303

[2] Szymanski, P., "Quelques Solutions exactes des equations de l'hydrodynamiquie due fluide visqueux dan les cas d'un tube cylindrique," Journal de Mathematiques Pures et Appliquies, Series 9, 11, (1932), pp. 67—107.

[3] Bird, R.B., W.E. Stewart, and E.N. Lightfoot, "Transport Phenomena," John Wiley & Sons, New York, (1960), pp. 126—130.

NOMENCLATURE

Symbols

p = pressure

r = radial coordinate, L

R = tube radius, L

t = time, t

v = velocity, L/t

z = rectangular coordinate, L

α = roots of Bessel function of the first kind, zero order

ρ = density, M/L^2

ξ = dimensionless position variable

ϕ = dimensionless volocity variable

θ = azmithual coordinate, radians

τ = dimensionless time variable

Subscripts

analytical	analytical solution
HERCOL	numerical solution
L	station "L"
max	maximum
z	z—component
0	station "0"

Mathematical Functions

J_0 = Bessel function of the first kind, zero order

J_1 = Bessel function of the first kind, first order

APPENDIX A

LISTING OF SOURCE CODE FOR CALCULATION OF UNSTEADY LAMINAR VELOCITY PROFILES

```
C  Name:          BSL126.FOR

C  Required:      HERCOL.FOR

C  Purpose:       Solve for unsteady laminar flow in a
C                 circular duct, Example 4.1-1 BSL

C  Keywords:      fluid flow, laminar, duct, unsteady

C  Type:          Program

C  Status:        Experimental

C  Reference:     Bird, R.B., W.E. Stewart, and E.N. Lightfoot,
C                 "Transport Phenomena", John Wiley & Sons, New York
C                 (1960), p. 126

C  Version:       011591

      PROGRAM HCTEST

      PARAMETER (NU1=81, NU2=4, LW=20)
      REAL RWORK(10000), XPTS(NU1), U(NU1, NU2)
      REAL WL(LW), WR(LW)
      REAL XO(NU1), UX(NU1, NU2), UERR(NU1,3), UMX, SECOND
      REAL RTOL, ATOL, T, TOUT, CPU1, CPU2, PI2, ALPHA, EPSA, EPSR
      INTEGER IWORK(1000), INFO(15), NODE(2), LUNIT, NPRT, NSYS
      INTEGER NPTS, LRW, LIW, MESS, L, NPDE, M, K, IDID, NODE2, I

      COMMON /FCOM/ NSYS
      COMMON /INICOM/ ALPHA
      COMMON /TOLCOM/ NODE2(2),EPSR,EPSA

C      INTEGER   NU1,NU2,LW

      DATA LRW, LIW/20000, 1000/
C
      OPEN(9,FILE='BSERRM.OUT',STATUS='UNKNOWN')
      OPEN(10,FILE='BSCALCS.OUT',STATUS='UNKNOWN')

C     DO 600, MM = 1,5

      MESS = 10

C  WRITE ERROR MESSAGES ON UNIT 9

      LUNIT = 9

      NSYS = 2

C  INITIAL VALUE OF THE TIME-LIKE VARIABLE

      T = 0.
```

```
C  FINAL VALUE OF THE INTEGRATION VARIABLE

      TOUT = 0.20

C  INFO IS USED TO SET OPTIONS

      DO 005, L = 1,15
        INFO(L) = 0
005   CONTINUE

C     ALPHA = 10**(MM + 1)
C     WRITE (MESS, 200) ALPHA

C  NUMBER OF ODE VARIABLES WL AT LEFT  BOUNDARY XPTS(1)

      NODE(1) = 0

C  NUMBER OF ODE VARIABLES WR AT RIGHT BOUNDARY XPTS(NPTS)

      NODE(2) = 0

C  RELATIVE AND ABSOLUTE ERROR TOLERANCES

      RTOL=1.0E-4
      ATOL=1.0E-4

C  NUMBER OF COMPONENTS IN THE VECTOR OF UNKNOWNS, U

         NPDE = 1

C  NUMBER OF BREAKPOINTS, XPTS(1)...XPTS(NPTS)

         NPTS = 11

C  SET UP THE MESH AND SPECIFY THE INITIAL VALUE OF U

      DO 010 I=1,NPTS

       XPTS(I) = REAL(I-1)/REAL(NPTS-1)

C  Bird, Stewart, and Lightfoot (4.1-28)

       U(I,1)  = 0.0
C  Bird, Stewart, and Lightfoot (4.1-27)

          UX(I,1) = 0.0

010     CONTINUE

      EPSR=RTOL
      EPSA=ATOL
```

```
      NODE2(1)=NODE(1)
      NODE2(2)=NODE(2)

C  WRITE OUT THE INPUT PARAMETERS
      WRITE (MESS,198)
      WRITE (MESS, 210) NPDE, NPTS, T, TOUT, RTOL, ATOL, NSYS,
     $         (INFO(L), L=1, 15)
      WRITE (MESS, 220) NODE

C  TIME THE HERCOL SUBROUTINE

      CPU1 = SECOND()

      CALL HERCOL(INFO,IDID,U,UX,NU1,NPTS,NPDE,WL,WR,NODE,XPTS,
     1        T,TOUT,RTOL,ATOL,RWORK,LRW,IWORK,LIW,LUNIT)

      CPU2 = SECOND() - CPU1

      IF(IDID .LT. 0) THEN
         WRITE (MESS,230) IDID,T
         GOTO 600
      END IF

      WRITE (MESS, 240) T, IDID, CPU2
      WRITE (MESS, 250) RWORK(7), (IWORK(L), L=11, 15), IWORK(8)


C  CALCULATE THE ERROR
      IFLAG = 0

      UMX = 0.0
      SQERR = 0.0

C  COMPUTE THE SQUARE ERROR

C     DO 020 I = 1,NPTS
C        VAL = THETA(XPTS(I), T, IFLAG)
C        UERR(I,1) = ABS(U(I,1) - VAL)
C        UMX = MAX(UMX,ABS(UERR(I,1)))
C        SQERR = SQERR + UERR(I,1)**2
020   CONTINUE

      WRITE (MESS,370) NSYS,T,UMX

      WRITE (MESS,398)

      DO 030 I = 1,NPTS


      WRITE (MESS,405) I, XPTS(I), U(I,NPDE)

C     WRITE (MESS,405) I, XPTS(I), U(I,NPDE), UX(I,NPDE), UERR(I,NPDE)

030   CONTINUE
```

```
C     WRITE (MESS,201) SQERR

600   CONTINUE

      CLOSE(9)
      CLOSE(10)
C  FORMAT STATEMENTS

198   FORMAT (1X,'+++++++++++++++++++++++++')
200   FORMAT (1X,'ALPHA = ',1PE12.4)
201   FORMAT (1X,' SQUARE ERROR = ',1PE12.4)
210   FORMAT (/' EX 4.1-2 BSL, P. 126     '/1X,'NPDE=', I2,' NPTS=',
     $  I3,' T=', 1P E10.3,' TOUT=', E10.3/1X,'RTOL=', E9.2,' ATOL=',
     $  E9.2, ' NSYS=', I2/1X,'INFO=', 15I4)
220   FORMAT (' NODE=', 2I5)
230   FORMAT(/' ***** HERCOL FAILED  IDID=',I6,' T=',1PE10.3)
240   FORMAT (' T=', 1P E10.3,'  IDID=', I5,' CPU=', E9.2)
250   FORMAT (3X,' H=', 1P E9.2,' NSTEP=', I4,' NFE=', I4,' NJE=', I4,
     $        ' NEF=', I3,' NCF=', I3,' NQ=', I1)
370   FORMAT(5X,'NSYS=',I2,'  T=',1PE10.3,' MAX ABS ERROR=',E10.3)
398   FORMAT (1X,'TABLE OF CALCULATED RESULTS')
405   FORMAT (1X,I4,2X,5(1PE12.4))
610   FORMAT(' UERR(M,I)    M=',I2/(5X,1P8E9.2))
620   FORMAT(' UERR(I)   '/(5X,1P8E9.2))

      END

C=============================================================
```

```
      SUBROUTINE FUN(T, XC, UT, U, UX, UXX, NCPT, NPDE, FT, IRES)

C     FOR THE HCTEST OF HERCOL

      REAL  T,XC(NCPT), U(NCPT, NPDE), UX(NCPT, NPDE),
     $        UXX(NCPT, NPDE), FT(NCPT, NPDE), UT(NCPT, NPDE)

      COMMON /FCOM/ NSYS

      INTEGER NCPT, NPDE, IRES
      INTEGER M, I
      INTEGER   NSYS

      DO 020 M = 1, NPDE

      DO 010 I = 1, NCPT

C  Bird, Stewart, and Lightfoot (4.1-21)

      FT(I, M) = XC(I)*UT(I,M) - XC(I) * UXX(I,M) - UX(I,M) - 4 * XC(I)

010     CONTINUE
020     CONTINUE

      RETURN
      END
C
C  =======================================================
C
      SUBROUTINE BDYLFT(T, UT, U, UX, NPDE, WT, W,  B,  IRES)
C
C     FOR THE HCTEST OF HERCOL
C
C
      REAL T, UT(NPDE), U(NPDE), UX(NPDE), WT(*),  W(*), B(*), PI2

      COMMON /FCOM/ NSYS

      INTEGER NPDE,  IRES
      INTEGER   NSYS

C  FINITE VALUE OF U AT LEFT-HAND BOUNDARY

        B(1) = UX(1)

      RETURN
      END

C  =======================================================

      SUBROUTINE BDYRHT(T, UT, U, UX, NPDE, WT, W,  B,  IRES)
```

```
        REAL T, UT(NPDE), U(NPDE), UX(NPDE), WT(*), W(*), B(*), PI2
      INTEGER NPDE,  IRES
      INTEGER     NSYS
      COMMON /FCOM/ NSYS
      COMMON /INICOM/ ALPHA

C  U = 0 AT RIGHT-HAND BOUNDARY

        B(1) = U(1)

      RETURN
      END
```

```
C  ==================================================

      SUBROUTINE JACOB(T, XC, UT, U, UX, UXX, WTL, WL,  WTR, WR,
     $         NCPT, NPDE, DFDU, DBLDW,NBL,NWL, DBLDU, DBRDW, NBR,NWR,
     1        DBRDU)

C     PROVIDE AN ANALYTIC JACOBIAN FOR THE HCTEST CODE

      REAL  XC(NCPT), UT(NCPT, NPDE), U(NCPT, NPDE),
     $       UX(NCPT, NPDE), UXX(NCPT, NPDE), WTL(*), WL(*), WTR(*),
     $       WR(*), DFDU(NCPT, NPDE, NPDE, 4), DBLDW(NBL, NWL, 2),
     $        DBLDU(NBL, NPDE, 3), DBRDW(NBR, NWR, 2),
     $        DBRDU(NBR, NPDE, 3), T

      COMMON /FCOM/ NSYS

      INTEGER NCPT, NPDE,NBL,NBR,NWL,NWR
      INTEGER K,M
      INTEGER    NSYS
C
      DBLDU(1, 1, 1) =  1.0E0
      DBLDU(1, 1, 2) = -1.0E0
      DBRDU(1, 1, 1) =  1.0E0
      DBRDU(1, 1, 2) = -1.0E0

      RETURN
      END

C  ========================================================
```

```
      SUBROUTINE SETTOL(RTOLK,ATOLK,RTOLW,ATOLW,NPTS,NPDE)

C     SET ERROR TOLERANCE ARRAY FOR TEST CASES

      REAL  RTOLK(NPTS,NPDE),ATOLK(NPTS,NPDE),
     1        RTOLW(*), ATOLW(*)

      REAL     EPSR,EPSA

      COMMON /TOLCOM/ NODE(2),EPSR,EPSA
      COMMON /FCOM/ NSYS

      INTEGER ND1,NPDE,NPTS,K,M
      INTEGER   NSYS,NODE
C
      DO 20 K=1,NPTS
      DO 20 M=1,NPDE

      RTOLK(K,M)=EPSR
      ATOLK(K,M)=EPSA

20    CONTINUE


      IF(NODE(1) .NE. 0) THEN

      DO 30 K=1,NODE(1)

      RTOLW(K)=EPSR
30    ATOLW(K)=EPSA

      END IF

      IF(NODE(2) .GT. 0)THEN
      ND1=NODE(1)

      DO 40 K=ND1+1,NODE(2)+ND1

      RTOLW(K)=EPSR
      ATOLW(K)=EPSA

40    CONTINUE

      END IF
      RETURN

      END
```

```
C  ==========================================================

      SUBROUTINE UINIT(XC,UC,NCPT,NPDE,WL,WR)

      REAL  XC(NCPT),UC(NCPT,NPDE),WL(*),WR(*),PI2
      REAL    ALPHA

      COMMON /INICOM/ ALPHA
      COMMON /FCOM/ NSYS

      INTEGER NCPT,NPDE,K,M
      INTEGER  NSYS

      DO 100 K = 1, NCPT

        UC(K, 1) = XC(K)**2

  100 CONTINUE

      RETURN
      END

C------------ END OF TEST ROUTINE -------------
```

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11. ABSTRACT (A 200-WORD OR LESS FACTUAL SUMMARY OF MOST SIGNIFICANT INFORMATION. IF DOCUMENT INCLUDES A SIGNIFICANT BIBLIOGRAPHY OR LITERATURE SURVEY, MENTION IT HERE.)

HERCOL, a computer code for the integration of second-order differential equations in one space dimension by Hermitian collocation was used to calculate the unsteady velocity profiles for laminar flow in a circular tube. The code was tested for stability and accuracy on this problem for which an analytical solution exists prior to application to a like problem in which the initial and boundary conditions preclude the existence of analytical solutions.

The test problem is one in which a pressure gradient is imposed on a fluid initially at rest in a circular tube; the fluid accelerates and at steady state has a parabolic velocity profile. A second example was constructed from the first; a pressure gradient equal but opposite in sign in imposed on the fluid with a fully developed parabolic velocity profile. At steady state, the velocity is again parabolic but in the opposite direction to that at the initial conditions.

Excellent agreement with the analytical solution was obtained in the first problem; in the second, the behaviour was as expected. This example is suitable for first-time users of the code.

12. KEY WORDS (6 TO 12 ENTRIES; ALPHABETICAL ORDER; CAPITALIZE ONLY PROPER NAMES; AND SEPARATE KEY WORDS BY SEMICOLONS)

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