

NBSIR 88-3095

DIELECTRIC MIXING RULES FOR BACKGROUND TEST SOILS

Richard G. Geyer

National Bureau of Standards
U.S. Department of Commerce
Boulder, Colorado 80303-3328

June 1988

FILE COPY
DO NOT REMOVE



Stimulating America's Progress
1913-1988

NBSIR 88-3095

DIELECTRIC MIXING RULES FOR BACKGROUND TEST SOILS

Richard G. Geyer

Electromagnetic Fields Division
Center for Electronics and Electrical Engineering
National Engineering Laboratory
National Bureau of Standards
Boulder, Colorado 80303-3328

June 1988

Sponsored by
U.S. Army Belvoir Research & Development Center
Fort Belvoir, Virginia 22060-5606



U.S. DEPARTMENT OF COMMERCE, C. William Verity, Secretary
NATIONAL BUREAU OF STANDARDS, Ernest Ambler, Director

CONTENTS

	<u>Page</u>
Abstract.1
1. INTRODUCTION1
2. DIELECTRIC PROPERTIES OF PURE WATER.3
3. SALINE WATER.4
4. HETEROGENEOUS SOIL MIXTURES.6
4.1 Function-Theoretic Rules.	6
4.1.1 General Water-Dominated Dielectric Behavior of Natural Soils.7
4.2 Generalized Heuristic Rules for Soil Mixtures.8
4.2.1 Dielectric Mixing Rule for Dry Soils.	11
4.2.2 Dielectric Mixing Rule for Lossy, Fluid-Saturated Soils.	11
4.2.2.1 Partially Saturated Soil Mixtures.	12
4.2.2.2 Saturated Soil Mixtures.	14
5. APPLICATION TO SOIL STANDARDS.15
6. CONCLUSIONS.16
7. REFERENCES.17

DIELECTRIC MIXING RULES FOR BACKGROUND
TEST SOILS

Richard G. Geyer

Electromagnetic Fields Division
National Bureau of Standards
Boulder, Co 80303

The bulk, or effective, dielectric constant of any background test medium (whether naturally occurring or synthetic) determines the electromagnetic visibility of buried objects. Heuristic mixing rules are considered that allow the prediction of complex dielectric behavior in linear, homogeneous, isotropic, and lossy multi-phase soil mixtures. Measurement results in bioelectromagnetic and microwave remote sensing suggest a refractive mixing model as that being most suited for dry soils or soil-water mixtures.

Key words: dielectric simulation; mixing rules; multi-phase; soils.

1. INTRODUCTION

Soils are mixtures of minerals that are the weathered by-products of rocks. Usually, natural soils contain some water so that the way in which dielectric constants combine is important in determining the bulk, or effective, dielectric constant of the soil at hand. The bulk dielectric constant of any soil (whether natural or synthetic) determines the electromagnetic visibility of buried land mines and, therefore, intrinsically affects the performance of any detection system. Once a range of suitable dielectric background test characteristics relatable to conditions actually encountered in the field is accepted, mixing rule(s) provide the easiest and most pragmatic approach to specification of "standard" test soils.

Geyer [1] has noted that some mixing rules have little broadband applicability for soils since they are based on systems where little or no conduction takes place. For example, when one of the components in a composite dielectric is conductive, large (frequency-, temperature-, and salinity-dependent) polarizations can occur that can be attributed to

interfacial polarizations at the boundaries of the conductive phase. A mixing rule that does not account for these interfacial polarizations has little broadband applicability. Despite the fact that most mixing rules fail in a broadband sense (since they do not account for all relaxation effects [1]), it is nonetheless useful to be able to predict the effective electromagnetic properties of soils of different lithologies and porosities even over narrower frequency ranges--100 MHz to 2.0 GHz in this case--that affect the signal levels measured by UHF land mine detection systems.

The purpose of a mixing rule is to allow estimation of the dielectric properties of a soil (either test standard or soil encountered in the field) from measurements of that soil's physical properties only. Its usefulness will be limited by its dependence on adjustable soil- and frequency-specific parameters. An efficacious mixing rule, then, will provide a convenient means for predicting a test soil's dielectric behavior in microwave land mine performance testing and in signal level determinations. It will also provide a physically based mixing model that is dependent on measurable soil parameters and can be used for subsequent development and research purposes.

Generally, all materials can be classified into one of three dielectric groups. The first of these, to which pure water and ice belong, is homogenous substances. A second group, in which ionic salts are dissolved in solution (usually water) is electrolytic solutions. The third group, heterogeneous mixtures, includes that of wet, lossy soils or multicomponent (lossy or nonlossy) media of interest here for test soil lanes. The merits of any chosen background material (standard) for mine test lanes depend not only on correlation with actual field conditions likely to be encountered but also on the ease with which its (complex) dielectric properties can be predicted and controlled for the operational frequency range of interest. Before discussing mixing rule(s) applicable to heterogeneous soil mixtures, it is necessary to review fundamental dielectric behavior of pure and saline water.

2. DIELECTRIC PROPERTIES OF PURE WATER

The frequency dependence of the complex dielectric constant of pure water for $\exp(j2\pi f\tau)$ time dependence is well-known and is given by the Debye equation [1,2]

$$\epsilon_W = \epsilon_{W\infty} + \frac{\epsilon_{W0} - \epsilon_{W\infty}}{1 + j2\pi f\tau_W} \quad (1)$$

where

$\epsilon_{W\infty}$ = high frequency (or optical) limit of ϵ_W (dimensionless),

ϵ_{W0} = static dielectric constant of pure water (dimensionless),

τ_W = relaxation time of pure water (seconds), and

f = electromagnetic frequency (Hz).

Equation (1) may be simply written

$$\epsilon_W = \epsilon'_W - j\epsilon''_W = \epsilon'_W (1 - j\tan\delta_W) \quad (2)$$

where $\tan\delta_W \equiv \epsilon''_W/\epsilon'_W$ is the loss tangent of water (temperature dependent).

Rationalization of eq (1) yields

$$\epsilon'_W = \epsilon_{W\infty} + \frac{\epsilon_{W0} - \epsilon_{W\infty}}{1 + (2\pi f\tau_W)^2} \quad (3)$$

and

$$\epsilon''_W = \frac{2\pi f\tau_W (\epsilon_{W0} - \epsilon_{W\infty})}{1 + (2\pi f\tau_W)^2} \quad (4)$$

The magnitude of the high frequency dielectric constant $\epsilon_{W\infty}$ has been determined by Lane and Saxton [3] to be

$$\epsilon_{W\infty} = 4.9 \quad (5)$$

and is practically temperature independent. The relaxation frequency of pure water, $f_{w0} = 1/2\pi\tau_w$, occurs in the microwave region and is temperature dependent. At 0°C, $f_{w0} \cong 9$ GHz and at 20°C, $f_{w0} \cong 17$ GHz. It can be seen from eqs (3) and (4) that ϵ_w'' has its maximum value at $f = f_{w0}$. A Cole-Cole plot of ϵ_w'' vs ϵ_w' , with f as a variable parameter, is shown in figure 1 for pure water at $T = 0^\circ\text{C}$. This plot takes the shape of a semicircle with end points defined by $\epsilon_w = \epsilon_{w0}$ in the low-frequency limit and by $\epsilon_w = \epsilon_{w\infty}$ in the high-frequency limit. The point on the circle at which ϵ_w'' is maximum occurs at the relaxation frequency, $f = f_{w0}$, whose coordinates are simply given by $\epsilon_w' = (\epsilon_{w0} + \epsilon_{w\infty})/2$ and $\epsilon_w'' = (\epsilon_{w0} - \epsilon_{w\infty})/2$.

The dielectric constant of nonconductive water, ϵ_{w0} , is a function of temperature. Klein and Swift [6] have generated a regression fit for $\epsilon_{w0}(T)$ from dielectric measurements conducted between 1 GHz and 3 GHz

$$\epsilon_{w0}(T) = \epsilon_{w0}'(T) = 88.045 - 0.4147 T + 0.0006295 T^2 \quad (6)$$

where T is in °C.

3. SALINE WATER

Although the dielectric properties of pure water and ice obey the Debye relaxation equations given above and are fairly well understood, ionic salts dissolved in water produce an electrolytic solution whose microwave dielectric properties may differ greatly from those of pure water. The salinity, S , of a solution is defined as the total mass of solid salt in grams dissolved in 1 kg of solution. Thus, S is normally expressed in parts per thousand on a weight basis. Little will be said about broadband dielectric characterization of electrolytic solutions, except that the real and imaginary parts of the dielectric constant of a saline water solution are given by [3,6]

$$\epsilon_{sw}' = \epsilon_{sw\infty} + \frac{\epsilon_{sw0} - \epsilon_{sw\infty}}{1 + (2\pi f \tau_{sw})^2} \quad (7)$$

and

$$\epsilon_{SW}'' = \frac{2\pi f \tau_{SW} (\epsilon_{SWO} - \epsilon_{SW\infty})}{1 + (2\pi f \tau_{SW})^2} + \frac{\sigma_i}{2\pi \epsilon_0 f} \quad (8)$$

where the subscript sw refers to saline water, σ_i is the ionic conductivity of the aqueous solution in S/m, and ϵ_0 is the permittivity of free space ($\epsilon_0 = 8.854 \times 10^{-12}$ F/m). Note that the form of the complex permittivity for saline water differs from the of pure water only in the loss term, ϵ_{SW}'' , where the added term due to ionic conductivity of the aqueous solution, $\sigma_i/\omega\epsilon_0$, is present. It is clear from eq (8) that the ionic conductivity of saline water has marked effect on the loss factor ϵ_{SW}'' below 10 GHz. Therefore, it is expected that high soil salinities will significantly affect the dielectric properties of wet soil. As Jedlicka [7] notes, few measurements and analyses have been reported relating soil salinity to effective soil dielectric constant. Consequently, the dependence to soil salinity to permittivity is not well understood. We do know, however, that the salinity of (free) pore water within a soil matrix depends directly on the cation exchange capacity of the matrix material. This fact should provide a direct relationship between soil type, amount of volumetric moisture present, and effective soil complex permittivity. Further work will enhance understanding between these physical soil parameters and measured electromagnetic properties.

Stogryn [8] points out that there is no evidence to indicate that $\epsilon_{SW\infty}$ depends on salinity; hence $\epsilon_{SW\infty} = \epsilon_{W\infty} = 4.9$. He also empirically determines both the temperature and salinity dependence of saline water by writing ϵ_{SWO} as a factorable product, i.e., $\epsilon_{SWO}(T,S) = \epsilon_{SWO}(T,0) F(T,S)$. Polynomial fits to the above relation have then been obtained [5,9] on measurements performed by Ho and Hall [10] and Ho, et. al. [11] for $4 \leq S \leq 35$ to obtain the following functional dependence of the dielectric constant of water with respect to salinity.

$$\epsilon_{SWO}' = \epsilon_{WO}'(T) - 0.1556 - 0.000413 S + 0.00000158 S^2 \quad (9)$$

and

$$\epsilon_{SWO}' \tan \delta_{SWO} = 5.66 + 0.00265 S - 0.0000045 S^2 \quad (10)$$

We note from eqs (9) and (10) that ϵ'_{SWO} is not an independent parameter. Equations (6), (9) and (10) will be used in subsequent development relating effective dielectric constant measurements (real part, ϵ' , and loss tangent, $\tan\delta$) of a fluid-saturated lossy soil mixture to the in situ fluid dielectric properties, as well as to the water-filled porosity of the soil mixture.

4. HETEROGENEOUS SOIL MIXTURES

4.1 Function-Theoretic Rules

In general, the average dielectric constant of a heterogeneous mixture consisting of two or more substances is related to the dielectric constants of the individual substances, their volume fractions, their spatial distributions, and their orientations relative to the direction of the incident electric field vector [12]. In order to determine the functional dependence of the average dielectric constant of a mixture to these variables, the average electric field within the mixture as a whole must be related to the electric fields within the inclusions [4]. The problem is that if the inclusions are randomly dispersed throughout the host medium, it is not generally possible to derive an exact solution for the fields within the inclusions since the mutual electromagnetic interactions of the inclusions are dependent on their positions with respect to each other. Tinga [13] gives a review of the various approximations that have been proposed for solving the interaction problem. These approximations vary from ones ignoring short-range interactions between inclusions (by restricting the validity of the dielectric mixing model to only those mixtures characterized by a low concentration of inclusions) to relations that account for first-order inclusion interactions via the solution of Maxwell's equations and appropriate boundary conditions [11,13]. In all cases, the dimensions of the inclusions are much smaller than the propagation wavelength in the host medium. Röttcher [14] gives a comprehensive review of dielectric mixing models that includes both empirical (or semi-empirical) formulations for specific mixtures as well as theoretical models developed for highly specialized media that contain either ellipsoidal particle inclusions (spheres, disks, needles) or confocal ellipsoidal inclusions. A limitation of much of this work is its restricted applicability to only specific mixtures and /or conditions.

4.1.1 General Water-Dominated Dielectric Behavior of Natural Soils

By definition, natural soils are mixtures of host matrix mineral(s), air, and water. Generally, the effective relative permittivity, ϵ_{eff}' , of the mixture rarely exceeds 8 in the microwave region--as long as there is no liquid water present within the mixture. Similarly, ϵ_{eff}'' usually does not exceed 1 in the absence of liquid water. By way of comparison, we see from figure 1 that ϵ_{w}' of liquid water is one order of magnitude larger than the relative permittivity of dry materials, whereas ϵ_{w}'' is two orders of magnitude larger than ϵ'' of dry materials, particularly at frequencies less than 2 GHz. Because of the large contrast between (complex) ϵ of the pore water and that of the host matrix in soils, the dielectric constant of the mixture is generally dominated by the dielectric behavior of water. For this reason, many investigators have generalized the Debye formulas given by eqs (3) and (4) to mixtures in the following way,

$$\epsilon_{\text{eff}}' = \epsilon_{\text{eff},\infty} + \frac{(\epsilon_{\text{eff},0} - \epsilon_{\text{eff},\infty})}{1 + (2\pi f \tau_{\text{eff}})^2} \quad (11)$$

$$\epsilon_{\text{eff}}'' = 2\pi f \tau_{\text{eff}} \frac{(\epsilon_{\text{eff},0} - \epsilon_{\text{eff},\infty})}{1 + (2\pi f \tau_{\text{eff}})^2} \quad (12)$$

where $\epsilon_{\text{eff},0}$, $\epsilon_{\text{eff},\infty}$ and τ_{eff} are functions of the dielectric constant of the matrix mineral(s), the water-volume fraction in the pore space of the matrix, and shape and orientation of the water inclusions. The dielectric constants of the matrix minerals are presumed to be nondispersive (frequency independent or lossless) as well as independent of the applied direction (polarization) of the incident electric field. The latter presumptions are borne out by many measurements so that the above formulas are often used in sea-ice and snow remote sensing [15,16]. However, caution must be exercised in the use of eqs (11) and (12) for soil mixtures over a broad range in frequency, since multiple relaxation phenomena not taken into account by these Debye-type rules can occur [1].

4.2 Generalized Heuristic Rules for Soil Mixtures

In the previous section it was stated that natural soils are mixtures of matrix minerals, air and water. As noted in [1], water in the pore spaces of surface soils consists of two phases. One of these phases is the bulk (free) pore water. The other is adsorbed water adjacent to the matrix grain surfaces that is usually several molecular layers in thickness. This adsorbed water is referred to as surface (or bound) water. The saturation level at which the dielectric constant becomes less sensitive to water saturation is entirely a function of the amount of surface water in soil pore spaces, a fact which explains the dependence of the dielectric constant on soil texture types (particle size and shape distributions) as well as on the intrinsic porosity of the soil mixture. Of course, the porosity is simply related to the bulk density, and the degree of conductive loss responsible for electromagnetic wave attenuation is dependent on the salinity of the pore water which, in turn, depends quite naturally on both the matrix cation exchange capacity and temperature. The fact that the electrical properties of water-saturated soils are frequency dependent is also not physically surprising since we would expect polarization, i.e., the orientation of polar molecules (molecules with asymmetric charge distributions) to occur in an applied electric field. The mobility of any polar molecule (such as water) will always depend on time rate of variation of that field.

It is well known that (bound) water adsorbed to a solid surface has a dielectric response significantly different from that of free water. This can be attributed to the fact that the molecular mobility of the water molecules has been reduced by physical bonding to the matrix grain surfaces [17,18,19]. Most of the measurement evidence to date seems to indicate that the restricted mobility causes a reduction of the static dielectric constant of water adsorbed to the surface from 80 for free water to about 6 for sorped water [18], greater than an order of magnitude reduction. Furthermore, surface-bound water exhibits a relaxation of about 10^4 Hz instead of 100 GHz, a relaxation frequency seven orders of magnitude lower than that of bulk water [1,20].

Thus the most general form of a heuristic, predictive dielectric mixing rule that might be used for soil specifications would separate the volumetric percentages of bound and free water. The soil mixture is then described electrically as a four-component system whose constituent components are linearly combined in terms of the respective volume fractions

$$\epsilon_{\text{soil}}^{\alpha} = v_m \epsilon_m^{\alpha} + v_a \epsilon_a^{\alpha} + v_{\text{bw}} \epsilon_{\text{bw}}^{\alpha} + v_{\text{fw}} \epsilon_{\text{fw}}^{\alpha} \quad (13)$$

where v represents volume fraction, the subscripts m , a , bw , and fw represent soil matrix, air, bound water, and free water, respectively, and where α is a constant.

Here we recognize that

$$\phi = \text{total porosity} = (\rho_m - \rho_b) / \rho_m = 1 - \rho_b / \rho_m$$

where ρ_m , ρ_b denote the matrix and bulk density, respectively.

$$v_m \equiv 1 - \phi \quad (14)$$

$$v_a \equiv \phi - S_w$$

$$S_w = v_{\text{bw}} + v_{\text{fw}}$$

The two terms involving bound and free water are often combined into a single term with an empirically determined multiplicative factor of the free water permittivity in the soil [6]. That is,

$$v_{\text{bw}} \epsilon_{\text{bw}}^{\alpha} + v_{\text{fw}} \epsilon_{\text{fw}}^{\alpha} \doteq S_w^{\nu} \epsilon_{\text{fw}}^{\alpha} \quad (15)$$

where the exponent, ν , of total water saturation, S_w , is empirically derived.

Substituting eqs (14) and (15) into (13) and recognizing that $\epsilon_a^{\alpha} \equiv 1$ yields the following relation in terms of porosity and water saturation

$$\epsilon_{\text{Soil}}^{\alpha} = (1 - \phi) \epsilon_m^{\alpha} + \phi - S_w + S_w^v \epsilon_{f_w}^{\alpha} . \quad (16)$$

In terms of the bulk and soil matrix densities, eq (16) may be written

$$\epsilon_{\text{Soil}}^{\alpha} = (1 - 1 + \rho_b/\rho_m) \epsilon_m^{\alpha} + 1 - \rho_b/\rho_m + (S_w^v \epsilon_{f_w}^{\alpha} - S_w)$$

or

$$\epsilon_{\text{Soil}}^{\alpha} = 1 + \frac{\rho_b}{\rho_m} (\epsilon_m^{\alpha} - 1) + (S_w^v \epsilon_{f_w}^{\alpha} - S_w) \quad (17)$$

where $\rho_b/\rho_m = v_m/v_b$ is the soil matrix volume fraction.

For $\alpha = 1$, eq (17) is known as a linear model; for $\alpha = 1/2$ as a refractive model (since $\epsilon^{1/2} \equiv n$ is the refractive index), and for $\alpha = 1/3$ as the cubic model.

The results of over 500 measurements made by Dobson et al. [6] on five differing soil types at nine frequencies (from 4.0 GHz to 18 GHz) and for volumetric moistures, S_w , ranging from 0.01 to total saturation gave values of v between 1.0 and 1.16 and suggested a refractive model ($\alpha = 1/2$) as that being most suited for soil-water mixtures. This was observed earlier by Shutko and Reutov [21]. Hence $S_w^v \doteq S_w$ and

$$\epsilon_{\text{Soil}}^{1/2} \doteq 1 + \frac{\rho_b}{\rho_m} (\epsilon_m^{1/2} - 1) + S_w (\epsilon_{f_w}^{1/2} - 1). \quad (18)$$

It has been noted [12] that combining the effects of bound and free water into one term (eq (15)) reduces the implicit dependence of ϵ_{Soil} on soil type; this is not surprising since clays are expected to have more bound water than, say, sand grains. It should be noted that ϵ_{Soil} and ϵ_{f_w} in eq (18) are in general complex quantities, since attenuative loss due to electrolytic conduction is fundamentally implicit in natural soils and therefore must be incorporated into test lane soils for realistic performance testing. In fact, the loss tangent of soil pore water, which depends on salinity (and temperature), is essentially an in situ parameter and cannot be ascertained correctly by pore fluid extraction. These complicating facts will be addressed.

4.2.1 Dielectric Mixing Rule for Dry Soils

For a dry soil mix eq (18) simplifies to

$$\epsilon_{\text{soil}}^{1/2} = 1 + \frac{\rho_h}{\rho_m} (\epsilon_{\text{matrix}}^{1/2} - 1) \quad (19)$$

since $S_w \approx 0$. Or, in terms of the porosity ϕ of the soil mix

$$\begin{aligned} \epsilon_{\text{soil}}^{1/2} &= (1 - \phi) \epsilon_{\text{matrix}}^{1/2} + \phi \epsilon_{\text{air}}^{1/2} \\ \epsilon_{\text{soil}}^{1/2} &= (1 - \phi) \epsilon_{\text{matrix}}^{1/2} + \phi \end{aligned} \quad (20)$$

since $\epsilon_{\text{air}}^{1/2} = 1 - j0$. In this case the soil mixture will have a real dielectric constant (or exhibit no conductive losses). As such, essentially no electromagnetic attenuation will be imposed on field behavior by the background soil medium in the frequency range of interest and detection systems operating at higher relative frequencies would be expected to perform best, all other factors being equal [1]. Examples of dielectric, nonlossy test soils are dry sands (SiO_2 or SiC). Frequency-domain measurements of the dielectric constant on these soils have been reported previously [1].

4.2.2 Dielectric Mixing Rule for Lossy, Fluid-Saturated Soils

For lossy, fluid-saturated soils the dielectric mixing rule becomes more complicated since both the bulk soil mix and the pore water become electrically lossy and attenuate electromagnetic fields. In this case the electromagnetic fields of a mine detection system operating at a higher relative frequency would suffer greater attenuation, i.e., the mine detector system would be characterized by a smaller (plane wave) skin depth. If the effective skin depth is too small relative to the burial depth of a buried mine, the detection system would perform more poorly than a system operating at a lower frequency. There are two cases to consider: one where background soils are only partially water saturated and the other where soils are fully saturated.

4.2.2.1 Partially Saturated Soil Mixtures

The complex dielectric constant of the soil may be written

$$\epsilon_{\text{soil}}^{1/2} = (1 - \phi) \epsilon_{\text{matrix}}^{1/2} + (\phi - S_w) \epsilon_{\text{air}}^{1/2} + S_w \epsilon_{\text{water}}^{1/2} \quad (21)$$

where

$$\begin{aligned} \epsilon_{\text{matrix}}^{1/2} &= \sqrt{\epsilon'_{\text{matrix}}} \quad (\text{nonlossy}) \\ \epsilon_{\text{air}}^{1/2} &\equiv 1 \quad (\text{nonlossy}) \\ \epsilon_{\text{water}}^{1/2} &= [\epsilon'_{\text{water}} - j \epsilon''_{\text{water}}]^{1/2} \quad (\text{lossy}) \\ &= [\epsilon'_{\text{water}} (1 - j \tan \delta_{\text{water}})]^{1/2}, \tan \delta_{\text{water}} \equiv \frac{\epsilon''_{\text{water}}}{\epsilon'_{\text{water}}} \\ \epsilon_{\text{soil}}^{1/2} &= [\epsilon'_{\text{soil}} - j \epsilon''_{\text{soil}}]^{1/2} \quad (\text{lossy}) \\ &= [\epsilon'_{\text{soil}} (1 - j \tan \delta_{\text{soil}})]^{1/2}, \tan \delta_{\text{soil}} \equiv \frac{\epsilon''_{\text{soil}}}{\epsilon'_{\text{soil}}} \end{aligned} \quad (22)$$

Use of the half-angle relation

$$\tan \delta = \frac{2 \tan \delta/2}{1 - \tan^2 \delta/2}$$

allows a complex dielectric constant $\epsilon = \epsilon' (1 - j \tan \delta)$ to be written as

$$\begin{aligned} \epsilon^{1/2} &= \frac{\sqrt{\epsilon'}}{[1 - \tan^2 \frac{\delta}{2}]^{1/2}} [1 - \tan^2 \frac{\delta}{2} - 2j \tan \frac{\delta}{2}]^{1/2} \\ &= \frac{\sqrt{\epsilon'}}{(1 - \tan^2 \frac{\delta}{2})^{1/2}} [(1 - j \tan \frac{\delta}{2})^2]^{1/2} \end{aligned} \quad (23)$$

or

$$\epsilon^{1/2} = \frac{\sqrt{\epsilon'}}{[1 - \tan^2 \frac{\delta}{2}]^{1/2}} (1 - j \tan \frac{\delta}{2}) .$$

Use of eqs (23) and (22) in (21) then yields

$$\begin{aligned} \frac{\sqrt{\epsilon'_{\text{soil}}}}{(1 - \tan^2 \frac{\delta_{\text{soil}}}{2})^{1/2}} (1 - j \tan \frac{\delta_{\text{soil}}}{2}) &= (1 - \phi) \sqrt{\epsilon'_{\text{matrix}}} + (\phi - S_w) \\ &+ \frac{S_w \sqrt{\epsilon'_{\text{water}}}}{(1 - \tan^2 \frac{\delta_{\text{water}}}{2})^{1/2}} [1 - j \tan \frac{\delta_{\text{water}}}{2}] \end{aligned}$$

Separating real and imaginary parts

$$\begin{aligned} \frac{\sqrt{\epsilon'_{\text{soil}}}}{(1 - \tan^2 \frac{\delta_{\text{soil}}}{2})^{1/2}} &= \sqrt{\epsilon'_{\text{matrix}}} + \phi (1 - \sqrt{\epsilon'_{\text{matrix}}}) - S_w \\ &+ \frac{S_w \sqrt{\epsilon'_{\text{water}}}}{(1 - \tan^2 \frac{\delta_{\text{water}}}{2})^{1/2}} \end{aligned} \quad (24)$$

and

$$\frac{\sqrt{\epsilon'_{\text{soil}}} \tan \frac{\delta_{\text{soil}}}{2}}{(1 - \tan^2 \frac{\delta_{\text{soil}}}{2})^{1/2}} = \frac{S_w \sqrt{\epsilon'_{\text{water}}} \tan \frac{\delta_{\text{water}}}{2}}{(1 - \tan^2 \frac{\delta_{\text{water}}}{2})^{1/2}} \quad (25)$$

Both eqs (24) and (25) must be satisfied simultaneously. From measurements ϵ'_{soil} , $\tan \delta_{\text{soil}}$, and $\epsilon'_{\text{matrix}}$ are known. The total porosity, ϕ , and the water and air saturations, S_w , and $\phi - S_w$, respectively, may be determined by using a value for the water salinity, S , which is based on the cation exchange capacity of the matrix soil type. The dielectric constant of the pore water, ϵ'_{water} , is computed from eqs (6) and (9). The loss tangent of the pore water is then calculated from eq (10). Substituting these values and measured ϵ'_{soil} and $\tan \frac{\delta_{\text{soil}}}{2}$ into eq (25) allows the determination of S_w from which the total porosity is computed (eq (24)).

4.2.2.2 Saturated Soil Mixtures

A more expedient approach is to relate the complex dielectric constant of the soil mixture to the water-filled porosity ϕ_w so that we may write

$$\epsilon_{\text{soil}}^{1/2} = (1 - \phi_w) \epsilon_{\text{matrix}}^{1/2} + \phi_w \epsilon_{\text{water}}^{1/2} \quad (26)$$

Use of eqs (22) and (23) now yields

$$\frac{\sqrt{\epsilon'_{\text{soil}}}}{(1 - \tan^2 \frac{\delta_{\text{soil}}}{2})^{1/2}} = (1 - \phi_w) \sqrt{\epsilon'_{\text{matrix}}} + \frac{\phi_w \sqrt{\epsilon'_{\text{water}}}}{[1 - \tan^2 \frac{\delta_{\text{water}}}{2}]^{1/2}} \quad (27)$$

and

$$\frac{\sqrt{\epsilon'_{\text{soil}}} \tan \frac{\delta_{\text{soil}}}{2}}{(1 - \tan^2 \frac{\delta_{\text{soil}}}{2})^{1/2}} = \frac{\phi_w \sqrt{\epsilon'_{\text{water}}} \tan \frac{\delta_{\text{water}}}{2}}{[1 - \tan^2 \frac{\delta_{\text{water}}}{2}]^{1/2}} \quad (28)$$

Again, ϵ'_{soil} , $\tan \delta_{\text{soil}}$, and $\epsilon'_{\text{matrix}}$ are known from measurements. A value for the salinity of the pore water, S , is then taken. The real part of the pore water permittivity, ϵ'_{water} , at temperature T ($^{\circ}\text{C}$), is then computed from

$$\epsilon'_{\text{water}} = \epsilon'_{w0}(T) - 0.1556 - 0.000413 S + 0.00000158 S^2 \quad (29)$$

where

$$\epsilon'_{w0}(T) = 88.045 - 0.4147 T + 0.0006295 T^2 \quad (30)$$

Next the loss tangent of the pore water is calculated from

$$\tan \delta_{\text{water}} = [\epsilon'_{\text{water}}]^{-1} [5.66 + 0.00265 S - 0.0000045 S^2] \quad (31)$$

This initial value of $\tan \delta_{\text{water}}$ is substituted into eq (27) to determine ϕ_w . A corrected value of $\tan \delta_{\text{water}}/2$ is determined from eq (28) which is then used to determine a new value for the water salinity and porosity, ϕ_w , until they converge to constant values. This approach is one commonly used in geophysics and reservoir engineering to determine both the water-filled porosity and pore water salinity of rocks from in situ dielectric measurements. For our purposes here, these are the fundamental parameters that allow us to predict,

based on a mixing relationship, how attenuative a background soil is. The important point is that the complex permittivity (both nonlossy and lossy parts) of the background test medium be the same in performance evaluations of mine detector systems operating at the same frequency. While various synthetic substances can be thought of that can simulate the real field situation of water-saturated soils in terms of the complex permittivity at a single frequency, there are few synthetics which, in combination, can simulate the dielectric behavior of water-saturated soils over a broad range in frequency.

5. APPLICATION TO SOIL STANDARDS

As an application to dielectric property prediction of soil mixtures, consider a two-phase mixture of air and dry sand, where sand consists of washed quartz grains having diameters ranging from 0.027 and 0.60 mm. From eq (19) we write

$$\epsilon_{\text{soil}}^{1/2} = 1 + \frac{\rho_b}{\rho_m} (\epsilon_{\text{matrix}}^{1/2} - 1).$$

For this mixture, Geyer [1] and Jesch [22] have reported a measured mean relative dielectric constant, ϵ'_{soil} , of 2.702 at 300 MHz and for a mean bulk density, ρ_b , of 1.540 g/cc. Standard deviations for the relative dielectric constant and mean bulk density were 0.0486 and 0.022 g/cc, respectively. Bussey [23,24] reports ϵ' of silica equal to 3.822, measured at 9 GHz with the use of cavity resonators. Von Hippel [25] reports ϵ' of (fused) silica at 25°C and at 300 MHz equal to 3.78. Von Hippel's results show fused silica essentially dispersionless over the frequency range of 100 Hz to 25 GHz, with $\tan\delta_{\text{silica}} = 0.5 \times 10^{-4}$ at 300 MHz. Clark [26] and Morey [27] give the matrix density of pure silica, ρ_m , to be 2.203 g/cc.

In summary, use of the above two-phase refractive dielectric mixing rule yields a predicted relative constant

$$\begin{aligned} \epsilon_{\text{soil, predicted}} &= [1 + 0.699 (\epsilon_{\text{matrix}}^{1/2} - 1)]^2 \\ &= [1 + 0.699 (3.780^{1/2} - 1)]^2 \\ \epsilon_{\text{soil, predicted}} &= 2.756. \end{aligned}$$

This result differs by 2.1 percent from observed measurements using an open-ended transmission line with a vector automatic network analyzer [1,22]. This difference is probably due to impurities in the quartz sand, which would affect both ρ_m and ϵ_{matrix} .

6. CONCLUSIONS

Refractive dielectric mixing rules provide a convenient means for predicting a (background) test soil's dielectric behavior in land mine performance testing. The test soil's dielectric behavior implicitly affects the electromagnetic visibility of a buried land mine. Although such mixing rules allow specification of multicomponent heterogeneous soils in terms of the effective nonlossy and attenuative behavior of the mix, they do not obviate the need for background media dielectric measurement. For lossy background media it is preferable to measure both the relative dielectric constant and the loss tangent in situ.

Additional research work should address possible in situ dielectric measurement techniques, as well as the relation between cation exchange capacities of the matrix materials in a fluid-saturated soil mixture, fluid salinities, and corresponding test medium attenuative loss tangents affecting mine detection system performance.

7. REFERENCES

- [1] Geyer, R. G. Experimental parameters for land mine detection performance evaluations. Report, U. S. Army Belvoir Research and Development Center, Fort Belvoir, VA (Proj. 7232457); 1987 November. 100 p.
- [2] Debye, P. Polar molecules. New York: Dover; 1929.
- [3] Lane, J.; Saxton, J. Dielectric dispersion in pure polar liquids at very high radio frequencies, III. The effect of electrolytes in solution. Proc., Roy. Soc. 214(A); 1952. 531-545.
- [4] Hasted, J. B. Liquid water: dielectric properties. Chapter 7 in Water - a comprehensive treatise, I, The physics and chemistry of water. New York: Plenum Press; 1972.
- [5] Klein, L. A.; Swift, C. T. An improved model for the dielectric constant of sea water at microwave frequencies. IEEE Trans. Antennas Propag. AP-25; 1977. 104-111.
- [6] Dobson, M. C.; Ulaby, F. T.; Hallikainen, M. T.; El-Rayes, M. A. Microwave dielectric behavior of wet soil, Part II: dielectric mixing models. IEEE Trans. Geosci. Remote Sensing. GE-23(1); 1985. 35-46.
- [7] Jedlicka, R. P. Saline soil dielectric measurements. M. S. Thesis New Mexico State University, NM; 1978.
- [8] Stogryn, A. Equations for calculating the dielectric constant of saline water. IEEE Trans. Microwave Theory Techn. MTT-19; 1971. 733-736.
- [9] Wharton, R. P.; Hazen, G.A.; Rau, R. N.; Best, D. L. Electromagnetic propagation logging: advances in technique and interpretation. SPE of AIME Annual Technical Conference and Exhibition (Paper SPE 9267); 1980 September. 21-24.
- [10] Ho, W.; Hall, W. F. Measurements of the dielectric properties of sea water and NaCl solutions at 2.65 GHz. J. Geophys. Res. 78; 1973. 6301-6315.
- [11] Ho, W. W.; Love, A. W.; VanMelle, M. J. Measurements of the dielectric properties of sea water at 1.43 GHz. NASA Contactor Report CR-2458. NASA Langley Research Center, Langley, VA; 1974.

- [12] Ulaby, F. T.; Moore, R. K.; Fung, A. K. Microwave remote sensing, Vol. III: from theory to applications. London, England: Addison-Wesley Publishing Co.; 1982.
- [13] Tinga, W. R. Generalized approach to multiphase dielectric mixture theory. J. Appl. Phys. 44; 1973. 3897-3902.
- [14] Böttcher, C. J. F. Theory of electric polarization. Amsterdam: Elsevier; 1952.
- [15] Hallikainen, M. Dielectric properties of sea ice at microwave frequencies, Report S94. Helsinki University of Technology, Radio Laboratory, Espoo, Finland; 1977.
- [16] Hallikainen, M.; Ulaby, F. T.; Abdelrazik, M. The dielectric behavior of snow in the 3- to 37-GHz range. IEEE International Geosci. Remote Sensing Symp. Digest, Strasbourg, France; 1984 August 27-30. 169-176.
- [17] Kurosaki, S. The dielectric behavior of water sorbed in silica gel. J. Phys. Chem. 58; 1954. 320-324.
- [18] Bockris, J.; Gileady, E.; Muller, K. Dielectric relaxation in the electric double layer. J. Chem. Physics. 44; 1966. 1445-1456.
- [19] McCafferty, E.; Zettlemayer, A. C. Adsorption of water vapor on α -Fe₂O₃. Faraday Soc. Discussions, Vol. 52; 1971. 239-254.
- [20] Hasted, J. B. The dielectric properties of water in Progress in dielectrics Vol. 3. J. B. Birks and J. Hart, ed. New York: John Wiley & Sons; 1961. 101-149.
- [21] Shutko, A. M.; Reutov, E. M. Mixture formulas applied in estimation of dielectric and radiative characteristics of soils and grounds at microwave frequencies. IEEE Trans. Geosci. Remote Sensing. GE-20; 1982. 29-32.
- [22] Jesch, R. L. Dielectric measurements of five different soil textural types as functions of frequency and moisture content. Nat. Bur. Stand. (U.S.) NBSIR 78-896; 1978. 26 p.
- [23] Bussey, H. E. International comparison of complex permittivity measurement at 9 GHz. IEEE Trans. Instr. Meas. IM-23(3); 1974. 235-239.

- [24] Bussey, H. E.; Gray, J. E.; Bamberger, E. C.; Rushton, E.; Russel, G.; Petley, B. W.; Morris, D. International comparison of dielectric measurements. IEEE Trans. Instr. Meas. IM-13; 1964. 305-311.
- [25] Von Hippel, A. R. Dielectric materials and applications. New York: The Technology Press of M.I.T. and John Wiley & Sons; 1954. 438 p.
- [26] Clark, S. P. Handbook of physical constants. Geol. Soc. Am. Memoir 97; 1966. 587 p.
- [27] Morey, L. Properties of glass. New York: Reinhold Press; 1954.

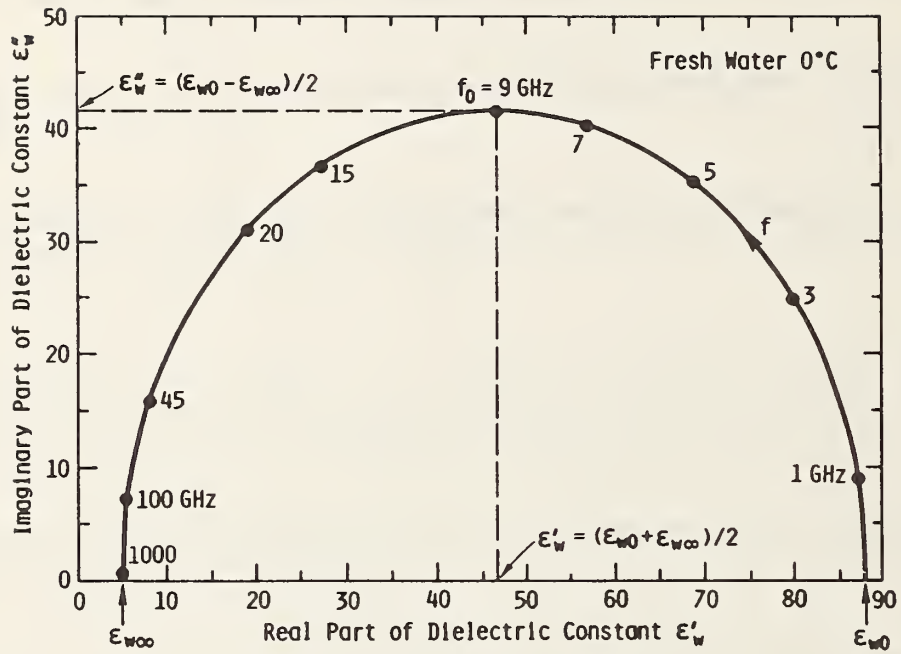


Figure 1. Cole-Cole Diagram for the dielectric constant of pure water at 0°C (after Hasted [4]).

U.S. DEPT. OF COMM. BIBLIOGRAPHIC DATA SHEET <i>(See instructions)</i>	1. PUBLICATION OR REPORT NO. NBSIR 88-3095	2. Performing Organ. Report No.	3. Publication Date June 1988
4. TITLE AND SUBTITLE <p style="text-align: center;">Dielectric Mixing Rules for Background Test Soils</p>			
5. AUTHOR(S) <p style="text-align: center;">Richard G. Geyer</p>			
6. PERFORMING ORGANIZATION <i>(If joint or other than NBS, see instructions)</i> NATIONAL BUREAU OF STANDARDS DEPARTMENT OF COMMERCE WASHINGTON, D.C. 20234		7. Contract/Grant No.	8. Type of Report & Period Covered
9. SPONSORING ORGANIZATION NAME AND COMPLETE ADDRESS <i>(Street, City, State, ZIP)</i> U.S. Army Belvoir Research & Development Center Fort Belvoir, Virginia 22060-5606			
10. SUPPLEMENTARY NOTES <p><input type="checkbox"/> Document describes a computer program; SF-185, FIPS Software Summary, is attached.</p>			
11. ABSTRACT <i>(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)</i> <p>The bulk, or effective dielectric constant of any background test medium (whether naturally occurring or synthetic) determines the electromagnetic visibility of buried objects. Heuristic mixing rules are considered that allow the prediction of complex dielectric behavior in linear, homogeneous, isotropic, and lossy multi-phase soil mixtures. Measurement results in bio-electromagnetic and microwave remote sensing suggest a refractive mixing model as that being most suited for dry soils or soil-water mixtures.</p>			
12. KEY WORDS <i>(Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)</i> dielectric simulation; mixing rules; multi-phase; soils			
13. AVAILABILITY <input checked="" type="checkbox"/> Unlimited <input type="checkbox"/> For Official Distribution. Do Not Release to NTIS <input type="checkbox"/> Order From Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402. <input checked="" type="checkbox"/> Order From National Technical Information Service (NTIS), Springfield, VA. 22161		14. NO. OF PRINTED PAGES <p style="text-align: center;">28</p>	15. Price

