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The Cooling Effect of A Single Evaporating Droplet on A Hot Semi-Infinite Metal Body

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FIGURE CAPTIONS

NOMENCLATURE

A, B, C,

A', B', C'   constants

c specific heat

D mass diffusivity (steam-air)

f generic function

h convective heat transfer coefficient

k thermal conductivity

L latent heat of vaporization (water)

q heat flux

r radial coordinate tangent to the solid surface originating at the center of the wetted region

R wetted region radius

t time

T temperature

T_o initial solid surface temperature prior to the droplet deposition

V droplet volume

V_i volume of influence

x molar fraction of steam in air

y coordinate normal to the solid surface originating at the center of the wetted region upward oriented (in the droplet region)

z coordinate normal to the solid surface originating at the center of the wetted region downward oriented (in the solid region)

α thermal diffusivity

β ratio of the wetted region radius to the radius of the spherical droplet of equivalent liquid volume

γ square root of the product of specific heat, density and thermal conductivity

v

δ shape parameter

ρ density

Subscripts

a   air property

c   cumulative or total

i   at the droplet exposed surface

o   initial value

s   solid property or at the exposed solid surface

u   at the solid surface under the droplet

w   liquid water property

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ABSTRACT

Transient cooling of solid surfaces by water droplet evaporation has been investigated through controlled experiments using a large heated aluminum cylinder. Quantitative prediction of droplet evaporation time and in-depth transient temperature distribution in the solid have been made. In the case studied, a single droplet is deposited on a horizontal non-porous surface with initial temperatures in the range of 75°C to 100°C. The liquid-vapor interfacial temperature and the water vapor molar fraction in the air at the exposed surface of the water droplet are deduced from the energy balance at the interface. Spatial and temporal integration of the overall droplet energy equation is used to predict the instantaneous evaporation rate and the droplet evaporation time. The boundary conditions for the wetted region proposed by Seki are used to obtain the transient temperature distribution for a semi-infinite solid. The region of the body affected by the droplet cooling is identified and its volume (normalized with the volume of the droplet) is plotted against the evaporation time. All data, regardless of the droplet volume or of the initial body surface temperature, lie within a narrow band about a straight line. This finding is the first important step to obtain a simple model for spray cooling based on local accurate description of the droplet-solid interactions. Modeling of spray cooling phenomena is the foundation for the construction of a thermal model for solid fuel fire extinguishment.

1. INTRODUCTION

The thermal behavior of a semi-infinite solid subjected to local surface cooling induced by a single evaporating droplet has been described in detail by Seki [1]. Numerous theoretical and experimental studies are found in the literature concerning the mechanism of evaporation of droplets impinging on a hot surface. A simple model is derived by Bonacina [2] which identifies an important geometrical parameter for the droplet-surface interaction. Grissom [3] investigated the cooling effect of a spray on a heated surface. The evaporation of single droplets is the objective of the works of Michiyoshi and Makino [4,5] which give a detailed insight into the thermal features of the evaporation process. Sadhal [6] provides an interesting analysis of the droplet-solid interactions for evaporation and condensation. The optical studies of the droplet configuration on a solid surface reported by Zhang [7] allow one to conclude that a spherical segment is a good geometrical representation of the actual droplet shape. In a previous paper di Marzo and Evans [8] derived a rigorous analytical model to predict the evaporation time, the evaporation rate and the local heat flux distribution at the droplet exposed surface during its evaporation.

In many of these studies, the full range of vaporization processes (including evaporation, nucleate boiling, film boiling, and transition boiling) are reported. This detailed study of the water droplet vaporization is intended to form the basis for a thermal model of solid fuel fire extinguishment, so this study is limited to investigations of the evaporation phenomena only.

The objective of this paper is to show that a simple relationship exists between the evaporation time and the volume of the portion of the solid affected by the cooling process induced by the droplet evaporation. In order to illustrate this finding, the basic heat transfer governing equation for the semi-infinite solid will be derived and merged with the previously developed model for the prediction of the evaporation time [8] to determine the cooling effect in the solid.

The high thermal conductivity of the heated aluminum block used in this study allows one to assume that the temperature, T_u, at the solid surface, under the droplet is constant in both space and time. This hypothesis was successfully tested by Seki [1] against experimental data. The experimental findings by Michiyoshi [4] infer also that the temperature of the solid surface under the droplet is constant during the evaporation process. This information coupled with the finding reported by Seki [1] provides a simple boundary condition for the wetted region of the solid surface.

2. MODEL FORMULATION

A spherical droplet upon impacting a solid surface spreads on it. The final configuration of the liquid varies a great deal and depends on a multitude of parameters. For the case of liquid water impinging a surface at near water saturation temperature, the shape can be regarded as a spherical segment [7,8].

In Fig. 1, a typical sequence of photographs of an evaporating droplet is presented. It is important to note that the wetted region of the semi-infinite solid surface remains constant in size throughout most of the evaporative process.

The ratio of the wetted area radius to the radius of an equivalent liquid volume spherical droplet is identified by the parameter β [2]. This parameter is the only experimental input used in the analysis. The complexity of the parameters influencing the initial droplet shape (surface tension, surface wettability, etc.) is such that this semi-empirical approach was selected to formulate a reasonably simple analysis. The effect of the droplet kinetic energy (before impact) on the parameter β is negligible for droplet release height of less than 1.7 centimeters. This study is concerned only with droplets released at less than 1.7 centimeters from the solid surface. These droplets are referred to as "softly" deposited droplets [2,3].

The cooling effect induced by the droplet evaporation is complex to model analytically. The boundary conditions of the energy equation written for the droplet should match the boundary conditions of the energy equation written for the solid. Namely, the heat flux and the temperature should be set equal at each location and at each time at the liquid-solid boundary.

The model proposed by Seki [1] is based on a simplifying assumption. He considers the solid-liquid boundary to be at uniform constant temperature. This assumption finds its justification in the large values of the thermal conductivity of the metals. The heat flux required for the droplet evaporation is induced by small temperature gradients in the solid. Since Michiyoshi

[4,5] confirmed experimentally that the solid-liquid interfacial temperature does not change appreciably in time, one can conclude that a uniform, constant temperature, T_u, under the droplet is a reasonable boundary condition. The advantages of such hypothesis is that now the energy equation for the solid can be integrated independently from the droplet behavior and only the overall evaporation time is required to obtain information on the solid cooling.

The solid thermal behavior is described by the energy equation and its boundary conditions, as previously derived by Seki [1]. In summary, for the system configuration depicted in Fig. 2, one can write

$$\frac{\partial T}{\partial t} = \alpha_s \left(\frac{\partial^2 T}{\partial z^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} \right) \tag{1}$$

with the following boundary conditions:

$$r = 0 \qquad\qquad \partial T/\partial r = 0 \tag{2}$$

$$r \to \infty \qquad\qquad \partial T/\partial r = 0 \tag{3}$$

$$z \to \infty \qquad\qquad q\ ha\ (T_o - Ta) = - k_s \partial T/\partial z \tag{4}$$

$$z = 0 \text{ and } r > R \qquad h_a(T_s - T_a) = - k_s \partial T/\partial z \tag{5}$$

$$z = 0 \text{ and } r \leq R \text{ for } t \leq 0 \text{ and } t > t_c$$
$$h_a(T_s - T_a) = - k_s \partial T/\partial z \tag{6}$$

$$z = 0 \text{ and } r \leq R \text{ for } 0 < t \leq t_c \qquad T = T_u \tag{7}$$

The evaporation time, t_c, is obtained from the model predictions as will be shown in the following section. Equation (1) is numerically integrated and the temperature distribution in the solid is obtained at each instant of time during the evaporation process and thereafter in the thermal recovery transient.

The spatial and temporal temperature distribution information generated by this process is overwhelming. In order to compact this information and to deduce useful correlations between the magnitude of the cooling effect and the droplet evaporation phenomena, a new quantity is introduced, namely the volume of influence. The volume of influence is the volume of the region where the temperature variation with respect to the steady state conduction temperature distribution prior to the droplet deposition exceeds a given value. The maximum possible temperature variation occurring in the solid is the difference between the solid surface temperature prior to the droplet deposition and the contact temperature (between the water and the solid) given by Seki [1] as

$$T_u = \frac{T_w \gamma_w + T_o \gamma_s}{\gamma_w + \gamma_s} \tag{8}$$

The volume of influence will be defined as the volume where the temperature variation exceeds ten percent of the maximum possible temperature difference. The results of the computations will be shown in terms of volume of influence and in particular in terms of the maximum volume of influence which is observed at the end of the evaporation process.

3. EVAPORATION TIME PREDICTION

In an earlier paper, di Marzo and Evans [8] showed that the droplet geometrical configuration is given by

$$y = \left[\frac{(1/\delta + \delta)^2}{4} - r^2 \right]^{1/2} - \frac{(1/\delta - \delta)}{2} \tag{9}$$

where δ is

$$\delta = R \left\{ \left[4/\beta^3 + (1 + 16/\beta^6)^{1/2} \right]^{1/3} + \left[4/\beta^3 - (1 + 16/\beta^6)^{1/2} \right]^{1/3} \right\} \tag{10}$$

and β is defined as the ratio of the radius of the wetted area to the radius of an equivalent liquid volume spherical droplet. The droplet configuration can be approximated to a spherical segment throughout all the evaporative process. It must be noted, however, that in the last portion of the process $(t > 0.95 \, t_c)$ the liquid film becomes very thin and the wetted surface suddenly receeds. The temperature at the exposed surface of the liquid droplet can be evaluated by a heat balance at the liquid vapor interface. By considering that the heat conducted through the liquid layer to the interface is equal to the energy required to evaporate the liquid droplet plus the small amount of energy convected away, one can write

$$\frac{k_w (T_u - T_i)}{y} = 0.624 \, h_a \left(\frac{D}{\alpha_a} \right)^{2/3} \frac{L}{c_a} \frac{x_i - x_a}{1 - x_i} + h_a (T_i - T_a) \tag{11}$$

where the mass transfer coefficient is evaluated by making use of the Chilton-Colburn analogy [9]. Note that the only unknown is x_i since the interfacial temperature, T_i, is the saturation temperature of steam at the pressure corresponding to its molar fraction.

The overall heat balance on the droplet allows one to calculate the instantaneous evaporation rate as

$$- \frac{dV}{dt} = \frac{2\pi \ (0.624) \ h_a}{\rho_w c_a} \ \frac{D}{\alpha_a}^{2/3} \int_o^R \frac{x_i - x_a}{1 - x_i} \ rdr \qquad (12)$$

where x_i is function both of y and t.

By integrating this equation over the droplet exposed surface at each instant of time and by reassessing the liquid inventory (and subsequently the shape of the droplet) at each time step, the overall evaporation time can be determined. Figure 3 illustrates the results obtained with the model in comparison with experimental data for water droplets evaporating on an aluminum cylindrical block. For a more complete description of the experimental apparatus and procedures, refer to Trehan [10]. A complete derivation of the evaporation time model is given by di Marzo in the aforementioned reference [8]. Comparison of the model results with experimental data obtained by Michiyohsi and Makino [4,5] show that the solid material does not influence the results provided that its thermal conductivity is such that uniform temperature at the solid surface under the droplet can be regarded as a reasonable assumption. In Fig. 4, various experimental data for different materials are compared with the theoretical calculations. Note that the evaporative heat flux is directly related to the evaporation time.

This droplet evaporation model provides detailed information on the local heat fluxes at the solid liquid-boundary during the evaporation. Therefore, a coupled solution of Eq. (11) and Eq. (1) could yield the desired results without imposing the constant temperature condition at the liquid-solid inter-

-8-

face. The drawback of this approach is that an iterative computation scheme needs to be set up to solve both the solid and droplet energy equations simultaneously.

By making use of a closed form analytical solution described by Carslaw and Jaeger [11], it was determined that the temperature nonuniformities in the liquid solid interfacial region amount to about 1°C. This result was obtained imposing, as a boundary condition of the solid energy equation, heat fluxes of the order of those required for the droplet evaporation.

In conclusion, one can argue that, with respect to the heat and mass transfer phenomena, three resistances in series can be envisioned where the first resistance in the solid is negligible with respect to the other two. The heat transfer resistance in the droplet is comparable to the mass transfer resistance in the air-vapor region. For the case of low conductivity solids, these three resistances are all of the same order of magnitude; therefore, the full coupled problem must be solved.

To elaborate further on these results, it is interesting to note that the evaporation time has an exponential behavior with respect to the initial solid surface temperature. This means that

$$\ln t_c = - A' \, T_o + f(V_o) \tag{13}$$

Furthermore, in order to find the intercept, $f(V_o)$, one can observe that also the volume show an exponential behavior hence:

$$f(V_o) = B' \ln(V_o) + C' \tag{14}$$

By combining these two equations, the following result is obtained

$$t_c = A \, V_o^B \, e - C \, T_o \tag{15}$$

The numerical values for the constant A, B, C were found to be respectively 880, 0.7 and 0.05 for water droplets on heated aluminum surfaces. The scatter of the experimental data about the model predictions is in the order of 15 percent.

4. SOLID THERMAL BEHAVIOR

The results of the numerical integration of Eq. (1) are illustrated in Fig. 5 through 7. The effectiveness of the volume of influence in compacting the cooling effect information is evident if one compares Fig. 5 and Figs. 6 and 7.

The volume of influence as a function of the initial solid surface temperature for various droplet volumes is depicted in Fig. 8. Again, an exponential pattern is observed both for the volume of influence curves as well as for the various droplet volumes. Consider, therefore, the plot of the volume of influence versus the evaporation time for the same droplet volume and initial solid surface temperature, as shown in Fig. 9. The uncertainties of the various parameters determining the evaporation time and the volume of influence are such that the points can be considered to lie in close proximity to the linear regression of the point values (for more details on the order of

approximation, see di Marzo [8]).

The expression for the linear fit is given by

$$V_i = V_o(0.021 \ t_c + 3) \tag{16}$$

By combining Eq. (15) and (16), one obtains

$$V_i = 18 \ V_o^{1.7} \ e - 0.05 \ T_o + 3 \ V_o \tag{17}$$

This closed form, simple, analytical result illustrates the power of the analysis. Note that the specific numerical constants are for aluminum. The same governing equations and boundary conditions are common for high thermal conductivity materials. Therefore, similar trends in the volume of influence are expected, thus leading to a linear dependence of the non-dimensional volume of influence with the evaporation time. The temperature spatial and temporal uniformity under the droplet is the condition that allows the uncoupling of the droplet evaporation process from the solid thermal behavior. In order to obtain this result the thermal conductivity of the solid must be larger than the thermal conductivity of the liquid water. The volume of influence is the basic information that allows one to predict the cooling effect in a semi-infinite body subjected to droplet evaporation. Further research based on these results will provide information on the cooling effect of an array of droplets impinging a surface, thus leading to a comprehensive model to predict spray cooling.

5. CONCLUSIONS

The energy equation, written in cylindrical coordinates for the semi-infinite metal body, provides information on the spatial and temporal temperature distribution in the body. More compact information is given in terms of the volume of influence. The boundary conditions for the energy equation are formulated by using the liquid-solid contact temperature suggested by Seki [1] coupled with the evaporation time calculated by di Marzo [8]. This analytical model to predict the evaporation time requires that the wetting parameter β must be determined by experiments.

The volume of influence is found to correlate linearly with the evaporation time and to be independent of the droplet volume and the initial solid surface temperature. A consequence of this finding is that a simple, closed form expression for the volume of influence can be obtained. This information is the first step towards the analysis of multi-droplet cooling effect thus leading to the modelling of spray cooling processes.

6. ACKNOWLEDGEMENTS

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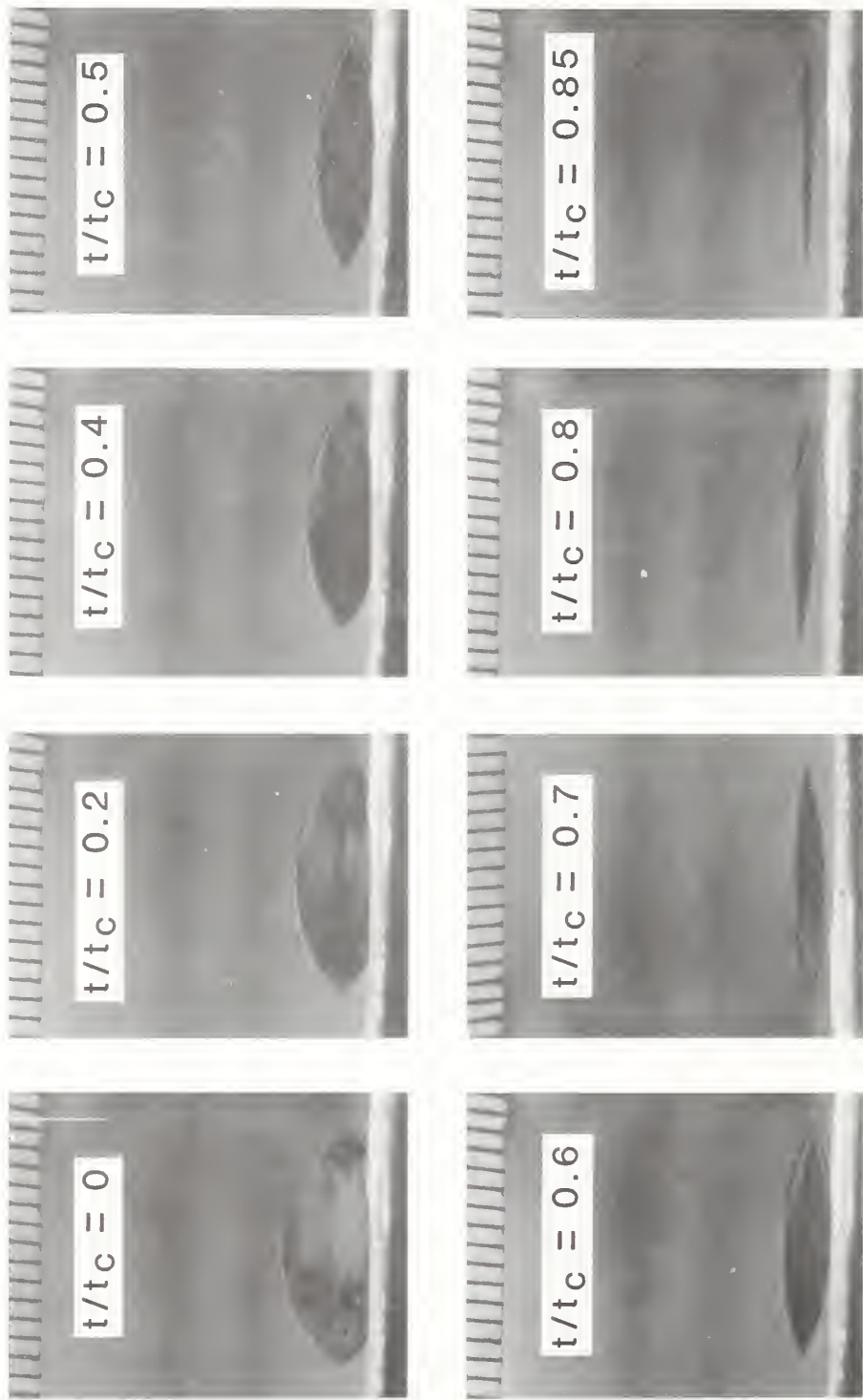[11] H.S. Carslaw, J.C. Jaeger, Conduction in Solids, Oxford University Press, Oxford, England, 1946.
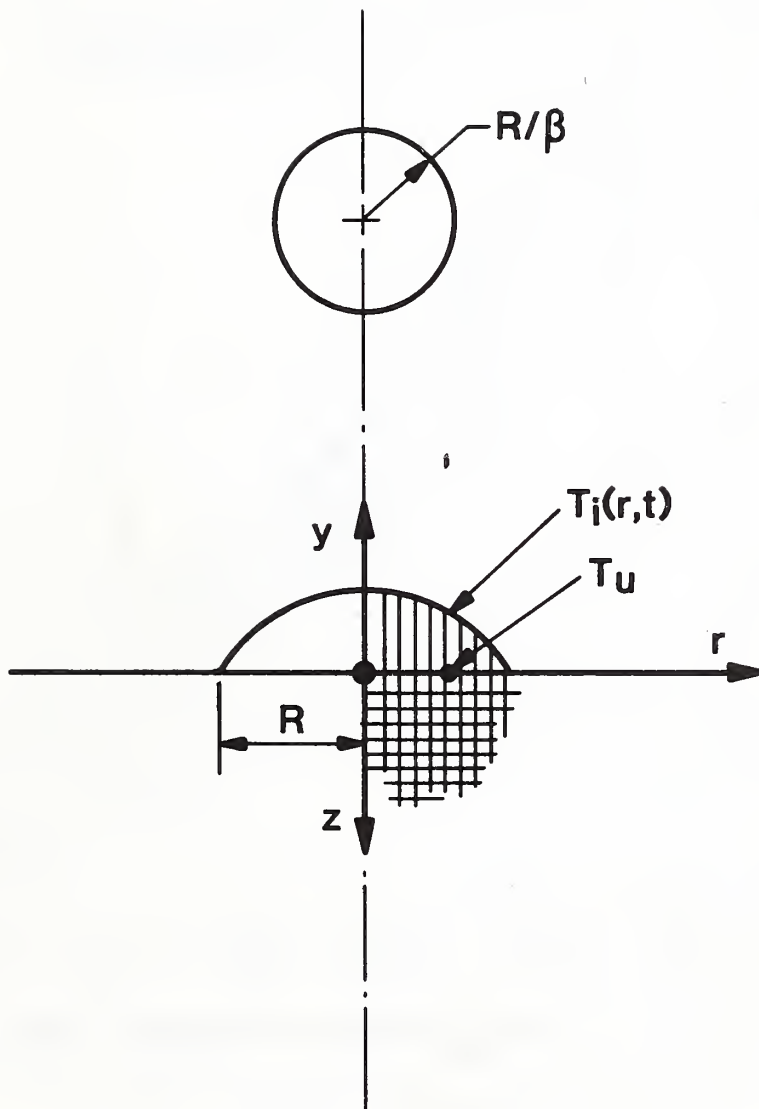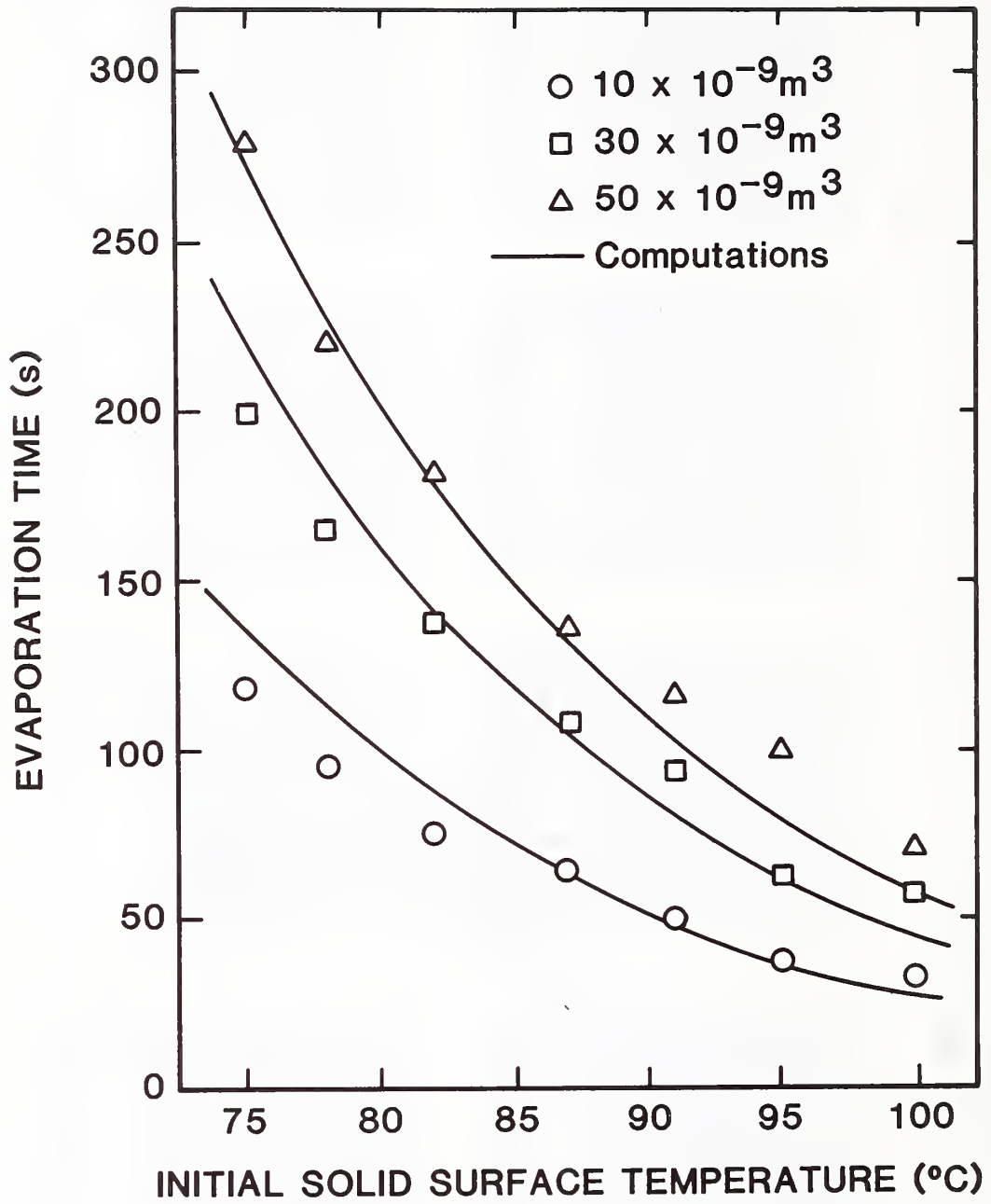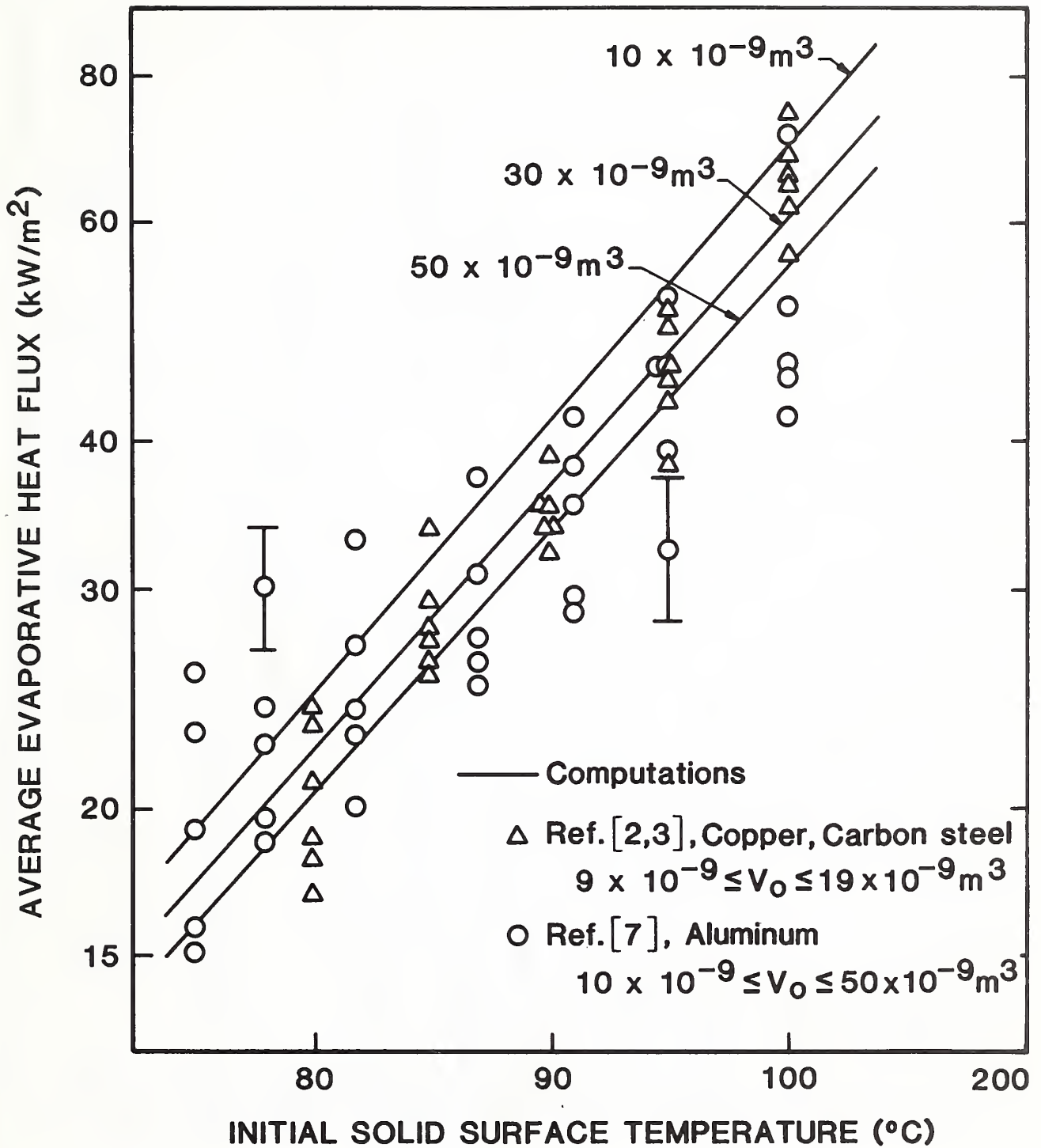
FIGURE ONE

FIGURE TWO

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FIGURE THREE

FIGURE FOUR

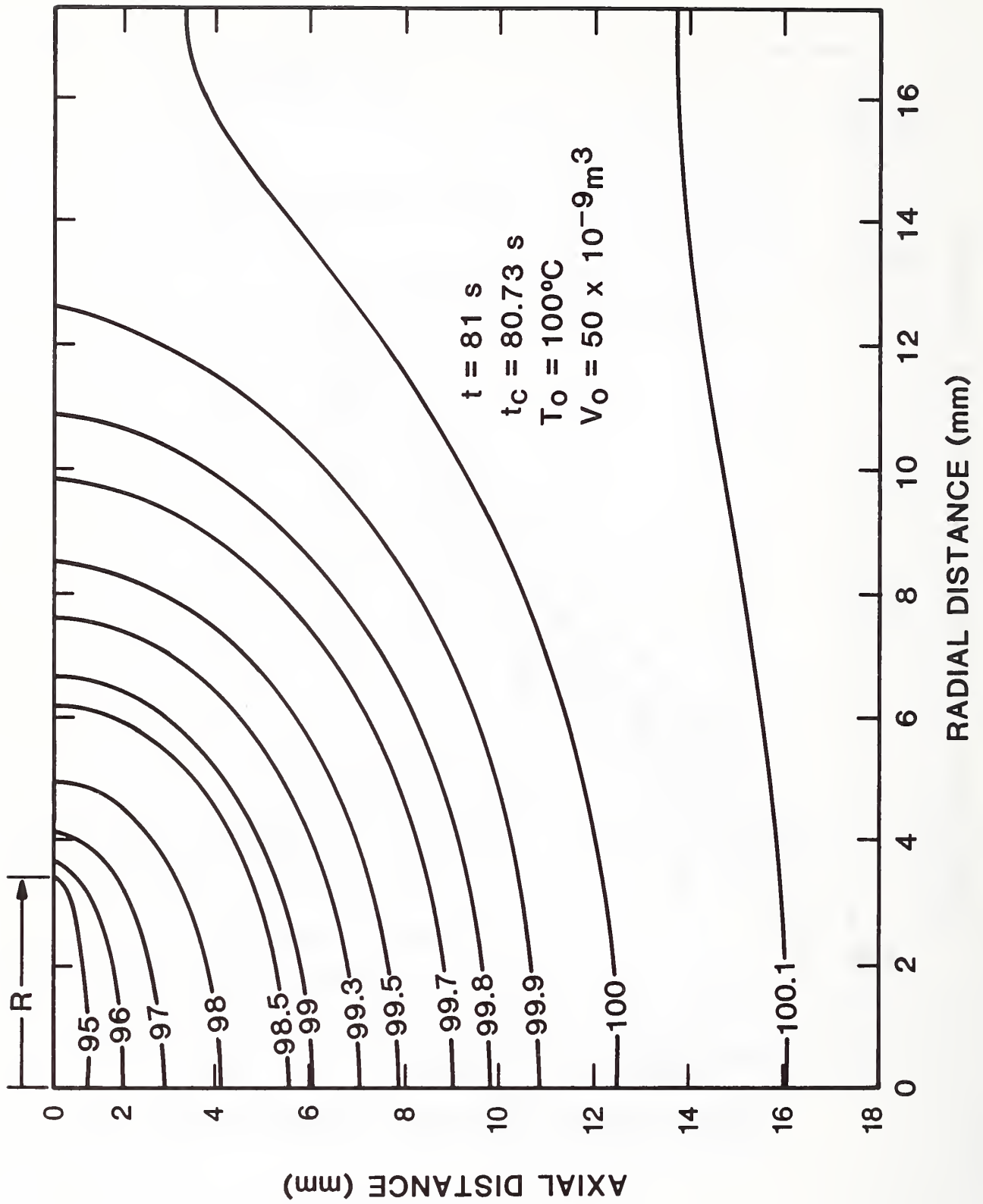17

The graph contains the following labels:

Y-axis: AXIAL DISTANCE (mm), with values 0, 2, 4, 6, 8, 10, 12, 14, 16, 18

X-axis: RADIAL DISTANCE (mm), with values 0, 2, 4, 6, 8, 10, 12, 14, 16

Contour labels: 95, 96, 97, 98, 98.5, 99, 99.3, 99.5, 99.7, 99.8, 99.9, 100, 100.1

R

Parameters shown in graph:
$t = 81$ s
$t_c = 80.73$ s
$T_o = 100°C$
$V_o = 50 \times 10^{-9} m^3$

FIGURE FIVE

18

FIGURE SIX

19

AXIAL DISTANCE (mm)

RADIAL DISTANCE (mm)

$T_o = 100°C$
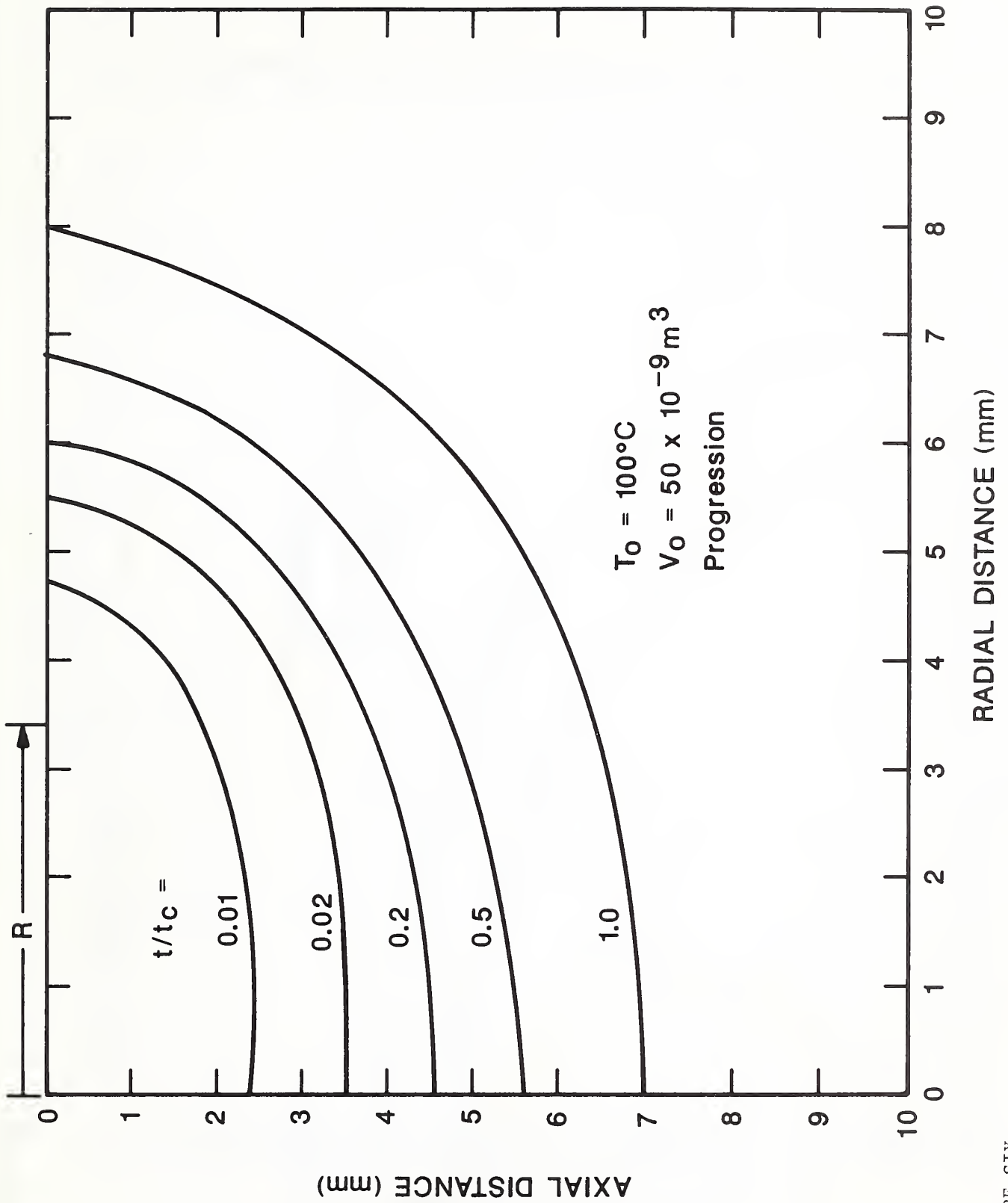
$V_o = 50 \times 10^{-9} m^3$

Regression

$t/t_c =$

2.0

1.5
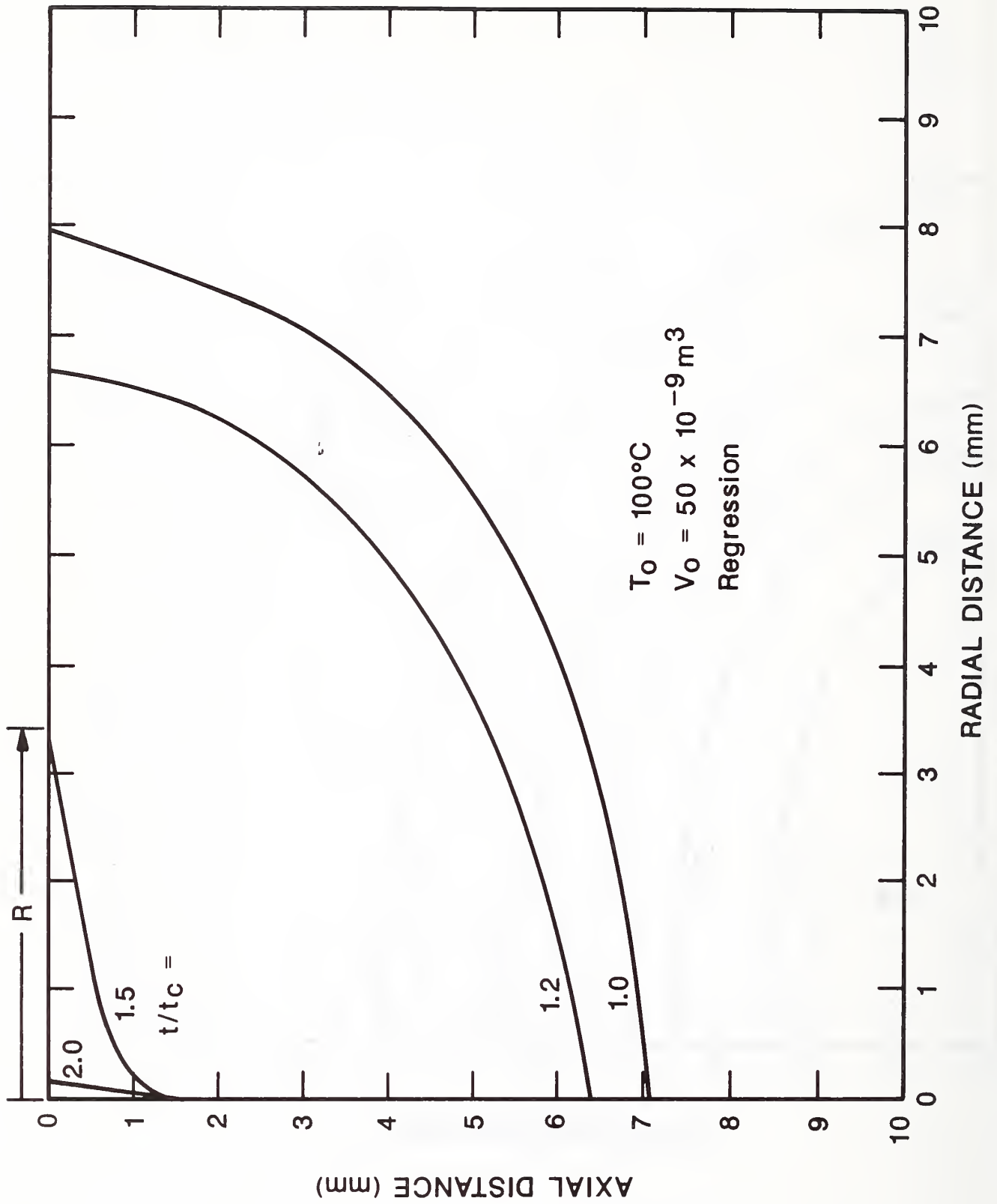
1.2
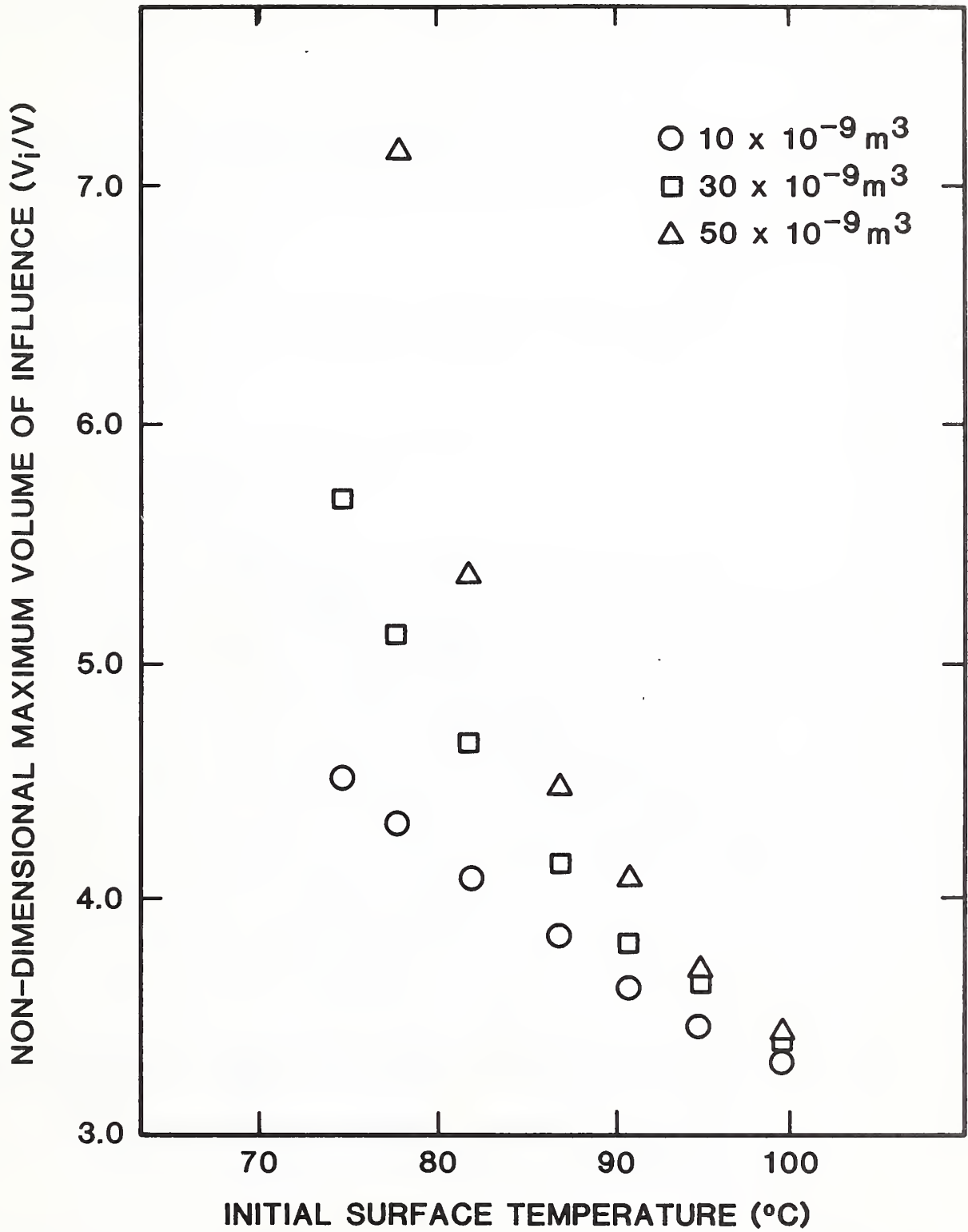
1.0

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FIGURE SEVEN
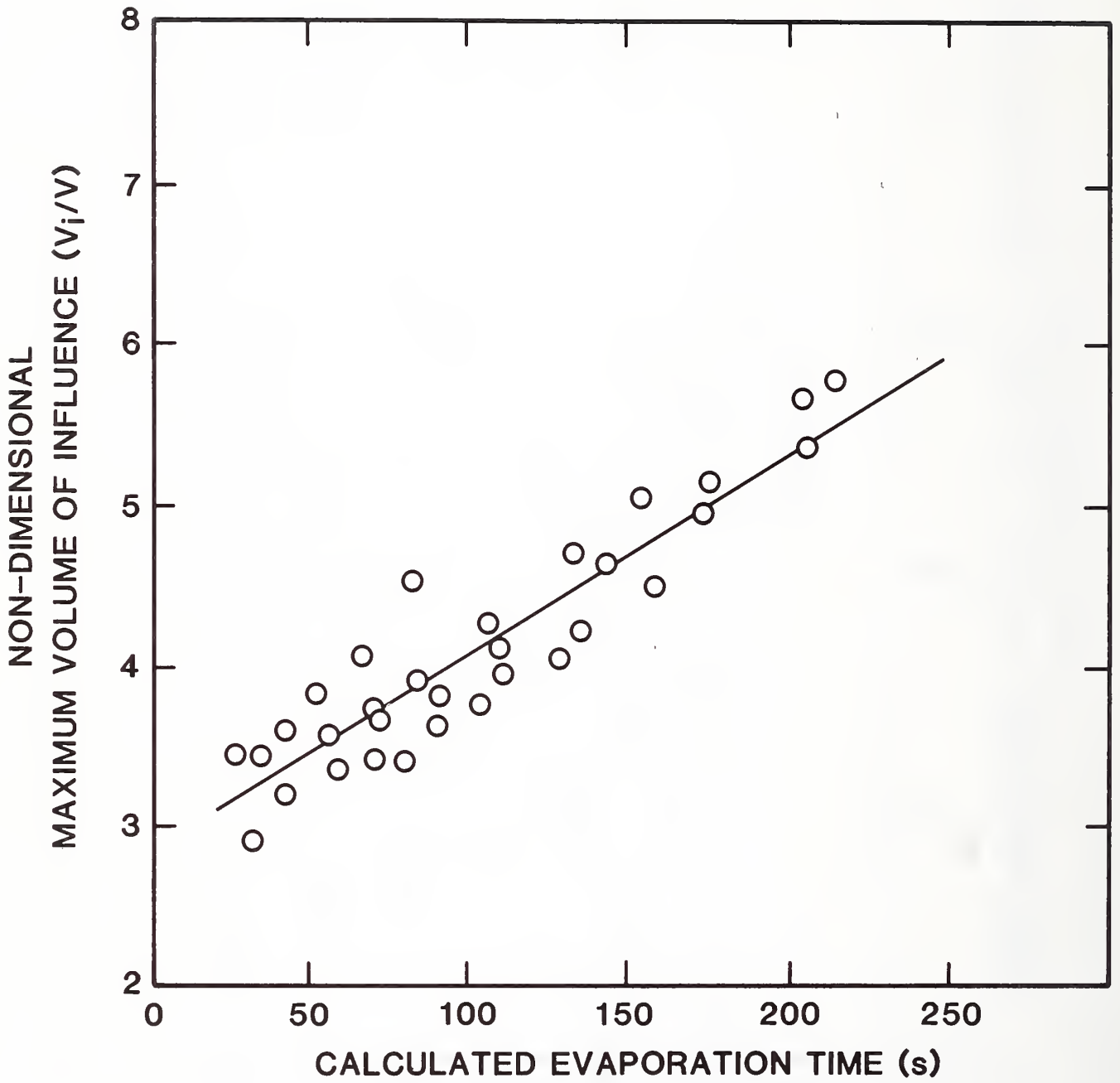
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FIGURE EIGHT

FIGURE NINE

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11. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)*

Transient cooling of solid surfaces by water droplet evaporation has been investigated through controlled experiments using a large heated aluminum cylinder. Quantitative prediction of droplet evaporation time and in-depth transient temperature distribution in the solid have been made. In the case studied, a single droplet is deposited on a horizontal non-porous surface with initial temperatures in the range of 75°C to 100°C. The liquid-vapor inter-facial temperature and the water vapor molar fraction in the air at the exposed surface of the water droplet are deduced from the coupled heat and mass transfer energy balance at the interface. Spacial and temporal integration of the overall droplet energy equation is used to predict the instantaneous evaporation rate and the droplet evaporation time. The boundary conditions for the wetted region proposed by Seki are used to obtain the transient temperature distribution for a semi-infinite solid. The region of the body affected by the droplet cooling is identified and its volume (normalized with the volume of the droplet) is plotted against the evaporation time. All data, regardless of the droplet volume or of the initial body surface temperature, lie within a narrow band about a straight line. This finding is the first important step to obtain a simple model for spray cooling based on local accurate description of the droplet-solid interactions. Modelling of spray cooling phenomena is the foundation for the construction of a thermal model for solid fuel fire extinguishment.

12. KEY WORDS *(Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)*

conduction; droplets; evaporation; extinguishment; fire models; spray quenching

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