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Modeling Window Optics for Building Energy Analysis

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U.S. DEPARTMENT OF COMMERCE, Malcolm Baldrige, *Secretary*
NATIONAL BUREAU OF STANDARDS, Ernest Ambler, *Director*

ABSTRACT

This report discusses modeling the optics of windows for the purposes of simulating building energy requirements or daylighting availability. The theory for calculating the optical performance of conventional windows is reviewed. The simplifications that might commonly be made in creating computational models are analyzed. Some of the possibilities for more complex windows are analyzed, and the type of model and data that would be necessary to simulate such windows in a building energy analysis program are determined. It is shown that the optical performance of different window types can be simulated with models which require varying amounts of memory or computing time. It is recommended that a building energy analysis program have all models available and use the most efficient for any given window.

SOLAR BUILDINGS RESEARCH AND DEVELOPMENT PROGRAM
CONTEXT STATEMENT
November 21, 1985

In keeping with the national energy policy goal of fostering an adequate supply of energy at a reasonable cost, the United States Department of Energy (DOE) supports a variety of programs to promote a balanced and mixed energy resource system. The mission of the DOE Solar Buildings Research and Development Program is to support this goal by providing for the development of solar technology alternatives for the building sector. It is the goal of the Program to establish a proven technology base to allow industry to develop solar products and designs for buildings which are economically competitive and can contribute significantly to building energy supplies nationally. Toward this end, the program sponsors research activities related to increasing the efficiency, reducing the cost, and improving the long term durability of passive and active solar systems for building water and space heating, cooling, and daylighting applications. These activities are conducted in four major areas: Advanced Passive Solar Materials Research, Collector Technology Research, Cooling Systems Research, and Systems Analysis and Applications Research.

Advanced Passive Solar Materials Research - This activity area includes work on new aperture materials for controlling solar heat gains, and for enhancing the use of daylight for building interior lighting purposes. It also encompasses work on low-cost thermal stroage materials that have high thermal storage capacity and can be integrated with conventional building elements, and work on materials and methods to transport thermal energy efficiently between any building exterior surface and the building interior by non--mechanical means.

Collector Technology Research - This activity area encompasses work on advanced low to medium temperature (up to 180^{o}F useful operating temperature) flat plate collectors for water and space heating applications, and medium to high temperature (up to 400^{o}F useful operating temperature) evacuated tube/-concentrating collectors for space heating and cooling applications. The focus is on design innovations using new materials and fabrication techniques.

Cooling Systems Research - This activity area involves research on high--performance dehumidifiers and chillers that can operate efficiently with the variable thermal outputs and delivery temperatures associated with solar collectors. It also includes work on advanced passive cooling techniques.

Systems Analysis and Applications Research - This activity area encompasses experimental testing, analysis, and evaluation of solar heating, cooling, and daylighting systems for residential and nonresidential buildings. This involves system integration studies, the development of design and analysis tools and the establishment of overall cost, performance, and durability targets for various technology or system options.

This report is an account of research conducted in the Systems Analysis area concerning the mathematical modeling of window optics for the purposes of simulating building energy requirements or daylighting availability.

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NOMENCLATURE

solar radiation - electromagnetic radiation in the ultraviolet, visible, and near infrared portions of the spectrun.
light - radiation in the visible portion of the electromagnetic spectrum.
optical interface - boundary between two media where radiation is reflected and transmitted.
optical layer - a pane of glass or a drape or shading device.
optical system - a window which is composed of one or more optical layers.

Commonly used variables:

- a - absorptance of a layer
- A - absorptance of a layer in a system
- ρ - reflectance of an interface
- r - reflectance of a layer
- R - reflectance of a system
- τ - transmittance of an interface
- t - transmittance of a layer
- T - transmittance of a system
- θ - angle of incidence
- λ - wavelength

Subscripts:

- n - layer number
- b - beam
- d - diffuse
- s - scattered

Superscripts:

- $+$ - for radiation going to the right (inside)
- $-$ - for radiation going to the left (outside)

Examples:

t^+_{nb} transmittance of layer n for beam radiation toward the inside

T^+_d transmittance of the system for diffuse radiation to the inside

a^-_{nb} absorptance of layer n for beam radiation to the outside

A^-_{nb} absorptance of layer n for beam radiation to the outside when the layer is part of an optical system

t^+_{ns} transmittance of layer n beam radiation which is incident from outside and scattered (becomes diffuse) at the inside

1. INTRODUCTION

Windows are thermally critical elements of the building envelope. They are difficult to insulate and usually produce much higher conductive gains and losses than corresponding wall areas. Many can be opened to provide ventilation, but such operable windows frequently become significant sources for infiltration. The unique characteristic of windows is the transmission of solar radiation directly into the building. This transmitted radiation usually produces a desirable energy gain in the winter and an undesirable gain in the summer. It also provides natural lighting inside the building which can be used to offset electric lighting requirements. Calculation of solar heat gain and of daylighting should be closely related since they both concern radiation from the sun, but there has been no unified approach to the optical modeling of windows.

Accuracy is usually the first goal in developing a computer model. This can include the ability to correctly predict both instantaneous values as well as long term totals or averages. For windows we are primarily interested in short term thermal and visual comfort considerations and long term energy requirements. Since perfect accuracy can never be achieved, the sensitivity of the model is important. That is, the model should be able to correctly rank any group of alternatives since selecting the correct alternative is the primary goal of any design process. This is related to generality, which is the ability to handle many different conditions. A model which is unable to simulate one of the alternatives under consideration does not help the building designer. Similarly, the model must use data that is available to the designer. Efficiency in terms of execution speed and computer memeory requirements should also be considered. Decreasing costs of computer memory and new operating systems make memory a less important consideration than it once was, but speed is always useful in that it allows more alternatives to be considered by allowing more simulations for a given cost or time.

Several models must be integrated to form an overall model for the analysis of solar gains and daylighting. First is the model of the angular distribution of the solar radiation and light that is incident on the window. This is usually classified as beam radiation which is direct from the sun and diffuse radiation which has been scattered or reflected by the sky, the ground, or surrounding objects. Another model must describe the distribution of radiation on the surfaces of the room to determine how much is reflected or absorbed. The optical model of the window establishes the connection between the outside and the inside. Heat gain calculations must also include a model of the conduction and convection of heat through the window.

Solar gain models are usually based on the ASHRAE shading coefficient procedure [1]. This procedure was developed twenty years ago for manual calculations. It's use for transient analysis of buildings with windows which are significantly different from the single pane reference window is questionable (see section 3.1). Some alternatives to shading coefficients have been included in the major public domain energy analysis programs. DOE-2 [2] includes detailed calculations for a few selected window types. The BLAST [3] and TARP [4] programs compute the detailed optics of windows consisting of up to four panes of glass.

Daylighting programs such as SUPERLIGHT [5], CEL-1 [6], and DALITE [7], include much more detail, particularly in the distribution of light from the sky and its distribution in the room. However, the detailed sky models apply only to clear or to completely overcast days. Partly cloudy days must be approximated. Some of these programs are restricted by lack of generality in their window models. Their primary drawback, however, is that the calculations are so detailed and time consuming that they cannot be directly incorporated into hourly energy analysis programs.

This report is concerned with the modeling of window optics for the purposes of simulating building energy requirements or daylighting availability. It reviews the theory for calculating the optical performance of conventional windows. It analyzes the simplifications that might be made in creating computational models. It then looks at some of the possibilities for more complex windows and determines the type of model and data that would be necessary to simulate such windows in a building energy analysis program. Such models require more data than is currently available to designers, so the requirements for additional test data are also discussed.

2. OPTICAL THEORY FOR IDEAL PANES OF GLASS

2.1 The Air-Glass Interface

The optical performance of ordinary window glass is well described by the classical optical theories [8] of Snell's law, the Fresnel equations, and Bouger's law. A pane of glass can be considered a partially transparent dielectric medium which has two smooth parallel glass-to-air interfaces. When a ray of electromagnetic radiation reaches a smooth interface between two dielectric media, part of the ray is reflected and part is refracted as shown in fig 1. The angle of reflection equals the angle of incidence, θ_1. The angle of refraction, θ_2, is given by Snell's law:

$$\sin\theta_1/\sin\theta_2 = v_1/v_2 \tag{1a}$$

where v_i is the velocity of light in each medium, which is given by $v = c/\sqrt{\varepsilon\mu}$ where c is the velocity of light in a vacuum, ε is the medium's dielectric constant (permittivity), and μ is its magnetic permeability. Defining the refractive index of a medium to be $n = c/v$ leads to the following version of Snell's law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{1b}$$

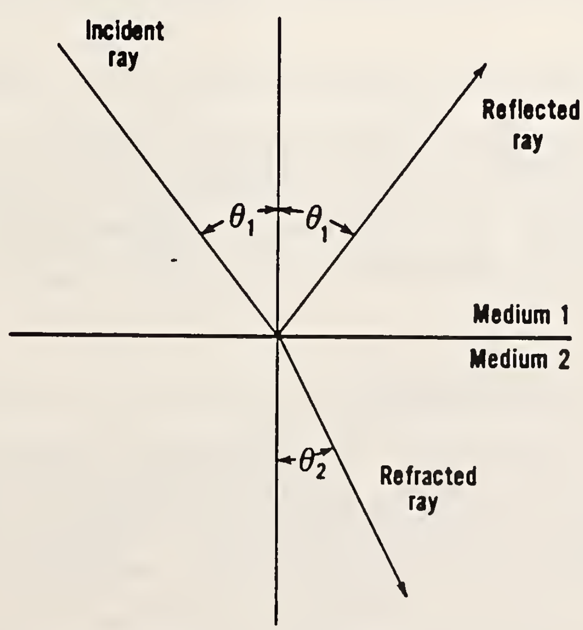


Figure 1. Reflection and Refraction at an Interface

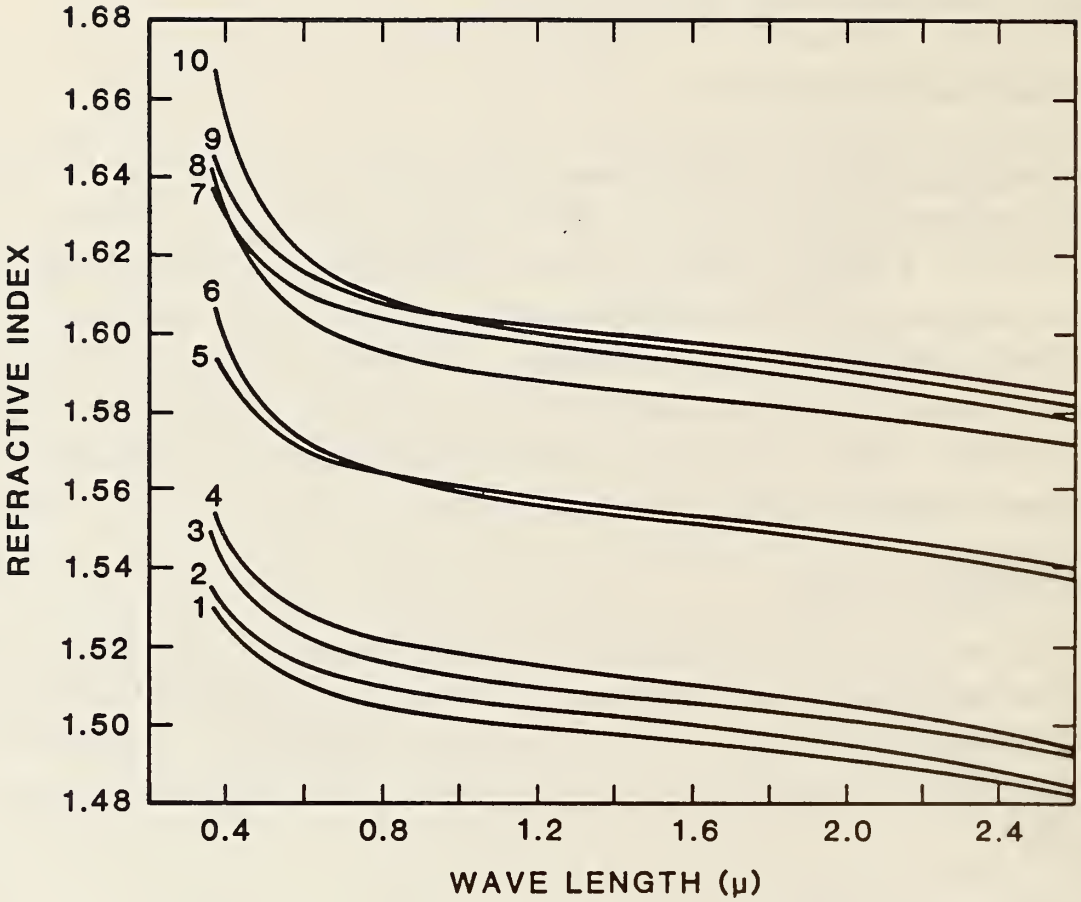


Figure 2. Wavelength Dependence of the Index of Refraction

The refractive index of air is effectively equal to 1 and for glass, since it is non-magnetic, $n = \sqrt{\varepsilon}$. The refractive index of glass is dependent on both the composition of the glass and the wavelength of the radiation. Fig 2 shows the relationship of index of refraction to wavelength for several optical glasses [9]. Trigonometric identities can convert Snell's law to a relation of cosines:

$$\cos\theta_2 = \sqrt{1 - (n_1^2/n_2^2)(1-\cos^2\theta_1)} \qquad (1c)$$

Electromagnetic theory shows that reflectance and transmittance at an interface depend on the polarization of the incident radiation. The two components of polarization of interest are pendicular to the plane of incidence (also called transverse electric or TE) and parallel to the plane of incidence (also called transverse magnetic or TM). Unpolarized radiation can be considered to consist of equal parts of both polarizations. The Fresnel formulae describe the amplitudes of the reflected (E_r) and transmitted (E_t) electric components of the electromagnetic wave relative to the incident (E_i) component.

$$(E_r/E_i)_{TE} = (n_1\cos\theta_1 - n_2\cos\theta_2) / (n_1\cos\theta_1 + n_2\cos\theta_2) \quad (2a)$$

$$(E_r/E_i)_{TM} = (n_2\cos\theta_1 - n_1\cos\theta_2) / (n_1\cos\theta_1 + n_2\cos\theta_2) \quad (2b)$$

$$(E_t/E_i)_{TE} = 2n_1\cos\theta_1 / (n_1\cos\theta_1 + n_2\cos\theta_2) \quad (2c)$$

$$(E_t/E_i)_{TM} = 2n_1\cos\theta_1 / (n_1\cos\theta_1 + n_2\cos\theta_2) \quad (2d)$$

However, instead of the ratio of amplitudes, we are more interested in the energy content of the radiation as given by Poynting's vector. The energy reflectance, ρ, is given by $\rho = (E_r/E_i)^2$ and the energy transmittance, τ, is given by $\tau = (E_t/E_i)^2(n_2\cos\theta_2)/(n_1\cos\theta_1)$. If we define

$$p_i = n_i\cos\theta_i \text{ for TE radiation} \quad (3a)$$

and

$$p_i = \cos\theta_i/n_i \text{ for TM radiation} \quad (3b)$$

for media, i = 1 and 2. Then for either polarization

$$\rho = [(p_1 - p_2)/(p_1 + p_2)]^2 \quad (3c)$$

and

$$\tau = (p_2/p_1)[2p_1/(p_1+p_2)]^2 \quad (3d)$$

It is faster to compute the transmittance from the reflectance and the requirement of conservation of energy: $\tau = 1 - \rho$.

2.2 Single Glass Pane

Glass absorbs radiation passing through it in proportion to the intensity of the radiation (Bouger's law). The fraction of radiation transmitted, τ_g, through a layer of glass is given by the exponential decay formula

$$\tau_g = e^{-kx} \quad (4)$$

where k is the extinction coefficient of the glass and x is the path length. The value of k is wavelength dependent but independent of polarization (in glass). The fraction of radiation absorbed is $\alpha_g = 1 - \tau_g$. For a glass pane of thickness L, the distance the radiation travels through the glass is $x = L/\cos\theta_2$.

A ray incident on a pane of glass will be both reflected and refracted at the front surface. Part of the refracted component will be absorbed in the glass before reaching the back surface where it is again reflected and refracted. This continues until all the radiation is either absorbed or escapes from the front or back surfaces of the glass. Fig 3 shows the first several reflections and refractions of the incident ray. Summation of the transmitted, reflected, and absorbed components of the incident ray give the total transmittance, t, reflectance, r, and absorptance, a, of the pane of glass. These values must be determined separately for both polarization components. ρ, τ (= $1 - \rho$), τ_g, and α_g (= $1 - \tau_g$) are functions of the angle of incidence and wavelength of the radiation. They are identical at both air-glass interfaces. The transmittance of the pane is given by

$$t = (1-\rho)^2\tau_g + (1-\rho)^2\rho^2\tau_g^3 + (1-\rho)^2\rho^4\tau_g^5 + \ldots$$

$$= (1-\rho)^2\tau_g \ (1 + \rho^2\tau_g^2 + \rho^4\tau_g^4 + \ldots)$$

$$= \tau^2 \ \tau_g \ / \ (1 - \rho^2\tau_g^2) \quad (5a)$$

The reflectance of a pane is given by

$$r = \rho + (1-\rho)^2\rho\tau_g^2 + (1-\rho)^2\rho^3\tau_g^4 + \ldots$$

$$= \rho + (1-\rho)^2\rho\tau_g^2 \ (1 + \rho^2\tau_g^2 + \rho^4\tau_g^4 + \ldots)$$

$$= \rho + \tau^2 \ \rho \ \tau_g^2 \ / \ (1 - \rho^2 \ \tau_g^2)$$

$$= \rho + \rho \ \tau_g \ t \quad (5b)$$

The absorptance of a pane is given by

$$a = [(1-\rho)-(1-\rho)\tau_g] + [(1-\rho)\rho\tau_g-(1-\rho)\rho\tau_g^2] + [(1-\rho)\rho^2\tau_g^2-(1-\rho)\rho^2\tau_g^3] + \ldots$$

$$= (1-\rho)(1-\tau_g)(1 + \rho\tau_g + \rho^2\tau_g^2 + \ldots)$$

$$= \tau \ \alpha_g \ / \ (1 - \rho \ \tau_g) \quad (5c)$$

The energy balance ($t + r + a = 1$) allows any one of the three components to be computed from the other two.

The extinction coefficient, k, is not commonly reported, however it can be easily computed from the thickness, refractive index, and the transmittance at normal incidence ($\theta = 0$). At normal incidence

$$\rho = [(n_1 - n_2)/(n_1 + n_2)]^2$$

$$= [(n - 1)/(n + 1)]^2 \quad (6a)$$

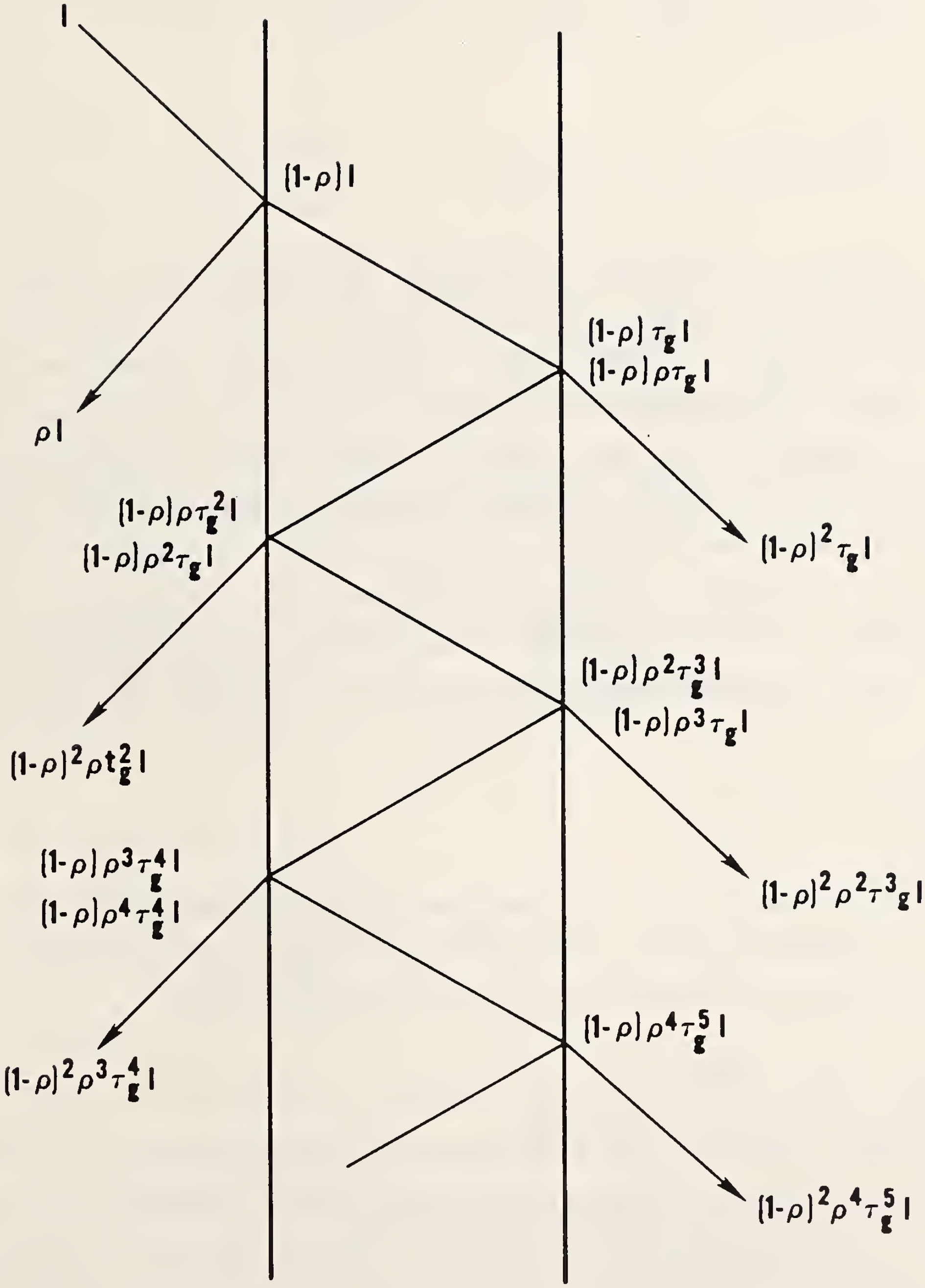


Figure 3. Multiple Reflections in a Pane of Glass

where n and 1 are the refractive indices of the glass and air, respectively. Eqn (5a) is a quadratic equation in τ_g, whose solution is

$$\tau_g = [\sqrt{(1-\rho)^4 + 4\rho^2t^2} - (1-\rho)^2] / 2\rho^2t \qquad (6b)$$

Therefore,

$$k = -\ln(\tau_g) / L \qquad (6c)$$

where L is the thickness of the glass.

The equations which were developed above apply to clear and heat absorbing glass. For these common types of glass, the optical properties of the pane do not depend on which side of the glass the initial ray is incident. However, thin film reflective layers do cause directional differences depending on which side of the glass is coated. Optical theory for thin films [10, 11] will not be reviewed in this report. The net result of thin film theory is that the film, or films, which are placed on a pane of glass, can be considered as a modified air-glass interface with transmission, refelction, and absorption in the interface. The basic energy conservation laws still apply for a ray traveling toward the inside or toward the outside of the pane:

$$t_b^+ + r_b^+ + a_b^+ = 1.0 \qquad (7a)$$

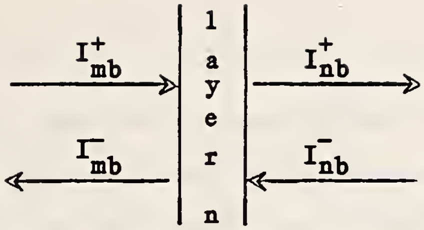
and

$$t_b^- + r_b^- + a_b^- = 1.0 \qquad (7b)$$

where t_b^+ always equals t_b^-, but r_b^+ need not equal r_b^-, and a_b^+ need not equal a_b^-. The + and − directions will be included in the equations that follow to maintain generality for glass panes with reflective films.

2.3 Multiple Pane Window

The optical properties of a multipane optical system for a given angle of incidence, wavelength, and polarization are quickly computed from the optical properties of the individual panes, eqns (5a-c). Instead of summing all the reflections and refractions of a single ray, a net radiation method is very effective. Consider the following sketch.

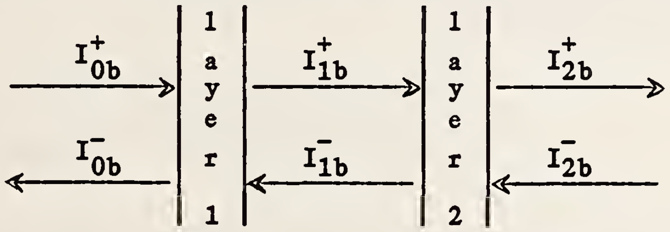


$$m = n - 1$$

$$I^+_{nb} = I^+_{mb} * t^+_{nb} + I^-_{nb} * r^-_{nb} \tag{8a}$$

$$I^-_{mb} = I^-_{nb} * t^-_{nb} + I^+_{mb} * r^+_{nb} \tag{8b}$$

I^+_{nb} is the sum of all radiation going to the right from layer n. It is composed of the portion of I^+_{mb} (the radiation incident on the left side) which is transmitted through the layer plus the portion of I^-_{nb} (the radiation incident on the right side) which is reflected from the layer, and similarly for I^-_{mb}. It is easy to formulate the relationships for a two-layer window as sketched below from the relationships of eqns (8a,b).



$$I^-_{0b} = r^+_{1b} * I^+_{0b} + t^-_{1b} * I^-_{1b} \tag{9a}$$

$$I^+_{1b} = t^+_{1b} * I^+_{0b} + r^-_{1b} * I^-_{1b} \tag{9b}$$

$$I^-_{1b} = r^+_{2b} * I^+_{1b} + t^-_{2b} * I^-_{2b} \tag{9c}$$

$$I^+_{2b} = t^+_{2b} * I^+_{1b} + r^-_{2b} * I^-_{2b} \tag{9d}$$

Equations (9a-d) plus equations defining I^+_{0b} and I^-_{2b} can be expressed in matrix form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -r^+_{1b} & 1 & 0 & -t^-_{1b} & 0 & 0 \\ -t^+_{1b} & 0 & 1 & -r^-_{1b} & 0 & 0 \\ 0 & 0 & -r^+_{2b} & 1 & 0 & -t^-_{2b} \\ 0 & 0 & -t^+_{2b} & 0 & 1 & -r^-_{2b} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} I^+_{0b} \\ I^-_{1b} \\ I^+_{1b} \\ I^-_{1b} \\ I^+_{2b} \\ I^-_{2b} \end{bmatrix} = \begin{bmatrix} X_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ X_2 \end{bmatrix} \tag{10}$$

To obtain the system properties for light traveling to the right, set X_1 to 1.0 and X_2 to 0.0 and solve the system of equations. This gives $I^+_{0b} = 1.0$ and $I^-_{2b} = 0.0$ and appropriate values for the other I's. These can be substituted into the following equations to determine the optical properties of the two-pane system.

$$T^+_b = I^+_{2b} / I^+_{0b} = I^+_{2b} \tag{11a}$$

$$R^+_b = I^-_{0b} \tag{11b}$$

$$A^+_{1b} = a^+_{1b} + a^-_{1b} * I^-_{1b} \tag{11c}$$

$$A^+_{2b} = a^+_{2b} * I^+_{1b} \tag{11d}$$

The system properties for light incident from the right, T^-_b, etc., can be determined by setting X_1 to 0.0 and X_2 to 1.0, solving for the I values, and substituting into the following equations.

$$T^-_b = I^-_{0b} / I^-_{2b} = I^-_{0b} \tag{12a}$$

$$R^-_b = I^+_{2b} \tag{12b}$$

$$A^-_{1b} = a^-_{2b} * I^-_{1b} \tag{12c}$$

$$A^-_{2b} = a^-_{2b} + a^+_{2b} * I^+_{2b} \tag{12d}$$

This analysis can be extended to any number of layers by increasing the size of the matrix and solving numerically. Note that the matrix is diagonally dominant which implies that the simultaneous equations can be solved iteratively or directly without pivoting. The number of layers in a window should be small enough that solution time is not a significant consideration for either method. There is a small gain in solution efficiency by doing LU decomposition [12, sec 2.5] and then solving with the two different right hand sides. BLAST [3] and TARP [4] use symbolic solutions of these and similar equations for up to four layers, which is faster, but greater generality is achieved by using the numeric solution method presented above.

2.4 Hemispheric (Diffuse) Performance

The formulae developed so far give the optical performance of a single or multipane window at a single angle of incidence of the incident radiation. It is sometimes useful to consider that the radiation is incident at equal intensity from all angles, that is, the radiation is perfectly diffuse. Coefficients for optical performance with respect to diffuse radiation are related to the coefficients for beam radiation in the following manner. The flux, dq, incident at a point due to radiation incident from a small solid angle, dΩ, is given by

$$dq = I \cos\theta \, d\Omega$$

$$= I \cos\theta \, (r \, d\theta) \, (r \sin\theta \, d\phi) \, / \, r^2 \qquad (13a)$$

where I is the intensity of incoming radiation, and the angles θ and ϕ are shown in fig 4. Assuming uniform intensity, the total incident flux is given by

$$q = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I \cos\theta \sin\theta \, d\theta \, d\phi = \pi I \qquad (13b)$$

The hemispherical transmittance is obtained by computing the fraction of the diffusely incident radiation which is transmitted through the window:

$$T_d = \frac{\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} T(\theta,\phi) \, I \cos\theta \sin\theta \, d\theta \, d\phi}{q} \qquad (13c)$$

If T is independent of ϕ, eqn (13c) reduces to

$$T_d = 2 \int_{\theta=0}^{\pi/2} T(\theta) \cos\theta \sin\theta \, d\theta \qquad (13d)$$

Evaluation of this integral is discussed in section 3.5 .

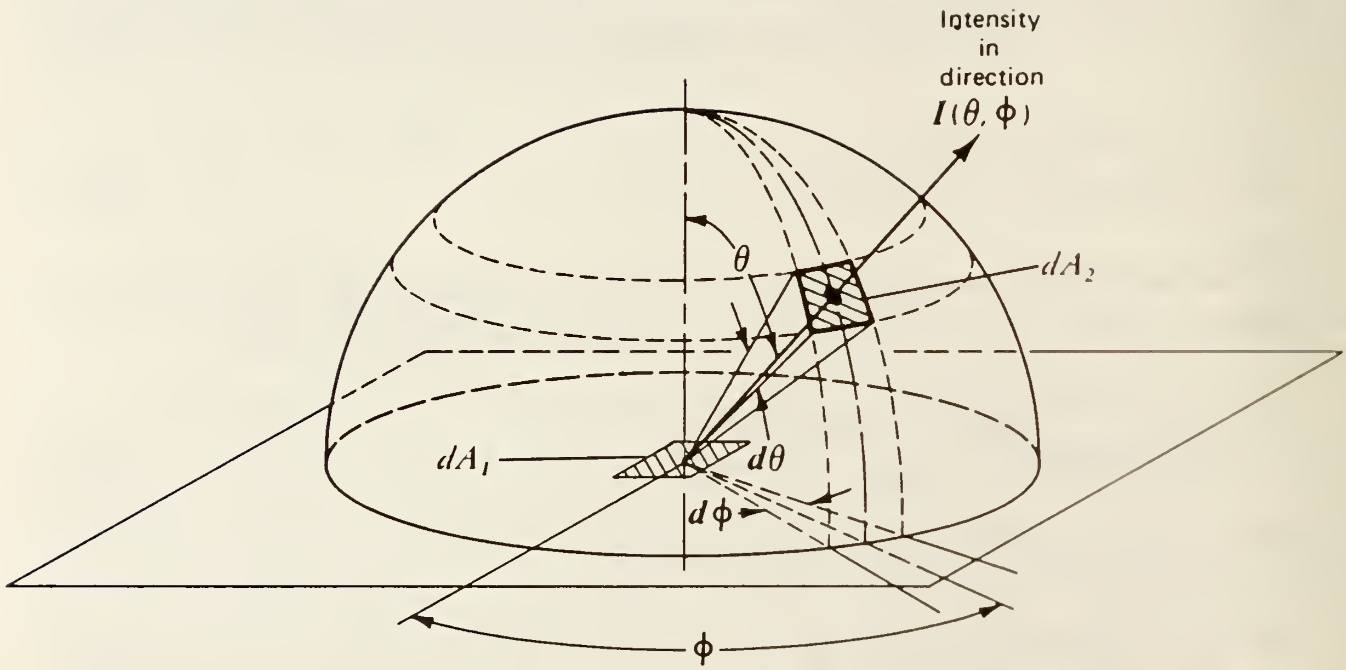


Figure 4. Coordinate System for Diffuse Radiation

3. APPROXIMATIONS FOR SIMPLE WINDOWS

In the following discussion, 'simple windows' will refer to those whose optical performance can be computed by the methods presented in the previous section.

3.1 Shading Coefficient Procedure

Before studying other approximations, it is useful to review the most common method for computing solar gains through windows, the ASHRAE shading coefficient method [1]. This method was developed in the early 1960's to provide a convenient method for manual calculation of solar gains in order to determine cooling loads for equipment sizing. Figure 5 shows the heat flows considered in determining the total heat admitted through a pane of glass into a room. The components of heat admission are

Total heat admission through glass	=	Sunlight transmitted through glass	+	Inward flow of absorbed solar radiation	+	Heat flow due to outdoor-indoor air temperature difference

which, by grouping the radiation terms, becomes

Total heat admission through glass	=	Solar Heat Gain	+	Conduction Heat Gain

The transmitted radiation causes a heat gain in the room by first being absorbed by room surfaces and then convected into the room air. There is a delay from the time heat is absorbed until the time it is released by convection depending on the thermal diffusivity of the surfaces. Some radiation absorbed in the glass is conducted to the inner surface and convected into the room air. Since there is usually little mass in the glass panes, there is negligible delay in this portion of the solar heat gain. Delay is also ignored in calculating the heat gain due to the outdoor-indoor temperature difference.

The shading coefficient (SC) of a given window is defined to be the window's solar heat gain divided by the solar heat gain of a reference glass (called a solar heat gain factor [SHGF]). Therefore, the total heat admission of a window is given by

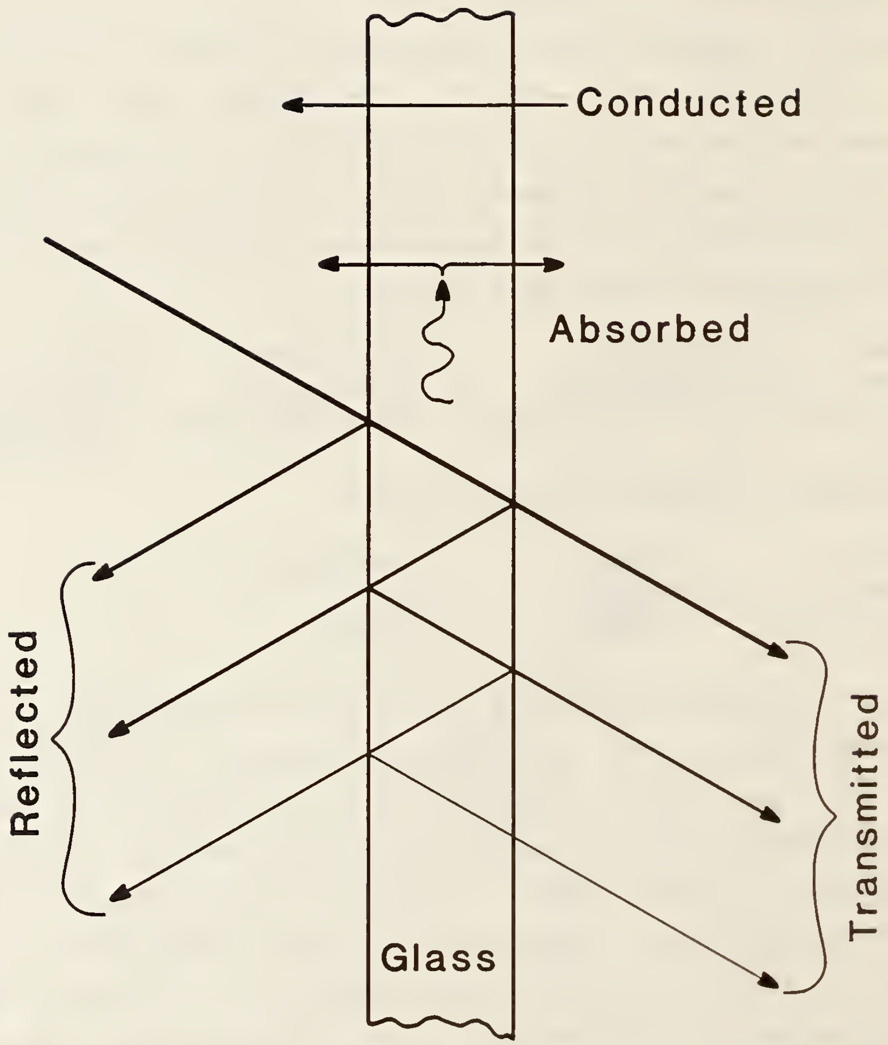


Figure 5. Heat Admission Through Glass

$$q = SC * SHGF + U (To - Ti) \tag{14}$$

where U is the overall heat transfer coefficient and (To - Ti) is the outdoor-indoor temperature difference.

Use of the SC method is based on an extensive set of SHGF tables for different latitudes and for different times of the day and the year. In a computer implementation the tables can be replaced by the formulae [1] which were used to generate them. The primary assumptions in the SC method are: (1) the convection coefficients are constant; (2) the relative proportions of diffuse and beam radiation are set for a clear day; (3) the angular dependence of the solar heat gain of the window is the same as for the reference glass; and (4) the proportions of radiation transmitted and absorbed are set for a low absorption glass (t = .86 at normal incidence). Each of these assumptions causes some loss of accuracy, or more especially sensitivity, for various windows and conditions. The first two assumptions can be overcome in a computer implementation of the original formulae by using convection coefficients and beam and diffuse radiation values appropriate for the actual weather conditions. However, convection coefficients are not well known, especially for the mixed free and forced convection regime. The last two assumptions cannot be improved unless detailed optical calculations are performed. The third assumption can fail completely for some windows (as in section 4.2). The last assumption can give the wrong portions of radiation which are absorbed and stored in the room walls or are absorbed and instantly convected from the glass, thereby giving an incorrect phasing of the heat gains. This inability to handle dynamic conditions is not surprising since the SC method was not developed for dynamic energy calculations.

Consideration of the radiation gains into the room indicates that the following optical performance values are needed for detailed window models: T_b^+ and T_d^+ for the window and A_{nb}^+ and A_{nd}^+ for each layer (pane) in the window. In addition to radiation gains, windows also allow radiation losses such as radiation from lights and solar radiation reflected from the surfaces of the room back out the windows. Since such radiation is diffuse, it requires the T_d^- and A_{nd}^- values. The magnitude of such losses is usually small.

3.2 Polarization

The performance of a pane of glass depends on the polarization of the incident radiation. Fig 6 shows the transmittance, reflectance, and absorptance of an ordinary pane of glass (t = .86 at normal incidence) for each polarization and also for the average of the two polarizations as a function of angle of incidence. Note that the polarization dependence is most significant at about 60° where $t_{TE} = .92$ and $t_{TM} = .68$. Beam solar radiation is usually unpolarized, which is equivalent to consisting of equal parts of both polarizations. The optical peformance for unpolarized radiation, therefore, is the average of the peformance at each polarization. Diffuse radiation from the clear sky is polarized, with the degree of polarization depending on the direction. Radiation reflected from clouds exhibits little polarization. Solar radiation can become polarized when it is specularly reflected from some surfaces, such as the glass of adjacent buildings or water. Building energy analysis programs have not generally considered such specular reflection energy gains, let alone the more detailed consideration of polarization effects.

In evaluating the performance of multipane systems eqn (10) should be evaluated separately for each polarization and the results combined in equal proportions for unpolarized radiation. Figs 7, 8, and 9 show for 2-, 3-, and 4- pane systems, respectively, the error caused by using the average properties for each pane instead of the exact procedure. The error in absorptance is not significant but the error in transmittance increases with the number of panes and transmittance is always less than the correct value. In addition, the error does become less significant as more radiation is absorbed in the panes. The implication of this polarization effect is that measurements of a single layer should be done for each polarization if that layer is to be mathematically combined with other layers in computing the performance of an optical system. Conversely, if data are not available for both polarizations, errors of the order indicated in figures 7 - 9 can be expected.

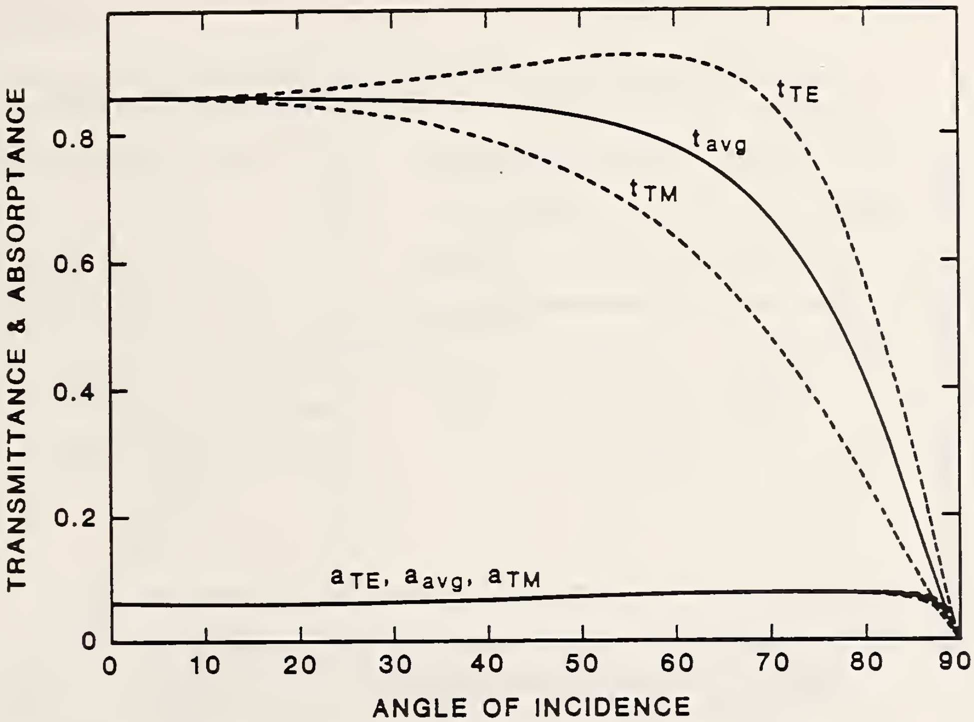


Figure 6. Polarization Performance of a Single Pane Window

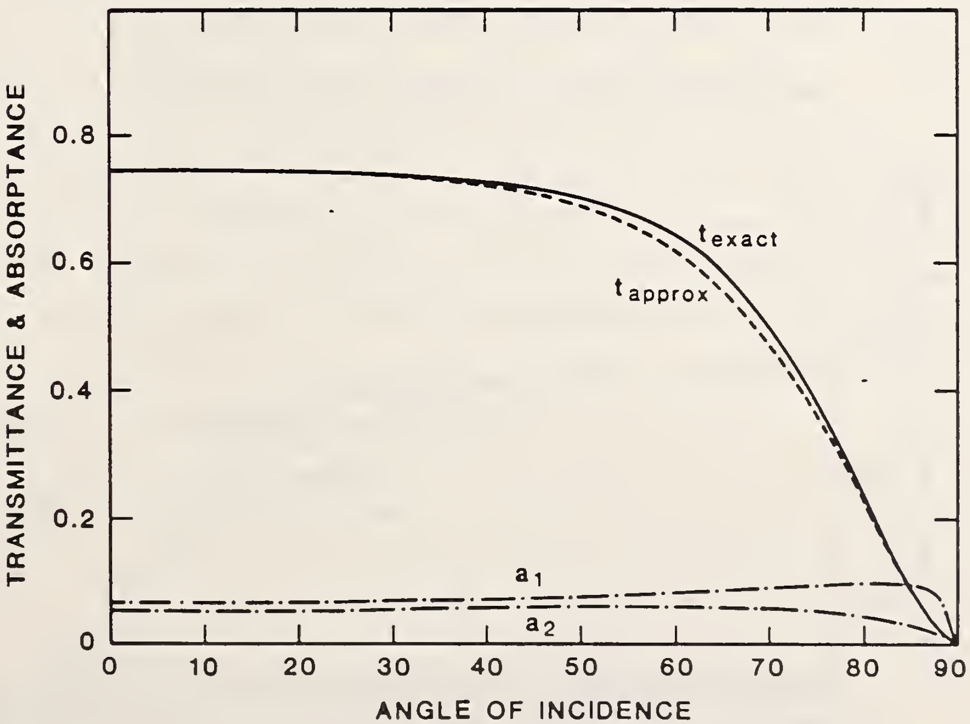


Figure 7. Polarization Performance of a 2-pane Window

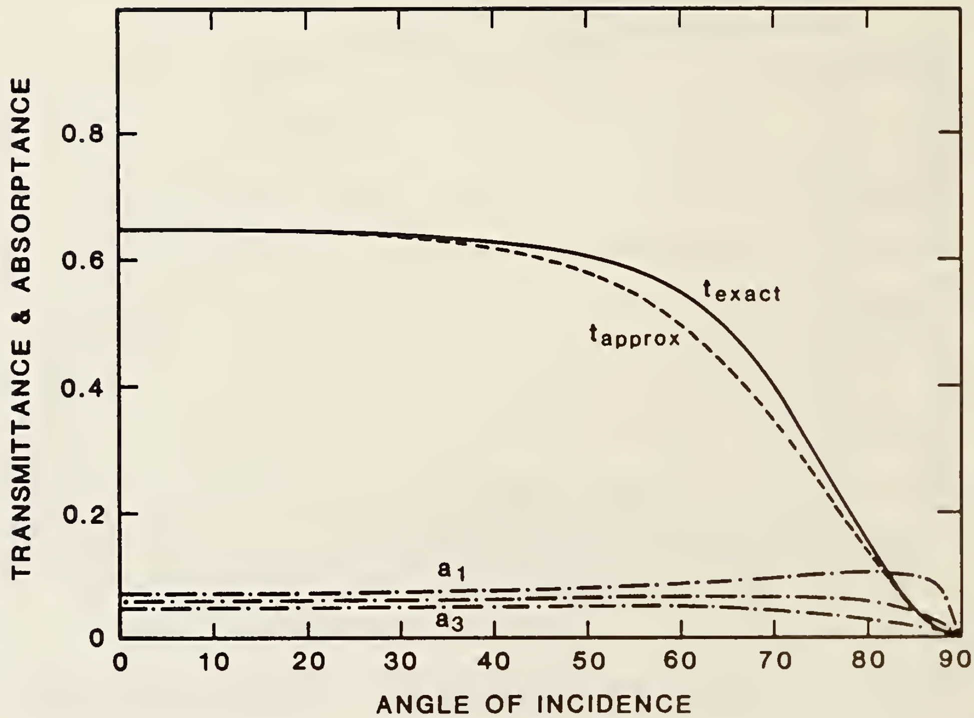


Figure 8. Polarization performance of a 3-pane Window

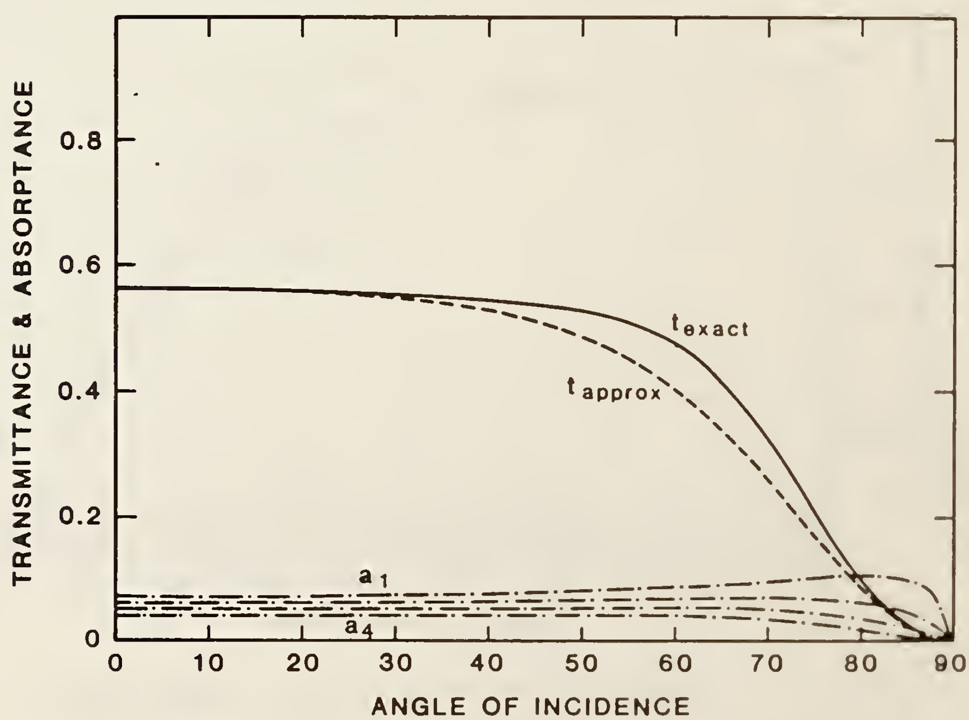


Figure 9. Polarization Performance of a 4-pane Window

3.3 Wavelength

A truly detailed window optical model should consider the wavelength dependence of optical properties. The formulae for optical properties given in Section 2 apply to a single wavelength, λ, of radiation. The total radiation incident on a window is composed of radiation of many wavelengths in varying proportions. Both the refractive index of glass and the extinction coefficient, n and k, are wavelength dependent properties. The average optical performance of a pane of glass is obtained by integrating the wavelength dependent performance, e.g. transmittance:

$$\bar{t} = \int t(\lambda)\, w(\lambda)\, d\lambda \Big/ \int w(\lambda)\, d\lambda \qquad (15a)$$

where $w(\lambda)$ is a weighting function for different wavelength distributions, e.g. full solar or visible. It has been common practice to model windows by using a single average value of refractive index and extinction coefficient.

In order to test this assumption it is necessary to do a detailed wavelength dependent model of several common window glasses. The wavelength integration is achieved by rectangular integration using 20 selected wavelengths from [14] for the solar energy distribution at sea level with air mass 2 (i.e., a solar altitude of 30°) listed in table 1. The spectral transmittances of five common glasses (3 mm (1/8 inch) regular sheet, 6 mm (1/4 inch) bronze heat absorbing, 6 mm (1/4 inch) gray heat absorbing, 6 mm (1/4 inch) green heat absorbing, and 3 mm (1/8 inch) low iron vey clear) were taken from the extinction coefficients listed in [13]. The refractive index of typical window glasses is closely represented by the following equation [13]

$$n = 1.5130 - 0.003169\lambda^2 + 0.003962/\lambda^2 \qquad (15b)$$

where the wavelength is given in μm.

Table 1. Wavelength Dependence of Optical Properties

Wavelength (µm)	index of refraction	transmittance clear	bronze	gray	green	low Fe
.390	1.539	.888	.463	.400	.683	.908
.444	1.532	.898	.433	.428	.750	.911
.481	1.529	.903	.425	.412	.796	.913
.511	1.527	.906	.441	.383	.794	.914
.543	1.526	.903	.510	.422	.776	.914
.574	1.524	.897	.540	.413	.728	.913
.606	1.523	.888	.546	.382	.668	.911
.639	1.521	.875	.536	.373	.600	.908
.669	1.520	.865	.545	.437	.539	.906
.705	1.519	.853	.564	.567	.488	.903
.745	1.518	.835	.522	.547	.413	.899
.786	1.517	.818	.472	.495	.344	.895
.831	1.517	.799	.429	.444	.287	.891
.877	1.516	.785	.391	.408	.247	.887
.959	1.514	.770	.359	.372	.210	.884
1.026	1.513	.765	.348	.358	.200	.883
1.105	1.512	.764	.344	.346	.198	.883
1.228	1.511	.773	.359	.355	.214	.885
1.497	1.508	.822	.494	.474	.358	.898
1.722	1.505	.846	.574	.544	.453	.904

Integration over the 20 wavelengths gives the detailed performance of the glass pane, in particular the transmittance at normal incidence, from which it is possible to generate a wavelength averaging approximation by using an average refractive index of 1.51955 and eqns (6a–c) to determine an average extinction coefficient. Figure 10 shows the transmittance for the five types of glass as computed with the wavelength dependent optical properties. The normal and hemispherical transmittances of the glasses are

	clear	bronze	gray	green	low Fe
normal	.843	.465	.428	.487	.901
hemispherical	.781	.402	.367	.429	.842

The hemispherical transmittance based on the wavelength averaging approximation were computed to be

hemispherical	.781	.402	.367	.423	.842
error	−.03%	−.18%	−.15%	−1.41%	0%

Only the performance of the green heat absorbing glass for wavelength averaging is shown in fig 10. The wavelength averaging approximation for the other glasses do not produce observable differences in the transmittance curves. Absorptance curves would show even less difference.

This test indicates that there is insignificant error in computing the optical performance of most common window glasses by using a wavelength averaging approximation instead of the detailed method, which is about 20 times slower. The least error occurs for glasses which do not have major wavelength dependencies, such as clear and low iron glasses. Glasses which do have major wavelength dependencies, such as those using thin reflective films, are likely to require detailed wavelength modeling.

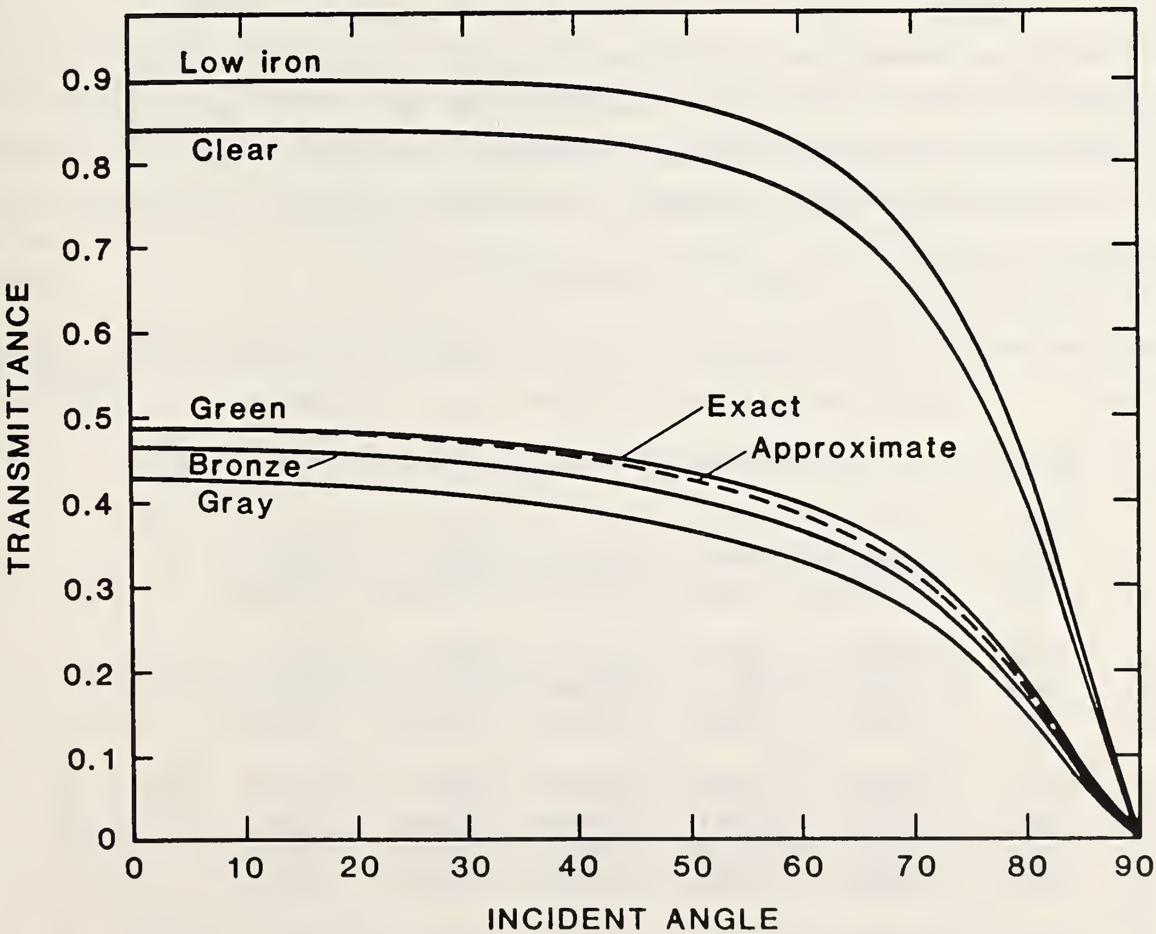


Figure 10. Wavelength Performance of 5 Common Window Glasses

3.4 Angular Performance

Although the optical theory for simple windows is straightforward and easy to implement for computation, it can become time consuming if the optical performance of every window in a building is to be evaluated hourly for a one-year simulation as is commonly done for energy analysis. In such a case it is important to save computation costs by using an approximation to the detailed theory. The most common such approximation is the polynomial expression for transmittance as a function of the cosine of the angle of incidence which has the form

$$T(\cos\theta) = c_1 \cos\theta + c_2 \cos^2\theta + c_3 \cos^3\theta + \ldots$$
$$= \sum_{i=1}^{N} c_i \cos^i\theta \qquad (16a)$$

and similarly for absorptance. This form is particularly convenient because $\cos\theta$ (rather than θ) is directly calculable for the sun's rays incident on a surface with any orientation; transmittance and absorptance are both zero at $\cos\theta = 0$; and the expression is quickly evaluated by the use of Horner's rule:

$$T(\cos\theta) = c_1(\cos\theta + c_2(\cos\theta + c_3(\cos\theta + \ldots))) \qquad (16b)$$

The coefficients, c_i, are computed by a least squares fit to the exact optical performance. High precision calculations are recommended because the

Table 2. RMS Error of the Polynomial Approximation for Window Optics

	Avg.	N = 2	N = 3	N = 4	N = 5	N = 6
T	.784	.03859	.00738	.00727	.00429	.00228
A1	.071	.01909	.01168	.00777	.00540	.00388
T	.655	.02077	.02117	.01038	.00257	.00061
A1	.078	.02574	.01576	.00984	.00636	.00432
A2	.058	.00670	.00240	.00131	.00083	.00048
T	.559	.02843	.02624	.00931	.00139	.00142
A1	.082	.02734	.01637	.00978	.00614	.00417
A2	.064	.01107	.00473	.00209	.00091	.00039
A3	.048	.00243	.00054	.00031	.00018	.00018
T	.482	.03390	.02797	.00793	.00346	.00225
A1	.084	.02786	.01648	.00960	.00596	.00411
A2	.067	.01245	.00525	.00120	.00067	.00028
A3	.053	.00482	.00085	.00040	.00040	.00039
A4	.041	.00161	.00162	.00060	.00015	.00015

simultaneous equations for the fit are relatively ill-conditioned.

Table 2 shows the accuracy for different numbers of coefficients in eqn (16a) for single through quadruple pane windows, each pane having a normal transmittance of .86 and a refractive index of 1.52 . The root mean square (rms) error is reported for the system transmittance and the absorptance in each pane. 'Avg.' refers to the hemispherically averaged performance values. Sixteen equally spaced values of cosθ were used to determine the curve fit, and the rms error is based on calculations at every one degree from 0^o to 89^o.

An alternative to the fitted polynomial is the interpolating polynomial [12, sec 3.4], formed by interpolation (e.g. Lagrangian) using two or more points. Table 3 shows the accuracy of interpolation using different numbers of points selected from values computed at 10^o intervals. In particular, N = 2 refers to linear interpolation which compares favorably with the 4th order polynomial curve fit. It is also possible to do interpolation with unevenly spaced angles for higher accuracy by using closer spacing where performance is changing rapidly with angle of incidence. Interpolation is not as fast as eqn (16b) because of the time required to determine the interpolation points. Maximum error in both methods occurs at high angles of incidence where other factors, such as small window setbacks and the actual thickness of the window, make the idealized theory inexact so computed errors at high θ may not be meaningful.

Table 3. Accuracy of Interpolation for Window Optics.

	Avg.	N = 2	N = 3	N = 4	N = 5	N = 6
T	.784	.00691	.00533	.00535	.00469	.00399
A1	.071	.01000	.00790	.00695	.00628	.00582
T	.655	.01415	.01162	.00923	.00704	.00566
A1	.078	.00466	.00302	.00232	.00182	.00153
A2	.058	.00231	.00141	.00113	.00097	.00087
T	.559	.00973	.00614	.00309	.00117	.00102
A1	.081	.00371	.00236	.00198	.00182	.00181
A2	.064	.00206	.00108	.00095	.00094	.00098
A3	.048	.00051	.00019	.00016	.00016	.00017
T	.482	.00799	.00422	.00132	.00196	.00245
A1	.084	.00363	.00228	.00195	.00185	.00187
A2	.067	.00196	.00125	.00131	.00078	.00079
A3	.053	.00078	.00068	.00076	.00078	.00079
A4	.041	.00052	.00037	.00022	.00014	.00013

In most cases the polynomial expression is the preferred method for representing window optical performance because of its speed, however some windows may be better modeled by interpolation when performance changes rapidly with respect to the angle.

3.5 Hemispheric Performance

There are several methods for computing the hemispheric performance, eqn (13d), for simple windows. One method ia s Simpson's rule numeric integration [12, sec 4.10] which requires evaluation of the window optical performance at n+1 equally spaced angles between 0^{o} and 90^{o}, where n is an even number. Figure 11 shows the computed hemispheric performance (T, R, and A for both polarizations) for values of n up to 20. There is no significant improvement in accuracy beyond n = 8 (values every 11.25^{o}). Computed hemispheric values given in previous sections are based on n = 10.

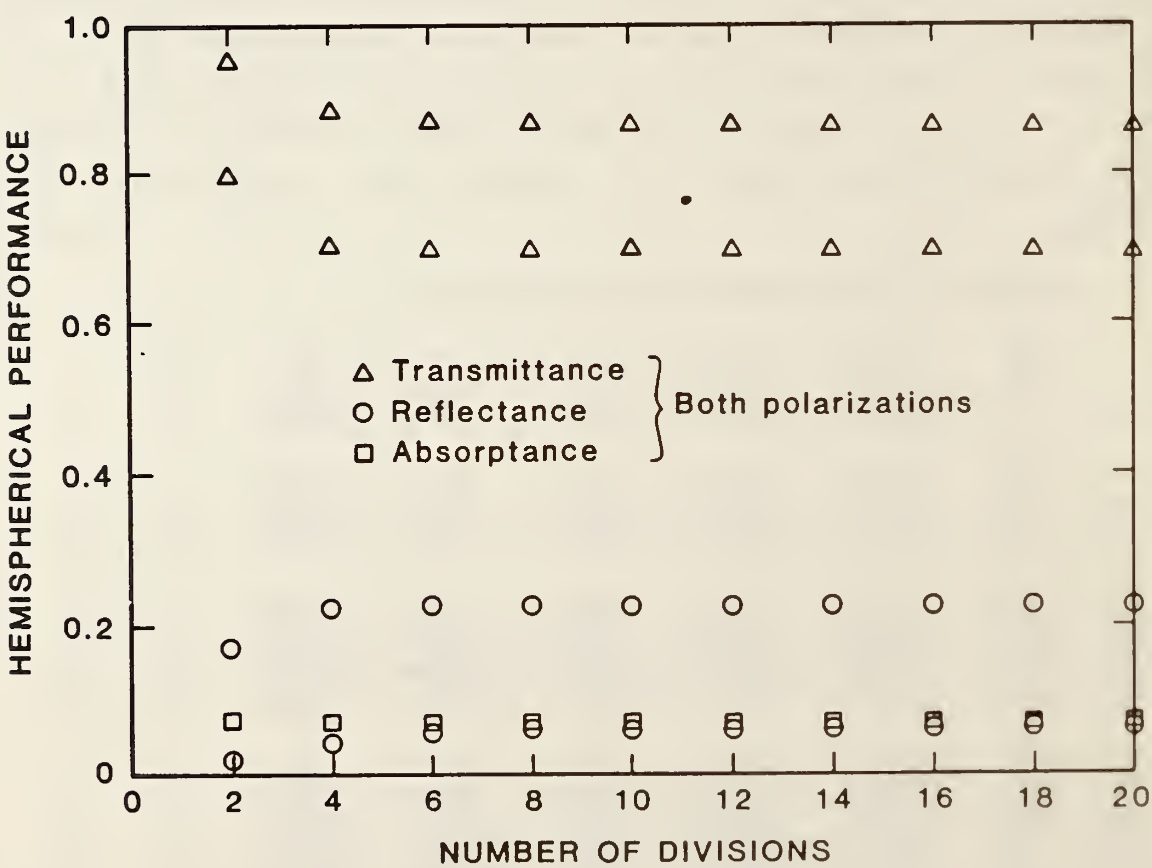


Figure 11. Hemispheric Performance Values by Simpson's Rule

A disadvantage of using Simpson's rule is that the window optical performance must be computed at evenly spaced angles which are not necessarily the angles convenient for angular performance fitting as described in the previous section. Performance fitting using evenly spaced interpolation values can provide the values necessary for a Simpson's rule integration. Unevenly spaced interpolation values and values taken at regular intervals of $\cos\theta$ do not. The unevenly spaced interpolation values can be used for a trapezoidal integration [12, sec 4.8] which has the same accuracy as shown in Table 3 for linear interpolation (N = 2) since trapezoidal integration is an application of linear interpolation.

By substituting the polynomial expression, eqn (16a), into eqn (13d) a very simple expression for hemispheric performance in terms of the polynomial coefficients is obtained:

$$T_d = \sum_{i=1}^{N} c_i/(i+2) \tag{17}$$

This expression gives the hemispheric performance much more accurately than the rms errors reported in the angular comparisons reported in Table 2. For example, for the single pane window with N = 2, eqn (17) gives $T_d = 0.789$ and $A_d = 0.072$, and for N = 3 it gives $T_d = 0.784$ and $A_d = 0.070$ versus values of 0.784 and 0.071, respectively, obtained by Simpson's rule.

Given the simplicity and the accuracy, eqn (17) should be used to evaluate hemispheric performance whenever the polynomial expression is being used to represent angular performance. The appropriate numeric integration should be used when angular performance is approximated by interpolation.

It may be noted that basic energy conservation applies to the hemispheric performance values just as it does to the angule dependent values in eqns (7a,b):

$$t_d^+ + r_d^+ + a_d^+ = 1.0 \tag{18a}$$

and

$$t_d^- + r_d^- + a_d^- = 1.0 \tag{18b}$$

However, this relationship cannot be used to develop an accurate method for computing the hemispheric performance of a multipane window from the hemispheric performance of its individual panes as done in eqns (8) through (12) for individual rays. There is an error because the portions of diffuse radiation which are transmitted through or reflected from a pane are no longer perfectly diffuse because of the angular performance characteristics of glass, i.e., more diffuse radiation is transmitted at low θ and more is reflected at high θ. The underprediction of transmission in a multipane system introduced by this approximation is about the same as the error caused by the polarization approximation discussed previously.

There is another assumption implicit in using T_d and A_d values to compute the performance of a window for non-direct incident radiation. This is the assumption that non-direct radiation around buildings is perfectly diffuse, which is almost never completely true although it is approached under some conditions. A detailed model of the non-direct radiation incident upon windows would have to include the non-uniform distribution of radiation from the sky under different weather conditions and the reflection of solar radiation from the ground and other objects seen from the window. This would be a complex calculation whose cost must be considered in relation to the additional accuracy and sensitivity which could be gained. The historic trend of every decreasing computation cost would indicate that even if such a calculation is not practical now, it may be in the future.

4. APPROXIMATIONS FOR COMPLEX WINDOWS

In the following discussion complex windows will refer to those whose optical performance cannot be computed by the methods presented in section 2.

4.1 Diffusing Layers

One of the most common window treatments is the drapery or curtain. Draperies can transmit from almost none to over one half of the incident radiation depending upon fabric color, thickness, and closeness of the weave. There is a fundamental difference from the optical performance of glass: some of the beam radiation transmitted and reflected by the drapery is diffusely scattered as radiation is reflected from individual fibers. If the thermal model of the room requires information on how much solar radiation strikes each surface, then it is important to know the amounts of radiation transmitted as beam and as diffuse radiation.

For the purposes of developing an optical model, a drapery will be considered an optical layer which interacts with a ray of radiation by transmitting part as beam radiation, transmitting another part as scattered radiation, reflecting part as beam radiation, reflecting another part as scattered radiation, and absorbing the remainder. This five-way distribution of the incident energy at layer n is expressed by

$$t_{nb}^{+} + t_{ns}^{+} + r_{nb}^{+} + r_{ns}^{+} + a_{nb}^{+} = 1.0 \qquad (19a)$$

$$t_{nb}^{-} + t_{ns}^{-} + r_{nb}^{-} + r_{ns}^{-} + a_{nb}^{-} = 1.0 \qquad (19b)$$

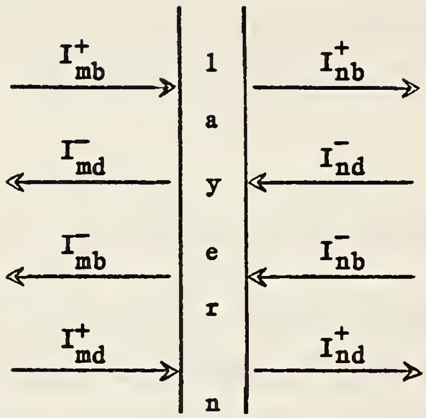
By assuming that the scattered radiation is perfectly diffuse, the diffuse performance values and relationships of a layer can also be used:

$$t_{nd}^{+} + r_{nd}^{+} + a_{nd}^{+} = 1.0 \qquad (20a)$$

$$t_{nd}^{-} + r_{nd}^{-} + a_{nd}^{-} = 1.0 \qquad (20b)$$

This model applies to other features besides draperies, such as a rough surfaced pane of glass intended specifically to diffuse the transmitted radiation. In most cases it will be necessary to determine the components of the optical performance in eqn (19) by measurements.

The optical relationships for a diffusing layer can be expressed in a manner similar to that developed in section 2.3 . Consider the sketch below.



m = n - 1

$$I^+_{nb} = I^+_{mb} * t^+_{nb} + I^-_{nb} * r^-_{nb} \qquad (21a)$$

$$I^-_{md} = I^-_{nb} * t^-_{ns} + I^+_{mb} * r^+_{ns} + I^-_{nd} * t^-_{nd} + I^+_{md} * r^+_{nd} \qquad (21b)$$

$$I^-_{mb} = I^-_{nb} * t^-_{nb} + I^+_{mb} * r^+_{nb} \qquad (21c)$$

$$I^+_{nd} = I^+_{mb} * t^+_{ns} + I^-_{nb} * r^-_{ns} + I^+_{md} * t^+_{nd} + I^-_{nd} * r^-_{nd} \qquad (21d)$$

This general layer model shows the values which must be measured to fully determine optical performance of a diffusing layer: the transmittance and reflectance for both beam and diffuse radiation for both directions of incidence. The general layer model also leads to a set of simultaneous equations which can be used to determine the optical performance of a system of layers:

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-r^+_{1b} & 0 & 1 & 0 & 0 & 0 & -t^-_{1b} & 0 & 0 & 0 & 0 & 0 \\
-r^+_{1s} & -r^+_{1d} & 0 & 1 & 0 & 0 & -t^-_{1s} & -t^-_{1d} & 0 & 0 & 0 & 0 \\
-t^+_{1b} & 0 & 0 & 0 & 1 & 0 & -r^-_{1b} & 0 & 0 & 0 & 0 & 0 \\
-t^+_{1s} & -t^+_{1d} & 0 & 0 & 0 & 1 & -r^-_{1s} & -r^-_{1d} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -r^+_{2b} & 0 & 1 & 0 & 0 & 0 & -t^-_{2b} & 0 \\
0 & 0 & 0 & 0 & -r^+_{2s} & -r^+_{2d} & 0 & 1 & 0 & 0 & -t^-_{2s} & -t^-_{2d} \\
0 & 0 & 0 & 0 & -t^+_{2b} & 0 & 0 & 0 & 1 & 0 & -r^-_{2b} & 0 \\
0 & 0 & 0 & 0 & -t^+_{2s} & -t^+_{2d} & 0 & 0 & 0 & 1 & -r^-_{2s} & -r^-_{2d} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
*
\begin{bmatrix}
I^+_{0b} \\ I^-_{0d} \\ I^-_{0b} \\ I^+_{0d} \\ I^+_{1b} \\ I^-_{1d} \\ I^-_{1b} \\ I^+_{1d} \\ I^+_{2b} \\ I^-_{2d} \\ I^-_{2b} \\ I^+_{2d}
\end{bmatrix}
=
\begin{bmatrix}
X_1 \\ Y_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ X_2 \\ Y_2
\end{bmatrix} \qquad (22)$$

The system optical performance values for beam radiation traveling to the right and to the left are obtained by setting X_1 and X_2, respectively to 1.0 and setting remaining elements of the right hand side of (22) to zero and solving the system of equations. If, in addition to the assumption of a perfectly diffusing layer, it can be assumed that the diffuse performance values are accurate for nearly diffuse radiation, then the system of equations (22) is accurate for a limited number of multilayer systems. These systems are g-s, s-g, s-g-s, and g-s-g where g represents a glass pane and s represents one or more scattering layers. This limitation occurs because pefectly diffuse radiation ceases to be perfectly diffuse after transmission through or reflection from a glass pane. The limitation on the number of glass panes can be overcome by treating a group of panes as a single pane when solving (22) and then determining the absorption values of the individual panes in the group. The system optical performance values for energy calculations must include the system inward scattering transmittance, T_s^+, in addition to the values for a simple window, T_b^+, A_{1b}^+, A_{2b}^+, etc.

One should avoid the temptation to compute performance for diffuse radiation by setting Y_1 or Y_2 to one, because of the diffuse radiation problem for glass panes, and should use integration of the beam radiation values as described in section 3.5 .

4.2 Slat Shading Devices

Another common window feature is the slat shading device, which may consist of vertical slats or of horizontal slats, either large (e.g. venetian blinds) or small (some types of sun screen). Figure 12 shows the general configuration and angles used to define a horizontal slat shading device. The regular geometry of such a device allows calculation of its optical performance. See Appendix A for an ideal (thin diffusely reflecting slats and no supporting structure) slat shading device. Some of the incident beam radiation is diffusely transmitted through and reflected from a slat shader so many of the ideas from the previous section are applicable. The critical new feature of a slat device is that its optical performance is dependent on the profile angle, Ω in fig 11, and not the angle of incidence, θ, or the azimuth angle, ϕ. At some profile angles there is no transmittance of beam radiation through the slats except as scattered radiation. The optical performance of the slat device can be described by functions (t_b^+, t_b^-, t_s^+, t_s^-, r_s^+, r_s^-) of the profile angle from −90° to +90°. (note: $r_b^+ = r_b^- = 0$) The difficulty comes when the slat device layer is combined with more conventional layers whose performance depend on the angle of incidence. The performance of such a system of layers is dependent on both Ω and ϕ.

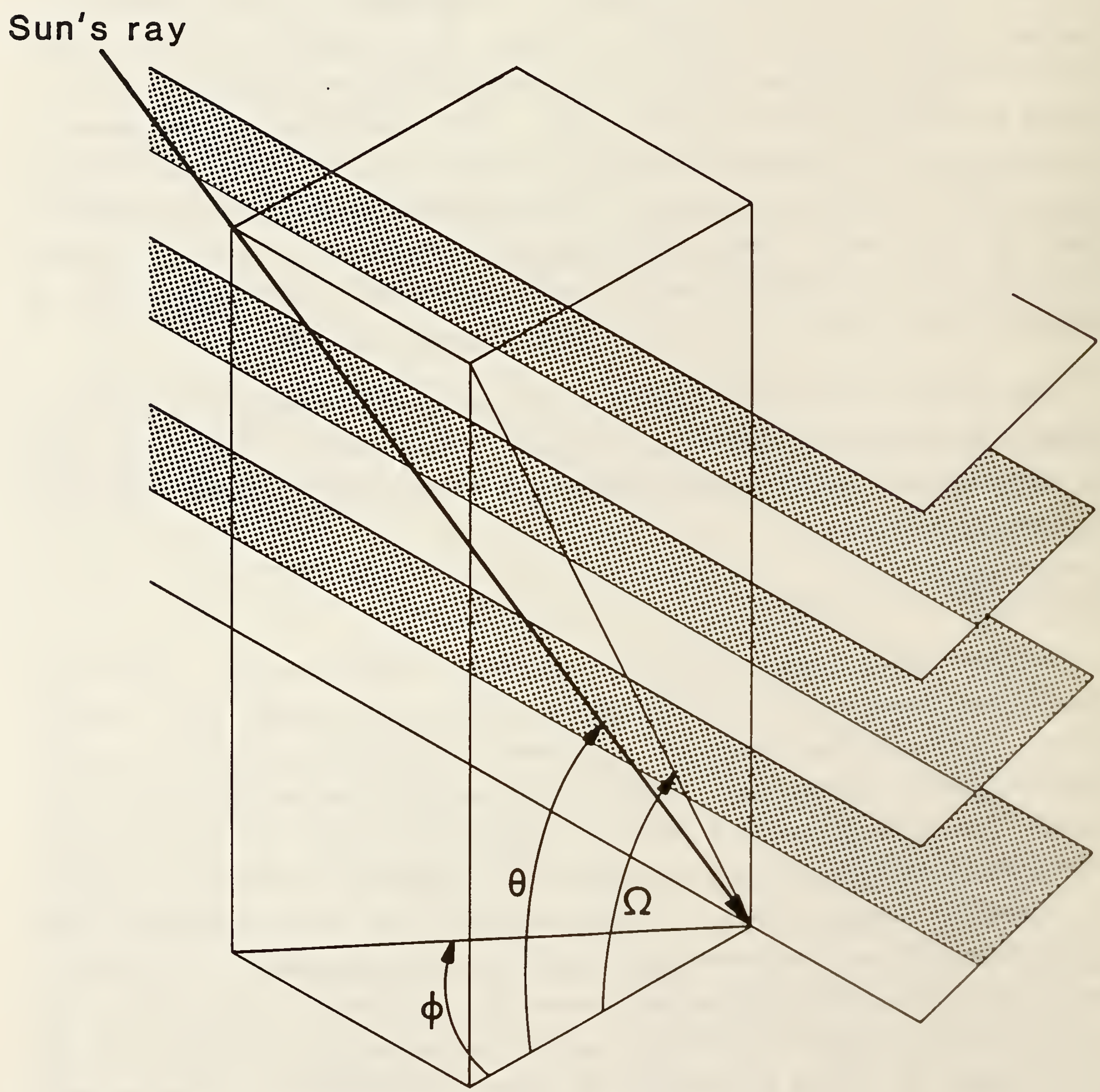


Figure 12. Horizontal Slat Shading Device

The most general method of fast calculation for energy analysis is a two-variable interpolation. This solution has a high storage cost. For example, establishing interpolation points at every 10^o in Ω from -80^o to $+80^o$ and every 10^o in ϕ from 0 to 80^o (values at 90^o can be assumed) requires a 17 by 9 array of values for each optical component, T_b^+, T_s^+, A_1^+, ..., needed. An even larger data structure would be required for a window with both horizontal and vertical slat layers. The calculation of hemispherical performance values must be based on numerical evaluation of eqn (13c).

4.3 Other Devices and Models

One of the important assumptions which has been made in developing the formulae for multilayer systems is that edge effects are unimportant. That is, the total thickness of the window is very small compared to its width and height. The error caused by this assumption has not been analyzed. Fortunately, transmittance is small at high angles of incidence where edge effects become important, and shadow casting features of the architecture often shade the window at high θ. Such edge effects play an important part in the performance of a common residential window type -- the colonial window consisting of many small panes separated by mullions. The performance characteristics of such a window should probably be determined by experimental measurements. The data structure developed for the window with a slat shading device can also handle such edge effects.

A more complicated form of the slat shading device has specularly reflecting surfaces on the upper sides of the slats. This has been proposed to reflect light upward to the ceiling for daylighting purposes. Simplified modeling would require additions to the data structure: both the fraction of beam light transmitted upward and the angle at which it is transmitted (if the light rays are being followed in detail).

An even more complicated window could be developed by the use of a holographic coating. This could allow incident radiation to be transmitted and scattered in arbirtary directions. A very general data structure could be required. One structure, continuing the description of radiation as beam and diffuse components, would give at every incidence direction (Ω, ϕ), the inward scattering transmittance, T_s^+, the inward beam transmittance, T_b^+, and the new direction (Ω_i, ϕ_i) of the beam. Even more general would be a data structure giving the portion of the incoming ray scatted in every direction (Ω_i, ϕ_i) for every incoming direction (Ω, ϕ). This is the 'bi-directional transmittance'. Values of Ω and o every 10^o would require an array of 17 by 17 by 17 by 17 (= 83521) elements, which is probably impractical for any building energy or daylighting analysis.

Another advanced window type would change its optical properties as a function of temperature or some other, perhaps electric, input. This should use the data structures developed above, except that the window would be described by multiple structures, one for each condition the window can have. The building analysis program would have to select which data structure to use depending on the conditions that control the window. This model also applies to some very conventional control mechanisms — human intervention such as changing the position of drapes and blinds.

5. SUMMARY AND CONCLUSIONS

This report has discussed the modeling of window optics for the purposes of simulating building energy requirements or daylighting availability. The theory for calculating the optical performance of conventional windows was reviewed. This included reflection and refraction at an air-to-glass interface, multiple reflections in a pane of glass, the net radiation method for computing the optical performance of a multipane window, and hemispheric performance. Special reflective films were not discussed.

The simplifications that might commonly be made in creating computational models were analyzed. Use of the ASHRAE shading coefficient procedure for transient analysis of buildings with windows which are significantly different for the single pane reference window is questionable. Windows with heat absorbing glass, reflective films, or shading slats are difficult to model with shading coefficients. Accurate optical analysis requires treating typical unpolarized radiation as consisting of equal parts of TE and TM polarized radiation. Failure to do this causes an error in transmittance which increases with the number of panes. The error does become less significant as more radiation is absorbed in the panes. It was shown that evaluating the optical performance of most common window glasses by using a wavelength averaging approximation instead of the detailed method is satisfactory. The least error occurs for glasses which do not have major wavelength dependencies, such as clear and most heat absorbing glasses. Glasses which do have major wavelength dependencies, such as those using thin reflective films, are likely to require detailed wavelength modeling.

Consideration of the radiation heat gains into the room indicates that the following optical performance values are needed for detailed window models: Transmittance into the room for beam and diffuse radiation (T_b^+ and T_d^+), absorptance of each pane (A_{nb}^+ and A_{nd}^+) for inward beam and diffuse radiation, and window transmittance and layer absorptances (T_d^- and A_{nd}^-) for outward diffuse radiation. Evaluation of daylighting performance requires the T_b^+ and T_d^+ values evaluated for the visible portion of the available solar radiation. In long term energy analysis simulations, it is desirable to quickly evaluate the performance for beam radiation as a function of angle of incidence. In most cases a polynomial expression is the preferred method because of its speed, however some windows may be better modeled by interpolation when performance changes considerably over small angles. The hemispheric performance (for diffuse radiation) can be quickly and accurately evaluated from the coefficients of the polynomial expression for beam performance. Other numeric integration methods may be used when angular performance is approximated by interpolation.

Some of the possibilites for more complex windows were analyzed, and the type of model and data that would be necessary to simulate such windows in a building energy analysis program were determined. The common drapery was modeled as a radiation scattering layer, and a theoretical method was presented for determining its optical performance in a multipane window. The window optical performance values for energy calculations must include the system inward scattering transmittance, T_s^+, in addition to the values for a simple window. The slat style shading device introduces a major complication because optical performance is dependent on two angles, profile and azimuth, so the data structure for quick calculation must reflect this fact. This is best handled by a two-variable interpolation. This methodology is applicable to other and more complex window types.

It was shown that the optics of different window types can be simulated with models which require varying amounts of memory or computing time. It is possible to develop a very general modeling method applicable to nearly all windows, but such a method would be inefficient in either computation time or data storage for the majority of window types. It is recommended that a building energy analysis program have all models available and use the most efficient for any given window.

Although theoretical approaches are possible for developing modeling data for most window configurations, the data should also be obtainable from experimental measurements. The measurements could also indicate the validity of some of the assumptions that must be made to obtain theoretical models. The applicability of available measurement techniques should be studied. There is also a need for a sensitivity analysis to determine the impact of different optical simulation methods on the computed loads and daylighting availability in buildings. This can be done for typical buildings and window types, but there is a limit to the usefulness of such analysis because simulation has its greatest benefit in the atypical cases. Similarly, the detailed analysis program will be used in developing simpler design tools capable of handling common window types, but the detailed program is necessary for predicting the performance of unusual windows.

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APPENDIX A: SIMPLE SLAT SHADING DEVICE MODEL

The following model describes an idealized slat shading device which is composed of thin flat slats which are T wide, with spacing S between slats, and with a tilt angle β from the vertical as shown in figure A1. Fig A1 shows three angles, α_1, α_2, and α_3, which define four regions which must be considered separately.

Region 1 ($0 < \alpha < \alpha_1$ as in fig A2): There is no direct transmission through the shade. The lower slat is partially irradiated.

Region 2 ($\alpha_1 < \alpha < \alpha_2$ as in fig A3): There is direct transmission through the shade. The lower slat is partially irradiated.

Region 3 ($\alpha_2 < \alpha < \alpha_3$ as in fig A4): There is direct transmission through the shade. The bottom side of the upper slat is partially irradiated.

Region 4 ($\alpha_3 < \alpha < \pi$ as in fig A5): There is no direct transmission through the shade. The bottom side of the upper slat is partially irradiated.

Consider only $\beta < \pi/2$, since other tilt angles can be considered by changing the origin for measuring angles. Then the lengths and angles in fig A1 are given by:

$$R = \sqrt{T^2 + S^2 - 2TS\cos\beta}$$

$$Q = \sqrt{T^2 + S^2 + 2TS\cos\beta}$$

$$\cos\alpha_1 = (R^2 + S^2 - T^2)/2RS$$

$$\cos\alpha_2 = -\cos\beta$$

$$\cos\alpha_3 = -(Q^2 + S^2 - T^2)/2QS$$

The non-zero view factors between the four surfaces (including the open ends: 1 and 4) are given by:

$$F_{12} = (S+T-R)/2S \qquad F_{13} = (S+T-Q)/2S \qquad F_{14} = (R+Q-2T)/2S$$

$$F_{21} = (S+T-R)/2T \qquad F_{23} = (R+Q-2S)/2T \qquad F_{24} = (S+T-Q)/2T$$

$$F_{31} = F_{24} \qquad F_{32} = F_{23} \qquad F_{34} = F_{21}$$

$$F_{41} = F_{14} \qquad F_{42} = F_{13} \qquad F_{43} = F_{12}$$

Figure A2 shows the condition for α in region 1. U is the length of the bottom slat which is in direct sun. The lengths in fig A2 are given by:

$$U = S\sin\alpha/\sin(\alpha+\beta)$$

$$V = S\sin\beta/\sin(\alpha+\beta)$$

$$W = T - U$$

$$X = \sqrt{S^2 + W^2 + 2SW\cos\beta}$$

The additional view factors are given by:

$F_{1a} = (S+U-V)/2S$ $\quad F_{1b} = (T+V-R-U)/2S$

$F_{3a} = (Q+V-S-X)/2T$ $\quad F_{3b} = (R+X-S-V)/2T$

$F_{4a} = (T+X-Q-W)/2S$ $\quad F_{4b} = (S+W-X)/2S$

$F_{a1} = (S+U-V)/2U$ $\quad F_{a3} = (Q+V-S-X)/2U$ $\quad F_{a4} = (T+X-Q-W)/2U$

$F_{b1} = (T+V-R-U)/2W$ $\quad F_{b3} = (R+X-S-V)/2W$ $\quad F_{b4} = (S+W-X)/2W$

Figure A3 shows the condition for α in region 2. X is the length of the open side which is in direct sun. X is given by:

$$X = T\sin(\alpha+\beta)/\sin\alpha$$

Figure A4 shows the condition for α in region 3 where X is given by:

$$X = -T\sin(\alpha+\beta)/\sin\alpha$$

Figure A5 shows the condition for α in region 4 where direct transmission is again blocked. U is the length of the top slat which is in direct sun. The lengths in fig A5 are given by:

$$U = -S\sin\alpha/\sin(\alpha+\beta)$$

$$V = U\sin\beta/\sin\alpha$$

$$W = T - U$$

$$X = \sqrt{S^2 + W^2 - 2SW\cos\beta}$$

The additional view factors are given by:

$F_{1a} = (S+U-V)/2S$ $\quad F_{1b} = (T+V-Q-U)/2S$

$F_{3a} = (R+V-S-X)/2T$ $\quad F_{3b} = (Q+X-S-V)/2T$

$F_{4a} = (T+X-R-W)/2S$ $\quad F_{4b} = (S+W-X)/2S$

$F_{a1} = (S+U-V)/2U$ $\quad F_{a3} = (R+V-S-X)/2U$ $\quad F_{a4} = (T+X-R-W)/2U$

$F_{b1} = (T+V-Q-U)/2W$ $\quad F_{b3} = (Q+X-S-V)/2W$ $\quad F_{b4} = (S+W-X)/2W$

In all four regions the angle of incidence of the ray on the slat (measured in the same plane as β and α) is given by:

$$\cos\theta = |\sin(\alpha+\beta)|$$

Calculation of the optical performance of the slat shading device can be accomplished by considering the parallelogram formed by surfaces 1 through 4 to be an enclosure and following the general enclosure theory presented by McCabe [A1]. In that theory all surfaces reflect diffusely, and any surface may be transparent. Since surface 1 and 4 of the slat shader are imaginary, they have have transmittances of 1 and reflectances of 0. The enclosure equations use the following additional quantities:

J = radiosity = flux of radiant energy leaving a surface diffusely,

G = diffuse irradiance,

G^o = diffuse irradiance on the outside of the surface,

H = beam irradiance,

ρ_d = reflectance for diffuse radiation,

ρ_b = reflectance for beam radiation, and

τ_d = transmittance for diffuse radiation.

Conservation of energy implies the following relationships:

$$J_j = \rho_{d,j} G_j + \rho_{b,j} H_j \qquad j \text{ opaque}$$

$$J_j = \rho_{d,j} G_j + \tau_{d,j} G_j^o \qquad j \text{ transparent}$$

and

$$G_j = \sum_k F_{j,k} J_k$$

Since J_j is a function of the J of all other surfaces in the enclosure, it must be computed by a set of simultaneous equations expressed in matrix for as:

$$\begin{bmatrix} 1-\rho_{d1}F_{11} & -\rho_{d1}F_{12} & -\rho_{d1}F_{13} & -\rho_{d1}F_{14} \\ -\rho_{d2}F_{21} & 1-\rho_{d2}F_{22} & -\rho_{d2}F_{23} & -\rho_{d2}F_{24} \\ -\rho_{d3}F_{31} & -\rho_{d3}F_{32} & 1-\rho_{d3}F_{33} & -\rho_{d3}F_{34} \\ -\rho_{d4}F_{41} & -\rho_{d4}F_{42} & -\rho_{d4}F_{43} & 1-\rho_{d4}F_{44} \end{bmatrix} * \begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{bmatrix} = \begin{bmatrix} \rho_{b1}H_1+\tau_{d1}G_1^o \\ \rho_{b2}H_2+\tau_{d2}G_2^o \\ \rho_{b3}H_3+\tau_{d3}G_3^o \\ \rho_{b4}H_4+\tau_{d4}G_4^o \end{bmatrix}$$

In this set of equations F_{jj} is always 0, and the optical performance of the shade is determined by setting different values for the right hand side (RHS) of the equations. The LHS matrix is diagonally dominant. When the incident ray is in regions 2 and 3, the direct transmittance is given by $t_b^+ = (S-X)/S$ and, of course, in regions 1 and 4 $t_b^+ = 0$. In addition, the set of simultaneous equations must be expanded to include subsurfaces a and b in these regions.

A critical aspect of the slat shader is that some of the beam radiation is scattered as it is reflected from the slats. This scattered radiation may be directly transmitted or reflected from the device, or it may be indirectly transmitted or reflected after further diffuse reflections between the slats. The scattering transmittance, τ_s^+, is obtained by setting the appropriate H component to $\cos\theta$ and all other components in the RHS to 0, computing the J_j, and then computing G_4, which is the scattering transmittance. The scattering reflectance is given by $\rho_s^+ = G_1$. The optical performance for radiation incident from the opposite side, $\tau_s^-(\alpha)$ and $\rho_s^-(\alpha)$, can be evaluated at the same time as $\tau_s^+(\pi-\alpha)$ and $\rho_s^+(\pi-\alpha)$ because of geometric similarity with the relative positions of surfaces 2 and 3 reversed.

The transmittance of the shader for diffuse radiation, τ_d^+, is obtained by setting $G_1{}^o$ to 1 and all other components in the RHS to 0, computing the J_j, and again computing G_4. The diffuse reflectance is given by G_1. The reverse diffuse transmittance, τ_d^-, is obtained by setting $G_4{}^o$ to 1 and solving for G_1.

Another major difference between the slat shader and the common pane of glass is that the optical performance of the shader is not symmetric as a function of incidence angle. This has a significant inpact on how the optical performance can be represented in a simple manner.

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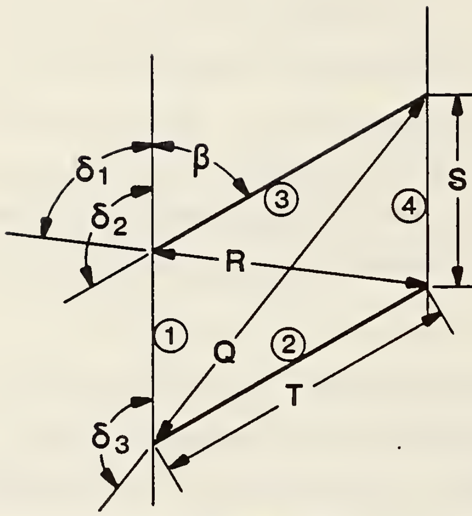

Figure A1

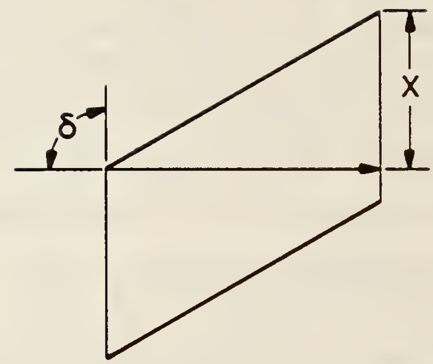

Figure A3

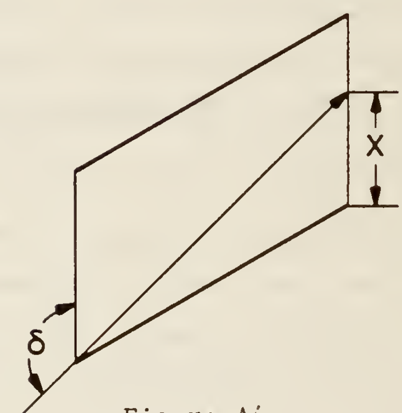

Figure A4

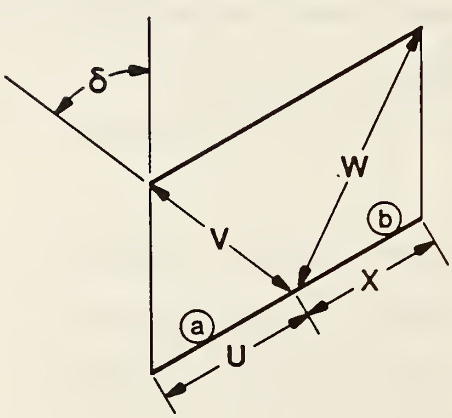

Figure A2

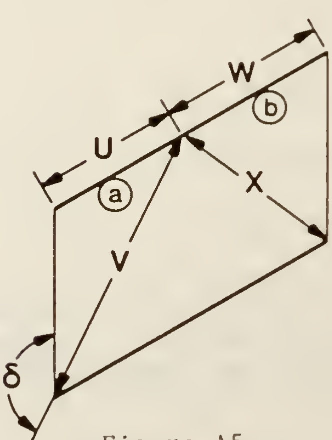

Figure A5

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10. SUPPLEMENTARY NOTES

☐ Document describes a computer program; SF-185, FIPS Software Summary, is attached.

11. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)*

This report discusses modeling the optics of windows for the purposes of simulating building energy requirements or daylighting availability. The theory for calculating the optical performance of conventional windows is reviewed. The simplifications that might commonly be made in creating computational models are analyzed. Some of the possibilities for more complex windows are analyzed, and the type of model and data that would be necessary to simulate such windows in a building energy analysis program are determined. It is shown that the optical performance of different window types can be simulated with models which require varying amounts of memory or computing time. It is recommended that a building energy analysis progam have all models available and use the most efficient for any given window.

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