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Implementation of Compressible Shoring Analysis for Multistory Concrete Construction

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U.S. DEPARTMENT OF COMMERCE, Malcolm Baldrige, ***Secretary***
NATIONAL BUREAU OF STANDARDS, Ernest Ambler, ***Director***

ABSTRACT

This report presents an analytical procedure for determining the loads on the shoring system and supporting slabs in multistory cast-in-place concrete construction. The procedure assumes that the slabs are supported by evenly distributed compressible shores and reshores and employs the stiffness method of analysis to solve for the loads on the shoring system and slabs as construction advances. The number of shores and reshores; shore, reshore and base support stiffnesses; casting rate; and concrete strength gaining characteristics are considered in the analysis. Details of the implementation of the shoring analysis in the form of a computer program are presented. The strategy for determining the next phase in the casting cycle is described and the details are given for formulating the stiffness equations and loads for each phase. Several example problems are presented to demonstrate the use of such a procedure in assisting to make critical decisions regarding planning of the casting schedule and determining when formwork can be safely removed.

KEY WORDS: Buildings; Computer models; Concrete construction; Concrete (reinforced); Formwork; Safety; Structural analysis

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1. INTRODUCTION

In cast-in-place concrete building construction, each freshly cast floor is supported by a system of shores and reshores which transfer the weight of the concrete, workers and equipment to the floors below. If the shoring system is not designed with sufficient strength and stability to carry the imposed loads, a collapse of the shoring system and, possibly the structure it supports, will result. Additionally, construction loads on the supporting slabs, imposed by the shoring system, may be greater than the service loads for which the slabs were designed. Since the supporting slabs have generally not reached their full design strength, their capacity may be exceeded resulting in failure.

In the course of casting a multistory concrete building, formwork is removed and shores and reshores are relocated as construction progresses. As a consequence, the structural load carrying system is constantly changing, making it difficult to assess the distribution of loads on the structure or shoring system at any given time. Decisions regarding the removal of forms and relocation of shores are often made without the benefit of an analysis. Premature removal of shores which lead to a punching shear failure in the slab was cited as a probable cause of the collapse of the Skyline Plaza apartment building in Fairfax, Virginia, in 1973 (1).

This report presents an analytical method which has been developed for determining the loads on the shoring system and supporting slabs as construction advances. The theory behind the method and comparisons with other analytical models and with field test data are covered in a separate report [1]. The procedure assumes that the slabs are supported by evenly distributed compressible shores or reshores. Forces on the slabs are computed by assuming that superimposed construction loads are distributed to the shoring system and interconnected floors in proportion to the relative stiffnesses of the slabs, shores, reshores and ground. This report focuses on the implementation of such an analysis for solution by computer.

1.1 BACKGROUND

The contractor is guided in the design of formwork by American Concrete Institute (ACI) Standard 347, Recommended Practice for Concrete Formwork (2). In the section addressing multistory structures, ACI Standard 347 states,

> "...No shoring should be removed until the concrete has gained sufficient strength to support the loads which will be transferred to the structure upon removal of such shoring."

While no procedure is specified for determining the loads that will be transferred to the structure, Formwork for Concrete (3), sponsored by ACI Committee 347, describes a procedure for analyz-

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1 Fattal, S. G., Gross, J. L. , and DeLorme, A. J., "An Analytical Model for the Design of Shoring Systems Used in Concrete Building Construction," NBS Building Science Series Report, in progress.

ing the shoring loads in multistory structures. This method, based on the work by Grundy and Kabaila (4), assumes that the shores are infinitely stiff relative to the slabs. Grundy and Kabaila point out, however, that wooden shores have sufficient compressibility to significantly modify the distribution of loads among the interconnected slabs. Including the compressibility of the shoring system tends to shift the slab loads to the uppermost interconnected floors as compared with an analysis assuming rigid shores. A complete finite element analysis, including compressible shores, for the investigation of the construction collapse of the Harbour Cay Condominium in Cocoa Beach, Florida (5) confirmed this effect.

The development of a simplified analytical procedure which accounts for the stiffness of shores and reshores was undertaken by the National Bureau of Standards (NBS). A computer program was developed, based on the stiffness method of analysis, which includes variable shore and reshore stiffness, nonrigid base support, and installation of reshores with precompression. This report describes, by means of logic flowcharts, the implementation of this analytical procedure on either a mainframe or desktop computer. The purpose is to make available to others a model on which to build and expand the concepts described herein.

1.2 SCOPE OF REPORT

Section 2 describes the sequence of steps (phases) in forming and shoring a typical floor of a multistory concrete building. The four phases of a casting cycle for each floor describe the structural configurations and loadings which must be considered in an analysis. In section 3, a model is presented for idealizing the structure and the fundamentals of a stiffness analysis of the interconnected floors are described. The details for implementing the stiffness analysis are given in Section 4. Here, the overall strategy, or program flow, is described and details are given for formulating the stiffness equations for each phase of casting. In addition, the formation of the loading condition for each phase is presented and the calculation of slab and shore forces is outlined. In Section 5, several example problems are solved to illustrate the application of such an analytical procedure to assist in making critical decisions regarding planning of the casting schedule and determining when formwork can be safely removed.

2. MULTISTORY CAST-IN-PLACE CONCRETE CONSTRUCTION

In multistory cast-in-place concrete construction, each freshly cast floor slab is supported by a system of shores and reshores which transfer the loads to the previously cast slabs below. The loads are distributed to the interconnected slabs in proportion to the relative stiffness of the slabs and the formwork system. This section describes the sequence of steps in forming and shoring each floor of a multistory concrete building and discusses the importance of accounting for the compressibility of the shoring system in an analysis.

2.1 CONSTRUCTION PHASES

A typical casting cycle for a scheme which uses both shores and reshores consists of the following four phases:

1) Install a story of shores and forms and cast the floor slab above
2) Remove the reshores from the lowest reshored story
3) Remove the forms and shores from the lowest shored story
4) Place reshores in the story in which the forms and shores were removed

It should be noted that ACI Standard 347 (2) suggests that reshores should be installed "snugly" under the slab just stripped so that the reshores are relatively load free upon installation. If deflection is to be controlled, however, the reshores may be installed with a precompression such that the total load due to precompression approaches that on the shores prior to their removal. The precompression of reshores is accounted for in the analysis described here by specifying a "precompression ratio", P, where $0 < P < 1$. The load in the reshores upon installation is computed as the load in the shores prior to their removal times this precompression ratio. By using a value of P equal to zero in the analysis, the installed reshores will be load free.

The four phases of a casting cycle noted above may be illustrated using the multistory building shown in Figures 2.1 through 2.4 . In these illustrations the fifth floor slab of a multistory building is cast and the formwork is advanced in preparation for casting the sixth floor. The floors are numbered from the bottom of the building to the top, beginning with the first floor above ground level. A story has the same number as the floor slab above it (e.g., the reshores in the third story support the third floor slab). The structure has two levels of shores (denoted by XX) and three levels of reshores (denoted by ||). The lowest level of reshores is grounded (or supported at ground level).

Figure 2.1 shows the configuration when the fifth floor is being cast (Phase 1). The weight of the fresh concrete is distributed to the interconnected floors and the foundation through the formwork system. The stiffness of the floor slab is taken to be zero since the concrete has no strength initially.

Phase 2, removal of the reshores from the first story, is shown in Figure 2.2. The removal of the first story reshores is equivalent to applying a load to the first floor slab, in a downward direction, which is equal to the compressive load in the reshores prior to their removal. Thus, the load applied to the lowest interconnected floor slab is that load acting on the reshores at the end of Phase 1. In this particular example, the load previously carried by reshores in the first story, is distributed to the remaining five interconnected floor slabs.

In Figure 2.3, the shores in the fourth story are removed (Phase 3). The load in the shores prior to their removal is applied to the two interconnected floors above the vacated story and the three interconnected floors below it. The direction of the load is down on the slab above the vacated story (removal of support) and up on the slab below the vacated story (removal of load). In general, two separate stiffness analyses are required for Phase 3.

Finally, Phase 4 is illustrated in Figure 2.4. Here the fourth story is reshored with a precompression load equal to the load in the shores prior to their removal in Phase 3 times the specified precompression ratio. As in Phase 3, two separate analyses are required to determine construction loads on the interconnected floors above and below the reshored story.

2.2 COMPRESSIBLE SHORING SYSTEM

The ACI 347 Standard tacitly endorses a shoring analysis which assumes both rigid shores and reshores and rigid base support (see Reference 2). The implications of these assumptions need to be clarified. If a rigid shoring system is used, the loads are distributed to the interconnected floors in proportion to the relative slab stiffnesses. ACI (Reference 2) recommends that all slabs have the same stiffness; that is, no account is taken of the lower stiffness of early-age concrete. If the system of shores and reshores is compressible rather than rigid, a greater portion of the loads resulting from casting a new floor will be distributed to the uppermost interconnected floors. These floors, being the most recently cast, may not have gained sufficient strength to carry the loads without distress or possible failure. It is, therefore, important to account for the effects of compressible shoring and reshoring on the distribution of construction loads in multistory cast-in-place concrete construction.

The flexural stiffness of an uncracked section may be taken to be directly proportional to the modulus of elasticity of concrete, E_c, which increases with age. Since the age of the concrete differs throughout the structure, a more realistic approach would be to distribute the loads between the slabs in proportion to their flexural stiffnesses based on E_c. With a variable modulus approach the lower floors, being older and therefore stiffer, carry a greater proportion of the load than would result from a constant modulus approach.

It should be noted that assuming a constant E_c and rigid shoring system does not produce the same effect as using a variable E_c and including the compressibility of the shoring system. Several points supporting this argument follow. First, the modulus of elasticity of concrete increases rather rapidly as compared with compressive strength and including the effects of a variable E_c does not significantly modify the load distribution except for very short casting cycles (4). Secondly, the construction loads which a slab experiences may be in excess of service loads, and since a slab will likely be cracked under such loads, the proportionality of flexural stiffness with respect to E_c is doubtful. Blakey and Beresford (6) have reported that the flexural stiffness of a cracked reinforced concrete section is mainly dependent on the percentage of reinforcement and not on E_c. It could, therefore, be concluded that slab stiffnesses are in fact relatively independent of the concrete modulus and that the assumption of a constant modulus cannot be counted on to offset the effects of compressible shoring.

An exact analysis of a multistory concrete building, with material properties that vary throughout the structure and with an elaborate system of discrete shores and reshores, each with an unknown preload, is a formidable task. Certain simplifications must be introduced to render the problem tractable. The simplified procedure described herein provides a working framework for expansion to more precise analyses.

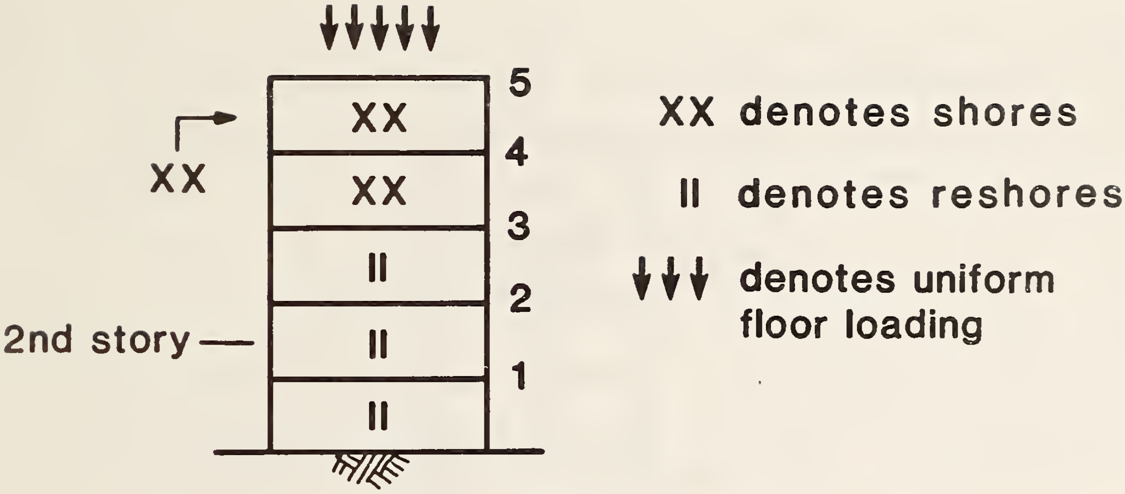

Figure 2.1 - Phase 1: Install Forms and Shores and Cast the Fifth Floor

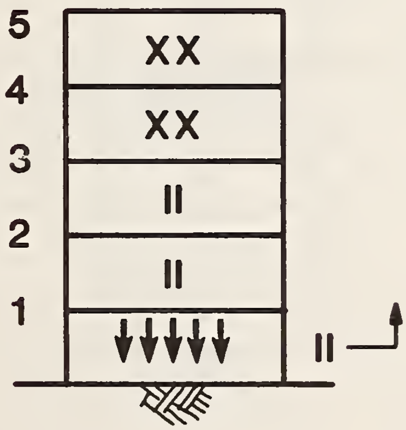

Figure 2.2 - Phase 2: Remove Reshores from the First Story

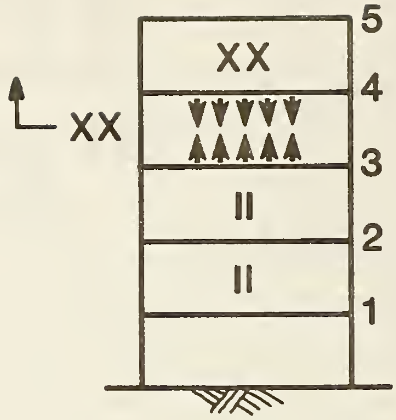

Figure 2.3 - Phase 3: Remove Shores from the Fourth Story

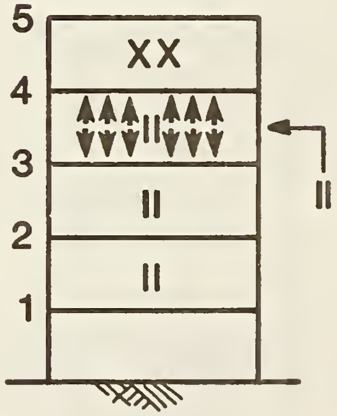

Figure 2.4 - Phase 4: Reshore the Fourth Story with
Precompression

3. COMPRESSIBLE SHORING ANALYSIS

This section describes a model for representing the dominant effects of a compressible shoring system while maintaining simplicity. To determine the load distribution, a stiffness analysis is employed which accounts for the slab stiffness at early-age, the stiffness of the ground which supports the first level of shores, and the stiffness of the shores and reshores. (The term "ground" is used here to mean "ground support system" which could be the soil or a slab on-grade.) The structure idealization is described first and then the details of the analysis including formulation of the stiffness equations are presented.

3.1 STRUCTURE IDEALIZATION

For analysis purposes, a multistory concrete structure under construction may be modeled as described below. The columns of the building are assumed to be axially and flexurally rigid. In addition, slab continuity between adjacent bays is ignored. These two assumptions reduces a multibay building to a series of single bay structures as shown in Figures 2.1 through 2.4. Each of the interconnected floors (floors connected by either shores or reshores) in the structure is represented by a single displacement degree of freedom, u_i. The stiffness of a given floor, k_n, is computed from the geometry of the slab and the elastic modulus of the early-age concrete if desired. The slab geometry comprises span length in two directions, slab thickness, and boundary conditions which reflect the edge fixities at adjacent bays or at the building perimeter. The stiffnesses of the shores, reshores, and ground are input to the analysis and may be different for each bay in the structure. Shores and reshores are assumed to be uniformly distributed and their stiffnesses, as well as the ground stiffness, are expressed in terms of load per unit area / unit deflection (e.g., psf/ft). A new loading condition is introduced to the system of interconnected floors when a floor is cast (Phase 1). The remaining operations (Phase 2 through Phase 4) produce a redistribution of forces within the system. Depending on the construction phase, the load applied to the structural model is the weight of the fresh concrete, the load carried by the level of shores or reshores to be removed, or the precompression in the installed reshores.

A schematic representation of the five story, single bay structure illustrated previously and the associated analytical model for Phase 1 of the construction cycle, are shown in Figures 3.1a and 3.1b, respectively. The floor slab stiffness is given by k_n where n is the floor number (numbered from bottom to top). Each floor slab may have a different stiffness due to its thickness or the strength gaining characteristics of concrete with time and temperature. The stiffness of the shores, reshores, and ground are given by k_s, k_r, and k_g, respectively. The loading for the configuration shown in Figure 3.1b represents the weight/unit area of the fresh concrete on the top floor.

3.2 STIFFNESS ANALYSIS

In general, one stiffness equation is written for each of the interconnected floors. The terms making up each stiffness equation depend on the combination of shores and reshores which support, or are supported by, the slab in question. If the shoring system is supported at ground level, an additional equation is included to account for the stiffness of the ground level support and the shores or reshores in the first story. The stiffness equations are numbered from the top of the interconnected floors down and are indicated by i, i = 1 to Q, where Q is equal to the number of interconnected floors. Each floor is directly connected, by either shores or reshores, to the floors immediately above it and below it. Thus, the i-th stiffness equation will, in general, have terms multiplying displacements u_{i-1}, u_i, and u_{i+1}. The concentrated force/unit area corresponding to the i-th equation is denoted by w_i.

A total of nine different configurations (cases) can be identified depending on the combination of shores and/or reshores which contribute to the stiffness of the particular floor or ground level. For example, a floor which supports a level of shores and which is itself supported by a level of reshores is identified here as Case 4. The nine cases and their associated stiffness equations are as follows:

Case 1 - Slab and shores in story below:

$$(k_s + k_n)\, u_i - k_s\, u_{i+1} = w_i \tag{3.1}$$

Case 2 - Slab and reshores in story below:

$$(k_r + k_n)\, u_i - k_r\, u_{i+1} = w_i \tag{3.2}$$

Case 3 - Slab and shores in stories above and below:

$$-k_s\, u_{i-1} + (2k_s + k_n)\, u_i - k_s\, u_{i+1} = w_i \tag{3.3}$$

Case 4 - Slab, shores in story above, and reshores in story below:

$$-k_s\, u_{i-1} + (k_s + k_r + k_n)\, u_i - k_r\, u_{i+1} = w_i \tag{3.4}$$

Case 5 - Slab and reshores in stories above and below:

$$-k_r\, u_{i-1} + (2k_r + k_n)\, u_i - k_r\, u_{i+1} = w_i \tag{3.5}$$

Case 6 - Slab and shores in story above:

$$-k_s\, u_{i-1} + (k_s + k_n)\, u_i = w_i \tag{3.6}$$

Case 7 - Slab and reshores in story above:

$$-k_r\ u_{i-1} + (k_r + k_n)\ u_i = w_i \tag{3.7}$$

Case 8 - Ground and shores in story above:

$$-k_s\ u_{i-1} + (k_s + k_g)\ u_i = w_i \tag{3.8}$$

Case 9 - Ground and reshores in story above:

$$-k_r\ u_{i-1} + (k_r + k_g)\ u_i = w_i \tag{3.9}$$

The relationship between the stiffness equation number, i, and the floor number, n, depends on the particular phase of the construction cycle and is defined in the next section. In addition, the specific form of the load vector, w_i, is given in Section 4.4 for each of the four phases.

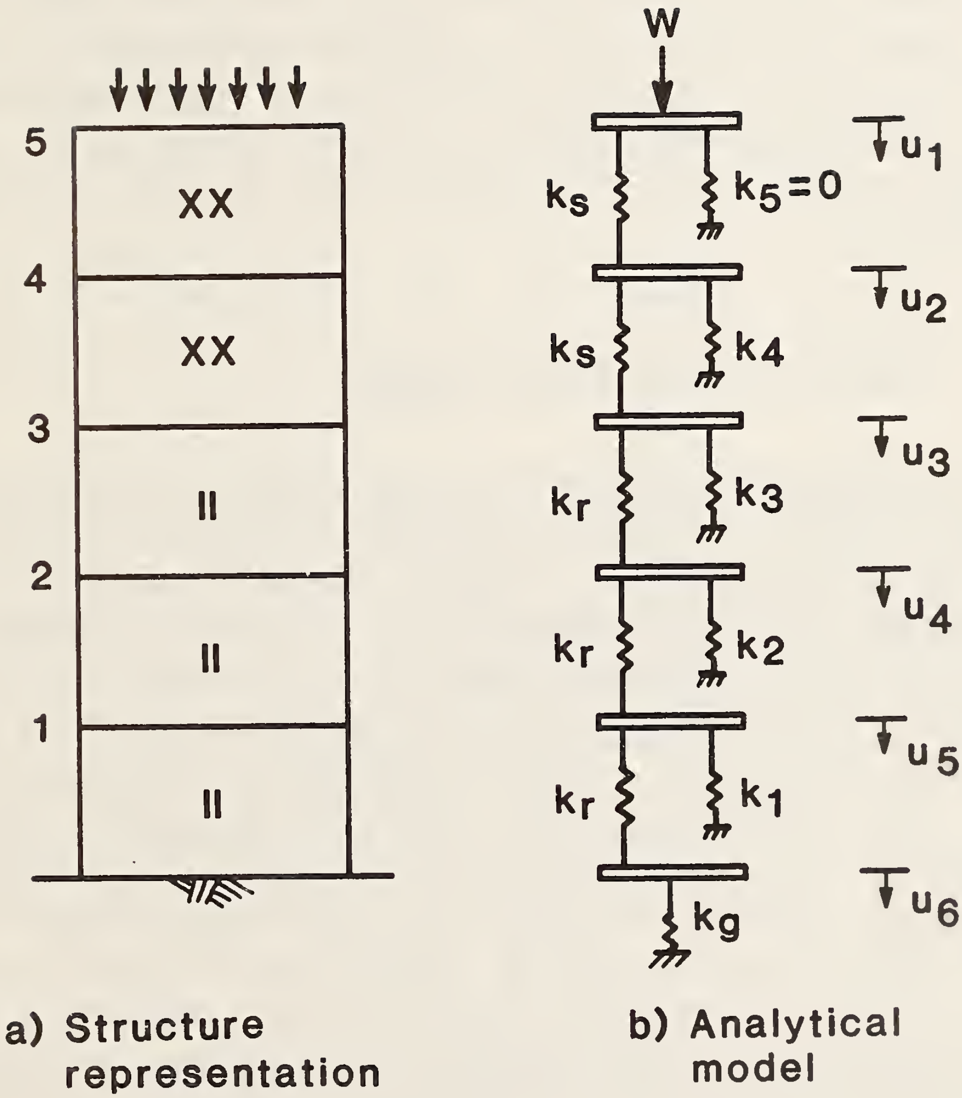

Figure 3.1 - Structure Idealization

4.0 IMPLEMENTATION OF SHORING ANALYSIS

This section, which discusses in detail the implementation of a compressible shoring analysis, is divided into several parts. First, a description of the overall flow of the program is presented. Here the logic is given for determining the next applicable phase of a casting cycle. Next, the details are given for computing the structural stiffness equations for the interconnected floors for each of the four construction phases. Then, the loading for each phase is covered. Finally, the computation of slab and shoring system forces from the known displacements is described.

4.1 DETERMINING CONSTRUCTION PHASE

The approach taken here to solve the problem of determining construction loads at all stages in the forming and shoring process for multistory cast-in-place construction, may be described as follows. Essentially, the analysis program "builds" the structure beginning with casting of the first floor and ending with casting of the top floor or roof. After each floor is cast, that is the load of the fresh concrete is distributed to the interconnected floors and shoring system, the program determines the next construction phase to be completed. The shores or reshores are removed or installed as applicable and a new distribution of forces within the structure is computed. This process is repeated until all four construction phases, as applicable, have been executed. The program then advances the shoring to the next floor level and applies the load of the fresh concrete. The above procedure is repeated until all of the floors have been cast. The loads resulting from the completion of any phase are added to those already existing on the structure and shoring system. Results are reported after each phase is completed and include the current structural configuration and updated loads on the floor slabs and shoring system.

The structure and shoring scheme are defined by the total number of stories in the completed structure, N_t, the number of shored stories, N_s, and number of reshored stories, N_r. Note that N_s and N_r describe the maximum number of shored and reshored stories and not the number of shored or reshored stories at any given time; thus, N_s and N_r do not change. N is used to indicate any floor level, while the floor most recently cast is designated by N_c (N_c = 1 to N_t). Two additional variables are necessary to define the configuration of the constructed building at any given time; the lowest shored story, L_s, and lowest reshored story, L_r. For example, consider the structural configuration shown in Figure 2.3 which illustrates the removal of the lowest level of shores, those in the fourth story, after casting the fifth floor. After completion of this phase, the structural configuration is defined by $N_c = 5$, $L_s = 5$, and $L_r = 2$.

A flowchart for determining the next phase of construction, in terms of the variables introduced above, is shown in Figure 4.1. The flowchart symbols conform to ANSI Standard X3.5 [7]. Within a decision symbol, the two quantities to be compared are separated by a colon (:). Alternative paths are indicated by labeled flowlines. For example, the first decision block in Figure 4.1 compares the variable L_s with zero (L_s:0). The logic of the program requires two alternative paths; one for "L_s equals zero", and the other for "L_s is not equal to zero". The two paths are indicated by flowlines labeled = and ≠, respectively. The direction of flow is indicated by arrowheads on all flowlines for clarity. The striped process symbols (those for Phase 1 through Phase 4) indicate that a more detailed representation is to be found in flowcharts to follow. In the more detailed flow-charts, an identification number is placed in the upper right corner of some of the symbols and is used for reference in the text. The flowchart in Figure 4.1 is straight-forward and no reference to specific blocks is required. A detailed description of its logic follows.

Phase 1 consists of installing the formwork and shoring for the next floor slab to be cast and then placing the concrete. In this step, the load of the freshly placed concrete is distributed to the interconnected floors through the formwork system. This phase is the first phase of each casting cycle and is always carried out. Before the first floor is cast the counter for the lowest shored story is set equal to one (L_s=1). This counter is then incremented each time the lowest level of shores is removed (see Phase 3). If the top floor of the building is cast (N_c=N_t), the program terminates.

Phase 2 consists of removing the reshores from the lowest reshored level. This step is omitted if there are no reshores in the structure (N_r=0), or if all the reshores are not yet in place (N_c<N_s+N_r). Otherwise, the lowest level of reshores is removed and the load carried by the reshores prior to their removal is distributed to the interconnected slabs. At the completion of this step, the counter for the lowest level of reshores is incremented by one (L_r=L_r+1).

Phase 3 consists of removing the lowest level of shores. It is carried out only if all shores are in place (N_c>N_s). The loading consists of the shore force prior to removal. This load is distributed to the interconnected floors in proportion to the relative stiffnesses of slabs, shores and reshores. When Phase 3 is completed, the counter for the lowest level of shores is incremented by one (L_s=L_s+1).

Phase 4 consists of installing reshores in the story which was vacated by the removal of shores (Phase 3). It will be executed provided that reshoring is specified (N_r>0). This step will have no structural consequence if the precompression is zero except for the addition of the reshores as structural elements for subsequent calculations. If, however, the precompression is greater than zero, a force distribution will result from this step. When this phase is executed for the first time (L_r=0), the counter for the lowest reshored level is set equal to one (L_r=1). This counter is incremented when the lowest level of reshores is removed (see Phase 2).

It should be noted that the values of the counters for the lowest shored level (L_s) and lowest reshored level (L_r) may not always reflect the true conditions prior to the completion of a casting cycle. For example, if there is only one reshored story (level n) and that level of reshores is removed in Phase 2, the counter will indicate that the lowest reshored story is n+1. This will not be correct until the vacated story is reshored in Phase 4. At the completion of any casting cycle, however, the counters will reflect the actual configuration.

4.2 STIFFNESS EQUATION FORMULATION

Once it has been established which phase in a casting cycle is to be executed, the forces in the slab and shoring system for that phase are computed. This is accomplished by forming the structural stiffness equations for the interconnected floors in terms of the unknown floor displacements, and by solving for the unknown displacements and back-substituting to find the forces. In this section, the logic is presented for forming the stiffness equations for each of the four phases by reference to the nine cases defined in Section 3. In addition, the special cases in which the slab and formwork forces are determined directly from equilibrium are identified. The solution of the stiffness equations for the displacement degrees of freedom is straight-forward and is not covered here. The formation of the loads for each phase is covered in Section 4.3 followed by a discussion of the computation of slab and shoring forces in Section 4.4.

For Phases 1 and 2, there is no interruption in support provided by shores and reshores from the top floor to the lowest level of shoring (refer to Figures 2.1 and 2.2) and, consequently, there is only one set of equations to solve. However, for Phase 3, depending on the combination of shores and reshores, two sets of equations may be required. One corresponds to the interconnected floors above the story where the shores are being removed, and the other to the floors below (refer to Figure 2.3). Likewise, Phase 4 may require solution for the levels above and below the reshored story (refer to figure 2.4).

4.2.1 Phase 1 - Casting of the Next Floor

Eight of the nine cases defined in Section 3 are applicable to Phase 1. Selection of the appropriate case depends on the combination of shores and/or reshores which connect to the particular floor or ground level. To illustrate all eight cases, four separate configurations must be considered: 1) shores are grounded, 2) shores are not grounded and there are no reshores, 3) reshores are grounded, and 4) reshores are not grounded. These four configurations are shown in Figures 4.2a through 4.2d. In addition, the eight cases (Case 1 and Cases 3 through 9) are indicated next to the corresponding floor or ground level. For example, Case 6 (see Figure 4.2b) refers to the situation where the lowest interconnected slab supports a level of shores. Selection of Cases 6, 7, 8, or 9 depends on the determination of which of the above four configurations is applicable.

To form the stiffness equations for the interconnected floors, the following steps must be performed:

1) Compute the number of equations (interconnected floors)
2) For each equation, select the appropriate case based on the configuration
3) Compute the floor number for determining the slab stiffness

The flowchart for forming the stiffness equations for Phase 1 is shown in Figure 4.3.

In Blocks 1.A through 1.C, the number of stiffness equations, Q, is computed. Block 1.D begins a "do loop" which is performed for each equation. Within this loop, the stiffness equations are formed by branching to the applicable case. To compute the stiffness of a particular slab the floor number must be known. The calculation of the floor number in terms of the equation number is given in Block 1.E. The remaining decision blocks determine which of the eight cases is applicable. Blocks 1.F and 1.G determine if the floor in question is the top floor (i=1), the lowest interconnected floor or ground (i=Q), or an intermediate floor (1<i<Q). Case 1 applies for the top floor. Block 1.H determines, for an intermediate floor, whether there are shores or reshores in the stories above and below it (Cases 3, 4, or 5). Block 1.I determines whether the shoring system is grounded or not. If not, Block 1.J determines whether there are shores or reshores supported by the lowest floor (Cases 6 and 7, respectively). If the shoring system is grounded, Block 1.K determines whether the first story contains shores or reshores (Case 8 or Case 9).

4.2.2 Phase 2 - Removal of the Lowest Reshores

Six different cases apply to Phase 2. Two separate configurations must be considered to illustrate all six: 1) there is more than one level of reshores, and 2) there is a single level of reshores. (Recall that Phase 2 will be skipped if there are no reshores or if not all the reshores are in place.) These two configurations are shown in Figures 4.4a and 4.4b. As before, the six cases (Case 1 and Cases 3 through 7) are indicated next to the corresponding floor. The steps in forming the stiffness equations are similar to those described for Phase 1. A flowchart for Phase 2 is shown in Figure 4.5.

In Block 2.A, the number of stiffness equations is computed. Block 2.B begins the do loop on the number of equations. Again, the stiffness equations are formed within this loop by branching to the applicable case. The calculation of the floor number is shown in Block 2.C. Blocks 2.D and 2.E determine if the floor in question is the top, intermediate, or lowest floor. Case 1 is selected if it is the top floor. Block 2.F determines the combination of shores or reshores which support or are supported by the intermediate floor (Cases 3, 4, or 5). Block 2.G establishes whether there are one or more levels of reshores. If there is a single level, Case 6 is selected; if more than one, Case 7 applies.

4.2.3 Phase 3 - Removal of the Lowest Shores

Phase 3 is slightly more involved than either Phase 1 or Phase 2. First, it involves, in most cases, two separate solutions. Determination of slab and shore loads above the story in which the shores are removed is designated Phase 3a. The calculation of slab and reshore loads below the vacated story is termed Phase 3b. Depending on the shoring scheme, a stiffness analysis may not be required. If only one floor is affected by the removal of shores for either Phase 3a or 3b, the load on that floor slab may be determined by equilibrium. A flowchart for Phase 3 which establishes whether loads are determined by equilibrium or by solution of stiffness equations, and whether Phase 3b is executed is shown in Figure 4.6.

Block 3.A determines whether a single floor or several floors above the vacated story are affected by the shore removal. A stiffness analysis is required only if more than one level is affected. Details of the stiffness analysis for Phase 3a are given below. Block 3.B checks whether the lowest level of shores is the first story. If it is, Phase 3b is skipped. If the lowest shored story is above the first story, then it must be determined how many floors interconnected by reshores are affected. This is accomplished by Block 3.C. Again, if only one floor is affected, the slab force is determined by equilibrium and if two or more floors are affected, a stiffness analysis is required. Formation of the stiffness equations for Phase 3b is covered below.

Phase 3a has only three possible forms of stiffness equations. A schematic diagram showing the corresponding cases (Cases 1, 3, and 6) is shown in Figure 4.7. The flowchart for Phase 3a is straight-forward and is shown in Figure 4.8. Block 3a.A sets the number of equations. The loop on number of equations is initialized in Block 3a.B. Block 3a.C computes the floor number. Blocks 3a.D and 3a.E determine whether the equation is for Case 1, Case 3, or Case 6.

Phase 3b has four different cases associated with the possible configurations (Cases 2, 5, 7, and 9). Distinction between Cases 7 and 9 is based on whether the reshores are grounded or not. Figures 4.9a and 4.9b illustrate the four possible cases. The flowchart for Phase 3b (see Figure 4.10) is identical to that of Phase 3a except for Block 3b.F which distinguishes between Case 7 and Case 9 on the basis of the lowest reshored story.

4.2.4 Phase 4 - Reshoring of the Vacated Story

Formulation of the stiffness equations for Phase 4 is essentially the same as that for Phase 3. The only difference is that, at the completion of Phase 3, the counter L_s is incremented by one (see Figure 4.1) so all expressions in Phase 4 which contain L_s differ from those in Phase 3. This affects Block 4.B in Figure 4.11 and Block 4b.A in Figure 4.15. The remaining figures (Figures 4.12, 4.13, and 4.14) are the same as the corresponding figures for Phase 3 and are included for completeness.

4.3 LOADING CONDITIONS

As noted earlier, a new loading condition is introduced when a floor is cast and the remaining operations in a casting cycle cause a redistribution of forces within the system of interconnected floors. Note that only the dead load is considered here. The live load, which consists of workmen, equipment, etc., acts only on the floor being cast and is assumed to be removed before shores or reshores are removed. This section describes the formation of loads consistent with the stiffness equation numbering as outlined in the previous section, that is, each term w_i ($i=1,\ldots,Q$) is defined. Solution of the stiffness equations for the loading described here produces the displacements needed for the calculation of the slab and shoring system forces. The sign convention assumed for the loads and displacements (see Figure 3.1) is positive downward.

For Phase 1, the load which is imposed is the weight of the slab being cast. It is equal to the volume of concrete placed times the unit weight of the concrete divided by the floor area. This load is denoted here by W_1 (the subscript on W refers to the phase). The load for each degree of freedom is defined by

$$w_1 = W_1 \tag{4.1a}$$

and

$$w_2,\ldots,w_Q = 0 \tag{4.1b}$$

The load for Phase 2 is that on the reshores in the lowest story prior to their removal and is denoted W_2. This load acts downward on the lowest interconnected floor. The loading is

$$w_1,\ldots,w_{Q-1} = 0 \tag{4.2a}$$

and

$$w_Q = W_2 \tag{4.2b}$$

Phase 3 generally requires two stiffness analyses. The loading condition for both is that carried by the lowest level of shores before the shores are removed and is denoted by W_3. This load acts down on the floor above the story in which the shores are removed (Phase 3a) and up on the floor below this story (Phase 3b). Therefore, the loading for Phase 3a is

$$w_1, \ldots, w_{Q-1} = 0 \tag{4.3a}$$

and

$$w_Q = W_3 \tag{4.3b}$$

For Phase 3b, the loading is

$$w_1 = -W_3 \tag{4.3c}$$

and

$$w_2, \ldots, w_Q = 0 \tag{4.3d}$$

Note that Q is, in general, not the same for Phases 3a and 3b. It is computed for each phase as indicated in the applicable flowchart (Figure 4.8 or 4.10).

If the precompression ratio, P, is specified to be greater than zero, the result of the reshoring operation (Phase 4) will be a redistribution of forces within the structure. If P=0, this operation will clearly have no effect. The load applied to both the floors above and below the reshored story is equal to the load on the shores in that story prior to their removal times P, and is denoted by W_4 ($W_4 = P \times W_3$). This load acts up on the upper floors and down on the lower floors. Therefore, the loading for Phase 4a is

$$w_1, \ldots, w_{Q-1} = 0 \tag{4.4a}$$

and

$$w_Q = -W_4 \tag{4.4b}$$

For Phase 4b, the loading is

$$w_1 = W_4 \tag{4.4c}$$

and

$$w_2, \ldots, w_Q = 0 \tag{4.4d}$$

Again, Q is computed separately for Phase 4a and Phase 4b.

4.4 SLAB AND SHORE LOAD CALCULATION

The calculation of both slab and shore loads is straightforward. The only problem is in determining the appropriate floor level for the particular degree of freedom (or equation number). For all phases except 3b and 4b, the floor number is given by

$$N = N_c + 1 - i \tag{4.5}$$

in which i is the equation number and N_c is the floor being cast. For phases 3b and 4b, the floor number is determined by

$$N = N_c - N_s + 1 - i \tag{4.6}$$

in which N_s is the number of shored stories.

The slab load is the product of the slab stiffness and the slab displacement (recall that stiffness is expressed in terms of force per area / displacement). Displacements are determined by solution of the stiffness equations for the appropriate loading. The calculation is repeated for each of the Q interconnected floors. The load on the n-th floor slab, $F^f{}_n$, is

$$F^f{}_n = k_n\, u_i \qquad (i=1,\ldots,Q) \tag{4.7}$$

in which k_n is the stiffness of the n-th floor, u_i is the slab displacement for the i-th equation, and n is computed from either equation 4.5 or 4.6 as appropriate. A slab load is positive if it acts downward.

The shore or reshore load for a given story is computed in a similar fashion. Note that, if there are Q interconnected floors, there are Q-1 stories which contain shores or reshores. The displacement of a shore is the difference between the displacements of the floors it connects. Thus, the load in the shores in the n-th story, $F^s{}_n$, is given by

$$F^s{}_n = k_s\, (u_i - u_{i+1}) \qquad (i=1,\ldots,Q-1) \tag{4.8}$$

in which k_s is the shore stiffness. If the story in question contains reshores, the load in the reshores, $F^r{}_n$, is

$$F^r{}_n = k_r\, (u_i - u_{i+1}) \qquad (i=1,\ldots,Q-1) \tag{4.9}$$

Note that, as computed here, positive shore or reshore loads are compressive.

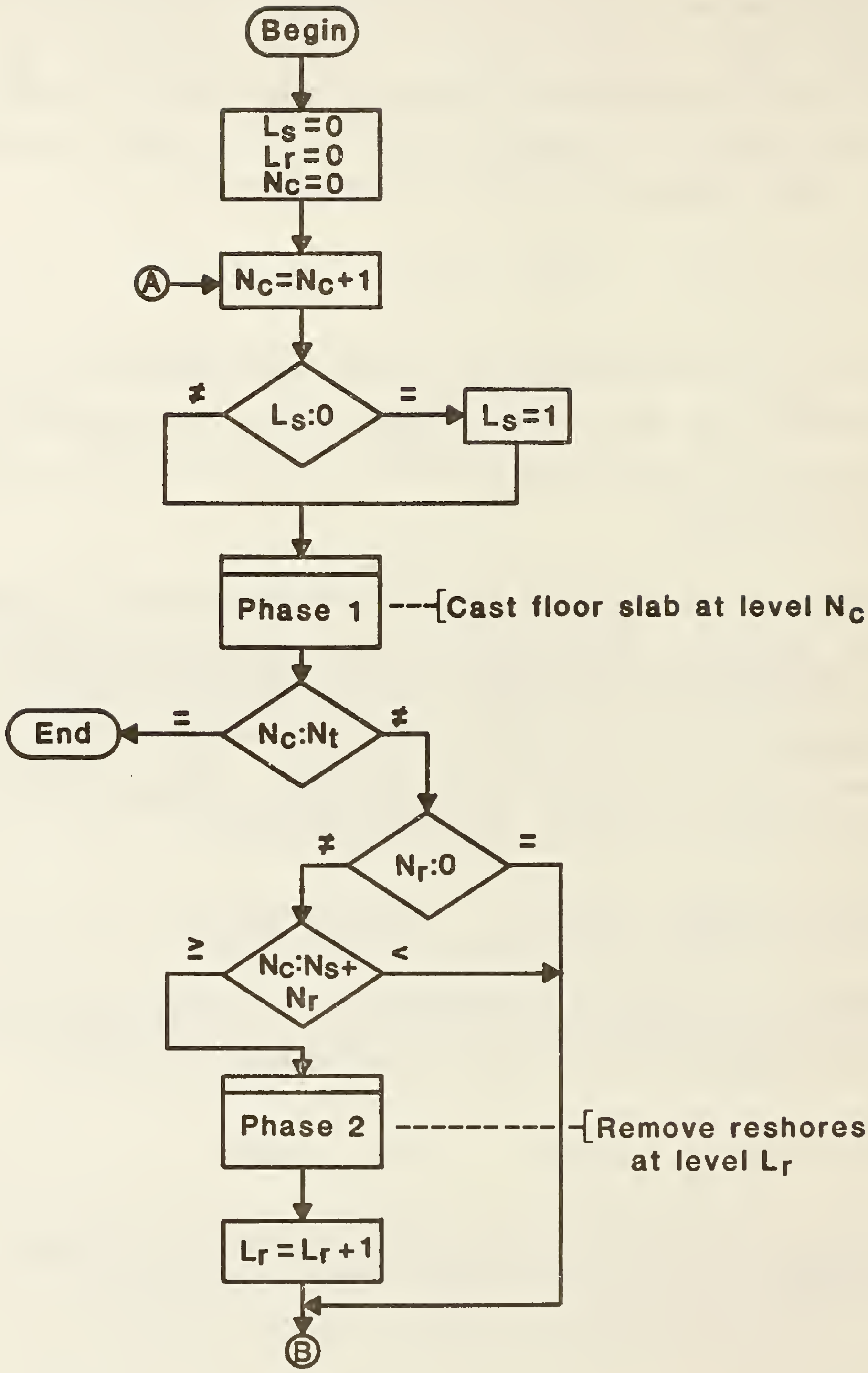

Figure 4.1 - Flowchart for Determining Next Phase

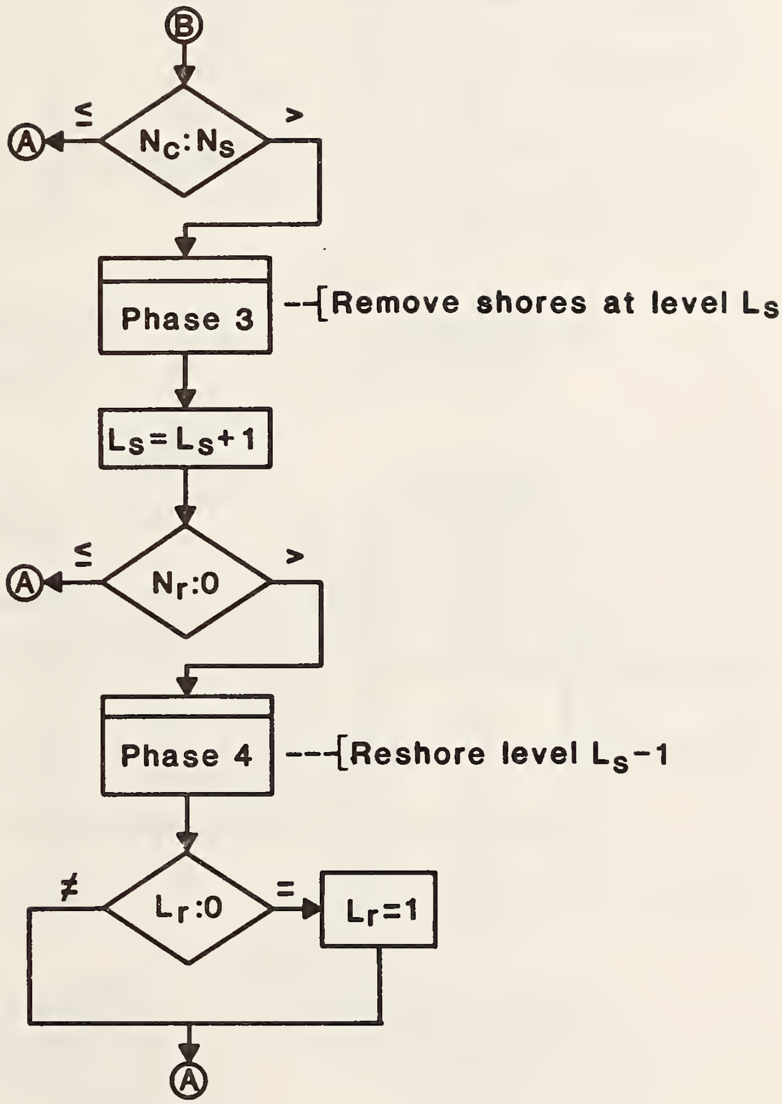

Figure 4.1 - Continued

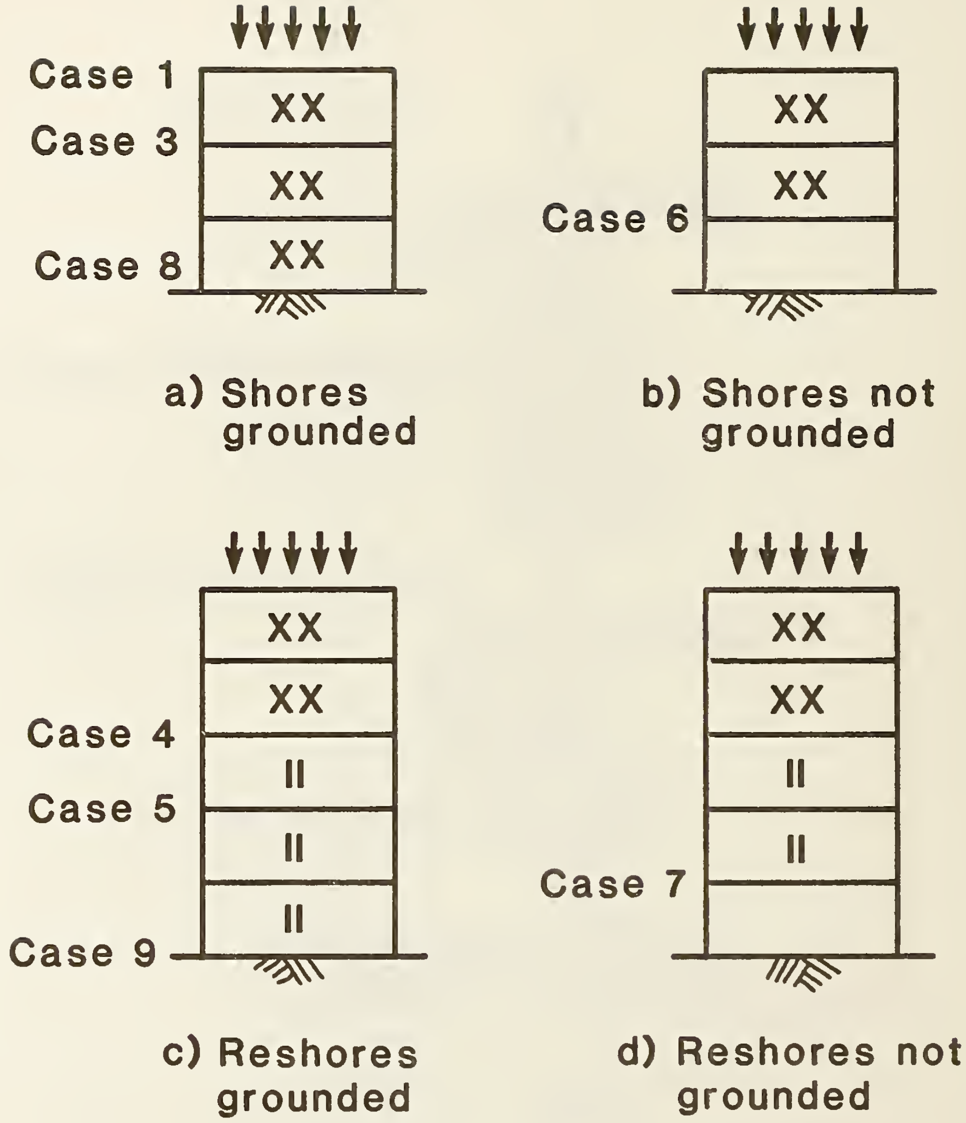

Figure 4.2 - Phase 1 Configurations

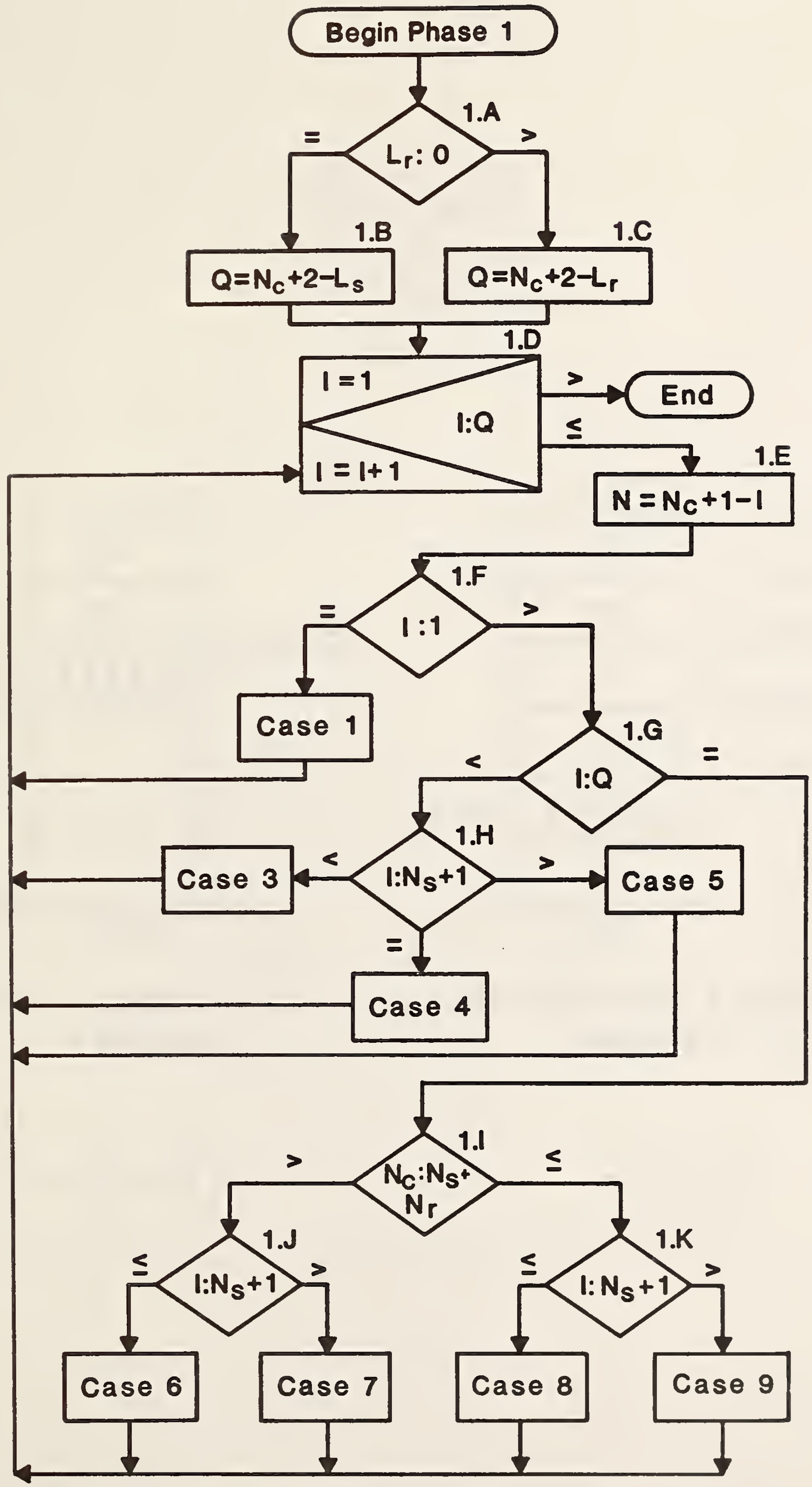

Figure 4.3 - Flowchart for Phase 1

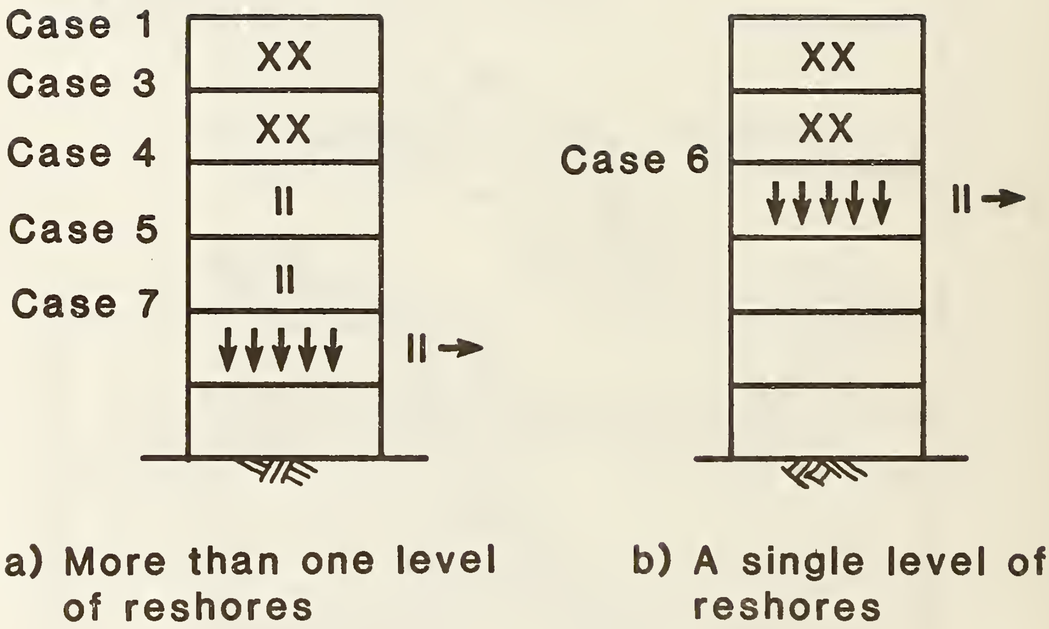

Figure 4.4 - Phase 2 Configurations

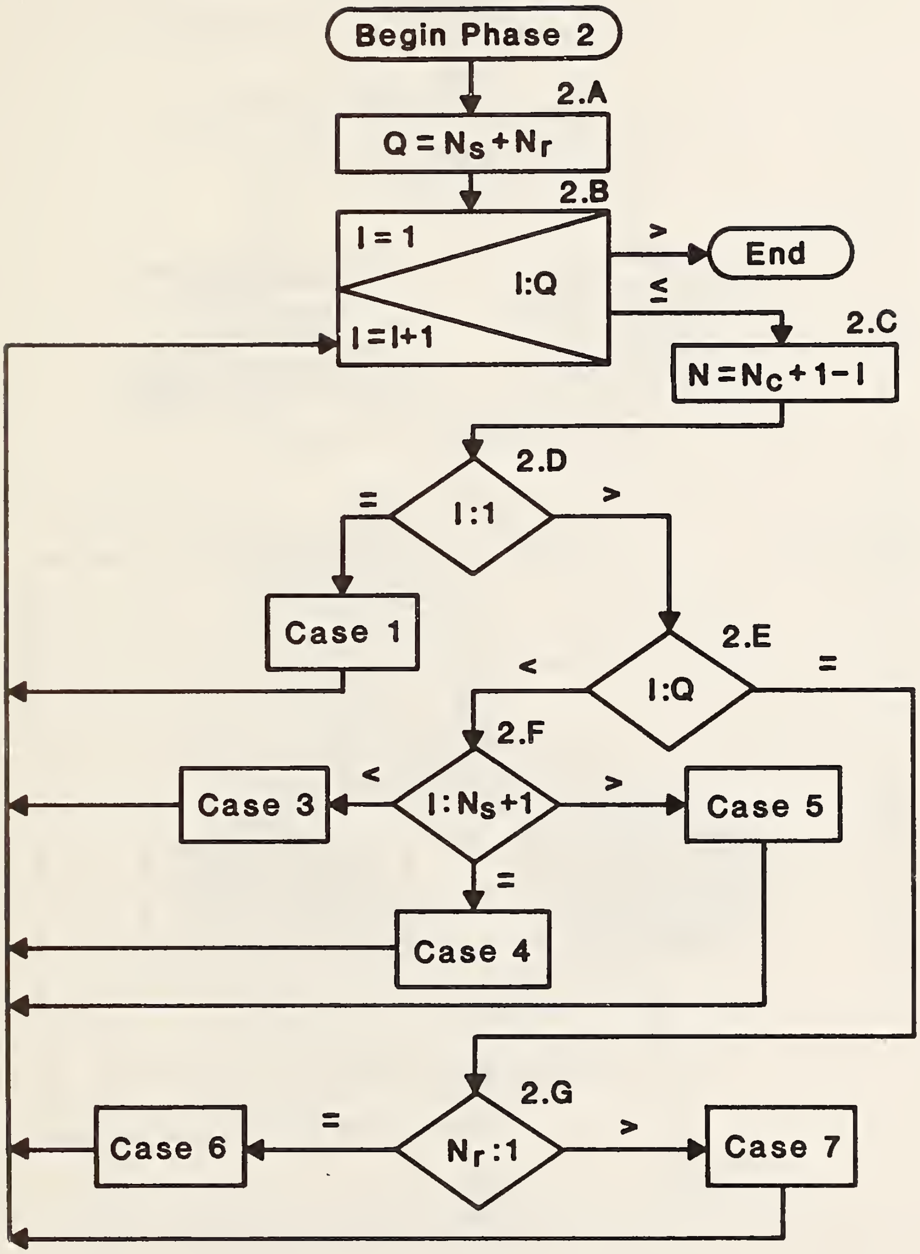

Figure 4.5 - Flowchart for Phase 2

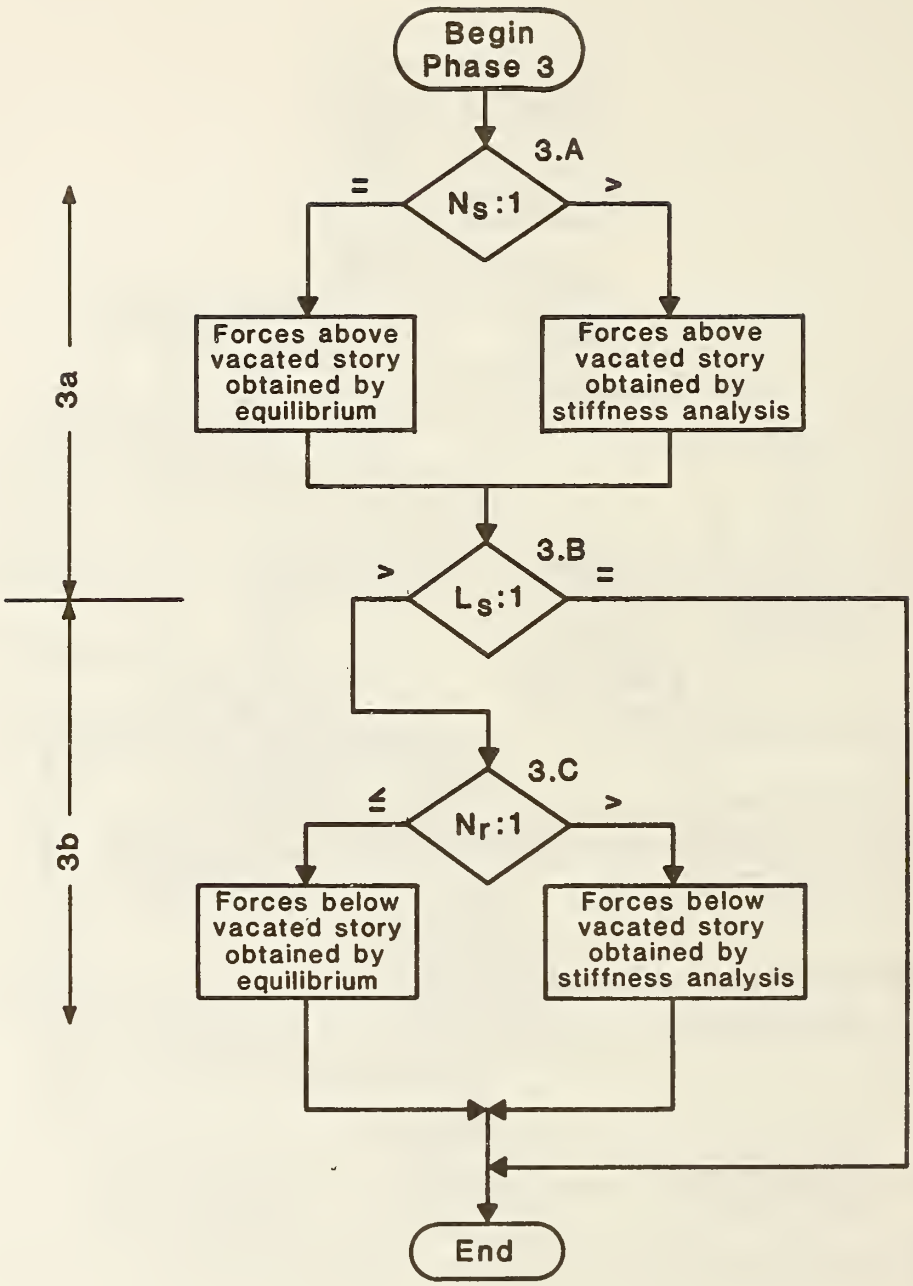

Figure 4.6 - Flowchart for Phase 3

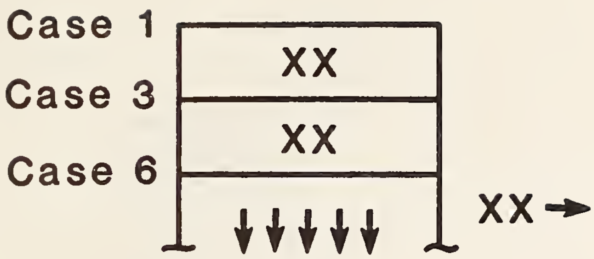

Figure 4.7 - Phase 3a Configuration

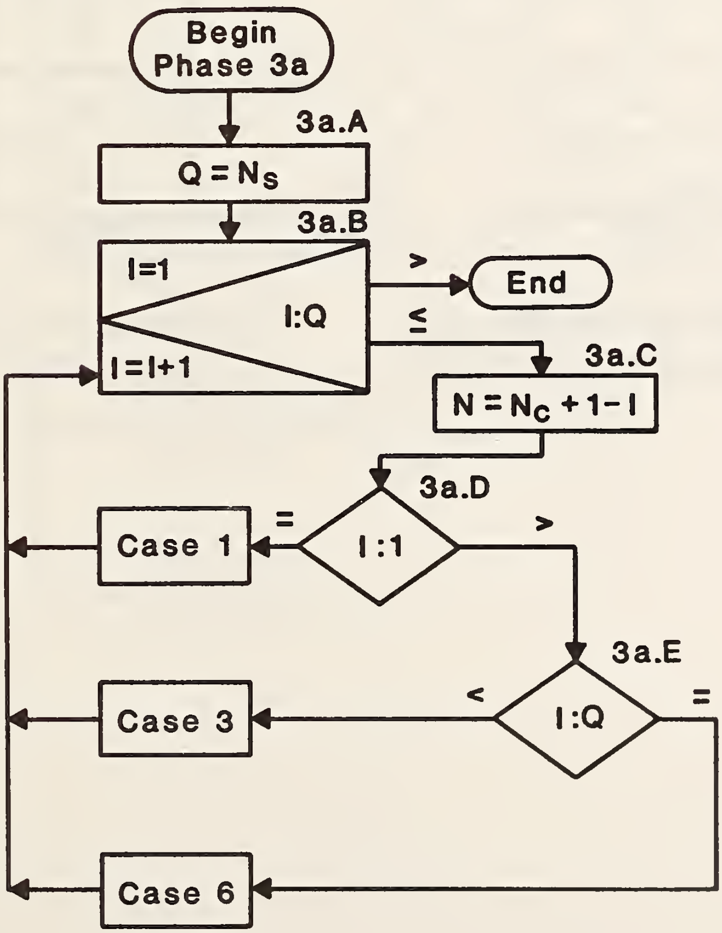

Figure 4.8 - Flowchart for Phase 3a

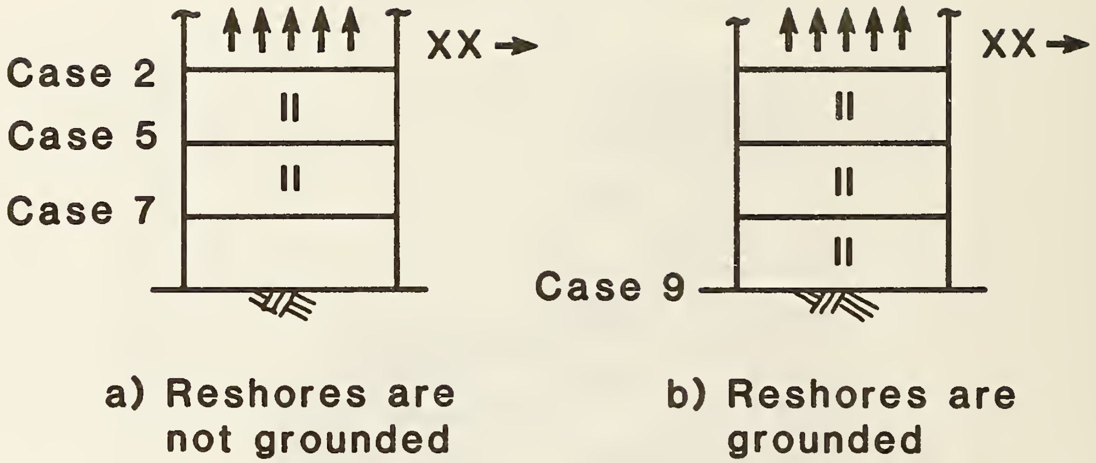

Figure 4.9 - Phase 3b Configurations

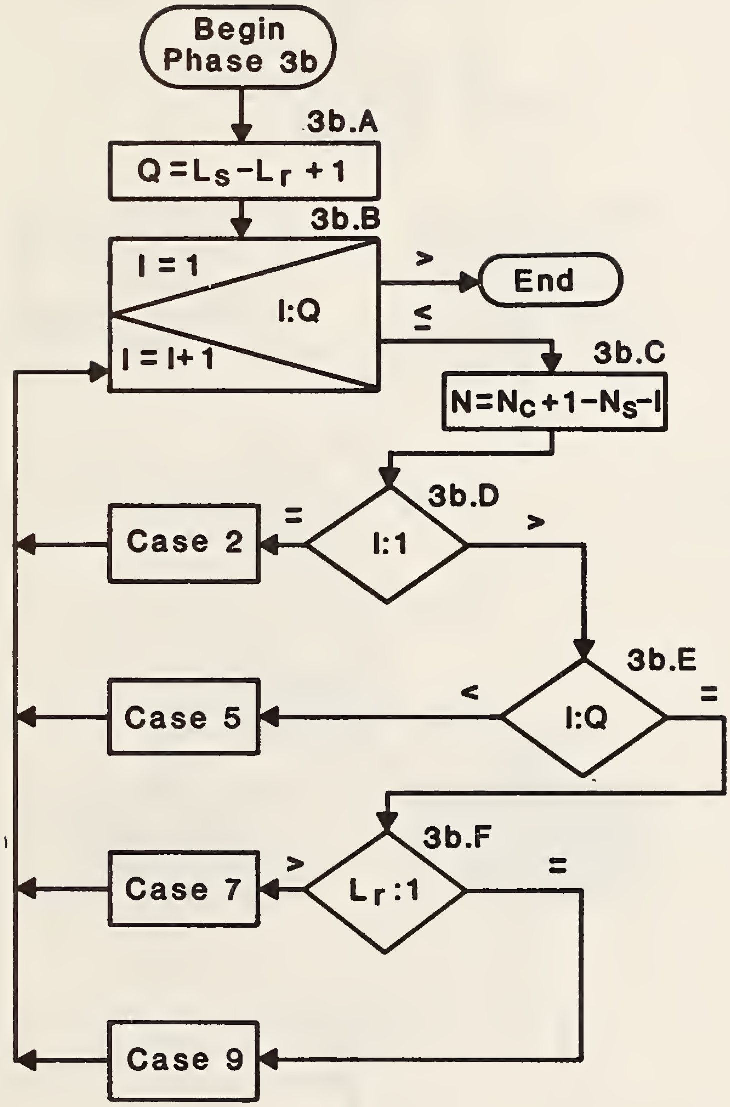

Figure 4.10 - Flowchart for Phase 3b

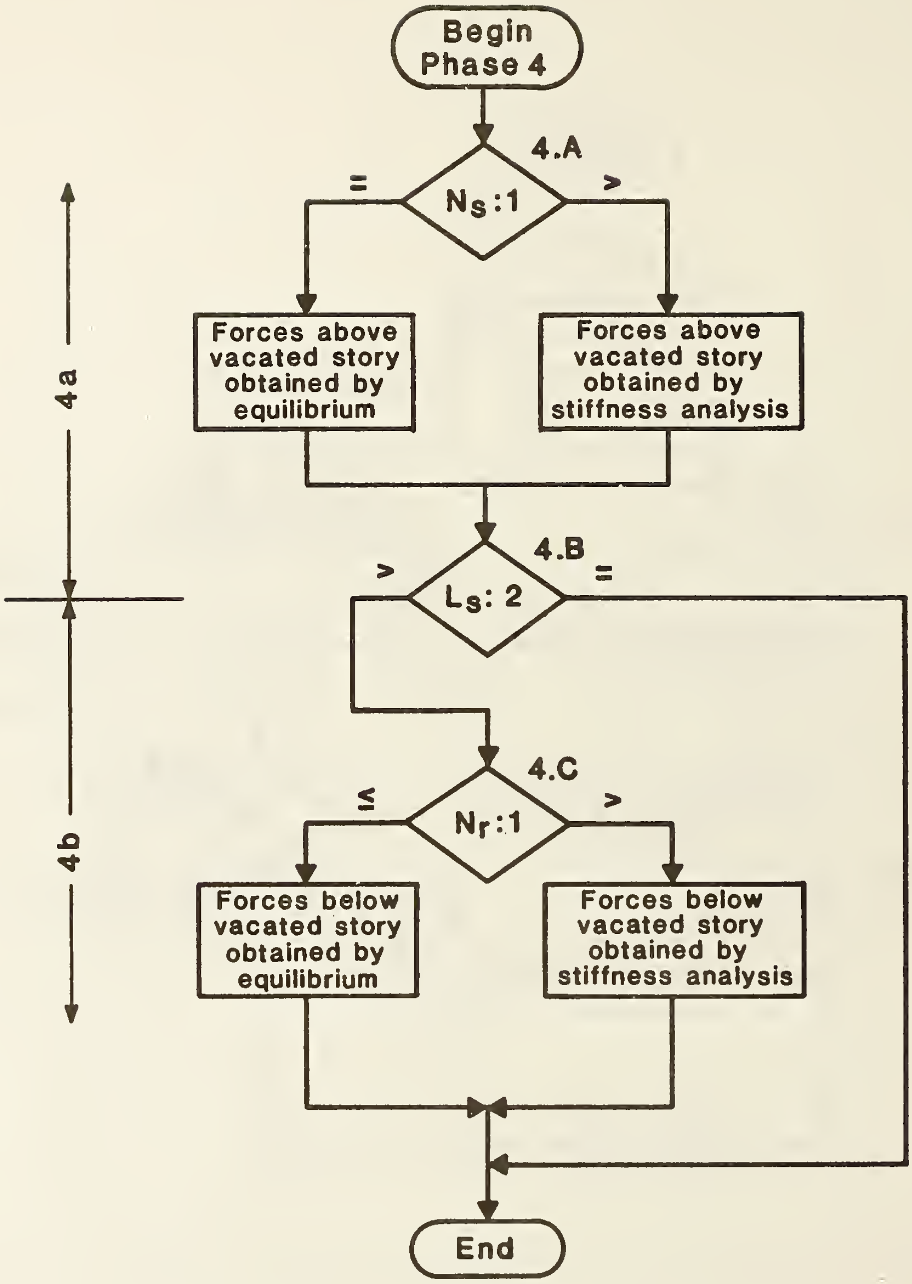

Figure 4.11 - Flowchart for Phase 4

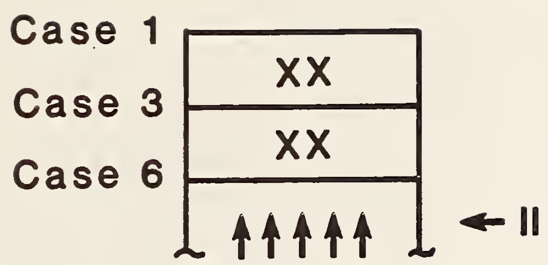

Figure 4.12 - Phase 4a Configuration

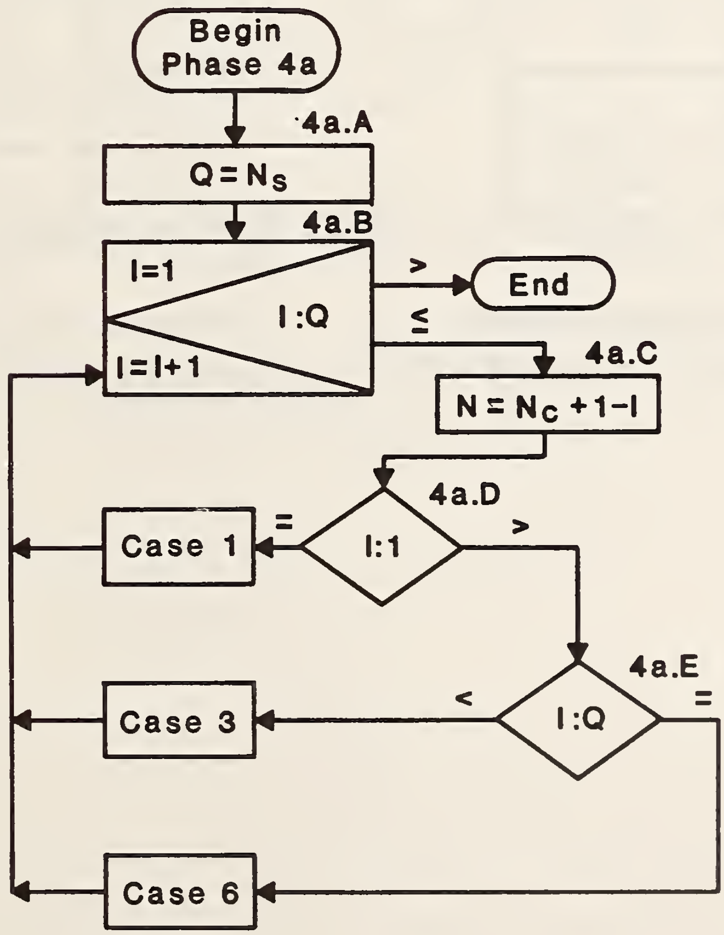

Figure 4.13 - Flowchart for Phase 4a

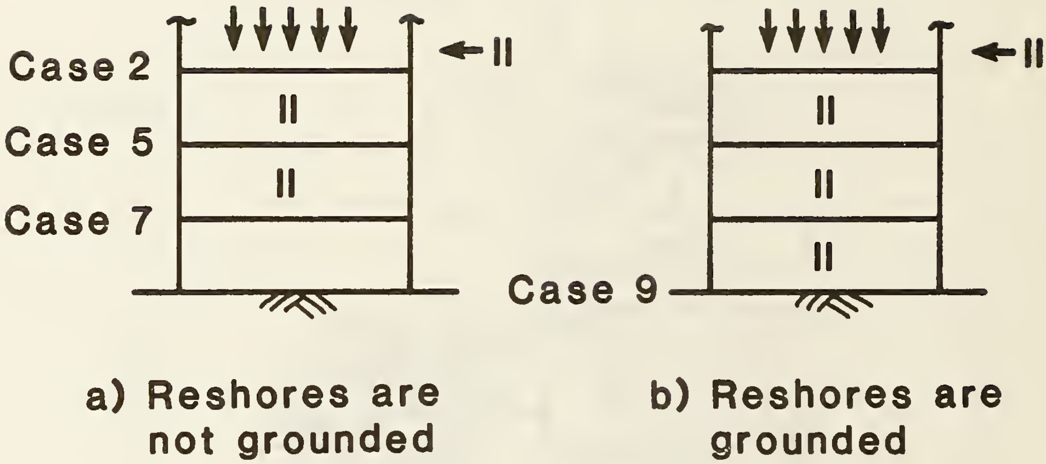

Figure 4.14 - Phase 4b Configurations

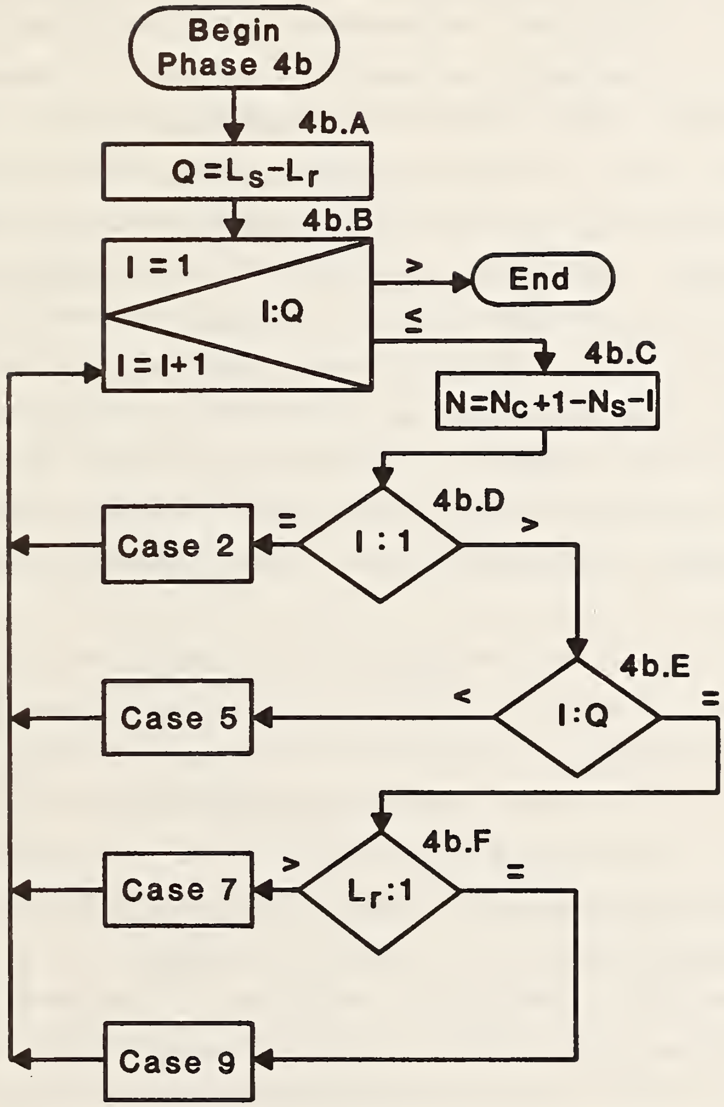

Figure 4.15 - Flowchart for Phase 4b

5. EXAMPLE PROBLEMS

In this section several examples of the application of the compressible shores analysis are presented. All the examples relate to the eight story structure shown in Figure 5.1. Only a single bay is considered since the analysis does not include continuity between bays. The slab stiffnesses are assumed not to vary with age (E_c is constant). Slab thickness is the same for all floors, thus, k_n (n = 1 to 8) is constant. For the examples where the shores, reshores, and ground are compressible a stiffness equal to twice the slab stiffness is used (k_s=k_r=k_g=$2k_n$). Rigid shores and base support are modeled by defining a very large stiffness for the shores, reshores, and ground relative to the slab stiffness (1000:1). A shoring scheme of two levels of shores and three levels of reshores is used for examples 1 and 3. The casting rate is seven days per floor for all three examples.

The results of an analysis are the floor slab loads and shore or reshore loads for each of the four phases of a casting cycle, as applicable, for each of the eight floors cast. Only the loads on the slabs are presented here for illustration purposes. The loads are expressed as a "load ratio" which is defined as the ratio of the load on the slab (construction load plus slab self weight) to the self weight of the slab. For example, a computed slab load ratio of 1.75 means that the slab load is equal to 1.75 times the self weight of the slab. The actual load on a slab may be computed by multiplying the computed load ratio by the self weight of the slab in psf.

A history of slab load ratio versus age of slab may be tabulated for each floor of the structure. Figure 5.2 shows a typical plot of the slab load ratio versus age of slab for stories one through five of the eight story building. The maximum of the tabulated slab load ratios for a given age is used for comparison purposes in the following examples. It is simply the envelope of the results shown in Figure 5.2.

5.1 COMPARISON OF RIGID AND COMPRESSIBLE SHORES

In this first example, the analysis results for the cases of rigid and compressible shores are compared. For both cases, the precompression ratio is taken as zero. Figure 5.3 shows a plot of the maximum slab load ratio versus the age of the slab. From this figure it is seen that a maximum slab load of 1.89D is computed where D is the self weight of the slab. From the complete analysis one can determine that this maximum occurs on the second floor slab during casting of the fourth floor (Phase 1). For the case of a rigid shoring system and base support (presently accepted procedure) a maximum slab load of 1.66D is found. This load occurs on the fifth floor slab during the removal of the lowest level of reshores (Phase 2) after casting the seventh floor. This one example indicates that, for shore, reshore and ground stiffnesses equal to twice the slab stiffness, a 14% increase in maximum slab load is realized as compared to an analysis assuming a rigid shoring system.

5.2 COMPARISON OF SHORING SCHEMES

In this example shoring schemes including either one, two, or three levels of shores and three levels of reshores are compared. Flexible shores, reshores, and base support are used in all cases and there is no precompression of reshores. It has been observed by Grundy and Kabaila (4) that increasing the number of levels of shores actually increases the maximum slab load. This holds true in the analysis for compressible shores as shown in Figure 5.4. For the case of a single level of shores, the maximum slab load was found to be 1.52D. The maximum occurred at an age of 15 days. For two levels of shores, the maximum was 1.89D occurring at an age of 14 days. And for three levels of shores, a maximum of 1.98D was found at an age of 21 days. Note that, although the maximum slab load increases with increased number of levels of shores, the age at which the maximum occurs also increases. If the strength-gaining characteristics of the concrete are known, it is possible to determine whether the slabs have sufficient capacity to carry the maximum construction load at the age at which the maximum occurs.

5.3 COMPARISON OF PRECOMPRESSION RATIOS

The third example explores the effects of varying the precompression ratio. The structure shown in Figure 5.1, with a compressible shoring system and base support, is analyzed for precompression ratios of 0.0 (no precompression), 0.5, and 1.0. Figure 5.5 compares the results of an analysis for no precompression and 50% precompression. It is seen that precompressing the reshores to 50% of the load in the shores prior to their removal reduces the maximum slab load at early ages and increases the maximum load at later ages. In this particular case the maximum slab load is reduced from 1.89D to 1.52D.

Figure 5.6 shows the effects of precompressing the reshores to 100% of the load in the shores. Here the results of 50% and 100% precompression are plotted. Additional precompression again reduces the maximum slab load at early ages and increases the load at later ages. In this case, however, it is seen that precompressing to 100% increases the maximum slab load to 1.97D. Note that this exceeds the maximum load encountered in the case of no precompression (1.89D). The age at which the maximum occurs (35 days) is much greater than the age at which the maximum occurs for no precompression (14 days). Information about the slab capacity as a function of the in-place strength of concrete at various ages is required to assess the relative safety of the two construction procedures.

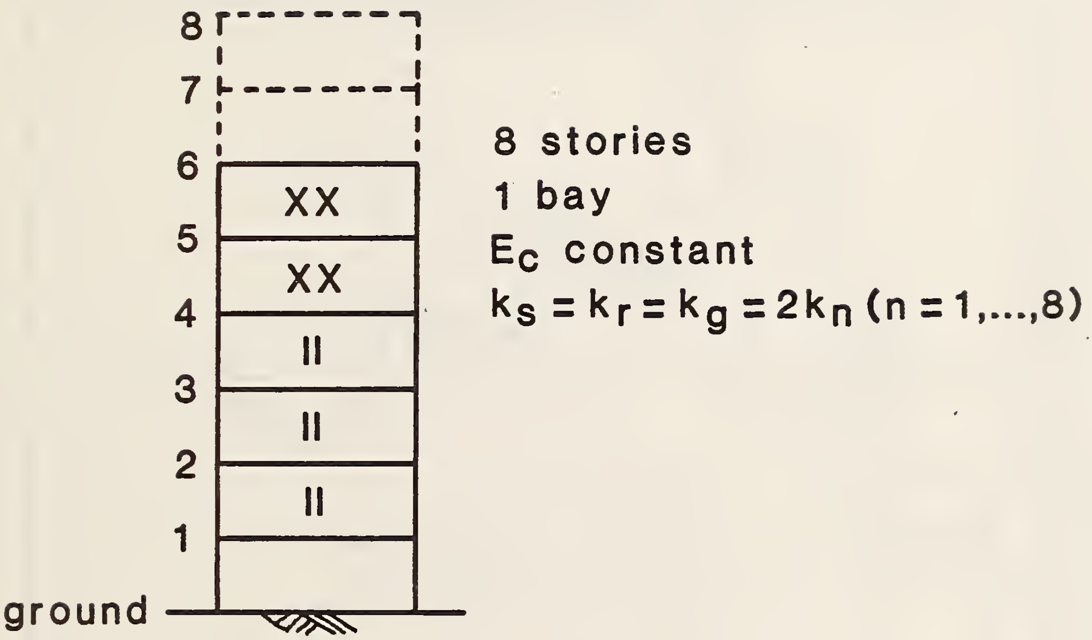

Figure 5.1 - Eight Story Building

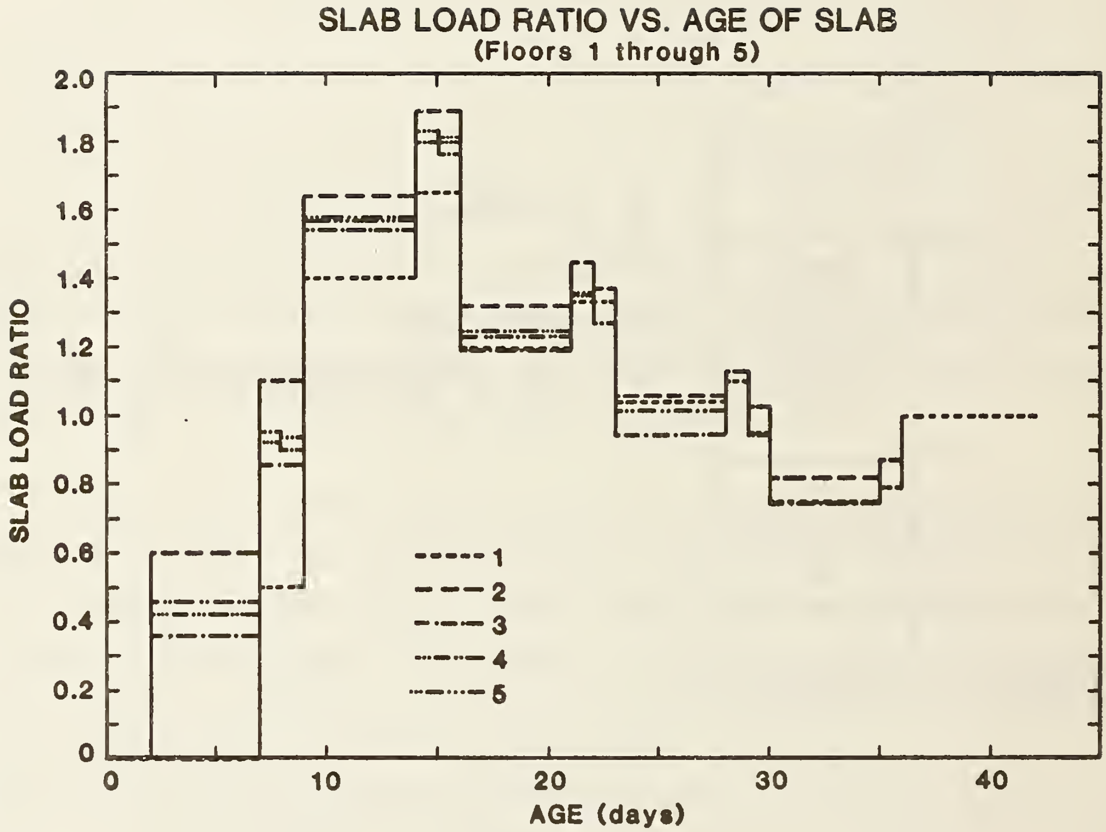

Figure 5.2 - Plot of Slab Load Ratio vs. Age of Slab

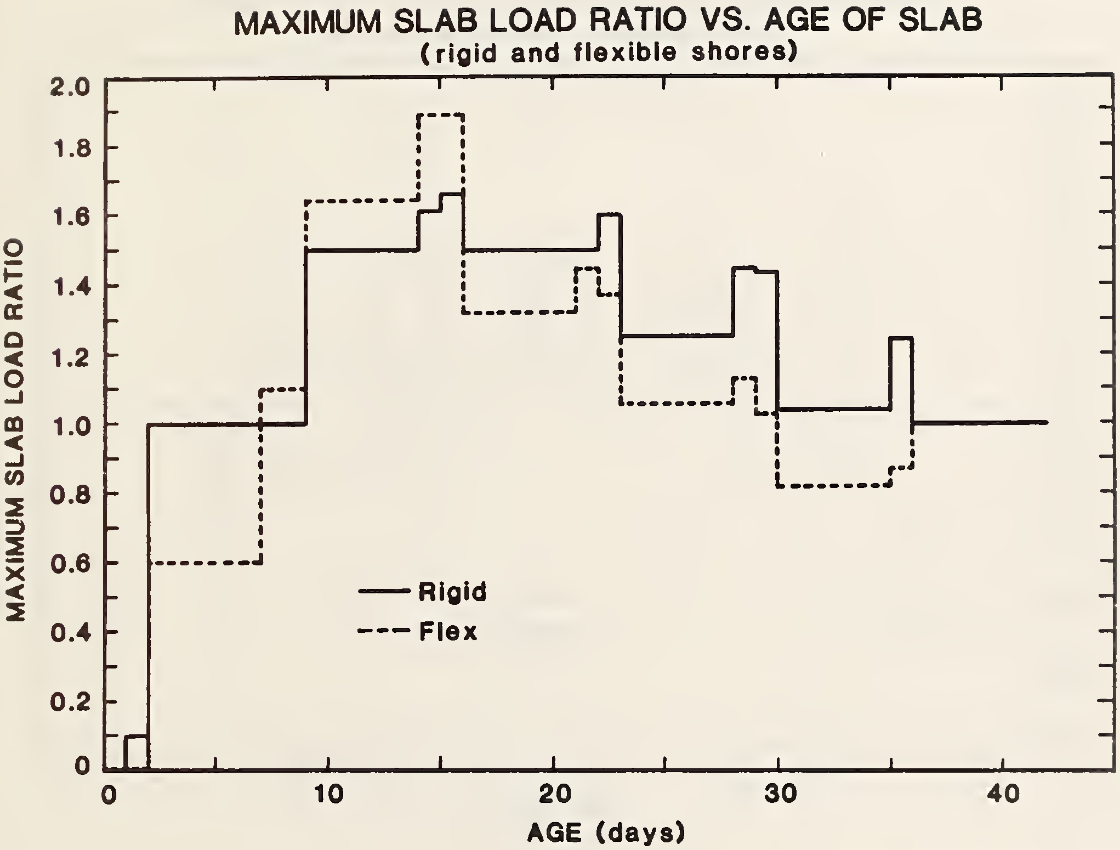

Figure 5.3 - Comparison of Rigid and Compressible Shores

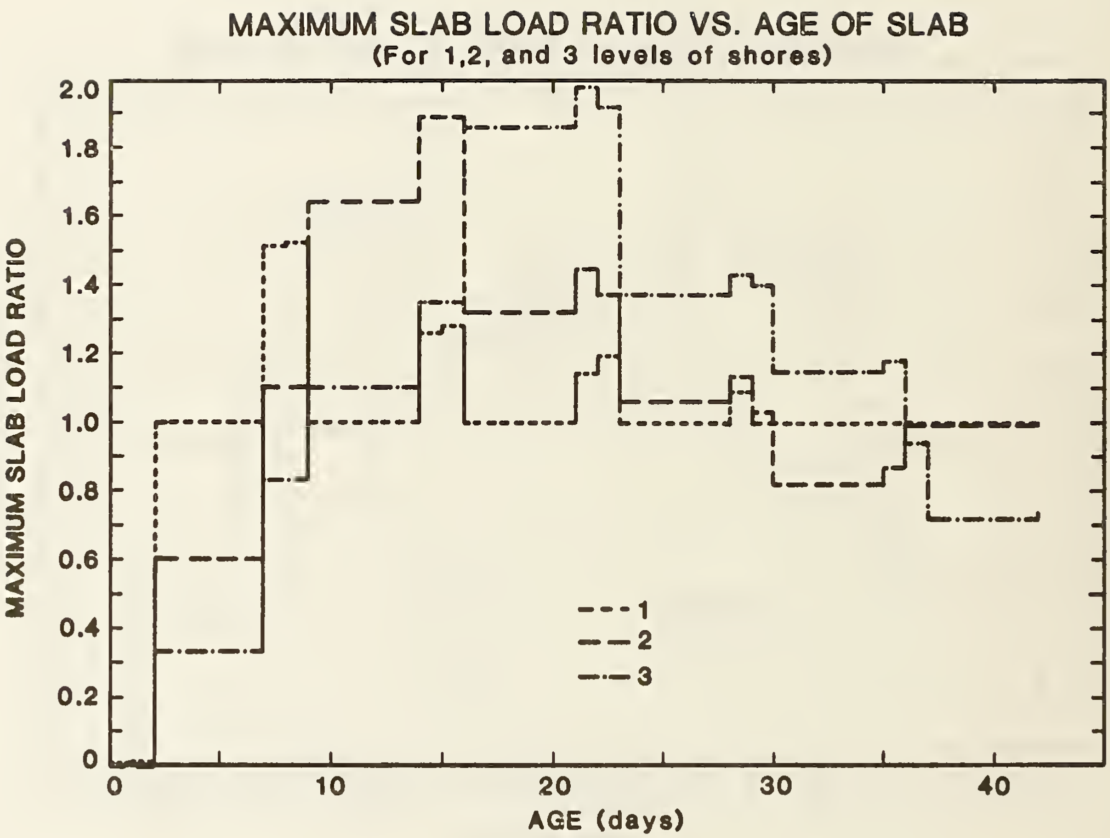

Figure 5.4 - Comparison of Shoring Schemes

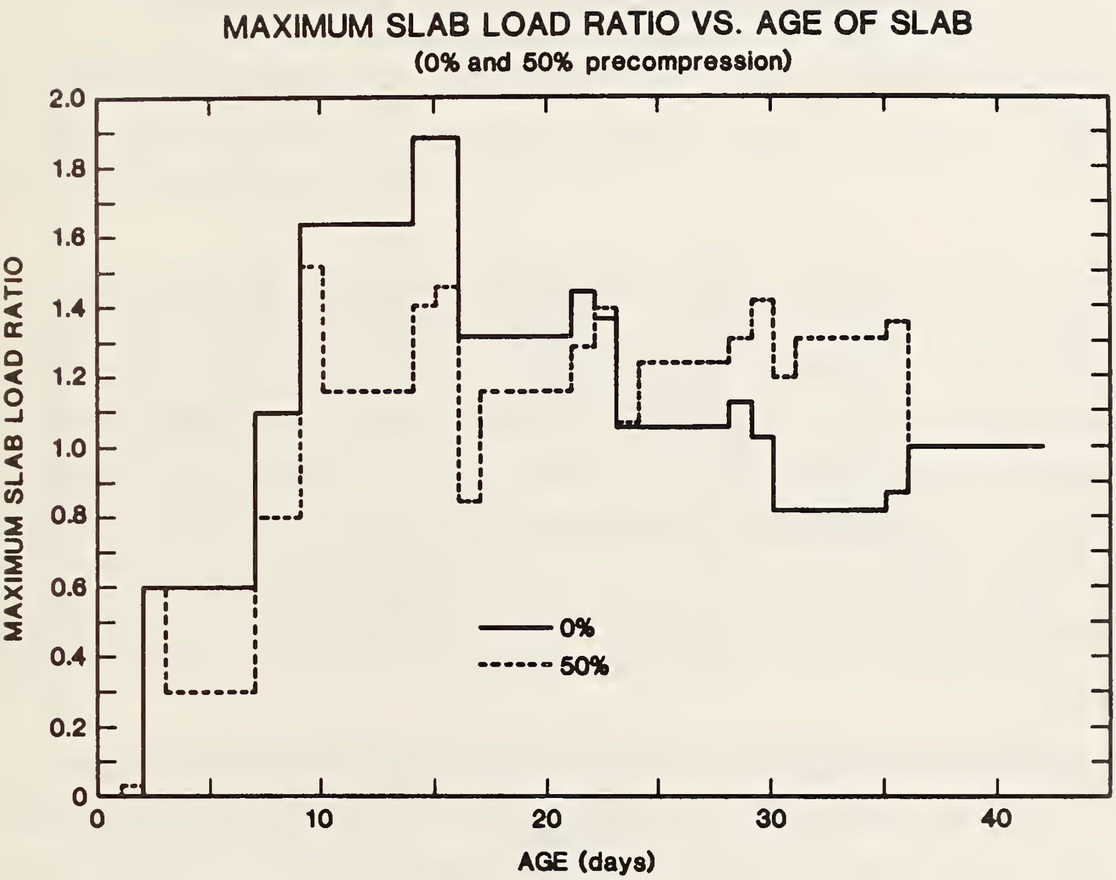

Figure 5.5 - Comparison of Precompression Ratios
(0% and 50% precompression)

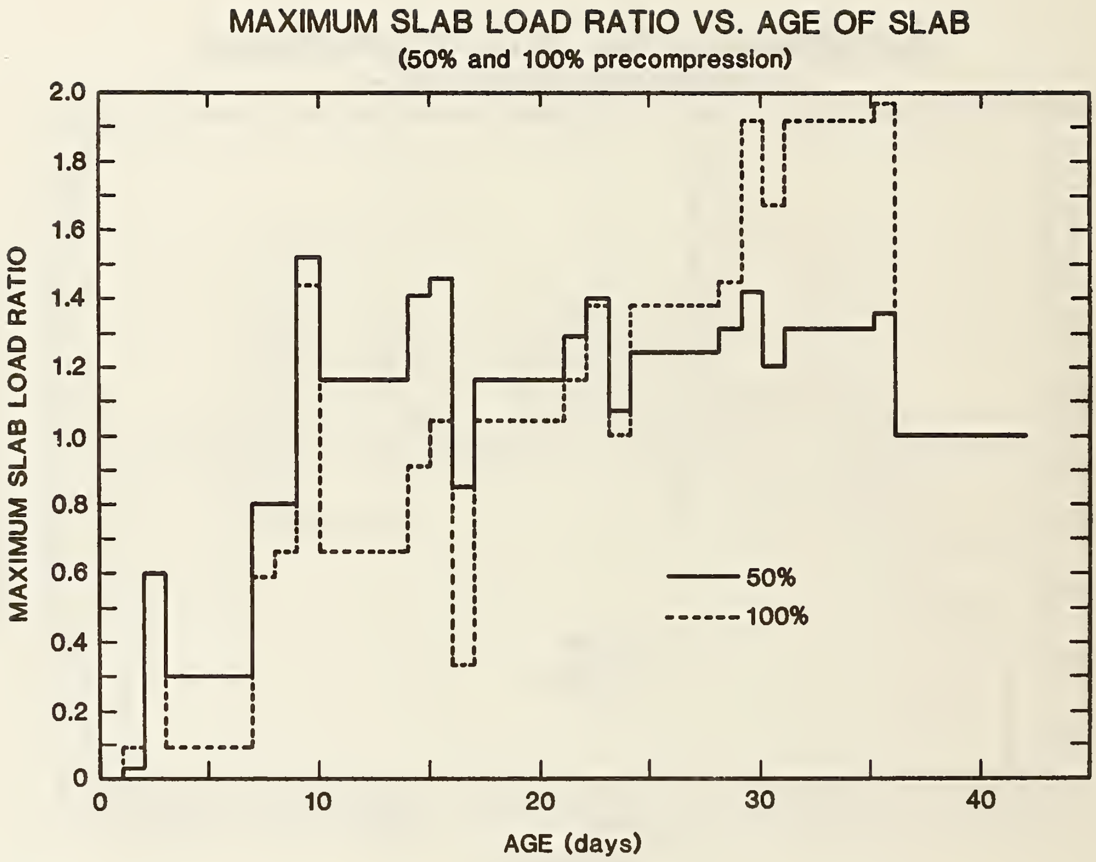

Figure 5.6 - Comparison of Precompression Ratios
(50% and 100% precompression)

6. SUMMARY

In this report, the importance of considering the compressibility of the shores in an approximate load analysis of multistory concrete construction was discussed. A model was described for computing the loads on the slabs and shores which accounts for the flexibility of the shoring system. The main focus of the report was a detailed description for implementing the proposed procedure. The overall flow of control was outlined and the explicit form of the stiffness equations and loading for each phase of construction was described. The information contained herein should be sufficient to permit the reader to develop a computer-based analysis.

A computer program implementing the procedure described herein was written at NBS. The program, written in FORTRAN, runs on a personal computer. It takes as input the structure geometry, concrete properties, and construction schedule. The program then computes the loads on slabs and shores as each floor is cast and the shores and reshores are advanced. Results are reported after each of the four construction phases. Maximum values are computed and listed along with their time of occurrence, after all floors have been cast.

To illustrate how such an analysis could be used to assist a construction engineer or contractor in determining the loads on the slabs of a multistory building, three example problems were presented. They demonstrate several of the program features including the influence of shore and reshore stiffness on the maximum slab loads, an evaluation of alternative construction schemes, and the effects of various amounts of precompression in the installed reshores.

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Appendix A - Notation

D = self weight of slab
F^f_n = Force on the n-th floor slab
F^r_n = Force on the reshores in the n-th story
F^s_n = Force on the shores in the n-th story
i = i-th stiffness equation
k_g = ground stiffness
k_n = stiffness of the n-th floor slab
k_r = reshore stiffness
k_s = shore stiffness
L_r = lowest reshored story
L_s = lowest shored story
N = n-th floor
N_c = current top floor
N_r = number of reshored stories
N_s = number of shored stories
N_t = total number of floors
P = precompression ratio
Q = number of stiffness equations
W_1 = weight of fresh concrete
W_2 = load in lowest reshores prior to their removal
W_3 = load in lowest shores prior to their removal
W_4 = reshore load (precompression)

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5. AUTHOR(S)

John L. Gross

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☐ Document describes a computer program; SF-185, FIPS Software Summary, is attached.

11. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)

This report presents an analytical procedure for determining the loads on the shoring system and supporting slabs in multistory cast-in-place concrete construction. The procedure assumes that the slabs are supported by evenly distributed compressible shores and reshores and employs the stiffness method of analysis to solve for the loads on the shoring system and slabs as construction advances. The number of shores and reshores; shore, reshore and base support stiffnesses; casting rate; and concrete strength gaining characteristics are considered in the analysis. Details of the implementation of the shoring analysis in the form of a computer program are presented. The strategy for determining the next phase in the casting cycle is described and the details are given for formulating the stiffness equations and loads for each phase. Several example problems are presented to demonstrate the use of such a procedure in assisting to make critical decisions regarding planning of the casting schedule and determining when formwork can be safely removed.

12. KEY WORDS (Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)

buildings; computer models; concrete construction; concrete (reinforced); formwork; safety; structural analysis

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