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NBSIR 84-2850

Recommended Practice for Measuring Simple and Discounted Payback for Investments in Buildings and Building Systems

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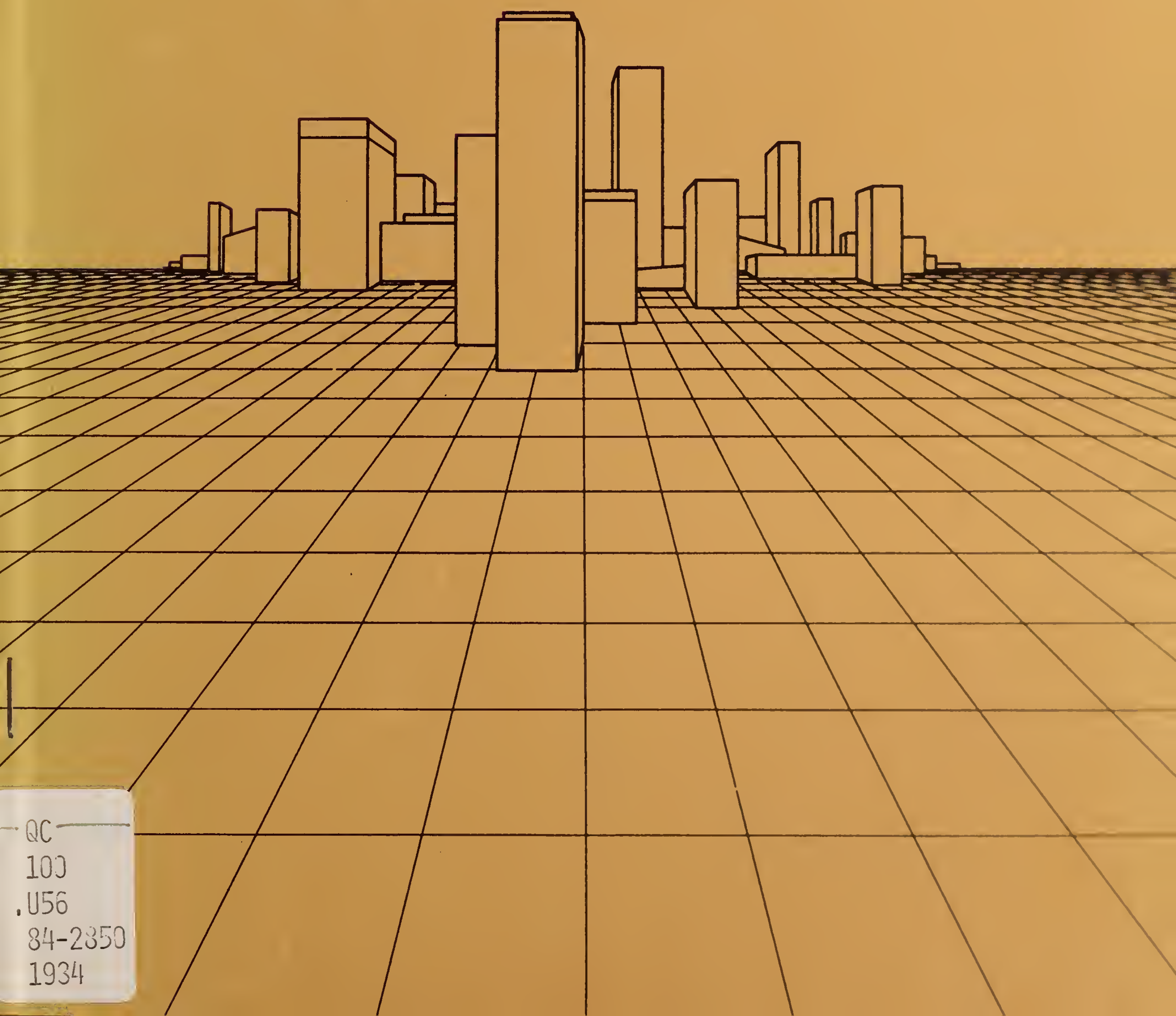

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RECOMMENDED PRACTICE FOR MEASURING SIMPLE AND DISCOUNTED PAYBACK FOR INVESTMENTS IN BUILDINGS AND BUILDING SYSTEMS

Harold E. Marshall

March 1984

Sponsored By:
U.S. DEPARTMENT OF COMMERCE
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U.S. DEPARTMENT OF COMMERCE, Malcolm Baldrige, *Secretary*
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ABSTRACT

This report describes how to calculate simple and discounted payback measures of economic performance of buildings and building systems. Formulas for calculating payback, applications for evaluating and selecting projects, and limitations in the use of payback analysis are discussed. The simple payback method measures the time between the date of initial project investment and the date when cumulative future earnings or savings on that investment, net of cumulative future costs, just pay off the investment. The discounted payback method measures the time between the date of initial project investment and the date when the present value of future earnings or savings, net of the present value of future costs, just equals the initial investment. This recommended practice will assist the private and public building communities in making cost-effective decisions in the design, operation, maintenance, and retrofit of buildings.

PREFACE

Rising costs of materials, labor, energy, and construction loans have caused architects, builders, engineers, building owners and operators, and code writers to look more closely at economic evaluation methods in selecting building designs and systems that are cost effective. Simple payback (SPB) and discounted payback (DPB) methods for measuring economic performance of buildings and building systems are commonly used because they are in many instances relatively simple to compute and easy to understand. Standardized approaches for calculating and applying the SPB and DPB methods are needed because traditional applications of these methods sometimes lead to uneconomic building choices.

The SPB method measures the time between the date of initial project investment and the date when cumulative future earnings or savings on that investment, net of cumulative future costs, just pay off the investment. The DPB method measures the time between the date of initial project investment and the date when the present value of future earnings or savings, net of the present value of future costs, just equals the initial investment. The SPB and DPB methods are used generally to decide if a single project is economically feasible (i.e., if it pays off within a predetermined maximum allowable payback period) and to choose among projects competing for the same purpose or the same budget (i.e., to choose projects with a short SPB before those with a long SPB). However, both payback methods have shortcomings which sometimes result in inefficient project choices. Each method ignores benefits and costs beyond the payback period, and the SPB method ignores the time value of money, treating, for example, a dollar saved five years from today as the equivalent to a dollar saved today.

This report describes how to calculate SPB and DPB and describes the circumstances under which each might be correctly applied in economic evaluations of buildings and building components. Project examples are also provided to illustrate the economic efficiency losses from using SPB and DPB incorrectly in making investment choices. Developing standardized SPB and DPB methods and providing guidelines for their application will help prevent misuse of these methods and contribute to improved cost effectiveness in building-related decisions.

This report was prepared by the National Bureau of Standards (NBS) in support of an ongoing standards development activity by the American Society of Testing and Materials (ASTM E-6, Performance of Building Constructions) and in response to requests from the building community for assistance in applying economic anaylsis in a uniform and practicable manner. The report has been submitted to ASTM E06.81, the Building Economics Subcommittee, to be used as the technical basis for the development of an ASTM standard practice for measuring SPB and DPB for building investments. It is the fourth in a series of reports to be submitted by NBS to ASTM E06.81, and it builds in part upon the previous three reports (NBSIR 83-2657, on net benefits and the internal rate of return; NBSIR 81-2397, on benefit-to-cost and savings-to-investment ratios; and NBSIR 80-2040, on life-cycle costing).[1] The series of NBS reports

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[1] Harold E. Marshall, Recommended Practice for Measuring Net Benefits and Internal Rates of Return for Investments in Buildings and Building Systems, National Bureau of Standards Interagency Report 83-2657, October, 1983; Harold E. Marshall and Rosalie T. Ruegg, Recommended Practice for Measuring Benefit/Cost and Savings-to-Investment Ratios for Buildings and Building Systems, National Bureau of Standards Interagency Report 81-2397, November 1981; and Rosalie T. Ruegg, Stephen R. Petersen, and Harold E. Marshall, Recommended Practice for Measuring Life-Cycle Costs of Buildings and Building Systems, National Bureau of Standards Interagency Report 80-2040, June 1980.

and ASTM recommended practices is aimed at producing a comprehensive set of measures of economic performance that will meet the needs of the private and public building sectors.

Thanks are due the members of ASTM who have participated in the Building Economics Subcommittee meetings and thereby have helped determine the framework of this paper. Special appreciation is extended to Robert E. Chapman, Joseph H. Engel, James G. Gross, Barbara C. Lippiatt, Stephen R. Petersen, Rosalie T. Ruegg, Larry Schindler, and Stephen F. Weber for their helpful technical and editorial comments on the paper; to Janet M. Cassard and Stefan D. Leigh for their assistance in generating some of the tabular and graphical material in the report; and to Laurene B. Linsenmayer for typing the manuscript.

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1. INTRODUCTION

The simple payback (SPB) method and discounted payback (DPB) method are part of a family of economic evaluation methods that provide measures of economic performance of an investment.[1] Included in this family of evaluation methods are life-cycle costing, benefit-to-cost and savings-to-investment ratios, net savings, and internal rates of return.

This is the fourth in a series of National Bureau of Standards (NBS) reports on recommended practices for applying economic evaluation methods to building decisions. The first report in the series was Recommended Practice for Measuring Life-Cycle Costs of Buildings and Building Systems, NBSIR 80-2040. The second was Recommended Practice for Measuring Benefit/Cost and Savings-to-Investment Ratios for Buildings and Building Systems, NBSIR 81-2397. The third was Recommended Practice for Measuring Net Benefits and Internal Rates of Return for Investments in Buildings and Building Systems, NBSIR 83-2657.

The SPB method measures the time between the date of initial project investment and the date when cumulative future earnings or savings on that investment, net of cumulative future costs, just pay off the investment.[2] In practice the SPB method is used sometimes as the single criterion for judging economic efficiency and sometimes as a supplementary criterion. One common use of the SPB method is to decide if a single project is economically feasible. If the time required to pay off the investment is less than some predetermined maximum allowable payback period (MAPP), then the project is presumed to be economical. Another common

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[1]The terms payoff and payout are sometimes used instead of payback.

[2]Note that future earnings as well as future costs are often not known with certainty and therefore, for purposes of analysis, will have to be estimated.

use of the SPB method is for choosing among projects competing for the same purpose or the same budget. Projects with a low SPB are preferred to projects with a high SPB because the investment is paid off more quickly. Choosing projects on the basis of the SPB, however, may lead to inefficient choices and fewer net benefits than might be earned following other economic evaluation methods.

The DPB method measures the time between the date of initial project investment and the date when the discounted present value of earnings or savings, net of discounted costs, just equals the initial investment. The DPB differs from SPB in that DPB accounts for the time value of money. The DPB method is used for the same types of decisions as the SPB method. The DPB also shares some of the shortcomings of the SPB method, and it, too, may lead to project choices that are not cost effective.

The purpose of this report is to establish a technical basis for the development of a recommended practice for calculating, interpreting, and applying SPB and DPB methods in the evaluation of building designs and systems. Special attention is given to applications, since the traditional use of payback methods sometimes leads to the construction of inefficient projects. All economic terminology used in the report is consistent with "Standard Definitions of Terms Relating to Building Economics."[1]

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[1]American Society for Testing and Materials, "Standard Definitions of Terms Relating to Building Economics," E833-83a, Annual Book of ASTM Standards, Vol 04.07 (Philadelphia, PA: ASTM, 1983).

Section 2 discusses the general procedures in carrying out a payback analysis. Objectives, alternatives, constraints, assumptions, data, and discounting are discussed as they pertain to the two payback methods. Readers are assumed to have a working knowledge of discounting and present value analysis. Section 3 describes the calculation procedures for measuring SPB, and section 4 does the same for DPB. Problem examples are used to illustrate the mathematical and graphical procedures for determining payback. Section 5 discusses under what circumstances SPB and DPB measures are appropriate for evaluating typical building investment problems. Recommendations of specific methods are made for different investment objectives. Limitations of the two methods and their economic efficiency implications are also discussed. Appendix A provides derivations of DPB with and without escalation of net cash flows. Appendix B, a bibliography, concludes the report.

2. PROCEDURES IN PAYBACK ANALYSIS

The recommended steps for making an economic analysis of buildings or building components are summarized as follows:

- Identify Objectives, Alternatives, and Constraints
- Select an Economic Evaluation Method
- Compile Data and Establish Assumptions
- Convert Cash Flows to a Common Time Basis
- Compute the Economic Measure and Compare Alternatives

Each of these steps, except the last one, is examined briefly in this section with respect to making a payback analysis. For elaboration on these steps, the three earlier reports in this series should be consulted. How to compute SPB and DPB is presented respectively in sections 3 and 4. How payback measures are used in comparing alternatives is discussed in section 5.

2.1 Objectives, Alternatives, and Constraints

The kind of building decision to be made must first be specified. The objectives of the decision maker must be made explicit. Alternative approaches for reaching the objectives and any constraints to reaching the objectives should be identified.

An example of a building investment problem that might be evaluated with payback methods is the decision on what heating system to install in an existing building. The objective of the building owner is to provide the required level of thermal comfort most efficiently. Feasible technical alternatives for providing the required level of thermal comfort might be replacement of the existing heating system with a high-efficiency oil furnace or an electric heat pump. Constraints might be a limit on investment funds and time limits by which the new heating equipment must be in operation.

2.2 Economic Evaluation Method

If the high-efficiency oil furnace and heat pump are technically feasible and meet all constraints, the building owner will want to evaluate the two alternatives to see which is preferred on economic grounds. The SPB or DPB method would indicate which of the two alternatives would pay off their investment cost first through cumulative future energy savings as compared with heating bills from the current heating system.

Note that other economic evaluation methods are appropriate for solving the problem cited above and that in many cases these other methods will lead to decisions that maximize the present value of net savings whereas application of payback methods will not. For examples of investment problems and their solutions by other methods, see the first three reports in this series.[1] Other applications and limitations of the SPB and DPB methods will be discussed in section 5.

2.3 Data and Assumptions

Data are needed to make SPB or DPB calculations. Some data can be collected from published and unpublished sources, some can be estimated, and some will

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[1] See Harold E. Marshall, Recommended Practice for Measuring Net Benefits and Internal Rates of Return; Harold E. Marshall and Rosalie T. Ruegg, Recommended Practice for Measuring Benefit/Cost and Savings-to-Investment Ratios; and Rosalie T. Ruegg, Stephen R. Petersen, and Harold E. Marshall, Recommended Practice for Measuring Life-Cycle Costs of Buildings and Building Systems.

have to be assumed.[1] Both engineering data (e.g., heating loads, equipment life, and equipment efficiencies) and economic data (e.g., tax rates, depreciation rates and periods, system costs, energy costs, and financing terms) will be needed.

Important values that must sometimes be assumed in using payback methods are the discount rate, project life, and price escalation rates. A maximum acceptable payback period (MAPP) must also be chosen if a cutoff period is used. If the SPB or DPB is longer than the MAPP, the investment is considered economically unattractive. The selection of a MAPP and its implications will be discussed in section 5.

Because the economic measure of a project's worth varies considerably depending on the assumptions, it is important to use care in making them and to consider the effects of different parameter values. Sensitivity analysis can be used to identify critical parameters and to test the outcome for a range of values in any given problem.

1For a more complete discussion of data and assumptions, see Rosalie T. Ruegg, Stephen R. Petersen, and Harold E. Marshall, Recommended Practice for Measuring Life-Cycle Costs of Buildings and Building Systems, Harold E. Marshall and Rosalie T. Ruegg, Simplified Energy Design Economics: Principles of Economics Applied to Energy Conservation and Solar Energy Investments in Buildings, National Bureau of Standards Special Report 544, January 1980; and Harold E. Marshall and Rosalie T. Ruegg, Energy Conservation in Buildings: An Economics Guidebook for Investment Decisions, National Bureau of Standards Handbook 132, May 1980. For data sources, see Rosalie T. Ruegg, Life-Cycle Cost Manual for the Federal Energy Management Program, National Bureau of Standards Handbook 135, December 1980, the chapter on "Data and Assumptions" (pp. 37-48), and some projections of energy prices in appendix C; Carol Chapman Rawie, Estimating Benefits and Costs of Building Regulations: A Step-by-Step Guide, National Bureau of Standards Report 81-2223, June 1981, the the end of chapters 1-3, for listings of sources of information on data relating to economic analyses of buildings; and Alphonse J. Dell'Isola and Stephen J. Kirk, Life-Cycle Cost Data (New York: McGraw Hill, 1983) for actual cost data on replacement, energy, and maintenance.

Calculating the DPB for the high-efficiency oil furnace and the electric heat pump, as described in section 2.1, provides an example of the types of data that might be needed. The annual heating requirements of the building, the efficiencies of the two heating systems, and the per unit energy content of the two energy sources are needed to calculate annual purchased energy requirements with the two systems. The present and future prices of electricity and oil as well as the discount rate are needed to convert the annual purchased energy requirements into an annual energy cost. Investment, maintenance, and perhaps replacement costs, as well as the resale value may be needed both for the oil system and for the heat pump to complete the data requirements to find the DPB from the heat pump substitution. Financing and taxes should be considered if the investment is made by a private building owner.

2.4 Conversion of Cash Amounts to Present Values

To calculate DPB, all benefits and costs estimated for future years must be converted to equivalent present values to account for the time value of money. Converting these cash amounts to present values (generally called discounting) is performed by applying discount formulas, or corresponding discount factors, to benefit and cost data associated with a given investment or design alternative.[1] Table 2.1 presents the most commonly used discounting equations, how they are used, and their algebraic form. This report does not

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[1]Cash amounts refer to both the annually recurring benefits and costs as well as the non-annually recurring benefits and costs that make up the cash flow associated with an investment.

explain discounting since the procedure has been explained in numerous publications.[1]

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[1]For an explanation of discounting and a complete set of discount factor tables, see the ASTM Standard Practice E917, "Recommended Practice for Measuring Life-Cycle Costs of Buildings and Building Systems, and the Adjunct Discount Factor Tables that accompany it (Philadelphia, Pennsylvania: ASTM, 1983).

Table 2.1 Discounting Equations

Name	Schematic Illustration	Application	Algebraic Form[a,b]
Single Compound-Amount (SCA) Equation	$\boxed{P} \longrightarrow \boxed{F?}$	To find F when P is known	$F = P \cdot [(1+i)^n]$
Single Present-Value (SPV) Equation	$\boxed{P?} \longleftarrow \boxed{F}$	To find P when F is known	$P = F \cdot \left[\dfrac{1}{(1+i)^n}\right]$
Uniform Sinking-Fund (USF) Equation	$\boxed{A?} + \boxed{A?} \cdots + \boxed{A?} \longleftarrow \boxed{F}$	To find A when F is known	$A = F \cdot \left[\dfrac{i}{(1+i)^n - 1}\right]$
Uniform Capital-Recovery (UCR) Equation	$\boxed{P} \longrightarrow \boxed{A?} + \boxed{A?} \cdots + \boxed{A?}$	To find A when P is known	$A = P \cdot \left[\dfrac{i(1+i)^n}{(1+i)^n - 1}\right]$
Uniform Compound-Amount (UCA) Equation	$\boxed{A} + \boxed{A} \cdots + \boxed{A} \longrightarrow \boxed{F?}$	To find F when A is known	$F = A \cdot \left[\dfrac{(1+i)^n - 1}{i}\right]$
Uniform Present-Value (UPV) Equation	$\boxed{P?} \longleftarrow \boxed{A} + \boxed{A} \cdots + \boxed{A}$	To find P when A is known	$P = A \cdot \left[\dfrac{(1+i)^n - 1}{i(1+i)^n}\right]$
Modified Uniform Present-Value (UPV*) Equation	$\boxed{P?} \longleftarrow \boxed{A_1} + \boxed{A_2} \cdots + \boxed{A_n}$	To find P when known A_0 is escalating at rate e	$P = A_0 \cdot \left(\dfrac{1+e}{i-e}\right) \cdot \left[1 - \left(\dfrac{1+e}{1+i}\right)^n\right]$

where:

- P = present sum of money,
- F = future sum of money equivalent to P at the end of n periods of time at i interest or discount rate,
- A = end-of-period payment (or receipt) in a uniform series of payments (or receipts) over n periods at i interest or discount rate,
- A_0 = initial value of a periodic payment (receipt) evaluated at the beginning of the study period,
- $A_t = A_0 \cdot (1+e)^t$, where $t = 1, \ldots, n$,
- n = number of interest or discount periods,
- i = interest or discount rate per period, and
- e = price escalation rate per period.

[a] Note that the USF, UCR, UCA, and UPV equations yield undefined answers when $i = 0$. The correct algebraic forms for this special case would be as follows: USF formula, $A = F/n$; UCR formula, $A = P/n$; UCA formula, $F = A \cdot n$; and UPV formula, $P = A \cdot n$. The UPV* equation also yields an undefined answer when $e = i$. In this case, $P = A_0 \cdot n$.

[b] The terms by which the known values are multiplied in these equations are the formulas for the factors found in discount factor tables. Using acronyms to represent the factor formulas, the discounting equations can also be written as $F = P \cdot SCA$, $P = F \cdot SPV$, $A = F \cdot USF$, $A = P \cdot UCR$, $F = A \cdot UCA$, $P = A \cdot UPV$, and $P = A_0 \cdot UPV^*$.

3. COMPUTATION OF SIMPLE PAYBACK (SPB)

The SPB method finds the amount of time (usually specified in years) between the date of initial project investment and the date when cumulative future earnings or savings on that investment, net of cumulative future costs, just pay off the investment. The minimum solution value of Y in equation 3.1 is the SPB.[1]

$$\sum_{t=1}^{Y} (B_t - C_t) - C_o = 0, \qquad (3.1)$$

where Y = number of years required for the project cash flows to offset the initial investment,

B_t = dollar value of benefits (including earnings, cost reductions or savings, and resale values, if any) in year t,

C_t = dollar value of costs (including operation, maintenance, and replacement) in year t,

$B_t - C_t$ = net cash flows in time period t,[2] and

C_o = initial project investment costs.

If the net cash flows are low, or the project life is short, or both, the project may not have a SPB; that is, it is undefined and the project will never pay off. (This is also true for DPB, as will be seen in section 4.)

3.1 Uniform Net Cash Flows

Direct solution of Y is not possible for equation 3.1. It is possible, however, for the case where the annual net cash flow is uniform; i.e., where $(B_t - C_t)$ is a constant, as shown in equation 3.2.

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[1]Benefits and costs in all computations should be in after-tax terms for all organizations that pay taxes.

[2]Cash flows are generally assumed to be end of year payments. The only exception is that cash flows are assumed to be spread evenly over the last year of payback so that partial years can be included in the payback measure.

$$SPB = Y = \frac{C_o}{(\overline{B - C})} \tag{3.2}$$

where $(B - C)$ = a uniform annual net cash flow.[1]

Another approach to solving for SPB when cash flows are uniform is to use the savings-to-investment ratio (SIR). The SIR relates the present value of annual savings to initial costs as shown in the general formula below:[2]

$$SIR = \frac{\sum_{t=1}^{N} \frac{(\overline{B_t - C_t})}{(1 + i)^t}}{C_o}, \tag{3.3}$$

where N = the total number of discounting periods in the study period.

When the net cash flow is a uniform annual amount, the SIR can be written as

$$SIR = \frac{UPV_{N,i} \cdot (\overline{B - C})}{C_o}. \tag{3.4}$$

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[1]Assuming that $(B_t - C_t)$ in equation 3.1 is a constant, $\sum_{t=1}^{Y} (B_t - C_t)$ becomes $Y\,(\overline{B - C})$. Rearrangement of terms in equation 3.1 yields $Y = \frac{C_o}{(\overline{B - C})}$.

[2]For a description of the SIR, see Harold E. Marshall and Rosalie T. Ruegg, Recommended Practice for Measuring Benefit/Cost and Savings-to-Investment Ratios.

By rearranging terms in equation 3.4 and substituting into equation 3.2, equation 3.5 is obtained for finding simple payback with the UPV factor and the SIR.

$$SPB = \frac{UPV_{N,i}}{SIR} . \tag{3.5}$$

Using equation 3.5 would be efficient only when values of UPV and SIR have already been obtained for other evaluation measures. Even then it involves at least as much effort as using equation 3.2., and therefore it would rarely be used.

3.2 Unequal Net Cash Flows

For problems with unequal annual cash flows, the most common approach to calculating SPB is to accumulate net future cash flows year by year and compare the cumulative amount at the end of each successive year to C_o. The payback year is the year in which the aggregate net cash flows just offset the initial investment costs.

This approach is illustrated with the problem outlined in table 3.1. A project with 5 years of unequal cash flows (column 2) is described in tabular form to facilitate computation of the SPB. Each additional year's net cash flow is added to the cumulative net cash flow of all preceding years (column 3). Investment costs (column 4) are subtracted from the cumulative net cash flow in each year to arrive at cumulative net benefits (column 5).

Table 3.1 SPB Problem

Years (t)	Net Cash Flow ($) $(B_t - C_t)$	Cumulative Net Cash Flow[a] ($) $\sum_{t=1}^{S} (B_t - C_t)$	Investment Cost ($) (C_o)	Cumulative Net Benefits[b] ($) $\sum_{t=1}^{S} (B_t - C_t) - C_o$
(1)	(2)	(3)	(4)	(5)=(3)-(4)
0			10,000	
1	4,000	4,000		-6,000
2	3,000	7,000		-3,000
3	2,500	9,500		-500
4	1,500	11,000		+1,000
5	1,000	12,000		+2,000

[a]S = the number of years over which net cash flows are accumulated.

[b]Net benefits in this column have not been discounted to account for the time value of money.

Inspection of table 3.1 shows that the investment cost is recouped sometime in the fourth year. Assuming that net cash flows are spread evenly over the fourth year, the SPB can be calculated as follows:

$$SPB = 3 \text{ years} + \frac{500}{1,500} \text{ years} = 3.33 \text{ years.}$$

Since the SPB is less than the number of years over which the project earns positive net benefits, we know that the project returns more undiscounted dollars than it costs initially.

Two economic measures derived from the payback (PB) method that are sometimes used are the "payoff period rate of return,"[1] calculated as (100/PB)%, and the "average yearly cash recovery,"[2] calculated as 1/PB. The payoff rate of return is equivalent to the percent form of the average yearly cash recovery. Both measures can be computed from the SPB measure as defined above or from the DPB measure to be explained in the next section. The shorter the payback, and therefore the higher the percent value, the more desirable would be the project.

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[1]E. J. Mishan, Cost-Benefit Analysis (New York: Praeger Publishers, 1982), p. 209.

[2]William R. Park, Cost Engineering Analysis (New York: John Wiley and Sons, 1973), p. 34.

4. COMPUTATION OF DISCOUNTED PAYBACK (DPB)

The DPB method finds the amount of time between the date of initial project investment and the date when the discounted present value of future earnings or savings, net of future discounted costs, just equals the initial investment. The DPB differs from the SPB in that the DPB, through discounting, accounts for the time value of money. The DPB can be determined mathematically or graphically.

4.1 Mathematical Solution

The minimum solution value of Y in equation 4.1 is the DPB.

$$\sum_{t=1}^{Y} \left[\frac{B_t - C_t}{(1 + i)^t} \right] - C_o = 0, \qquad (4.1)$$

where i = discount rate.

Direct solution of Y is not possible using equation 4.1.

4.1.1 Uniform Net Cash Flows

For the case where $(B_t - C_t)$ is a constant, Y can be isolated for direct solution by using logarithms, as illustrated in equation 4.2.[1]

$$DPB = Y = \frac{\log \left[\dfrac{1}{1 - (SPB \cdot i)} \right]}{\log \ (1 + i)} \quad \text{when } i \neq 0, \qquad (4.2)$$

$$DPB = SPB \quad \text{when } i = 0, \text{ and} \qquad (4.3)$$

DPB is undefined when $(SPB \cdot i) \overline{>} 1$.

[1]See Appendix A for the derivation of equation 4.2.

When $i = 0$, the stream of constant cash flows is in effect simply added, thereby yielding DPB = SPB. When $(SPB \cdot i) \overline{>} 1$, equation 4.2 cannot be solved because SPB is undefined.[1] However, we can infer certain values for DPB and for the present value of net benefits (PVNB) when $(SPB \cdot i) \overline{>} 1$. For example, as $(SPB \cdot i)$ approaches 1, we can infer that the DPB approaches infinity. Since $\lim_{Z \to \infty} \log Z = \infty$, and the $\lim_{(SPB \cdot i) \to 1} \left[\frac{1}{1 - (SPB \cdot i)}\right] = \infty$, then the $\lim_{(SPB \cdot i) \to 1} DPB = \infty$.

We can also show that the PVNB = 0 when $(SPB \cdot i) = 1$ and the study period is infinite. Since $SPB = C_o/(\overline{B - C})$, as shown in equation 3.2, and $i = (\overline{B - C})/PVCF$, where PVCF = present value of net cash flows,[2] then by substitution

$$(SPB \cdot i) = \frac{C_o}{(\overline{B - C})} \cdot \frac{(\overline{B - C})}{PVCF} = \frac{C_o}{PVCF} .$$

When $(SPB \cdot i) = 1$, then $C_o/PVCF$ also equals 1, and C_o must equal the PVCF that represents the present value of the $(\overline{B - C})$ annuity in perpetuity. Thus the PVNB (i.e., PVCF $- C_o$) must equal zero.

When $(SPB \cdot i) > 1$, no period of time is long enough for the project to pay back, and the PVNB will be negative.

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[1] When $(SPB \cdot i) = 1$, the divisor in the numerator of equation 4.2 is a zero, and when $(SPB \cdot i) > 1$, the numerator becomes a negative number.

[2] The formula for an annuity is $(P \cdot i) = A$, where P is a present value, i is the interest rate, and A is the uniform annual amount that will be earned in perpetuity. Letting P = PVCF and $A = (\overline{B - C})$, by rearrangement of terms we get $i = (\overline{B - C})/PVCF$.

An application of equation 4.2 is presented for the following investment problem. What would be the DPB for a project investment of $12,000, earning uniform annual net cash flows of $4,500 for 6 years? A 10% discount rate applies. Equation 4.2 would yield the following:

$$Y = \frac{\log \left[\dfrac{1}{1 - \dfrac{\$12{,}000 \cdot .10}{\$4{,}500}} \right]}{\log 1.10} = \frac{\log 1.3636}{\log 1.1000} = \frac{.1347}{.0414} = 3.25.$$

Since the DPB is less than the number of years over which the project earns constant net benefit returns, we know that the project is cost effective. If the DPB were longer than the period over which the project earns positive net benefits, then it would not be cost effective because the PVNB would be negative.

4.1.2 Unequal Net Cash Flows

For problems with unequal annual net cash flows, the most common approach to calculating DPB is to accumulate the present value of net cash flows year-by-year until the sum just offsets the original investment costs. The year in which the two become equal will be the DPB.

This approach is illustrated with the problem outlined in table 4.1. A project with 7 years of unequal cash flows (column 2) is evaluated at a discount rate of 12%. The net cash flow in each year is discounted at 12% to present value (column 3). Each year's addition to the present value is accumulated in column 4. The present value of net benefits (column 6) is derived by subtracting the investment costs from the cumulative, discounted, future net cash flows. The present value of net cash flows just offsets investment costs in the fifth year. The DPB can be calculated as follows:

$$\text{DPB} = 4 \text{ years} + \frac{3{,}011}{7{,}944} \text{ year} = 4.38 \text{ years.}$$

Since the DPB is less than the period over which the project earns positive net benefits, we know it is cost effective in the sense that the PVNB > 0.

Table 4.1 DPB Problem With Unequal Annual Cash Flows

Years (t)	Net Cash Flows ($) $(B_t - C_t)$	Discounted Net Cash Flows[a] ($) $\left[\frac{B_t - C_t}{(1+i)^t}\right]$	Cumulative Discounted Net Cash Flows ($) $\sum_{t=1}^{S}\left[\frac{B_t - C_t}{(1+i)^t}\right]$	Investment Cost ($) (C_o)	Cumulative PVNB ($) $\sum_{t=1}^{S}\left[\frac{B_t - C_t}{(1+i)^t}\right] - C_o$
(1)	(2)	(3)	(4)	(5)	(6) = (4) - (5)
0				50,000	
1	10,000	8,929	8,929		-41,071
2	20,000	15,944	24,873		-25,127
3	15,000	10,677	35,550		-14,450
4	18,000	11,439	46,989		-3,011
5	14,000	7,944	54,933		+4,933
6	12,000	6,080	61,013		+11,013
7	8,000	3,619	64,632		+14,632

[a]The discount rate = 12%.

An alternative approach to calculating DPB for a project with unequal annual cash flows is to find the time period for which the SIR = 1.0.[1] The DPB is the number of years required for the present value of future net savings to just cover initial costs. The procedure is to select a trial payback (P), calculate the SIR, and if the SIR is not close to one, select another P calculated from equation 4.4 and repeat the process. When the SIR approximates one, the last trial payback is in fact the true payback.[2]

Basing the P on previous results helps reduce the iterations required to find the P for which SIR = 1.0. In most cases, the following approach will provide P's that will converge on DPB within three or four iterations. Equation 4.4 yields the trial payback values.

$$P_i = \left[\frac{SIR_{i-1} - 1}{SIR_{i-1} - SIR_{i-2}} \right] \cdot \left[P_{i-2} - P_{i-1} \right] + P_{i-1} \, , \qquad (4.4)$$

where P_i = ith trial value of DPB,

i = index starting with the value one for the first calculation of P_i,

SIR_i = SIR computed for a study period of value P_i,

SIR_o = SIR computed for the original study period,

SIR_{-1} = 0,

P_o = original study period, and

P_{-1} = 0.

---

[1]Note that this approach will also work for projects with uniform net cash flows.

[2]Note that the trial P's are in fact the values of N used to calculate the SIR in equations 3.3 and 3.4.

The general approach can be summarized as follows:

- o The project SIR is calculated.
- o If the SIR is not close to 1.0, try another P calculated from equation 4.4.
- o Continue until the SIR is close to 1.0. The P for that SIR will approximate the DPB.

The method can be illustrated with the problem described in table 4.1. Assuming an intial study period of seven years, the SIR_{i-1} will be 1.29 (i.e., \$64,632/\$50,000). Applying equation 4.4, the first trial value of the DPB would be calculated as follows:

$$P_1 = \left[\frac{1.29 - 1.0}{1.29 - 0}\right] \cdot \left[0 - 7\right] + 7 = 5.43.$$

Substituting the value 5.43 for the study period N in equation 3.3 yields a new SIR of 1.15, (\$54,933 + .43 (\$6,080))/\$50,000. Since the SIR value exceeds 1.0, a second trial period is computed with equation 4.4. Using 1.15 as $SIR_{i=1}$ and 5.43 as $P_{i=1}$,

$$P_2 = \left[\frac{1.15 - 1.0}{1.15 - 1.29}\right] \cdot \left[7 - 5.43\right] + 5.43 = 3.75.$$

Substituting the value of 3.75 for the study period N in equation 3.3 yields a new SIR of .88, ((\$35,550) + .75 (\$11,439))/\$50,000. Since the SIR is still not close to 1.0, a third trial period is computed.

$$P_3 = \left[\frac{.88 - 1.0}{.88 - 1.15}\right] \cdot \left[5.43 - 3.75\right] + 3.75 = 4.50.$$

Substituting 4.50 into equation 3.3 yields a SIR of 1.02, (($46,989) + .5 ($7,944))/$50,000. This SIR is very close to 1.00 and the corresponding trial payback, 4.50, is close to the value computed earlier of 4.38.

The SIR-based approach is most useful when mathematical solutions of closed form are not available and when it is used either (1) with a standardized worksheet in a step-by-step procedure that simplifies the calculations or (2) in a computer application.

4.1.3 DPB With Escalation

The solution value of Y in equation 4.5 is the DPB with escalation (DPB_E).

$$\sum_{t=1}^{Y} \left(\frac{1 + e}{1 + i}\right)^{t} (B_t - C_t) - C_o = 0, \qquad (4.5)$$

where e = constant escalation rate of net cash flows (e.g., energy savings).

It accounts for annual net cash flows that escalate at a constant rate e.[1] Direct solution of Y is not possible using equation 4.5.

By using logarithms, however, as was done in equation 4.2, Y can be isolated for direct solution, as illustrated in equation 4.6.[2]

---

[1]In recent years investments in energy conservation projects typically involved escalating savings from reduced energy consumption due to energy prices rising faster than other prices.

[2]For a derivation of equation 4.6, see Appendix A.

$$DPB_E = Y = \frac{\log\left[1 + (SPB)\left(1 - \dfrac{1}{\dfrac{1+e}{1+i}}\right)\right]}{\log\left(\dfrac{1+e}{1+i}\right)} \quad \text{when } e \neq i, \qquad (4.6)$$

$$DPB_E = SPB \quad \text{when } e = i, \text{ and} \qquad (4.7)$$

$$DPB_E \text{ is undefined when } (SPB)\left(1 - \dfrac{1}{\dfrac{1+e}{1+i}}\right) \leq -1,$$

where DPB_E = discounted payback year with cash flow escalation.[1]

When $e > i$, the term $(1 + e)/(1 + i) > 1.0$, and therefore $DPB_E < SPB$. That is, the stream of net cash flows will be inflated more by escalation than it is reduced by discounting, resulting in a shorter DPB_E than SPB.[2] When $e < i$, the term $(1 + e)/(1 + i) < 1.0$, and thus $DPB_E > SPB$. That is, the discounting outweighs the escalation, resulting in a longer DPB_E than SPB. When $e = i$, the discounting just cancels out the escalation effects, and $DPB_E = SPB$. As long as the DPB_E is less than the period over which the project yields returns, the project is cost effective in the general sense that the $PVNB > 0$.

---

[1]A variation of equation 4.6 sometimes used is the following:

$$DPB_E = \frac{\log\left[\dfrac{(\overline{B - C})\,(1 + e)}{C_o\,(e - i) + (\overline{B - C})\,(1 + e)}\right]}{\log\left[\dfrac{1 + i}{1 + e}\right]}, \qquad \text{when } e \neq i.$$

[2]This assumes that cash flows are not inflated by the escalation factor when SPB is calculated.

When $(SPB)\left(1 - \frac{1}{\frac{1+e}{1+i}}\right) \leq -1.0$, then DPB_E is undefined because the logarithm of either zero or a negative number is undefined. If the value is -1.0, however, the DPB_E can be inferred to be approaching infinity, following the logic described in section 4.1 for DPB. When this term is -1.0, it can also be inferred that the PVNB = 0 when the study period is infinity. If $(SPB)\left(1 - \frac{1}{\frac{1+e}{1+i}}\right) < -1.0$, the project never pays for itself, even if it continues to earn net benefits forever, and the PVNB will be negative.

Equation 4.6 can be illustrated with the following problem. An energy conservation investment of \$40,000 yielding energy savings initially worth \$8,000 annually is to be evaluated with an 8% energy price escalation and a 12% discount rate. Applying equation 4.6 yields the following:

$$DPB_E = \frac{\log\left[1 + \left(\frac{\$40{,}000}{\$8{,}000}\right)\left(1 - \frac{1}{\frac{1.08}{1.12}}\right)\right]}{\log \frac{1.08}{1.12}}$$

$$= \frac{\log\ [1 + 5\ (\ -.0370)]}{\log\ .9643}$$

$$= \frac{\log\ .8150}{\log\ .9643}$$

$$= 5.63 \text{ years.}$$

If the conservation project yields energy savings for a period longer than 5.63 years, the PVNB of the project will be positive.

4.2 Graphical Solution[1]

The DPB for projects with uniform annual net cash flows or flows that increase at a constant rate can be found by using graphs. Since equation 4.2 for DPB and equation 4.6 for DPB_E are functions in part of SPB, and since the value of SPB is often desired along with that of DPB, the DPB graphs used here are presented as functions of SPB.

4.2.1 Uniform Net Cash Flows

Figure 4.1 plots DPB up to 10 years as a function of SPB for SPB values of zero to 4 years and discount rate values of 1% to 25% in 2% increments. Figure 4.2 is similar to figure 4.1 except that even values of the discount rate, from 2% to 24%, are used to plot DPB.[2] Figures 4.3 and 4.4 are the same respectively as figures 4.1 and 4.2, except that SPB values range from 4 to 12 years and DPB values range from 4 to 24 years. All of the curves are derived from equation 4.2. The procedure for finding DPB is to solve first for SPB using equation 3.2, and then to find the corresponding value of DPB on the curve for the given discount rate.

Taking the DPB problem from section 4.1.1, we can use the graphical approach to find the DPB for a \$12,000 investment earning uniform annual net cash flows of \$4,500 for 6 years. A 10% discount rate is used. The SPB is 2.7 (i.e., \$12,000 ÷ \$4,500). Thus figure 4.2 is used. Finding the value 2.7 on the

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[1]Graphs can be constructed that provide values of SPB for different combinations of initial costs and uniform cash flows. But given the ease with which SPB can be calculated from equation 3.2, there is no need for such graphs.

[2]The odd/even designation of the curves has no significance beyond making the values along the curves easier to read.

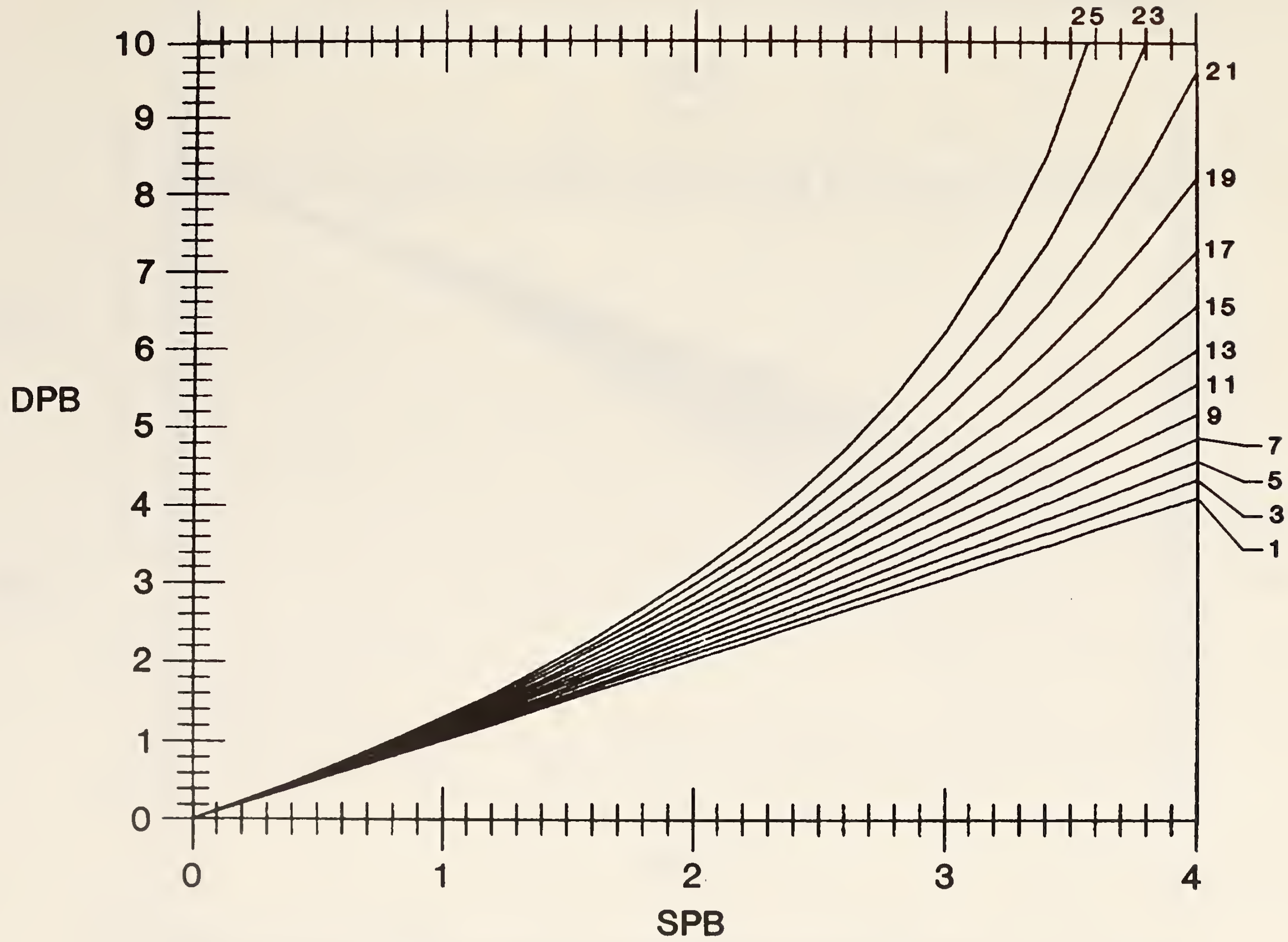

Figure 4.1. Discounted payback for SPB=0 to 4 years, i=1 to 25% (odd)

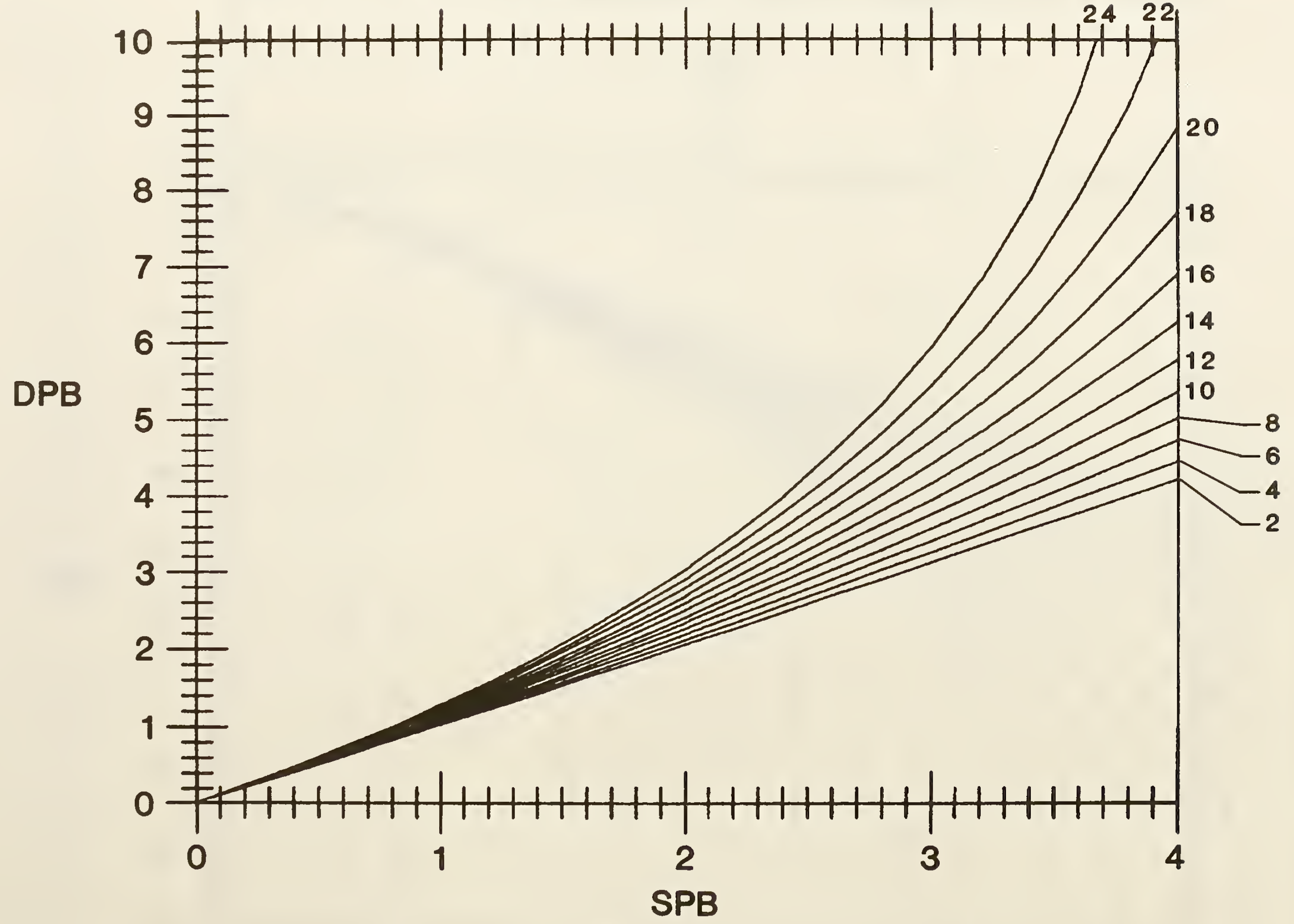

Figure 4.2. Discounted payback for SPB=0 to 4 years, i=2 to 24% (even)

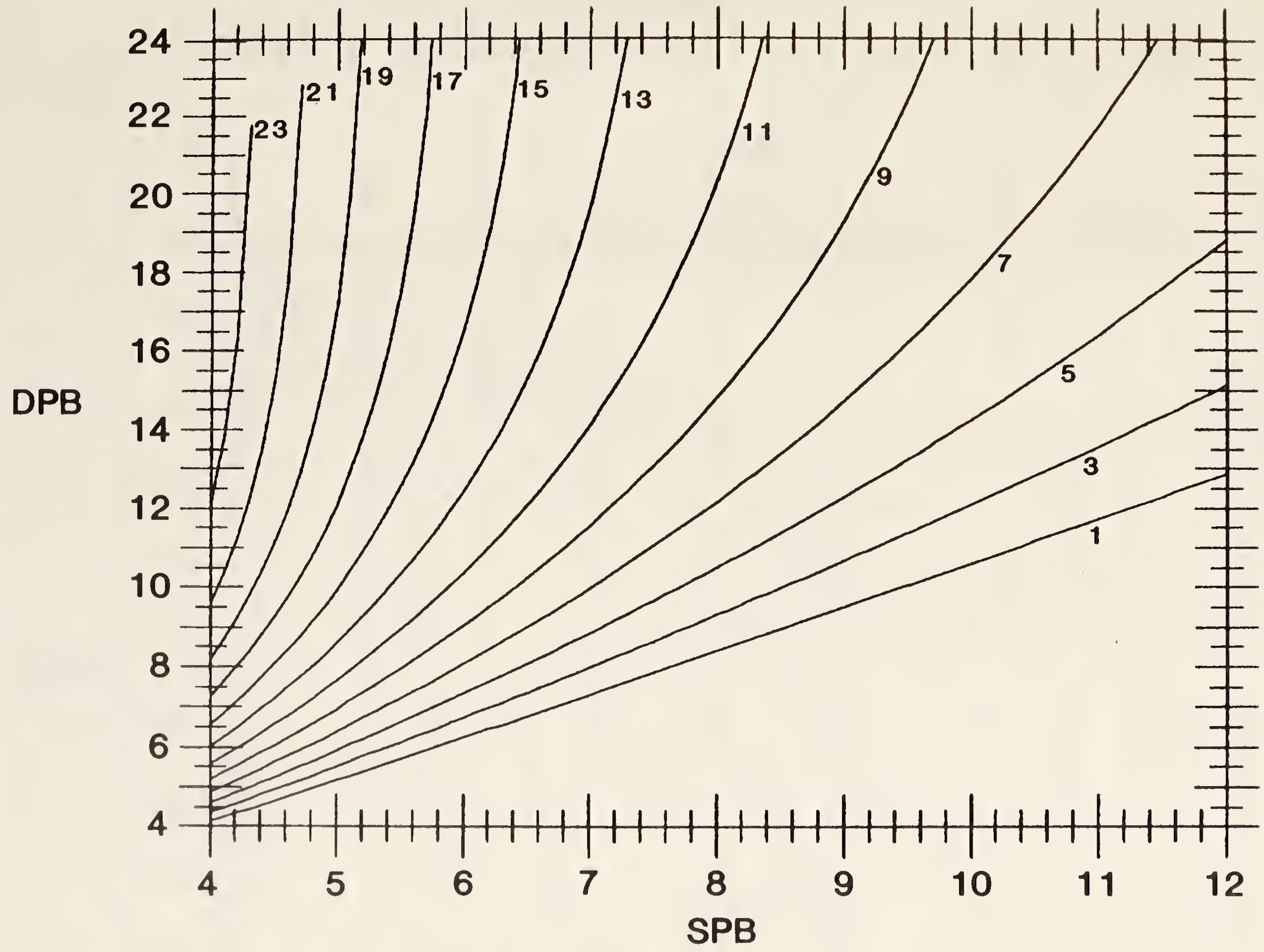

Figure 4.3. Discounted payback for SPB=4 to 12 years, i=1 to 23% (odd)

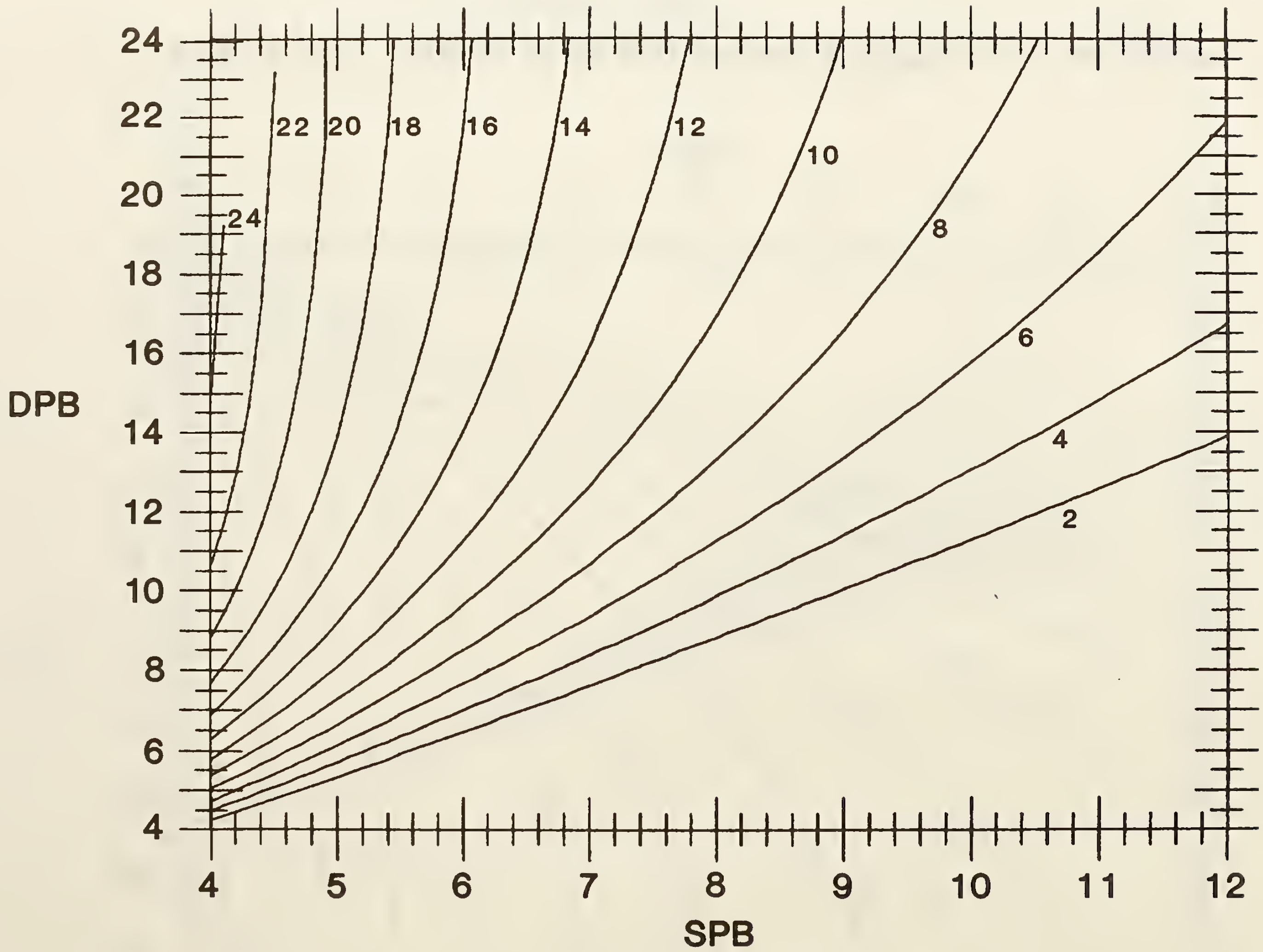

Figure 4.4. Discounted payback for SPB = 4 to 12 years, i = 2 to 24% (even)

horizontal SPB axis, a vertical line is drawn from that point to find its intersection with the DPB curve for 10%. Extending a line horizontally from that intersection to the vertical axis indicates a DPB value of approximately 3.3 corresponding to the SPB value 2.7.

The DPB > SPB for all values of $i > 0$. Thus the functions in figures 4.1 through 4.4 will all lie above the straight line function (not shown on the graphs) that would equate SPB to DPB when $i = 0$. Furthermore, due to the cumulative discounting effect, the DPB functions will increase at an increasing rate.

When the DPB value is greater than the limit of the vertical axis, such as is the case for SPB = 9 and i = 11% in figure 4.3, then the DPB value cannot be read from the graph and would have to be computed from equation 4.2.

As explained in section 4.1, any project for which $(SPB \cdot i) > 1$ will have an undefined DPB. Thus the project never pays off and there is no DPB. For example, in figure 4.3, a project evaluated with a SPB = 4.8 and a discount rate of 21% would never pay off, and consequently the DPB curve is truncated before it reaches a DPB value corresponding to an SPB of 4.8 on the horizontal axis.

4.2.2 DPB With Escalation

Two types of graphs are presented here for finding DPB_E. The first shows DPB_E as a function of the constant k, where $k = (1 + e)/(1 + i)$. The second type of graph relates DPB_E directly to SPB for specific escalation rates and discount rates. Each type of graph is presented because each has advantages over the other under certain circumstances.

Figures 4.5 through 4.8 present a family of DPB_E curves labeled with their k values. Each curve is derived from equation 4.6, where k has been substituted for (1 + e)/(1 + i). Figures 4.5 and 4.6 present respectively DPB_E for odd and even k values over the range k = .77 through 1.17, for SPB's up to 4 years. Figures 4.7 and 4.8 are the same respectively as figures 4.5 and 4.6, except that SPB values range from 4 to 12 years, and k values have a lower bound of .81.

The major advantage of plotting DPB_E for each value of k is that few graphs are required to describe many combinations of e and i. The use of figures 4.5 through 4.8 can be simplified by finding the value of k in table 4.2, which provides a matrix of k values for all likely combinations of e and i.

Using again the problem example for DPB_E in section 4.1.3, we illustrate the graphical approach in finding the DPB_E for a $40,000 investment initially yielding annual net savings of $8,000, with an energy price escalation rate of 8% and a discount rate of 12%. Since the SPB is 5 (i.e., $40,000 ÷ $8,000), we know the DPB_E will be found either in figure 4.7 or 4.8, which cover the SPB range 4 to 12 years. By consulting the matrix of table 4.2, we find a k value of .96 in the cell intersection for e = 8 and i = 12. Since the last digit of k is an even number, we look to figure 4.8 (even) for the DPB_E for 5 years and k = .96. The answer is approximately 5.7.

The DBP_E = SPB whenever k = 1, since the escalation and discounting effects cancel each other. When $k < 1$, $DPB_E > SPB$, and the curves will increase at an increasing rate. When $k > 1$, $DPB_E < SPB$, and the curves will increase at a decreasing rate.

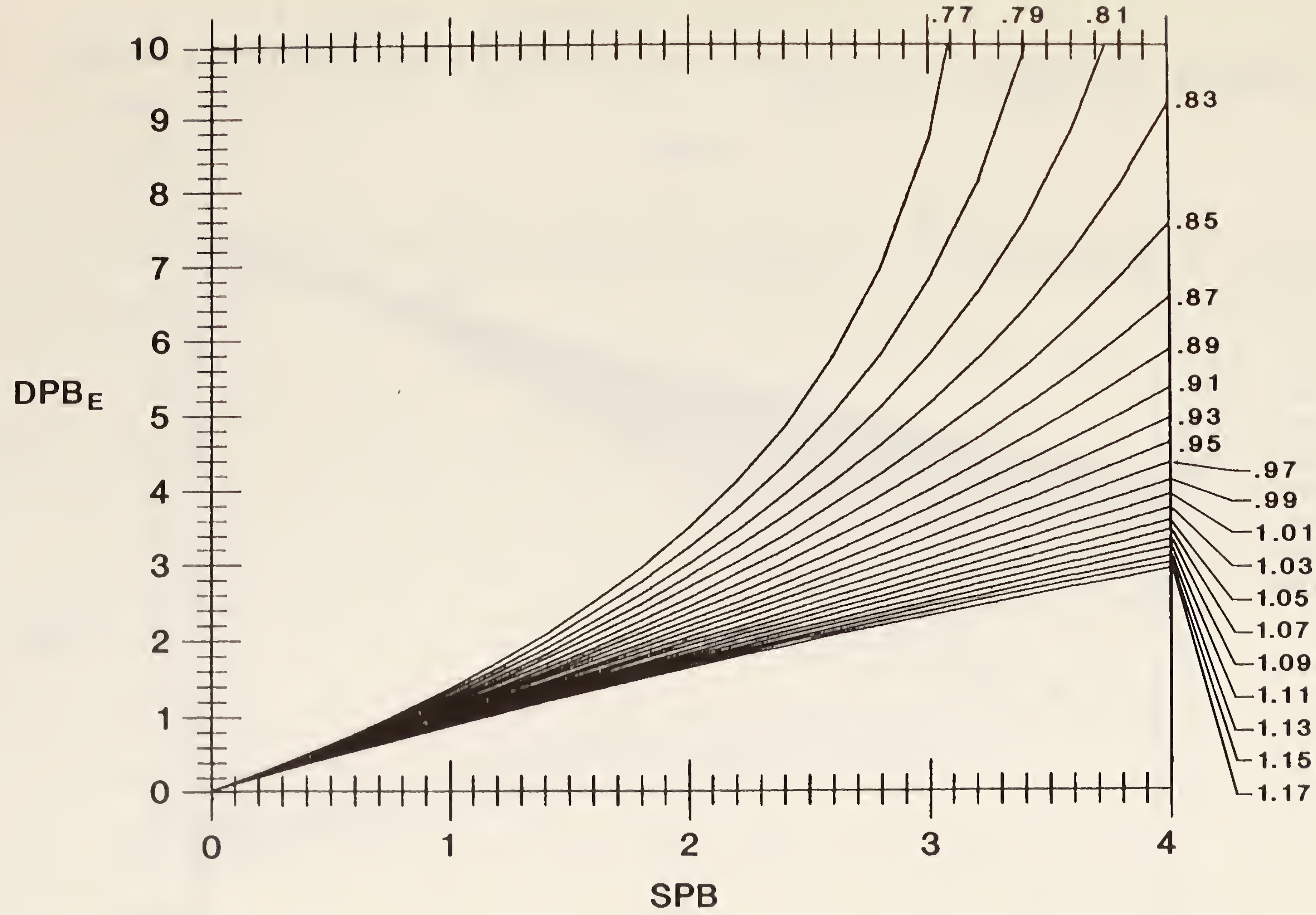

Figure 4.5. Discounted payback with escalation for SPB=0 to 4 years, k=.77 to 1.17 (odd)

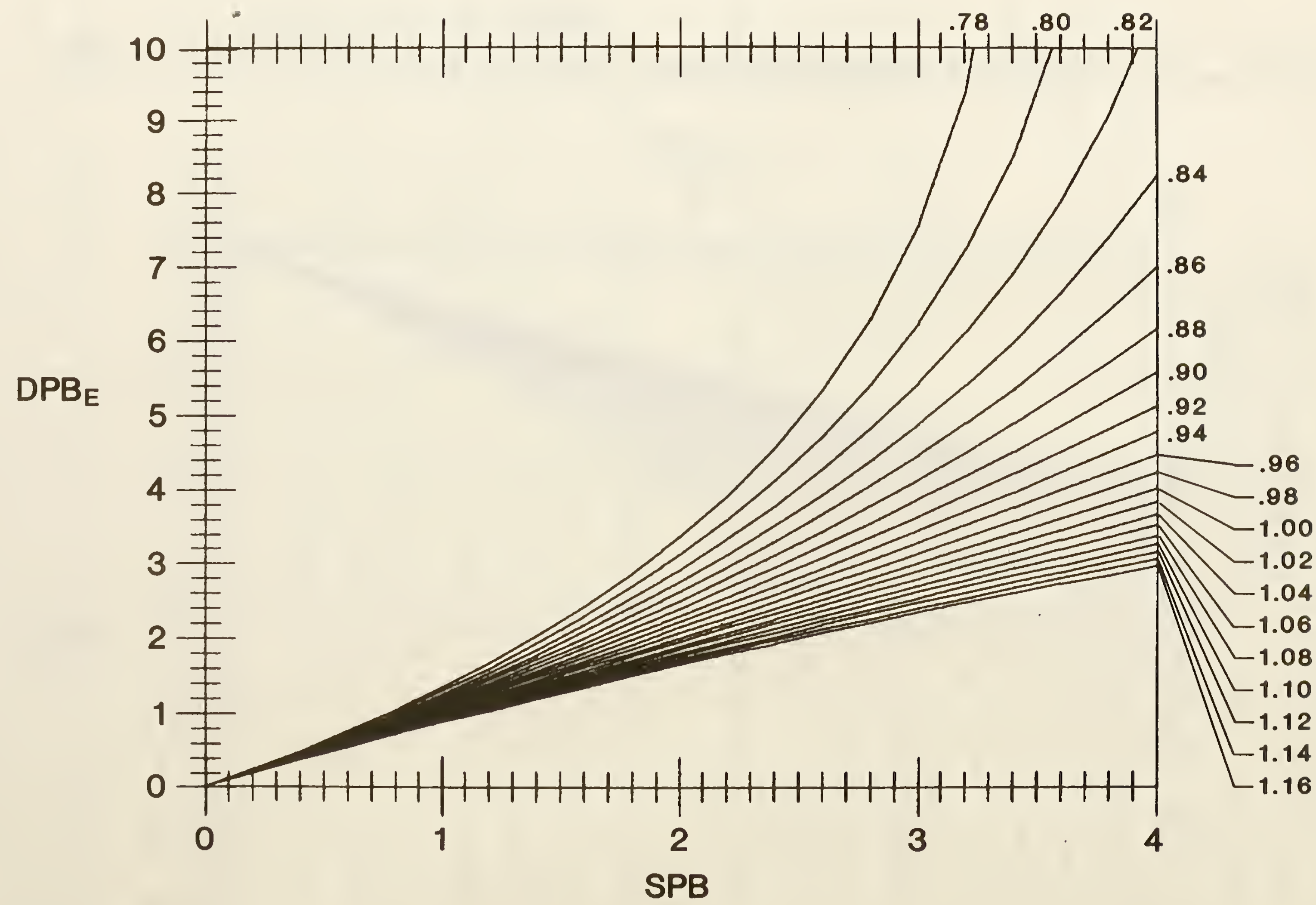

Figure 4.6. Discounted payback with escalation for SPB=0 to 4 years, k=.78 to 1.16 (even)

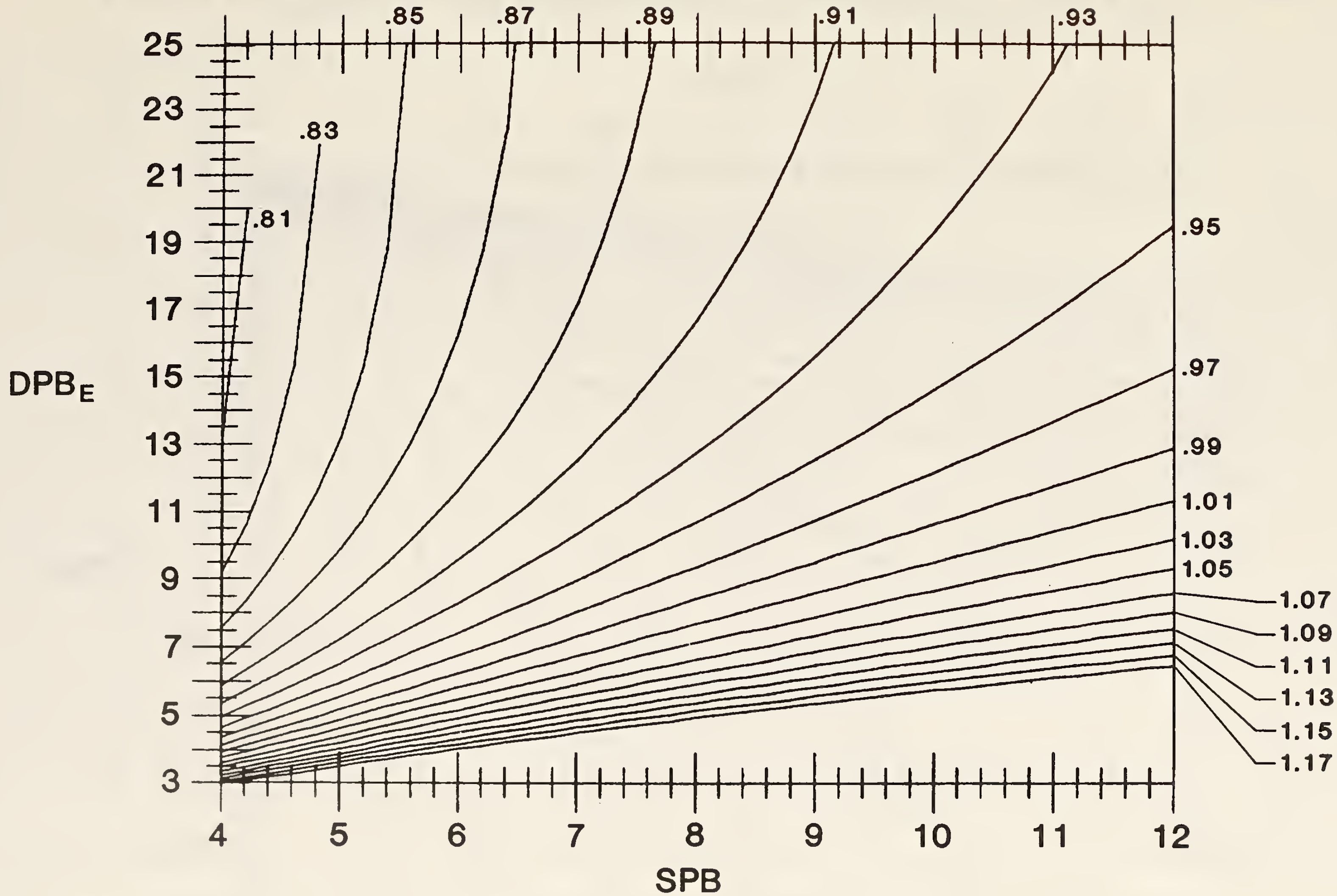

Figure 4.7. Discounted payback with escalation for SPB = 4 to 12 years, k = .81 to 1.17 (odd)

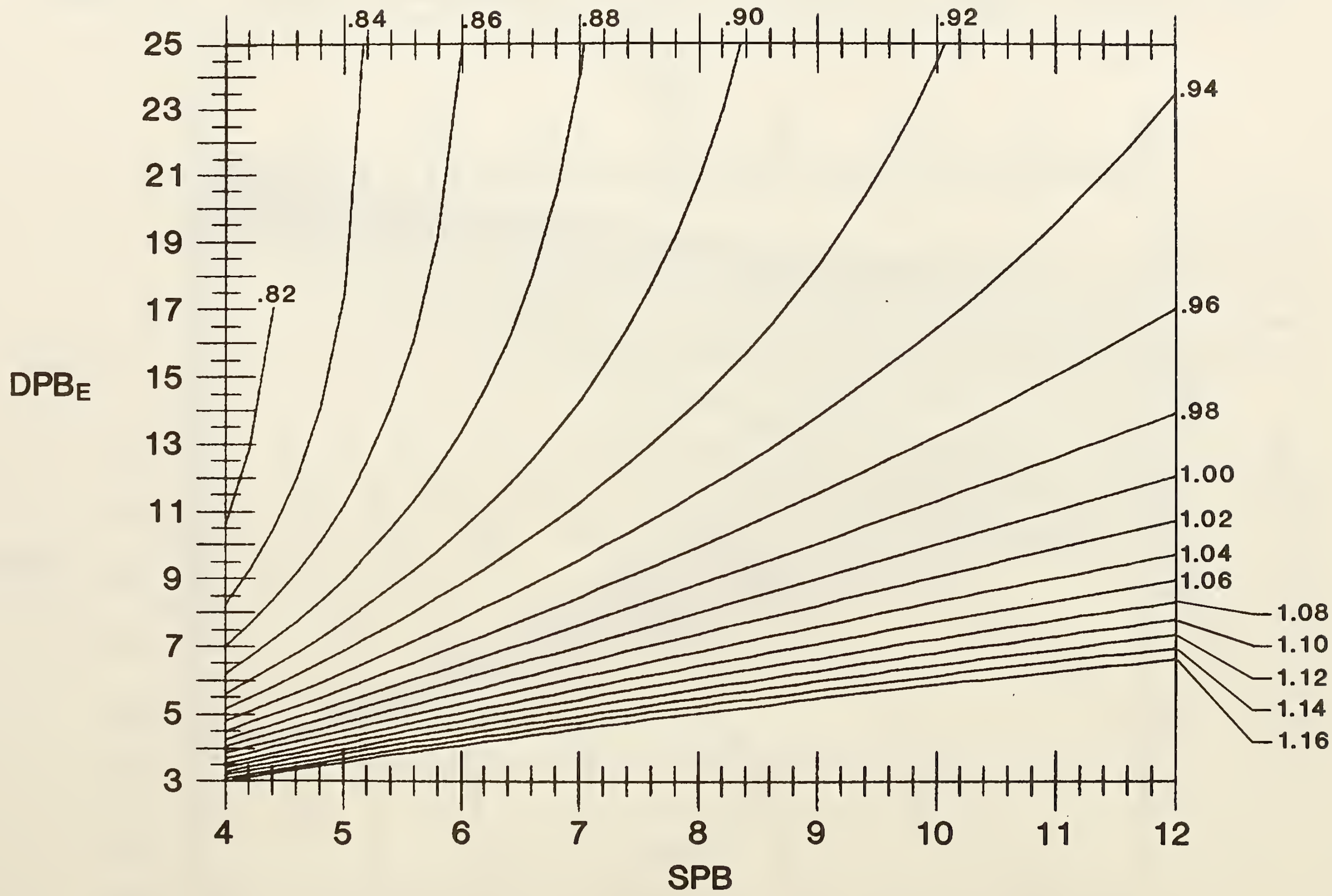

Figure 4.8. Discounted payback with escalation for SPB=4 to 12 years, k=.82 to 1.16 (even)

Table 4.2. Matrix of k values for combinations of e and i

i % \ e %	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
-2	0.98	0.99	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.13	1.14	1.15	1.16	1.17
-1	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.13	1.14	1.15	1.16
0	0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.13	1.14	1.15
1	0.95	0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.13	1.14
2	0.94	0.95	0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.13
3	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.09	1.10	1.11	1.12
4	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.09	1.10	1.11
5	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.09	1.10
6	0.91	0.92	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.08
7	0.90	0.91	0.92	0.93	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.07
8	0.89	0.90	0.91	0.92	0.93	0.94	0.94	0.95	0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.06
9	0.88	0.89	0.90	0.91	0.92	0.93	0.94	0.94	0.95	0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04	1.05	1.06
10	0.87	0.88	0.89	0.90	0.91	0.92	0.93	0.94	0.95	0.95	0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04	1.05
11	0.86	0.87	0.88	0.89	0.90	0.91	0.92	0.93	0.94	0.95	0.95	0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04
12	0.86	0.87	0.88	0.88	0.89	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03
13	0.85	0.86	0.87	0.88	0.88	0.89	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.96	0.97	0.98	0.99	1.00	1.01	1.02
14	0.84	0.85	0.86	0.87	0.88	0.89	0.89	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.96	0.97	0.98	0.99	1.00	1.01
15	0.83	0.84	0.85	0.86	0.87	0.88	0.89	0.90	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.97	0.98	0.99	1.00
16	0.83	0.84	0.84	0.85	0.86	0.87	0.88	0.89	0.90	0.91	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.97	0.98	0.99
17	0.82	0.83	0.84	0.85	0.85	0.86	0.87	0.88	0.89	0.90	0.91	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.97	0.98
18	0.81	0.82	0.83	0.84	0.85	0.86	0.86	0.87	0.88	0.89	0.90	0.91	0.92	0.92	0.93	0.94	0.95	0.96	0.97	0.97
19	0.81	0.82	0.82	0.83	0.84	0.85	0.86	0.87	0.87	0.88	0.89	0.90	0.91	0.92	0.92	0.93	0.94	0.95	0.96	0.97
20	0.80	0.81	0.82	0.82	0.83	0.84	0.85	0.86	0.87	0.88	0.88	0.89	0.90	0.91	0.92	0.93	0.93	0.94	0.95	0.96
21	0.79	0.80	0.81	0.82	0.83	0.83	0.84	0.85	0.86	0.87	0.88	0.88	0.89	0.90	0.91	0.92	0.93	0.93	0.94	0.95
22	0.79	0.80	0.80	0.81	0.82	0.83	0.84	0.84	0.85	0.86	0.87	0.88	0.89	0.89	0.90	0.91	0.92	0.93	0.93	0.94
23	0.78	0.79	0.80	0.80	0.81	0.82	0.83	0.84	0.85	0.85	0.86	0.87	0.88	0.89	0.89	0.90	0.91	0.92	0.93	0.93
24	0.77	0.78	0.79	0.80	0.81	0.81	0.82	0.83	0.84	0.85	0.85	0.86	0.87	0.88	0.89	0.90	0.90	0.91	0.92	0.93
25	0.77	0.78	0.78	0.79	0.80	0.81	0.82	0.82	0.83	0.84	0.85	0.86	0.86	0.87	0.88	0.89	0.90	0.90	0.91	0.92

When the DPB_E value is greater than the limit of the vertical axis, such as would be the case for a SPB = 8.4 and a k = .90 in figure 4.8, then the DPB value cannot be read from the graph and would have to be computed from equation 4.6.

When the DPB_E is undefined, i.e., when the term $(SPB)(1 - 1/k) < -1$, the project never pays off, so there will be no reading from the graph and there will be no solution to equation 4.6.

The second type of DPB_E graph is illustrated with figures 4.9 and 4.10. Each figure presents a family of DPB_E curves for a given rate of discount. Equation 4.6 is used to plot the curves. Figure 4.9, for example, yields the DPB_E curves for a discount rate of 7% and for escalation rates of 0%, 2%, 4%, 6%, 8%, and 10%. Each curve is labeled with its escalation rate. Figure 4.10 does the same for a discount rate of 10%. An advantage of this approach over the k graphs in figures 4.5 through 4.8 is that no k factor has to be determined and the DPB_E can be found directly from the graph once the SPB is calculated. However, the disadvantage is that many graphs would be required to cover the feasible combinations of e and i that are covered just with the figures 4.5 through 4.8.

To read the graphs in figures 4.9 and 4.10, find the figure for the given discount rate, extend a vertical line for the given SPB value, and find the corresponding DPB_E where the vertical line intersects the DPB_E curve for the given escalation rate. For example, if we had a SPB = 10, an i = 7%, and an escalation rate of zero, we would find a DPB_E of about 17.8 years.

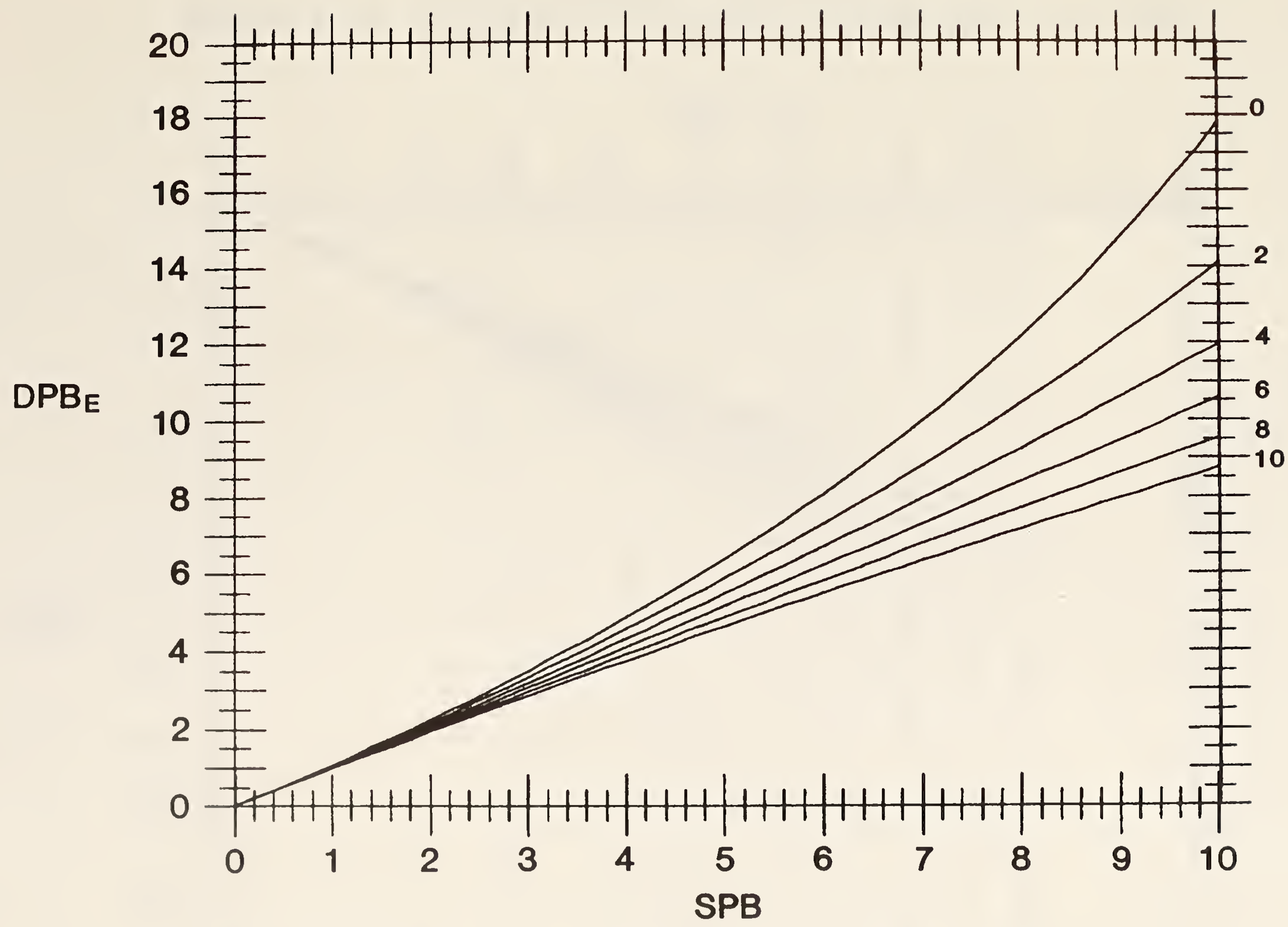

Figure 4.9. Discounted payback for SPB=0 to 10 years, e=0 to 10% (even), i=7%

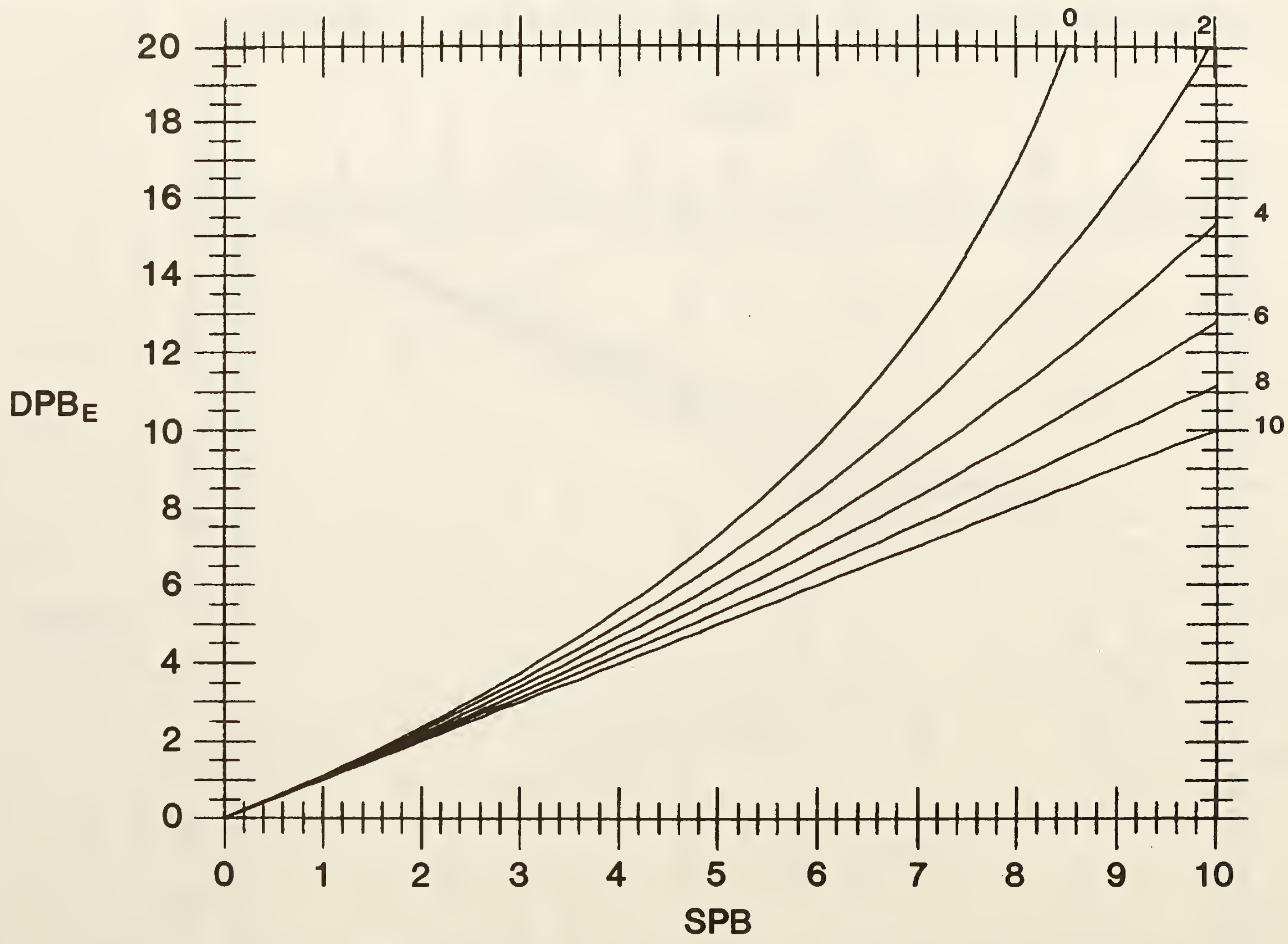

Figure 4.10. Discounted payback for SPB=0 to 10 years, e=0 to 10% (even), i=10%

For $e < i$, $DPB_E > SPB$, and the curves will increase at an increasing rate and lie above the straight-line function (not shown in the figures) where $DPB_E = SPB$ and $e = i$. When $e > i$, $DPB_E < SPB$, and the curves will increase at a decreasing rate and lie beneath the straight-line function where $DPB_E = SPB$.

Figures 4.9 and 4.10 will give slightly different answers for the same SPB's than will figures 4.5 through 4.8 because the value of k in table 4.2 on which the k plots are based is rounded to 2 decimal places whereas the corresponding value of $(1 + e)/(1 + i)$ in figures 4.9 and 4.10 is computed to 8 decimal places in making the plots. These differences in answers will become greater as k decreases and the curves in figures 4.5 through 4.8 increase at an increasing rate. If a greater degree of accuracy is required using the k matrix, the k values in table 4.2 could be carried out to more decimal places and more k plots at these intermediate values could be generated. Alternatively, equation 4.6 could be used to solve directly for DPB_E with k calculated to whatever number of decimal places is desired.

5. RECOMMENDED APPLICATIONS AND LIMITATIONS

This section discusses whether the SPB or DPB is a better measure of a project's economic worth. Recommendations regarding the applicability of payback methods are made for typical investment problems where there is some controversy as to which economic evaluation method is appropriate. Limitations of the payback method in general are described. The use of payback as a supplementary method of economic evaluation is discussed.

5.1 SPB or DPB

Because SPB is quick and easy to compute, it is appropriately used as a screening "first measure" of project worth when time, resources, or expertise are not available for calculating a DPB measure. The SPB measure also may have some value as an accounting measure in that it is computed in current, undiscounted dollars and indicates when actual net cash flows just balance investment costs.

Time and resources permitting, the DPB measure will be preferred to the SPB measure because DPB accounts for the time value of money. During a period when inflation, real earning opportunities for capital, or both result in a positive discount rate, a given cash amount in the future will be worth less than that same amount today. Thus the DPB measure will be longer than the SPB measure for the same set of cash flows over time. Using a SPB measure under these circumstances suggests a shorter payback than the project really has. The DPB measure rather than the SPB measure is required to determine when the present value of future net cash flows equals the initial investment.

5.2 Payback Versus Other Economic Methods

Table 5.1 lists some typical economic questions pertaining to investment projects and indicates whether payback measures are appropriate for answering those questions. Payback information may be helpful in answering the first two questions, but will generally lead to economically poor investment decisions if used to answer the last three questions.

Table 5.1 Economic Questions and Usefulness of Payback

	Use Payback? Yes	Use Payback? No
1. How to compare benefits with costs?	X	
2. How to find the break-even year?	X	
3. How large an investment to make?		X
4. How can projects competing for the same purpose be compared?		X
5. How can different-purpose projects competing for the same budget be compared?		X

The first question, how can benefits be compared with costs, can be reliably answered under some circumstances with payback measures. For example, if a project is being examined to determine if it is cost effective in the general sense that PVNB > 0, the DPB can be used to test for cost effectiveness. That is, if the DPB is less than the period over which positive net benefits are earned, then the project is cost effective. If negative cash flows occur beyond the payback year, however, one of the following methods would be needed to determine if the project were cost effective: net benefits analysis (to see if the present value of net benefits exceeds zero), the adjusted internal rate of return, AIRR (to see if the AIRR is greater than a minimum acceptable rate of return), or the savings-to-investment ratio (SIR) or benefit-to-cost ratio (BCR) method (to see if the SIR or BCR is greater than 1.0).[1] Thus payback as the sole measure of a project's worth is limited in usefulness to those cases where the stream of benefits clearly dominates all costs over the project's life.

Another approach sometimes used in payback analysis is to establish a maximum acceptable payback period (MAPP) as a cutoff value for accepting or rejecting projects. For example, an arbitrary time of 3 years might be determined by the management of a firm as the MAPP they will allow. Thus all projects with paybacks greater than 3 years, regardless of their cost effectiveness as measured by the PVNB, will be rejected.

Table 5.2 illustrates the PVNB implications of using a MAPP to decide whether a project is to be accepted or rejected. If the cutoff payback value were 2.5, for example, no project with a payback longer than 2.5 years would be accepted. Thus project A, with SPB of 3.1 and DPB of 3.9, would be rejected. Yet project A is cost effective because it has a positive PVNB. Rejecting it would result in an economic efficiency loss of $30. Project B, on the other hand, meets the MAPP criterion in that its paybacks are less than 2.5 years. However,

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[1]For a description of net benefits analysis and the AIRR, see Harold E. Marshall, Recommended Practice for Measuring Net Benefits and Internal Rates of Return. For a description of the SIR and BCR, see American Society for Testing and Materials, "Standard Practice for Measuring Benefit-to-Cost and Savings-to-Investment Ratios for Buildings and Building Systems," E-964, Annual Book of ASTM Standards (Philadelphia, PA: ASTM, 1984).

Table 5.2 Using Payback to Accept or Reject Projects[a]

Projects	$(B_t - C_t)$ ($) Years/1	2	3	4	C_o ($)	PVNB ($)	SPB	DPB
A	325	325	325	325	1,000	30	3.1	3.9
B	800	500	-100	-300	1,000	-140	1.4	1.7

[a]A 10% discount rate is used to compute $PVNB = \sum_{t=1}^{N} \left[\frac{(B_t - C_t)}{(1 + i)^t} \right] - C_o$.

accepting project B will result in an economic efficiency loss of $140 in PVNB due to negative cash flows beyond the payback year.[1] Since accept/reject decisions made with the payback method may lead to such efficiency losses, the other evaluation methods listed above are recommended for making these decisions whenever benefits do not obviously dominate costs over the project's life.

The second question, how to find the break-even year, can be answered correctly by using the payback method. The number of years required for a

[1]We recognize that a manager concerned with efficiency would not implement project B as described in this simple example because losses in the final two years make the project cost ineffective. However, the manager must look beyond the payback year to see this, and that is why the other methods that look at the whole study period are recommended over payback.

project to just break even (i.e., the number of years for earnings or savings to pay off all project costs) is called the break-even year.[1] Hence it is the same as the payback year. Although the break-even (payback) year is helpful to an investor in deciding whether or not to undertake a project, it does not show that a project is cost effective except under the special conditions described above.

The third question in table 5.1, how large an investment to make, cannot be answered properly using payback measures. That is, sizing a project so as to minimize payback would lead to inefficient projects (i.e., projects that do not maximize the PVNB). The net benefits method, on the other hand, is an appropriate method.

Table 5.3 illustrates the efficiency implications of using payback instead of the net benefits method in choosing project size. Both payback measures favor size C, requiring a $1,000 investment and yielding a PVNB of $751. However, the PVNB is maximized at size D for an investment of $2,000 which yields a PVNB of $1,073.[2] Choosing size C on the basis of payback would result in an economic loss of $322 ($1,073 -$751).

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[1]Note that break-even analysis in general can be applied to any variable that affects the net benefits of a project. For example, the break-even wage rate might be computed for a labor-intensive construction job to determine the wage rate at which the builder would just break even.

[2]It is assumed for this example that there is no budget limitation regarding initial project investment costs.

Table 5.3 Project Sizing With Payback and Net Benefits[a]
(Ten Year Project Life)

Size (1)	$(B_t - C_t)$ Each Year (%) (2)	(C_o) ($) (3)	PVNB ($) (4)	SPB (5)	DPB (6)
C	285	1,000	751	3.5	4.5
D	500	2,000	1,073	4.0	5.4
E	1,000	8,000	-1,855	8.0	16.9

[a]The discount rate used is 10%.

The fourth question, how can projects competing for the same purpose be compared, again is best answered by the net benefits method. Since benefits and costs beyond the payback year are ignored, selecting the project with the minimum payback biases project selection against projects with a high proportion of net benefits accruing in the future. Thus the project with the shortest payback period may not be the one that maximizes the PVNB.

Table 5.4 illustrates how payback can lead to selection of an inefficient project when two projects are competing for the same purpose. Project F earns all of its benefits in the last 3 years, whereas project G earns most of its benefits in the first 2 years. Both projects are cost effective in the sense that they earn in present value terms more than they cost (i.e., the PVNB is greater than zero). However, project F is the more efficient choice since it yields a larger PVNB than project G.[1] If minimum payback were the criterion for choosing, project G

[1]In this and subsequent illustrations risk is assumed to be the same among project alternatives and not to increase as a function of time.

would be preferred, because its payback is shorter than F's. The economic loss from choosing according to the minimum payback would in this case be \$402 (\$555 - \$153).

Table 5.4 Project Acceptance, PVNB Versus SPB or DPB[a]

	$(B_t - C_t)$ (\$)								
Projects	Years/1	2	3	4	5	C_o	PVNB	SPB	DPB
F	0	0	1,000	1,000	1,000	1,500	555	3+	5+
G	700	600	10	10	10	1,000	153	2+	2+

[a]A 10% discount rate is used to compute $PVNB = \sum_{t=1}^{N} \left[\frac{(B_t - C_t)}{(1 + i)^t} \right] - C_o$.

The last question in table 5.1, how can different-purpose projects competing for the same budget be compared, can be answered properly with the savings-to-investment ratio (SIR) method or adjusted internal rate of return (AIRR) method. Either of these methods when correctly applied will generally lead to a set of projects that maximizes the PVNB for the limited budget.

Table 5.5 illustrates the losses in net benefits that would result if a limited budget of \$1,800 were allocated among a set of different-purpose projects on the basis of minimum paybacks. Four projects with earnings from 2 to 8 years are considered. Uniform annual cash flows are shown in column 3, initial investment costs are shown in column 4, and the annual values of initial costs (converted at a 10% discount rate) are shown in column 5.[1]

[1]The equation for converting present values (P) to annual values (A) is the Uniform Capital-Recovery (UCR) Equation in table 2.1.

Table 5.5 Ranking Different-Purpose Projects

Projects	Years (N)	$\overline{(B_t - C_t)}$ ($)	C_o ($)	Annual Value of C_o ($)	AVNB ($)	SIR	SPB	DPB[a]
(1)	(2)	(3)	(4)	(5)	(6)=(3)-(5)	(7)=(3)÷(5)	(8)=(5)÷(3)	(9)
H	8	750	1,000	187	563	4.0	1.3	1.5
I	5	308	500	132	176	2.3	1.6	1.9
J	3	643	800	322	321	2.0	1.2	1.4
K	2	864	1,000	576	288	1.5	1.2	1.3

[a]DPB is computed from equation 4.2.

Given the budget of $1,800, projects J and K would be chosen if minimum paybacks were the criterion for choosing projects. The AVNB (column 6) for J and K would be $609. At first glance, projects J and K might seem attractive in their quick recovery of first cost. But two other combinations would yield greater returns. For example, if the same $1,800 were allocated to H and I, on the basis of the highest SIR's, an AVNB of $739 would result, with $300 left in the budget. Payoff of the initial costs would take a little longer, but the extra benefits in subsequent years make selection of projects H and I worth $739 as compared to the J and K combined worth of only $609 on an annual basis. Following the SPB or DPB method here would result in a net efficiency loss of $130 ($739 - $609) per annum. Alternatively, to get the most from the $1,800, projects H and J could be selected by moving to the project with the next lowest value of SIR which exhausts the budget.[1] The AVNB would then be $884. Choosing J and K instead of H and J would result in a net efficiency loss of $275 ($884 - $609).

[1]For an explanation of how to apply the SIR technique under limited budget conditions, see ASTM Standard Practice for Measuring Benefit-to-Cost and Savings-to-Investment Ratios, section 10.

5.3 Payback As a Supplementary Method

Most of the questions in table 5.1 have been marked "No" regarding the applicability of payback. It received an unqualified yes only in finding the break-even value. In fact, much of the engineering economics literature dismisses payback from serious consideration as a reliable economic decision tool. For example, Park, using the term payout for payback, states the following:

> In summary, the payout period is often used and referred to as a "quick-and-dirty" means of looking at the relative attractiveness of investment proposals. But by itself the payout time is <u>so</u> dirty that the analyst cannot see what lies underneath--or worse still, sees the wrong thing. Therefore the payout period should never be used as the sole criterion in investment evaluation and can seldom be justified for any use other than as an interesting, even if irrelevant, piece of information.[1]

Grant and Ireson also criticize the use of payback, as described in the following statement:

> Except for the special case where funds are so limited that <u>no</u> outlay can be made unless the money can be recovered in an extremely short time ..., the payout time is <u>never</u> an appropriate way to compare a group of proposed investments.[2]

However, the primary contribution of payback lies not in its use as a method for making major decisions, but in its use as a supplementary method of economic evaluation. That is, it gives one kind of information that, in conjunction with other economic measures, helps determine the economic desirability of one or more projects.

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[1] William R. Park, <u>Cost Engineering Analysis</u>, pp. 35 and 36.

[2] Eugene L. Grant and W. Grant Ireson, <u>Principles of Engineering Economy</u>, 5th ed. (New York: The Ronald Press Co., 1970) p. 528.

As a supplementary method, payback is probably most helpful as a screening method for evaluating investment candidates that have limited lives beyond which potential returns may become irrelevant. The payback measure helps define the feasible set of projects to which additional economic methods can be applied. For example, an investor who is considering investments in a foreign country might establish a MAPP of 2 years if nationalization, revolution, political instability, or other conditions that might diminish returns on the investment were likely to occur within several years. A manufacturer of building components, for example, might establish a MAPP of only 3 or 4 years for a product line that could potentially yield profits for 10 years, because the uncertainty and risk of such factors as obsolescence, competitive products, and shifting market conditions might threaten the manufacturer's profit potential after 3 or 4 years.

Numerous other reasons can be found for the widespread applications of payback in practice. It is easy to determine and to understand intuitively. It can be used to indicate how long an investor's capital is at risk in terms of how many years are required before payoff. It serves as an index to short-run earnings per share of stock. It helps to identify projects that will be unusually profitable or unprofitable early in their life. And finally, with tight capital conditions, investors often want to be assured of short paybacks in addition to high rates of return or high PVNB before they will part with their capital.

The basic limitations of payback remain, however. It ignores net benefits beyond the payback year. This imposes a bias against long-term projects such as public works or commercial projects with significant research and development startups in favor of projects with quick payoffs. The SPB measure further distorts economic evaluations by ignoring the time value of money. Thus to make economically efficient choices among competing projects and among designs/sizes for a single project, payback as an evaluation method is generally appropriate only when used as a supplementary method with other economic evaluation methods.

APPENDIX A

DERIVATION OF DPB AND DPB_E[1]

A.1 DPB (Equation 4.2)

We know that the solution value of Y in equation 4.1 is the DPB. By rearranging terms in equation 4.1 we get

$$\sum_{t=1}^{Y} \frac{1}{(1+i)^t} (B_t - C_t) = C_o.$$

Given that $B_t - C_t$ is a constant,

$$(\overline{B-C}) \sum_{t=1}^{Y} \frac{1}{(1+i)^t} = C_o,$$

$$\sum_{t=1}^{Y} \frac{1}{(1+i)^t} = \frac{C_o}{(\overline{B-C})} \quad \text{, and}$$

$$\sum_{t=1}^{Y} \frac{1}{(1+i)^t} = \text{SPB}.$$

Since $\sum_{t=1}^{Y} \frac{1}{(1+i)^t} = \frac{(1+i)^Y - 1}{i\,(1+i)^Y}$, the UPV factor formula from table 2.1,

$$\text{SPB} = \frac{(1+i)^Y - 1}{i\,(1+i)^Y} \quad .$$

---

[1]Thanks are due Robert E. Chapman and Stephen F. Weber for their help in the derivation of these equations.

By rearranging terms,

$$i \cdot SPB = \frac{(1+i)^Y - 1}{(1+i)^Y},$$

$$i \cdot SPB = 1 - \frac{1}{(1+i)^Y}, \text{ and}$$

$$\frac{1}{(1+i)^Y} = 1 - (i \cdot SPB).$$

Taking logarithms of both sides of the equation,

$$-Y \log (1+i) = \log (1 - i \cdot SPB),$$

$$Y \log (1+i) = -\log (1 - i \cdot SPB),$$

$$Y \log (1+i) = \log \left[\frac{1}{1 - (i \cdot SPB)}\right], \text{ and}$$

$$Y = \frac{\log \left[\frac{1}{1 - (SPB \cdot i)}\right]}{\log (1+i)}.$$

A.2 DPB_E (Equation 4.6)

We know that the solution value of Y in equation 4.5 is the DPB_E. Given that $(B_t - C_t)$ is a constant, we can rearrange equation 4.5 as follows:

$$\overline{(B - C)} \sum_{t=1}^{Y} \left(\frac{1+e}{1+i}\right)^t = C_o,$$

$$\sum_{t=1}^{Y} \left(\frac{1+e}{1+i}\right)^t = \frac{C_o}{\overline{(B - C)}}, \text{ and therefore}$$

$$\sum_{t=1}^{Y}\left(\frac{1+e}{1+i}\right)^{t} = SPB.$$

Since $$\sum_{t=1}^{Y}\left(\frac{1+e}{1+i}\right)^{t} = \left(\frac{1+e}{i-e}\right)\left[1-\left(\frac{1+e}{1+i}\right)^{Y}\right],$$

the UPV* factor formula from table 2.1,

$$SPB = \left(\frac{1+e}{i-e}\right)\left[1-\left(\frac{1+e}{1+i}\right)^{Y}\right].$$

By rearranging terms,

$$\left(\frac{i-e}{1+e}\right) SPB = \left[1-\left(\frac{1+e}{1+i}\right)^{Y}\right],$$

$$1-\left(\frac{i-e}{1+e}\right) SPB = \left(\frac{1+e}{1+i}\right)^{Y},$$

$$Y \log\left(\frac{1+e}{1+i}\right) = \log\left[1-\left(\frac{i-e}{1+e}\right) SPB\right], \text{ and}$$

$$Y = \frac{\log\left[1-\left(\frac{i-e}{1+e}\right)SPB\right]}{\log\left(\frac{1+e}{1+i}\right)}$$

$$Y = \frac{\log\left[1+SPB\left(\frac{e-i}{1+e}\right)\right]}{\log\left(\frac{1+e}{1+i}\right)}$$

$$\text{Since} \quad \frac{e - i}{1 + e} = \frac{(1 + e) - (1 + i)}{1 + e} = 1 - \left(\frac{1 + i}{1 + e}\right) = 1 - \left(\frac{1}{\frac{1 + e}{1 + i}}\right) ,$$

$$Y = \frac{\log \left[1 + SPB\left(1 - \frac{1}{\frac{1 + e}{1 + i}}\right)\right]}{\log \left(\frac{1 + e}{1 + i}\right)} .$$

APPENDIX B

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11. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)

This report describes how to calculate simple and discounted payback measures of economic performance of buildings and building systems. Formulas for calculating payback, applications for evaluating and selecting projects, and limitations in the use of payback analysis are discussed. The simple payback method measures the time between the date of initial project investment and the date when cumulative future earnings or savings on that investment, net of cumulative future costs, just pay off the investment. The discounted payback method measures the time between the date of initial project investment and the date when the present value of future earnings or savings, net of the present value of future costs, just equals the initial investment. This recommended practice will assist the private and public building communities in making cost-effective decisions in the design, operation, maintenance, and retrofit of buildings.

12. KEY WORDS (Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)

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