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**U.S. DEPARTMENT OF COMMERCE, Malcolm Baldrige,** ***Secretary***
**NATIONAL BUREAU OF STANDARDS, Ernest Ambler,** ***Director***

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Abstract

Numerical solutions have been obtained for steady natural convection in a square cavity. The numerical method used was developed for unsteady, incompressible, viscous fluid flow. The similarity parameters were chosen to match those of an international comparison exercise. Results are presented and compared with those obtained by other researchers using different methods.

Key Words: Cavity flow; fluid dynamics; natural convection; numerical methods.

## Introduction

The idea of carrying out a comparison exercise for numerical schemes based on the problem of steady natural convection in a square cavity with vertical sides which are maintained at different temperatures was proposed in 1979 [1]. The problem is of interest due to the range of detailed flow patterns found in this simple geometry. The applicable equations here are the Navier-Stokes and continuity equations of fluid flow. The Navier-Stokes equations include a body force term, which requires the solution of an additional equation for temperature. These equations are all simplified by the Boussinesq approximation. There are no singularities in the boundary conditions, as occur in the driven cavity problem. The results of this study could be used as a model problem solution for checking computer codes developed for practical elliptic fluid flow computations.

Two recent papers summarize the contributions received from participating researchers [2] and a bench mark solution [3] with which their results were compared. This report describes a solution obtained at NBS using a numerical scheme developed for unsteady, incompressible, viscous fluid flow problems.

In the following section, the comparison problem and similarity parameters will be presented. The solution procedure will then be described. Results and comparisons with the bench mark solution and those of other contributors are also included.

## The Comparison Problem

The basic problem considered here is the steady two-dimensional flow of a Boussinesq fluid in an upright square cavity. The details are shown in figure 1. Each side of the cavity has length D, and both velocity components are zero on the walls. The vertical sides are at temperatures $T_1$ and $T_2$, with the normal temperature gradient being zero on the top and bottom walls. The fluid has thermal diffusivity $\kappa$ and kinematic viscosity $\nu$, with Prandtl number $Pr = \frac{\nu}{\kappa}$. The Rayleigh number for this flow is defined as $Ra = \frac{\beta g \Delta T D^3}{\kappa\nu}$, where $\beta$ is the coefficient of expansion of the fluid, g is gravitational acceleration, and $\Delta T = T_1 - T_2$. In this study, all lengths are nondimensionalized with respect to D; all velocities with respect to $\frac{\kappa}{D}$; time with respect to $\frac{D^2}{\kappa}$; and p, the ratio of pressure to reference density, with respect to $\frac{\kappa^2}{D^2}$. The nondimensional temperature is $T = \frac{T' - T_2}{T_1 - T_2}$, where

T' is the dimensional value. Therefore, $0 \leq x \leq 1$, $0 \leq z \leq 1$, $T = 1$ at $x = 0$ and $T = 0$ at $x = 1$.

The velocity and temperature fields are to be computed with Prandtl number 0.71 for Rayleigh numbers of $10^3$, $10^4$, $10^5$, and $10^6$. The comparison also includes Nusselt numbers, or nondimensional values of heat flux in the horizontal direction.

## Numerical Modeling

The two-dimensional Navier-Stokes, continuity, and temperature equations for a viscous Boussinesq fluid are

$$\frac{\partial u}{\partial t} + \frac{\partial (u^2)}{\partial x} + \frac{\partial (uw)}{\partial z} = -\frac{\partial p}{\partial x} + Pr\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\frac{\partial w}{\partial t} + \frac{\partial (uw)}{\partial x} + \frac{\partial (w^2)}{\partial z} = -\frac{\partial p}{\partial z} + Pr\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}\right) + RaPrT$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial T}{\partial t} + \frac{\partial (uT)}{\partial x} + \frac{\partial (wT)}{\partial z} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} ,$$

where u and w are velocity components in the x- and z- directions, respectively.

The basic numerical scheme employed here uses quadratic upwind differencing for convection and an explicit Leith-type of temporal differencing [4]. A fast direct method is used to solve the Poisson equation for pressure at each time step. The solution is accomplished on a staggered mesh in which pressures and temperatures are defined at cell centers and normal velocities at cell faces.

The boundary conditions on the four cavity walls are that normal and tangential velocities are zero. The temperatures on the vertical walls are specified. Any values for grid points just outside the walls are obtained by quadratic extrapolation using the wall value and two points just inside. The extrapolated values are used in computing derivatives at the wall. At the top and bottom walls, $\frac{\partial T}{\partial z} = 0$, so the points just outside those walls have the same temperatures as those just inside the walls.

The only mesh employed in this study was a 50 x 50 uniform grid with $\Delta x = \Delta z = 0.02$. The initial conditions for the computations were either $u = 0$, $w = 0$, $T = 1-x$ for $Ra = 10^3$, or the results of a previous computation at a lower Rayleigh number. The time step, $\Delta t$, was set at a maximum of $7.5 \cdot 10^{-5}$ so that the diffusion coefficient in the temperature equation was less than 0.5 and the Courant number less than one. Computation times on the NBS UNIVAC 1100/82 required to obtain steady-state solutions ranged between 5 and 7 hours. At steady-state, the averages of the absolute values of the time-derivatives of the velocities [$\frac{(u^n-u^{n-1})}{\Delta t}$ and $\frac{(w^n-w^{n-1})}{\Delta t}$, where n indicates time level] throughout the cavity were $O(10^{-2})$ for the lower three Rayleigh numbers and $O(10^{-1})$ for $Ra = 10^6$. The maximum change in u or w during the final time step was $O(10^{-4})$ for $Ra = 10^6$ and $O(10^{-6})$ for the lowest Rayleigh numbers. The average of the absolute values of the time-derivatives of temperature was $O(10^{-3})$ or less. The summation of the absolute values of the mass flux residuals was $O(10^{-5})$ for $Ra = 10^3$ and increased by an order of magnitude for each tenfold increase in Ra.

## Results and Discussion

Computations have been carried out for the four Rayleigh numbers in the comparison exercise. The results are shown in table I. The quantities listed are those requested for comparison purposes:

$u_{max}$, z - the maximum u-component of velocity along the line x = 0.5 and its location;

$w_{max}$, x - the maximum w-component of velocity along the line z = 0.5 and its location;

$Nu_0$ - the average Nusselt number on the wall of the cavity at x = 0;

$Nu_{max}$, z - the maximum value of the local Nusselt number on the wall at x = 0 and its location;

$Nu_{min}$, z - the minimum value of the local Nusselt number on the wall at x = 0 and its location;

$\overline{Nu}$ - the average Nusselt number throughout the cavity;

$Nu_{1/2}$ - the average Nusselt number along the line x = 0.5.

The values for $u_{max}$, z, $w_{max}$, and x are grid point values (no interpolation). The Nusselt number, $Nu(x,z) = uT - \frac{\partial T}{\partial x}$, was calculated at each of the mesh points. The quantities $Nu_{max}$ and $Nu_{min}$ and their locations are values of Nu(0, z) at the z grid points. The average Nusselt number along a line x = constant, such as $Nu_0$ or $Nu_{1/2}$, was obtained by integration: $Nu_x = \int_0^1 Nu(x,z)\, dz$. The integrals were evaluated by the rectangular rule. Average values obtained at each value of x were integrated by the trapezoidal rule to find $\overline{Nu} = \int_0^1 Nu_x dx$.

Contour plots of temperature T, velocity components u and w, and vorticity are presented in figures 2-5 for each of the four Rayleigh numbers. These illustrate the diagonal symmetry of the problem:

$T(x,z) = 1 - T(1-x, 1-z)$, $u(x,z) = -u(1-x, 1-z)$, $w(x,z) = -w(1-x, 1-z)$. Also, the development of the thermal boundary layer as Ra increases is clearly seen. The same qualitative features appear in the bench mark solution (figures 4-7 in reference 3).

Figure 6 presents velocity vector plots for the four cases, where the line segments indicate the local flow direction. They are analogous to the streamline plots in figure 3 of reference 3. Secondary recirculating rolls appear for Ra $\geq 10^5$ as observed ~~previously~~ [1,5].

The quantitative results presented here in table I can be compared with those of the bench mark solution given in table I of reference 2. The percentage differences (with respect to the bench mark solution) are shown here in table II. The field locations where the parameters of table II occur have been omitted from the comparisons as was done in reference 2. Also, stream function was not computed in the present work. The percentage differences are seen to be small, within about 1% for the lower three Rayleigh numbers.

It is interesting to compare these results with those in tables VII-X of reference 2 which contain the contributed results. For purposes of comparison, the absolute values of the percentage errors in Nu, $Nu_{max}$, $Nu_{min}$, $u_{max}$, and $w_{max}$ were averaged for each solution at each Rayleigh number. Nu is either $\overline{Nu}$ or $Nu_{1/2}$, if available, and has been compared with $Nu_{1/2}$ in the bench mark solution. The errors at each Rayleigh number were then averaged to give an overall average error for each contributed solution.

The average errors in the present work ranged from 0.1% at Ra = $10^3$ to 2.1% at Ra = $10^6$. The error averaged over the four Rayleigh numbers is 0.8%. Not all contributors computed all quantities or submitted solutions for all Rayleigh numbers. Of the 26 solutions from reference 2 that had all quantities, four had a smaller average error than the present work, placing these results in the top 20% of the solutions.

## Conclusion

Solutions have been presented for laminar natural convection in a square cavity. The computer code was adapted from one used for unsteady, incompressible, viscous fluid flow problems. The results obtained here compare well with the bench mark solution given in reference 3. The present results also compare well with the contributed solutions in reference 2. Although the errors in the present solution increased with increasing Rayleigh number, it should be noted that all four solutions were obtained using the same grid. Mesh refinement would have been preferable as Ra increased but was not possible due to limitations on computer resources.

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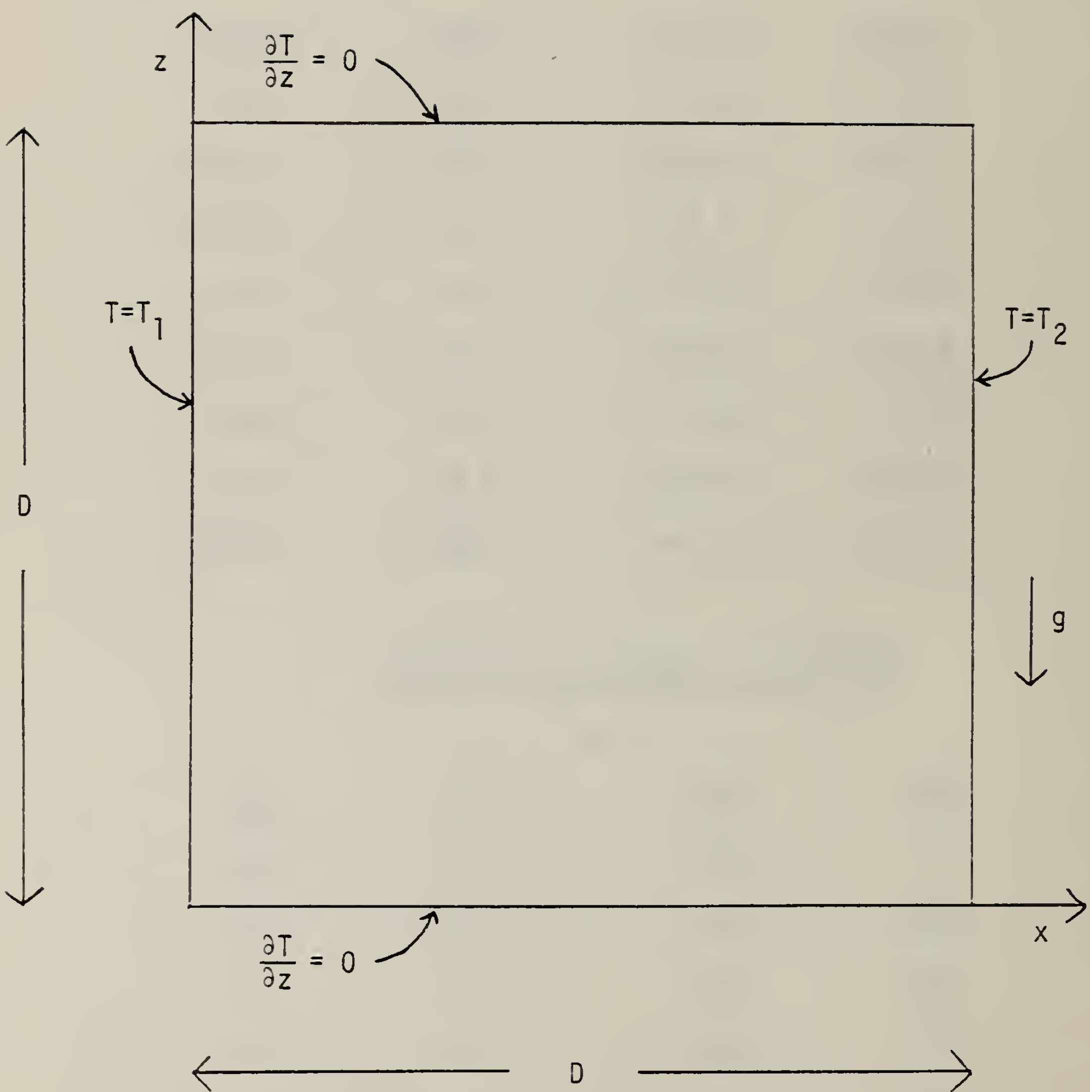
5. Mallinson, G. D. and de Vahl Davis, G., Three-dimensional natural convection in a box: a numerical study, J. Fluid Mech., Vol. 83, p. 1, 1977.

Table I. Results

| | Ra $10^3$ | $10^4$ | $10^5$ | $10^6$ |
|---|---|---|---|---|
| $u_{max}$ | 3.646 | 16.126 | 34.62 | 64.86 |
| z | 0.81 | 0.83 | 0.85 | 0.85 |
| $w_{max}$ | 3.689 | 19.507 | 67.98 | 211.86 |
| x | 0.17 | 0.11 | 0.07 | 0.03 |
| $Nu_o$ | 1.118 | 2.244 | 4.526 | 8.902 |
| $Nu_{max}$ | 1.507 | 3.538 | 7.812 | 18.809 |
| z | 0.09 | 0.15 | 0.07 | 0.03 |
| $Nu_{min}$ | 0.691 | 0.586 | 0.732 | 0.997 |
| z | 0.99 | 0.99 | 0.99 | 0.99 |
| $\overline{Nu}$ | 1.118 | 2.244 | 4.523 | 8.893 |
| $Nu_{1/2}$ | 1.118 | 2.244 | 4.524 | 8.898 |

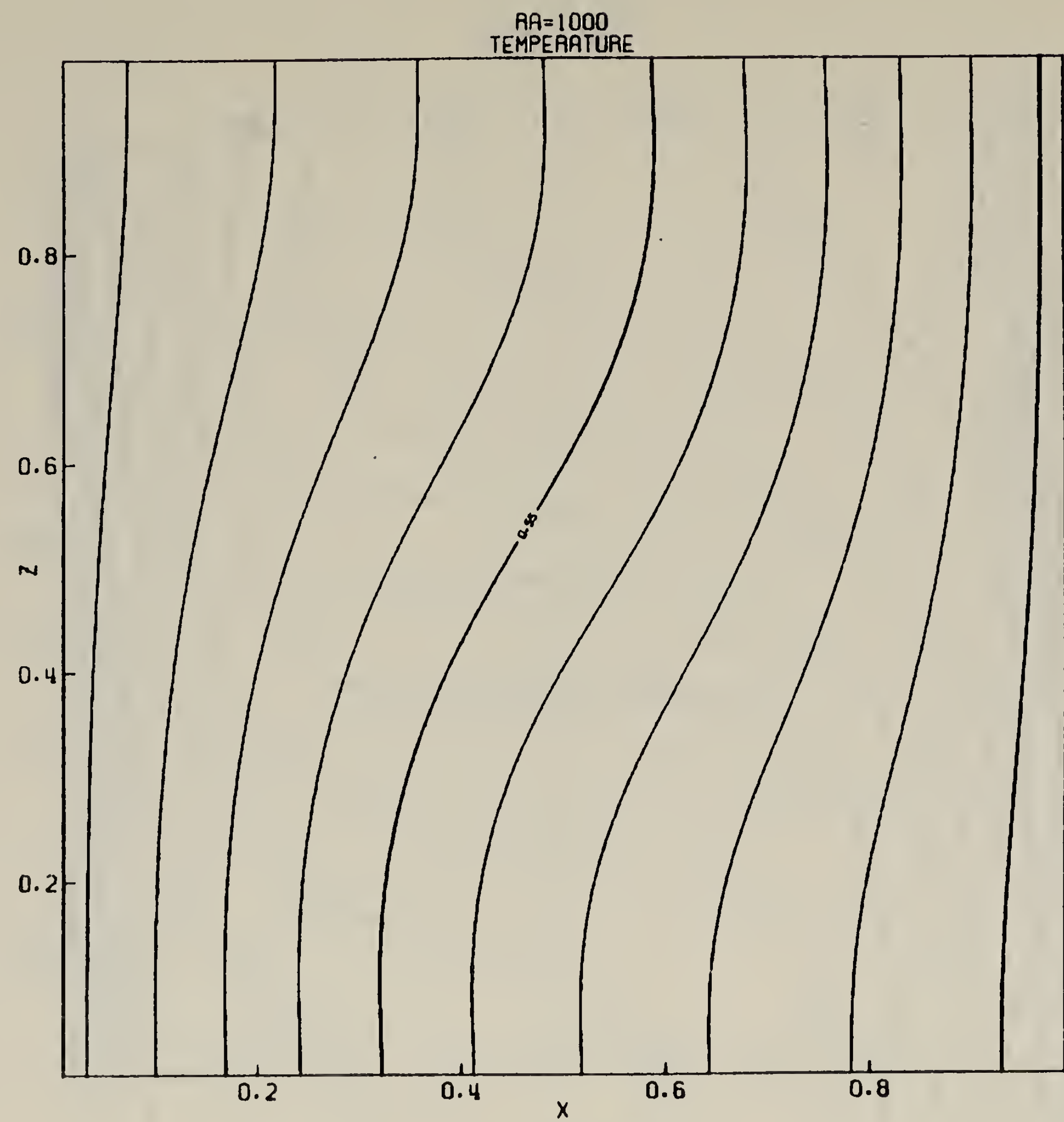
Table II. Percentage Differences

| | Ra $10^3$ | $10^4$ | $10^5$ | $10^6$ |
|---|---|---|---|---|
| $Nu_o$ | 0.1 | 0.3 | 0.4 | 1.0 |
| $\overline{Nu}$ | 0.0 | 0.0 | 0.1 | 1.1 |
| $Nu_{1/2}$ | 0.0 | 0.0 | 0.1 | 1.1 |
| $Nu_{max}$ | 0.1 | 0.3 | 1.2 | 4.9 |
| $Nu_{min}$ | -0.1 | 0.0 | 0.4 | 0.8 |
| $u_{max}$ | -0.1 | -0.3 | -0.3 | 0.4 |
| $w_{max}$ | -0.2 | -0.6 | -0.9 | -3.4 |



z
$\frac{\partial T}{\partial z} = 0$
$T=T_1$
$T=T_2$
D
g
x
$\frac{\partial T}{\partial z} = 0$
D


Fig. 1

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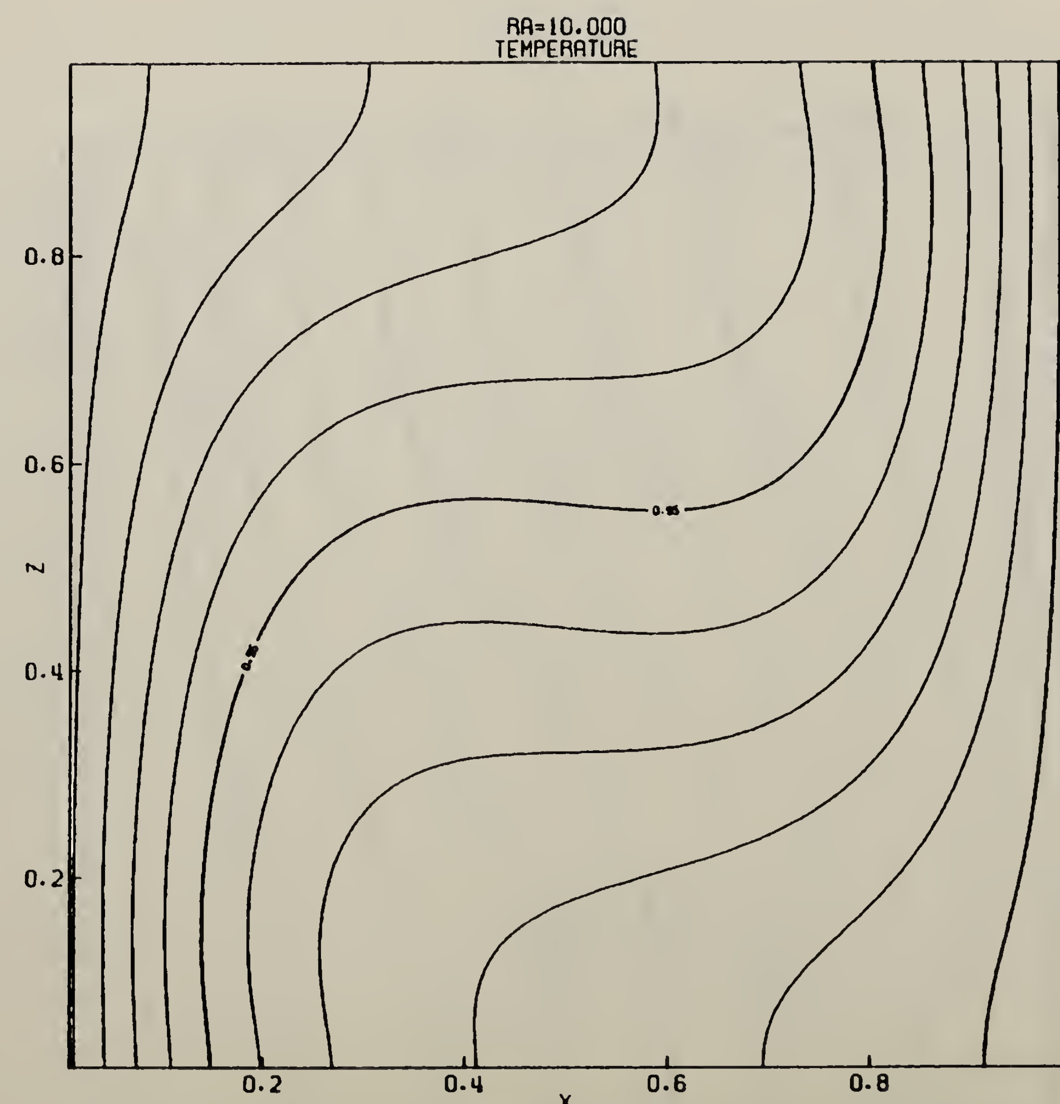



Fig. 2b

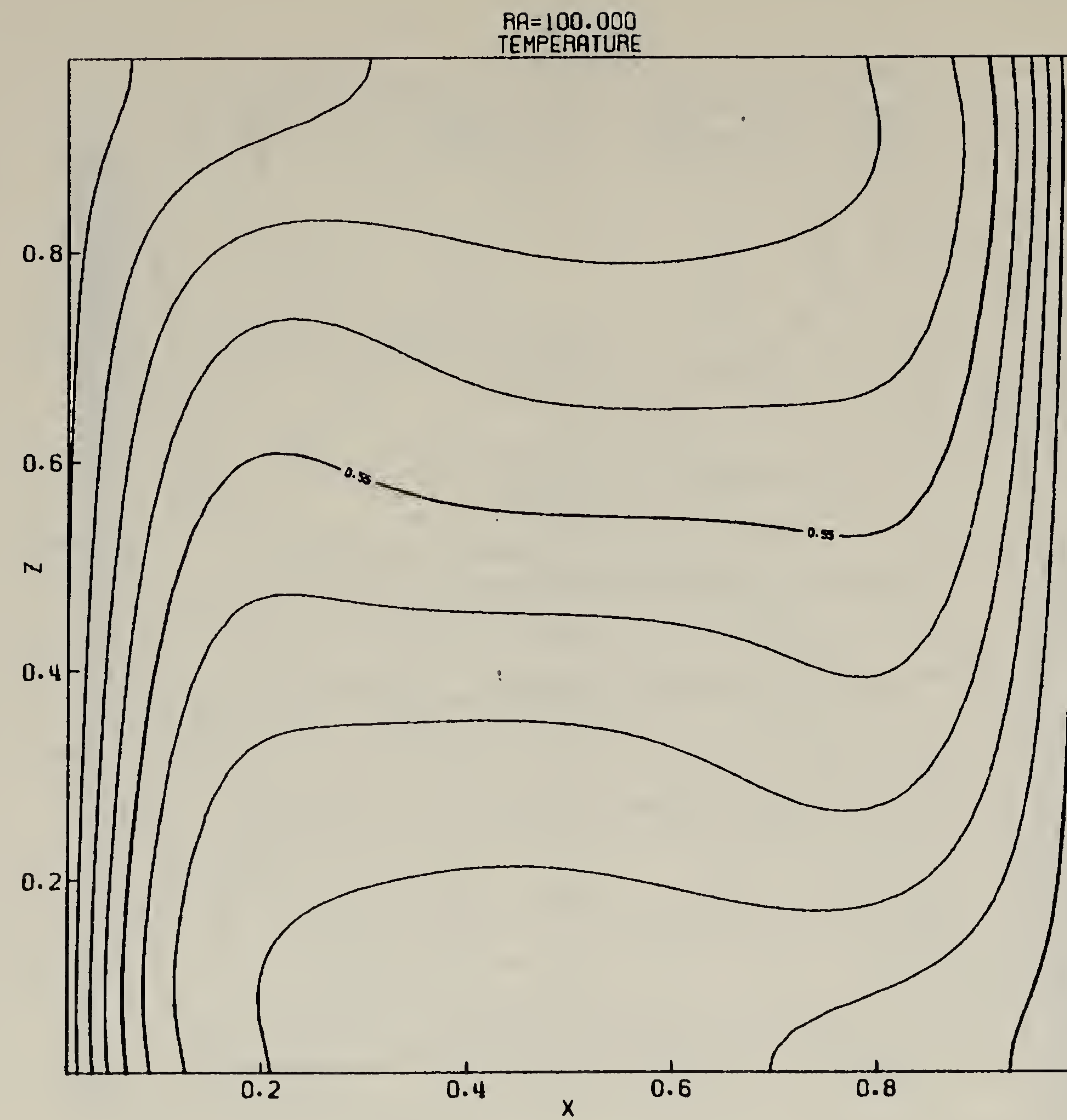



Fig. 2c

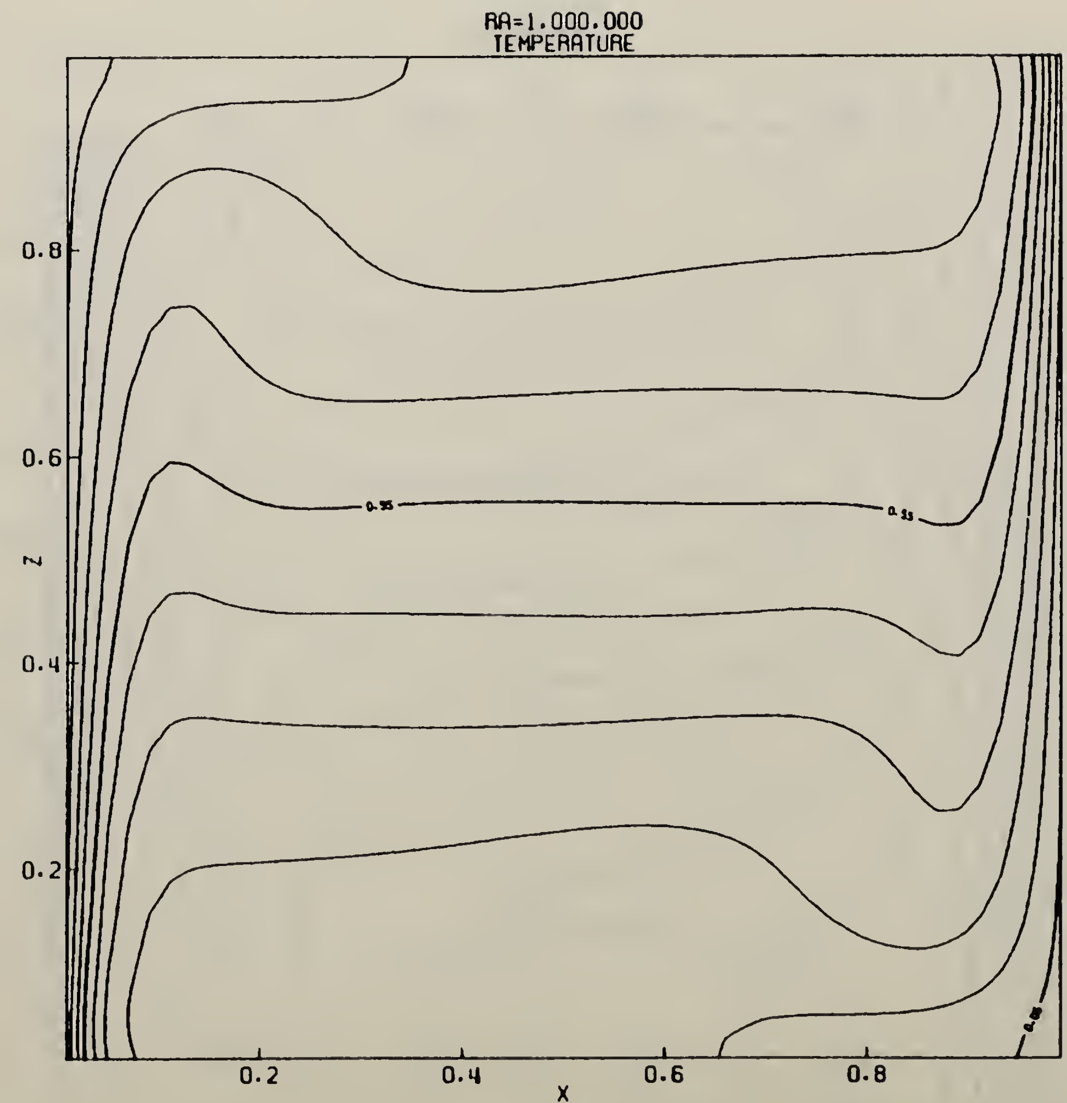
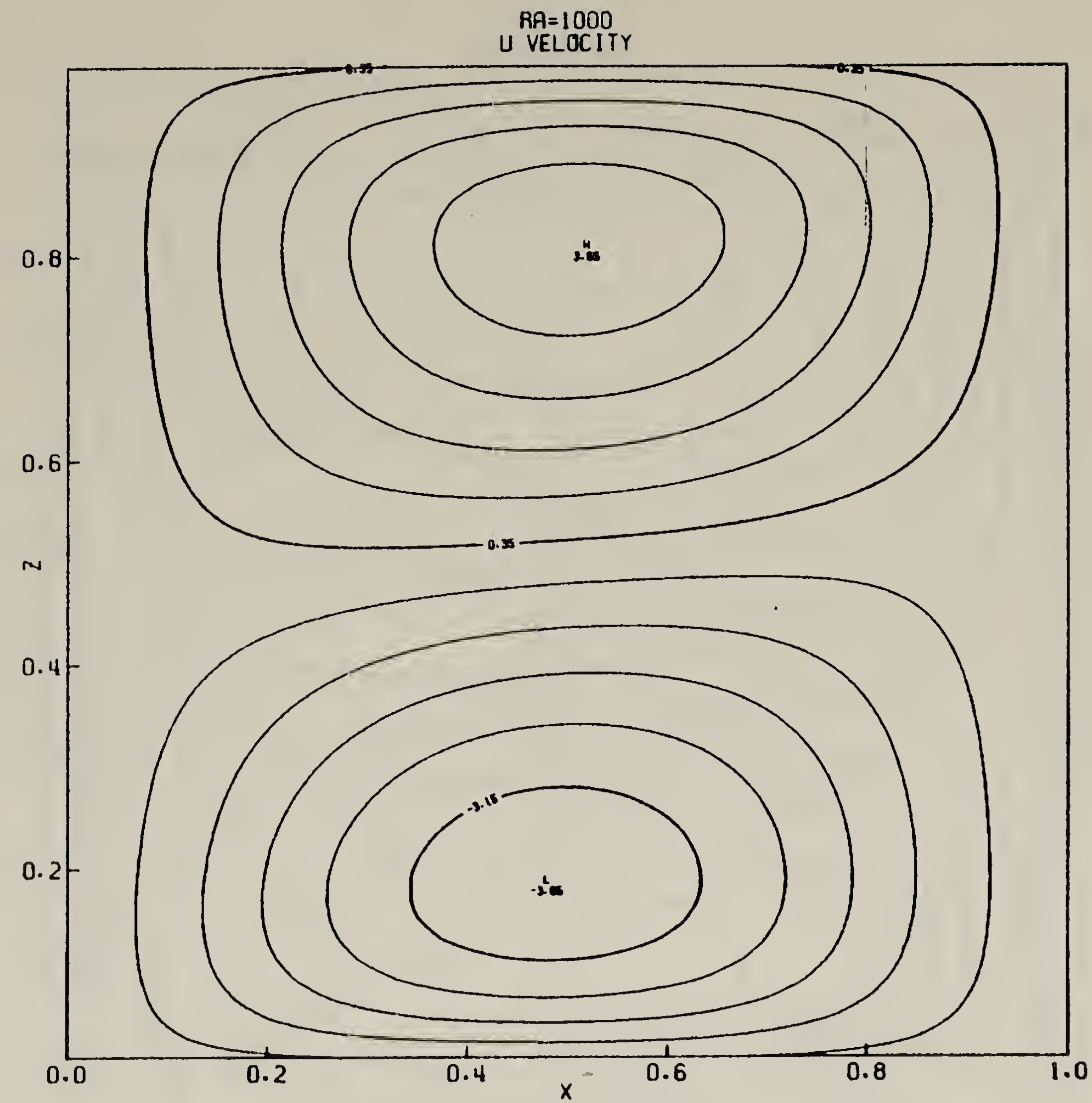



Fig. 2d

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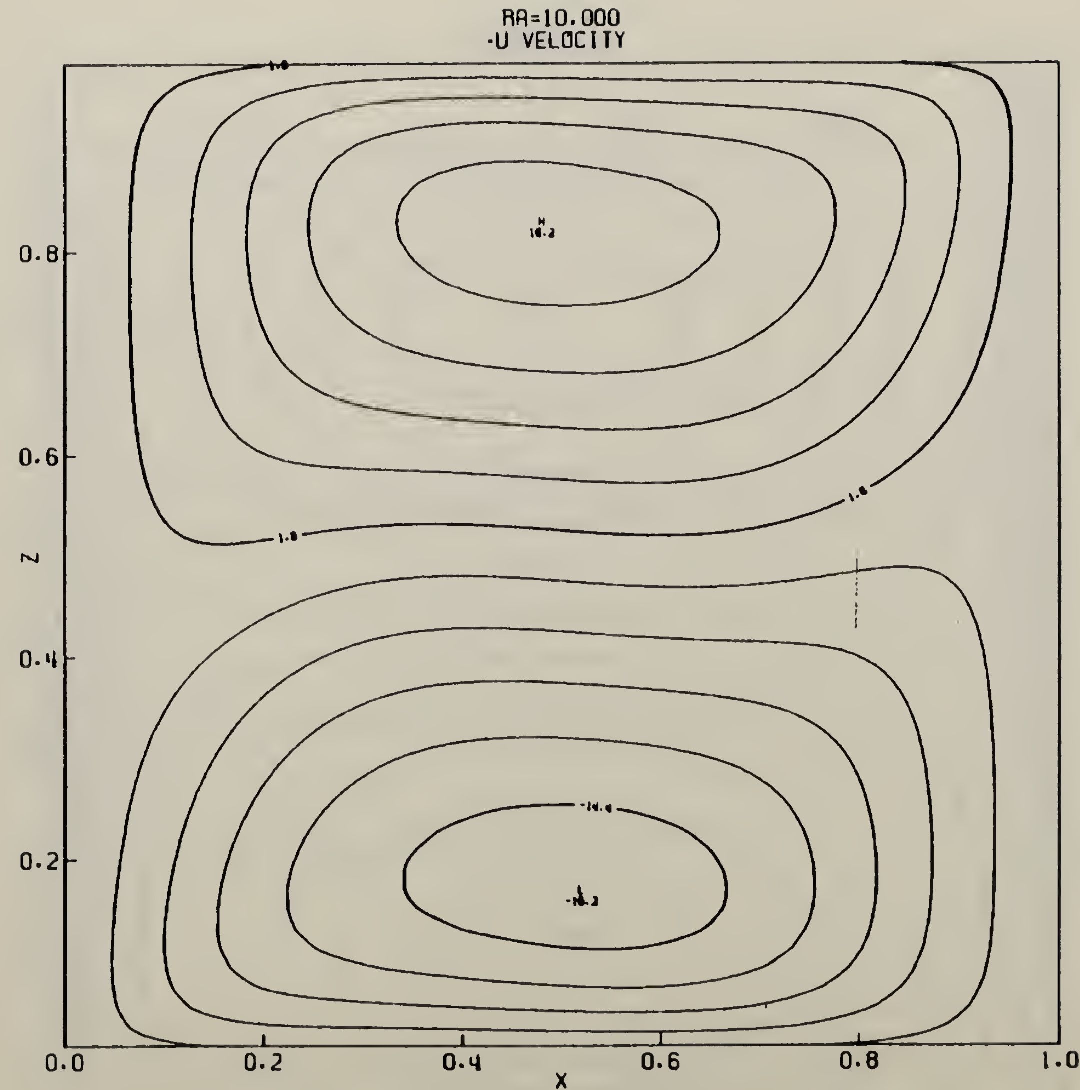
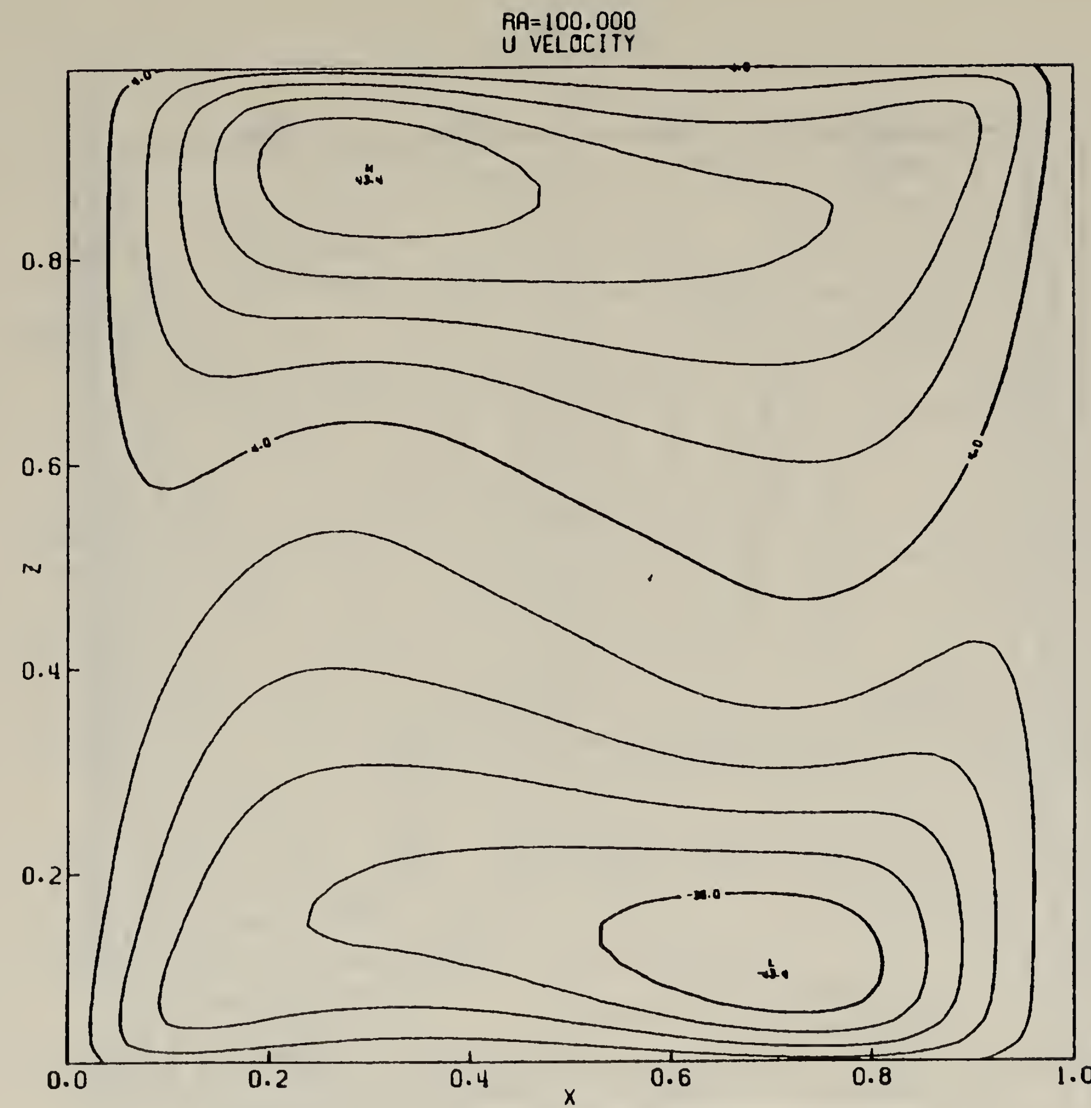



Fig. 3b

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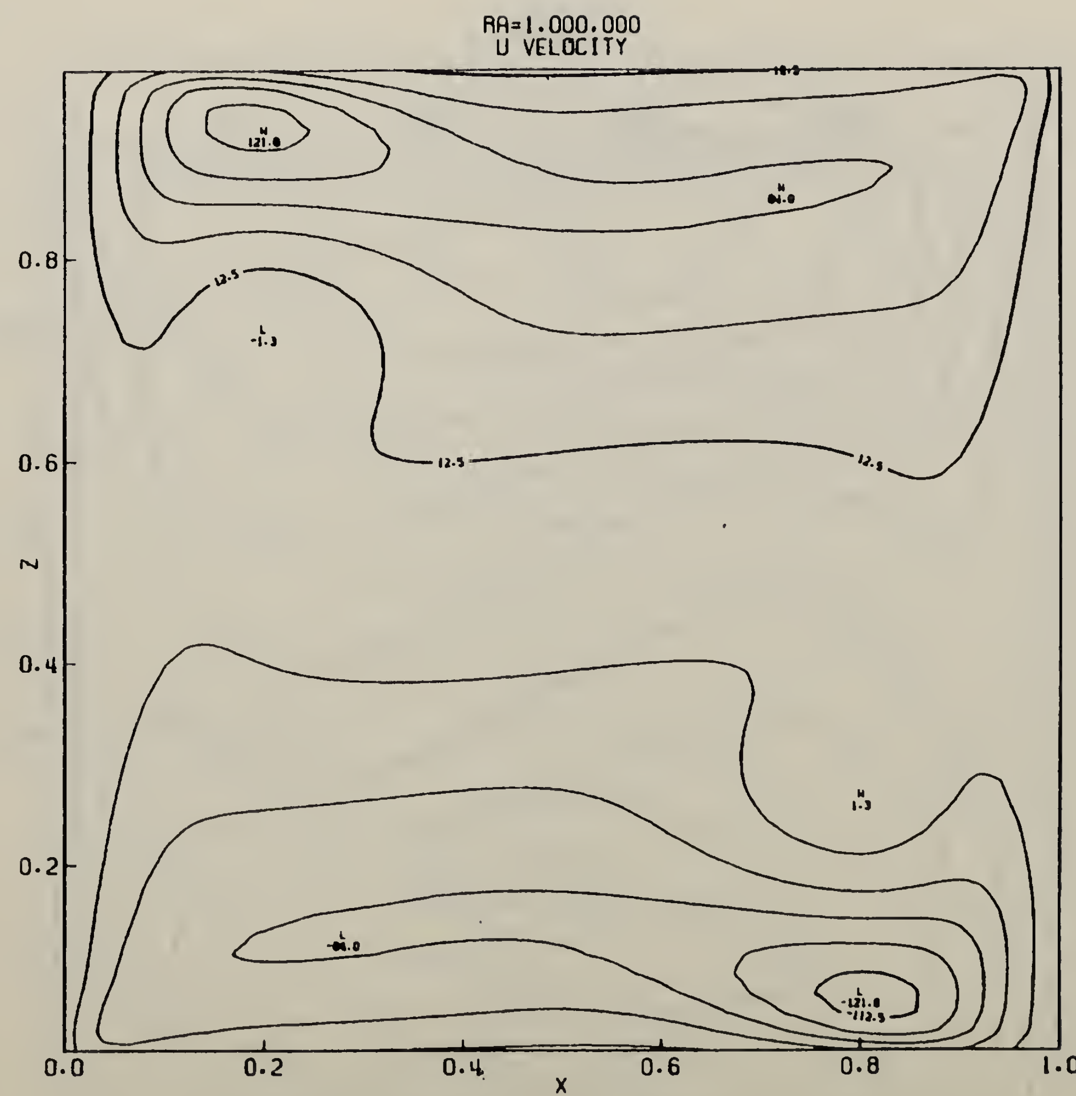


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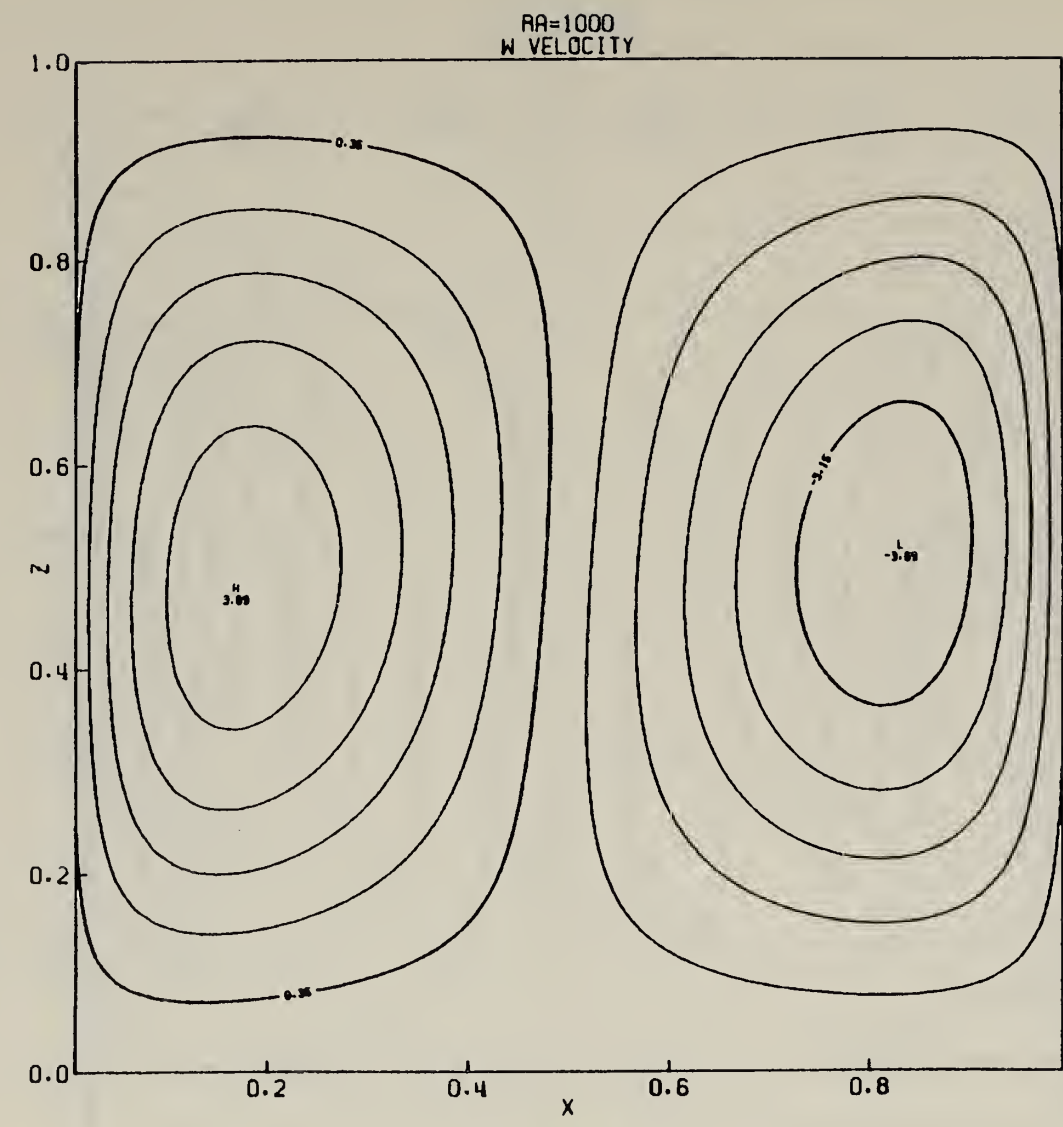
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H
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L
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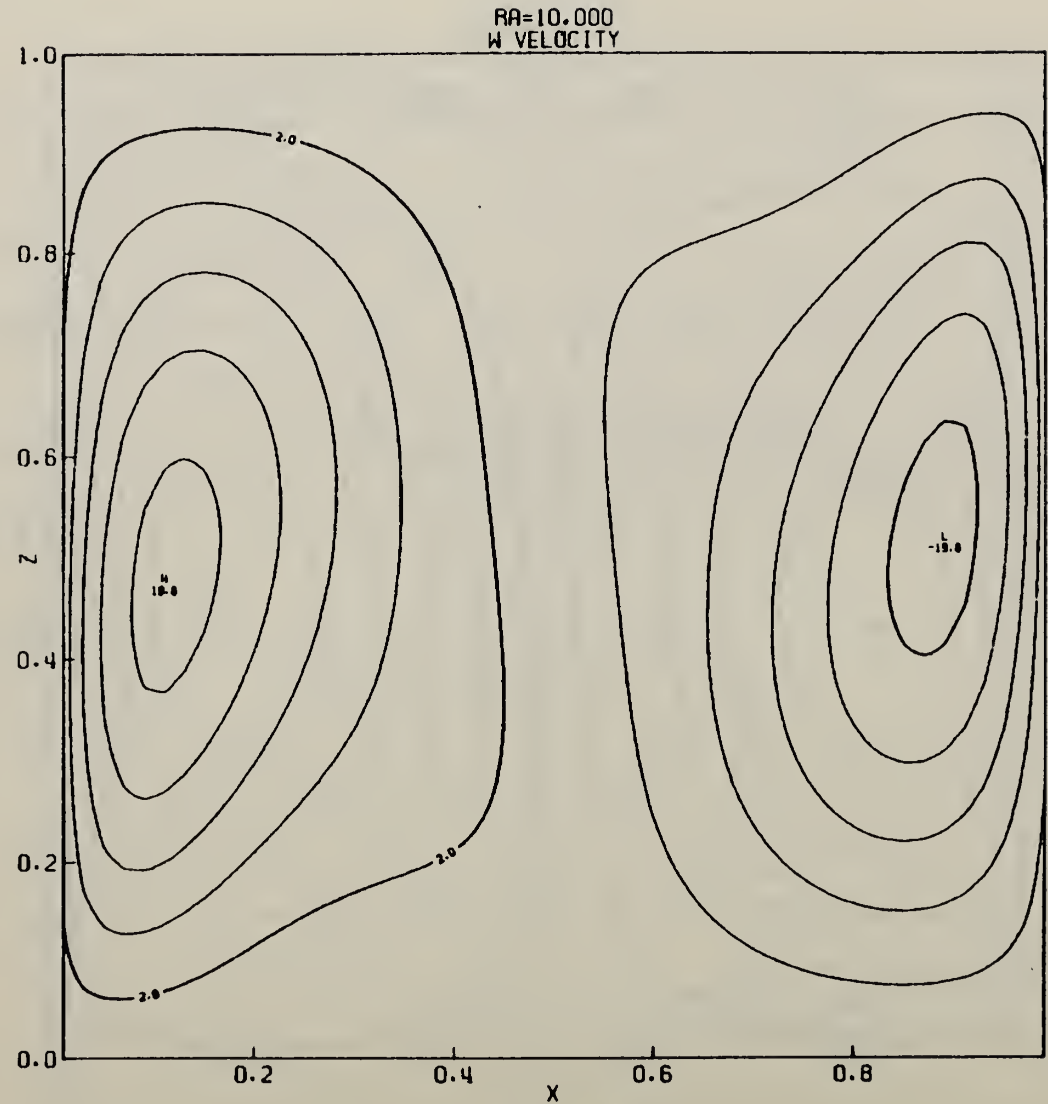
Fig. 4a

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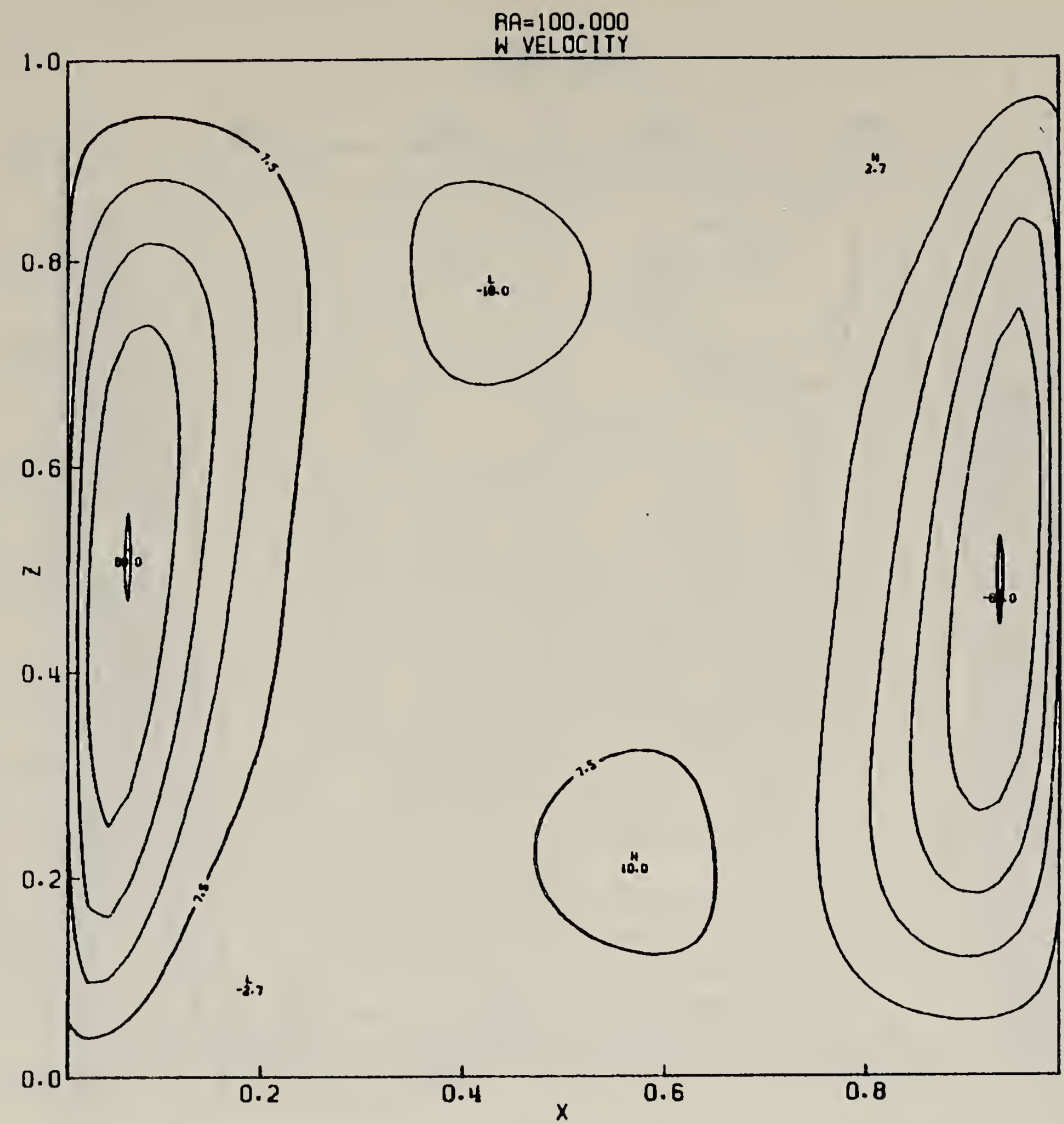
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H
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H
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L
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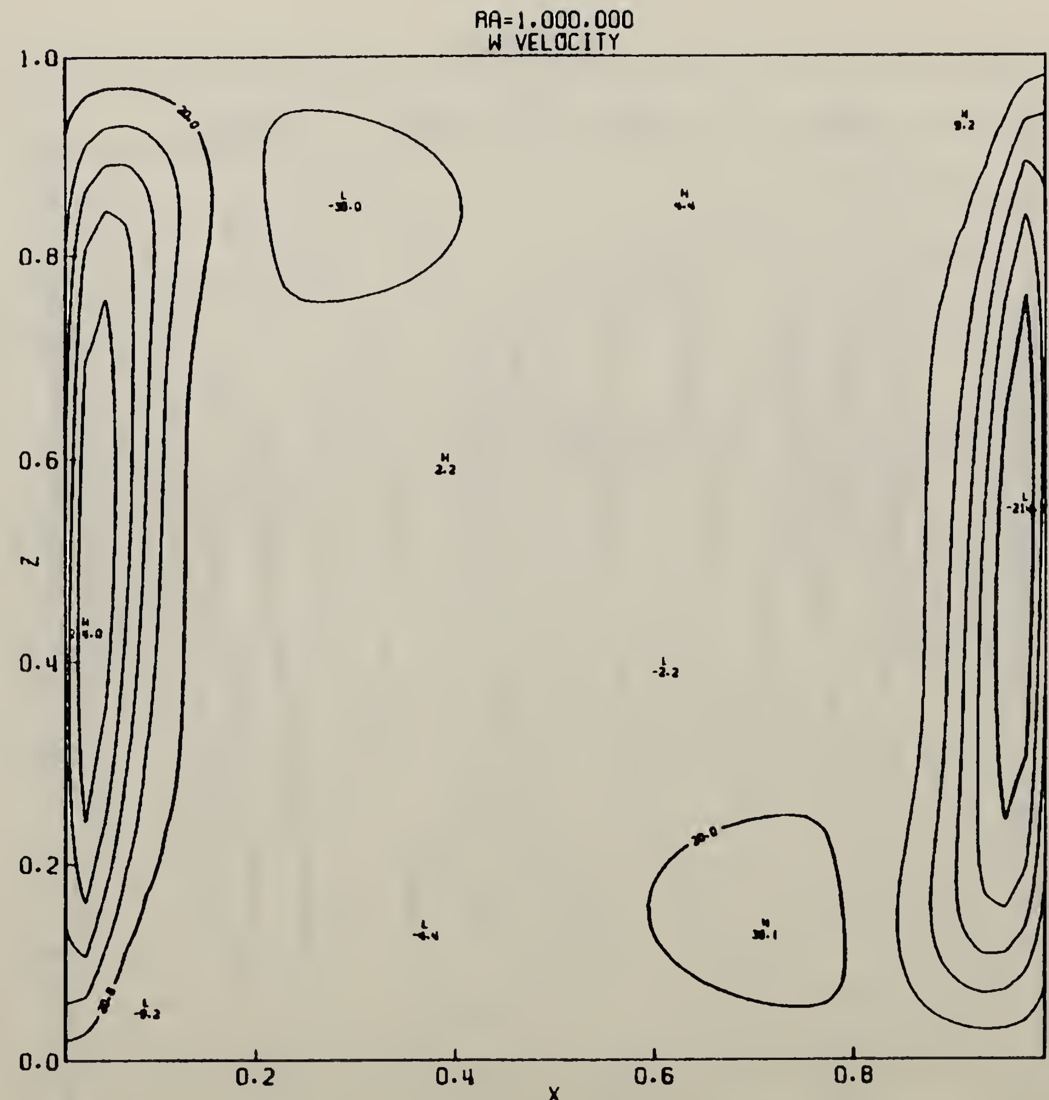
Fig. 4c

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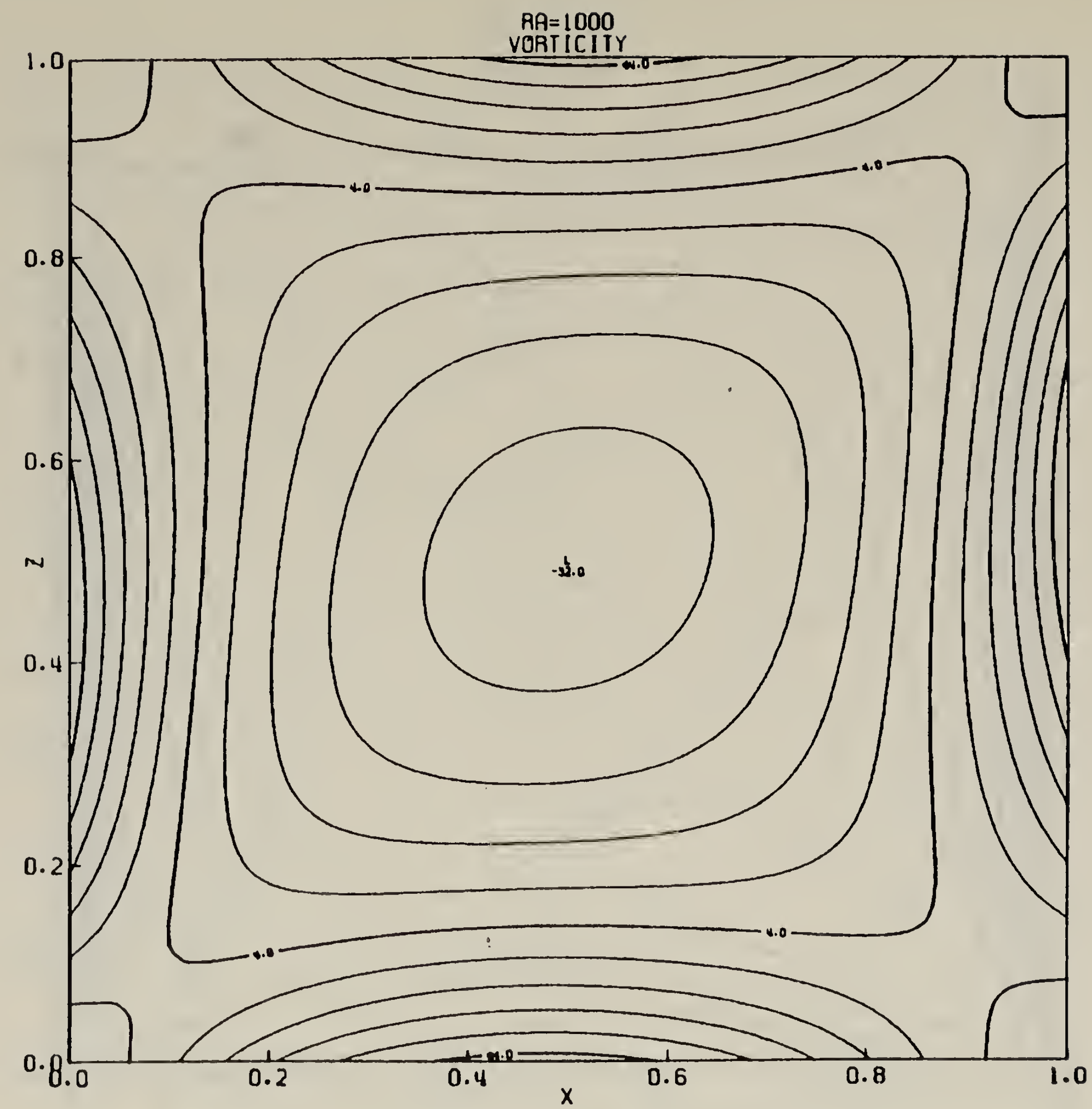
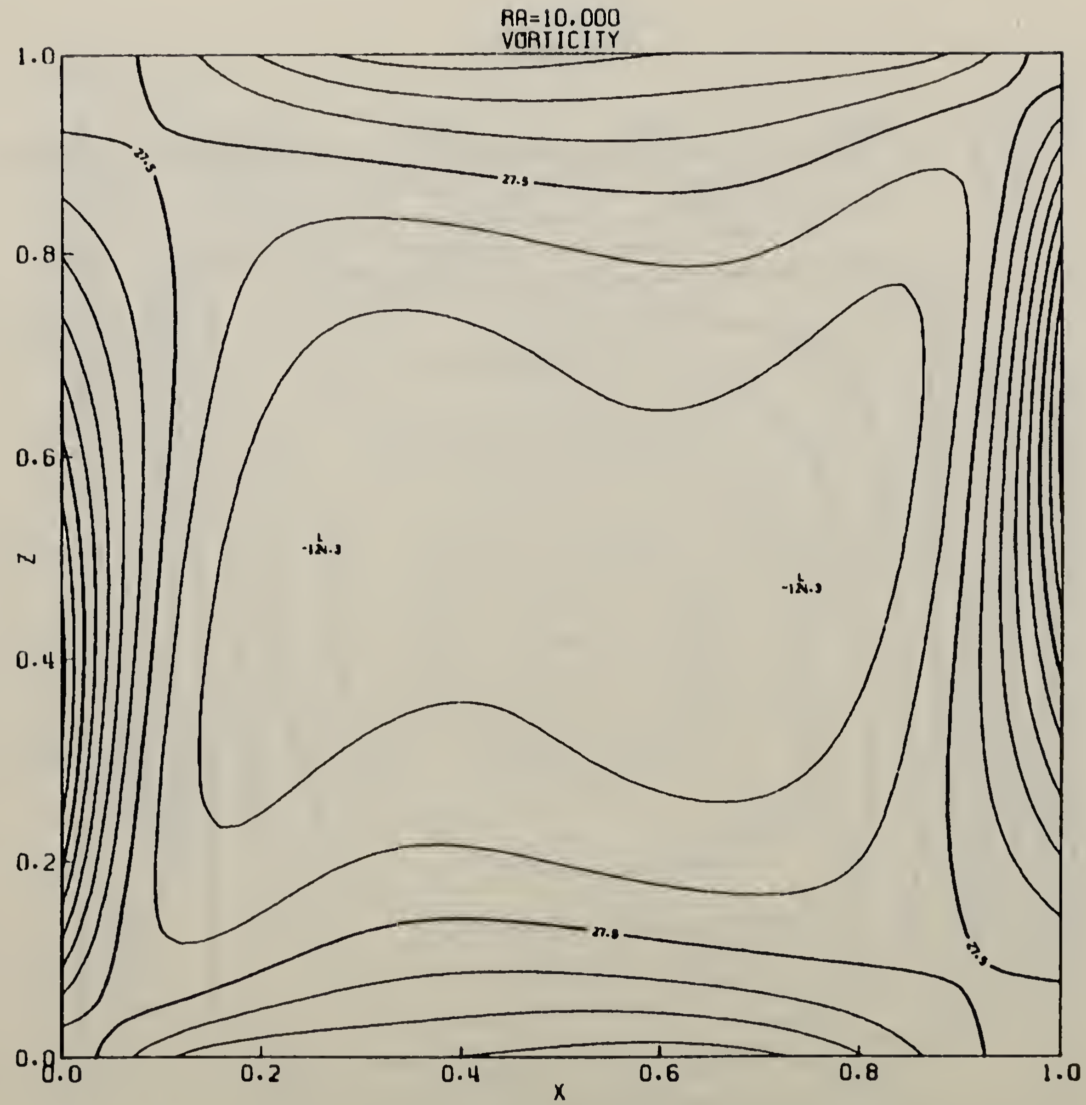



Fig. 5a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L
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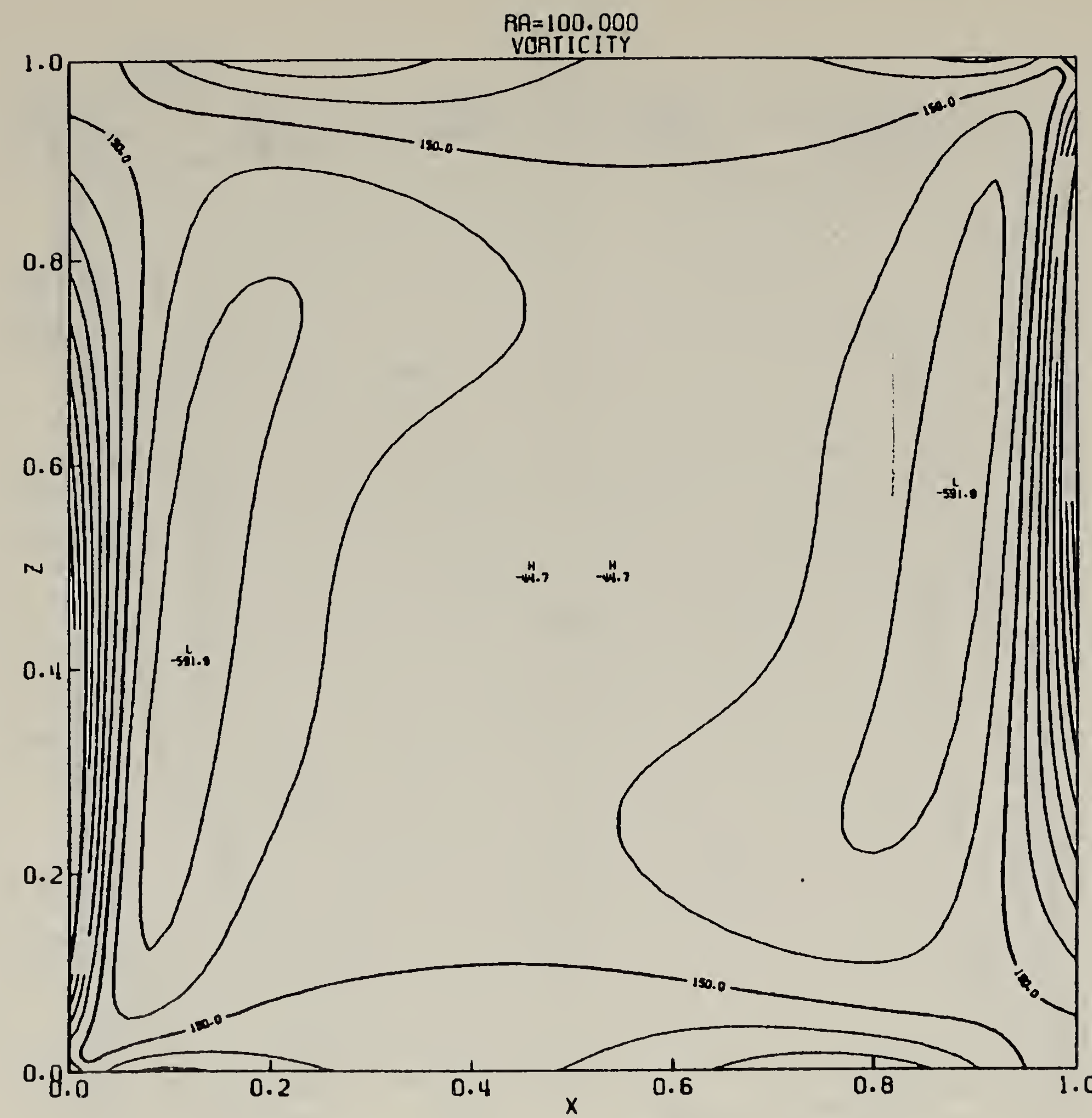
Fig. 5b

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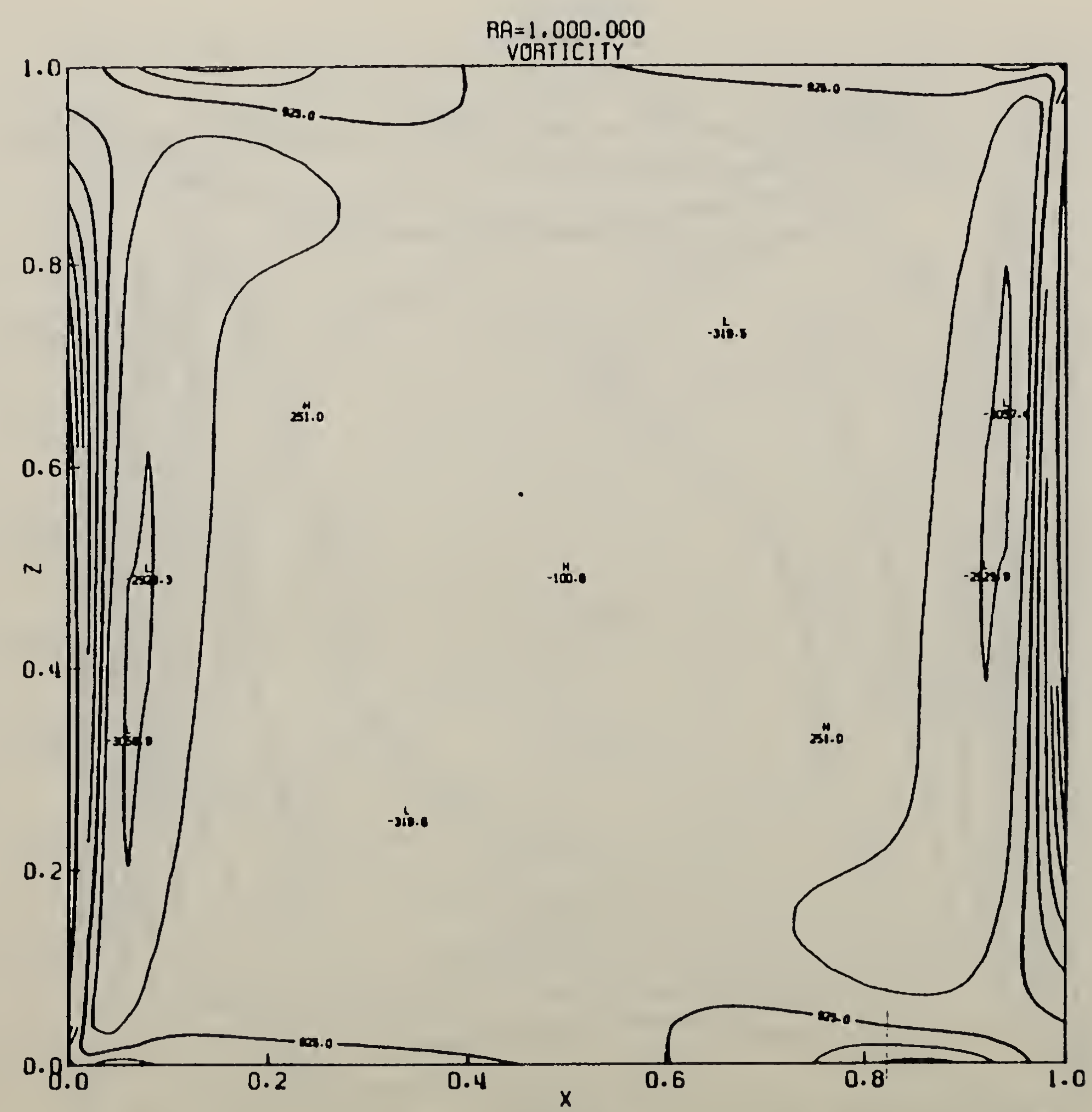
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0.8
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H
251.0

Fig. 5d

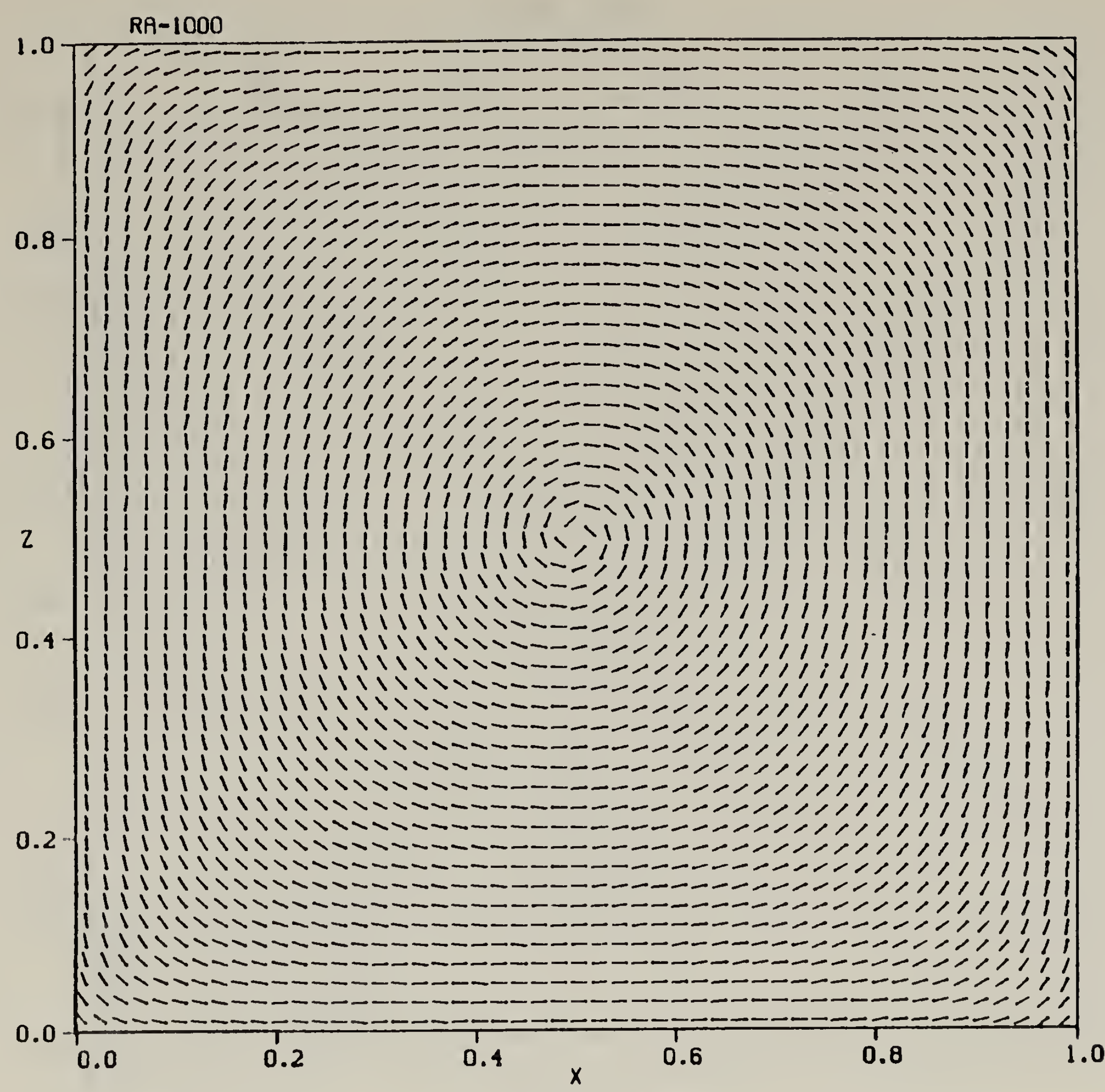



Fig. 6a

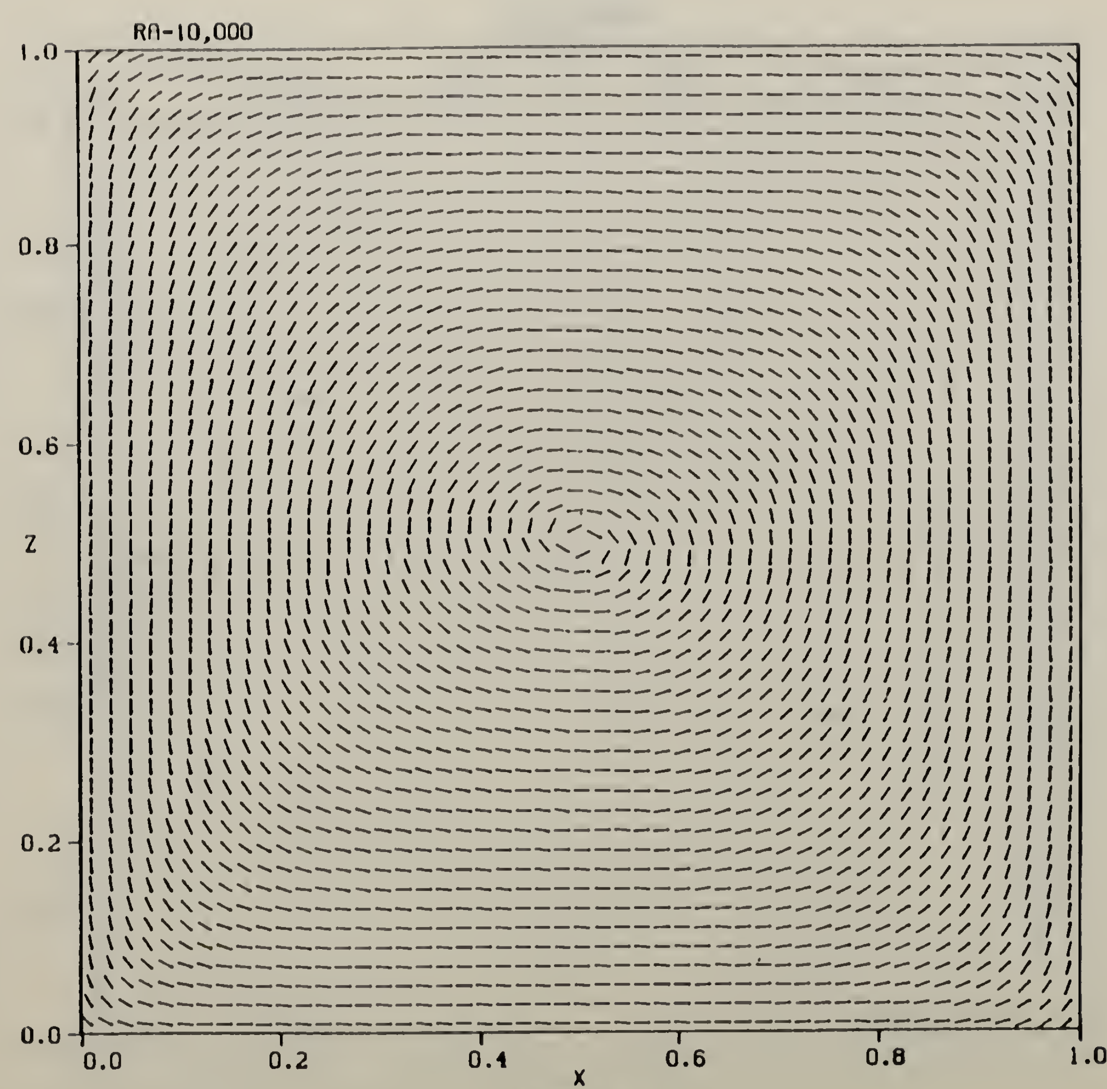
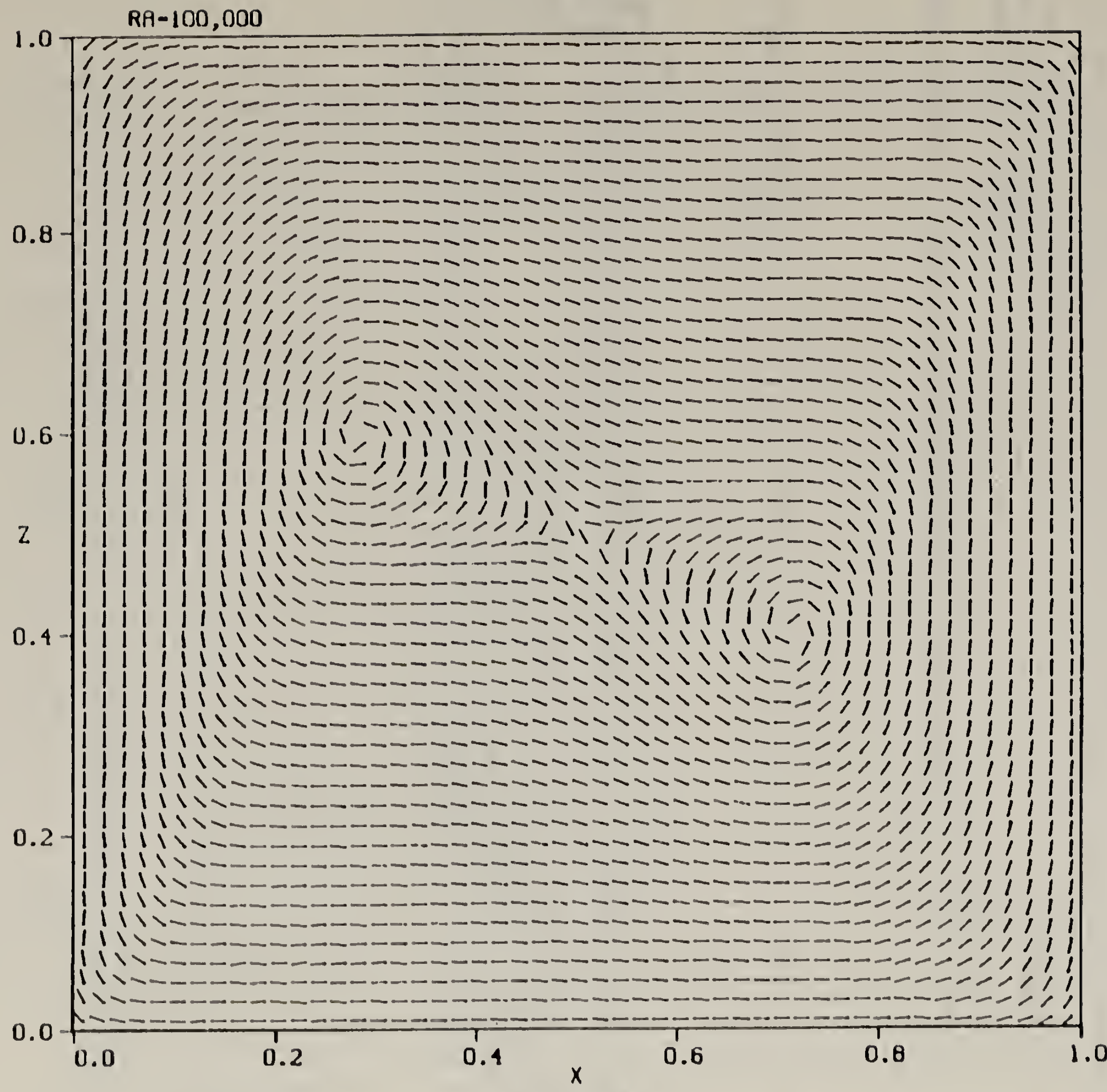



Fig. 6b

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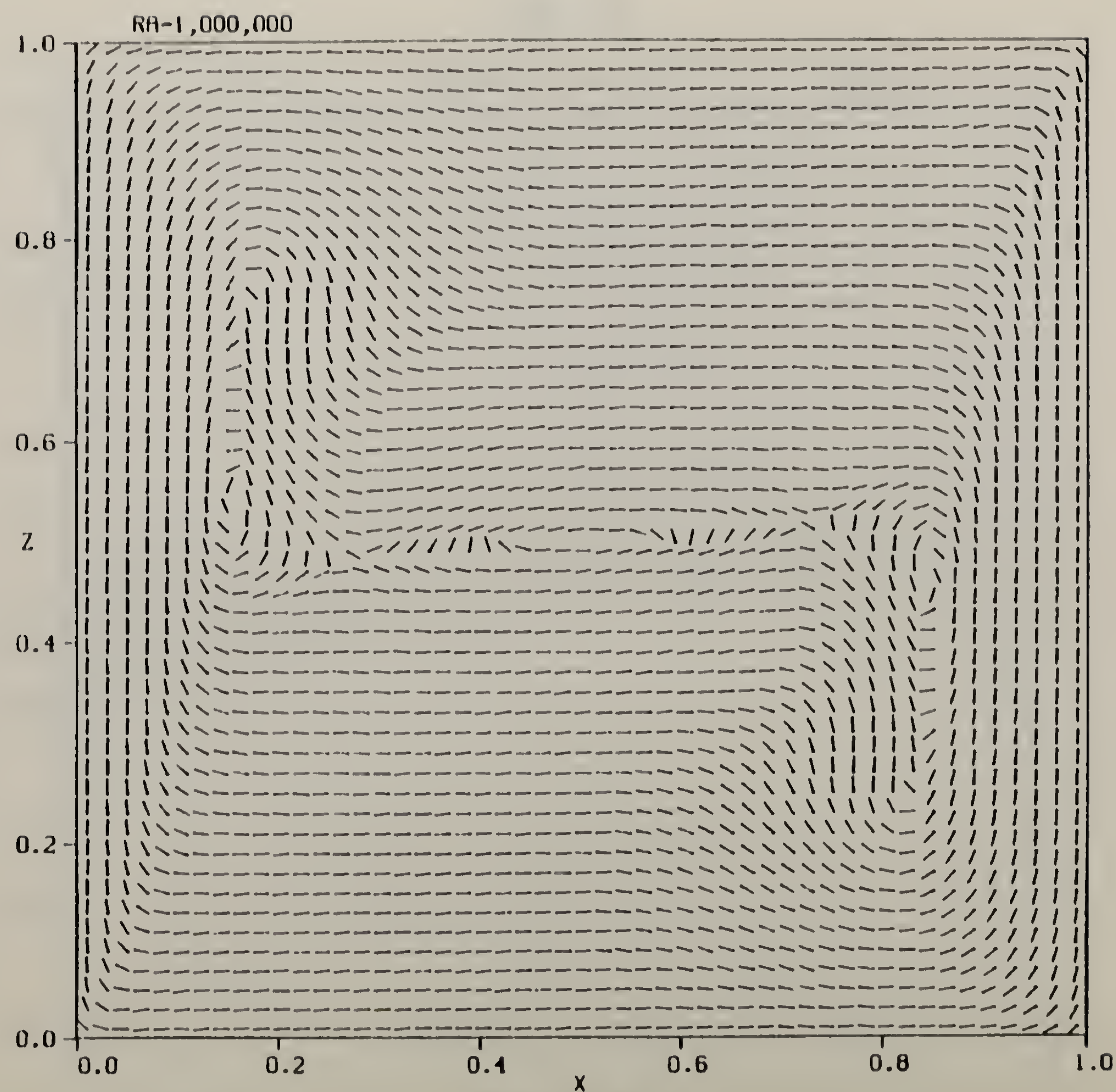


Fig. 6d

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E. F. Moore and R. W. Davis

6. PERFORMING ORGANIZATION (*If joint or other than NBS, see instructions*)

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☐ Document describes a computer program; SF-185, FIPS Software Summary, is attached.

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Numerical solutions have been obtained for steady natural convection in a square cavity. The numerical method used was developed for unsteady, incompressible, viscous fluid flow. The similarity parameters were chosen to match those of an international comparison exercise. Results are presented and compared with those obtained by other researchers using different methods.

12. KEY WORDS (*Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons*)

cavity flow; fluid dynamics; natural convection; numerical methods

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