NBSIR 83-2652

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U.S. DEPARTMENT OF COMMERCE, Malcolm Baldrige, *Secretary*
NATIONAL BUREAU OF STANDARDS, Ernest Ambler, *Director*

Calculation of Compressibility Factor for Air Over the Ranges of Pressure, Temperature, and Relative Humidity of Interest in Flowmeter Calibration

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A simple yet precise equation has been developed to enable calculation (using programmable calculators) of the compressibility factor, Z, for air from measurements of pressure, temperature, and humidity. The compressibility factor, a factor which accounts for the non-ideality of air in real-gas equations of state, is conventionally computed using virial coefficients. In the present paper, an equation is fitted to tabulated values of Z. The deviation between calculated and tabulated values is of the order of 0.01% or less; this does not imply, however, that the <u>accuracy</u> of calculated values is of this order.

Key Words: Air; compressibility factor; extrapolation formulas; flowmeter calibration; pressure; relative humidity; temperature.

1. Introduction

In the calculation of the flowrate of air through various metering devices (Venturi tubes, orifice plates, and nozzles, for example), the density of air enters. Since air is not an "ideal" gas, the compressibility factor (Z) a measure of the nonideality is introduced in the real gas equation for density (ρ). The compressibility factor is conventionally computed using the virial equation of state of an air-water vapor mixture (1,2). In this paper a simple interpolation formula is developed which enables the engineer to conveniently make <u>precise</u> calculations of Z using small readily available programmable calculators.

2. Real Gas Equation for Density of Air

The real gas equation for the density, ρ, of dry air is [1]

$$\rho = \frac{PM_a}{RTZ} , \qquad (1)$$

where P is the total pressure; M_a is the apparent molecular weight of dry air; R is the molar gas constant; T is absolute temperature; and Z is the compressibility factor. The magnitude of the non-ideality of the gas is reflected in the departure of Z from 1. For moist air, a mixture of dry air and water vapor, the real gas equation for ρ is [1]

$$\rho = \frac{PM_a}{RTZ} [1 + (\varepsilon - 1) \frac{e'}{P}] , \quad (2)$$

where ε is the ratio of the molecular weight of water vapor to the apparent molecular weight of dry air, and e' is the effective vapor pressure of water in moist air. The value of Z depends on P, T, and relative humidity of the air.

3. Development of the Equations

To develop the equation for Z, unpublished data for dry air provided by Wexler and Hyland [4] and data for moist air in table 2 of reference [1] were used. The equation was developed in two parts: the first includes the dependence of Z on pressure and temperature; the second part is a correction due to the effect of relative humidity (RH).

At each of nine values of P (covering the range 1 to 40 atmospheres), values of (1-Z) were fitted by least squares to an equation quadratic in temperature, t, in °C. At each value of pressure, 19 values of (1-Z) covering the temperature range 0°C to 54°C at intervals of three degrees were used. The resulting equations are:

$$\text{at } P = 1 \text{ atm}, (1-Z) \times 10^6 = 581.98 - 12.471\, t + 0.063156\, t^2, \quad (3)$$

$$\text{at } P = 5 \text{ atm}, (1-Z) \times 10^6 = 2860.3 - 62.056\, t + 0.31525\, t^2, \quad (4)$$

$$\text{at } P = 10 \text{ atm}, (1-Z) \times 10^6 = 5594.3 - 123.32\, t + 0.62894\, t^2, \quad (5)$$

$$\text{at } P = 15 \text{ atm}, (1-Z) \times 10^6 = 8195.8 - 183.24\, t + 0.93145\, t^2, \quad (6)$$

$$\text{at } P = 20 \text{ atm}, (1-Z) \times 10^6 = 10670.6 - 243.22\, t + 1.2517\, t^2, \quad (7)$$

$$\text{at } P = 25 \text{ atm}, (1-Z) \times 10^6 = 13006.4 - 301.64\, t + 1.5587\, t^2, \quad (8)$$

$$\text{at } P = 30 \text{ atm}, (1-Z) \times 10^6 = 15204.0 - 358.89\, t + 1.8613\, t^2, \quad (9)$$

$$\text{at } P = 35 \text{ atm}, (1-Z) \times 10^6 = 17261.7 - 415.00\, t + 2.1607\, t^2, \quad (10)$$

$$\text{at } P = 40 \text{ atm}, (1-Z) \times 10^6 = 19175.0 - 469.72\, t + 2.4534\, t^2, \quad (11)$$

The general form of eqs (3) through (11) is

$$(1-Z) \times 10^6 = a(P) + b(P)t + c(P)t^2, \quad (12)$$

where the dependence of Z on P is assigned to the coefficients, a, b, and c. Each of these coefficients was fitted by least squares to an equation quadratic in P (in atm). The resulting equations are

$$a(P) = -10.864 + 588.26\ P - 2.7106\ P^2, \tag{13}$$

$$b(P) = 0.33297 - 12.585\ P + 2.0659 \times 10^{-2}\ P^2, \tag{14}$$

$$c(P) = -2.4925 \times 10^{-3} + 6.3706 \times 10^{-2}\ P - 5.5619 \times 10^{-5}\ P^2 \tag{15}$$

An equation expressing the dependence of Z for dry air on t and P is synthesized by substituting eqs (13) through (15) in eq (12):

$$\begin{aligned}(1-Z) \times 10^6 = &-10.864 + 588.26\ P - 2.7106\ P^2 \\ &+ 0.33297\ t - 12.585\ Pt + 2.0659 \times 10^{-2}\ P^2t \\ &- 2.4925 \times 10^{-3}\ t^2 + 6.3706 \times 10^{-2}\ Pt^2 \\ &- 5.5619 \times 10^{-5}\ P^2t^2\end{aligned} \tag{16}$$

To develop an expression for the correction for the effect of RH on Z, values of Z for temperatures 19°C to 26°C at 1-degree intervals for RH of 0, 25, 50, 75, and 100 percent were used. The five mean values of the difference (one corresponding to each value of RH) between Z for moist air and Z for dry air, $\Delta(1-Z)$, were fitted by least squares to an equation quadratic in RH. The intercept of the equation was forced sufficiently close to 0 by weighting the 0 percent RH point. The resulting equation is

$$(1-Z) \times 10^6 = -0.35\ (RH) - 5.0 \times 10^{-3}\ (RH)^2, \tag{17}$$

where RH is in percent.

By adding eq (17) to eq (16) and rearranging, the equation expressing the dependence of Z on P, t, and RH results:

$$\begin{aligned}Z = &\ 1.00001 - 5.8826 \times 10^{-4}\ P + 2.7106 \times 10^{-6}\ P^2 \\ &- 3.3297 \times 10^{-7}\ t + 1.2585 \times 10^{-5}\ Pt - 2.0659 \times 10^{-8}\ P^2t \\ &+ 2.4925 \times 10^{-9}\ t^2 - 6.3706 \times 10^{-8}\ Pt^2 + 5.5619 \times 10^{-11}\ P^2t^2 \\ &- 3.5 \times 10^{-7}\ (RH) - 5.0 \times 10^{-9}\ (RH)^2,\end{aligned} \tag{18}$$

where P is in atm, t is in °C, RH is in percent, and Z is a pure number.

For P in PSI, t in °C, and RH in percent,

$$Z = 1.00001 - 4.0029 \times 10^{-5} P + 1.2551 \times 10^{-8} P^2$$
$$- 3.3297 \times 10^{-7} t + 8.5636 \times 10^{-7} Pt - 9.5827 \times 10^{-11} P^2t$$
$$+ 2.4925 \times 10^{-9} t^2 - 4.3349 \times 10^{-9} Pt^2 + 2.5753 \times 10^{-13} P^2t^2$$
$$- 3.5 \times 10^{-7} (RH) - 5.0 \times 10^{-9} (RH)^2 \qquad (19)$$

For P in MPa, t in °C, and RH in percent,

$$Z = 1.00001 - 5.8057 \times 10^{-3} P + 2.6402 \times 10^{-4} P^2$$
$$- 3.3297 \times 10^{-7} t + 1.2420 \times 10^{-4} Pt - 2.0158 \times 10^{-6} P^2t$$
$$+ 2.4925 \times 10^{-9} t^2 - 6.2873 \times 10^{-7} Pt^2 + 5.4174 \times 10^{-9} P^2t^2$$
$$- 3.5 \times 10^{-7} (RH) - 5.0 \times 10^{-9} (RH)^2 \qquad (20)$$

The values of Z for dry air calculated using eq (18) have been compared with the data used to develop the equation. The mean of the 171 differences between calculated and data values is 6×10^{-7}, and the estimate of standard deviation is 2.6×10^{-5} which corresponds to 0.0026%. At a pressure of 1 atmosphere the estimate of standard deviation is 2.1×10^{-6} which corresponds to 0.00021%. The estimate of overall uncertainty for the data on which the calculation of Z is based [4] ranges from 0.00093% to 0.0011% at a pressure of 0.987 atmosphere over the temperature range of interest; at a pressure of 49.3 atmospheres the estimate of overall uncertainty ranges from 0.054% to 0.075%. The estimate of standard deviation for the difference between the calculated RH correction (the last two terms in eq (18)) and that inferred from the data on which it is based is 1×10^{-5} which corresponds to 0.001% in Z. Since the RH correction was fitted over the range 19°C to 26°C it will have a greater deviation outside this range. However at the extremes of the temperature range of interest here, approximately 0°C to 50°C, the deviation is only of the order of 0.01% in Z.

4. Conclusions

A simple yet precise equation has been developed to enable calculation of the compressibility factor, Z, for air from measurements of pressure, temperature, and humidity. The deviation between calculated and data values of Z is of the order of 0.01% or less; this does not imply, however, that the accuracy of calculated values is of this order.

5. Acknowledgment

The author gratefully acknowledges the assistance of R. W. Hyland in providing access to the compressibility factor data, the computational assistance of C. P. Reeve, and the typing of the manuscript by Susan Johnson.

6. References

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NBS-114A (REV. 2-8C)

U.S. DEPT. OF COMM. BIBLIOGRAPHIC DATA SHEET (See instructions)	1. PUBLICATION OR REPORT NO. NBSIR 83-2652	2. Performing Organ. Report No.	3. Publication Date March 1983

4. TITLE AND SUBTITLE

Calculation of Compressibility Factor for Air Over the Ranges of Pressure, Temperature, and Relative Humidity of Interest in Flowmeter Calibration

5. AUTHOR(S)

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6. PERFORMING ORGANIZATION *(If joint or other than NBS, see instructions)*

NATIONAL BUREAU OF STANDARDS
DEPARTMENT OF COMMERCE
WASHINGTON, D.C. 20234

7. Contract/Grant No.

8. Type of Report & Period Covered

Final

9. SPONSORING ORGANIZATION NAME AND COMPLETE ADDRESS *(Street, City, State, ZIP)*

10. SUPPLEMENTARY NOTES

☐ Document describes a computer program; SF-185, FIPS Software Summary, is attached.

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A simple yet precise equation has been developed to enable calculation (using programmable calculators) of the compressibility factor, Z, for air from measurements of pressure, temperature, and humidity. The compressibility factor, a factor which accounts for the non-ideality of air in real-gas equations of state, is conventionally computed using virial coefficients. In the present paper, an equation is fitted to tabulated values of Z. The deviation between calculated and tabulated values is of the order of 0.01% or less; this does not imply, however, that the accuracy of calculated values is of this order.

12. KEY WORDS *(Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)*

Air; compressibility factor; extrapolation formulas; flowmeter calibration; pressure; relative humidity; temperature.

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☒ Order From National Technical Information Service (NTIS), Springfield, VA. 22161

14. NO. OF PRINTED PAGES

7

15. Price

$7.00

USCOMM-DC 6043-P80