The Photodisintegration of the Deuteron, 1982

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Measurements of the deuteron's photodisintegration cross section made over a thirty year time span are evaluated in terms of the cross section for E1 and M1 transitions calculated in the effective range approximation. The energy range covered is from threshold to 44 MeV. Data that do not depend on a knowledge of a bremsstrahlung spectrum's intensity or spectral distribution are shown to be described very well by the effective-range expressions. The values of the deuteron's electric polarizability is shown to be in fair agreement with a value derived from the observed deviation from Rutherford scattering of deuterons by $^{208}\text{Pb}$. A comparison is made of the experimental data with three recent calculations of the photodisintegration cross section.

This short analysis of the data available for the deuteron's photodisintegration cross section was instigated both by the recent controversy regarding the interpretation of these data$^{1,2}$, and by the evaluation of the deuteron's electric polarizability from a measurement of the deviation from Rutherford scattering in the scattering of deuterons by $^{208}\text{Pb}$. The data considered for this analysis cover the energy range from the threshold for the reaction, 2.2246 MeV, to 44 MeV. As well as being the energy region where the cross section is of most value for technological applications, it is also the region that establishes the magnitude of the $\sigma_2$ sum rule for the deuteron, i.e. the magnitude of the integral:

$$\sigma_2 = \int \frac{\sigma(E_\gamma)}{E_\gamma^2} \, dE_\gamma,$$

where $\sigma(E_\gamma)$ is the cross section for the absorption of photons by the deuteron.
As was first pointed out by Migdal\textsuperscript{4} and reemphasized 17 years later by Levinger\textsuperscript{5}, the static electric polarizability of the nucleus is proportional to the integral $a_0$ of the cross section for electric-dipole transitions. Sixteen years later, Ericson and H"ufner\textsuperscript{6} pointed out that in addition to depending on the electric polarizability the low energy photon scattering and therefore the value of $a_0$ should depend on the magnetic susceptibility of the nucleus. Finally Friar\textsuperscript{7} in his complete treatment of the scattering of photons by nuclei showed that the value of the $a_0$ sum rule depends not only upon the electric polarizability and the diamagnetic susceptibility of the nucleus but also on the nuclear paramagnetic susceptibility, the individual nucleon polarizabilities and susceptibilities, and various center of mass corrections. By far the largest contributions to the $a_0$ integral for the deuteron come from the cross sections for the electric- and magnetic-dipole transitions, i.e., from the deuteron's electric polarizability and magnetic susceptibility (diamagnetic). All other terms in the general expression are negligible compared with these. The objective of these notes is to show that for energies below 50 MeV the reliable data for the deuteron's photodisintegration cross section are very well described by the sum of simple analytic functions giving the cross section for M1 and E1 transitions. It is therefore appropriate to use these expressions both for technological applications as well as to evaluate the $a_0$ integral for the deuteron.

The measurements of the cross section for the photodisintegration of the deuteron fall into three classes: 1., those in which monoenergetic photon sources were used, 2., those using accelerators but where it was not necessary to know either the detailed shape or absolute intensity of a bremsstrahlung spectrum produced in the laboratory, and 3., those in which
the resulting cross section depended directly on the spectral shape and absolute intensity of a laboratory-produced bremsstrahlung spectrum. Most of the measurements made with monoenergetic photon sources were published 30 or more years ago. In all cases where two or more measurements were made at the same energy, the data were consistent within the stated uncertainties in the measurements. At these energies only a single weighted mean value for the cross section is used in this analysis. In the experimental bibliography at the end of this report, all of these measurements are designated by a single reference code, (57Hu80). The original sources of these data, grouped by photon energy, are given under this code in the bibliography. Individual reference codes are used to indicate the sources of the data for the group 2 and 3 measurements.

The simplest theory for the photodisintegration of the deuteron is that based on the effective-range approximation. It results in easy to evaluate analytic functions which give the cross section for M1 and E1 transitions. Other than fundamental constants the only parameters of the n-p system that enter these expressions are the magnetic moments of the neutron and proton, \( \mu_n \) and \( \mu_p \), the binding energy of the deuteron, \( B \), the very low energy singlet and triplet scattering lengths for the n-p system, \( a_s \) and \( a_t \) respectively, and the related scattering lengths \( r_{0s} \) and \( r_{0t} \). These expressions are:

\[
\sigma(E1) = \sigma(\text{Bethe-Peierls}) N
\]

\[
\sigma(\text{Bethe-Peierls}) = \frac{8 \pi e^2 \hbar}{3 M c} B^{1/2} \left( \frac{E_Y - B}{E_Y^3} \right)^{3/2}
\]

\[
\sigma(M1) = \frac{2 \pi e^2}{\hbar c} \left( \frac{\hbar}{M c} \right)^2 (\mu_p - \mu_n)^2 \frac{(1-\gamma a_s)^2 (B(E_Y-B))^{1/2}}{E_Y(1 + \frac{M}{\hbar^2} a_s^2 (E_Y-B))}
\]
where $a_s$ is the singlet S scattering length, $-23.719 \pm 0.013$ fm, $B$ is the deuteron binding energy, $2.2246$ MeV, $\mu_p - \mu_n = 4.7059$, and $\gamma^2 = MB/h^2$. The mass $M$ is the average of the neutron and proton masses, $938.9$ MeV. The approximate effective-range normalization factor $N(ER) = 1/(1 - \gamma r_{ot})$ is $1.682 \pm 0.013$. This factor results from using the value of the triplet effective range, $r_{ot} = 1.750 \pm 0.005$ fm, derived from the measured triplet scattering length, $a_t = 5.414$ fm. The approximation used here is that the effective-range derived directly from the low-energy, neutron-proton scattering data in a model independent way is the appropriate range to use in normalizing the deuteron's ground-state wave function, see, e.g., Brown and Jackson.\(^8\) Note that no effective-range normalization factor appears in the expression for the magnetic-dipole cross section. The parameters used here are taken from the evaluation of neutron-proton scattering by Lomon and Wilson.\(^9\)

In the following discussion the experimental data are compared with two functions for the total photodisintegration cross section based on the electric- and magnetic-dipole cross sections given by equations (3) and (4). The two functions were obtained by summing equations (3) and (4) multiplied by different normalization factors. The first function, $F(1)$, was obtained by applying a normalization factor only to equation (3). The usual effective range expression normalization, $N(ER) = 1.682$, was not used. Instead a factor was determined by fitting the total cross section at 10, 20, and 40 MeV to the values resulting from the detailed calculation of Partovi.\(^10\) The resulting normalization factor was $N(P) = 1.637$. With this normalization the total cross section given by the above expressions differ from the Partovi values by $-1.45$, $-0.03$, and $+1.42$ percent at 10, 20, and 40 MeV respectively. For the second function, $F(2)$, the same normalization factor, $N(P) = 1.637$, was applied to the electric-dipole cross section. In addition a factor of
M = 1.36 was applied to the magnetic-dipole cross section given by equation (4). The magnitude of this factor was determined by fitting the total cross section to the low energy points plotted in figure 1. The cross section given by F(2) differs from the Partovi values at 10, 20, and 40 MeV by -0.87, +0.46, and +1.86 percent respectively. Only for energies below 4 MeV is the cross section given by F(2) larger than that given by F(1) by more than two percent.

The experimental cross section data are compared with the function F(1) in figures 1 to 4. In figures 1 and 2 the function F(2) is also plotted. Note that in figures 1 and 2 cross section values are plotted while in figures 3 and 4 relative cross sections are given, i.e., the ratio of the measured cross section to that given by F(1) is plotted. The data in figures 1-3 are all from experiments in which the cross-section measurements did not require a knowledge of either the absolute intensity or the detailed spectral distribution of a laboratory-produced bremsstrahlung spectrum, i.e., they are all from class 1 or 2 experiments as defined above. Note the good agreement between the measured values and the calculated curves. For the class 1 and 2 experiments the average deviation of the experimental points from the values given by F(1) is +1.2 ± 5.5 percent. Based on the 31 data points between 1.504 and 38.7 MeV the chi-square for F(1) is 60. Using F(2), the average deviation of the points is reduced to +0.4 ± 4.6 percent and the chi-square is reduced to 12.

The data from the class 3 measurements, i.e., those which required information on both the spectral distribution as well as the absolute intensity of a laboratory produced bremsstrahlung spectrum, are plotted in figure 4. Note the rather poor agreement with the calculated cross section, F(1), shown by the points plotted in this figure. Note particularly the
different trends with photon energy. Some of the measurements indicate a positive slope for the relative cross section with increasing photon energy while other measurements indicate a decreasing slope. The mean deviation from the calculated cross section for the 43 class 3 measurements is $-9.3 \pm 7.8$ percent. This result is not surprising in view of the known problems associated with measurements making use of bremsstrahlung beams. Unless considerable care is taken to determine both the absolute intensity and spectral distribution of the bremsstrahlung beam used in a specific experimental geometry such experiments can easily have 10 to 20 percent uncertainties both in the absolute value of a cross section at a specific energy as well as in the relative cross section obtained by using photons from different parts of the same bremsstrahlung spectrum. Taken together the class 3 experiments give little information on the appropriateness of using the energy dependence given by the effective-range expressions to describe the deuteron's photodisintegration cross section below 50 MeV. In contrast the effective-range cross sections with a slight change in normalization give a very good representation of the data from the class 1 and 2 experiments. The values of $\sigma_{-2}$ obtained using the functions $F(1)$ and $F(2)$ are, respectively, $0.686 \pm 0.041$ and $0.712 \pm 0.036$ mb/MeV. These integrals were carried out to 44 MeV. Since the total cross section peaks at 4.4 MeV little is lost as a result of this cut off; 97 percent of the integral to 44 MeV is obtained by integrating to only 15 MeV.

The value of $\sigma_{-2}$ obtained from the "fit" to the empirical data contains contributions from all of the multipoles that contribute to the photodisintegration cross section. The quantity required to evaluate the static electric polarizability of the deuteron is $\sigma_{-2}(E1)$ which while responsible for most of the observed value does not account for all of it in any of the
theories of the deuteron. The measured value of $\sigma_{-2}$ given by the total cross section must then be corrected for the contribution of these other multipoles. The most important contribution results from magnetic-dipole transitions. All calculations result in a cross section for M1 transitions that peaks sharply just above the photodisintegration threshold (2.2246 MeV) and dominates over the electric-dipole cross section for energies up to 2.4 MeV, see e.g., the M1 cross section given in figure 1. While this cross section is small, the fact that it peaks so sharply at such a low energy results in it making a contribution of about 10 percent to the total $\sigma_{-2}$. Note that the data plotted in figures 1 and 2 require that there be a component in the total cross section that peaks strongly in the threshold region.

In figure 5 the total cross section given by the sum of equations 2 and 4 with $N = 1.682$, the effective-range value, is compared with the results of recent calculations as well as the low-energy experimental data points. Except for the energy region below 2.5 MeV the calculated cross section using $N = 1.637$ would be indistinguishable from the curve labeled JOENPEA. The curve labeled LEE results from a calculation to study weak parity-nonconservation effects in the disintegration of the deuteron by polarized low-energy photons and electrons.\textsuperscript{11} The calculation makes use of parity-admixed deuteron bound and continuum states. Note that this cross section curve maintains about the same positive slope above 2.9 MeV, the energy above which the slopes of the other curves plotted in figure 5 start to decrease. No multipole decomposition of this cross section is given. Both electric- and magnetic-dipole cross sections are given by Chetouani, et al.\textsuperscript{12} and by Joenpera.\textsuperscript{13} The main difference between these two calculations is in the wave functions used for the deuteron's ground state.
Chetouani et al. derived two ground-state wave functions by solving the Schrödinger equation resulting from two different "realistic", nonlocal, separable, nucleon-nucleon interactions that gave "equivalent satisfactory fits to the deuteron data and to the phase shifts up to 350 MeV". Except for a slight difference in the magnitude of the E1 cross section between 4 and 15 MeV these two wave functions gave equivalent results for the photodisintegration cross section. The maximum difference between the two cross sections was less than 4 percent at about 5 MeV. The ground-state wave function used by Joenperä was that derived by McGee. Both calculations used plane-wave continuum states except for the $^1S_0$ state where the singlet phase shift, $\delta_0$, was taken into account. Joenperä included the "shape-dependent" term in the expansion for ctg $\delta_0$ (see, e.g., ref. 8). The total cross section calculated by Chetouani, et al. remains about 8 percent higher than the cross section $F(1)$ plotted in figures 1 and 2 up to 10 MeV and then falls to join this curve at about 20 MeV. At 2.23 MeV the total cross section calculated by Joenperä is 1.4 times that given by $F(1)$ but falls rapidly so that between 2.7 and 6 MeV it is higher than that given by $F(1)$ in figures 1 and 2 by less than 1.5 percent. Above 6 MeV it rises again and is higher than the plotted curve by a factor of 1.14 at 8 MeV, the highest energy calculated.

While the cross-section expressions given by $F(1)$ and $F(2)$ result in a total cross section that agrees very well with the empirical values of the photodisintegration cross section it does not necessarily follow that the break up of the total cross section into its E1 and M1 components is correctly given by these expressions. While there is theoretical and some weak experimental evidence that the magnetic-dipole contribution to the total cross section is correctly given by equation (4), there is no evidence
that this component could be larger by a factor of 1.36 as was necessary to obtain the cross section given by \( F(2) \) which fits the low energy data. There have been eight photoproton angular-distribution measurements made at five photon energies between 2.5 and 2.89 MeV which indicate that the ratio \( \sigma(M1)/\sigma(E1) \) is not given correctly by any of the expressions used to describe the total cross section. In table 1 these data are compared with the ratios given by several calculations of the magnetic- and electric-dipole cross sections. The ratios of the values listed for \( \sigma(M1)/\sigma(E1) \) at 2.89 MeV to the values at 2.504 MeV is the same for all of the calculations to within 1.5 percent, i.e., to a good approximation the functional dependence on photon energy for the M1 and E1 cross sections is essentially the same for all calculations, e.g., that given by equations 3 and 4 above. The ratio given by these equations is 0.263, a much smaller value than the ratio of 0.45 \( \pm 0.10 \) given by the experimental data. As was first pointed out by Hulthén and Nagel\(^\text{15}\) it is difficult to simultaneously resolve the discrepancies between theory and experiment for both the angular distribution and total cross section data at these low energies. It is interesting to note that if the difference between the total cross sections \( F(2) \) and \( F(1) \), \( \Delta\sigma = 0.36 \sigma(M1) \), is assumed to be due to electric-dipole rather than magnetic-dipole transitions, a somewhat better agreement can be obtained between the calculated and experimental values for the ratio \( \sigma(M1)/\sigma(E1) \). Based on the five measured values of \( \sigma(M1)/\sigma(E1) \) the chi-square is reduced from 93.5 for \( F(1) \) to 27.4 for a total cross section given by \( F(1) + \Delta\sigma(E1) \).

While not directly giving a quantitative confirmation of the magnitude of the M1 strength concentrated just above the photodisintegration threshold, the early 41.5 MeV, 180 degree electron-scattering data\(^\text{16}\) tend to support an M1 strength that is consistent with that given by equation 4. While
differing considerably in the magnitude of the magnetic-dipole cross section they give at 2.23 MeV, ranging from 0.6 (ref. 15) to 1.4 (ref. 13) times the value given by equation 3, the values for the integral \( \sigma_{-2} \) up to 27 MeV given by various calculations range only from 0.0702 (ref. 15) to 0.0773 (ref. 12) mb/MeV. The value resulting from using equation 3 for the M1 cross section is 0.0721 mb/MeV. This value, with a conservative estimate of ±10 percent for its uncertainty has been used to obtain a value for \( \sigma_{-2}(E1) \):

\[
\sigma_{-2}(E1) = \sigma_{-2}(\text{tot}) - \sigma_{-2}(M1)
\]

\[
= 0.686 \pm 0.041 - 0.0713 \pm 0.0072
\]

\[
= 0.615 \pm 0.042 \text{ mb/MeV}
\]

The most this integral could be increased by is 0.026 mb/MeV. This would involve making the unlikely assumption that the entire difference between the functions \( F(1) \) and \( F(2) \) is due to electric dipole transitions. The relationship between the polarizability \( \alpha \) and \( \sigma_{-2} \) is

\[
\alpha = \frac{\hbar c}{2\pi^2} \sigma_{-2}(E1)
\]

or,

\[
\alpha(\text{fm}^3) = 0.9995 \sigma_{-2}(\text{mb/MeV})
\]

The value of the polarizability derived in a rather direct manner from the photodisintegration cross section, \( \alpha(pd) = 0.62 \pm 0.04 \text{ fm}^3 \) is in fair agreement with the value obtained by Rodning, et al. \(^3\) from the scattering of deuterons by \(^{209}\text{Pb} \), \( \alpha(sc) = 0.70 \pm 0.05 \text{ fm}^3 \).
The weakest points in the above discussion are all associated with questions concerning the magnitude and energy dependence of the magnetic- and electric-dipole photodisintegration cross sections between threshold and 10 MeV. The available data all come from measurements made at only eight energies with monoenergetic photon sources over 30 years ago. Both total cross section and angular distribution measurements making use of modern photon-source calibration and cross-section measurement technologies would be extremely valuable. In order to firmly establish the magnitude of both the empirical $\sigma_2$ integral as well as the contribution of magnetic-dipole transitions to it, careful total cross section and angular distribution measurements at as many photon energies as feasible between threshold and 5 MeV are required. The energy region up to 3 MeV is particularly important.
Fig. 1 The low energy photodisintegration cross section for the deuteron. The data points were obtained from measurements made with monoenergetic photon sources. The total cross section functions $F(1)$ and $F(2)$ are defined in the text. The lower curve is the magnetic-dipole cross section given by equation 4. References for the data points are given in the Bibliography of Experimental Data.
Fig. 2 Cross section for the photodisintegration of the deuteron. The data shown did not require a knowledge of either the intensity or shape of a bremsstrahlung spectrum. The curves give the total cross section given by expressions $F(1)$ and $F(2)$ described in the text. See Bibliography of Experimental Data for sources of data indicated by reference codes given in the upper right corner of figure.
Fig. 3 Ratio of experimental cross section values for the photodisintegration of the deuteron to the cross section given by F(l). The data shown did not require a knowledge of either the intensity or shape of a bremsstrahlung spectrum. See Bibliography of Experimental Data for sources of data indicated by reference codes given in the upper right corner of figure.
Fig. 4 Ratio of experimental cross section values for the photodisintegration of the deuteron to those given by equations 2 and 4 with N=1.637. The experimental data points all required a knowledge of both the intensity and shape of a bremsstrahlung spectrum. See Bibliography of Experimental Data for sources of data indicated by reference codes given in the upper right corner of figure.
Fig. 5 Comparison of calculations of the deuteron's total photodisintegration cross section for low energies. The curve labeled ER $N=1.682$ is that given by equations 2 and 4 with the indicated value of $N$. The curve for $N=1.637$ would be indistinguishable from that labeled JOENPERÄ for energies above 2.5 MeV. The curves labeled LEE, CHETOUANI, and JOENPERÄ are from references 11, 12, and 13 respectively.
Table 1. Ratio of magnetic-dipole to electric-dipole cross sections

<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>Hulthén (Nagel)</th>
<th>Eqs. 2+4 (N=1.637)</th>
<th>Chetouani et al.</th>
<th>Joenperä</th>
<th>Data</th>
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<tbody>
<tr>
<td>2.504</td>
<td>.660</td>
<td>.722</td>
<td>.693</td>
<td>.750</td>
<td>.600 ± .02&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>2.618</td>
<td>.395</td>
<td>.415</td>
<td>.395</td>
<td>.431</td>
<td>.360 ± .008&lt;sup&gt;a&lt;/sup&gt;</td>
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<tr>
<td>2.71</td>
<td>.286</td>
<td>.303</td>
<td>.293</td>
<td>.322</td>
<td>.49 ± .07&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>2.757</td>
<td>.250</td>
<td>.266</td>
<td>.257</td>
<td>.285</td>
<td>.265 ± .006&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td>2.89</td>
<td>.175</td>
<td>.190</td>
<td>.185</td>
<td>.200</td>
<td>.27 ± .06&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

<sup>c</sup>Weighted mean: G. R. Bishop, et al., Phys. Rev. <i>83</i>, 1052 (1951);
E. P. Meiners, Jr., Phys. Rev. <i>76</i>, 259 (1949);
N. O. Lassen, Phys. Rev. <i>75</i>, 1099 (1949);
B. Hammermesh and A. Wattenberg, Phys. Rev. <i>76</i> 1408 (1949).
BIBLIOGRAPHY OF EXPERIMENTAL DATA

Ref. Code | Energy Range (MeV)
---|---
57Hu80 L. Hulthén, M. Sugawara; Encyclopedia of Physics 39, 1, 2.504 - 17.6 edited by S. Flugge (Springer, Berlin, Germany, 1957)

Most of the monoenergetic photon-beam measurements reported in the following papers are listed in table 4 of this review.

G. R. Bishop, et al., Phys. Rev. 80, 211 (1950) 2.504
S. A. Colgate, Phys. Rev. 83, 1262 (1951) 2.62
G. R. Bishop, et. al., Phys. Rev. 80, 211 (1950)

A. H. Snell, Phys. Rev. 80, 632 (1950) 2.757
C. A. Barnes, et al., Phys. Rev. 86, 359 (1952) 4.45 ± .04
C. A. Barnes, et al., Phys. Rev. 86, 359 (1952) 7.39 ± .15
C. A. Barnes, et al., Phys. Rev. 80, 326 (1950) 12.5 ± .21
C. A. Barnes, et al., Phys. Rev. 90, 1069 (1950) 17.6 ± .2
P.V.C. Hough, Phys. Rev. 80, 1069 (1950)

55Al1 L. Allen, Jr., Phys. Rev. 98, 705 (1955) 22.7 - 64.0
58Wh1 A. Whetstone and J. Halpern, Phys. Rev. 109, 2072 (1958) 9.5 - 22.5
79Bo10 M. Bosman, et al., Phys. Lett. 82B, 212 (1979) 20.8 - 38.6
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Measurements of the deuteron's photodisintegration cross section made over a thirty year time span are evaluated in terms of the cross section for E1 and M1 transitions calculated in the effective range approximation. The energy range covered is from threshold to 44 MeV. Data that do not depend on a knowledge of a bremsstrahlung spectrum's intensity or spectral distribution are shown to be described very well by the effective-range expressions. The values of the deuteron's electric polarizability is shown to be in fair agreement with a value derived from the observed deviation from Rutherford scattering of deuterons by $^{208}$Pb. A comparison is made of the experimental data with three recent calculations of the photodisintegration cross section.

cross section, deuteron, dipole, electric, magnetic, nuclear, photodisintegration, polarizability