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Electromagnetic Nuclear Interactions: I. Introduction, Operators, and Sum Rules

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U.S. DEPARTMENT OF COMMERCE, Malcolm Baldrige, ***Secretary***
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FOREWORD

This report is the first of three in a series that will constitute a monograph on the subject of nuclear reactions initiated by photons and electrons. The present report (I) covers introductory examples, the electromagnetic operators, and sum rules. Subsequent reports will cover: (II) few nucleon systems and giant resonances in complex nuclei, and (III) intermediate and high energy reactions.

ELECTROMAGNETIC NUCLEAR REACTIONS

PREFACE

This monograph is aimed at the student or research worker in theoretical or experimental nuclear physics who wishes to acquire a working knowledge of the basic concepts and methods of electromagnetic reactions. The point of view is phenomenological with emphasis on understanding the data in terms of simple quantum models of the many body system. Prior knowledge assumed of the reader is a familiarity with the basic concepts of quantum mechanics and a general familiarity with nuclear physics. An advanced undergraduate course in these subjects would be an adequate preparation for reading this book.

There has been no book dedicated to the subject of electromagnetic nuclear reactions since Levinger's *Nuclear Photo-Disintegration* published in 1960. That monograph gave an overview of the field at a time of transition from betatron based measurements to electron linacs. The intervening twenty years has seen the growth of electron scattering, electrodisintegration, the neutron time-of-flight technique, monoenergetic photon generation by tagged bremsstrahlung and positron annihilation-in-flight, radiative particle capture experiments with tandem van de Graaffs, semiconductor detectors, sophisticated magnetic analysis systems, computer assisted data acquisition and numerous other technical advances. Theoretical advances over the last two decades have also been enormous: the schematic model with particle-hole interaction, the refinement of the shell model to include correlations, unification of collective models of rotation, vibration, and giant resonances, the recognition and treatment of meson exchange currents and isobar configurations in nuclear wave functions, Faddeev solution of the three and four nucleon equation of motion, and the accessibility of high speed, large memory computers for nuclear structure and reaction computations. Nuclear structure information gleaned from real and virtual photon reactions has become the foundation for single particle and collective motion analysis of hadronic and energetic weak interaction nuclear studies.

Photonuclear reactions are at the threshold of a new phase of development. High duty factor electron accelerators coming into use permit for the first time in this field the widespread use of coincidence experiments to measure correlations of reaction products with an incident electromagnetic probe of precisely controlled energy and momentum. This technique has long been profitably used in hadronicaly induced reactions, but the technique holds even higher promise for electromagnetically induced reactions because of the well understood interaction and single particle nature of the probe. Correlation information is of great interest in two other fields of nuclear physics: intermediate energy hadronic reactions (especially the pion-nucleus reactions), and heavy ion reactions. It is, therefore, an appropriate time to synthesize the state of common understanding of photonuclear reactions that has emerged from the "single arm" measurements made with pulsed electron linacs and radiative capture experiments.

The scope of this book differs from the excellent treatises of Überall on *Electron Scattering from Complex Nuclei* and of Eisenberg and Greiner *Excitation Mechanisms of the Nucleus-Electromagnetic and Weak Interactions*. My aim is more modest. I have tried to present the different facets of electromagnetic nuclear reactions in a unified way. The formalism of quantum electrodynamics is not developed, but nonrelativistic operators are introduced and used to compute reaction cross sections.

The first chapter differs in theme from those that follow. It is meant to lure the reader not yet convinced he or she can or wants to read an entire book on photonuclear reactions. In it examples are given of the successful application of simple quantum mechanics to the understanding of some basic electromagnetic reactions in nuclear physics.

The second chapter on the electromagnetic operator plunges into the nature of the three basic transition operators at a level of development suitable for the rest of the book. The connection between electromagnetic and energetic weak interactions is shown. Reaction rates of weak processes can often be predicted from known rates of the electromagnetic.

Sum rules are introduced in the third chapter as tools with which to analyze photon and electron reactions. A collection of expressions frequently used is derived from the closure relation, dispersion relation, and momentum integrals.

Chapter IV looks at how two, three, and four nucleon systems and their virtual meson constituents are used as a natural laboratory in which to test understanding of how real and virtual photons operate on interacting nucleons. Traditionally, electromagnetic measurements on few nucleon targets have been used to verify the structure and reaction predictions of two-nucleon force models based on hadronic reaction theories and data.

Nuclear collective motion induced by oscillating electric and magnetic fields gives rise to giant multipole resonances; these form the subject of the fifth chapter. The electric dipole giant resonance was the first universal collective property observed and understood in nuclei across the periodic table. The systematics of its energy, strength, width and decay properties occupied the attention of photonuclear scientists for many years. Attention has now turned to other possible giant resonances.

In Chapter VI we focus our attention on non-resonant reactions. Photon interactions between 40 and 140 MeV are often characterized as direct reactions with quasi-free absorption on one nucleon for virtual photons or on two nucleons for real photons. The study of these reactions gives information about how average nuclear properties affect the absorption and dissipation of electromagnetic energy. A connection with macroscopic physics is made by discussing the optical properties of nuclear matter.

Finally reactions above the meson production threshold are reviewed. The dominance of the spin 3/2, isospin 3/2 nucleon resonance on nuclear phenomena is discussed. At energies above this new giant resonance the hadronic structure of the photon itself comes into play. Finally, the dispersion relation is used again to demonstrate that photon reactions at all energies are mathematically related.

A note on units and dimensions used in the book: natural units (= c = 1) are used in the equations. SI (system international) dimensions are used for data except where some other dimension is so common it would be confusing to change. Energies and masses are usually quoted in MeV while lengths and momenta are in fm (femtometer or Fermi) and fm^{-1}.

ELECTROMAGNETIC NUCLEAR REACTIONS

I. INTRODUCTORY EXAMPLES

A. Single Particle Wave Functions

B. The Current Operator

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E. The Magnetic Moment Operator

F. Application to the M1 Cross Section of Deuterium

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I. INTRODUCTORY EXAMPLES

Electrons and photons have been favorite probes for studying the structure of physical systems. Only the wavelengths λ (corresponding to energies $\omega = 2\pi/\lambda$) change as one proceeds in scale from condensed matter studies to molecules, atoms, nuclei, and finally elementary particles. Nuclei vary in size from 10^{-12} to 10^{-13} cm. To study details of nuclear structure and transitions one needs photon and electron energies up to several hundred MeV.

In this first chapter, five simple electromagnetic reactions are examined. Characteristic cross section data are shown together with calculations based on single-particle Hamiltonians. These examples serve as an introduction to topics to be treated at greater length in subsequent chapters and demonstrate how electromagnetic phenomena test nuclear wave functions through the three basic interactions: current, magnetic moment, and charge.

A. SINGLE PARTICLE WAVE FUNCTIONS

A useful approximation to the wave functions of bound or ejected nucleons in a photonuclear reaction is given by the solution of the Schrödinger equation

$$\left[\frac{p^2}{2M} + V(\vec{r})\right] \psi(\vec{r}) = E\, \psi(\vec{r}) \tag{1A-1}$$

for a proton (mass M, charge e) acted on by a potential V whose range is very short. If we make the extreme assumption that $V(\vec{r}) = V_0 r\, \delta(\vec{r})$ the s-wave ($\ell = 0$) radial equation reduces to

$$\frac{1}{2M} \frac{d^2 u}{d r^2} + V_o \frac{\delta(r)}{r} u = Eu$$

$$u = r \psi(\vec{r}) \tag{1A-2}$$

which has the normalized solution* for the bound state E = -B

$$u_o = (2\alpha)^{\frac{1}{2}} \exp(-\alpha r) Y_{oo}(\hat{r}) \tag{1A-3}$$

with $\alpha = (2MB)^{\frac{1}{2}}$ and

$$u^{(o)} = \frac{1}{p} e^{-i\delta} \sin(pr + \delta), \; \delta = \tan^{-1}\left(\frac{-p}{\alpha}\right) \tag{1A-4}$$

for the s-wave scattering state with momentum p and kinetic energy $E = \frac{p^2}{2M}$. For $\ell > 0$ a centripetal term $-\frac{\ell(\ell+1)}{r^2}$ is added to the potential and $u^{(\ell)}$ becomes r times the ℓ^{th} component part of the plane wave $\exp(i\vec{p} \cdot \vec{r})$, i.e.,

$$u^{(\ell)} = i^{\ell}(4\pi(\ell+1))^{\frac{1}{2}} r \, j_{\ell}(pr) \, Y_{\ell o}(\hat{r}) \quad . \tag{1A-5}$$

B. THE CURRENT OPERATOR

A real photon is characterized by a unit electric polarization vector $\hat{e}$ and an energy-momentum four-vector $k_{\mu} = (\vec{k},\omega)$ with $|\vec{k}| = \omega$. The first order interaction energy of the photon vector potential $\vec{A}$ with the current $\vec{j}$ of a proton at point $\vec{r}$ is

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*The strength of the zero-range potential is fixed at $V_o = -(2M)^{-1}$, but the binding energy B is a free parameter. An alternative (Be35) to solving the zero range potential problem is to work with the equation for free motion, but with the boundary condition $-\frac{1}{u}\frac{du}{dr}\Big|_{r=0} = \alpha$.

$$H_{int} = \vec{A} \cdot \vec{j} = (\hat{\varepsilon} \exp(i\vec{k} \cdot \vec{r})) \cdot \left(e \frac{\vec{p}}{M}\right) . \tag{1B-1}$$

The photon flux is usually given a boson normalization factor $(2\pi/\omega)^{\frac{1}{2}}$. For our present purposes we will replace the exponential by unity and confine ourselves to the long wavelength approximation $kr << 1$. This electric dipole operator $\hat{\varepsilon} \cdot \vec{p}$ carries an intrinsic unit of angular momentum. Thus, if the initial proton wave function has zero orbital angular momentum, the final state must have $\ell = 1$ if the proton absorbs the photon.

The relevant radial functions are plotted in fig. (1B-1). The transition amplitude

$$T = \langle\psi_{final}|H_{int}|\psi_{initial}\rangle \tag{1B-2}$$

becomes

$$T_{E1} = - i \frac{e}{M} \left(\frac{2\alpha}{4\pi}\right)^{\frac{1}{2}} \int e^{-ip\cdot r} (\vec{\varepsilon} \cdot \vec{\nabla}) \frac{e^{-\alpha r}}{r} d^3 r \tag{1B-3}$$

having replaced the momentum operator by

$$\vec{p} = -i \vec{\nabla} .$$

The angular integration insures that only the p-wave part of ψ_{final} contributes to T_{E1}. The integral can be evaluated by applying* the gradient to the plane wave giving

$$T_{E1} = \frac{e}{M} \left(\frac{2\alpha}{4\pi}\right)^{\frac{1}{2}} \frac{4\pi\hat{\varepsilon} \cdot \vec{p}}{(\alpha^2 + p^2)} . \tag{1B-4}$$

$$* \int e^{-i\vec{p}\cdot\vec{r}} \hat{\varepsilon}\cdot\vec{\nabla} f(\vec{r}) d^3r = i\hat{\varepsilon}\cdot\vec{p} \int e^{-i\vec{p}\cdot\vec{r}} f(\vec{r}) d^3r$$

For future reference we note the zero-range electric dipole transition amplitude is also easily evaluated with momentum space wave functions as

$$T_{E1} = \int \delta(\vec{p} - \vec{p}_o)\, \frac{e}{M}\, \hat{\varepsilon} \cdot \vec{p}_o \; \frac{(4\pi 2\alpha)^{\frac{1}{2}}}{(\alpha^2 + p_o^2)}\, d^3 p_o \qquad (1B\text{-}5)$$

$$= \left(\frac{2\alpha}{4\pi}\right)^{\frac{1}{2}} \frac{e}{M}\, \frac{4\pi\, \hat{\varepsilon} \cdot \vec{p}}{(\alpha^2 + p^2)} \; .$$

The dipole operator $\hat{\varepsilon}\cdot\vec{p}/M$ can be expressed in another form

$$\hat{\varepsilon}\cdot \frac{\vec{p}}{M} = i(E_f - E_i)\hat{\varepsilon}\cdot\vec{r} \qquad (1B\text{-}6)$$

where $E_{i,f}$ are the initial and final nuclear excitation energies. This replacement is useful when wave functions are expressed in $\vec{r}$-space. Fig. 1B-1C shows the dipole radial integrand in the zero range model using this form of the operator.

C. PHOTOABSORPTION CROSS SECTION

The differential cross section for absorbing a photon and emitting the bound proton is

$$d\sigma = \frac{2\pi}{(\text{flux})} \left| T \right|^2_{\substack{\text{pol.} \\ \text{avg.}}} \frac{dn}{d\omega} \, , \qquad (1C\text{-}1)$$

the flux for a photon is $(\omega/2\pi)$ and $\frac{dn}{d\omega}$ is the density of final nucleon states. An average over the two polarization states of the photon is to be performed on $|T|^2$. Figure (1C-1) shows how the two photon polarizations relate to the angle θ the ejected proton makes with the incident photon momentum. The average of $\hat{\varepsilon} \cdot \vec{p}$ is

$$\left|\hat{\varepsilon} \cdot \vec{p}\right|^2_{\substack{\text{pol.} \\ \text{avg.}}} = \frac{1}{2}\,[(\hat{\varepsilon}_x \cdot \vec{p})^2 + (\hat{\varepsilon}_y \cdot \vec{p})^2]$$

$$= \frac{1}{2}\, p^2\, [1 - (\hat{k} \cdot \hat{p})^2]$$

$$= \frac{1}{2}\, p^2 \sin^2\theta \; . \qquad (1C\text{-}2)$$

The density of final nucleon states

$$\frac{dn}{d\omega} = \frac{d^3 p}{(2\pi)^3 \; d\omega} \tag{1C-3}$$

is evaluated using energy conservation

$$\omega = \frac{p^2}{2M} + B \tag{1C-4}$$

to give

$$\frac{dn}{d\omega} = \frac{Mp \; d\Omega}{(2\pi)^3} \quad . \tag{1C-5}$$

Combining all factors the angular distribution of the photoprotons as a function of photon energy is

$$\frac{d\sigma}{d\Omega} = 2e^2 \frac{\alpha}{M} \frac{p}{\omega} \; \frac{p^2 \sin^2\theta}{(\alpha^2 + p^2)^2} \quad . \tag{1C-6}$$

The total cross section* (using $p^2 + \alpha^2 = 2M\omega$) is then

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*The corresponding retarded expressions which include the momentum dependence (i.e. all multipoles) of the photon operator eq. (1B-1) are:

$$\frac{d\sigma}{d\Omega} = 2e^2 \frac{\alpha}{M} \frac{p}{\omega} \; \frac{p^2 \sin^2\theta}{[\alpha^2 + (\vec{p}-\vec{k})^2]^2} \tag{1C-6'}$$

and

$$\sigma(\omega) = 2e^2 \frac{\alpha}{M} \frac{p}{\omega} \left[\frac{\pi}{2} \; \frac{(\alpha^2 + p^2 + k^2)}{pk^3} \; \ell n \left(\frac{\alpha^2 + (p + k)^2}{\alpha^2 + (p - k)^2} \right) \; - \frac{2\pi}{k^2} \right] \tag{1C-7'}$$

$$\sigma(\omega) = \frac{4\pi\, e^2}{3}\,\frac{\alpha(2M(\omega - B))^{3/2}}{(M\omega)^3}$$

$$= \frac{8\pi}{3}\;\frac{2e^2}{BM}\;\;\frac{(x - 1)^{3/2}}{x^3} \qquad \text{(1C-7)}$$

with $x = \omega/B$ and is shown plotted in fig. (1C-2). Note the peak of σ is at twice the threshold energy B and approaches zero asymptotically as $\omega^{-3/2}$. Although the cross section rises and falls with ω the reaction is not a resonance because the transition amplitude eq. (1B-4) does not have a singularity on or near the photon positive energy axis.

The shape of $\sigma(\omega)$ does not change appreciably if finite range potentials are chosen for $V(\vec{r})$, e.g., Yukawa or Woods-Saxon shapes. Furthermore, it will be shown in Chapter III that the area under $\sigma(\omega)$ is independent of the shape of $V(\vec{r})$. Integrating eq. (1C-7) gives

$$\int_B^\infty \sigma(\omega)\; d\omega = 2\pi^2\,\frac{e^2}{M} \quad . \qquad \text{(1C-8)}$$

This is an example of a *sum rule.*

D. APPLICATION TO THE ELECTRIC DIPOLE CROSS SECTION OF DEUTERIUM

These results may be immediately applied to the photodisintegration of the deuteron by making two adjustments to eqs. (1C-6) and (1C-7). First, np relative coordinates* are used to calculate T_{E1}. Second, the asymptotic normalization of the ground state wave function must be adjusted to account for the finite range r_t of the np potential. In the theory of finite range potentials (Be50) one replaces the zero-range normalization constant $(2\alpha)^{\frac{1}{2}}$ by $(2\alpha/(1 - \alpha r_t))^{\frac{1}{2}}$.

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*$\vec{r} = \vec{r}_p - \vec{r}_n$, $\vec{p} = \frac{1}{2}(\vec{P}_p - \vec{P}_n)$

This adjustment provides the proper asymptotic normalization for the ground state. The ψ_o of a finite range potential is finite at r = o. However, the exponential shape of the zero range ψ_o is correct for any finite range potential when $r > r_t$. With these two adjustments the effective range calculation for the photodisintegration of deuterium reads

$$\sigma_D(\omega) = \frac{8\pi}{3} \frac{e^2\alpha}{(1 - \alpha r_t)} \frac{[M(\omega - B)]^{3/2}}{(M\omega)^3} . \qquad (1D-1)$$

The result is compared with D (γ,p) n data in fig. (1C-3) with $\alpha = 0.232\ fm^{-1}$ and a triplet range evaluated from low energy neutron-proton elastic scattering data $r_t = 1.76$ fm.

E. THE MAGNETIC MOMENT OPERATOR

The nucleon spin operator $\vec{\sigma} = (\sigma_{+1}, \sigma_{-1}, \sigma_3)$ is important for many electromagnetic reactions. The magnetic field $\vec{B}$ of the photon interacts with the nucleon magnetic moment $\vec{m}$ to give

$$H_{int} = \vec{m} \cdot \vec{B} \qquad (1E-1)$$

using

$$\vec{B} = \vec{\nabla} \times \vec{A} = \vec{\nabla} \times \hat{\varepsilon} \exp(ik\cdot\vec{r}) = i(\vec{k}\times\hat{\varepsilon}) \exp(i\vec{k}\cdot\vec{r}) \qquad (1E-2)$$

and

$$\vec{m} = \frac{e\mu}{2M} \vec{\sigma} \qquad (1E-3)$$

gives

$$H_{int} = \left(\frac{e\mu}{2M}\vec{\sigma}\right) \cdot (i(\vec{k}x\hat{\varepsilon})\exp(i\vec{k}\cdot\vec{r})) \tag{1E-4}$$

In analogy to our previous example of the E1 photo effect on a bound s-wave nucleon let us calculate the long wavelength approximation $[\exp(i\vec{k}\cdot\vec{r})\to 1]$ for the magnetic-dipole (M1) transition from an s-wave bound state to the s-wave continuum. This transition depends on the spin structure of the wave functions,

$$\Psi_{initial} = (2\alpha)^{\frac{1}{2}} \frac{e^{-\alpha r}}{r} Y_{oo}(\hat{r}) \chi_m^{\frac{1}{2}} \tag{1E-5}$$

$$\Psi_{final} = e^{-i\delta} (4\Pi)^{\frac{1}{2}} \frac{\sin(pr+\delta)}{pr} Y_{oo}(\hat{r}) \chi_{m'}^{\frac{1}{2}} , \tag{1E-6}$$

where $\chi_{\pm\frac{1}{2}}^{\frac{1}{2}}$ are the up (+) or down (-) nonrelativistic spin functions. The spin-flip amplitude is

$$T_{M1} = \frac{e\mu}{2M} \langle \chi_{m'}^{\frac{1}{2}} \left| (\vec{k}x\hat{\varepsilon})\cdot\vec{\sigma} \right| \chi_m^{\frac{1}{2}} \rangle$$

$$x\ (4\Pi)^{\frac{1}{2}} e^{-i\delta} \int |Y_{oo}|^2 d\Omega \int_0^\infty dr \frac{\sin(pr+\delta)}{p} e^{-\alpha r} (2\alpha)^{\frac{1}{2}} . \tag{1E-7}$$

The spin matrix element may be evaluated by standard techniques to give the polarization average

$$\frac{1}{2} \sum_{m',\lambda,m} |\langle \chi_{m'}^{\frac{1}{2}} |(\vec{k}x\hat{\varepsilon})\cdot\vec{\sigma}| \chi_m^{\frac{1}{2}} \rangle|^2 \tag{1E-8}$$

$$= k^2 = \omega^2 .$$

We note the radial integral is zero because of orthogonality if $\tan\delta = -p/\alpha$. Therefore, if the transition is to go at all, the phase shift in the spin-flip final state must be governed by a parameter different from $\alpha = (2M\,B)^{\frac{1}{2}}$; call it β so that

$$\tan\delta = -p/\beta \quad . \tag{1E-9}$$

For zero-range potentials the parameters α and β are the inverse of the scattering lengths for the two spin states. The spatial part of the transition amplitude is

$$T_{M1} = \frac{e\mu}{2M}\,(4\Pi)^{\frac{1}{2}}\,(2\alpha)^{\frac{1}{2}}\,\omega\,\frac{[\cos\delta + \frac{\alpha}{p}\sin\delta]}{(\alpha^2 + p^2)}\,e^{-i\delta}$$

$$= \frac{e\mu}{2M}\,(4\Pi)^{\frac{1}{2}}\,(2\alpha)^{\frac{1}{2}}\,\omega\,\frac{(\beta-\alpha)}{(\alpha^2+p^2)(\beta^2+p^2)^{\frac{1}{2}}}\,e^{-i\delta} \quad . \tag{1E-10}$$

The M1 differential cross section is

$$\frac{d\sigma_{M1}}{d\Omega} = \frac{4\Pi^2}{\omega}\,\left|T_{M1}\right|^2_{\substack{\text{pol.}\\ \text{avg.}}}\,\frac{Mp}{(2\Pi)^3}$$

The angular distribution of the ejected particle is isotopic,

$$\frac{d\sigma_{M1}}{d\Omega} = \frac{e^2\mu^2\alpha\omega p(\alpha-\beta)^2}{M(\alpha^2+p^2)^2(\beta^2+p^2)} \qquad (1E\text{-}11)$$

giving a total magnetic dipole cross section

$$\sigma_{M1} = \frac{\pi e^2\mu^2(2MB)^{\frac{1}{2}}(2M(\omega-B)^{\frac{1}{2}}(\alpha-\beta)^2}{M^3\omega[2M(\omega-B)+\beta^2]} \qquad (1E\text{-}12)$$

Figure (1E-1) illustrates the zero-range model wave functions and their product. The lack of a spatial part to the M1 operator in the long wavelength limit causes the integrand in eq. (1E-7) to peak at r=0. This behavior is in contrast to the E1 integrand which peaks at a relatively large value of r (see fig. (1B-1c)).

The magnetic dipole cross section for the zero range model is shown in fig. (1E-2) for three values of the final state potential strength parameter β.

F. APPLICATION TO THE MAGNETIC DIPOLE CROSS SECTION OF DEUTERIUM

The ${}^3S_1 \rightarrow {}^1S_0$ transition* from the deuteron bound state to the spin singlet s-wave continuum near the np breakup threshold is a good example of the action of the magnetic moment part of the photon operator. Both the neutron and proton contribute through their magnetic moments (presumed to equal their free values $\mu_p = 2.79$ and $\mu_n = -1.91$ in units of e/2M). The spin matrix element yields

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*The notation is ${}^{2S+1}L_J$ where S is the magnitude of the vector sum of the neutron and proton spins $\vec{S} = \frac{1}{2}(\vec{\sigma}_p + \vec{\sigma}_n)$, L is the orbital quantum name (S,P,D,F, etc. for L = 0, 1, 2, 3, etc.) and J is the total np angular momentum.

$$\left|\langle {}^1S_0 \left| (\hat{k}\times\hat{\varepsilon})\cdot[\mu_p\vec{\sigma}_p+\mu_n\vec{\sigma}_n]\right| {}^3S_1\rangle\right|^2_{\substack{\text{pol.}\\ \text{avg.}}}$$

$$= \frac{1}{3}(\mu_p-\mu_n)^2 \; . \qquad (1F\text{-}1)$$

The spatial part of the transition amplitude is obtained from eq. (1E-11) with the replacement of M and p by their reduced values to give*

$$\frac{d\sigma_D(M1)}{d\Omega} = \frac{e^2(\mu_p-\mu_n)^2\alpha\omega p(\alpha-\beta)^2}{6M(\alpha^2+p^2)^2(\beta^2+p^2)} \qquad (1F\text{-}2)$$

and

$$\sigma_D(M1) = \frac{2\pi e^2(\mu_p-\mu_n)^2(MB)^{\frac{1}{2}}(M(\omega-\beta))^{\frac{1}{2}}(\alpha-\beta)^2}{3\,M^3\,\omega\,[M(\omega-B)+\beta^2]} \qquad (1F\text{-}3)$$

with α^2 = MB for the M1 contribution to D+γ→p+n. The parameters appropriate for the bound deuteron (B = 2.22 MeV) are $\alpha = 0.232\ \text{fm}^{-1}$, $r_t = 1.76$ fm while β of the singlet s-wave continuum is the inverse of the singlet np scattering length $\beta = a_s^{-1} = -0.042\ \text{fm}^{-1}$. The M1 photoabsorption cross section is much smaller than the E1 except within a few keV of threshold.

The reaction which is most sensitive to the M1 transition is the inverse of the photoabsorption viz. radiative capture n+p → D + γ of very slow neutrons. For neutrons thermalized to room temperature (T = 300 K) the lab momentum p_n = 2p is

$$\frac{p_n^2}{2M} = kT = 0.026\ \text{eV} \qquad (1F\text{-}4)$$

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*Note the effective range correction to the normalization factor $(1-\alpha\, r_t)^{-1}$ is not used for the S→S transition because the overlap integral peaks at r=0. However other effective range corrections modify eqs. 1F-2 and 3 slightly (No 65).

and the $^1S_o \to {}^3S_1$, transition is dominant. The relation between the photo-disintegration and radiative capture cross sections for this transition is[*] given by

$$\sigma_{cap} = \frac{3}{2}\,\frac{\omega^2}{p^2}\,\sigma_{dis} \quad . \tag{1F-5}$$

For 300 K neutrons on hydrogen eqs. (1F-3,5) give σ_{cap} = 300 mb whereas measurement yields 334 mb (No 65). The 10% disagreement between these two numbers was a puzzle for more than 20 years. The missing 10% of theoretical cross section has now been found to be due to explicit meson and nucleon resonance effects in the interaction of photons with nucleons in the presence of other nucleons as discussed in Chapter IV. These two (or more) nucleon effects, collectively called *meson exchange corrections* are explored in more detail in the next chapter.

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[*]In general the detailed balance for the photoreaction A+γ ⇄ B+C is

$$\sigma_{cap}(p) = \frac{(2J_A+1)2}{(2J_B+1)(2J_C+1)} \left(\frac{\omega}{p}\right)^2 \sigma_{dis}(\omega)$$

where the J's are the spins of the respective particles and ω and p are center-of-momentum variables.

G. PHOTON ABSORPTION IN A RESONANCE

A nucleon bound in a harmonic oscillator potential,

$$V(r) = \frac{1}{2} M \omega_0 r^2 \qquad (1G\text{-}1)$$

where ω_0 is the energy spacing between major shells, can absorb a photon by making a transition to the next higher unfilled shell. The cross section for E1 absorption on a proton is given by

$$\sigma(\omega) = \frac{4\pi^2 e^2}{\omega} \left| \langle \psi_f \; | \hat{\varepsilon} \cdot \vec{p}/M | \; \psi_i \rangle \right|^2 . \qquad (1G\text{-}2)$$

The density of final states for a resonance transition is unity. As an example an S→P transition has an amplitude

$$T_{E1} = eN_pN_s \int Y^*_{10} (\hat{r}) \left[-i\hat{\varepsilon}\cdot\hat{r} \right] Y_{00} (r) \, d\Omega \int_0^\infty \frac{r}{b} \, e^{-r^2/2b^2} \frac{\partial}{\partial r} \, e^{-r^2/2b^2} \, r^2 dr$$

$$= \frac{1}{\sqrt{2}} \frac{e}{b} \qquad (1G\text{-}3)$$

where $\vec{p} = -i \vec{\nabla}$ has been used for the current operator and $b^{-2} = M\omega_0$ relates the oscillator parameter to the energy level spacing. Evaluation of the cross section gives a spike at the inter shell spacing

$$\sigma(\omega) = \frac{2\pi^2 e^2}{M} \, \delta \, (\omega - \omega_0) . \qquad (1G\text{-}4)$$

The harmonic potential is a good approximation for bound single-particle states in a nucleus, but it lacks the damping due to decay channels for continuum states. If we add a damping term to the oscillator Hamiltonian $\omega_0 \rightarrow \omega_0 - i \, \Gamma/2$ the photon absorption cross section has an energy dependence given by the Lorentz form

$$\sigma(\omega) = \frac{4\pi e^2}{M} \frac{\omega^2 \Gamma}{(\omega_0^2-\omega^2)^2+\omega^2\Gamma^2} \tag{1G-5}$$

preserving the area

$$\int_0^\infty \sigma(\omega)\, d\omega = \frac{2\pi^2 e^2}{M} \ . \tag{1G-6}$$

Note the area is independent of ω_0 and Γ.

Complex nuclei have particles in many oscillator levels, but nucleons can only make dipole transitions across one major shell to unfilled orbitals. Thus only the last neutron and proton shells contribute to electric dipole absorption. Since all photon transitions occur with $\omega = \omega_0$ the total cross section is a superposition of single-particle Lorentz lines. Figure 1G-1 shows the total absorption cross section for gold and a fit to the data by a Lorentz resonance line.

The phenomenon of the concentration of electric dipole absorption cross section around one photon energy is called the E1 giant resonance. The interpretation of the resonance as a collection of independent harmonic oscillator transitions suffers from the fact that the observed resonance frequency is at twice the energy as that inferred from other measurements of ω_0. This problem is discussed in Chapter V.

H. ELECTRON SCATTERING - THE CHARGE OPERATOR

When high energy electrons scatter from the nucleus the electromagnetic quanta exchanged in the interaction are very similar to real photons. As will be discussed in the next chapter, one may assume only one virtual quantum with an energy above a few keV is exchanged between the electron and the nucleus. This quantum differs in two respects from a real photon. First, the momentum transfer to the nucleus

$$\vec{k} = \vec{k}_1 - \vec{k}_2 \tag{1H-1}$$

(where $\vec{k}_{1,2}$ are the initial and final electron momenta) is always greater than the energy transfer to the nucleus

$$\omega = E_1 - E_2 \simeq k_1 - k_2 \ . \tag{1H-2}$$

This is in contrast to a real photon for which energy and magnitude of momentum transfer are equal, $\omega = |\vec{k}|$. In this respect, electron scattering gains a new degree of freedom over photoabsorption since the momentum transfer in a reaction can be varied independently of the energy transfer. This feature of the electron scattering quantum gives it the name "virtual photon."

The second difference between real and virtual photons is that the virtual quanta have a component of the vector potential A_z parallel to the direction of propagation $\vec{k}$ of the virtual quantum and a non-zero fourth component ϕ in the four-vector potential $A_\mu = (\vec{A},\phi)$. The scalar field ϕ is the Coulomb potential created by the electron at the position of the nucleus. Thus, in addition to the transverse current and magnetic moment interaction terms $\vec{j}\cdot\vec{A}_\perp$ and $\vec{m}\cdot(\vec{\nabla}\times\vec{A})$, the virtual photon has a longitudinal current interaction $\vec{j}\cdot\vec{A}_{||}$ and a Coulomb interaction $e\phi$ with each proton.

To round out our illustrative examples of electromagnetic interactions of a nucleon bound in a potential, we calculate the Coulomb contribution to electron scattering from a point proton bound by an energy B in a zero-range potential.

The operator for Coulomb interaction with a proton is simply

$$H_{int} = e\phi = e \exp(i\vec{k}\cdot\vec{r}) \ . \qquad (1H\text{-}3)$$

Since one of the final nuclear states possible with the Coulomb transition operator is the ground state, elastic electron scattering is possible. The transition amplitude is the Fourier transform of the ground state density

$$T_{el} = \langle\psi_o(r) \ |\exp\ (i\vec{k}\cdot\vec{r})|\ \psi_o(r)\rangle \ . \qquad (1H\text{-}4)$$

For the zero-range wave function

$$T_{el} = \frac{2\alpha}{k} \tan^{-1} \frac{k}{2\alpha} \quad . \qquad (1H\text{-}5)$$

The continuum states in the spin independent zero range model are

$$\psi(\vec{p},\vec{r}) = e^{i\vec{p}\cdot\vec{r}} + \left[\frac{e^{-i\delta}\sin(pr+\delta)}{pr} - \frac{\sin pr}{pr}\right] \qquad (1H\text{-}6)$$

giving an inelastic amplitude*

$$T_{inel} = \langle\psi(\vec{p},\vec{r})\,|\exp(i\vec{k}\cdot\vec{r})|\,\psi_o(r)\rangle$$

---

*Note that although the s-wave continuum state (without spin flip) is orthogonal to the bound state, the spatial dependence of the s-wave part of the operator makes the contribution to the overall amplitude non-zero.

$$= (2\alpha)^{\frac{1}{2}} (4\pi)^{\frac{1}{2}} \left[\frac{1}{\alpha^2 + (\vec{k}-\vec{p})^2} \right.$$

$$+ \frac{(e^{-i\delta}\cos\delta - 1)}{4pk} \quad \ell n \quad \frac{\alpha^2 + (p+k)^2}{\alpha^2 + (p-k)^2}$$

$$\left. + \frac{e^{-i\delta}\sin\delta}{2pk} \left(\tan^{-1}\left(\frac{2\alpha k}{\alpha^2+p^2-k^2} \right) + \varepsilon\pi \right) \right] \qquad (1H\text{-}7)$$

where

$$\tan\delta = -\,p/\alpha, \ \sin\delta = -p/(p^2+\alpha^2)^{\frac{1}{2}}, \ \cos\delta = \alpha/(p^2+\alpha^2)^{\frac{1}{2}} \qquad (1H\text{-}8)$$

and

$$\varepsilon = \begin{Bmatrix} 0 \text{ for } \alpha^2 + p^2 - k^2 \geq 0 \\ 1 \text{ for } \alpha^2 + p^2 - k^2 < 0 \end{Bmatrix} .$$

The electron scattering Coulomb cross section (whose derivation will be discussed in the next chapter) has an elastic part

$$\frac{d\sigma_{el}}{d\Omega} = \sigma_o \left| T_{el} \right|^2 \qquad (1H\text{-}9)$$

where σ_o is the Mott cross section

$$\sigma_o = \left[\frac{e^2 \cos\theta/2}{2k_1^2 \sin^2\theta/2} \right]^2 \qquad (1H\text{-}10)$$

and an inelastic part*

$$\frac{d^3\sigma}{d\Omega dk_2} = \sigma_o \left| T_{inel} \right|^2 \frac{d^3p}{(2\pi)^3} \ \delta(\omega - B - \frac{p^2}{2M}) \ . \qquad (1H\text{-}11)$$

$^*\int p^2 \delta(\omega - B - \frac{p^2}{2M})\, dp = Mp$

If the recoil proton is not detected the scattered electron spectrum is

$$\frac{d^2\sigma}{d\Omega dk_2} = \sigma_o \left|F_{inel}(k,\omega)\right|^2 = \sigma_o \int \left|T_{inel}\right|^2 \frac{Mp\ d\Omega_p}{(2\pi)^3} . \qquad (1H\text{-}12)$$

Figure (1H-1) shows $|F_{inel}(k,\omega)|^2$ the inelastic response function. At higher momentum transfer ($k \gg \alpha$) the peak shifts to $p=k$ making $\omega_{peak} = \frac{k^2}{2m} + B$. The scattering is then called *quasi-free*.

As the momentum transfer increases, the elastic form factor $F_{el}^2(k) = |T_{el}|^2$ decreases while the area under the inelastic spectrum increases. A remarkable sum rule governs the interplay of elastic and inelastic scattering from a particle bound in a well

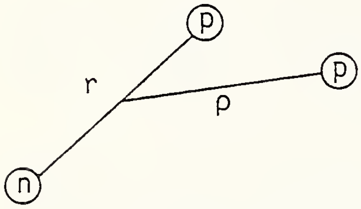
$$F_{el}^2(k) + \int_B^\infty \left|F_{inel}(k,\omega)\right|^2 d\omega = 1. \qquad (1H\text{-}13)$$

This results from the completeness relation of the eigenstates of a Hamiltonian. A nuclear system with more than one proton has a slightly more complicated sum rule (see Chapter III).

I. APPLICATION TO THE ELECTRODISINTEGRATION OF ^3He

The breakup of ^3He into p + d and p + p + n by electrons is dominated by the Coulomb interaction for scattering angles away from the extreme forward and backward directions. A calculation which treats the ^3He ground state as a proton bound by a zero-range potential to a deuteron can be used to compute the ^3He (e,e') p d cross section near the two-body breakup threshold at $\omega_2 = B_{pd} = 5.5$ MeV. Above $\omega_3 = B_{ppn} = 7.7$ MeV, however, the ^3He (e,e') p p n reaction also contributes to the inelastic electron scattering cross section.

The three nucleon breakup channel can be calculated by treating the deuteron as a proton bound to a neutron by another zero-range potential with $B_{np} = 2.2$ MeV. The ^{3}He ground state wave function is then a product of zero range wave functions



$$\psi_o = \frac{(4\gamma\alpha)^{\frac{1}{2}}}{r\rho} \exp(-\gamma r-\alpha\rho) \ . \qquad (1I-1)$$

The pd and ppn breakup channels are represented in this model by products of continuum wave functions (eqs. 1A-4,5) in the r and ρ coordinates. The electron scattering calculation using the charge operator acting on the two protons can then be carried out as described in the previous section. The result is compared with the data in fig. (1I-1). The initial sharp rise in the inelastic scattering cross section is explained by the theory as due to s-wave p + d breakup and is in agreement with the known p + d s-wave scattering length.

This reaction is treated in Chapter IV using more realistic three-nucleon wave functions, but it is interesting that the qualitative features of the data can be understood in this simple zero-range model.

J. THRESHOLD PION PHOTOPRODUCTION

The liberation of pions from nucleons is the dominant reaction of photons above $\omega = m_\pi \simeq 140$ MeV. Figure (1J-1) indicates the relevant kinematic variables, fluxes, and coupling constants. The nonrelativistic amplitude near threshold for charged pion photoproduction

$$T_{\gamma\pi}{}^{\mp} = \frac{ef}{m_\pi} \langle N' | \sqrt{2}\, \tau^{\pm}\, \hat{\varepsilon}\cdot\vec{\sigma} \left(1 \pm \frac{\omega}{2M}\right) | N \rangle \qquad (1J-1)$$

involves the nucleon spin operator and the isopin raising and lowering operators. The (+) on the right hand side of (1J-1) goes with $\gamma n \rightarrow p\pi^-$ and the (-) with $\gamma p \rightarrow n\pi^+$. The center-of-momentum differential cross section can be calculated from

$$d\sigma = \frac{2\pi}{|V_\gamma - V_N|} \left(\frac{2\pi}{\omega}\right)\left(\frac{2\pi}{q_o}\right) |T_{\gamma\pi}|^2_{\substack{\text{pol.}\\ \text{avg.}}} \frac{d^3q}{(2\pi)^3 dW} \qquad (1J\text{-}2)$$

where $W = \omega\left(1 + \frac{\omega}{2M}\right)$ is the total CM energy available. Reduced mass factors appear from the initial $\left(1 + \frac{\omega}{M}\right)$ and final $\left(1 + \frac{m_\pi}{M}\right)$ relative momenta. The resulting expression near threshold is

$$\frac{d\sigma}{d\Omega}(\gamma,\pi^{\mp}) = \left(\frac{2^2 e^2 f}{m_\pi{}^2}\right) \frac{\left(1 \pm \frac{\omega}{2M}\right)^2}{\left(1 + \frac{\omega}{M}\right)\left(1 + \frac{m_\pi}{M}\right)} \left(\frac{q}{k}\right) \qquad (1J\text{-}3)$$

using $f^2 = 0.08$

$$\frac{k}{q}\ \frac{d\sigma}{d\Omega} \begin{cases} 15\ \mu b/sr\ (\gamma p \to n\pi^+) \\ 20\ \mu b/sr\ (\gamma n \to p\pi^-) \end{cases} . \qquad (1J\text{-}4)$$

Figure (1J-2) shows a comparison of eq. (1J-3) with the data.

In Chapter VII, we shall explore the influence of nuclear structure and pion-nucleus interactions on the (γ,π) process.

CONCLUSION

We have tried to show in this introductory chapter some examples of how the simplicity of the electromagnetic charge, current and magnetic moment operators permit us to calculate the cross sections of real and virtual photon reactions. Many fine points have been glossed over, but the agreement with data on hydrogen and helium photoreactions is impressive. In the following chapters we shall elaborate on: the electromagnetic operators for real and virtual photons, sum rules, more realistic wave functions for few nucleon system, many-nucleon systems and collective motion, direct reactions and the influence of the nuclear medium on elementary processes, the optical properties of nuclear matter, the explicit role of mesons in photoabsorption, and the high ω dependence of photonuclear reactions.

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FIGURE CAPTIONS

Fig. (1B-1) a) Bound state wave function with $\alpha = 0.694\ \mathrm{fm}^{-1}$ corresponding to 10 MeV binding. R marks the nuclear radius. b) The $\ell = 0$ and $\ell = 1$ continuum wave functions with $p = 0.694\ \mathrm{fm}^{-1}$ corresponding to 10 MeV kinetic energy, $\delta = -0.785$ radians. c) Integrand of the electric dipole amplitude at 20 MeV photon energy. Note the main contribution to the radial integral comes from outside the bound state radius R.

Fig. (1C-1) Coordinate system for calculating the average of the square of the transition amplitude over the two photon polarization vectors using the relation

$$(\hat{\varepsilon}_x \cdot \vec{p})^2 + (\hat{\varepsilon}_y \cdot \vec{p})^2 + (\hat{k}\cdot\vec{p})^2 = p^2$$

Fig. (1C-2) Electric dipole cross section for a photon bound by energy B in a zero-range potential well, eq. (1C-7). The cross section is in units of $(2e^2/BM)$ and the photon energy is in units of B.

Fig. (1C-3) Comparison of expression (1D-1) with data for the reaction $D(\gamma,p)n$ (Sk74).

Fig. (1E-1) Radial wave functions for an s-wave transition. The parameters are: $\alpha = 0.694\ \mathrm{fm}^{-1}$, $\beta = -\alpha$, $p = 0.694\ \mathrm{fm}^{-1}$, $\delta = +0.785$ rad. The units are: μ_o in $\mathrm{fm}^{-\frac{1}{2}}$, $\mu^{(o)}$ in fm, and the overlap integrand $\mu^{(o)}\ \mu_o$ in $\mathrm{fm}^{+\frac{1}{2}}$.

Fig. (1E-2) Magnetic dipole cross section for a nucleon with magnetic moment μ bound by energy B in a zero-range potential well, eq. (1E-3). The cross section is in units of $(2e^{2/}BM)$ and the photon energy in units of B.

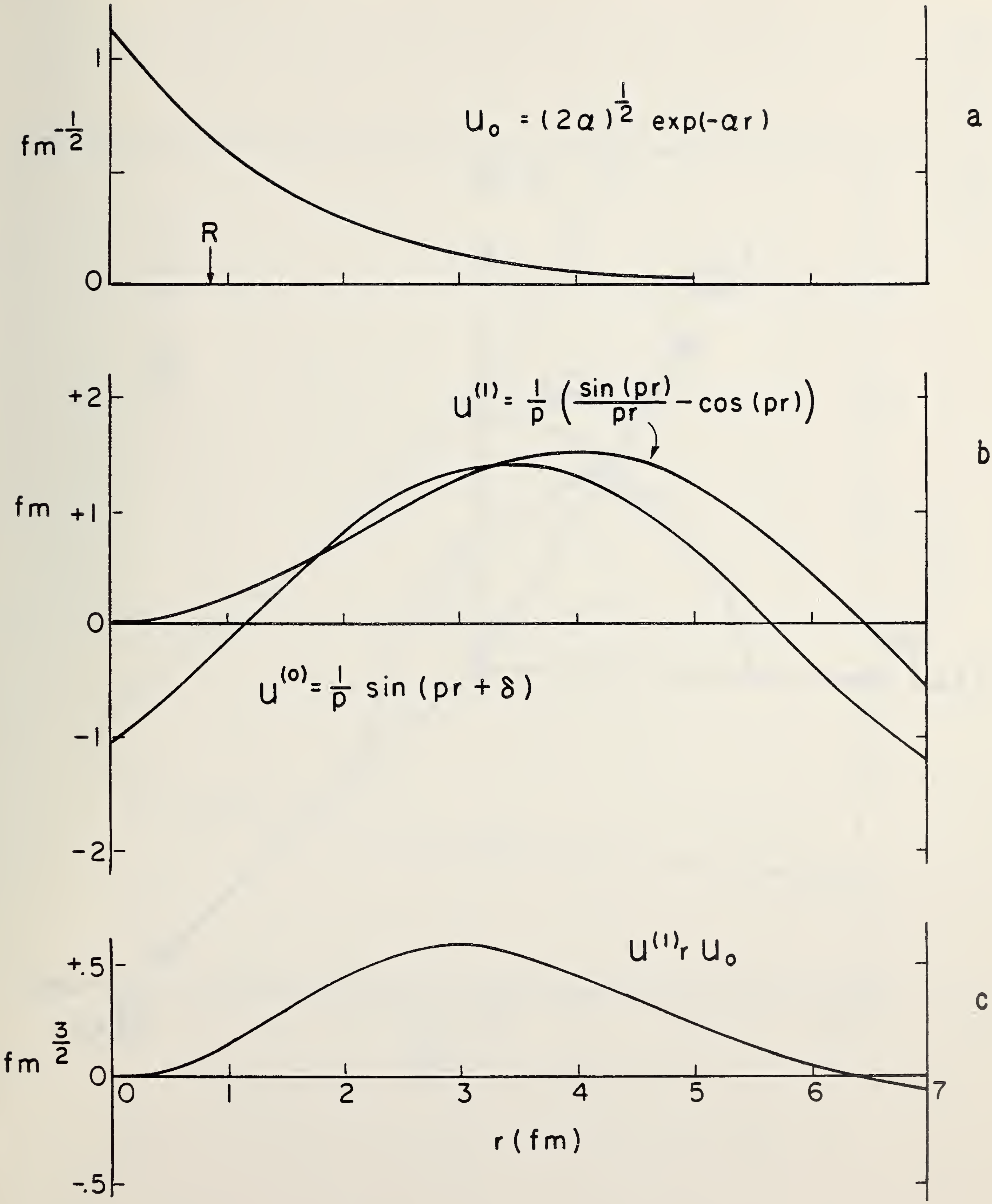
Fig. (1G-1) Photon cross section of gold as measured by neutron emission. A Lorentz resonance with parameters $\omega_0 = 13.82$ MeV, $\Gamma = 3.84$ MeV, $\sigma(\omega_0) = 560$ mb gives a good representation of the data (Be76).

Fig. (1H-1) Inelastic Coulomb form factor in the zero-range model as a function of energy transfer for two values of the momentum transfer with $B = \frac{\alpha^2}{2M} = 10$ MeV.

Fig. (1I-1) Cross section for the reaction $^3He(e,e')$ compared with a calculation in which the Coulomb interaction is used with zero-range wave functions for the ^3He ground state (Ka 75).

Fig. 1J-1) Feynman-type diagram for threshold photoproduction of charged pions. The nucleon is treated nonrelativistically but relativistic kinematics are used for the pion.

Fig. (1J-2) The differential cross section in the center-of-momentum system (Ad68) compared with the theoretical expression of eq. (1J-3).



$U_o = (2\alpha)^{\frac{1}{2}} \exp(-\alpha r)$
$fm^{-\frac{1}{2}}$
R
a
$U^{(1)} = \frac{1}{p}\left(\frac{\sin(pr)}{pr} - \cos(pr)\right)$
$U^{(0)} = \frac{1}{p}\sin(pr + \delta)$
fm
b
$U^{(1)} r\, U_o$
$fm^{\frac{3}{2}}$
c
r (fm)

FIG. 1B-1

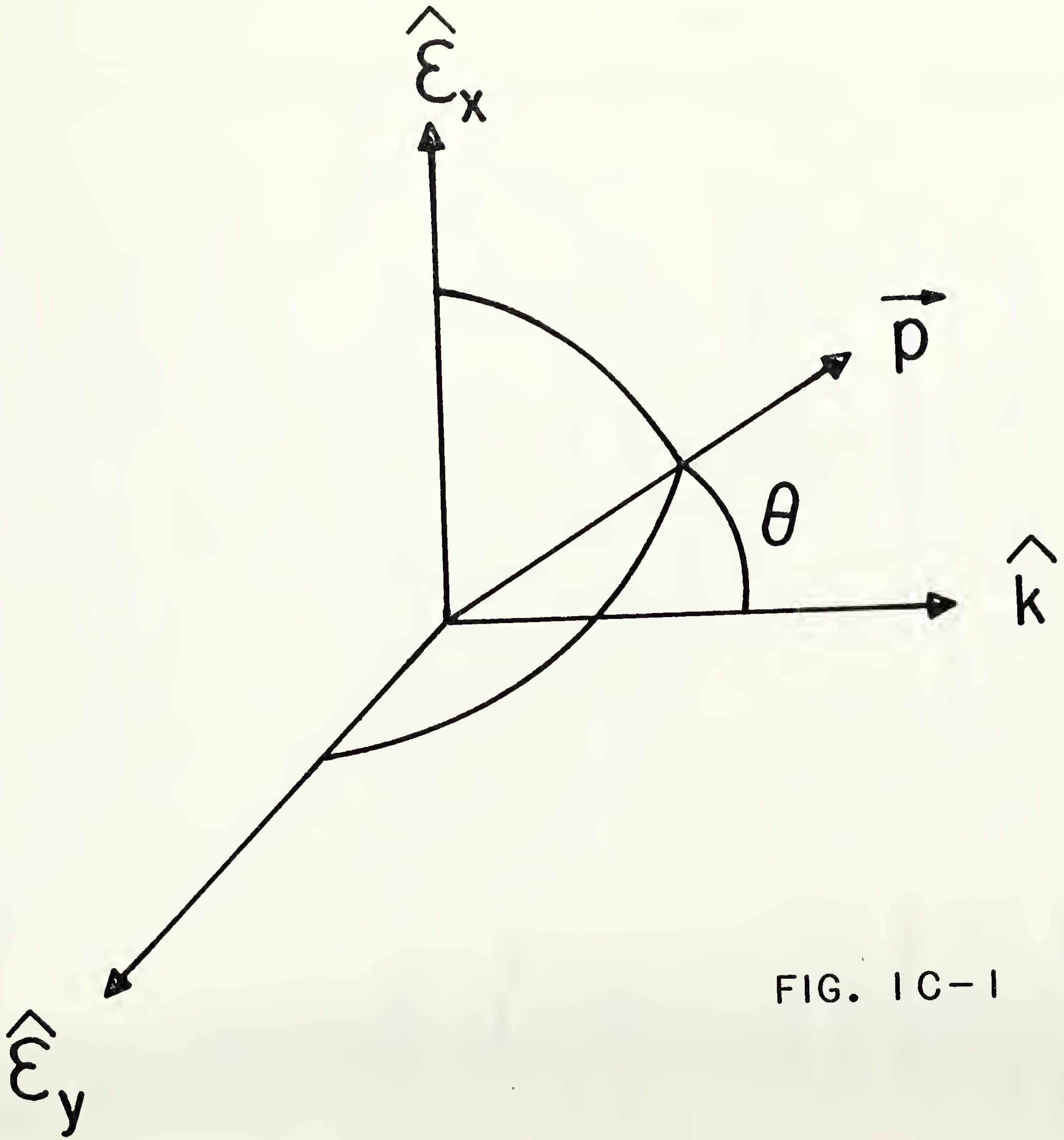


FIG. 1C-1

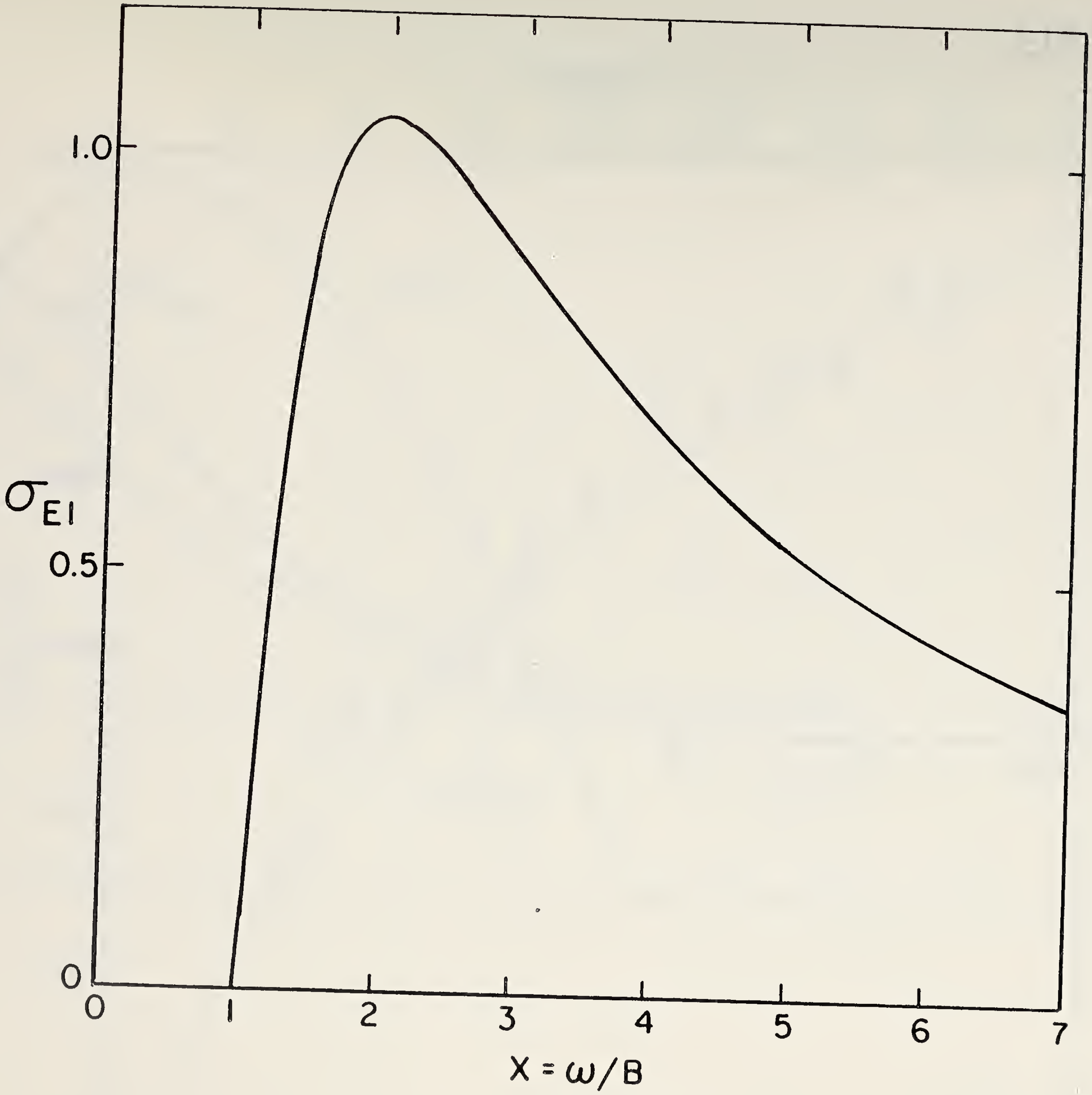


FIG. IC-2

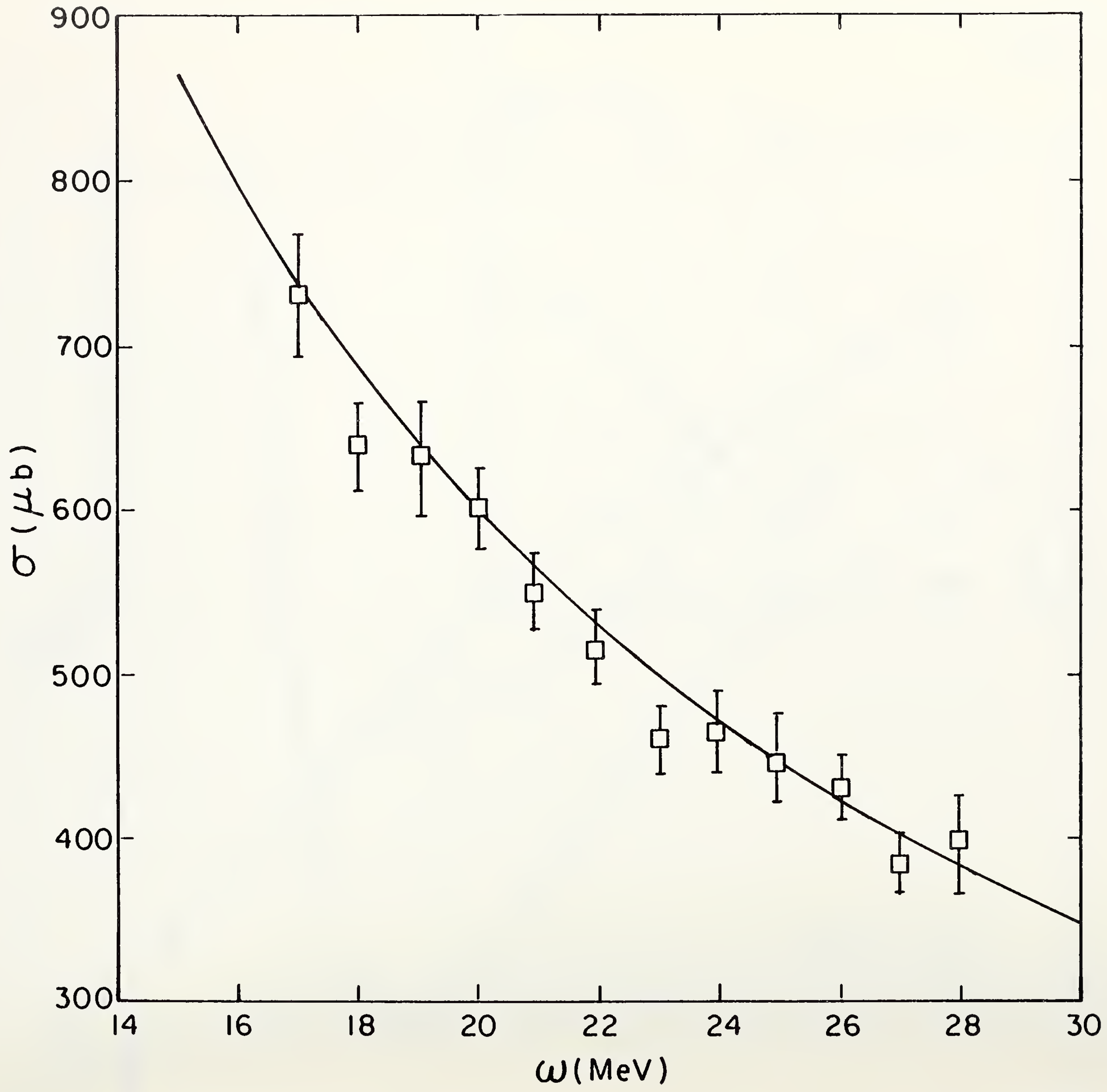


FIG. 1C-3

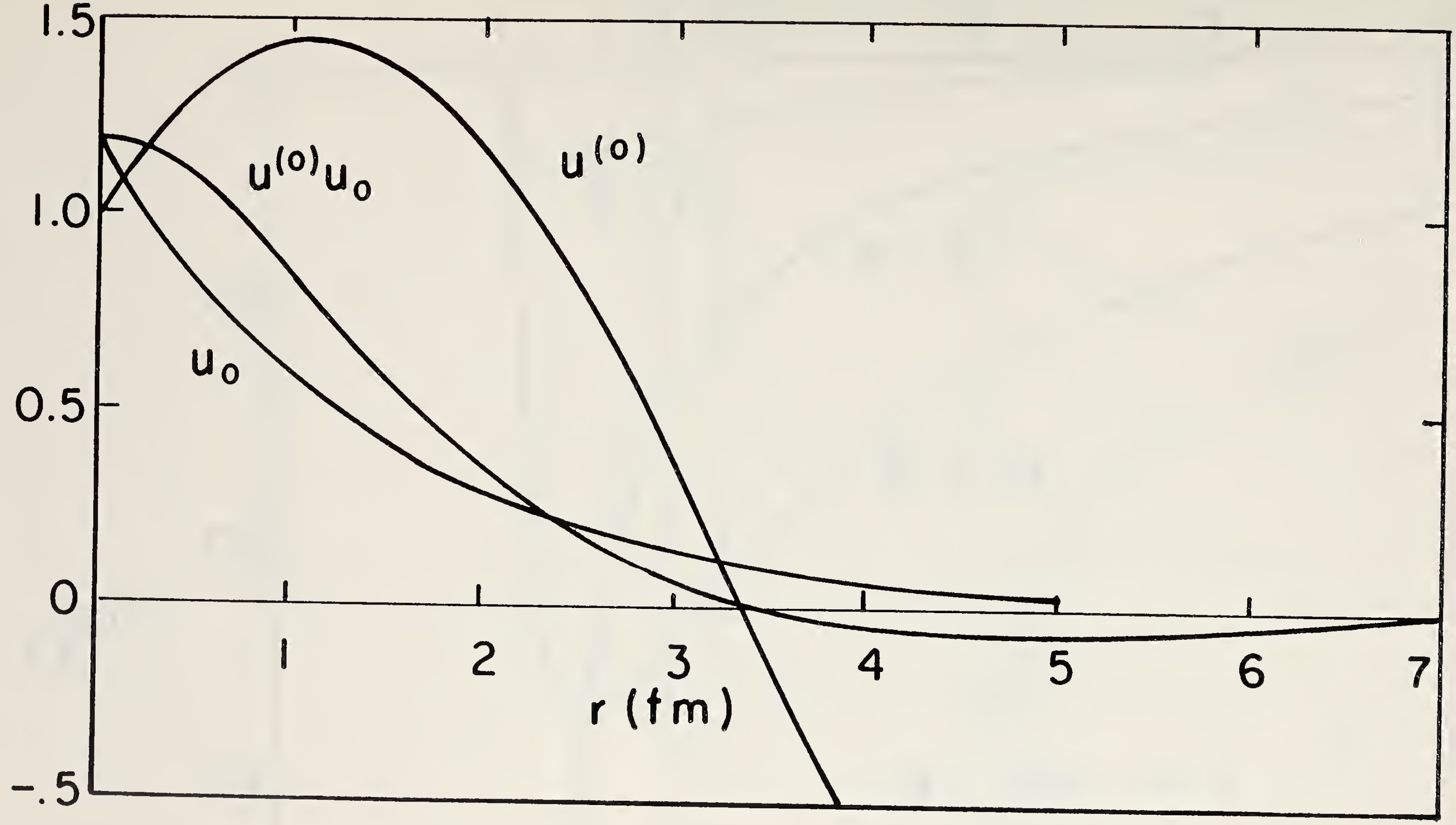


FIG. IE-I

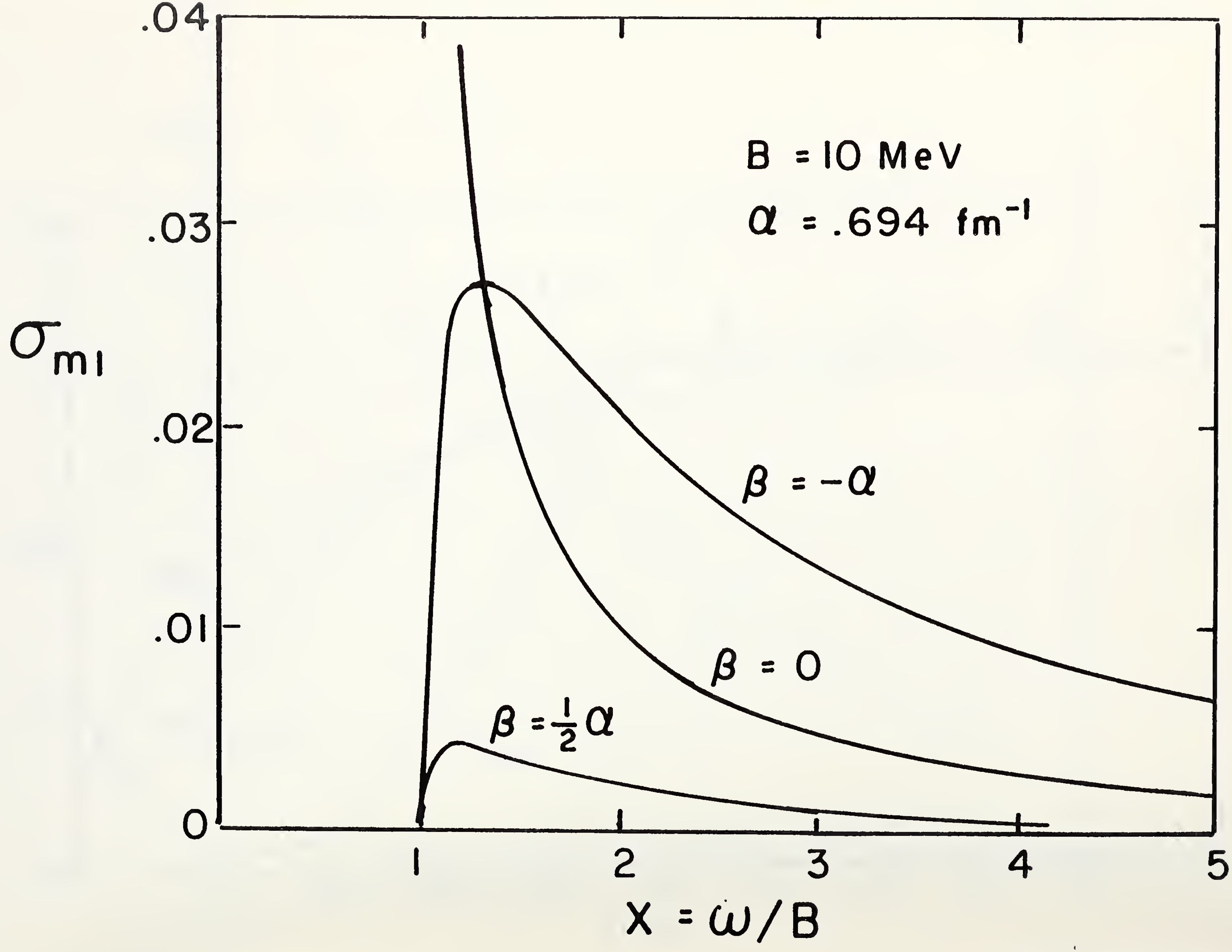


FIG. 1E-2

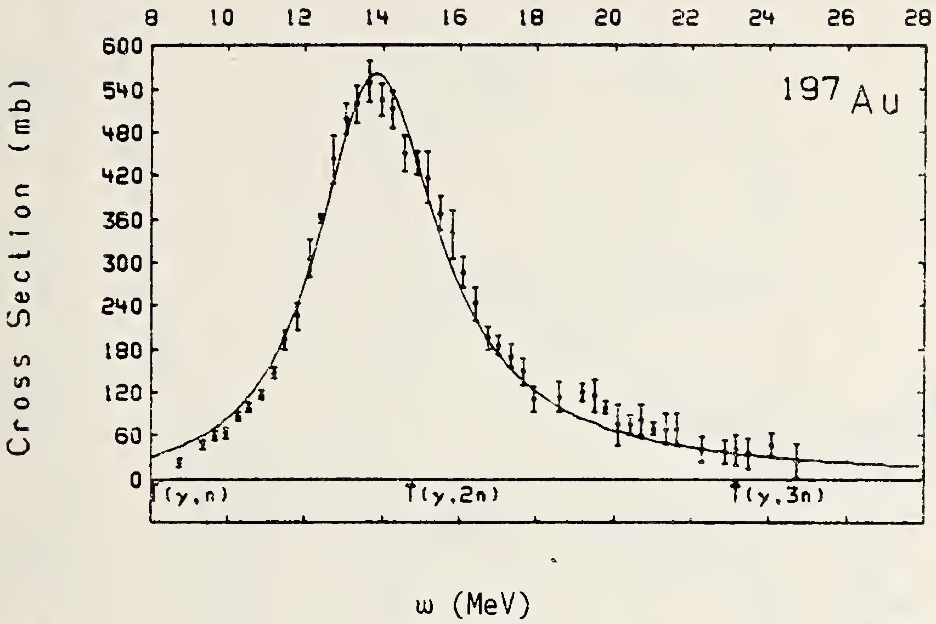


Fig. (1G-1). Photon cross section of gold as measured by neutron emission. A Lorentz resonance with parameters ω_0 = 13.82 MeV, Γ = 3.84 MeV, $\sigma(\omega_0)$ = 560 mb gives a good representation of the data (Be76).

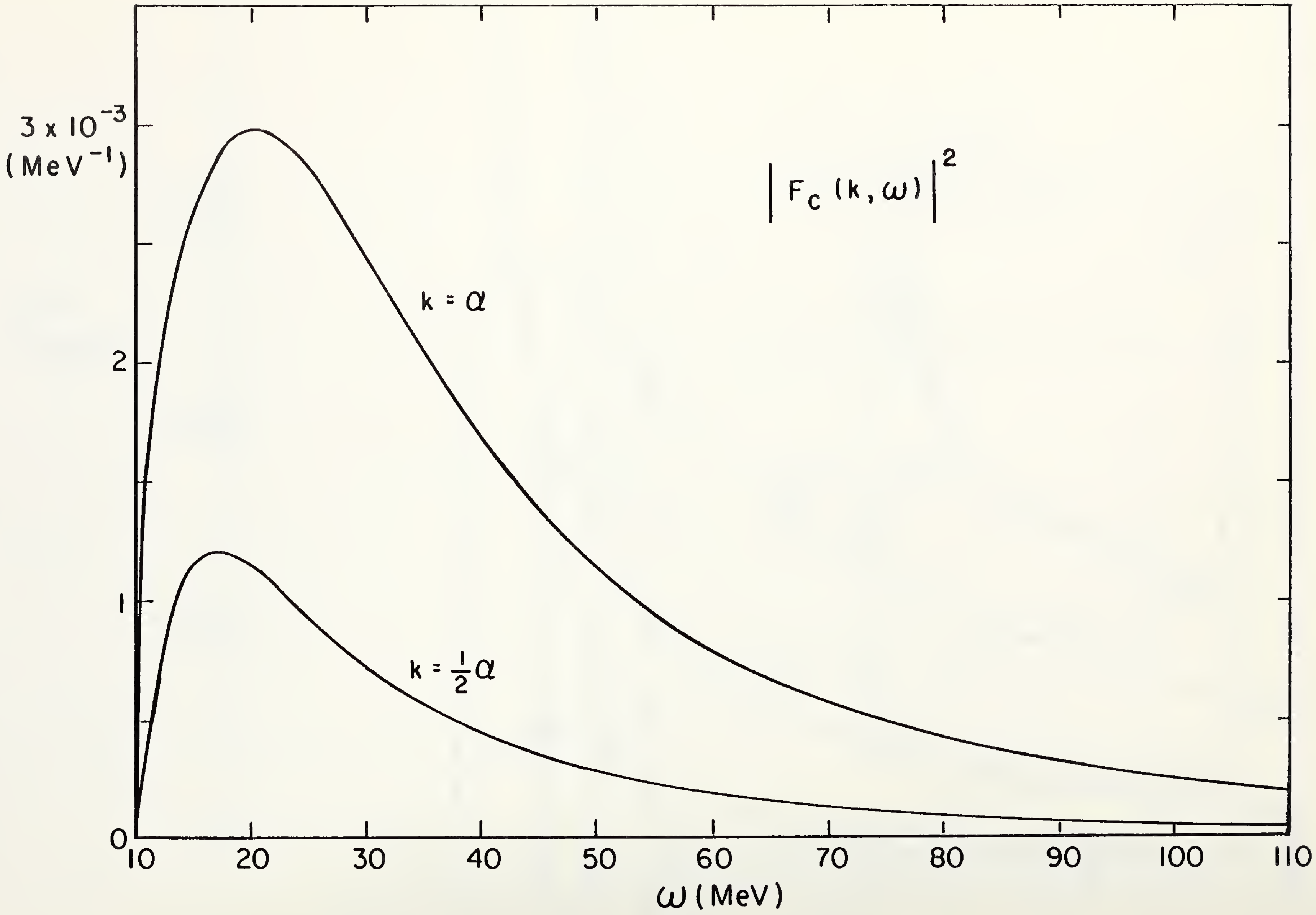

3 x 10^-3
(MeV^-1)
2
1
0
10
20
30
40
50
60
70
80
90
100
110
ω (MeV)
|F_c (k, ω)|^2
k = α
k = 1/2 α

FIG. 1H-1

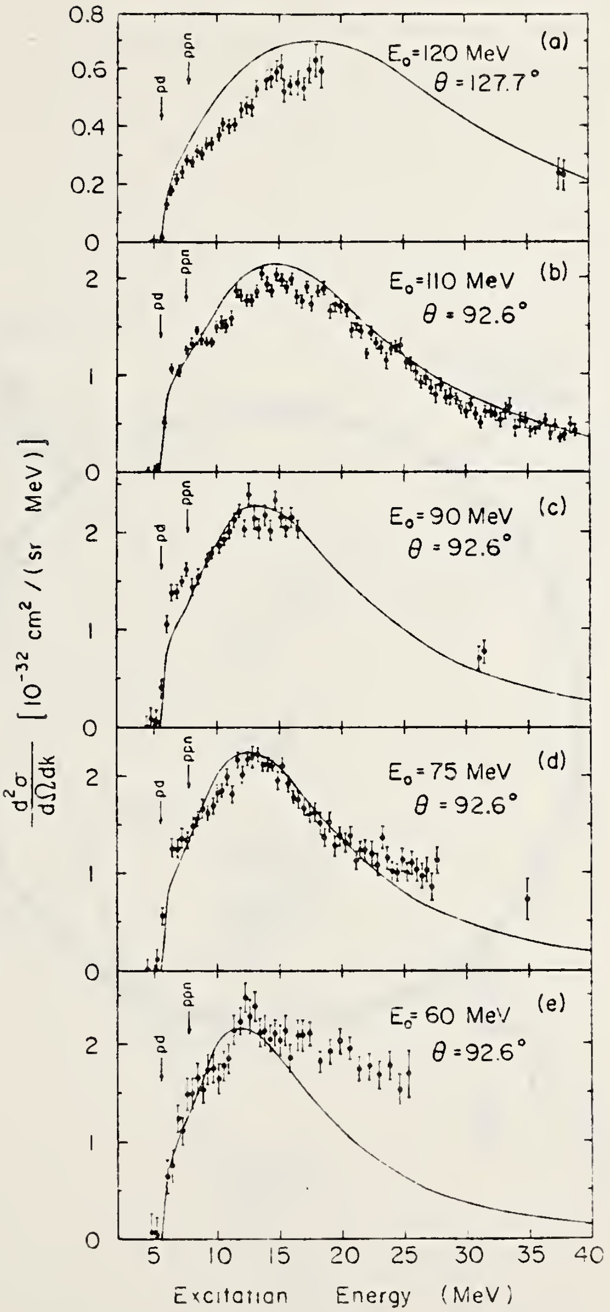
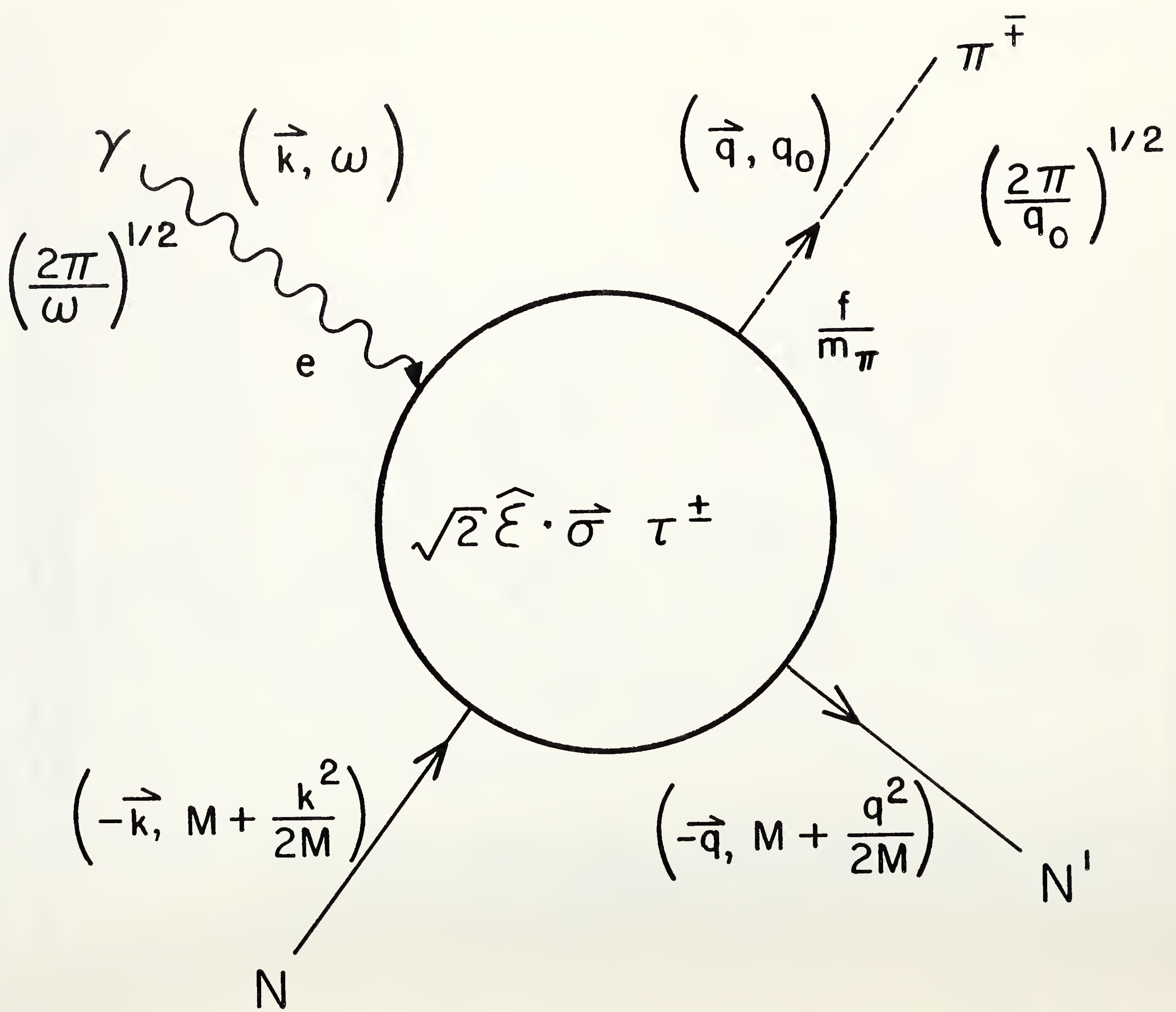
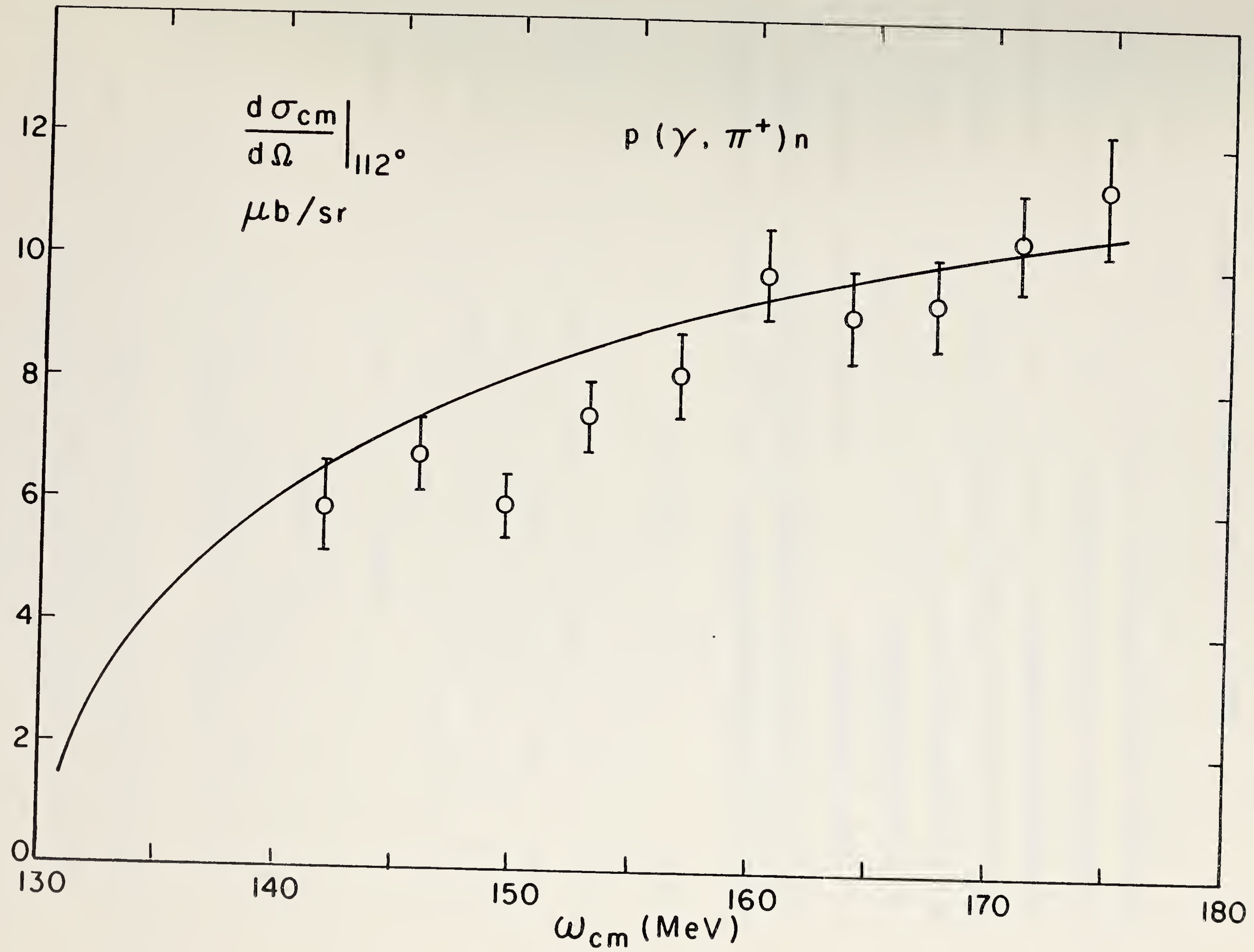


Fig. (1I-1) Cross section for the reaction ^{3}He(e,e') compared with a calculation in which the Coulomb interaction is used with zero-range wave functions for the ^{3}He ground state (Ka75).



γ
$(\vec{k}, \omega)$
$\left(\frac{2\pi}{\omega}\right)^{1/2}$
e
$\pi^{\mp}$
$(\vec{q}, q_0)$
$\left(\frac{2\pi}{q_0}\right)^{1/2}$
$\frac{f}{m_\pi}$
$\sqrt{2}\,\hat{\varepsilon}\cdot\vec{\sigma}\ \tau^{\pm}$
$\left(-\vec{k},\ M+\frac{k^2}{2M}\right)$
$\left(-\vec{q},\ M+\frac{q^2}{2M}\right)$
N
N'

FIG. 1J-1



$\frac{d\sigma_{cm}}{d\Omega}\Big|_{112°}$
μb/sr
p (γ, π⁺) n
12
10
8
6
4
2
0
130
140
150
160
170
180
ω_{cm} (MeV)

FIG. 1J-2

CHAPTER II. THE ELECTROMAGNETIC OPERATOR

A. Real photon transition operators

B. Virtual photon transition operators

C. Meson exchange currents

D. Coulomb distortion and radiative processes

E. Multipole analysis

F. Long wavelength expansions

G. Coincidence measurements

H. Relation of weak and electromagnetic interactions

I. Real and virtual bremsstrahlung

CHAPTER II. THE ELECTROMAGNETIC OPERATOR

The formal development of the electromagnetic interaction operator with nuclear constituents follows from quantum field theory and the Feynman analysis of quantum electrodynamics (Bj64). In practice, photonuclear calculations are performed with the nonrelativistic reduction of the elementary currents to give operators for use with model single-particle wave functions (De66,Do75). The intent of the present chapter is to introduce enough background to give physical insight for calculations to be discussed in subsequent chapters.

A. REAL PHOTON TRANSITION OPERATORS

The interaction energy of the electromagnetic field (described by the vector potential $A_\mu(\vec{r},t) = (\vec{A},\phi)$ with a nucleon current $j_\mu(\vec{r},t) = (\vec{j},\rho)$ is

$$H_\gamma = j_\mu A_\mu \tag{2A-1}$$

where ϕ is the Coulomb potential and ρ the charge density. For a real photon, i.e., a wave packet of electric ($\vec{E}=-\partial\vec{A}/\partial t-\vec{\nabla}\phi$) and magnetic ($\vec{B}=\vec{\nabla}\times\vec{A}$) fields for which the energy ω and momentum k are equal in magnitude and the $\vec{E}$, $\vec{B}$, and $\vec{k}$ vectors form an orthogonal set, the ϕ can be chosen zero (transverse gauge). The vector potential of a plane wave is written as

$$\vec{A} = \hat{\varepsilon}\ \exp\ (i\vec{k}\cdot\vec{r}-i\omega t) \tag{2A-2}$$

where $\hat{\varepsilon}$ is a unit vector in the direction of the electric field and at right angles to the direction of propagation ($\hat{\varepsilon}\cdot\vec{k}=0$). The magnetic field unit vector is ($\hat{k}$ x $\hat{\varepsilon}$).

The single nucleon current has two sources $\vec{j} = \vec{j}_p + \vec{m}_\sigma$ when treated non-relativistically. The orbital current due to the velocity of the proton

$$\vec{j}_p = \frac{e\vec{p}}{M} \tag{2A-3}$$

and the magnetic moment of the proton or neutron

$$\vec{m}_\sigma = \frac{e\mu\vec{\sigma}}{2M} \tag{2A-4}$$

where the nucleon magnetic moment is expressed in nuclear magnetons (e/2M). The orbital current has an interaction energy $\vec{j}_p \cdot \vec{A}$ while the magnetic moment interaction is $\vec{m}_\sigma \cdot \vec{B} = \vec{m}_\sigma \cdot (\vec{\nabla} x \vec{A})$. The total interaction energy of the nucleon with a plane wave* photon is

$$H_\gamma = \left[\frac{e\hat{\varepsilon}\cdot\vec{p}}{M} + i\,\frac{e\mu}{2M}\,(\vec{k}x\hat{\varepsilon})\cdot\vec{\sigma}\right] \exp(i\vec{k}\cdot\vec{r}-i\omega t). \tag{2A-5}$$

$$= \left[\frac{eP_\lambda}{M} + i\,\frac{e\mu}{2M}\,(\vec{\sigma}x\vec{k})_\lambda\right] \exp(i\vec{k}\cdot\vec{r}-i\omega t) \quad .$$

The photon absorption cross section is given by

$$d\sigma = 2\pi\left(\frac{4\pi}{2\omega}\right)\overline{\sum}\left|\langle\Psi_f\right|\sum_j H_\gamma(j)\left|\Psi_i\rangle\right|^2\frac{dn}{d\omega} \tag{2A-6}$$

where the bar over the sum is a short hand for averaging over spin components of the initial state $\Psi_i(J_i,M_i)$, summing over the final state components $\psi_f(J_f,M_f)$, and averaging over the two values of photon polarization $\lambda = \pm 1$ for helicity states (or $\lambda = \hat{x},\hat{y}$ for plane polarization)

$$\overline{\sum} \equiv \frac{1}{2}\,\frac{1}{(2J_i+1)} \quad \sum_{M_f,\lambda,M_i} \quad . \tag{2A-7}$$

---

* Useful identities

$$\vec{\nabla}(e^{i\vec{k}\cdot\vec{r}}) = i\vec{k}e^{i\vec{k}\cdot\vec{r}}$$

$$\vec{\nabla}\cdot(\hat{\varepsilon}e^{i\vec{k}\cdot\vec{r}}) = i(\hat{\varepsilon}\cdot\vec{k})e^{i\vec{k}\cdot\vec{r}}$$

$$\vec{\nabla}x(\hat{\varepsilon}e^{i\vec{k}\cdot\vec{r}}) = i(\vec{k}x\hat{\varepsilon})e^{i\vec{k}\cdot\vec{r}}$$

$$(\vec{k}x\hat{\varepsilon})\cdot\vec{\sigma} = \hat{\varepsilon}\cdot(\vec{\sigma}x\vec{k})$$

The factor $(4\pi/2\omega)$ is the square of the photon normalization factor and $dn/d\omega$ is the density of final nuclear states. The leading 2π is from the Fermi Golden Rule for transition rates. One may associate the transition amplitude $\langle\psi_f|H_\gamma|\psi_i\rangle$ with the diagram shown in fig. (2A-1).

We saw in Chapter I how model wave functions for ψ_i and ψ_f and approximations for H_γ lead to cross section expressions for a number of reactions that agree with measurements.

The usefulness of photon reactions derives in large measure from the simplicity and reliability of eqs. (2A-5,6). Compared to strong interactions the photon has the virtues of not being distorted as it approaches a nucleon inside the nucleus and interacting with only one nucleon at a time. This last property follows from the smallness of the coupling constant $e^2 = (137)^{-1}$.

The expressions (2A-5,6) can be derived by a nonrelativistic reduction of the S-matrix expressions of quantum electrodynamics. The nucleon current is

$$j_\mu = e\bar{u}[F_1(k_\mu^2)\gamma_\mu + \frac{i\kappa}{2M} F_2\,(k_\mu^2)\,\sigma_{\mu\nu}\,k_\nu]u \qquad (2A\text{-}8)$$

where u is the nucleon two-component spinor, $F_{1,2}$ the Dirac and Pauli form factors, κ the nucleon anomalous magnetic moment, γ_μ the Dirac matrix, and $\sigma_{\mu\nu} = i/2\,[\gamma_\mu,\gamma_\nu]$ (Pe74).

Elastic photon scattering by a nucleus involves terms of second order in e from $j_\mu A_\mu$. The amplitude $f(\omega,\theta)$ for this process can be calculated from the three diagrams of fig. 2A-2. Diagram 1a is the coherent scattering by the total charge of the nucleus. At moderate energies this is the energy-independent Thomson amplitude

$$f_a = -\frac{(Ze)^2}{AM}\;F\,(\vec{k}_1-\vec{k}_2) \qquad (2A\text{-}9)$$

modulated by the nuclear form factor. In diagrams 1b and 1c the intermediate nucleus is excited to state n> with energy E_n,

$$f_b = \sum_n \frac{\langle f|H_\gamma|n\rangle\langle n|H_\gamma|i\rangle}{E_n + \omega}$$

$$f_c = \sum_n \frac{\langle f|H_\gamma|n\rangle\langle n|H_\gamma|i\rangle}{E_n - \omega} \qquad \text{(2A-10)}$$

The coherent nuclear scattering of photons interferes with other extranuclear and atomic processes: Delbrück scattering (associated with e^+e^- pair production) and elastic scattering from electrons (atomic Rayleigh scattering). The Delbrück and atomic Rayleigh amplitudes are peaked very strongly forward and can often be neglected at backward angles.

In principle the photon scattering cross section could be computed with model wave functions for Ψ_i, Ψ_n, Ψ_f; however in practice one uses the optical theorem relating the imaginary part of the forward scattering amplitude f to the photon absorption cross section for each multipole L,

$$4\pi \text{ Im } f^L(\omega) = \omega\, \sigma^L(\omega)\ , \qquad \text{(2A-11)}$$

and a dispersion relation (see Chapter III) between Re $f^L(\omega,0)$ and Im $f^L(\omega,0)$ to compute each f^L. Then

$$\frac{d\sigma}{d\Omega}(\omega,\theta) = \sum_{L,L'} g_{LL'}(\cos\theta)\, f^L f^{L'} \qquad \text{(2A-12)}$$

where $g_{LL'}$ is a geometric factor (Ei70). The absorption cross section is easier to compute than $f(\omega)$ with model wave functions since the sum over intermediate states is not required.

Photon scattering and absorption and their relation can be seen in the S-matrix expressions

$$f^L(\omega,\theta) = \frac{1}{2ik}(2L+1)\,P_L(\cos\theta)\,[S_L(\omega)-1] \qquad (2A-13)$$

$$\sigma^L_{tot} = \frac{\pi}{k^2}(2L+1)\,2(1-\mathrm{Re}S_L(\omega)) \ . \qquad (2A-14)$$

Both processes tend to be dominated by resonances (poles in $f(\omega,0)$ in the lower half of the complex energy plane $E_n = \omega_o - i\Gamma/2$). A Breit-Wigner form gives

$$S_L - 1 = \frac{i\Gamma}{(\omega_o - i\Gamma/2) - \omega} \qquad (2A-15)$$

B. VIRTUAL PHOTON TRANSITION OPERATORS

The analysis of electron scattering in the one-photon exchange approximation leads to a factorization of the reaction amplitude into a purely leptonic part (the $e\gamma e'$ vertex) times a hadronic part (the $N\gamma N'$ vertex) (De66). The electromagnetic quantum that propagates between these vertices has a momentum greater than its energy ($k>\omega$). It is called a virtual photon to distinguish it from a real photon for which $k=\omega$. The distance over which a virtual quantum propagates can be estimated from the uncertainty principle. The mismatch between momentum k and energy ω is allowed to travel a distance $\Delta z \cong (k-\omega)^{-1}$ before attaching to a nucleon. Typically this distance is smaller than the nuclear radius implying the lepton vertex is inside the nucleus, that is, the impact parameter for electron scattering is a few fm or less. It is assumed that the $e\gamma e'$ vertex is unaffected by the nuclear medium.

Electron scattering from a nucleon in a nucleus can be considered as a generalization of the real photon-nucleon interaction. Using the kinematic variables k and ω defined by fig. (2B-1) the vector potential $A_\mu = (\vec{A},\phi)$ now has a non-zero Coulomb part and $\vec{A}$ has a longitudinal component $\hat{k}\cdot\vec{A}$ plus two transverse components ($\lambda=\pm1$) which add incoherently. The interaction energies become

$$H_e = j_\mu A_\mu = \left[e\rho + e\,\frac{\hat{k}\cdot\vec{p}}{M} + e\,\frac{p_\lambda}{M} + i\,\frac{e\mu}{2M}\,(\vec{\sigma}\times\vec{k})_\lambda \right] \exp(i\vec{k}\cdot\vec{r}) \qquad (2B\text{-}1)$$

The first term on the right of (2B-1) is the Coulomb interaction, the second the longitudinal current, the third the transverse current, and the fourth the magnetic moment interaction. The Coulomb and longitudinal current operators are not independent. By current conservation* one can replace their sum by one operator

$$\rho + \frac{\hat{k}\cdot\vec{p}}{M} = (1 - \frac{\omega}{k})\rho \qquad (2B\text{-}2)$$

A nonrelativistic reduction of $j_\mu A_\mu$ in powers of k/M and p/M yields the following expression for the inelastic electron scattering cross section on a nuclear target in the one-photon exchange approximation with $m_e = 0$ (Mc62,De66)

---

*$\vec{\nabla}\cdot\vec{j}_p + \frac{\partial\rho}{\partial t} = 0$, $\vec{k}\cdot\vec{j}_p + \omega\rho = 0$, $\vec{\nabla}\cdot\vec{m}_\sigma = 0$.

$$\frac{d^2\sigma}{d\Omega dk_2}(k,\omega,\theta) = \sigma_o \left\{ \frac{k_\mu^4}{k^4} \left|F_c(k,\omega)\right|^2 \right.$$

$$\left. + \left(\frac{k_\mu^2}{2k^2} + \tan^2\frac{\theta}{2}\right) [\left|F_p(k,\omega) + F_\sigma(k,\omega)\right|^2] \right\}$$

$$\sigma_o = \left[\frac{e^2\cos\theta/2}{2\,k_1\sin^2\theta/2}\right]^2 \left(1 + \frac{2k_1\sin^2\theta/2}{M_T}\right)^{-1} f_N^2(k_\mu^2) \quad ,$$

$$|F_c|^2 = \frac{1}{2J_i+1} \sum_{M_f,M_i} \left|\langle J_f M_f\right| \sum_j^A e_j \exp(i\vec{k}\cdot\vec{r}_j)\left|J_i M_i\rangle\right|^2 \left(\frac{dn}{dE_x}\right) \quad ,$$

$$|F_p + F_\sigma|^2 = \frac{1}{2J_i+1} \sum_{M_f,M_i} \sum_{\lambda=\pm1} \left|\langle J_f M_f\right| \sum_j^A \frac{e_j(p_j)_\lambda}{M} \exp i\vec{k}\cdot\vec{r}_j$$

$$+ \sum_j^A \frac{i\mu_j}{2M} (\vec{\sigma}_j x\vec{k})_\lambda \exp i\vec{k}\cdot\vec{r}_j \left|J_i M_i\rangle\right|^2 \left(\frac{dn}{dE_x}\right) \qquad (2B\text{-}3)$$

where σ_o contains the Mott cross section, target mass M_T recoil factor, and the average charge and magnetic nucleon form factor given approximately by

$$f_N(k_\mu^2) = G_E^P = \frac{G_\mu^P}{\mu_p} = \frac{G_M^n}{\mu_n} \simeq \left[1 + \frac{k_\mu^2}{a^2}\right]^{-2} \qquad (2B\text{-}4)$$

with* $a = 855$ MeV and $G_E^n = 0$. The density of nuclear states dn/dE_x is unity for a discrete level. The charge projection operator $e_j = 1/2(1+\tau_z)_j$ gives 1 for a proton and 0 for a neutron. The Coulomb, current, and magnetic moment form factors $F_{c,p,\sigma}$ are averaged and summed over initial and final angular momentum states, but in contrast to a real photon, the polarization projections $\lambda = \pm1$ of the transverse components of p and σ are summed (not averaged). The Coulomb part has the $\lambda = 0$ longitudinal component.

---

*The dipole form corresponds to an exponential distribution in space. The radius parameter associated with this value of a^2 is $\langle r^2\rangle^{1/2} = 0.81$ fm.

The single-particle expression given in eq. (2B-1) is an approximation since small terms of order $(k/M)^2$ and $(p/M)^2$ were neglected. A form correct through order $(k/M)^2$ and $(p/M)^2$ is (De66)

$$\rho = F_1 - \left(\frac{k^2}{8M^2} - \frac{i\vec{k}\cdot(\vec{\sigma}\times\vec{p})}{4M^2}\right)(F_1 + 2\kappa F_2)$$

$$\vec{j}_p = \frac{\vec{p}\,'+\vec{p}}{2M} F_1 \qquad (2B\text{-}5)$$

$$\vec{j}_\sigma = i\,\frac{\vec{\sigma}\times\vec{k}}{2M}(F_1+\kappa F_2)$$

The non-relativistic operators through the order $(k/M)^4$ are given in (Gi80). The Dirac and Pauli nucleon form factors are related to the electric and magnetic form factors by

$$G_E = F_1 + \eta\,\kappa\,F_2$$
$$G_M = F_1 + \kappa\,F_2 \qquad (2B\text{-}6)$$

where $\eta = k_\mu^2/4M^2$.

For reference, we give the fully-relativistic elastic and inelastic electron-nucleon scattering cross sections,

$$\frac{d\sigma}{d\Omega} = \left(\frac{k_2}{k_1}\right)\left[\frac{e^2\cos\theta/2}{2k_1\sin^2\theta/2}\right]^2 \left\{G_E^2/(1+\eta) + G_M^2\,2\eta\left(\frac{1}{2(1+\eta)} + \tan^2\frac{\theta}{2}\right)\right\} \qquad (2B\text{-}7)$$

$$\frac{d^2\sigma}{d\Omega d\omega} = \left[\frac{e^2\cos\theta/2}{2k_1\sin^2\theta/2}\right]^2\left[W_2(k_\mu^2,\omega) + 2W_1(k_\mu^2,\omega)\ \tan^2\frac{\theta}{2}\right] \qquad (2B\text{-}8)$$

An alternative expression for the inelastic electron scattering cross section (for nucleons or nuclei) can be given in terms of two fictitious total absorption cross sections σ_T and σ_L for virtual photons (Pe74)

$$\frac{d^2\sigma}{d\Omega d\omega} = \Gamma_T \, \sigma_T \, (k_\mu^2,\omega) + \Gamma_L \, \sigma_L \, (k_\mu^2,\omega)$$

$$\Gamma_T = \frac{e^2}{4\pi^2} \, \frac{K}{k_\mu^2} \, \frac{E_2}{E_1} \left(\frac{2}{1 - \varepsilon} \right) \tag{2B-9}$$

$$\Gamma_L = \varepsilon \, \Gamma_T \; , \; \varepsilon = \left[1 + \frac{2k^2}{k_\mu^2} \tan^2\theta/2 \right]^{-1} \quad .$$

The Γ's can be interpreted as the number of virtual photons associated with the inelastic scattering process, ε is related to the polarization of the photons and $K = \omega - k_\mu^2/2M_T$ is the energy of a real photon leading to the same hadronic final state. The cross sections have the low k limit

$$\sigma_T \, (k \to \omega,\omega) = \sigma_\gamma(\omega)$$
$$\sigma_L \, (k \to \omega,\omega) = 0$$

i.e., the transverse term becomes the real photon absorption cross section at the same energy transfer ω.

C. MESON EXCHANGE CURRENTS (MEC)

Other currents in the nucleus are due to charged mesons (mostly pions) and to isobars, excited states of the nucleon, (mainly the delta). The electromagnetic $\vec{E}$ and $\vec{B}$ fields couple to the meson and isobar currents to give contributions to $j_\mu A_\mu$ beyond the single-nucleon impulse approximation discussed in the previous two sections (Ri79). The electromagnetic interaction operators are constructed by making a nonrelativistic reduction of those two-nucleon, one-pion diagrams considered to be the most important. The products of vertex functions and intermediate propagators are taken.

The vertex operator for the NπN interaction, fig. (2C-1a), is

$$\frac{f}{m_\pi} \vec{\sigma}\cdot\vec{q}\,\vec{\tau} \qquad (2C\text{-}1)$$

where f is the coupling constant ($f^2 = 0.081$), $\vec{\sigma}$ and $\vec{\tau}$ the nucleon spin and isospin operators, and q the pion momentum. Fig. (2C-1b) shows the NπΔ vertex in which the $J^\pi = 3/2^-$, T = 3/2 isobar resonance (M_Δ = (1236 - i60)MeV) is formed. The interaction operator is

$$\frac{f_\Delta}{m_\pi} \vec{S}\cdot\vec{q}\,\vec{T} \qquad (2C\text{-}2)$$

where $\vec{S}$ and $\vec{T}$ are the spin and isospin operators for a 1/2 to 3/2 transition, $f_\Delta = \frac{6\sqrt{2}}{5} f$ in the quark model (Ri 79).

The long range (r > 2 fm) two-nucleon force is thought to arise from one-pion exchange (OPE), fig. 2C-2a, with an interaction potential given by the product of the two NπN vertices divided by the pion propagator

$$V_{OPE} = -\frac{f^2}{m_\pi^2}\,\frac{(\vec{\sigma}_1\cdot\vec{q})(\vec{\tau}_1\cdot\vec{\tau}_2)(\vec{\sigma}_2\cdot\vec{q})}{q^2 + m_\pi^2}\;. \qquad (2C\text{-}3)$$

The intermediate range potential (1 < r < 2 fm) is thought to arise from the intermediate-delta two-pion exchange shown in fig. 2C-2b. Four vertices and four intermediate propagators are now involved.

The principal electromagnetic couplings to the pion and delta currents are shown in fig. 2C-3. Diagram 3a is called a pair* current; its operator is (Ho73)

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*The name "pair" comes from the nucleon-antinucleon intermediate state at the γNπ vertex before the nonrelativistic reduction. This diagram is also called the "sea gull" or "gauge" term.

$$\hat{\varepsilon}\cdot\vec{j}_{pair} = \frac{ef^2}{m_\pi^2}(\vec{\tau}_1 x\vec{\tau}_2)_z\left[\frac{(\hat{\varepsilon}\cdot\vec{\sigma}_1)(\vec{\sigma}_2\cdot\vec{q}_2)}{q_2^2+m_\pi^2} - \frac{(\hat{\varepsilon}\cdot\vec{\sigma}_2)(\vec{\sigma}_1\cdot\vec{q}_1)}{q_1^2+m_\pi^2}\right]F_1^{\ V}(k) \qquad (2C\text{-}4)$$

where $F_1^{\ V}$ is the nucleon isovector form factor.

Diagram 3b is the pion current

$$\hat{\varepsilon}\cdot j_\pi = \frac{ef^2}{m_\pi^2}(\vec{\tau}_1 x\vec{\tau}_2)_z\left[\frac{\hat{\varepsilon}\cdot(\vec{q}_1-\vec{q}_2)(\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{q}_2)}{(q_1^2+m_\pi^2)\ (q_2^2+m_\pi^2)}\right]F_\pi(k) \qquad (2C\text{-}5)$$

where F_π is the pion form factor.

The creation of a virtual delta shown in fig. 2C-3c is given by (Mo78) a magnetic coupling

$$(\hat{\varepsilon}x\vec{k})\cdot\vec{j}_\Delta = \frac{eff_\Delta}{m_\pi^2}\left[\frac{\mu(\vec{T}_1\cdot\vec{\tau}_2)T_1^{\ z}(\hat{\varepsilon}x\vec{k})\cdot\vec{S}_1(\vec{q}_2\cdot\vec{\sigma}_2)(\vec{q}_2\cdot\vec{S}_1)}{(M_\Delta^2-M^2)\quad(q_2^2+m_\pi^2)}\right]F_\Delta(k) \qquad (2C\text{-}6)$$

where F_Δ is the delta transition form factor.

These three operators have the following features:

(a) They are transverse and isovector in character. Their forms follow from the dominant photopion production mechanisms discussed in Chapter VII and the NN pion exchange potential.

(b) They are two-nucleon operators and are to be evaluated between two-nucleon wave functions in analogy to the NN potential where meson coordinates are also suppressed.

(c) The charge distribution of the nucleus is not altered by these isovector currents. The pions can be thought of as moving instantaneously between the nucleons.

The connection between the OPE potential and pion exchange currents can be seen by considering the expression for total current conservation expressed in energy-momentum space

$$\vec{k}\cdot\vec{j} = -\ \omega\rho = -i\ [K+V,\ \rho] \tag{2C-7}$$

where K and V are the one-nucleon kinetic and the two-nucleon potential energy operators. The charge density is given by

$$\rho(\vec{r}) = e \sum_j \tfrac{1}{2}(1+\tau_z)_j\ \delta(\vec{r}-\vec{r}_j). \tag{2C-8}$$

When the commutator of ρ is taken with the OPE potential eq. 2C-3, using

$$[(\vec{\tau}_1\cdot\vec{\tau}_2),\tau_{1z}] = \tau_-^1\ \tau_+^2 - \tau_+^1\ \tau_-^2 = 2i\ (\vec{\tau}_1 x \vec{\tau}_2)_z \tag{2C-9}$$

one finds that if $F_\pi = F_1^V$ in eqs. (2C-4 and 5) then

$$\vec{k}\cdot(\vec{j}_{pair} + \vec{j}_\pi) = -i\ [V_{OPE},\ \rho] \text{ and } \vec{k}\cdot\vec{j}_{sp} = -i\ [K,\ \rho]\ . \tag{2C-10}$$

The commutator of the two-nucleon potential with the charge density is related to the two-nucleon pion currents while the commutator of the one-nucleon kinetic energy with the charge density is related to the single-particle current j_{sp}.

The isoscalar MEC are less certain. They are usually smaller and of the same order as relativistic corrections (p^2/M^2). Currents due to higher mass mesons and isobars are thought to lead to smaller transition amplitudes because of the anticorrelation of two-nucleons at small interparticle distances.

Additional one-body currents are generated by resident nucleon isobars, fig. 2C-3d.

D. COULOMB DISTORTION AND RADIATIVE PROCESSES

Eq. (2B-3) for electron-nucleus scattering is derived using plane waves for the incident and scattered electrons. Although this approximation is valid for few nucleon systems, it becomes increasingly inaccurate for larger Z. The distortion of the electron wave by the Coulomb field of the nucleus can be found by using the Dirac equation with the monopole part of the nuclear charge distribution. The equation is expanded in partial waves and solved numerically. The resulting asymptotic phase shifts are summed to obtain the complex electron scattering amplitude as a function of scattering angle (Ub71).

Additional distortions due to deformation of the charge distribution, magnetic moments of the nucleus (for $J > 0$), and virtual excitation and de-excitation of intermediate nuclear states are not included in the usual treatment, but are estimated to be small.

An approximate correction to the plane wave Born approximation (PWBA) is the replacement of the asymptotic value of the momentum transfer by its value inside the nucleus

$$k_{eff} = k\ (1 + 1.16\ Ze^2/(k_1 R)) \qquad \text{(2D-1)}$$

where R is the charge radius. The "experimental form factor" $(d^2\sigma/\sigma_0)^{\frac{1}{2}}$ plotted as a function of k_{eff} will correspond more closely to the nuclear response $F_L{}^2(k,\omega) + (\frac{1}{2}+\tan^2\theta/2)F_T{}^2(k,\omega)$.

For inelastic electron scattering the distorted wave Born approximation (DWBA) is the most widely used approach. In the DWBA the incoming and outgoing electron waves are calculated using the phase shift method discussed above while the nuclear excitation is calculated treating the virtual photon as a plane wave $\exp(i\vec{k}\cdot\vec{r}_j)$. The distortion mixes the Coulomb and transverse response functions so that a model independent separation of the two form factors cannot be made.

A further complication in the calculation of the electron scattering cross section for a nuclear transition is the radiation of real and virtual photons during the scattering process (Ma69, Mo69). These photons affect the shape of $d^2\sigma$ for a discrete transition (with internal nuclear excitation E_x) adding a radiative tail that extends from

$$k_2 = [k_1 - E_x\ (1 + E_x/2M_T)]/(1 + 2k_1 \sin^2\frac{\theta}{2}/M_T) \qquad \text{(2D-2)}$$

to lower energies. This tail complicates the extraction of the transition strength of the level (which in real photon scattering would be fit by a Lorentz line) and forms a background under all nuclear excitations of higher E_x. The estimation of discrete peak strength (radiative correction) is usually accomplished by fitting the data near the peak with an exponential form.

The estimation of the background cross section underlying a peak from bremsstrahlung accompanying transitions of lower nuclear excitation (radiation tail subtraction), as given by evaluating fig. 2D-1, can be expressed approximately by

$$\frac{d^2\sigma}{d\Omega d\omega} = \sum_x \left[\frac{d\sigma}{d\Omega}(k_1-k_\gamma)\ \mathcal{F}(k_1) + \frac{d\sigma}{d\Omega}(k_1)\ \mathcal{F}(k_2+k_\gamma)\right], \qquad (2D-3)$$

where $d\sigma/d\Omega$ is the calculated radiationless cross section for exciting a specific level and $\mathcal{F}$ is the radiation function giving the spectrum of real photons. (See Section I.) The summation adds the tails of all nuclear transitions with E_x less than the level in question.

In addition to the radiative corrections to the data, ionization energy loss, bremmsstrahlung in the field of another nucleus (thick target effects), and detector resolution must be taken into account before comparison with nuclear model calculations (Be71).

E. MULTIPOLE ANALYSIS

The notational simplicity of the nucleon charge, current, and moment operators

$$[e,\ e\vec{p}/M,\ ie\mu(\vec{\sigma}\times\vec{k})/2M]\ \exp(i\vec{k}\cdot\vec{r})$$

must be sacrificed when these operators are to be sandwiched between single-particle wave functions of definite angular momentum. The (e,e') cross section for a discrete transition $J_i \rightarrow J_f$ (analogous to eq. (2B-3)) is written in terms of multipole amplitudes as

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_o}{(2J_i+1)} \sum_J \left\{ \frac{k_u^4}{k^4} \left| \langle J_f \| C^{[J]} \| J_i \rangle \right|^2 \right.$$

$$\left. + \left(\frac{k_u^2}{2k^2} + \tan^2 \frac{\theta}{2} \right) \left[\left| \langle J_f \| \mathcal{E}^{[J]} \| J_i \rangle \right|^2 + \left| \langle J_f \| \mathcal{M}^{[J]} \| J_i \rangle \right|^2 \right] \right\} \qquad \text{(2E-1)}$$

The reduced matrix elements of the Coulomb $C^{[J]}$, electric $\mathcal{E}^J$, and magnetic $\mathcal{M}^{[J]}$ multipole operators are formed by decomposing the operators in eq. (2B-3) into partial waves carrying angular momentum J and definite parity.

The usual formula for the expansion of a plane wave into spherical Bessel functions and spherical harmonics is

$$\exp(i\vec{k}\cdot\vec{r}) = \sum_L (4\pi)^{\frac{1}{2}} \hat{L} (i)^L j_L(kr) Y_{L0}(\hat{r}) \qquad \text{(2E-2)}$$

where $\hat{L} = (2L+1)^{\frac{1}{2}}$ and the z-axis has been chosen along k. In this section, we will work with the contrastandard elements $Y_M^{[L]}$ which are related to ordinary spherical harmonics by

$$Y_M^{[L]} = (-i)^L Y_{LM}(\theta,\phi) \; . \qquad \text{(2E-3)}$$

This notation permits us to treat the angular momentum properties of operators* on the same footing as wave functions (Fa 59, Da71,Oc72a,Le73).

Defining the Coulomb multipole operator as

$$C_o^{[J]} = (4\Pi)^{\frac{1}{2}} \sum_j (-1)^J e_j \, j_J(kr_j) \, Y_o^{[J]}(\hat{r}_j) \quad , \qquad \text{(2E-4)}$$

the Coulomb form factor can be written as

$$|F_c(k)|^2 = \left| \langle \psi_f | \sum_j e_j \exp(i\vec{k}\cdot\vec{r}_j) | \psi_i \rangle \right|^2$$

$$= \frac{1}{2J_i+1} \sum_{J,M_f,M_i} (\hat{J})^2 \left| \langle J_f M_f | C_o^{[J]} | J_i M_i \rangle \right|^2 \quad . \qquad \text{(2E-5)}$$

---

*Components of vector operators are written as $v_\lambda = -iv_\lambda^{[1]}$.

Using the Wigner-Eckart theorem*

$$|F_c(k)|^2 = \frac{1}{2J_i+1} \sum_J |\langle J_f \| C^{[J]} \| J_i \rangle|^2 \quad . \qquad (2E-6)$$

The current and moment operators are more complicated because they involve the coupling of a $Y^{[L]}$ (from the plane wave) with one unit of angular momentum (from the vectors p or σxk) to form an operator with total angular momentum J,

$$\begin{aligned} e^{i\vec{k}\cdot\vec{r}} p_\lambda &= (4\pi)^{\frac{1}{2}} \sum_L (i)^L \hat{L}\, j_L(kr)\, Y_{L0}(\hat{r})\, p_\lambda \\ &= (4\pi)^{\frac{1}{2}} \sum_L (-1)^{L+\frac{1}{2}} \hat{L}\, j_L\, Y_0^{[L]}\, p_\lambda^{[1]} \\ &= (4\pi)^{\frac{1}{2}}\, i \sum_{LJ} \hat{L}\, \hat{J} \begin{pmatrix} L & 1 & J \\ 0 & \lambda & -\lambda \end{pmatrix} j_L\, [Y^{[L]} \times p^{[1]}]_\lambda^{[J]} \; . \end{aligned} \qquad (2E-7)$$

In the last step the uncoupled product of two elements was replaced by its expansion in terms of elements coupled to a definite angular momentum.**

Similarly, for the moment operator

$$e^{i\vec{k}\cdot\vec{r}} (\vec{\sigma}\times\vec{k})_\lambda = (4\pi)^{\frac{1}{2}}\, k\lambda \sum_{LJ} \hat{L}\, \hat{J} \begin{pmatrix} L & 1 & J \\ 0 & \lambda & -\lambda \end{pmatrix} j_L\, [Y^{[L]} \times \sigma^{[1]}]_\lambda^{[J]} \qquad (2E-8)$$

where we have replaced the vector cross product by an angular momentum coupling

$$(\vec{\sigma}\times\vec{k})_\lambda = 2^{\frac{1}{2}} [\sigma^{[1]} \times k^{[1]}]_\lambda^{[1]} = \lambda\, k\, \sigma_\lambda^{[1]} \; . \qquad (2E-9)$$

---

* $\langle J_f M_f | \theta_\lambda^{[J]} | J_i M_i \rangle = (-1)^{J_f - M_f} \begin{pmatrix} J_f & J & J_i \\ -M_f & \lambda & M_i \end{pmatrix} \langle J_f \| \theta^{[J]} \| J_i \rangle$

** $A_{m_1}^{[j_1]}\, B_{m_2}^{[j_2]} = \sum_{J,\lambda} \langle j_1 m_1 j_2 m_2 | J\lambda \rangle\, [A^{[j_1]} \times B^{[j_2]}]_\lambda^{[J]}$

If we hold J fixed, L takes on three values: J+1, J, J-1. The parity of each current term is $\pi = (-1)^{L+1}$, but each moment term is $\pi = (-1)^L$ since $\vec{\sigma}$ is an axial vector. Therefore, to form an operator of definite parity, we combine the L = J ± 1 current terms with the L = J moment term to form an operator of parity $\pi = (-1)^J$. This combination is called an electric operator $E^{[J]}$ because the classical electric multipole field has this parity. The $\pi = (-1)^{J+1}$ or magnetic combination $M^{[J]}$ is formed by the sum of the L = J ± 1 moment terms with the L = J current term.

Define*

$$E_\lambda^{[J]} = i\ (-1)^J \left(\frac{2}{2J+1}\right)^{\frac{1}{2}} \frac{(4\pi)^{\frac{1}{2}}}{M}$$

$$\times \Bigg\{ e(\widehat{J+1})\ \hat{j} \begin{pmatrix} J+1 & 1 & J \\ 0 & \lambda & -\lambda \end{pmatrix} j_{J+1}\ [Y^{[J+1]} \times p^{[1]}]_\lambda^{[J]}$$

$$+ e(\widehat{J-1})\ \hat{j} \begin{pmatrix} J-1 & 1 & J \\ 0 & \lambda & -\lambda \end{pmatrix} j_{J-1}\ [Y^{[J-1]} \times p^{[1]}]_\lambda^{[J]}$$

$$+ \frac{\mu k}{2}\ \hat{J}\hat{J}\lambda \begin{pmatrix} J & 1 & J \\ 0 & \lambda & -\lambda \end{pmatrix} j_J\ [Y^{[J]} \times \sigma^{[1]}]_\lambda^{[J]} \Bigg\}, \quad (2E\text{-}10)$$

---

*The single particle label j and its summation has been suppressed in e_j, p_j, σ_j, $j(kr_j)$, and $Y(\hat{r}_j)$ for notational simplicity.

$$M_\lambda^{[J]} = i(-1)^J \left(\frac{2}{2J+1}\right)^{\frac{1}{2}} \frac{(4\pi)^{\frac{1}{2}}}{M}$$

$$\times \left\{ \frac{\mu k}{2} \ (\widehat{J+1}) \ \hat{J} \ \lambda \begin{pmatrix} J+1 & 1 & J \\ 0 & \lambda & -\lambda \end{pmatrix} j_{J+1} \ [Y^{[J+1]} \times \sigma^{[1]}]_\lambda^{[J]} \right.$$

$$+ \frac{\mu k}{2} (\widehat{J-1}) \ \hat{J} \ \lambda \begin{pmatrix} J-1 & 1 & J \\ 0 & \lambda & -\lambda \end{pmatrix} j_{J-1} \ [Y^{[J-1]} \times \sigma^{[1]}]_\lambda^{[J]}$$

$$\left. + e \ \hat{J} \ \hat{J} \begin{pmatrix} J & 1 & J \\ 0 & \lambda & -\lambda \end{pmatrix} j_J \ [Y^{[J]} \times p^{[1]}]_\lambda^{[J]} \right\}. \qquad (2E\text{-}11)$$

Evaluating the 3-j coefficients, we obtain

$$E_\lambda^{[J]} = -i \ \frac{(4\pi)^{\frac{1}{2}}}{M}$$

$$\times \left\{ e \left(\frac{J}{J+1}\right)^{\frac{1}{2}} j_{J+1} \ [Y[J+1] \times p[1]]_\lambda^{[J]} \right.$$

$$+ e \left(\frac{J+1}{2J+1}\right)^{\frac{1}{2}} j_{J-1} \ [Y^{[J-1]} \times p^{[1]}]_\lambda^{[J]}$$

$$\left. + \frac{\mu k}{2} \ j_J \ [Y^{[J]} \times \sigma^{[1]}]_\lambda^{[J]} \right\}, \qquad (2E\text{-}12)$$

$$M_\lambda^{[J]} = -i \ \frac{(4\pi)^{\frac{1}{2}}}{M}$$

$$\times \lambda \left\{ \frac{\mu k}{2} \left(\frac{J}{2J+1}\right)^{\frac{1}{2}} j_{J+1} \ [Y^{[J+1]} \times \sigma^{[1]}]_\lambda^{[J]} \right.$$

$$+ \frac{\mu k}{2} \left(\frac{J+1}{2J+1}\right)^{\frac{1}{2}} j_{J-1} \ [Y^{[J-1]} \times \sigma^{[1]}]_\lambda^{[J]}$$

$$\left. + e \ j_J \ [Y^{[J]} \times p^{[1]}]_\lambda^{[J]} \right\} . \qquad (2E\text{-}13)$$

The $E_\lambda^{[J]}$ and $M_\lambda^{[J]}$ operators never connect to the same final state because they differ in parity*. The transverse form factor is then

$$|F_T(k)|^2 = \sum_\lambda |\langle\psi_f| \sum_j \left\{ \frac{e_j p_{j\lambda}}{M} + \frac{i\mu_j(\vec{\sigma}_j x\vec{k})_\lambda}{2M} \right\} \exp(i\vec{k}\cdot\vec{r}_j)|\psi_i\rangle|$$

$$= \frac{1}{2J_i+1} \sum_{J,\lambda,M_f,M_i} \frac{(\hat{J})^2}{2} \{|\langle J_f M_f|E_\lambda^{[J]}|J_i M_i\rangle|^2 + |\langle J_f M_f|M_\lambda^{[J]}|J_i M_i\rangle|^2\} \qquad (2E\text{-}14)$$

the Wigner-Eckart theorem and summation** over M_f, M_i, and λ then yield

$$|F_T(k)|^2 = \frac{1}{2J_i+1} \sum_J \left\{ |\langle J_f|| E^{[J]} || J_i\rangle|^2 + |\langle J_f|| M^{[J]} || J_i\rangle|^2 \right\}. \qquad (2E\text{-}15)$$

The matrix elements may be further reduced in isospin by using the Wigner-Eckart theorem in isospin space,

$$\langle J_f T_f T_z || O_0^{[T]} || J_i T_z T_z\rangle = (-1)^{T_f - T_z} \begin{pmatrix} T_f & T & T_z \\ -T_z & 0 & T_z \end{pmatrix} \langle J_f T_f ||| O^{[T]} ||| J_i T_z\rangle. \quad (2E\text{-}16)$$

Here, $O_0^{[1]}$ is the isovector part of $C^{[J]}$, $E^{[J]}$ or $M^{[J]}$ and $O_0^{[0]}$ the isoscalar component.

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*The Coulomb, electric and magnetic operators given in (De66) and (Do75) are related to those presented here by

$$C_o^{[J]} = (4\pi)^{\frac{1}{2}} M_{J0}^{Coul}, \; E_\lambda^{[J]} = (4\pi)^{\frac{1}{2}} T_{J\lambda}^{el}, \; M_\lambda^{[J]} = (4\pi)^{\frac{1}{2}} T_{J\lambda}^{mag}.$$

** $$\sum_{M_f,M_i,\lambda} \begin{pmatrix} J_f & J & J_i \\ -M_f & \lambda & M_i \end{pmatrix}^2 = \frac{2}{2J+1}$$

The total absorption cross section for real photons integrated over a resonance line can also be written in terms of the transverse multipole operators

$$\sigma_\gamma(\omega) = \frac{2\pi^2 e^2}{\omega} \frac{1}{2J_i+1} \sum_J \left\{ |\langle J_f \| \mathcal{E}^{[J]} \| J_i \rangle|^2 + |\langle J_f \| \mathcal{M}^{[J]} \| J_i \rangle|^2 \right\}_{k \to \omega} \tag{2E-17}$$

Table 2-1 reviews the terminology used in the classification of electromagnetic single-particle operators.

We are now required to compute the reduced matrix elements of the operators

$$j_J(kr)\, Y^{[J]},\ j_L(kr)\, [Y^{[L]} \times p^{[1]}]^{[J]},\ j_L(kr)\, [Y^{(L]} \times \sigma^{[1]}]^{[J]} \quad . \tag{2E-18}$$

This type of analysis is most easily performed using the diagrammatic recoupling technique of ref. (Da71). Writing a single-particle state as

$$\psi_m^{[j]} \doteq R_\ell(r)\, [Y^{[\ell]}(\hat{r}) \times \chi^{[\frac{1}{2}]}]_m^{[j]} \tag{2E-19}$$

where χ is the spin state and R_ℓ the radial wave function of orbital ℓ, the reduced matrix elements are:

$$(i)^{\ell'-J-\ell} \langle \psi^{[j']} \| j_J\, Y^{[J]} \| \psi^{[j]} \rangle = (-)^{j+J+\frac{1}{2}} \frac{\hat{J}\, \hat{j}'\, \hat{j}\, \hat{\ell}'\, \hat{\ell}}{(4\pi)^{\frac{1}{2}}} \begin{Bmatrix} \ell' & j' & \frac{1}{2} \\ j & \ell & L \end{Bmatrix} \begin{pmatrix} \ell' & J & \ell \\ 0 & 0 & 0 \end{pmatrix} \langle R_{\ell'} | j_J(kr) | R_\ell \rangle, \tag{2E-20}$$

$$(i)^{(\ell'-L-1-\ell)} \langle \psi^{[j']} \| j_L\, [Y^{[L]} \times p^{[1]}]^{[J]} \| \psi^{[j]} \rangle$$

$$= (-)^{\ell'+j-1} \frac{\hat{J}\, \hat{L}\, \hat{j}'\, \hat{\ell}'\, \hat{\ell}}{(4\pi)^{\frac{1}{2}}} \begin{Bmatrix} \ell' & j' & \frac{1}{2} \\ j & \ell & J \end{Bmatrix}$$

$$\times \left[\begin{Bmatrix} L & 1 & J \\ \ell & \ell' & \ell+1 \end{Bmatrix} \begin{pmatrix} \ell & L & \ell+1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell+1 & 1 & \ell \\ 0 & 0 & 0 \end{pmatrix}^{-1} \left(\frac{\ell+1}{2\ell+1}\right) \langle R_{\ell'} | j_L \left(\frac{\partial}{\partial r} - \frac{\ell}{r}\right) | R_\ell \rangle \right.$$

$$\left. + \begin{Bmatrix} L & 1 & J \\ \ell & \ell' & \ell-1 \end{Bmatrix} \begin{pmatrix} \ell' & L & \ell-1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell-1 & 1 & \ell \\ 0 & 0 & 0 \end{pmatrix}^{-1} \left(\frac{\ell}{2\ell+1}\right) \langle R_{\ell'} | j_L \left(\frac{\partial}{\partial r} + \frac{\ell+1}{r}\right) | R_\ell \rangle \right], \tag{2E-21}$$

$$(i)^{(\ell' - L - 1 - \ell)} \langle \psi^{[j']} \| j_L \, [Y^{[L]} \times \sigma^{[1]}]^{[J]} \| \psi^{[j]} \rangle$$

$$= (-)^{\ell'+2j'} \frac{6^{\frac{1}{2}} \hat{j} \, \hat{L} \, \hat{j}' \, \hat{j} \, \hat{\ell}' \, \hat{\ell}}{(4\pi)^{\frac{1}{2}}} \begin{Bmatrix} \ell' & \ell & L \\ \frac{1}{2} & \frac{1}{2} & 1 \\ j' & j & J \end{Bmatrix} \begin{pmatrix} \ell' & L & \ell \\ 0 & 0 & 0 \end{pmatrix} \langle R_{\ell'} | j_L(kr) | R_\ell \rangle \, . \qquad \text{(2E-22)}$$

The point to be gleaned from these complicated expressions is that the form factor of a transition to a state of specific ℓ, j, J is given by the product of vector coupling coefficients and a radial integral of $j_L(kr)$. The momentum dependence of the transition resides here. This fact is extensively used in spectroscopic studies to assign quantum numbers to discrete states.

Shell model calculations often involve harmonic oscillator wave functions because of their simplicity and integrability. As shown by eqs. (2E-20, 21, 22) one is left with radial integrals of three types:

Charge and Moment:

$$\langle R_{n'\ell'} \mid j_J \, (kr) \mid R_{n\ell} \rangle \qquad \text{(2E-23)}$$

Current:

$$\langle R_{n'\ell'} \mid j_L \, (kr) \left(\frac{\partial}{\partial r} - \frac{\ell}{r} \right) | R_{n\ell} \rangle \qquad \text{(2E-24)}$$

and

$$\langle R_{n'\ell'} \mid j_L \, (kr) \left(\frac{\partial}{\partial r} + \frac{\ell+1}{r} \right) | R_{n\ell} \rangle \, . \qquad \text{(2E-25)}$$

Oscillator radial wave functions are of the form

$$R^2_{n\ell}(r) = P(z) \exp(-z) \qquad \text{(2E-26)}$$

where P is a polynomial in $z = (r/b)^2$ and b is the oscillator parameter. The derivatives in (2E-24, 25) operating on $R_{n\ell}$ give back the same form with ℓ changed by one unit. The remaining integral can be evaluated as

$$\langle R_{n'\ell'} | j_L(kr) | R_{n\ell} \rangle = P'(y) \exp(-y) \, . \qquad \text{(2E-27)}$$

Where P' is now another polynomial (confluent hypergeometric function) in the variable $y = (bk/2)^2$. The function P' depends on the multipolarity of the transition operator, the principle quantum number of the shell, the orbital angular momentum, and the total angular momentum of both single particle wave functions. These polynomials can be found in ref. (Do79a).

Shell model wave functions require an additional correction for the motion of the center-of-potential about the center-of-momentum of the nucleons. For the harmonic oscillator this correction is given by multiplying all transition amplitudes by the form factor

$$f_{CM}(k) = \exp(y/A) \quad . \tag{2E-28}$$

We illustrate this formalism with an example to show how the sum over particles $\sum_j$ works and to show the effect of considering a transition between states of definite isospin T. ^{12}C has a ground state with $J^{\pi}=0^+$ and T=0 and an excited state at 15.1 MeV with $J^{\pi}=1^+$ and T=1. The only electromagnetic transition operator that can connect these two states is one with J=1, π=+ which is the magnetic dipole operator $\mathcal{M}^{[1]}$.

If the ^{12}C ground state is taken as the configuration $(1s_{\frac{1}{2}})^4\,(1p_{3/2})^8$ and the 1^+ excited state as $(1s_{\frac{1}{2}})^4\,(1p_{3/2})^7\,(1p_{\frac{1}{2}})^1$, then the transition can be pictured as simply one nucleon in the p-shell recoupling its orbital angular momentum and spin from $j = 1 + \frac{1}{2}$ to $j' = 1 - \frac{1}{2}$. Both neutrons and protons in the p-shell can make this transition. Because of the way single particle states are normalized (total state functions normalized to 1 and single particle wave function also normalized to 1) we have the simple relation for closed shell nuclei

$$<\psi^{[J_f]}\| \sum_i O^{[J]}_{(i)} \| \psi^{[J_i]}>$$

$$= \sum_{jj'} <\psi^{[j']}\| O^{[J]} \| \psi^{[j]}> \tag{2E-29}$$

i.e., the sum over single particle operators between the total state wave functions is the same as the sum of possible single particle transitions. In our example j = 3/2 and j' = ½ so that only one reduced matrix element* is involved $< \left[Y^{[1]} \times \tilde{\chi}^{[\frac{1}{2}]}\right]^{[\frac{1}{2}]} \| \mathcal{M}^{[1]} \| \left[Y^{[1]} \times \chi^{[\frac{1}{2}]}\right]^{[3/2]} >$. Electromagnetic transitions can, in general, change the isospin quantum number of a nuclear state by 0 or 1, an isoscalar or isovector transition. The fundamental interaction operators can be divided into their isoscalar and isovector components by the replacement of $e_j = \frac{1}{2}(1 + \tau_j^z)$ where $\tau^z|p> = +|p>$ in the Coulomb and current operators and $\mu_j = \frac{1}{2}(\mu_s + \tau_j^z \mu_v)$ where $\mu_s = \mu_p + \mu_n = 0.88$ and $\mu_v = \mu_p - \mu_n = 4.70$ in the moment operator. In the present example only the isovector parts of the operators will contribute to the transition. The single particle operators are therefore multiplied by the factor $\frac{1}{2}\tau^z$ and μ_j is replaced by μ_v for all particles. The isospin part of the single particle matrix element is always of the form

$$\sum M(j) <1\ 0| \tfrac{1}{2}\, \tau_j^z |0\ 0> = \frac{M}{2^{\frac{1}{2}}} \qquad \text{(2E-30)}$$

where the M(j) are the single-particle space-spin parts of the matrix element and $M = \sum_j M(j)$. Thus, the additional specification of the isospin quantum numbers of the initial and final states reduces the transition amplitude. Explicit evaluation of the 3-, 6-, and 9-j coefficients and the use of harmonic oscillator radial wave functions gives (with η = kb, b being the oscillator parameter)

$$<1p_{\frac{1}{2}} \| j_0(kr) \left[Y^{[0]} \times \sigma^{[1]}\right]^{[1]} \| 1p_{3/2}> = -i \frac{4}{(3)^{\frac{1}{2}}} (1 - \frac{1}{6}\eta^2) \frac{e^{-\eta^2/4}}{(4\pi)^{\frac{1}{2}}}$$

$$<1p_{\frac{1}{2}} \| j_2(kr) \left[Y^{[2]} \times \sigma^{[1]}\right]^{[1]} \| 1p_{3/2}> = +i \frac{\eta^2}{3(6)^{\frac{1}{2}}} \frac{e^{-\eta^2/4}}{(4\pi)^{\frac{1}{2}}} \qquad \text{(2E-31)}$$

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*The symbol $\tilde{\chi}$ denotes the transpose of χ.

$$\langle 1p_{\frac{1}{2}} \| j_1(kr) \left[Y^{[1]} \times p^{[1]} \right]^{[1]} \| 1p_{3/2} \rangle = -i \, \frac{(2)^{\frac{1}{2}}}{3} \, k \, \frac{e^{-\eta^2/4}}{(4\pi)^{\frac{1}{2}}} \qquad \text{(2E-30)}$$

$$\left| (1p_{\frac{1}{2}} \| \mathcal{M}^{[1]} \| 1p_{3/2} \rangle \right|^2 = \frac{k^2}{M^2} \, \frac{1}{3} \left[1 - \mu_v \left(2 - \frac{k^2 b^2}{4} \right) \right]^2 e^{-k^2 b^2/2} \; . \qquad \text{(2E-31)}$$

Taking the isospin factor into account

$$\frac{d\sigma}{d\Omega} (0^+ \rightarrow 1^+) = \sigma_o \left(\frac{k_\mu^{\;2}}{2k^2} + \tan^2\theta/2 \right) \frac{k^2}{M^2} \, \frac{1}{9} \left[1 - \mu_v \left(2 - \frac{k^2 b^2}{4} \right) \right]^2$$

$$e^{-k^2 b^2/2 + k^2 b^2/24} \; f_N^2(k_\mu^2) \; . \qquad \text{(2E-32)}$$

This equation has the momentum dependence of the nucleon form factor f_N and center-of-mass correction for the oscillator model. Other examples can be found in ref. Do79a.

F. LONG WAVELENGTH EXPANSIONS

The multipole expressions for current and magnetic moment operators given by (2E-15) can often be simplified for real photons by expanding the spherical Bessel functions and keeping the first nonvanishing term

$$j_L(kr) = \frac{(kr)^L}{1\cdot 3\cdot 5\cdots(2L+1)}\left[1 - \frac{(kr)^2}{2(2L+3)} + O(kr)^4\right]. \qquad (2F\text{-}1)$$

This approximation will be valid for $kr \ll 1$, (e.g., $r \lesssim 5$ fm, $k = \omega \lesssim 40$ MeV). In particular the electric-dipole current operator is often used in its unretarded form (replacing $\exp(i\vec{k}\cdot\vec{r}_j)$ by unity)

$$H_{E1} = \sum_j e_j \frac{\hat{\varepsilon}\cdot\vec{p}_j}{M} \qquad (2F\text{-}2)$$

and the magnetic-dipole moment term as

$$H_{M1} = \sum_j \frac{i\mu_j}{2M}(\hat{\varepsilon}\times\vec{k})\cdot\vec{\sigma}_j + \frac{ie}{2M}\, j\,(\hat{\varepsilon}\times\vec{k})\cdot\vec{\ell}_j \qquad (2F\text{-}3)$$

where $\vec{\ell}_j = (\vec{r}\times\vec{p})_j$.

It is often convenient to replace the nucleon momentum operator by the position operator

$$\langle f \left|\frac{\vec{p}}{M}\right| o\rangle = \langle f \left|\frac{d\vec{r}}{dt}\right| o\rangle = i\langle f\,|[H,\vec{r}]|\,o\rangle = i\,\omega\langle f|\vec{r}|o\rangle \qquad (2F\text{-}4)$$

by letting the nuclear Hamiltonian operate on the initial and final states, $\omega = E_f - E_o$ (nuclear recoil has been neglected). This replacement is sometimes referred to as Siegert's theorem. Then

$$H_{E1} = i\,\omega \sum_j e_j\, \hat{\varepsilon}\cdot\vec{r}_j \qquad (2F\text{-}5)$$

$$d\sigma_\gamma = 4\pi^2 e^2 \omega \overline{\sum} \left|\langle f| \sum_j e_j\, \hat{\varepsilon}\cdot\vec{r}_j\, |o\rangle\right|^2 \frac{dn}{d\omega} \qquad (2F\text{-}6)$$

where $e_j = 1$ for a proton, 0 for a neutron. Using isospin notation ($\tau^z|n\rangle = -|n\rangle$, $\tau^z|p\rangle = +|p\rangle$) one can write $e_j = \frac{1}{2}(1+\tau^z)_j$ and specify the $\vec{r}_j$ as center-of-mass coordinates. (2F-6) can be written in the familiar form

$$d\sigma_\gamma = 4\pi^2 e^2 \omega \overline{\sum} \left| \langle f \left| \tfrac{1}{2} \sum_j \hat{\varepsilon} \cdot \vec{r}_j \, \tau_j^z \right| o \rangle \right|^2 \frac{dn}{d\omega} \quad . \tag{2F-7}$$

Neutrons, although uncharged, can make single-particle electric dipole transitions. As can be seen in the expression for the dipole operator in center-of-mass coordinates $D = \tfrac{1}{2} \sum_j^A \hat{\varepsilon} \cdot \vec{r}_j \, \tau_j^z$. The dipole moment is created by the shift of the center-of-charge $C = \tfrac{1}{2} \sum \vec{r}_j \, \tau_j^z$ away from the center-of-mass $\vec{R} = \sum_j \vec{r}_j = 0$. Neutrons are as effective as protons in making this shift.

Two relations between form factors and cross sections can be seen in the low momentum transfer limit. The transverse form factors $F_p(k,\omega)$ and $F_\sigma(k,\omega)$ become the same as the real photon transition amplitudes as $k \to \omega$. This gives the following connection between inelastic electron scattering and the photon absorption cross section

$$\left| F_T \, (k,\omega) \right|^2_{k \to \omega} = \frac{\omega \, \sigma_\gamma(\omega)}{2\pi^2 e^2} \tag{2F-8}$$

for transverse transitions of any multipolarity.

In this same limit the current and Coulomb operators can be related through current conservation to give

$$\left| F^L_{T,p}(k,\omega) \right|^2_{k \to \omega} = \left(\frac{\omega}{k}\right)^2 \frac{L+1}{L} \left| F^L_c(k,\omega) \right|^2 \tag{2F-9}$$

G. COINCIDENCE MEASUREMENTS

When a nuclear decay product x (x = γ, n, p, π, etc.) is detected in coincidence with an inelastically scattered electron in the reaction A(e,e'x) the cross section becomes a three-fold differential and, in general, depends on four form factors which are functions of k, ω, p_x, $\hat{k} \cdot \hat{p}_x$. The Born approximation result can be written (Am79) as an extension of the single arm expression (2B-9)

$$\frac{d^3\sigma(e,e'x)}{d\Omega d\omega d\Omega_x} = \Gamma_T \frac{d\sigma_v}{d\Omega_x} \tag{2G-1}$$

The $d\sigma_v$ is the angular distribution of x about the direction of momentum transfer k

$$\frac{d\sigma_v}{d\Omega_x} = \frac{e^2}{16\pi}\frac{Px}{K}\frac{1}{MW}\left[\varepsilon \frac{k_\mu^2}{\omega^2} M_{zz} + \frac{1}{2}(M_{xx} + M_{yy}) + \frac{1}{2}(M_{xx}-M_{yy})\ \varepsilon \cos 2\phi_x + \frac{1}{2}(M_{zx}+M_{xz})\left(2\frac{k_\mu^2}{\omega^2}\varepsilon(1+\varepsilon)\right)^{\frac{1}{2}} \cos\phi_x\right] \qquad (2G\text{-}2)$$

The azimuthal angle ϕ_x is defined in fig. (2G-1). W is the total energy available ($\omega + M_T$). The M_{ij} are products of transition amplitudes in the transverse (x and y) and longitudinal (z) directions. For real photons the first and fourth terms vanish and ε becomes the degree of linear polarization. One regains the single arm (e,e') cross section (2B-9) by integrating (2G-2) over the direction of the unobserved product x

$$\sigma_T + \varepsilon\sigma_L = \int_0^{2\pi} d\phi_x \int_0^{\pi} \frac{d\sigma_v}{d\Omega_x} \sin\theta_x\, d\theta_x \qquad (2G\text{-}3)$$

then the two interference terms with $\cos 2\phi_x$ and $\cos\phi_x$ vanish.

Coincidence measurements of the type (e,e'x) are often limited by the accidental detection of a scattered electron with a decay product x from an unrelated event. If the average counting rates of the e' and x detectors are N_e and N_x respectively, the accidental rate is given by

$$N_{acc} = \left(\frac{\tau}{d.f.}\right) N_{e'} N_x \qquad (2G-4)$$

where τ is the resolving time of the coincidence circuit (typically $\lesssim 10^{-8}$ s) and d.f., the duty factor, is the fraction of the time the electron beam is on the target (typically $\lesssim 10^{-2}$ for pulsed linacs). The average true coincidence rate for detecting a scattered electron and the associated decay product is

$$N_{coin} = N_{e'} \frac{\Delta\Omega_x}{4\pi} \frac{\Delta E_x}{E_x} \qquad (2G-5)$$

where $\Delta\Omega_x$ is the solid angle acceptance of the product detector and $\Delta E_x/E_x$ is the fractional energy acceptance. A coincidence measurement can be considered feasible when

$$\frac{N_{coin}}{N_{acc}} \geq 1 \qquad (2G-6)$$

This usually requires d.f. = 1 unless there exits a strong energy and angular correlation between the scattered electron and the decay product (as there is for example in quasi-free electron-nucleon scattering).

H. RELATION OF WEAK AND ELECTROMAGNETIC INTERACTIONS

Inelastic electron scattering in the one-photon exchange approximation is an example of the larger class of semileptonic reactions that share the common feature that the leptonic and strong interaction vertices can be evaluated separately. The charge changing weak interactions are: beta decay ($e^-\bar{\nu}_e$), ($e^+\nu_e$), lepton capture (ℓ^-, ν_ℓ) and neutrino reactions (ν_ℓ,ℓ^-), ($\bar{\nu}_\ell,\ell^+$), $\ell = \mu$ or e.

The weak transition operators have a richer structure than the electromagnetic. Besides the vector currents γ_μ there are axial vector currents from $\gamma_\mu\gamma_5$,

$$j_\mu = \bar{u}\,[F_1\,\gamma_\mu + F_2\,\sigma_{\mu\nu}\,k_\nu]\,\tau_\pm\,u$$

$$j_{\mu5} = \bar{u}\,[F_A\,\gamma_5\,\gamma_\mu - i\,F_p\,\gamma_5\,\gamma_\mu]\,\tau_\pm\,u \quad . \qquad (2H\text{-}1)$$

The conserved vector current hypothesis identifies the weak vector coupling constants $F_{1,2}$ as the same functions of k_μ^2 as the nucleon electromagnetic form factors. The axial vector weak coupling constants and form factors are

$$F_A = -\,1.24\,(1 + k_\mu^2/b^2)^{-2}\ ,\quad b^2 = (855\ \text{MeV})^2$$

$$F_p = 2M\,F_A/(k_\mu^2 + m_\pi^2)\ . \qquad (2H\text{-}2)$$

The overall weak coupling constant (the analog of e^2) is

$$G = 1 \times 10^{-5}/M^2 \quad . \qquad (2H\text{-}3)$$

The weak interaction transition operators are similar in structure to the electromagnetic single particle operators

$$H_{weak} = (1,\vec{\sigma},\vec{p},\vec{\sigma}\cdot\vec{p})\ \tau_\pm\ \exp(i\vec{k}\cdot\vec{r}) \qquad (2H\text{-}4)$$

and their multipole decomposition is carried out as in the previous section for the H_e operators (Oc72b). The experimental confirmation that the nuclear part of a weak interaction amplitude has the same numerical value as the corresponding $<\Psi_f|H_e|\Psi_i>$ electromagnetic form factor has been verified by many examples.

The correspondence between weak and electromagnetic amplitudes is useful for predicting the cross sections of energetic weak interactions. For example, the reaction $D(\nu_e, e^-)pp$ (see fig. 2H-1), has been measured (Wi80) and found to be in agreement with the expression

$$\frac{d^2\sigma}{d\Omega dE_2} = \frac{4}{3} \frac{G^2}{(2\pi)^2} k_2 E_2 \frac{Mp}{(2\pi)^2} \left\{ \left(\frac{1}{2} + \frac{1}{2} \sin^2 \frac{\theta}{2} \right) \left[F_A^2 + \left(\frac{k\mu_v}{2M} \right)^2 \right] \right. \tag{2H-5}$$

$$\left. - 2 \sin (\theta/2)\, F_A \left(\frac{k\mu_v}{2M} \right) \quad |<{}^1S_o|j_o \left(\frac{kr}{2} \right)|{}^3S_1>|^{\,2} \right\}$$

where the last factor can be deduced from model fits to the electromagnetic ${}^3S_1 \rightarrow {}^1S_0$ magnetic dipole transition from D(e,e')np data. The neutrino breakup of the deuteron is closely related to the weak interaction $p+p \rightarrow d+e^+ + \nu_e$ which governs the rate of proton fusion in the sun and other stars (a reaction that cannot be measured directly in the laboratory).

The weak neutral current due to Z^0 exchange between a lepton and nucleon is responsible for neutrino elastic scattering and for a weak component in electron and muon scattering from nucleons. Although the weak scattering amplitude is much smaller than the electromagnetic amplitude the parity nonconservation of the weak interaction gives rise to an a asymmetry in the scattering of polarized electrons (Pr78). Transitions between discrete nuclear states due to neutral currents have total reaction cross sections for energetic neutrinos in the range $10^{-40} - 10^{-41} cm^2$ (Do79b).

I. REAL AND VIRTUAL BREMSSTRAHLUNG

Three closely related cross sections that involve the radiation process are shown in figs. 2B-1 and 2D-1. Diagrams 2D-1a and b are the emission of a high energy real photon ($\vec{k},\omega=|\vec{k}|$) by an electron undergoing elastic scattering in the Coulomb field of a nucleus (momentum transfer $\vec{K}=(\vec{k}_1-\vec{k}-\vec{k}_2)$, energy transfer $\omega=K^2/2M_T\cong 0$). The integrated-over-angle spectrum of the real photons is

$$\frac{d\sigma}{dk} = Z^2 \left(\frac{4e^6}{m_e^2}\right) \frac{1}{k} \left[(1 + \varepsilon^2 - \frac{2}{3}\varepsilon)\,\ell n\,\frac{183}{Z^{1/3}} + \frac{\varepsilon}{9}\right] \tag{2I-1}$$

where $\varepsilon = E_2/E_1$. This is the completely-screened Bethe-Heitler form valid for E_1=100-1000 MeV on high Z radiators (Ko59,Ma73).

Most real photon measurements are done using bremsstrahlung spectra. The number of photons per MeV per microampere of electron beam incident on a radiator whose thickness is one percent of a radiation length (r.l.) is shown in fig. 2I-1. Radiators much thicker than 1% r.l. generally do not generate much additional flux in the forward direction due to multiple scattering of the electrons in the target.

The disadvantage of bremsstrahlung as a source of photons is that photons of all energies are incident on the target at once. Other schemes to generate monoenergetic photons are: tagged bremsstrahlung (Oc62), positron annihilation in flight (Be79), and Compton scattered laser light (Fe79).

The energy spectrum and angular distribution of the scattered electron after bremsstrahlung is given in the peaking approximation ($\hat{k} = \hat{k}_1$ or $\hat{k}_2$) as

$$\frac{d^2\sigma}{d\Omega dE_2} = \mathcal{F}(E_1)\,\frac{d\sigma}{d\Omega}\,(E_2,\theta) + \mathcal{F}(E_2)\,\frac{d\sigma}{d\Omega}\,(E_1,\theta) \tag{2I-2}$$

where $d\sigma/d\Omega$ is the elastic scattering cross section at the appropriate incident electron energy and

$$\mathcal{F}(E) = \frac{e^2}{\pi k}\left[(1 + \varepsilon^2)\ \ell n\,\frac{2E}{m_e} - \varepsilon\right] \qquad (2I\text{-}3)$$

an unscreened expression valid for $\theta >> m_e/E_{1,2}$. Wide angle bremsstrahlung forms a background to A(e,e') measurements (Mo69,Ma69).

Inelastic electron scattering observing only the scattered electron was discussed in Section B. Now, however, we are interested in the cross section for this reaction when the scattered electron is not observed i.e., $A(e,x)A'e'$ where x is some nuclear decay product (γ',N,π, etc.). To obtain this cross section we need to integrate over the angular distribution of k_2 at fixed ω. Eq. (2B-3) was derived using the simplifying assumption that the mass of the electron could be neglected. This is accurate except for electrons that are scattered in a very forward direction $\theta \simeq m_e/E_{1,2}$. A more accurate formula (but still treating the electrons as plane waves) is

$$\frac{d^2\sigma}{d\Omega_e dE_2} = \sigma_C\ (\theta)\ F_C^2(q,\omega) + \sigma_T(\theta)\ F_T^2(q,\omega) \qquad (2I\text{-}4)$$

where the transition form factors are multiplied by "Coulomb" and "transverse Mott cross sections"

$$\sigma_C = \frac{2\alpha^2}{k_\mu^{\ 4}}\ \frac{k_2}{k_1}\ \frac{k_\mu^{\ 4}}{k^4}\ (E_1\ E_2 + \vec{k}_1\cdot\vec{k}_2 + m_e^2) \qquad (2I\text{-}5)$$

$$\sigma_T = \frac{2\alpha^2}{k_\mu^{\ 4}}\ \frac{k_2}{k_1}\ (E_1E_2 - (\vec{k}_1\cdot\hat{k})\ (\vec{k}_2\cdot\hat{k}) - m_e^2) \qquad (2I\text{-}6)$$

and

$$E_i = (\vec{k}_i^2 + m^2)^{\frac{1}{2}},\ k_\mu^2 = 2(E_1E_2 - \vec{k}_1\cdot\vec{k}_2 - m^2) = k^2 - \omega^2$$

$$\omega = E_1 - E_2,\ \vec{k} = \vec{k}_1 - \vec{k}_2,\ k_i^2 = - m_e^2 \quad .$$

When $m_e = 0$, we obtain the familiar forms

$$\sigma_c = \sigma_M \frac{k_\mu{}^4}{k^4},\ \sigma_T = \sigma_M\left(\frac{k_\mu{}^2}{2k^2} + \tan^2\theta/2\right),\ \sigma_M = \frac{\alpha^2\cos^2\theta/2}{4k_1^2\sin^4\theta/2} \qquad (2I\text{-}7)$$

$$k_\mu{}^2 = 4k_1k_2\sin^2\theta/2\ . \qquad (2I\text{-}8)$$

The Coulomb and transverse Mott cross sections are plotted in fig. 2I-2 for $k_1 = 100$ MeV, $k_2 = 80$ MeV. We note the strong forward peaking of σ_T at

$$\theta_{max} \cong \frac{m_e\omega}{k_1k_2} \quad . \qquad (2I\text{-}9)$$

This peaking is responsible for the fact that the integrated over angle inelastic electron scattering cross section $\frac{d\sigma}{dE_2}$ comes mainly from the far forward transverse component (i.e., like a real photon).

For electric transitions at angles away from the extreme forward and backward scattering angles Coulomb transitions dominate current transitions, both because $\sigma_c > \sigma_T$ and because, in general at low momentum transfer, $F_c > F_T$ as can be seen in the relation (2F-9).

When the angular integral over the unobserved electron is made to obtain the (e,x) cross section at fixed excitation energy

$$\frac{d\sigma_{e,x}}{d\omega} = \int [\sigma_c(\theta)\ F_c{}^2(k,\omega) + \sigma_T(\theta)\ F_T{}^2(k,\omega)]\ d\Omega \qquad (2I\text{-}10)$$

most of the contribution to the integral comes from small values of θ. In this region $k \to \omega$ i.e., the virtual photon is almost real. If the (e,x) reaction is dominated by one multipole L the integral can be evaluated using low-momentum transfer approximations for F_C and F_T. As noted in Section E the momentum dependence of F_C and F_T (and, therefore, the angular dependence resides in the radial integrals. The form factors can be expanded at low k as*

$$\langle R_{\ell'} | j_L(kr)\ f(r) | R_\ell \rangle \simeq \frac{k^L}{(2L+1)!!} \langle R_{\ell'} | r^L\ f(r) | R_\ell \rangle. \qquad (2I\text{-}11)$$

Thus eq. (2F-8) can be rewritten as

$$|F_T^{\,L}(k,\omega)|^2_{k\to\omega} = \frac{\omega\sigma_\gamma(\omega)}{2\pi^2 e^2} \left(\frac{k}{\omega}\right)^{2L} \qquad (2I\text{-}12)$$

Low k transverse form factors are usually dominated by the current part so eq. (2F-9) can be written as

$$|F_C^{\,L}(k,\omega)|^2_{k\to\omega} = \frac{L}{L+1} \left(\frac{k}{\omega}\right)^{2L+2} \frac{\omega\sigma_\gamma(\omega)}{2\pi^2 e^2}\ . \qquad (2I\text{-}13)$$

Now the integral eq. (2I-10) can be evaluated. The result (Gi71,Dr76,Sh79) is usually expressed as

---

*Monopole transitions are an exception since the leading term is zero by orthogonality; the next term $(kr)^2$ applies then.

$$\frac{d\sigma^{L}_{e,x}}{d\omega} = \sigma^{L}_{\gamma}(\omega)\ N^{L}\ (E_1,\omega) \qquad (2I\text{-}14)$$

where N^L is interpreted as the spectrum of virtual photons seen by the nucleus during inelastic electron scattering. For electric dipole transitions

$$N^{E1}\ (E_1,k) = \frac{e^2}{\pi k}\left[(1+\varepsilon^2)\ \ell n\ \frac{2E_1E_2}{m_e k} - 2\varepsilon\right] \qquad (2I\text{-}15)$$

i.e., a function similar to that found for real bremsstrahlung. Other multipoles differ in terms linear in ε.

When Coulomb distortion of the electron waves is taken into account (So77) the N^L spectra show a great sensitivity to L. Fig. 2I-3 shows the intensity $kN(E_1,k)$ vs k for a Z = 28 target. This sensitivity has been exploited to look for E_2 giant resonances in the presence of the much stronger E_1 giant resonance (Ha80).

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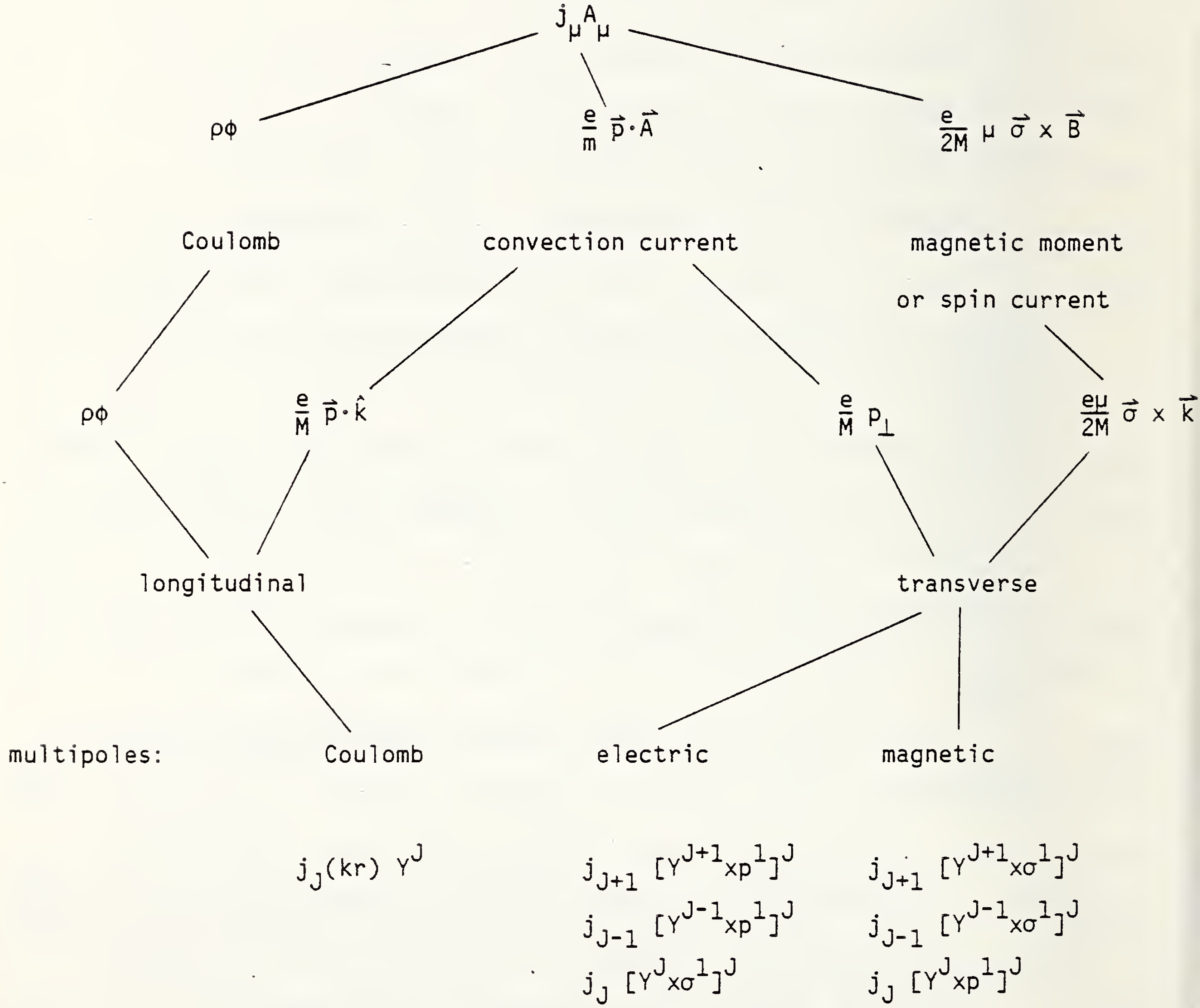
Table 2-1. Classification Scheme of Electromagnetic Operators.

Four-vector interaction energy



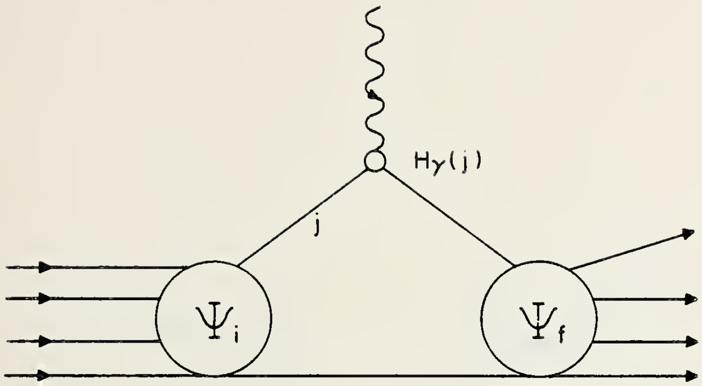


Fig. 2A-1 Diagram for the single particle amplitude for absorption of a photon.

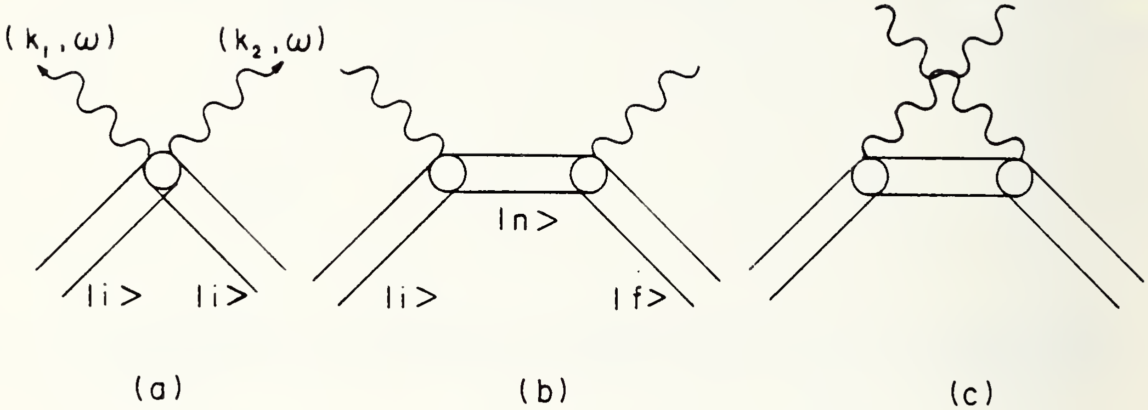


Fig. 2A-2 Diagrams for computing the photon scattering amplitude.

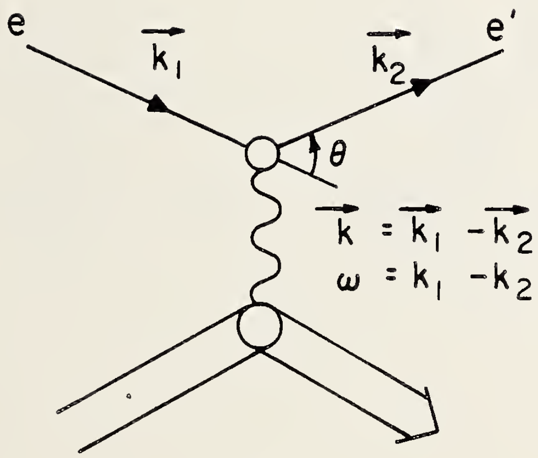


Fig. 2B-1 Diagram for computing the electron scattering amplitude.

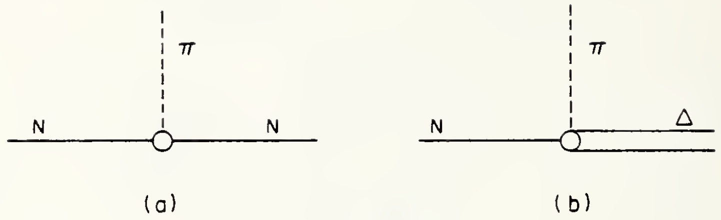


Fig. 2C-1 Pion-nucleon vertices (a) NΠN, (b) NΠΔ.

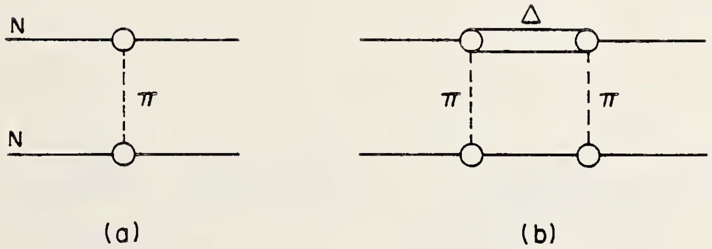


Fig. 2C-2 Two-nucleon interactions (a) one-pion exchange, (b) two-pion exchange.

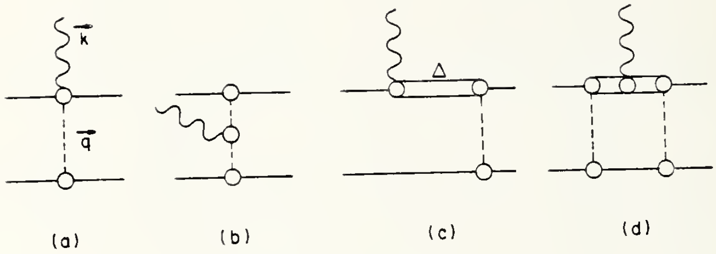


Fig. 2C-3 Principal electromagnetic couplings to the pion and delta currents, (a) pair or gauge term, (b) pion current term, (c) delta creation, (d) delta current.

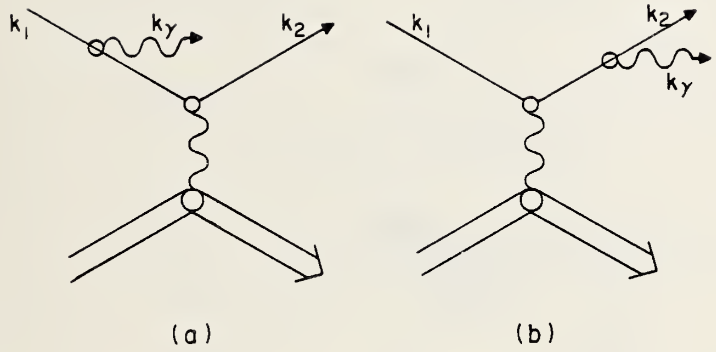


Fig. 2D-1 Two diagrams which contribute to bremsstrahlung and the radiation tail of electron scattering.

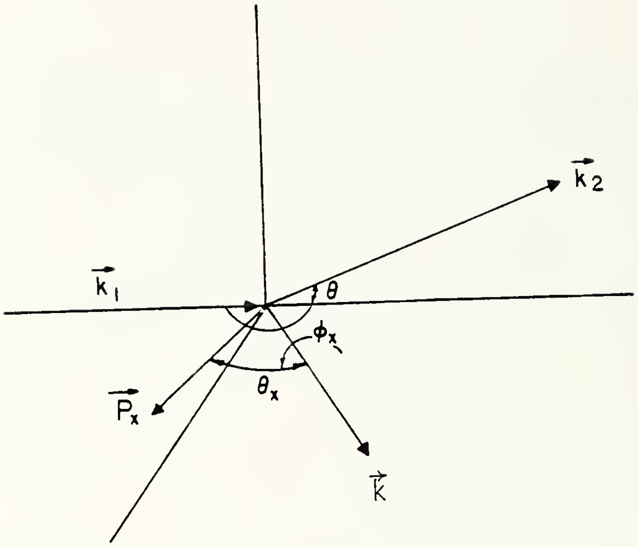


Fig. 2G-1 Coordinate system for the kinematic variables of the reaction (e,e'x). $\vec{k}_1$, $\vec{k}_2$, and $\vec{k}$ lie in the horizontal plane. The nuclear reaction product x is measured relative to the momentum transfer $\vec{k}$ and the horizontal plane.

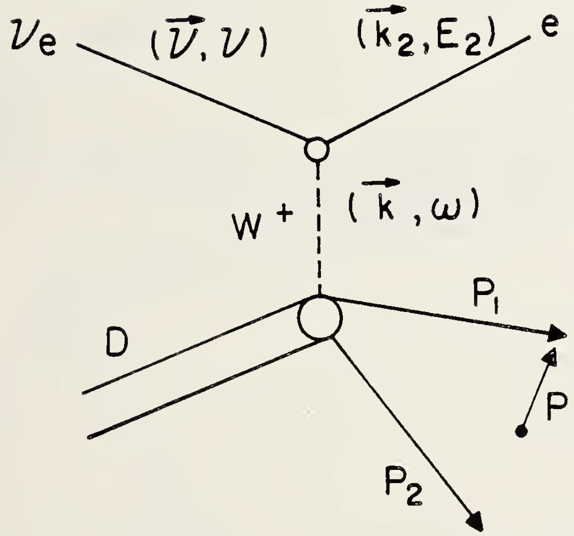


Fig. 2H-1 Variables for the reaction $D(\nu_e,e)pp$.

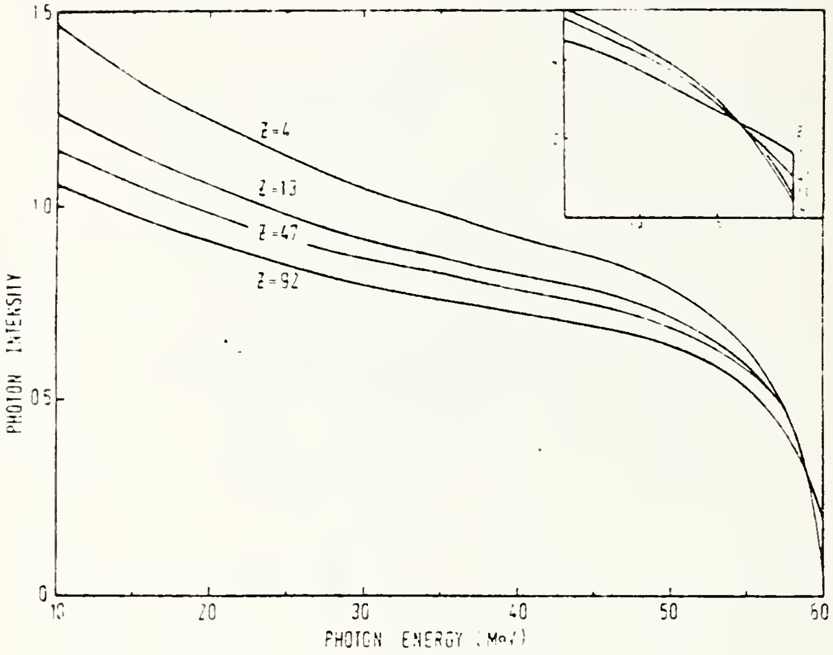


Fig. 2I-1 Bremsstrahlung intensity spectra of 4 elements as radiators. The ordinate is the number of photons per MeV per radiation length multiplied by the photon energy in MeV. The endpoint region is illustrated in the inset (Ma73).

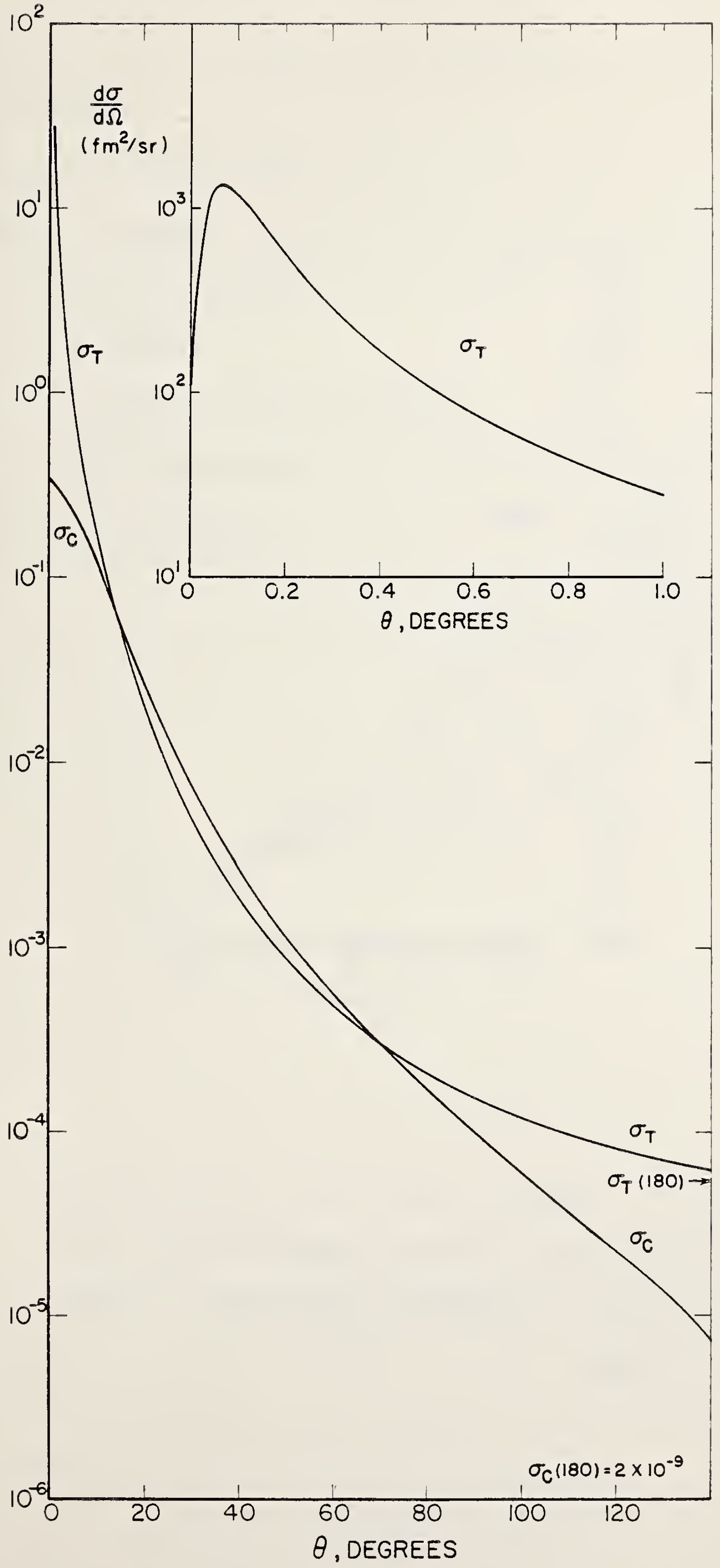


FIG. 21-2

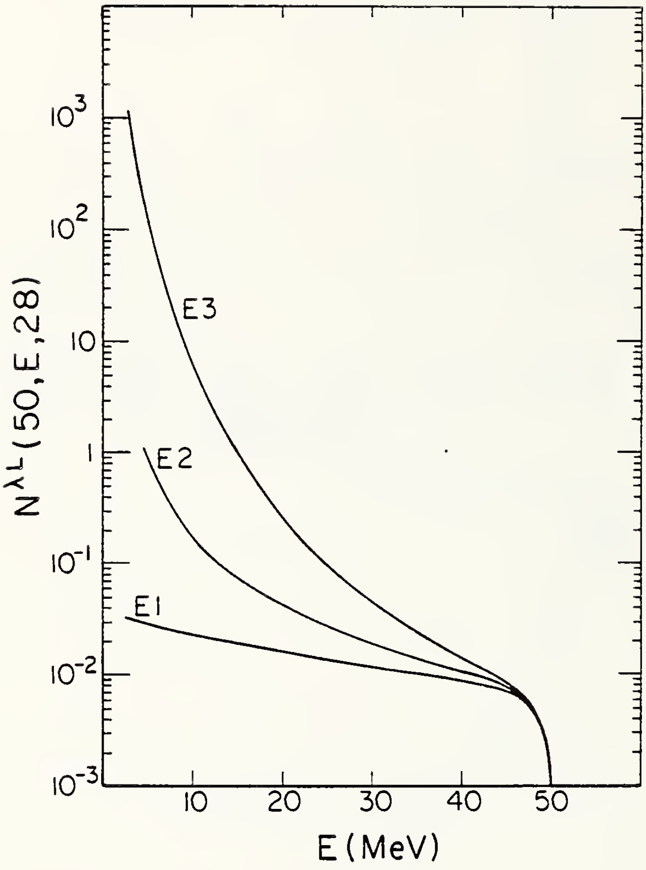


Fig. 2I-3 Intensity spectra versus photon energy of E1, E2, E3 virtual photons generated when 50 MeV electrons are inelastically scattered by a nickel target (Ha80).

CHAPTER III. SUM RULES

A. Closure Sum Rules

1. Electric Dipole Sum Rules

2. Siegert's Theorem

3. Other Electric Sum Rules

4. Magnetic Sum Rules

5. Electron Scattering Closure Rules

B. Dispersion Relation Sum Rules

C. Momentum Transfer Sum Rules

CHAPTER III. SUM RULES

The electromagnetic interaction in the nucleus can be treated as a perturbation because of the small coupling constant $e = (137)^{-\frac{1}{2}}$. A real or virtual photon is absorbed on a single nucleon (or meson) without interacting with the other nuclear constituents. Let us denote the charge, current, or moment operators generically by Ⓗ. The transition amplitude

$$T_{fi} = \langle \Psi_f(\vec{r}_1,\vec{r}_2,\ldots\vec{r}_A) \Big| \sum_j^A Ⓗ_j\,(\vec{r}_j,\vec{p}_j) \Big| \Psi_i(\vec{r}_1,\vec{r}_2,\ldots\vec{r}_A)\rangle \quad (3\text{-}1)$$

will have only one term active at a time out of the A possible choices. In contrast, the multiple interactions of hadronic probes (p,n,π, etc.) displace more than one nucleon during their interactions with the nucleus.

Sum rules are statements about integrals over electromagnetic cross sections. They are used to relate reaction data to ground state properties or to relate data from different types of reactions. Sum rules are also used to test the internal consistency of theoretical models of nuclear reactions. The discussion in this chapter will cover sum rules based on closure over excited nuclear states, dispersion relation sum rules, and momentum transfer sum rules for electron scattering. Review articles on aspects of this subject can be found in refs. (Oc73, We74, No78, Ar79).

A. CLOSURE SUM RULES

Completeness requires the eigenstates of a Hamiltonian $(H|\Psi_n\rangle = E_n|\Psi_n\rangle)$ sum to unity when written as a projection operator

$$\sum_n |\Psi_n\rangle\langle\Psi_n| = 1 \,. \quad (3A\text{-}1)$$

This closure property can be used to eliminate intermediate states in a sum over the absolute square of a transition amplitude if the operator is independent of the transition energy $\omega = E_f - E_o$,

$$\sum_f |\langle f|\text{\textcircled{H}}|0\rangle|^2 = \sum_f \langle 0|\text{\textcircled{H}}^+|f\rangle\langle f|\text{\textcircled{H}}|0\rangle = \langle 0|\text{\textcircled{H}}^+\text{\textcircled{H}}|0\rangle \quad \text{(3A-2)}$$

where $\text{\textcircled{H}}^+$ is the Hermitian conjugate[+] of $\text{\textcircled{H}}$ (interchange of rows and columns and complex conjugation of the matrix). Frequently, some power of the transition energy appears in the amplitude. It can be removed by successive application of the relation

$$\langle f|\omega^n \text{\textcircled{H}}|0\rangle = \langle f|\omega^{n-1}[H,\text{\textcircled{H}}]|0\rangle \ . \quad \text{(3A-3)}$$

The time derivative of a matrix element whose operator does not explicitly depend on time is given by its commutator with H,

$$\frac{d}{dt}\langle f|\text{\textcircled{H}}|0\rangle = i\langle f|[H,\text{\textcircled{H}}]|0\rangle \ . \quad \text{(3A-4)}$$

This allows the replacement of a particle momentum operator $\vec{p} = M\, d\vec{r}/dt$ by the position operator

$$\langle f|\frac{\vec{p}}{M}|0\rangle = i\langle f|[H,\vec{r}]|0\rangle = i\omega\langle f|\vec{r}|0\rangle \quad \text{(3A-5)}$$

by letting H operate on the initial and final states. Equation (3A-4) leads to the useful theorem

$$\sum_f \omega|\langle f|\text{\textcircled{H}}|0\rangle|^2 = \frac{1}{2}\langle 0|[\text{\textcircled{H}}^+, [H,\text{\textcircled{H}}]]|0\rangle \ . \quad \text{(3A-6)}$$

Equations (3A-2 and 6) are the progenitors of most closure sum rules.

[+]Reversing the order of a matrix element brings in the Hermitian conjugate of the operator $\langle 0|\text{\textcircled{H}}^+|f\rangle = \langle f|\text{\textcircled{H}}|0\rangle^*$ where (*) is complex conjugation (change of sign of imaginary components). Although the Hamiltonian is Hermitian $H^+=H$, as are all operators that generate physical observables, the commutator of two Hermitian operators is anti-Hermitian $\langle 0|[H,\text{\textcircled{H}}]^+|f\rangle = -\langle 0|[H,\text{\textcircled{H}}]|f\rangle = -\langle f|[H,\text{\textcircled{H}}]|0\rangle^* = -\langle f|[H,\text{\textcircled{H}}]|0\rangle$ since $(H\,\text{\textcircled{H}})^+ = \text{\textcircled{H}}^+ H^+$.

1. Electric Dipole Sum Rules

The oldest and most venerable sum rule is that for the electric dipole cross section

$$\sigma_o \equiv \int_0^\infty \sigma_\gamma(\omega)d\omega = 2\pi^2 e^2 <0|[D, [H,D]]|0> \qquad (3A\text{-}7)$$

where $D = \frac{1}{2}\sum_j \hat{\varepsilon}\cdot\vec{r}_j \tau_j^z = D^+$ is the long wavelength approximation to the E1 operator. Equation (3A-7) is derived as follows: writing the energy integral of eq. (2F-7) as

$$\sigma_o = 4\pi^2 e^2 \sum_f <f|D|0>^* \omega <f|D|0> \qquad (3A\text{-}8)$$

and removing the energy dependence (replacing ωD by $[H,D]$ symmetrically) one obtains

$$\sigma_o = 2\pi^2 e^2 \sum_f \{<f|[H,D]|0>^* <f|D|0> + <f|D|0>^* <f|[H,D]|0>\} . \qquad (3A\text{-}9)$$

Turning around the matrix elements $<0|D|f> = <f|D|0>$ and $<0|[H,D]|f> = -<f|[H,D]|0>^*$ and applying closure gives

$$\sigma_o = 2\pi^2 e^2 \{<0|D[H,D] - [H,D]D|0>\} \qquad (3A\text{-}10)$$

and eq. (3A-7) results.

The double commutator of the dipole operator with the Hamiltonian can be evaluated

$$H = \sum_j \frac{p_j^2}{2M} + \frac{1}{2} \sum_{j \neq k} V(\vec{r}_j, \vec{r}_k) \equiv K + V \; . \qquad (3A\text{-}11)$$

The kinetic energy term gives the Thomas-Reiche-Kuhn (TRK) formula*

$$\sigma_o^{TRK} = \frac{2\pi^2 e^2}{M} \frac{NZ}{A} \cong 60 \frac{NZ}{A} \text{ (MeV-mb)} \; . \qquad (3A\text{-}12)$$

The potential energy contribution depends on details of the effective interaction of nucleons in the nuclear medium and on the two-nucleon wave function. The long range one-pion-exchange potential, eq. (2C-3), gives a contribution since τ_j^z in the dipole operator does not commute with $\vec{\tau}_j \cdot \vec{\tau}_k$ in the OPE potential, see eq. (2C-9). Meson exchange currents also contribute to $<0|[D, [V,D]]|0>$. The net effect of the potential contribution to σ_o is usually written as a multiplicative constant $(1 + \kappa)$ times the TRK result. In practice the upper limit of the energy integral is taken as the threshold for pion production $\omega = m_\pi = 140$ MeV.

Approximating the real photon operator by D is not as bad an approximation as one might expect. The full current single-particle operator $\hat{\varepsilon} \cdot \frac{\vec{p}}{M} \exp(i\vec{k} \cdot \vec{r})$ leads to

$$\sigma_o = 2\pi^2 e^2 \sum_f \frac{1}{\omega} \left| <f| \frac{p_x}{M} e^{ikz} |0> \right|^2 \; . \qquad (3A\text{-}13)$$

where $Px = \hat{\varepsilon} \cdot \vec{p}$ and $z = \hat{k} \cdot \vec{r}$.

---

* $[x, p_x] = i$, $[x, \frac{p^2}{2M}] = i \frac{p_x}{M}$, $[x, [\frac{p^2}{2M}, x]] = \frac{1}{M}$, $[\frac{p^2}{2M}, [\frac{p^2}{2M}, x^2]] = -2 \frac{p_x^2}{M^2}$,
$[\hat{\varepsilon} \cdot \vec{p} \; , \; \hat{k} \cdot \vec{r}] = 0$.

The exponential can be expanded as

$$e^{\pm ikz} = 1 \pm ikz - \frac{1}{2} k^2 z^2 + \ldots + \frac{(\pm ikz)^n}{n!} .$$

The current operator can then be expanded as

$$\frac{p_x}{M} \exp(ikz) = \omega \left(ix + \frac{ip_x z}{M} - ½ \left[H, \frac{p_x z^2}{M}\right] + \ldots\right) \equiv \omega \hat{\varepsilon} \cdot \vec{J} . \quad (3A\text{-}14)$$

Closure gives

$$\sigma_o = 2 \pi^2 e^2 \langle 0 | [(\hat{\varepsilon} \cdot \vec{J})^+, [H, \hat{\varepsilon} \cdot \vec{J}]] | 0 \rangle . \quad (3A\text{-}15)$$

Using only the kinetic energy parts of H in the commutator one finds

$$\sigma_o = \sigma_o^{TRK} \left[1 + \frac{2}{3} \frac{\langle p^2 \rangle}{M^2} + 0 \left(\frac{p^4}{M^4}\right)\right] . \quad (3A\text{-}16)$$

The average momentum in nuclear matter is p = 208 MeV/c leading to a 3% correction to σ_o (Fr75).

Relativistic and potential energy corrections to σ_o^{TRK} decrease the sum by a term proportional to $\langle p^2 \rangle / M^2$ so the net change is probably smaller than the correction quoted above. Magnetic moment contributions to σ_o are also at the few percent level since the leading term in the cross section is k^2/M^2 times the current term.

Dipole sum rules with other energy weightings can be constructed

$$\sigma_n \equiv \int_0^\infty \omega^n \sigma_o(\omega)\, d\omega = 4\pi^2 e^2 \sum_f \omega^{n+1} |\langle f | D | 0 \rangle|^2 \quad (3A\text{-}17)$$

using closure. The most useful are n = -2, -1. The n = -2 sum rule is given in terms of the nuclear polarizibility α defined classically as the induced dipole moment divided by the electric field strength. Quantum mechanically

$$\alpha = 2e^2 \sum_f \omega^{-1} |\langle f|D|0\rangle|^2 = \left. \frac{\mathrm{Re}\, f(\omega) - \mathrm{Re}\, f(0)}{\omega^2} \right|_{\omega \to 0} \tag{3A-18}$$

where Re f(ω) is the real part of the photon elastic forward scattering amplitude (as will be discussed in Section 3B). Thus

$$\sigma_{-2} \equiv \int_0^\infty \frac{\sigma_\gamma(\omega) d\omega}{\omega^2} = 2\,\pi^2 \alpha \quad . \tag{3A-19}$$

In the harmonic oscillator model $V^{HO}(r) = (M\omega_o^{\ 2}) r^2$; the polarizability is the inverse of the spring constant

$$\alpha^{HO} = \frac{NZ}{A} \frac{1}{M\,\omega_o^{\ 2}} \quad . \tag{3A-20}$$

In general, the polarizability is related to the nuclear symmetry energy constant K (≅ 25 MeV) entering the semiempirical mass formula as $K(N-Z)^2/A$. A simple model (Le60) gives

$$\alpha = \frac{e^2 A}{40K} \langle R^2 \rangle \frac{\sigma_o}{\sigma_o^{TRK}} \quad . \tag{3A-21}$$

The n = -1 dipole sum rule

$$\sigma_{-1} \equiv \int_0^\infty \frac{\sigma(\omega) d\omega}{\omega} = 4\pi^2\, e^2 \langle 0|D^2|0\rangle = \pi^2 e^2 \langle 0| \sum_{i,j} \tau_i^{\ z} \tau_j^{\ z} x_i x_j |0\rangle \quad . \tag{3A-22}$$

This sum is sensitive to position and isospin correlation among the nucleons in the ground state generated by the residual interaction among nucleon pairs

$$V^{res}(\vec{r}_1, \vec{r}_2) = V(\vec{r}_1, \vec{r}_2) - V(\vec{r}_1) - V(\vec{r}_2) \tag{3A-23}$$

where $V(\vec{r})$ is the average one-body potential well (e.g., a harmonic oscillator or Woods-Saxon radial dependence). In general one expects $\sigma_{-1} \sim A^{4/3}$ and the data bears this out. The ratio $\sigma_{-1}/A^{4/3}$ is twice that predicted by the oscillator model without residual interactions.

The n = +1 sum is readily evaluated by using the $\hat{\varepsilon}\cdot p$ form of the dipole operator

$$\sigma_{+1} \equiv \int_0^\infty \omega\, \sigma_\gamma(\omega) d\omega = \frac{\pi^2 e^2}{M^2} \langle 0 | \sum_{ij} \tau_i^z \tau_j^z\, p_i^x p_j^x | 0 \rangle \qquad (3A\text{-}24)$$

which is seen to be the momentum analog of σ_{-1}. High energy photon absorption cross sections fall off as $\omega^{-3/2}$ (deuteron-like) so the integral is not convergent (neither is that of a Lorentz line). This fact limits the usefulness of σ_{+1}.

The dipole cross section can be split up among final state channels that differ in J_f or T_f (for nuclear ground states having J_i or $T_i \neq 0$). Sum rules relating the σ_{-1} moments of different channels have been derived (Ha72). For nuclei with $T_z = \frac{1}{2}$ the dipole resonance may have $T_f = \frac{1}{2}$ or 3/2. As will be discussed in the next chapter the location of the $T_f = \frac{1}{2}$, E1 resonance is at a lower energy than the T_f = 3/2 resonance. A relation exists between the σ_{-1} moments of these two channels and the body radii,

$$\sigma_{-1}\left(\frac{1}{2}\right) - \frac{1}{2}\,\sigma_{-1}\left(\frac{3}{2}\right) = \frac{1}{3}\,\pi^2 e^2\,(N R_n^{\,2} - Z\, R_p^{\,2}) \qquad (3A\text{-}25)$$

where $R^2_{n,p}$ are the mean square radii of the neutrons or protons in the nuclear ground state. This relation (and a more complex one for $T_z > \frac{1}{2}$) is a useful boundary condition for identifying isospin split E1 giant resonances.

2. Siegert's Theorem

The replacement of the current operator by the position operator, $\vec{p}/M \rightarrow i\omega\vec{r}$ for the electric-dipole operator in the long wavelength approximation can also be made for other electric multipoles. This leads to the relation

$$|\langle J_f \| \mathcal{E}^{[J]} \| J_i \rangle|^2 = \left(\frac{J+1}{J}\right) |\langle J_f \| C^{[J]} \| J_i \rangle|^2 \qquad (3A\text{-}26)$$

for $k = \omega$, $\omega R << 1, J>0$. The ability to replace the current density by the charge density is known as Siegert's theorem (Si37). Although the true operator for real photons is $\mathcal{E}^{[J]}$, the more mathematically tractable Coulomb operator $C^{[J]}$ is useful for model calculations and sum rule derivations. In the long wavelength approximation the single particle multipole operator is

$$C_0^{[J]} = \frac{(-1)^J \hat{J} (4\pi)^{\frac{1}{2}}}{(2J+1)!!} \sum_j e_j (kr_j)^J Y_{J,0}(\hat{r}_j) \quad . \qquad (3A\text{-}27)$$

The derivation of eq. (3A-26) follows from replacing the momentum term in

$$e^{i\vec{k}\cdot\vec{r}} p_\lambda = \sum_L (i)^L \hat{L} j_L(kr) Y_{L,0}(\hat{r}) p_\lambda \qquad (3A\text{-}28)$$

with

$$p_\lambda/M = \hat{\varepsilon}\cdot\vec{p}/M \rightarrow i\,\omega\hat{\varepsilon}\cdot\vec{r} = i\omega r \left(\frac{8\pi}{3}\right)^{\frac{1}{2}} Y_{1,\lambda}(\hat{r}) \ . \qquad (3A\text{-}29)$$

The two spherical harmonics are combined to one $Y_{J,\lambda}$ by the addition theorem and the $L = J-1$ component selected as the lowest order term in k. After the polarization average eq. (3A-26) results.

The content of Siegert's theorem is that by current conservation each transition current is accompanied by a transition charge density given by $\vec{k}\cdot\vec{j}(\vec{r}) = -\omega\rho(r)$ and one can compute either transition amplitude to compare with real photon data. An advantage to using the charge density operator rather than the current density is that the matrix elements of $\rho(\vec{r})$ are not affected by meson currents in the nucleus. Mesons are pictured as moving instantaneously in the nonrelativistic approximation so that charge always resides on the nucleons, but meson currents exist between nucleons.

3. Other Electric Sum Rules

The reduced transition probability of a multipole transition is often defined as

$$B(CJ,\omega) = \frac{1}{2J_i+1} \sum_{M_f,M_i} |\langle J_f M_f| \sum_j e_j r_j^J \; Y_{JM}(\hat{r}_j) | J_i M_i \rangle|^2$$

$$= \frac{1}{2J_i+1} |\langle J_f || \sum_j e_j r_j^J \; Y_J || J_i \rangle^2 \; . \qquad (3A\text{-}30)$$

The photon absorption cross section (integrated over a discrete level) is then given by

$$\sigma(\omega) = (2\pi)^3 e^2 \omega^{2J-1} \left[\frac{(J+1)}{J} \; \frac{1}{[(2J+1)!!]^2} \right] B(CJ,\omega) \qquad (3A\text{-}31)$$

Sum rules are formed by taking various energy moments of $\sigma(\omega)$. The relation (for $J_i = 0$)

$$\sum_f \omega \; B(CJ,\omega) = \frac{1}{2} \langle 0 | \sum_j [e_j r_j^J Y^*_{JM}, [H, e_j r_j^J Y_{JM}]] | 0 \rangle \qquad (3A\text{-}32)$$

follows from closure on eq. (3A-30). Using the identity

$$\sum_M [r^J Y^*_{JM}, \; [p^2, r^J Y_{JM}]] = \frac{J(2J+1)^2}{2\pi} \; r^{2J-2} \qquad (3A\text{-}33)$$

the kinetic energy part of the double commutator is evaluated leading to a TRK-like energy weighted sum rule

$$\sum_f \omega \; B(CJ,\omega) = \frac{J(2J+1)^2}{4\pi} \; \frac{Z}{2M} \; r^{2J-2} \qquad (3A\text{-}34)$$

or

$$\sigma_o \; (EJ) = \int \sigma_{EJ}(\omega) d\omega \cong \frac{(J+1)}{[(2J-1)!!]^2} \; \frac{\pi^2 e^2}{M} \; Z \; (\omega_R r)^{2J-2} \qquad (3A\text{-}35)$$

where ω_R is the resonance energy. This result does not include contributions from the potential.

The sum in eq. (3A-34) can be divided into a part from the isoscalar part of the transition operator and the isovector part (for a $T_z = 0$ ground state). For example the E2 sum rule for photon absorption reads

$$\sigma_{-2}\,(E2) = \int_0^\infty \frac{\sigma(E2)}{\omega^2}\, d\omega = \frac{\pi^2 e^2}{3M}\left[\frac{Z^2}{A} + \frac{NZ}{A}\right] r^2 \qquad \text{(3A-36)}$$

where the terms in the bracket are the isoscalar and isovector contributions respectively.

4. Magnetic Sum Rules

The magnetic dipole operator in the long wavelength limit

$$\mathcal{M} = i\, \hat{\varepsilon}\cdot \sum_j \left(\frac{\mu_j \vec{\sigma}_j}{2M} + e_j \vec{\ell}_j\right) \qquad \text{(3A-37)}$$

has a spin and an orbital contribution. The M1 cross section for photon absorption is

$$\sigma_{M1}(\omega) = 4\pi^2 e^2\, \omega\, \left|\langle f|\mathcal{M}|i\rangle\right|^2\, \frac{dn}{d\omega}\,. \qquad \text{(3A-38)}$$

As with the electric dipole, closure leads to (Ku 63)

$$\sigma_0(M1) = \int_0^\infty \sigma_{M1}(\omega)d\omega = 2\pi^2 e^2 \langle 0|[\mathcal{M}^+,[H,\mathcal{M}]]|0\rangle\,. \qquad \text{(3A-39)}$$

In contrast to $\sigma_0(E1)$ however the kinetic energy part of H commutes with the M1 operator because there is no spatial dependence to $\mathcal{M}$. This feature makes the $\sigma_0(M1)$ sum rule more model dependent than the $\sigma_0(E1)$ rule.

The potential terms which do not commute with $\mathcal{M}$ are the NN spin-dependent terms and the single particle spin-orbit term

$$V_s\, \vec{\sigma}_j\cdot\vec{\sigma}_k + V_{so}\, \vec{\ell}_j\cdot\vec{\sigma}_j\,. \qquad \text{(3A-40)}$$

The spin-orbit term is usually the larger and contributes to σ_o (M1) mainly through isovector transition as

$$V_{so}\ (\mu_p-\mu_n-\tfrac{1}{2})^2\ \langle 0|\sum_j \ell_j\cdot\sigma_j|0\rangle \qquad \text{(3A-41)}$$

since $(\mu_p+\mu_n)/(\mu_p-\mu_n) = 0.19$. This term has its maximum value for nuclei which have within-orbit spin-flip transitions $\ell,j\to\ell,j-1$. Closed shell nuclei with both spin-orbit partners have $\langle 0|\sum_j \vec{\ell}_j\cdot\vec{\sigma}_j|0\rangle = 0$.

5. Electron Scattering Closure Rules

Rules for the integrals of $F_L^2(k,\omega)$ and $F_T^2(k,\omega)$ over ω are a continuation of the real photon results into the region of virtual photons. Since two dynamical variables k and ω are involved, one must choose a path of integration in the (k,ω) plane. Here we discuss $\int d\omega$ with k constant.

When closure is applied to the electron scattering cross section eq. (2B-3) at constant k one obtains

$$\int_o^k \frac{d^2\sigma}{d\omega d\Omega}\bigg|_k \frac{d\omega}{\sigma_o} = \sum_f \Bigg\{\left(1-\frac{\omega^2}{k^2}\right)^2 \left|\langle f|\sum_j e_j\, e^{\vec{k}\cdot\vec{r}_j} + DF|0\rangle\right|^2$$

$$+\left(\frac{1}{2}\left(1-\frac{\omega^2}{k^2}\right)+\tan^2\frac{\theta}{2}\right)\sum_\lambda \left|\langle f|\sum_j e_j\, (p_j)_\lambda\, e^{\vec{k}\cdot\vec{r}_j}|0\rangle\right.$$

$$\left. + i\,\langle f|\sum_j \frac{\mu_j}{2M}\,(\vec{k}\times\vec{\sigma}_j)_\lambda\, e^{\vec{k}\cdot\vec{r}_j}|0\rangle\right|^2\Bigg\}$$

$$\equiv F_L^2\,(k^2) + (\tfrac{1}{2}+\tan^2\tfrac{\theta}{2})\,[F_{T,\,p}^2(k^2) + F_{T,\,\sigma}^2(k^2)] \qquad \text{(3A-42)}$$

where DF is the Darwin-Foldy term from eq. (2B-5). Note the sum includes elastic scattering. The sum over final states of the form factors is facilitated, without much loss in accuracy, by neglecting the terms in

ω^2/k^2 or by using an average $<\omega^2>$. This step is justified because the matrix elements are usually small when $\omega^2/k^2 = 1$. The sum rule is expected to hold in the range of momentum transfer 0.25 $fm^{-1} < k < 2.5\ fm^{-1}$. The lower limit is determined by the minimum excitation energy ($\omega \cong k$) for which one has a fairly complete set of final states available ($\cong$ 50 MeV). The upper limit follows from the fact that the interaction operator $j_\mu A_\mu$ is correct only through order k^2/M^2. With these approximations closure gives

$$F_L^2\,(k^2) = <0\Big|\sum_{j,k}^{A} \left(e_j + \frac{k^2}{8M^2}(e_j - 2\mu_j)\right)\left(e_k + \frac{k^2}{8M^2}(e_k - 2\mu_k)\right) e^{i\vec{k}\cdot(\vec{r}_j - \vec{r}_k)}\Big|0> \quad ,$$

$$F_{T,p}^2(k^2) = <0\Big|\frac{2}{M^2}\sum_{j,k}^{A} e_j e_k p_j^x p_k^x\, e^{i\vec{k}\cdot(\vec{r}_j - \vec{r}_k)}\Big|0> \quad ,$$

$$F_{T,\sigma}^2(k^2) = <0\Big|\frac{k^2}{2M}\sum_{j,k}^{A} \mu_j \mu_k \sigma_j^x \sigma_k^x\, e^{i\vec{k}\cdot(\vec{r}_j - \vec{r}_k)}\Big|0> \quad . \qquad (3A-43)$$

The z-axis has been taken in the direction of k, and the x and y matrix elements in the transverse direction have been taken equal (Dr58, Mc62, Cz67, Oc70).

The charge sum has a particularly simple form (neglecting for simplicity the k^2/M^2 terms).

$$F_L^2\,(k^2) = Z + Z\,(Z-1)\,f_2\,(k^2) \qquad (3A-44)$$

where

$$Z(Z-1)\,f_2\,(k^2) = \sum_{j\neq k} <0\big|e_j e_k\, e^{i\vec{k}\cdot(\vec{r}_j - \vec{r}_k)}\big|0> \qquad (3A-45)$$

is the proton pair-correlation form factor. This function is constrained by

$$f_2(0) = 1 \ , \ f_2(\infty) = 0 \ . \qquad (3A\text{-}46)$$

$f_2(k^2)$ modulates the total charge scattering between the coherent (z^2) and incoherent (z) limits.

The transverse current and moment sums are

$$F^2_{T,p}\,(k^2) = \frac{2}{3}\, Z\, \frac{\langle p^2\rangle}{M^2} + \frac{2Z(Z-1)}{M^2}\, f_2{}^{p}(k^2) \qquad (3A\text{-}47)$$

$$F^2_{T,\sigma}(k^2) = \frac{k^2}{2M^2}\,[Z\,\mu_p{}^2 + N\,\mu_n{}^2 + A(A-1)\,f_2{}^{\sigma}(k^2)] \qquad (3A\text{-}48)$$

where $f_2{}^{p}$ and $f_2{}^{\sigma}$ are similar to the charge function f_2 with $e_j e_k$ replaced by $e_j p_j{}^x e_k p_k{}^x$ and $\mu_j \sigma_j{}^x \mu_k \sigma_k{}^x$, respectively. At their moment limits the F^2 tend toward their coherent and incoherent values as shown in Table 3-1.

An energy weighted sum rule, for inelastic scattering only, can be formed. The Coulomb part is

$$\int_0^k \frac{d^2\sigma}{d\omega d\Omega}\bigg|_k \frac{\omega d\omega}{\sigma_o} = \sum_f \omega\, \Big|\langle f|\sum_j e_j\, e^{i\vec{k}\cdot\vec{r}_j}|0\rangle\Big|^2 \ . \qquad (3A\text{-}49)$$

For the kinetic energy term

$$\int_0^k \frac{d^2\sigma}{d\omega d\Omega}\bigg|_k \frac{\omega d\omega}{\sigma_o} = Z\,\frac{k^2}{M}\,\frac{(A-1)}{A}\left[1 - \frac{Z-1}{A-1}\, f_2(k^2)\right] . \qquad (3A\text{-}50)$$

This expression is the analog of the TRK sum for real photons and reduces to $k^2\sigma_o{}^{TRK}/2\pi^2e^2$ in the $k\to0$ limit. The enhancement factor $(1 + \kappa)$ of real photon reactions will apply to electron scattering at low k, but will approach unity at high k as the coherence disappears (To80).

B. DISPERSION RELATION SUM RULES

A second method for deriving integrals over the photoabsorption cross section uses the mathematical properties of a real photon's forward scattering amplitude $f(\omega)$ in the lab system. The absolute square of this complex function of photon energy is the zero degree elastic photon scattering cross section

$$\left.\frac{d\sigma_{el}}{d\Omega}\right|_{0^\circ} = \left|f(\omega)\right|^2 = (\mathrm{Re}\ f)^2 + (\mathrm{Im}\ f)^2. \qquad (3B\text{-}1)$$

The imaginary part of f is related to the total absorption cross* section through the optical theorem

$$\mathrm{Im}\ f(\omega) = \frac{\omega\ \sigma\ (\omega)}{4\pi} \qquad (3B\text{-}2)$$

which states that the depletion of the incident photon wave $\exp(i\vec{k}\cdot\vec{r})$, due to absorption by the target nucleus, is accounted for by an out-of-phase (therefore imaginary) forward scattering wave. The total wave function is

$$\exp(i\vec{k}\cdot\vec{r}) + f(\theta)\ \exp(ikr)/r\ . \qquad (3B\text{-}3)$$

The dispersion relation expresses Re f in terms of an integral over Im f. Thus, given the total absorption cross section $\sigma(\omega)$, we can calculate the differential elastic scattering cross section in the forward direction at all energies. To find $d\sigma/d\Omega$ at other angles one needs further information about the multipolarity of the absorption and its related scattering process.

The derivation of the dispersion relation between Re f and Im f starts by allowing the photon energy to become complex (although all final results will be for real positive values of ω). Cauchy's theorem from function theory states that the real and imaginary parts of a normalizable function with no singularities

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*Total absorption includes elastic scattering.

in the upper half plane of its complex argument are related through a principal value integral*

$$\mathrm{Re}\ f(\omega) = \frac{1}{\pi}\ P \int_{-\infty}^{+\infty} \frac{\mathrm{Im}\ f(\omega')\ d\omega'}{\omega' - \omega} . \qquad (3B\text{-}4)$$

Our scattering amplitude $f(\omega)$ satisfies this criterion of having no singularities (poles or cuts) for $\mathrm{Im}\ \omega > 0$. This follows from the principle of causality which states that a scattered wave cannot propagate faster than the speed of light. In reference (To56) this is shown to be equivalent to the statement that $f(\mathrm{Im}\ \omega > 0)$ has no singularities.

Two steps are needed to put eq. (3B-4) in a practical form: the negative frequencies must be eliminated, and the integral must be convergent when $\mathrm{Im}f(\omega')$ is replaced by $\omega'\sigma(\omega')$ using the optical theorem (3B-2). For zero-spin nuclei $f(-\omega) = f^*(\omega)$, i.e., the imaginary part changes sign for negative frequencies, but the real part does not (by time reversal arguments). The amplitude for spin one-half targets, such as the proton and neutron, has two components, the non-spin flip f_1 and the spin flip f_2,

$$f(\omega) = \langle\chi_f \left| \hat{\varepsilon}_2^{\,*}\cdot\hat{\varepsilon}_1\ f_1(\omega) + i\vec{\sigma}\cdot(\hat{\varepsilon}_2^{\,*} \times \hat{\varepsilon}_1)\ f_2(\omega)\right| \chi_i\rangle \qquad (3B\text{-}5)$$

where $\hat{\varepsilon}_1$ and $\hat{\varepsilon}_2$ are the polarization vectors of the incident and scattered photons. The operator $\vec{\sigma}$ is evaluated between the initial and final target spin states $\chi_{i,f}$. The forward elastic scattering cross section averaged over photon and target polarizations is given by

$$\left.\frac{d\sigma_{el}}{d\Omega}\right|_{0°} = |f_1|^2 + |f_2|^2 . \qquad (3B\text{-}6)$$

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*Recall $P\ (\frac{1}{x}) = \frac{1}{x+i\varepsilon} + i\pi\delta(x)$ so that poles on the real axis are passed in the upper complex plane.

f_1 and f_2 satisfy the following crossing relations,

$$f_1(-\omega) = f_1^*(\omega)\ ,\ f_2(-\omega) = -f_2^*(\omega) \tag{3B-7}$$

and optical theorems

$$\text{Im } f_1 = \frac{\omega}{4\pi}(\sigma_a + \sigma_p)/2 = \frac{\omega}{4\pi}\sigma_t \tag{3B-8}$$

$$\text{Im } f_2 = \frac{\omega}{4\pi}(\sigma_a - \sigma_p)/2 \tag{3B-9}$$

where σ_a and σ_p are the total absorption cross sections for photon and target helicities antiparallel and parallel and σ_t is the total cross section for unpolarized photons. We shall mainly be interested in the spin-averaged forward amplitude f_1 (hereafter called f) although f_2 is needed to construct the scattering cross section.

The negative frequencies of (3B-4) are eliminated by using Im $f(-\omega) = -\text{Im } f(\omega)$ to obtain

$$\text{Re } f(\omega) = \frac{2}{\pi} P\int_0^\infty \frac{\omega' \text{ Im } f(\omega')\, d\omega'}{\omega'^2 - \omega^2}\ . \tag{3B-10}$$

It is known from measurements of σ on nucleons and nuclei at $\omega > 2$ GeV that σ is constant or a slowly rising function of photon energy so that integrals of the form $\int_0^\infty \sigma(\omega)\, d\omega/\omega^n$ must have $n>1$ to be convergent. This can be achieved by evaluating the disperson relation (3B-10) at a fixed value of ω for which Re $f(\omega)$ is known. For example, the scattering amplitude at zero frequency has the well known real Thomson limit equal to $-(\text{charge})^2/\text{mass}$. For a nucleus

$$f(o) = -\frac{(Ze)^2}{AM} = \frac{2}{\pi} P\int_0^\infty \frac{\text{Im } f(\omega')\, d\omega'}{\omega'}\ . \tag{3B-11}$$

If (3B-11) is subtracted from (3B-10)

$$\text{Re } f(\omega) - \text{Re } f(o) = \frac{2\omega^2}{\pi} P \int_0^\infty \frac{\text{Im } f(\omega')\, d\omega'}{\omega'(\omega'^2 - \omega^2)}$$

$$= \frac{\omega^2}{2\pi^2} P \int_0^\infty \frac{\sigma(\omega') - \sigma(o)}{\omega'^2 - \omega^2}\, d\omega' \quad . \qquad (3B\text{-}12)$$

Equation (3B-12) is now in a practical form for application to photoabsorption data at finite ω since $\int_0^\infty \sigma(\omega)\, d\omega/\omega^2$ is convergent.

The nucleon spin-flip amplitude f_2 is thought to satisfy an unsubtracted dispersion relation. The low energy limit of f_2 for nucleons is

$$\lim_{\omega \to o} \text{Re } f_2(\omega)/\omega = - (e^2/M)\ (\kappa^2/2M) \qquad (3B\text{-}13)$$

where κ is the anomalous magnetic moment.

In reference (Ma74) eight sum rules are derived for elastic scattering and photon absorption for the nucleon and J=0,½ nuclei. We discuss here a few of the more useful expressions. In the previous section on closure sum rules the static polarizability α was introduced (induced dipole moment divided by the field strength). The magnetic polarizability α_m is given by the low energy limit of f_2, the spin-flip amplitude (3B-13), whereas the electric polarizability α_e is a coefficient in the expansion of the spin averaged amplitude

$$\text{Re } f(\omega) = f(o) + \alpha_e\, \omega^2 + O(\omega^4) \quad . \qquad (3B\text{-}14)$$

Taking the $\omega \to 0$ limit of (3B-12) the following sum rule for the electric polarizability is obtained

$$\int_0^\infty \frac{\sigma(\omega) - \sigma(o)}{\omega^2}\, d\omega = 2\pi^2\, \alpha_e \ . \qquad (3B\text{-}15)$$

In the past the assumption was made that Re $f(\infty)$ for a nucleus was Z times the Thomson amplitude of the proton (Ge54). (3B-12) would then lead to the TRK sum rule (3A-12). However, we now know that this assumption is incorrect for reasons discussed in Chapter VII. Equation (3B-12) may be used to predict the spin-averaged forward elastic scattering cross section in the giant resonance region.

A useful model that leads to an analytic expression for Re $f(\omega)$ is the classical absorption and scattering of an electromagnetic wave on a damped harmonic oscillator. The absorption cross section is the Lorentz expression

$$\sigma(\omega) = \sigma_0 \frac{\omega^2 \Gamma^2}{(\omega^2-\omega_0^2)^2 + \omega^2 \Gamma^2} \qquad (3B\text{-}16)$$

where σ_0 is the absorption cross section at the resonant energy ω_0 and $\Gamma \ll \omega_0$ is the width. The corresponding scattering amplitude* is

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*Note this form of $f(\omega)$ appears to have poles in the upper complex energy plane ($\omega = \omega_0 + i\Gamma/2$) which violates causality. However, this expression comes from combining the contributions of two scattering diagrams (see fig. 2A-2b &c), whose poles are in the lower plane by

$$f(\omega) \sim \frac{1}{\omega - (-\omega_0 - i\frac{\Gamma}{2})} - \frac{1}{\omega - (\omega_0 - i\frac{\Gamma}{2})}$$

Using only the first term leads to a Breit-Wigner resonance form.

$$\mathrm{Im}\ f(\omega) = \frac{\sigma_o}{4\pi} \frac{\omega^3 \Gamma^2}{(\omega^2-\omega_o^2)^2 + \omega^2\Gamma^2} \tag{3B-17}$$

$$\mathrm{Re}\ f(\omega) = \frac{\sigma_o}{4\pi} \frac{\omega^2(\omega_o^2-\omega^2)\Gamma}{(\omega^2-\omega_o^2)^2 + \omega^2\Gamma^2} + \text{Thomson} . \tag{3B-18}$$

In the long wavelength approximation (for a J = 0 nucleus)

$$\frac{d\sigma_{el}}{d\Omega} = \left|f(\omega,\theta)\right|^2 = \left\{ \left(\frac{\sigma_o}{4\pi}\right)^2 \frac{\omega^4\Gamma^2}{(\omega^2-\omega_o^2)^2 + \omega^2\Gamma^2} + \frac{2\sigma_o}{4\pi} \frac{\omega^2(\omega_o^2-\omega^2)\ \Gamma\ \mathrm{Re}\ f(o)}{(\omega^2-\omega_o^2)^2 + \omega^2\Gamma^2} + (\mathrm{Re}\ f(o))^2 \right\} \left(\frac{1+\cos^2\theta}{2}\right) \tag{3B-19}$$

with $\mathrm{Re}\ f(o) = -(Ze)^2/AM$. Note the scattering cross section becomes constant at $\omega \gg \omega_o$.

Im f depends on σ at one energy, but Re f depends on an integral over $\sigma(\omega)$ at all energies. The reason is Im f comes from processes in which the intermediate state satisfies energy conservation, but Re f comes from transitions between nuclear states whose energy difference is not equal to ω. These states contribute to absorption at frequencies other than ω. The integral sums these contributions to Re $f(\omega)$.

C. MOMENTUM TRANSFER SUM RULES

A second type of electron scattering sum rule involves the integration of form factors over momentum transfer (Oc75). One such rule relates the total volume under the Coulomb form factor surface (elastic plus inelastic) to the nuclear Coulomb energy E_c

$$\int\left[\int_0^\infty \left|F_c(k,\omega)\right|^2 d\omega - (Z\ f_p{}^2(k) + N\ f_n{}^2(k))\right]\frac{d^3k}{4\pi^2k^2} = \frac{\pi}{e^2}E_c \qquad (3C\text{-}1)$$

where $f_{p,n}(k)$ are the nucleon charge form factors and

$$E_c = e^2\langle 0|\sum_{j>k}\int \frac{f_j(\vec{r}-\vec{r}_i)f_k(\vec{r}-\vec{r}_j)}{|\vec{r}-\vec{r}'|}d^3rd^3r'|0\rangle \qquad (3C\text{-}2)$$

is the Coulomb energy of interacting nucleons in the nuclear ground state. An alternate statement of this rule is that the momentum integral of the proton pair correlation function is

$$Z\ (\ Z - 1)\int f_2\ (k)\ \frac{d^3k}{4\pi k^2} = \frac{\pi}{e^2}E_c\ .$$

Another simple rule relates (in plane wave approximation) the momentum integral of the elastic form factor to the central charge density of the nucleus

$$\int F_c(k)\ d^3k = (2\pi)^3\ \rho(o). \qquad (3C\text{-}3)$$

For inelastic Coulomb form factors from a spin-zero ground state,

$$F_J(k) = (4\pi)^{\frac{1}{2}}\ \hat{J}\int_0^\infty \rho_J(r)\ j_J(kr)r^2dr \quad , \qquad (3C\text{-}4)$$

a number of sum rules can be formed using the identity

$$\int_0^\infty k^n\ j_J(kr)\ dk = \frac{C_{n,J}}{r^{n+1}} \qquad (3C\text{-}5)$$

where

$$C_{n,J} \equiv \pi^{\frac{1}{2}}\ 2^{n-1}\ \frac{\Gamma(\frac{1}{2}(J+n+1))}{\Gamma(\frac{1}{2}(J-n+2))} \qquad (3C\text{-}6)$$

for $-(J+1) < n < J+2$, $J = +$ integer, n continuous. The k^n moment of the transition form factor can be related to $r^{-(n+1)}$ moments of the elastic or transition charge density

$$\int_0^\infty k^n F_J(k)dk = (4\pi)^{\frac{1}{2}} \hat{J} C_{n,J} \int_0^\infty \frac{\rho_J(r)r^2dr}{r^{n+1}}$$

$$= (4\pi)^{\frac{1}{2}} \hat{J} C_{n,J} \overline{r^{-(n+1)}} \quad . \qquad (3C-7)$$

For example, the r^{-1} and r^{-2} moments can be formed for elastic scattering and monopole transitions.

The n = -1 moments are particularly interesting

$$\int_0^\infty \frac{F_J(k)}{k} dk = (4\pi)^{\frac{1}{2}} \hat{J} C_{-1,J} \int_0^\infty \rho_J(r)r^2dr \ . \qquad (3C-8)$$

This sum rule applies to $J \geqq 1$ transitions. (For inelastic monopole transitions $\int_0^\infty \rho_0(r)r^2dr = 0$.) The first few coefficients are:

$C_{-1,1} = \pi/4$, $C_{-1,2} = \pi/3$, $C_{-1,3} = \pi/16$. The integral on the right of eq. (3C-8) represents the net amount of nuclear charge which participated in the transition. If we define a transition charge Q_J as

$$Q_J \equiv e(4\pi)^{\frac{1}{2}} \int_0^\infty \rho_J(r)r^2dr$$

then

$$Q_J = \frac{e}{C_{-1,J}\hat{J}} \int_0^\infty \frac{F_J(k)}{k} dk \quad . \qquad (3C-9)$$

The net charge change Q_J is a measure of the strength of the transition. The restriction "net charge" is necessary since $\rho_J(r)$ can have positive and negative regions.

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Table 3-1. Momentum limits of the total form factors.

k	Charge	Current	Moment
0	Z^2	$\frac{\sigma_1(E1)}{2\,\pi^2 e^2}$	$\frac{k^2}{2M^2}\left[\frac{J+1}{3J}\,\mu_A{}^2 + \frac{\sigma^V_{-1}(M1)}{\pi^2 e^2/M^2}\right]$
∞	Z	$\frac{2}{3}\,Z\,\frac{\langle p^2\rangle}{M^2}$	$\frac{k^2}{2\,M^2}\,(Z\mu_p{}^2 + N\mu_n{}^2)$

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