COMPUTATION OF ANTENNA SIDE-LOBE COUPLING IN THE NEAR FIELD USING APPROXIMATE FAR-FIELD DATA

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Glossary of Symbols for The Text

a: radius of the antenna aperture.
a_T, a_R: radius of the transmitting and receiving antennas, respectively.
a_0: incident wave-amplitude in the transmitting antenna feed.
A_MAX: the maximum amplitude of the far field in the direction of the axis of separation.
A_TMAX, A_RMAX: the maximum amplitude of the far field in the direction of the axis of separation for the transmitting and receiving antennas, respectively.
b'_0: the emergent wave-amplitude in the receiving antenna feed.
C': \[ \frac{1}{n Z (1 - r_1 r_2)} \] .
d: the separation distance between antennas.
D_T, D_R: the diameter of the smallest sphere which circumscribes the transmitting and receiving antennas, respectively.
E: the electric field.
f'_T, f'_R: the far field of the receiving and transmitting antennas, respectively, with components f'_x, f'_y, f'_z and f_x, f_y, f_z.
G: the antenna gain.
G_T, G_R: the gain of the transmitting and receiving antennas, respectively.
J_0: the Bessel function of first order.
k: the propagation vector with components k_x, k_y, k_z.
k_k = \frac{2\pi}{\lambda} .
k_0: k(D_T + D_R)/2d.
K: k\hat{e}_x + k\hat{e}_y .
K: \frac{rK \cdot K}{|rK|^2} .
\text{input}: the input power to the antenna.
\text{position vector with components } x, y, z.
\text{magnitude of } r.
R: \hat{x} \hat{e}_x + \hat{y} \hat{e}_y .
S: the relative side-lobe level in the direction of the separation axis.
S_T, S_R: the value of S for the transmitting and receiving antennas, respectively.
Z_0: the wave impedance of free space.
\gamma: the z-component of k.
\Gamma_L: the reflection coefficient of the receiving load.
\Gamma_0: the reflection coefficient of the receiving antenna.
\eta_0: the characteristic admittance for the propagated mode in the waveguide feed of the receiving antenna.
\lambda: the wavelength.
\phi_T, \theta_T, \psi_T: the Eulerian angles of the transmitting antenna.
\phi_R, \theta_R, \psi_R: the Eulerian angles of the receiving antenna.
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Computer programs, in particular CUPLNF and CUPLZ, are presently in existence to calculate the coupling loss between two antennas provided that the amplitude and phase of the far field are available. However, for many antennas the complex far field is not known accurately. In such cases it is nevertheless possible to specify approximate far fields from a knowledge of the side-lobe level of each antenna along the axis of separation, and the electrical size of each antenna. To determine the effectiveness of using approximate side-lobe level data instead of the detailed far fields, we chose as our test antennas two hypothetical, linearly polarized, uniformly illuminated circular antennas for which the exact far fields are given by a simple analytic expression. The exact far fields are supplied to the program CUPLNF to compute the exact near-field coupling loss. Approximate fields are supplied to a new program ENVLP developed for the purpose of computing the approximate near-field coupling loss. The comparison of the results from ENVLP to those of CUPLNF indicates that the use of approximate far fields gives an estimate of the coupling loss which is good to about ±5 dB. In addition, the plane-wave transmission formula for coupling between two antennas is used to estimate upper-bound values of coupling loss. These upper bounds are compared with the maximum coupling losses obtained from programs CUPLNF and ENVLP.

Key words: antenna coupling; antenna theory; coupling loss; near-field measurements.
1. Introduction

Three years ago, the Antenna Systems Metrology Group, Electromagnetic Fields Division of the National Bureau of Standards, under the sponsorship of the Electromagnetic Compatibility Analysis Center, completed the development of a highly efficient computer program, CUPLNF, for calculating the coupling loss between two antennas given the far fields of each antenna [1,2]. For antennas arbitrarily oriented and separated in free space, CUPLNF computes the coupling loss at a single frequency as a function of antenna displacement in a plane transverse to the axis of separation of the antennas. Multiple reflections between antennas are neglected but no other restrictive assumptions are involved. CUPLNF automatically computes electric fields from the transmitting antenna when a "virtual antenna" of uniform far field replaces the receiving antenna. Coupling loss for antennas hundreds of wavelengths in diameter is computed in a few minutes of CPU time within the central memory core, e.g., of a CDC 6600.¹

The major limitation of CUPLNF is its requirement for the magnitude and phase of the electric far-field components of each antenna within the solid angle subtended by the antennas. For example, if two antennas are oriented so that coupling occurs mainly through their side-lobe region, the complex vector far field of each antenna covering this angular side-lobe region must be supplied to the program CUPLNF.

In practice, one may not have such detailed information on the side-lobe far fields to estimate the coupling loss between two antennas co-sited in their near field. Often, one has some knowledge of the side-lobe levels of one's antennas, even if detailed phase and amplitude information of the field components is not available. Thus a natural and important question to answer is whether this limited information, specifically side-lobe level of each antenna near their axis of separation, can be used to estimate the antenna coupling in the near field using a new program, ENVLP. Of course, as the separation distance between the antennas approaches the mutual Rayleigh distance, i.e., the antennas lie in each others far field, the ordinary far-field formula for coupling between two antennas can be used to estimate coupling loss. For separation distances much less than the mutual Rayleigh distance, the far-field formula would not be expected to give realistic estimates of coupling. It is in this near-field region that we set out to decide if ENVLP will give reasonable values of side-lobe coupling (±6 dB accuracy) by utilizing only side-lobe levels

¹The specific computer is identified in this paper to adequately describe the computer program. Such identification does not imply recommendation or endorsement by the National Bureau of Standards nor does it imply that the computer identified is necessarily the best available for the purpose.
along the axis of separation of the antennas (in addition to their size, separation distance, and feed characteristics).

To determine the efficacy of ENVLP using approximate side-lobe level information instead of the detailed far fields, we chose as our test antennas two hypothetical, linearly polarized, uniformly illuminated circular aperture antennas for which the exact far fields are given by a fairly simple analytic expression involving functions no more complicated than the first order Bessel function. With these hypothetical antennas, the exact far fields could be supplied to the program CUPLNF to compute near-field coupling loss without introducing the errors incumbent with measured far-field data. An approximate side-lobe far field can be substituted for the exact far field and a comparison of the near-field coupling loss computed with the exact and approximate far fields can be made simply and straightforwardly.

At first thought it may seem overly optimistic to want to obtain reasonable estimates of near-field coupling having only side-lobe level information, i.e., having only the envelope of the magnitude of the far fields in the direction of separation. Fortunately, however, we can also estimate the phase variation of the side lobes of most microwave antennas merely from the frequency of operation and the radii of the antennas, more precisely, from the radius in wavelengths of the smallest sphere which circumscribes all significantly radiating parts of each antenna.

Also, even though polarization characteristics of the side-lobe far fields may be unknown, we can assume polarization match of the far-fields of the antennas over the relevant range of integration. This assumption of polarization match tends to produce an upper-bound estimate of coupling loss, which may be of appreciable value to the user. We also assume the user has a knowledge of (1) the separation distance between the antennas, and (2) the reflection coefficients in the waveguides feeding the antennas—in all a rather minimal amount of information about one's antennas.

In Appendix B, we document program ENVLP, which is a modification of the program CUPLNF requiring as input this minimal amount of input data, i.e., the antennas' radii, separation distance, reflection coefficients, and relative side-lobe levels in the direction of separation.

Finally, we work directly with the plane-wave transmission formula for coupling between two antennas to estimate upper-bound values of coupling loss, comparing these upper bounds with the maximum coupling losses obtained from program CUPLNF and program ENVLP using exact and approximate far-field input. We anticipate these simple upper-bound formulas being especially valuable when the major requirement for co-sited antennas is that the coupling interference or fields lie below a certain threshold.
2. Side-Lobe Coupling of Two Antennas Using Their Exact 
and Approximate Far Fields

2.1 Statement and Formulation of the Problem

Yaghjian [1] showed that the coupling between two antennas in free space, 
neglecting multiple reflections, is given by

- \[
\frac{b'_0}{a_0} = -\frac{C'}{k} \int \int \frac{f'(-k) \cdot f(k) e^{i y d \cdot k \cdot R}}{k < \gamma} \, dk, \quad (1)
- \[
\text{where } C' = \frac{1}{n_o Z_0 (1 - \Gamma_0 \Gamma_L)}, \text{ and } 1/(1 - \Gamma_0 \Gamma_L) \text{ is the mismatch factor of the receiving }
\text{antenna, } n_o \text{ is the characteristic admittance of the propagated mode in the waveguide feed of the receiving }
\text{antenna, } Z_0 \text{ is the wave impedance of free space, } k \text{ is the}
\text{magnitude of the propagation vector, } K = k_x e_x + k_y e_y \text{ is the transverse part of the}
\text{propagation vector, } \gamma = (k^2 - k'^2)^{1/2}, \text{ } dk \text{ is an abbreviation for } dk_x dk_y, \text{ and the}
\text{coordinates } (R, z = d) \text{ give the position of the origin } O' \text{ fixed in the receiving}
\text{antenna with respect to the } (x, y, z) \text{ coordinate system fixed at } O \text{ in the transmitting}
\text{antenna (see fig. 1). The functions } f \text{ and } f' \text{ are the electric far fields of the}
\text{transmitting and receiving antennas, respectively, without the presence of the other,}
\Gamma_0 \text{ and } \Gamma_L \text{ are the reflection coefficients of the receiving antenna and receiving load}
\text{respectively, } a_0 \text{ is the amplitude of the input to the transmitting antenna, and } b'_0 \text{ is}
\text{the amplitude of the output of the receiving antenna. Formula (1) assumes the}
\text{receiving antenna is reciprocal, although the formula easily generalizes to include}
\text{non-reciprocal antennas. Yaghjian [1] also showed that for most practical purposes}
\text{the integration range in (1) could be limited to } K/k < (D_T + D_R)/2d \text{ for } R = 0 \text{ provided}
\text{(D_T + D_R)/2 < d < (D_T + D_R)^2/\lambda}. \text{ } D_T \text{ is the diameter of the smallest sphere which}
circumscribes the transmitting antenna and } D_R \text{ is the diameter of the smallest sphere}
circumscribing the receiving antenna. (If } D_T \text{ or } D_R \text{ is less than twice the wavelength,}
\text{ } \gamma, D_T, \text{ or } D_R \text{ is replaced by } 2\lambda). \text{ For coupling in the transverse plane over the range}
- (D_T + D_R) < R < D_T + D_R \text{ the integration range should be doubled [1].}

There are some situations where detailed information of the far fields (f', f) is
lacking and it is desirable to get an estimate of the coupling between two antennas.
For example, both amplitude and phase information may not be available. We would
like, therefore, to consider some possible methods of approximating the coupling and
in particular of finding a good estimate for the maximum coupling between two antennas
in the general direction of the separation axis (the separation axis is drawn from a
point centrally located on the transmitting antenna to a point centrally located on the receiving antenna for the antennas in their initial position, \( R = 0 \).

To find the maximum coupling in the general direction of the separation axis we shall calculate the coupling over a range of values of \( R \) (\( R \) is perpendicular to the separation axis). Specifically, we shall move the receiving antenna in the \( x \) and \( y \) directions, over a range from \( -(D_T + D_R) \) to \( +(D_T + D_R) \) for both the \( x \) and \( y \) directions. In order to save computer time and cost we will hold \( y = 0 \) while varying \( x \) (the \( XO \) cut) and hold \( x = 0 \) while varying \( y \) (the \( YO \) cut).

If we are to determine how accurate our approximate results for coupling loss are, we need to compare them to the results that we would have obtained if we knew and used the exact far fields of the antennas for finding the coupling quotients. For this purpose we will use two hypothetical circular antennas and compare results obtained using the exact far fields to those obtained using approximate far fields. These results will be compared for different values of the separation distance, diameters of the antennas, frequency, and orientation of the antennas.

2.2 The Hypothetical Circular Antenna

2.2.1 The Exact Far Field

We consider the case of a uniformly illuminated circular aperture with the transverse electric field polarized in the \( x \)-direction. Then the far field for a point in the \((\theta, \phi)\) direction is given by [3]:

\[
\begin{align*}
    f_x &= \frac{ka}{\sqrt{\pi}} \cos \theta \frac{J_1(ka \sin \theta)}{ka \sin \theta} \quad (2a) \\
    f_y &= 0 \quad (2b) \\
    f_z &= -\frac{ka}{\sqrt{\pi}} \sin \theta \cos \phi \frac{J_1(ka \sin \theta)}{ka \sin \theta} \quad (2c)
\end{align*}
\]

For this formula, the circular aperture lies in the \( xy \) plane with the center of the aperture at the origin. The symbols, \( \theta, \phi \) denote the usual angles in spherical coordinates, \( k \) is the magnitude of the propagation vector, \( a \) is the radius of the aperture, and \( J_1 \) is the Bessel function of first order. An example of this pattern can be seen in figure 2, where \( ka = 20.94 \).
2.2.2 The Approximate Far Field for Approximation-1

Approximation-1 is specifically geared to the uniformly illuminated circular aperture antenna. For this approximation we replace

\[ \frac{J_1(ka \sin \theta)}{ka \sin \theta} \text{ by } \frac{2}{\pi k a \sin \theta} \cos(ka(\theta - \theta_0)) \frac{ka \sin \theta}{\pi k a \sin \theta} , \]

where \( \theta_0 \) is the direction of the separation axis relative to the preferred antenna coordinate system. \( \theta_0 \) equals \( \theta_T \) for the transmitting antenna and \( \theta_R \) for the receiving antenna (see the sketches on figures 6 through 20). This electric field pattern can be found for \( \theta_0 = 60^\circ \) in figure 3. The values of the field are given relative to the main beam at \( \theta_0 = 0^\circ \) in figure 2, i.e. the field is normalized to a main beam equal to 1.

The above approximation may be justified as follows. \( \frac{J_1(ka \sin \theta)}{ka \sin \theta} \) may be replaced using the asymptotic expansion for \( J_1 \). This yields:

\[ \frac{J_1(ka \sin \theta)}{ka \sin \theta} \sim \frac{2}{\pi k a \sin \theta} \cos(ka \sin \theta - 3\pi/4) \frac{ka \sin \theta}{\pi k a \sin \theta} . \]

The cosine term represents the variation of the sidelobe while the rest of the term is the amplitude of the envelope. We replace the terms involving the amplitude of the envelope by their values at \( \theta_0 \) and expand the cosine term about \( \theta_0 \). Thus,

\[ \cos(ka \sin \theta - 3\pi/4) \sim \cos(k a \sin \theta_0 - 3\pi/4 + ka(\theta - \theta_0) \cos \theta_0) . \]

The \( \cos \theta_0 \) term can be lumped with \(-3\pi/4\) into a phase factor which we will ignore since we are finding only an average coupling over a solid angle. The \( \cos \theta_0 \) term has been replaced by 1 because actual antennas do not display this \( \cos \theta_0 \) dependence of sidelobe width peculiar to the hypothetical finite planar aperture distribution.

2.2.3 The Approximate Far Field for Approximation-2

The approximation-2 far field is a general approximation that can be made for a large class of coupling cases. For this approximation we replace \( f'(-k) \cdot f(k) \) by

\[ A_{T\text{MAX}} \cos k_A x_T \cos k_B y_T A_{R\text{MAX}} \cos k_A x_R \cos k_B y_R . \]

\( A_{T\text{MAX}} \) and \( A_{R\text{MAX}} \) are the maximum magnitudes of the far fields in the general direction of the separation axis (i.e., the side-lobe level) for the transmitting and receiving antennas, respectively, \( k_A = k \sin \theta \cos \phi \), and \( k_B = k \sin \theta \sin \phi \). Hence, \( A_{T\text{MAX}} \) and \( A_{R\text{MAX}} \) are the magnitudes of the approximate electric far fields in the direction of the
separation axis and are given in absolute SI (mksA) units. In Appendix A we derive $A_{T \text{MAX}}$ and $A_{R \text{MAX}}$ in terms of the side-lobe levels and the gains of the transmitting and receiving antennas. $a_T$ and $a_R$ are the radii of the smallest spheres circumscribing the transmitting antenna and the receiving antenna, respectively.\footnote{The far side-lobe region of the far-field pattern is predominantly caused by diffraction from edge points of the antenna. These edge points are usually separated by a distance of approximately an antenna diameter and this leads to a side-lobe pattern which has a null approximately every $\lambda/2a$ radians, as numerous hypothetical and measured far-field patterns confirm (see for example Johnson et al. [4], and Newell and Crawford [5]). We assume a smooth variation in the form of a cosine, and that the $k_x$ and $k_y$ variation is separable. This leads to the $\cos k_x a \cos k_y a$ dependence about the axis of separation - a fairly reasonable approximation provided the antenna is not highly elongated (i.e., the length in one direction is not much greater than the length in the other direction). In the case of a highly elongated antenna the value of $a$ is approximately half the longer side so that we obtain a variation which is good in the direction of the long side but poor in the direction of the short side. For such elongated antennas, a better approximation can be obtained by using a variation of the form $\cos k_x a_x \cos k_y a_y$, where $a_x$ is half the length of the long side and $a_y$ is half the length of the short side, assuming the $x$ and $y$ axes are aligned with the long and short sides of the antenna, respectively.}

We have chosen the above approximation because for many antennas the components of the far-field patterns vary roughly as $\cos k_x a \cos k_y a$, where $a$ is the radius of the smallest sphere circumscribing all the significantly radiating parts of the antenna.\footnote{Note that for approximation-2 we assume that the polarization of the receiving and transmitting antennas are matched and thus approximation-2 tends to yield an upper bound. The approximation-2 pattern for $\phi = 0^\circ$ or $90^\circ$ and the separation axis in the direction of $\theta = 60^\circ$ can be found in figure 4. The values of the field are given relative to the main beam at $\theta = 0^\circ$ in figure 2.} Note that for approximation-2 we assume that the polarization of the receiving and transmitting antennas are matched and thus approximation-2 tends to yield an upper bound. The approximation-2 pattern for $\phi = 0^\circ$ or $90^\circ$ and the separation axis in the direction of $\theta = 60^\circ$ can be found in figure 4. The values of the field are given relative to the main beam at $\theta = 0^\circ$ in figure 2.

2.2.4 Limitations on the Approximations

Both approximation-1 and approximation-2 are valid over a relatively narrow range of angles. In particular, the approximations should be valid (under the above stated limitations) if the amplitude of the envelope of the far field of each antenna varies by not more than about 3 dB over the range of angles mutually subtended by the transmitting and receiving antennas. Thus, for any case in which the integration of equation (1) must be performed over a large range of angles, approximation-1 and approximation-2 may be poor. This will, in general, include those cases involving coupling with the main beam. It will also include electrically small broad beam antennas and the unusual cases where both the receiving and transmitting antennas are identical and also have identical Eulerian angles [1] (see figure 5 for a definition of these angles). In addition, for $\theta_0 = 0^\circ$ the approximation of the far field for
approximation-1 goes to infinity. Thus, approximation-1 is never good for
the main beam. Finally, d must be in the range \( \frac{D_T + D_R}{2} < d < \frac{(D_T + D_R)^2}{\lambda} \).

2.3 Results and Comparisons

The results using the exact far fields and the approximation-1 and approximation-2
far fields in equation (1) were obtained using computer programs based on CUPLNF [2].
The program for approximation-2 (titled ENVLP) was written for general use and can be
found documented in Appendix B.

Results were obtained for various values of antenna orientation, antenna
diameter, separation distance and frequency, and are summarized in table 2.1 and in
figures 6 through 20.

Column 1 of table 2.1 is an identifier which corresponds to the identifier in
the caption of figures 6 through 20. Columns 2 to 4 give the Eulerian angles of the
transmitting antenna; columns 5 through 7 give the Eulerian angles of the receiving
antenna; columns 8 and 9 give the diameters of the transmitting and receiving anten-
as, respectively, in units of wavelength. Columns 10 and 11 give the side-lobe
levels relative to the main beam and within the integration range (as specified above)
for the transmitting and receiving antennas, respectively. Column 12 gives the
separation distance in terms of the mutual Rayleigh distance; column 13 gives the
frequency in hertz. Columns 14 through 16 give the maximum coupling amplitude as
given by the use of the exact far field, the approximation-1 far field, and the
approximation-2 far field, respectively. The maximum coupling magnitude implied by
the far-field formula is given in column 17.

The far-field formula which is used to determine the maximum coupling is

\[
\frac{b'_o}{a_o} = \frac{A_{MAX} A_{MAX}}{\eta Z_0 (1 - \Gamma \Gamma^*)} \frac{\lambda}{d}. \tag{3}
\]

Column 18 gives the maximum coupling amplitude as implied by the upper-bound
equations (9), which will be discussed in section 3. Finally, columns 19, 20, and 21
contain the RMS mean coupling for the exact, approximation-1, and approximation-2 far
fields, respectively.

Figures 6 through 20 are plots of the magnitude of the coupling quotients in the
x-direction at \( y = 0 \) (the XO cut) for the exact far fields (---), approximation-1 (----),
and approximation-2 (-----). The YO cut is not shown because it is similar to the XO
cut. Each figure corresponds to one of the cases in table 2.1.
<table>
<thead>
<tr>
<th>Case</th>
<th>$\theta_1$</th>
<th>$\theta_T$</th>
<th>$\psi_T$</th>
<th>$\theta_R$</th>
<th>$\psi_R$</th>
<th>$D_T$</th>
<th>$D_R$</th>
<th>$S_T$</th>
<th>$S_R$</th>
<th>$d$</th>
<th>Frequency</th>
<th>Exact</th>
<th>Approximation-1</th>
<th>Approximation-2</th>
<th>Maximum Coupling Magnitude</th>
<th>Field Formula</th>
<th>Equation (9)</th>
<th>RHS Mean Coupling Magnitude</th>
<th>Approximation-1</th>
<th>Approximation-2</th>
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<td>60</td>
<td>0</td>
<td>60</td>
<td>180</td>
<td>6.67</td>
<td>6.67</td>
<td>31.4</td>
<td>31.4</td>
<td>0.19</td>
<td>$1.0 \times 10^{10}$</td>
<td>-65.6</td>
<td>-66.6</td>
<td>-69.0</td>
<td>-62.4</td>
<td>-58.4(c)</td>
<td>-69.0</td>
<td>-71.2</td>
<td>-73.1</td>
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<tr>
<td>ALPHA-4</td>
<td>0</td>
<td>50</td>
<td>0</td>
<td>60</td>
<td>180</td>
<td>6.67</td>
<td>6.67</td>
<td>31.4</td>
<td>31.4</td>
<td>0.19</td>
<td>$1.0 \times 10^{10}$</td>
<td>-66.7</td>
<td>-66.0</td>
<td>-66.3</td>
<td>-59.7</td>
<td>-56.0(c)</td>
<td>-71.5</td>
<td>-69.6</td>
<td>-70.4</td>
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<tr>
<td>ALPHA-7</td>
<td>0</td>
<td>70</td>
<td>0</td>
<td>60</td>
<td>180</td>
<td>6.67</td>
<td>6.67</td>
<td>31.4</td>
<td>31.4</td>
<td>0.19</td>
<td>$1.0 \times 10^{10}$</td>
<td>-66.9</td>
<td>-67.7</td>
<td>-71.2</td>
<td>-64.6</td>
<td>-60.1(c)</td>
<td>-73.6</td>
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An inspection of table 2.1 will show that the maximum coupling predicted by approximation-1 compares favorably with the result given by the exact far fields. With the exception of case Omega, the difference in the maximum coupling loss between the exact far-field result and the result from approximation-1 is less than 5 dB while for case Omega, the difference is about 10 dB. The explanation for the large difference for case Omega is that the wavelength for this case is so large that the electric field pattern lies near the first null (see, e.g., figure 2); thus, approximation-1 which is based on the envelope of the Bessel function is very poor in this case.

We find that with the exception of two cases (Beta-3 and Delta-1) the maximum coupling loss predicted by approximation-2 is within 10 dB of that given by the use of the exact far fields. For Beta-3, \( D_T = 0.2\, \text{m}, D_R = 0.8\, \text{m}, \) and \( d = 1.0\, \text{m}; \) thus, for this case the coupling integration covers a broad angular region and the polarization does not match at the wider angles as it does at the center. This means that approximation-2, which assumes perfect polarization match over the entire range of integration, will appreciably overestimate the coupling for this case. In the case of Delta-1, even though the approximation-2 estimate of the maximum coupling is about 13 dB low, this is a relatively good estimate of coupling since the exact coupling lies below -116 dB.

If we now examine figures 6 through 20 and columns 19 through 21, we find that, in general, the RMS mean level of coupling generally agrees to within about ±5 dB of the exact for approximation-2 and considerably closer for approximation-1. On the other hand, approximate and exact coupling loss at individual points can differ by a substantially greater amount (occasionally more than 20 dB).

It is of interest to compare our results to those obtained from the far-field formula, equation (3). It can be seen that, in general, if the separation distance between the antennas is greater than about one-quarter of a mutual Rayleigh distance, \((D_T + D_R)^2/\lambda,\) the far-field formula gives results closer to the exact results than do approximation-1 and approximation-2. On the other hand, if the separation distance is less than one-tenth of a mutual Rayleigh distance, the results from approximation-1 and approximation-2 are in general substantially better than those given by the far-field formula. For separation distances between one-tenth and one-quarter of a mutual Rayleigh distance both the far-field formula and approximation-1 or approximation-2 yield values of coupling loss of about the same degree of accuracy.

2.4 Conclusions

We conclude that approximation-1 and approximation-2 can be used in the computer programs to obtain a reasonable estimate of the maximum coupling in the general direction of the separation axis. It should be emphasized that while we can use
approximation-1 and approximation-2 to find the maximum coupling or the mean coupling over a narrow range of directions in the general direction of the separation axis we do not necessarily get a good estimate in the exact direction of the separation axis. Approximation-1 is limited to uniformly illuminated circular aperture antennas while approximation-2 can be used for general antennas. In using either approximation-1 or approximation-2 care must be used so as to stay within the limitations stated in section 2.2.4.

In the event that the separation distance is greater than about one-quarter of a Rayleigh distance the far-field formula (equation (3)) gives better results than either approximation-1 or approximation-2 and the far-field formula should be used in those cases to estimate the maximum coupling.

The real advantage of the computer program for approximation-2 (Program ENVLP) is that it estimates the coupling loss between arbitrary antennas arbitrarily oriented in the near field of each other from a mere knowledge of (1) the separation distance between antennas, (2) the side-lobe level of each antenna in the direction of the axis of separation, and (3) the radius of each antenna.

3. Mathematical Upper Bounds Derived from Equation (1)

3.1 Assumptions and Integration

It is possible to derive an upper bound to the magnitude of \( b'_0/a_0 \) if we make some simplifying assumptions that will allow us to integrate equation (1); initially, let us make the following extremely crude assumptions which will allow us to immediately derive an upper bound for coupling quotient by performing the integration in equation (1). Afterwards a smaller, more realistic upper bound will be derived.

\[
\frac{f'(-k) \cdot f(k)}{A_{TMAX} A_{RMAX}} \tag{4a}
\]

\[
\gamma = \gamma_{min} = k \left( 1 - \frac{(D_T + D_R)^2}{4d^2} \right)^{1/2} \tag{4b}
\]

\[
e^{j\gamma d} = e^{jkR} = 1. \tag{4c}
\]

\( A_{TMAX} \) and \( A_{RMAX} \) are the maximum values of \( f \) and \( f' \), respectively, near the axis of separation.

With the assumptions of equations (4) we find after integrating equation (1) from \( \frac{-k(D_T + D_R)}{2d} \) to \( \frac{+k(D_T + D_R)}{2d} \) that
By using this integration range we limit the validity of equation (5) to
\((D_T + D_R)/2 < d < (D_T + D_R)^2/\lambda\).
Equation (5) gives an absolute upper bound to the magnitude of the coupling quotient, neglecting multiple reflections, provided that coupling does not occur through the main beam of either antenna, that \((D_T + D_R)/2 < d < (D_T + D_R)^2/\lambda\) and that the integration range we have used \([i.e., -k(D_T + D_R)/2d to +k(D_T + D_R/2d)]\) is adequate (as explained in section 2.2.4).
As \(d\) approaches the mutual Rayleigh distance we expect equation (5) to approach the far-field formula, equation (3). We can see that this is indeed the case if we allow \(1 - \frac{(D_T + D_R)^2}{4d^2}\) to approach 1 (note that \((D_T + D_R)^2\) will be much less than \(4d^2\) at the mutual Rayleigh distance) and replace one of the \(d\)'s in \(d^2\) by \((D_T + D_R)^2/\lambda\). We thus obtain
\[
\left| \frac{b^*}{a} \right| \leq \left| \frac{A_{\text{MAX}} A_{\text{RMAX}}}{n Z_0 (1 - r_{\text{MAX}})} \right| \left[ \frac{(D_T + D_R)^2}{\lambda} \right]
\]
which is just the far-field formula. If we compare equation (5) to the far-field formula for separation distances less than a mutual Rayleigh distance, equation (5) always gives a larger value.
Instead of the assumptions of equations (4), we can derive more realistic upper bounds by making the more realistic approximations
\[
f'(k) \cdot f(k) = A_{\text{MAX}} \cos \left( k \frac{D_T}{2} + \phi_1 \right) \cos \left( k \frac{D_T}{2} + \phi_3 \right) A_{\text{RMAX}} \cos \left( k \frac{D_R}{2} + \phi_2 \right)
\]
\[
X \cos \left( k \frac{D_R}{2} + \phi_4 \right)
\]
\[
\gamma = \gamma_{\text{MIN}} = k \left( 1 - \frac{(D_T + D_R)^2}{4d^2} \right)^{1/2}
\]
\[
e^{i\gamma d} = e^{ia}
\]
where $\alpha$ is a constant and the $\phi$'s are arbitrary phase shifts. The last assumption (6c), is made because $e^{i\gamma d}$ varies more slowly than the cosine terms or the $e^{iK\cdot R}$ term for the integration range being considered. Under assumptions (6), equation (1) becomes

$$
\left| \frac{b'_0}{a_0} \right| \sim \left| \frac{A_{\text{TMAX}} A_{\text{RMAX}} e^{i\alpha}}{n Z (1 - t \pi L) k^2 \left( 1 - (D_T + D_R)^2 \right)^{1/2}} \int_{-k_0}^{k_0} \int_{-k_0}^{k_0} \cos(k_{\chi} x + \phi_1) \cos(k_{\chi} x / 2 + \phi_2) \cos(k_{\chi} y + \phi_3) \cos(k_{\chi} y / 2 + \phi_4) \frac{d\chi}{2}\frac{d\chi}{2} \right|,
$$

where $k_0 = k(D_T + D_R)/2d$.

The integration of equation (7) is rather lengthy, but straightforward, and can be written after some rearrangement, as

$$
\left| \frac{b'_0}{a_0} \right| \sim \left| \frac{A_{\text{TMAX}} A_{\text{RMAX}} e^{i\alpha}}{n Z (1 - t \pi L) k^2 \left( 1 - (D_T + D_R)^2 \right)^{1/2}} \left[ \frac{1}{4} e^{i(\phi_2 + \phi_1)} \sin \left( \frac{D_T + D_R}{2} + x \right) k_0 \right] \right|
$$

$$
+ e^{i(\phi_2 - \phi_1)} \sin \left( \frac{D_R - D_T}{2} + x \right) k_0 + e^{-i(\phi_2 - \phi_1)} \sin \left( \frac{D_R - D_T}{2} + x \right) k_0
$$

$$
+ e^{-i(\phi_2 + \phi_1)} \sin \left( \frac{D_T + D_R}{2} - x \right) k_0 + e^{i(\phi_2 + \phi_1)} \sin \left( \frac{D_T + D_R}{2} - x \right) k_0
$$

$$
+ e^{i(\phi_4 - \phi_3)} \sin \left( \frac{D_R - D_T}{2} + y \right) k_0 + e^{-i(\phi_4 - \phi_3)} \sin \left( \frac{D_R - D_T}{2} + y \right) k_0
$$

$$
+ e^{-i(\phi_4 + \phi_3)} \sin \left( \frac{D_T + D_R}{2} - y \right) k_0 + e^{i(\phi_4 + \phi_3)} \sin \left( \frac{D_T + D_R}{2} - y \right) k_0
\right|.
$$

We wish to find the maximum value of equation (8). First, we notice that each term in the two pairs of brackets, [ ], is of the form $e^{i(\phi + \phi)}(\sin B k_0)/B$. We might be tempted to say that the maximum value of a term of this form is
but this would be true only if \( |Bk_0| > 1 \). For \( |Bk_0| < 1 \) this term has a maximum value of \( k_0 \) at \( B = 0 \). We note that there are four values of \( x \) and four values of \( y \) for which \( B > 0 \). They are

\[
x, y = \frac{(D_T + D_R)}{2}, -\frac{(D_T - D_R)}{2}, \frac{(D_T - D_R)}{2}, \text{ and } +\frac{(D_T + D_R)}{2}.
\]

In general, only one term of the \( x \) bracket and one term of the \( y \) bracket at a time have \( |Bk_0| < 1 \). However, if either \( D_T \) or \( D_R \) or both are sufficiently small more than one term in the \( x \) bracket and more than one term in the \( y \) bracket can have \( |Bk_0| < 1 \). The maximum value of equation (8) as a function of phase will occur when each of the phases is an integral number of \( 2\pi \) radians.

Keeping this in mind and substituting for \( k_0 \), we have the following five expressions for the maximum coupling:

\[
\left| \frac{b_0'}{a_0} \right| < \frac{A_{\text{MAX}}^A_{\text{RMAX}}}{n Z O (1 - \Gamma_o L) \left(1 - \frac{(D_T + D_R)^2}{4d^2}\right)^{1/2}} \left[ \frac{k(D_T + D_R)}{2d} + \frac{1}{D_T} + \frac{1}{D_R} + \frac{1}{|D_T - D_R|} \right]_2
\]

for \( |D_T^2 - D_R^2| > \frac{4d}{k} \), \( D_T^2 > \frac{4d}{k} \), \( D_R^2 > \frac{4d}{k} \)

\[
\left| \frac{b_0'}{a_0} \right| < \frac{A_{\text{MAX}}^A_{\text{RMAX}}}{n Z O (1 - \Gamma_o L) \left(1 - \frac{(D_T + D_R)^2}{4d^2}\right)^{1/2}} \left[ \frac{2k(D_T + D_R)}{2d} + \frac{1}{|D_T - D_R|} + \frac{1}{\text{Max}(D_T, D_R)} \right]_2
\]

for \( |D_T^2 - D_R^2| > \frac{4d}{k} \), \( \text{Max}(D_T^2, D_R^2) > \frac{4d}{k} \) \( \min(D_T^2, D_R^2) < \frac{4d}{k} \)

\[
\left| \frac{b_0'}{a_0} \right| < \frac{A_{\text{MAX}}^A_{\text{RMAX}}}{n Z O (1 - \Gamma_o L) \left(1 - \frac{(D_T + D_R)^2}{4d^2}\right)^{1/2}} \left[ \frac{2k(D_T + D_R)}{2d} + \frac{1}{D_T} + \frac{1}{D_R} \right]_2
\]

for \( |D_T^2 - D_R^2| < \frac{4d}{k} \), \( D_T^2 > \frac{4d}{k} \), \( D_R^2 > \frac{4d}{k} \)
\[
\frac{b'_0}{a'_0} \leq \frac{A_{\text{MAX}} A_{\text{RMAX}}}{n_0 Z \left(1 - \frac{\Gamma_{\text{LR}}}{\Gamma_{\text{TR}}}\right)} \left(1 - \frac{(D_T + D_R)^2}{4d^2}\right)^{1/2} \left[\frac{3k(D_T + D_R)}{2d} + \frac{1}{\text{Max}(D_T, D_R)}\right]
\]

(9d)

for \(|D_T^2 - D_R^2| \leq \frac{4d}{k}, \min(D_T^2, D_R^2) \leq \frac{4d}{k}, \max(D_T^2, D_R^2) > \frac{4d}{k}\)

\[
\frac{b'_0}{a'_0} \leq \frac{A_{\text{MAX}} A_{\text{RMAX}}}{n_0 Z \left(1 - \frac{\Gamma_{\text{LR}}}{\Gamma_{\text{TR}}}\right)} \left(1 - \frac{(D_T + D_R)^2}{4d^2}\right)^{1/2} \frac{(D_T + D_R)^2}{d^2}
\]

(9e)

for \(|D_T^2 - D_R^2| \leq \frac{4d}{k}, D_T^2 \leq \frac{4d}{k}, D_R^2 \leq \frac{4d}{k}\)

Of course, in equation (9e), the latter two conditions imply the first. Notice that equation (9e) gives the same result as does equation (5).

3.2 Comparison of Upper Bound from Formulas to the Exact Upper Bound

The results obtained from equations (9) for each case in section 2 is given in the last column of table 2.1. The small letter in parentheses in column 18 of table 2.1 indicates which set of conditions of equations (9) is applicable for each case. There is a minimum of one case for each set of conditions in equations (9). We notice that equations (9) give results which differ from the exact results by more than 20 dB in some cases. However, we further notice that equations (9) always give results which are greater than the results found using the exact far fields.

3.3 Conclusion

We conclude that equations (9) can be used to obtain an upper bound to the coupling between two antennas neglecting multiple reflections, provided that coupling does not take place through the main beam of either antenna and provided that the integration range we have chosen is valid, i.e., it is adequate to integrate \(k_X\) and \(k_Y\) only over the range \(-k(D_T - D_R)/2d\) to \(+k(D_T + D_R)/2d\) and we limit \(d\) to the range \((D_T + D_R)/2 < d < (D_T + D_R)^2/\lambda\). It has been shown that this range of integration is adequate for determining side-lobe coupling for nearly all realistic microwave antennas [1].
4. Summary and Concluding Remarks

In this report we have performed a feasibility study to see if it is possible to get an estimate of the maximum coupling between two antennas when the details of the far-field amplitude and phase are unknown. We conclude from a study using hypothetical uniformly illuminated circular antennas that approximation-1 and approximation-2 are especially useful methods of estimating the maximum coupling of two antennas when the separation distance is less than one-tenth of a Rayleigh distance. Approximation-1 is limited to uniformly illuminated circular aperture antennas. Approximation-2 replaces the dot product of the fields in equation (1) by the product of the maximum field magnitudes in the general direction of the separation axis multiplied by cosine functions of $k_x$ and $k_y$ and is an approximation applicable to general antennas. Approximation-2 is used in computer program ENVLP, which is documented in Appendix B.

For distances greater than about one-quarter of a mutual Rayleigh distance, the far-field formula [equation (3)] used with $A_{TMAX}$ and $A_{RMAX}$ gives an increasingly accurate estimate of the maximum coupling between two antennas that is generally more accurate than the estimates provided by approximation-1 and approximation-2. For distances between one-tenth and one-quarter of a mutual Rayleigh distance approximation-2 and the far-field formula give equally reasonable estimates. For separation distances less than about one-tenth of a mutual Rayleigh distance approximation-2, i.e., program ENVLP, gives substantially more accurate values of coupling.

In section 3 we derived a set of expressions [equations (9)] that allow one to determine an upper bound to the coupling between two antennas at arbitrary separation distance. For many applications the upper-bound expression may be adequate, especially when the requirement is simply that coupling lies below a given value.

The appeal of approximation-2 and the corresponding computer program ENVLP, is also their simplicity and the few input parameters that they require. In particular, the near-field coupling is computed between arbitrary antennas from a mere knowledge of (1) the separation distance of the antennas, (2) the side-lobe level of each antenna along the axis of separation, and (3) the radius of each antenna.

Having obtained encouraging results for approximation-2 and the upper-bound equations (9) for hypothetical antennas we suggest that these results be tested experimentally using real antennas, and that a similar approximation be formulated and tested for the computer program CUPLZ.
The authors wish to thank Carl Stubenrauch of NBS for the original suggestion of using approximate far fields as well as for helpful discussions and suggestions throughout the work. Helpful discussions were also held with Allen Newell and Andrew Repjar of NBS. In addition, we wish to gratefully acknowledge the Electromagnetic Compatibility Analysis Center of the Defense Department for its financial support of this work.
5. References


6. List of Figures

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<td>The approximation-1 far-field pattern of the x-component of electric field as a function of θ with λ = 3 cm and a = 10 cm for a uniformly illuminated circular antenna.</td>
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<td>Figure 6</td>
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Figure 4. The approximation-2 far-field pattern as a function of θ, with $\phi = 0^\circ$ or $90^\circ$, $\lambda = 3$ cm, $a = 10$ cm for a uniformly illuminated circular antenna. (At $\phi = 0^\circ$, $k_x = k \sin \theta$, $k_y = 0$; at $\phi = 90^\circ$, $k_y = k \sin \theta$, $k_x = 0$).
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Figure 13. The coupling quotient for the XO cut using the exact, approximation-1 and approximation-2 far fields: Case Gamma-1.
Figure 14. The coupling quotient for the XO cut using the exact, approximation-1 and approximation-2 far fields: Case Gamma-2.
FREQ = 10 GHz  
\[ d = 1666.7 \lambda \]

\[ D_T = 28.87 \lambda \]  \[ D_R = 53.33 \lambda \]
\[ \phi_T = 0^\circ \]  \[ \phi_R = 0^\circ \]
\[ \theta_T = 60^\circ \]  \[ \theta_R = 60^\circ \]
\[ \psi_T = 0^\circ \]  \[ \psi_R = 180^\circ \]

Figure 15. The coupling quotient for the XO cut using the exact, approximation-1 and approximation-2 far fields: Case Delta-1.
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Figure 17. The coupling quotient for the X0 cut using the exact, approximation-1 and approximation-2 far fields: Case Epsilon-1.
Figure 18. The coupling quotient for the XO cut using the exact, approximation-1 and approximation-2 far fields: Case Epsilon-2.
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Figure 20. The coupling quotient for the XO cut using the exact, approximation-1 and approximation-2 far fields: Case Omega.
Appendix A. Derivation of $A_{TMAX}$ and $A_{RMAX}$ in Terms of Antenna Gains and Relative Side-Lobe Levels

In this section we derive the amplitudes, $A_{TMAX}$ and $A_{RMAX}$, of the approximate pattern for the transmitting and receiving antennas, respectively (see section 2.2.3) in terms of the antenna gains, $G_T$ and $G_R$, the side-lobe levels in dB, $S_T$ and $S_R$, the reflection coefficients of the antennas, $\Gamma_{OT}$ and $\Gamma_{OR}$, the characteristic admittances for the propagated mode in the waveguide feeds of the antennas, $\eta_{OT}$ and $\eta_{OR}$, and the wave impedance of free space, $Z_0$.

The magnitude of the vector $\mathbf{f}$ (which is just $A_{MAX}$) in equation (1) of section 2.1 in the direction $\mathbf{r}$ is given by [1]:

$$|\mathbf{f}(\mathbf{r})| = \frac{|E(\mathbf{r})|}{|a_0|} = A_{MAX} \quad (A1)$$

where $E(\mathbf{r})$ is the electric field in the direction $\mathbf{r}$, and $a_0$ is the amplitude of the incident mode of the waveguide feed to the antenna. $A_{MAX}$ is either $A_{TMAX}$ or $A_{RMAX}$ depending on whether the transmitting or receiving antenna is being considered. The relative side-lobe level $S$ in the direction $\mathbf{r}$ ($S_T$ for the transmitting antenna and $S_R$ for the receiving direction), is:

$$S = -20 \log \left( \frac{|E(\mathbf{r})|}{|E(\mathbf{r}_0)|} \right) \quad (A2)$$

where $\mathbf{r}_0$ is the direction of the main beam. It is well known that the gain for an antenna, $G$ ($G_T$ for the transmitting antenna and $G_R$ for the receiving antenna), is given in dB by

$$G = 10 \log \left[ \frac{4\pi |E(\mathbf{r}_0)|^2 r^2}{Z_0 P_{input}} \right] \quad (A3)$$

Assuming a single propagating mode in the waveguide feeding the antenna, the input power to an antenna, $P_{input}$, can be expressed as [6]:

$$P_{input} = \frac{1}{2} \eta_0 |a_0|^2 (1 - |\Gamma_0|^2) \quad (A4)$$

where $\eta_0$ is $\eta_{OT}$ for the transmitting antenna and $\eta_{OR}$ for the receiving antenna and $\Gamma_0$ is $\Gamma_{OT}$ for the transmitting antenna and $\Gamma_{OR}$ for the receiving antenna.

Substituting (A4) into (A3) for $P_{input}$ we find that the gain is

$$G = 10 \log \left[ \frac{4\pi |E(\mathbf{r}_0)|^2 r^2}{Z_0 \eta_0 |a_0|^2 (1 - |\Gamma_0|^2)} \right]. \quad (A5)$$
We solve for \( \frac{|E(r_0)|}{|a_0|} r \) in (A5) to get

\[
\frac{|E(r_0)|}{|a_0|} r = \left( \frac{n_oZ_o}{4\pi} (1 - |\Gamma_o|^2) \right)^{1/2} 10^{G/20}.
\]  

(A6)

Substitution of \( |E(r_0)| \) from (A2) into (A6) gives

\[
\frac{|E(r)|}{|a_0|} 10^{S/20} = A_{\text{MAX}} 10^{S/20} = \left( \frac{n_oZ_o}{4\pi} (1 - |\Gamma_o|^2) \right)^{1/2} 10^{G/20}.
\]  

(A7)

Thus, \( A_{\text{MAX}} \) turns out to be simply

\[
A_{\text{MAX}} = \left( \frac{n_oZ_o}{4\pi} (1 - |\Gamma_o|^2) \right)^{1/2} 10^{((G - S)/20)}
\]  

(A8)

and in particular,

\[
A_{\text{TMAX}} = \left( \frac{n_oZ_o}{4\pi} (1 - |\Gamma_{0T}|^2) \right)^{1/2} 10^{((G_T - S_T)/20)}
\]  

(A9a)

\[
A_{\text{RMAX}} = \left( \frac{n_oZ_o}{4\pi} (1 - |\Gamma_{0R}|^2) \right)^{1/2} 10^{((G_R - S_R)/20)}.
\]  

(A9b)

In summary, equations (A9) give the magnitude of the approximate electric far-field pattern of the transmitting and receiving antenna along their axis of separation in terms of the gain (in dB), side-lobe level (in dB) along the axis of separation, the antenna input reflection coefficient, the characteristic admittance of the waveguide feeding the antenna, and, of course, the impedance of free space.
Appendix B. Documentation of ENVLP, the Computer Program to Estimate Coupling Between Two Antennas

This appendix documents the program ENVLP which estimates the coupling loss between two antennas given their radii, separation distance, and side-lobe levels along their axis of separation. Subroutines used by this program have also been used by CUPLNF (except for CRTPLT3) and are documented in NBSIR 80-1630[2].

B.1 General Overview of Computer Program ENVLP

The techniques used by ENVLP for evaluating the coupling loss are basically the same as those used by the computer program CUPLNF [2]. The flow chart for ENVLP is presented below to give the reader a general understanding of the program package.

**Purpose:** To calculate an estimate of the coupling loss between two antennas given the radii of the antennas, their separation distance, and their side-lobe levels along their separation axis.

**Method:** Evaluate equation (1) of the main text using approximation-2 (see section 2.2.3) to estimate the dot product of the far fields.

**General Discussion:** The main program ENVLP divides into the following sections:

1. General information,
2. Specification statements,
3. Definition and reading of input data,
4. Far-field coupling computation,
5. Limits of integration and number of integration points,
6. Filling the input matrices to the FFT subroutine FOURT, and
7. Calculation of maximum, minimum, mean coupling; printout and plotting of the XO and YO cuts.

**General Information:** This section provides a general description to the program user of what the program does and also defines the more important parameters of the program. A reading of this section will be sufficient for most users to begin using the program.

**Specification Statements:** This section dimensions arrays, places arrays in common, and declares complex variables.
FLOW CHART FOR PROGRAM ENVLP

1. Read in antenna parameters

2. Is the separation distance greater than the Rayleigh Distance?
   - Yes: Go to step 6
   - No: Go to step 3

3. Enter integration limit loop

4. Enter increment size loop

5. Calculate approximate field dot product

6. Sum in x and y directions and do 1D FFT's to find coupling in xo and yo cuts

7. Find the maximum, mean, RMS mean coupling, and the standard deviation for the xo and yo cuts

8. Print and plot coupling for xo and yo cuts

9. Is increment size loop satisfied?
   - Yes: Go to step 8
   - No: Go to step 10

10. Is integration limit loop satisfied?
    - Yes: Stop
    - No: Go to step 2

Stop
Definition and Reading of Input Data: This section defines basic constants such as the speed of light and reads from data cards the antenna parameters. The required data cards are:

Card 1 Col. 1-10 An alphanumeric identifier of the user's choice used to identify the case being computed; the identifier appears at the top of the printout, (HEAD(2)).

Card 2 Col. 5 The maximum value that the user wishes XLIM to assume; it should be 1 or 2. XLIM adjusts the integration range (see below), (ILIMAX).

Col. 10 The maximum value that the user wishes BFAC to assume; it should be 1 or 2. BFAC adjusts the integration step size (see below), (IBFMAX).

Card 3 Col. 1-10 The separation distance in meters, (ZO).

Col. 11-20 The frequency in Hz.

Card 4 Col. 1-10 The radius in meters of the smallest sphere circumscribing the effective transmitting antenna, (RADT).

Col. 11-20 The gain of the transmitting antenna in dB, (GAINR).

Col. 21-30 The relative side-lobe level in dB of the transmitting antenna along the axis of separation (ST).

Col. 31-40 The real part of the input reflection coefficient, GAMMAOT, for the transmitting antenna in free space.

Col. 41-50 The imaginary part of the input reflection coefficient, GAMMAOT, for the transmitting antenna in free space.

Card 5 Col. 1-10 The radius in meters of the smallest sphere circumscribing the effective receiving antenna, (RADR).

Col. 11-20 The gain of the receiving antenna in dB, (GAINR).

Col. 21-30 The relative side-lobe level in dB of the receiving antenna along the separation axis (SR).

Col 31-40 The real part of the input reflection coefficient, GAMMAOR, for the receiving antenna in free space.

Col. 41-50 The imaginary part of the input reflection coefficient, GAMMAOR, for the receiving antenna in free space.

Col. 51-60 The real part of the reflection coefficient, GAMMALR, for the passive termination on the receiving antenna.

Col. 61-70 The imaginary part of the reflection coefficient, GAMMALR, for the passive termination on the receiving antenna.

Card 6 Col. 1-10 The characteristic admittance of the propagating mode in the waveguide feed of the transmitting antenna, (ETAT). If
ETAT=0, the program will set ETAT=1/CAPZ0.

Col. 11-20 The characteristic admittance of the propagating mode in the waveguide feed of the receiving antenna, (ETAR). If ETAR=0, the program will set ETAR=1/CAPZ0.

**Far-Field Coupling Computation:** This section computes the coupling between two antennas directly from their far fields if their separation distance is greater than a mutual Rayleigh distance (equal to the square of the sum of the effective antenna diameters, divided by the wavelength; see [1]).

**Limits of Integration and Number of Integration Points:** For XLIM=1, the limits of integration are computed using the specification of section 2.1 of the main text. These limits can be doubled to see if a wide enough integration range has been included by setting XLIM=2. Strictly speaking, approximation-2 (see section 2.2.3) is only good for the XLIM=1 integration range. However, if the results of the XLIM=2 integration range for the mean coupling differs by a huge amount (e.g., 10 dB) from the mean coupling result for XLIM=1, it is questionable whether ENVLP gives a good estimate of the coupling loss between the two antennas being considered.

This section also chooses the integration increment size small enough to prevent aliasing. The increment size can be reduced by increasing BFAC. Normally, BFAC is set equal to 1 and convergence is tested by halving the integration increment size, i.e., letting BFAC=2.

**Filling the Input Matrices to the FFT Subroutine FOURT:** This section calculates the dot product of the far fields in a square array using approximation-2. The dot products are then summed in the \( k_y(k_x) \) direction for each value of \( k_x(k_y) \) and placed in the array AX(AY). FOURT then performs a one dimensional FFT on the array AX(AY) to obtain the values of the coupling quotient along the XO(YO) cut.

**Calculation of Maximum, Minimum, Mean Coupling; Printout and Plotting of the XO and YO Cuts:** This section computes the maximum, minimum, mean, and RMS mean coupling for the XO and YO cuts. It further computes the standard deviation in the coupling for each cut. It prints out the values of the coupling quotient for each cut and then plots the values of the coupling quotient for each cut from -(DIAMR+DIAMT) to +(DIAMR+DIAMT).
Symbol Dictionary (in alphabetical order):

ABL = Intermediate value for defining the range of $k_x/k$ and $k_y/k$. The range of ABL beyond XKLIM is zero filled.

ACLCUT = A real array used to store the magnitude of the coupling quotient along XO and YO cuts.

AMAX = The maximum value of the coupling quotient for either the XO or YO cut.

AMEAN = The mean value of the coupling quotient for either the XO or YO cut.

AMINX,(AMINY) = The minimum value of the coupling quotient for the XO(YO) cut.

ARMAX,(ATMAX) = The side-lobe level for the receiving (transmitting) antenna.

AX,(AY) = Complex arrays used to store first the far-field product then the coupling quotient along the XO(YO) cut.

(A1,A2),(B1,B2) = The limits of integration of $k_x/k$ and $k_y/k$ respectively.

BFAC = Variable which adjusts the integration increments and should be about 1 or 2; making BFAC larger tests for convergence by making the integration increments proportionally smaller.

C = The increment size for the XO and YO cuts when the far-field formula is used.

CAPZ0 = The wave impedance of free space.

CEE = The speed of light in a vacuum in meters/second.

CLCUTXA(CLCUTYA) = The values of the coupling quotients in the XO(YO) cuts stored for plotting.

COEF = $2 \pi FMM/\eta/CAPZ0/XK$ for the far-field formula, $-FMM*C1*C2/\eta/CAPZ0$ otherwise.

C1,C2 = The $k_x/k$ and $k_y/k$ increments respectively.

DIAMR,(DIAMT) = Twice the larger of RADR (RADT) or WAVLTH.

DIAMSUM = DIAMR plus DIAMT.

DKOK = The approximate $k_x/k$ and $k_y/k$ increments.

DX,(DY) = The increments in XO(YO) over which the coupling quotient is computed by the FFT.

ETAT,(ETAR) = The characteristic admittance for the propagated mode for the waveguide feed of the transmitting (receiving) antenna.

FDOTFP = The dot product of the electric far-field pattern of the two antennas.

FMM = The mismatch factor, $1/(1-\Gamma_o \Gamma)$, for the receiving antenna.

FREQ = Frequency in Hz.
FX,FY = Intermediate complex variables.
GAINT,(GAINR) = The gain in dB of the transmitting (receiving) antenna.
GAMMALR,GAMMAOR,GAMMAOT = The reflection coefficients of the receiving load,
  receiving antenna, and transmitting antenna, respectively.
HEAD = Integer array identifier.
IBFAC = Loop index for varying BFAC.
IBFMAX = The maximum value assumed by BFAC; it should equal to 1 or 2.
ILIMAX = The maximum value assumed by XLIM; it should be 1 or 2.
IXLIM = Loop index for varying XLIM.
J1,J2 = Loop indices used in the filling of AX(AY) before the FFT.
M1,M2 = Loop indices used in completing the coupling quotient computation
  after the FFT.
N = The number of points in an XO(YO) cut when the far-field formula
  is used.
NN1,NN2 = Integer arrays of dimensions one used in the call to the FFT
  subroutine FOURT.
NSX,(NSY) = The number of values of the coupling quotients to be plotted in
  the XO(YO) cut.
N1,N2 = The number of \( k_x \) and \( k_y \) integration points respectively.
N1MAX,(N2MAX) = Integers determining the maximum of the XO(YO) range over which
  the coupling is printed.
N1MIN,(N2MIN) = Integers determining the minimum of the XO(YO) range over which
  the coupling is printed.
N10,N20 = Intermediate integers used to determine (N1MIN,N1MAX) and
  (N2MIN,N2MAX).
PI = \( \pi = 3.14159... \)
RADR,(RADT) = The radius of the smallest sphere circumscribing the receiving
  (transmitting) antenna in meters.
RMS = The RMS mean coupling quotient for the XO or YO cut.
RO = The total distance between the two antennas = \( (X^2 + Y^2 + Z^2)^{1/2} \).
SDEV = The standard deviation in the coupling quotient for the XO or YO
  cut.
SR,ST = The relative side-lobe levels, in dB, of the receiving and
  transmitting antennas along their axis of separation.
SUM, SUMRMS, SUMSQ = Intermediate variables used to calculate AMEAN, RMS and SDEV,
  respectively.
SUM2 = Summation variable used in the filling of the AX and AY matrices.
TEMP = An intermediate variable used to calculate ATMAX and ARMAX.
TSUM21 = Summation variable used to compute the coupling quotient at \( X_0 = 0 \) by summing directly without the use of the FFT.

WAVLNGTH = Wavelength in meters.

WORK = Complex array required by the FFT subroutine FOURT.

\( X_0(Y_0) \) = Array containing the abscissa value for the plots of the \( X_0(Y_0) \) cuts.

\( XK = 2\pi/\lambda. \)

\( XKLIM = \) Variable which limits the range of \( k_x/k \) and \( k_y/k \) integration when its value is less than \( XKMAX. \)

\( XKMAX = \) An upper limit (less than 1.0 and usually chosen at .9) on \( XKLIM; \) except for close antennas \( XKLIM \) will usually be less than \( XKMAX. \)

\( XKMIN = \) Sum of the diameters of the two antennas divided by their separation distance.

\( XKOK = \) The square root of the sum of the squares of \( XXXOK \) and \( XKYOK. \)

\( XXXOK, XKYOK = \) \( k_x/k \) and \( k_y/k, \) respectively.

\( XLIM = \) Variable used for adjusting \( XKLIM; \) making \( XLIM \) larger tests to see if a large enough integration range has been included; \( XLIM \) should equal 1 or 2.

\( XNX, XNY, XNZ = \) Variables used for incrementing \( k_x/k, k_y/k, \) and \( y/k, \) respectively.

\( XYO = \) Intermediate variable used for calculating \( RO. \)

\( XO,YO,ZO = X,Y,Z \) coordinates of the origin of the receiving antenna in the mutual coupling coordinate system of the transmitting antenna; specifically \( ZO \) is the separation distance.

\( ID, IFL, LU, NOFRAME = \) Variables used by the CRT plotting routine CRTPLT3, which is a specialized routine for the NOAA/NBS CYBER 170/750.

**Subroutines Not Available in the FORTRAN Library:**

FOURT (a standard FFT subroutine documented with program CUPLNF in NBSIR 80-1630[2].)

PLT12OR (printout plotting routine documented in NBSIR 80-1630[2].)

CRTPLT3 (a CRT plotting routine specifically written for the NOAA/NBS Cyber 170/750; we suggest you substitute your own subroutine).

**Note:** If the electric far field, including phase as well as amplitude, is available use the program CUPLNF documented in NBSIR 80-1630[2].

**B.2 Computer Code and Sample Output**

A copy of the computer code for program ENVLP and a sample output for the case BETA-1 are found below.
PROGRAM FVLP(INPUT,OUTPUT)


******************************************************************************WARNING******************************************************************************

DO NOT USE THIS PROGRAM FOR COUPLING INVOLVING THE MAIN BEAM. CAUTION SHOULD ALSO BE USED WHEN APPLYING THIS PROGRAM TO IDENTICAL ANTENNAS AS THE PROGRAM MAY NOT GIVE VALID RESULTS IF THESE IDENTICAL ANTENNAS HAVE THE SAME FUTURE ANGLES.

THE COUPLING QUOTIENT IS COMPUTED ALONG X0 AND Y0 PERPENDICULAR LINES OR CUTS.

AX0,AY0, AND Z0 SHOULD BE DIMENSIONED OF THE LARGER OF (M1,M2). APICUT SHOULD BE DIMENSIONED AT LEAST 2 GREATER THAN THE LARGER OF (M1,M2).

HEADER IS AN ARRAY WHICH CONTAINS AN ALPHANUMERIC IDENTIFIER TO IDENTIFY THE CASE BEING COMPUTED. IT IS PLACED AT THE TOP OF THE PRINTOUT AND EACH MICROFILM FRAME. HHEADER() CAN BE SPECIFIED AS ANY TEN CHARACTER WORD THE USER WISHES.

FREQ IS THE FREQUENCY IN HERTZ.

FREQ0, FTA0, FTAP0 ARE THE CHARACTERISTIC ADMITTANCES FOR THE PROPAGATED MODES IN THE WAVEGUIDE FEEDS OF THE TRANSMITTING AND RECEIVING ANTENNAS, RESPECTIVELY.

SGAMMA0, SGAMMAO0, SGAMMAOL ARE THE REFLECTION COEFFICIENTS OF THE TRANSMITTING ANTENNA, RECEIVING ANTENNA, AND THE RECEIVING LOAD.

RESPECTIVELY.

CAPT0 IS THE WAVE IMPEDANCE OF FREE SPACE.

XY0, Y0, Z0 ARE THE COORDINATES OF THE ORIGIN OF THE RECEIVING ANTENNA IN THE REPRESENTED RECTANGULAR SYSTEM OF THE TRANSMITTING ANTENNA.

THE REPRESENTED COORDINATE SYSTEMS OF EACH ANTENNA ARE THE COMMON MUNAL COUPLING COORDINATE SYSTEMS OF THE ANTENNAS. 7C MUST BE SPECIFIED, BUT THE RANGE OF X0 AND Y0 ARE DETERMINED IMPLICITLY BY THE REQUIREMENTS OF THE FFT ALGORITHM FOUR.

Z0 IS THE SEPARATION DISTANCE IN THE DIRECTION OF THE SEPARATION AXIS.

RPT0 = RADIUS OF SMALLEST SPHERE WHICH CIRCUMSCRIBES THE TRANSMITTING ANTENNA.

RPT0 = RADIUS OF SMALLEST SPHERE WHICH CIRCUMSCRIBES THE RECEIVING ANTENNA.

SRT0 = TWICE THE LARGER OF RPT OR WAVELENGTH.

SRT0 = TWICE THE LARGER OF RPT OR WAVELENGTH.
ST, ST ARE THE SIDELOBE LEVELS OF THE TRANSMITTING AND RECEIVING ANTENNAS, RESPECTIVELY. THEY ARE GIVEN IN DECIBELS BELOW THE MAIN BEAM AND ARE POSITIVE FOR SIDELOBES LESS THAN THE MAIN BEAM.

GAIN, GAIN ARE THE GAIN OF THE TRANSMITTING AND RECEIVING ANTENNAS, RESPECTIVELY.

REAUDIO THE INTEGRATION INCREMENTS, AND SHOULD BE APPROXIMATELY 1 OR 2. MAKING REAUDIO LARGER TESTS WHETHER
CONVERGENCE HAS BEEN REACHED.

IF A REAUDIO GIVES THE MAXIMUM VALUE THAT REAUDIO ASSUMES.

XLIM ADJUSTS THE NONZERO-FILL FUNCTION OF THE INTEGRATION RANGE.

XLIM SHOULD BE 1 FOR THIS PROGRAM. HOWEVER, IF MAKING
XLIM EQUAL TO 2 CHANGES THE MEAN VALUE OF THE COUPLING LOSS BY A
HUGE AMOUNT (SAY 10DB), THIS WOULD INDICATE THAT THE COMPUTED
COUPLING LOSS UNDER THE ASSUMED APPROXIMATE FAR FIELD IS
UNRELIABLE. INCREASING XLIM INCREASES THE LIMITS OF INTEGRATION
AND AUTOMATICALLY DECREASES THE INTEGRATION INCREMENTS
PREVENTING ALIASING.

IF A XLIM GIVES THE MAXIMUM VALUE THAT XLIM ASSUMES.

A1X+A2X A2X OFFSET THE TOTAL INT[WITH ZERO-FILL] INTEGRATION RANGES
(KY/K FROM -A1 TO APPRX A2) AND (KX/K FROM -A1 TO APPRX A2),
IN INCREMENTS OF (A1+A2)/N OR (A1-A2)/N APPROX. EQUAL TO DKOK.

DKOK = WAVELENGTH/(2*(DIAMT+DIAMR)*FAAC*XLIM).

IF SOPT ((KX/K)**2+(KY/K)**2) IS .GF. XLIM THE SPECTRUM
IS OFF EQUAL TO ZERO. (APPEND TABLE 7F D_FILLING IS AN OPTION
DESIGNED TO ALLOW FINER INCREMENTS DX AND DY AT THE WHICH
COUPLING FUNCTION IS COMPUTED BY THE FFT.)

XLIM MUST BE EQUAL TO OR LESS THAN 1 BECAUSE
THE PROGRAM NEGLECTS THE EVANSSENT MODES. IN ORDER NOT TO GET
TOO CLOSE TO THE 1/2AMAP SINGULARITY, IT IS SAFER TO CHOOSE XLIM
NO LARGER THAN XLIM NO ABOUT .9.

THE YO AND YO INCREMENTS ARE DX=WAVELENGTH/(A1+A2) AND
DY=WAVELENGTH/(A1-A2).

THE RANGE OF BOTH YO AND YO IS GIVEN APPROXIMATELY BY
-(STAMT+DIAM)+FAAC+XLIM TO +(DIAMT+DIAM)+FAAC+XLIM, BUT ONLY
-(STAMT+DIAM) TO +(DIAMT+DIAM) APPROXIMATELY IS PRINTED AND
PLTTFD WHEN XLIM+FAAC IS GREATER THAN OR EQUAL TO 1.

IN THE PLOTS, -1 OF THE ABSCISSA CORRESPONDS TO -(STAMT+DIAM) AND
+1 TO (STAMT+DIAM).

COMMON/NODE/NODE
COMMON/D1MT/D1MT/F2L/F2L
DEPNSTM CLCUTY(A2000).*CLCUTY(A2000)
DEPNSTM HFACT
COMPLEX WORK(2000)
COMPLEX EMAC,CONF
COMPLEX GAMMA,T,GAMMACR,GAMMALR
COMPLEX FY,FY
COMMON BLOCKS
C
DIMENSION WN(10), MN(10)
DIMENSION ALCUT(7)
C
C
ND FRAME=0
C
C
IT IS AN ARRAY WHICH CONTAINS NAME AND PHONE NUMBER ID FOR
C MICROFILM USE.
IN(1)=10, IN(2)=10, IN(3)=10, IN(4)=10
IF ARP(1)=10, CASE
READ 3, HFAD(2)
3 FONMAT(10)
PRINT 4, HFAD(2)
4 FONMAT(11), CASE 4, A100
NP ? 1=4, 7
2 HFADNT=10
C
C
THE ARRAY HEAD PUTS A HEADER ON EACH FRAME OF MICROFILM.
C
C
C
C
GENERAL PARAMETERS
READ 700,70,FPFO
PRINT 700,70, FPFO
READ 775, ILMAX, IRFMAX
PRINT 780, ILMAX, IRFMAX
PI=4, *ATAN(1.0)
CAP70=276, 77
CFF=7, 007025 FR
WAVELENGTH=CEF/FPFO
XX=2, *PI/WAVELENGTH
C
C
C
PARAMETERS OF THE ANTENNAS
READ RD0, RANT, GAIN, ST, GAMMA0
PRINT R25, RANT, GAIN, ST, GAMMA0
READ RD0, RANT, GAMMA0, GAMMA0, GAMMA0, GAMMA0
PRINT R0, RANT, GAIN, SR, GAMMA0, GAMMA0
READ R75, FTTAT, FTAR
C
C
C
C
FMP=1, -CAMA00GAMMALR
IF (FTAT = FC.0) FTAT=1./*CAP70
IF (FTAR = FC.0) FTAR=1./*CAP2C
PRINT R0, FTAT, FTAR
TFMP=FTAT*CAP70* [1, -CASA(GAMMA)10]*10.0, (((GAIN-5)/20.))
ATMAX=CO8(TMP)*10.0, (((GAIN-5)/20.))
TFMP=FTAR*CAP70* [1, -CASA(GAMMA)10]*10.0, (((GAIN-5)/20.))
PRINT R0, ATMAX, AMAX
C
C
C
C
MAT=2, *MAX1(WAVELENGTH, RANT)
MAT=2, *MAX1(WAVELENGTH, RANT)
PROGRAM ENVLP 74/175 CPT=1

DIAMSUM=DIAMT+DIAMP
PRINT 17
17 FORMAT(1*E)

175 PRINT 7, (DIAMSUM)**2/WAVLNGTH
7 FORMAT(I,4X,INITIAL RAYLEIGH DISTANCE = (DIAMSUM)**2/SQUARED/WAVLNGTH =
10F12.5,4X,MTERS/*/"
PRINT 111C

1110 FORMAT(I,7X,"THIS RESULT IS FOR AN APPROXIMATE CALCULATION")

120 IF 70 IS GREATER THAN DIAMSUM**2/WAVLNGTH, THE COUPLING CAN
BE COMPUTED APPROX. FROM THE FAR-FIELDS OF THE ANTENNA WITHOUT
INTEGRATION AS FOLLOWS.

130 IF(70.0,LT, 0.0 DIAMSUM**2/WAVLNGTH) GO TO 50
PRINT 40

40 LIMITS FOR ELIPSE DIRECTLY FROM THE FAR-FIELDS WITH
THE LIMIT FOR X AND Y CUTS FROM -DIAMSUM TO +DIAMSUM, 0/1)

140 N=21
MSY=N'S MSX=N
C=2.*DIAMSUM/(N-1.0)
COFF=2.*PI*FIN/FSTAR/CAP70/XK

ON 60 J=1,N
195 XIC=DIAMSUM*(J-1,0)*C
196 SPTR=SPRT(70.0+2.*XYO**2)
197 FNTFEP=AUX*APMAX*CNFF
198 AX(J)=FNTFEP*PO*CXP(CMPLX(0.0,XYO*PO+PI/2.0))
199 AY(J)=FNTFEP*PO*CXP(CMPLX(0.0,XYO*PO+PI/2.0))

200 CONTINUE

50 PRINT 510

510 AMAX=0.0, AMINX=10.0, AMINY=10.0, SUM0=0.0, SUMSO=0.0
SUPRS=0.0

ON 70 J1=1,N
205 ACLCUT(J1)=CARS(AX(J1))
XK=DIAMSUM*(J1-1.0)*C

IF(ACLICUT(J1),GT,0.0)AMAX=ACLICUT(J1)
IF(ACLICUT(J1),LT,0.0)AMINX=ACLICUT(J1)

210 CONTINUE

70 CONTINUE

AMAN=SUM/N

219 SUM=SUM+C(AFAN-ACLICUT(J1))**2/(N-1)

77 CONTINUE

ACLICUT(J1)=PO**ALG10(ACLICUT(J1))
CLCUTX(J1)=ACLICUT(J1)

77 CONTINUE

PRINT 515,ACLICUT(J1),X(J1)

220 SF=SPRT(SUM0)
PS=SPRT(SPRT0)
SF=SF**ALG10(1.0+SF/AFAN) & AFAN=PO**ALG10(AFAN)
AMAX=PO**ALG10(AMAX) & RMS=PO**ALG10(PS)
AMINT=ABALG10(AFAN)

221 CONTINUE

PRINT 620,AFAN, SF, AMAX
PRINT 621, RMS
PRINT AF

AMAX=0.0, & AFAN=0.0, SUM0=0.0, SUMSO=0.0.
PROGRAM FHVLP

CALCULATION FOR 70 LFS THAN THE RAYLEIGH DISTANCE FOLLOWS.
LIMITS OF INTEGRATION AND NUMBER OF POINTS.
IF YOU WISH TO CHANGE XLFAC OR RFAC CHANGE THE CORRESPONDING LOPP.
PARAMETER TLTMAX,TFMAX.

DO 1000 IXLM=1,TLTMAX
IXLM=IXLM
DO 1000 IYLFAC=1,IBFMAX
RFAC=RFAC

THE PRECEDING STATEMENT IS INCLUDED HERE IN CASE IT IS EVER
EVER WHEN 70 IS GREATER THAN THE MUTUAL
RAYLEIGH DISTANCE (DTAMSUM#2/WAVLETH).
N2=N1(1)
IF(IYL1M,F0.1)NSX=N1-1
IF(IYL1M,F0.1)NSY=N2-1

C
C1=(A1/A2)/N1
C2=(A1/A2)/N2
CNEFF=FNU*C1*C2/FTAR/CAZ

C
C FILLING THE AX AND AY MATRICES USED IN FOUPT
TSUM21=0.0001
DO 10 J2=1,N2
AY(J2)=0.0001
10 CONTINUE
DO 200 J1=1,N1
XYX=1/(J1-1.)
XYXK=XYX-1
SUP2=0.0001
DO 100 J2=1,N2
XYN=C2*(J2-1.)
100 CONTINUE
XYK=XY(NK)!
XYK=XYK
IF(XKYOK) GO TO 11C
XYK=XYK+XYKNK+2)
11C CONTINUE
XYK=XYK*COS(XKYOK*KRAD)*COS(XKYOK*KRAD)
FOFTFP=FOFTFP*COS(XKYOK*KRAD)*COS(XKYOK*KRAD)
SC, THE COS FUNCTIONS IN FOFTFP ARE A ROUGH APPROXIMATION TO THE
\[ \cos(x) \text{ and } \cos(y) \text{ DIRECTIONS}\]
FOFTFP=FOFTFP*COS(XKYOK*KRAD)*COS(XKYOK*KRAD)
SUP2=FOFTFP*SYN2
AY(J2)=FOFTFP*SYN2
310 CONTINUE
AX(J1)=-SUM2*(-1)***J1
TSUM2=SUM2+TSUM21
320 CONTINUE
DO 200 J2=1,N2
AY(J2)=AY(J2)+(-1)**J2
200 CONTINUE
C
C THE AX MATRIX HAS DATA COLLAPSED IN THE Y-DIRECTION AND THE AY
C MATRIX HAS THE SAME FOR THE X-DIRECTION, THUS FOR AX THE KY
C INTEGRATION HAS BEEN COMPLETED WHILE FOR AY THE KY INTEGRATION
C HAS BEEN COMPLETED.
C
C ***FOUPT IS A SUBROUTINE THAT PERFORMS A FAST FOURIER TRANSFORM
C FOR COMPLEX DATA, IF YOU HAVE CUPLEF YOU SHOULD HAVE FOUPT.
C
C
C IMPLEMENTS AFT ON THE MATRIX AX IS EQUIVALENT TO DOING THE
C KY INTEGRATION WHILE AFT ON THE MATRIX AY IS EQUIVALENT TO DOING THE
C KY INTEGRATION.
C
CALL FOUPT(AX,N1,1,*1,*1,WORK)
CALL FOUPT(AY,N2,1,*1,*1,WORK)
DO 400 M1=1,N1
X0=-(N1/2.+M1-1.)*WAVLTH/(A1+A2)
FO=CFXP(CPLX(0.,-XK*A1*X0))
A1=M1)FO=CNEFF*AX(M1)
400 CONTINUE
DO 340 M2=1,N2
Y0=-(N2/2.+M2-1.)*WAVLTH/(P1+P2)
340 CONTINUE
345 CONTINUE

350 PRINTOUT
   PRINT 3,Y1IM,PFA
   FNORM Y1M+Y1F+2,5+X+0PFA=0F12.5//
   PRINT 15,N1,N2
   FORMAT (1X,N1,N2=16,5+X,THTY ROTH SHOULD BE EVEN//)
   PRINT 25,YAVGLTH,PIA,RAI,N2
   FORMAT (1X,YAVGLTH,PIA,RAI,N2=244,5+X)
   Z0 =**F12.5* MFTRS RESPECTIVELY

355 NY=YAVGLTH/(A1+A2). NY=WAVGLTH/(A1+A2
   PRINT 55,60=NY,60=NY/(1/2.-1.1,IDY
   55 FORMAT (1X,Y0 RANGES FROMF12.5* TOF12.5* IN INCREMENTS OFF12.5*,
           15* MFTRS/=//)
   PRINT 65,60=NY,60=NY/(1/2.-1.1),IDY
   65 FORMAT (1X,Y0 RANGES FROMF12.5* TOF12.5* IN INCREMENTS OFF12.5*,
           15* MFTRS/=//)

360 PRINT 75,-A1/2-(A1+A2)/N1,A1+A2/N1
   75 FORMAT (1X,THE INTEGRATION VARIABLE K/K RANGES FROMF12.5* TOF
            112.5* IN INCREMENTS OFF12.5//)

365 PRINT 85,-R1/2-(R1+R2)/N2,R1+R2/N2
   85 FORMAT (1X,THE INTEGRATION VARIABLE K/K RANGES FROMF12.5* TOF
            112.5* IN INCREMENTS OFF12.5//)

370 PRINT R7,YAVGLTH
   87 FORMAT (1X,THE SPECTRUM IS FPRO FILLED BYBRD SO(1X2+KY2)=K TIME
            15*F12.5/=//)
   PRINT 95,TSUM21=COFF
   95 FORMAT (1X,THE COUPLING OVIDTENT AT X0=0 AND Y0=0, SUMMED DIRECT
            1LY WITHOUT THE FFT, EQUALS2F12.5//)

375 PRINTOUT OF X0 AND Y0 CENTERLINE CUTS RESPECTIVELY
   PRINT 27
   27 FORMAT (1X,X0-CUTF/,)
   PRINT 25,(X0(JJ1),J1=1,N1)
   PRINT 20

380 PRINT 29,Y0-CUTF/,
   PRINT 25,(AY(J2),J2=1,N2)
   PRINT 29

385 PRINT OF MAGNITUDE OF X0 AND Y0 CENTERLINE CUTS
   IF(X1M*PFA=1.1 GO TO 1500
   IF Powers FROM HERE TO 1500 ON THE PRINTING, PLOTTING, AND
   STATISTICAL ANALYSIS FOR X1M*PFA EQUAL TO 1 IT COULD BE USED IF
   ONE MISTRES TO PRINTOUT ANY PLOT FOR X1M*PFA LESS THAN 1 OR
   FOR ALL POINTS WHICH ARE COMPUTED WHEN X1M*PFA IS GREATER
   THAN 1.
   FOR X0=PFA*X1M GREATER THAN 1 SKIP TO 1500 WHERE A SHORTENED
   PRINTOUT AND STATISTICAL ANALYSIS ARE DONE.
   PRINT 510
   510 FORMAT (1X,MAGNITUDE (XC-CUTF/)A0X=0. $ AMINY=10. $ AMINY=10. $ SUM=0. $ SUM50=0.
   SUPRM=0.
   IF (NO J1=1,N1)
   A[CUT(JJ1-1)+CAR(C,JJ1))
   YC=/(N1/2.+J1-11)*YAVGLTH/(A1+A2)
400  IF (ACLCUT(J1-1).GT.AMAX) AMAX=ACLCUT(J1-1)
   IF (ACLCUT(J1-1).LT.AMINX) AMINX=ACLCUT(J1-1)
   Y(J1-1)=Y(J1-1)*TAN(SUM)
   SUPRMS(SUPRMS+ACLCUT(J1-1))=0/N1
   END CONTINUE

405  SUPRMS(SUPRMS+ACLCUT(J1-1))=0/N1

500  CONTINUE

519  ENDAT(IY,F12.5) XQ=F12.5)
   AMFAN=SUM(N1-1)
   DO 520 J1=2,N1
   SUPRMS=SUPRMS+(AMFAN-ACLCUT(J1-1))**2/(N1-2)
   AC栎UT(J1-1)=20.*ALOG10(ACLCUT(J1-1))
   CCLUTX(J1-1)=ACLCUT(J1-1)
   CONTINUE

520  PRINT 519,ACLCUT(J1-1:XJ(J1-1)

610  PRINT(1X,30,MAGNITUDE(YQ-CUT)**/)
   AMAX=0.0
   $AMAN=0.0
   $SUM=0.0
   $SUMR=0.0
   NC=600 J2=2,N2
   AC栎UT(J2-1)=CAREAY(J2-1)
   YQ=(-N/3)+J2-1.)*WAVELTV/(N1+R2)
   IF (ACLCUT(J2-1).GT.AMAX) AMAX=ACLCUT(J2-1)
   IF (ACLCUT(J2-1).LT.AMINX) AMINX=ACLCUT(J2-1)
   SUPRMS=SUPRMS+ACLCUT(J2-1)**2/N2
   SUPRMS=SUPRMS+ACLCUT(J2-1)*Y(J2-1)=YO/463

600  CONTINUE

615  PRINT (1X,F12.5) XQ=F12.5)
   AMFAN=SUM(N2-1)
   DO 625 J2=2,N2
   SUPRMS=SUPRMS+(AMFAN-ACLCUT(J2-1))**2/(N2-2)
   AC栎UT(J2-1)=20.*ALOG10(ACLCUT(J2-1))
   CCLUTX(J2-1)=ACLCUT(J2-1)
   CONTINUE

620  PRINT 519,ACLCUT(J2-1:XJ(J2-1)

625  PRINT(1X,30,CONTINUE

630  PRINT(SUMS)
   RMS=SQRT(SUMS)
   AMAX=20.*ALOG10(AMAX) $ RMS=20.*ALOG10(RMS)
   SDF=20.*ALOG10(AMAN) $ SDF=20.*ALOG10(AMAN)
   AMAN=20.*ALOG10(AMAN)
   AMAN=20.*ALOG10(AMAN)
   PRINT 620,AMAN,SDF,AMAX
   PRINT 621,RMS
   PRINT 17

450  GC TO 1000

1=CO CONTINUE

C C C SHORTEN PRTNTOUT AND STATISTICAL ANALYSIS FOR XLIM*BFAC
C C C GREATER THAN 1 FOLLOWS.

457  PRINT '(A10,2F10.0)
   XI=N1/(XLIM*BFAC)+.00001
N1/MAX=N1/2+1-N10/2
N1/MAY=N1/2+1+N10/2

460 SUP=C0, AMAY=O.
DP 501 J1=N1/MAY
ACLUT(J1-N1/MAY)=CARSA(YJ1))
Y0=-(N1/2+J1-1)*WAVGLTH/(N1+B2)
 SUP=SUM*ACLUT(J1-N1/MAY)

465 IF (ACLUT(J1-N1/MAY)<0. GT. AMAY) AMAY=ACLUT(J1-N1/MAY)
& ACLUT(J1-N1/MAY)<20.*ALOG10&ACLUT(J1-N1/MAY)
Y0=Y0/NIAM&NI2
PRINT 515,ACLUT(J1-N1/MAY),X0

501 CONTINUE
AMFAN=SUM/(N1/MAX-N1/MAY+1)
AMFAN=70.*ALOG10(AMFAN)
AMAY=70.*ALOG10(AMAY)
PRINT 600,AMFAN,AMAY
PRINT 910
N2=NI*(11#*RFAC)+COCC01
N2/MIN=N2/2+1-N20/2.
SUP=O. % AMAY=O.
DP 601 J2=N2/MIN
ACLUT(J2-N2/MAY)=CARSA(YJ2))
Y0=-(N2/2+J2-1)*WAVGLTH/(N1+B2)
 SUP=SUM*ACLUT(J2-N2/MAY)

490 IF (ACLUT(J2-N2/MAY<0. GT. AMAY) AMAY=ACLUT(J2-N2/MAY+1)
ACLUT(J2-N2/MAY)=20.*ALOG10&ACLUT(J2-N2/MAY)
Y0=Y0/NIAM&NI2
PRINT 615,ACLUT(J2-N2/MAY),Y0

601 CONTINUE
AMFAN=SUM/(N2/MAX-N2/MAY+1)
AMFAN=70.*ALOG10(AMFAN)
AMAY=70.*ALOG10(AMAY)
PRINT 999,AMFAN,AMAY
PRINT 917

1000 CONTINUE
C

2000 CONTINUE
C

PLTTING
C

** WARNING **
C
C PLTPILO IS A CPT PLOTTING ROUTINE SPECIALIZED FOR THE NOAA/NRS
C CYBER 170/750 MACHINE. PLT120P IS A PRINTOUT PLOTTING ROUTINE.
C IT MAY BE NECESSARY TO SUPPLY YOUR OWN PLOTTING ROUTINES TO
C PLTCLCUTXY,CLCUTYA AGAINST X, Y. IT E, THE COUPLING QUOTIENT VERSUS
C YO AND Y0. IF YOU HAVE THE PROGRAM CUPLNE YOU SHOULD HAVE PLT120R
C
C WFAP(3)=10W XO-CUT
C
509 CALL CPTPILO(X,Y,CLCUTXY,1.0,-1.0,0.0,-150.0,NSY,WFAP,1.1,1.0,0.1)
CALL PLT120P(X,Y,CLCUTXY,1.0,-1.0,0.0,-150.0,NSY,1H*,1.1)
PRINT 925
WFAP(3)=10W XO-CUT
C
510 CALL CPTPILO(X,Y,CLCUTXY,1.0,-1.0,0.0,-150.0,NSY,WFAP,1.1,1.0,0.1)
CALL PLT120P(X,Y,CLCUTXY,1.0,-1.0,0.0,-150.0,NSY,1H*,1.1)
PRINT 925
C
C ENDMAS
620 FORMAT(1X,*THE MEAN COUPLING AMPLITUDE IS *F10.4* DB WITH A ST
1AMPARD DEVIATION OF *F10.4* DB FOR 1X AND A MAXIMUM COUPLING A
MPITUDE OF *F10.4* DB)
630 FORMAT(1X,*THE RMS MEAN COUPLING AMPLITUDE IS *F10.4* DB*)
700 FORMAT(2F10.4)
750 FORMAT(1X,*70=%E10.4,5%,#F10.4* E10.4)
775 FORMAT(7T5)
780 FORMAT(1X,*ILIMAX=%,1%5%,#TARMAX=%,15)
900 FORMAT(F10.4,F10.4,2(F10.4,F10.4))
A25 FORMAT(1X,*RADE=%F10.4,5%,#GAINT=%F10.4,5%,#ST=%F10.4,5%,
1 *GAMMA0=%,2(F10.4,5%))
A50 FORMAT(1X,*RACT=%F10.4,5%,#GAINR=%F10.4,5%,#SR=%F10.4,5%,
1 *GAMMARD=%,2(F10.4,5%))
A75 FORMAT(F2F10.4)
A90 FORMAT(1X,*ETAT=%F10.4,5%,#ETAR=%F10.4)
900 FORMAT(1X,*AYMAX=%F10.4,5%,#ARYAX=%F10.4)
925 FORMAT(///,39%,#MAGNITUDES OF COUPLING QUOTIENT FOR xo-CUT*)
950 FORMAT(///,39%,#MAGNITUDES OF COUPLING QUOTIENT FOR xo-CUT*)
CALL DEFND
529
530
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<th>Case</th>
<th>RFA-1</th>
<th>Freq</th>
<th>Z0</th>
<th>TLIMAX</th>
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YO-CUT Table:

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The mean coupling amplitude is -73.1294 dB with a standard deviation of 3.1728 dB and a maximum coupling amplitude of -64.0472 dB. The RMS mean coupling amplitude is -72.3835 dB.

MAGNITUDE (y0-cut)

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<tr>
<td>-0.8444769</td>
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</table>

The mean coupling amplitude is -73.1286 dB with a standard deviation of 3.1728 dB and a maximum coupling amplitude of -66.0472 dB.

The RMS mean coupling amplitude is -72.3838 dB.
MAGNITUDES OF COUPLING QUOTIENT FOR XO-CUT
Appendix C.* Subroutine FOURT(DATA, NN, NDIM, ISIGN, IFORM, WORK)

Purpose: To compute the discrete Fourier transform of the array DATA using the fast Fourier transform algorithm.

Arguments:
DATA is a multidimensional complex array whose real and imaginary parts are adjacent in storage, such as FORTRAN IV places them.
NN is an array giving the lengths of the array in each dimension.
NDIM is the number of dimensions of the array DATA, hence the number of elements in array NN.
ISIGN is +1 for a forward transform -1 for a reverse transform.
IFORM If all imaginary parts of the input array are zero (input array is real), set IFORM = 0 to reduce running time by approximately 40 percent, otherwise set IFORM = +1.
WORK if all dimensions of DATA are not integral powers of 2, specify array WORK in calling routine with dimension greater than largest non $2^k$ dimension, otherwise set WORK = 0.

Methods: Using the Fast Fourier transform algorithm, FOURT replaces the array DATA with its discrete Fourier transform given by

\[
\text{TRANSFORM}(K1,K2,\ldots) = \sum_{J1=1}^{NN(1)} \sum_{J2=1}^{NN(2)} \text{DATA}(J1,J2) e^{i \frac{2\pi ISIGN (J1-1)(K1-1)}{NN(1)} + \frac{(J2-1)(K2-1)}{NN(2)} + \ldots}.
\]

For a more complete description of the subroutine and its usage, see the comments included at the beginning of its listing or the supplementary comments by the programmer, Norman Brenner of MIT.

Uses external library functions COS, SIN, FLOAT, and MAXO.

Note: Brenner, Norman, "FOUR2 and FOURT program description," private communication, 1968.

*This appendix is taken from appendix A.1.11 of Stubenrauch and Yaghjian [2].
THE COOLEY-TUKEY FAST FOURIER TRANSFORM IN USASI BASIC FORTRAN

1. THE FORTY-EIGHT POINT (DATA, HH, WDIP, ISIGN, IFORM, WORK)

2. FORTRAN IV

3. TRANSFORM((X,...)) = SUM([DATA(J1),...]*EXP(-i*Pi*SORT(-1))FORTRAN IV)

4. J=1,J1,1/J(J2-1),J2,J1,1/J(J2-1)MM(12).SUMMED FOR ALL

5. J1, K BETWEEN 1 AND NMM(J1), J2 BETWEEN 1 AND NMM(J2), ETC.

6. THERE IS NO LIMIT TO THE NUMBER OF SUBSCRIPTS. DATA IS A

7. MULTIDIMENSIONAL COMPLETE ARRAY WHOSE REAL AND IMAGINARY

8. PARTS ARE ADJACENT IN STORAGE, SUCH AS FORTRAN IV PLACES THEM.

9. IF ALL IMAGINARY PARTS ARE ZERO (DATA ARE DISGUESSED REAL), SET

10. IFORM TO ZERO TO CUT THE RUNNING TIME BY UP TO FOURTY PERCENT.

11. OTHERWISE, IFORM = 1, THE LENGTHS OF ALL DIMENSIONS ARE

12. STORED IN ARRAY NHH, LENGTH NMM(J1). THEY MAY BE ANY POSITIVE

13. INTEGRAL. TWO THE PROGRAM RUNS FASTER ON COMPOSITE INTEGERS, AND

14. ESPECIALLY FAST ON NUMBERS RICH IN FACTORS OF TWO. ISIGN = 1

15. NO = -1. IF A -1 TRANSFORM IS FOLLOWED BY A +1 ONE FOR A +1

16. ONE -1, THE ORIGINAL DATA REAPPEAR, MULTIPLIED BY iOT (NMM(J1))

17. NMM(J2),...). TRANSFORM VALUES ARE ALWAYS COMPLEX, AND ARE RETURNED

18. IN ARRAY DATA, REPLACING THE INPUT. IN ADDITION, IF ALL

19. DIMENSIONS ARE NOT POWERS OF TWO, ARRAY WORK MUST BE SUPPLIED,

20. COMPLETE OR LENGTH POLAR TO THE LONGEST NON 2000 DIMENSION.

21. OTHERWISE, REPLACE WORK BY ZER0 IN THE CALLING SEQUENCE.

22. NORMAL FORTRAN DATA ORDERING IS EXPECTED, FIRST SUBSCRIPT VARYING

23. FASTEST. ALL SUBSCRIPTS REGAT TO ONE.

24. RUNNING TIME IS MUCH SHORTER THAN THE NAIVE NTTOT=2, BEING

25. GIVEN BY THE FOLLOWING FORMULA, OF COMPOSE NTTOT INTO

26. POKP @ POKP @ POKP @ ... LET SUM2 = SUM2, SUMF = SUMF + SUMF

27. + ... AND 6P = K3 + K5 + ... THE TIME TAKEN BY A MUL-1-

28. DIMENSIONAL TRANSFORM ON THOSE DATA IS T = T0 + NTTOT(T1)

29. T0 = TIME SUMF=SUMF, T1 = TIME OF THE CDC 3300 (FLOATING POINT ADD TIME

30. OF SIX MICROSECONDS), T0 = 3000 + NTTOT(3000+3000+3000)

31. 3200(M) MICROSECONDS ON COMPLEX DATA. IN ADDITION, THE

32. ACCURACY IS GREATLY IMPROVED, AS THE RNG RELATIVE ERROR IS

33. COMPUTED BY nSUM(Factors(j)>1j5, WHERE 0 IS THE NUMBER

34. OF TERMS IN THE FLOATING POINT FRACTION AND FACTORS(j) ARE THE

35. PRIME FACTORS OF NTTOT.

36. PROGRAM BY规范 BRENHALL FROM THE BASIC PROGRAM BY CHARLES

37. BALOH. RALPH ALFRED SUGGESTED THE IDEA FOR THE DIGIT REVERSAL.

38. WIT LINCOLN LABORATORY, AUGUST 1967. THIS IS THE FASTEST AND MOST

39. VERSATILE VERSION OF THE FFT KNOWN TO THE AUTHOR. SHORTER PROGR

40. RAMS FOURS AND FOURS RESTRICT DIMENSION LENGTHS TO POWERS OF TWO.

41. SEE ---- IEEE AUDIO TRANSACTIONS (JUNE 1969) SPECIAL ISSUE ON FFT.

42. FORTRAN IV

43. THE DISCRETE FOURIER TRANSFORM PLACES THREE RESTRICTIONS UPON THE

44. FOUR

45. 1. THE NUMBER OF INPUT DATA AND THE NUMBER OF TRANSFORM VALUES

46. MUST BE THE SAME.

47. 2. BOTH THE INPUT DATA AND THE TRANSFORM VALUES MUST REPRESENT

48. FORM OF SPACED POINTS IN THEIR RESPECTIVE DOMAINS OF TIME AND

49. FREQUENCY. CALLING THESE SPACINGS DELTAT AND DELTAF, IT MUST BE

50. TRUE THAT DELTAF=FRQ/(NMM(J1)*DELTAT). OF COURSE, DELTAT NEED NOT

51. BE THE SAME AND EVERY DIMENSION.

52. 3. CONCEPTUALLY AT LEAST, THE INPUT DATA AND THE TRANSFORM OUTPUT

53. REPRESENT SINGLE CYCLES OF PERIODIC FUNCTIONS.

54. EXAMPLE 1. THREE-DIMENSIONAL FORWARD FOURIER TRANSFORM OF A

55. COMPLEX ARRAY DIMENSIONED 32 BY 29 BY 13 IN FORTRAN IV.

56. FOUR

57. DIMENSION DATA(32,25,13), WORK(1001,4NH(3)

58. C

59. COMPLEX DATA

60. DATA(32,25,13)

61. C

62. DATA(1,J,K) = COMPLEX VALUE

63. CALL FOURTRDATA(1,J,K,1,1,WORK)

64. C

65. EXAMPLE 2. ONE-DIMENSIONAL FORWARD TRANSFORM OF A REAL ARRAY OF

66. LENGTH 66 IN FORTRAN IV.

67. DIMENSION DATA(2,64)

68. DATA(1,64)

69. CALL FOURTRDATA(1,J,1,1,WORK)
DIMENSION DATA (1), NN (1), IFACT (32), WORK (1)

40 WP = 0.
41 WT = 0.
42 MTP = 0.
43 WTP = 0.
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149
\begin{verbatim}
I = 9
DO 290 J = 2, NWTOT
DATA (JH) = DATA (H)
290 I = I + 2
300 IF (IIPNG = NPI1) THEN 300, 300, 330
301 IIPNG = NPI1
310 IF (IIPNG = NPI2 * (1 + NPREV / 2)) THEN 310
C
C  \textbf{SHUFFLE ON THE FACTORS OF TWO IN N. AS THE SHUFFLING}
C  CAN BE DONE BY SIMPLE INTERCHANGE, NO WORKING ARRAY IS NEEDED}
C
320 IF (NMTWO = NPI1) THEN 320, 320, 330
330 NPREV = NPI2 / 2
J = 1
DO 340 I = 1, NPI1, NON2
IF (I = J) THEN 340, 340, 350
DO 350 J = J + J3 - J2
TEMPI = DATA (J)
DATA (J) = DATA (J3)
DATA (J3 + 1) = DATA (J3)
DATA (J3) = TEMPI
350 IF (J > M) THEN 350, 350, 360
360 M = NPI2
370 IF (J = 1) THEN 370, 370, 380
380 J = J - M
Q = Q / 30
390 IF (Q = NON2) THEN 390, 390, 390
C
C  \textbf{READ LSN FOR FACTORS OF TWO. PERFORM FOURIER TRANSFORMS OF}
C  \textbf{FOURT 155}
C  \textbf{FOURT 156}
C  \textbf{FOURT 157}
C  \textbf{FOURT 158}
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C  \textbf{FOURT 230}
C  \textbf{FOURT 231}
\end{verbatim}

950 \texttt{KSTEP = 4 \* KDF} \\
\texttt{DN = 30, K1 = KMIN, MTOT, KSTEP} \\
\texttt{K2 = K1 \* KDF} \\
\texttt{K3 = K2 \* KDIF} \\
\texttt{K4 = K3 \* KDF} \\
\texttt{IF (KDF = -NON?1560, 960, 999) FOURT 237} \\
\texttt{U20 = DATA (K1) \* DATA (K2) FOURT 238} \\
\texttt{U21 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 239} \\
\texttt{U22 = DATA (K3 \* DATA (K4) FOURT 240} \\
\texttt{U23 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 241} \\
\texttt{U24 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 242} \\
\texttt{U25 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 243} \\
\texttt{U26 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 244} \\
\texttt{U27 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 245} \\
\texttt{U28 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 246} \\
\texttt{U29 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 247} \\
\texttt{U30 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 248} \\
\texttt{U31 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 249} \\
\texttt{U32 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 250} \\
\texttt{U33 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 251} \\
\texttt{U34 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 252} \\
\texttt{U35 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 253} \\
\texttt{U36 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 254} \\
\texttt{U37 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 255} \\
\texttt{U38 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 256} \\
\texttt{U39 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 257} \\
\texttt{U40 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 258} \\
\texttt{U41 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 259} \\
\texttt{U42 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 260} \\
\texttt{U43 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 261} \\
\texttt{U44 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 262} \\
\texttt{U45 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 263} \\
\texttt{U46 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 264} \\
\texttt{U47 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 265} \\
\texttt{U48 = DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 266} \\
\texttt{G..N = 420} \\
\texttt{U50 = U41 \* U42 FOURT 267} \\
\texttt{U51 = U43 \* U44 FOURT 268} \\
\texttt{DATA (K1) = U30 \* U31 FOURT 269} \\
\texttt{DATA (K2) = U32 \* U33 FOURT 270} \\
\texttt{DATA (K3) = U34 \* U35 FOURT 271} \\
\texttt{DATA (K4) = U36 \* U37 FOURT 272} \\
\texttt{DATA (K1) = U38 \* U39 FOURT 273} \\
\texttt{DATA (K2) = U40 \* U41 FOURT 274} \\
\texttt{DATA (K3) = U42 \* U43 FOURT 275} \\
\texttt{DATA (K4) = U44 \* U45 FOURT 276} \\
\texttt{DATA (K1) \* DATA (K2) \* DATA (K3 \* DATA (K4) FOURT 277} \\
\texttt{KMIN = A \* (KMIN - J3 + J3) FOURT 278} \\
\texttt{KDF = KSTEP} \\
\texttt{IF (KDF = -MR0) 560, 640} \\
\texttt{440 CONTINUE} \\
\texttt{M = MMAX - D} \\
\texttt{IF (KMIN) 650, 660, 660} \\
\texttt{450 TEMPQ = WR} \\
\texttt{WR = -U1} \\
\texttt{U1 = TEMPQ} \\
\texttt{G..N = 450} \\
\texttt{460 TEMPQ = WR} \\
\texttt{WR = U1} \\
\texttt{U1 = TEMPQ} \\
\texttt{G..N = 440} \\
\texttt{470 IF (D = LMAX) 670, 680, 690} \\
\texttt{A40 TEPQ = WR} \\
\texttt{WR = WR + USTP0 = UI + WSTPI + WR} \\
\texttt{980 WI = WI + WSTRE + TEPQ + WSTPI + WI} \\
\texttt{990 WI = V + TEPQ + WSTP = WSTPI + WI} \\
\texttt{995 G..N = 490} \\
\texttt{C} \\
\texttt{C CHECK IF FACTORS NOT EQUAL TO TWO. APPLY THE TWIDDLE FACTOR} \\
\texttt{C IF (STP(15) = 2) \* 2 = 1) (J2-1 \* DJ2-1) \* (NP1 \* IFP1), THEN} \\
\texttt{C IF (J2-1 \* DJ2-1) \* IFP1 = J2-1 \* DJ2-1 \* (NP1 \* IFP1), THEN} \\
\texttt{C CONJUGATE SYMMETRIES.} \\
\texttt{C} \\
\texttt{700 IF (KMIN = NP2-10, 100, 990} \\
\texttt{710 IFP1 = MN2} \\
\texttt{720 IF = 1}
CONTINUE

IF (ICASE = 1) GOTO 990, 990, 990

990 WP = WSTTP + 1,

WI = WSTPI

GO TO 910

910 TEMP = WP

WI = TEMP + WSTPI + WP

IF (ICASE = 3) GOTO 910, 930

970 IF (IP1 = 2) GOTO 990, 930, 930

990 K = 1

IF (12 = 19 + WP - WP1)

DATA (I2) = WEND (K)

DATA (I2 + 1) = WEND (K + 1)

K = K + P

GO TO 990

405 C COMPLETE A REAL TRANSFORM IN THE 1ST DIMENSION, N ODD, BY COM-

C JUGATE SYMMETRIES AT EACH STAGE.

410

JMAX = 19 + WP - WP1

NO G70 J3 = 13, J96AX, WP1

JMAX = J3 + WP - J970

NO G70 J2 = J2, J96AX, J97P

JMAX = J2 + J97P - WP1

JIC7 = J3 + J97P + JSTP - J7

NO G70 J1 = J2, JIC7, J9P2

C = 1 + J1 - J7

DATA (J1) = WEND (K)

DATA (J1 + 1) = WEND (K + 1)

TF (J1 - J7)700, 970, WP0

470 JIC7 = JIC7 + JSTP - J7

CONTINUE

IF = IF + 1

IF1 = IFP2

IF (IP1 = WP1) 9900, 990, 990

495 C COMPLETE A REAL TRANSFORM IN THE 1ST DIMENSION, N EVEN, BY COM-

C JUGATE SYMMETRIES.

500 CN T0 (1960, 1960, 1960, 1960), ICASE

1050 WHALE = 1

N = N + N

TMFTA = TMDP1 / ELIFAT (4)

TF (1915)700, 1010, 1010

495

1010 TMFTA = TMFTA

1060 SINTH = SIN (TMFTA / P)

WSTTP = 2, SINTH + SINTH

WSTPI = SINTH (TMFTA)

WI = WSTTP + 1

WI = WSTPI

IMIN = 3

JMIN = P + WHALE - 1

GO TO 1090

1080 J = JMIN

DD JMIN 1 = JMIN, WSTP, WP

3HD = (DATA (I + 1) - DATA (J)) / P.

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1040  IF (1STGN)1070, 1090, 1090
1070  DO 1090  J = I*1, NTO, NR2
1090  NR2 = NR2 + NR2
    NTO = NTO + NTO
    J = NTO + 1
  IMAX = NTO / 2 + 1
1100  IMIN = IMAX - 2 - NH Alf
    I = IMIN
    GO TO 1170
1110  DATA (J) = DATA (I)
    DATA (J + 1) = DATA (I + 1)
1120  IF (1) = J - 1170, 1130
1130  DATA (J) = DATA (IMAX) - DATA (IMIN + 1)
    DATA (J + 1) = 0.
    IF (1) = J - 11170, 1170
1140  DATA (J) = DATA (I)
    DATA (J + 1) = DATA (I + 1)
1150  IF (1 = J - 2
    J = 2
1160  DATA (J) = DATA (IMAX) + DATA (IMIN + 1)
    DATA (J + 1) = 0.
    IMAX = IMIN
    GO TO 1170
1170  DATA (1) = DATA (1) + DATA (2)
    DATA (2) = 0.
    GO TO 1260
C
C COMPLETE A REAL TRANSFORM FOR THE END OR 3RD DIMENSION BY
C CONJUGATE SYMMETRIES.

1180  IF (1180 = 11180, 1160, 1118)
1190  DO 1210  I = 1, NTO, NR2
1200  IMAX = I + NR2 - NR2
    DO 1200  I = 1, NTO, NR2
    IMIN = I + I180
    IMAX = I + NR2 - 2
    IMAX = ? + T3 * NR1 - IMIN
    IF (1 = I1B1120, 1120, 1200
1210  J = IMAX + NR0
    DATA (1) = DATA (J)
    DATA (1) = DATA (J + 1)
    DATA (1) = DATA (J + 1)
1220  J = J - ?
1230  J = J + IMAX + NR0
    DATA (1) = DATA (J)
    DATA (1) = DATA (J + 1)
1240  J = J - NR0
C
C END OF LOOP ON EACH DIMENSION
C
1250  NR0 = NR1
    NR1 = NR2
1260  RETURN

FMAIN
Appendix D.* Subroutine PLT120R(X,Y,XMAX,XMIN,YMAX,YMIN,LAST,ISYMBOL,NO,MOST)

Purpose: To make a page plot of array Y versus array X.

Arguments:

X = Array containing abscissa values of the function to be plotted.
Y = Array containing ordinate values of the function to be plotted.
XMIN = Minimum abscissa value.
XMAX = Maximum abscissa value.
YMIN = Minimum ordinate value.
YMAX = Maximum ordinate value.
LAST = Number of points to be plotted.
ISYMBOL = A Hollerith variable containing the plotting symbol, e.g., to plot with the symbol "X" ISYMBOL = 1HX.
NO = Number of plot on page.
MOST = Total number of plots to be made on one page.

Discussion: This subroutine produces a "quick and dirty" plot of Y versus X on the page printer. The size of the plotting area is 50 x 120 units. Multiple plots may be made on a single page. A page eject is performed before the first plot of a series is begun, but no eject is performed after completion of a series. This allows a title to be printed at the bottom of the plot. The subroutine uses inline function FLOAT.

*This appendix is taken from Appendix A.1.12 of Stubenrauch and Yaghjian [2].
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4HIT 1
5HIT 1
6HIT 1
7HIT 1
8HIT 1
9HIT 1
10HIT 1
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12HIT 1
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Computation of Antenna Side-Lobe Coupling in the Near Field Using Approximate Far-Field Data

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Computer programs, in particular CUPLNF and CUPLZ, are presently in existence to calculate the coupling loss between two antennas provided that the amplitude and phase of the far field are available. However, for many antennas the complex far field is not known accurately. In such cases it is nevertheless possible to specify approximate far fields from a knowledge of the side-lobe level of each antenna along the axis of separation, and the electrical size of each antenna. To determine the effectiveness of using approximate side-lobe level data instead of the detailed far fields, we chose as our test antennas two hypothetical, linearly polarized, uniformly illuminated circular antennas for which the exact far fields are given by a simple analytic expression. The exact far fields are supplied to the program CUPLNF to compute the exact near-field coupling loss. Approximate fields are supplied to a new program ENVLP developed for the purpose of computing the approximate near-field coupling loss. The comparison of the results from ENVLP to those of CUPLNF indicates that the use of approximate far fields gives an estimate of the coupling loss which is good to about ±5 dB. In addition, the plane-wave transmission formula for coupling between two antennas is used to estimate upper-bound values of coupling loss. These upper bounds are compared with the maximum coupling losses obtained from programs CUPLNF and ENVLP.

antenna coupling; antenna theory, coupling loss; near-field measurements.

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