NBSIR 82-1674

COMPUTATION OF ANTENNA SIDE-LOBE COUPLING IN THE NEAR FIELD USING APPROXIMATE FAR-FIELD DATA

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U.S. DEPARTMENT OF COMMERCE, Malcolm Baldrige, Secretary

NATIONAL BUREAU OF STANDARDS, Ernest Ambler, Director

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Glossary of Symbols for The Text

a: radius of the antenna aperture.
a_T, a_R: radius of the transmitting and receiving antennas, respectively.
a_o: incident wave-amplitude in the transmitting antenna feed.
A_{MAX}: the maximum amplitude of the far field in the direction of the axis of separation.
A_{TMAX}, A_{RMAX}: the maximum amplitude of the far field in the direction of the axis of separation for the transmitting and receiving antennas, respectively.
b_o': the emergent wave-amplitude in the receiving antenna feed.
C': $\dfrac{1}{\eta_o Z_o (1 - \Gamma_o \Gamma_L)}$.
d: the separation distance between antennas.
D_T, D_R: the diameter of the smallest sphere which circumscribes the transmitting and receiving antennas, respectively.
$\underline{E}$: the electric field.
$\underline{f}', \underline{f}$: the far field of the receiving and transmitting antennas, respectively, with components f_x', f_y', f_z' and f_x, f_y, f_z.
G: the antenna gain.
G_T, G_R: the gain of the transmitting and receiving antennas, respectively.
J_1: the Bessel function of first order.
$\underline{k}$: the propagation vector with components k_x, k_y, γ.
k: $\sqrt{\underline{k} \cdot \underline{k}} = 2\pi/\lambda$.
k_o: $k(D_T + D_R)/2d$.
$\underline{K}$: $k_x \hat{e}_x + k_y \hat{e}_y$.
K: $\sqrt{\underline{K} \cdot \underline{K}}$.
P_{input}: the input power to the antenna.
$\underline{r}$: position vector with components x, y, z.
r: magnitude of $\underline{r}$.
$\underline{R}$: $x\hat{e}_x + y\hat{e}_y$.
S: the relative side-lobe level in the direction of the separation axis.
S_T, S_R: the value of S for the transmitting and receiving antennas, respectively.
Z_o: the wave impedance of free space.
γ: the z-component of $\underline{k}$.
Γ_L: the reflection coefficient of the receiving load.
Γ_o: the reflection coefficient of the receiving antenna.
η_o: the characteristic admittance for the propagated mode in the waveguide feed of the receiving antenna.
λ: the wavelength.
ϕ_T, θ_T, ψ_T: the Eulerian angles of the transmitting antenna.
ϕ_R, θ_R, ψ_R: the Eulerian angles of the receiving antenna.

COMPUTATION OF ANTENNA SIDE-LOBE COUPLING IN THE NEAR FIELD USING APPROXIMATE FAR-FIELD DATA

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Computer programs, in particular CUPLNF and CUPLZ, are presently in existence to calculate the coupling loss between two antennas provided that the amplitude and phase of the far field are available. However, for many antennas the complex far field is not known accurately. In such cases it is nevertheless possible to specify approximate far fields from a knowledge of the side-lobe level of each antenna along the axis of separation, and the electrical size of each antenna. To determine the effectiveness of using approximate side-lobe level data instead of the detailed far fields, we chose as our test antennas two hypothetical, linearly polarized, uniformly illuminated circular antennas for which the exact far fields are given by a simple analytic expression. The exact far fields are supplied to the program CUPLNF to compute the exact near-field coupling loss. Approximate fields are supplied to a new program ENVLP developed for the purpose of computing the approximate near-field coupling loss. The comparison of the results from ENVLP to those of CUPLNF indicates that the use of approximate far fields gives an estimate of the coupling loss which is good to about ±5 dB. In addition, the plane-wave transmission formula for coupling between two antennas is used to estimate upper-bound values of coupling loss. These upper bounds are compared with the maximum coupling losses obtained from programs CUPLNF and ENVLP.

Key words: antenna coupling; antenna theory; coupling loss; near-field measurements.

1. Introduction

Three years ago, the Antenna Systems Metrology Group, Electromagnetic Fields Division of the National Bureau of Standards, under the sponsorship of the Electromagnetic Compatibility Analysis Center, completed the development of a highly efficient computer program, CUPLNF, for calculating the coupling loss between two antennas given the far fields of each antenna [1,2]. For antennas arbitrarily oriented and separated in free space, CUPLNF computes the coupling loss at a single frequency as a function of antenna displacement in a plane transverse to the axis of separation of the antennas. Multiple reflections between antennas are neglected but no other restrictive assumptions are involved. CUPLNF automatically computes electric fields from the transmitting antenna when a "virtual antenna" of uniform far field replaces the receiving antenna. Coupling loss for antennas hundreds of wavelengths in diameter is computed in a few minutes of CPU time within the central memory core, e.g., of a CDC 6600.[1]

The major limitation of CUPLNF is its requirement for the magnitude and phase of the electric far-field components of each antenna within the solid angle mutually subtended by the antennas. For example, if two antennas are oriented so that coupling occurs mainly through their side-lobe region, the complex vector far field of each antenna covering this angular side-lobe region must be supplied to the program CUPLNF.

In practice, one may not have such detailed information on the side-lobe far fields to estimate the coupling loss between two antennas co-sited in their near field. Often, one has some knowledge of the side-lobe levels of one's antennas, even if detailed phase and amplitude information of the field components is not available. Thus a natural and important question to answer is whether this limited information, specifically side-lobe level of each antenna near their axis of separation, can be used to estimate the antenna coupling in the near field using a new program, ENVLP. Of course, as the separation distance between the antennas approaches the mutual Rayleigh distance, i.e., the antennas lie in each others far field, the ordinary far-field formula for coupling between two antennas can be used to estimate coupling loss. For separation distances much less than the mutual Rayleigh distance, the far-field formula would not be expected to give realistic estimates of coupling. It is in this near-field region that we set out to decide if ENVLP will give reasonable values of side-lobe coupling (±6 dB accuracy) by utilizing only side-lobe levels

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[1] The specific computer is identified in this paper to adequately describe the computer program. Such identification does not imply recommendation or endorsement by the National Bureau of Standards nor does it imply that the computer identified is necessarily the best available for the purpose.

along the axis of separation of the antennas (in addition to their size, separation distance, and feed characteristics).

To determine the efficacy of ENVLP using approximate side-lobe level information instead of the detailed far fields, we chose as our test antennas two hypothetical, linearly polarized, uniformly illuminated circular aperture antennas for which the exact far fields are given by a fairly simple analytic expression involving functions no more complicated than the first order Bessel function. With these hypothetical antennas, the exact far fields could be supplied to the program CUPLNF to compute near-field coupling loss without introducing the errors incumbent with measured far-field data. An approximate side-lobe far field can be substituted for the exact far field and a comparison of the near-field coupling loss computed with the exact and approximate far fields can be made simply and straightforwardly.

At first thought it may seem overly optimistic to want to obtain reasonable estimates of near-field coupling having only side-lobe level information, i.e., having only the envelope of the magnitude of the far fields in the direction of separation. Fortunately, however, we can also estimate the phase variation of the side lobes of most microwave antennas merely from the frequency of operation and the radii of the antennas, more precisely, from the radius in wavelengths of the smallest sphere which circumscribes all significantly radiating parts of each antenna.

Also, even though polarization characteristics of the side-lobe far fields may be unknown, we can assume polarization match of the far-fields of the antennas over the relevant range of integration. This assumption of polarization match tends to produce an upper-bound estimate of coupling loss, which may be of appreciable value to the user. We also assume the user has a knowledge of (1) the separation distance between the antennas, and (2) the reflection coefficients in the waveguides feeding the antennas--in all a rather minimal amount of information about one's antennas.

In Appendix B, we document program ENVLP, which is a modification of the program CUPLNF requiring as input this minimal amount of input data, i.e., the antennas' radii, separation distance, reflection coefficients, and relative side-lobe levels in the direction of separation.

Finally, we work directly with the plane-wave transmission formula for coupling between two antennas to estimate upper-bound values of coupling loss, comparing these upper bounds with the maximum coupling losses obtained from program CUPLNF and program ENVLP using exact and approximate far-field input. We anticipate these simple upper-bound formulas being especially valuable when the major requirement for co-sited antennas is that the coupling interference or fields lie below a certain threshold.

2. Side-Lobe Coupling of Two Antennas Using Their Exact and Approximate Far Fields

2.1 Statement and Formulation of the Problem

Yaghjian [1] showed that the coupling between two antennas in free space, neglecting multiple reflections, is given by

$$\frac{b_o'}{a_o} = \frac{-C'}{k} \iint\limits_{K<k} \frac{\underline{f}'(-\underline{k})\cdot\underline{f}(\underline{k})e^{i\gamma d} e^{i\underline{K}\cdot\underline{R}}}{\gamma} d\underline{K} , \tag{1}$$

where $C' = \frac{1}{\eta_o Z_o (1 - \Gamma_o \Gamma_L)}$, and $1/(1 - \Gamma_o \Gamma_L)$ is the mismatch factor of the receiving antenna, η_o is the characteristic admittance of the propagated mode in the waveguide feed of the receiving antenna, Z_o is the wave impedance of free space, k is the magnitude of the propagation vector, $\underline{K} = k_x e_x + k_y e_y$ is the transverse part of the propagation vector, $\gamma = (k^2 - K^2)^{1/2}$, $d\underline{K}$ is an abbreviation for $dk_x dk_y$, and the coordinates ($\underline{R}$, z = d) give the position of the origin 0' fixed in the receiving antenna with respect to the (x,y,z) coordinate system fixed at 0 in the transmitting antenna (see fig. 1). The functions $\underline{f}$ and $\underline{f}'$ are the electric far fields of the transmitting and receiving antennas, respectively, without the presence of the other, Γ_o and Γ_L are the reflection coefficients of the receiving antenna and receiving load respectively, a_o is the amplitude of the input to the transmitting antenna, and b_o' is the amplitude of the output of the receiving antenna. Formula (1) assumes the receiving antenna is reciprocal, although the formula easily generalizes to include non-reciprocal antennas. Yaghjian [1] also showed that for most practical purposes the integration range in (1) could be limited to $K/k < (D_T + D_R)/2d$ for $R \cong 0$ provided $(D_T + D_R)/2 < d < (D_T + D_R)^2/\lambda$. D_T is the diameter of the smallest sphere which circumscribes the transmitting antenna and D_R is the diameter of the smallest sphere circumscribing the receiving antenna. (If D_T or D_R is less than twice the wavelength, γ, D_T, or D_R is replaced by 2λ.) For coupling in the transverse plane over the range $-(D_T + D_R) < R < D_T + D_R$ the integration range should be doubled [1].

There are some situations where detailed information of the far fields ($\underline{f}'$, $\underline{f}$) is lacking and it is desirable to get an estimate of the coupling between two antennas. For example, both amplitude and phase information may not be available. We would like, therefore, to consider some possible methods of approximating the coupling and in particular of finding a good estimate for the maximum coupling between two antennas in the general direction of the separation axis (the separation axis is drawn from a point centrally located on the transmitting antenna to a point centrally located on the receiving antenna for the antennas in their initial position, $\underline{R} = 0$).

To find the maximum coupling in the general direction of the separation axis we shall calculate the coupling over a range of values of $\underline{R}$ ($\underline{R}$ is perpendicular to the separation axis). Specifically, we shall move the receiving antenna in the x and y directions, over a range from $-(D_T + D_R)$ to $+(D_T + D_R)$ for both the x and y directions. In order to save computer time and cost we will hold y = 0 while varying x (the XO cut) and hold x = 0 while varying y (the YO cut).

If we are to determine how accurate our approximate results for coupling loss are, we need to compare them to the results that we would have obtained if we knew and used the exact far fields of the antennas for finding the coupling quotients. For this purpose we will use two hypothetical circular antennas and compare results obtained using the exact far fields to those obtained using approximate far fields. These results will be compared for different values of the separation distance, diameters of the antennas, frequency, and orientation of the antennas.

2.2 The Hypothetical Circular Antenna

2.2.1 The Exact Far Field

We consider the case of a uniformally illuminated circular aperture with the transverse electric field polarized in the x-direction. Then the far field for a point in the (θ,ϕ) direction is given by [3]:

$$f_x = \frac{ka}{\sqrt{\pi}} \cos\theta \frac{J_1(ka \sin\theta)}{ka \sin\theta} \tag{2a}$$

$$f_y = 0 \tag{2b}$$

$$f_z = - \frac{ka}{\sqrt{\pi}} \sin\theta \cos\phi \frac{J_1(ka \sin\theta)}{ka \sin\theta} \, . \tag{2c}$$

For this formula, the circular aperture lies in the xy plane with the center of the aperture at the origin. The symbols, θ,ϕ denote the usual angles in spherical coordinates, k is the magnitude of the propagation vector, a is the radius of the aperture, and J_1 is the Bessel function of first order. An example of this pattern can be seen in figure 2, where ka = 20.94.

2.2.2 The Approximate Far Field for Approximation-1

Approximation-1 is specifically geared to the uniformly illuminated circular aperture antenna. For this approximation we replace

$$\frac{J_1(ka\sin\theta)}{ka\sin\theta} \quad \text{by} \quad \sqrt{\frac{2}{\pi ka\sin\theta_o}}\;\frac{\cos(ka(\theta-\theta_o))}{ka\sin\theta_o}\,,$$

where θ_o is the direction of the separation axis relative to the preferred antenna coordinate system. θ_o equals θ_T for the transmitting antenna and θ_R for the receiving antenna (see the sketches on figures 6 through 20). This electric field pattern can be found for $\theta_o = 60°$ in figure 3. The values of the field are given relative to the main beam at $\theta_o = 0°$ in figure 2, i.e. the field is normalized to a main beam equal to 1.

The above approximation may be justified as follows. $J_1(ka\sin\theta)/ka\sin\theta$ may be replaced using the asymptotic expansion for J_1. This yields:

$$\frac{J_1(ka\sin\theta)}{ka\sin\theta} \sim \sqrt{\frac{2}{\pi ka\sin\theta}}\;\frac{\cos(ka\sin\theta - 3\pi/4)}{ka\sin\theta}\,.$$

The cosine term represents the variation of the sidelobe while the rest of the term is the amplitude of the envelope. We replace the terms involving the amplitude of the envelope by their values at θ_o and expand the cosine term about θ_o. Thus, $\cos(ka\sin\theta - 3\pi/4) \sim \cos(ka\sin\theta_o - 3\pi/4 + ka(\theta-\theta_o)\cos\theta_o)$. The $ka\sin\theta_o$ can be lumped with $-3\pi/4$ into a phase factor which we will ignore since we are finding only an average coupling over a solid angle. The $\cos\theta_o$ term has been replaced by 1 because actual antennas do not display this $\cos\theta_o$ dependence of sidelobe width peculiar to the hypothetical finite planar aperture distribution.

2.2.3 The Approximate Far Field for Approximation-2

The approximation-2 far field is a general approximation that can be made for a large class of coupling cases. For this approximation we replace $\underline{f}'(-\underline{k})\cdot\underline{f}(\underline{k})$ by

$$A_{TMAX}\cos k_x a_T \cos k_y a_T\, A_{RMAX}\cos k_x a_R \cos k_y a_R\,.$$

A_{TMAX} and A_{RMAX} are the maximum magnitudes of the far fields in the general direction of the separation axis (i.e., the side-lobe level) for the transmitting and receiving antennas, respectively, $k_x = k\sin\theta\cos\phi$, and $k_y = k\sin\theta\sin\phi$. Hence, A_{TMAX} and A_{RMAX} are the magnitudes of the approximate electric far fields in the direction of the separation axis and are given in absolute SI (mksA) units. In Appendix A we derive A_{TMAX} and A_{RMAX} in terms of the side-lobe levels and the gains of the transmitting and receiving antennas. a_T and a_R are the radii of the smallest spheres circumscribing the transmitting antenna and the receiving antenna, respectively.

We have chosen the above approximation because for many antennas the components of the far-field patterns vary roughly as $\cos k_x a \cos k_y a$, where a is the radius of the smallest sphere circumscribing all the significantly radiating parts of the antenna.[2] Note that for approximation-2 we assume that the polarization of the receiving and transmitting antennas are matched and thus approximation-2 tends to yield an upper bound. The approximation-2 pattern for $\phi = 0°$ or $90°$ and the separation axis in the direction of $\theta = 60°$ can be found in figure 4. The values of the field are given relative to the main beam at $\theta = 0°$ in figure 2.

2.2.4 Limitations on the Approximations

Both approximation-1 and approximation-2 are valid over a relatively narrow range of angles. In particular, the approximations should be valid (under the above stated limitations) if the amplitude of the envelope of the far field of each antenna varies by not more than about 3 dB over the range of angles mutually subtended by the transmitting and receiving antennas. Thus, for any case in which the integration of equation (1) must be performed over a large range of angles, approximation-1 and approximation-2 may be poor. This will, in general, include those cases involving coupling with the main beam. It will also include electrically small broad beam antennas and the unusual cases where both the receiving and transmitting antennas are identical and also have identical Eulerian angles [1] (see figure 5 for a definition of these angles). In addition, for $\theta_0 = 0$ the approximation of the far field for

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[2] The far side-lobe region of the far-field pattern is predominantly caused by diffraction from edge points of the antenna. These edge points are usually separated by a distance of approximately an antenna diameter and this leads to a side-lobe pattern which has a null approximately every $\lambda/2a$ radians, as numerous hypothetical and measured far-field patterns confirm (see for example Johnson et al. [4], and Newell and Crawford [5]). We assume a smooth variation in the form of a cosine, and that the k_x and k_y variation is separable. This leads to the $\cos k_x a \cos k_y a$ dependence about the axis of separation - a fairly reasonable approximation provided the antenna is not highly elongated (i.e., the length in one direction is not much greater than the length in the other direction). In the case of a highly elongated antenna the value of a is approximately half the longer side so that we obtain a variation which is good in the direction of the long side but poor in the direction of the short side. For such elongated antennas, a better approximation can be obtained by using a variation of the form $\cos k_x a_x \cos k_y a_y$ where a_x is half the length of the long side and a_y is half the length of the short side, assuming the x and y axes are aligned with the long and short sides of the antenna, respectively.

approximation-1 goes to infinity. Thus, approximation-1 is never good for the main beam. Finally, d must be in the range $\frac{D_T + D_R}{2} < d < \frac{(D_T + D_R)^2}{\lambda}$.

2.3 Results and Comparisons

The results using the exact far fields and the approximation-1 and approximation-2 far fields in equation (1) were obtained using computer programs based on CUPLNF [2]. The program for approximation-2 (titled ENVLP) was written for general use and can be found documented in Appendix B.

Results were obtained for various values of antenna orientation, antenna diameter, separation distance and frequency, and are summarized in table 2.1 and in figures 6 through 20.

Column 1 of table 2.1 is an identifier which corresponds to the identifier in the caption of figures 6 through 20. Columns 2 to 4 give the Eulerian angles of the transmitting antenna; columns 5 through 7 give the Eulerian angles of the receiving antenna; columns 8 and 9 give the diameters of the transmitting and receiving antennas, respectively, in units of wavelength. Columns 10 and 11 give the side-lobe levels relative to the main beam and within the integration range (as specified above) for the transmitting and receiving antennas, respectively. Column 12 gives the separation distance in terms of the mutual Rayleigh distance; column 13 gives the frequency in hertz. Columns 14 through 16 give the maximum coupling amplitude as given by the use of the exact far field, the approximation-1 far field, and the approximation-2 far field, respectively. The maximum coupling magnitude implied by the far-field formula is given in column 17.

The far-field formula which is used to determine the maximum coupling is

$$\left|\frac{b'_o}{a_o}\right| = \left|\frac{A_{TMAX} A_{RMAX}}{\eta_o Z_o (1 - \Gamma_o \Gamma_L)} \frac{\lambda}{d}\right| . \qquad (3)$$

Column 18 gives the maximum coupling amplitude as implied by the upper-bound equations (9), which will be discussed in section 3. Finally, columns 19, 20, and 21 contain the RMS mean coupling for the exact, approximation-1, and approximation-2 far fields, respectively.

Figures 6 through 20 are plots of the magnitude of the coupling quotients in the x-direction at $y = 0$ (the XO cut) for the exact far fields (—), approximation-1 (....), and approximation-2 (— —). The YO cut is not shown because it is similar to the XO cut. Each figure corresponds to one of the cases in table 2.1.

Table 2.1 Results of Calculations

Column 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Case													Maximum Coupling Magnitude					RMS Mean Coupling Magnitude		
	ϕ_T (- - -	θ_T	ψ_T in degrees	ϕ_R	θ_R - -	ψ_R -)	D_T (λ)	D_R (λ)	S_T dB (relative to the main beam)	S_R dB	d (units of mutual Rayleigh distance)	Frequency (Hz)	Exact dB	Approximation-1 dB	Approximation-2 dB	Far-Field Formula dB	Equation (9) dB	Exact dB	Approximation-1 dB	Approximation-2 dB
ALPHA-1	0	60	0	0	60	180	6.67	6.67	31.4	31.4	0.19	1.0×10^{10}	-65.6	-66.6	-69.0	-62.4	-58.4(c)	-69.0	-71.2	-73.1
ALPHA-4	0	50	0	0	60	180	6.67	6.67	27.4	31.4	0.19	1.0×10^{10}	-66.7	-65.0	-66.3	-59.7	-56.0(c)	-71.5	-69.6	-70.4
ALPHA-7	0	70	0	0	60	180	6.67	6.67	33.6	31.4	0.19	1.0×10^{10}	-66.9	-67.7	-71.2	-64.6	-60.1(c)	-73.6	-72.2	-75.3
BETA-1	0	60	0	0	60	180	6.67	13.33	30.7	39.5	0.08	1.0×10^{10}	-69.8	-71.5	-66.0	-63.7	-62.5(a)	-74.7	-76.5	-72.4
BETA-2	0	60	0	0	60	180	6.67	20.00	28.8	43.1	0.05	1.0×10^{10}	-70.5	-71.4	-63.8	-61.9	-56.9(a)	-77.4	-76.1	-70.0
BETA-3	0	60	0	0	60	180	6.67	26.67	26.6	43.6	0.03	1.0×10^{10}	-71.2	-73.9	-60.3	-57.4	-49.8(a)	-78.0	-81.4	-66.5
BETA-4	0	60	0	0	60	180	13.33	13.33	37.8	37.8	0.05	1.0×10^{10}	-74.4	-73.1	-64.4	-63.2	-47.5(c)	-81.7	-77.2	-71.3
GAMMA-1	0	60	0	0	60	180	3.33	3.33	22.8	22.8	0.38	5.0×10^{9}	-55.4	-55.3	-56.7	-51.1	-50.5(c)	-58.2	-58.4	-61.9
GAMMA-2	0	60	0	0	60	180	13.33	13.33	40.8	40.8	0.01	2.0×10^{10}	-80.8	-78.6	-78.6	-75.1	-65.3(c)	-85.2	-83.1	-83.6
DELTA-1	0	60	0	0	60	180	26.67	53.33	51.6	60.6	0.26	1.0×10^{10}	-116.1	-116.8	-129.3	-115.7	-113.0(b)	-122.3	-121.9	-133.7
DELTA-2	0	60	0	0	60	180	6.67	13.33	33.4	42.4	0.58	1.0×10^{10}	-85.2	-87.5	-91.8	-86.3	-84.8(d)	-88.5	-90.1	-95.8
EPSILON-1	0	60	0	0	60	180	6.67	6.67	28.8	28.8	0.09	1.0×10^{10}	-60.8	-61.8	-50.3	-51.1	-41.6(c)	64.5	-66.8	-56.0
EPSILON-2	0	60	0	0	60	180	6.67	6.67	31.8	31.8	0.38	1.0×10^{10}	-71.0	-73.1	-75.4	-71.2	-70.9(c)	-77.1	-79.0	-80.2
EPSILON-3	0	60	0	0	60	180	6.67	6.67	32.8	32.8	0.75	1.0×10^{10}	-74.5	-78.6	-80.6	-78.2	-74.4(e)	-79.8	-82.6	-84.6
OMEGA	0	60	0	0	60	180	1.33	1.33	14.3	14.3	0.94	2.0×10^{9}	-48.1	-38.8	-41.9	-42.2	-41.5(e)	-51.9	-41.2	-48.0

An inspection of table 2.1 will show that the maximum coupling predicted by approximation-1 compares favorably with the result given by the exact far fields. With the exception of case Omega, the difference in the maximum coupling loss between the exact far-field result and the result from approximation-1 is less than 5 dB while for case Omega, the difference is about 10 dB. The explanation for the large difference for case Omega is that the wavelength for this case is so large that the electric field pattern lies near the first null (see, e.g., figure 2); thus, approximation-1 which is based on the envelope of the Bessel function is very poor in this case.

We find that with the exception of two cases (Beta-3 and Delta-1) the maximum coupling loss predicted by approximation-2 is within 10 dB of that given by the use of the exact far fields. For Beta-3, $D_T = 0.2$m, $D_R = 0.8$m, and $d = 1.0$ m; thus, for this case the coupling integration covers a broad angular region and the polarization does not match at the wider angles as it does at the center. This means that approximation-2, which assumes perfect polarization match over the entire range of integration, will appreciably overestimate the coupling for this case. In the case of Delta-1, even though the approximation-2 estimate of the maximum coupling is about 13 dB low, this is a relatively good estimate of coupling since the exact coupling lies below -116 dB.

If we now examine figures 6 through 20 and columns 19 through 21, we find that, in general, the RMS mean level of coupling generally agrees to within about ±5 dB of the exact for approximation-2 and considerably closer for approximation-1. On the other hand, approximate and exact coupling loss at individual points can differ by a substantially greater amount (occasionally more than 20 dB).

It is of interest to compare our results to those obtained from the far-field formula, equation (3). It can be seen that, in general, if the separation distance between the antennas is greater than about one-quarter of a mutual Rayleigh distance, $(D_T + D_R)^2/\lambda$, the far-field formula gives results closer to the exact results than do approximation-1 and approximation-2. On the other hand, if the separation distance is less than one-tenth of a mutual Rayleigh distance, the results from approximation-1 and approximation-2 are in general substantially better than those given by the far-field formula. For separation distances between one-tenth and one-quarter of a mutual Rayleigh distance both the far-field formula and approximation-1 or approximation-2 yield values of coupling loss of about the same degree of accuracy.

2.4 Conclusions

We conclude that approximation-1 and approximation-2 can be used in the computer programs to obtain a reasonable estimate of the maximum coupling in the general direction of the separation axis. It should be emphasized that while we can use approximation-1 and approximation-2 to find the maximum coupling or the mean coupling over a narrow range of directions in the general direction of the separation axis we do not necessarily get a good estimate in the exact direction of the separation axis. Approximation-1 is limited to uniformly illuminated circular aperture antennas while approximation-2 can be used for general antennas. In using either approximation-1 or approximation-2 care must be used so as to stay within the limitations stated in section 2.2.4.

In the event that the separation distance is greater than about one-quarter of a Rayleigh distance the far-field formula (equation (3)) gives better results than either approximation-1 or approximation-2 and the far-field formula should be used in those cases to estimate the maximum coupling.

The real advantage of the computer program for approximation-2 (Program ENVLP) is that it estimates the coupling loss between arbitrary antennas arbitrarily oriented in the near field of each other from a mere knowledge of (1) the separation distance between antennas, (2) the side-lobe level of each antenna in the direction of the axis of separation, and (3) the radius of each antenna.

3. Mathematical Upper Bounds Derived from Equation (1)

3.1 Assumptions and Integration

It is possible to derive an upper bound to the magnitude of b_0'/a_0 if we make some simplifying assumptions that will allow us to integrate equation (1); initially, let us make the following extremely crude assumptions which will allow us to immediately derive an upper bound for coupling quotient by performing the integration in equation (1). Afterwards a smaller, more realistic upper bound will be derived.

$$\underline{f}'(-\underline{k}) \cdot \underline{f}(\underline{k}) = A_{TMAX}A_{RMAX} \tag{4a}$$

$$\gamma = \gamma_{min} = k\left(1 - \frac{(D_T + D_R)^2}{4d^2}\right)^{1/2} \tag{4b}$$

$$e^{i\gamma d} = e^{i\underline{K}\cdot\underline{R}} = 1\ . \tag{4c}$$

A_{TMAX} and A_{RMAX} are the maximum values of $\underline{f}$ and $\underline{f}'$, respectively, near the axis of separation.

With the assumptions of equations (4) we find after integrating equation (1) from $\frac{-k(D_T + D_R)}{2d}$ to $\frac{+k(D_T + D_R)}{2d}$ that

$$\left|\frac{b'_o}{a_o}\right| \leqslant \left|\frac{1}{\eta_o Z_o(1 - \Gamma_o\Gamma_L)} \frac{A_{TMAX} A_{RMAX}}{\left(1 - \frac{(D_T + D_R)^2}{4d^2}\right)^{1/2}} \frac{(D_T + D_R)^2}{d^2}\right| . \tag{5}$$

By using this integration range we limit the validity of equation (5) to $(D_T + D_R)/2 < d < (D_T + D_R)^2/\lambda$.

Equation (5) gives an absolute upper bound to the magnitude of the coupling quotient, neglecting multiple reflections, provided that coupling does not occur through the main beam of either antenna, that $(D_T + D_R)/2 < d < (D_T + D_R)^2/\lambda$ and that the integration range we have used [i.e., $-k(D_T + D_R)/2d$ to $+k(D_T + D_R/2d)$] is adequate (as explained in section 2.2.4).

As d approaches the mutual Rayleigh distance we expect equation (5) to approach the far-field formula, equation (3). We can see that this is indeed the case if we allow $\left(1 - \frac{(D_T + D_R)^2}{4d^2}\right)^{1/2}$ to approach 1 (note that $(D_T + D_R)^2$ will be much less than $4d^2$ at the mutual Rayleigh distance) and replace one of the d's in d^2 by $(D_T + D_R)^2/\lambda$. We thus obtain

$$\left|\frac{b'_o}{a_o}\right| \leqslant \left|\frac{A_{TMAX} A_{RMAX}}{\eta_o Z_o(1 - \Gamma_o\Gamma_L)} \frac{\lambda}{d}\right| ,$$

which is just the far-field formula. If we compare equation (5) to the far-field formula for separation distances less than a mutual Rayleigh distance, equation (5) always gives a larger value.

Instead of the assumptions of equations (4), we can derive more realistic upper bounds by making the more realistic approximations

$$\underline{f}'(\underline{k}) \cdot \underline{f}(\underline{k}) = A_{TMAX}\cos\left(k_x\frac{D_T}{2} + \phi_1\right)\cos\left(k_y\frac{D_T}{2} + \phi_3\right)A_{RMAX}\cos\left(k_x\frac{D_R}{2} + \phi_2\right) \times \cos\left(k_y\frac{D_R}{2} + \phi_4\right) \tag{6a}$$

$$\gamma = \gamma_{min} = k\left(1 - \frac{(D_T + D_R)^2}{4d^2}\right)^{1/2} \tag{6b}$$

$$e^{i\gamma d} = e^{i\alpha}, \tag{6c}$$

where α is a constant and the ϕ's are arbitrary phase shifts. The last assumption (6c), is made because $e^{i\gamma d}$ varies more slowly than the cosine terms or the $e^{i\underline{K}\cdot\underline{R}}$ term for the integration range being considered. Under assumptions (6), equation (1) becomes

$$\left|\frac{b'_o}{a_o}\right| \sim \left| \frac{A_{TMAX}A_{RMAX}e^{i\alpha}}{\eta_o Z_o(1 - \Gamma_o\Gamma_L)k^2\left(1 - \frac{(D_T + D_R)^2}{4d^2}\right)^{1/2}} \int_{-k_o}^{k_o}\int_{-k_o}^{k_o} \cos\left(k_x\frac{D_T}{2} + \phi_1\right)\cos\left(k_x\frac{D_R}{2} + \phi_2\right)\right.$$

$$\left. \times \cos\left(k_y\frac{D_T}{2} + \phi_3\right)\cos\left(k_y\frac{D_R}{2} + \phi_4\right)(\cos k_x + i\sin k_x X)(\cos k_y y + i\sin k_y y)\,dk_x\,dk_y\right|, \qquad (7)$$

where $k_o = k(D_T + D_R)/2d$.

The integration of equation (7) is rather lengthy, but straightforward, and can be written after some rearrangement, as

$$\left|\frac{b'_o}{a_o}\right| \sim \left| \frac{A_{TMAX}A_{RMAX}e^{i\alpha}}{\eta_o Z_o(1 - \Gamma_o\Gamma_L)k^2\left(1 - \frac{(D_T + D_R)^2}{4d^2}\right)^{1/2}} \left\{\frac{1}{4}\left[e^{i(\phi_2 + \phi_1)}\frac{\sin\left(\left(\frac{D_T + D_R}{2} + x\right)k_o\right)}{\left(\frac{D_T + D_R}{2} + x\right)}\right.\right.\right.$$

$$+ e^{i(\phi_2 - \phi_1)}\frac{\sin\left(\left(\frac{D_R - D_T}{2} + x\right)k_o\right)}{\left(\frac{D_R - D_T}{2} + x\right)} + e^{-i(\phi_2 - \phi_1)}\frac{\sin\left(\left(\frac{D_T - D_R}{2} + x\right)k_o\right)}{\left(\frac{D_T - D_R}{2} + x\right)}$$

$$\left. + e^{-i(\phi_2 + \phi_1)}\frac{\sin\left(\left(\frac{D_T + D_R}{2} - x\right)k_o\right)}{\left(\frac{D_T + D_R}{2} - x\right)}\right]\left[e^{i(\phi_4 + \phi_3)}\frac{\sin\left(\left(\frac{D_T + D_R}{2} + y\right)k_o\right)}{\left(\frac{D_T + D_R}{2} + y\right)}\right.$$

$$+ e^{i(\phi_4 - \phi_3)}\frac{\sin\left(\left(\frac{D_R - D_T}{2} + y\right)k_o\right)}{\left(\frac{D_R - D_T}{2} + y\right)} + e^{-i(\phi_4 - \phi_3)}\frac{\sin\left(\left(\frac{D_T - D_R}{2} + y\right)k_o\right)}{\left(\frac{D_T - D_R}{2} + y\right)}$$

$$\left.\left.\left. + e^{-i(\phi_4 + \phi_3)}\frac{\sin\left(\left(\frac{D_T + D_R}{2} - y\right)k_o\right)}{\left(\frac{D_T + D_R}{2} - y\right)}\right]\right\}\right| . \qquad (8)$$

We wish to find the maximum value of equation (8). First, we notice that each term in the two pairs of brackets, [ ], is of the form $e^{\pm(\phi_i \pm \phi\ )}(\sin BK_o)/B$. We might be tempted to say that the maximum value of a term of this form is

(1/B) but this would be true only if $|Bk_o| > 1$. For $|Bk_o| < 1$ this term has a maximum value of k_o at $B = 0$. We note that there are four values of x and four values of y for which $B \to 0$. They are

$x, y = -\frac{(D_T + D_R)}{2}, -\frac{(D_T - D_R)}{2}, +\frac{(D_T - D_R)}{2}$, and $+\frac{(D_T + D_R)}{2}$. In general, only one term of the x bracket and one term of the y bracket at a time have $|Bk_o| < 1$. However, if either D_T or D_R or both are sufficiently small more than one term in the x bracket and more than one term in the y bracket can have $|Bk_o| < 1$. The maximum value of equation (8) as a function of phase will occur when each of the phases is an integral number of 2π radians. Keeping this in mind and substituting for k_o, we have the following five expressions for the maximum coupling:

$$\left|\frac{b'_o}{a_o}\right| \lesssim \left|\frac{A_{TMAX}A_{RMAX}}{\eta_o Z_o (1 - \Gamma_o\Gamma_L)\left(1 - \frac{(D_T + D_R)^2}{4d^2}\right)^{1/2} 4k^2}\left[\frac{k(D_T + D_R)}{2d} + \frac{1}{D_T} + \frac{1}{D_R} + \frac{1}{|D_T - D_R|}\right]^2\right| \tag{9a}$$

for $|D_T^2 - D_R^2| > \frac{4d}{k}$, $D_T^2 > \frac{4d}{k}$, $D_R^2 > \frac{4d}{k}$

$$\left|\frac{b'_o}{a_o}\right| \lesssim \left|\frac{A_{TMAX}A_{RMAX}}{\eta_o Z_o (1 - \Gamma_o\Gamma_L)\left(1 - \frac{(D_T + D_R)^2}{4d^2}\right)^{1/2} 4k^2}\left[\frac{2k(D_T + D_R)}{2d} + \frac{1}{|D_T - D_R|} + \frac{1}{\mathrm{Max}(D_T,D_R)}\right]^2\right| \tag{9b}$$

for $|D_T^2 - D_R^2| > \frac{4d}{k}$, $\max(D_T^2,D_R^2) > \frac{4d}{k}$ $\min(D_T^2,D_R^2) \lesssim \frac{4d}{k}$

$$\left|\frac{b'_o}{a_o}\right| \lesssim \left|\frac{A_{TMAX}A_{RMAX}}{\eta_o Z_o (1 - \Gamma_o\Gamma_L)\left(1 - \frac{(D_T + D_R)^2}{4d^2}\right)^{1/2} 4k^2}\left[\frac{2k(D_T + D_R)}{2d} + \frac{1}{D_T} + \frac{1}{D_R}\right]^2\right| \tag{9c}$$

for $|D_T^2 - D_R^2| \lesssim \frac{4d}{k}$, $D_T^2 > \frac{4d}{k}$, $D_R^2 > \frac{4d}{k}$

$$\left|\frac{b'_o}{a_o}\right| \lesssim \left| \frac{A_{TMAX}A_{RMAX}}{\eta_o Z_o (1 - \Gamma_o \Gamma_L)\left(1 - \frac{(D_T + D_R)^2}{4d^2}\right)^{1/2} 4k^2} \left[\frac{3k(D_T + D_R)}{2d} + \frac{1}{\mathrm{Max}(D_T, D_R)}\right]^2 \right| \tag{9d}$$

for $|D_T^2 - D_R^2| \lesssim \frac{4d}{k}$, $\min(D_T^2, D_R^2) \lesssim \frac{4d}{k}$, $\max(D_T^2, D_R^2) > \frac{4d}{k}$

$$\left|\frac{b'_o}{a_o}\right| \lesssim \left| \frac{A_{TMAX}A_{RMAX}}{\eta_o Z_o (1 - \Gamma_o \Gamma_L)\left(1 - \frac{(D_T + D_R)^2}{4d^2}\right)^{1/2}} \frac{(D_T + D_R)^2}{d^2} \right| \tag{9e}$$

for $|D_T^2 - D_R^2| \lesssim \frac{4d}{k}$, $D_T^2 \lesssim \frac{4d}{k}$, $D_R^2 \lesssim \frac{4d}{k}$

Of course, in equation (9e), the latter two conditions imply the first. Notice that equation (9e) gives the same result as does equation (5).

3.2 Comparison of Upper Bound from Formulas to the Exact Upper Bound

The results obtained from equations (9) for each case in section 2 is given in the last column of table 2.1. The small letter in parentheses in column 18 of table 2.1 indicates which set of conditions of equations (9) is applicable for each case. There is a minimum of one case for each set of conditions in equations (9). We notice that equations (9) give results which differ from the exact results by more than 20 dB in some cases. However, we further notice that equations (9) always give results which are greater than the results found using the exact far fields.

3.3 Conclusion

We conclude that equations (9) can be used to obtain an upper bound to the coupling between two antennas neglecting multiple reflections, provided that coupling does not take place through the main beam of either antenna and provided that the integration range we have chosen is valid, i.e., it is adequate to integrate k_x and k_y only over the range $-k(D_T - D_R)/2d$ to $+k(D_T + D_R)/2d$ and we limit d to the range $(D_T + D_R)/2 < d < (D_T + D_R)^2/\lambda$. It has been shown that this range of integration is adequate for determining side-lobe coupling for nearly all realistic microwave antennas [1].

4. Summary and Concluding Remarks

In this report we have performed a feasibility study to see if it is possible to get an estimate of the maximum coupling between two antennas when the details of the far-field amplitude and phase are unknown. We conclude from a study using hypothetical uniformly illuminated circular antennas that approximation-1 and approximation-2 are especially useful methods of estimating the maximum coupling of two antennas when the separation distance is less than one-tenth of a Rayleigh distance. Approximation-1 is limited to uniformly illuminated circular aperture antennas. Approximation-2 replaces the dot product of the fields in equation (1) by the product of the maximum field magnitudes in the general direction of the separation axis multiplied by cosine functions of k_x and k_y and is an approximation applicable to general antennas. Approximation-2 is used in computer program ENVLP, which is documented in Appendix B.

For distances greater than about one-quarter of a mutual Rayleigh distance, the far-field formula [equation (3)] used with A_{TMAX} and A_{RMAX} gives an increasingly accurate estimate of the maximum coupling between two antennas that is generally more accurate than the estimates provided by approximation-1 and approximation-2. For distances between one-tenth and one-quarter of a mutual Rayleigh distance approximation-2 and the far-field formula give equally reasonable estimates. For separation distances less than about one-tenth of a mutual Rayleigh distance approximation-2, i.e., program ENVLP, gives substantially more accurate values of coupling.

In section 3 we derived a set of expressions [equations (9)] that allow one to determine an upper bound to the coupling between two antennas at arbitrary separation distance. For many applications the upper-bound expression may be adequate, especially when the requirement is simply that coupling lies below a given value.

The appeal of approximation-2 and the corresponding computer program ENVLP, is also their simplicity and the few input parameters that they require. In particular, the near-field coupling is computed between arbitrary antennas from a mere knowledge of (1) the separation distance of the antennas, (2) the side-lobe level of each antenna along the axis of separation, and (3) the radius of each antenna.

Having obtained encouraging results for approximation-2 and the upper-bound equations (9) for hypothetical antennas we suggest that these results be tested experimentally using real antennas, and that a similar approximation be formulated and tested for the computer program CUPLZ.

The authors wish to thank Carl Stubenrauch of NBS for the original suggestion of using approximate far fields as well as for helpful discussions and suggestions throughout the work. Helpful discussions were also held with Allen Newell and Andrew Repjar of NBS. In addition, we wish to gratefully acknowledge the Electromagnetic Compatability Analysis Center of the Defense Department for its financial support of this work.

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6. List of Figures

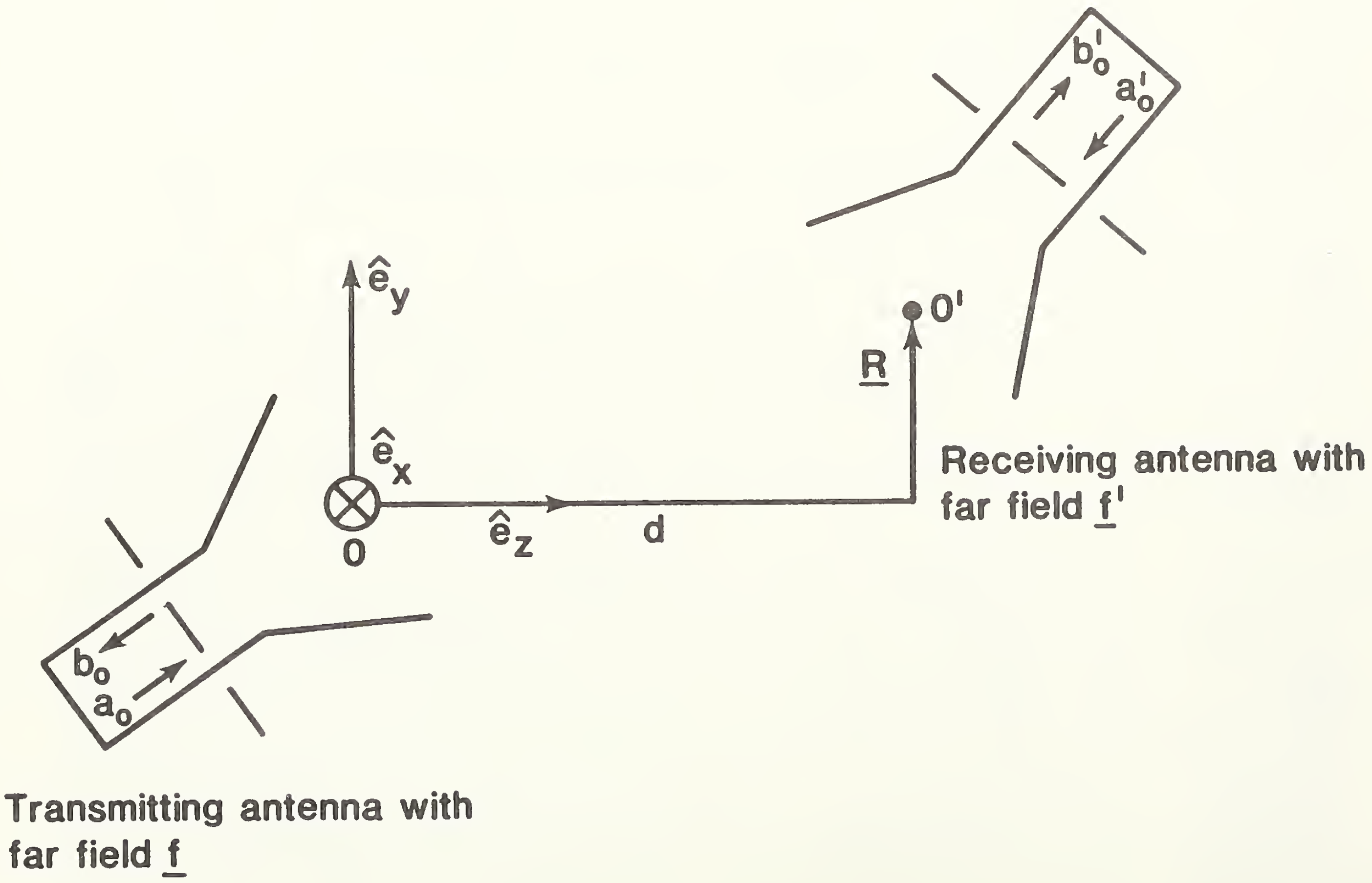


Figure 1. Schematic of two antennas arbitrarily oriented and separated in free space.

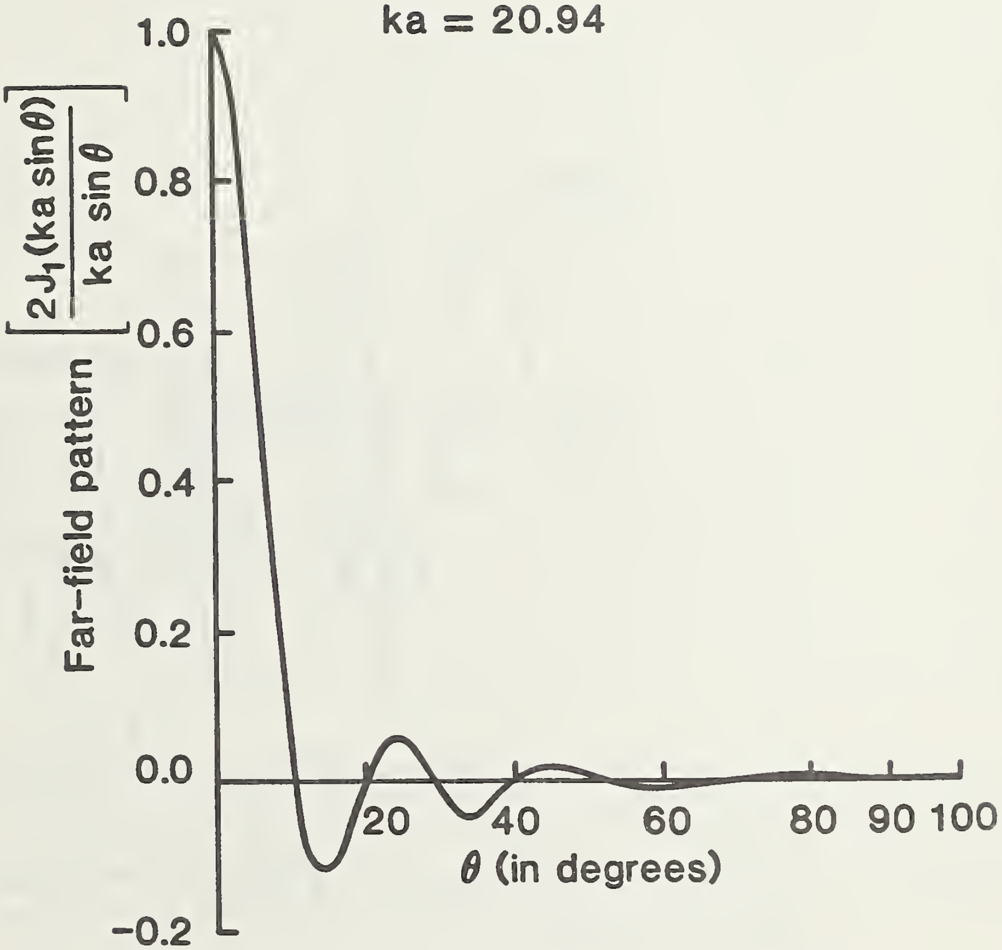


Figure 2. The exact far-field pattern of the x-component of electric field as a function of θ with λ = 3 cm and a = 10 cm for a uniformly illuminated circular antenna.

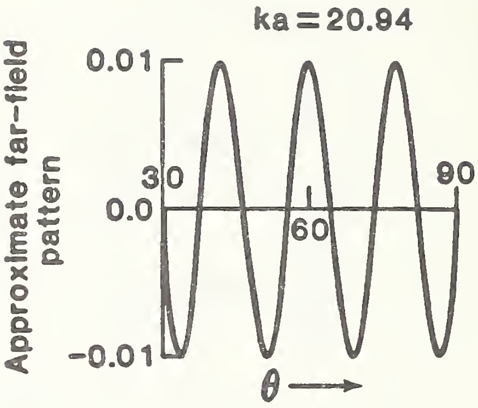


Figure 3. The approximation-1 far-field pattern of the x-component of electric field as a function of θ with λ = 3 cm and a = 10 cm for a uniformly illuminated circular antenna.

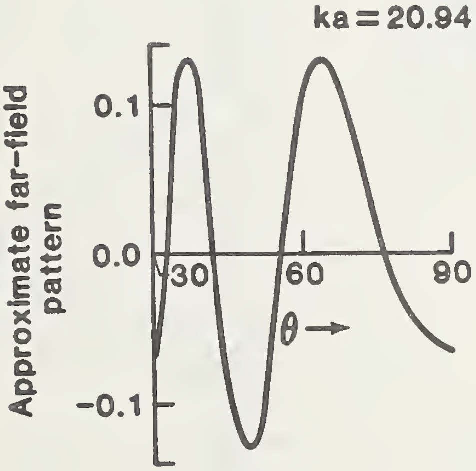


Figure 4. The approximation-2 far-field pattern as a function of θ, with $\phi = 0°$ or $90°$, $\lambda = 3$ cm, $a = 10$ cm for a uniformly illuminated circular antenna. (At $\phi = 0°$ $k_x = k\sin\theta$, $k_y = 0$; at $\phi = 90°$, $k_y = k\sin\theta$, $k_x = 0$).

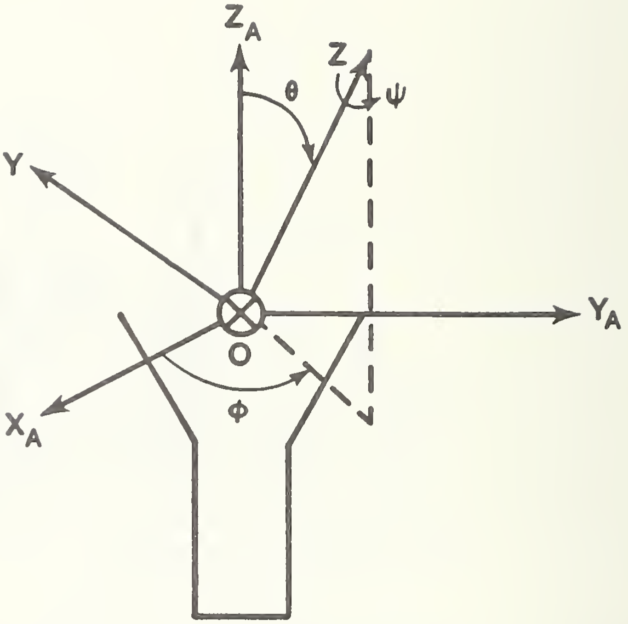


Figure 5. Eulerian angles (ϕ,θ,ψ) needed to rotate the fixed axes x_A, y_A, z_A to the coupling axes x,y,z of figure 1.

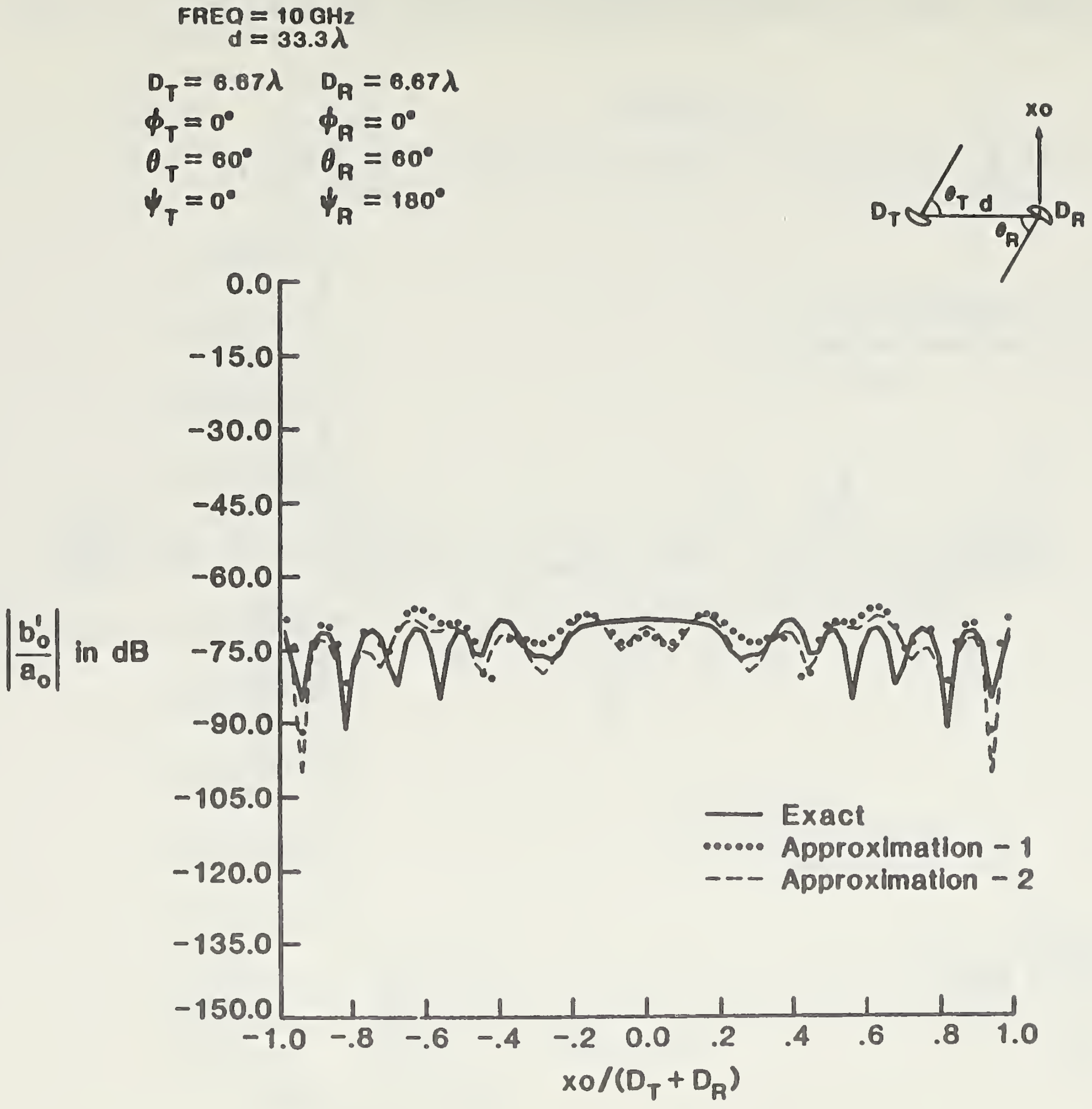


Figure 6. The coupling quotient for the XO cut using the exact, approximation-1 and approximation-2 far fields: Case Alpha-1.

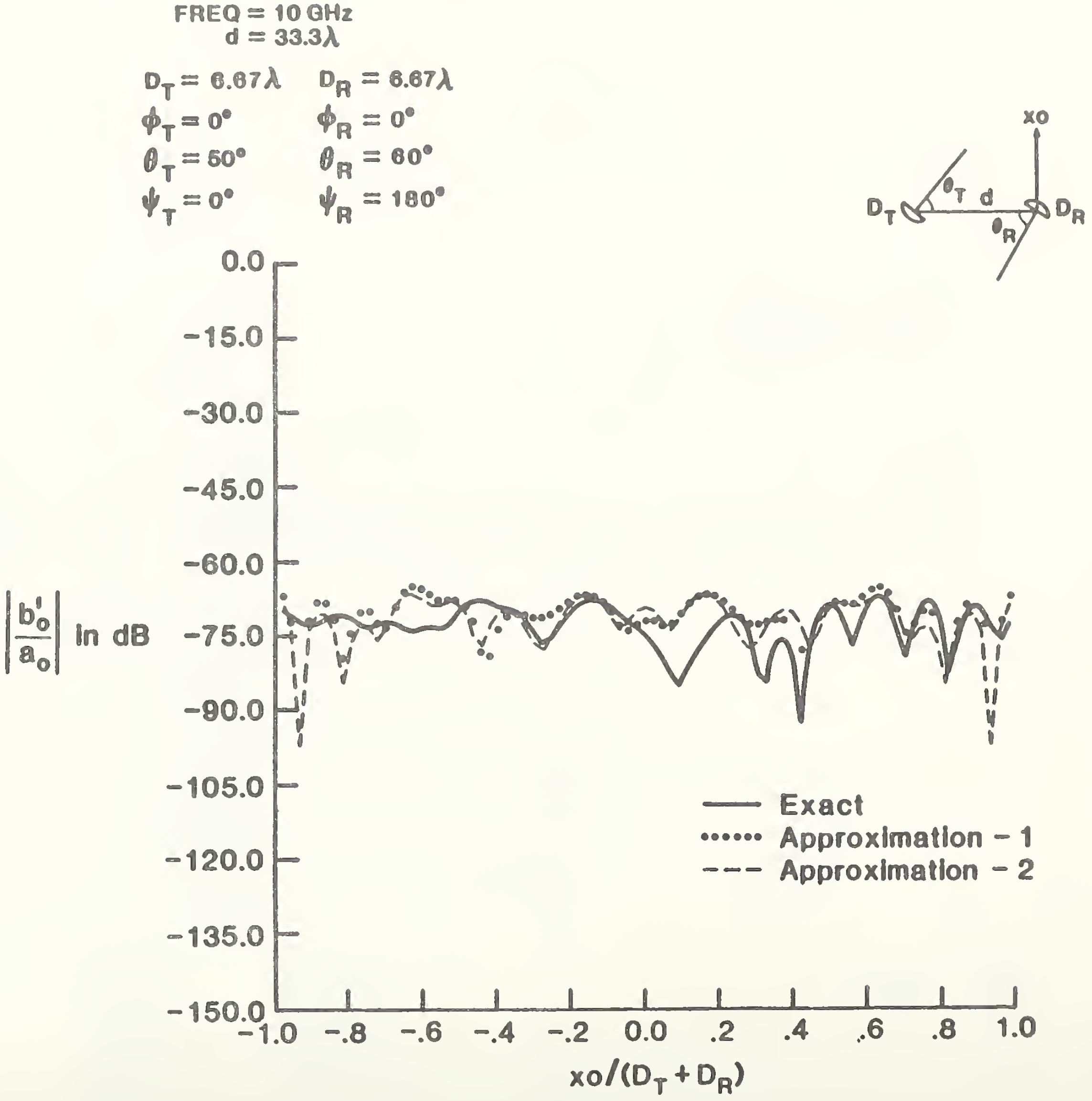


Figure 7. The coupling quotient for the XO cut using the exact, approximation-1 and approximation-2 far fields: Case Alpha-4.

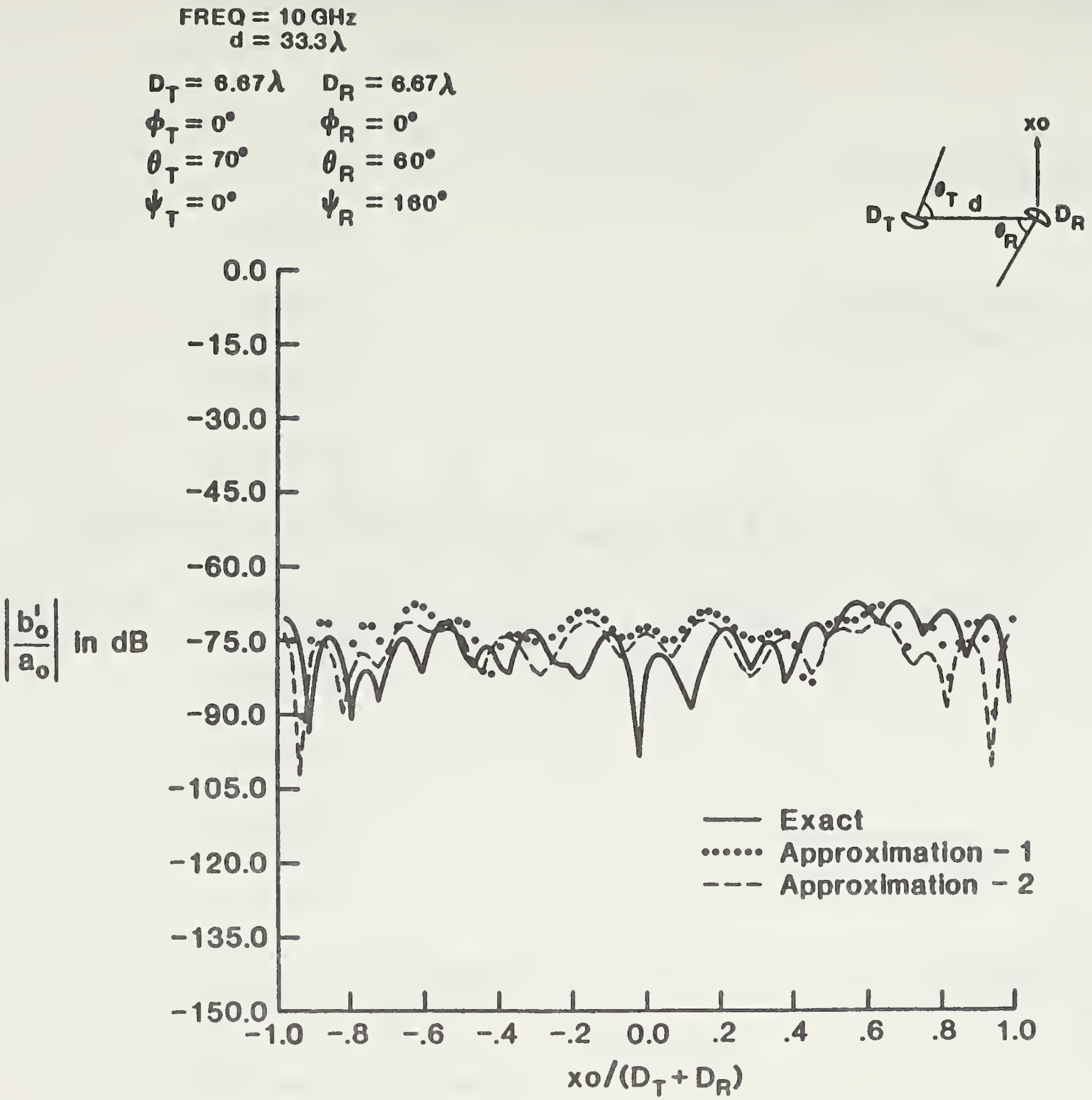
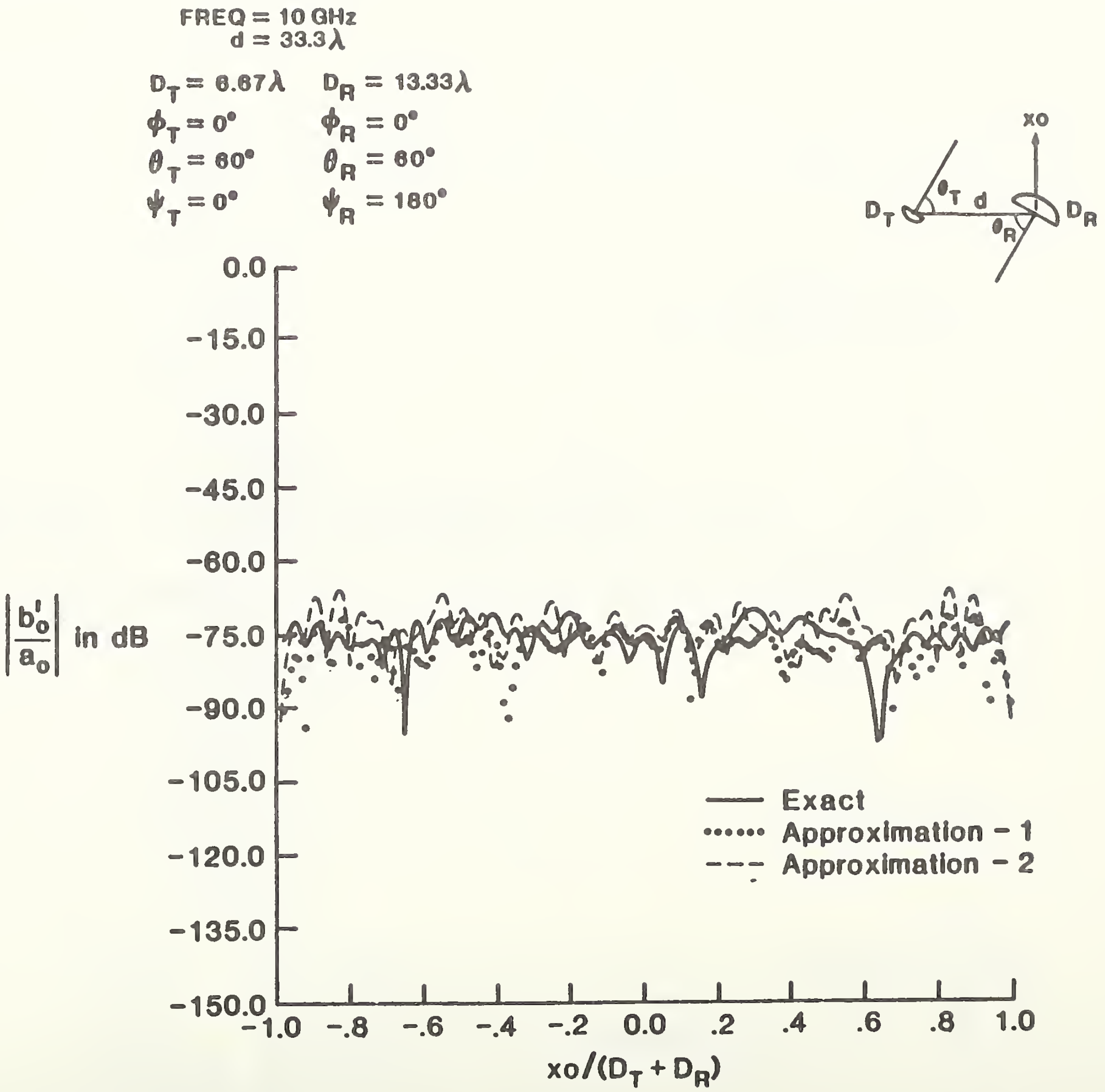


Figure 8. The coupling quotient for the XO cut using the exact, approximation-1 and approximation-2 far fields: Case Alpha-7.



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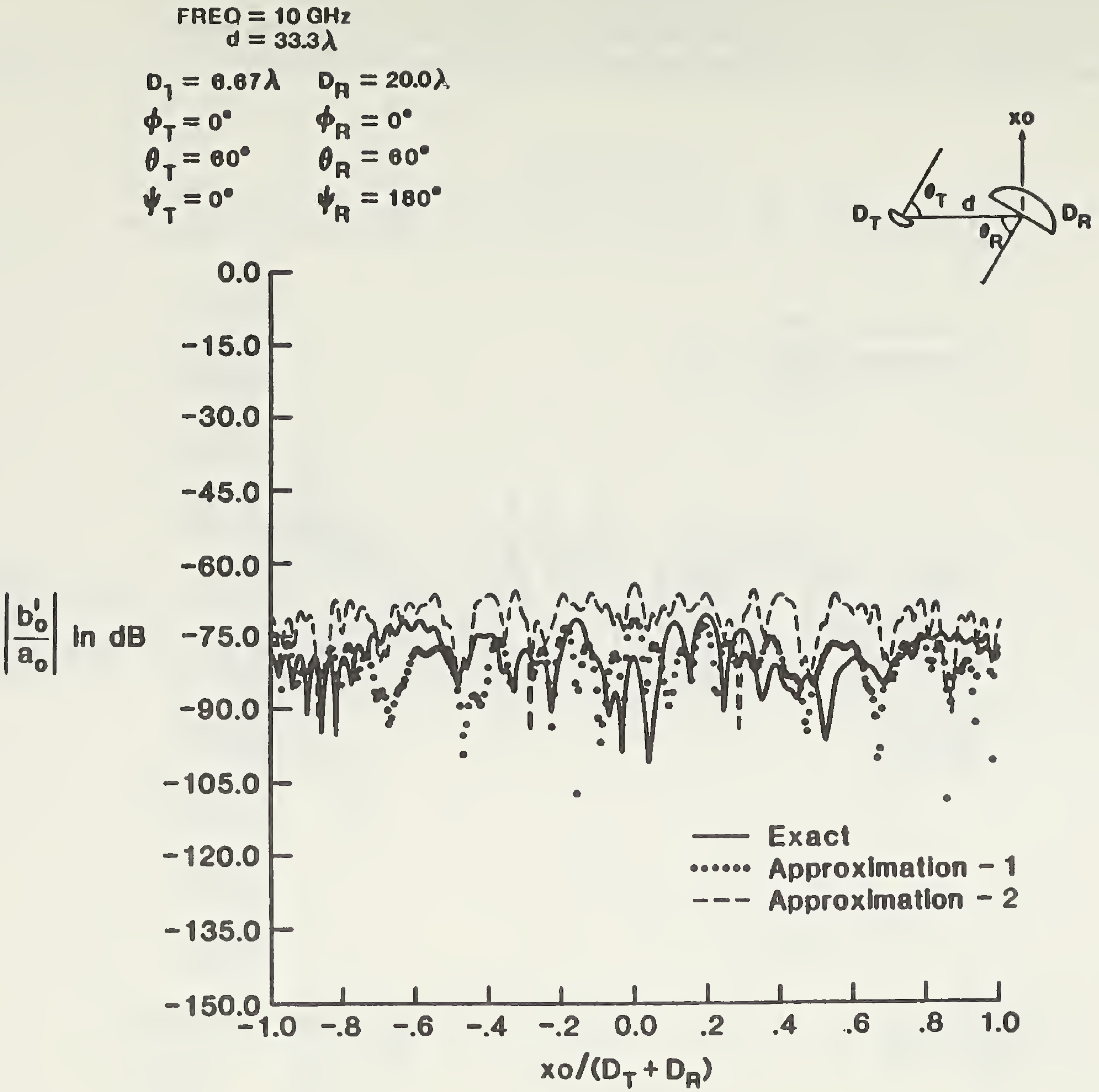

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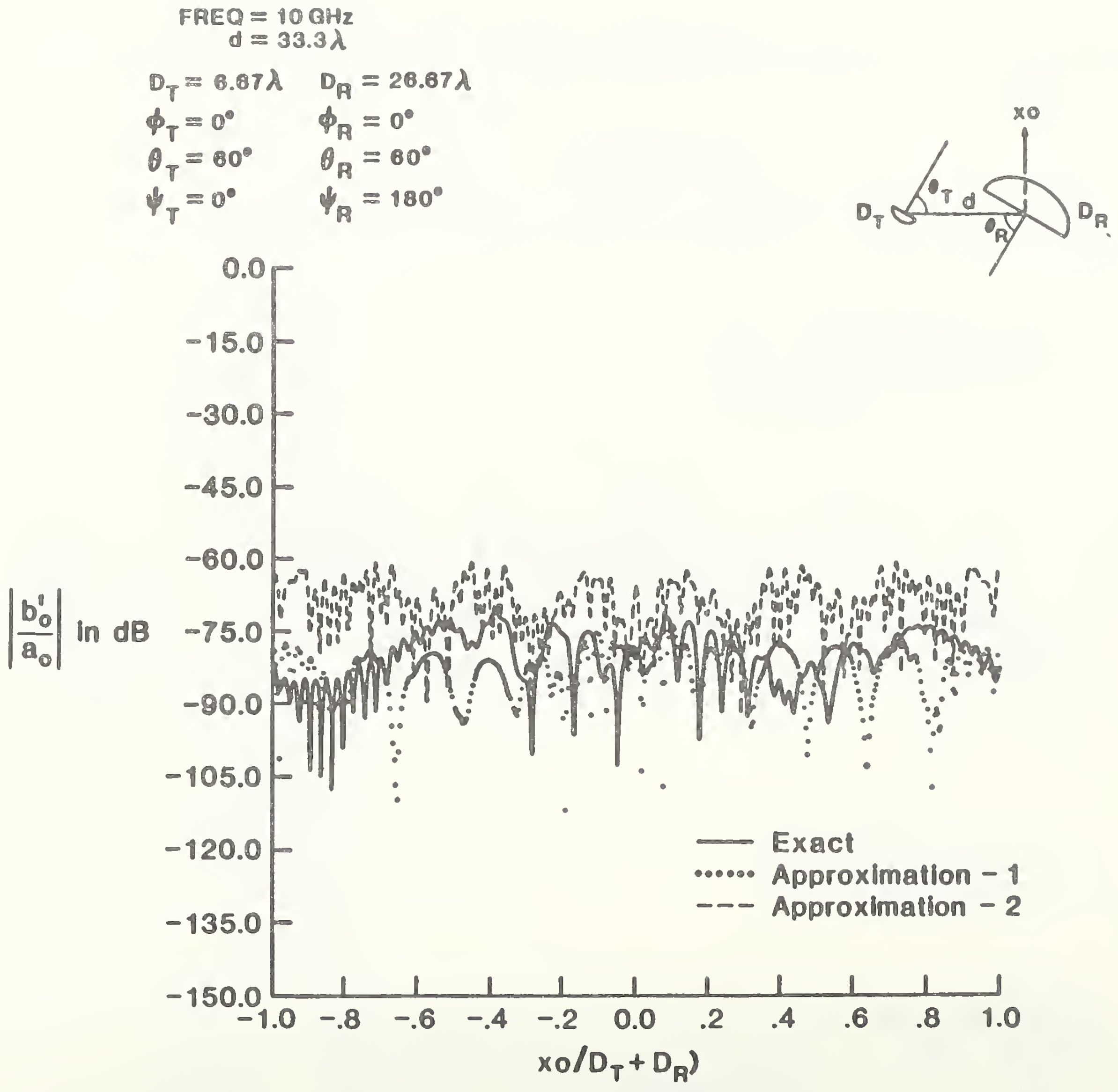
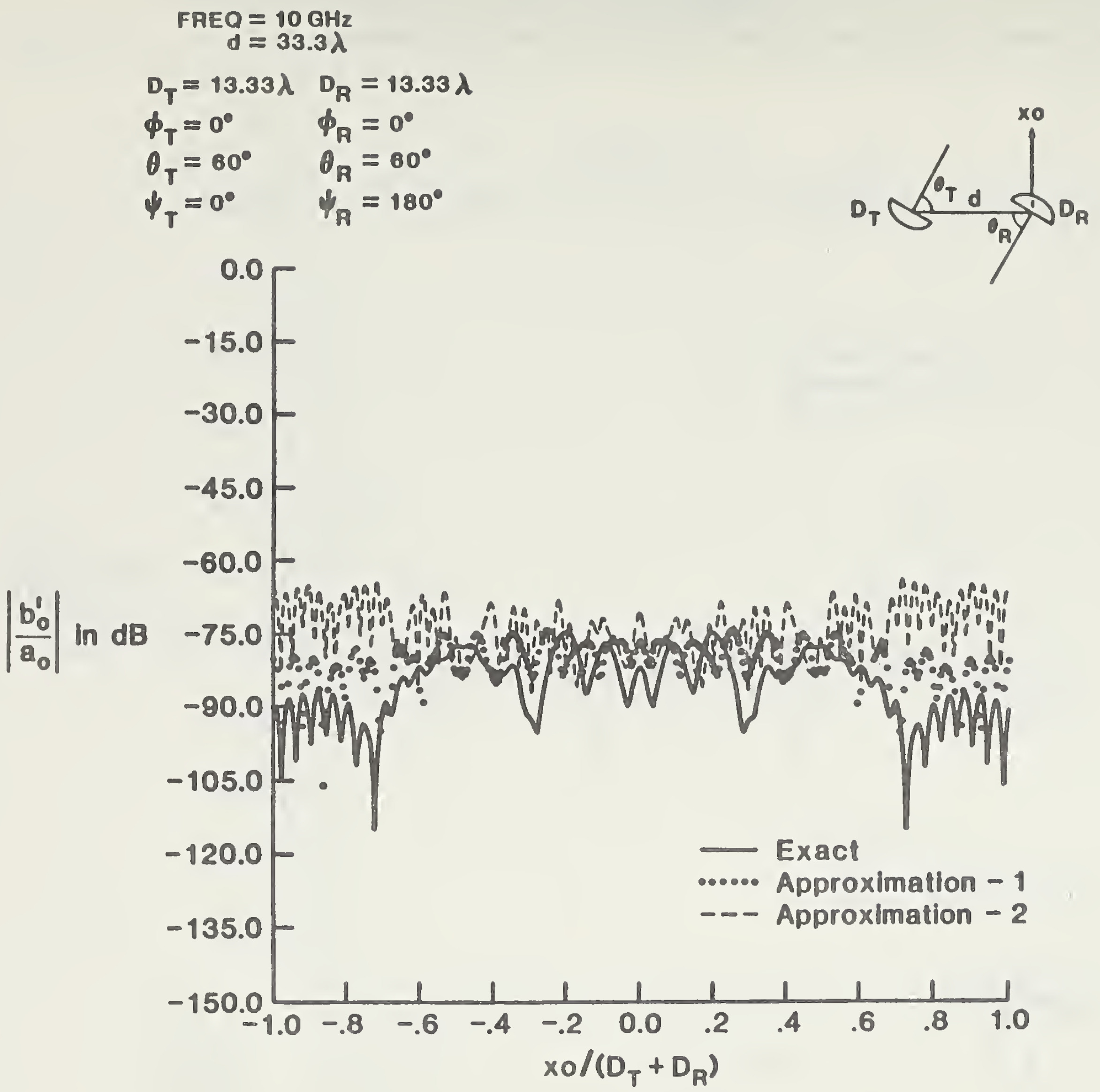


Figure 11. The coupling quotient for the XO cut using the exact, approximation-1 and approximation-2 far fields: Case Beta-3.



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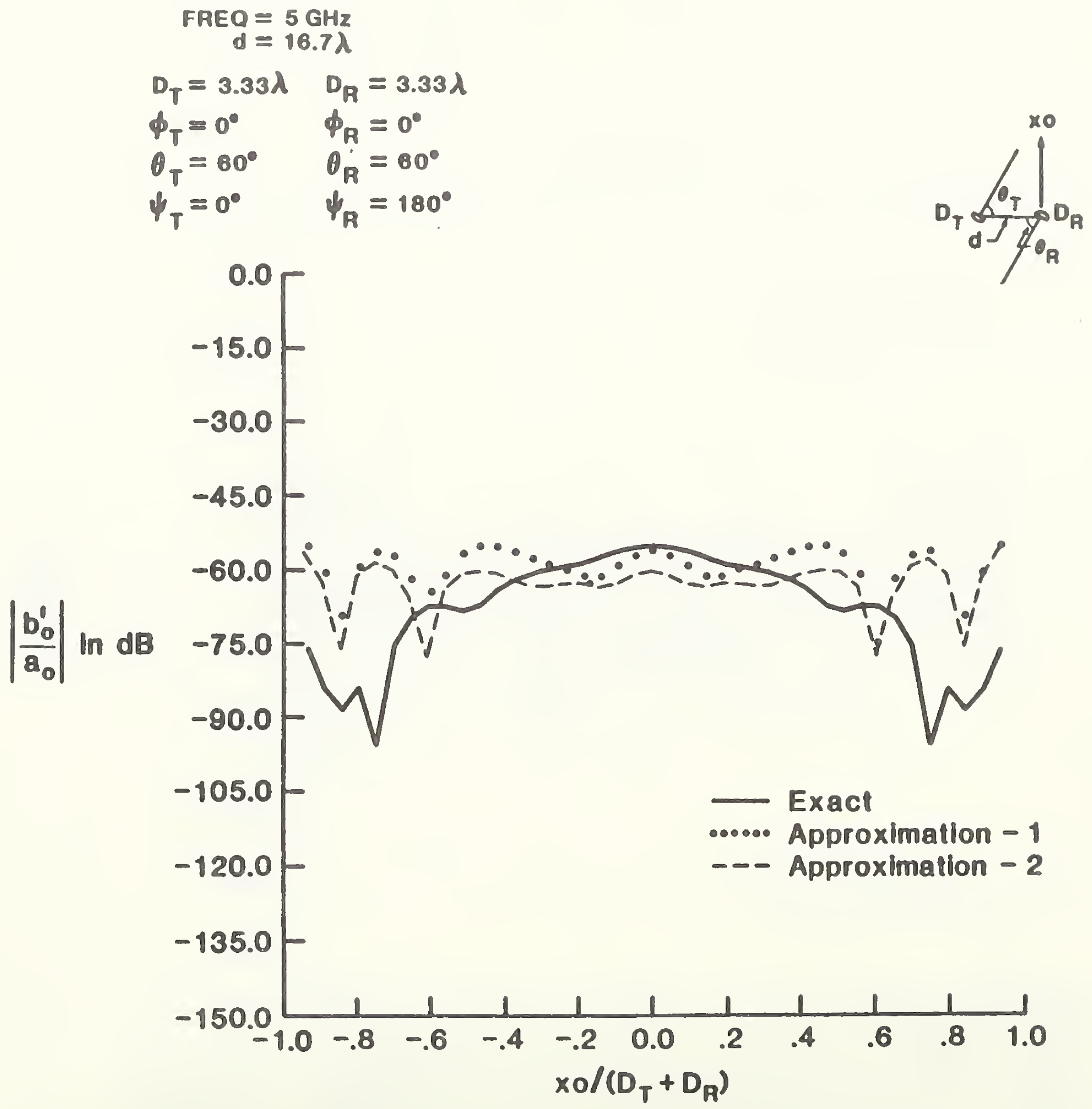

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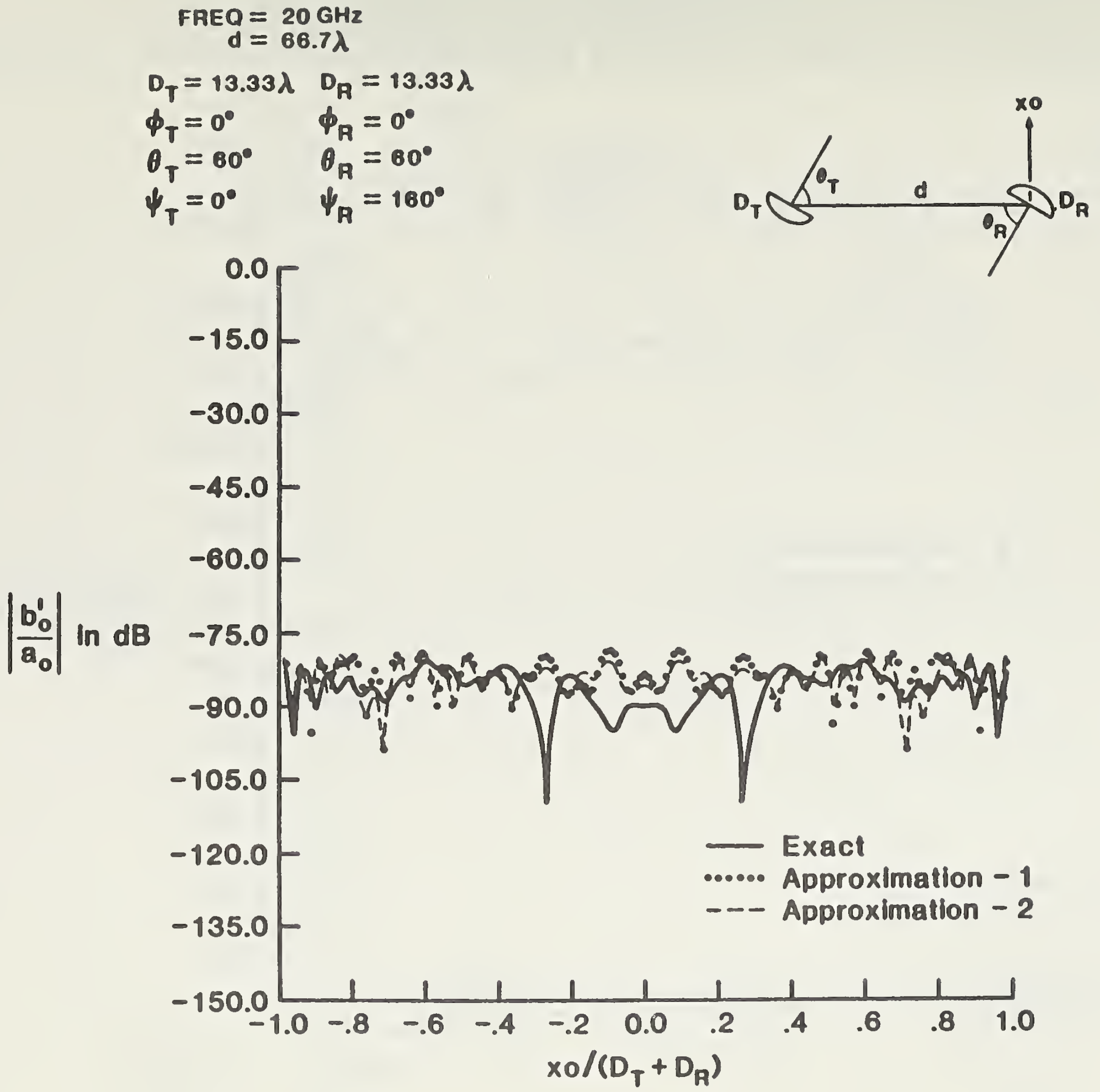

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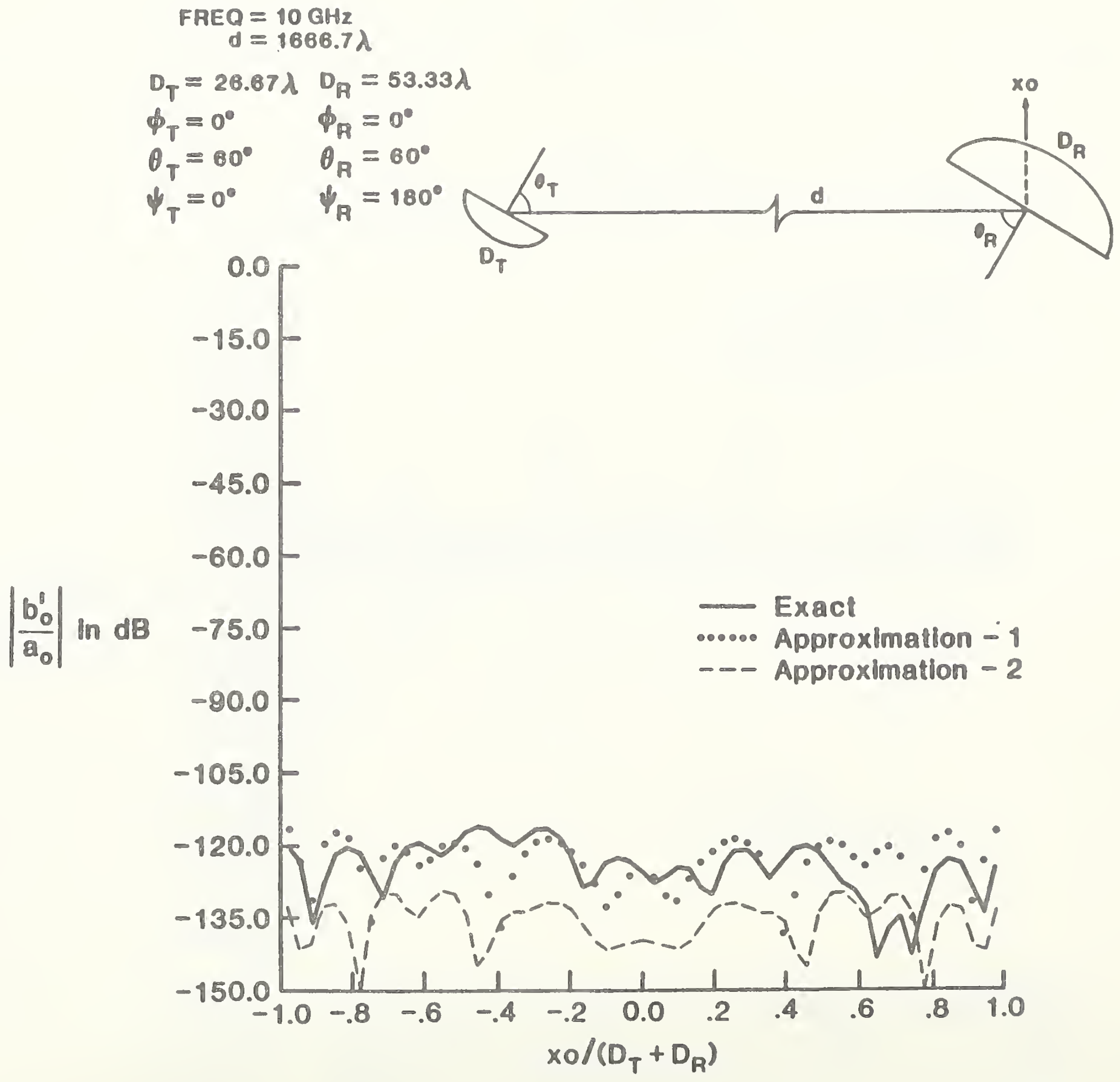

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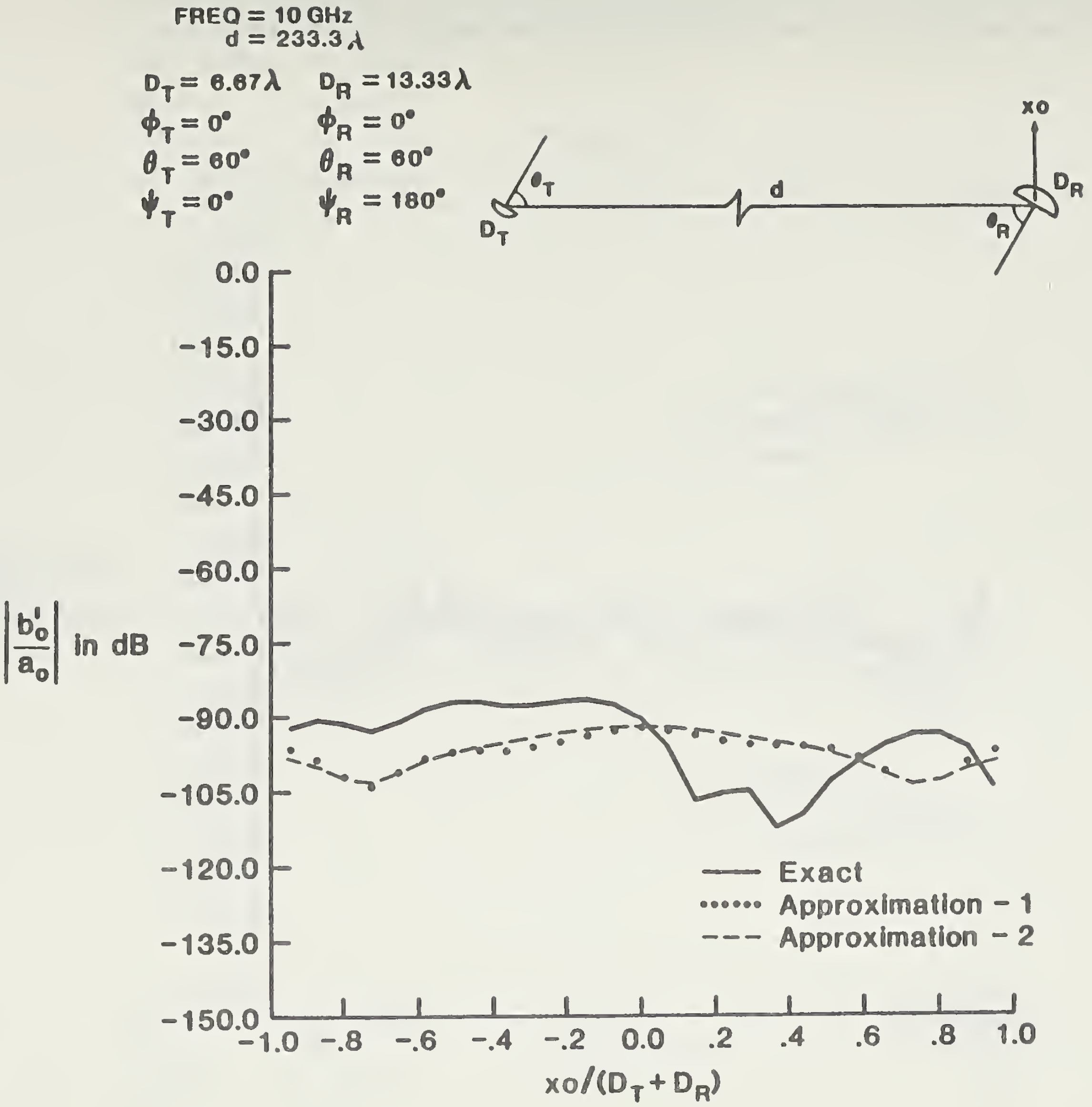

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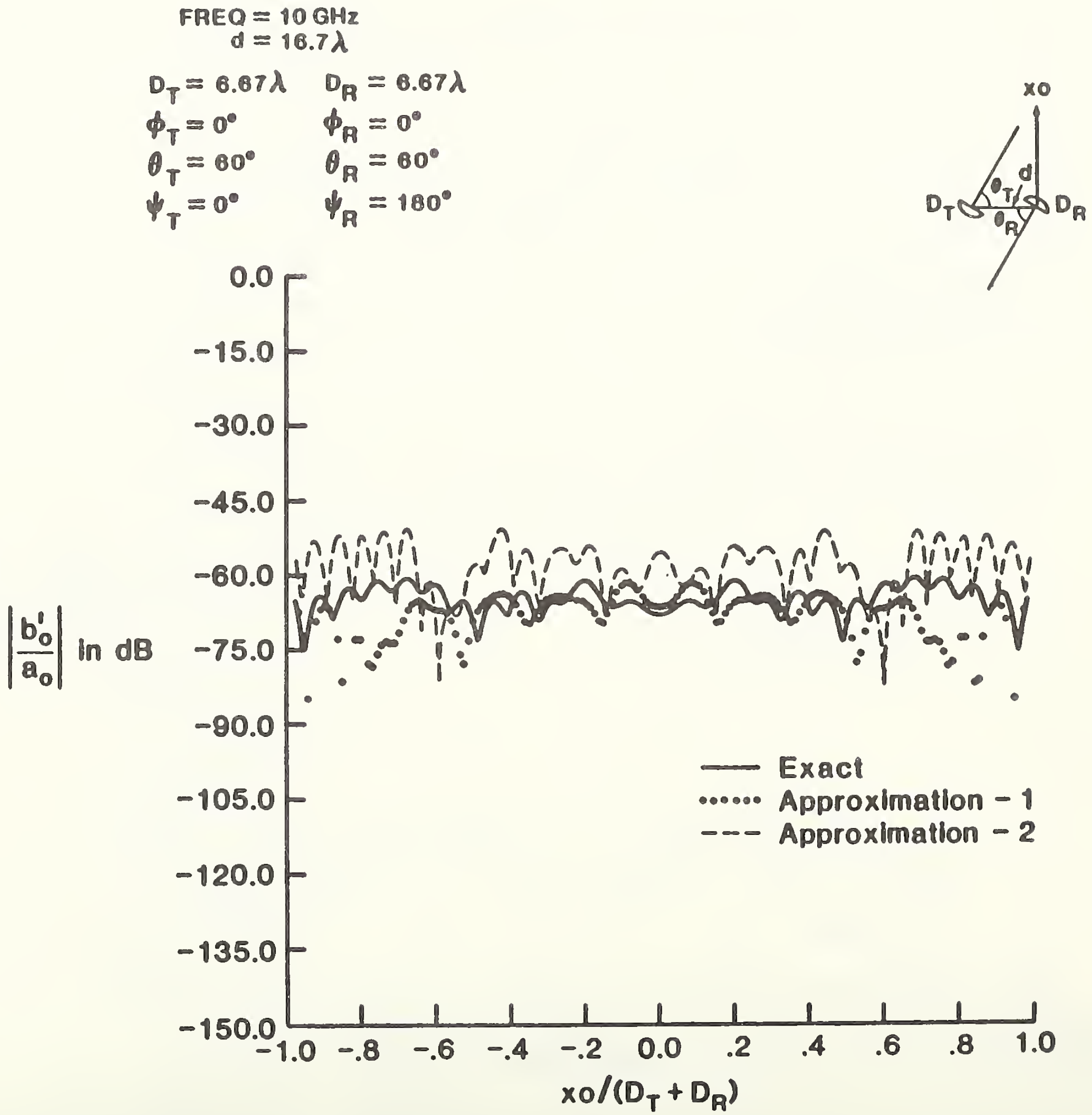

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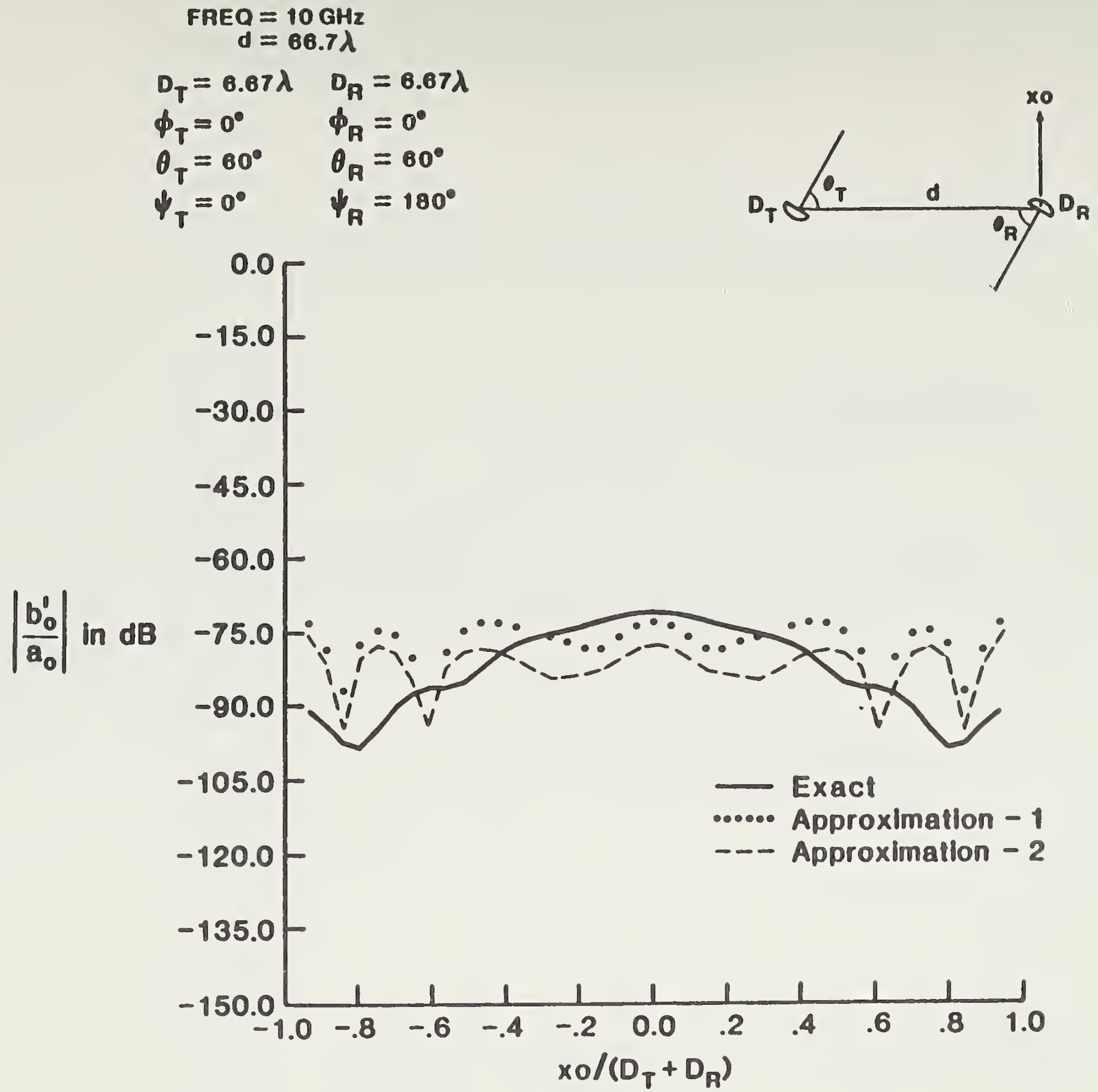

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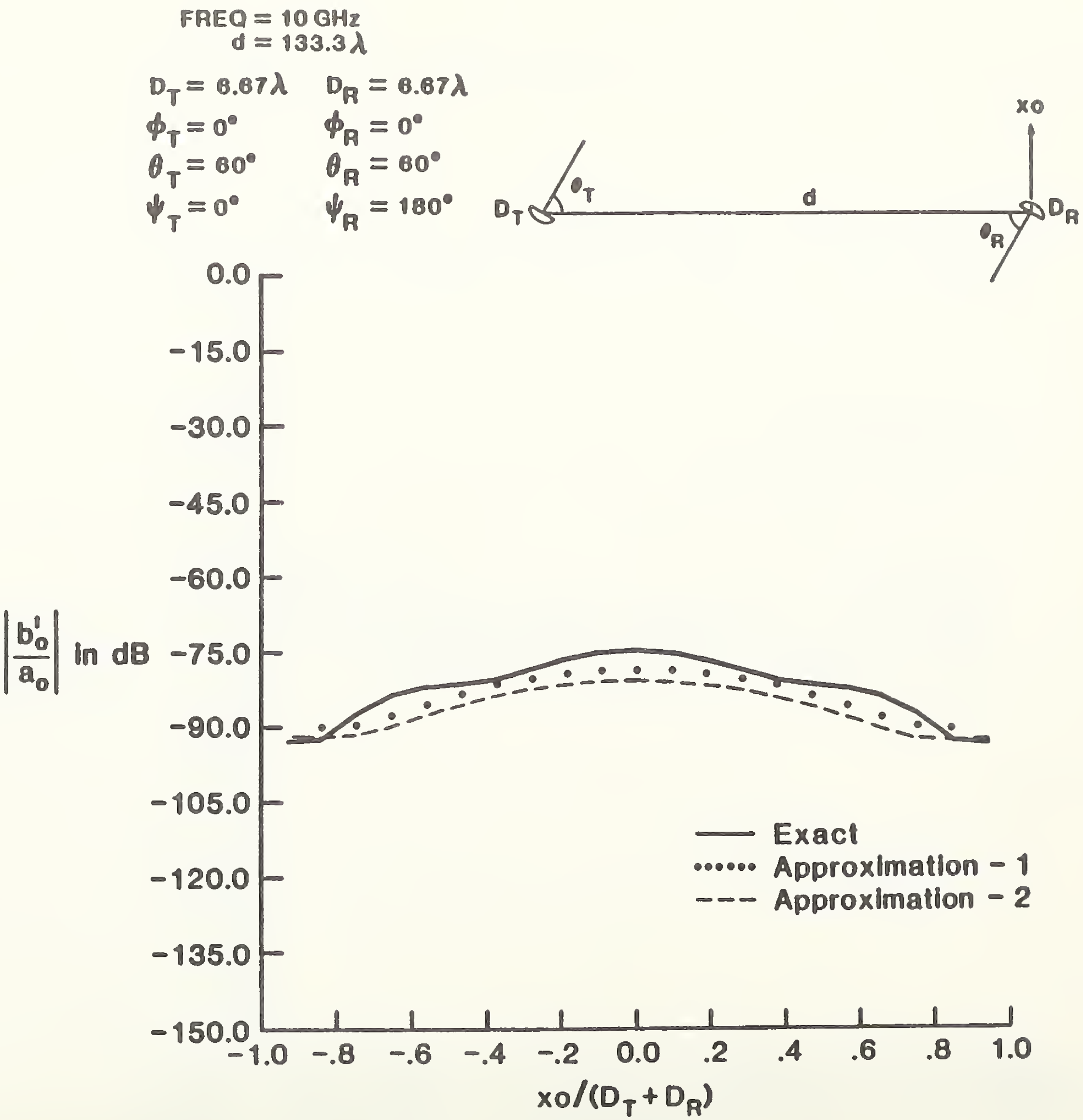

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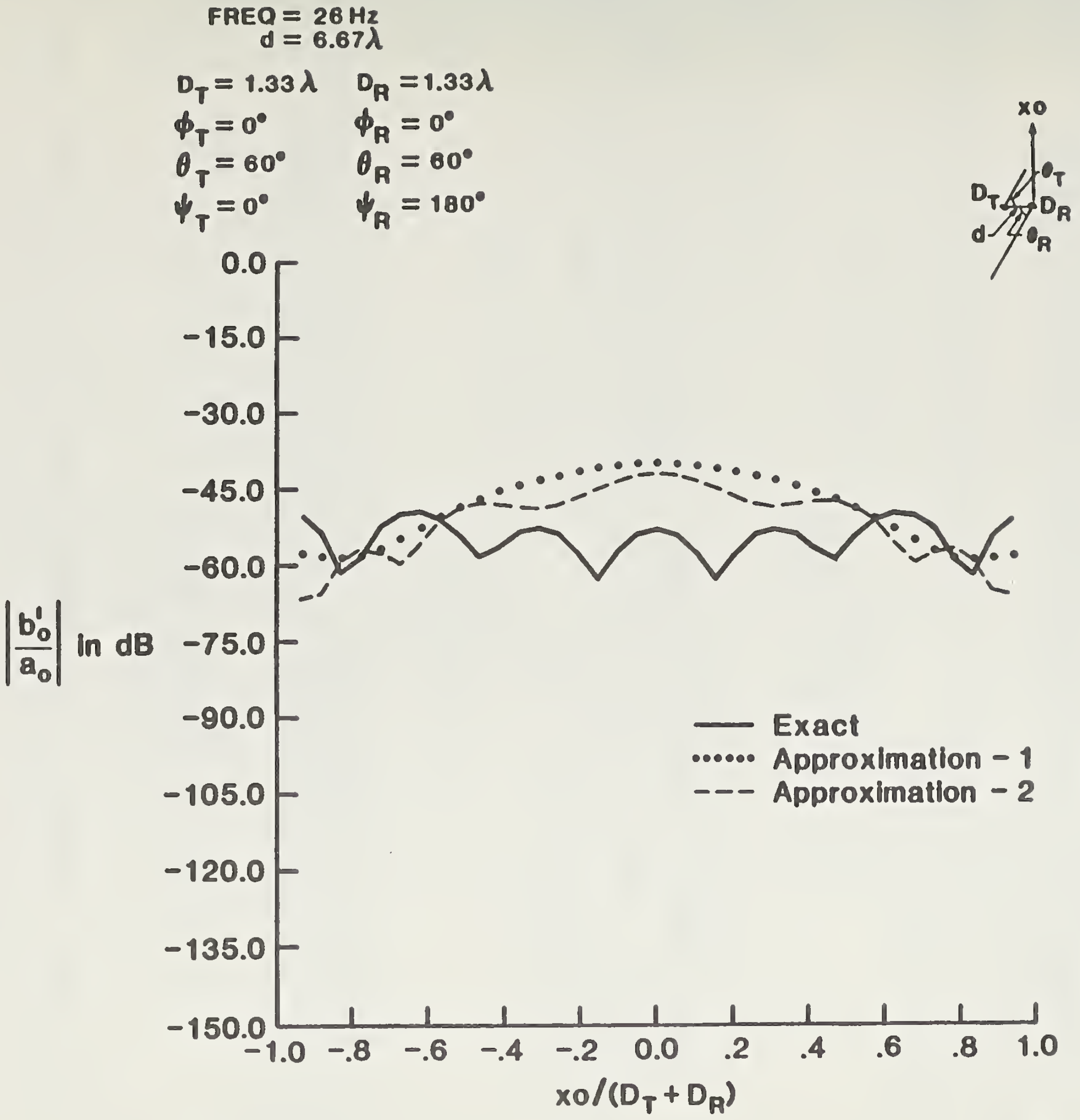

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Appendix A. Derivation of A_{TMAX} and A_{RMAX} in Terms of Antenna Gains and Relative Side-Lobe Levels

In this section we derive the amplitudes, A_{TMAX} and A_{RMAX}, of the approximate pattern for the transmitting and receiving antennas, respectively (see section 2.2.3) in terms of the antenna gains, G_T and G_R, the side-lobe levels in dB, S_T and S_R, the reflection coefficients of the antennas, Γ_{oT} and Γ_{oR}, the characteristic admittances for the propagated mode in the waveguide feeds of the antennas, η_{oT} and η_{oR}, and the wave impedance of free space, Z_o.

The magnitude of the vector $\underline{f}$ (which is just A_{MAX}) in equation (1) of section 2.1 in the direction $\underline{r}$ is given by [1]:

$$|\underline{f}(\underline{r})| = \frac{|\underline{E}(\underline{r})|r}{|a_o|} = A_{MAX} \tag{A1}$$

where $\underline{E}(\underline{r})$ is the electric field in the direction $\underline{r}$, and a_o is the amplitude of the incident mode of the waveguide feed to the antenna. A_{MAX} is either A_{TMAX} or A_{RMAX} depending on whether the transmitting or receiving antenna is being considered. The relative side-lobe level S in the direction $\underline{r}$ (S_T for the transmitting antenna and S_R for the receiving direction), is:

$$S = -20 \log\left(\frac{|\underline{E}(\underline{r})|}{|\underline{E}(\underline{r}_o)|}\right) \tag{A2}$$

where $\underline{r}_o$ is the direction of the main beam. It is well known that the gain for an antenna, G (G_T for the transmitting antenna and G_R for the receiving antenna), is given in dB by

$$G = 10 \log\left[\frac{4\pi|\underline{E}(\underline{r}_o)|^2 r^2}{2\, Z_o P_{input}}\right] . \tag{A3}$$

Assuming a single propagating mode in the waveguide feeding the antenna, the input power to an antenna, P_{input}, can be expressed as [6]:

$$P_{input} = \frac{1}{2}\, \eta_o |a_o^2| (1 - |\Gamma_o|^2) \tag{A4}$$

where η_o is η_{oT} for the transmitting antenna and η_{oR} for the receiving antenna and Γ_o is Γ_{oT} for the transmitting antenna and Γ_{oR} for the receiving antenna. Substituting (A4) into (A3) for P_{input} we find that the gain is

$$G = 10 \log\left[\frac{4\pi|\underline{E}\,(\underline{r}_o)|^2 r^2}{Z_o \eta_o |a_o|^2 (1 - |\Gamma_o|^2)}\right]. \tag{A5}$$

We solve for $\frac{|\underline{E}(\underline{r}_o)|}{|a_o|} r$ in (A5) to get

$$\frac{|\underline{E}(\underline{r}_o)|r}{|a_o|} = \left(\frac{\eta_o Z_o}{4\pi} (1 - |\Gamma_o|^2)\right)^{1/2} 10^{G/20}. \tag{A6}$$

Substitution of $|\underline{E}(\underline{r}_o)|$ from (A2) into (A6) gives

$$\frac{|\underline{E}(\underline{r})|r}{|a_o|} 10^{S/20} = A_{MAX}\ 10^{S/20} = \left(\frac{\eta_o Z_o}{4\pi} (1 - |\Gamma_o|^2)\right)^{1/2} 10^{G/20}. \tag{A7}$$

Thus, A_{MAX} turns out to be simply

$$A_{MAX} = \left(\frac{\eta_o Z_o}{4\pi} (1 - |\Gamma_o|^2)\right)^{1/2} 10^{((G - S)/20)} \tag{A8}$$

and in particular,

$$A_{TMAX} = \left(\frac{\eta_{oT} Z_o}{4\pi} (1 - |\Gamma_{oT}|^2)\right)^{1/2} 10^{((G_T - S_T)/20)} \tag{A9a}$$

$$A_{RMAX} = \left(\frac{\eta_{oR} Z_o}{4\pi} (1 - |\Gamma_{oR}|^2)\right)^{1/2} 10^{((G_R - S_R)/20)}. \tag{A9b}$$

In summary, equations (A9) give the magnitude of the approximate electric far-field pattern of the transmitting and receiving antenna along their axis of separation in terms of the gain (in dB), side-lobe level (in dB) along the axis of separation, the antenna input reflection coefficient, the characteristic admittance of the waveguide feeding the antenna, and, of course, the impedance of free space.

Appendix B. Documentation of ENVLP, the Computer Program to Estimate Coupling Between Two Antennas

This appendix documents the program ENVLP which estimates the coupling loss between two antennas given their radii, separation distance, and side-lobe levels along their axis of separation. Subroutines used by this program have also been used by CUPLNF (except for CRTPLT3) and are documented in NBSIR 80-1630[2].

B.1 General Overview of Computer Program ENVLP

The techniques used by ENVLP for evaluating the coupling loss are basically the same as those used by the computer program CUPLNF [2]. The flow chart for ENVLP is presented below to give the reader a general understanding of the program package.

Purpose: To calculate an estimate of the coupling loss between two antennas given the radii of the antennas, their separation distance, and their side-lobe levels along their separation axis.

Method: Evaluate equation (1) of the main text using approximation-2 (see section 2.2.3) to estimate the dot product of the far fields.

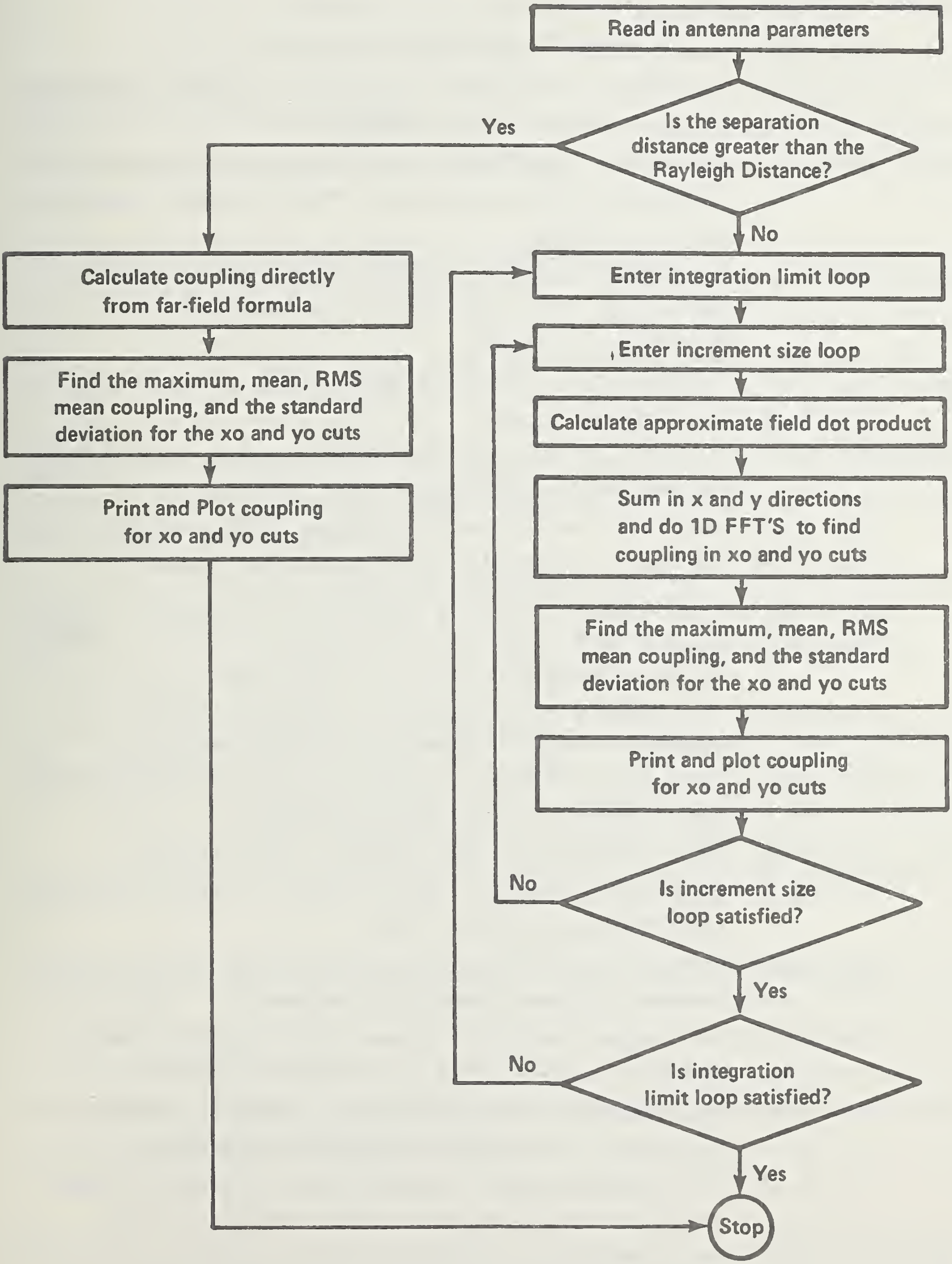
General Discussion: The main program ENVLP divides into the following sections:

1. General information,
2. Specification statements,
3. Definition and reading of input data,
4. Far-field coupling computation,
5. Limits of integration and number of integration points,
6. Filling the input matrices to the FFT subroutine FOURT, and
7. Calculation of maximum, minimum, mean coupling; printout and plotting of the XO and YO cuts.

General Information: This section provides a general description to the program user of what the program does and also defines the more important parameters of the program. A reading of this section will be sufficient for most users to begin using the program.

Specification Statements: This section dimensions arrays, places arrays in common, and declares complex variables.

FLOW CHART FOR PROGRAM ENVLP



Definition and Reading of Input Data: This section defines basic constants such as the speed of light and reads from data cards the antenna parameters. The required data cards are:

Card 1	Col. 1-10	An alphanumeric identifier of the user's choice used to identify the case being computed; the identifier appears at the top of the printout, (HEAD(2)).
Card 2	Col. 5	The maximum value that the user wishes XLIM to assume; it should be 1 or 2. XLIM adjusts the integration range (see below), (ILIMAX).
	Col. 10	The maximum value that the user wishes BFAC to assume; it should be 1 or 2. BFAC adjusts the integration step size (see below), (IBFMAX).
Card 3	Col. 1-10	The separation distance in meters, (ZO).
	Col. 11-20	The frequency in Hz.
Card 4	Col. 1-10	The radius in meters of the smallest sphere circumscribing the effective transmitting antenna, (RADT).
	Col. 11-20	The gain of the transmitting antenna in dB, (GAINT).
	Col. 21-30	The relative side-lobe level in dB of the transmitting antenna along the axis of separation (ST).
	Col. 31-40	The real part of the input reflection coefficient, GAMMAOT, for the transmitting antenna in free space.
	Col. 41-50	The imaginary part of the input reflection coefficient, GAMMAOT, for the transmitting antenna in free space.
Card 5	Col. 1-10	The radius in meters of the smallest sphere circumscribing the effective receiving antenna, (RADR).
	Col. 11-20	The gain of the receiving antenna in dB, (GAINR).
	Col. 21-30	The relative side-lobe level in dB of the receiving antenna along the separation axis (SR).
	Col 31-40	The real part of the input reflection coefficient, GAMMAOR, for the receiving antenna in free space.
	Col. 41-50	The imaginary part of the input reflection coefficient, GAMMAOR, for the receiving antenna in free space.
	Col. 51-60	The real part of the reflection coefficient, GAMMALR, for the passive termination on the receiving antenna.
	Col. 61-70	The imaginary part of the reflection coefficient, GAMMALR, for the passive termination on the receiving antenna.
Card 6	Col. 1-10	The characteristic admittance of the propagating mode in the waveguide feed of the transmitting antenna, (ETAT). If

ETAT=0, the program will set ETAT=1/CAPZO.

Col. 11-20 The characteristic admittance of the propagating mode in the waveguide feed of the receiving antenna, (ETAR). If ETAR=0, the program will set ETAR=1/CAPZO.

Far-Field Coupling Computation: This section computes the coupling between two antennas directly from their far fields if their separation distance is greater than a mutual Rayleigh distance (equal to the square of the sum of the effective antenna diameters, divided by the wavelength; see [1]).

Limits of Integration and Number of Integration Points: For XLIM=1, the limits of integration are computed using the specification of section 2.1 of the main text. These limits can be doubled to see if a wide enough integration range has been included by setting XLIM=2. Strictly speaking, approximation-2 (see section 2.2.3) is only good for the XLIM=1 integration range. However, if the results of the XLIM=2 integration range for the mean coupling differs by a huge amount (e.g., 10 dB) from the mean coupling result for XLIM=1, it is questionable whether ENVLP gives a good estimate of the coupling loss between the two antennas being considered.

This section also chooses the integration increment size small enough to prevent aliasing. The increment size can be reduced by increasing BFAC. Normally, BFAC is set equal to 1 and convergence is tested by halving the integration increment size, i.e., letting BFAC=2.

Filling the Input Matrices to the FFT Subroutine FOURT: This section calculates the dot product of the far fields in a square array using approximation-2. The dot products are then summed in the $k_y(k_x)$ direction for each value of $k_x(k_y)$ and placed in the array AX(AY). FOURT then performs a one dimensional FFT on the array AX(AY) to obtain the values of the coupling quotient along the XO(YO) cut.

Calculation of Maximum, Minimum, Mean Coupling; Printout and Plotting of the XO and YO Cuts: This section computes the maximum, minimum, mean, and RMS mean coupling for the XO and YO cuts. It further computes the standard deviation in the coupling for each cut. It prints out the values of the coupling quotient for each cut and then plots the values of the coupling quotient for each cut from -(DIAMR+DIAMT) to +(DIAMR+DIAMT).

Symbol Dictionary (in alphabetical order):

ABL = Intermediate value for defining the range of k_x/k and k_y/k. The range of ABL beyond XKLIM is zero filled.

ACLCUT = A real array used to store the magnitude of the coupling quotient along X0 and Y0 cuts.

AMAX = The maximum value of the coupling quotient for either the X0 or Y0 cut.

AMEAN = The mean value of the coupling quotient for either the X0 or Y0 cut.

AMINX,(AMINY) = The minimum value of the coupling quotient for the X0(Y0) cut.

ARMAX,(ATMAX) = The side-lobe level for the receiving (transmitting) antenna.

AX,(AY) = Complex arrays used to store first the far-field product then the coupling quotient along the X0(Y0) cut.

(A1,A2),(B1,B2) = The limits of integration of k_x/k and k_y/k respectively.

BFAC = Variable which adjusts the integration increments and should be about 1 or 2; making BFAC larger tests for convergence by making the integration increments proportionally smaller.

C = The increment size for the X0 and Y0 cuts when the far-field formula is used.

CAPZO = The wave impedance of free space.

CEE = The speed of light in a vacuum in meters/second.

CLCUTXA(CLCUTYA) = The values of the coupling quotients in the X0(Y0) cuts stored for plotting.

COEF = 2.*PI*FMM/ETA/CAPZO/XK for the far-field formula, -FMM*C1*C2/ETA/CAPZO otherwise.

C1,C2 = The k_x/k and k_y/k increments respectively.

DIAMR,(DIAMT) = Twice the larger of RADR (RADT) or WAVLGTH.

DIAMSUM = DIAMR plus DIAMT.

DKOK = The approximate k_x/k and k_y/k increments.

DX,(DY) = The increments in X0(Y0) over which the coupling quotient is computed by the FFT.

ETAT,(ETAR) = The characteristic admittance for the propagated mode for the waveguide feed of the transmitting (receiving) antenna.

FDOTFP = The dot product of the electric far-field pattern of the two antennas.

FMM = The mismatch factor, $1/(1-\Gamma_o\Gamma_L)$, for the receiving antenna.

FREQ = Frequency in Hz.

FX,FY = Intermediate complex variables.

GAINT,(GAINR) = The gain in dB of the transmitting (receiving) antenna.

GAMMALR,GAMMAOR,GAMMAOT = The reflection coefficients of the receiving load, receiving antenna, and transmitting antenna, respectively.

HEAD = Integer array identifier.

IBFAC = Loop index for varying BFAC.

IBFMAX = The maximum value assumed by BFAC; it should equal to 1 or 2.

ILIMAX = The maximum value assumed by XLIM; it should be 1 or 2.

IXLIM = Loop index for varying XLIM.

J1,J2 - Loop indices used in the filling of AX(AY) before the FFT.

M1,M2 = Loop indices used in completing the coupling quotient computation after the FFT.

N = The number of points in an XO(YO) cut when the far-field formula is used.

NN1,NN2 = Integer arrays of dimensions one used in the call to the FFT subroutine FOURT.

NSX,(NSY) = The number of values of the coupling quotients to be plotted in the XO(YO) cut.

N1,N2 = The number of k_x and k_y integration points respectively.

N1MAX,(N2MAX) = Integers determining the maximum of the XO(YO) range over which the coupling is printed.

N1MIN,(N2MIN) = Integers determining the minimum of the XO(YO) range over which the coupling is printed.

N1O,N2O = Intermediate integers used to determine (N1MIN,N1MAX) and (N2MIN,N2MAX).

PI = $\pi = 3.14159...$

RADR,(RADT) = The radius of the smallest sphere circumscribing the receiving (transmitting) antenna in meters.

RMS = The RMS mean coupling quotient for the XO or YO cut.

RO = The total distance between the two antennas = $(X^2 + Y^2 + Z^2)^{1/2}$.

SDEV = The standard deviation in the coupling quotient for the XO or YO cut.

SR,ST = The relative side-lobe levels, in dB, of the receiving and transmitting antennas along their axis of separation.

SUM,SUMRMS,SUMSQ = Intermediate variables used to calculate AMEAN, RMS and SDEV, respectively.

SUM2 = Summation variable used in the filling of the AX and AY matrices.

TEMP = An intermediate variable used to calculate ATMAX and ARMAX.

TSUM21 = Summation variable used to compute the coupling quotient at X0 = 0 by summing directly without the use of the FFT.

WAVLGTH = Wavelength in meters.

WORK = Complex array required by the FFT subroutine FOURT.

X,(Y) = Array containing the abscissa value for the plots of the X0(Y0) cuts.

XK = $2\pi/\lambda$.

XKLIM = Variable which limits the range of k_x/k and k_y/k integration when its value is less than XKMAX.

XKMAX = An upper limit (less than 1.0 and usually chosen at .9) on XKLIM; except for close antennas XKLIM will usually be less than XKMAX.

XKMIN = Sum of the diameters of the two antennas divided by their separation distance.

XKOK = The square root of the sum of the squares of XKXOK and XKYOK.

XKXOK,XKYOK = k_x/k and k_y/k, respectively.

XLIM = Variable used for adjusting XKLIM; making XLIM larger tests to see if a large enough integration range has been included; XLIM should equal 1 or 2.

XNX,XNY,XNZ = Variables used for incrementing k_x/k, k_y/k, and γ/k, respectively.

XY0 = Intermediate variable used for calculating R0.

X0,Y0,Z0 = X,Y,Z coordinates of the origin of the receiving antenna in the mutual coupling coordinate system of the transmitting antenna; specifically Z0 is the separation distance.

ID, IFL, LU, NOFRAME = Variables used by the CRT plotting routine CRTPLT3, which is a specialized routine for the NOAA/NBS CYBER 170/750.

Subroutines Not Available in the FORTRAN Library:

FOURT (a standard FFT subroutine documented with program CUPLNF in NBSIR 80-1630[2].)
PLT120R (printout plotting routine documented in NBSIR 80-1630[2].)
CRTPLT3 (a CRT plotting routine specifically written for the NOAA/NBS Cyber 170/750; we suggest you substitute your own subroutine).

Note: If the electric far field, including phase as well as amplitude, is available use the program CUPLNF documented in NBSIR 80-1630[2].

B.2 Computer Code and Sample Output

A copy of the computer code for program ENVLP and a sample output for the case BETA-1 are found below.

```
      PROGRAM ENVLP(INPUT,OUTPUT)
C
C
C     THIS PROGRAM COMPUTES THE APPROXIMATE MAXIMUM COUPLING QUOTIENT
C      BETWEEN A TRANSMITTING ANTENNA ON THE LEFT AND A RECEIVING
C     ANTENNA ON THE RIGHT,GIVEN THE SIDELOBE LEVEL IN THE DIRECTION OF
C     THE SEPARATION AXIS(THE SEPARATION AXIS IS DRAWN FROM A POINT
C     CENTRALLY LOCATED ON THE TRANSMITTING ANTENNA TO A POINT CENTRALLY
C     LOCATED ON THE RECEIVING ANTENNA), THE ANTENNA RADII,AND THE
C     SEPARATION DISTANCE ALONG THE SEPARATION AXIS.
C
C      *********************************WARNING**************************
C      DO NOT USE THIS PROGRAM FOR COUPLING INVOLVING THE MAIN BEAM.
C      CAUTION SHOULD ALSO BE USED WHEN APPLYING THIS PROGRAM TO
C      IDENTICAL ANTENNAS AS THE PROGRAM MAY NOT GIVE VALID RESULTS IF
C      THESE IDENTICAL ANTENNAS HAVE THE SAME EULER ANGLES.
C
C     THE COUPLING QUOTIENT IS COMPUTED ALONG XO AND YO PERPENDICULAR
C     LINES OR CUTS.
C
C     AX,AY,AND WORK SHOULD BE DIMENSIONED .GE. THE LARGER OF (N1,N2).
C     ACICUT SHOULD BE DIMENSIONED AT LEAST 2 GREATER THAN THE LARGER
C     OF (N1,N2).
C
C     HEAD IS AN ARRAY WHICH CONTAINS AN ALPHANUMERIC IDENTIFIER
C     TO IDENTIFY THE CASE BEING COMPUTED. IT IS PLACED AT THE TOP OF
C     THE PRINTOUT AND EACH MICROFILM FRAME. HEAD(2) CAN BE SPECIFIED
C     AS ANY TEN CHARACTER WORD THE USER WISHES.
C
C     FREQ IS THE FREQUENCY IN HERTZ.
C
C     ETAT,ETAR ARE THE CHARACTERISTIC ADMITTANCES FOR THE PROPAGATED
C     MODES IN THE WAVEGUIDE FEEDS OF THE TRANSMITTING AND RECEIVING
C     ANTENNAS,RESPECTIVELY.
C
C     GAMMAOT,GAMMAOP,GAMMAOL ARE THE REFLECTION COEFFICIENTS OF THE
C     TRANSMITTING ANTENNA,RECEIVING ANTENNA, AND THE RECEIVING LOAD,
C     RESPECTIVELY.
C
C     CAPZO IS THE WAVE IMPEDANCE OF FREE SPACE.
C
C     (XO,YO,ZO) ARE THE COORDINATES OF THE ORIGIN OF THE RECEIVING
C     ANTENNA IN THE REORIENTED RECTANGULAR SYSTEM OF THE TRANSMITTING
C     ANTENNA.
C     THE REORIENTED CORDINATE SYSTEMS OF EACH ANTENNA ARE THE COMMON
C     MUTUAL COUPLING COORDINATE SYSTEMS OF THE ANTENNAS.
C     ZO MUST BE SPECIFIED,BUT THE RANGE OF XO AND YO ARE DETERMINED
C     IMPLICITTLY BY THE REQUIREMENTS OF THE FFT ALGOTITHM FOURT.
C     ZO IS THE SEPARATION  DISTANCE IN THE DIRECTION OF THE SEPARATION
C     AXIS.
C
C     RADT=RADIUS OF SMALLEST SPHERE WHICH CIRCUMSCRIBES THE
C     TRANSMITTING ANTENNA.
C     RADR=RADIUS OF SMALLEST SPHERE WHICH CIRCUMSCRIBES THE
C     RECEIVING ANTENNA.
C     DIAMT=TWICE THE LARGER OF RADT OR WAVLGTH.
C     DIAMR=TWICE THE LARGER OF RADR OR WAVLGTH.
```

```
C
C     ST,SR ARE THE SIDELOBE LEVELS OF THE TRANSMITTING AND RECEIVING
C     ANTENNAS,RESPECTIVELY.THEY ARE GIVEN IN DECIBELS BELOW THE
C     MAIN BEAM AND ARE POSITIVE FOR SIDELOBES LESS THAN THE MAIN BEAM.
C     GAINT,GAINR ARE THE GAIN OF THE TRANSMITTING AND RECEIVING
C     ANTENNAS,RESPECTIVELY.
C
C     BFAC ADJUSTS THE INTEGRATION INCREMENTS, AND SHOULD BE
C     APPROXIMATELY 1 OR 2.  MAKING BFAC LARGER TESTS WHETHER
C     CONVERGENCE HAS BEEN REACHED.
C     IBFMAX GIVES THE MAXIMUM VALUE THAT BFAC ASSUMES.
C     XLIM ADJUSTS THE NONZERO-FILL PORTION OF THE INTEGRATION RANGE.
C     *******XLIM SHOULD BE 1 FOR THIS PROGRAM. HOWEVER IF MAKING
C     XLIM EQUAL TO 2 CHANGES THE MEAN VALUE OF THE COUPLING LOSS BY A
C     HUGE AMOUNT(SAY 10DB), THIS WOULD INDICATE THAT THE COMPUTED
C     COUPLING LOSS UNDER THE ASSUMED APPROXIMATE FAR FIELDS IS
C     UNRELIABLE. INCREASING XLIM INCREASES THE LIMITS OF INTEGRATION
C     AND AUTOMATICALLY DECREASES THE INTEGRATION INCREMENTS
C     PROPORTIONALLY TO PREVENT ALIASING.
C     ILIMAX GIVES THE MAXIMUM VALUE THAT XLIM ASSUMES.
C
C     A1,A2,B1,B2 DEFINE THE TOTAL(WITH ZERO-FILL)INTEGRATION RANGES
C     (KX/K FROM -A1 TO APPROX.A2) AND (KY/K FROM -B1 TO APPROX.B2),
C     IN INCREMENTS OF (A1+A2)/N1 OR (B1+B2)/N2 APPROX. EQUAL TO DKOK.
C     DKOK=WAVLGTH/(2*(DIAMT+DIAMR)*BFAC*XLIM).
C
C     IF SQRT((KX/K)**2+(KY/K)**2) IS .GE. XKLIM THE SPECTRUM
C     IS SET EQUAL TO ZERO. (APPRECIALBLE ZERO FILLING IS AN OPTION
C     DESIGNED TO ALLOW FINER INCREMENTS DX AND DY AT WHICH THE
C     COUPLING QUOTIENT IS COMPUTED BY THE FFT.)
C     XKLIM MUST BE EQUAL TO OR LESS THAN 1 BECAUSE
C     THE PROGRAM NEGLECTS THE EVANESCENT MODES. IN ORDER NOT TO GET
C     TOO CLOSE TO THE 1/GAMMA SINGULARITY, IT IS SAFER TO CHOOSE XKLIM
C     NO LARGER THAN XKMAX= ABOUT .9.
C
C     THE XO AND YO INCREMENTS ARE DX=WAVLGTH/(A1+A2) AND
C     DY=WAVLGTH/(B1+B2).
C
C     THE RANGE OF BOTH XO AND YO IS GIVEN APPROXIMATELY BY
C     -(DIAMT+DIAMR)*BFAC*XLIM TO +(DIAMT+DIAMR)*BFAC*XLIM, BUT ONLY
C     -(DIAMT+DIAMR) TO +(DIAMT+DIAMR) APPROXIMATELY IS PRINTED AND
C     PLOTTED WHEN XLIM*BFAC IS GREATER THAN OR EQUAL TO 1.
C     IN THE PLOTS,-1 OF THE ABSCISSA CORRESPONDS TO -(DIAMT+DIAMR) AND
C     +1 TO (DIAMT+DIAMR).
C
C
C
      COMMON/FRAME/NOFRAME
      COMMON/DDIO/ID(4),IEL,LU
      DIMENSION CLCUTXA(2000),CLCUTYA(2000)
      DIMENSION HEAD(7)
      DIMENSION X(2000),Y(2000)
      COMPLEX AX(2000),AY(2000)
      COMPLEX WORK(2000)
      COMPLEX EMM,COEF
      COMPLEX GAMMAOT,GAMMACR,GAMMALR
      COMPLEX EX,EY
```

```
      COMPLEX FOOTER
      COMPLEX TSUM21,SUM2
C
      DIMENSION NN1(1),NN2(1)
      DIMENSION ACLCUT(2010)
C
C
C
      NOFRAME=0
C     ID IS AN ARRAY WHICH CONTAINS NAME AND PHONE NUMBER ID FOR
C     MICROFILM USE.
      ID(1)=10HFRANCIS,M.
      ID(2)=10H     X-5484
      ID(3)=10H
      ID(4)=10H
      HEAD(1)=10H      CASE
      READ 3,HEAD(2)
    3 FORMAT(A10)
      PRINT 4,HEAD(2)
    4 FORMAT(1X,* CASE *,A10)
      DO 2 I=4,7
    2 HEAD(I)=10H
C     THE ARRAY HEAD PUTS A HEADER ON EACH FRAME OF MICROFILM.    .
C
C
C
C     GENERAL PARAMETERS
      READ 700,70,FREQ
     \PRINT 750,70,FREQ
      READ 775,ILIMAX,IRFMAX
      PRINT 780,ILIMAX,IRFMAX
      PI=4.*ATAN(1.0)
      CAP70=376.73
      CEE=2.997925 E8
      WAVLGTH=CEE/FREQ
      XK=2.*PI/WAVLGTH
C
C
C     PARAMETERS OF THE ANTENNAS
      READ 800,RADT,GAINT,ST,GAMMAOT
      PRINT 825,RADT,GAINT,ST,GAMMAOT
      READ 800,RADR,GAINR,SR,GAMMAOR,GAMMALR
      PRINT 850,RADR,GAINR,SR,GAMMAOR,GAMMALR
      READ 875,FTAT,FTAR
C
      EMM=1.-GAMMAOR*GAMMALR
      IF(FTAT.EQ.0.)FTAT=1./CAP70
      IF(FTAR.EQ.0.)FTAR=1./CAP70
      PRINT 880,FTAT,ETAR
      TEMP=FTAT*CAP70*(1.-CABS(GAMMAOT)**2)/4./PI
      ATMAX=SQRT(TEMP)*10.**((GAINT-ST)/20.)
      TEMP=FTAR*CAP70*(1.-CABS(GAMMAOR)**2)/4./PI
      ARMAX=SQRT(TEMP)*10.**((GAINR-SR)/20.)
      PRINT 900,ATMAX,ARMAX
C
      DIAMT=2.*AMAX1(WAVLGTH,RADT)
      DIAMR=2.*AMAX1(WAVLGTH,RADR)
```

```
      DIAMSUM=DIAMT+DIAMP
      PRINT 17
   17 FORMAT(*1*)
      PRINT 7, (DIAMSUM)**2/WAVLGTH
    7 FORMAT(1X,*MUTUAL RAYLEIGH DISTANCE = (DIAMSUM)SQUARED/WAVLGTH =
     1*F12.5* METERS*//)
      PRINT 1110
 1110 FORMAT(1X,*THIS RESULT IS FOR  AN APPROXIMATE CALCULATION*)
C
C
C     IF Z0 IS GREATER THAN   DIAMSUM**2/WAVLGTH, THE COUPLING CAN
C     BE COMPUTED APPROX. FROM THE FAR-FIELDS OF THE ANTENNA WITHOUT
C     INTEGRATION AS FOLLOWS.
      IF(Z0.LT.    DIAMSUM**2/WAVLGTH) GO TO 50
      PRINT 40
   40 FORMAT(1X,* Z0 IS GT  (DIAMSUM)SQUARED/WAVLGTH SO THE COUPLING QUO
     1TIENT IS COMPUTED APPROXIMATELY*/* DIRECTLY FROM THE FAR-FIELDS WI
     2THOUT INTEGRATION FOR X AND Y CUTS FROM -DIAMSUM TO +DIAMSUM.*//)
      N=21
      NSX=N $ NSY=N
      C=2.*DIAMSUM/(N-1.)
      COEF=2.*PI*FMN/FTAR/CAPZ0/XK
      DO 60 J=1,N
      XYO=-DIAMSUM+(J-1.)*C
      RO=SQRT(Z0**2+XYO**2)
      FOCTEP=ATMAX*ARMAX*COEF
      AX(J)=FOCTEP/RO*CEXP(CMPLX(0.,XK*RO+PI/2.))
      AY(J)=FOCTEP/RO*CEXP(CMPLX(0.,XK*RO+PI/2.))
   60 CONTINUE
      PRINT 510
      AMAX=0. $ AMINX=10. $ AMINY=10. $ SUM=0. $ SUMSQ=0.
      SUMRMS=0.
      DO 70 J1=1,N
      ACLCUT(J1)=CABS(AX(J1))
      XO=-DIAMSUM+(J1-1.)*C
      IF(ACLCUT(J1).GT.AMAX)AMAX=ACLCUT(J1)
      IF(ACLCUT(J1).LT.AMINX)AMINX=ACLCUT(J1)
      SUM=SUM+ACLCUT(J1)
      SUMRMS=SUMRMS+ACLCUT(J1)**2/N
      X(J1)=XO/DIAMSUM
   70 CONTINUE
      AMEAN=SUM/N
      DO 77 J1=1,N
      SUMSQ=SUMSQ+(AMEAN-ACLCUT(J1))**2/(N-1)
      ACLCUT(J1)=20.*ALOG10(ACLCUT(J1))
                 CLCUTXA(J1)=ACLCUT(J1)
      PRINT 515,ACLCUT(J1),X(J1)
   77 CONTINUE
      SDEV=SQRT(SUMSQ)
      RMS=SQRT(SUMRMS)
      SDEV=20.*ALOG10(1.+SDEV/AMEAN) $ AMEAN=20.*ALOG10(AMEAN)
      AMAX=20.*ALOG10(AMAX) $ RMS=20.*ALOG10(RMS)
      AMINX=20.*ALOG10(AMINX)
      PRINT 620,AMEAN,SDEV,AMAX
      PRINT 621,RMS
      PRINT 610
      AMAX=0.            $ AMEAN=0. $ SUM=0. $ SUMSQ=0.
```

```
      SUMRMS=0.
      DO 80 J2=1,N
      ACLCUT(J2)=CABS(AY(J2))
      Y0=-DIAMSUM+(J2-1.)*C
      IF(ACLCUT(J2).GT.AMAX)AMAX=ACLCUT(J2)
      IF(ACLCUT(J2).LT.AMINY)AMINY=ACLCUT(J2)
      SUMRMS=SUMRMS+ACLCUT(J2)**2/N
      SUM=SUM+ACLCUT(J2)
      Y(J2)=Y0/DIAMSUM
   80 CONTINUE
      AMEAN=SUM/N
      DO 90 J2=1,N
      SUMSQ=SUMSQ+(AMEAN-ACLCUT(J2))**2/(N-1)
      ACLCUT(J2)=20.*ALOG10(ACLCUT(J2))
                CLCUTYA(J2)=ACLCUT(J2)
      PRINT 615,ACLCUT(J2),Y(J2)
   90 CONTINUE
      SDEV=SQRT(SUMSQ)
      RMS=SQRT(SUMRMS)
      SDEV=20.*ALOG10(1.+SDEV/AMEAN) $ AMEAN=20.*ALOG10(AMEAN)
      AMAX=20.*ALOG10(AMAX) $ RMS=20.*ALOG10(RMS)
      AMINY=20.*ALOG10(AMINY)
      PRINT 620,AMEAN,SDEV,AMAX
      PRINT 621,RMS
      GO TO 2000
C
C
   50 CONTINUE
C      CALCULATION FOR Z0 LESS THAN THE RAYLEIGH DISTANCE FOLLOWS.
C     LIMITS OF INTEGRATION AND NUMBER OF POINTS.
C      IF YOU WISH TO CHANGE XLIM OR BFAC CHANGE THE CORRESPONDING LOOP
C      PARAMETER ILIMAX,IBFMAX.
      DO 1000 IXLIM=1,ILIMAX
      XLIM=IXLIM
      DO 1000 IBFAC=1,IBFMAX
      BFAC=IBFAC
      XKMAX=.9
      XKMIN=DIAMSUM/Z0
      XKLIM=XLIM*XKMIN
      XKLIM=AMIN1(XKLIM,XKMAX)
C     XKLIM DEFINES THE NONZERO-FILL LIMITS OF INTEGRATION.
      ABL=4.*XKLIM
      A1=ABL  $A2=ABL  $B1=ABL  $B2=ABL
C     FROM XKLIM TO ABL THERE IS ZERO FILLING. NEXT WE COMPUTE
C     NUMBER OF POINTS.
      DKOK=WAVLGTH/(2.*DIAMSUM)
      DKOK=AMIN1(DKOK,DIAMSUM/Z0/2.)
C     THE PRECEEDING STATEMENT IS INCLUDED HERE IN CASE IT IS EVER
C     DESIRED TO COMPUTE THE COUPLING QUOTIENT THROUGH INTEGRATION
C     EVEN WHEN Z0 IS GREATER THAN THE MUTUAL
C     RAYLEIGH DISTANCE (DIAMSUM**2/WAVLGTH).
      DKOK=DKOK/(BFAC*XLIM)
      NN1(1)=(A1+A2)/DKOK
      NN1(1)=2*((NN1(1)+1)/2)
      NN2(1)=(B1+B2)/DKOK
      NN2(1)=2*((NN2(1)+1)/2)
      N1=NN1(1)
```

```
      N2=NN2(1)
      IF(IXLIM.EQ.1)NSX=N1-1
      IF(IXLIM.EQ.1)NSY=N2-1
C
      C1=(A1+A2)/N1
      C2=(B1+B2)/N2
      COEF=-FMM*C1*C2/ETAR/CAPZO
C
C     FILLING THE AX AND AY MATRICES USED IN FOURT
      TSUM21=(0.,0.)
      DO 10 J2=1,N2
      AY(J2)=(0.,0.)
   10 CONTINUE
      DO 200 J1=1,N1
      XNX=C1*(J1-1.)
      XKXOK=XNX-A1
      SUM2=(0.,0.)
      DO 100 J2=1,N2
      XNY=C2*(J2-1.)
      XKYOK=XNY-B1
      XKOK=SQRT(XKXOK**2+XKYOK**2)
      IF(XKOK.GE.XKLIM) GO TO 100
      XNZ=SQRT(1.-XKXOK**2-XKYOK**2)
      FOOTFP=ATMAX*ARMAX*COS(XK*XKXOK*RADT)*COS(XK*XKXOK*RADR)
      FOOTFP=FOOTFP*COS(XK*XKYOK*RADT)*COS( XK*XKYOK*RADR)
C     THE COS FUNTIONS IN FOOTFP ARE A ROUGH APPPROXIMATION TO THE
C     SIDELOBE FIELD PATTERN AND RESULT IN SYMMETRY BETWEEN THE
C     XO AND YO DIRECTIONS.
      FOOTFP=FOOTFP*CEXP(CMPLX(0.,XK*XNZ*ZO))/XNZ
      SUM2=FOOTFP+SUM2
      AY(J2)=FOOTFP+AY(J2)
  100 CONTINUE
      AX(J1)=-SUM2*(-1)**J1
      TSUM21=SUM2+TSUM21
  200 CONTINUE
      DO 12 J2=1,N2
      AY(J2)=-AY(J2)*(-1)**J2
   12 CONTINUE
C     THE AX MATRIX HAS DATA COLLAPSED IN THE Y-DIRECTION AND THE AY
C     MATRIX HAS THE SAME FOR THE  X-DIRECTION. THUS,FOR AX THE KY
C     INTEGRATION HAS BEEN COMPLETED,WHILE FOR  AY THE KX INTEGRATION
C     HAS BEEN COMPLETED.
C     ****FOURT IS A SUBROUTINE THAT PERFORMS A FAST FOURIER TRANSFORM
C     FOR COMPLEX DATA. IF YOU HAVE CUPLNE YOU SHOULD HAVE FOURT.
C     HERE FOURT OPERATING ON THE MATRIX AX IS EQUIVALENT TO DOING THE
C     KX INTEGRAL,WHILE FOURT OPERATING ON AY IS EQUIVALENT TO DOING THE
C     KY INTEGRAL.
      CALL FOURT(AX,NN1,1,+1,+1,WORK)
      CALL FOURT(AY,NN2,1,+1,+1,WORK)
      DO 400 M1=1,N1
      XO=(-N1/2.+M1-1.)*WAVLGTH/(A1+A2)
      FX=CEXP(CMPLX(0.,-XK*A1*XO))
      AX(M1)=FX*COEF*AX(M1)
  400 CONTINUE
      DO 300 M2=1,N2
      YO=(-N2/2.+M2-1.)*WAVLGTH/(B1+B2)
      FY=CEXP(CMPLX(0.,-XK*B1*YO))
```

```
      AY(M2)=FY*COFF*AY(M2)
  300 CONTINUE
C
C
C     PRINTOUT
      PRINT 5,XLIM,BFAC
    5 FORMAT(1X,*XLIM=*F12.5,5X,*BFAC=*F12.5//)
      PRINT 15,N1,N2
   15 FORMAT(1X,*N1=* I6,5X,*N2=*  I6,5X,*THEY BOTH SHOULD BE EVEN*//)
      PRINT 35,WAVLGTH,RADT,RADR,ZO
   35 FORMAT(1X,*WAVLGTH,RADT,RADR,AND ZO =*4F12.5*  METERS RESPECTIVELY
     1*//)
      DX=WAVLGTH/(A1+A2)  $DY=WAVLGTH/(B1+B2)
      PRINT 55,-DX*N1/2.,DX*(N1/2.-1.),DX
   55 FORMAT(1X,*XO RANGES FROM*F12.5*  TO*F12.5*  IN INCREMENTS OF*F12.
     15*  METERS*//)
      PRINT 65,-DY*N2/2.,DY*(N2/2.-1.),DY
   65 FORMAT(1X,*YO RANGES FROM*F12.5*  TO*F12.5*  IN INCREMENTS OF*F12.
     15*  METERS*//)
      PRINT 75,-A1,A2-(A1+A2)/N1,(A1+A2)/N1
   75 FORMAT(1X,*THE INTEGRATION VARIABLE KX/K RANGES FROM*F12.5*  TO*F
     1 12.5*  IN INCREMENTS OF*F12.5//)
      PRINT 85,-B1,B2-(B1+B2)/N2,(B1+B2)/N2
   85 FORMAT(1X,*THE INTEGRATION VARIABLE KY/K RANGES FROM*F12.5*  TO*F
     1 12.5*  IN INCREMENTS OF*F12.5//)
      PRINT 87,XKLIM
   87 FORMAT(1X,*THE SPECTRUM IS ZERO FILLED BEYOND SQRT(KX2+KY2)=K TIME
     1S*F12.5//)
      PRINT 95,TSUM21*COFF
   95 FORMAT(1X,//* THE COUPLING QUOTIENT AT XO=0 AND YO=0,SUMMED DIREC
     1TLY WITHOUT THE FFT, EQUALS*2F12.5)
C
C     PRINTOUT OF XO AND YO CENTERLINE CUTS RESPECTIVELY
      PRINT 27
   27 FORMAT(1X,*XO-CUT*/)
      PRINT 25,(AX(J1),J1=1,N1)
      PRINT 29
   29 FORMAT(1X,//* YO-CUT*/)
      PRINT 25,(AY(J2),J2=1,N2)
   25 FORMAT(1X,(10F12.5))
C
C     PLOT OF MAGNITUDE OF XO AND YO CENTERLINE CUTS
      IF(XLIM*BFAC .GT. 1.) GO TO 1500
C     THESE CARDS FROM HERE TO 1500 DO THE PRINTING,PLOTTING,AND
C     STATISTICAL ANALYSIS FOR XLIM*BFAC EQUAL TO 1.IT COULD BE USED IF
C     ONE DESIRES TO PRINTOUT AND PLOT FOR XLIM*BFAC LESS THAN 1, OR
C     FOR ALL POINTS WHICH ARE COMPUTED WHEN XLIM*BFAC IS GREATER
C     THAN 1.
C     FOR BFAC*XLIM GREATER THAN 1 SKIP TO 1500,WHERE A SHORTENED
C     PRINTOUT AND STATISTICAL ANALYSIS ARE DONE.
      PRINT 510
  510 FORMAT(1X,//* MAGNITUDE (XO-CUT)*/)
      AMAX=0. $ AMINX=10. $ AMINY=10. $ SUM=0. $ SUMSQ=0.
      SUMRMS=0.
      DO 500 J1=2,N1
      ACLCUT(J1-1)=CABS(AX(J1))
      XO=(-N1/2.+J1-1.)*WAVLGTH/(A1+A2)
```

```
      IF(ACLCUT(J1-1).GT.AMAX)AMAX=ACLCUT(J1-1)
      IF(ACLCUT(J1-1).LT.AMINX)AMINX=ACLCUT(J1-1)
      X(J1-1)=XO/DIAMSUM
      SUMRMS=SUMRMS+ACLCUT(J1-1)**2/N1
      SUM=SUM+ACLCUT(J1-1)
  500 CONTINUE
  515 FORMAT(1X,F12.5*  XO=*F12.5)
      AMEAN=SUM/(N1-1)
      DO 520 J1=2,N1
      SUMSQ=SUMSQ+(AMEAN-ACLCUT(J1-1))**2/(N1-2)
      ACLCUT(J1-1)=20.*ALOG10(ACLCUT(J1-1))
                 CLCUTXA(J1-1)=ACLCUT(J1-1)
      PRINT 515,ACLCUT(J1-1),X(J1-1)
  520 CONTINUE
      SDEV=SQRT(SUMSQ)
      RMS=SQRT(SUMRMS)
      SDEV=20.*ALOG10(1.+SDEV/AMEAN) $ AMEAN=20.*ALOG10(AMEAN)
      AMAX=20.*ALOG10(AMAX) $ RMS=20.*ALOG10(RMS)
      AMINX=20.*ALOG10(AMINX)
      PRINT 620,AMEAN,SDEV,AMAX
      PRINT 621,RMS
      PRINT 610
  610 FORMAT(1X,//* MAGNITUDE (YO-CUT)*/)
      AMAX=0.             $ AMEAN=0. $ SUM=0. $ SUMSQ=0.
      SUMRMS=0.
      DO 600 J2=2,N2
      ACLCUT(J2-1)=CABS(AY(J2))
      YO=(-N2/2.+J2-1.)*WAVLGTH/(R1+R2)
      IF(ACLCUT(J2-1).GT.AMAX)AMAX=ACLCUT(J2-1)
      IF(ACLCUT(J2-1).LT.AMINY)AMINY=ACLCUT(J2-1)
      SUMRMS=SUMRMS+ACLCUT(J2-1)**2/N2
      SUM=SUM+ACLCUT(J2-1)
      Y(J2-1)=YO/DIAMSUM
  600 CONTINUE
  615 FORMAT(1X,F12.5*  YO=*F12.5)
      AMEAN=SUM/(N2-1)
      DO 625 J2=2,N2
      SUMSQ=SUMSQ+(AMEAN-ACLCUT(J2-1))**2/(N2-2)
      ACLCUT(J2-1)=20.*ALOG10(ACLCUT(J2-1))
                 CLCUTYA(J2-1)=ACLCUT(J2-1)
      PRINT 615,ACLCUT(J2-1),Y(J2-1)
  625 CONTINUE
      SDEV=SQRT(SUMSQ)
      RMS=SQRT(SUMRMS)
      AMAX=20.*ALOG10(AMAX) $ RMS=20.*ALOG10(RMS)
      SDEV=20.*ALOG10(1.+SDEV/AMEAN) $ AMEAN=20.*ALOG10(AMEAN)
       AMINY=20.*ALOG10(AMINY)
      PRINT 620,AMEAN,SDEV,AMAX
      PRINT 621,RMS
      PRINT 17
      GO TO 1000
 1500 CONTINUE
C
C     SHORTENED PRINTOUT AND STATISTICAL ANALYSIS FOR XLIM*BFAC
C     GREATER THAN 1 FOLLOWS.
      PRINT 510
      N1C=N1/(XLIM*BFAC)+.000001
```

```
            N1MIN=N1/2+1-N10/2.
            N1MAX=N1/2+1+N10/2.
            SUM=0. $ AMAX=0.
            DO 501 J1=N1MIN,N1MAX
            ACLCUT(J1-N1MIN+1)=CABS(AX(J1))
            XO=(-N1/2.+J1-1.)*WAVLGTH/(A1+A2)
            SUM=SUM+ACLCUT(J1-N1MIN+1)
            IF(ACLCUT(J1-N1MIN+1).GT.AMAX)AMAX=ACLCUT(J1-N1MIN+1)
            ACLCUT(J1-N1MIN+1)=20.*ALOG10(ACLCUT(J1-N1MIN+1))
            XO=XO/DIAMSUM
            PRINT 515,ACLCUT(J1-N1MIN+1),XO
        501 CONTINUE
            AMEAN=SUM/(N1MAX-N1MIN+1)
            AMEAN=20.*ALOG10(AMEAN)
            AMAX=20.*ALOG10(AMAX)
            PRINT 900,AMEAN,AMAX
            PRINT 610
            N20=N2/(XLIM*RFAC)+.000001
            N2MIN=N2/2+1-N20/2.
            N2MAX=N2/2+1+N20/2.
            SUM=0. $ AMAX=0.
            DO 601 J2=N2MIN,N2MAX
            ACLCUT(J2-N2MIN+1)=CABS(AY(J2))
            YO=(-N2/2.+J2-1.)*WAVLGTH/(B1+B2)
            SUM=SUM+ACLCUT(J2-N2MIN+1)
            IF(ACLCUT(J2-N2MIN+1).GT.AMAX)AMAX=ACLCUT(J2-N2MIN+1)
            ACLCUT(J2-N2MIN+1)=20.*ALOG10(ACLCUT(J2-N2MIN+1))
            YO=YO/DIAMSUM
            PRINT 615,ACLCUT(J2-N2MIN+1),YO
        601 CONTINUE
            AMEAN=SUM/(N2MAX-N2MIN+1)
            AMEAN=20.*ALOG10(AMEAN)
            AMAX=20.*ALOG10(AMAX)
            PRINT 900,AMEAN,AMAX
            PRINT 17
       1000 CONTINUE
      C
       2000 CONTINUE
      C
      C     PLOTTING
      C     **********WARNING*********
      C     CRTPLT3 IS A CRT PLOTTING ROUTINE SPECIALIZED FOR THE NOAA/NBS
      C     CYBER 170/750 MACHINE. PLT120R IS A PRINTOUT PLOTTING ROUTINE.
      C     IT MAY BE NECESSARY TO  SUPPLY YOUR OWN PLOTTING ROUTINES TO
      C     PLOT CLCUTXA,CLCUTYA AGAINST X,Y(I.E.,THE COUPLING QUOTIENT VERSUS
      C     XO AND YO). IF YOU HAVE THE PROGRAM CUPLNF YOU SHOULD HAVE PLT120R
      C
            HEAD(3)=10H XO-CUT
            CALL CRTPLT3(X,CLCUTXA,1.0,-1.0,0.,-150.,NSX,HEAD,1,1,1,0,0,1)
            CALL PLT120R(X,CLCUTXA,1.0,-1.0,0.,-150.,NSX,1H*,1,1)
            PRINT 925
            HEAD(3)=10H YO-CUT
            CALL CRTPLT3(Y,CLCUTYA,1.0,-1.0,0.,-150.,NSY,HEAD,1,1,1,0,0,1)
            CALL PLT120R(Y,CLCUTYA,1.0,-1.0,0.,-150.,NSY,1H*,1,1)
            PRINT 950
      C
      C     FORMATS
```

```
  620 FORMAT(1X,*THE MEAN COUPLING AMPLITUDE IS *,F10.4,*  DB  WITH A ST
     1ANDARD DEVIATION OF  *,F10.4,*  DB*,/,1X,*AND A MAXIMUM COUPLING A
     2MPLITUDE OF  *,F10.4,*  DB*)
  621 FORMAT(1X,*THE RMS MEAN COUPLING AMPLITUDE IS  *,F10.4,*  DB*)
  700 FORMAT(2F10.4)
  750 FORMAT(1X,*70=*,E10.4,5X,*FREQ=*,E10.4)
  775 FORMAT(2I5)
  780 FORMAT(1X,*ILIMAX=*,I5,5X,*IREFMAX=*,I5)
  800 FORMAT(F10.4,2F10.4,2(F10.4,F10.4))
  825 FORMAT(1X,*RADT=*,F10.4,5X,*GAINT=*,F10.4,5X,*ST=*,F10.4,5X,
     1 *GAMMAOT=*,2(F10.4,5X))
  850 FORMAT(1X,*RADR=*,E10.4,5X,*GAINR=*,F10.4,5X,*SR=*,F10.4,5X,
     1 *GAMMAOR=*,2(F10.4,5X),*GAMMALR=*,2(F10.4,5X))
  875 FORMAT(2F10.4)
  880 FORMAT(1X,*FTAT=*,F10.4,5X,*FTAR=*,F10.4)
  890 FORMAT(1X,*ATMAX=*,E10.4,5X,*ARMAX=*,F10.4)
  900 FORMAT(1X,*THE MEAN COUPLING AMPLITUDE IS *,F10.4,* DB AND THE MAX
     1IMUM COUPLING AMPLITUDE IS *,F10.4,* DB*)
  925 FORMAT(///,39X,*MAGNITUDES OF COUPLING QUOTIENT FOR XO-CUT*)
  950 FORMAT(///,39X,*MAGNITUDES OF COUPLING QUOTIENT FOR YO-CUT*)
      CALL DDEND
      END
```

```
 CASE  BETA-1
ZO= .1000E+01      FREQ= .1000E+11
ILIMAX=     1      IRFMAX=     1
RADT= .1000E+00      GAINT=   26.4200      ST=   30.7000      GAMMAOT=     0.0000            0.0000
RADR= .2000E+00      GAINR=   32.4400      SR=   39.5000      GAMMAOR=     0.0000            0.0000    GAMMALR=    0.0000         0.0000
ETAT=      .0027     ETAR=      .0027
ATMAX= .1723E+00      ARMAX= .1251E+00
```

```
MUTUAL RAYLEIGH DISTANCE = (DIAMSUM)SQUARED/WAVLGTH =      12.00831 METERS

THIS RESULT IS FOR  AN APPROXIMATE CALCULATION
XLIM=     1.00000     RFAC=     1.00000

N1=  192     N2=  192     THEY BOTH SHOULD BE EVEN

WAVLGTH,RADT,RADR,AND Z0 =       .02998      .10000      .20000      1.00000  METERS RESPECTIVELY

X0 RANGES FROM     -.59959  TO      .59334  IN INCREMENTS OF       .00625  METERS

Y0 RANGES FROM     -.59959  TO      .59334  IN INCREMENTS OF       .00625  METERS

THE INTEGRATION VARIABLE KX/K RANGES FROM    -2.40000  TO     2.37500  IN INCREMENTS OF      .02500

THE INTEGRATION VARIABLE KY/K RANGES FROM    -2.40000  TO     2.37500  IN INCREMENTS OF      .02500

THE SPECTRUM IS ZERO FILLED BEYOND SQRT(KX2+KY2)=K TIMES      .60000


THE COUPLING QUOTIENT AT X0=0 AND Y0=0,SUMMED DIREC TLY WITHOUT THE FFT, EQUALS  .11665E-03  .18407E-03
X0-CUT

   .31179E-04  .16125E-04  .84167E-05  .20238E-04 -.48909E-04  .34388E-04 -.11226E-03  .61104E-04 -.14687E-03  .97258E-04
  -.12584E-03  .12968E-03 -.41590E-04  .13755E-03  .85567E-04  .10224E-03  .23151E-03  .19949E-04  .33960E-03 -.90570E-04
   .37534E-03 -.19109E-03  .31942E-03 -.23860E-03  .17884E-03 -.20541E-03 -.14083E-04 -.94719E-04 -.21061E-03  .57179E-04
  -.35948E-03  .19455E-03 -.42181E-03  .26615E-03 -.38240E-03  .24825E-03 -.25418E-03  .15546E-03 -.74367E-04  .33802E-04
   .10658E-03 -.60610E-04  .23999E-03 -.87131E-04  .29343E-03 -.37943E-04  .25993E-03  .60585E-04  .15915E-03  .16235E-03
   .30065E-04  .22385E-03 -.82529E-04  .22312E-03 -.14285E-03  .16665E-03 -.13538E-03  .82469E-04 -.68678E-04  .44537E-05
   .29346E-04 -.43014E-04  .12269E-03 -.52625E-04  .18070E-03 -.31690E-04  .18876E-03  .66693E-05  .15263E-03  .51974E-04
   .95011E-04  .99102E-04  .46233E-04  .14448E-03  .32421E-04  .17980E-03  .65555E-04  .18934E-03  .13869E-03  .15465E-03
   .22782E-03  .65437E-04  .25972E-03 -.69405E-04  .32314E-03 -.21814E-03  .27988E-03 -.33428E-03  .17228E-03 -.37440E-03
   .24287E-04 -.31808E-03 -.12449E-03 -.17962E-03 -.23065E-03 -.50428E-05 -.26120E-03  .14571E-03 -.20557E-03  .22354E-03
  -.80681E-04  .21049E-03  .73440E-04  .12609E-03  .20691E-03  .16701E-04  .27781E-03 -.66574E-04  .26795E-03 -.90814E-04
   .19012E-03 -.54147E-04  .83149E-04  .16868E-04 -.37688E-05  .82999E-04 -.31444E-04  .11264E-03  .12769E-04  .96424E-04
   .10965E-03  .49554E-04  .21670E-03  .17785E-05  .28711E-03 -.18766E-04  .29087E-03  .77642E-06  .22841E-03  .53371E-04
   .13051E-03  .11783E-03  .44383E-04  .17069E-03  .12198E-04  .19740E-03  .51994E-04  .19696E-03  .14954E-03  .17860E-03
   .26472E-03  .15332E-03  .34921E-03  .12613E-03  .36690E-03  .93484E-04  .30804E-03  .47373E-04  .19090E-03 -.16695E-04
   .51578E-04 -.92990E-04 -.72455E-04 -.16450E-03 -.15669E-03 -.20881E-03 -.19552E-03 -.20825E-03 -.19833E-03 -.15925E-03
  -.17939E-03 -.75848E-04 -.14808E-03  .14643E-04 -.10488E-03  .82132E-04 -.44869E-04  .10572E-03  .34461E-04  .81777E-04
   .12631E-03  .24673E-04  .21358E-03 -.39809E-04  .27485E-03 -.85210E-04  .29435E-03 -.94464E-04  .27019E-03 -.65188E-04
   .21615E-03 -.85236E-05  .15560E-03  .57222E-04  .11070E-03  .11479E-03  .92469E-04  .15424E-03  .97072E-04  .17488E-03
   .11017E-03  .18252E-03  .11665E-03  .18407E-03  .11017E-03  .18252E-03  .97072E-04  .17488E-03  .92469E-04  .15424E-03
   .11070E-03  .11479E-03  .15560E-03  .57222E-04  .21615E-03 -.85236E-05  .27019E-03 -.65188E-04  .29435E-03 -.94464E-04
   .27485E-03 -.85210E-04  .21358E-03 -.39809E-04  .12631E-03  .24673E-04  .34461E-04  .81777E-04 -.44869E-04  .10572E-03
  -.10488E-03  .82132E-04 -.14808E-03  .14643E-04 -.17939E-03 -.75848E-04 -.19833E-03 -.15925E-03 -.19552E-03 -.20825E-03
  -.15669E-03 -.20881E-03 -.72455E-04 -.16450E-03  .51578E-04 -.92990E-04  .19090E-03 -.16695E-04  .30804E-03  .47373E-04
   .36690E-03  .93484E-04  .34921E-03  .12613E-03  .26472E-03  .15332E-03  .14954E-03  .17860E-03  .51994E-04  .19696E-03
   .12198E-04  .19740E-03  .44383E-04  .17069E-03  .13051E-03  .11783E-03  .22841E-03  .53371E-04  .29087E-03  .77642E-06
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  -.37688E-05  .82999E-04  .83149E-04  .16868E-04  .19012E-03 -.54147E-04  .26795E-03 -.90814E-04  .27781E-03 -.66574E-04
   .20691E-03  .16701E-04  .73440E-04  .12609E-03 -.80681E-04  .21049E-03 -.20557E-03  .22354E-03 -.26120E-03  .14571E-03
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 .29343E-03 -.37943E-04  .23999E-03 -.87131E-04  .10658E-03 -.60610E-04 -.74367E-04  .33802E-04 -.25418E-03  .15546E-03
-.38240E-03  .24825E-03 -.42181E-03  .26615E-03 -.35948E-03  .19455E-03 -.21061E-03  .57179E-04 -.14083E-04 -.94719E-04
 .17884E-03 -.20541E-03  .31942E-03 -.23860E-03  .37534E-03 -.19109E-03  .33960E-03 -.90570E-04  .23151E-03  .19949E-04
 .89567E-04  .10224E-03 -.41590E-04  .13755E-03 -.12584E-03  .12968E-03 -.14687E-03  .97258E-04 -.11226E-03  .61104E-04
-.48909E-04  .34388E-04  .84167E-05  .20238E-04
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 .37534E-03 -.19109E-03  .31942E-03 -.23860E-03  .17884E-03 -.20541E-03 -.14083E-04 -.94719E-04 -.21061E-03  .57179E-04
-.35948E-03  .19455E-03 -.42181E-03  .26615E-03 -.38240E-03  .24825E-03 -.25418E-03  .15546E-03 -.74367E-04  .33802E-04
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 .30065E-04  .22385E-03 -.82529E-04  .22312E-03 -.14285E-03  .16665E-03 -.13538E-03  .82469E-04 -.68678E-04  .44537E-05
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-.48909E-04  .34388E-04  .84167E-05  .20238E-04
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  -77.86874  XO=     -.96808
  -75.08232  XO=     -.95767
  -74.86105  XO=     -.94726
  -76.85109  XO=     -.93685
  -77.33389  XO=     -.92644
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-69.08230   X0=    -.90562
-67.51055   X0=    -.89521
-67.98716   X0=    -.88480
-71.29727   X0=    -.87439
-80.37624   X0=    -.86398
-73.22167   X0=    -.85358
-67.77090   X0=    -.84317
-66.04224   X0=    -.83276
-66.82239   X0=    -.82235
-70.51726   X0=    -.81194
-81.75674   X0=    -.80153
-78.22981   X0=    -.79112
-71.85826   X0=    -.78071
-70.57781   X0=    -.77030
-71.47323   X0=    -.75989
-72.86606   X0=    -.74948
-72.92306   X0=    -.73907
-72.47245   X0=    -.72866
-73.17158   X0=    -.71825
-75.99846   X0=    -.70784
-83.24539   X0=    -.69743
-85.66809   X0=    -.68702
-77.49031   X0=    -.67662
-74.72942   X0=    -.66621
-74.47624   X0=    -.65580
-75.85086   X0=    -.64539
-77.24729   X0=    -.63498
-76.38057   X0=    -.62457
-74.76514   X0=    -.61416
-73.96330   X0=    -.60375
-73.65004   X0=    -.59334
-72.50378   X0=    -.58293
-70.23874   X0=    -.57252
-68.18145   X0=    -.56211
-67.21074   X0=    -.55170
-67.69914   X0=    -.54129
-69.92410   X0=    -.53088
-73.20917   X0=    -.52047
-72.73901   X0=    -.51006
-70.48391   X0=    -.49965
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-76.71789   X0=    -.46843
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-70.88261   X0=    -.44761
-70.96656   X0=    -.43720
-74.08054   X0=    -.42679
-81.42766   X0=    -.41638
-81.60961   X0=    -.40597
-78.64044   X0=    -.39556
-80.24083   X0=    -.38515
-78.39247   X0=    -.37474
-73.28244   X0=    -.36433
-70.82046   X0=    -.35392
-70.72599   X0=    -.34351
-72.59480   X0=    -.33310
-75.09800   X0=    -.32269
-75.07176   X0=    -.31228
-74.07662   X0=    -.30187
-73.81974   X0=    -.29146
-72.65532   X0=    -.28106
-70.28788   X0=    -.27065
-68.60580   X0=    -.26024
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-74.35075   YO=       -.22901
-79.46638   YO=       -.21860
-74.90662   YO=       -.20819
-71.66524   YO=       -.19778
-70.88326   YO=       -.18737
-71.89122   YO=       -.17696
-74.20969   YO=       -.16655
-76.54797   YO=       -.15614
-77.50886   YO=       -.14573
-78.79745   YO=       -.13532
-81.03746   YO=       -.12491
-77.80865   YO=       -.11450
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-70.81946   YO=       -.09369
-70.19688   YO=       -.08328
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-74.90244   YO=       -.03123
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-73.42448   YO=       -.01041
-73.23388   YO=       0.00000
-73.42448   YO=        .01041
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-71.12082   YO=        .07287
-70.19688   YO=        .08328
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-77.50886   YO=        .14573
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-74.35075   YO=        .22901
-70.12637   YO=        .23942
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-70.28788   YO=        .27065
-72.65532   YO=        .28106
-73.81974   YO=        .29146
-74.07662   YO=        .30187
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-75.09800   YO=        .32269
-72.59480   YO=        .33310
-70.72599   YO=        .34351
-70.82046   YO=        .35392
-73.28244   YO=        .36433
-78.39247   YO=        .37474
-80.24082   YO=        .38515
-78.64044   YO=        .39556
-81.60961   YO=        .40597
-81.42766   YO=        .41638
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    -73.65619  YO=        .45802
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    -72.93096  YO=        .47884
    -70.35141  YO=        .48924
    -70.48391  YO=        .49965
    -72.73901  YO=        .51006
    -73.20917  YO=        .52047
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    -67.69914  YO=        .54129
    -67.21074  YO=        .55170
    -68.18145  YO=        .56211
    -70.23874  YO=        .57252
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    -73.65004  YO=        .59334
    -73.96330  YO=        .60375
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    -76.38057  YO=        .62457
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    -75.85086  YO=        .64539
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    -77.49031  YO=        .67662
    -85.66809  YO=        .68702
    -83.24539  YO=        .69743
    -75.99846  YO=        .70784
    -73.17158  YO=        .71825
    -72.47245  YO=        .72866
    -72.92306  YO=        .73907
    -72.86606  YO=        .74948
    -71.47323  YO=        .75989
    -70.57781  YO=        .77030
    -71.85826  YO=        .78071
    -78.22981  YO=        .79112
    -81.75674  YO=        .80153
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    -66.82239  YO=        .82235
    -66.04224  YO=        .83276
    -67.77090  YO=        .84317
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    -80.37634  YO=        .86399
    -71.29727  YO=        .87439
    -67.98716  YO=        .88480
    -67.51055  YO=        .89521
    -69.08230  YO=        .90562
    -72.67659  YO=        .91603
    -77.33389  YO=        .92644
    -76.85109  YO=        .93685
    -74.86105  YO=        .94726
    -75.08232  YO=        .95767
    -77.86874  YO=        .96808
    -84.46768  YO=        .97849
    -93.18390  YO=        .98890
THE MEAN COUPLING AMPLITUDE IS   -73.1286  DB  WITH A STANDARD DEVIATION OF      3.1728  DB
AND A MAXIMUM COUPLING AMPLITUDE OF    -66.0422  DB
THE RMS MEAN COUPLING AMPLITUDE IS    -72.3835  DB


MAGNITUDE (YO-CUT)

    -93.18390  YO=      -.98890
    -84.46768  YO=      -.97849
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-77.86874   YO=     -.96808
-75.08232   YO=     -.95767
-74.86105   YO=     -.94726
-76.85109   YO=     -.93685
-77.33386   YO=     -.92644
-72.67656   YO=     -.91603
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-67.51055   YO=     -.89521
-67.98716   YO=     -.88480
-71.29727   YO=     -.87439
-80.37634   YO=     -.86399
-73.22167   YO=     -.85358
-67.77090   YO=     -.84317
-66.04224   YO=     -.83276
-66.82239   YO=     -.82235
-70.81726   YO=     -.81194
-81.75674   YO=     -.80153
-78.22981   YO=     -.79112
-71.85826   YO=     -.78071
-70.87781   YO=     -.77030
-71.47323   YO=     -.75989
-72.86606   YO=     -.74948
-72.92306   YO=     -.73907
-72.47245   YO=     -.72866
-73.17198   YO=     -.71825
-75.99846   YO=     -.70784
-83.24539   YO=     -.69743
-85.66809   YO=     -.68702
-77.49031   YO=     -.67662
-74.72942   YO=     -.66621
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-75.85086   YO=     -.64539
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-76.38057   YO=     -.62457
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-73.96330   YO=     -.60375
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-68.18145   YO=     -.56211
-67.21074   YO=     -.55170
-67.69914   YO=     -.54129
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-72.73901   YO=     -.51006
-70.48391   YO=     -.49965
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-76.71789   YO=     -.46843
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-73.28244   YO=     -.36433
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-70.72595   YO=     -.34351
-72.59480   YO=     -.33310
-75.09800   YO=     -.32269
-75.07176   YO=     -.31228
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-68.43582  YO=    -.24983
-70.12637  YO=    -.23942
-74.35075  YO=    -.22901
-79.46638  YO=    -.21860
-74.90662  YO=    -.20819
-71.66524  YO=    -.19778
-70.88326  YO=    -.18737
-71.89122  YO=    -.17696
-74.20969  YO=    -.16655
-76.54797  YO=    -.15614
-77.50886  YO=    -.14573
-78.79745  YO=    -.13532
-81.03746  YO=    -.12491
-77.80865  YO=    -.11450
-73.26066  YO=    -.10409
-70.81946  YO=    -.09369
-70.19688  YO=    -.08328
-71.12082  YO=    -.07287
-73.29834  YO=    -.06246
-75.60896  YO=    -.05205
-75.94629  YO=    -.04164
-74.90244  YO=    -.03123
-73.97861  YO=    -.02082
-73.42448  YO=    -.01041
-73.23388  YO=    0.00000
-73.42448  YO=     .01041
-73.97861  YO=     .02082
-74.90244  YO=     .03123
-75.94629  YO=     .04164
-75.60896  YO=     .05205
-73.29834  YO=     .06246
-71.12082  YO=     .07287
-70.19688  YO=     .08328
-70.81946  YO=     .09369
-73.26066  YO=     .10409
-77.80865  YO=     .11450
-81.03746  YO=     .12491
-78.79745  YO=     .13532
-77.50886  YO=     .14573
-76.54797  YO=     .15614
-74.20969  YO=     .16655
-71.89122  YO=     .17696
-70.88326  YO=     .18737
-71.66524  YO=     .19778
-74.90662  YO=     .20819
-79.46638  YO=     .21860
-74.35075  YO=     .22901
-70.12637  YO=     .23942
-68.43582  YO=     .24983
-68.60580  YO=     .26024
-70.28788  YO=     .27065
-72.65532  YO=     .28106
-73.81974  YO=     .29146
-74.07662  YO=     .30187
-75.07176  YO=     .31228
-75.09800  YO=     .32269
-72.59480  YO=     .33310
-70.72599  YO=     .34351
-70.82046  YO=     .35392
```
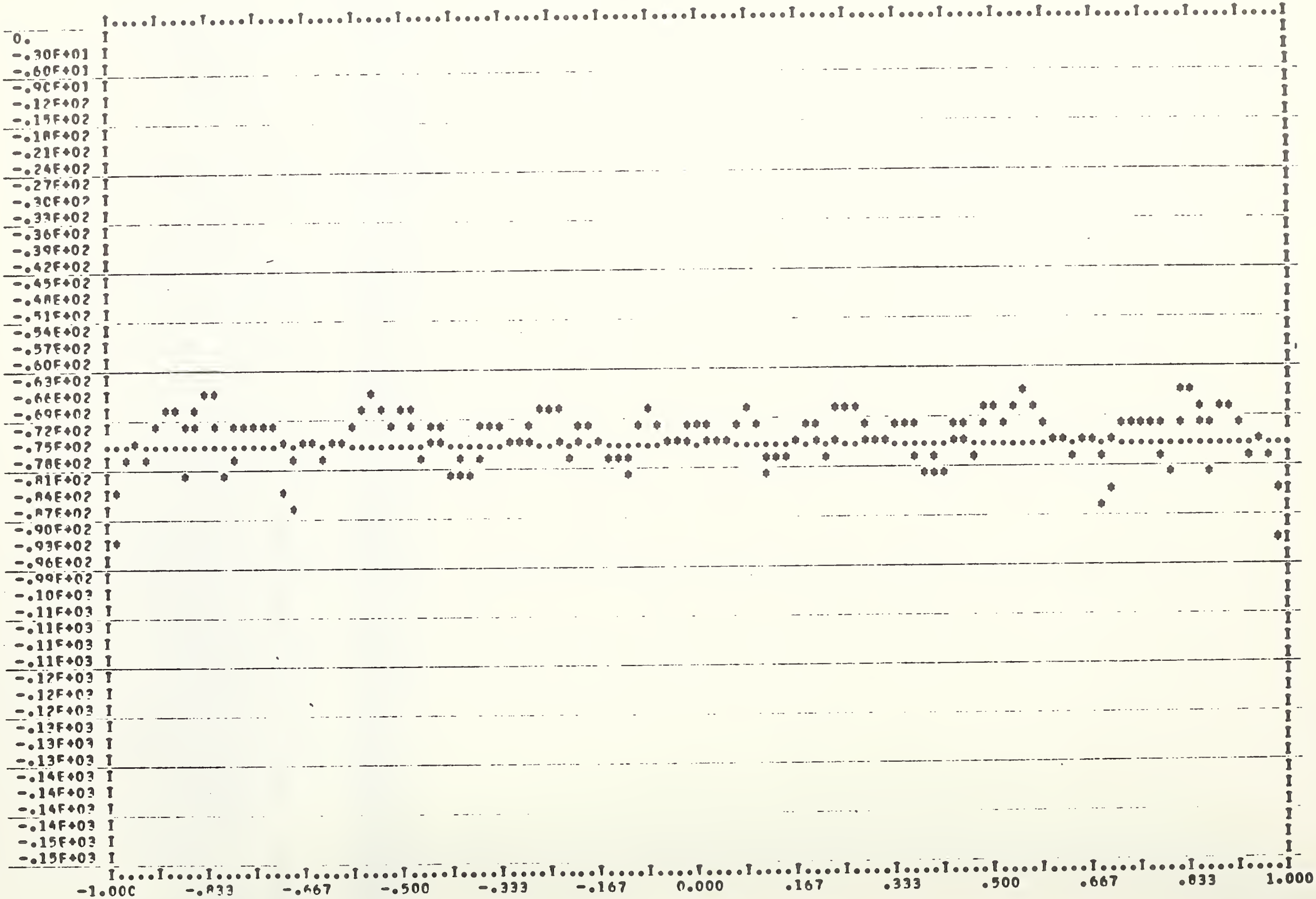
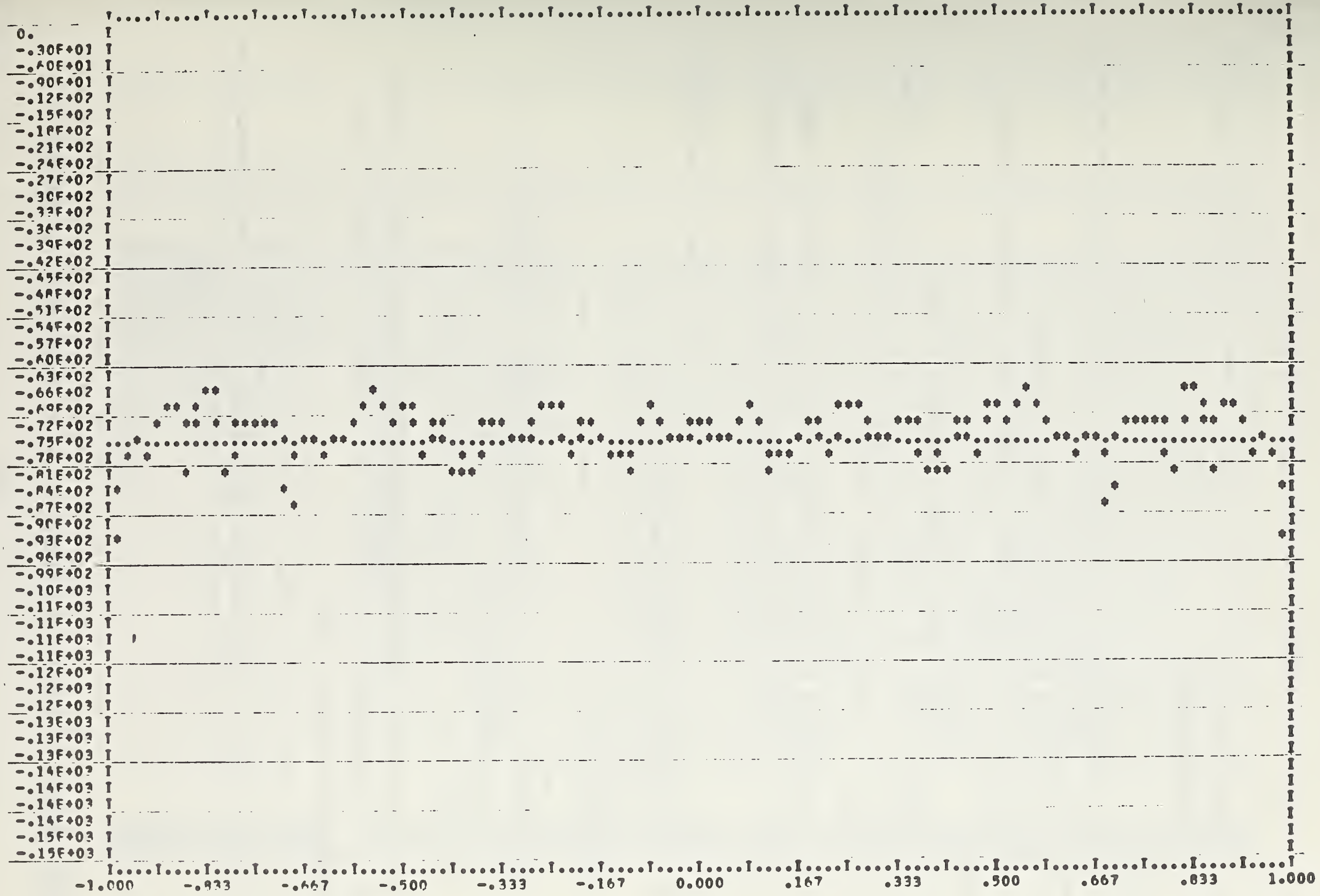

```
    -73.28244  YO=      .36433
    -78.39247  YO=      .37474
    -80.24083  YO=      .38515
    -78.64044  YO=      .39556
    -81.60961  YO=      .40597
    -81.42766  YO=      .41638
    -74.09054  YO=      .42679
    -70.96656  YO=      .43720
    -70.88261  YO=      .44761
    -73.65619  YO=      .45802
    -76.71789  YO=      .46843
    -72.93996  YO=      .47884
    -70.35141  YO=      .48924
    -70.48391  YO=      .49965
    -72.73901  YO=      .51006
    -73.20917  YO=      .52047
    -69.92410  YO=      .53088
    -67.69914  YO=      .54129
    -67.21074  YO=      .55170
    -68.18145  YO=      .56211
    -70.23874  YO=      .57252
    -72.50378  YO=      .58293
    -73.65004  YO=      .59334
    -73.96330  YO=      .60375
    -74.76514  YO=      .61416
    -76.38057  YO=      .62457
    -77.24729  YO=      .63498
    -75.85086  YO=      .64539
    -74.47624  YO=      .65580
    -74.72942  YO=      .66621
    -77.49031  YO=      .67662
    -85.66809  YO=      .68702
    -83.24539  YO=      .69743
    -75.59846  YO=      .70784
    -73.17158  YO=      .71825
    -72.47245  YO=      .72866
    -72.92306  YO=      .73907
    -72.86606  YO=      .74948
    -71.47323  YO=      .75989
    -70.57781  YO=      .77030
    -71.85826  YO=      .78071
    -78.22981  YO=      .79112
    -81.75674  YO=      .80153
    -70.51726  YO=      .81194
    -66.82239  YO=      .82235
    -66.04224  YO=      .83276
    -67.77090  YO=      .84317
    -73.22167  YO=      .85358
    -80.27634  YO=      .86399
    -71.29727  YO=      .87439
    -67.98716  YO=      .88480
    -67.51055  YO=      .89521
    -69.08230  YO=      .90562
    -72.67659  YO=      .91603
    -77.33380  YO=      .92644
    -76.85109  YO=      .93685
    -74.86105  YO=      .94726
    -75.08232  YO=      .95767
    -77.86874  YO=      .96808
    -84.46768  YO=      .97849
    -93.18390  YO=      .98890
 THE MEAN COUPLING AMPLITUDE IS   -73.1286  DB  WITH A STANDARD DEVIATION OF      3.1728  DB
 AND A MAXIMUM COUPLING AMPLITUDE OF    -66.0422  DB
 THE RMS MEAN COUPLING AMPLITUDE IS    -72.3835  DB
```



MAGNITUDES OF COUPLING QUOTIENT FOR XO-CUT



MAGNITUDES OF COUPLING QUOTIENT FOR YO-CUT

Appendix C.* Subroutine FOURT(DATA,NN,NDIM,ISIGN,IFORM,WORK)

Purpose: To compute the discrete Fourier transform of the array DATA using the fast Fourier transform algorithm.

Arguments:

DATA is a multidimensional complex array whose real and imaginary parts are adjacent in storage, such as FORTRAN IV places them.

NN is an array giving the lengths of the array in each dimension.

NDIM is the number of dimensions of the array DATA, hence the number of elements in array NN.

ISIGN is +1 for a forward transform -1 for a reverse transform.

IFORM If all imaginary parts of the input array are zero (input array is real), set IFORM = 0 to reduce running time by approximately 40 percent, otherwise set IFORM = +1.

WORK if all dimensions of DATA are not integral powers of 2, specify array WORK in calling routine with dimension greater than largest non 2^k dimension, otherwise set WORK = 0.

Methods: Using the Fast Fourier transform algorithm, FOURT replaces the array DATA with its discrete Fourier transform given by

$$\mathrm{TRANSFORM}(K1,K2,\ldots) = \sum_{J1=1}^{NN(1)} \sum_{J2=1}^{NN(2)} \mathrm{DATA}\,(J1,J2)\, e^{i\,2\pi\mathrm{ISIGN}\left\{\frac{(J1-1)(K1-1)}{NN(1)} + \frac{(J2-1)(K2-1)}{NN(2)} + \ldots\right\}} .$$

For a more complete description of the subroutine and its usage, see the comments included at the beginning of its listing or the supplementary comments by the programmer, Norman Brenner of MIT.

Uses external library functions COS, SIN, FLOAT, and MAXO.

Note: Brenner, Norman, "FOUR2 and FOURT program description," private communication, 1968.

---

*This appendix is taken from appendix A.1.11 of Stubenrauch and Yaghjian [2].

```
      SUBROUTINE FOURT (DATA, NN, NDIM, ISIGN, IFORM, WORK)             FOURT
C                                                                       FOURT
C     THE COOLEY-TUKEY FAST FOURIER TRANSFORM IN USASI BASIC FORTRAN    FOURT
C                                                                       FOURT
C     TRANSFORM(K1,K2,...) = SUM(DATA(J1,J2,...)*EXP(ISIGN*2*PI*SQRT(-1)FOURT
C     *((J1-1)*(K1-1)/NN(1)+(J2-1)*(K2-1)/NN(2)+...))), SUMMED FOR ALL  FOURT
C     J1, K1 BETWEEN 1 AND NN(1), J2, K2 BETWEEN 1 AND NN(2), ETC.      FOURT
C     THERE IS NO LIMIT TO THE NUMBER OF SUBSCRIPTS.  DATA IS A         FOURT
C     MULTIDIMENSIONAL COMPLEX ARRAY WHOSE REAL AND IMAGINARY           FOURT
C     PARTS ARE ADJACENT IN STORAGE, SUCH AS FORTRAN IV PLACES THEM.    FOURT
C     IF ALL IMAGINARY PARTS ARE ZERO (DATA ARE DISGUISED REAL), SET    FOURT
C     IFORM TO ZERO TO CUT THE RUNNING TIME BY UP TO FORTY PERCENT.     FOURT
C     OTHERWISE, IFORM = +1.  THE LENGTHS OF ALL DIMENSIONS ARE         FOURT
C     STORED IN ARRAY NN, OF LENGTH NDIM.  THEY MAY BE ANY POSITIVE     FOURT
C     INTEGERS, THO THE PROGRAM RUNS FASTER ON COMPOSITE INTEGERS, AND  FOURT
C     ESPECIALLY FAST ON NUMBERS RICH IN FACTORS OF TWO.  ISIGN IS +1   FOURT
C     OR -1.  IF A -1 TRANSFORM IS FOLLOWED BY A +1 ONE (OR A +1        FOURT
C     BY A -1) THE ORIGINAL DATA REAPPEAR, MULTIPLIED BY NTOT (=NN(1)*  FOURT
C     NN(2)*...).  TRANSFORM VALUES ARE ALWAYS COMPLEX, AND ARE RETURNEDFOURT
C     IN ARRAY DATA, REPLACING THE INPUT.  IN ADDITION, IF ALL          FOURT
C     DIMENSIONS ARE NOT POWERS OF TWO, ARRAY WORK MUST BE SUPPLIED,    FOURT
C     COMPLEX OF LENGTH EQUAL TO THE LARGEST NON 2**K DIMENSION.        FOURT
C     OTHERWISE, REPLACE WORK BY ZERO IN THE CALLING SEQUENCE.          FOURT
C     NORMAL FORTRAN DATA ORDERING IS EXPECTED, FIRST SUBSCRIPT VARYING FOURT
C     FASTEST.  ALL SUBSCRIPTS BEGIN AT ONE.                            FOURT
C                                                                       FOURT
C     RUNNING TIME IS MUCH SHORTER THAN THE NAIVE NTOT**2, BEING        FOURT
C     GIVEN BY THE FOLLOWING FORMULA.  DECOMPOSE NTOT INTO              FOURT
C     2**K2 * 3**K3 * 5**K5 * ....  LET SUM2 = 2*K2, SUMF = 3*K3 + 5*K5 FOURT
C     + ... AND NF = K3 + K5 + ....  THE TIME TAKEN BY A MULTI-         FOURT
C     DIMENSIONAL TRANSFORM ON THESE NTOT DATA IS T = T0 + NTOT*(T1+    FOURT
C     T2*SUM2+T3*SUMF+T4*NF).  ON THE CDC 3300 (FLOATING POINT ADD TIME FOURT
C     OF SIX MICROSECONDS), T = 3000 + NTOT*(500+43*SUM2+68*SUMF+       FOURT
C     320*NF) MICROSECONDS ON COMPLEX DATA.  IN ADDITION, THE           FOURT
C     ACCURACY IS GREATLY IMPROVED, AS THE RMS RELATIVE ERROR IS        FOURT
C     BOUNDED BY 3*2**(-B)*SUM(FACTOR(J)**1.5), WHERE B IS THE NUMBER   FOURT
C     OF BITS IN THE FLOATING POINT FRACTION AND FACTOR(J) ARE THE      FOURT
C     PRIME FACTORS OF NTOT.                                            FOURT
C                                                                       FOURT
C     PROGRAM BY NORMAN BRENNER FROM THE BASIC PROGRAM BY CHARLES       FOURT
C     RADER.  RALPH ALTER SUGGESTED THE IDEA FOR THE DIGIT REVERSAL.    FOURT
C     MIT LINCOLN LABORATORY, AUGUST 1967.  THIS IS THE FASTEST AND MOSTFOURT
C     VERSATILE VERSION OF THE FFT KNOWN TO THE AUTHOR.  SHORTER PRO-   FOURT
C     GRAMS FOUR1 AND FOUR2 RESTRICT DIMENSION LENGTHS TO POWERS OF TWO.FOURT
C     SEE-- IEEE AUDIO TRANSACTIONS (JUNE 1967), SPECIAL ISSUE ON FFT.  FOURT
C                                                                       FOURT
C     THE DISCRETE FOURIER TRANSFORM PLACES THREE RESTRICTIONS UPON THE FOURT
C     DATA.                                                             FOURT
C     1.  THE NUMBER OF INPUT DATA AND THE NUMBER OF TRANSFORM VALUES   FOURT
C     MUST BE THE SAME.                                                 FOURT
C     2.  BOTH THE INPUT DATA AND THE TRANSFORM VALUES MUST REPRESENT   FOURT
C     EQUISPACED POINTS IN THEIR RESPECTIVE DOMAINS OF TIME AND         FOURT
C     FREQUENCY.  CALLING THESE SPACINGS DELTAT AND DELTAF, IT MUST BE  FOURT
C     TRUE THAT DELTAF=2*PI/(NN(I)*DELTAT).  OF COURSE, DELTAT NEED NOT FOURT
C     BE THE SAME FOR EVERY DIMENSION.                                  FOURT
C     3.  CONCEPTUALLY AT LEAST, THE INPUT DATA AND THE TRANSFORM OUTPUTFOURT
C     REPRESENT SINGLE CYCLES OF PERIODIC FUNCTIONS.                    FOURT
C                                                                       FOURT
C     EXAMPLE 1.  THREE-DIMENSIONAL FORWARD FOURIER TRANSFORM OF A      FOURT
C     COMPLEX ARRAY DIMENSIONED 32 BY 25 BY 13 IN FORTRAN IV.           FOURT
C     DIMENSION DATA(32,25,13),WORK(50),NN(3)                           FOURT


C     COMPLEX DATA                                                      FOURT
C     DATA NN/32,25,13/                                                 FOURT
C     DO 1 I=1,32                                                       FOURT
C     DO 1 J=1,25                                                       FOURT
C     DO 1 K=1,13                                                       FOURT
C   1 DATA(I,J,K)=COMPLEX VALUE                                         FOURT
C     CALL FOURT(DATA,NN,3,-1,1,WORK)                                   FOURT
C                                                                       FOURT
C     EXAMPLE 2.  ONE-DIMENSIONAL FORWARD TRANSFORM OF A REAL ARRAY OF  FOURT
C     LENGTH 64 IN FORTRAN II.                                          FOURT
C     DIMENSION DATA(2,64)                                              FOURT
C     DO 2 I=1,64                                                       FOURT
C     DATA(1,I)=REAL PART                                               FOURT
C   2 DATA(2,I)=0.                                                      FOURT
C     CALL FOURT(DATA,64,1,-1,0,0)                                      FOURT
C                                                                       FOURT
```

```
      DIMENSION DATA (1), NN (1), IFACT (32), WORK (1)
      WR = 0.
      WI = 0.
      WSTPR = 0.
      WSTPI = 0.
      TWOPI = 6.283185307
      IF (NDIM - 1)1280, 100, 100
  100 NTOT = 2
      DO 110 IDIM = 1, NDIM
      IF (NN (IDIM))1280, 1280, 110
  110 NTOT = NTOT * NN (IDIM)
C
C     MAIN LOOP FOR EACH DIMENSION
C
      NP1 = 2
      DO 1270 IDIM = 1, NDIM
      N = NN (IDIM)
      NP2 = NP1 * N
      IF (N - 1)1280, 1260, 120
C
C     FACTOR N
C
  120 M = N
      NTWO = NP1
      IF = 1
      IDIV = 2
  130 IQUOT = M / IDIV
      IREM = M - IDIV * IQUOT
      IF (IQUOT - IDIV)210, 140, 140
  140 IF (IREM)160, 150, 160
  150 NTWO = NTWO + NTWO
      M = IQUOT
      GO TO 130
  160 IDIV = 3
  170 IQUOT = M / IDIV
      IREM = M - IDIV * IQUOT
      IF (IQUOT - IDIV)230, 180, 180
  180 IF (IREM)200, 190, 200
  190 IFACT (IF) = IDIV
      IF = IF + 1
      M = IQUOT
      GO TO 170
  200 IDIV = IDIV + 2
      GO TO 170
  210 IF (IREM)230, 220, 230
  220 NTWO = NTWO + NTWO
      GO TO 240
  230 IFACT (IF) = M
C
C     SEPARATE FOUR CASES--
C        1. COMPLEX TRANSFORM OR REAL TRANSFORM FOR THE 4TH, 5TH,ETC.
C           DIMENSIONS.
C        2. REAL TRANSFORM FOR THE 2ND OR 3RD DIMENSION.  METHOD--
C           TRANSFORM HALF THE DATA, SUPPLYING THE OTHER HALF BY CON-
C           JUGATE SYMMETRY.
C        3. REAL TRANSFORM FOR THE 1ST DIMENSION, N ODD.  METHOD--
C           TRANSFORM HALF THE DATA AT EACH STAGE, SUPPLYING THE OTHER
C           HALF BY CONJUGATE SYMMETRY.
C        4. REAL TRANSFORM FOR THE 1ST DIMENSION, N EVEN.  METHOD--
C           TRANSFORM A COMPLEX ARRAY OF LENGTH N/2 WHOSE REAL PARTS
C           ARE THE EVEN NUMBERED REAL VALUES AND WHOSE IMAGINARY PARTS

C           ARE THE ODD NUMBERED REAL VALUES.  SEPARATE AND SUPPLY
C           THE SECOND HALF BY CONJUGATE SYMMETRY.
C
  240 NON2 = NP1 * (NP2 / NTWO)
      ICASE = 1
      IF (IDIM - 4)250, 300, 300
  250 IF (IFORM)260, 260, 300
  260 ICASE = 2
      IF (IDIM - 1)270, 270, 300
  270 ICASE = 3
      IF (NTWO - NP1)300, 300, 280
  280 ICASE = 4
      NTWO = NTWO / 2
      N = N / 2
      NP2 = NP2 / 2
      NTOT = NTOT / 2
```

```
      I = 3
      DO 290 J = 2, NTOT
      DATA (J) = DATA (I)
  290 I = I + 2
  300 I1RNG = NP1
      IF (ICASE - 2)320, 310, 320
  310 I1RNG = NP0 * (1 + NPREV / 2)
C
C     SHUFFLE ON THE FACTORS OF TWO IN N.  AS THE SHUFFLING
C     CAN BE DONE BY SIMPLE INTERCHANGE, NO WORKING ARRAY IS NEEDED
C
  320 IF (NTWO - NP1)700, 700, 330
  330 NP2HF = NP2 / 2
      J = 1
      DO 390 I2 = 1, NP2, NON2
      IF (J - I2)340, 360, 360
  340 I1MAX = I2 + NON2 - 2
      DO 350 I1 = I2, I1MAX, 2
      DO 350 I3 = I1, NTOT, NP2
      J3 = J + I3 - I2
      TEMPR = DATA (I3)
      TEMPI = DATA (I3 + 1)
      DATA (I3) = DATA (J3)
      DATA (I3 + 1) = DATA (J3 + 1)
      DATA (J3) = TEMPR
  350 DATA (J3 + 1) = TEMPI
  360 M = NP2HF
  370 IF (J - M)390, 390, 380
  380 J = J - M
      M = M / 2
      IF (M - NON2)390, 370, 370
  390 J = J + M
C
C     MAIN LOOP FOR FACTORS OF TWO.  PERFORM FOURIER TRANSFORMS OF
C     LENGTH FOUR, WITH ONE OF LENGTH TWO IF NEEDED.  THE TWIDDLE FACTOR
C     W=EXP(ISIGN*2*PI*SQRT(-1)*M/(4*MMAX)).  CHECK FOR W=ISIGN*SQRT(-1)
C     AND REPEAT FOR W=ISIGN*SQRT(-1)*CONJUGATE(W).
C
      NON2T = NON2 + NON2
      IPAR = NTWO / NP1
  400 IF (IPAR - 2)440, 420, 410
  410 IPAR = IPAR / 4
      GO TO 400
  420 DO 430 I1 = 1, I1RNG, 2
      DO 430 J3 = I1, NON2, NP1
      DO 430 K1 = J3, NTOT, NON2T
      K2 = K1 + NON2
      TEMPR = DATA (K2)
      TEMPI = DATA (K2 + 1)
      DATA (K2) = DATA (K1) - TEMPR
      DATA (K2 + 1) = DATA (K1 + 1) - TEMPI
      DATA (K1) = DATA (K1) + TEMPR
  430 DATA (K1 + 1) = DATA (K1 + 1) + TEMPI
  440 MMAX = NON2
  450 IF (MMAX - NP2HF)460, 700, 700
  460 LMAX = MAX0 (NON2T, MMAX / 2)
      IF (MMAX - NON2)500, 500, 470
  470 THETA = - TWOPI * FLOAT (NON2) / FLOAT (4 * MMAX)
      IF (ISIGN)490, 480, 480
  480 THETA = - THETA
  490 WR = COS (THETA)


      WI = SIN (THETA)
      WSTPR = - 2. * WI * WI
      WSTPI = 2. * WR * WI
  500 DO 690 L = NON2, LMAX, NON2T
      M = L
      IF (MMAX - NON2)520, 520, 510
  510 W2R = WR * WR - WI * WI
      W2I = 2. * WR * WI
      W3R = W2R * WR - W2I * WI
      W3I = W2R * WI + W2I * WR
  520 DO 640 I1 = 1, I1RNG, 2
      DO 540 J3 = I1, NON2, NP1
      KMIN = J3 + IPAR * M
      IF (MMAX - NON2)530, 530, 540
  530 KMIN = J3
  540 KDIF = IPAR * MMAX
```

```
 550  KSTEP = 4 * KDIF                                                    FOURT     232
      DO 630 K1 = KMIN, NTOT, KSTEP                                       FOURT     233
      K2 = K1 + KDIF                                                      FOURT     234
      K3 = K2 + KDIF                                                      FOURT     235
      K4 = K3 + KDIF                                                      FOURT     236
      IF (MMAX - NON2)560, 560, 590                                       FOURT     237
 560  U1R = DATA (K1) + DATA (K2)                                         FOURT     238
      U1I = DATA (K1 + 1) + DATA (K2 + 1)                                 FOURT     239
      U2R = DATA (K3) + DATA (K4)                                         FOURT     240
      U2I = DATA (K3 + 1) + DATA (K4 + 1)                                 FOURT     241
      U3R = DATA (K1) - DATA (K2)                                         FOURT     242
      U3I = DATA (K1 + 1) - DATA (K2 + 1)                                 FOURT     243
      IF (ISIGN)570, 580, 580                                             FOURT     244
 570  U4R = DATA (K3 + 1) - DATA (K4 + 1)                                 FOURT     245
      U4I = DATA (K4) - DATA (K3)                                         FOURT     246
      GO TO 620                                                           FOURT     247
 580  U4R = DATA (K4 + 1) - DATA (K3 + 1)                                 FOURT     248
      U4I = DATA (K3) - DATA (K4)                                         FOURT     249
      GO TO 620                                                           FOURT     250
 590  T2R = W2R * DATA (K2) - W2I * DATA (K2 + 1)                         FOURT     251
      T2I = W2R * DATA (K2 + 1) + W2I * DATA (K2)                         FOURT     252
      T3R = WR * DATA (K3) - WI * DATA (K3 + 1)                           FOURT     253
      T3I = WR * DATA (K3 + 1) + WI * DATA (K3)                           FOURT     254
      T4R = W3R * DATA (K4) - W3I * DATA (K4 + 1)                         FOURT     255
      T4I = W3R * DATA (K4 + 1) + W3I * DATA (K4)                         FOURT     256
      U1R = DATA (K1) + T2R                                               FOURT     257
      U1I = DATA (K1 + 1) + T2I                                           FOURT     258
      U2R = T3R + T4R                                                     FOURT     259
      U2I = T3I + T4I                                                     FOURT     260
      U3R = DATA (K1) - T2R                                               FOURT     261
      U3I = DATA (K1 + 1) - T2I                                           FOURT     262
      IF (ISIGN)600, 610, 610                                             FOURT     263
 600  U4R = T3I - T4I                                                     FOURT     264
      U4I = T4R - T3R                                                     FOURT     265
      GO TO 620                                                           FOURT     266
 610  U4R = T4I - T3I                                                     FOURT     267
      U4I = T3R - T4R                                                     FOURT     268
 620  DATA (K1) = U1R + U2R                                               FOURT     269
      DATA (K1 + 1) = U1I + U2I                                           FOURT     270
      DATA (K2) = U3R + U4R                                               FOURT     271
      DATA (K2 + 1) = U3I + U4I                                           FOURT     272
      DATA (K3) = U1R - U2R                                               FOURT     273
      DATA (K3 + 1) = U1I - U2I                                           FOURT     274
      DATA (K4) = U3R - U4R                                               FOURT     275
 630  DATA (K4 + 1) = U3I - U4I                                           FOURT     276
      KMIN = 4 * (KMIN - J3) + J3                                         FOURT     277
      KDIF = KSTEP                                                        FOURT     278
      IF (KDIF - NP2)550, 640, 640                                        FOURT     279
 640  CONTINUE                                                            FOURT     280
      M = MMAX - M                                                        FOURT     281
      IF (ISIGN)650, 660, 660                                             FOURT     282
 650  TEMPR = WR                                                          FOURT     283
      WR = - WI                                                           FOURT     284
      WI = - TEMPR                                                        FOURT     285
      GO TO 670                                                           FOURT     286
 660  TEMPR = WR                                                          FOURT     287
      WR = WI                                                             FOURT     288
      WI = TEMPR                                                          FOURT     289
 670  IF (M - LMAX)680, 680, 510                                          FOURT     290
 680  TEMPR = WR                                                          FOURT     291
      WR = WR * WSTPR - WI * WSTPI + WR                                   FOURT     292


 690  WI = WI * WSTPR + TEMPR * WSTPI + WI                                FOURT     293
      IPAR = 3 - IPAR                                                     FOURT     294
      MMAX = MMAX + MMAX                                                  FOURT     295
      GO TO 490                                                           FOURT     296
C                                                                         FOURT     297
C     MAIN LOOP FOR FACTORS NOT EQUAL TO TWO.  APPLY THE TWIDDLE FACTOR   FOURT     298
C     W=EXP(ISIGN*2*PI*SQRT(-1)*(J2-1)*(J1-J2)/(NP2*IFP1)), THEN          FOURT     299
C     PERFORM A FOURIER TRANSFORM OF LENGTH IFACT(IF), MAKING USE OF      FOURT     300
C     CONJUGATE SYMMETRIES.                                               FOURT     301
C                                                                         FOURT     302
 700  IF (NTWO - NP2)710, 990, 990                                        FOURT     303
 710  IFP1 = NON2                                                         FOURT     304
      IF = 1                                                              FOURT     305
      NP1HF = NP1 / 2                                                     FOURT     306
 720  IFP2 = IFP1 / IFACT (IF)                                            FOURT     307
      J1RNG = NP2                                                         FOURT     308
```

```
      IF (ICASE - 3)740, 730, 740
  730 J1RNG = (NP2 + IFP1) / 2
      J2STP = NP2 / IFACT (IF)
      J1RG2 = (J2STP + IFP2) / 2
  740 J2MIN = 1 + IFP2
      IF (IFP1 - NP2)750, 800, 800
  750 DO 790 J2 = J2MIN, IFP1, IFP2
      THETA = - TWOPI * FLOAT (J2 - 1) / FLOAT (NP2)
      IF (ISIGN)770, 760, 760
  760 THETA = - THETA
  770 SINTH = SIN (THETA / 2.)
      WSTPR = - 2. * SINTH * SINTH
      WSTPI = SIN (THETA)
      WR = WSTPR + 1.
      WI = WSTPI
      J1MIN = J2 + IFP1
      DO 790 J1 = J1MIN, J1RNG, IFP1
      I1MAX = J1 + I1RNG - 2
      DO 780 I1 = J1, I1MAX, 2
      DO 780 I3 = I1, NTOT, NP2
      J3MAX = I3 + IFP2 - NP1
      DO 780 J3 = I3, J3MAX, NP1
      TEMPR = DATA (J3)
      DATA (J3) = DATA (J3) * WR - DATA (J3 + 1) * WI
  780 DATA (J3 + 1) = TEMPR * WI + DATA (J3 + 1) * WR
      TEMPR = WR
      WR = WR * WSTPR - WI * WSTPI + WR
  790 WI = TEMPR * WSTPI + WI * WSTPR + WI
  800 THETA = - TWOPI / FLOAT (IFACT (IF))
      IF (ISIGN)820, 810, 810
  810 THETA = - THETA
  820 SINTH = SIN (THETA / 2.)
      WSTPR = - 2. * SINTH * SINTH
      WSTPI = SIN (THETA)
      KSTEP = 2 * N / IFACT (IF)
      KRANG = KSTEP * (IFACT (IF) / 2) + 1
      DO 980 I1 = 1, I1RNG, 2
      DO 980 I3 = I1, NTOT, NP2
      DO 910 KMIN = 1, KRANG, KSTEP
      J1MAX = I3 + J1RNG - IFP1
      DO 880 J1 = I3, J1MAX, IFP1
      J3MAX = J1 + IFP2 - NP1
      DO 880 J3 = J1, J3MAX, NP1
      J2MAX = J3 + IFP1 - IFP2
      K = KMIN + (J3 - J1 + (J1 - I3) / IFACT (IF)) / NP1HF
      IF (KMIN - 1)830, 830, 850
  830 SUMR = 0.
      SUMI = 0.
      DO 840 J2 = J3, J2MAX, IFP2
      SUMR = SUMR + DATA (J2)
  840 SUMI = SUMI + DATA (J2 + 1)
      WORK (K) = SUMR
      WORK (K + 1) = SUMI
      GO TO 880
  850 KCONJ = K + 2 * (N - KMIN + 1)
      J2 = J2MAX
      SUMR = DATA (J2)
      SUMI = DATA (J2 + 1)
      OLDSR = 0.
      OLDSI = 0.
      J2 = J2 - IFP2

  860 TEMPR = SUMR
      TEMPI = SUMI
      SUMR = TWOWR * SUMR - OLDSR + DATA (J2)
      SUMI = TWOWR * SUMI - OLDSI + DATA (J2 + 1)
      OLDSR = TEMPR
      OLDSI = TEMPI
      J2 = J2 - IFP2
      IF (J2 - J3)870, 870, 860
  870 TEMPR = WR * SUMR - OLDSR + DATA (J2)
      TEMPI = WI * SUMI
      WORK (K) = TEMPR - TEMPI
      WORK (KCONJ) = TEMPR + TEMPI
      TEMPR = WR * SUMI - OLDSI + DATA (J2 + 1)
      TEMPI = WI * SUMR
      WORK (K + 1) = TEMPR + TEMPI
      WORK (KCONJ + 1) = TEMPR - TEMPI
```

```
  880 CONTINUE                                                          FOURT   386
      IF (KMIN - 1)890, 890, 900                                        FOURT   387
  890 WR = WSTPR + 1.                                                   FOURT   388
      WI = WSTPI                                                        FOURT   389
      GO TO 910                                                         FOURT   390
  900 TEMPR = WR                                                        FOURT   391
      WR = WR * WSTPR - WI * WSTPI + WR                                 FOURT   392
      WI = TEMPR * WSTPI + WI * WSTPR + WI                              FOURT   393
  910 TWOWR = WR + WR                                                   FOURT   394
      IF (ICASE - 3)930, 920, 930                                       FOURT   395
  920 IF (IFP1 - NP2)950, 930, 930                                      FOURT   396
  930 K = 1                                                             FOURT   397
      I2MAX = I3 + NP2 - NP1                                            FOURT   398
      DO 940 I2 = I3, I2MAX, NP1                                        FOURT   399
      DATA (I2) = WORK (K)                                              FOURT   400
      DATA (I2 + 1) = WORK (K + 1)                                      FOURT   401
  940 K = K + 2                                                         FOURT   402
      GO TO 980                                                         FOURT   403
C                                                                       FOURT   404
C     COMPLETE A REAL TRANSFORM IN THE 1ST DIMENSION, N ODD, BY CON-    FOURT   405
C     JUGATE SYMMETRIES AT EACH STAGE.                                  FOURT   406
C                                                                       FOURT   407
  950 J3MAX = I3 + NP2 - NP1                                            FOURT   408
      DO 970 J3 = I3, J3MAX, NP1                                        FOURT   409
      J2MAX = J3 + NP2 - J2STP                                          FOURT   410
      DO 970 J2 = J3, J2MAX, J2STP                                      FOURT   411
      J1MAX = J2 + J1RG2 - IFP2                                         FOURT   412
      J1CNJ = J3 + J2MAX + J2STP - J2                                   FOURT   413
      DO 970 J1 = J2, J1MAX, IFP2                                       FOURT   414
      K = 1 + J1 - I3                                                   FOURT   415
      DATA (J1) = WORK (K)                                              FOURT   416
      DATA (J1 + 1) = WORK (K + 1)                                      FOURT   417
      IF (J1 - J2)970, 970, 960                                         FOURT   418
  960 DATA (J1CNJ) = WORK (K)                                           FOURT   419
      DATA (J1CNJ + 1) = - WORK (K + 1)                                 FOURT   420
  970 J1CNJ = J1CNJ - IFP2                                              FOURT   421
  980 CONTINUE                                                          FOURT   422
      IF = IF + 1                                                       FOURT   423
      IFP1 = IFP2                                                       FOURT   424
      IF (IFP1 - NP1)990, 990, 720                                      FOURT   425
C                                                                       FOURT   426
C     COMPLETE A REAL TRANSFORM IN THE 1ST DIMENSION, N EVEN, BY CON-   FOURT   427
C     JUGATE SYMMETRIES.                                                FOURT   428
C                                                                       FOURT   429
  990 GO TO (1260, 1100, 1260, 1000), ICASE                             FOURT   430
 1000 NHALF = N                                                         FOURT   431
      N = N + N                                                         FOURT   432
      THETA = - TWOPI / FLOAT (N)                                       FOURT   433
      IF (ISIGN)1020, 1010, 1010                                        FOURT   434
 1010 THETA = - THETA                                                   FOURT   435
 1020 SINTH = SIN (THETA / 2.)                                          FOURT   436
      WSTPR = - 2. * SINTH * SINTH                                      FOURT   437
      WSTPI = SIN (THETA)                                               FOURT   438
      WR = WSTPR + 1.                                                   FOURT   439
      WI = WSTPI                                                        FOURT   440
      IMIN = 3                                                          FOURT   441
      JMIN = 2 * NHALF - 1                                              FOURT   442
      GO TO 1050                                                        FOURT   443
 1030 J = JMIN                                                          FOURT   444
      DO 1040 I = IMIN, NTOT, NP2                                       FOURT   445
      SUMR = (DATA (I) + DATA (J)) / 2.                                 FOURT   446

      SUMI = (DATA (I + 1) + DATA (J + 1)) / 2.                         FOURT   447
      DIFR = (DATA (I) - DATA (J)) / 2.                                 FOURT   448
      DIFI = (DATA (I + 1) - DATA (J + 1)) / 2.                         FOURT   449
      TEMPR = WR * SUMI + WI * DIFR                                     FOURT   450
      TEMPI = WI * SUMI - WR * DIFR                                     FOURT   451
      DATA (I) = SUMR + TEMPR                                           FOURT   452
      DATA (I + 1) = DIFI + TEMPI                                       FOURT   453
      DATA (J) = SUMR - TEMPR                                           FOURT   454
      DATA (J + 1) = - DIFI + TEMPI                                     FOURT   455
 1040 J = J + NP2                                                       FOURT   456
      IMIN = IMIN + 2                                                   FOURT   457
      JMIN = JMIN - 2                                                   FOURT   458
      TEMPR = WR                                                        FOURT   459
      WR = WR * WSTPR - WI * WSTPI + WR                                 FOURT   460
      WI = TEMPR * WSTPI + WI * WSTPR + WI                              FOURT   461
 1050 IF (IMIN - JMIN)1030, 1060, 1090                                  FOURT   462
```

```
 1060 IF (ISIGN)1070, 1090, 1090                                      FOURT     463
 1070 DO 1080 I = IMIN, NTOT, NP2                                     FOURT     464
 1080 DATA (I + 1) = - DATA (I + 1)                                   FOURT     465
 1090 NP2 = NP2 + NP2                                                 FOURT     466
      NTOT = NTOT + NTOT                                              FOURT     467
      J = NTOT + 1                                                    FOURT     468
      IMAX = NTOT / 2 +.1                                             FOURT     469
 1100 IMIN = IMAX - 2 * NHALF                                         FOURT     470
      I = IMIN                                                        FOURT     471
      GO TO 1120                                                      FOURT     472
 1110 DATA (J) = DATA (I)                                             FOURT     473
      DATA (J + 1) = - DATA (I + 1)                                   FOURT     474
 1120 I = I + 2                                                       FOURT     475
      J = J - 2                                                       FOURT     476
      IF (I - IMAX)1110, 1130, 1130                                   FOURT     477
 1130 DATA (J) = DATA (IMIN) - DATA (IMIN + 1)                        FOURT     478
      DATA (J + 1) = 0.                                               FOURT     479
      IF (I - J)1150, 1170, 1170                                      FOURT     480
 1140 DATA (J) = DATA (I)                                             FOURT     481
      DATA (J + 1) = DATA (I + 1)                                     FOURT     482
 1150 I = I - 2                                                       FOURT     483
      J = J - 2                                                       FOURT     484
      IF (I - IMIN)1160, 1160, 1140                                   FOURT     485
 1160 DATA (J) = DATA (IMIN) + DATA (IMIN + 1)                        FOURT     486
      DATA (J + 1) = 0.                                               FOURT     487
      IMAX = IMIN                                                     FOURT     488
      GO TO 1100                                                      FOURT     489
 1170 DATA (1) = DATA (1) + DATA (2)                                  FOURT     490
      DATA (2) = 0.                                                   FOURT     491
      GO TO 1260                                                      FOURT     492
C                                                                     FOURT     493
C     COMPLETE A REAL TRANSFORM FOR THE 2ND OR 3RD DIMENSION BY       FOURT     494
C     CONJUGATE SYMMETRIES.                                           FOURT     495
C                                                                     FOURT     496
 1180 IF (I1RNG - NP1)1190, 1260, 1260                                FOURT     497
 1190 DO 1250 I3 = 1, NTOT, NP2                                       FOURT     498
      I2MAX = I3 + NP2 - NP1                                          FOURT     499
      DO 1250 I2 = I3, I2MAX, NP1                                     FOURT     500
      IMIN = I2 + I1RNG                                               FOURT     501
      IMAX = I2 + NP1 - 2                                             FOURT     502
      JMAX = 2 * I3 + NP1 - IMIN                                      FOURT     503
      IF (I2 - I3)1210, 1210, 1200                                    FOURT     504
 1200 JMAX = JMAX + NP2                                               FOURT     505
 1210 IF (IDIM - 2)1240, 1240, 1220                                   FOURT     506
 1220 J = JMAX + NP0                                                  FOURT     507
      DO 1230 I = IMIN, IMAX, 2                                       FOURT     508
      DATA (I) = DATA (J)                                             FOURT     509
      DATA (I + 1) = - DATA (J + 1)                                   FOURT     510
 1230 J = J - 2                                                       FOURT     511
 1240 J = JMAX                                                        FOURT     512
      DO 1250 I = IMIN, IMAX, NP0                                     FOURT     513
      DATA (I) = DATA (J)                                             FOURT     514
      DATA (I + 1) = - DATA (J + 1)                                   FOURT     515
 1250 J = J - NP0                                                     FOURT     516
C                                                                     FOURT     517
C     END OF LOOP ON EACH DIMENSION                                   FOURT     518
C                                                                     FOURT     519
 1260 NP0 = NP1                                                       FOURT     520
      NP1 = NP2                                                       FOURT     521
 1270 NPREV = N                                                       FOURT     522
 1280 RETURN                                                          FOURT     523

      END                                                             FOURT     524
```

Appendix D.* Subroutine PLT120R(X,Y,XMAX,XMIN,YMAX,YMIN,LAST,ISYMBOL,NO,MOST)

Purpose: To make a page plot of array Y versus array X.

Arguments:

X = Array containing abscissa values of the function to be plotted.
Y = Array containing ordinate values of the function to be plotted.
XMIN = Minimum abscissa value.
XMAX = Maximum abscissa value.
YMIN = Minimum ordinate value.
YMAX = Maximum ordinate value.
LAST = Number of points to be plotted.
ISYMBOL = A Hollerith variable containing the plotting symbol, e.g., to plot with the symbol "X" ISYMBOL = 1HX.
NO = Number of plot on page.
MOST = Total number of plots to be made on one page.

Discussion: This subroutine produces a "quick and dirty" plot of Y versus X on the page printer. The size of the plotting area is 50 x 120 units. Multiple plots may be made on a single page. A page eject is performed before the first plot of a series is begun, but no eject is performed after completion of a series. This allows a title to be printed at the bottom of the plot. The subroutine uses inline function FLOAT.

---

*This appendix is taken from Appendix A.1.12 of Stubenrauch and Yaghjian [2].

```
      SUBROUTINE PLT120R(X, Y, XMAX, XMIN, YMAX, YMIN, LAST, ISYMBOL, NOPLT120R
     1, MOST)                                                           PLT120R
C     MODIFIED 11/4/68                                                  PLT120R
      DIMENSION X(1), Y(1), ZX(13), GRAPH(121, 51)                      PLT120R
      INTEGER GRAPH, COLUMNS, BLANK, BORDER                             PLT120R
      DATA (LINES = 51), (COLUMNS = 121)                                PLT120R
      KMAX = COLUMNS / 10 + 1                                           PLT120R
      IF (NO .NE. 1) GO TO 190                                          PLT120R
      YLAR = YMAX                                                       PLT120R
      YSMA = YMIN                                                       PLT120R
      XLAR = XMAX                                                       PLT120R
      XSMA = XMIN                                                       PLT120R
      BORDER = 1HI                                                      PLT120R
      BLANK = 1H                                                        PLT120R
      MATRIX = COLUMNS * LINES                                          PLT120R
      IF (MATRIX .LT. 1) GO TO 120                                      PLT120R
      DO 100 I = 1, MATRIX                                              PLT120R
  100 GRAPH(I) = BLANK                                                  PLT120R
  120 CONTINUE                                                          PLT120R
      IF (LINES .LT. 1) GO TO 140                                       PLT120R
      DO 130 I = 1, LINES                                               PLT120R
  130 GRAPH(1, I) = GRAPH(COLUMNS, I) = BORDER                          PLT120R
  140 CONTINUE                                                          PLT120R
      IF (COLUMNS .LT. 1) GO TO 160                                     PLT120R
      DO 150 I = 1, COLUMNS                                             PLT120R
  150 GRAPH(I, 26) = 1H.                                                PLT120R
  160 CONTINUE                                                          PLT120R
      XSCALE = (XLAR - XSMA) / (COLUMNS - 1.)                           PLT120R
      YSCALE = (YLAR - YSMA) / (LINES - 1.)                             PLT120R
      IF (KMAX .LT. 1) GO TO 180                                        PLT120R
      DO 170 K = 1, KMAX                                                PLT120R
  170 ZX(K) = 10. * FLOAT(K - 1) * XSCALE + XSMA                        PLT120R
  180 CONTINUE                                                          PLT120R
  190 IF (LAST .LT. 1) GO TO 250                                        PLT120R
      DO 240 I = 1, LAST                                                PLT120R
      IF (X(I) .GT. XLAR .OR. X(I) .LT. XSMA) GO TO 240                 PLT120R
      IF (Y(I) .GT. YLAR .OR. Y(I) .LT. YSMA) GO TO 240                 PLT120R
      IX = (X(I) - XSMA) / XSCALE + 1.5                                 PLT120R
      IY = (Y(I) - YSMA) / YSCALE + .5                                  PLT120R
      IY = LINES - IY                                                   PLT120R
      GRAPH(IX, IY) = ISYMBOL                                           PLT120R
  240 CONTINUE                                                          PLT120R
  250 CONTINUE                                                          PLT120R
      IF (NO .NE. MOST) RETURN                                          PLT120R
      PRINT 1500                                                        PLT120R
      YES = YLAR + YSCALE                                               PLT120R
      IF (LINES .LT. 1) GO TO 270                                       PLT120R
      DO 260 I = 1, LINES                                               PLT120R
      YES = YES - YSCALE                                                PLT120R
      PRINT 1510, YES, (GRAPH(J, I), J = 1, COLUMNS)                    PLT120R
  260 CONTINUE                                                          PLT120R
  270 CONTINUE                                                          PLT120R
      PRINT 1520                                                        PLT120R
      PRINT 1530, ZX                                                    PLT120R
      RETURN                                                            PLT120R
 1500 FORMAT  (1H1,9X,24(5HI....)1HI)                                   PLT120R
 1510 FORMAT  (1H ,F8.2,1X,121A1)                                       PLT120R
 1520 FORMAT  (1H ,9X,24(5HI....)1HI)                                   PLT120R
 1530 FORMAT  (1H ,2X,13(1X,F9.3))                                      PLT120R
      END                                                               PLT120R
```

NBS-114A (REV. 2-80)

U.S. DEPT. OF COMM. **BIBLIOGRAPHIC DATA SHEET** *(See instructions)*	1. PUBLICATION OR REPORT NO. NBSIR 82-1674	2. Performing Organ. Report No.	3. Publication Date August 1982

4. TITLE AND SUBTITLE

Computation of Antenna Side-Lobe Coupling in the Near Field Using Approximate Far-Field Data

5. AUTHOR(S)

M. H. Francis and A. D. Yaghjian

6. PERFORMING ORGANIZATION *(If joint or other than NBS, see instructions)*

NATIONAL BUREAU OF STANDARDS
DEPARTMENT OF COMMERCE
WASHINGTON, D.C. 20234

7. Contract/Grant No.

8. Type of Report & Period Covered

9. SPONSORING ORGANIZATION NAME AND COMPLETE ADDRESS *(Street, City, State, ZIP)*

Dept. of Defense
IITRI/ECAC
North Severn
Annapolis, MD 21402

10. SUPPLEMENTARY NOTES

☐ Document describes a computer program; SF-185, FIPS Software Summary, is attached.

11. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)*

Computer programs, in particular CUPLNF and CUPLZ, are presently in existence to calculate the coupling loss between two antennas provided that the amplitude and phase of the far field are available. However, for many antennas the complex far field is not known accurately. In such cases it is nevertheless possible to specify approximate far fields from a knowledge of the side-lobe level of each antenna along the axis of separation, and the electrical size of each antenna. To determine the effectiveness of using approximate side-lobe level data instead of the detailed far fields, we chose as our test antennas two hypothetical, linearly polarized, uniformly illuminated circular antennas for which the exact far fields are given by a simple analytic expression. The exact far fields are supplied to the program CUPLNF to compute the exact near-field coupling loss. Approximate fields are supplied to a new program ENVLP developed for the purpose of computing the approximate near-field coupling loss. The comparison of the results from ENVLP to those of CUPLNF indicates that the use of approximate far fields gives an estimate of the coupling loss which is good to about ±5 dB. In addition, the plane-wave transmission formula for coupling between two antennas is used to estimate upper-bound values of coupling loss. These upper bounds are compared with the maximum coupling losses obtained from programs CUPLNF and ENVLP.

12. KEY WORDS *(Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)*

antenna coupling; antenna theory, coupling loss; near-field measurements.

13. AVAILABILITY

☒ Unlimited
☐ For Official Distribution. Do Not Release to NTIS
☐ Order From Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402.
☒ Order From National Technical Information Service (NTIS), Springfield, VA. 22161

14. NO. OF PRINTED PAGES

84

15. Price

$10.50

USCOMM-DC 6043-P80

☆U.S. GOVERNMENT PRINTING OFFICE:1981—580-327 / 383