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COMPUTATION OF TWO-DIMENSIONAL TIME-DEPENDENT NATURAL CONVECTION OF COMPRESSIBLE FLUID IN A RECTANGULAR ENCLOSURE

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NOMENCLATURE

Symbol

C_p	specific heat, [J kg^{-1} K^{-1}]
F	arbitrary value, eq.(15)
g	acceleration due to gravity, [ms^{-1}]
k	thermal conductivity, [W m^{-1} K^{-1}]
P	pressure, [Pa]
P_o	pressure at original point, $= R \rho_o T_o$
ΔP	pressure difference, $= P - P_o$
Q	total heat is added into the enclosure, [W]
R	gas constant, [J kg^{-1} K^{-1}]
T	temperature, [K]
ΔT	temperature difference, $= T_w - T_o$
u	velocity component in x direction, [m s^{-1}]
v	velocity component in y direction [m s^{-1}]
x	coordinate along the vertical wall, [m]
Δx	grid mesh size in the x direction, [m]
y	coordinate along the horizontal wall, [m]
Δy	grid mesh size in the y direction, [m]

Greek Letters

β	coefficient of volumetric expansion, [K^{-1}]
γ	specific heat ratio, C_p/C_v
μ	absolute viscosity, [Pa s]
ρ	density, [kg m^{-3}]
τ	time, [s]
$\Delta\tau$	time step, [s]

Subscripts and Superscript

o	value at the wall except the heated wall
i	subscript denoting the i th grid point in the x direction
j	subscript denoting the j th grid point in the y direction
m	average value
n	superscript denoting the time at τ_n
r	restricted value
w	value at the heating surface

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Studies of natural convection processes generally assume an incompressible fluid wherein the density is a function of temperature only (the Boussinesq approximation). However, local pressure gradients caused by rapid variations in the heated wall temperature cannot be described within this approximation. These time-varying gradients cause fluid motions which perturb the quasi-static natural convection process. In this study, we describe a numerical analysis procedure which includes compressibility effects and allows computation of transient fluid motions during onset of natural convection. Details of the computational procedure and preliminary results for one geometry are given.

Key words: compressible fluid motion; convection; finite difference approximation; heat transfer; natural convection; nonlinear convection; numerical integration; transient fluid motion; transient heat transfer.

INTRODUCTION

For two-dimensional time-dependent laminar natural convection about a heated surface, many numerical solutions are known in the literature.[2,3,5,6] These solutions use Boussinesq's approximation in which the thermophysical properties are taken to be constant and the fluid is assumed incompressible, except when considering the body force term in the equation of motion. These assumptions lead to a numerical formulation in which the pressure terms are eliminated from the equation of motion and a stream function is defined which satisfies the equation of continuity. Spiegel and Veroni's (4) have presented the conditions under which the Boussinesq approximation is applicable for thermal convection in compressible fluids; although the Boussinesq approximation is valid for a number of natural convecting problems, it is of uncertain accuracy for studies of supercritical fluid motion where the thermophysical properties of these fluids are strongly dependent on temperature and pressure. However, pressure changes generated by pulsed thermal input, which are significant in determining the fluid motion in early time periods, cannot be described within such a formulation.

From this point of view, the authors have attempted to get computational results using the program PDETWO (7) for two-dimensional time-dependent natural convection of a compressible fluid in a rectangular enclosure. However, stability problems arose in computational work, perhaps associated with implicit time integration. Also, the cross differential term $\partial^2/(\partial x \partial y)$ in momentum equations is neglected in the PEDETWO program, and it was not known whether this could lead to a significant error.

In this paper, computational analysis is described on the laminar natural convection heat transfer from an isothermal wall to compressible fluid within a rectangular enclosure, taking into account the variation of thermophysical properties and cross differential terms in momentum equations. Calculated results for both pressure and buoyancy effects in early time periods for air are shown.

MATHEMATICAL FORMULATION AND PHYSICAL DESCRIPTION

Consider the motion of a viscous fluid within a rectangular enclosure as shown in Fig. 1. The fluid is initially motionless with a uniform temperature T_0. The enclosure walls are also at this temperature. The temperature difference $T_W - T_0$ is initiated at time 0 to induce the flow within the enclosure. The variations in all relevant physical properties are taken into account. However kinetic energy, internal heat sources and irreversible viscous dissipation in the energy equation are not considered.

The governing equation for an compressible fluid with variable properties will be as follows.(1)

continuity equation:

$$\frac{\partial \rho}{\partial \tau} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \qquad (1)$$

momentum equations:

$$\rho\left(\frac{\partial u}{\partial \tau} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} - \rho g - \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y}\right) \qquad (2)$$

$$\rho\left(\frac{\partial v}{\partial \tau} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial P}{\partial y} - 0 - \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y}\right) \qquad (3)$$

where viscous forces on element are

$$\tau_{xx} = -2\mu\frac{\partial u}{\partial x} + \frac{2}{3}\mu\left\{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right\} \qquad (5)$$

$$\tau_{yy} = -2\mu\frac{\partial v}{\partial y} + \frac{2}{3}\mu\left\{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right\} \qquad (6)$$

$$\tau_{xy} = \tau_{yx} = -\mu\left\{\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right\} \qquad (7)$$

energy equation:

$$\rho C_p \left\{ \frac{\partial T}{\partial \tau} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right\} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \beta_T \left\{ \frac{\partial P}{\partial \tau} + u\frac{\partial P}{\partial x} + v\frac{\partial P}{\partial y} \right\} \quad (8)$$

where $\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p$ is the volumetric coefficient of thermal expansion of the fluid.

The initial conditions are:

$\tau \leq 0$

$$u = v = o \quad (9)$$

$$T = T_o \quad (10)$$

$$\frac{\partial P}{\partial x} = -\rho g \quad (11)$$

and boundary conditions are:

$\tau > 0$

$u = v = o$ at all walls (12)

$T = T_o$ at all walls except heated surface (13)

$T = T_w$ at heated surface (14)

NUMERICAL SOLUTION OF THE EQUATION

Finite difference formulation

The left hand sides of eqs. (1), (2) and (3) can be manipulated by noting that for arbitrary variable F as follows.

$$\rho\frac{DF}{D\tau} = \frac{\partial}{\partial \tau}(\rho F) + \frac{\partial}{\partial x}(\rho u F) + \frac{\partial}{\partial y}(\rho v F) \quad (15)$$

Using eq. (15), continuity eq. (1) can be written

$$\frac{\partial(\rho)}{\partial\tau} + \frac{\partial\{(\rho)u\}}{\partial x} + \frac{\partial\{(\rho)v\}}{\partial y} = 0 \tag{16}$$

momentum eq. (2) is

$$\frac{\partial(\rho u)}{\partial\tau} + \frac{\partial\{(\rho u)u\}}{\partial x} + \frac{\partial\{(\rho u)v\}}{\partial y}$$
$$= -\frac{\partial P}{\partial x} - \rho g$$
$$+ \frac{4}{3}\left(\frac{\partial\mu}{\partial x}\frac{\partial u}{\partial x} + \mu\frac{\partial^2 u}{\partial x^2}\right) - \frac{2}{3}\left(\frac{\partial\mu}{\partial x}\frac{\partial v}{\partial y}\right)$$
$$+ \frac{\partial\mu}{\partial y}\frac{\partial u}{\partial y} + \mu\frac{\partial^2 u}{\partial y^2} + \frac{\partial\mu}{\partial y}\frac{\partial v}{\partial x} + \frac{1}{3}\mu\frac{\partial^2 v}{\partial x\partial y} \tag{17}$$

$$\frac{\partial(\rho v)}{\partial\tau} + \frac{\partial\{(\rho v)u\}}{\partial x} + \frac{\partial\{(\rho v)v\}}{\partial y}$$
$$= -\frac{\partial P}{\partial y} + \frac{4}{3}\left(\frac{\partial\mu}{\partial y}\frac{\partial v}{\partial y} + \mu\frac{\partial^2 v}{\partial y^2}\right) - \left(\frac{2}{3}\frac{\partial\mu}{\partial y}\frac{\partial u}{\partial x}\right)$$
$$+ \frac{\partial\mu}{\partial x}\frac{\partial v}{\partial x} + \mu\frac{\partial^2 v}{\partial x^2} + \frac{\partial\mu}{\partial x}\frac{\partial u}{\partial y} + \frac{1}{3}\mu\frac{\partial^2 u}{\partial x\partial y} \tag{18}$$

and energy eq. (8) is also written as follows.

$$\left\{1 - \frac{\beta T}{\rho C_p}\left(\frac{\partial P}{\partial T}\right)\right\}\frac{\partial T}{\partial\tau}$$
$$= T\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - \frac{\partial(uT)}{\partial x} - \frac{\partial(vT)}{\partial y}$$
$$+ \frac{1}{\rho C_p}\left\{k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\partial k}{\partial x}\frac{\partial T}{\partial x} + \frac{\partial k}{\partial y}\frac{\partial T}{\partial y}\right\}$$
$$+ \frac{\beta T}{\rho C_p}\left[-\left(\frac{\partial P}{\partial\rho}\right)\left\{\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y}\right\}\right.$$
$$\left. + \frac{\partial(uP)}{\partial x} + \frac{\partial(vP)}{\partial y} - P\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right] \tag{19}$$

The numerical scheme shown in Fig. 2 employs a 12x12 rectangular grid system in the x and y direction with a total of 144 grid points and Δx, Δy are the grid size in both x and y direction respectively. This numerical method is based on the simplest explicit finite difference approximations to the governing differential equations which will be obtained at a finite number of grid points having coordinate $x = i\Delta x$ and $Y = j\Delta Y$ except next to the wall where the mesh size is one-half the mesh within the cavity. All grid points are evaluated at discrete times τ_n. The values of all physical properties at each grid point should be thought of as average value over a small volume of fluid in Fig. 2.

The finite difference approximations on the derivatives of the arbitrary variable F at the grid point (i, j) in advancing from time τ_n to the new level $\tau_{n+1} = \tau_n + \Delta\tau$ may be written as follows.

$$\frac{\partial F}{\partial \tau} = \frac{F^{n+1} - F^{n}}{\Delta \tau} \tag{21}$$

The finite difference scheme for spatial derivatives uses central differences except next to the wall. The first order difference is

$$\frac{\partial F}{\partial x} = \frac{(F^{n}_{i+1,j} - F^{n}_{i-1,j})}{2\Delta x} \quad ; \tag{22}$$

the second order finite difference is

$$\frac{\partial^2 F}{\partial x^2} = \frac{(F^{n}_{i+1,j} - 2F^{n}_{i,j} + F^{n}_{i-1,j})}{\Delta x^2} \; ; \tag{23}$$

the cross finite difference is

$$\frac{\partial^2 F}{\partial x \partial y} = \frac{(F_{i+1,j+1} - F^n_{i-1,j+1} - F^n_{i+1,j-1} + F^n_{i-1,j-1})}{4\Delta x \Delta y} \tag{24}$$

The nonlinear convection terms cause the main difficulties to achieve a stable numerical method. For some difference methods, the rate of heat removal may differ from the rate of heat addition at steady state. Torrance (5) tested several methods for differencing the convection term. In this calculation, Torrance's V method (5) is employed as follows.

$$\frac{\partial(Fu)}{\partial x} = \begin{cases} \dfrac{(F^n_{i,j}\overline{U}^n_{i,j} - F^n_{i-1,j}\,\overline{U}^n_{i-1,j})}{\Delta x}, & (\overline{U}^n_{i,j} \text{ and } \overline{U}^n_{i-1,j} > 0) \quad (25) \\ \dfrac{(F^n_{i+1,j}\,\overline{U}^n_{i,j} - F^n_{i,j}\,\overline{U}^n_{i-1,j})}{\Delta x}, & (\overline{U}^n_{i,j} \text{ and } \overline{U}^n_{i-1,j} \leq 0) \quad (26) \\ \dfrac{(\overline{F}^n_{i+1,j}\overline{U}^n_{i,j} - \overline{F}^n_{i-1,j}\,\overline{U}^n_{i-1,j})}{\Delta x}, & (\overline{U}^n_{i,j} \quad \overline{U}^n_{i-1,j} \quad < 0) \quad (27) \end{cases}$$

where

$$\overline{F}^n_{i\underline{+}1,j} = \begin{cases} F^n_{i\underline{+}1,j} & (\overline{U}^n_{i,j} < 0,\ \overline{U}^n_{i-1,j} > 0) \\ F^n_{i,j} & (\overline{U}^n_{i,j} > 0,\ \overline{U}^n_{i-1,j} < 0) \end{cases}$$

$$\overline{U}^n_{m,n} = \frac{1}{2}(U^n_{m+1,n} + U^n_{m,n})$$

When these approximations are introduced into eqs. (16), (17), (18) and (19) we obtain

$$\rho^{n+1}_{i,j} = \rho^n_{i,j} - \Delta\tau \frac{(\rho U)^{n+1}_{i+1,j} - (\rho U)^{n+1}_{i-1,j}}{2\Delta x} + \frac{(\rho v)^{n+1}_{i,j+1} - (\rho v)^{n+1}_{i,j-1}}{2\Delta y} \tag{28}$$

$$(\rho U)^{n+1}_{i,j} = (\rho U)^n_{i,j} + \Delta\tau\Bigg[-\frac{\{(\rho U)^n_{i,j}\,\overline{U}^n_{i,j} - (\rho U)^n_{i-1,j}\,\overline{U}^n_{i-1,j}\}}{\Delta x}$$

$$- \frac{\{(\rho U)^n_{i,j}\,\overline{V}^n_{i,j} - (\rho U)^n_{i,j-1}\,\overline{V}^n_{i,j-1}\}}{\Delta y}$$

$$- \frac{(P^n_{i+1,j} - P^n_{i-1,j})}{2\Delta x} - \rho^n_{i,j}\, g$$

$$+ \frac{4}{3}\left\{\frac{(\mu^n_{i+1,j} - \mu^n_{i-1,j})}{2\Delta x}\frac{(U^n_{i+1,j} - U^n_{i-1,j})}{2\Delta x}\right\}$$

$$+ \mu^n_{i,j}\left\{\frac{(U^n_{i+1,j} - 2U^n_{i,j} + U^n_{i-1,j})}{\Delta x^2}\right\}$$

$$- \frac{2}{3}\left\{\frac{(\mu^n_{i+1,j} - \mu^n_{i-1,j})}{2\Delta x}\frac{(V^n_{i,j+1} - V^n_{i,j-1})}{2\Delta y}\right\}$$

$$+ \left\{\frac{(\mu^n_{i,j+1} - \mu^n_{i,j-1})}{2\Delta y}\frac{(U^n_{i,j+1} - U^n_{i,j-1})}{2\Delta y}\right\}$$

$$+ \mu^n_{i,j}\left\{\frac{(U^n_{i,j+1} - 2U^n_{i,j} + U^n_{i,j-1})}{\Delta y^2}\right\}$$

$$+ \left\{\frac{(\mu^n_{i,j+1} - \mu^n_{i,j-1})}{2\Delta y}\frac{(V^n_{i+1,j} - V^n_{i-1,j})}{2\Delta x}\right\}$$

$$+ \frac{1}{3}\mu^n_{i,j}\left\{\frac{(V^n_{i+1,j+1} - V^n_{i-1,j+1} - V^n_{i+1,j-1} + V^n_{i-1,j-1})}{4\Delta x\Delta y}\right\}\Bigg] \quad (29)$$

$$(\rho V)^{n+1}_{i,j} = (\rho V)^n_{i,j} + \Delta\tau\Bigg[-\frac{\{(\rho V)^n_{i,j}\,\overline{U}^n_{i,j} - (\rho V)^n_{i-1,j}\,\overline{U}^n_{i-1,j}\}}{\Delta x}$$

$$- \frac{\{(\rho V)^n_{i,j}\,\overline{V}^n_{i,j} - (\rho V)^n_{i,j-1}\,\overline{V}^n_{i,j-1}\}}{\Delta y}$$

$$- \frac{(P^n_{i,j+1} - P^n_{i,j-1})}{2\Delta y}$$

$$+ \frac{4}{3}\left\{\frac{(\mu^n_{i,j+1} - \mu^n_{i,j-1})}{2\Delta y}\frac{(V^n_{i,j+1} - V^n_{i,j-1})}{2\Delta y}\right.$$

$$\left. + \mu^n_{i,j}\frac{(V^n_{i,j+1} - 2V^n_{i,j} + V^n_{i,j-1})}{\Delta y^2}\right\}$$

$$- \frac{2}{3}\left\{\frac{(\mu^n_{i,j+1} - \mu^n_{i,j-1})}{2\Delta y}\frac{(U^n_{i+1,j} - U^n_{i-1,j})}{2\Delta x}\right\}$$

$$+ \left\{\frac{(\mu^n_{i+1,j} - \mu^n_{i-1,j})}{2\Delta x}\frac{(V^n_{i+1,j} - V^n_{i-1,j})}{2\Delta x}\right\}$$

$$+ \mu^n_{i,j}\frac{(V^n_{i+1,j} - 2V^n_{i,j} + V^n_{i-1,j})}{\Delta x^2}$$

$$+ \left\{\frac{(\mu^n_{i+1,j} - \mu^n_{i-1,j})}{2\Delta x}\frac{(U^n_{i,j+1} - U^n_{i,j-1})}{2\Delta y}\right\}$$

$$\left. + \frac{1}{3}\mu^n_{i,j}\frac{(U^n_{i+1,j+1} - U^n_{i-1,j+1} - U^n_{i+1,j-1} + U^n_{i-1,j-1})}{4\Delta x\Delta y}\right] \tag{30}$$

$$T^{n+1}_{i,j} = T^n_{i,j} + \frac{\Delta\tau}{C}\left[T^n_{ij}\left\{\frac{(U^n_{+1,j} - U^n_{i-1,j})}{2\Delta x} + \frac{(V^n_{i,j+1} - V^n_{i,j-1})}{2\Delta y}\right\}\right.$$

$$- \frac{(\overline{U}^n_{i,j}T^n_{ij} - \overline{U}^n_{i-1,j}T^n_{i-1,j})}{\Delta x} - \frac{(\overline{V}^n_{i,j}T^n_{ij} - \overline{V}^n_{i,j-1}T^n_{i,j-1})}{\Delta y}$$

$$+ \frac{1}{\rho^n_{i,j}Cp^n_{ij}}\left\{k^n_{i,j}\left(\frac{(T^n_{i+1,j} - 2T^n_{i,j} + T^n_{i-1,j})}{\Delta x^2} + \frac{(T^n_{i,j+1} - 2T^n_{ij} + T^n_{i,j-1})}{\Delta y^2}\right)\right.$$

$$\left. + \frac{(k^n_{i+1,j} - k^n_{i-1,j})}{2\Delta x}\frac{(T^n_{i+1,j} - T^n_{i-1,j})}{2\Delta x} + \frac{(k^n_{i,j+1} - k^n_{i,j-1})}{2\Delta y}\frac{(T^n_{i,j+1} - T^n_{i,j-1})}{2\Delta y}\right\}$$

$$+ \frac{\beta^n_{i,j}\cdot T^n_{i,j}}{\rho^n_{i,j}\cdot Cp^n_{i,j}}\left[-\left(\frac{\partial P}{\partial\rho}\right)\left\{\frac{((\rho U)^{n+1}_{i+1,j} - (\rho U)^{n+1}_{i-1,j})}{2\Delta x} + \frac{((\rho V)^{n+1}_{i,j+1} - (\rho V)^{n+1}_{i,j-1})}{2\Delta y}\right\}\right.$$

$$+ \frac{(\overline{U}^n_{i,j}P^n_{i,j} - \overline{U}^n_{i-1,j}P^n_{i-1,j})}{\Delta x} + \frac{(\overline{V}^n_{i,j}P^n_{i,j} - \overline{V}^n_{i,j-1}P^n_{i,j-1})}{\Delta y}$$

$$\left.\left. - P^n_{ij}\left\{\frac{(U^n_{i+1,j} - U^n_{i-1,j})}{2\Delta x} + \frac{(V^n_{i,j+1} - V^n_{i,j-1})}{2\Delta y}\right\}\right]\right] \tag{31}$$

where for an ideal gas

$$\frac{\partial \rho}{\partial T} = - \frac{\rho^n_{i,j}}{T^n_{i,j}}$$

$$\frac{\partial P}{\partial T} = R \cdot \rho^n_{i,j}$$

$$\frac{\partial P}{\partial \rho} = R \cdot T^n_{i,j}$$

$$\beta = \frac{1}{T^n_{i,j}}$$

$$C = \left\{ 1 - \frac{\beta \cdot T^n_{i,j} \cdot R}{Cp^n_{i,j}} \right\}$$

Although the finite difference of convection terms in eqs. (29), (30) and (31) are shown for only one condition of eq. (25), appropriate finite difference of convection terms should be used in actual computation work.

NUMERICAL PROCEDURE

The calculation proceeds by explicitly advancing ρ, n, v and T with difference forms of eqs. (28), (29), (30) and (31). Also pressure P is calculated explicitly from an equation of state using ρ and T. Fluid within enclosure is initially at a uniform temperature To and at rest. Here, for preliminary studies, we consider a rectangular enclosure of height X_{max} (0.1m), width Y_{max} (0.05m) and vertical heated wall (0.04m) which is located in the middle part of left side wall, as shown in figure 1.

During any one time-step, all values appearing in the right side of eqs. (28), (29), (30) and (31) are treated as constants. In the first place, the new $(\rho U)^{n+1}_{i,j}$ and $(\rho V)^{n+1}_{i,j}$ at all interior grid points may be obtained from successive momentum eqs. (29) and (30). Then new density $\rho^{n+1}_{i,j}$ should be calculated from continuity eq. (28) substituting $(\rho U)^{n+1}_{i,j}$ and $(\rho V)^{n+1}_{i,j}$ into eq.

(28). A new temperature $T_{i,j}^{n+1}$ is obtained from energy eq. (31) using the value of $\rho_{i,j}^{n+1}$ just computed. Finally new velocities $U_{i,j}^{n+1}$ and $V_{i,j}^{n+1}$ are calculated mathematically from $(\rho U)_{i,j}^{n+1}$ and $(\rho V)_{i,j}^{n+1}$ using the value of $\rho_{i,j}^{n+1}$ as follows:

$$U_{i,j}^{n+1} = \frac{(\rho U)_{i,j}^{n+1}}{\rho_{i,j}^{n+1}} \tag{32}$$

New pressure $P_{i,j}^{n+1}$ are calculated from the equation of state using new temperatures and densities which are already computed at interior grid points, though not at the wall. Pressures at the wall are obtained by quadratic extrapolations. This process is repeated in time, provided the time-step is sufficiently small. The time-step $\Delta\tau_r$ has been restricted to 10^{-5}s or less [5] in this computational work. This value corresponds to less than the time interval for a sound wave to propagate across the mesh size y as follows

$$\Delta\tau_r \leq \frac{\Delta y}{\sqrt{\gamma RT}} \tag{33}$$

DISCUSSION OF RESULTS

Numerical calculations have been carried out for air within rectangular enclosure (0.1 x 0.05 m^2) in a time periods from 0 to 40 [ms]. The fluid conditions and the imposed temperature differences correspond to a Grashof number of approximately 8.8 x 10^4. Figure 3-(1) ~ (10) shows the velocity vectors at intervals of 0.03 ms from 0.03 to 0.30 ms, where the dominant motion is normal to the heated wall (and gravity). The absence of a vector at a given grid point means that the magnitude of the calculated velocity at that point was less than 5% of the maximum velocity at any of the grid points at that instant of time. The disturbance-front separating the region of non-zero fluid velocities from the region of essentially static fluid is seen to move away from the heated wall with the velocity of sound. At t = 0.15 ms (figure 3-(5)), the disturbance front reaches the right side wall. In figures 3-(6) to (10), the fluid motion becomes complicated by the sum of many phases and amplitudes of motion with multiple reflection from all walls. No gravitational effects can be seen.

Figure 4 ~ (1) to (10) shows the velocity vectors at later times, from 20.03 to 20.30 ms. The gravitational contributions to the fluid motion, which causes assymmetry around the horizontal center line, is still quite small compared to the motions induced by the initial expansion wave away from the heated surface. In order to distinguish the growth of natural convection, the velocity components $U_{5,2}$ and $U_{8,2}$ are shown in Fig. 11 for increasing time. $U_{5,2}$ and $U_{8,2}$ are the vertical components of velocity at grid points (5,2) and (8,2) respectively. These grids points are symmetrical located about the horizontal center line of the heated surface. The data are plotted at every hundreth time step, which causes the apparent sawtooth character. Nevertheless, the superposition of various amplitudes and phases, mentioned above, is clearly evident. The dotted lines indicate the values of $-U_{5,2}$, i.e. symmetrical values of $U_{5.2}$ about the zero velocity. In the early time periods, less than about 5 [ms], the velocity components $U_{8,2}$ and $-U_{5,2}$ are equal within 1% or better. After that time they begin to deviate, and this is a manifestation of buoyancy force. The difference between $U_{8,2}$ and $-U_{5,2}$, illustrated with shadow, represents the growth of natural convection. The values of the shadow of deduced from Fig. 11 are shown in Fig. 12. Also the difference of horizontal components at both points are shown in same figure with the dotted curve. The magnitudes of velocity component at upper point of (8,2) are larger than one at lower point of (5,2) for either horizontal or vertical component, and this is what should be expected on physical grounds in a natural convection heat transfer. It is of interest to note that the natural convection flow near the heated wall induced by the buoyancy force develop continuously and smoothly as shown in Fig. 12 and stream lines, which always close for incompressible fluid flow, will not do so in this case.

On the other hand, the temperature field in the vicinity of the heated wall is essentially that of pure conduction for this range, and isotherms are practically symmetrical to the heated wall.

The pressure and velocity fluctuation at a near-mid point of the enclosure are shown in Figs. 5 and 6 for time to 1.0 millisecond. The relation between pressure and velocity fluctuation is not distinct in Fig. 5 and 6 for this physical model which has the heated wall at the middle part of the left side wall as in Fig. 2. In order to reduce the influence of reflection at upper and lower wall, the calculation was repeated with the heating surface extending over the complete left wall of the cavity. The calculated pressure and horizontal component of velocity are shown in Fig. 7 and 8. It is quite evident that the frequency of pressure fluctuation is two times that of the velocity. The average pressure in the enclosure increases as heat is added, and is proportional to the heated area, as seen in Fig. 5 and Fig. 7:

$$\frac{\partial Pm}{\partial \tau} \propto Q$$

The results of pressure and velocity fluctuation in later time periods from 30.5 to 32.0 [ms] are shown in Figs. 9 and 10 respectively. From these figures we are not directly able to make clear the correlation between pressure and velocity fluctuation. Therefore further study, using spectrum analysis (or something similar) should be required to fully understand this phenomena where the fluid motion is the sum of many phases and amplitudes of motion with multiple reflection from the walls.

Numerical procedures for solutions of heat transfer equations in the time-dependent domain may fall into two categories, explicit and implicit. These two types of difference equations have previously been studied in which explicit difference equations are simple to solve, but which require a large number of time steps of limited size, and implicit difference equations do not limit the time step but which do require iteration at each time step in the solution. Therefore, explicit procedures are convenient under conditions where a sufficiently large time step, consistent with computational stability, can be used. In order to examine the accuracy of this computational results, the smaller time step $\Delta\tau$ of 10^{-4} [ms] which is one tenth of normal time step have been used for early time periods from 0 to 0.3 [ms]. The agreement with the solutions of $v_{7,6}$ for this calculation of small time step and the prior one is better than 0.1%.

The numerical calculations have been performed on a large digital computer. The execution time at each time step is approximately 33 CP millisecond.

ACKNOWLEDGEMENT

This work was performed during the time that H. Yamashita was a guest worker in the Thermophyscial Properties Division of NBS. Cooperative studies using the method developed here will continue after Prof. Yamashita returns to Fukuoka University in Japan.

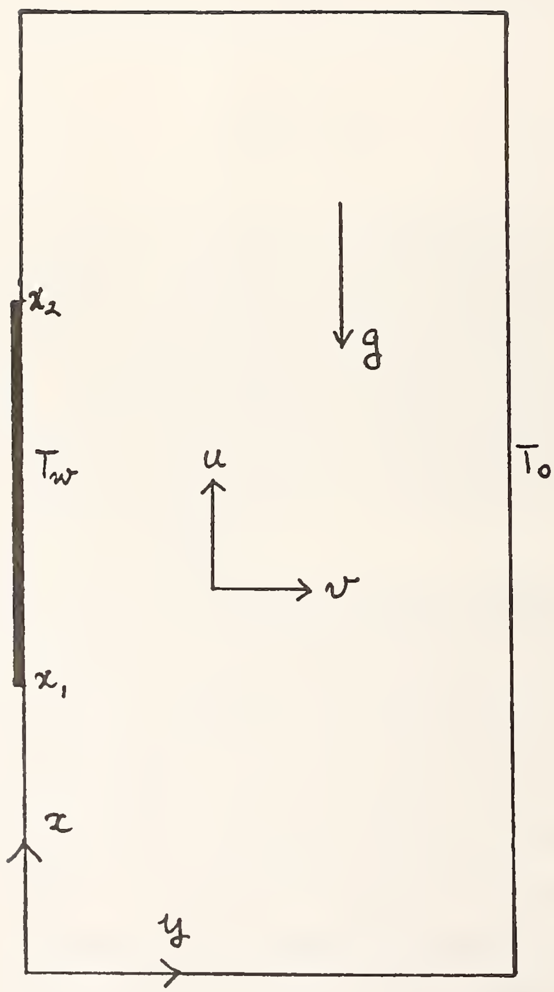

Figure 1. Physical model and coordinates.

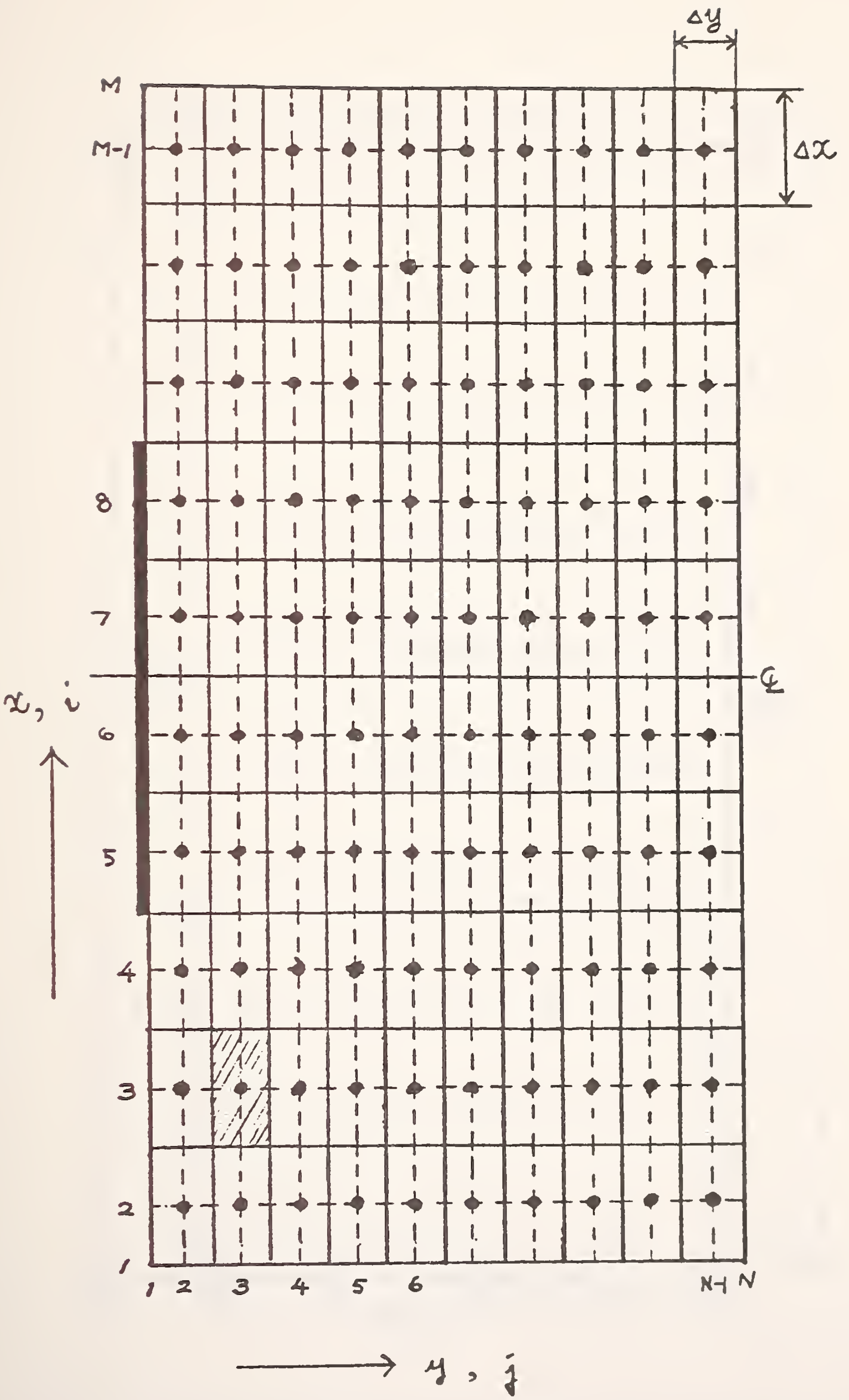

Figure 2. Schematic diagram of the numerical method.

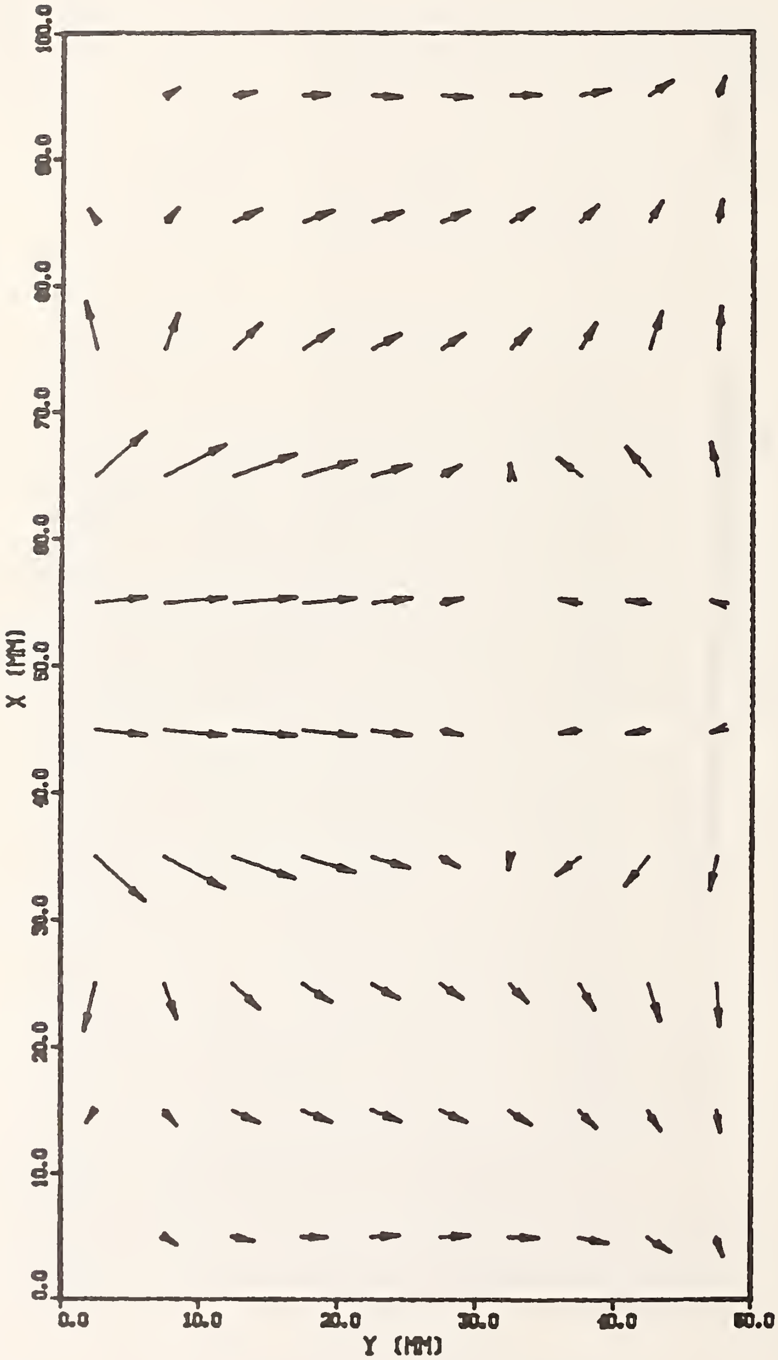

Figure 3-(7). Velocity field at t=0.21 ms. V_{max}=0.25 mm/s.

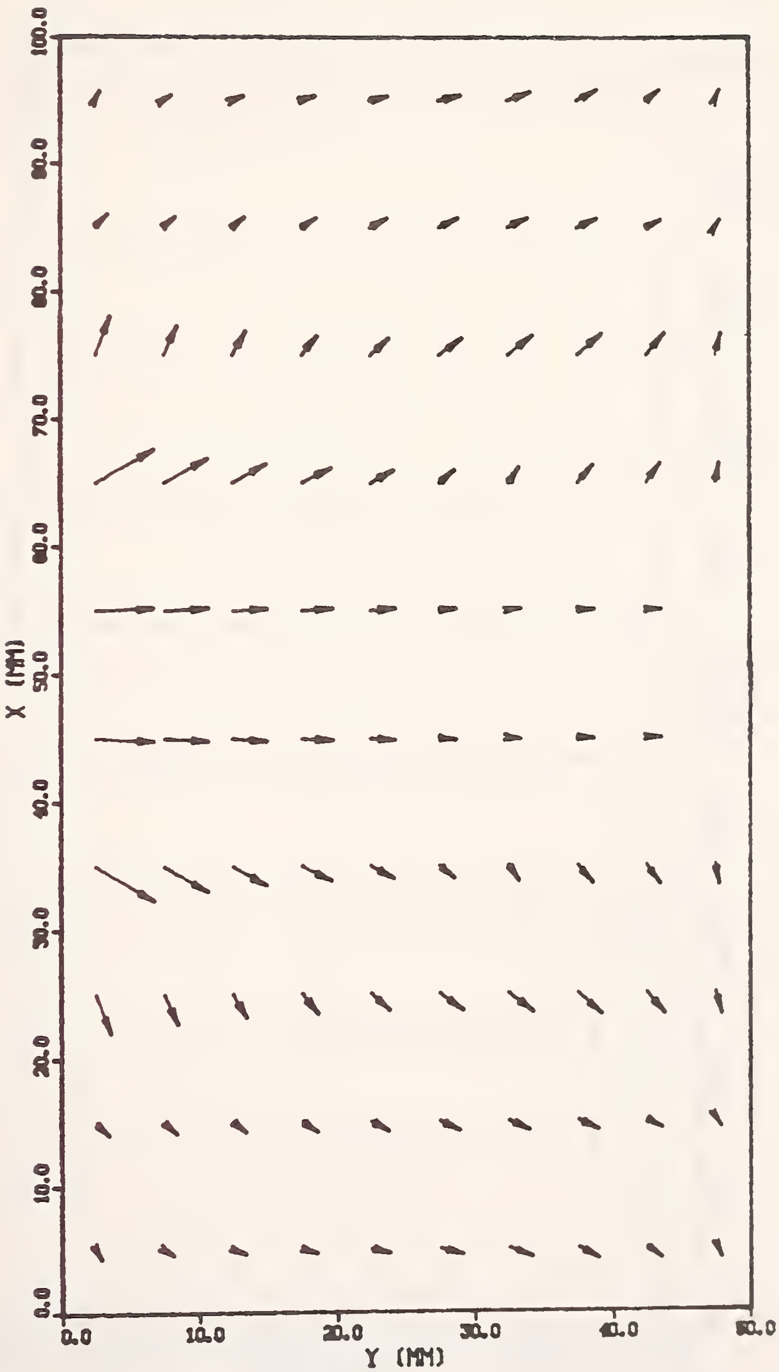

Figure 3-(6). Velocity field at t=0.18 ms. V_{max}=0.427 mm/s.

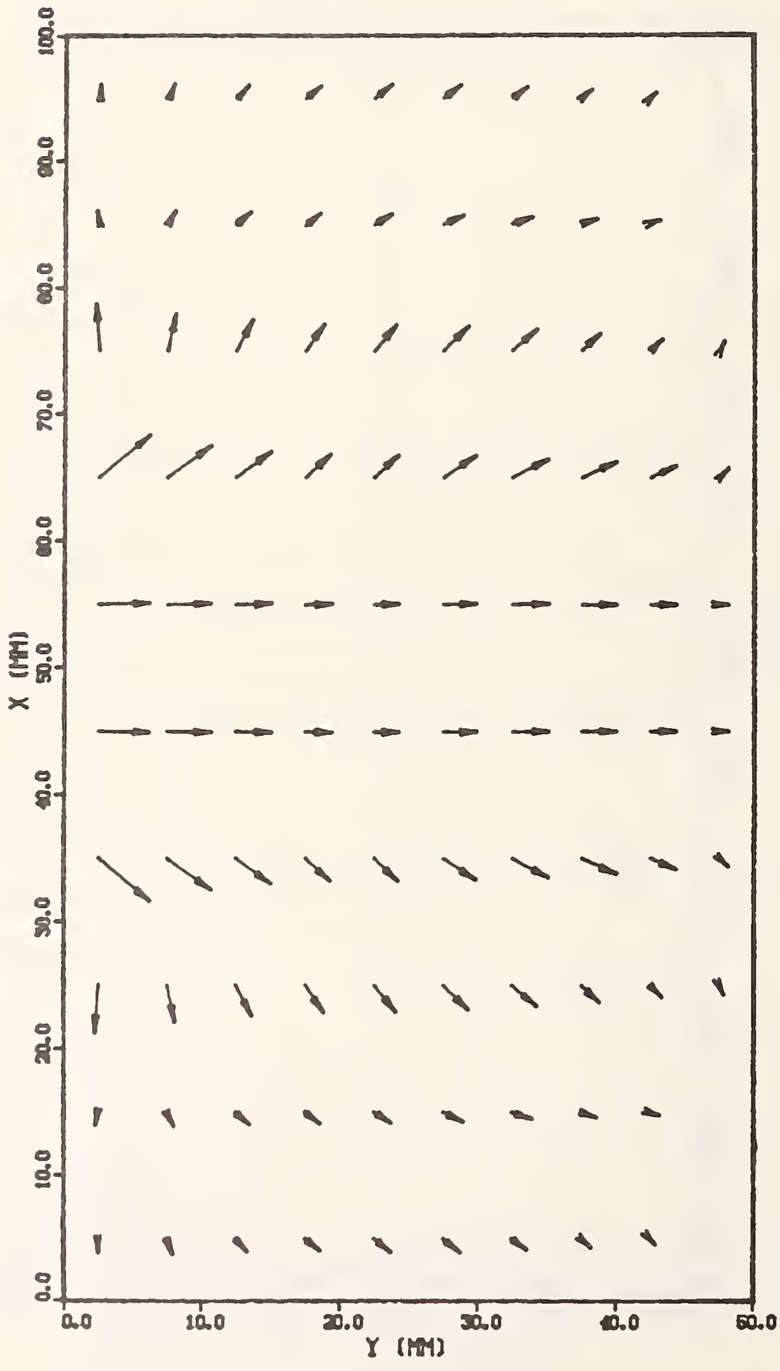

Figure 3-(5). Velocity field at t=0.15 ms. V_{max}=0.428 mm/s.

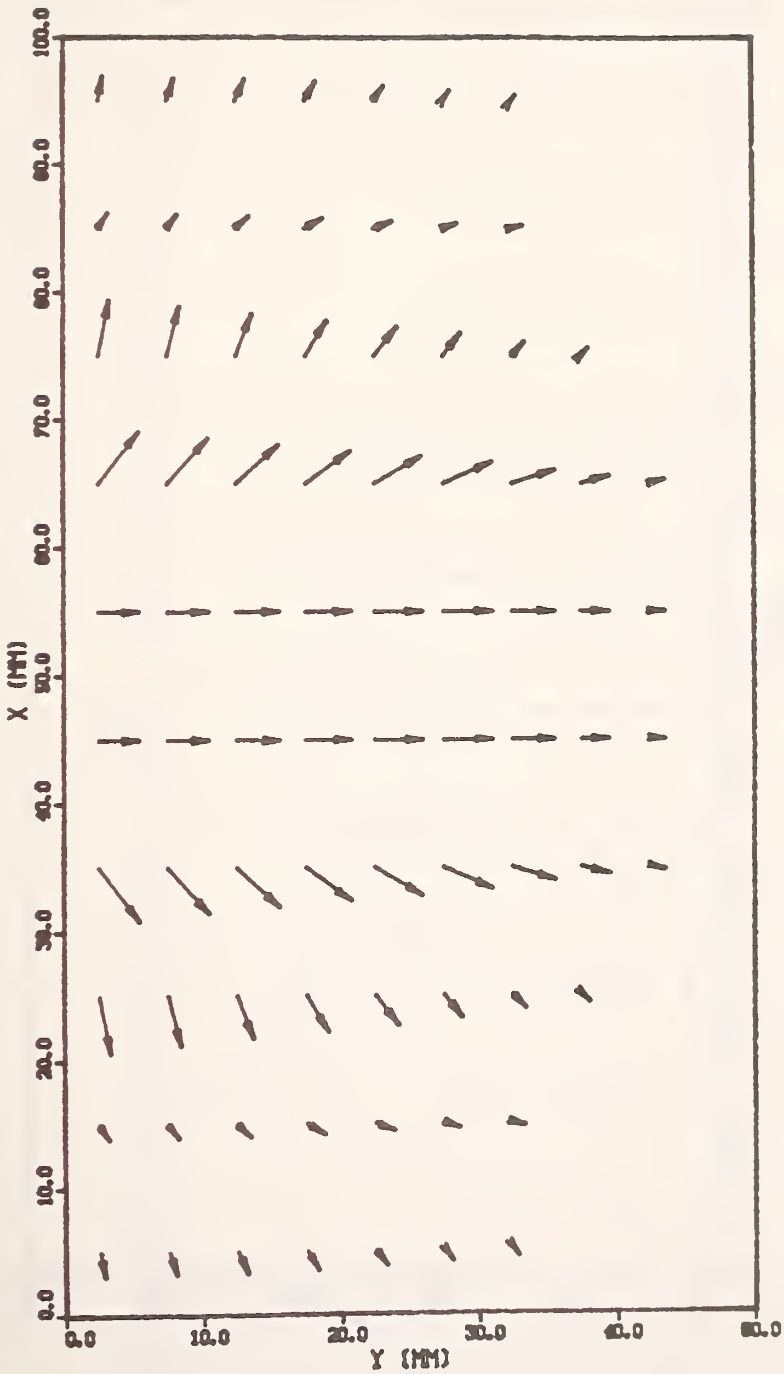

Figure 3-(4). Velocity field at t=0.12 ms. V_{max}=0.376 mm/s.

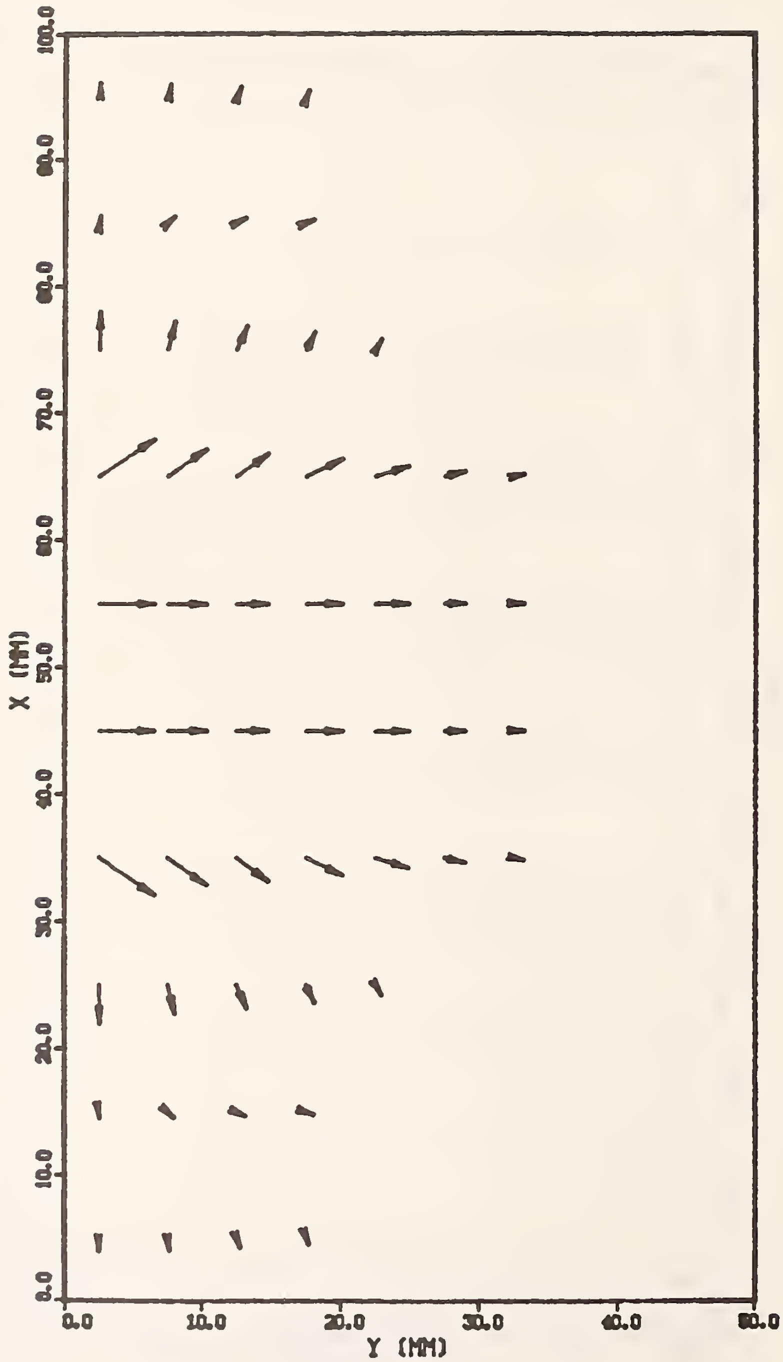

Figure 3-(3). Velocity field at t=0.09 ms. V_{max}=0.566 mm/s.

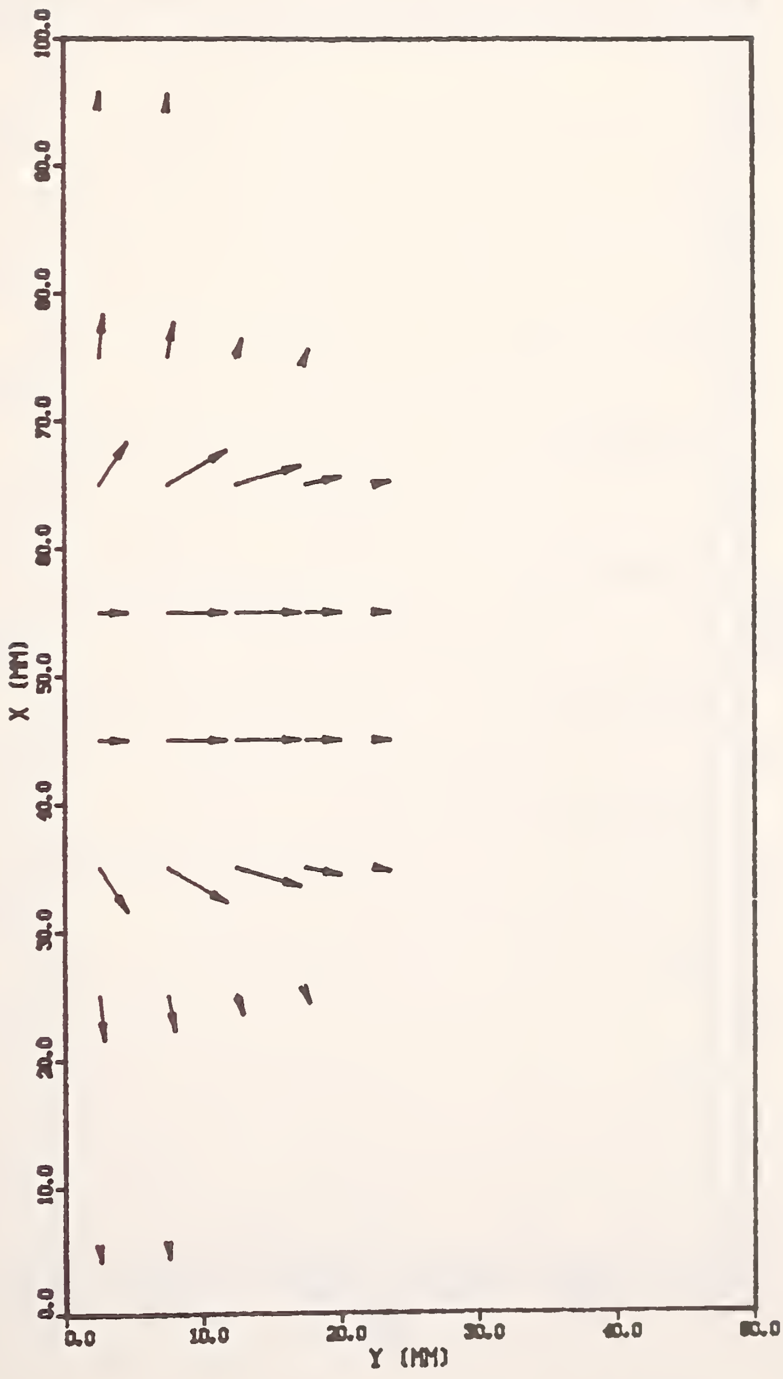

Figure 3-(2). Velocity field at t=0.06 ms. V_{max}=0.368 mm/s.

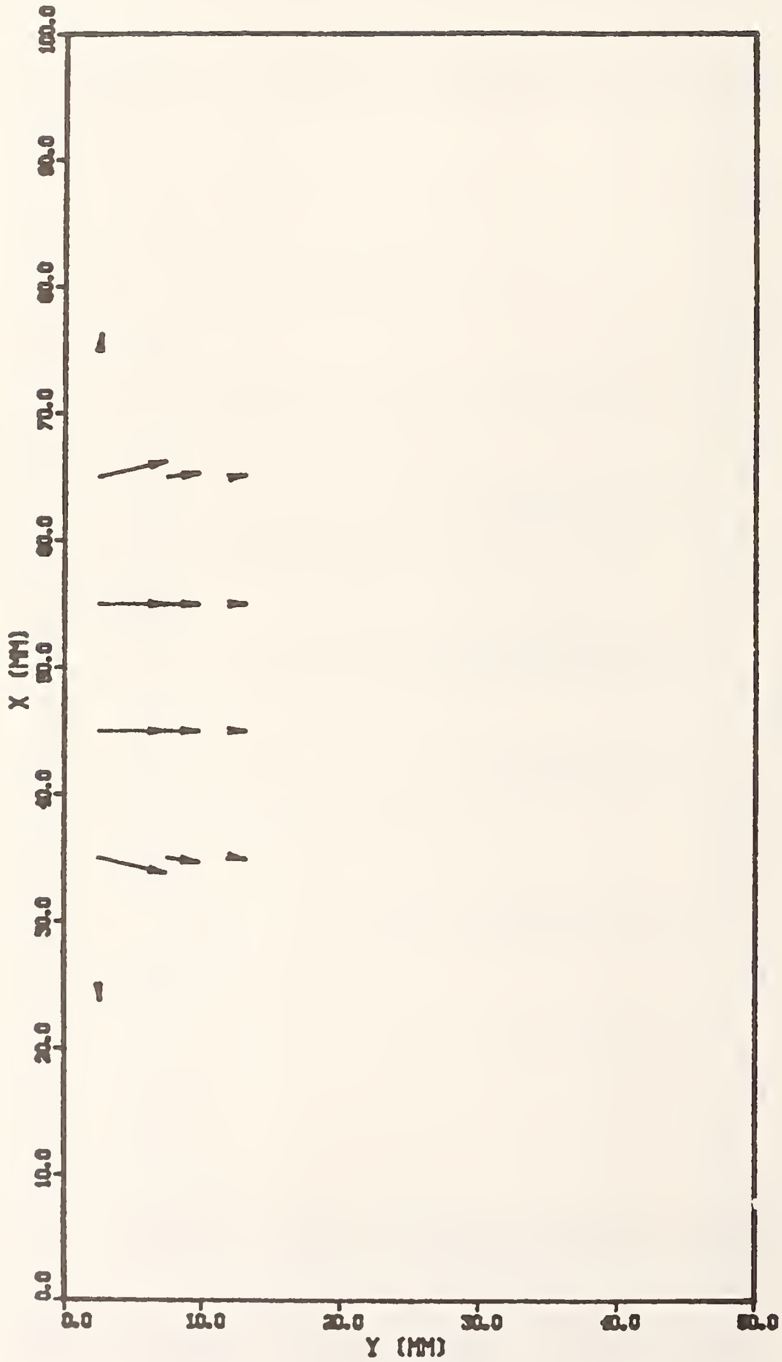

Figure 3-(1). Velocity field at t=0.03 ms. V_{max}=0.521 mm/s.

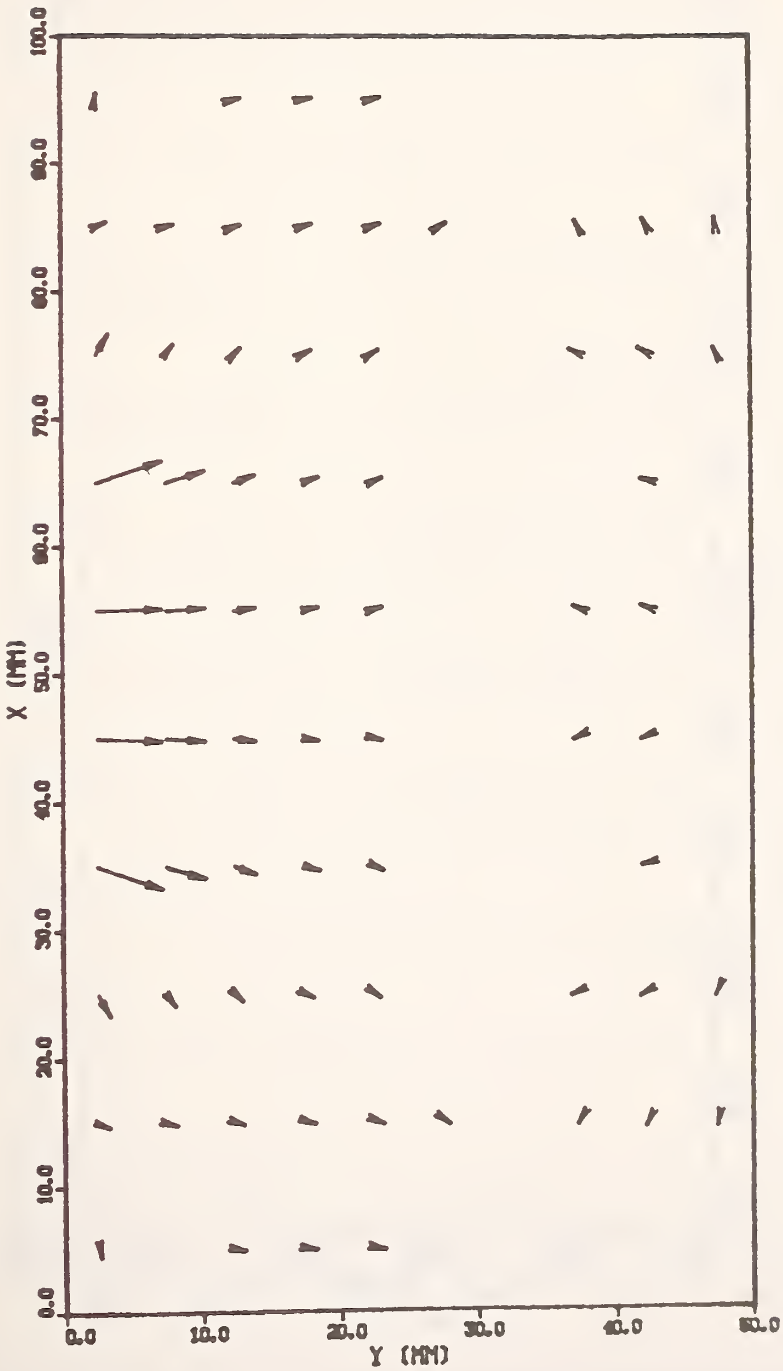

Figure 3-(8). Velocity field at t=0.24 ms. V_{max}=0.498 mm/s.

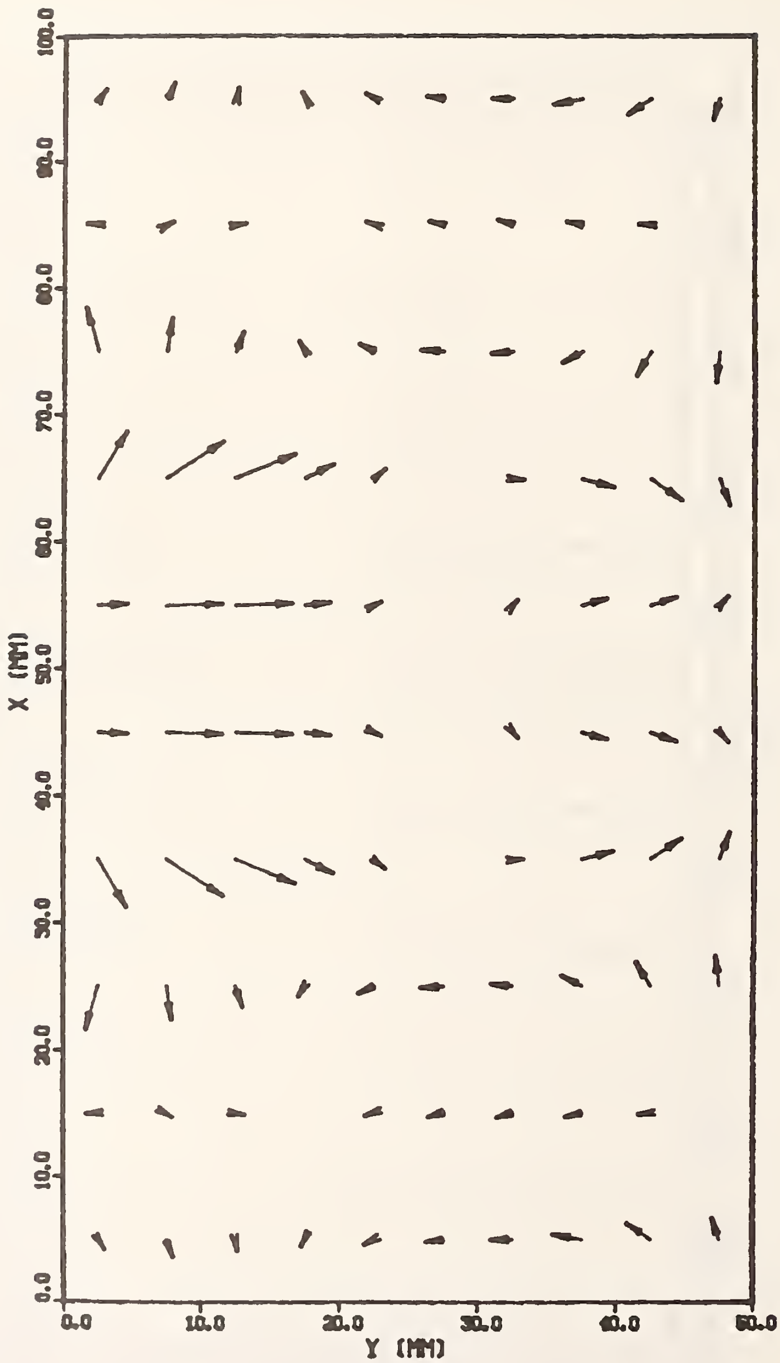

Figure 3-(9). Velocity field at t=0.27 ms. V_{max}=0.214 mm/s.

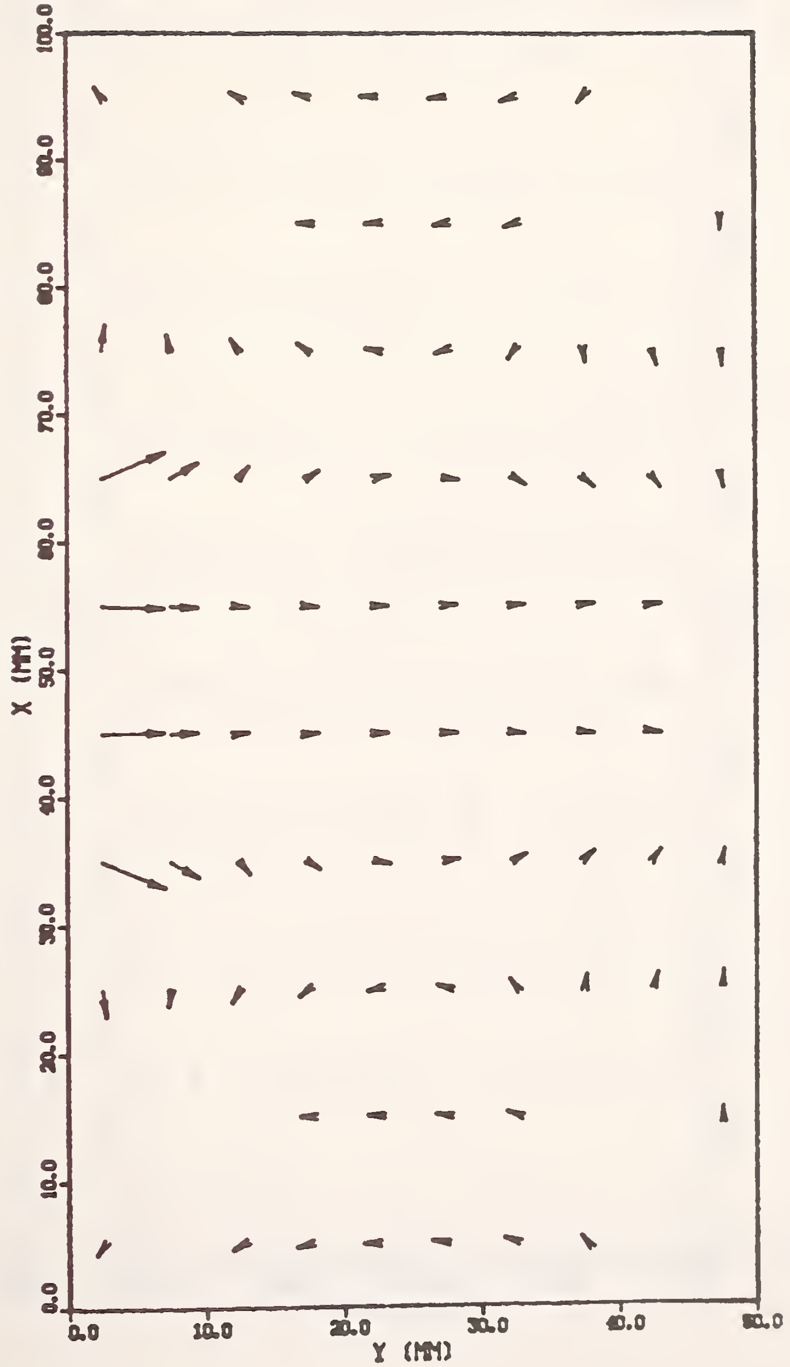

Figure 3-(10). Velocity field at t=0.30 ms. V_{max}=0.544 mm/s.

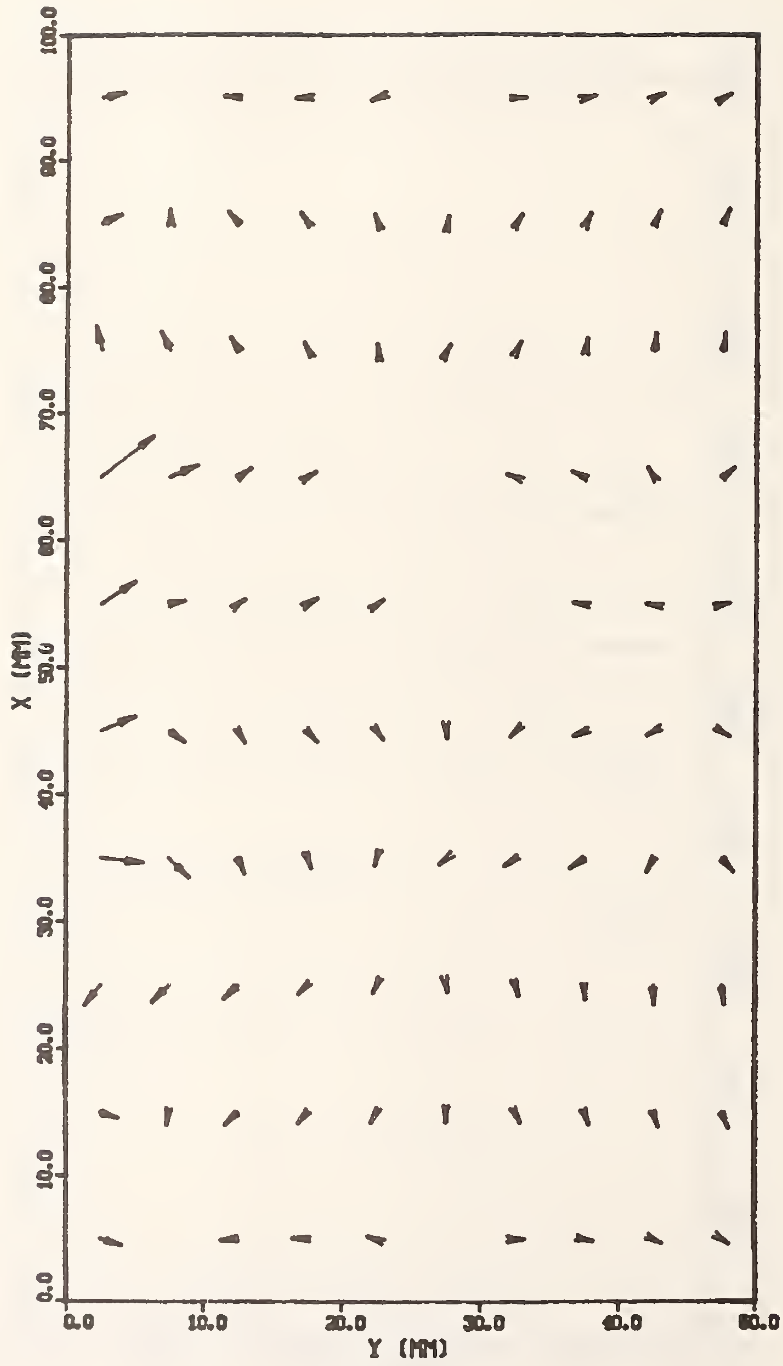

Figure 4-(1). Velocity field at t=20.03 ms. V_{max}=0.343 mm/s.

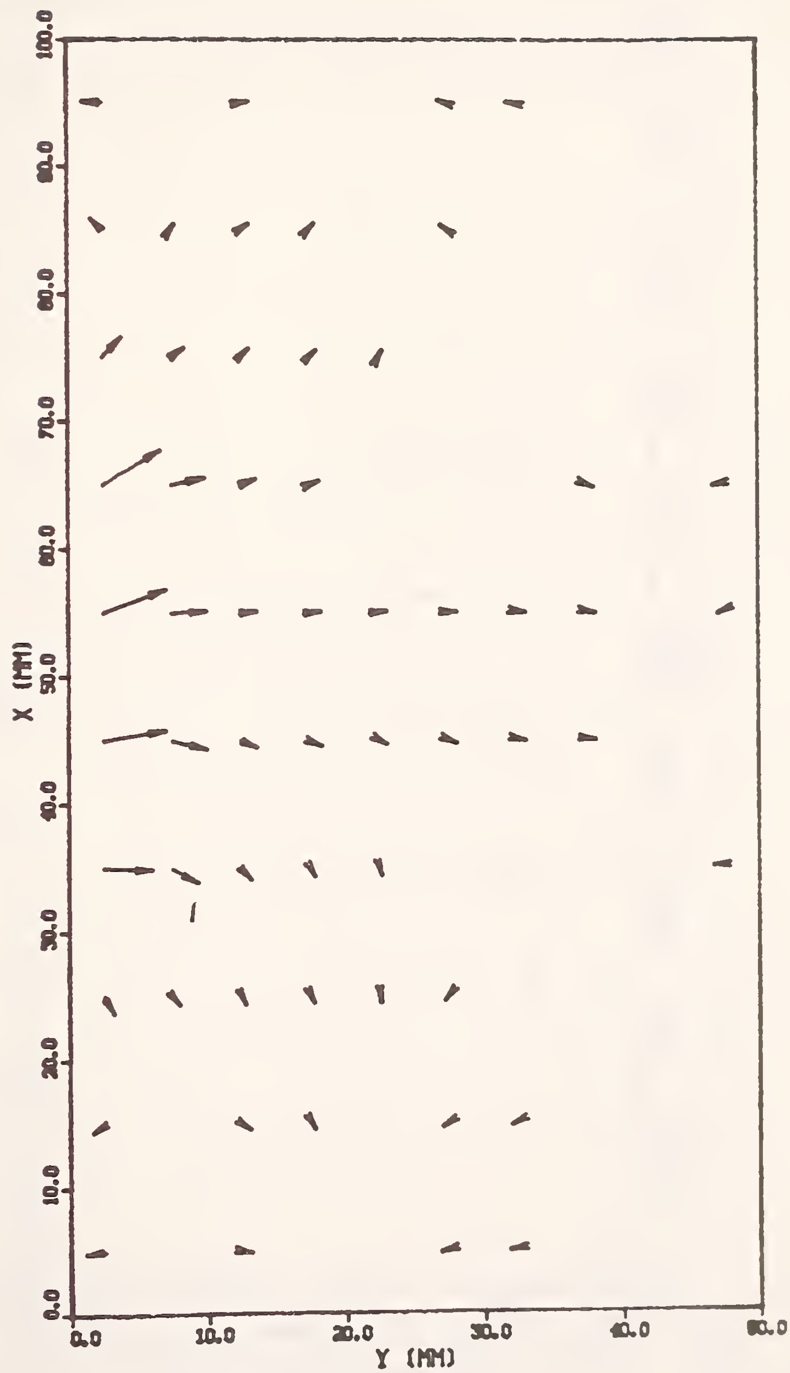

Figure 4-(2). Velocity field at t=20.06 ms. V_{max}=0.380 mm/s.

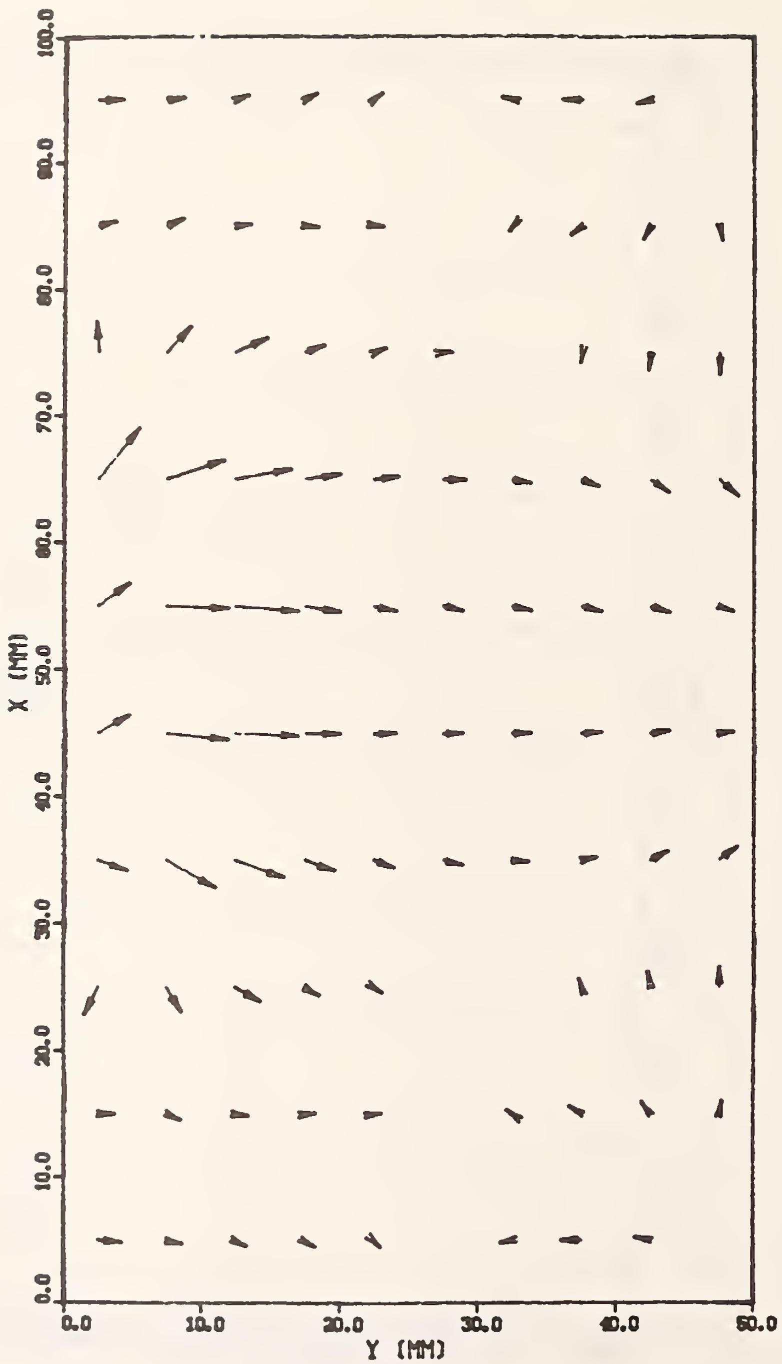

Figure 4-(3). Velocity field at t=20.09 ms. V_{max}=0.310 mm/s.

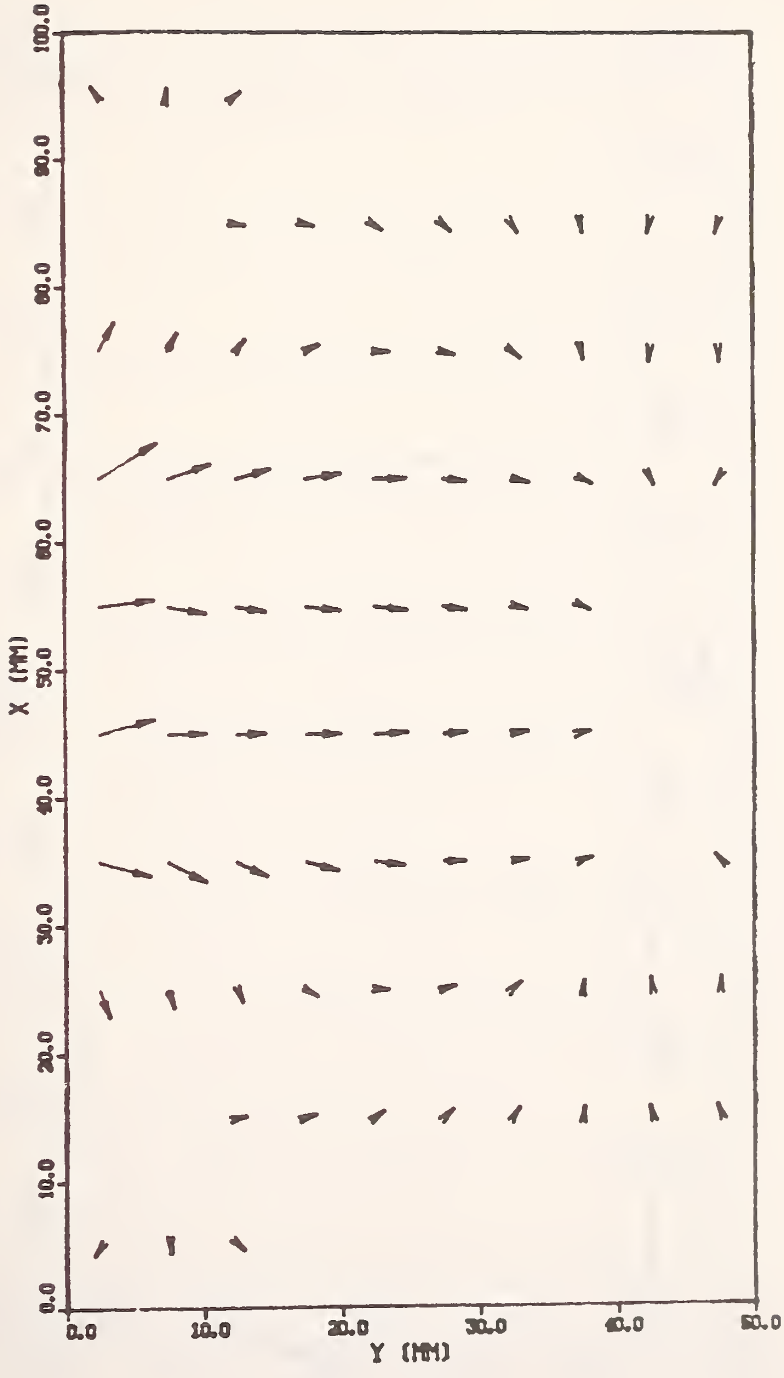

Figure 4-(4). Velocity field at t=20.12 ms. V_{max}=0.567 mm/s.

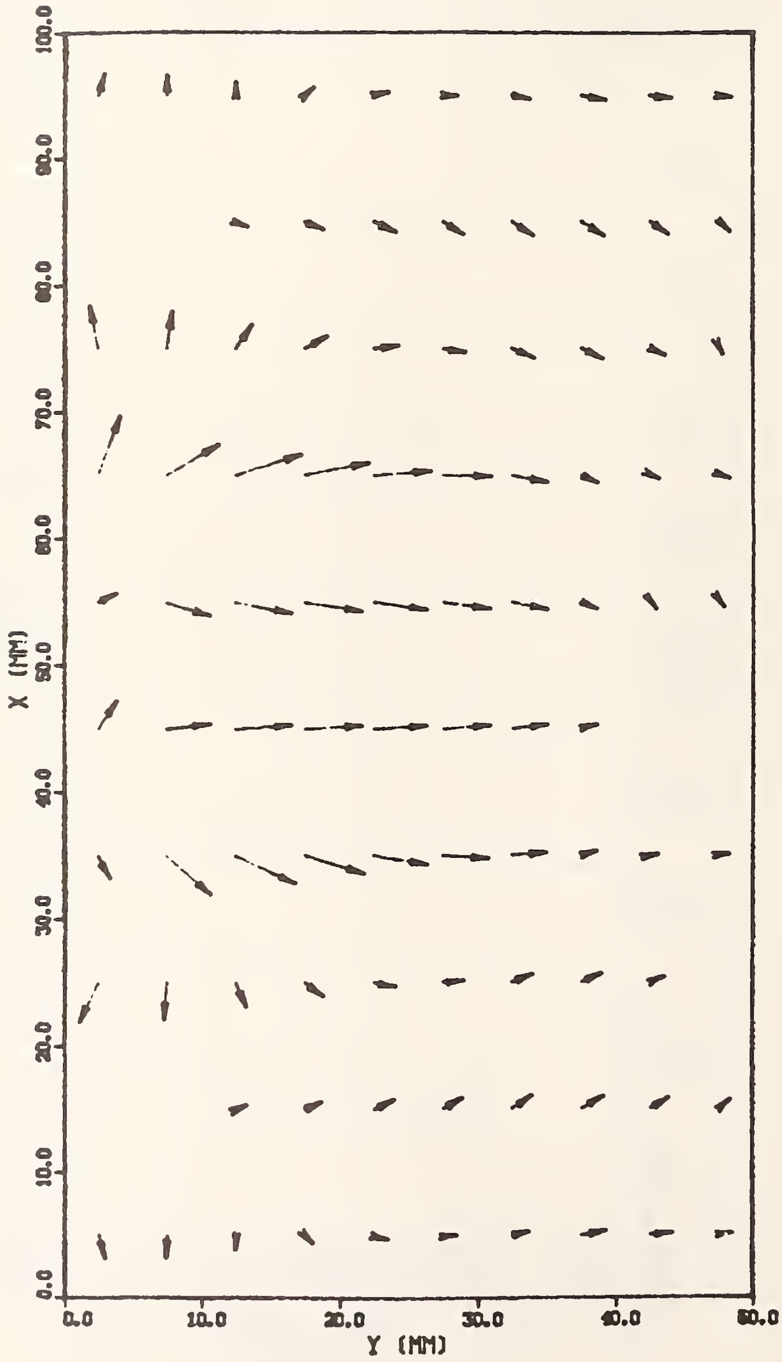

Figure 4-(5). Velocity field at t=20.15 ms. V_{max}=0.339 mm/s.

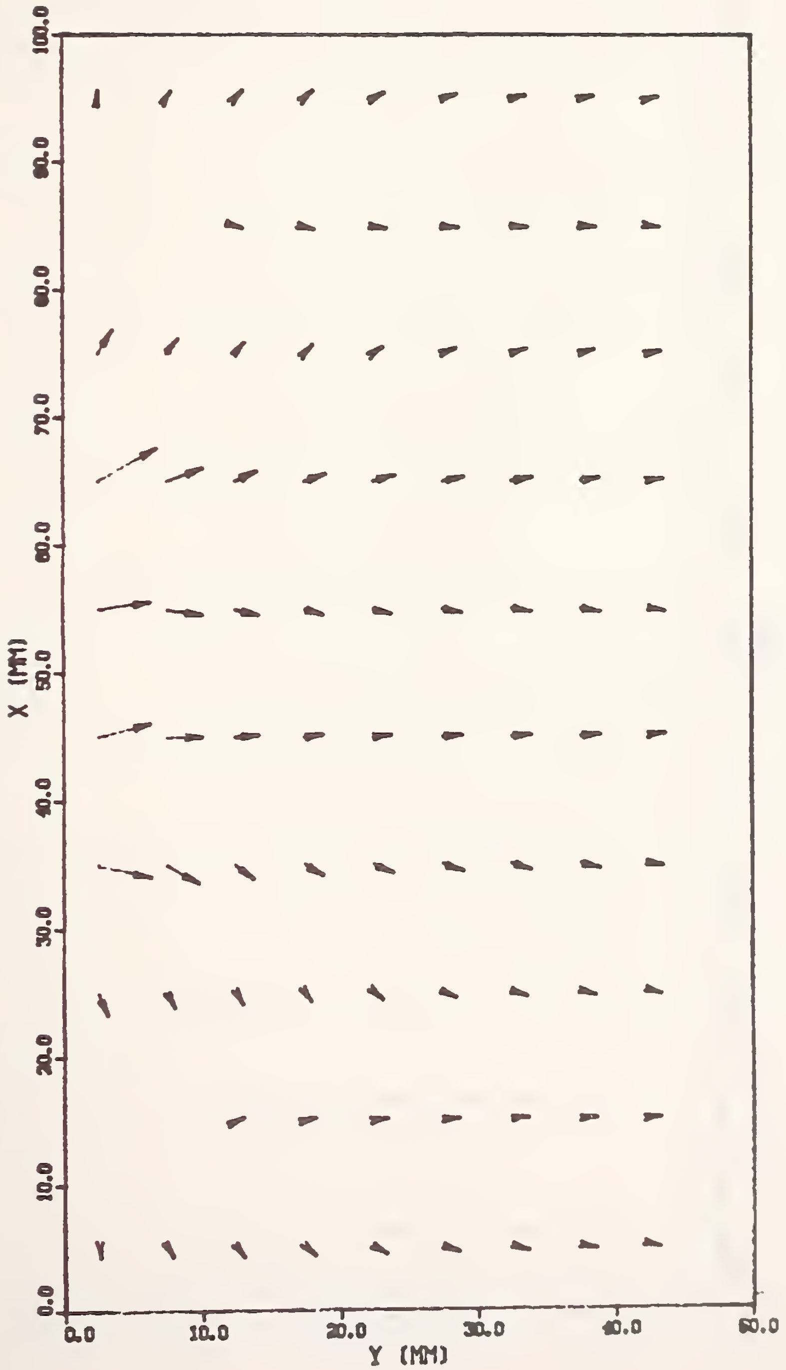

Figure 4-(6). Velocity field at t=20.18 ms. V_{max}=0.588 mm/s.

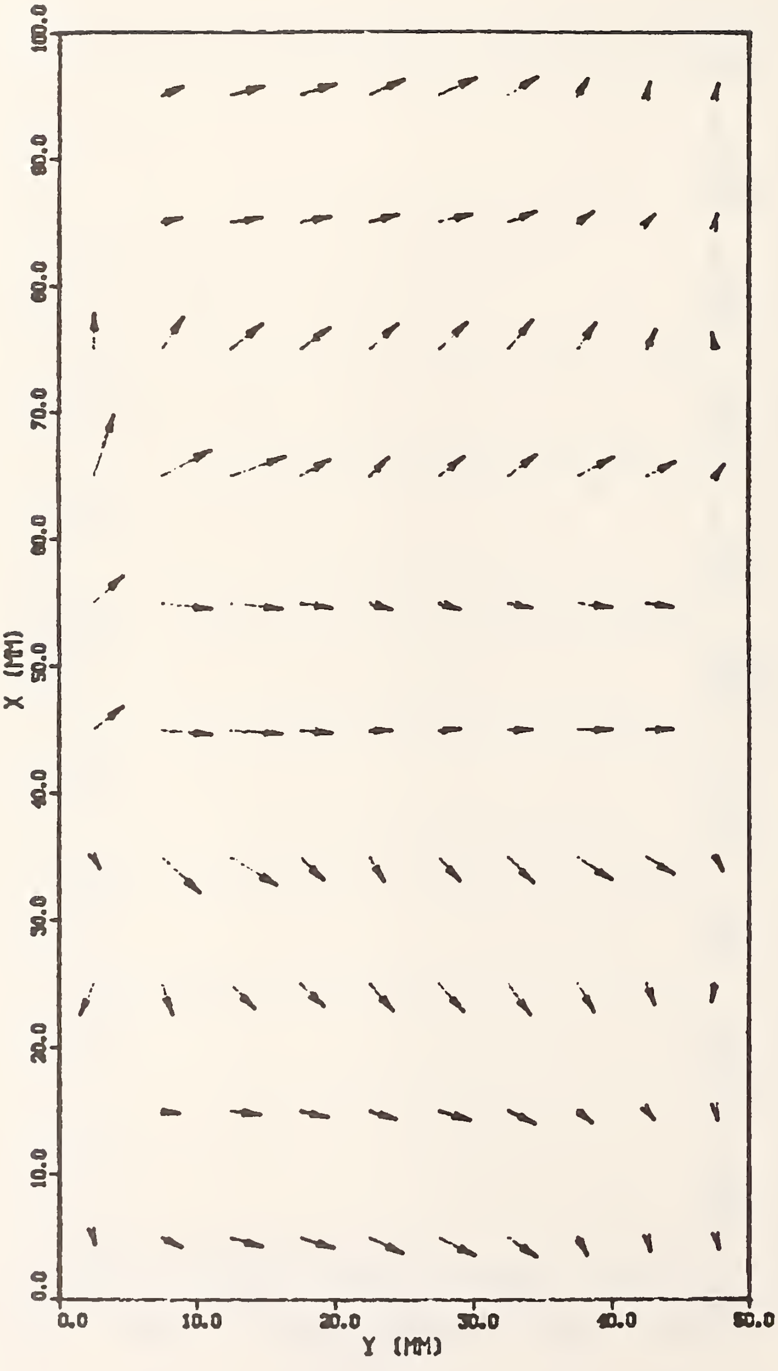

Figure 4-(7). Velocity field at t=20.21 ms. V_{max}=0.260 mm/s.

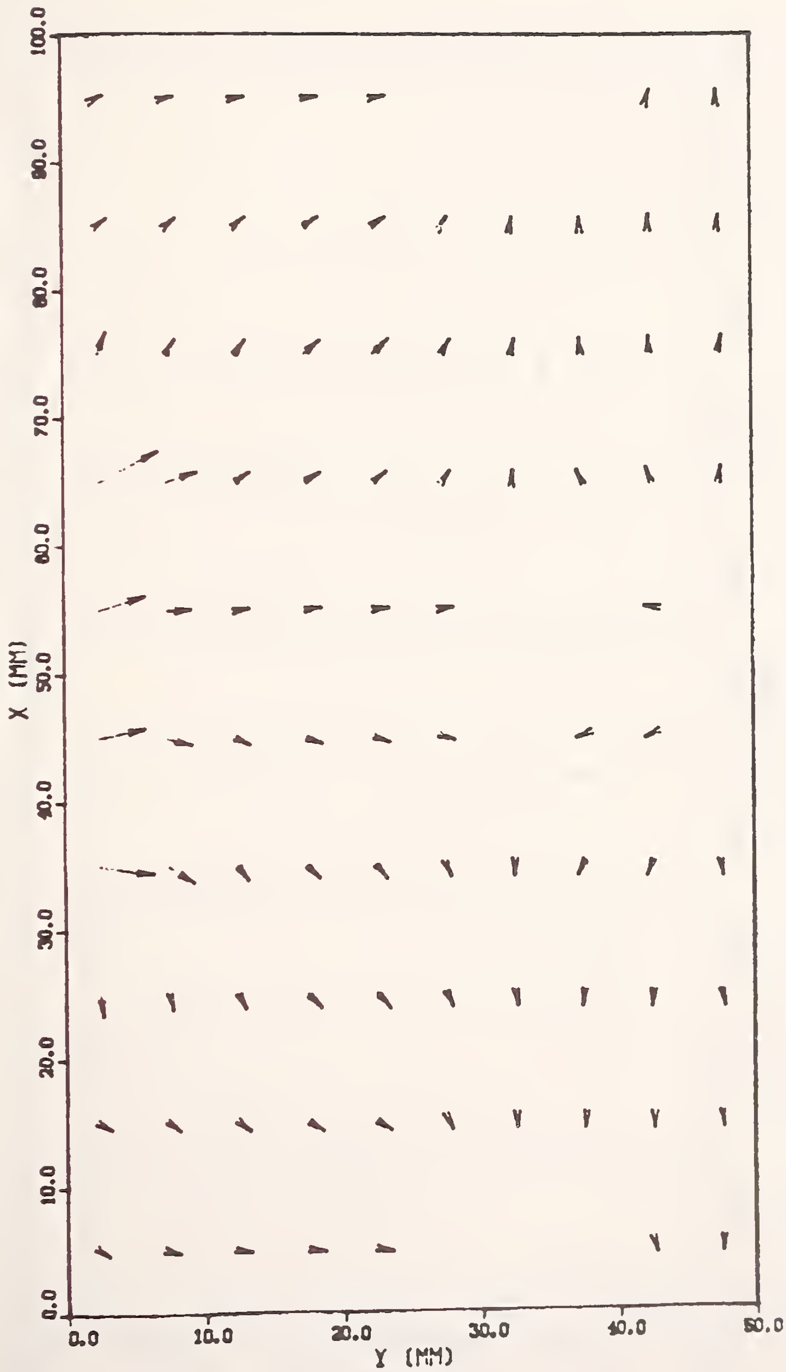

Figure 4-(8). Velocity field at t=20.24 ms. V_{max}=0.527 mm/s.

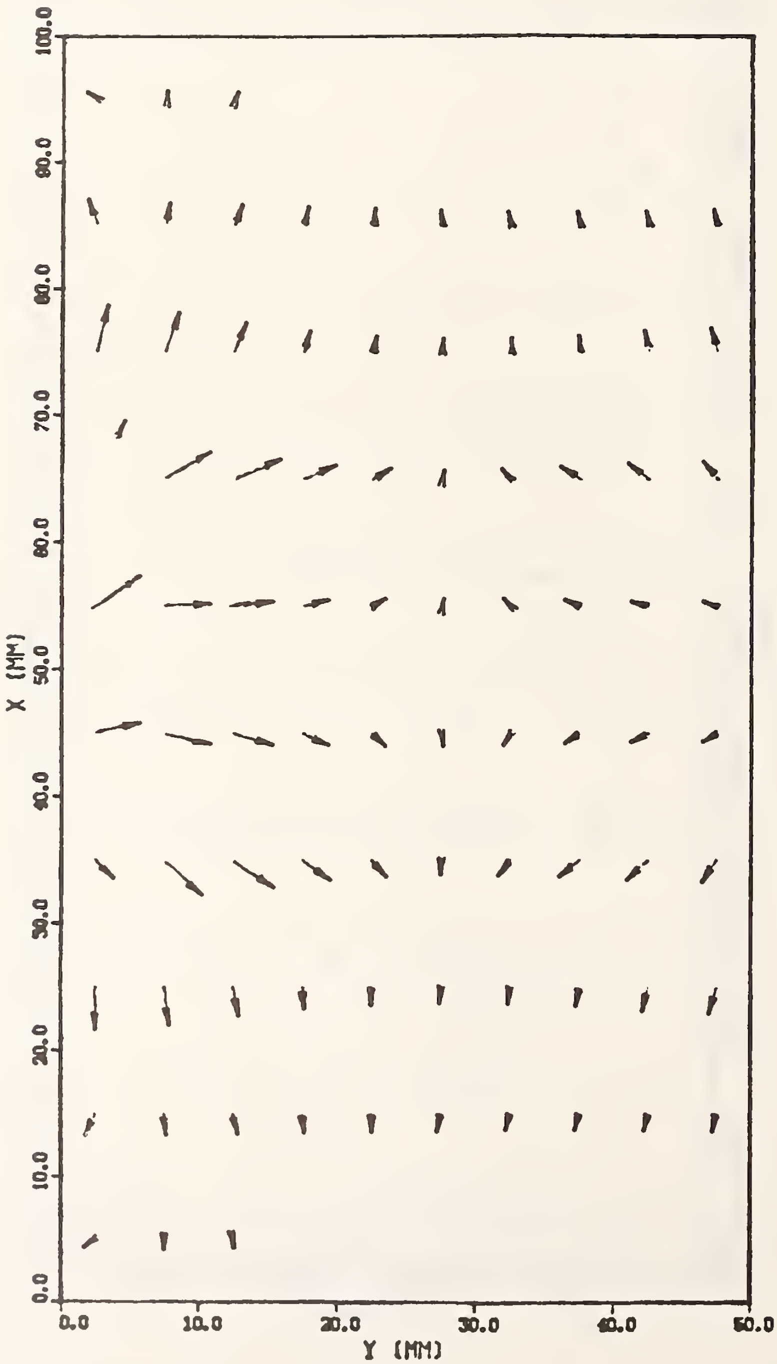

Figure 4-(9). Velocity field at t=20.27 ms. V_{max}=0.325 mm/s.

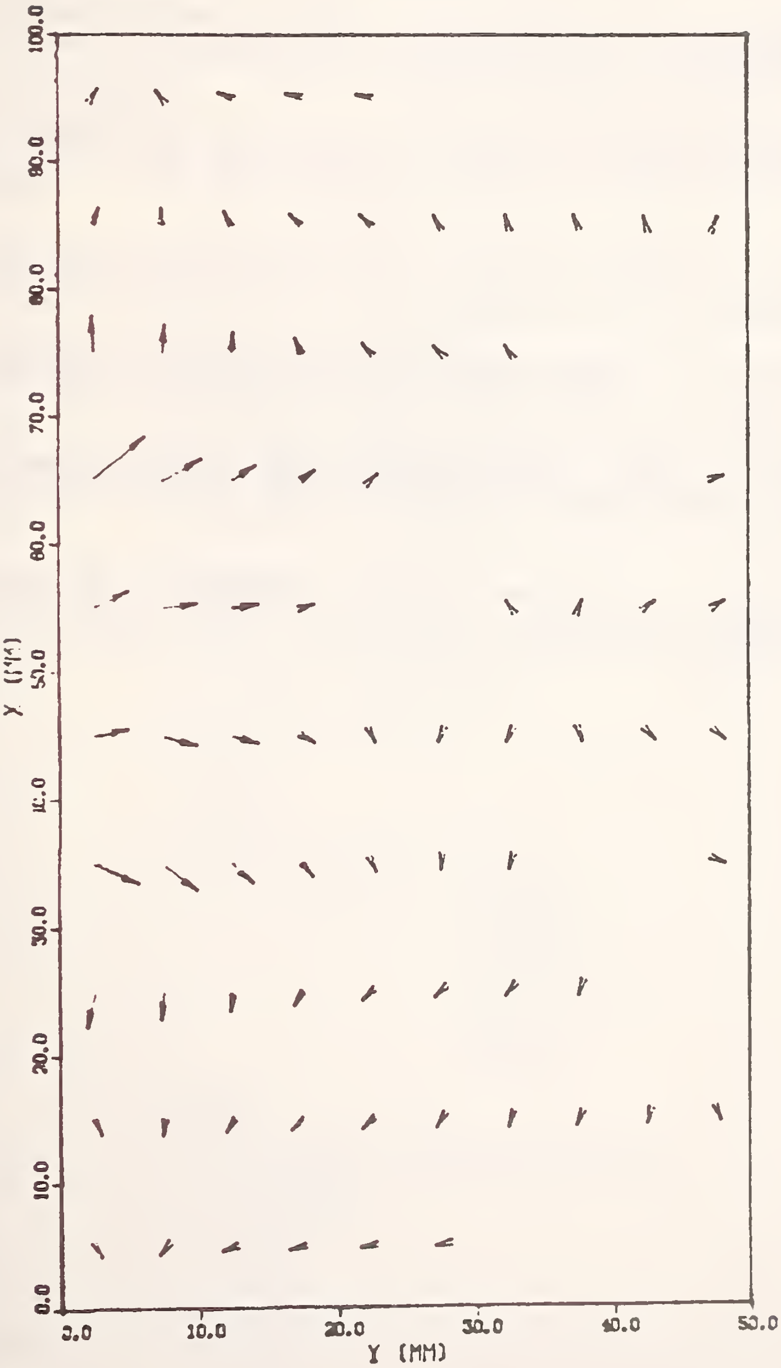

Figure 4-(10). Velocity field at t=20.30 ms. V_{max}=0.531 mm/s.

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Appendix B

List of variables [Main Program]

These are founded on computer program list of 1.

Line No.	Program Symbol	Definition
43	A	=g/(RT), to use initial density distribution
35	ADATE	Date of computation
127	AFPX	Dummy argument, see sub. AVEVELO
127	AFPY	" "
35	ATIME	Time of computation
130	AUPX	Average velocity $\bar{u}(i) = 0.5[u(i+1,j) + u(i,j)]$
131	DUPY	Average velocity $\bar{u}(j) = 0.5[u(i,j+1) + u(i,j)]$
135	AVPX	Average velocity $\bar{v}(i) = 0.5[v(i+1,j) + v(i,j)]$
136	AVPY	Average velocity $\bar{v}(j) = 0.5[v(i,j+1) + v(i,j)]$
107	CODX	$\partial k/\partial x$
108	CODY	$\partial k/\partial y$
78	COND	Thermal conductivity, k
78	CP	Specific heat, C_p
97	CPDX	$\partial C_p/\partial x$
98	CPDY	$\partial C_p/\partial y$
	DFDX	Dummy argument of $\partial/\partial x$
	DFDXX	" $\partial^2/\partial x^2$
	DFDXY	" $\partial^2/\partial x\partial y$
	DFDY	" $\partial/\partial y$
	DFDYY	" $\partial^2/\partial y^2$
78	DPDR	$\partial P/\partial \rho$
198	DPUDX	$= \partial(Pu)/\partial x$ in energy equation
198	DPVDY	$= \partial(Pv)/\partial x$ "
78	DRDT	$= \partial\rho/\partial T$
180	DTDX	$= \partial T/\partial x$
193	DTDXX	$= \partial^2 T/\partial x^2$
181	DTDY	$= \partial T/\partial y$
194	DTDYY	$= \partial^2 T/\partial y^2$
73	DTIME	$= \Delta\tau$ time step

Line No.	Program Symbol	Definition
197	DTUDX	$= \partial(Tu)/\partial x$ difference of convection term
197	DTVDY	$= \partial(Tv)/\partial x$
32	DTW	$= T_w - T_o$, temperature difference
	DUDX	$= \partial u/\partial x$
	DUDXX	$= \partial^2 u/\partial x^2$
	DUDXY	$= \partial^2 u/\partial x \partial y$
	DUDY	$= \partial u/\partial y$
	DUDYY	$= \partial^2 u/\partial y^2$
	DURDX	$= \partial(\rho u)/\partial x$
	DURDY	$= \partial(\rho u)/\partial y$
	DVDX	$= \partial v/\partial x$
	DVDXX	$= \partial^2 v/\partial x^2$
	DVDXY	$= \partial^2 v/\partial x \partial y$
	DVDY	$= \partial v/\partial y$
	DVDYY	$= \partial^2 v/\partial y^2$
	DVRDX	$= \partial(\rho v)/\partial x$
	DVRDX	$= \partial(\rho v)/\partial y$
	DX	= dx
	DY	= dy
	G	= g, acceleration of gravity
	I	i th grid point in x direction
	ICO	= 0, printout at first time step
74,225	ICOUNT	counter on the number of time-step for printout
232	II	integer of changing of I th order for printout
59	IS1	The lowest grid point of heated surface
59	IS2	The highest grid point of heated surface
225	IWRITE	Controller integer value of printout
	J	j th grid point in y direction
	K	K has 1,2 and 3. K=3: newest one; K=1: old one
	M	M th grid point is correspond to x_{max}
	M1	= M-1
	M2	= M-2
	N	N th grid point corresponds to y_{max}
	N1	= N-1
	N2	= N-2

Line No.	Program Symbol	Definition
	PRDX	=
	PRDY	=
	PRESS	pressure, P
	R	density, ρ
149	RU	= ρu
	RUDX	= $\partial(\rho u)/\partial x$
157	RV	= ρv
	RVDY	= $\partial(\rho v)/\partial y$
	T	temperature
32	TB	static temperature or wall temp. except heated wall
269	TMAX	limitation of computing time
61	TW	= T_w, temperature at heated wall
73	TYME	= $\tau + \Delta\tau$, increment time in computation
199	T1	some term in energy equation
200	T2	"
201	T3	"
202	T4	"
204	T5	"
205	T6	"
206	T7	"
	U	velocity component in x direction
	UDV	is not used, only dimension
	UVMG	is not used, only dimension
143	U1	some term in mementum equation
144	U2	"
145	U3	"
147	U4	"
	V	velocity component in y direction
145	VIDX	= $\partial\mu/\partial x$
147	VIDY	= $\partial\mu/\partial y$
	VISC	= μ, viscosity
151	V1	some term in momentum equation
152	V2	"
153	V3	"
155	V4	"
45	X	= x
30	XMAX	= x_{max}
31	YMAX	= y_{max}

[Subroutine]

Line No.	Program Symbol	Definition
***	PROPER	Subprogram Name [calculation for properties]
	COND	thermal conductivity = k
	CP	specific heat, = C_p
	DPDR	= $\partial P/\partial \rho$
	DPDT	= $\partial P/\partial T$
	DRDT	= $\partial \rho/\partial T$
	M3	= M-3
	N3	= N-3
	PRESS	pressure, = P
	R	density, = ρ
	T	temperature, = T
	VISC	absolute viscosity, = μ
***	FIRSTDF	Subprogram Name [difference for first order]
	DFDX	F is dummy value, = $\partial F/\partial x$
	DFDY	= $\partial F/\partial y$
	DX	mesh size in x direction
	DY	mesh size in y direction
	F	dummy argument value
***	SECONDDF	Subprogram Name [difference for second order]
	DFDXX	= $\partial^2 F/\partial x^2$
	DFDYY	= $\partial^2 F/\partial y^2$
	DXX	= $(\Delta x)^2$
	DYY	= $(\Delta Y)^2$
	QDXX	= $1/4\ (\Delta x)^2$
	QDYY	= $1/4\ (\Delta Y)^2$
***	CROSSDF	Subprogram Name [cross difference]
	DXY	= $\Delta x \cdot \Delta y$
***	CONVDF	Subprogram Name [difference for convection term]
	AUPX	see page 1 of this appendix
	AUPY	"
	AVPX	"
	AVPY	"
	DUFDX	= $\partial(uF)/\partial x$ F is dummy argument
	DVFDY	= $\partial(vF)/\partial y$ "
***	AVEVELO	Subprogram Name [calculation of average velocity]
	AFPX	= AUPX or AVPX
	AFPY	= AUPY or AVPY

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5. AUTHOR(S)

H. Yamashita and V. D. Arp

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Studies of natural convection processes generally assume an incompressible fluid wherein the density is a function of temperature only (the Boussinesq approximation). However, local pressure gradients caused by rapid variations in the heated wall temperature cannot be described within this approximation. These time-varying gradients cause fluid motions which perturb the quasi-static natural convection process. In this study, we describe a numerical analysis procedure which includes compressibility effects and allows computation of transient fluid motions during onset of natural convection. Details of the computational procedure and preliminary results for one geometry are given.

12. KEY WORDS (Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)

compressible fluid motion; convection; finite difference approximation; heat transfer; natural convection; non-linear convection; numerical integration; transient fluid motion; transient heat transfer.

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