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# Seasonal Heat Loss Calculation for Slab-On-Grade Floors

U.S. DEPARTMENT OF COMMERCE
National Bureau of Standards
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Washington, DC 20234

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Abstract

In order to facilitate an efficient slab-on-grade heat transfer calculation on a comprehensive energy analysis program such as DOE-2, BLAST AND NBSLD, heat transfer calculations for slab-on-grade floors are reviewed. The computational procedure based on the Lachenbruch method is studied in depth to generate monthly average temperatures at a given depths below the floor slab. The data generated by the Lachenbruch method are then used to develop a simplified procedure for determining the monthly average earth temperatures below the floor slab. These monthly average temperature data can be used for the hourly response factor analysis of floor-slab heat transfer.

Key Words: building heat transfer; DOE-2 Energy Analysis computer program; monthly average earth temperature; thermal response factors.

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## Nomenclature

Unless otherwise defined separately in the text, the following standard symbols are used throughout this paper.

A = $T_R - T_M$

a = half length of a rectangular slab, ft

b = half width of a rectangular slab, ft

B = amplitude of annual cycle of monthly normal outdoor temperature, °F

c = specific heat, Btu/lbm, °F

C = amplitude of annual indoor temperature cycle, °F

$C_R$ = common ratio of the slab response factors

d = number of days elapsed after April 1

e = edge distance, ft

F = slab heat transfer factor, Btu/h•ft

k = thermal conductivity of soil, Btu/h•ft•F

$\ell$ = thickness of soil, ft

N = number of the slab response factor terms

P = perimeter length of the slab, ft

$\overline{P}$ = period of temperature cycle, hr

Q = local floor heat flux, Btu/h•ft$^2$

q = integrated floor heat loss, Btu/h

R = $\sqrt{(x-a)^2 + (y-b)^2}$, ft

r = $\sqrt{R^2 + Z^2}$, ft

T = temperature of a field point, °F

$T_m$ = mean temperature of the annual cycle of the monthly normal outdoor temperature, °F

$T_R$ = mean temperature of the slab in the building, which is assumed equal to the house temperature, °F

$T_R'$ = slab-soil interface temperature for insulated floor slab, °F

$T_Z$ = mean temperatures of earth at a given depth Z, °F

$T_i$ = the house temperature, °F

$T_o$ = outdoor temperature, °F

$U_F$ = overall thermal conductance of floor slab, Btu/hr•ft²•°F

$U_G$ = overall thermal conductance of soil layer, Btu/hr•ft²•°F

V = $T-T_m$ = temperature difference, °F

$X_j, Y_j$, CR = floor slab thermal response factors, Btu/hr•ft²•°F

$\alpha$ = thermal diffusivity of soil, ft²/h

$\beta = r\sqrt{\frac{\omega}{2\alpha}}$

$\omega$ = angular frequency of the annual cycle $dy^{-1}$

$= \frac{2\pi}{365}$

$\theta$ = time, hr

$\Omega$ = a solid angle subtended by the slab with respect to a field point

$\psi$, $\phi$ = functions defined in the text

$\rho$ = density, lbm/ft³

$\lambda$ = angle subtended by the slab

CONVERSION FACTORS TO METRIC (SI) UNITS

| Physical Quantity | Symbol | To Convert From | To | Multiply By |
|---|---|---|---|---|
| Length | $\ell$ | ft | m | $3.05 \times 10^{-1}$ |
| Area | A | $ft^2$ | $m^2$ | $9.29 \times 10^{-2}$ |
| Volume | $\forall$ | $ft^3$ | $m^3$ | $2.83 \times 10^{-2}$ |
| Temperature | T | Fahrenheit | Celsius | $t_c=(t_f-32)/1.8$ |
| Temp. Diff. | $\Delta T$ | Fahrenheit | Kelvin | $K=(\Delta T_F)/1.8$ |
| Density | $\rho$ | $lbm/ft^3$ | $kg/m^3$ | 16.0 |
| Thermal Conductivity | k | $Btu \cdot in/h \cdot ft^2 \cdot ft^2 \cdot °F$ | $W/m \cdot K$ | $1.442 \times 10^{-1}$ |
| Thermal Transmittance (or Conductance) | U | $Btu/h \cdot ft^2 °F$ | $W/m^2 \cdot K$ | 5.68 |
| Thermal Resistance | R | $h \cdot ft^2 \cdot °F/Btu$ | $m^2 \cdot K/W$ | 0.176 |
| Heat Flux Rate | q/A | $Btu/h \cdot ft^2$ | $W/m^2$ | 3.15 |
| Heat Flow | q | Btu/h | W | $2.93 \times 10^{-1}$ |
| Thermal Diffusivity | $\alpha$ | $ft^2/h$ | $m^2/h$ | $9.29 \times 10^{-2}$ |

# Seasonal Heat Loss Calculation for Slab-on-Grade Floors

T. Kusuda, M. Mizuno,* J. W. Bean

## 1. INTRODUCTION

Although heat transfer calculations for slab-on-grade floors are a major factor in residential energy analysis, an accurate calculation methodology has not been available. This is because of the complex three-dimensional and time-dependent nature of the heat conduction process involved in the earth-contact building heat transfer process. In addition, there are fundamental questions as to appropriate choices of the thermophysical properties of soil and soil temperature, both of which are by nature not well defined as far as the soil around the building is concerned. This is because moisture condition in the soil is constantly changing and affecting the temperature profile as well as the thermal property values. Exact analysis dealing with this simultaneous transfer of heat and moisture is beyond the scope of this paper.

The 1977 ASHRAE Handbook procedure [1] for the slab-on-grade floor heat transfer calculation is simply to use a set of numbers representing the perimeter heat loss coefficient $F_2$ as follows:

$$q = F_2 \cdot P \cdot (T_i - T_o)$$

where

$q$ = floor heat loss

$P$ = perimeter of exposed edges

$T_i$ = inside air temperature

$T_o$ = outside air temperature

Data for $F_2$ are available for the floor with no edge insulation, with 1-inch edge insulation, and 2-inch edge insulation, which are 0.81, 0.55, and 0.50 respectively. These data are derived from the old experimental measurements conducted by the National Bureau of Standards [2] and, separately, by the University of Illinois [3]. It is difficult for building heat transfer analysts to apply these ASHRAE data for general purpose evaluation of a variety of slab-on-grade heat losses with respect to sizes, shapes, and physical properties of soils. Moreover, they are valid only under a steady set of test conditions for specific floor types and are not suitable for hourly or seasonal heat transfer calculations.

A conventional approach for the heat transfer analysis of the slab-on-grade problem is to use numerical calculation techniques such as the finite difference and/or finite element solution of the heat conduction equation. Although

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extremely powerful and in many cases the only recourse available for the real-world problem, the three dimensional finite difference solution requires a large number of grid points and lengthy computer time because the heat conduction domain influenced by slab-on-grade structure is extremely large. Moreover, the time span in the order of several years is required before steady annual cycle of ground temperature is achieved. Akasaka [4], for example, used a two-dimensional region of 3 m x 6 m represented by a finite difference grid of 30 x 60, which required more than 3,000 iterations to obtain a near-steady-state solution. It is doubtful that general design data suitable for three-dimensional heat transfer analysis can be obtained by numerical calculations unless one has access to a large memory, high speed and low cost computing services.

Several authors have, in the past, attempted to develop theoretical bases for estimation of heat loss from slab-on-grade floors, most notable among them being a pioneering work of A. H. Lachenbruch, who used Green's Function [5]. Because of the complexity of the mathematical formulation, Lachenbruch developed an elaborate graphical procedure and demonstrated the procedure for a rectangular slab of 30 ft x 100 ft. The most striking finding of Lachenbruch's calculation was that it takes more than three years before the temperature beneath the house experiences a steady periodic annual cycle. B. Adamson [6] used the Lachenbruch procedure for a 10 m x 10 m slab and generated the steady periodic annual temperature profiles under the slab-on-grade floors as well as the heat flux along the slab surface.

Adamson's calculations show that the heat flow paths are practically semi-circular, with the center of the circle being at the perimeter edge of the slab during the winter season, and they are practically parallel and normal to the slab surface during the summer. Adamson's results also show that the heat flux is practically constant from edge to edge during the summer, and that an extremely large edge heat loss occurs during the winter.

Admittance and transfer parameters based on the frequency response (Fourier series) solutions were developed by Muncey and Spencer [7]. They developed a method for determining steady-state thermal resistance values of slabs-on-grade of many different shapes, details of which are given in Appendix A. Peavy [8] also analyzed a two-dimensional slab-on-grade problem.

In this report, the Lachenbruch procedure will be extended for generating the monthly average earth temperature beneath the floor slab, which is used in energy analysis simulation programs such as NBSLD, DOE-2 and/or BLAST [8]. The graphic solution method developed by Lachenbruch is converted into an efficient digital computer simulation procedure. The computer program is then used to determine average soil temperatures at a given depth below the floor slab. This average earth temperature below the slab, which is considered constant at least during a month for the hour-by-hour floor heat transfer calculation, has been used in the response factor calculation [9] of the building energy analysis as follows.

$$Q_t = \sum_{j=1}^{N} X_j T_{R,t-j} - \sum_{j=1}^{N} Y_j T'_{R,t-j} + CR \cdot Q_{t-1}$$

where $Q_t$ = hourly floor heat loss at time t

$Q_{t-1}$ = hourly floor heat loss at time t-1

$X_j$ and $Y_j$ are thermal response factors predetermined for the floor slab composite and include the thermal resistance at the floor surface

$T_{R,t-j}$ = room temperature above the floor slab at time t-j

$T'_{R,t-j}$ = slab-soil interface temperatures at time t-j

CR = common ratio of the floor slab thermal response factors [9]

The value of $T'_{R,t-j}$ usually remains unchanged (thus depicted as $T'_R$ hereafter) for the hour by hour calculation but would vary depending on the season and is dependent upon other parameters such as floor shape, thermal properties and soil.

The rationale of this approach is to interface the hourly simulation of the building floor heat transfer process with the slowly changing earth temperature surrounding the building. It has been a difficult problem to assign an appropriate slab-floor ground temperature $T'_R$ for the hour-by-hour slab-floor heat transfer calculation. Various approximations such as an arithmetic average of indoor temperature and well-water temperature, ten-foot-depth average earth temperature, or a monthly average outdoor temperature, have been used in the past simply because of the lack of more appropriate data. Major objective of the paper is then to provide a means to determine improved data for $T'_R$.

## 2. LACHENBRUCH SOLUTION

Figure 1 shows the mathematical system of a heated slab (presented by S) on the ground surface, while the equation below is the basic heat conduction equation and its boundary conditions.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial \theta} \qquad (1)$$

at $z = 0$, $T = T_R$ on S

$$T = T_m + B\sin(\frac{2\pi\theta}{\bar{P}}) \text{ outside S}$$

$z \to \infty \quad T = T_m$

where

$T$ = earth temperature

$\alpha$ = thermal diffusivity of earth

$\theta$ = time

$T_R$ = room temperature

$T_m$ = annual average temperature of earth

$B$ = amplitude of annual cycle of monthly average surface temperatures

$\bar{P}$ = period of annual temperature cycle

A general solution for equation (1) may be expressed by the Green's Function form as follows:

$$T(x,y,z,\theta) = \frac{z}{8(\pi\alpha)^{3/2}} \int_0^\theta \left[\int_S\int \frac{\phi(x',y',t)e^{\frac{-r^2}{4\alpha(\theta-t)}}}{(\theta-t)^{5/2}} dx',dy'\right]dt \qquad (2)$$

where $\phi(x',y',t)$ = temperature distribution over S

$$r = [(x-x')^2 + (y-y')^2 + z^2]^{1/2}$$

In equation (2), the temperature of the soil under a slab S is represented by a triple integral with respect to time t and the surface temperature profile $\phi(x',y',t)$ of the slab S. Applying this general solution to a specific boundary condition indicated in figure, 1, Lachenbruch obtained the soil temperature solution in the form of equation (3).

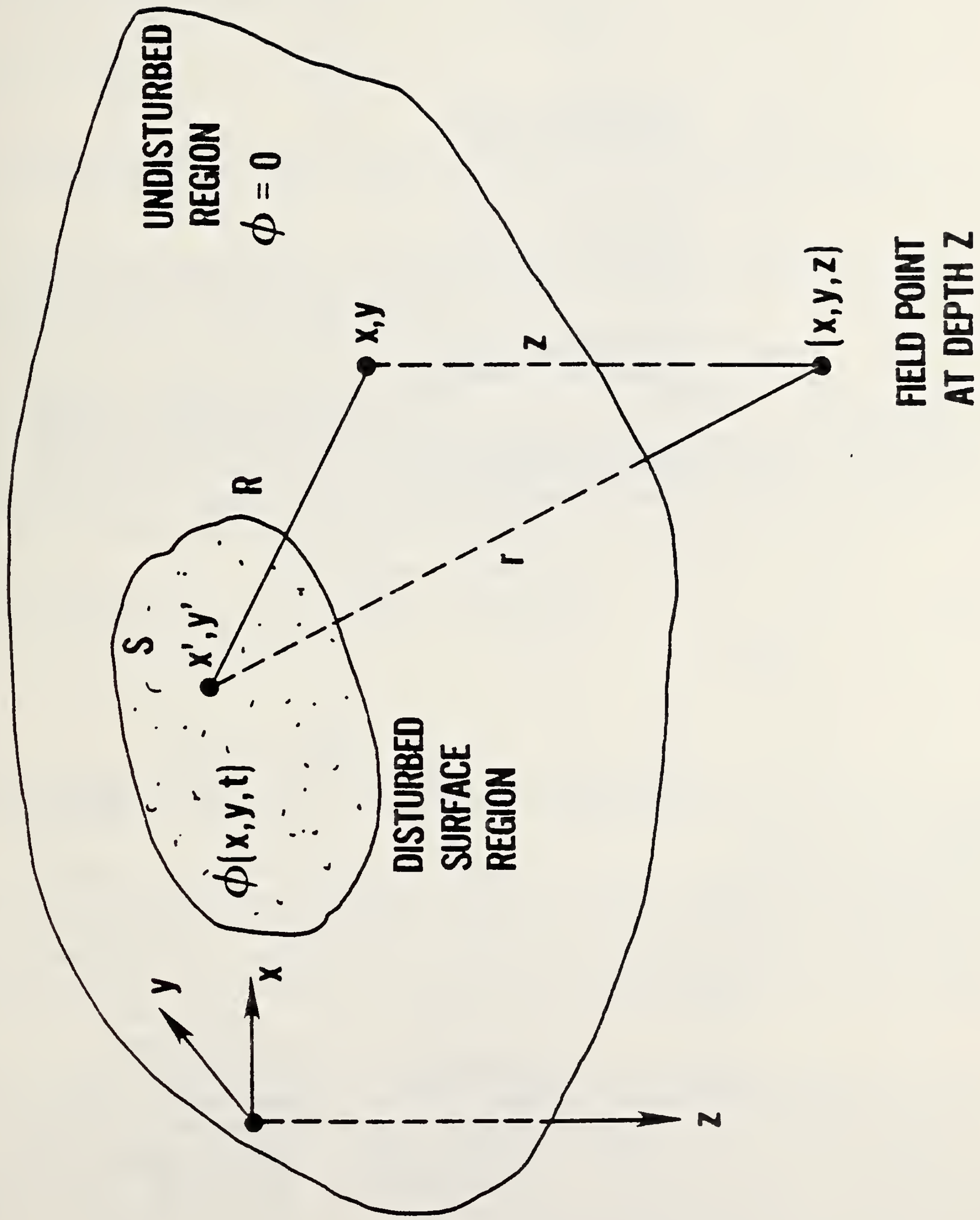



Figure 1. Boundary condition for heat conduction equation used by Lachenbruch

$$T = T_m + \left[\frac{T_R - T_m}{2\pi}\right] \Omega + Be^{-z\sqrt{\frac{\omega}{2\alpha}}} \sin\left(\omega\theta - z\sqrt{\frac{\omega}{2\alpha}}\right) + \frac{B}{2\pi} \int_\Omega \int \psi(\beta, \theta) d\Omega \tag{3}$$

where

$$\Omega = \int_S \int d\Gamma = \int_S \int \frac{z dx' \, dy'}{r^3} \quad : \text{ solid angle subtended by S}$$

$$= \tan^{-1} \frac{(x+a)(y+b)}{z\sqrt{(x+a)^2+(y+b)^2+z^2}}$$

$$-\tan^{-1} \frac{(x-a)(y+b)}{z\sqrt{(x-a)^2+(y+b)^2+z^2}}$$

$$-\tan^{-1} \frac{(x+a)(y-b)}{z\sqrt{(x+a)^2+(y-b)^2+z^2}}$$

$$+\tan^{-1} \frac{(x-a)(y-b)}{z\sqrt{(x-a)^2+(y-b)^2+z^2}}$$

$$\psi(\beta, \theta) = e^{-\beta}\{(1+\beta)\sin(\omega\theta-\beta) + \beta\cos(\omega\theta-\beta)\}$$

$$\beta = r\sqrt{\frac{\omega}{2\alpha}}$$

$$\omega = \frac{2\pi}{P}$$

Differentiating equation (3) with respect to z, the surface heat flux may be obtained as follows:

$$Q\,(x,y,o,\theta) = -\,k\left(\frac{dT}{dz}\right)_{z=0}$$

$$= k\sqrt{\frac{\varpi}{2\alpha}}\,B(\sin\,\omega\theta + \cos\,\omega\theta)$$

$$+ \frac{k}{2\pi}(T_R - T_m)\left\{\frac{\sqrt{(x+a)^2+(y+b)^2}}{(x+a)(y+b)} - \frac{\sqrt{(x-a)^2+(y+b)^2}}{(x-a)(y+b)} - \frac{\sqrt{(x+a)^2+(y-b)^2}}{(x+a)(y-b)} + \frac{\sqrt{(x-a)^2+(y-b)^2}}{(x-a)(y-b)}\right\}$$

$$- \frac{k}{2\pi}^{B} \int_S \int \frac{\psi(\beta_o, \theta) dS}{R_o^3} \tag{4}$$

where $\beta_o = R_o\sqrt{\frac{\omega}{2\alpha}}$

$R_o = \sqrt{(x-a)^2 + (y-b)^2}$

$ds = dx'dy'$

$$q = \frac{1}{S}\int\int_s Q dx'dy'$$

Since the major purpose of this paper is to determine the average earth temperature below the floor slab, only the evaluation of equation (3) will be discussed.

In order to evaluate equation (3), however, it is necessary to perform the integration of $\psi(\beta,\theta)$ over an entire solid angle $\Omega$ subtended by the surface slab with respect to a given field point (x, y, z).

Figures 2-1 and 2-2 show a scheme to evaluate this integral by a superposition of annular segments emanating from a point (x,y) in the ground. The scheme was originally developed by Lachenbruch on the basis that analytical integration of $\psi(\beta, \theta)$ is available for an annular region between radii $R_i$ and $R_{i-1}$ emanating from a point x, y, z. Equation (5) shows a general form of the integration using the superposition of segments of annular region solutions.

$$\frac{B}{2\pi}\int\int_{\Omega} \psi(\beta,\theta)d\Omega = \frac{B}{2\pi}\sum_{i=1}^{\infty} \lambda_i\{\phi(R_i) - \phi(R_{i-1})\} \qquad (5)$$

where

$$\phi(R) = \frac{z}{\sqrt{z^2+R^2}}\, e^{-\sqrt{z^2 + R^2}\sqrt{\frac{\omega}{2\alpha}}} \sin(\omega\theta - \sqrt{z^2+R^2}\sqrt{\frac{\omega}{2\alpha}}).$$

Although it requires a relatively large number of annular spaces of fine width over the slab, this scheme is extremely efficient for covering a large area outside the building where the angular effect ($\lambda$ = central angle substantiated by the slab boundary with respect to a projected point in question) vanishes, as the annular region is extended beyond the slab. The value $\phi(R)$ also vanishes quickly as R becomes large.

An efficient computer program has been developed to determine monthly earth temperature profile under heated slabs of different shapes and different temperatures. Figures 3-1 through 3-4 show results of sample calculations where depth isotherms across the floor center line are indicated in terms of ratio of $T - T_m$ with respect to $T_R - T_m$ of equation (3). In these figures, the amplitude of the monthly average outdoor air temperature B are chosen to be 1.0 and 1.5 of $(T_R-T_m)$. The subject floor is 20 ft square and thermal diffusivity of soil is also varied to cover from 0.027 and 0.039 $ft^2/hr$, which represent average and wet soil conditions respectively.

It is interesting to note that:

1. Earth temperature outside the region directly below the floor is also affected as far as one house width.

2. A large temperature gradient exists near the floor edge during the winter months.

3. Summer heat transfer is practically one-dimensional and normal to the floor slab without having much of the edge heat flow phenomena of winter.

4. Earth temperature disturbance extends beyond 30 ft depth.

Figure 4 is the annual average earth temperature beneath the slab, which represents the second or the steady-state heat conduction term of equation (3).

The zero temperature region indicated in figure 4 is the annual average temperature of undisturbed earth, which is numerically equal to $T_m$. Figures 5-1 and 5-2 show the monthly earth temperatures with respect to depth at selected distances away from the building for wet and average soil conditions and for the temperature amplitudes of 1.0 and 1.5 $(T_R - T_m)$. These figures show that the annual cycle of the earth temperatures is not significantly affected even at a point as close as 5 ft away from the building.

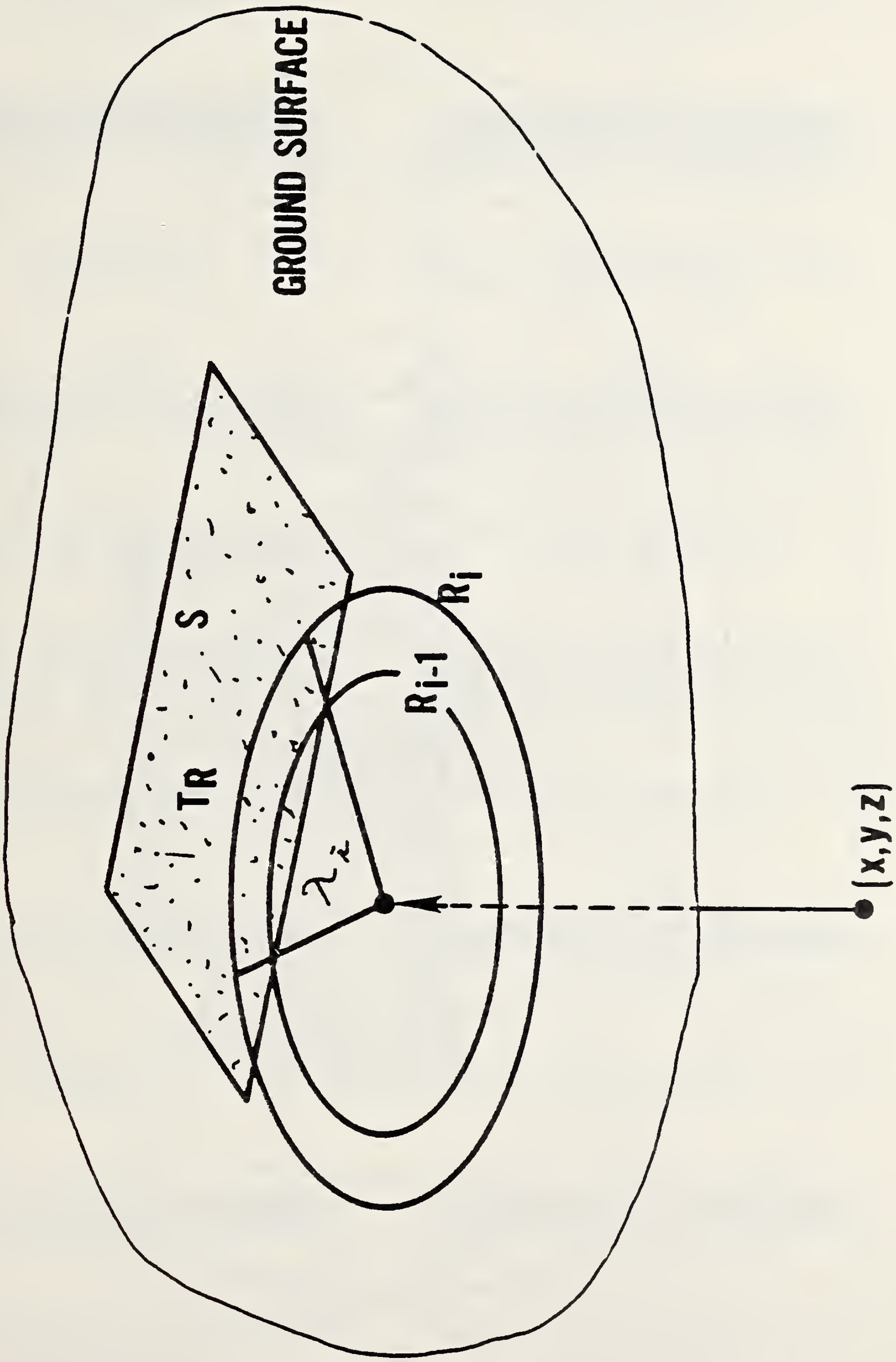



Figure 2-1. Calculation algorithms for slab-on grade problem based on the superposition of a solution for annular segments

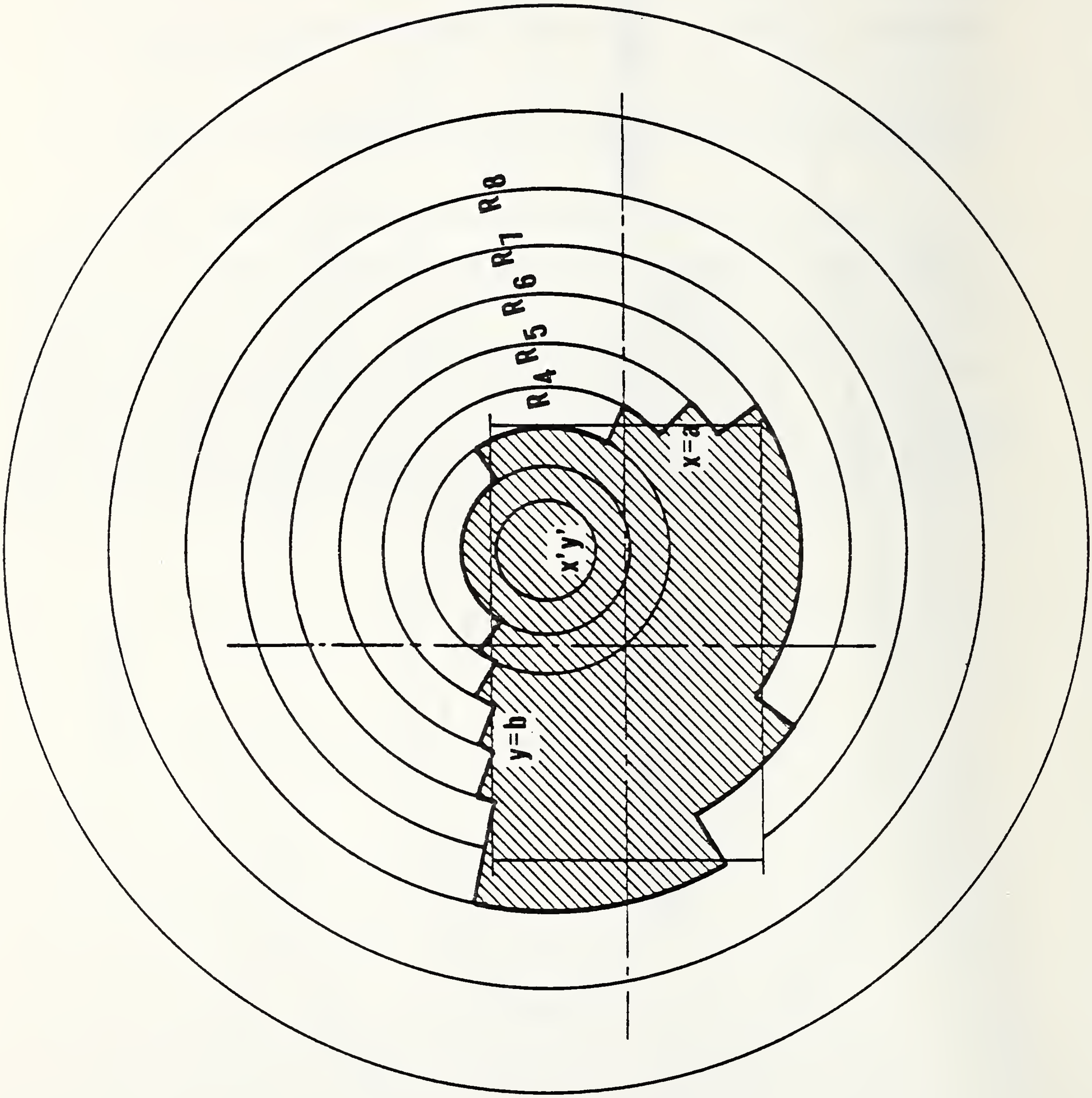



Figure 2-2. [illegible]mation of a rectangular region by annular segments

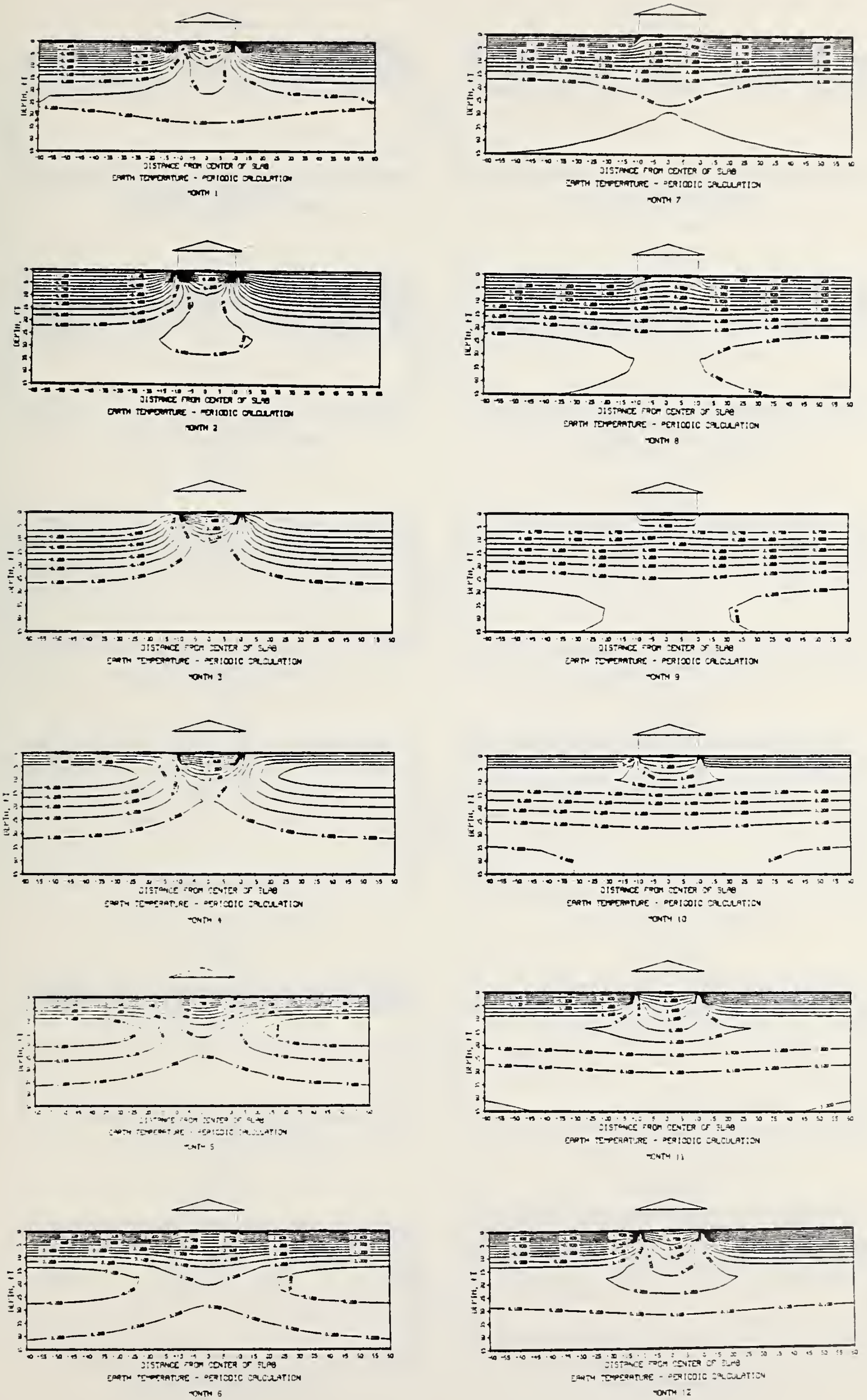



Figure 3-1. Annual earth temperature expressed in $(T-T_R)/A$ beneath slab-on-grade floor; $\alpha$ = (.93 $ft^2$/day) and b/a = 1.5

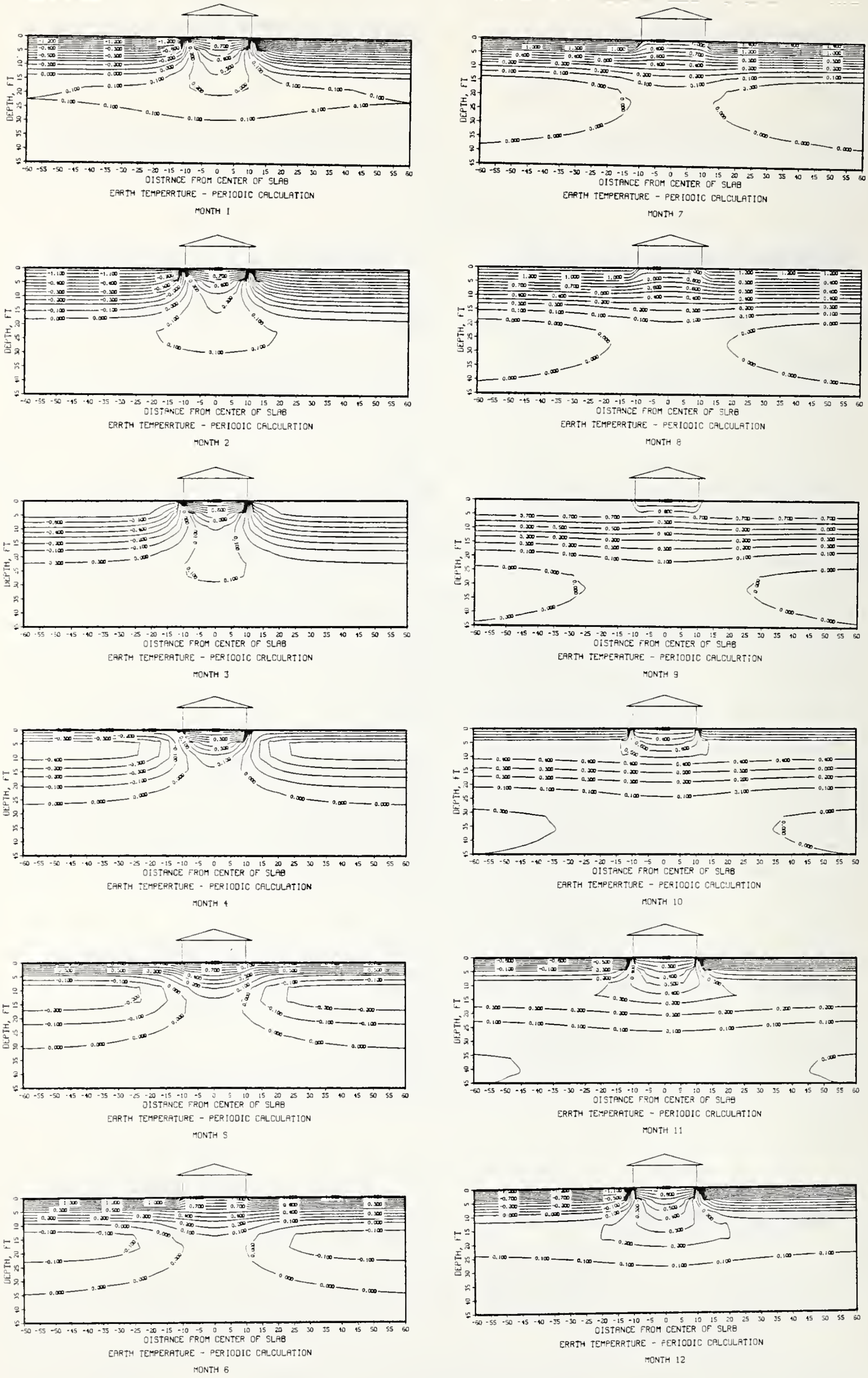



Figure 3-2. Annual earth temperature expressed in $(T-T_R)/A$ beneath slab-on-grade floor: $\alpha$ = (.65 $ft^2$/day), b/a = 1.5

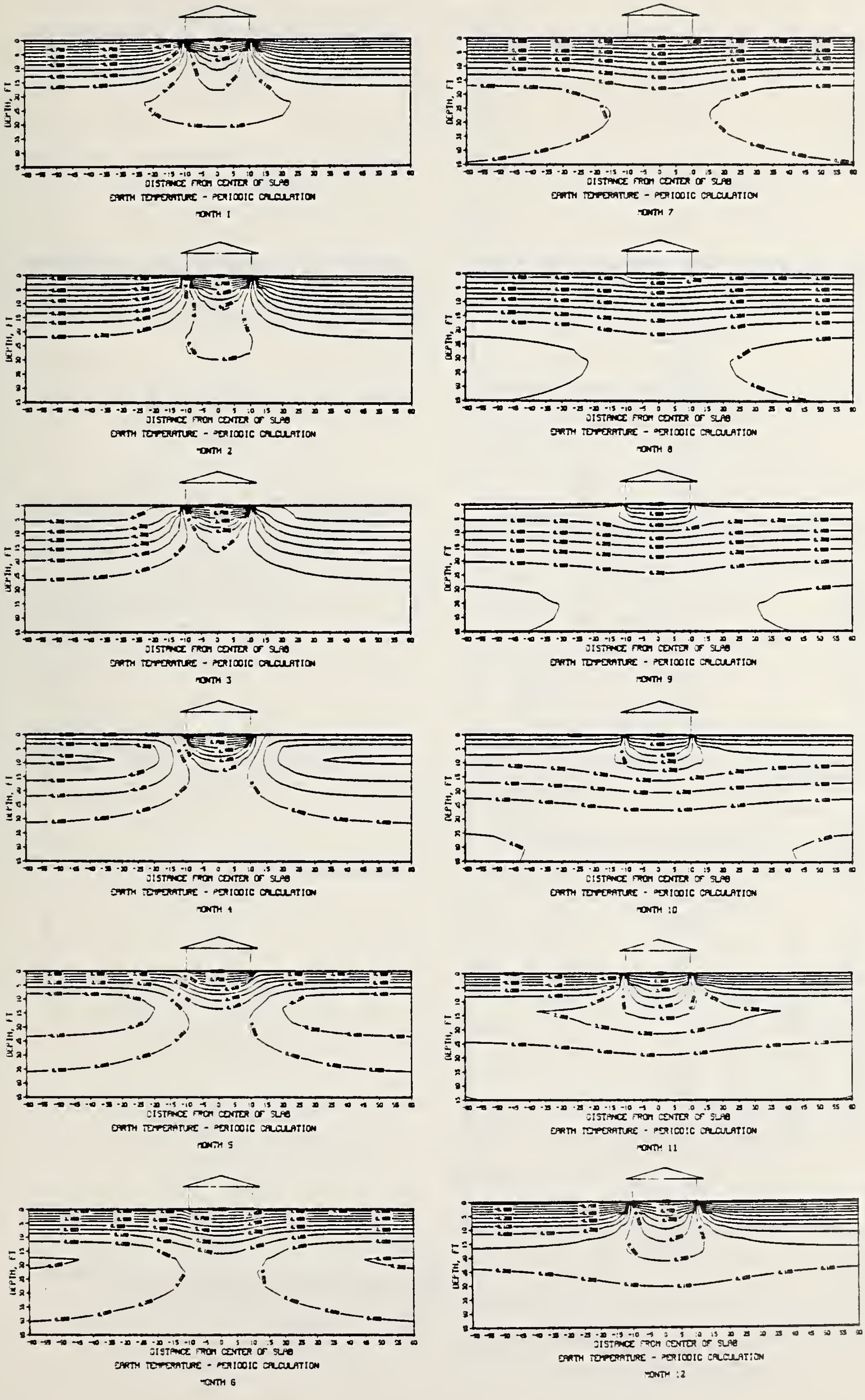



Figure 3-3. Annual earth temperature expressed in $(T-T_R)/A$ beneath slab-on-grade floor: $\alpha$ = (.93 $ft^2$/day), b/a = 1.0

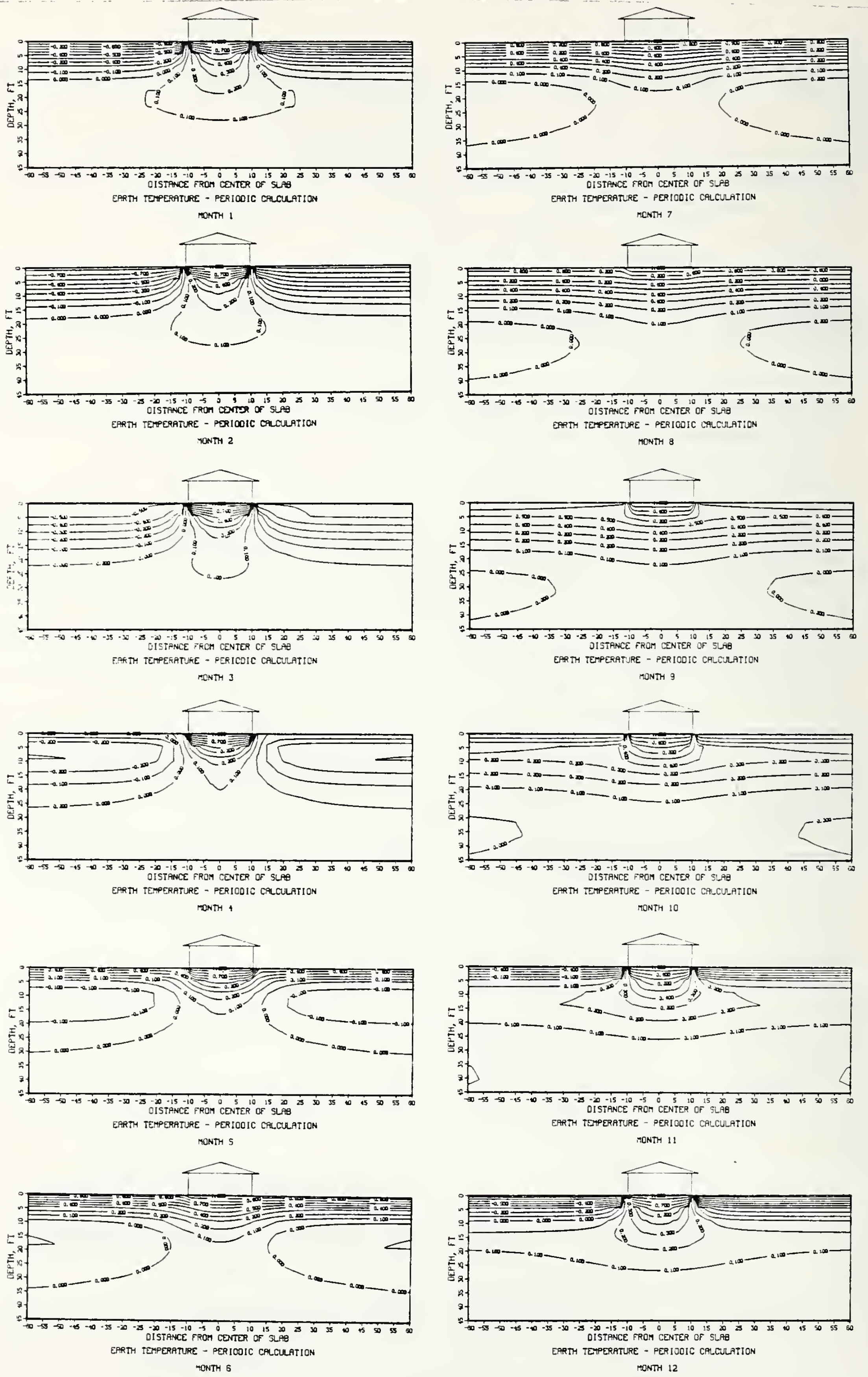



Figure 3-4. Annual earth temperature expressed in $(T-T_R)/A$ beneath slab-on-grade floor: $\alpha$ = (.65 $ft^2$/day), b/a = 1.0

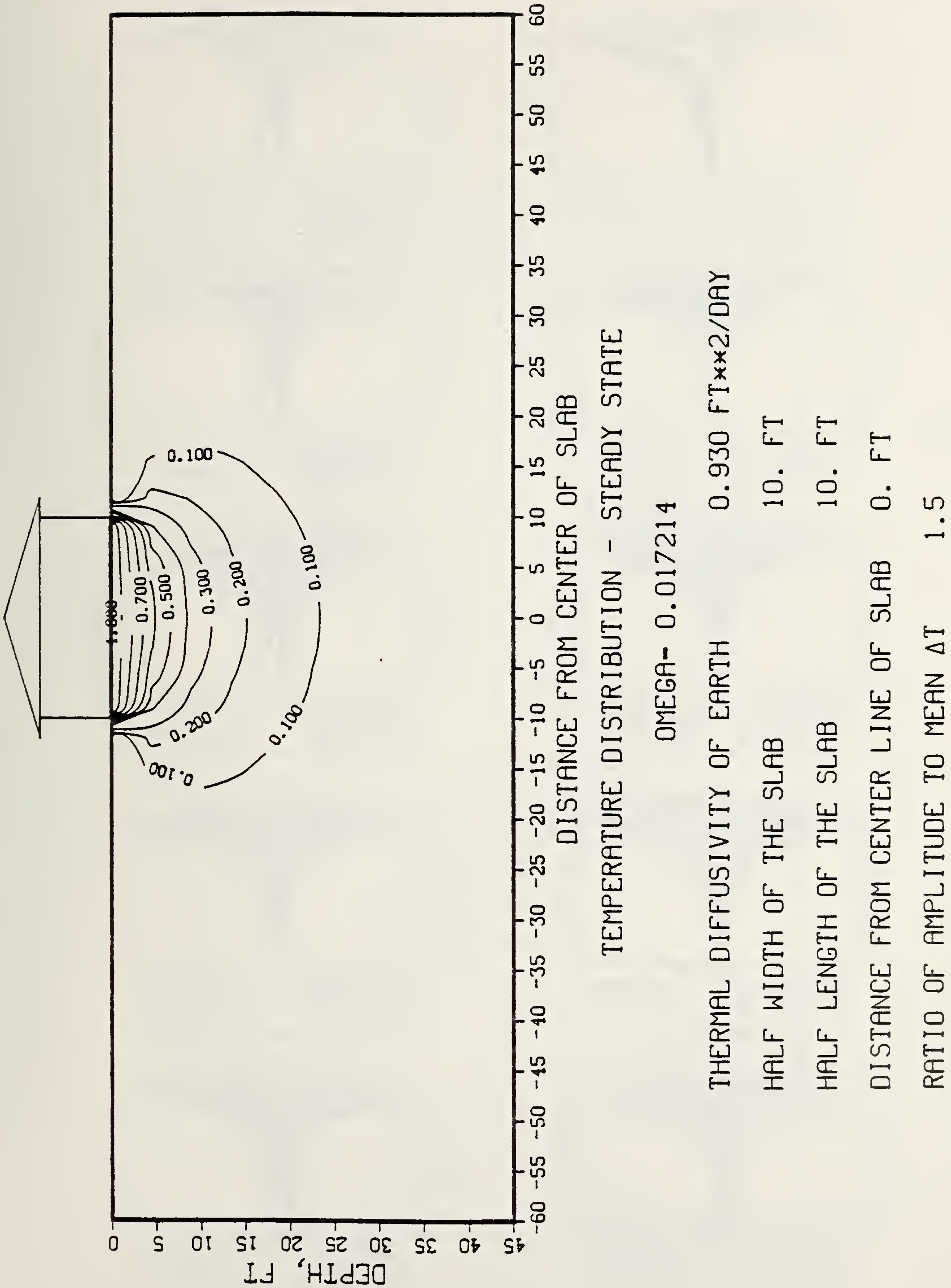



Figure 4-1. Annual average earth temperature expressed in $(T-T_R)/T_R-T_M)$ beneath the slab-on-grade floor

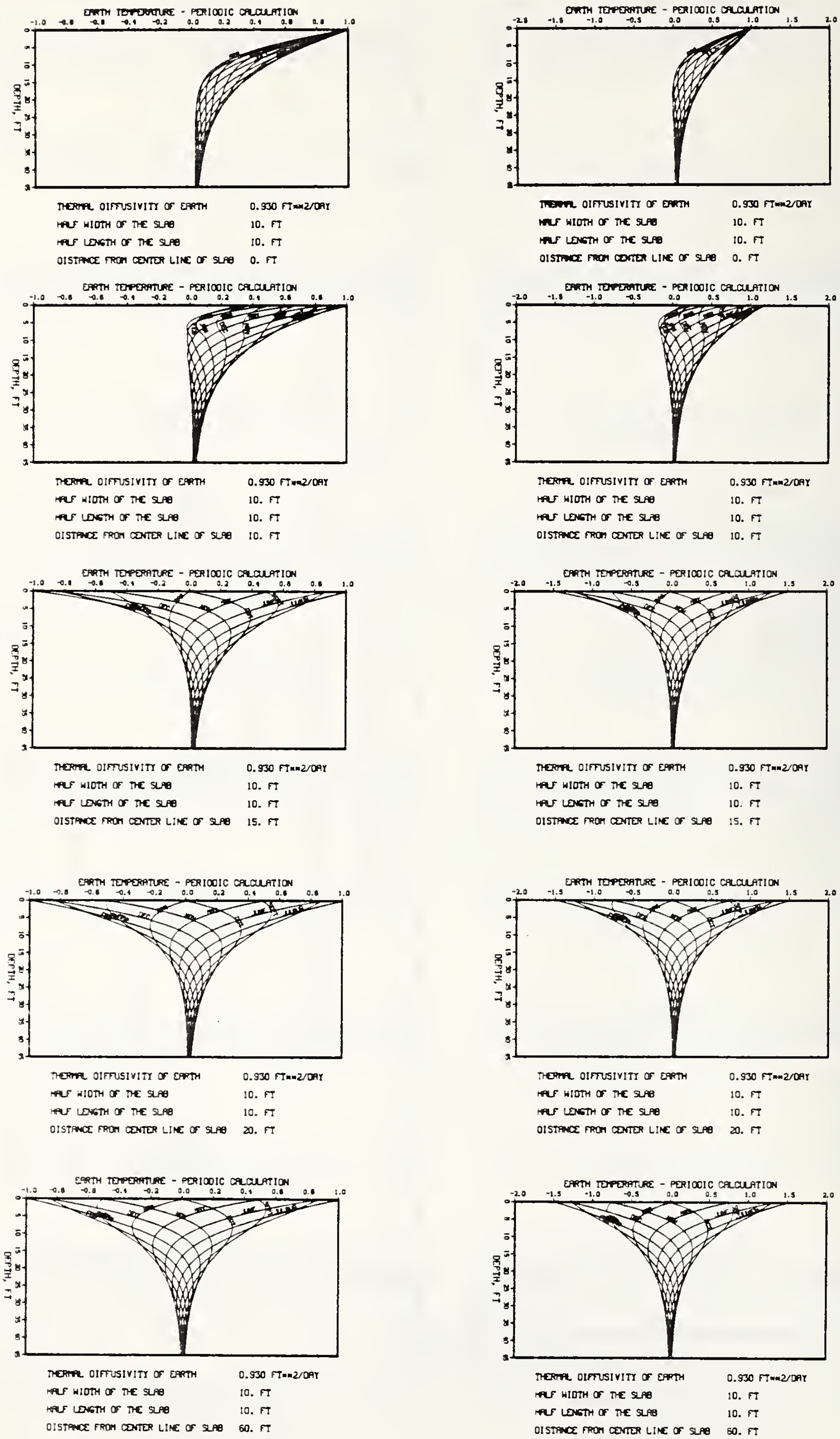



Figure 5-1. Earth temperature-depth profiles at selected distances away from the slab-on-grade floor: $\alpha$ = (.93 $ft^2$/day)

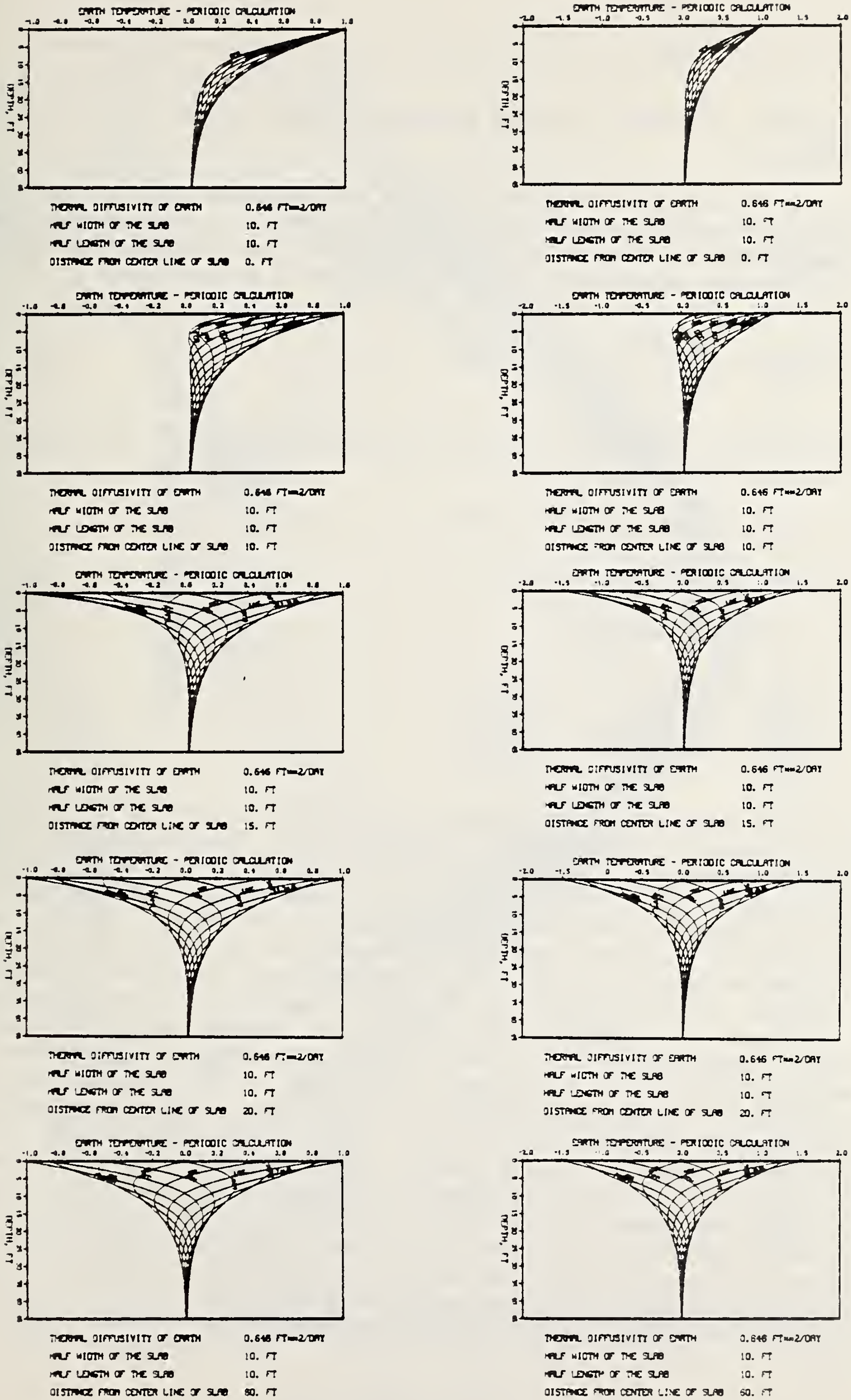



Figure 5-2. Earth temperature-depth profiles at selected distances away from the slab-on-grade floor for $\alpha$ = (.65 $ft^2$/day)

## 3. DETERMINATION OF MONTHLY AND AREA AVERAGE TEMPERATURE FOR BUILDING ENERGY ANALYSIS

In order to use the solution thus obtained in the previous sections for the floor-slab heat transfer calculation, the monthly and area average earth temperature z ft below the floor slab is needed. The data are obtained by integrating the earth temperature beneath the slab area in such a manner that

$$\bar{T}_{(z)} = \frac{1}{ab} \int_0^a \int_0^b T(x,y,z)dxdy \qquad (6)$$

The data for $\bar{T}(z)$ are obtained for several selected values for $\alpha$, $T_R-T_m$, B, a and b, such as follows.

$\alpha$ = 0.015, 0.025 and 0.035 $ft^2/h$

$T_R-T_m$ = -10, 7.5, 15, 20 and 25°F

B = 3, 10, 20, and 30°F

a x b =

| | | |
|---|---|---|
| 10x10 | 10x6.67 | 10x5 |
| 15x15 | 15x10 | 15x7.5 |
| 20x20 | 20x13.33 | 20x10 |
| 30x30 | 30x20 | 30x15 |

Figure 6-1, 2, 3 shows the sample results of such calculations for $T_R - T_m$ = 20°F, and B = 30, 20 and 10°F, respectively, for z = 1 ft. They represents the area average earth temperature 1 ft below the floor slab of 20 ft x 40 ft, 30 ft x 40 ft and 40 ft x 40 ft. These figures show a surprisingly small effect of soil thermal diffusivity upon the sub-slab temperature for all seasons, and show a larger effect of floor aspect ratio during the winter. The use of these figures for the estimation of floor heat loss, however, requires the knowledge of annual average soil temperature $T_m$ (which is very close to the deep underground or the well water temperature), and annual maximum and minimum monthly normal outdoor temperatures, all of which are readily available from the U.S. Weather Record Center [10]. For example, according to [9], the annual average temperature, annual maximum and minimum for the monthly normal (30-year average) data for Washington, D.C. are 56.8°F, 77.8°F and 36.5°F, respectively. From this, one can arrive at $T_m$ = 56.5°F, and B = (77.8-36.5)/2 = 20.65°F or say 20°F. Using figure 6-2, one can then estimate the seasonal floor slab heat loss for the slab having temperature of 56.5 + 20 = 76.5°F, provided that the thermal properties of the slab are very similar to those of the soil beneath the floor. If one assumes that the soil has the thermal diffusivity of 0.025 $ft^2/h$, thermal conductivity 0.5 Btu/h•ft•F, and the floor-slab is 30 ft x 40 ft (aspect ratio 0.75), the following floor heat loss calculation can be made from figure 6-2.

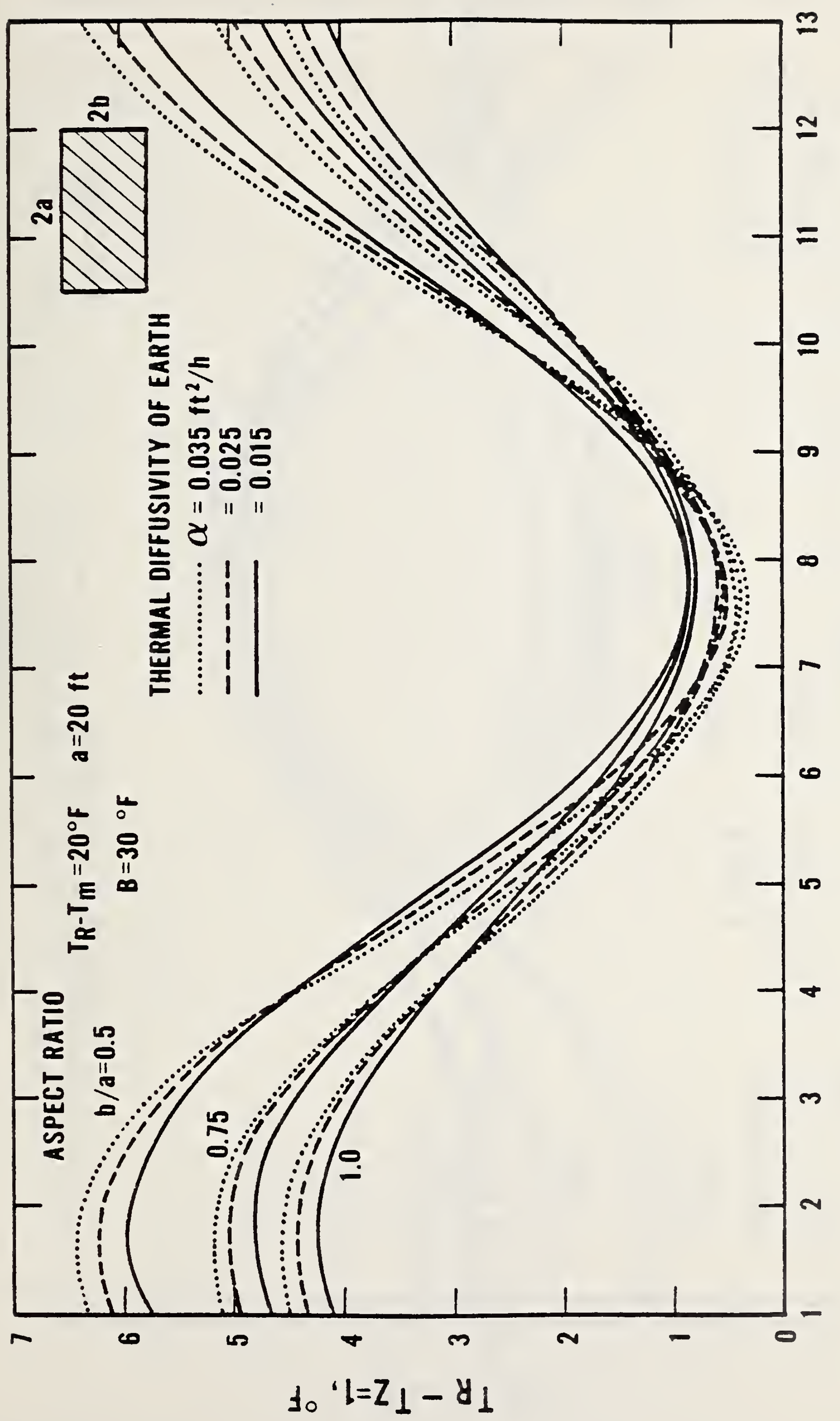



Figure 6-1. Average earth temperature 1 ft below the floor slab for B=30

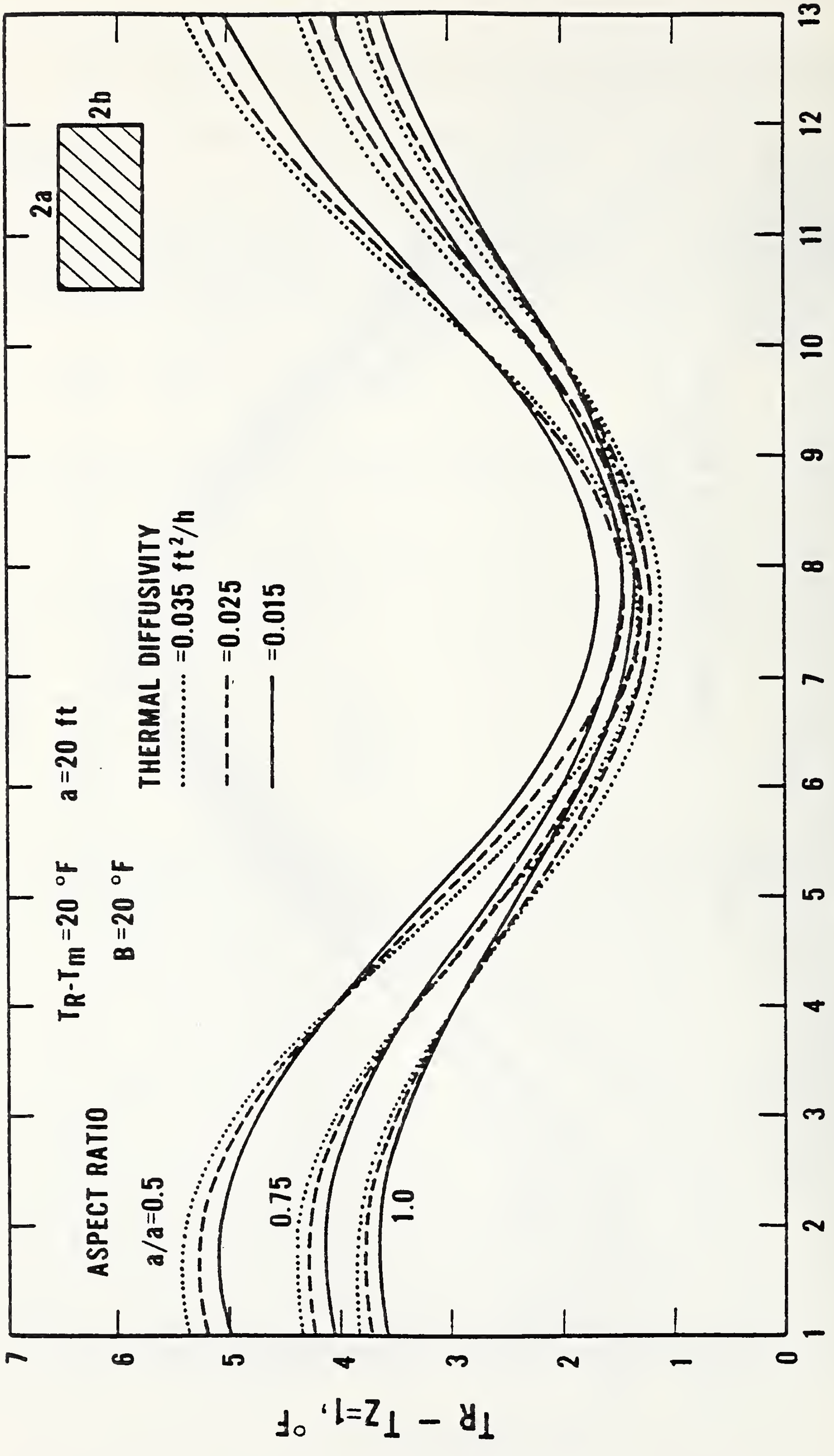



Figure 6-2. Average earth temperature 1 ft below the floor slab for B=20

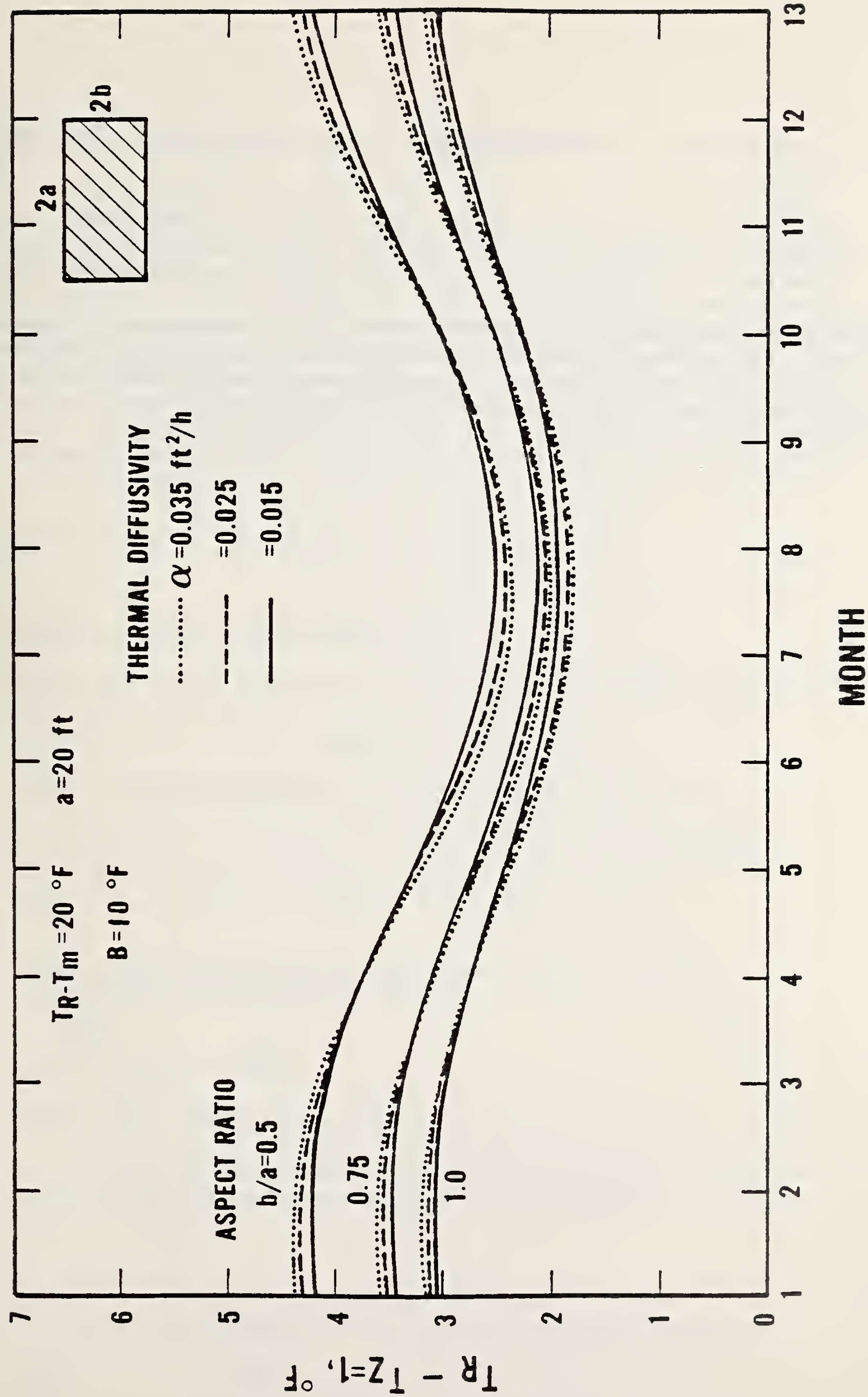



Figure 6-3. Average earth temperature 1 ft below the floor slab for B=10

| Month | $T_R - T_{z=1}$ | $Q = 0.5(T_R - T_z)(1200)$ |
|---|---|---|
| January | 4.3 | 2580 Btu/h |
| April | 3.4 | 2040 Btu/h |
| July | 1.3 | 780 Btu/h |
| October | 2.3 | 1380 Btu/h |

When the slab temperature is different from 76.5°F, because of different house temperature and/or floor insulation, the procedure becomes more complicated. The following section describes a general procedure applicable to floor slabs having different thermal properties than the soil beneath them.

## 4. SIMPLIFIED PROCEDURE FOR ESTIMATING SEASONAL HEAT LOSS FROM THE SLAB-ON-GRADE FLOOR

The seasonal heat loss calculation procedure developed herein for the slab-on-grade floor is based on the seasonal and area average soil temperature beneath the floor, room temperature, and floor system thermal conductance (inclusive of the soil layer beneath the slab). The sub-floor soil temperature data are derived from the extensive Lachenbruch-type calculations discussed in the previous sections for various combinations of floor aspect ratios, annual cycles of monthly normal temperature, and soil thermal diffusivity. Also included in the analysis is the annual cyclic variation of indoor temperature that simulates the different thermostat settings encountered during summer and winter. In the procedure derived herein, the average sub-floor temperature $\bar{T}_z$ at a depth of z is expressed in a fashion similar to equation (3), by

$$\bar{T}_z = T_m + (T_R - T_m)\,\theta'_{1m} + B\,\theta'_{2m} + (C - B)\,\theta'_{3m} \qquad (7)$$

where

$T_m$ = annual average outdoor temperature

B = amplitude of the annual cycle of the monthly normal temperature

C = amplitude of the house temperature cycle

$\theta'_{1m}$ = annual average temperature rise function found in figure 7

$\theta'_{1m}$ may also be approximated by the following empirical relationship

$$\theta'_{1m} = e^{-\left(\frac{z}{a}\right)^{0.8}} \cdot \left\{3.312 - 3.324\left(\frac{b}{a}\right) + 1.476\left(\frac{b}{a}\right)^2\right\}$$

$\theta'_{2m}$ = annual cycle of undisturbed earth temperature, which is determined by the following equation

$$\theta'_{2m} = e^{-z\sqrt{\frac{\omega}{2\alpha}}} \sin\left(\omega t - z\sqrt{\frac{\omega}{2\alpha}}\right) \qquad (8)$$

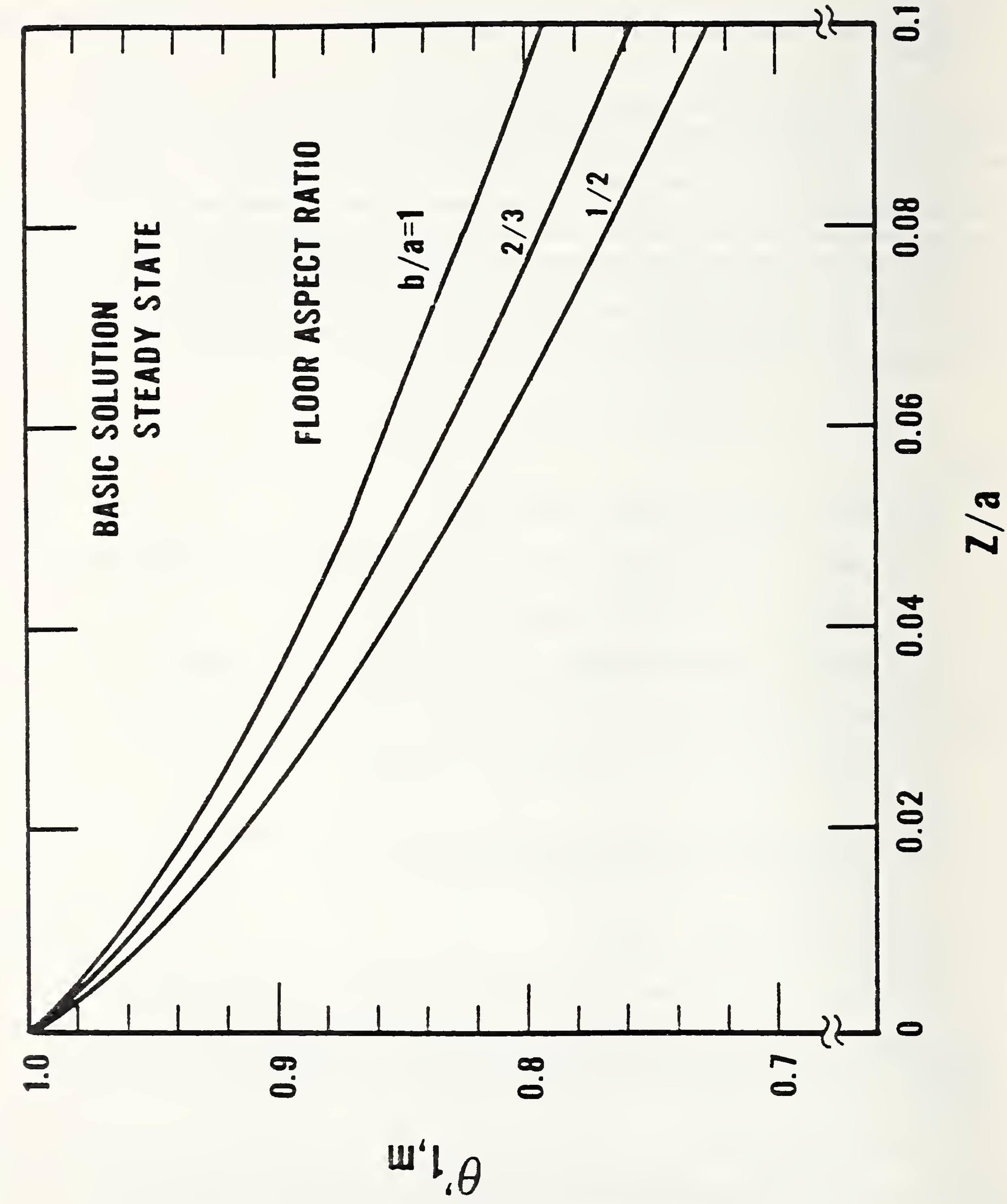
$\theta'_{3m}$ = annual cyclic temperature effect upon the sub-slab temperature.

The value for $\theta'_{3m}$ will be determined by the following equation using two factors, $\beta$ and $\psi$, which are shown in figures 8-1, 8-2, and 8-3, for three different aspects ratios

$$\theta'_{3m} = \beta \sin(\omega t - \psi) \qquad (9)$$

$\beta$ and $\psi$ also can be approximated by the following empirical equations

$$\beta = e^{-\left(\frac{z}{a}\right)^m} \left\{A_1 + B_1\left(\frac{a^2 w}{\alpha}\right)^n\right\}$$




-1. Average temperature function under steady-state conditions

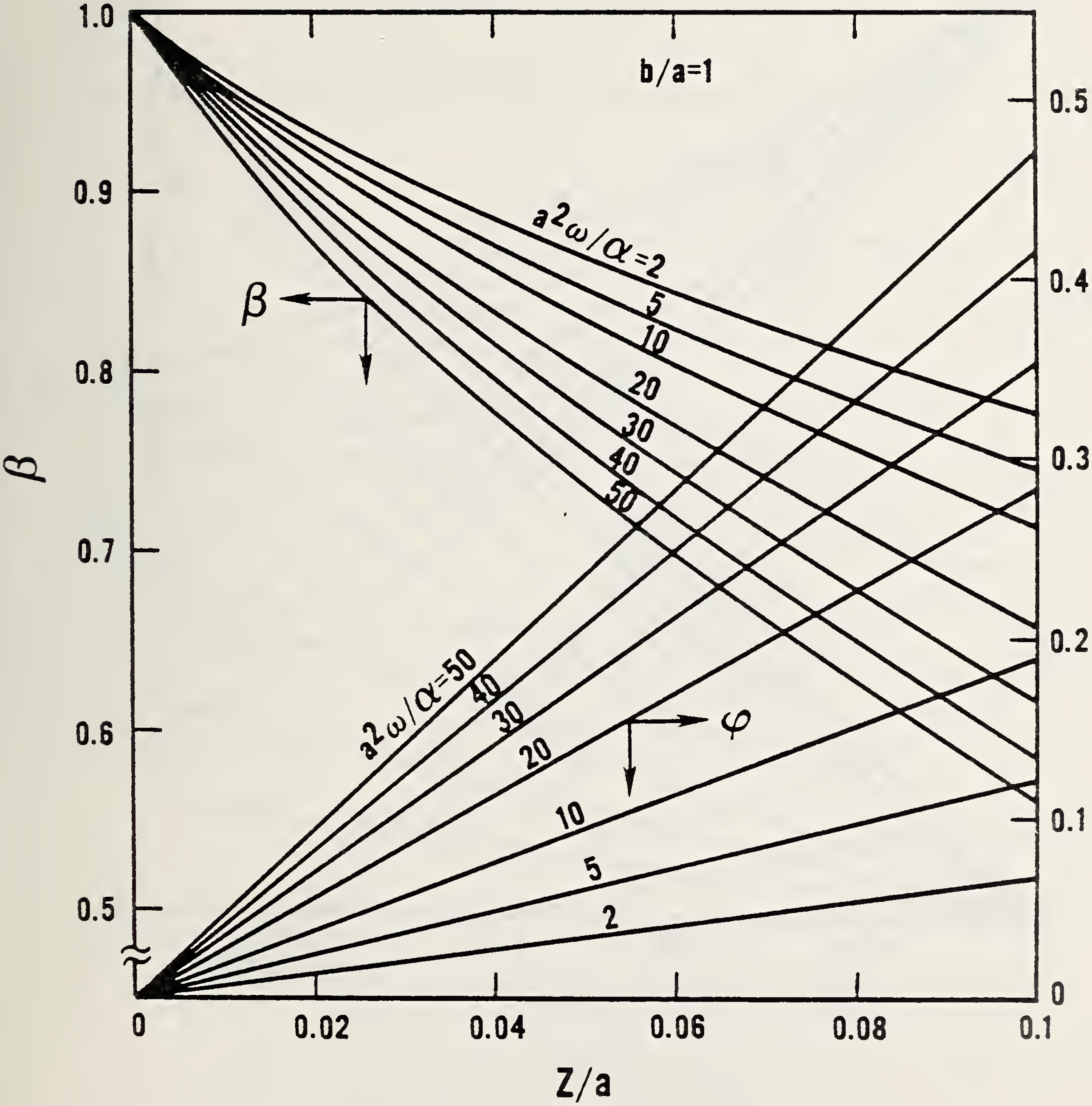



Figure 8-1. Average sub-grade temperature coefficients for the floor aspect ratio 1:1

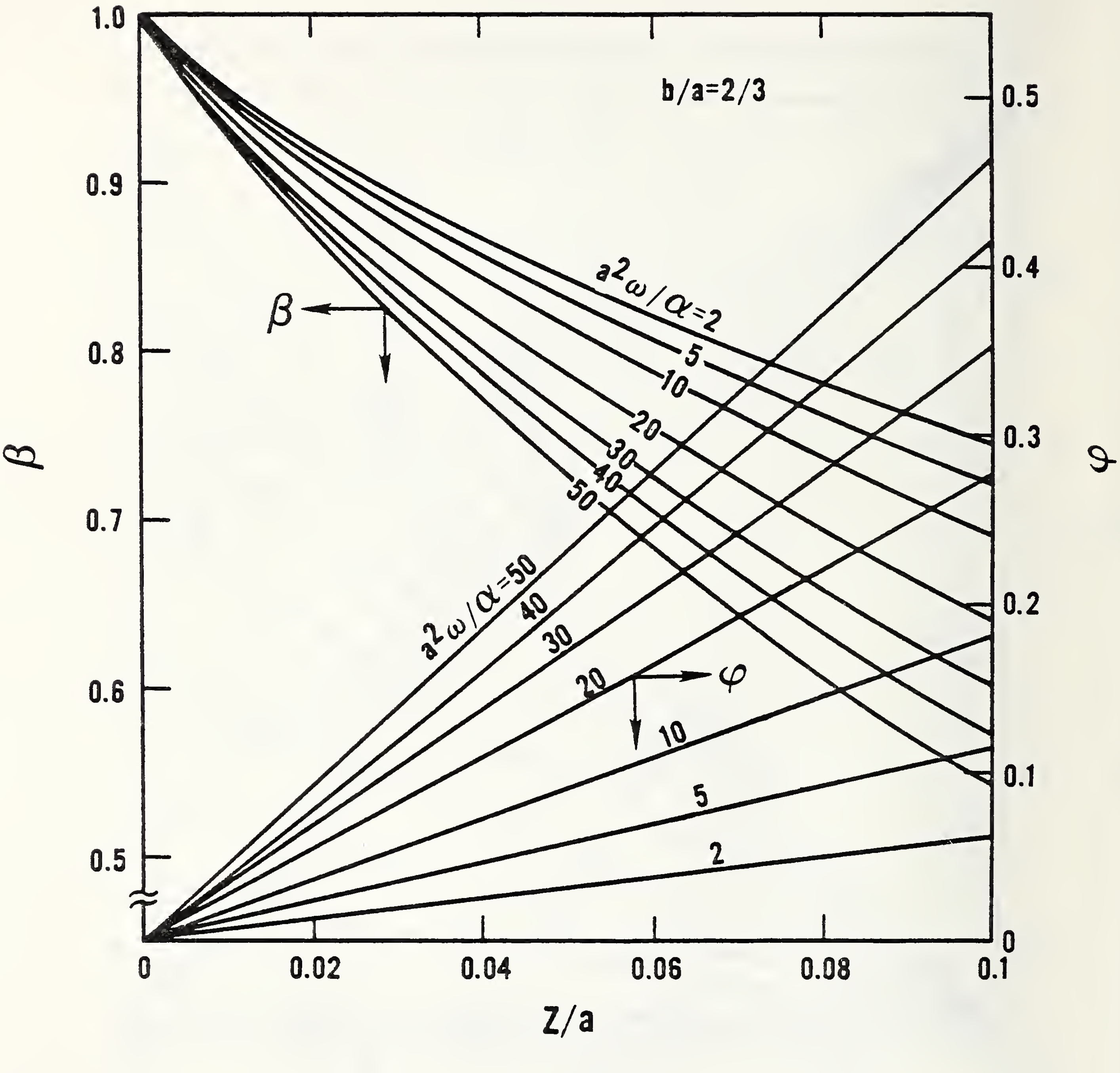



Figure 8-2. Average sub-grade temperature coefficients for the floor aspect ratio 2:3

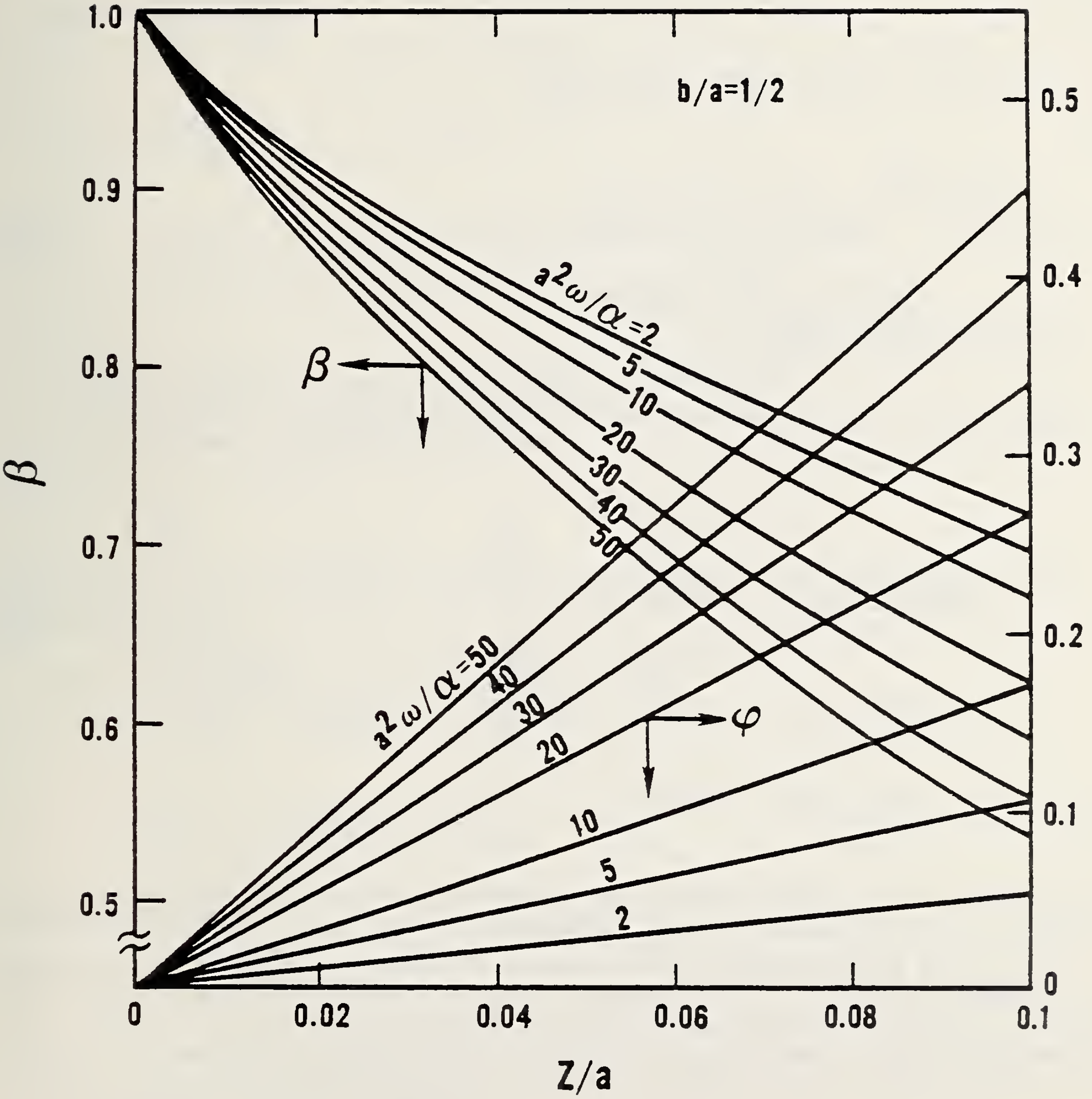



Figure 8-3. Average sub-grade temperature coefficients for the floor aspect ratio 1:2

where $A_1 = 2.919 - 3.029 (\frac{b}{a}) + 1.362 (\frac{b}{a})^2$

$B_1 = 0.1957 + 0.0936 (\frac{b}{a}) + 0.0144 (\frac{b}{a})^2$

$n = 0.6773 + 0.0141 (\frac{b}{a}) - 0.0426 (\frac{b}{a})^2$

$m = (\frac{a^2 w}{\alpha})^{0.035} \{0.756 + 0.046 (\frac{b}{a})\}$

$\psi = \{C_1 + D_1(\frac{b}{a})\} (\frac{a^2 w}{\alpha})^{E_1 - F_1(\frac{b}{a})}$

where values of $C_1$, $D_1$, $E_1$ and $F_1$ may be found in table as a function of $\frac{a^2 w}{\alpha}$

| | $C_1$ | $D_1$ | $E_1$ | $F_1$ |
|---|---|---|---|---|
| $1.8 < \frac{a^2 w}{\alpha} < 7$ | 0.236 | 0.168 | 0.800 | 0.111 |
| $7 < \frac{a^2 w}{\alpha} < 16$ | 0.278 | 0.207 | 0.713 | 0.119 |
| $16 < \frac{a^2 w}{\alpha} < 45$ | 0.372 | 0.170 | 0.623 | 0.067 |

The floor heat loss is then calculated by knowing the overall thermal conductance $U_G$ of the soil layer of thickness $\ell$ such that

$$Q = U_G(T_R - \bar{T}_z)A \tag{10}$$

and $$U_G = \frac{k}{\ell} . \tag{11}$$

Insulated slab-floor

While the preceding analysis assumes that the floor slab has the same thermal properties as those of the soils below (which is approximately correct for uninsulated concrete floor over relatively wet soil), many floor systems include thermal insulation with their thermal property data being considerably different from those of the sub-soil. When dealing with an insulated floor, one must account for an appreciable temperature drop across it. Thus the values of $T_R$

and C in equation (7) must be adjusted to take this fact into account. A suggested method is to estimate them by the following equations:

$$T_R' = \frac{U_F T_R + U_G \bar{T}_z}{U_F + U_G} \tag{12}$$

$$C' = \frac{U_F C}{U_F + U_G} \tag{13}$$

where $T_R'$ = annual average temperature of soil-floor interface

$C'$ = amplitude of the annual cycle of soil-floor interface

$U_F$ = insulated floor system thermal conductance

$U_G$ = subsoil thermal conductance

$= \frac{k}{z}$

Substituting $T_R'$ and $C'$ for $T_R$ and C of equation (7), and rearranging the terms, one obtains

$$\bar{T}_z = \frac{U_F + U_G}{U_F + U_G(1 - \theta'_{1m})} \left[ T_m + \left( \frac{U_F T_R}{U_F + U_G} - T_m \right) \theta'_{1m} \right.$$

$$\left. + B\,\theta'_{2m} + \left( \frac{U_F C}{U_F + U_G} - B \right) \theta'_{3m} \right] \tag{14}$$

Sample Calculations

Determine the January and August heat loss from a slab-on-grade floor of a 40 ft x 30 ft house in Washington, D.C. Assume that the floor is carpeted with underlay over the 4-in concrete slab, and that soil is of medium moisture content. In addition, assume that 1-in polystyrene thermal insulation board is placed between the soil and concrete slab. The house is thermostated at 78°F in summer and 68°F in winter.

1. Floor system thermal resistance

| | $hr \cdot ft^2 \cdot F/Btu$ |
|---|---|
| inside surface resistance | 0.6 |
| carpet | 1.0 |
| underlay | 1.4 |
| 4" concrete | 0.3 |
| 1" polystyrene board | 3.5 |
| | 6.8 |

$$U_F = \frac{1}{6.8} = 0.147 \qquad Btu/hr \cdot ft^2 \cdot F$$

2. Assume soil thermal property as follows:

thermal conductivity $k = 0.5$ Btu/hr•ft•F
thermal diffusivity $\alpha = 0.025\ ft^2/h$
or $U_G = 0.5$ Btu/hr•ft•F for $z = 1$ ft

3. Surface temperature condition (see previous example)

$T_m = 56.5°F$
$B = 20.65°F$

4. Indoor temperature condition

$T_R = (78 + 68)/2 = 73$

$C = (78 - 68)/2 = 5$

5. Since $2a = 40$, $2b = 30$, $a = 20$ and $b = 15$
aspect rates $b/a = 15/20 = 0.75$
depth parameter $z/a = 1/20 = 0.050$
$\theta'_{1m} = 0.85$ from figure 7.

6. Elapsed time after April 1 will be

$$\omega t = \frac{2\pi}{365}(275) = 4.73 \text{ for January 1}$$

$$\frac{2\pi}{365}(122) = 2.100 \text{ for August 1}$$

$$z\sqrt{\frac{\omega}{2\alpha}} = (1)\sqrt{\frac{2\pi}{365 \times 0.025 \times 24}} = 0.170$$

$$\theta'_{2m} = e^{-0.17} \sin(4.73 - 0.17) = -0.834 \text{ for January 1}$$

$$= e^{-0.17} \sin(2.10 - 0.17) = 0.790 \text{ for August 1}$$

7. $$\frac{a^2 w}{\alpha} = \frac{(20)^2 (\frac{2\pi}{365})}{0.025 \times 24} = 11.5$$

from figure 8-2 for $z/a = 0.050$

$\beta = 0.84$
$\psi = 0.10$

8. Using equation (9)

$$\theta'_{3m} = 0.81 \sin(4.73 - 0.10) = -0.807 \text{ for January 1}$$

$$= 0.81 \sin(2.11 - 0.10) = 0.733 \text{ for August 1}$$

9. $$\frac{U_F T_R}{U_F + U_G} = \frac{(0.147)(73)}{0.147 + 0.5} = 16.6$$

$$\frac{U_F C}{U_F + U_G} = \frac{(0.147)(5)}{0.147 + 0.5} = 1.14$$

$$\frac{U_F + U_G}{U_F + U_G(1 - \theta'_{1m})} = \frac{0.147 + 0.5}{0.147 + 0.5(1 - 0.85)}$$

$$= \frac{0.647}{0.222} = 2.91$$

10. For January 1

$$\bar{T}_{z=1} = 2.91[56.5 + (16.6 - 56.5)(0.85) + (20.65)(-0.834) + (1.14 - 20.65)(-0.807)]$$

$$= 2.91[56.6 - 33.9 - 17.2 + 15.74]$$

$$= 2.91[21.14] = 61.5°F$$

For August 1

$T_{z=1}$ = 2.91[56.5 + (16.6 - 56.5)(0.85) + (20.65)(0.79)

+ (1.14 - 20.65)(0.733)]

= 2.91[56.5 - 33.9 + 16.31 - 14.30]

= (2.91)(24.61) = 71.6

11. House temperature

January 1 : 73 + 5 sin(4.73) = 68°F
August 1 : 73 + 5 sin(2.10) = 79°F

12. Floor/soil thermal conductance

$$\frac{1}{U} = \frac{1}{U_F} + \frac{1}{U_G} = 6.8 + 2 = 8.8$$

U = 0.1136

13. Floor heat loss is then

January : (0.11367) (68 - 61.5)(1200) = 886 Btu/h
August : (0.1136)(77 - 71.8)(1200) = 736 Btu/h

If similar calculations are performed for the non-insulated slab floor whereby the slab thermal properties are nearly equal to those of the soil below, equation (7) may be used directly to find the sub-soil temperature 1 ft below the floor surface as follows.

January 1:

$T_{z=1}$ : 56.5 + (73 - 56.5)(0.85) + (20.65)(-0.834) + (5 - 20.65)(-0.807)

= 56.5 + 14.03 - 17.22 + 12.63 = 65.9

For August 1:

$T_{z=1}$ : 56.5 + (73 - 56.5)(0.85) + (20.65)(0.79) + (5 - 20.65)(0.733)

= 56.5 + 14.03 + 16.30 - 11.47 = 75.4

The floor heat loss would then be

for January : (0.5)(68 - 65.9)(1200) = 1260 Btu/h

for August : (0.5)(77 - 75.4)(1200) = 960 Btu/h

These calculations show that the subfloor temperature is lower when the floors are insulated than otherwise. Comparison of the uninsulated floor heat loss with that determined by the ASHRAE Handbook equation could yield

$$q = (0.81)(140)(68 - 36.5) = 3572 \text{ Btu/h}$$

when the winter outdoor temperature is assumed 36.5°F.

## 5. SUMMARY AND DISCUSSION

Monthly depth profiles of the earth temperature beneath the floor slab have been determined by using the Lachenbruch method. The results were used to develop a simplified procedure for calculating the area average temperature beneath the slab floor area. The area-averaged earth temperature can in turn be used to approximate the monthly floor heat loss. In addition, the area-averaged sub-floor earth temperature determined by this method should be good reference data for the response factor-type evaluation of hourly floor heat transfer.

Caution must be exercised, however, for the use of average sub-floor soil temperatures in the calculation of the floor heat loss, because the method ignores several important facts.

1. The floor foundation effect: The floor slab is usually mounted on a concrete foundation of complex shape around its edge. These concrete foundations could cause additional complexity in the temperature field around the floor slab because of differences in thermal conductivity from the surrounding soil.

2. The edge insulation: Some slab-on-grade floors employ insulation around their edges, which would yield different temperature fields than predicted by the procedure developed in this paper.

3. Building wall thickness effect: The analysis employed in the proposed procedure assumes that the slab temperature is uniform from one end of the floor to another, and assumes an abrupt temperature drop at the edge from the indoor condition to outdoor condition. In reality, the slab temperature would undergo a gradual change near the edge, and the transition temperature profile from the indoor condition to the outdoor condition depends heavily upon the thickness and thermal characteristics of the wall over the slab edge. If the wall is thicker and better insulated, the temperature transition would be steeper than the thinner and poorly insulated wall.

   The Lachenbruch procedure employed in this paper could simulate the nonuniform slab temperature as long as its profile is known. Muncey/Spencer [6] relation can indeed take into account the building wall problem by assuming prescribed slab-to-outdoor temperature distribution functions.

One of the dubious aspects of this area-averaged sub-floor earth temperature concept is that the depth for which the area averaging is to be performed is not well defined. If the depth is too great, the edge heat loss effect will not be adequately reflected. On the other hand, if the depth is too small, the edge effect will be overated with the reason discussed above, in reference to the wall thickness effect. In fact, a preliminary result of the heat flux measurement on the NBS test house (in Gaithersburg, MD) during January of 1981, indicated that the edge effect on a 20 ft x 20 ft slab-on-grade house is not as severe as that calculated by equation (4). It appears that the use of area-averaged earth temperatures 1 ft below the slab provides a good base for the seasonal slab-heat-loss calculation, although further and detailed validation effort is needed.

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## Muncey/Spencer Solution

In 1977, Muncey/Spencer published a result of many years' work [6] that involves the frequency domain or Fourier series solution of the same differential equation which was solved by Lachenbruch by Green's Function [4]. The following equations show the Muncey/Spencer representations of slab surface temperature, slab heat flux, Q, and room temperature above the slab, X, respectively:

$$V = \frac{T(x,y,0,\theta)}{T_R} = \Sigma\Sigma\ T_{m,n}\cos\left(\frac{\pi mx}{fu}\right)\cos\left(\frac{\pi ny}{gv}\right)e^{j\omega\theta} \tag{A-1}$$

$$Q(x,y,0,\theta) = \Sigma\Sigma\ W_{m,n}\cos\left(\frac{\pi mn}{fu}\right)\cos\left(\frac{\pi ny}{gv}\right)e^{j\omega\theta} = \frac{1}{R_a}\,[X - V] \tag{A-2}$$

where $R_a$ = thermal resistance of the slab surface

$$X(x,y,0,\theta) = \Sigma\Sigma\ Z_{m,n}\cos\left(\frac{\pi mx}{fu}\right)\cos\left(\frac{\pi ny}{gv}\right)e^{j\omega\theta} \tag{A-3}$$

where u, v, f and g are defined graphically in figure A-1.

Moreover, it was recommended that the summation of Fourier terms be carried out for m and n from 1 through up to 16.

The integrated average floor heat loss is then

$$q = \int_{-v}^{v}\int_{-u}^{u} Q(X,Y,0,\theta)\,dx\,dy \tag{A-4}$$

$$= \sum_{mn}\Sigma\ W_{m,n}\cos\left(\frac{\pi mx}{fu}\right)\cos\left(\frac{\pi ny}{gv}\right)e^{j\omega\theta}$$

$$= \frac{W_{o,o}}{fg} + \sum_{m=1}^{16}\frac{2W_{m,o}}{fg}\left(\frac{\sin\frac{\pi m}{f}}{\frac{\pi m}{f}}\right)^2 + \sum_{n=1}^{16}\frac{2W_{o,n}}{fg}\left(\frac{\sin\frac{\pi n}{g}}{\frac{\pi n}{g}}\right)^2$$

$$+ \sum_{m=1}^{16}\sum_{n=1}^{16}\frac{4W_{m,n}}{fg}\left(\frac{\sin\frac{\pi m}{f}}{\frac{\pi m}{f}}\right)^2\left(\frac{\sin\frac{\pi n}{g}}{\frac{\pi n}{g}}\right)^2$$

where

$$W_{m,n} = \frac{\sqrt{\left(\frac{\pi m}{fu}\right)^2 + \left(\frac{\pi n}{gv}\right)^2 + j\frac{\omega\rho c}{k}}\ k\cdot Z_{m,n}}{1 + \sqrt{\left(\frac{\pi m}{fu}\right)^2 + \left(\frac{\pi n}{gv}\right)^2 + j\frac{\omega\rho c}{k}}\ k\cdot R_a}$$

$$Z_{o,o} = \frac{1}{fg}$$

$$Z_{m,o} = \frac{2}{\pi mg} \sin(\frac{\pi m}{f})$$

$$Z_{o,n} = \frac{2}{\pi nf} \sin(\frac{\pi m}{g})$$

$$Z_{m,n} = \frac{4mn}{\pi^2} \sin(\frac{\pi m}{f}) \sin(\frac{\pi n}{g})$$

$$\alpha = \frac{k}{c\rho} \text{ or } \frac{\omega c\rho}{k} = \frac{\omega}{\alpha}$$

$$j = \sqrt{-1}$$

The heat flux solution (A-4), which is also the overall heat transfer coefficient between the in- and outdoor temperature, was obtained on the basis that room temperature X, instead of the slab surface temperature V, was kept constant over the slab, or $-u \leq x \leq u$ and $-v \leq y \leq v$ (Figure A-1).

Applying the solution (A-4) into many different slab programs, Muncey and Spencer were able to develop a standard slab thermal resistance curve $R_s$, such as shown in Figure A-2, for a 10 m x 10 m square slab on the soil having thermal conductivity of 1 W/m • K. This $R_s$ can be used to determine the thermal resistance R of other slabs by the following relationship:

$$R\ (\beta k, \alpha P, \ell, R_a) = F \cdot \left(\frac{\alpha}{\beta}\right) R_s \left(k, P, \ell, \frac{R_{a\beta}}{\alpha}\right)$$

where

$\alpha$ = perimeter ratio with respect to the standard slab

$\beta$ = thermal conductivity ratio with respect to the standard slab

$\ell$ = wall thickness (edge distance)

F = shape correction factor, which is a function of $\frac{A}{[P/4]^2}$ (Figure A-3)

A = slab area

P = slab perimeter length

This particular Muncey/Spencer solution is, however, not applicable for the evaluation of seasonal slab-on-grade heat transfer.

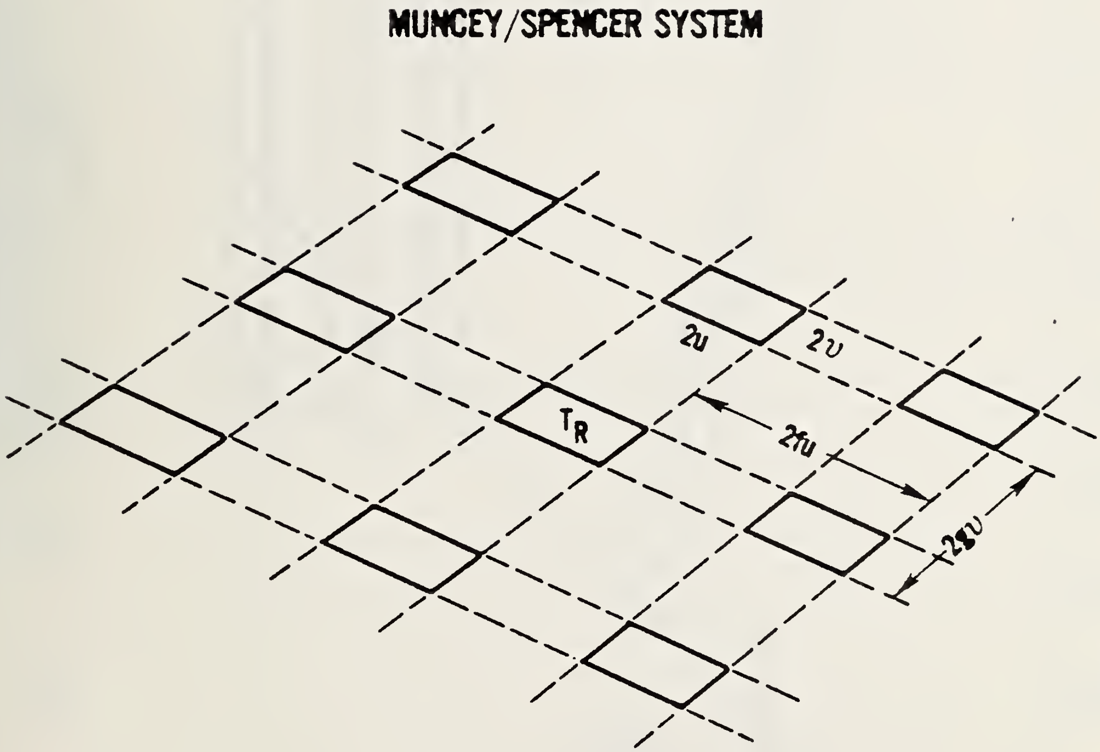


Figure A-1. Muncey/Spencer slab model[5]

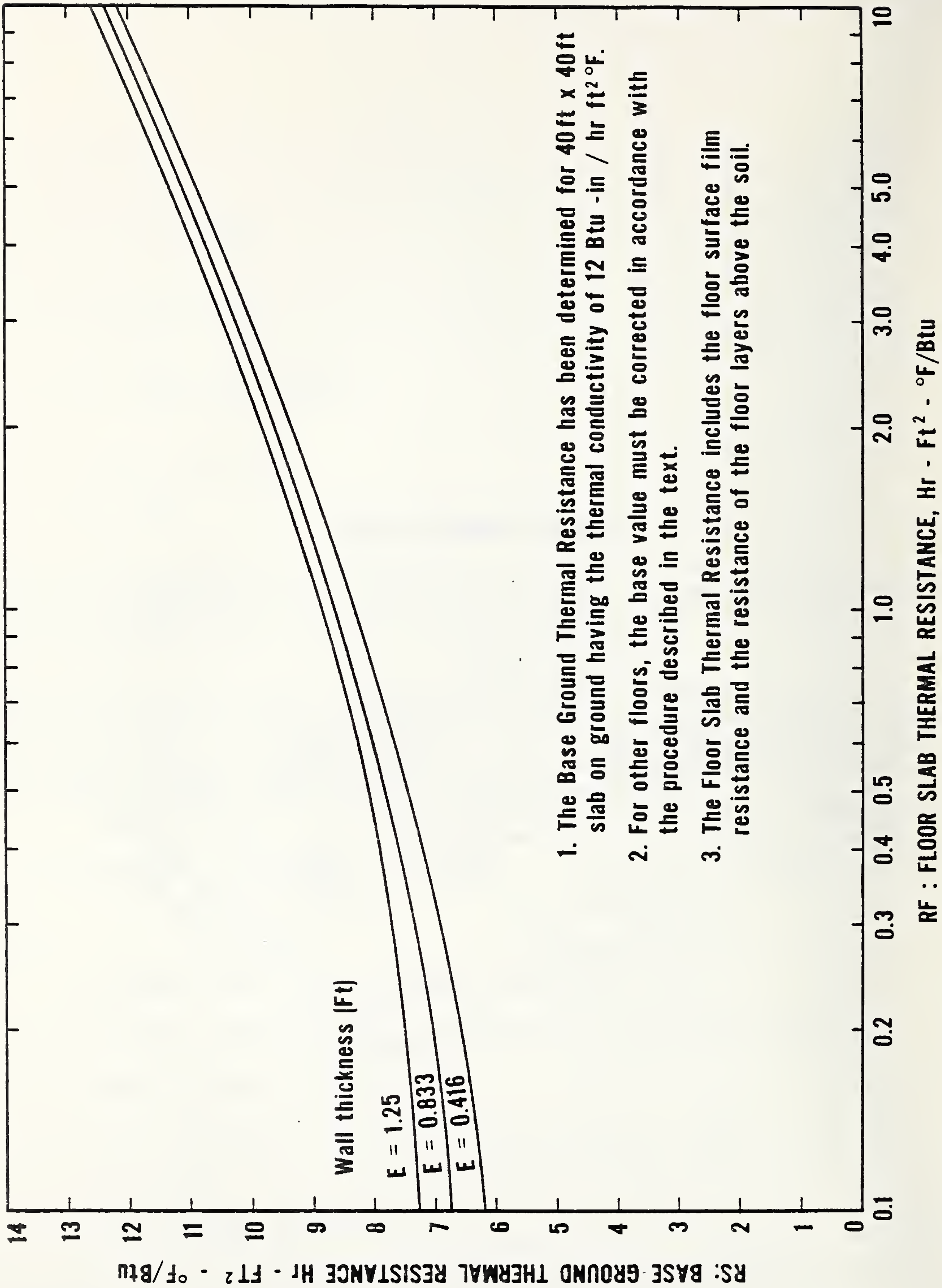


Figure A-2. Thermal resistances for a square slab of 40 m perimeter for a ground conductivity of 1.0 W/m·K for various values of edge distance e and air film resistance H.

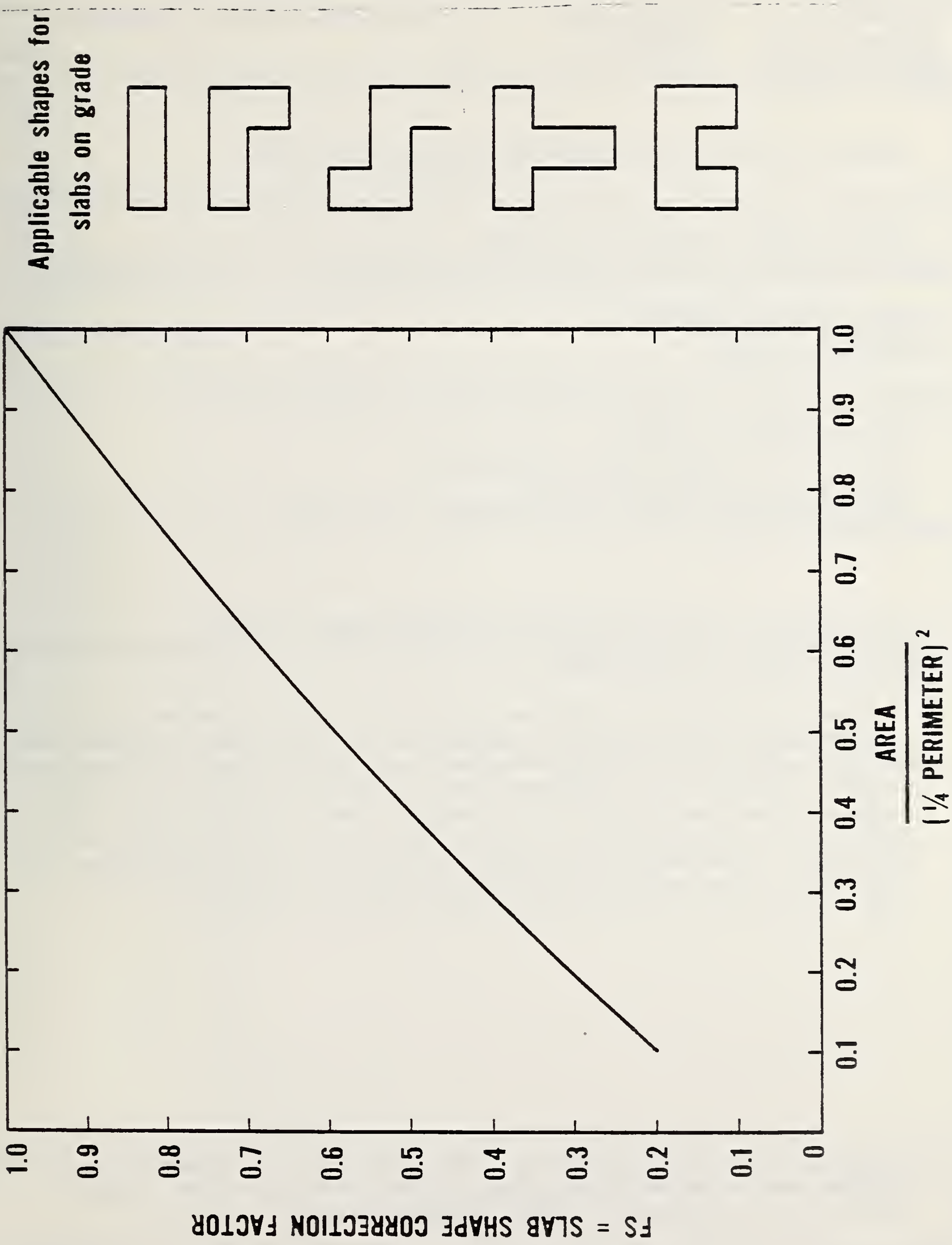


Figure A-3. Correction factor for non-square slabs

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Tamami Kusuda; M. Mizuno; J.W. Bean

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10. SUPPLEMENTARY NOTES

☐ Document describes a computer program; SF-185, FIPS Software Summary, is attached.

11. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)

In order to facilitate an efficient slab-on-grade heat transfer calculation on a comprehensive energy analysis program such as DoE-2, BLAST and NBSLD, heat transfer calculations for slab-on-grade floors are reviewed. The computational procedure based on the Lachenbruch method is studied in depth to generate monthly average temperatures at a given depth below the floor slab. The data generated by the Lachenbruch method are then used to develop a simplified procedure for determining the monthly average earth temperatures below the floor slab. These monthly average temperature data can be used for the hourly response factor analysis of floor-slab heat transfer.

12. KEY WORDS (Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)

Building heat transfer; DoE-2 Energy Analysis computer program; monthly average earth temperature; thermal response factors.

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