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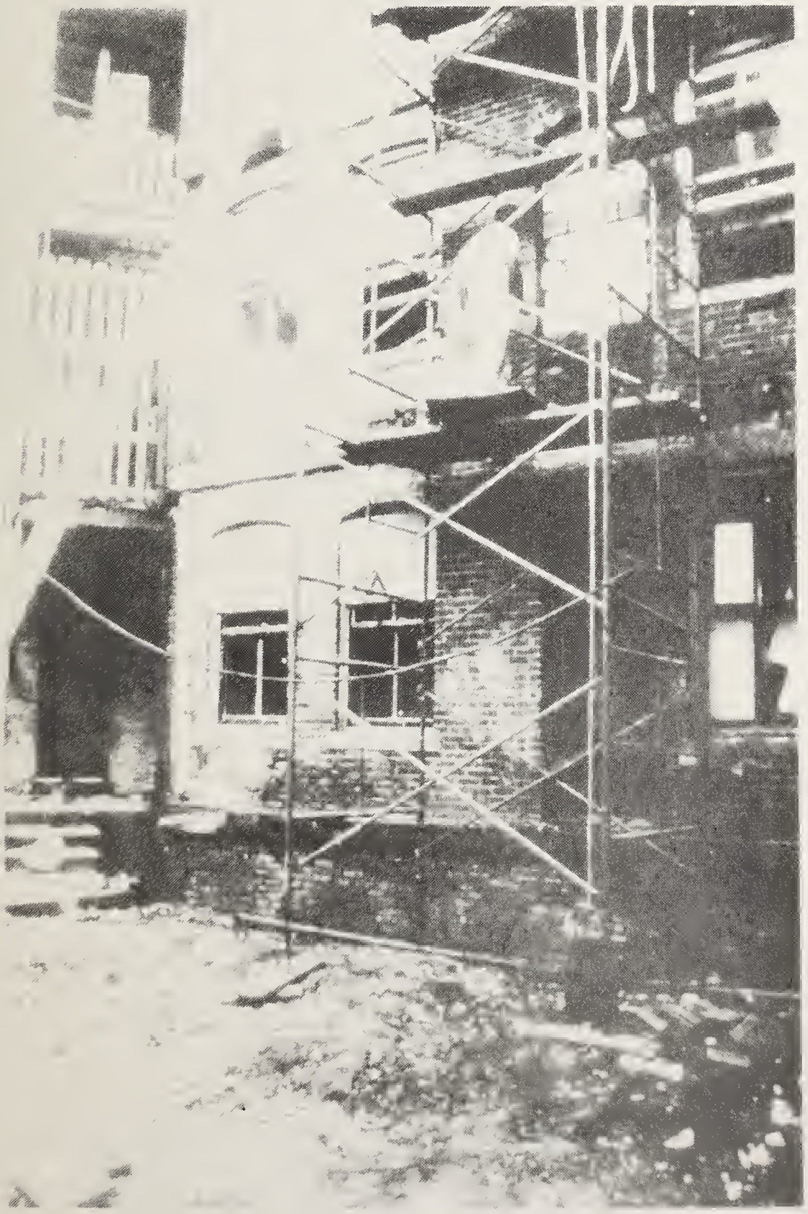
Application of an Equivalency Methodology to Building Rehabilitation: A Pilot Study



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APPLICATION OF AN EQUIVALENCY METHODOLOGY TO BUILDING REHABILITATION: A PILOT STUDY

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ABSTRACT

With increased emphasis on the re-use of existing buildings, new approaches must be developed to assist regulators in making code-related decisions. The application of performance criteria to building rehabilitation provides flexibility in the use of technically sound design alternatives in lieu of prescriptive provisions which may be restrictive. This report presents the results of a pilot study on the application of an equivalency methodology in achieving regulatory compliance. The use of such a methodology is particularly attractive in this area because prescriptive type provisions have been shown to constrain rehabilitation activities and, in some cases, may be mutually contradictory. Regulatory requirements were chosen so as to explicitly incorporate conflicting requirements as affecting the design of windows and doors -- illumination, ventilation, egress and security. The methodology is computerized to allow the selection of least-cost means of achieving compliance with these requirements. A prototypical townhouse is evaluated using the pilot equivalency methodology and optimal compliance strategies are identified and compared with the cost of prescriptive compliance. The results of the study produced potential savings ranging from 20 to 30 percent depending on the initial conditions of the building.

Key Words: applied economics; building codes; health and safety; housing; mathematical programming; rehabilitation; renovation.

PREFACE

This research was conducted under the joint sponsorship of the Building Economics and Regulatory Technology Division of the Center for Building Technology (CBT) and the Operations Research Division of the Center for Applied Mathematics (CAM), National Engineering Laboratory, National Bureau of Standards (NBS). The work was initiated as part of the building rehabilitation technology program in CBT. It is an extension of the work being conducted within NBS by CAM and CBT on the Fire Safety Evaluation System developed by the Center for Fire Research.

The application of prescriptive code provisions to existing buildings has been shown to constrain the rehabilitation process. These constraints, coupled with the rapidly rising costs of new construction, have aggravated the Nation's building problems. This report develops a pilot equivalency methodology which includes a computerized procedure for identifying the least-cost means of achieving compliance with selected regulatory attributes. The focus of the study is on how these regulatory attributes affect and are affected by various window and door compliance strategies in buildings being rehabilitated. An objective of the study was to demonstrate the feasibility of applying equivalency methodologies to regulatory areas where conflicting requirements may be present (e.g., window hardware may improve security performance but hinder egress). Application of the computerized procedure was tested on a prototypical townhouse residence. The results of the computer study indicate that potential cost savings associated with the use of such an approach to code compliance are substantial when compared to the more traditional prescriptive approach.

Special appreciation is extended to Messrs. Patrick W. Cooke and Phillip T. Chen who provided support in the formulation of the prototypical building and its associated retrofit costs, as well as, in the development of code requirements and related performance values. Special appreciation is also extended to James G. Gross, Building Economics and Regulatory Technology Division, and Christoph J. Witzgall, Operations Research Division, whose stimulating discussions provided guidance and encouragement throughout this effort. The significant contributions of Mary Chaney, secretary to the Rehabilitation Technology Group, and Mary L. Ramsburg and Lisa Gray of the CBT Word Processing Center in the typing of this report are gratefully acknowledged.

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EXECUTIVE SUMMARY

Throughout the United States, increasing concern is being expressed for the need to more efficiently utilize the existing building stock. Studies of the impact of the current regulatory process on existing buildings have concluded that application of prescriptive code requirements developed for new construction to such buildings presents a constraint to rehabilitation.

The purpose of this report is to demonstrate how an equivalent performance approach can be adapted, in a fairly general but rigorous manner, to the problem of identifying and selecting cost effective code compliance strategies for rehabilitating existing buildings. The primary objective is to determine the feasibility of applying equivalency methodologies to regulatory areas which adversely impact rehabilitation and to highlight the advantages of such an approach to code compliance over the prescriptive approach. The major disadvantage of the performance approach is that performance is difficult to measure unless it can be tied to some prescriptive solution. In order to get around this obstacle, previous researchers have focused on the development of an equivalency methodology for fire safety known as the Fire Safety Evaluation System (FSES). This system separates the levels of performance into a series of discrete steps, or states, each of which can be tied to a prescriptive solution. An equivalency methodology is constructed which is sufficiently general that it is possible to define a distinct level of performance (or performance score) associated with each code or code requirement under consideration.

A pilot study was formulated to extend the previous work on the FSES to regulatory areas other than fire safety. The procedure identifies the least-cost means of achieving compliance with selected regulatory requirements which affect windows and doors in residential buildings being rehabilitated. The specific regulatory areas considered are security, egress, ventilation, and illumination.

Equivalency methodologies can be easily adapted to computer optimization. The specific method may vary according to the problem. For purpose of exposition however, it is sufficient to focus on one technique known as linear programming which is used in this pilot study. In its usual context, linear programming deals with the problem of allocating limited resources among competing activities in an optimal way. At the foundation of any linear programming problem is a mathematical model which describes the problem of concern. The term "linear" refers to the requirement that all mathematical functions in the model are linear. The term "program" is used in the general sense in that it refers to a plan rather than a computer program _per se_.

A case application was designed to test the equivalency methodology and to demonstrate how the procedure would be applied in practice. This includes the selection of regulatory requirements, development of sample worksheets, and the formulation of a prototypical design. Criteria for the design of the case study included the following: (1) regulatory requirements were selected such that an improvement in one area could result in a negative effect in another area; (2) regulatory requirements were selected such that an improvement in one area could affect another area in the same way; and (3) building components were selected such that their design would be affected by the regulatory requirements. Egress and security would be affected in a conflicting way by improvement in window and door security hardware (criterion 1); ventilation and illumination would be similarly affected by changes in window size (criterion 2); and window and doors can contribute to the level of compliance of all four regulatory requirements (criterion 3).

Worksheets capable of defining a level of performance were developed for each set of regulatory requirements. This task was accomplished by subdividing each building component into a set of states which cover all possible situations expected to be encountered in practice. For example, window type includes slider, casement, single hung, double hung and fixed as states. Associated with each component/state pair is a value which represents the contribution of this pair to meeting the overall objective; i.e., attaining the level of performance required by the regulatory requirement under consideration. The state values in the worksheets were assigned by the project team based on their knowledge of the subject areas. There was no attempt made to utilize broader technical input in setting these values since the purpose of the study was to evaluate the applicability of equivalency methodologies for the regulatory areas considered and was not intended to be a comprehensive technical treatment.

A prototypical building was synthesized in a manner which provided a test of the equivalency methodology formulated in the case study. A typical inner city residential townhouse was selected with various initial window and door conditions. "In-place prices" for accomplishing these various retrofits were identified. The computer program which was developed based on the model equivalency methodology was applied to this building with the various initial window and door conditions (type, size, number, location, etc.).

Potential savings associated with the use of an equivalency based approach to compliance versus the more traditional prescriptive approach were then estimated for eight buildings with different initial conditions. Generally, the cost of code compliance based on the equivalency approach is about three-quarters of those associated with strict prescriptive compliance. If the number of regulatory requirements and building components are increased to reflect the actualities of building rehabilitation activities, this percentage may either increase or decrease. However, based on the studies in the fire safety area, fairly substantial savings, i.e., 20 percent or more, would be expected.

The study demonstrated the feasibility of applying equivalency techniques to regulatory requirements which may have conflicting impacts on building rehabilitation. A mathematical optimization model can be used to identify least-cost means of achieving compliance with significant cost savings.

SI CONVERSION UNITS

The conversion factors and units contained in this report are in accordance with the International System of Units (abbreviated SI for Systeme International d'Unites). The SI was defined and given official status by the 11th General Conference on Weights and Measures which met in Paris, France in October 1960. For assistance in converting U.S. customary units to SI units, see ASTM E 380, ASTM Standard Metric Practice Guide, available from the American Society for Testing and Materials, 1916 Race Street, Philadelphia, PA 19103. The conversion factors for the units found in this Standard are are follows:

Length

1 in = 0.0254* meter

1 ft = 0.3048* meter

1 mil = 0.001* in

1 yd = 0.9144* meter

Area

1 in^2 = 6.4516* x 10^{-4} $meter^2$

1 ft^2 = 0.0929 $meter^2$

1 yd^2 = 0.836 $meter^2$

Volume

1 in^3 = 1.639 x 10^{-5} $meter^3$

1 liter = 1.00* x 10^{-3} $meter^3$

1 gallon = 3.785 liters

Temperature

°C = 5/9 (Temperature °F-32)

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* Exactly

1. INTRODUCTION

1.1 BACKGROUND

Throughout the United States, increasing concern is being expressed for the need to utilize more fully the existing building stock. The re-use of old buildings has grown far beyond the preservation movement which spawned it with all existing structures now viewed as assets that can be reclaimed. Soaring construction costs and Federal incentives are creating a positive environment for building rehabilitation. During the late 1970's, national policy has promoted a program to rehabilitate older city buildings instead of tearing them down and erecting new ones, a process that had destroyed many inner city neighborhoods. The Department of Commerce reported that in the third quarter of 1980, seasonally adjusted annual expenditures for residential alteration and repair reached a new high of almost $50 billion.[1] The Architectural Record reported in October of 1979 that expenditures for nonresidential additions, alterations, and major replacements are expected to increase from $15 billion in 1978 to as much as $30 billion annually by the mid 1980's.[2]

Anderson[3] gives the following reasons for the recycling boom: (1) from start to finish a recycling project may take only half as long to complete as a similarly sized new development, (2) recycling costs are competitive with new construction, (3) recycled space is much in demand, and (4) attitudes of city officials and lenders are changing. Because of these reasons, more and more decaying and abandoned houses, warehouses, schools, railroad stations, and other buildings once slated for demolition are being turned into money-making properties while having positive impacts on the neighborhood.

Several studies of the impact of the current regulatory process on building rehabilitation have concluded that application of prescriptive code provisions for new construction to existing buildings presents a constraint to rehabilita-

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[1] U.S. Department of Commerce Construction Reports, "Residential Alterations and Repairs, Annual 1979," issued April 1980.

[2] Philip E. Kidd, Value of Nonresidential Rehabilitation Will Double by Mid-1980's, Architectural Record (pg. 61), October 1979.

[3] Timothy Anderson, Recycling: Old Buildings Big New Opportunity, Professional Builder (pp. 197-199), January 1979.

tion.[1,2,3,4,5] This is manifested in two ways: (1) most codes contain administrative provisions which state that a building's conformance with the requirements of the code for new construction should increase in relation to the dollar amount of rehabilitation planned; and (2) when the occupancy classification of an existing building changes the building must be made to conform to the code requirements for the new occupancy. Applying codes for new construction to existing buildings, with their generally prescriptive format, presents difficulties since: (1) they may not address the types of construction present in many older buildings; (2) innovative solutions are limited since building officials feel they do not have a legal or technical basis for approving code deviations; and (3) they are structured to follow the design process associated with new construction as contrasted to the analytical procedure required in building rehabilitation.

Existing buildings constructed to some code or standard that reflect the state-of-the-art at the time will probably not comply with current codes. However, if they have performed their intended use without exposing the public to any undue hazard, one could conclude that they should be considered for re-use. This philosophy has become increasingly accepted over the past several years with several significant rehabilitation code and guideline development activities taking place.

A. Code Provisions for Existing Buildings by the State of Massachusetts

Massachusetts has published a new Article 22 of the State Building Code entitled "Repair, Alterations, Additions and Changes in Use of Existing Buildings."[6] This article allows rehabilitation of existing buildings

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[1] Saundra A. Berry, Ed., Proceedings of the National Conference on Regulatory Aspects of Building Rehabilitation, National Bureau of Standards, Special Publication 549, August 1979.

[2] James G. Gross, James H. Pielert and Patrick W. Cooke, Impact of Building Regulations on Rehabilitation -- Status and Technical Needs, National Bureau of Standards, Technical Note 998, May 1979.

[3] Impact of Building Codes on Housing Rehabilitation, Hearings Before the Committee on Banking, Housing and Urban Affairs -- United States Senate, March 24, 1978.

[4] N. John Habraken, et al., An Investigation of Regulatory Barriers to the Re-Use of Existing Buildings, National Bureau of Standards, GCR 78-139, January 1978.

[5] Robert J. Kapsch, Building Codes: Preservation and Rehabilitation, National Bureau of Standards, Special Publication 473 (pp. 437-452), June 1977.

[6] State of Massachusetts, Article 22 - Repair, Alteration, Additions and Changes in Use of Existing Buildings, Massachusetts State Building Codes Commission, October 1980.

without necessarily meeting all new construction code requirements, as long as, minimum health and safety provisions are met. The following rules apply: (1) ordinary repairs (paint and wallpaper) require no permit; (2) repairs (maintenance) made with similar materials are allowed; (3) alterations are allowed with similar materials, but any new system must comply with the code for new construction (e.g., egress, fire protection, plumbing, electrical, mechanical, energy conservation, and building enclosure system); and (4) additions must conform to the code for new construction including height and area limits. A recent NBS report documents the development of Article 22 and provides four case studies of its application.[1]

Complete or partial changes in use are regulated according to the change in hazard index which depends on the use group of the building occupancy. Appendix T of of the Massachusetts State Building Code contains a detailed listing of occupancies and related hazard indexes. The change in the hazard index from the existing occupancy to the proposed occupancy determines the level of code compliance under the following requirements:

(a) Hazard index stays the same or decreases (Article 22 - Section 2203.0) - Eight items of code compliance are mandatory including floor loads, structural loads (excluding seismic), number of exits, capacity of exits, exit signs and lights, means of egress lighting, fire alarm system, and smoke enclosure of stairways.

(b) Hazard index increases by one (Article 22 - Section 2204.0) - Change-in-use must conform to the code for new construction with the following exceptions: fire limits, area and height limits, accessibility for physically handicapped, earthquake resistance and soil liquefaction, mortar, and certain requirements related to fire and party walls.

(c) Hazard index increases by two or more (Article 22 - Section 2205.0) - change-in-use must conform to the code for new construction.

Section 2206.1 of Article 22 provides that "where compliance with the provisions of the code for new construction, required by this article, is impractical because of structural or construction difficulties or regulatory conflicts, compliance alternatives may be accepted by the building official." Appendix T of Article 22 contains some acceptable compliance

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[1] James H. Pielert, Removing Regulatory Restraints to Building Rehabilitation: The Massachusetts Experience, National Bureau of Standards, Special Publication 623, October 1981.

alternatives. The development and implementation of Article 22 has been discussed in several publications.[1,2,3]

B. Rehabilitation Guidelines by Department of Housing and Urban Development

The U.S. Department of Housing and Urban Development (HUD) has developed Rehabilitation Guidelines to help communities resolve building code problems encountered in implementing rehabilitation programs.[4] The Rehabilitation Guidelines are divided into eight volumes:

Volume 1 - Setting and Adopting Standards for Building Rehabilitation
Volume 2 - Municipal Approval of Building Rehabilitation
Volume 3 - Statutory Guideline for Building Rehabilitation
Volume 4 - Managing Official Liability Associated With Building Rehabilitation
Volume 5 - Egress Guideline for Residential Rehabilitation
Volume 6 - Electrical Guideline for Residential Rehabilitation
Volume 7 - Plumbing DWV Guideline for Residential Rehabilitation
Volume 8 - Fire Ratings of Archaic Materials and Assemblies

The information contained in Volumes 1 through 4 is most closely related to the subject of this report. Potential problems caused by application of the "25-50 percent rule" and the change-in-occupancy triggering mechanism are recognized and guidance is provided on a more realistic approach. Locally developed rehabilitation codes which no longer use these triggering mechanisms are described in the appendices of Volume 1, including codes for Massachusetts, San Francisco, Denver, Washington, D.C., Los Angeles and Detroit. Such performance oriented approaches make use of the technical guidelines in Volumes 5 through 8.

1.2 PURPOSE

The purpose of this pilot study is to demonstrate how the equivalent performance approach can be adapted, in a fairly general but rigorous manner, to the problem of identifying and selecting cost effective code compliance strategies when rehabilitating existing buildings. The primary objective is to determine the feasibility of applying equivalency methodologies to regulatory areas which adversely impact rehabilitation and to highlight the advantages of such an approach to code compliance over the prescriptive approach. The pilot study

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[1] James G. Gross, James H. Pielert, and Patrick W. Cooke, op. cit.

[2] Sandra A. Berry, Ed., op. cit.

[3] Proceedings of the Conference on Building Rehabilitation Research and Technology for the 1980's, National Conference of States on Building Codes and Standards, August 1980.

[4] Rehabilitation Guidelines, U.S. Department of Housing and Urban Development, October 1980.

considers the design of windows and doors in buildings being rehabilitated as affected by the regulatory areas of security, egress, ventilation, and illumination.

1.3 SCOPE AND APPROACH

The general plan of this report is to describe briefly previous work on the application of equivalency methodologies to regulatory applications; to discuss the mathematical concepts and a computer program based on the equivalency methodology; and to identify and discuss the relevant engineering considerations which must be exercised in order to use this approach in solving a particular problem. Specifically, this report is organized as follows:

Chapter 2 describes the general application of the equivalency approach to the building rehabilitation process including: (1) development of equivalence methodologies based on a management tool known as the Delphi Method; and (2) the structures of such equivalency methodologies including the technical bases. The advantages of this approach to code compliance over the more restrictive prescriptive approach are highlighted. An optimization model which minimizes construction and materials costs is utilized. The chapter concludes with the presentation of a graphical approach for obtaining optimal compliance strategies.

Chapter 3 presents a case application of the equivalency methodology to a residential rehabilitation. The rationale for the selection of regulatory requirements in the area of health and safety is discussed and specific requirements applied to window and door retrofits are presented. Specific worksheets are included which define the state values (level of performance) for each regulatory requirement associated building components. Details of the prototypical residential building are then presented. The chapter concludes with some preliminary results of the study where a computer program was exercised to identify the least-cost means of achieving compliance to the selected regulatory requirements for the prototypical building. The cost of strict compliance, as well as the costs of the alternative solutions, are then compared in order to demonstrate the flexibility available to decision makers who use the equivalency methodology.

Chapter 4 summarizes the major findings of the study and sets forth several recommendations for continued development of the methodology including technical and mathematical considerations.

Appendix A presents a discussion incorporating a cost engineering approach with the economic theory of cost functions in arriving at a cost estimating procedure which is sensitive to the technical considerations of the rehabilitation process as well as local market conditions. Appendix B presents the mathematical formulation of the equivalency methodology for the prototypical building discussed in Chapter 3.

It should be pointed out that the pilot study on the use of equivalency methodologies discussed in the report was conducted to assess the feasibility of the approach as applied to regulatory areas other than fire safety in the building rehabilitation process. There was no attempt made to subject the worksheets shown in Chapter 3 to a comprehensive Delphi-type review because of funding and time constraints. However, it is believed that the engineering and mathematical treatment was of sufficient rigor to have demonstrated the feasibility of the approach and to justify continued development.

2.0 APPLICATION OF THE EQUIVALENCY APPROACH TO THE BUILDING REHABILITATION PROCESS: SOME BASIC GUIDELINES AND EXTENSIONS

The purpose of this chapter is to demonstrate how the equivalent performance approach can be applied, in a fairly general but rigorous manner, to the problem of identifying and/or selecting efficient investment strategies for rehabilitating existing buildings. The objective of this exercise is to highlight the advantages of such an approach to code compliance over the more restrictive prescriptive approach. A previous study[1] has shown that there are major advantages of the equivalent performance approach which make it highly desirable for application to the rehabilitation process: it provides decision makers with greater latitude in making choices, has cost reducing potential, and permits innovation.

The major disadvantage is that performance is difficult to measure unless it can be tied to some prescriptive solution. In order to get around this obstacle, previous researchers have focused on the development of a system known as an equivalency methodology.[2,3] This system separates the levels of performance into a series of discrete steps, or states, each of which can be tied to a prescriptive solution. Furthermore, the equivalency methodology approach is sufficiently general that it is possible to define a distinct quantitative level of performance (or performance score) associated with each regulatory requirement under consideration. Thus, the use of an equivalency methodology:

(1) provides a mechanism for demonstrating code compliance; and

(2) promotes greater latitude in making choices by permitting substitutions among building components.

It is the second reason which lends support to the claim that this approach will reduce both cost and cost variability. This relationship may be seen more clearly by noting that greater freedom in making retrofit choices will permit the investor to avoid not only one or more "expensive" retrofits but also some of those situations which are more risk prone. This would imply that the variance about any estimate where a substitution was made could be reduced.

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[1] Robert E. Chapman, Cost Estimation and Cost Variability in Residential Rehabilitation, National Bureau of Standards, Building Science Series 129, November 1980.

[2] Harold E. Nelson and A. J. Shibe, A System for Fire Safety Evaluation of Health Care Facilities, National Bureau of Standards, NBSIR 78-1555, November 1978.

[3] Robert E. Chapman, Phillip T. Chen and William G. Hall, Economic Aspects of Fire Safety in Health Care Facilities: Guidelines for Cost-Effective Retrofits, National Bureau of Standards, NBSIR 79-1902, November 1979.

Consequently, the end result would be a reduction in both the budget estimate and its variability.

The discussion which follows is divided into two parts. The first deals with the development of equivalency methodologies based on a management tool known as the Delphi Method and the second part develops the structure of a generalized equivalency methodology.

2.1 THE MODEL DEVELOPMENT PROCESS: AN APPLICATION OF THE DELPHI METHOD

In essence, an equivalency methodology or model is a formal mathematical representation of the states(levels of performance) associated with building rehabilitation. As with a conceptual model of a physical process or process control situation, there are both internal constraints and external trade-offs involved in the formulation. Resolution of the trade-off problem is strongly dependent upon a complete understanding of the way in which the model is to be utilized. The model must be useful and in a form which is usable.

The internal constraints involve items such as validity (Does the model reflect, with sufficient accuracy, the physical situation?); generality (Does the validity hold over the expected range of physical situations? Is an inordinate volume of input required?); calibration (Can the required parameter or coefficients be determined?); and technological state-of-the-art issues (Are variables of measurable dimensions and quantities? Can they be quantified?). Computational tractability or feasibility is also an important internal factor; it is, however, of lesser importance here than in the external trade-off decision.

The use envisioned for the model impacts upon all of the model's internal attributes. Use, in turn, depends primarily on the user class, where the user class is actually a target market for a very specialized product. The user class (institutionally and individually) must be understood and characterized in terms of its expectations, orientations, motivations and capabilities.

2.1.1 Evaluative Models

There are two general types of equivalency model formulations. The first and simplest of these is an evaluative model. Its only output is an assessment of the expected performance of a given configuration of components and/or subsystems, or possibly a given sequence of configurations. The performance is in terms of desirable or mandatory functions of the building. Any considerations other than performance, such as cost and aesthetics, must be accommodated external to the model. The required calculations may be done by hand, by calculator, or by a small computer and the user may be relatively unsophisticated in computer science.

The extent to which the use and user can dominate the model formulation will be illustrated by several scenarios for an evaluative model. These are not intended to be exhaustive but they are moderately realistic.

As a first scenario, consider a model whose purpose is to determine if a particular combination of materials, components, and/or subsystems is adequate with respect to some law, regulation, or standard. The primary users are regulatory agencies who use the model to produce a basis for acceptance or rejection of a proposed design. For these uses and users, the desirable model characteristics are:

1) Validity must be very good; a mistake in the accept-reject decision may have costly consequences.

2) Generality should apply to all building types over which the agency has authority to allow for broad application. It may be attained by a family of models or by parameterization of a single model. A large volume of perhaps rather detailed data is acceptable.

3) Calibration is not a particular problem. The components and subsystems are homogeneous; most of the information required is standard engineering data.

4) Technological state-of-the-art issues do not pose problems.

5) Computational tractability is not important for this use. A tremendous amount of computation for the evaluation of a single case can be tolerated.

For the second scenario, consider an evaluative model used by an architectural/engineering (A/E) firm as an aid to evaluate a fairly large number of conceptual designs, say several hundred, from among the millions of potential configurations. For this use and user, the desirable model characteristics are somewhat different. They are:

1) Validity requirements can be relaxed considerably. The user applies the model as a filter to select a smaller group of candidate configurations for further analysis. Accuracy to perhaps as little as one significant figure should be adequate.

2) Generality is much the same as in the first scenario except that large volumes of data (per case) should be avoided.

3) Calibration is again no problem.

4) Technological state-of-the-art is of even less importance than in the first scenario.

5) Computational tractability is a moderately critical factor. Long, involved calculations are burdensome and conducive to error, and there are two orders of magnitude more cases to evaluate.

A third scenario for an evaluative model deals with an existing facility. Depending upon the user, the model characteristics of validity, generality, and computational tractability are as those in scenarios one and two. The other model characteristics differ:

1) Calibration becomes considerably more complicated. It may be difficult or even impossible to estimate calibration parameters either empirically or theoretically. Obsolete technologies, heterogeneous components, and performance degradation due to age, use, or abuse may be encountered.

2) The technological state-of-the-art may pose problems of measurement of the component or subsystem properties. These difficulties are similar to those expected in calibration.

2.1.2 Optimization Models

The second general type of model is an <u>optimization or optimum seeking model</u>. Functionally, it differs from the evaluative model in that it determines the component/subsystem configuration which is acceptable and optimizes some objective. The most obvious example is to determine the least-cost configuration which maintains the functional integrity of the building. Other objectives are possible, the requirement being that they are quantifiable in commensurate units, but considered here is only the case in which the objective is to minimize costs. The optimization model must include the evaluative model, additional explicit constraints, a costing submodel, an optimization procedure, and a procedure for producing a class of alternate solutions all integrated into a single model. It is more complex, more expensive to develop and exercise, and requires much more mathematical and computer science sophistication. As with the evaluative model, the use and user class impact upon the model formulation are the same as described in scenarios one through three. However, there are some more stringent requirements upon the developer.

A costing sub-model is essential to quantify the costs of various retrofit options. The complexity of the costing model may vary from a mechanism for direct input to rather sophisticated costing functions (see appendix A for a detailed discussion of this issue).

The evaluation portion of the model must be augmented by formal statements of certain requirements relating to such things as aesthetics and engineering judgment which are not explicit in the performance requirements. Otherwise, the optimum configuration with respect to the performance criteria may be non-feasible with respect to some implicit construction, architectural, engineering, or aesthetic requirements. To some extent, if the non-performance requirements are difficult to quantify, these relations may be replaced by augmentation via a set of near-optimum solutions. In effect this strategy evades the formal statement of these requirements at the expense of producing extra sets of solutions for critical examination.

In addition to the optimum solution, a class of near optimum or "good" solutions should be generated. These are required to answer many "what if" questions of value to the user; they should provide a basis for accommodation of non-construction costs and whatever requirements are not explicit in the model.

Ideally, the alternate solutions should contain all configurations reasonably close to the optimum with respect to the value of the objective; each candidate subsystem should appear in at least one solution. The class of solutions identified combined with engineering judgment should provide for the selection of the best overall configuration.

The optimization model is much more costly to develop and exercise than an evaluative model. However, in principle, most of the extra burden can be absorbed by the model developer. Consequently, it is practical to produce software which is "user-friendly" to the extent that the additional requirements placed upon the user are primarily additional computational resources. While some user-supplied information is required, it does not require any change in user orientation, skills, or capability.

The procedures required can be characterized as cost-effective constrained optimization models. The obvious candidate methodologies are mathematical programming procedures. These include linear programming, integer programming, and various forms of non-linear programming.[1] The procedure to be used is dependent upon the form of the relations between the costs, the subsystem characteristics, and the performance requirements. The task of developing an optimization model could range from almost trivial to impossible depending on the form and structure of these relations.

2.1.3 Application of the Delphi Method

The model development process outlined earlier is the output of a consensus generating technique known as the Delphi Method. The Delphi Method in its present sense and as applied to the development of an equivalency methodology, consists of a panel of experts charged with achieving each of six objectives relative to the application of any resulting model. These objectives are safety, flexibility, workability, economy, choice, and accountability.

The first objective, safety, is the primary reason for regulations and consequently any alternative to prescriptive compliance must ensure that the health and well-being of the individuals occupying the building are not compromised. The second objective, flexibility, is designed to provide options which can be categorized according to some rule (e.g., cost, aesthetics, function, mission, etc.). The third objective, workability, implies that the system can in fact be used (e.g., no new education of inspectors or designers is needed since the system uses present well-understood principles.) The fourth objective, economy, explicitly recognizes that the introduction of alternatives to prescriptive compliance permits the intent of the regulation to be met at some savings in cost to the building owner. Greater economy in design and contruction should also result because an equivalency methodology permits innovations in the building process to be introduced more easily. The fifth objective, choice, is the building owners option; unlike the prescriptive compliance approach, the equivalency methodology produces classes of solutions which

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[1] An introduction to these techniques is given in Saul Gass, Linear Programming: Methods and Applications, Fourth Edition, McGraw-Hill, New York, 1975.

satisfy the regulation(s) under consideration. Building owners may have a variety of reasons for undertaking building renovation; by providing a class of solutions all of which satisfy the regulation(s), the building owner can select the retrofit alternative which best satisfies his needs. The sixth objective, accountability, is a requirement of the authority having jurisdiction. Since an equivalent solution is an alternative to prescriptive compliance, the building official(s) responsible for enforcing the regulation(s) must be assured that the solutions provided by the model are based on sound technical considerations and satisfy all of the requirements associated with the prescriptive solution. Another area of accountability concerns the granting of waivers. Traditionally, waivers have been granted when prescriptive compliance can be achieved only at undue cost to the building owner. It is believed that waivers will be granted on a more systematic basis under an equivalency methodology than in the past.

The task of achieving the six objectives just described is illustrated in figure 2.1. The figure shows the stages in the development of an equivalency methodology. It is designed to highlight the needs associated with analysis of the complex interactions expected in the model development process. The initial step, which is essential for success, involves the postulation of a model which governs the way in which the various regulatory requirements interact (i.e., are they independent, complementary or conflicting?). The next step is to select a universe of key components which span all regulatory requirements (e.g., window size and glazing type). Once all components have been identified, it is then necessary to subdivide each one into a set of conditions or states which span all likely possibilities. At this stage it is important to point out that the set of parameters must be large enough to include the component/state pair which corresponds to prescriptive compliance for each regulatory requirement under consideration. The next step is to assign weights to each component/state pair which best reflects the relative degree of risk or safety associated with that state. More specifically, negative values should be used to reflect greater risks whereas positive values should be used to reflect contributions toward a higher level of safety within the area under consideration. Ideally, the system would treat a value of zero as "safety neutral". In order to reduce the complexity of this step in the process, it is recommended that separate weights be assigned to each regulatory requirement under consideration. The postulation of a model in a way in which the requirements interact as a first step ensures that such an approach is technically valid. The final step is to reassess the weight assigned to each state for consistency with the interaction model, the list of components and the list of states. In the event that one or more of the previously mentioned attributes is found to be lacking or inconsistent, the sequence of steps following the inconsistency up to and including the entire process should be repeated. The level of performance associated with each regulatory requirement is calculated by first identifying a unique state for each component which corresponds to prescriptive compliance. This process is repeated for each regulatory requirement. The score, or weight, associated with this state is then recorded on a worksheet. The interaction model is then used to evaluate the level of performance associated with prescriptive compliance. The set of scores which results provides constraints which must be met or exceeded in order for the solution to satisfy the intent

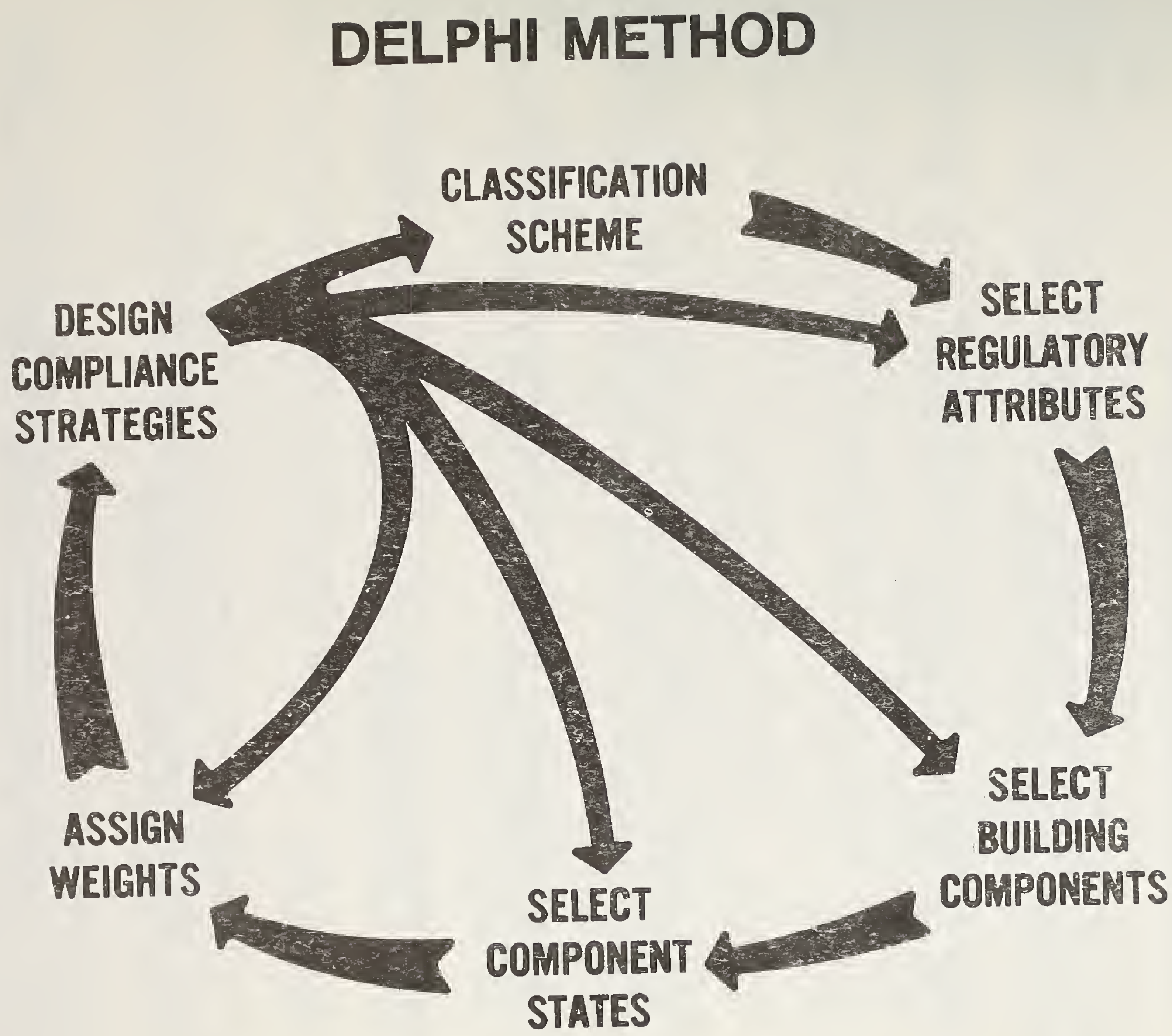

Figure 2.1 Stages in the Development of an Equivalency Methodology

of the regulatory requirements. Upon achievement of consensus on the entire process it will be beneficial for the members of the Delphi group to act as liaison among the various groups which formulate and promulgate building regulations. This step is necessary to demonstrate that an equivalency methodology is based on a sound technical basis which is broad enough to avoid problems in implementation.

2.2 STRUCTURE OF A GENERALIZED EQUIVALENCY METHODOLOGY

This section contains a series of tables (matrices) which reveal the basic structure for a generalized equivalency methodology. Prior to the development of this generalized methodology, however, it is necessary to define several items and state the assumptions upon which the analysis rests. The following terms will be used throughout the discussion.

(1) Building Component - any portion of the building or a building system for which a prescriptive solution is or can be defined for the regulatory requirements under consideration.

(2) State - a discrete level of performance for a particular building component.

(3) State Value - a numeric score associated with the level of performance of a particular state for a given requirement.

(4) State Variable - a variable which takes on a value of 1 if the building component is in that state and a value of 0 if it is not. It may take on positive integer values if the state admits multiples (e.g., the number of doors).

(5) Regulatory Requirement - the level of performance required by each requirement under consideration. It is assigned a numeric score which later becomes a constraint.

(6) Score Assessment - a numeric score associated with the level of performance provided within the building for each requirement under consideration. It is the sum of all state value/state variable products.

(7) Retrofit Cost - the cost of moving from any given or initial state to any other state.[1]

The following assumptions will be made in the discussion which follows:

(1) All regulatory requirements must be satisfied simultaneously in order for the building to be deemed in compliance.

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[1] Appendix A outlines how the engineering economics concept of a cost function coupled with probabilistic methods can be used to develop a cost structure for the problem.

(2) The score assessment is the sum of all state values for those building components which affect the regulatory requirements under consideration.

(3) With respect to a given requirement, each building component can be in one and only one state. (This assumption can be relaxed to accomodate design considerations.)

(4) The state of a building component is governed by a worst case condition for that component. (This assumption can be relaxed without loss of generality.)

As one might suspect, the structure of the generalized equivalency methodology will be rather complex. Fortunately, previous efforts in the area of fire safety have demonstrated that such a methodology can be adapted to computer optimization. The specific method which will be used in this exposition is the mathematical programming technique known as linear programming.[1] In its usual context, linear programming deals with the problem of allocating limited resources among competing activities in an optimal way. At the foundation of any linear programming problem is a mathematical model which describes the problem of concern. The term "linear" refers to the requirement that all mathematical functions in the model are linear.[2] The term "program" is used in the general sense in that it refers to a plan rather than a computer program *per se*.

The foundation of the linear programming model is shown in table 2.1, the building component/state score matrix. An examination of table 2.1 reveals that there are n building components, m states (levels of performance) associated with each building component[3] and p regulatory requirements associated with each component/ state pair. The state value associated with the i^{th} component, j^{th} state and k^{th} code is denoted V_{ijk} (see the cell removed from the building component/ state score matrix in table 2.1). As a general point of departure, it will be assumed that negative state values represent undesirable circumstances whereas positive state values represent desirable circumstances.

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[1] The requirement that all functions be linear is relaxed in appendix B where the general mathematical structure of the problem is presented.

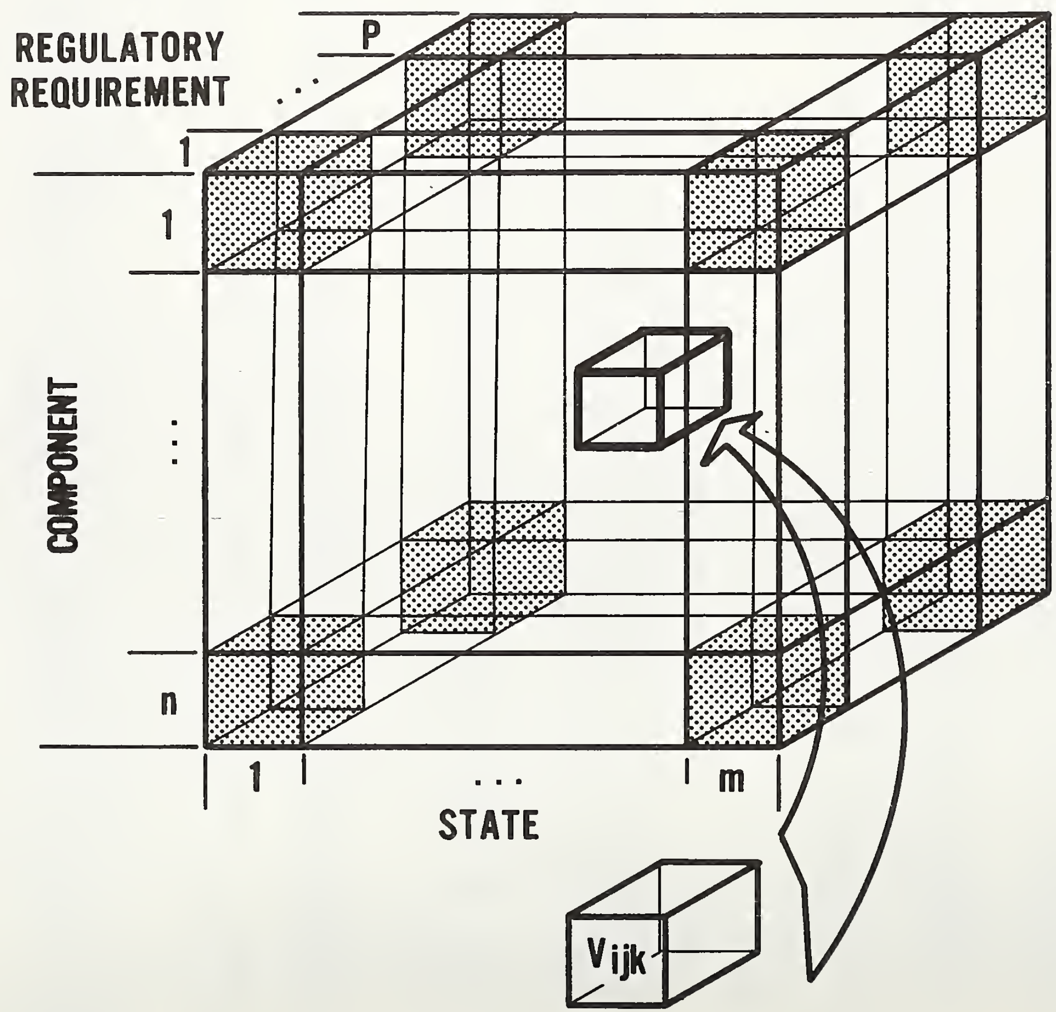
[2] A linear function is defined as a function of the form

$$f(X) = a_o + a_1x_1 + \dots + a_jx_j + \dots + a_nx_n$$

where a_j are coefficents not all zero and the x_j are variables. The geometrical representation of a linear function is a straight line, a plane, or a hyperplane. For example, $f(x) = a + bx$, a straight line, is a linear function whereas $g(x) = c + dx^2$, a parabola, is not.

[3] In reality there need not be an equal number of states for each building component. This stronger assumption is used only for purposes of expository clarity. It will be shown in appendix B that an unequal number of states can easily be accommodated in the mathematical formulation of the problem.

Table 2.1 Building Component/State Score Matrix



A state value of 0 is assumed to be neutral. Based on assumption 2 and the previous discussion it can be seen that a movement from the initial state, j_c, to some other state j, represents a potential retrofit if V_{ijk} is greater than V_{ij_ck}.[1]

Thus far, the mechanics of actually calculating the level of performance provided within the building for a particular code have not been discussed. Tables 2.2 and 2.3 provide the format for performing these calculations. Table 2.2 is a vertical slice of the building component/state score matrix. Note that table 2.2 is a matrix with n rows, one for each building component, and m columns, one for each possible state. An examination of table 2.2 would reveal that some cells of the matrix are shaded. This is due to the fact that each building component can be in one and only one state. In view of this fact and for expository clarity we can, without loss of generality, replace V_{11k} with S_{1k}, V_{2mk} with S_{2k}, and so on through $V_{n,m-1,k}$ with S_{nk}.[2] In order to perform the score assessment calculation, one would record the state values, S_{ik}, in the appropriate spaces in table 2.3. Once this task has been completed, it is only necessary to sum each column and enter the resultant score in the appropriate space in the row labeled "Total" in table 2.3. These scores are labeled P_1, P_2, ... P_p, indicating the level of performance provided within the building building for each of the p regulatory requirements under consideration.

The regulatory requirements are given in table 2.4. These numeric values are determined by identifying the state in table 2.2 which includes or is the prescriptive solution defined in each requirement. These values are then entered in the appropriate spaces in table 2.3 and summed in order to get the regulatory

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[1] It is of course possible that a movement to a higher state in one building component will affect the performance of another building component with respect to the same regulatory requirement. "Component interdependence" can usually be handled through the use of dummy state variables representing dummy states. A more important issue however, is "requirement interdependence" whereby a retrofit in one area may represent a retrograde in another. This relationship (if and when it exits) serves to constrain the feasible region to the class of bounded polyhedrons.

[2] At a more rigorous level, S_{ik} may be defined as follows:

$$S_{ik} = \sum_{j=1}^{m} V_{ijk} \cdot X_{ij}$$

where V_{ijk} = the state value associated with the i^{th} component, the j^{th} state and the k^{th} regulatory requirement; and

X_{ij} = the state variable

Since X_{ij} can only take on values of 0 or 1, S_{ik} is uniquely determined by the value of j, j', for which X_{ij} takes on a value of 1.

Table 2.2 Calculating the Score for a Particular Regulatory Requirement

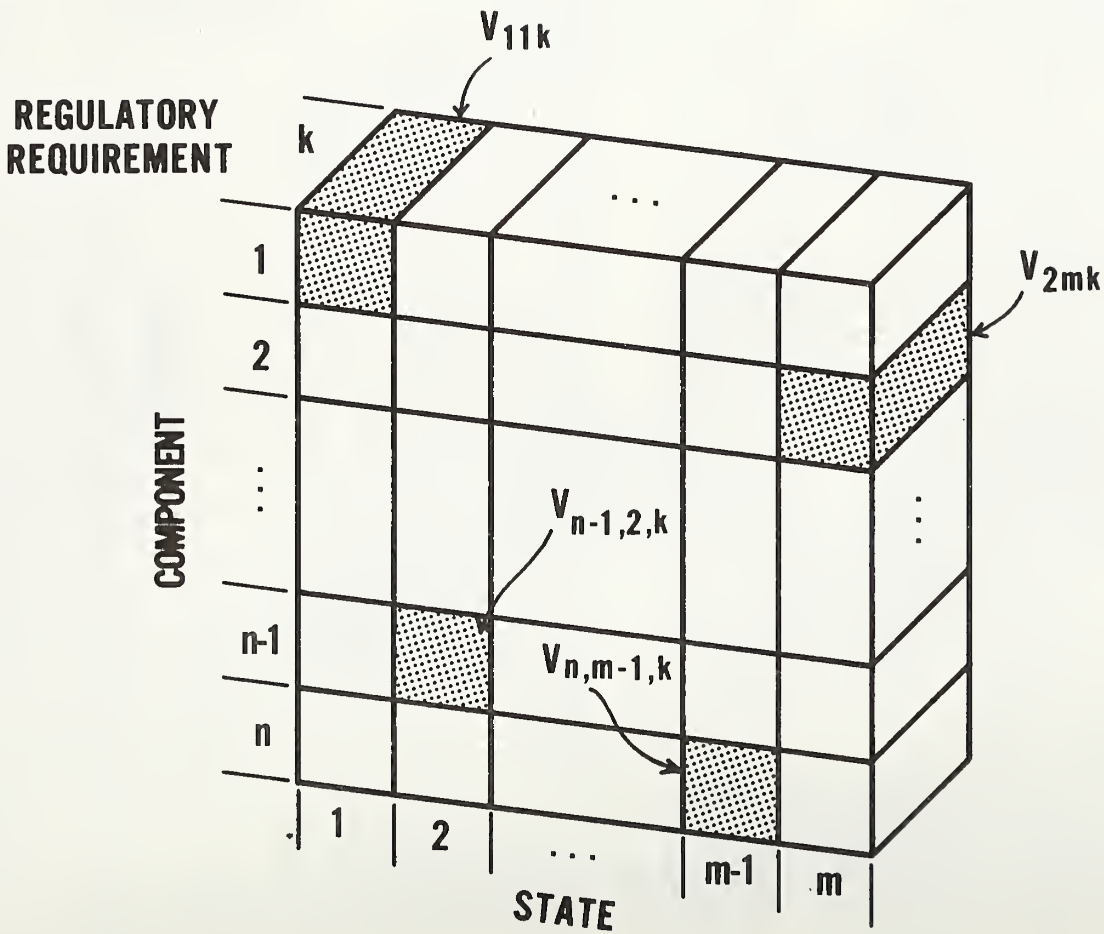

Table 2.3 Score Assessment Matrix

		Regulatory Requirement				
		1	2	. . .	P-1	P
Building Component	1	S_{11}	S_{12}	. . .	S_{1P-1}	S_{1P}
	2	S_{21}	S_{22}	. . .	S_{2P-1}	S_{2P}
	3	S_{31}	S_{32}	. . .	S_{3P-1}	S_{3P}
	⋮	⋮	⋮	⋱	⋮	⋮
	n-1	$S_{n-1,1}$	$S_{n-1,2}$	. . .	$S_{n-1,P-1}$	$S_{n-1,P}$
	n	S_{n1}	S_{n2}	. . .	S_{nP-1}	S_{nP}
Total		$P_1 =$	$P_2 =$	. . .	$P_{P-1} =$	$P_P =$

$$P_k = \sum_{i=1}^{n} S_{ik} = S_{1k} + S_{2k} + \cdots + S_{n-1,k} + S_{nk}$$

Table 2.4 Regulatory Requirement Matrix

	Regulatory requirement				
	1	2	3	. . .	p
Score	R_1	R_2	R_3	. . .	R_p

Table 2.5 Equivalency Test Matrix

		≥ 0	
Regulatory requirement	P – R = ☐	Yes	No
1	P_1 – R_1 ☐ – ☐ = ☐		
2	P_2 – R_2 ☐ – ☐ = ☐		
:	: : :		
p	P_p – R_p ☐ – ☐ = ☐		

requirements, R_1, R_2, ..., R_p. Note that the equivalency methodology is sufficiently general to accommodate variations in the level of performance required by different regulatory jurisdictions.

The final step, determining if the building is in compliance with each requirement, is outlined in table 2.5. This step is taken by transferring the first set of scores P_1, P_2, ... P_p from table 2.3 to the boxes labeled P_1, P_2, ... P_p, in table 2.5. The second set of scores R_1, R_2, ... R_p, is then transferred from table 2.4 to the boxes labeled R_1, R_2, ... R_p. Regulatory equivalency is tested by determining if the differences between the first set of numbers, P_1, P_2, ... P_p, and the second set of numbers, R_1, ... R_p, are <u>all</u> greater than or equal to zero.

In the event that one or more of the differences is negative, the building is deemed not to be in compliance with the regulatory requirement. At this point it becomes necessary to define a plan of correction or, more simply, a compliance strategy. Although retrofits can be defined based on score improvement alone, such an approach overlooks the potential of the linear programming procedure alluded to earlier. In order to illustrate how such a procedure would be applied in practice, a scaled down version of the generalized equivalency methodology was applied to a prototypical building. The results of this case study are the subject of Chapter 3; a mathematical statement of the optimization model is given in appendix B.

2.3 OPTIMAL COMPLIANCE STRATEGIES: A GRAPHICAL APPROACH

This section draws upon the mathematical formulation of the problem discussed in the previous section. A way in which alternative solutions are generated will also be presented. In order to focus on the main concepts, a geometrical, rather than an algebraic, approach will be used.

In order to illustrate how the mathematical programming approach used in the pilot study operates, an example is used in which it is assumed that the performance for each of the four regulatory requirements discussed in section 3.1 is a function of the level (e.g., size, hardware), of two techniques (e.g. building components). Limiting the discussion to two dimensions permits the basic concepts to be illustrated graphically. Problems like the generalized equivalency methodology just described would, however, be many dimensional. In the discussion which follows, the level of each technique is denoted by L_1 and L_2 respectively. In all cases, the level of technique "1" will be shown along the horizontal axis and the level of technique "2" along the vertical axis. More precisely, any movement out along the horizontal axis indicates higher levels for technique "1". Similarly, any movement up along the vertical axis indicates higher levels for technique "2". Consequently, if one were to construct a straight line which passed through the origin, any movement outward along that line (i.e., a move in the north easterly direction) would indicate a movement to higher levels for both techniques.

Turning now to figure 2.2, it can be seen that four sets of lines have been constructed. Each of these lines represents a regulatory requirement. All points on a given line have equal performance. The points of intersections of

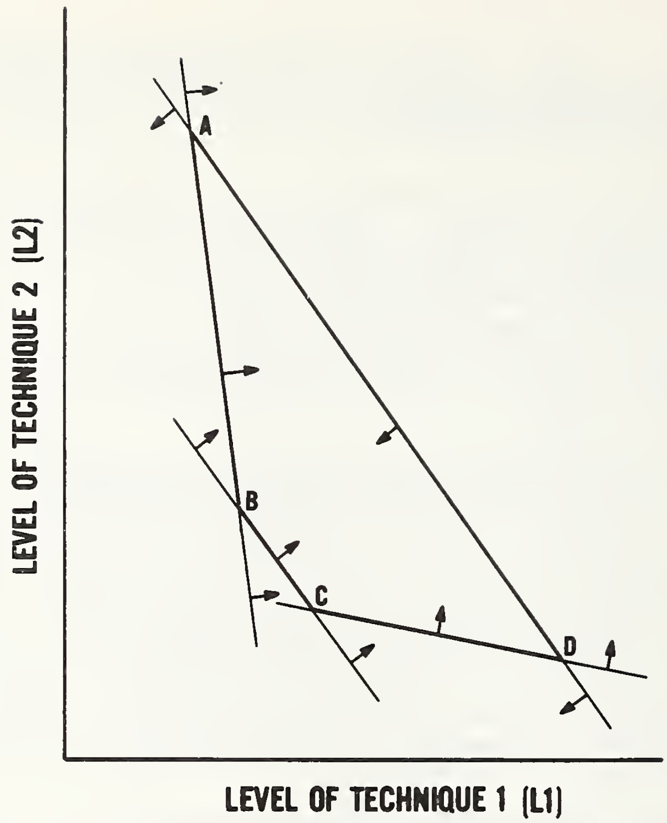

Figure 2.2 A Graphical Representation of the Constraints Associated with the Four Regulatory Requirements

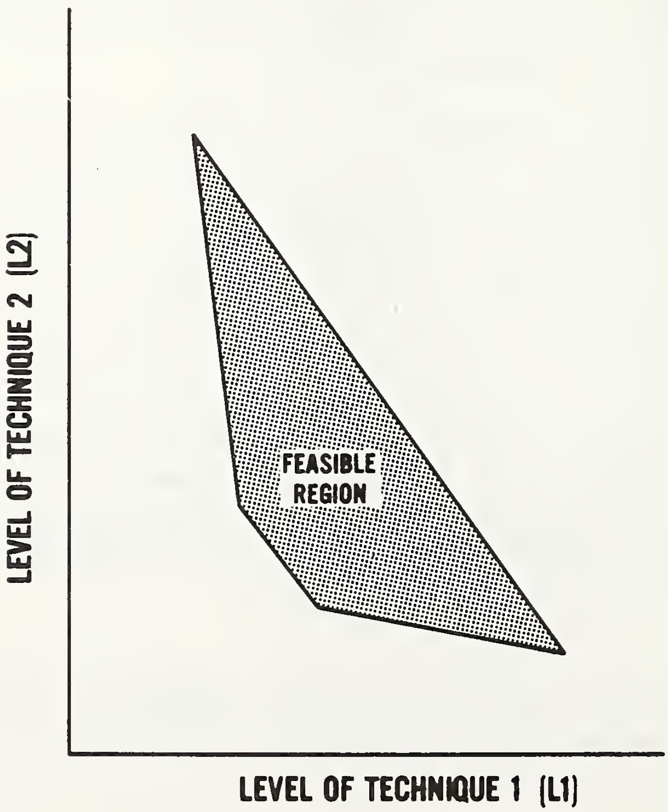

Figure 2.3 The Feasible Region Associated with the Four Regulatory Requirements

the lines are denoted as A, B, C and D. These points also serve to define a quadrilateral. Notice that associated with each line is a series of arrows. These arrows indicate the portion of the quadrant (all possible combinations of techniques "1" and "2" where L_1, $L_2 \geq 0$) within which that regulatory requirement is met or exceeded. A closer examination of figure 2.2 reveals that one of the sets of arrows points away from the other three sets. This is a graphical representation of a conflicting requirement. As discussed in section 3.1, the scores associated with security tended to conflict with some of the other regulatory requirements. If one takes the intersection of each set of potential solutions, the quadrilateral ABCD and its interior results. In mathematical terminology ABCD is referred to as a convex set since for each pair of points in the entire collection, the entire line segment joining these two points is also in the collection.

This intersection is shown in figure 2.3 by the lightly shaded region representing all the combinations of the various levels of techniques "1" and "2" which match or exceed each of the four regulatory attributes discussed earlier. The lightly shaded region in figure 2.3 is referred to as the feasible region since all points which lie along its boundary or within are technically feasible. The boundry excluding the vertices represents those combinations of the two techniques which exactly satisfy one of the regulatory requirements, while the vertices represent those combinations which exactly satisfy two regulatory requirements.

A graphical solution to the least-cost means of achieving compliance is shown in figure 2.4. The solution is illustrated via a series of equal cost lines. An equal cost line shows all the combinations of technique "1" and technique "2" which cost the same. It is drawn based on the assumption that the unit prices for technique "1" and technique "2" are constant. Higher equal cost lines imply greater costs are being incurred. By referring to figure 2.4, it can be seen that the first equal cost line, C_0, does not touch the shaded region, implying that it is not possible to achieve compliance with the funds being allocated. The second equal cost line, C_1, just touches a vertex of the feasible region. Since no other equal cost line, which is lower also touches the feasible region, it can be asserted that the point of contact is the least-cost combination of the two techniques which complies with all four regulatory requirements. Figure 2.4 also illustrates that if slightly more money were spent it would still be possible to hold the level of performance for one regulatory requirement constant (i.e., the requirement associated with line segment AB as illustrated in figure 2.2) and increase the other (i.e., the requirement associated with line segment BC.) Such a solution would lie on the boundary of the feasible region (i.e., where C_2 intersects the line segment AB). However, from the figure, it can be seen that it may be preferable to use those dollars to achieve a higher-level of performance for both requirements. Such a strategy would result in a point lying entirely within the feasible region such as "B'". Points like "B'" may have other desirable requirements, for example, they may refer to a standard product rating such as size or weight.

Notice that the move from point B to B' resulted in an increase in the overall performance for three of the four regulatory requirements. This is because the

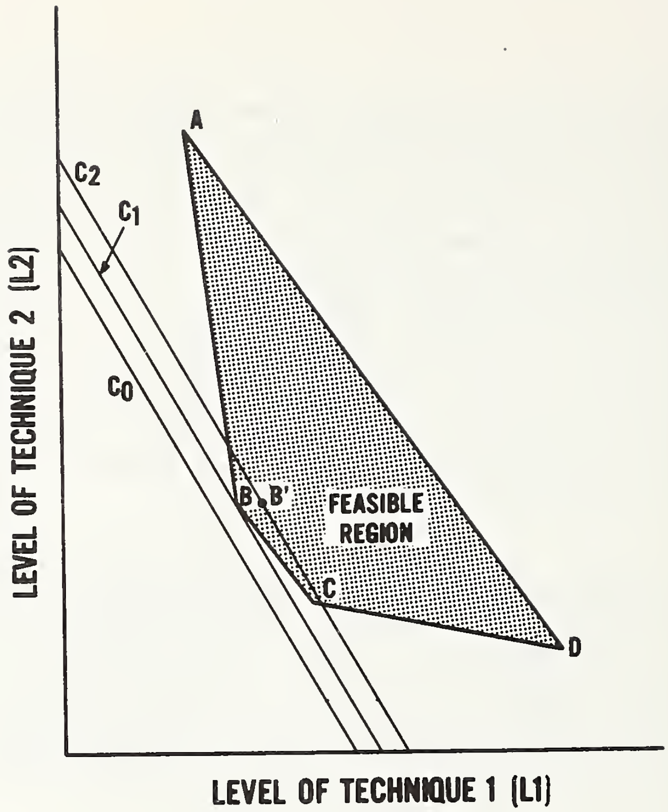

Figure 2.4 The Selection of the Optimal Compliance Strategy

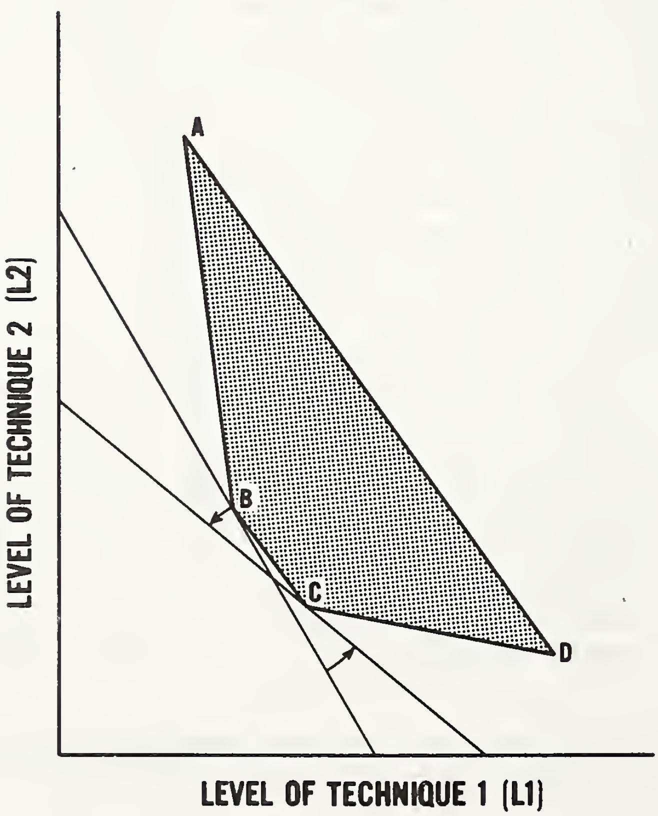

Figure 2.5 The Generation of an Alternate Compliance Strategy

constraint gradients associated with line segments AB, BC and CD (see figure 2.2) form acute angles with the cost gradient. The score associated with the fourth regulatory requirement, line segment AD, is reduced however. This is due to the conflict between security and the other three regulatory requirements (i.e., egress, ventilation and illumination). Conflicting requirements serve to constrain the size and shape of the feasible region and may, in some cases, reduce the potential savings associated with the use of an equivalency methodology. Generally speaking, these problems are most serious when building components are only available in a small number of fixed sizes or types. The implications of this additional constraint will be explored shortly.

As mentioned earlier, optimization models should also produce a set of alternate solutions which contain some configurations reasonably close to the optimum with respect to costs but differ from the optimum solution in terms of the post retrofit states. One way in which alternate solutions may be generated is shown graphically in figure 2.5. This procedure which generates two classes of alternate solutions was used in a previous study[1]. Only the first class is illustrated in figure 2.5. The optimal solution derived earlier is denoted by "B" in the figure. Let us define i as representing the technique index ($i = 1,2$) and j as representing the various states ($j = 1, \ldots, j^i_{max}$, where j^i_{max} is the maximal state of the i^{th} technique). Now if we define X_{ij} to be a state variable and it appears in the optimal solution, it is equal to one, otherwise it is equal to zero. Suppose X_{ij} appears in the optimal solution as a retrofit, then the cost, C_{ij}, of going from X_{ijc}, the preretrofit state, to X_{ij}, the post-retrofit state is made arbitrarily high.[2] This step guarantees that X_{ij} can no longer be in the optimal solution. Graphically this would correspond to a rotation of the equal cost line. The new solution would thus be the point at which the lowest of the new class of equal cost lines just touches the feasible region. Such a point is designed as "C" in figure 2.5. Exactly analagous is the case where X_{ij} did not appear in the optimal solution (i.e., $X_{ij} = 0$). In this case, the cost of going from X_{ijc} to X_{ij}, C_{ij}, is held fixed and all C_{ik}, $k \neq j$, are made arbitrarily high. This step guarantees that X_{ij} will appear in the optimal solution as a retrofit. The number of solutions generated in this class is equal to the number of variables in the original problem minus the number of retrofit states which are excluded for technical or other related reasons.

The second class of alternatives is determined by the number of techniques. For example, suppose X_{ij} appears in the optimal solution as a retrofit, then the cost of going from X_{ijc} to $X_{ij'}$, for all j', $j_c < j' \leq j^i_{max}$, is made arbitrarily high. This step guarantees that no potential retrofits associated with that technique can appear in the optimal solution. The number of solutions generated is equal to the number of techniques which moved to a higher state (were retrofitted) in the optimal solutions.

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[1] Robert E. Chapman, Phillip T. Chen and William G. Hall, op. cit.

[2] An operational definition of an arbitrarily high retrofit cost could be one for which its cost is n + 1 times the highest individual retrofit cost, where n is the number of building components.

The two classes of alternative solutions described above are useful because they are "close" to the optimal solution. "Close" is put within quotation marks because through reference to figure 2.5 it can be seen that the alternative solutions are adjacent vertices of the feasible region. Thus "close" means a small change in the basis (component/state pair combinations) rather than a small change in the objective function (retrofit costs). Although these solutions will be more costly than the optimal solution, some of them will be extremely close (in dollar terms) to the least-cost combination of retrofits.

All of the discussion so far has proceeded as if the levels of the techniques were continuously variable. In some cases this may not be correct since building materials often come in fixed sizes, weights, thickness, etc. Thus what must be solved is an integer program rather than a continuous program. There are, of course, classes of problems for which the continuous solution is actually the integer solution. In general, this is not true; figure 2.6 illustrates a case in which the solution to the continuous problem is not the solution to the integer problem. In figure 2.6 the grid points which satisfy all four regulatory requirements are shown as dots within the quadrilateral ABCD. Note that points "A" and "B" are not shown as dots since they are not grid points (i.e., both techniques do not take on integer values). Now if the cost of technique "1" with respect to technique "2" were quite high, producing an equal cost line steeper than the line segment AB, the optimal solution for the continuous problem would be point "A". Since point "A" is not a grid point, we may seek to round it up with respect to one technique, as shown by point "a", or with respect to both techniques, as shown by point "a'". Unfortunately, due to the conflicting requirements, neither of these points lies within the feasible region. If we now choose the nearest grid point to "A", point "A'", it is fairly easy to see (by sweeping an equal cost line rightward) that a point like "A"" would be less costly. If the optimal solution were at point "B", an ad hoc integerization would probably produce a feasible solution such as "B'" which may or may not be the optimal integer solution. Regardless of the integer characteristics of the optimal solution, it represents a lower bound on the total retrofit cost. It is obvious that the introduction of an additional constraint (i.e., each value for each variable must be integer) cannot improve the cost.

Fortunately, several arguments can be advanced which permit this general class of problems to be solved as continuous linear programs. The reason that a linear programming approach is attractive is due to: (1) the "near integer" property of the solutions; (2) the judicious selection of the family of alternative solutions; and (3) the computational cost, efficiency, and reliability of the algorithm. The near integer property occurs because of the structure of the constraint matrix. One criterion used in the design of the alternate solution family was that any variable appearing as a fraction in the optimal solution must appear in at least one of the alternates as a one and be suppressed in at least one of the alternates. Other criteria were that the alternates

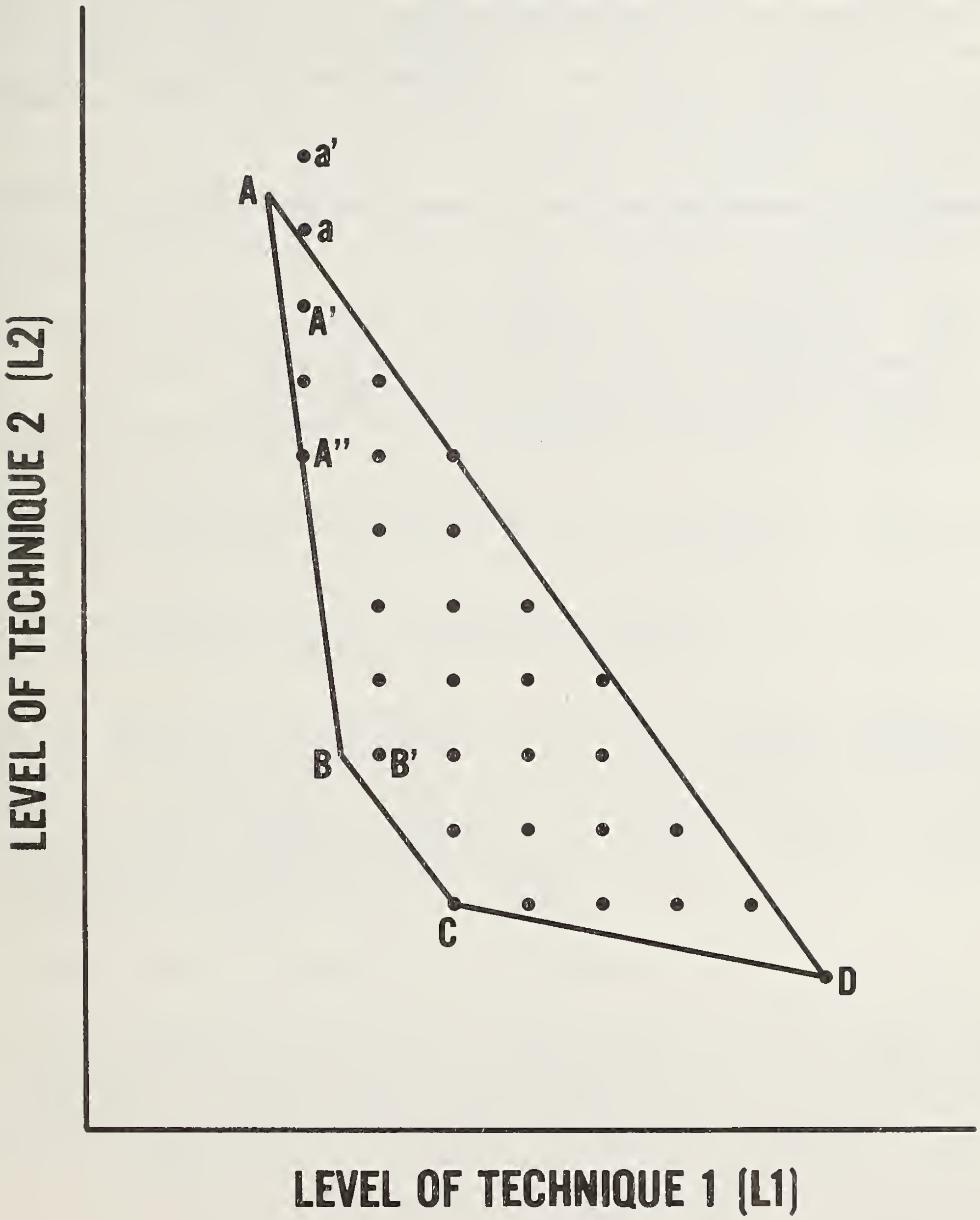

Figure 2.6 The Selection of Compliance Strategies when Performance Levels Vary Discretely

should be amenable to generation via a systematic but comprehensive specification, that they meet (at least our conception of) user needs, and that the number of alternates be adequate but not burdensome. Parametric linear programming techniques provide a convenient vehicle for achieving these objectives. To solve the optimization problem as an integer model is actually possible. The reasons for selecting a continuous model are completely pragmatic. Computation time for an integer solution can be expected to be an order of magnitude higher, however.

Thus the solutions presented, while not guaranteed to include the optimum integer solution, do contain:

(1) a bounding value on the retrofit cost;
(2) costs corresponding to the alternates most likely to be considered by the user; and
(3) one or more costs close to the integer optimum.

3.0 CASE APPLICATION OF THE EQUIVALENCY METHODOLOGY TO RESIDENTIAL REHABILTATION

It was necessary to design a case application which would test the innovative equivalency methodology formulated in this report. This included the selection of regulatory requirements and a prototypical building which could be evaluated to determine optimal compliance strategies based on cost considerations.

3.1 SELECTION OF REGULATORY REQUIREMENTS

Since this was a pilot study with time and funding constraints, it was necessary to select a narrow range of regulatory requirements and building components. This selection was based on an NBS study which provided a matrix showing the relation between these factors.[1] Figure 3.1 reproduces this matrix except that building components under "Enclosure and Space" have been specifically identified.

Criteria for the design of this pilot study included the following:

1. Regulatory requirements would be selected such that an improvement in one area could result in a negative effect in another area.

2. Regulatory requirements would be selected such than an improvement in one area could affect another area in the same way.

3. Building components would be selected such that they could contribute to compliance with the regulatory requirements.

As shown by the shaded areas in figure 3.1, the code requirements selected are ventilation, illumination, egress and security and the building components are doors and windows. Egress and security would be affected in a conflicting way by improvement in window and door security hardware (criterion 1); ventilation and illumination would be similarly affected by changes in window size (criterion 2); and window and doors can contribute to the level of compliance of all four regulatory requirements (criterion 3).

3.1.1 Applicable Building Regulatory Requirements

The selection of regulatory requirements which affect the design of windows and doors in existing buildings was based on a recent NBS study by Cooke[2]. The study was undertaken to identify and compare the levels of selected code provisions contained in seven model codified documents which deal with the occupancy, maintenance, and rehabilitation of existing residential buildings. The code

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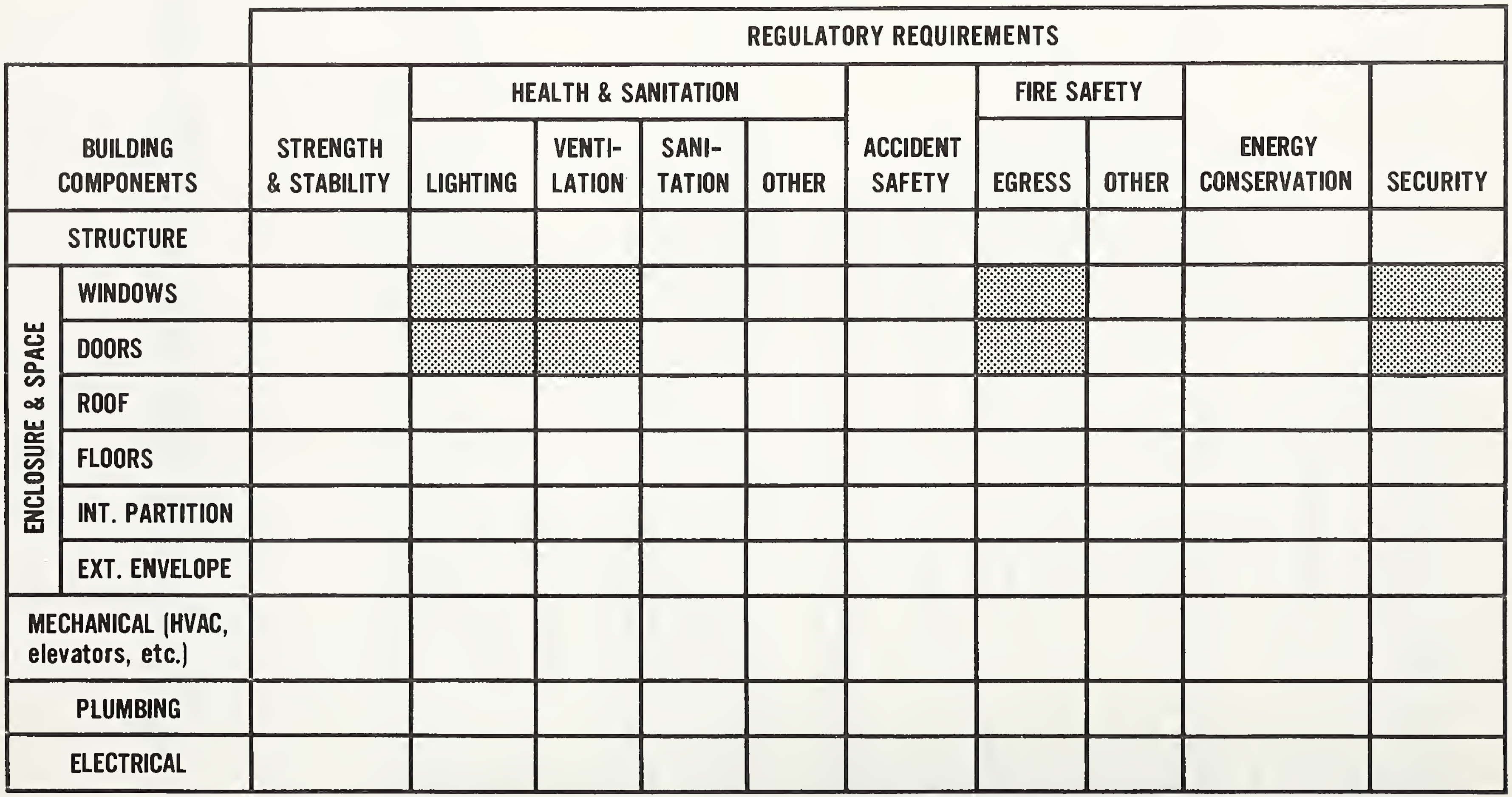
[1] James G. Gross, James H. Pielert and Patrick W. Cooke, op. cit.

[2] Patrick W. Cooke, Comparison of Selected Codes and Standards Relating to Existing Residential Buildings, National Bureau of Standards, NBSIR 80-2081, July 1980.

Figure 3.1. Relationship of Regulatory Requirements and Building Components



NOTE: Shaded areas represent regulatory requirements and building components treated in the pilot study provisions are compared and analyzed in sixteen major code areas. Areas considered which are relevant to this case study are windows and doors, means of egress/exits, light and illumination, and ventilation. Hardware and physical security are covered in the section on "windows and doors."

A conclusion of the report is that windows and doors do not receive extensive coverage in most of the codes studied with regard to such factors as exterior protection, energy conservation, maintenance of glazing, and physical security. Practically all the codes directly integrate their ventilation requirements with those provisions for light, since, in many cases, the natural ventilation requirements are a function of the window area requirements for natural lighting. Except for a very few instances, however, no specific illumination levels are provided. The study finds that regulatory requirements for providing and maintaining means of egress from existing buildings are in some cases expressed in such generalized terminology as to be vague and, in other cases, the provisions are very specific (e.g., clearances, headroom). However, requirements are not consistent among the codes and several significant differences exist regarding number of required exits and clearances for emergency egress.

Because of the inconsistency of the coverage of the regulatory requirements being considered in this case study as shown by the Cooke report, it was decided to develop a hybrid list of requirements for windows and doors which could be used to evaluate the equivalency methodology which was formulated for illumination, egress, ventilation and security. An additional resource in establishing these requirements were those used by the City of Baltimore in rehabilitating abandoned city owned housing.[1]

Table 3.1 shows the window characteristics and related regulatory requirements which have been selected for the prototypical building. The characteristics are of two general categories: (1) configuration/construction (size, type, hardware, glazing, screen type, and percent of floor area); and (2) vertical location in the exterior envelope (height above grade and height of sill). The right column of the table indicates the prescriptive regulatory level for each of the characteristics.

Table 3.2 shows similar information for exterior doors. Configuration/construction characteristics include size, type of glass, hardware, composition, number of exterior doors, and the presence of storm doors. The prescriptive regulatory levels were selected in the same way as described above for windows.

3.1.2 Sample Worksheets Developed for the Pilot Study

The regulatory requirements highlighted in figure 3.1 and outlined in the previous section are presented in greater detail in exhibits 3.1 through 3.4. Each exhibit focuses on a single regulatory requirement; 3.1 is concerned with security, 3.2 with egress, 3.3 with ventilation, and 3.4 with illumination.

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[1] Housing Authority of Baltimore City - Construction and Buildings Inspection, Construction Standards for Housing Renewal Program, August 1978.

Table 3.1 Window Characteristics and Related Regulatory Requirements

CHARACTERISTICS		PRESCRIPTIVE LEVEL
Size	<20 in. Height or < 20 in. Width 20-40 in. Height and > 20 in. Width 40-60 in. Height and > 20 in. Width* >60 in. Height and > 20 in. Width	° at least one window per habitable room ° minimum 5.7 sq. ft. net clear opening ° 20 in. minimum width of opening
Type	slider casement single hung double hung* fixed	° double hung windows shall be used ° units shall be fully weatherstripped and counter balanced for easy operation
Hardware	latch locking latch* dead bolt	° hardware to be openable from inside ° units shall be easily opened and held in position by window hardware
Glazing	single double (1 frame)* double (2 frames) triple (2 frames)	° provide double glazing or equivalent
% Floor Area	<5% 5-10%* >10%	° illumination: aggregate glazing area of 10% of floor area ° clear ventilation area of not less than 5% of floor area
Above Grade	≤0 0-5 ft.* 5 ft.-8 ft. >8 ft.	° dwellings below fourth floor shall be provided with exterior door or window of such dimension to be used as means of emergency egress ° habitable rooms below grade to satisfy same standards as for rooms below grade-particularly with regard to light and ventilation
Height of Sill	0-24 in. 24 in.-44 in.* >44 in.	° minimum sill height of 24 in. ° maximum sill height of 44 in.
Screen	Yes* No	° provide screens that will effectively prevent the entrance of insects

* Prescriptive Regulatory Level Selected for Prototypical Building

Table 3.2 Exterior Door Characteristics and Related Regulatory Requirements

CHARACTERISTICS		PRESCRIPTIVE LEVEL
Size (Area)	<18 ft.^{2} 18-21 ft.^{2}* 21-25 ft.^{2} >25 ft.^{2}	Entrance doors shall be 1 3/4 in. x 3 ft. - 0 x 7 ft.-0 unless existing openings are smaller and conditions prevent the increase of opening.
Glass	1 lite 3 lite* 6 lite 9 lite none	Rear and side doors shall have three horizontal lites at the top section.
Hardware	lock set* dead bolt(1 key) dead bolt(2 key)	Exit doors to be easily opened from inside without a key.
Composition	hollow core solid wood* metal	Exterior doors shall be of solid core or solid lumber construction.
Number	1 2* ≥3	Two exits to the outside per dwelling for egress.
Storm Door	Yes* No	All entrance doors shall have installed a combination aluminum storm and screen door.

* Prescriptive Regulatory Level Selected for Prototypical Building

Exhibit 3.1 Sample Worksheet for Evaluating Security Requirements

COMPONENT	SECURITY				
1 WINDOW SIZE	< 20"W or < 20"H	> 20"W 20"-40"H	> 20"W 40"-60"H	> 20"W > 60"H	
	7	2	0	–2	
2 WINDOW TYPE	SLIDER	CASEMENT	S. HUNG	D. HUNG	FIXED
	–3	–2	0	0	10
3 WINDOW HARDWARE	LATCH	LOCKING LATCH	DEAD BOLT		
	–5	0	5		
4 WINDOW GLAZING	SINGLE	DOUBLE(1F)	DOUBLE(2F)	TRIPLE (2F)	
	0	0	3	5	
5 WINDOW SCREEN	YES	NO			
	0	–1			
6 WINDOW HEIGHT ABOVE GRADE	≤ 0'	0'-5'	5'-8'	> 8'	
	–3	0	2	5	
7 WINDOW SILL HEIGHT	0"-24"	24"-44"	> 44"		
	0	0	0		
8 DOOR SIZE	< 18	18-21	21-25	> 25	
	1	0	–2	–3	
9 GLASS IN DOOR	NONE	1 LITE	3 LITES	6 LITES	9 LITES
	6	1	0	–2	–4
10 DOOR HARDWARE	LOCKSET	1 KEY (DB)	2 KEY (DB)		
	0	2	6		
11 DOOR COMPOSITION	HOLLOW CORE	SOLID WOOD	METAL		
	–2	0	2		
12 SECOND DOOR	YES	NO			
	0	–5			

Exhibit 3.2 Sample Worksheet for Evaluating Egress Requirements

COMPONENT	EGRESS				
1 WINDOW SIZE	< 20"W or < 20"H	> 20"W 20"-40"H	> 20"W 40"-60"H	> 20"W > 60"H	
	–10	–1	0	2	
2 WINDOW TYPE	SLIDER	CASEMENT	S. HUNG	D. HUNG	FIXED
	2	–4	0	0	–10
3 WINDOW HARDWARE	LATCH	LOCKING LATCH	DEAD BOLT		
	5	0	–4		
4 WINDOW GLAZING	SINGLE	DOUBLE(1F)	DOUBLE(2F)	TRIPLE (2F)	
	0	0	–2	–5	
5 WINDOW SCREEN	YES	NO			
	0	3			
6 WINDOW HEIGHT ABOVE GRADE	≤ 0'	0'-5'	5'-8'	> 8'	
	–10	0	–1	–5	
7 WINDOW SILL HEIGHT	0"-24"	24"-44"	> 44"		
	4	4	–5		
8 DOOR SIZE	< 18	18-21	21-25	> 25	
	–1	0	1	2	
9 GLASS IN DOOR	NONE	1 LITE	3 LITES	6 LITES	9 LITES
	0	0	0	0	0
10 DOOR HARDWARE	LOCKSET	1 KEY (DB)	2 KEY (DB)		
	0	–2	–6		
11 DOOR COMPOSITION	HOLLOW CORE	SOLID WOOD	METAL		
	0	0	0		
12 SECOND DOOR	YES	NO			
	0	5			

Exhibit 3.3 Sample Worksheet for Evaluating Ventilation Requirements

COMPONENT	VENTILATION				
1 WINDOW SIZE	< 20"W or < 20"H	> 20"W 20"-40"H	> 20"W 40-60"H	> 20"W > 60"H	
	−2	−1	0	2	
2 WINDOW TYPE	SLIDER	CASEMENT	S. HUNG	D. HUNG	FIXED
	−1	5	−1	0	−10
3 WINDOW HARDWARE	LATCH	LOCKING LATCH	DEAD BOLT		
	2	0	−3		
4 WINDOW GLAZING	SINGLE	DOUBLE(1F)	DOUBLE(2F)	TRIPLE (2F)	
	0	0	0	−2	
5 WINDOW SCREEN	YES	NO			
	0	−5			
6 WINDOW HEIGHT ABOVE GRADE	≤ 0'	0'-5'	5'-8'	> 8'	
	−5	0	1	2	
7 WINDOW SILL HEIGHT	0"-24"	24"-44"	> 44"		
	−1	0	1		
8 DOOR SIZE	< 18	18-21	21-25	> 25	
	−1	0	1	2	
9 GLASS IN DOOR	NONE	1 LITE	3 LITES	6 LITES	9 LITES
	0	0	0	0	0
10 DOOR HARDWARE	LOCKSET	1 KEY (DB)	2 KEY (DB)		
	0	0	0		
11 DOOR COMPOSITION	HOLLOW CORE	SOLID WOOD	METAL		
	0	0	0		
12 SECOND DOOR	YES	NO			
	0	−4			

Exhibit 3.4 Sample Worksheet for Evaluating Illumination Requirements

COMPONENT	ILLUMINATION				
1 WINDOW SIZE	< 20"W or < 20"H	> 20"W 20"-40"H	> 20"W 40"-60"H	> 20"W > 60"H	
	–3	–1	0	4	
2 WINDOW TYPE	SLIDER	CASEMENT	S. HUNG	D. HUNG	FIXED
	1	0	0	0	0
3 WINDOW HARDWARE	LATCH	LOCKING LATCH	DEAD BOLT		
	0	0	0		
4 WINDOW GLAZING	SINGLE	DOUBLE(1F)	DOUBLE(2F)	TRIPLE (2F)	
	0	0	–1	–2	
5 WINDOW SCREEN	YES	NO			
	0	1			
6 WINDOW HEIGHT ABOVE GRADE	≤ 0'	0'-5'	5'-8'	> 8'	
	–5	0	1	2	
7 WINDOW SILL HEIGHT	0"-24"	24"-44"	> 44"		
	0	0	0		
8 DOOR SIZE	< 18	18-21	21-25	> 25	
	0	0	0	0	
9 GLASS IN DOOR	NONE	1 LITE	3 LITES	6 LITES	9 LITES
	–2	–1	0	+2	+4
10 DOOR HARDWARE	LOCKSET	1 KEY (DB)	2 KEY (DB)		
	0	0	0		
11 DOOR COMPOSITION	HOLLOW CORE	SOLID WOOD	METAL		
	0	0	0		
12 SECOND DOOR	YES	NO			
	0	0			

The four exhibits are designed as worksheets capable of defining a level of performance for each regulatory requirement. This task is accomplished by subdividing each building component into a set of states which cover all possible situations expected to be encountered in practice. For example, window type includes slider, casement, single hung, double hung and fixed as states. Associated with each component/state pair is a value which represents the contribution of this pair to meeting the overall objective; i.e., attaining the level of performance required by the regulatory requirement under consideration.[1] In the context of the pilot study, negative state values detract from the performance goal whereas positive state values contribute towards the attainment of the performance goal. A state value of zero is goal neutral. It should be recognized that a zero value can occur both for a particular component/state pair or for all states within a component. An example of the second issue can be seen in exhibit 3.4 where all classes of window hardware and door hardware are goal neutral with respect to illumination requirements.

The level of performance provided within the building for each regulatory requirement is calculated by first recording the number of units occurring within each component/state pair. The state values for all units within each component are then summed. The level of performance for the regulatory requirement under consideration is then computed by summing across all components. As shown in appendix B, the unit counts are subject to a set of constraints which require the total number of units to be equal across components 1 through 7 (window components) and across components 8 through 12 (door components). These constraints are designed to impose a set of consistency requirements. Compliance with each regulatory requirement is then checked by determining if the performance score associated with all component/state pairs matches or exceeds the performance score associated with prescriptive compliance. It is important to point out that a satisfactory retrofit strategy is one in which <u>all</u> performance scores match or exceed that which results from prescriptive compliance.

An examination of exhibits 3.1 through 3.4 reveals that improvements in security often exert a negative impact on egress, ventilation or illumination (e.g., window size and glazing). This conflicting requirement is thought to be fairly common in rehabilitation activities and was discussed earlier as a driving force in the design of the pilot study. Another issue relates to the close score relationship between ventilation and illumination, where the same general trend in state values is seen across most components. The final issue relates to the basic structure of the system. As pointed out earlier, each component was selected so that it contributed to goal attainment. (More precisely, the system was constructed so that each component contributed in an additive or linear fashion.) In order to illustrate the flexibility of the equivalency approach, however, it was necessary to embed the pure prescriptive

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[1] The state values shown in exhibits 3.1 through 3.4 were assigned by the project team based on their knowledge of the subject areas. There was no attempt made to utilize a formal Delphi technique in setting these values. The purpose of the study was to evaluate the applicability of equivalency methodologies for the regulatory areas considered and was not intended to be a comprehensive technical treatment.

solution into the system. This was done by insuring that for each component one or more states bracketed the state corresponding to prescriptive compliance. In this manner some state values exceeded and some were exceeded by the prescriptive state's value. Combining this feature with anticipated rehabilitation costs permits one to identify optimal compliance strategies. An illustration of this feature was given in section 2.3. A detailed mathematical treatment is given in appendix B.

3.2 SELECTION AND ASSESSMENT OF A PROTOTYPICAL DESIGN

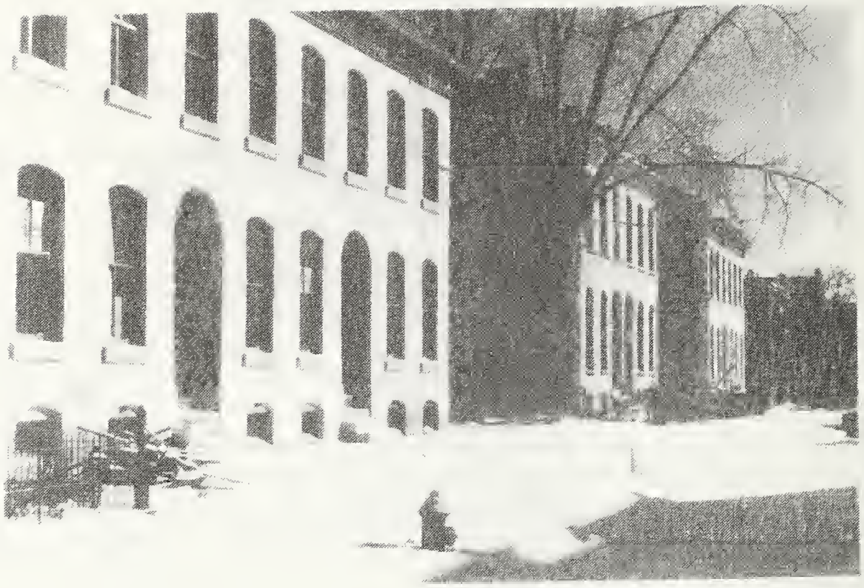
A prototypical design was selected to demonstrate the usefulness of the equivalency methodology developed. Specific aspects of the design include:

1. applicable building regulatory requirements (discussed earlier);
2. configuration of the prototypical building to be rehabilitated; and
3. typical window and door retrofits and related costs.

Typical inner city residential buildings in several major cities in the United States were visited and inspected in order to identify the relevant engineering and architectural design information required for the synthesis of a prototypical building. Figures 3.2 and 3.3 shows some of the typical buildings inspected in St. Louis and Baltimore. Photographs and sketches of candidate buildings were collected and analyzed and discussions were held with building officials and contractors involved with the rehabilitation of such buildings. Staff of the Construction and Building Inspection Department of the Housing Authority of Baltimore City were particularly helpful in allowing visits to housing undergoing rehabilitation in the city.

A prototypical building was then synthesized in a manner which provided a test of the equivalency methodology formulated in this case study. Figure 3.4 shows the basement plan, first floor plan and second floor plan of the prototypical single family townhouse, or rowhouse with residential units on both sides. The first floor contains a living room, dining room, kitchen and den; the second floor has three bedrooms and a bath; and the basement contains a utility room and storage area. There is a deck extending in the rear of the house at the first floor level. The building is assumed to be at least 50 years old, all exterior walls are brick bearing wall construction, and roof, floor and interior wall construction is timber. The occupancy of the building is assumed to comply with housing code requirements for structures of that size.

Figure 3.5 illustrates the front and rear elevations of the dwelling including the locations of windows and doors. All windows are of the steel casement type of the sizes and types indicated; doors are wood of the sizes indicated. All windows and doors are assumed to be in a severely deteriorated condition (e.g., not secure, lack of weathertightness) requiring replacement including all frames and jambs. Costs are estimated on this basis and include necessary masonry work in the exterior walls. Since this study is concerned with only the window and door components, the type and condition of the other building structural and mechanical systems need not be identified.



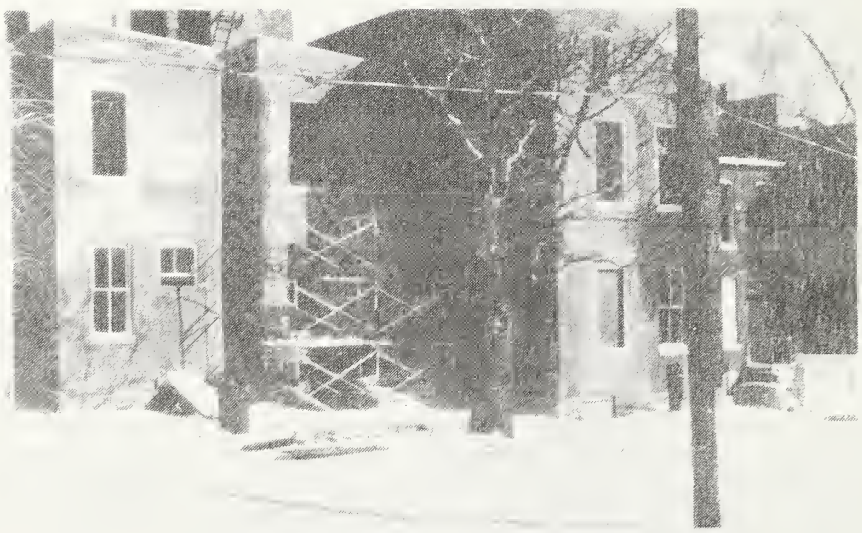
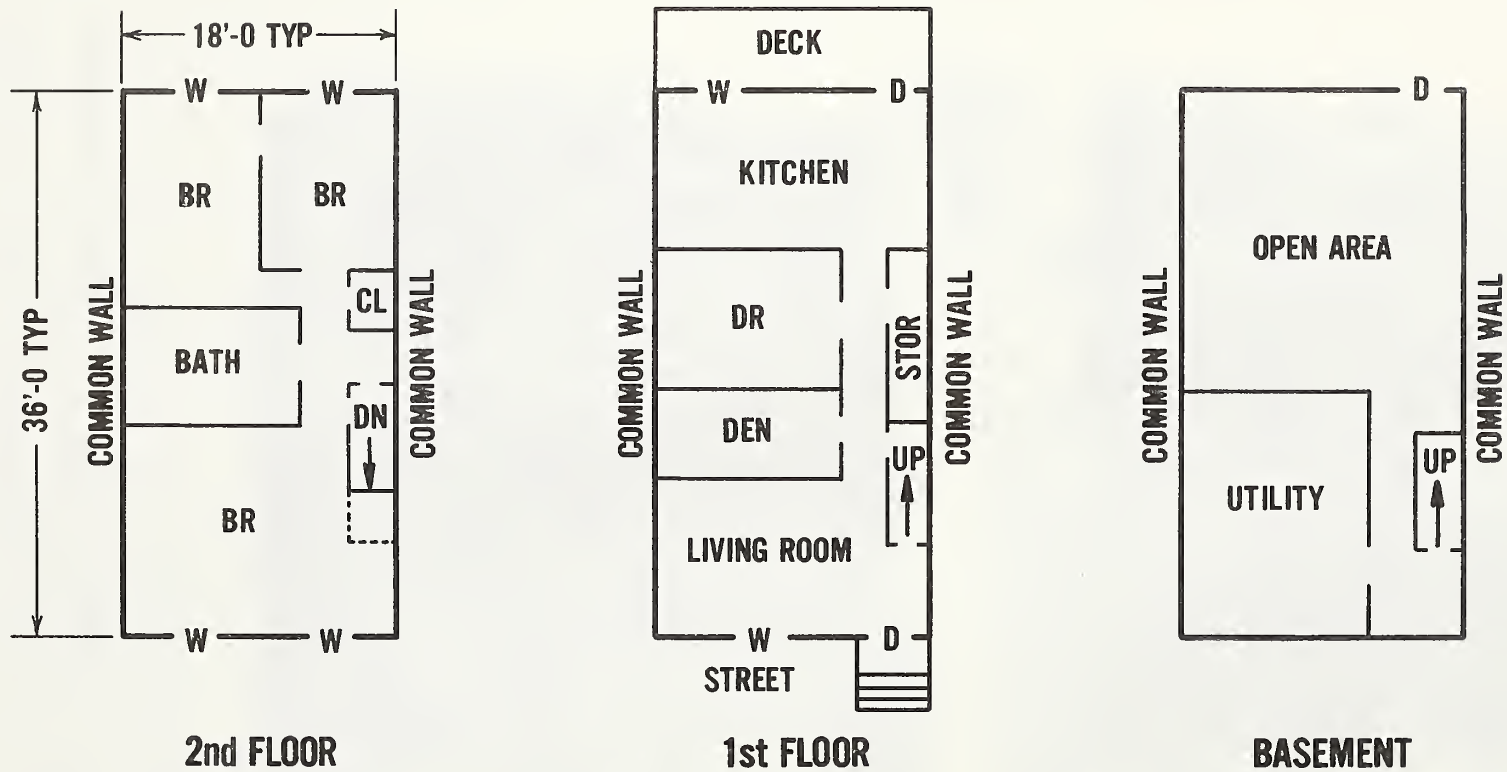


Figure 3.2 Typical Buildings Inspected in St. Louis, Missouri

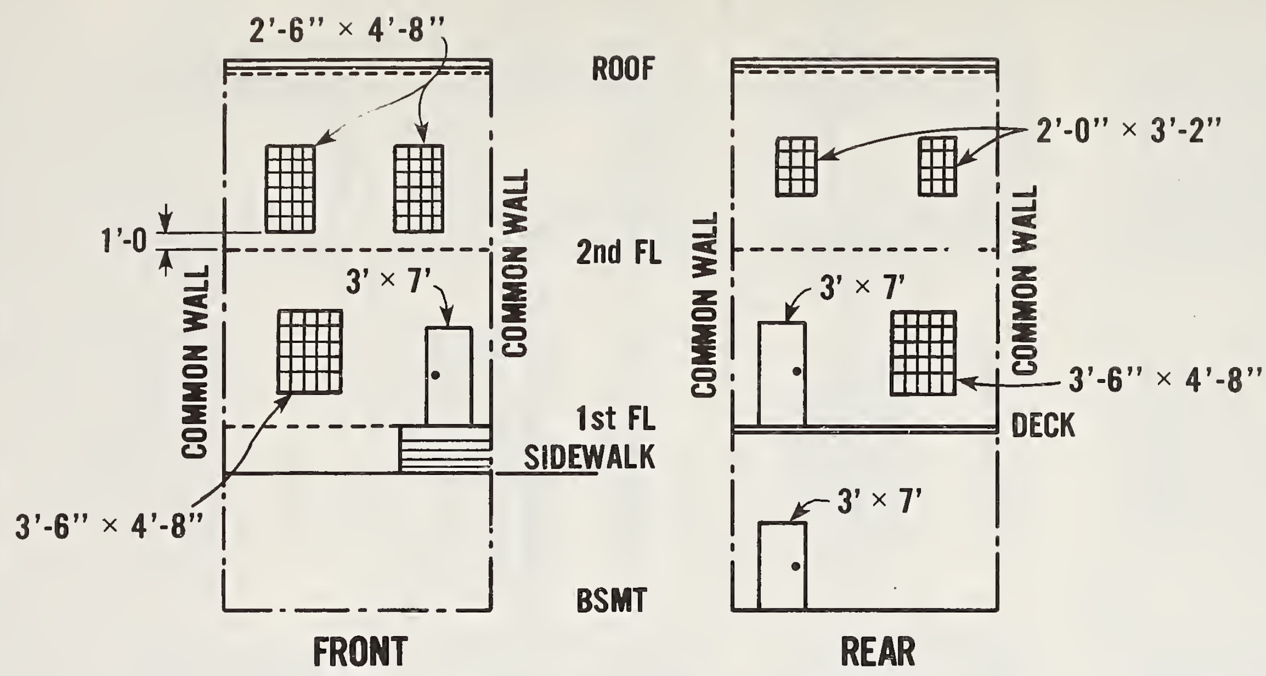
Figure 3.3 Typical Buildings Inspected in Baltimore, Maryland

Figure 3.4 Floor Plans for Prototypical Building



LIVING SPACE: 1300 sq. ft. (1st & 2nd fl.)
BASEMENT: 650 sq. ft.
OPEN AREA: 350 sq. ft.
DECK: 140 sq. ft.
D = Door
W = Window

Figure 3.5 Front and Rear Elevations of Prototypical Building



EXISTING CONSTRUCTION:

FLAT ROOF
BRICK EXTERIOR
STEEL CASEMENT WINDOWS
WOOD DOOR

It is necessary to identify potential window and exterior door retrofits and associated costs in testing the equivalency methodology. Since the case study was designed to evaluate the methodology and is not intended for use in actual rehabilitation projects, cost data were developed in-house by cost engineers. Sources of these data include the City of Baltimore Construction and Building Inspection Department, Building Construction Cost Data 1980[1] published by Robert S. Means Company, Inc., and discussions with designers and contractors involved with residential rehabilitation. Since very little cost data on actual rehabilitation projects were obtained, the resulting cost data should not be considered as representing current rehabilitation experience. It is felt that data used were developed in a consistent manner and that the trends and comparisons presented are valid.

Window and exterior door retrofits are classified into the following five categories as shown on figure 3.6

1. Adding a component where there is no existing opening,

2. blocking up a component opening including removal of existing component and closing of the opening in a manner compatible with the exterior facade,

3. replacing an existing component with one of the same size (component type may or may not be the same),

4. replacing an existing component with one of a smaller size (component type may or may not be the same), and

5. replacing an existing component with one of a larger size (component type may or may not be the same).

"In-place prices" for accomplishing these retrofits include the following factors; removal of existing component, addition or enlargement of an opening, waste removal, building materials, labor, installation, and finishing, refinishing an adjacent area, and mark-up for contractor overhead and profit.

3.3 SOME PRELIMINARY RESULTS

The computer program outlined in appendix B was applied to the prototypical building described in section 3.2. The purpose of this section is to summarize in a qualitative manner those costs associated with prescriptive compliance and the optimal cost solution.

Table 3.3 summarizes the window and door details for the initial prototypical building, the retrofitted building based on a prescriptive solution, and the configuration based on a near optimal cost solution. The costs associated with the optimal cost solution are about 29 percent less than those for the

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[1] Building Construction Cost Data 1980, Robert Snow Means Company, Inc., New York, 1979.

Figure 3.6 Typical Window and Door Retrofits

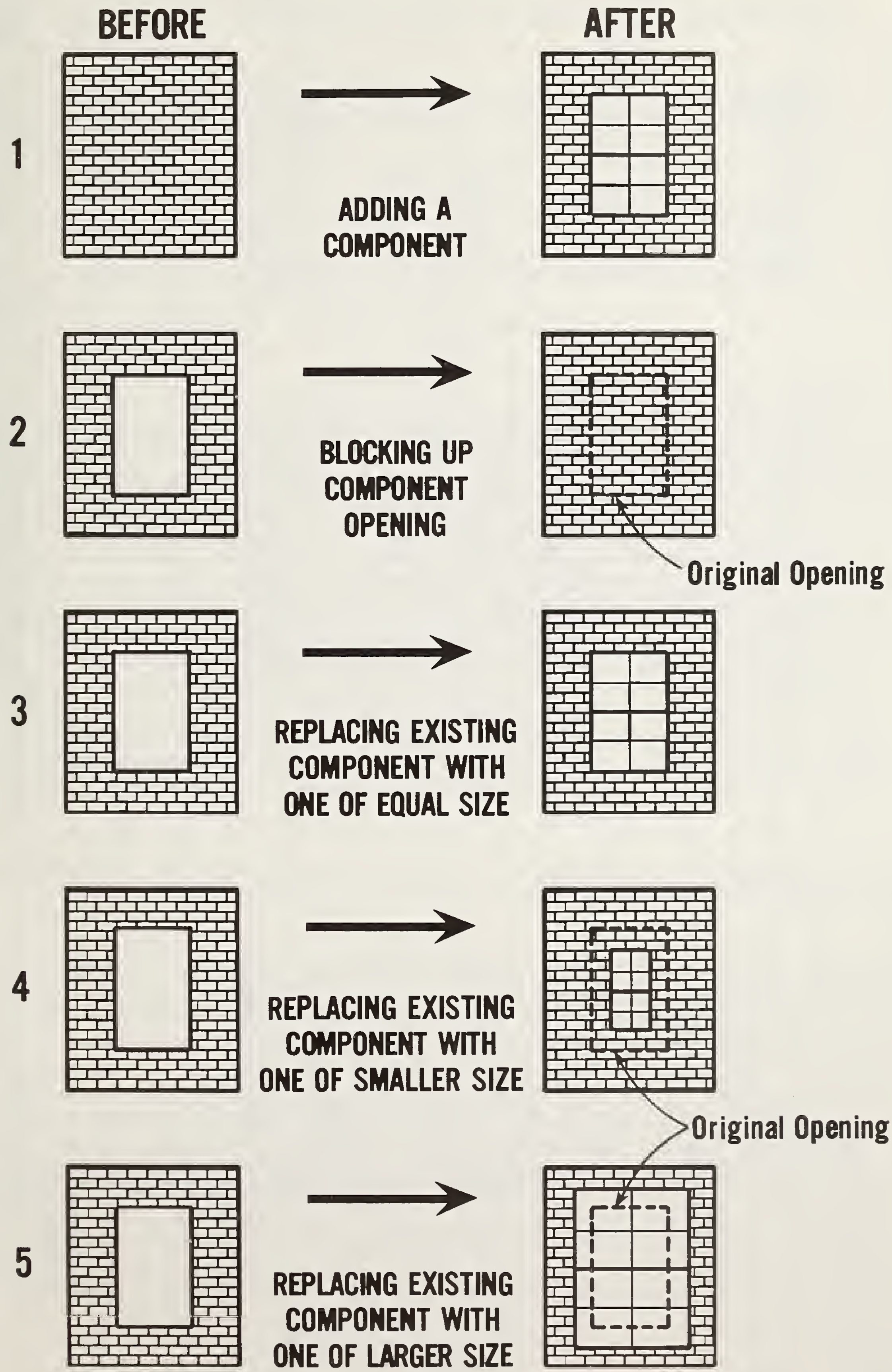

Table 3.3 Comparison of Initial, Prescriptive and a Near Optimal Solution for the Prototypical Building 2/

Component	Floor	Initial Window and Door Configuration 1/	Near Optimal Solution 4/	Prescriptive Solution
Windows	1	Front (1 opening >20"W, 40"-60"H) Rear (1 opening >20"W, 40"-60"H)	2 Casement, double glazed with single frame with dead bolt hardware (40"-60"H)	2 Double hung, double glazed windows with locking latch hardware (40"-60")
	2	Front (2 openings >20"W, 40"-60"H) Rear (2 openings >20"W, 20"-40"H)	2 Double hung, double glazed with double frame with latch hardware (40"-60"H) 2 Double hung, double glazed with single frame with latch frame hardware (20"-40"H)	4 Double hung, double glazed windows with locking latch hardware (40"-60"H) 3/
Screens	1	None	3 screens	3 screens
	2	None	3 screens	3 screens
Main Doors	1	2 openings (18-21 ft.^{2})	3 Metal doors with no glass with 1 key, dead bolt hardware (18-21 ft.^{2})	3 Wood doors with 3 glass lites with lockset hardware (18-21 ft.^{2})
	Basement Walkout	1 opening (18-21 ft.^{2})		
Second Door	1	None	None	3 Metal storm doors
	Basement Walkout	None		

1/ Refer to figures 3.4 and 3.5 for details.

2/ Window sill height and window height above grade assumed the same for all three solutions.

3/ Enlargement of two rear window openings required.

4/ Cost of near optimal solutions is 29 percent less than the prescriptive solution.

prescriptive solution. By referring to table 3.3, and comparing these two solutions, it can be seen that these savings are possible because the near optimal soution:

1. selects windows which conform to existing openings, thereby, avoiding the costly process of enlarging openings to accept windows which meet the minimum prescriptive level,

2. does not require upgraded security hardware on second floor windows over that normally provided,

3. upgrades the main exterior doors to metal construction with no glass and dead bolt (1 key) hardware, and

4. requires no second doors.

It is not possible to specifically identify the impact of these retrofit items on each of the regulatory areas under consideration because of the complex interactions in the optimization model. However, the state value excess over requirement (P_p - R_p shown in table 2.5) for the near optimal solution are 25 for security, 15 to egress, and zero for ventilation and illumination.

Other initial building configurations were analyzed with the computerized equivalency approach and similar cost savings in the 25 to 30 percent range were obtained. It should be noted that these figures represent only the four regulatory requirements; security, egress, ventilation and illumination. If the number of requirements and building components are increased to reflect the actualities of building rehabilitation activities this percentage may either increase or decrease. Also, broad based technical input was not possible in developing components states and state values shown on the worksheets. Application of Delphi techniques in formulating such data may effect results. Based on studies in the fire area, however, fairly substantial savings would still be expected.

Computer generated solutions such as those just discussed are designed to produce: (1) an optimal retrofit strategy; and (2) a series of alternative retrofit strategies.

Note that optimal as used here implies the minimum retrofit cost based on the original configuration. However, if the investor opts for a change in configuration, the new minimum retrofit cost solution is not neccessarily similar to the original configuration. Consequently, the solution corresponding to the minimum percentage of prescriptive compliance costs may deviate from the original configuration. By superimposing constraints on the rehabilitation process, the user can substantially increase the costs of prescriptive compliance. Since the computer generated retrofits address these user-imposed constraints in the most cost-effective manner, it is possible that the costs as a percent of prescriptive compliance may actually decline.

Furthermore, if all retrofit strategies are ranked on the basis of construction cost only, they do not include any recurring or nonrecurring costs which will take place in the future. Differences in the length of the renovation period are only treated through their effect on construction cost. In reality, longer renovation periods will result in lost revenues or increased housing expenses if the dwelling is to be rented or occupied by the investor. By the same token, computer printouts would not include a measure of aesthetic quality or professional design judgment, unless these attributes were reflected in the retrofit cost of upgrading to a particular state. For these reasons, it is important that the user of such computer programs carefully review the complete set of retrofit strategies with respect to any of the additional objectives outlined above in order to select the one which is optimal for the case at hand. For example, if one of the attributes under consideration was related to energy performance, one might opt for a higher initial cost in order to increase savings on future energy bills. Similar statements can be made about building aesthetics.

The previous discussion was not intended to leave the reader with a feeling that the final selection was an arbitrary one. On the contrary, different investors have different objectives which are not always reflected in renovation costs. In some cases, the additional constraints placed on the problem by the potential investor are purely subjective, and hence, not amenable to mathematical optimization. In other cases the costs (and benefits) of a particular retrofit strategy can be quantified. It is these cases where a clear-cut economic rationale can be applied. Stated more simply, if future costs (e.g., fire insurance, energy, water) are unlikely to vary across retrofit strategies, then other things being equal, the potential investor would be advised to select the least-cost solution. Any differences in amenities provided by one retrofit strategy over any other can be evaluated by the investor's willingness to pay for that amenity. On the other hand, if future costs are likely to vary across retrofit strategies, then a well proven building investment tool such as life-cycle costing should be used. Furthermore, if any differences in amenities exist between the retrofit strategy which minimizes life-cycle costs and any other, the economic viability of that strategy can be assessed by the user's willingness to pay for that amenity or set of amenities.

4. SUMMARY AND RECOMMENDATIONS FOR FURTHER RESEARCH

4.1 SUMMARY

The application of prescriptive type code provisions designed for new construction has been identified as a constraint when applied to the rehabilitation of existing buildings. The study which has been presented in this report addresses the need for a systematic procedure to evaluate the impact of a wide range of regulatory requirements on a building being considered for rehabilitation. This research is an extension of Fire Safety Evaluation System developed by the NBS Center for Fire Research (CFR); the Center for Applied Mathematics and the Center for Building Technology were a source of technical support to CFR in implementing the Fire Safety Evaluation System.

A computer based pilot equivalency methodology has been developed for identifying the least-cost means of achieving compliance with selected regulatory requirements applied to existing buildings. It has been shown that the use of a mathematical optimization model along with the Delphi Method to generate consensus on technical issues can be used to consider regulatory requirements which have conflicting impacts on building component design (e.g., window hardware criteria have a different effect on egress performance as compared to security performance). A case application of an equivalency methodology to the design of window and door retrofits in a residential rehabilitation project has been presented. This included the selection of the prototypical design, cost data for various retrofits, and regulatory requirements related to health and safety concerns including egress, security, ventilation and illumination.

A computer program developed as part of the study permits the effects of these specific regulatory requirements on various window and door compliance strategies to be identified and assessed. Comparing the costs of optimal compliance strategies with those of prescriptive compliance indicated potential saving ranging from 20 to 30 percent depending on the initial condition of the building.

4.2 RECOMMENDATIONS FOR FURTHER RESEARCH

The pilot study was conducted to assess the feasibility of applying an equivalency methodology to regulatory areas other than fire safety in the building rehabilitation process. The study was carried out with technical expertise available within the project team with no attempt made to use a broad base Delphi group of technical experts. With this in mind, the following recommendations are presented for broadening the scope of the pilot equivalency methodology and improving the technical bases.

A. Develop the Equivalency Methodology to Include a Broader Range of Regulatory Requirements and Building Components

This study has demonstrated the feasibility of the equivalency approach presented and the need for continued development. A comprehensive Delphi approach should be applied in selecting additional regulatory requirements and in providing technical expertise in defining performance levels. Interested members of the building community might be informed of the research and their support sought in providing technical assistance and implementation of results.

B. Develop Revised Classification Schemes for Building Component/ Regulatory Requirement Relationships

The classifications shown in Exhibits 3.1 through 3.4 are based primarily on dimensional and hardware considerations which may not truly represent component performance for the regulatory requirement and may cause problems from a computational perspective. For example, the window size classification shown in figure 3.1 for security is based on the width and height of the component. A better approach may be similar to that used to classify flame spread ratings of interior finish which is designated either Class A, B or C based on results of tests conducted using standard procedures. A result of the Delphi activity would be to provide technical guidance in presenting performance classifications in a more appropriate manner.

C. Illustrate Ways in Which Mathematical Models Can be Used as Instruments of Measurement

Engineering principles occupy a central role in the successful development of equivalency methodologies. A related but more fundamental concept is the use of mathematical models as instruments of measurement. More precisely, the performance of a system is a function of the levels of its subsystems and components. The axioms of logic and mathematics which provide a basis for the building of models impart a structure which ensures that the measurement of performance is carried out in a consistent manner. This structure also permits one to measure attainment or nonattainment of an engineering goal (e.g., compliance or noncompliance with a standard). Furthermore, since known goals can be embedded in the model (e.g., prescriptive compliance) a given model is capable of measuring attainment against a wide variety of goals. Thus given the model one need only specify the status quo, use the model to determine the level of performance associated with the status quo, and then set the goal at that level of performance. In this sense the model is acting as an instrument of measurement, one which is capable of comparing alternatives in an objective manner.

D. Use Econometric Techniques to Estimate a Set of Cost and Production Functions for Residential Rehabilitation Activities

The duality relationship between cost functions and production functions is an important criterion in choosing a technically sound and logically consistent method for estimating building rehabilitation costs. Unfortunately, the development of a series of cost functions for rehabilitation activities can only be accomplished through the application of econometric techniques to actual rehabilitation cost data. Past empirical studies have defined the type and nature of such cost data. It would be helpful if these guidelines could be followed in collecting a complete set of rehabilitation cost data. The use of probabilistic methods is an important tool for assessing the riskiness of a potential rehabilitation project. At the present time, little empirical evidence exists on the likely candidates for the probability distributions associated with a given factor in the rehabilitation process. It would be useful if such a study were undertaken in conjunction with the analysis of rehabilitation cost data. The use of equivalency methodologies is expected to have a substantial effect on the cost structure of the firm carrying out rehabilitation activities. However, due to the duality relationship mentioned earlier it can be asserted that equivalancy methodologies will also affect the production choices of the firm via the production function. More precisely, on theoretical grounds it can be shown that the regulatory structure facing the firm affects the ease with which inputs can be substituted for one another. Empirical studies relating the elasticity of substitution of the firm's production function to the regulatory structure facing the firm would thus be an important step towards understanding how regulations affect productivity.

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APPENDIX A. COST ESTIMATION AND COST VARIABILITY: AN APPROACH BASED ON COST FUNCTIONS AND PROBABILISTIC CONCEPTS

The purpose of this appendix is to show how the requirements of having a sound cost engineering approach can be integrated with the economic theory of cost functions. Combining both aspects of the problem results in a cost estimating procedure which is sensitive to the technical considerations of the renovation process as well as local market conditions. Furthermore, the development of such a procedure facilitates the treatment of risk through the use of probabilistic methods.

The cost function approach is highly desirable due to the duality relationship between the cost function and the production function associated with the physical process. (The term production function as used in this study refers to an explicit relationship between a set of inputs (i.e., labor, materials, and capital) and technological factors which taken together produce a given output, e.g., square feet of floor area renovated.)

This attribute has been documented in numerous economic and engineering economics articles.[1,2,3] Through reference to the duality relationship, it is possible to assert that the cost function tells us the least-cost way of renovating Q square feet of floor area. In an actual empirical study, the cost function associated with the underlying process would be derived from the production function by solving a constrained optimization problem.

In the discussion which follows, it is assumed that the prices of all inputs are independent. Under this assumption the quantity of output, Q, the square feet of floor area renovated, and the cost of output, C, may be expressed as:

$$Q = q(X_1, \ldots, X_n, \tau_o, \tau_1, \ldots, \tau_m) \qquad \text{A.1}$$

$$C = \sum_{j=1}^{n} X_j P_j \qquad \text{A.2}$$

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[1] Eugene Silberberg, The Structure of Economics: A Mathematical Analysis, McGraw-Hill Publishing Company, New York,1978.

[2] Gerald L. Musgrave and Robert H. Rasche, "Estimation of Cost Functions," The Engineering Economist, Vol. 22, No. 3, 1973.

[3] For an authoritative source on this subject see John S. McConnaughey, Production Functions in Contract Construction for the United States, 1972 (unpublished), Ph.D. dissertation, Michigan State University, 1976.

where

Q = total square feet of floor area to be renovated;

q = the basic functional relationship;

X_j = quantity of the j^{th} input (materials, labor, equipment) required to perform the job;

n = number of inputs considered;

τ_o = basic construction technology factor;

τ_i = construction technology factor associated with i^{th} building system or subsystem (e.g., plumbing, mechanical, electrical);

m = number of systems or subsystems considered;

C = the total cost (expected bid price) of the job; and

P_j = the unit cost of the j^{th} input.

The cost function associated with equation A.1 consists of three distinct factors: (1) a technology factor; (2) a size factor; and (3) a market factor. The technology factor is defined by the underlying construction/renovation process. That is, certain basic construction techniques (technologies) interact with the condition of the building's systems and subsystems in defining an approach which is feasible in the engineering sense.

The size factor may be expressed as the product of the number of structures being renovated, N, and the average number of square feet per structure renovated, q. This division is important because it permits the existence, or lack of existence, of economies of scale to be tested.

The market factor reflects the influence that supply and demand conditions in the local construction market for key labor, material, and equipment inputs have on the overall cost of the job.

The cost function based on these attributes may thus be expressed as:

$$C^* = c(TF, SF, MF) \qquad (A.3)$$

where C^* = the cost minimizing solution

c = the basic functional relationship

TF = the technology factor;

SF = the size factor; and

MF = the market factor

Statistical and/or econometric techniques would then be used to estimate the parameters associated with the basic functional form c.

Another related form of construction cost estimation which has grown rapidly as low-cost computer software packages have become available is probabilistic cost estimation.[1] The term probabilistic is a reflection of the fact that a probability distribution can be associated with each key factor in the renovation process. Once these probability distributions have been specified for each factor, it becomes possible to perform a monte carlo simulation of the cost estimation process.[2] The usual output of such a simulation is what is known as a cost profile. A cost profile may be defined as a graphical or tabular portrayal, for the given values of the data input, of the probability of overrunning any given budget estimate. Probabilistic cost estimating procedures are quite attractive because they can be applied to either an average cost method or a parametric cost estimating procedure. Since the application of probabilistic procedures to average costs is simpler than for a whole series of parameters, the discussion in this section will focus upon average cost methods.

Although no theoretical limitation exists on which probability distribution can be used in the simulation of the cost estimating process, most actual applications rely on four basic distributions. These distributions are: (1) the uniform; (2) the normal; (3) the log normal; and (4) the triangular.

These four probability distributions are plotted graphically in figure A.1. In each case the value of the random variable (average total cost for a particular subtask) is plotted along the horizontal (x) axis. The value taken on by the density function, f(x), is shown along the vertical axis on figure A.1. The mathematical form of each density function is given in table A.1. A nonmathematical interpretation of the density function is that it provides a measure of the frequency with which a certain event will take place for a given "small interval" along the x axis. Note that in figure A.1 the uniform and the triangular distribution both have well defined starting and stopping points. These distributions might be appropriate if average total cost was known to be at least $a but no more than $c. It is also important to point out that the log normal (of necessity) and the triangular (by construction) distributions can be skewed. That is, the tails of the distribution are of unequal length. For the distributions as drawn, it reflects the possibility that an extremely high value of the random variable (average total cost) can occur with non-zero probability. Such cost patterns are a common occurrence in the construction industry, where the most likely cost (the point where f(x) is a maximum) for a particular subtask may be rather low but due to the riskiness of the process, costs may take on a very large value with non-zero probability.

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[1] Michael Curran, "A Scientific Approach to Bidding: Range Estimating," Constructor, January 1975, pp. 27-33.

[2] Phillip F. Ostwald, Cost Estimating for Engineering and Management, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1974.

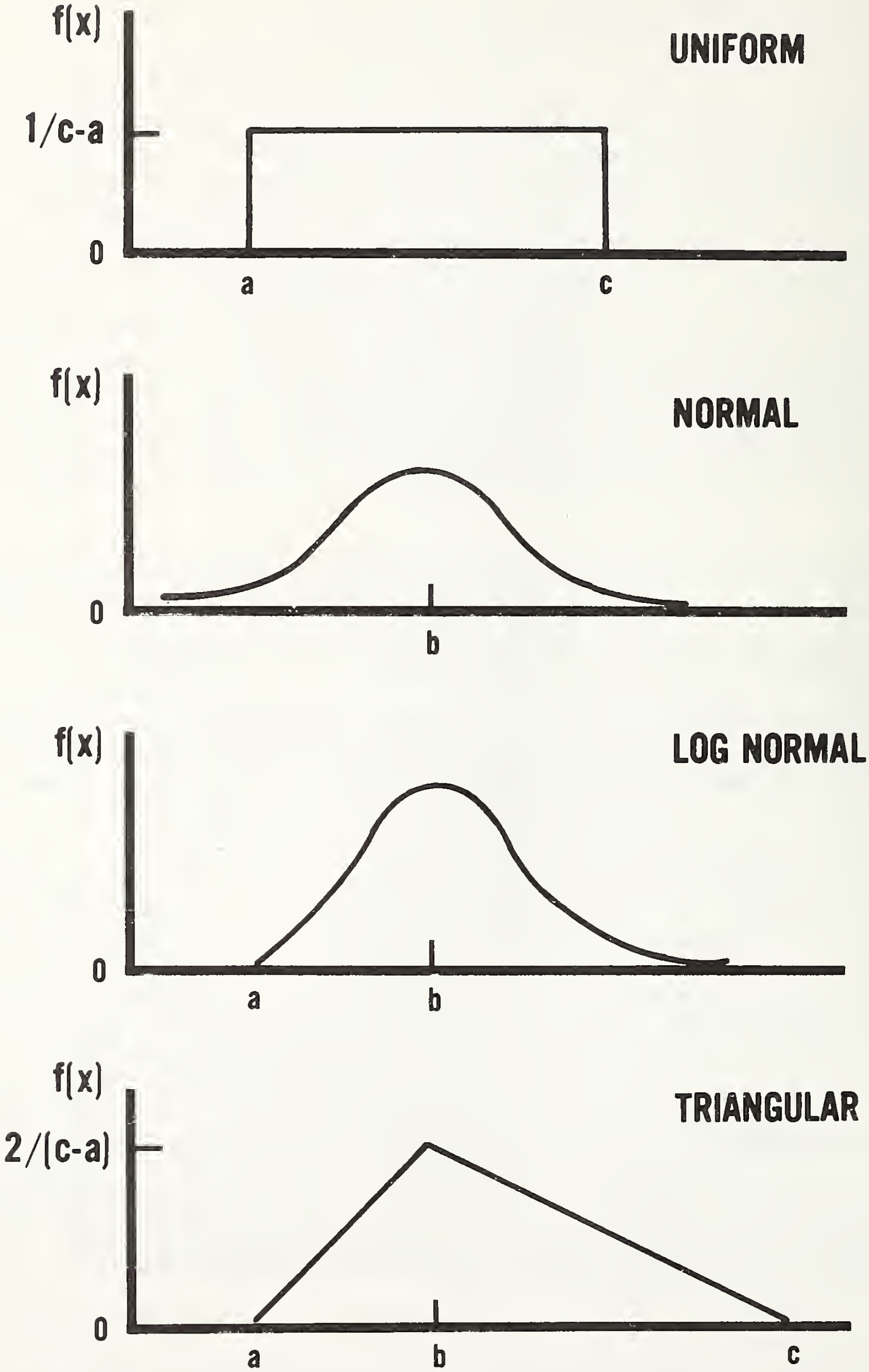

Figure A.1 Commonly used Probability Distributions for Key Construction Factors

Table A.1 Density Functions for Commonly Used Probability Distributions[a]

UNIFORM	
$f(x) = 1/(c-a)$	in the interval [a, c]
0	otherwise
NORMAL	
$f(x) = \exp(-(x-m)^2/2\sigma^2)/\sigma(2\pi)^{1/2}$	$-\infty < x < \infty$
LOG NORMAL	
$f(x) = 0$	$x \leq a$
$\exp(-(\ln(x-a)-m)^2/2\sigma^2)/(x-a)\sigma(2\Pi)^{1/2}$	$x \geq a$
TRIANGULAR	
$f(x) = 2x/(c-a)(b-a)$	in the interval [a, b]
$2x/(c-a)(c-b)$	in the interval [b, c]
0	othewise

[a] Source: Marek Fisz, Probability Theory and Mathematical Statistics, Third Edition, John Wiley and Sons, New York, 1963.

The crucial step which must be taken prior to the application of any probabilistic cost estimation procedure is to determine which distribution is appropriate for the case at hand. The first step is to collect cost data from similar projects or contact local building material suppliers and contractors. The actual cost of each subitem should then be recorded. Once all data has been collected it will then be necessary to group them into intervals. (A general rule of thumb states that at least five five data points should be in each interval.)

The number of times average total cost occurred within the interval may then be used to construct a histogram. The histogram may then be compared to sample standard histograms such as appear in the text by Hastings and Peacock.[1] Based on these comparisons, a class of distributions which might fit the data can be hypothesized. The next step is to develop maximum likelihood estimators for the parameters of the distribution (e.g., the mean and variance) under the assumption that the hypothesized distribution is correct.[2] The final step is to perform a goodness-of-fit test on the data. If the fit is unacceptable it will be necessary to hypothesize a new class of distributions and reestimate the parameters of the distribution.

It is important to recognize that if probability distributions are used to estimate renovation costs, the underlying assumptions implicit in that application must be understood. It is crucial to recognize that the distributions are ideal theoretical constructions whereas the data are from the "real world" and hence can not be "ideal." In some cases it may in fact be impossible to obtain a sample of actual average total cost figures for a particular subtask. In this case expert judgment can be used to choose the distribution. In such cases past researchers have recommended the use of the triangular distribution because it explicitly allows for low-probability high-cost events.[3] Furthermore, it may be completely defined by only three points: (1) the minimum point; (2) the maximum point; and (3) the mode or most likely point. (If one believed that costs were equally likely to be distributed throughout the interval then the uniform distribution would be appropriate. The uniform distributions may be completely defined by only two points: (1) the minimum point; and (2) the maximum point.)

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[1] N. Hastings and S.B. Peacock, Statistical Distributions, John Wiley and Sons, New York, 1975.

[2] The likelihood function of a sample, given a parameter, is the product of the density function with respect to the parameter at each sample point. For an excellent discussion see John Freund, Mathematical Statistics, Second Edition, McGraw-Hill, New York, 1970.

[3] Phillip T. Chen and Robert E. Chapman, "Budget Estimates for Placement of Plant and Facility Equipment at the National Bureau of Standards," ASHRAE Transactions 1981, Vol. 87, Pt. 1, pp. 1243-59.

The previous discussion has touched upon how the distributions would be applied in an actual monte carlo simulation to estimate the costs of residential rehabilitation. Although only two of the four distributions were discussed it can be easily shown that the normal and log normal distributions may be applied almost as simply in actual cost studies. The basic difference between the normal--log normal and uniform--triangular distributions is that the upper tail of the normal and log normal distributions extends to infinity. This is not true for both lower tails since the log normal distribution has a minimum point at a (a is greater than or equal to zero). The lower tail on the normal distribution has no minimum point however. Thus, in applying probabilistic cost estimating techniques, it is useful to specify three sets of numbers when a normal or log normal distribution enters the process. Each set of three numbers may be either estimated from actual data or based on the opinions of constructon experts. The first number needed is the "optimistic" estimate of average total cost. This estimate is called optimistic because there is only a 10 percent (subjective) probability that average total cost will fall below it. Mathematically, the optimistic estimate is the 10th percentile point of the distribution. The second number needed is the "middle ground" estimate of average total cost. The term middle ground is used because in 50 percent of the cases average total cost will fall below it and in 50 percent of the cases average total cost will fall above it. Mathematically, the middle ground estimate is the median of the distribution. The third number needed is the "pessimistic" estimate of average total cost. This estimate is called pessimistic because there is only a 10 percent (subjective) probability that average total cost will fall above it. Mathematically, the pessimistic estimate is the 90th percentile point of the distribution. In essence, once the distribution is specified the user need enter (at most) three numbers for each subtask into the computer software package. These numbers are summarized in table A.2.

Table A.2 Input Requirements for a Standard Probabilistic Cost Estimating Procedure

Distribution	Numerical Inputs
Uniform	1. Minimum Cost 2. Maximum Cost
Triangular	1. Minimum Cost 2. Maximum Cost 3. Modal or Most Likely Cost
Normal and Log Normal	1. Optimistic: 10^{th} Percentile 2. Middle Ground: 50^{th} Percentile 3. Pessimistic: 90^{th} Percentile

Assuming that the user has correctly specified the probability distribution and correctly input the information identified in table A.2, the software package will perform a monte carlo simulation. The term monte carlo is used to indicate that the process is patterned after several popular games of chance. Basically what the computer program does is estimate the cost of each subtask sequentially. This is done by generating a random number. Each random number will correspond to a value between 0 and 1. The random number is then associated with the parent probability distribution (i.e., the average total cost distribution for that subtask). Since each random number is between 0 and 1 it can be interpreted as the probability that the value of the random variable will be less than or equal to a specified amount. If we denote the random number as R, the random variable as χ and the specified amount as X then the expression can be written formally as

$$Pr\ (\chi \leq X) = R.$$

Since we are concerned with the average total cost of the subtask, the relevant cost is X. Thus, if there are n (say 100) subtasks we will get random numbers and n separate X's. Suppose there are N (say 1000) iterations. Then for the first iteration we get.

$$Pr\ (\chi \leq X_{1j}) = R_{1j} \qquad j = 1, \ldots n.$$

The estimated total cost for the overall job on the first iteration, TC_1, is thus

$$TC_1 = \sum_{j=1}^{n} X_{1j}.$$

Similarly, the estimated total cost for the overall job on the second iteration, TC_2, would be

$$TC_2 = \sum_{j=1}^{n} X_{2j}.$$

and for the i^{th} iteration

$$TC_i = \sum_{j=1}^{n} X_{ij}.$$

The computer will generate N estimates of total cost for the overall job. These estimates are then ranked from least costly to most costly. For example, the least costly estimate (i.e., the first order statistic) is denoted as $TC_{(1)}$. The parentheses are used to distinguish the first order statistic from the estimated cost on the first iteration. Since the total costs are ranked from least to most costly, it is possible to compute the probability that total cost will be less than or equal to a specified dollar amount. For example,

$$Pr\ (tc \leq TC_{(k)}) = k/N,$$

where tc = the random variable total cost

$TC_{(k)}$ = the k^{th} order statistic for the total cost; and

N = the number of iterations.

Conversely, the probability of exceeding $TC_{(k)}$, the projected budget, may be expressed as

$$Pr\ (tc > TC_{(k)}) = 1 - k/N$$

The process described above is most easily understood through reference to a cost profile. An example of a cost profile is shown in figure A.2..

Note that the total cost of the job is shown along with vertical axis. In this case, the probability of overrunning a specified budget is shown along the horizontal axis. For example, the probability of overrunning a $35,000 budget is 15 percent whereas the probability of overrunning a $30,000 budget is 30 percent. Thus, the perspective investor can specify a given level of risk (in terms of probability of overrun) and then choose a budget which will satisfy this constraint. In addition to dealing with risk, the use of probabilistic cost estimation permits the investor to more effectively manage any funds held for contingencies. That is, the investor may proceed with a basic contract which permits contingencies in terms of better quality products (say floor coverings) to be installed should the cost of the job fall below some agreed upon figure.

It is important to point out that the term monte carlo is quite appropriate since there is still some element of chance remaining. In essence there are no absolutes. Total costs will either exceed or fall below the projected budget. The measure of risk is only approximate. In particular, different sequences of random numbers can yield different estimates. Thus, one should be cautioned against demanding a one percent risk. On one simulation the figure might be $45,000 and on another $55,000. This is because each simulation (N iterations) yields a single estimate of the cost profile. Therefore, in order to get a more meaningful measure of the true risk being assumed, one should replicate the simulation using a different starting random number (seed). Should the investor be using the simulation to choose between two alternative methods of renovation, replications should certainly be performed. For an excellent discussion of this topic the interested reader is referred to the article by Law.[1]

[1] Averill M. Law, "Confidence Intervals in Discrete Event Simulation: A Comparison of Replication and Batch Means," Naval Research Logistics Quarterly, Vol. 24, 1977.

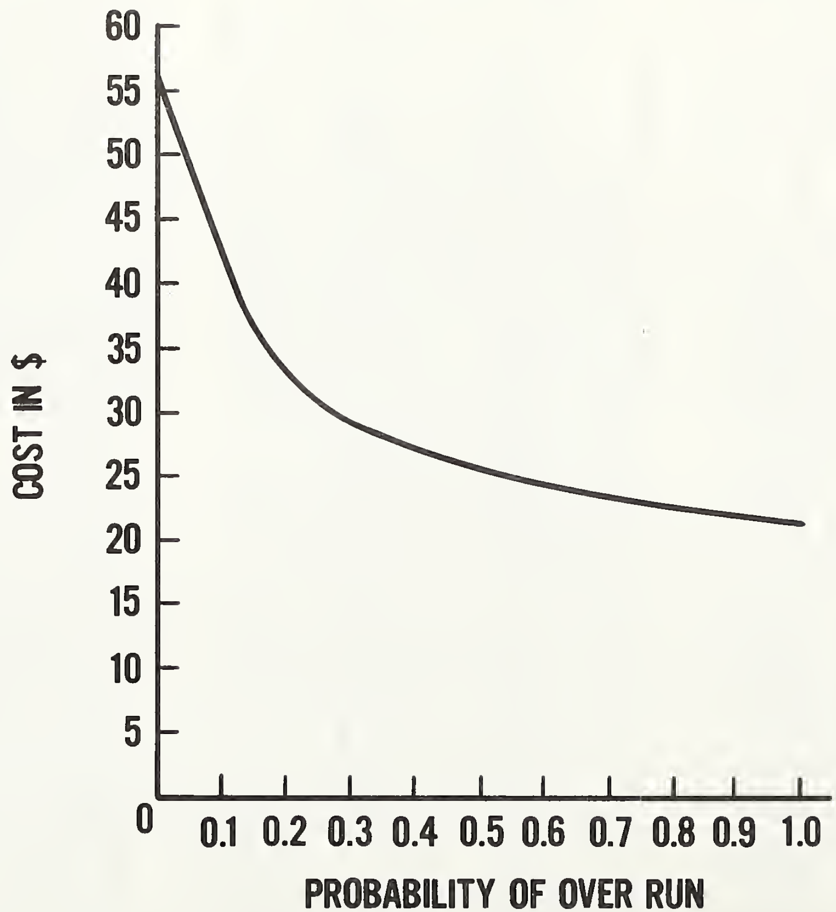

Figure A.2 Cost Profile for a Renovation Project

Before proceeding with a discussion of the mechanics of the cost model, it is useful to review the ways in which the inherent riskiness of the building renovation process may be assessed. There are three basic methods of treating risk in construction cost estimation. These three methods are: (1) judgmental modifications; (2) defining a confidence interval about the estimate; and (3) probabilistic (monte carlo simulation) concepts. Each method will be treated in turn. In the discussion which follows, unless explicitly stated, it will be assumed that the cost estimating technique being discussed is based on average total costs.

Judgmental Modifications

By far the most popular method of risk assessment is to make use of judgmental modifications. Unfortunately, the use of such a method can only be viewed as a placebo since no a priori grounds can be established which would enable the user to associate percentage changes in risk due to a particular judgmental modification. In reality, judgmental modifications are more a means for adjusting the cost estimate to reflect a change in some technical attribute rather than a measure of the interaction of technical attributes and economic forces in the market place. Thus, experts applying judgmental modification methods are, in a sense, superimposing a parametric cost estimating procedure on top of an average total cost procedure. Although this concept may seem appropriate, it is important to point out that parametric or regression based techniques obey certain rules of the real numbers. This is because most cost estimating relationships are fitted using a continuum of data. An expert who modifies the average total cost procedure, however, is usually only capable of establishing a relative order or rank among different scenarios. Such an approach makes use of what is known as ordinal level data.[1]

Unfortunately, ordinal level data need not obey all the rules of the real number system which a regression based procedure obeys by construction. Since the estimator of the rehab project is concerned with the absolute rather than the relative cost of the project, it does not appear that any level of significance can be attached to the "reduction in risk" associated with the judgmental modification method.

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[1] Statisticians have defined four levels of measurement in data. The first level, nominal, makes no assumption about the values being assigned to the data. The second level, ordinal, assumes that it is possible to rank-order all categories but that any numeric values assigned to the categories does not imply that any other properties of the real numbers follow. The third level, interval, implies that the distance between categories are defined in terms of fixed and equal units. The fourth level, ratio, implies that a zero point is inherently defined by the measurement scheme. In the context of the previous discussion, parametric procedures are usually estimated at the interval or ratio levels. For an excellent discussion on the levels of measurement see Norman H. Nie, C. Hadlia Hull, Jean G. Jenkins, Karin Steinbrenner, and Dale Bent, SPSS: Statistical Package for the Social Sciences, Second Edition, McGraw-Hill Book Company, New York, 1975.

Defining a Confidence Interval About the Estimate

The second method for treating risk involves fitting a confidence interval about the estimate. To use this method however, an estimate of the standard deviation of the predicted value must be known in order to define the upper and lower limits on the confidence interval. Since the construction cost guidebooks, the traditional source of cost data, do not provide measures of variability associated with their average total cost estimates, it is not possible to fit a confidence interval about the estimate. Even if the guidebooks published such information, there is some question about the effects of aggregation on the variance of the sample. This criticism stems from the fact that observations are drawn from cities across the nation. Consequently, the structure of costs in St. Louis might so differ from Boston that the variances about the means would not be homogeneous. In this case, pooling information about the variance of the sample would not justified. The thrust of this criticism is that in order to fit confidence intervals about the average total cost figures in the guidebooks, it would be desirable to have sample variances for each city.

Although the use of confidence intervals helps in assessing the impacts of risk on the rehab decision, they are lacking in some respects. For example, a 95 percent confidence interval implies that if samples were taken over and over under identical circumstances from the same population, then 95 out of 100 intervals would contain the true mean of the population. If the sample is not random or the desired estimate is for an atypical case, then the estimate of the mean may be biased. This implies that the concept of a confidence interval loses some of its meaning. The previous statement is reinforced by the fact that no discussions of sampling and nonsampling errors are given in the guidebooks. Consequently, it is not possible to quantify the bias, if any, associated with these average total cost estimates. Under these circumstances fitting a confidence interval should be viewed as a rather academic exercise.

Closely related to the concept of a confidence interval is a technique known as sensitivity analysis. This technique derives its name from measuring the cost sensitivity to a change in one or more factors involved in the process. That is, it permits one to determine how "sensitive" the average total cost estimate is to a change in one or more of the key factors. More succinctly, the sensitivity of the estimate to a change in a factor may be defined by simply differentiating the cost estimation equation.

Probabilistic (Monte Carlo Simulation) Concepts

The third and most comprehensive method for treating risk draws upon probabilistic concepts. Probabilistic concepts are the most appropriate for treating risk because they allow the estimator to "fit" a distribution about each factor and then run through a whole series of "what if" questions. Although it is possible to manually fit a confidence interval about an average total cost estimate, most probabilistic methods rely on computer software for support. The major advantages of probabilistic methods are: (1) they do explicitly treat risk, and (2) they do not suffer from some of the criticisms voiced about confidence intervals. In particular, if one has strong *a priori* beliefs about the structure of rehab costs, it is still possible to apply probabilistic methods even if the average total cost estimate in the guide is biased. More pragmatically, it is possible to adjust for the bias through a judicious choice of a distribution so that the behavior of the distribution parallels the critical steps in the process.

If one wishes to combine parametric and probabilistic methods, however, it would be necessary to fit a probability distribution about each factor rather than just about the average total cost estimate. It is fairly easy to see that this approach is superior to the rather simplistic average total cost approach since costs are "sensitive" to a change in several factors. The "degree" of sensitivity is, however, dependent upon the relative weight of that factor in determining the renovation cost for the specific task. In the case where the relationship is linear (or log linear), the application of probabilistic methods is rather simple and straightforward. The computer software package would first perform a monte carlo simulation on the cost estimating equation (relationship) until a cost profile for the task results.

The user can then choose a particular level of risk which he is willing to accept. In order to compute a confidence interval about the desired level, the user must replicate the simulation a certain number of times (the number of replications is dependent on the desired "tightness" of the confidence interval).

Probabilistic methods can also be used to perform a type of sensitivity analysis. The type of sensitivity analysis may be either qualitative, in the sense that it is based solely on judgmental modification, or quantitative. For example, one might not be sure about the true distribution of a particular factor. The monte carlo simulation could then be repeated under the assumption that the factor was distributed differently. By the same token, uncertainty about the condition of the building can be incorporated by either shifting the entire distribution upward or by requiring the distribution to be more skewed. Such an approach could thus complement expert judgment about the physical process. In any event by using the baseline estimate as a reference point, it will be possible to attach a percentage change in the risk being borne by the investor due to a change in a particular factor or group of factors.

Given the basic framework of the equivalency methodology discussed in the main body of the text, it becomes possible to use cost functions and probabilistic methods to develop a cost structure for the problem. The structure of costs is, however, complicated by two factors. First, through reference to table 2.1 it can be seen that there are nm (n rows and m columns) possible states. Since the installation of a new door for security purposes would cost the same as the installation of that door for egress purposes, we can assert that the retrofit cost associated with a particular component/state transition is independent of the regulatory requirement under consideration. Even with this simplification, however, the likelihood of one cost function being able to treat all of these cases is quite remote. Thus, in the discussion which follows, it will be assumed that there are N distinct cost functions, $1 < N \leq nm$. The second complication is of a more subtle nature.

From the previous discussion we know that each cost function contains a set of key factors. Since we now have N cost functions, it is likely that some of them will contain factors in common. This implies that greater care must be exercised in applying probabilistic methods. For example, several building components might be associated with certain categories of plumbing system retrofits all of which require a skilled plumber. We shall assume that the same plumbing contractor would perform whichever tasks are selected by the investor and would use the same staff in carrying them out. Now if each retrofit were treated independently using probabilistic methods, it would be possible to get differing wages from the same plumber doing different tasks. Such a state of affairs does not accurately reflect the way in which construction services are contracted. Fortunately, the solution to the problem is rather simple. To see this, denote the key factors contained in the first cost function as K_1, in the second cost function K_2, and so on. The universe of key factors, K, is thus the union of all key factors in the N cost functions. This relationship may be expressed mathematically as

$$K = \bigcup_{a=1}^{n} K_a$$

Now if there are L distinct key factors in K, then we may express K mathematically as

$$K = \{ k_1, k_2, \ldots k_L \}$$

As in the earlier discussion, each element of K has associated with it an estimated mean k_i and probability distribution.[1]

Once all elements of K have been identified and the relevant information about their probability distributions have been put into the software package, it becomes possible to generate a meaningful set of cost estimates. The cost estimates desired for this problem are shown in table A.3. From the table it can be seen that a cost estimate is needed for each building component/state pair. Since the goal of this operation is to reduce both cost and variability, probabilistic methods should be used in conjunction with the linear programming procedure.

In order to apply probabilistic methods to the problem at hand, it is first necessary to select a random number for each factor, refer to the probability distribution of that factor, and select the appropriate value of the random variable. Once all L random variables have been determined, it is necessary to plug them into the appropriate cost function in order to get an estimated cost for each potential retrofit. Just as in the previous discussion, this process is reiterated until a cost profile for each potential retrofit is defined (see figure A.2). The next step is for the user to define a risk level, say 20 percent. The software package will then survey each potential retrofit and pick

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[1] In the event that k_i is a fixed constant, say $\widehat{k_i}$ square feet of floor area, then the probability distribution associated with k_i is the point distribution.

Table A.3 Cost Matrix Used in Linear Programming Approach

Retrofit Cost

	State				
Building Component	1	2	3	. . .	m
1	C_{11}	C_{12}	C_{13}	. . .	C_{1m}
2	C_{21}	C_{22}	C_{23}	. . .	C_{2m}
3	C_{31}	C_{32}	C_{33}	. . .	C_{3m}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
n	C_{n1}	C_{n2}	C_{n3}	. . .	C_{nm}

off the retrofit cost which has a 20 percent change of being overrun. The program would then automatically select the maximum value, multiply it by some constant, say n+1, and use it as the "arbitrarily high" cost for all retrofits which are deemed to be non-feasible. The result of this exercise is that table A.3 is now filled with all the appropriate cost figures. These values can now be used as inputs for the linear programming procedure.

What is particularly attractive about this approach is that the linear programming algorithm will produce the least-cost solution for achieving compliance to all p regulatory requirements. However, by construction this solution has only a 20 percent chance of being overrun. Thus, we have obtained the least-cost solution for a given level of risk.

APPENDIX B. A MATHEMATICAL PROGRAMMING APPROACH TO EQUIVALENCY-BASED REGULATIONS

This appendix is designed to supplement the discussions in chapters 2 and 3 on identifying and pursuing optimal compliance strategies. Its focus is on how the worksheets presented in chapter 3 can be used as a basis for the development of a linear programming procedure which ranks compliance strategies according to retrofit cost. A secondary focus is on some of the technical challenges associated with implementing a generalized equivalency methodology. Since the worksheets occupy a central role in the cost-optimization model, a moderately detailed description of them is required for exposition of the model. The primary use of such a set of worksheets is to determine how combinations of various levels, or states, of several widely known building components can be used to provide a level of performance equivalent to that prescribed in a set of building codes or standards. Four major regulatory areas are basic to the evaluation system developed in the pilot study. They are:

(1) Security: the ability of the building and its associated components to resist entry of individuals intent upon theft or violence.

(2) Egress: the ability of the building and its associated components to facilitate an orderly evacuation of the building should such a need arise.

(3) Ventilation: the ability of the building and its associated components to facilitate the natural expulsion of stale air and the circulation of fresh air.

(4) Illumination: the ability of the building and its associated components to promote the use of daylighting.

It is important to point out that the design of the evaluation system developed in the pilot study is intended to ensure that the failure of a single device or method will not result in a major failure of the entire system. This concept of system redundancy is an integral part of several major codes and hence was deemed appropriate for inclusion in the pilot study.

In order to promote a more complete understanding of the cost-optimization model, the framework will first be developed from a descriptive viewpoint. Each of the components of the model will then be reintroduced and developed more fully. An explicit algebraic statement of the model will be given at the end of the appendix.

The problem actually solved may be stated as:

minimize the total retrofit cost subject to the following constraints[1]:

(1) conservation of units equations;
(2) accessory matching equations; and
(3) performance requirements.

The Objective Function

The problem as stated has a single objective[2], namely the minimization of total retrofit cost. The total retrofit cost is merely the sum of all the individual retrofit costs. Recall that retrofit cost was defined in chapter 2 as the cost of moving from any given or initial state to any other state. Unfortunately, some minor modifications to this definition had to be made due to a cost allocation problem. More precisely, window size, window type and window glazing were each identified as building components in the evaluation system. Since replacement windows usually come as units (i.e., the size, type, and glazing characteristics are determined simultaneously) it was impossible to allocate the retrofit cost in any meaningful manner to the individual size-type-glazing components. Similar cost allocation problems resulted from door systems. This complication required the redefinition of several variables which had the effect of increasing the number of variables so as to accomodate all possible combinations. Since these variables are used throughout the remainder of this appendix, they are numbered individually and defined in table B.1 according to system (i.e., windows or doors), preretrofit state (input) and postretrofit state (output). Table B.1 therefore provides a ready reference for each variable's code number used in the algebraic statement of the problem.

The Constraints

The Conservation of units equations are five identities which define the maximum number of windows and doors as well as the number of windows and doors of various sizes that already exist within the dwelling unit.

The accessory matching equations are four identities which contain the allocation of accessories (window hardware, window screens, door hardware, and second doors). The equations are designed to ensure that each accessory is counted once and only once.

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[1] An additional set of constraints which is also important for this problem are the nonnegativity requirements. This set of constraints requires each and every state variable to be greater than or equal to zero. These constraints are noted for completeness in exposition but will not be discussed in any detail.

[2] The problem may also be extended and formulated as a multiobjective program, where there are a combination of objectives to be minimized or maximized. For the reader interested in this approach to the problem, the texts by Cohon and Ignizio are strongly recommended.

Table B.1 Definitions of the Variables Used in The Computer Program

Variable Number X_i	Input State	Output State	Comments
1	Window Blank	Window Blank	Window Place Holder
2	Window 20-40" H	Window Blank	"Brick Up" Option
3	Window 40-60" H	Window Blank	"Brick Up" Option
4	Window Blank	20-40" H Slider Single	Add A New Window
5	Window Blank	20-40" H Slider Double	Add A New Window
6	Window Blank	20-40" H Casement Single	Add A New Window
7	Window Blank	20-40" H Casement Double	Add A New Window
8	Window Blank	20-40" H Casement Triple	Add A New Window
9	Window Blank	20-40" H D. Hung Single	Add A New Window
10	Window Blank	20-40" H D. Hung Triple	Add A New Window
11	Window Blank	20-40" H D. Hung Triple	Add A New Window
12	Window 20-40" H	20-40" H Slider Single	Replace with Same Size
13	Window 20-40" H	20-40" H Slider Double	Replace with Same Size
14	Window 20-40" H	20-40" H Casement Single	Replace with Same Size
15	Window 20-40" H	20-40" H Casement Double	Replace with Same Size
16	Window 20-40" H	20-40" H Casement Triple	Replace with Same Size
17	Window 20-40" H	20-40" H D. Hung Single	Replace with Same Size
18	Window 20-40" H	20-40" H D. Hung Double	Replace with Same Size
19	Window 20-40" H	20-40" H D. Hung Triple	Replace with Same Size
20	Window 40-60" H	20-40" H Slider Single	Replace with Smaller Size
21	Window 40-60" H	20-40" H Slider Double	Replace with Smaller Size
22	Window 40-60" H	20-40" H Casement Single	Replace with Smaller Size
23	Window 40-60" H	20-40" H Casement Double	Replace with Smaller Size
24	Window 40-60" H	20-40" H Casement Triple	Replace with Smaller Size
25	Window 40-60" H	20-40" H D. Hung Single	Replace with Smaller Size
26	Window 20-60" H	20-40" H D. Hung Double	Replace with Smaller Size
27	Window 40-60" H	20-40" H D. Hung Triple	Replace with Smaller Size
28	Window Blank	40-60" H Slider Single	Add A New Window
29	Window Blank	40-60" H Slider Double	Add A New Window
30	Window Blank	40-60" H Casement Single	Add A New Window
31	Window Blank	40-60" H Casement Double	Add A New Window
32	Window Blank	40-60" H Casement Triple	Add A New Window
33	Window Blank	40-60" H D. Hung Single	Add A New Window
34	Window Blank	40-60" H D. Hung Double	Add A New Window
35	Window Blank	40-60" H D. Hung Triple	Add A New Window
36	Window 20-40" H	40-60" H Slider Single	Replace With Larger Size
37	Window 20-40" H	40-60" H Slider Double	Replace With Larger Size
38	Window 20-40" H	40-60" H Casement Single	Replace With Larger Size
39	Window 20-40" H	40-60" H Casement Double	Replace With Larger Size
40	Window 20-40" H	40-60" H Casement Triple	Replace With Larger Size
41	Window 20-40" H	40-60" H D. Hung Single	Replace With Larger Size
42	Window 20-40" H	40-60" H D. Hung Double	Replace With Larger Size
43	Window 20-40" H	40-60" H D. Hung Triple	Replace With Larger Size
44	Window 40-60" H	40-60" H Slider Single	Replace With Same Size
45	Window 40-60" H	40-60" H Slider Double	Replace With Same Size
46	Window 40-60" H	40-60" H Casement Single	Replace With Same Size
47	Window 40-60" H	40-60" H Casement Double	Replace With Same Size
48	Window 40-60" H	40-60" H Casement Triple	Replace With Same Size
49	Window 40-60" H	40-60" H D. Hung Single	Replace With Same Size
50	Window 40-60" H	40-60" H D. Hung Double	Replace With Same Size
51	Window 40-60" H	40-60" H D. Hung Triple	Replace With Same Size
52	No Window Hardware	Latch	Hardware Place Holder
53	No Window Hardware	Locking Latch	Hardware Installation
54	No Window Hardware	Maximum Window Hardware	Hardware Installation
55	No Window Screen	No Window Screen	Screen Placeholder
56	No Window Screen	Install Window Screen	Screen Installation
57	Door Blank	Door Blank	Door Placeholder
58	Door 18-21 ft^2	Door Blank	"Brick Up" Option
59	Door Blank	18-21 ft^2 Hollow 0 Lite	Add A New Door
60	Door Blank	18-21 ft^2 Hollow 1 Lite	Add A New Door
61	Door Blank	18-21 ft^2 Hollow 3 Lite	Add A New Door
62	Door Blank	18-21 ft^2 Hollow 6 Lite	Add A New Door
63	Door Blank	18-21 ft^2 Hollow 9 Lite	Add A New Door
64	Door Blank	18-21 ft^2 Solid 0 Lite	Add A New Door
65	Door Blank	18-21 ft^2 Solid 1 Lite	Add A New Door
66	Door Blank	18-21 ft^2 Solid 3 Lite	Add A New Door
67	Door Blank	18-21 ft^2 Solid 6 Lite	Add A New Door
68	Door Blank	18-21 ft^2 Solid 9 Lite	Add A New Door
69	Door Blank	18-21 ft^2 Metal 0 Lite	Add A New Door
70	Door Blank	18-21 ft^2 Metal 1 Lite	Add A New Door
71	Door Blank	18-21 ft^2 Metal 3 Lite	Add A New Door
72	Door Blank	18-21 ft^2 Metal 6 Lite	Add A New Door
73	Door Blank	18-21 ft^2 Metal 9 Lite	Add A New Door
74	Door 18-21 ft^2	18-21 ft^2 Hollow 0 Lite	Replace With Same Size
75	Door 18-21 ft^2	18-21 ft^2 Hollow 1 Lite	Replace With Same Size
76	Door 18-21 ft^2	18-21 ft^2 Hollow 3 Lite	Replace With Same Size
77	Door 18-21 ft^2	18-21 ft^2 Hollow 6 Lite	Replace With Same Size
78	Door 18-21 ft^2	18-21 ft^2 Hollow 9 Lite	Replace With Same Size
79	Door 18-21 ft^2	18-21 ft^2 Solid 0 Lite	Replace With Same Size
80	Door 18-21 ft^2	18-21 ft^2 Solid 1 Lite	Replace With Same Size

Table B.1 (cont.)

Variable Number X_i	Input State	Output State	Comments
81	Door 18-21 ft^2	18-21 ft^2 Solid 3 Lite	Replace With Same Size
82	Door 18-21 ft^2	18-21 ft^2 Solid 6 Lite	Replace With Same Size
83	Door 18-21 ft^2	18-21 ft^2 Solid 9 Lite	Replace With Same Size
84	Door 18-21 ft^2	18-21 ft^2 Metal 0 Lite	Replace With Same Size
85	Door 18-21 ft^2	18-21 ft^2 Metal 1 Lite	Replace With Same Size
86	Door 18-21 ft^2	18-21 ft^2 Metal 3 Lite	Replace With Same Size
87	Door 18-21 ft^2	18-21 ft^2 Metal 6 Lite	Replace With Same Size
88	Door 18-21 ft^2	18-21 ft^2 Metal 9 Lite	Replace With Same Size
89	Door Blank	21-25 ft^2 Hollow 0 Lite	Add A New Door
90	Door Blank	21-25 ft^2 Hollow 1 Lite	Add A New Door
91	Door Blank	21-25 ft^2 Hollow 3 Lite	Add A New Door
92	Door Blank	21-25 ft^2 Hollow 6 Lite	Add A New Door
93	Door Blank	21-25 ft^2 Hollow 9 Lite	Add A New Door
94	Door Blank	21-25 ft^2 Solid 0 Lite	Add A New Door
95	Door Blank	21-25 ft^2 Solid 1 Lite	Add A New Door
96	Door Blank	21-25 ft^2 Solid 3 Lite	Add A New Door
97	Door Blank	21-25 ft^2 Solid 6 Lite	Add A New Door
98	Door Blank	21-25 ft^2 Solid 9 Lite	Add A New Door
99	Door Blank	21-25 ft^2 Metal 0 Lite	Add A New Door
100	Door Blank	21-25 ft^2 Metal 1 Lite	Add A New Door
101	Door Blank	21-25 ft^2 Metal 3 Lite	Add A New Door
102	Door Blank	21-25 ft^2 Metal 6 Lite	Add A New Door
103	Door Blank	21-25 ft^2 Metal 9 Lite	Add A New Door
104	Door 18-21 ft^2	21-25 ft^2 Hollow 0 Lite	Replace With Larger Size
105	Door 18-21 ft^2	21-25 ft^2 Hollow 1 Lite	Replace With Larger Size
106	Door 18-21 ft^2	21-25 ft^2 Hollow 3 Lite	Replace With Larger Size
107	Door 18-21 ft^2	21-25 ft^2 Hollow 6 Lite	Replace With Larger Size
108	Door 18-21 ft^2	21-25 ft^2 Hollow 9 Lite	Replace With Larger Size
109	Door 18-21 ft^2	21-25 ft^2 Solid 0 Lite	Replace With Larger Size
110	Door 18-21 ft^2	21-25 ft^2 Solid 1 Lite	Replace With Larger Size
111	Door 18-21 ft^2	21-25 ft^2 Solid 3 Lite	Replace With Larger Size
112	Door 18-21 ft^2	21-25 ft^2 Solid 6 Lite	Replace With Larger Size
113	Door 18-21 ft^2	21-25 ft^2 Solid 9 Lite	Replace With Larger Size
114	Door 18-21 ft^2	21-25 ft^2 Metal 0 Lite	Replace With Larger Size
115	Door 18-21 ft^2	21-25 ft^2 Metal 1 Lite	Replace With Larger Size
116	Door 18-21 ft^2	21-25 ft^2 Metal 3 Lite	Replace With Larger Size
117	Door 18-21 ft^2	21-25 ft^2 Metal 6 Lite	Replace With Larger Size
118	Door 18-21 ft^2	21-25 ft^2 Metal 9 Lite	Replace With Larger Size
119	Door Blank	>25 ft^2 Hollow 0 Lite	Add A New Door
120	Door Blank	>25 ft^2 Hollow 1 Lite	Add A New Door
121	Door Blank	>25 ft^2 Hollow 3 Lite	Add A New Door
122	Door Blank	>25 ft^2 Hollow 6 Lite	Add A New Door
123	Door Blank	>25 ft^2 Hollow 9 Lite	Add A New Door
124	Door Blank	>25 ft^2 Solid 0 Lite	Add A New Door
125	Door Blank	>25 ft^2 Solid 1 Lite	Add A New Door
126	Door Blank	>25 ft^2 Solid 3 Lite	Add A New Door
127	Door Blank	>25 ft^2 Solid 6 Lite	Add A New Door
128	Door Blank	>25 ft^2 Solid 9 Lite	Add A New Door
129	Door Blank	>25 ft^2 Metal 0 Lite	Add A New Door
130	Door Blank	>25 ft^2 Metal 1 Lite	Add A New Door
131	Door Blank	>25 ft^2 Metal 3 Lite	Add A New Door
132	Door Blank	>25 ft^2 Metal 6 Lite	Add A New Door
133	Door Blank	>25 ft^2 Metal 9 Lite	Add A New Door
134	Door 18-21 ft^2	>25 ft^2 Hollow 0 Lite	Replace With Larger Size
135	Door 18-21 ft^2	>25 ft^2 Hollow 1 Lite	Replace With Larger Size
136	Door 18-21 ft^2	>25 ft^2 Hollow 3 Lite	Replace With Larger Size
137	Door 18-21 ft^2	>25 ft^2 Hollow 6 Lite	Replace With Larger Size
138	Door 18-21 ft^2	>25 ft^2 Hollow 9 Lite	Replace With Larger Size
139	Door 18-21 ft^2	>25 ft^2 Solid 0 Lite	Replace With Larger Size
140	Door 18-21 ft^2	>25 ft^2 Solid 1 Lite	Replace With Larger Size
141	Door 18-21 ft^2	>25 ft^2 Solid 3 Lite	Replace With Larger Size
142	Door 18-21 ft^2	>25 ft^2 Solid 6 Lite	Replace With Larger Size
143	Door 18-21 ft^2	>25 ft^2 Solid 9 Lite	Replace With Larger Size
144	Door 18-21 ft^2	>25 ft^2 Metal 0 Lite	Replace With Larger Size
145	Door 18-21 ft^2	>25 ft^2 Metal 1 Lite	Replace With Larger Size
146	Door 18-21 ft^2	>25 ft^2 Metal 3 Lite	Replace With Larger Size
147	Door 18-21 ft^2	>25 ft^2 Metal 6 Lite	Replace With Larger Size
148	Door 18-21 ft^2	>25 ft^2 Metal 9 Lite	Replace With Larger Size
149	No Door Hardware	Ordinary Lock	Hardware Placeholder
150	No Door Hardware	Dead Bolt 1 Key	Hardware Installation
151	No Door Hardware	Dead Bolt 2 Keys	Hardware Installation
152	No Second Door	Install Second Door	Door Installation
153	No Second Door	No Second Door	Door Placeholder
154	# of Rooms	# of Rooms	Room Placeholder
155	# of Doors	# of Doors	Door Placeholder
156	Egress Coefficient	Egress Coefficient	Surplus Value
157	Security Coefficient	Security Coefficient	Surplus Value
158	Illumination Coefficient	Illumination Coefficient	Surplus Value
159	Ventilation Coefficient	Ventilation Coefficient	Surplus Value
160	Percent Illumiination	Percent Illumination	Surplus Value
161	Percent Ventilation	Percent Ventilation	Surplus Value

The last eight constraints define the <u>performance requirements</u>. The first two constraints define the minimum number of doors and windows permissible. The third and forth constraints define the total window and door area. These figures are used to ensure that the ratio of window area to floor area exceeds the code requirements for ventilation and illumination. The levels for each regulatory requirement are calculated by first identifying the component/state pair which corresponds to prescriptive compliance, recording the state value and summing across all components.

The discussion which follows provides an explicit algebraic statement of the model.

The Objective Function

Minimize the total retrofit costs

$$C_{TOTAL} = \sum_{i=1}^{161} c_i X_i$$

where c_i = the cost of retrofitting from the initial state to state i

The Constraints

A. Conservation of Units Equations

A.1 $$X_1 + \sum_{i=4}^{11} X_i + \sum_{i=28}^{35} X_i = 20$$

Constraint A.1 requires that the number of windows not created (i.e., a transition from a blank (no window) to a blank) plus the number of new windows (i.e., a transition from a blank to a window 20 to 40 inches high or 40 to 60 inches high) remain fixed. The right hand side variable (20) is a limit on the number of new windows; it is conceptually, an arbitrarily large number.

A.2 $$X_{57} + \sum_{i=59}^{73} X_i + \sum_{i=89}^{103} X_i + \sum_{i=119}^{133} X_i = 20$$

Constraint A.2 requires that the number of doors not created plus the number of new doors remain fixed. The right hand side variable is a limit on the number of new doors.

A.3 $$X_2 + \sum_{i=12}^{19} X_i + \sum_{i=36}^{43} X_i = W_2$$

This constraint conserves the number of windows 20 to 40 inches high. W_2 is the number of such windows existing; it is input for each problem.

A.4 $$X_3 + \sum_{i=20}^{27} X_i + \sum_{i=44}^{51} X_i = W_3$$

This constraint conserves the number of windows 40 to 60 inches high. W_3, the number of such windows, is an input.

A.5 $$X_{58} + \sum_{i=74}^{88} X_i + \sum_{i=104}^{118} X_i + \sum_{i=134}^{148} X_i = D_1$$

This constraint conserves the number of doors which are 18 to 21 square feet in size. D_1, the number of such doors, is an input.

B. Accessory Matching Equations

B.1 $$\sum_{i=4}^{51} X_i - \sum_{i=52}^{54} X_i = 0$$

Constraint B.1 requires that the number of window hardware systems (latch, locking latch, maximal locking hardware) equals the number of windows.

B.2 $$\sum_{i=4}^{51} X_i - \sum_{i=55}^{56} X_i = 0$$

Constraint B.2 requires that the number of window screen systems (the number with and the number without) equals the number of windows.

B.3 $$\sum_{i=59}^{148} X_i - \sum_{i=149}^{151} X_i = 0$$

Constraint B.3 requires that the number of door hardware systems (lockset, deadbolt (1 key), deadbolt (2 key) equals the number of doors.

B.4 $$\sum_{i=59}^{148} X_i - \sum_{i=152}^{153} X_i = 0$$

This constraint requires that the number of second door systems (the number with and the number without) equals the number of doors.

C. Performance Requirements

C.1 $$X_{152} + X_{153} - X_{155} = D_m$$

Constraint C.1 requires a minimum of D_m doors. D_m is input for each problem; X_{155} is an explicit surplus variable.

C.2 $$\sum_{i=4}^{51} X_i - X_{154} = W_m$$

Constraint C.2 requires a minimum of W_m windows. W_m is input for each problem; X_{154} is an explicit surplus value.

C.3 $$\sum_{i=1}^{159} A_i^V X_i - X_{161} = F^V$$

This constraint requires the effective ventilation area of all doors and windows to meet or exceed a minimum area requirement. A_i^V is the effective ventilation area in square feet per item in state i. The variable X_{161} is an explicit surplus and F^V is the required area in square feet. F^V is equal to a prescribed fraction of floor area (see table 3.1).

C.4 $$\sum_{i=1}^{159} A_i^I X_i - X_{160} = F^I$$

This constraint requires the effective illumination area of all doors and windows to meet or exceed a minimum area requirement. A_i^I is the effective illumination area in square feet per item in state i. The variable X_{160} is an explicit surplus and F^I is the required area in square feet. F^I is equal to a prescribed fraction of floor area (see table 3.1).

C.5 $$\sum_{i=1}^{161} S_i^S X_i - X_{157} = R^S$$

Constraint C.5 defines the level of performance necessary to attain the security requirement, R^S. The variable X_{157} is an explicit surplus; the S_i^S terms in the equation represent state values indexed by code attribute, S is for security.

C.6 $$\sum_{i=1}^{161} S_i^E X_i - X_{156} = R^E$$

Constraint C.6 defines the level of performance necessary to attain the egress requirement, R^E. The variable X_{156} is an explicit surplus; the S_i^E terms in the equation represent state values indexed by code attribute, E is for egress.

C.7 $$\sum_{i=1}^{161} S_i^V X_i - X_{159} = R^V$$

Constraint C.7 defines the level of performance necessary to attain the ventilation requirement, R^V. The variable X_{159} is an explicit surplus; the S_i^V terms in the equation represent state values indexed by code attribute, V is for ventilation.

C.8 $$\sum_{i=1}^{161} S_i^I X_i - X_{158} = R^I$$

This constraint defines the level of performance necessary to attain the illumination requirement, R^I. The variable X_{158} is an explicit surplus; the S_i^I terms in the equation represent state values indexed by code attribute, I is for illumination.

Some Comments on a Generalized Equivalency Methodology

The generalization of the pilot equivalency methodology outlined in this report poses technical challenges both in the computational aspects of the problem and in several theoretical issues in operations research. The computational aspects of a generalized equivalency methodology involve: (1) fixed charge problems; (2) large integer programming formulations; and (3) nonlinear constraints. The theoretical aspects are associated with the task of systematically generating alternate solutions for large integer programming problems.

As more emphasis is placed on the accuracy of the cost estimates, it is likely that fixed charge problems will be encountered. For example, a contractor may quote a figure of $1,000 plus $350 for each replacement window. In this case, $1,000 is the fixed charge. Although the algorithm used in this report can not handle the fixed charge problem directly, algorithms do exist which can handle it as well as explicitly model economies of scale. (The existence of economies of scale implies that the objective function is nonlinear. A piecewise linear approximation to the objective function is possible however.)

The structure of the problem solved in this report was such that a linear programming formulation could be used, and almost all variables which appeared in the solution took on integer values. Depending on the structure of the generalized equivalency methodology this result may or may not occur. If it does not occur, it will be necessary to rely on an integer programming formulation.

The problem can, of course, be solved using an integer programming formulation but only at the expense of greatly increased computational costs. The last computational problem is associated with nonlinear constraints. In the general formulation of the problem it may occur that some building components interact in a nonadditive manner. The mixture of additive and multiplicative interactions implies that the regulatory requirement constraints are nonlinear. In some cases, a transformation of variables or an ingenious redefinition of the regulatory requirement constraints can induce linearity. Under such circumstances a linear programming formulation would still be appropriate. However, it may not be satisfactory from an engineering point of view to redefine, transform or rely on pricewise linear approximations to the regulatory requirement constraints. In these cases it would be necessary to use a nonlinear programming formulation. Such a decision would again be at the expense of greatly increased computational costs.

The generation of alternate solutions to large integer programming problems is an important theoretical issue in operations research. At the present time, most integer programming codes use an approach known as the branch and bound technique. In essence this technique constructs a tree, selects a node to branch on, and places a bound on it through a process known as fathoming. If the code is written such that a branch is only partially fathomed and the nodes representing potential solutions are stored, it may be possible to generate a class of "good" solutions. Unfortunately, there seems to be no priori reason for believing that this set of solutions would be as systematic as the one generated with the linear programming formulation. Furthermore, this approach may terminate before the optimal solution is found.

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10. SUPPLEMENTARY NOTES

[ ] Document describes a computer program; SF-185, FIPS Software Summary, is attached.

11. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)

With increased emphasis on the re-use of existing buildings, new approaches must be developed to assist regulators in making code related decisions. The application of performance criteria to building rehabilitation provides flexibility in the use of technically sound design alternatives in lieu of prescriptive provisions which may be restrictive. This report presents the results of a pilot study on the application of an equivalency methodology in achieving regulatory compliance. The use of such a methodology is particularly attractive in this area because prescriptive type provisions have been shown to constrain rehabilitation activities, and in some cases, may be mutually contradictory. Regulatory requirements were chosen so as to explicitly incorporate conflicting requirements as affecting the design of windows and doors -- illumination, ventilation, egress and security. The methodology is computerized to allow the selection of least-cost means of achieving compliance with these requirements. A prototypical townhouse is evaluated using the pilot equivalency methodology and optimal compliance strategies are identified and compared with the cost of prescriptive compliance. The results of the study produced potential savings ranging from 20 to 35 percent depending on the initial conditions of the building.

12. KEY WORDS (Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)

Applied economics; building codes; health and safety; housing; mathematical programming; rehabilitation; renovation.

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