NBSIR 80-1967

The Calibration of Angle Blocks by Intercomparison

Charles P. Reeve

Statistical Engineering Division
Center for Applied Mathematics
National Bureau of Standards
U.S. Department of Commerce
Washington, DC 20234

April 1980

Final

U.S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

NBSIR 80-1967

THE CALIBRATION OF ANGLE BLOCKS BY INTERCOMPARISON

Charles P. Reeve

Statistical Engineering Division
Center for Applied Mathematics
National Bureau of Standards
U.S. Department of Commerce
Washington, DC 20234

April 1980

Final

U.S. DEPARTMENT OF COMMERCE, Philip M. Klutznick, *Secretary*
Luther H. Hodges, Jr., *Deputy Secretary*
Jordan J. Baruch, *Assistant Secretary for Productivity, Technology, and Innovation*
NATIONAL BUREAU OF STANDARDS, Ernest Ambler, *Director*

TABLE OF CONTENTS

1. Introduction

The calibration of angle blocks is a routine service provided by the National Bureau of Standards. Test blocks normally come in sets of 16 nominal sizes:

1, 3, 5, 20, 30 seconds,*
1, 3, 5, 20, 30 minutes, and
1, 3, 5, 15, 30, 45 degrees.

Periodically all tests sets which have been received are measured against two sets of NBS master angle blocks by intercomparing all blocks of the same nominal angle. The two sets of master blocks are called reference blocks and check blocks in accordance with the role they play in the measurement process. The reference blocks have been calibrated by the "absolute" method and serve as "ground zero" for establishing the values of the other blocks. The check blocks also have known values but are treated as unknowns during calibration. Their calibrated values are compared to their historical values in order to maintain control over the measurement process.

As part of the routine calibration each face of the test blocks is measured for flatness and for squareness to its top and bottom surfaces. (These measurements may be omitted on test blocks which have previously been calibrated at NBS.)

The purpose of this paper is to describe each phase of the calibration process and give a detailed description of the mathematical model for the intercomparison scheme. Three statistical tests for process control are described and an example is given.

There is not an abundance of literature on angle block calibration available. However, general (and often brief) discussions of angle blocks may be found in [2,3,4,5,6,7,8].**

2. Measurement of Flatness

The two polished faces of each test block may be denoted by $\underline{b}$ (base) and $\underline{h}$ (hypotenuse) as shown in Figure 2.1. The flatness of each face is measured in a light box using a Fizeau-type interferometer. An optical flat is placed over the base and adjusted until a series of interference fringes is observed as shown in Figure 2.2. A moveable hairline is adjusted to touch the ends of one fringe and the ratio a/b is estimated where $\underline{a}$ is the maximum deviation of the fringe from the

*Customary U.S. units are used in this report rather than the recognized metric (SI) units. The well established ongoing calibration procedures described here employ customary units exclusively. The conversion to SI units will be made at a future date.

**Numbers in brackets indicate references listed at the end of this paper.

hairline and b is the fringe separation. The out-of-flatness is then given by

$$f_b = \frac{\lambda a}{2b} \tag{2-1}$$

where λ is the wavelength of the monochromatic light in the light box. The value f_h is similarly derived for the hypotenuse, and the value

$$f_{max} = \max(f_b, f_h) \tag{2-2}$$

is reported as the maximum out-of-flatness of the angle block faces. Currently this value is rounded off to the nearest microinch.

The measurement of flatness is not intended to be of the highest accuracy. The maximum out-of-flatness figure is an indicator of the quality of the angle block and, depending on its magnitude, may indicate a source of long-term variability in the measured values of the angle between the faces.

3. Measurement of Squareness

The squareness of the base and hypotenuse to the top and bottom is measured for each test block. Let the four interior angles between these faces be given by $\{90° + \beta_i, i=1,4^*\}$ as shown in Figure 3.1. An autocollimator which is set to read vertical angle is mounted near the anvil upon which the angle block rests. The block is wrung to the anvil so that one of the four surfaces of interest faces the autocollimator (as does the base in Figure 3.1). The autocollimator is adjusted vertically to read near the center of its scale and the reading s_1 is recorded. The block is then rotated 90° and rewrung to the anvil so that the adjacent surface faces the autocollimator. The reading s_2 is is recorded. The rotation pattern is continued in the same direction until readings s_3 and s_4 have been recorded. The readings take the form (ignoring possible error terms)

$$s_i = \Delta - \beta_i \ , \tag{3-1}$$

for $i = 1,4$ where Δ is some initial reading of the autocollimator. Assuming that the surfaces are true planes then $\beta_1 + \beta_2 + \beta_3 + \beta_4 = 0$. Incorporating this equation with Equation 3-1 and solving for the unknown parameters gives

$$\begin{aligned} \Delta &= (s_1 + s_2 + s_3 + s_4)/4 \text{ and} \\ \beta_i &= \Delta - s_i \end{aligned} \tag{3-2}$$

for $i = 1,4$. The value

$$\beta_{max} = \max(|\beta_1|, |\beta_2|, |\beta_3|, |\beta_4|) \tag{3-3}$$

*Throughout this paper the abbreviated notation $i = p,q$ indicates that i takes on the consecutive integer values p, p+1,..., q-1, q.

DESIGNATION OF FACES

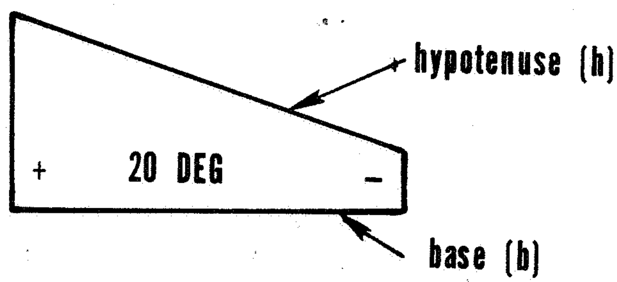


Figure 2.1

MEASUREMENT OF FLATNESS

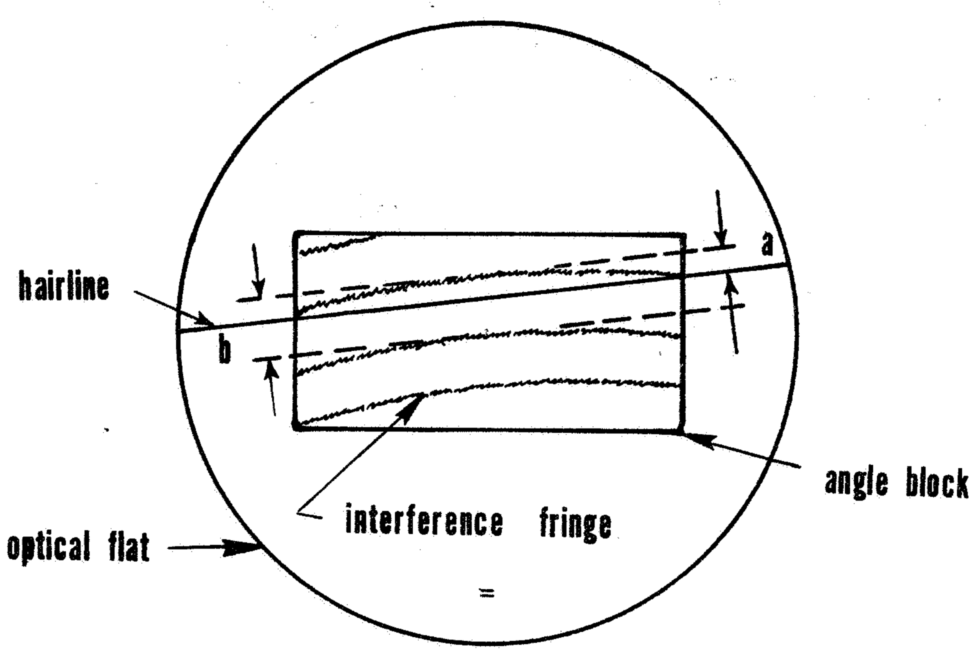


Figure 2.2

MEASUREMENT OF SQUARENESS
(END VIEW OF ANGLE BLOCK)

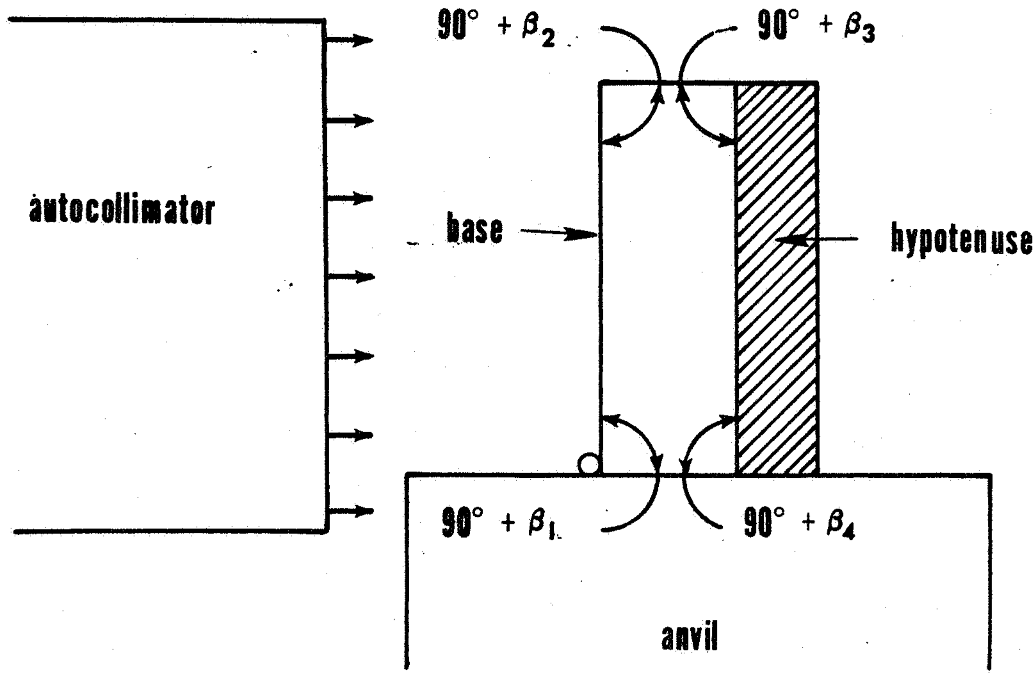


Figure 3.1

is then reported as the maximum out-of-squareness of the angle block surfaces. Currently this value is rounded off to the nearest arc second.

The measurement of squareness is not intended to be of the highest accuracy. As in the case of flatness measurement, the maximum out-of-squareness figure indicates the quality of the angle block and a possible source of long-term variability.

4. Measurement of Angle between Faces

4.1 Setup Procedures

Two autocollimators and a serrated anvil are mounted on a surface plate as shown in Figure 4.1. The anvil has three stops which enable the angle blocks to be inserted at the same position repeatedly. Since it is usually most convenient to measure the smallest blocks first, a 1" block is inserted on the anvil and the two autocollimators are aligned so that the block is centered in their respective fields of view and each reads near the center of its scale. After this point autocollimator A does not have to be realigned, but autocollimator B must be realigned for each different nominal angle. The autocollimators are connected to a digital voltmeter which displays the difference between their readings. The voltmeter is set so that a larger block size gives a larger reading.

In order to keep track of the blocks during measurement, small pieces of masking tape can be stuck to the side of the surface plate in front of the anvil and numbered from 1 to n where n is the total number of blocks to be intercompared. (Normally there is one reference block, one check block, and n-2 test blocks although an extra check block is sometimes included.) The corresponding identification numbers which are engraved on each block are also written on the pieces of tape, and the blocks are then set in the proper positions. The blocks should be checked to see that they are free of lint and smudges. The anvil should also be cleared of lint and dust particles so that the blocks will sit flat during measurement.

4.2 Intercomparison Scheme

Two series of measurements are made on each set of blocks of the same nominal size. In the first series the blocks are in the "top-up" position, and in the second series they are in the "bottom-up" position (Figure 4.2). In the "top-up" position the inscribed block size reads from + to - while in the "bottom'up" position it reads from - to +.

The intercomparison scheme is illustrated in Figure 4.3 for n = 7. The check block and each of the test blocks is compared to the reference block and to the two blocks ahead of it in a counterclockwise direction. The reference block must always appear in position 1 (center). The check block may appear anywhere on the perimeter, but for the sake of

CALIBRATION SET-UP

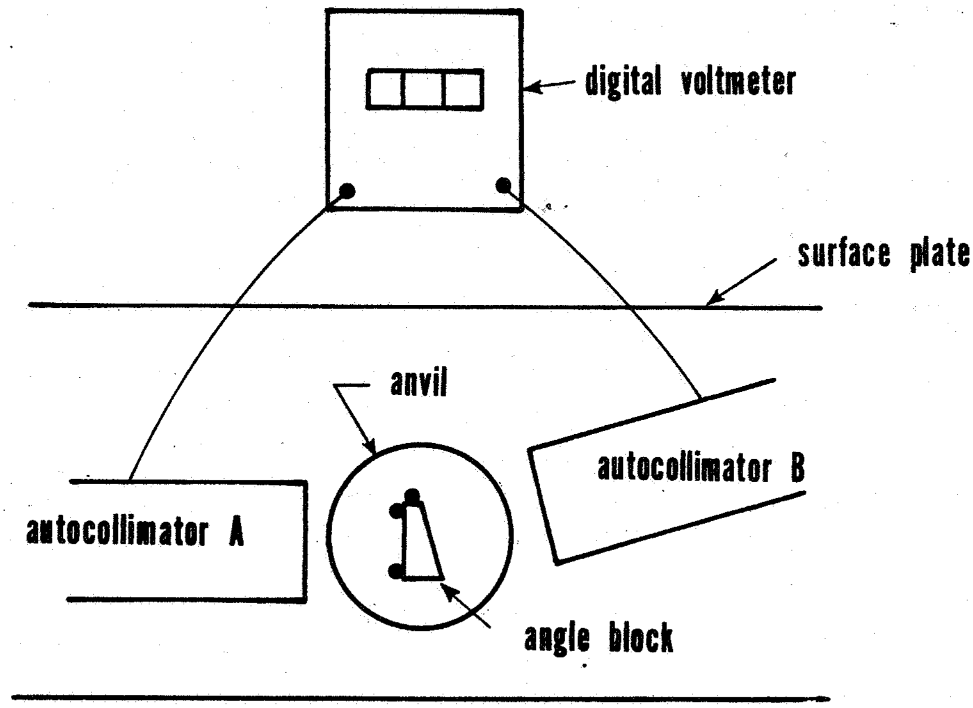


Figure 4.1

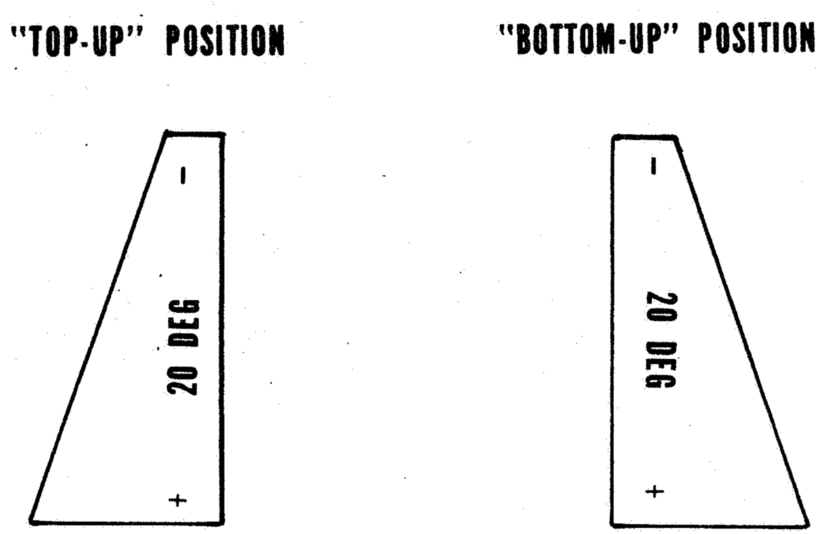


Figure 4.2

uniformity it is normally put in position 2. Angular differences between the blocks are obtained by making n-1 groups of seven measurements each. From each group three differences are derived in the manner described in the following section.

4.3 Measurement Equations

For each block size the first group of seven measurements is obtained by inserting the blocks on the anvil in the order 2-3-2-1-2-4-2. Let the corresponding readings of the digital voltmeter be given by the vector $y_1 = (y_{11}\ y_{12}\ y_{13}\ y_{14}\ y_{15}\ y_{16}\ y_{17})'$ where the symbol ' indicates vector (or matrix) transposition. Let the deviations from nominal angle of the n blocks be given by $\alpha = (\alpha_1\ \alpha_2 \ldots \alpha_n)'$. The first group of seven measurement equations is then given by

$$\begin{aligned}
y_{11} &= \Delta + \alpha_2 \phantom{{}+ 0d} + \varepsilon_{11} \\
y_{12} &= \Delta + \alpha_3 + d + \varepsilon_{12} \\
y_{13} &= \Delta + \alpha_2 + 2d + \varepsilon_{13} \\
y_{14} &= \Delta + \alpha_1 + 3d + \varepsilon_{14} \\
y_{15} &= \Delta + \alpha_2 + 4d + \varepsilon_{15} \\
y_{16} &= \Delta + \alpha_4 + 5d + \varepsilon_{16} \\
y_{17} &= \Delta + \alpha_2 + 6d + \varepsilon_{17}
\end{aligned} \tag{4-1}$$

where Δ is some initial reading of the digital voltmeter, d is a linear drift factor, and the ε_{ij} 's are independent error values from a distribution with mean zero and variance σ_w^2 (see section 4.5 for a discussion of σ_w^2). Let three new computed observations for the first group be given by the vector $z_1 = (z_{11}\ z_{12}\ z_{13})'$ where

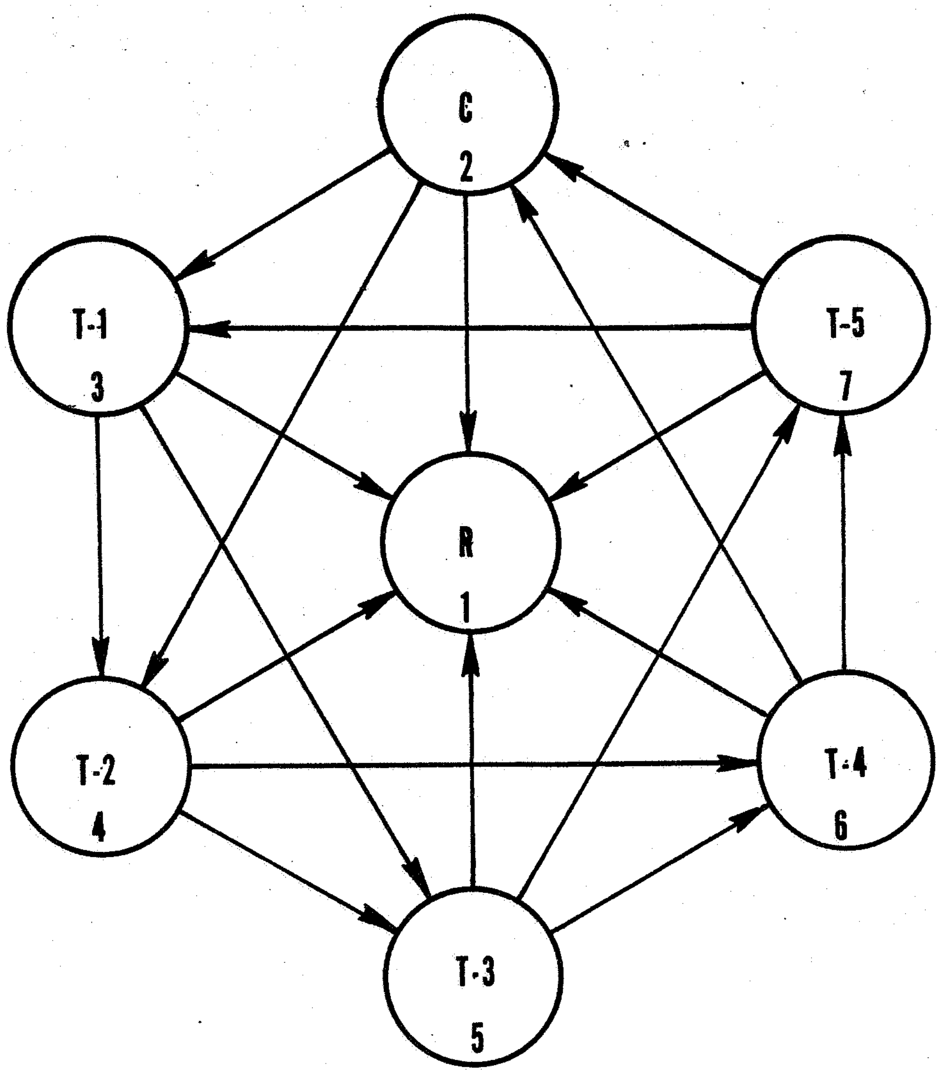
$$\begin{aligned}
z_{11} &= (y_{11} - 2y_{12} + y_{13})/2 \\
z_{12} &= (y_{13} - 2y_{14} + y_{15})/2 \\
z_{13} &= (y_{15} - 2y_{16} + y_{17})/2 \; .
\end{aligned} \tag{4-2}$$

Then in matrix notation $z_1 = My_1$ where

$$M = \frac{1}{2}\begin{bmatrix} 1 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 \end{bmatrix} . \tag{4-3}$$

Since $\mathrm{Var}(y_1) = \sigma_w^2 I$ then $\mathrm{Var}(z_1) = \sigma_w^2 V$ where

INTERCOMPARISON SCHEME FOR 7 ANGLE BLOCKS



T=TEST BLOCK
C=CHECK BLOCK
R=REFERENCE BLOCK

Figure 4.3

$$V = MM' = \frac{1}{4}\begin{bmatrix} 6 & 1 & 0 \\ 1 & 6 & 1 \\ 0 & 1 & 6 \end{bmatrix}. \qquad (4\text{-}4)$$

(Note that in forming z_1 the Δ and d terms drop out.)

The remaining n-2 groups of measurements are taken and transformed in a similar manner to give the complete 3(n-1) vector of new observations $z = (z_1' \; z_2' \; \ldots \; z_{n-1}')'$.

4.4 Method of Solution

Let E(z) and Var(z) denote the statistical expectation and variance of the random vector z just described. Then the least squares estimation [1,10] of the angular values takes the form

$$E(z) = X\alpha \qquad (4\text{-}5)$$

where X is the (3n-3) x n matrix

$$X = \left[\begin{array}{cccccccc} 0 & 1 & -1 & 0 & 0 & & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & & 0 & 0 \\ \hline 0 & 0 & 1 & -1 & 0 & & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & & 0 & 0 \\ \hline & & & & \vdots & & & \\ \hline 0 & -1 & 0 & 0 & 0 & & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 & & 0 & 1 \end{array}\right], \qquad (4\text{-}6)$$

and $Var(z) = W\sigma_W^2$ where W is a (3n-3) x (3n-3) block diagonal matrix given by

$$W = \begin{bmatrix} V & 0 & 0 \\ 0 & V & 0 \\ & \ddots & \\ 0 & 0 & V \end{bmatrix}. \qquad (4\text{-}7)$$

The normal equations (incorporating the restraint $\alpha_1 = m$) take the form

$$\begin{bmatrix} X'W^{-1}X & a \\ a' & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \lambda \end{bmatrix} = \begin{bmatrix} X'W^{-1}z \\ m \end{bmatrix} \tag{4-8}$$

where $a' = (1\ 0\ \ldots\ 0)$ and λ is a Lagrangian multiplier entering in the minimization process. The least squares estimates are given by

$$\begin{bmatrix} \hat{\alpha} \\ \lambda \end{bmatrix} = \begin{bmatrix} X'W^{-1}X & a \\ a' & 0 \end{bmatrix}^{-1} \begin{bmatrix} X'W^{-1}z \\ m \end{bmatrix} = \begin{bmatrix} C & \pm \\ \pm' & 0 \end{bmatrix} \begin{bmatrix} X'W^{-1}z \\ m \end{bmatrix} \tag{4-9}$$

where $\pm' = (1\ 1\ \ldots\ 1)$ and C is the variance-covariance matrix of the estimate $\hat{\alpha}$. The predicted values of the observations are given by

$$\hat{z} = X\hat{\alpha}\ , \tag{4-10}$$

and the deviations by

$$d = z - \hat{z}\ . \tag{4-11}$$

The estimate of σ_w is given by

$$s_w = \sqrt{d'W^{-1}d/(2n-2)} \tag{4-12}$$

and the within-series standard deviation of the estimates by

$$\sigma_{w,\alpha_i} = \sigma_w\sqrt{C_{ii}} = k_n\sigma_w\text{*} \tag{4-13}$$

for i = 2,n.

Let the estimates $\hat{\alpha}$ from equation 4-9 be called $\hat{\alpha}^t$ if the blocks were in the "top-up" position and $\hat{\alpha}^b$ if they were in the "bottom-up" position. Then the final angular values assigned to the blocks are given by

$$\hat{\bar{\alpha}} = \frac{\hat{a}^t + \hat{a}^b}{2} \tag{4-14}$$

where $\hat{\bar{\alpha}} = (\hat{\bar{\alpha}}_1, \hat{\bar{\alpha}}_2, \ldots, \hat{\bar{\alpha}}_n)'$.

*By the symmetry of the intercomparison design, $C_{11} = 0$ and $C_{22} = C_{33} = \ldots = C_{nn}$. Thus the value of C_{ii} (i = 2,n) depends only on n, the number of angle blocks in the design. The values of $k_n = \sqrt{C_{ii}}$ and $k_n^2 = C_{ii}$ are tabulated in Table 1.

4.5 Sources of Error

There are three known types of error in the reported test block values $\hat{\bar{\alpha}}_2, \ldots, \hat{\bar{\alpha}}_n$. The first type, called long-term systematic error, is due primarily to the uncertainty in the accepted value of the reference block. This uncertainty, denoted by E, affects the "top-up" and "bottom-up" values equally and thus must be attached to the reported values given in equation 4-14. For the current set of reference blocks E is believed not to exceed 0.20 second for each block size. Errors of this type from other sources are assumed to be negligible.

The second type of error, called within-series random error, is due to the "noise" in the measurement system which is caused primarily by fluctuations in atmosphereic conditions and instabilities in the autocollimators. The true within-series standard deviation (or, more precisely, the standard deviation of the within-series error values) σ_w is estimated from each individual series by s_w as described in the previous section. The s_w values from all series done over the first several years of operation of the measurement system were pooled to give the best available estimate of σ_w. (In the pooling process the s_w values were weighted according to their degrees of freedom.) The value obtained was

$$\sigma_w = 0.040 \text{ second} \tag{4-15}$$

which can reasonably be considered the true value since it is based on N > 2000 degrees of freedom.

The within-series standard deviation of the reported value $\hat{\bar{\alpha}}_i$ is then easily computed from Equations 4-13 and 4-14 to be

$$\sigma_{w,\hat{\bar{\alpha}}_i} = \frac{k_n \sigma_w}{\sqrt{2}} \tag{4-16}$$

(see Table 1 for values of k_n).

The third type of error, called between-series random error (or short-term systematic error), is due to changes in the system when the blocks are flipped from "top-up" to "bottom-up" positions. (The presence of this type of error was first detected at NBS when we began measuring angle blocks in the two opposite orientations. In many cases the differences between "top-up" and "bottom-up" values were much larger than could be explained by the within-series random error.) Some probable causes for these changes are

(1) imperfections in the master and test block geometries such as out-of-flatness and out-of-squareness of the faces,

(2) misalignment of the autocollimators (i.e., reading some vertical angle), and

(3) the presence of burrs on the anvil which tilt the blocks differently when they are in different orientations.

A discussion of the estimation and propagation of this type of error is given in the Appendix. One result is that the between-series standard deviation of the reported value $\hat{\hat{\alpha}}_i$ is $\sigma_{b,\hat{\hat{\alpha}}_i} = \sigma_b$ (Equation A-11 in the Appendix).

The uncertainty of the reported value $\hat{\hat{\alpha}}_i$ is taken to be the sum of random and systematic components of error. The random component is obtained by combining the within-series and between-series standard deviations in quadrature and then taking the three standard deviation limit. Thus the total uncertainty of $\hat{\hat{\alpha}}_i$ is

$$U_{\hat{\hat{\alpha}}_i} = 3\sqrt{\frac{1}{2} k_n^2\sigma_w^2 + \sigma_b^2} + E \qquad (4\text{-}17)$$

4.6 Statistical Tests

Two statistical tests are applied to each series of measurements in order to maintain control over the measurement process.

The (two-sided) z test compares the computed value of the check block, $\hat{\alpha}_c$, to its historical value, a_c. The test is implemented by forming the statistic

$$z = \frac{\hat{\alpha}_c - a_c}{\sqrt{k_n^2\sigma_w^2 + 2\sigma_b^2}} \qquad (4\text{-}18)$$

and comparing it to the appropriate critical value. Note that the denominator in the above expression is the <u>total</u> standard deviation of $\hat{\alpha}_c$ as given by Equations 4-13 and A-10 (Appendix). The number of degrees of freedom associated with σ_w^2 and σ_b^2 (N and M respectively) are very large, so the denominator can be reasonably considered a "true" standard deviation. Thus, under the hypothesis that the true value of the check standard is a_c, z has the standard normal distribution (mean zero and variance one). The critical value at the .01 level of significance is $z_{.995} = 2.58$. If $|z| > 2.58$ the usual statistical procedure is to reject the above hypothesis. However, a large z-value may be caused by something other than a change in the true value of the check block. In fact, it may be caused by a change in the true value of the reference block or by the malfunctioning of a component in the measurement system. Whichever the case, a large z value indicates that something extraordinary has happened, and the usual procedure is to repeat the series of measurements.

The (one-sided) F test compares the computed within-series standard deviation, s_w, to the true within-series standard deviation, σ_w. The test is implemented by forming the statistic

$$F_1 = \frac{s_w^2}{\sigma_w^2} \tag{4-19}$$

and comparing it to the appropriate critical value. Under the hypothesis that $s_w^2 = \sigma_w^2$, F_1 has the Snedecor F distribution with 2n-2 and N degrees of freedom. The critical value at the .01 level of significance is $F_{.99}(2n-2,N)$ which is available in tables (see Table 2, column A). If $F_1 > F_{.99}(2n-2,N)$ then the above hypothesis is rejected because the observed system variability is extraordinarily large. This indicates the malfunctioning of a component of the measurement system or a blunder of some sort by the operator. As before, the usual procedure is to repeat the series of measurements.

If a given series passes both the z and F tests, then that series is said to be "in statistical control".

A third statistical test is performed on each pair of "top-up" and "bottom-up" series in order to control the between-series variability. The test is implemented by forming the statistic

$$F_2 = \frac{s_b^2}{\sigma_b^2} \tag{4-20}$$

(where s_b^2 is computed as in the Appendix) and comparing it to the appropriate critical value. Under the hypothesis that $s_b^2 = \sigma_b^2$, F_2 has the F distribution with n-1 and M degrees of freedom (see Table 2, column B). If $F_2 > F_{.99}(n-1,M)$ then the above hypothesis is rejected. A large value of F_2 may be caused either by a malfunctioning of the measurement system or by one or more blocks being of inferior quality. If the operator decides the latter case is true then he may accept the results and note the problem in the appropriate Report(s) of Calibration. Otherwise both series should be repeated.

5. Example

In August 1974 an intercomparison was made between one set of reference blocks, one set of check blocks, and five sets of test blocks. The data sheet for the 1° blocks is shown in Figure 5.1 as an example.

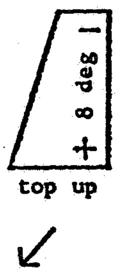
ANGLE BLOCK CALIBRATION - INTERCOMPARISON OF 7 BLOCKS

Nominal Angle	Check Std. Lot # L9325	Comments:
1 3 5 15 20 30 45	Observer W. G.	
sec min deg	Date 8/15/74	

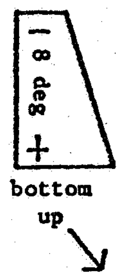
Do not pause during any group of 7 measurements. OK to pause between groups.

Block Identification

1 NBS-1
2 NBS-7
3 BAC-16
4 UCC-36
5 HAC-24
6 NBS-25
7 512-C



top up



bottom up

Top up

I.	2	2.75
	3	2.92
	2	2.70
	1	2.92
	2	2.68
	4	2.93
	2	2.71
II.	3	2.89
	4	2.85
	3	2.86
	1	2.78
	3	2.78
	5	3.27
	3	2.79
III.	4	2.77
	5	3.25
	4	2.71
	1	2.70
	4	2.65
	6	1.96
	4	2.65
IV.	5	3.14
	6	1.92
	5	3.08
	1	2.57
	5	3.09
	7	3.14
	5	3.04
V.	6	1.87
	7	3.05
	6	1.84
	1	2.47
	6	1.74
	2	2.22
	6	1.73
VI.	7	2.96
	2	2.21
	7	2.93
	1	2.40
	7	2.94
	3	2.40
	7	2.94

Bottom up

V.	6	1.38
	7	2.53
	6	1.33
	1	2.05
	6	1.43
	2	1.84
	6	1.41
VI.	7	2.55
	2	1.89
	7	2.66
	1	2.14
	7	2.61
	3	2.14
	7	2.65
I.	2	2.03
	3	2.26
	2	2.07
	1	2.23
	2	2.06
	4	2.36
	2	2.04
II.	3	2.23
	4	2.31
	3	2.29
	1	2.15
	3	2.26
	5	2.45
	3	2.26
III.	4	2.27
	5	2.41
	4	2.25
	1	2.11
	4	2.21
	6	1.42
	4	2.22
IV.	5	2.40
	6	1.49
	5	2.30
	1	2.05
	5	2.34
	7	2.59
	5	2.33

USCOMM-NBS-DC 3/75

Figure 5.1

Calibration of 1° Angle Blocks - August 1974

Block	Series 1*	Series 2**	Avg	Diff (1-2)
	Deviation from Nominal Angle			
NBS-1 (reference)	-.15"	-.15"	---	---
NBS-7 (check)	-.36"	-.32"	-.34"	-.04"
BAC-16	-.14"	-.07"	-.10"	-.07"
UCC-36	-.15"	-.02"	-.08"	-.13"
HAC-24	+.35"	+.11"	+.23"	+.24"
NBS-25	-.83"	-.79"	-.81"	-.04"
512-C	+.39"	+.38"	+.38"	+.01"
Uncertainty (eq. 4-17)			0.40"	
Accepted value of check standard	-.39"	-.39"		
Computed value of check standard	-.36"	-.32"		
z statistic (eq. 4-18)	-1.17	-2.63		
$z_{.995}$	2.58	2.58		
Accepted within-series standard deviation (σ_W)	.040"	.040"		
Computed within-series standard deviation (s_W)	.018"	.026"		
F_1 statistic (eq. 4-19)	0.21	0.42		
$F_{.99}(12,\infty)$	2.18	2.18		
Accepted between-series standard deviation (σ_b)	.063			
Computed between-series standard deviation (s_b)	.078			
F_2 statistic (eq. 4-20)	1.55			
$F_{.99}(6,650)$	2.84			

* top-up

** bottom-up

Figure 5.2

The estimated angular values and associated statistics computed from these observations are shown in Figure 5.2. The three statistical tests show both series of measurements to be "in statistical control". The listed uncertainty value, as computed from Equation 4-17, applies to each of the average values enclosed in the box.

6. Conclusion

The lumping of test sets into a single large intercomparison scheme has proven to be much more economical than calibrating one set at a time. The data reduction process has been fully computerized for several years. The output includes a Report of Calibration for each set of test blocks and for the check blocks.

The larger angle blocks may alternately be calibrated by the "absolute" method whereby each test block is compared to several indexing table angles that sum to exactly 360°. There is no reference block involved, hence the test block is measured more accurately. The drawback is that the number of measurements required becomes cumbersome for blocks of 5° or less. Currently this method is used only to calibrate the NBS reference blocks or single large test blocks.

In closing it seems fitting to speculate on how angle blocks might be measured more accurately in the future by the intercomparison method. An examination of the three components of error in the uncertainty statement (Equation 4-17) is revealing. For the case n=7 (as in the example in Section 5) the numerical value of the uncertainty is

$$
\begin{aligned}
U_{\hat{\bar{\alpha}}_i} &= 3\sqrt{\frac{1}{2}(.4815)(0.040)^2 + (0.063)^2} + 0.20 \\
&= 3\sqrt{0.000385 + 0.003969} + 0.20 \qquad (6\text{-}1) \\
&= 0.20 + 0.20 \\
&= 0.40
\end{aligned}
$$

The within-series random error component is by far the smallest of the three, and it is doubtful that any further reduction could be made there. On the other hand, the between-series component seems to be an area where real improvements are possible. There has not yet been a full-scale investigation at NBS into the cause of differences between "top-up" and "bottom-up" values. One might expect large differences to be positively correlated with the maximum out-of-flatness and out-of-squareness of the blocks. However, visual examination of the data has never given a hint of such a correlation. Further investigation into the problem is needed. If the differences could be explained and removed then the between-series component of error could be drastically reduced.

The systematic component consists solely of the uncertainty in the reference blocks. This component could be reduced to some extent by painstakingly recalibrating the reference blocks "absolutely". However, in order to obtain realistic uncertainty values the reference blocks would have to be calibrated in both the "top-up" and "bottom-up" positions. It is likely that differences between values obtained in the two positions would be larger than expected, thus the uncertainty values of the reference blocks would be inflated. A reduction in this component of error then seems to depend on the reduction in the between-series component.

It now seems clear that under the present measurement system the main obstacle blocking the path toward more accurate angle block calibrations is the between-series beast which rears its ugly head in two places. The removal of this obstacle, if it is indeed removable, would be a significant accomplishment.

Table 1

Values of k_n and k_n^2 for selected values of n

n	$k_n = \sqrt{C_{ii}}$	$k_n^2 = C_{ii}$
4	.7949	.6318
5	.7465	.5572
6	.7111	.5057
7	.6939	.4815
8	.6824	.4657

Table 2

Critical values of two F distributions at the .01 level of significance for selected values of n

No. of Blocks in series	Column A	Column B
n	$F_{.99}(2n-2, \infty^{*})$	$F_{.99}(n-1, 650^{**})$
4	2.80	3.82
5	2.51	3.35
6	2.32	3.05
7	2.18	2.84
8	2.07	2.68

*Since $N > 2000$ it is reasonable to take $N = \infty$ in tabulating the critical values.

**The value of M is 650 as of the writing of this paper.

References

1. Cameron, J.M., The Use of the Method of Least Squares in Calibration, NBSIR 74-587, September, 1974.

2. Farago, Francis T., Handbook of Dimensional Measurement, Industrial Press, Inc., 1968.

3. Handbook of Industrial Metrology, Society of Manufacturing Engineers, Prentice-Hall, Inc., 1967.

4. Knoyle, C.H., Production of Combination Angle Gages, The Engineer, Sept. 1, 1950, p. 237-8, Machy (London).

5. Miller, L., Engineering Dimensional Metrology, Edward Arnold Ltd., London, 1962.

6. Moore, Wayne R., Foundations of Mechanical Accuracy, The Moore Special Tool Company, 1970.

7. Parsons, S.A.J., Metrology and Gauging, MacDonald and Evans Ltd., London, 1970.

8. Thomas, Geoffrey G., Engineering Metrology, Butterworth and Company, Ltd., London, 1974.

9. Veale, R.C., and Reeve, C.P., A Survey of the Temporal Stability of Angle Blocks, NBSIR 74-601, November, 1974.

10. Zelen, Marvin, Linear Estimation and Related Topics, Chapter 17 of Survey of Numerical Analysis, edited by John Todd, McGraw-Hill, 1957.

Appendix

Estimation and Propagation of Between-Series Error

A.1. Estimation of Between Series Standard Deviation

The true between-series standard deviation σ_b is estimated from each pair of series in which the same set of blocks are measured in opposite orientations. The least squares estimates of the angular values, as given in Equation 4-9 may be expressed as

$$\hat{\alpha}_i^t = (\alpha_i + \delta_i^t - \delta_1^t) + \theta_i^t \quad \text{and}$$

$$\hat{\alpha}_i^b = (\alpha_i + \delta_i^b - \delta_1^b) + \theta_i^b \tag{A-1}$$

for the "top-up" and "bottom-up" positions respectively where i = 2,n. The α_i's (without superscript) are the "true" angular values of the blocks, and the θ's are error values representing within-series variability. The θ's have mean zero, and

$$\text{Var}\begin{bmatrix}\theta_2^t \\ \theta_3^t \\ \cdot \\ \cdot \\ \cdot \\ \theta_n^t\end{bmatrix} = \text{Var}\begin{bmatrix}\theta_2^b \\ \theta_3^b \\ \cdot \\ \cdot \\ \cdot \\ \theta_n^b\end{bmatrix} = \tilde{C}\sigma_w^2 \tag{A-2}$$

where the (n-1) x (n-1) matrix $\tilde{C}$ is the variance-covariance matrix of the estimates $\hat{\alpha}_2, \hat{\alpha}_3, \ldots, \hat{\alpha}_n$ which is obtained by deleting the first row and column from the matrix C given in Equation 4-9.

The δ's are independent error values from a population with mean zero and variance σ_b^2 representing between-series variability. The subscript indicates the block to which the error is attached (with the reference block being 1). The δ values are assumed to remain constant throughout their respective series and thus do not contribute to the within-series error. The standard deviation of the δ values, σ_b, is estimated by considering the n-1 vector of differences from a pair of series

$$h = \begin{bmatrix} \hat{\alpha}_2^t - \hat{\alpha}_2^b \\ \hat{\alpha}_3^t - \hat{\alpha}_3^b \\ \vdots \\ \hat{\alpha}_n^t - \hat{\alpha}_n^b \end{bmatrix} \qquad \text{(A-3)}$$

Now from Equations A-1,

$$\hat{\alpha}_i^t - \hat{\alpha}_i^b = \delta_i^t - \delta_1^t - \delta_i^b + \delta_1^b + \theta_i^t - \theta_i^b \qquad \text{(A-4)}$$

for i = 2,n, thus

$$\mathrm{Var}(\hat{\alpha}_i^t - \hat{\alpha}_i^b) = 4\sigma_b^2 + 2C_{ii}\sigma_w^2 \qquad \text{(A-5)}$$

and

$$\mathrm{Cov}(\hat{\alpha}_i^t - \hat{\alpha}_i^b, \hat{\alpha}_j^t - \hat{\alpha}_j^b) = 2\sigma_b^2 + 2C_{ij}\sigma_w^2 \qquad \text{(A-6)}$$

for i = 2,n; j = 2,n; and $i \neq j$. Let I denote the (n-1) x (n-1) identity matrix and J denote the (n-1) x (n-1) matrix of one's. Then the variance-covariance matrix of the vector h is given by

$$2(I + J)\sigma_b^2 + 2\tilde{C}\sigma_w^2 = 2(I + J + \tilde{C}r)\sigma_b^2 = H\sigma_b^2 \qquad \text{(A-7)}$$

where $r = \dfrac{\sigma_w^2}{\sigma_b^2}$, a known constant. The estimate of σ_b from a pair of series is then given by

$$s_b = \sqrt{\frac{h'H^{-1}h}{n-1}} \qquad \text{(A-8)}$$

where n-1 is the associated degrees of freedom.

Note that during the ongoing measurement process the ratio r is known because σ_w and σ_b are assumed known. However, before σ_b was determined the value of r was unknown. To initially determine σ_b the estimates s_b from each series done on the measurement system during the first several years of operation were computed according to Equations A-7 and A-8 with $r = 0$. These estimates were then pooled (using weights n-1) to give an estimate of σ_b. The value of r was recomputed using this estimate of σ_b, and the process was repeated in an iterative fashion until the estimates of σ_b converged. The value obtained was

$$\sigma_b = 0.063 \quad \text{second} \tag{A-9}$$

which can reasonably be considered the true between-series standard deviation since it is based on M = 650 degrees of freedom.

A.2 Propagation of Between-Series Error

The between-series standard deviation of $\hat{\alpha}_i$, the estimated angular value of the i^{th} block from a <u>single series</u>, is easily computed from Equations A-1 to be

$$\sigma_{b,\hat{\alpha}_i^t} = \sigma_{b,\hat{\alpha}_i^b} = \sqrt{2}\sigma_b \ . \tag{A-10}$$

This term appears in the statistic given in Equation 4-18.

The between-series standard deviation of $\hat{\bar{\alpha}}_i$, the estimated mean angular value of the i^{th} block from a <u>pair of series</u>, is computed from Equations 4-14 and A-1 to be

$$\sigma_{b,\hat{\bar{\alpha}}_i} = \sigma_b$$

This term appears in the expression for the total uncertainty of the reported angular values given in Equation 4-17.