

Reference

NBS
Publi-
cations

NATL INST. OF STAND. & TECH



A11106 978667

NBSIR 78-898

**ANALYTICAL AND NUMERICAL TECHNIQUES FOR ANALYZING
AN ELECTRICALLY SHORT DIPOLE WITH A NONLINEAR LOAD**

Motohisa Kanda



Electromagnetic Fields Division
National Engineering Laboratory
National Bureau of Standards
Boulder, Colorado 80303

November 1978

QC

100

.U56

78-898

APR 17 1979

157-400-318
80 80
1036
N.D. 17-898

NBSIR 78-898

121

ANALYTICAL AND NUMERICAL TECHNIQUES FOR ANALYZING AN ELECTRICALLY SHORT DIPOLE WITH A NONLINEAR LOAD

Motohisa Kanda

Electromagnetic Fields Division
National Engineering Laboratory
National Bureau of Standards
Boulder, Colorado 80303

November 1978



NBS Interagency Report # 78-898

U.S. DEPARTMENT OF COMMERCE, Juanita M. Kreps, Secretary

Sidney Harman, Under Secretary

Jordan J. Baruch, Assistant Secretary for Science and Technology

NATIONAL BUREAU OF STANDARDS, Ernest Ambler, Director

Contents

Abstract-----	1
1. Introduction-----	1
2. Theory-----	2
2.1 Analytical Technique-----	3
2.2 Numerical Technique-----	5
3. Results-----	7
4. Conclusion-----	8
Acknowledgments-----	8
References-----	9
List of Figures-----	10

ANALYTICAL AND NUMERICAL TECHNIQUES
FOR ANALYZING AN ELECTRICALLY SHORT DIPOLE WITH A NONLINEAR LOAD

Motohisa Kanda

Electromagnetic Fields Division
National Bureau of Standards
Boulder, Colorado 80303

An electrically short dipole with a nonlinear dipole load is analyzed theoretically using both the analytical and numerical techniques. The analytical solution is given in terms of the Anger function of imaginary order and imaginary argument, and is derived from the nonlinear differential equation for the Thévenin's equivalent circuit of a dipole with a diode. The numerical technique used was to solve the nodal equation using a time-stepping finite difference equation method. The nonlinear resistance of the diode is treated using an iteration technique. A comparison between the analytical and numerical solutions is given.

Key words: Anger function; electrically-short dipole; iteration method; nonlinear differential equation; nonlinear load; time-stepping finite difference equation technique.

1. Introduction

It is well accepted that microwave radiation can produce biological effects. Although it is very difficult to determine the biological hazards associated with electromagnetic (EM) fields, the biological effects resulting from the EM fields can be adequately described from knowledge of one or more parameters that characterize the EM field. One of the advocated field parameters for quantifying hazardous EM fields is an electric energy density which can easily be calculated from the electric field strength [1]. For this reason, the National Bureau of Standards has recently developed a broadband, isotropic electric energy density meter (EDM). The EDM consists of three orthogonal, electrically short dipoles with diode detectors between the arms of the dipoles. To synthesize such an EDM in terms of its frequency response and its dynamic range, one needs to analyze theoretically an electrically short dipole with a nonlinear load such as a diode.

Traditionally, the characteristics of an antenna with a nonlinear load have been analyzed in the frequency domain by considering the spectral components of the solutions at harmonic frequencies [2]. For example, Sarker and Weiner [3] have used the Volterra series analysis to obtain the scattering due to nonlinearly loaded antennas. The nonlinear transfer function of a nonlinearly loaded antenna was determined at several harmonic frequencies. However, the calculation of the nonlinear transfer function is generally very tedious, particularly when the circuit model of the nonlinear load is complicated and its nonlinearity is strong. For this reason, recent analyses of the nonlinearly loaded antennas have been considered using direct time-domain techniques. Schuman [4] has described the application of the time-domain method of moments technique to determine the scattering current on a thin wire with discrete nonlinear resistive loading. Lin and Tesche [5,6] have used frequency-domain data to compute the time-dependent currents and voltages across the nonlinear load by means of the Laplace transform. A unified numerical procedure was recently proposed by Landt [7]. The antenna characteristics were derived from a time-domain electric field integral equation, whereas the nonlinear network analysis was performed in a simple time-domain nodal analysis.

In this paper, two techniques for analyzing an electrically short dipole with a nonlinear load are described. The first technique, described in Section 2.1, gives an analytical solution to the first-order, nonlinear time-domain differential equation in terms of Anger functions. The second technique, described in Section 2.2, is a time-stepping, finite difference solution technique for obtaining the numerical solution to the time-domain nonlinear differential equation. The nonlinear effect due to a diode is solved by a conventional iteration method. This numerical technique gives the physical insight for the

nonlinear load effects on antennas in terms of the time-domain waveform, and also permits the consideration of certain problems which are too complicated to be treated by an analytical technique.

The nonlinear effects on an electrically short dipole are first investigated in Section 3 in the time domain using a time-stepping, finite difference solution technique. The frequency responses and the dynamic ranges of the dipole with a nonlinear diode are then compared using the two different techniques described above.

2. Theory

Using the frequency-domain concept of the effective length and the driving point impedance of an electrically short dipole without a nonlinear load, the Thévenin's equivalent circuit for a diode with a nonlinear load is shown in Fig. 1. The element $v_i(t)$ is the induced, open-circuit voltage at the dipole terminal and is given by

$$v_i(t) = e_{inc}(t) \cdot h_e, \quad (1)$$

where e_{inc} is the normal, incident electric field strength and h_e is the effective length of the dipole. The element C_a is the equivalent driving point capacitance of the dipole. A parallel combination of a linear capacitance C_d and a nonlinear resistance R_d represents a simplified model of a diode.

For an electrically short dipole antenna (i.e., $kh < 1$ where k is free-space wave number), the effective length h_e and the driving point capacitance C_a of the antenna are given by [8]

$$h_e = \frac{h(\Omega - 1)}{2(\Omega - 2 + \ln 4)} \text{ meter}, \quad (2)$$

and

$$C_a = \frac{4\pi\epsilon_0 h}{(\Omega - 2 - \ln 4)} \text{ farads}. \quad (3)$$

The symbols have the following meanings: h is half the physical length of a dipole antenna in meters, ϵ_0 is the free-space permittivity in farads/meter, and Ω is the antenna thickness factor (i.e., $\Omega = 2 \ln(2h/a)$ where a is the antenna radius in meters).

When $h = 0.02$ m and $a = 2.84 \times 10^{-5}$ m (i.e., the antenna thickness factor $\Omega = 14.50$), the effective length h_e and the equivalent antenna input capacitance of the antenna C_a become, respectively, 9.72×10^{-3} m and 0.2×10^{-12} F.

When an electrically short dipole is terminated with a nonlinear load such as a beam lead Schottky barrier diode, the effect of loading on the antenna can be analyzed using the simple equivalent circuit shown in Fig. 1, which consists of a parallel combination of a nonlinear resistance R_d and a linear capacitance C_d . Here, the nonlinear resistance R_d of the diode is characterized by its V-i characteristic, i.e.,

$$i(t) = I_s (e^{\alpha v_o(t)} - 1) \quad (4)$$

The symbols have the following meanings: $i(t)$ is the current (ampere) and $V_o(t)$ is the voltage (volt) across the diode junction; I_s is the saturation current which is assumed to be 2×10^{-9} A; and $\alpha = \frac{q}{nkT} \approx 38 \text{ V}^{-1}$ where q is the electronic charge (1.6×10^{-19} coulomb), n is the diode ideality factor (~ 1.05), k is Boltzmann's constant (1.38×10^{-23} J/K), and T is the temperature (~ 290 K).

The junction capacitance C_j and the package capacitance C_p are combined and are shown as C_d in Fig. 1. The exact value of C_d varies from diode to diode and is very difficult to determine. In this paper, C_d is assumed to be constant and equal to 0.34 pF for the beam lead Schottky barrier diode. In a more elaborate diode model, the junction capacitance C_j is nonlinear and is a function of the built-in potential V_b as

$$C_j(V) = \frac{C_j(0)}{\left(1 - \frac{V}{V_b}\right)^{1/2}} \quad (5)$$

for a step junction. The package capacitance C_p is generally constant. The more general treatment of an antenna with linear and nonlinear loads, such as a nonlinear resistance and a nonlinear junction capacitance as well as a linear package inductance and a linear series resistance of a diode, is being pursued.

In this paper, a simple nonlinear resistance R_d with a constant diode capacitance C_d for the beam lead Schottky barrier diode shown in Fig. 1 is used in the following sections for analyzing the loading effect of an electrically short dipole with a nonlinear diode using analytical and numerical techniques.

2.1. Analytical Technique

Using thévenin's equivalent nonlinear circuit shown in Fig. 1, the voltage equation and the corresponding current equation are given below:

$$v_i(t) + \frac{q_a(t)}{C_a} = \frac{q_d(t)}{C_d} = v_o(t) \quad (6)$$

and

$$\frac{dq_a(t)}{dt} + \frac{dq_d(t)}{dt} + i(t) = 0 \quad , \quad (7)$$

where q_a and q_d are the charges on C_a and C_d , respectively, and i is the current through the nonlinear resistance R_d .

By substituting

$$y(t) = e^{av_o(t)} \quad , \quad (8)$$

one gets

$$\frac{dy(t)}{dt} + ay^2(t) + y(t)f(t) = 0 \quad (9)$$

where

$$a = \frac{\alpha I_s}{C_a + C_d}, \quad (10)$$

$$f = \frac{-\alpha}{(1 + C_d/C_a)} \left[\frac{I_s}{C_a} + \frac{dv_1(t)}{dt} \right] \quad (11)$$

Equation (9) given above is a first-order, nonlinear differential equation. The detailed mathematical steps for solving the equation have been performed by P. F. Wacker. When the induced voltage $v_1(t)$ is a periodic sinusoid, i.e., $v_1(t) = V_1 \sin \omega t$ the detected dc voltage averaged over a complete cycle \bar{V}_0 is given by

$$V_0 = -\frac{1}{\alpha} \ln \frac{2\pi T J_{jT}(jU)}{e^{2\pi T} - 1}, \quad (12)$$

where $J_{jT}(jU)$ is the Anger function of imaginary order (jT) and imaginary argument (jU); T is the normalized period as

$$T = \frac{\alpha I_s}{\omega(C_a + C_d)} \quad (13)$$

and U is the normalized induced voltage as

$$U = \frac{\alpha V_1 C_a}{C_a + C_d}. \quad (14)$$

Using the series representation for the Anger function, one gets

$$V_0 = -\frac{1}{\alpha} \ln(S_1 - T S_2) \quad (15)$$

where

$$S_1 = 1 + \sum_{\substack{m=1 \\ \text{even} \\ k=2 \\ \text{even}}}^{\infty} \frac{U^m}{\pi (k^2 + T^2)} \quad (16)$$

and

$$S_2 = \sum_{\substack{m=1 \\ \text{odd} \\ k=1 \\ \text{odd}}}^{\infty} \frac{U^m}{\pi (k^2 + T^2)}. \quad (17)$$

At high frequencies where

$$T = \frac{\alpha I_s}{\omega(C_a + C_d)} \ll 1, \quad (18)$$

then one can show that, for small V_i

$$V_o = -\frac{\alpha}{4} \left(\frac{V_i}{1 + C_d/C_a} \right)^2, \quad (19)$$

and, for large V_i

$$V_o = -\frac{V_i}{1 + C_d/C_a}. \quad (20)$$

Equation (19) indicates that, for the small induced steady-state voltage V_i , the output dc voltage V_o is a square-law function of the induced voltage V_i . On the other hand, eq (20) indicates that, for large induced voltage V_i , the output dc voltage is proportional to the induced voltage V_i .

2.2 Numerical Technique

A time-stepping difference equation technique can be used for solving the nonlinear network shown in Fig. 1. The basic idea of a time-stepping, finite difference equation technique is briefly discussed below. More detailed discussion on this subject is given by Calahan [9].

The linear and nonlinear elements are converted into resistance-current source equivalent networks in the nodal equation method. For instance, in a regular R, C, and L circuit, we have

$$v_n = R i_n \quad (21)$$

$$i_{n+1} = C \frac{v_{n+1} - v_n}{\tau} \quad (22)$$

and

$$v_{n+1} = L \frac{i_{n+1} - i_n}{\tau} \quad (23)$$

where τ is a sampling time interval. Once initial v_o or i_o is given, one can determine v_1 and i_1 , then v_2 and i_2 , etc.; such a method is, therefore, called a time-stepping, finite difference equation method.

In order to deal with nonlinear equations which result from nonlinear elements such as a diode, the general approach to a solution of such problems is by iteration. The basic technique used is discussed below. Given a nonlinear system $f(i,v)$, the solution of $f(i,v) = 0$ yields the solution for the system response.

First $f(i,v)$ is expanded at an initial solution

$$\begin{pmatrix} i^0 \\ v^0 \end{pmatrix};$$

that is,

$$f(i,v) = f(i^0, v^0) + J[f(i,v)] \begin{pmatrix} \Delta i^0 \\ \Delta v^0 \end{pmatrix} \quad (24)$$

where J is the Jacobian of f and has the form

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial i_1} & \dots & \frac{\partial f_1}{\partial v_{n+m}} \\ \vdots & & \vdots \\ \frac{\partial f_{n+m}}{\partial i_1} & \dots & \frac{\partial f_{n+m}}{\partial v_{n+m}} \end{pmatrix} \quad (25)$$

Now $f(i,v) = 0$ determines

$$\begin{pmatrix} \Delta i^0 \\ \Delta v^0 \end{pmatrix}$$

as

$$J[f(i^0, v^0)] \begin{pmatrix} \Delta i^0 \\ \Delta v^0 \end{pmatrix} = -f(i^0, v^0) \quad (26)$$

The solution of

$$\begin{pmatrix} \Delta i^0 \\ \Delta v^0 \end{pmatrix}$$

updates the initial value of

$$\begin{pmatrix} i^0 \\ v^0 \end{pmatrix},$$

via

$$\begin{pmatrix} i^{n+1} \\ v^{n+1} \end{pmatrix} = \begin{pmatrix} i^n \\ v^n \end{pmatrix} + \begin{pmatrix} \Delta i^n \\ \Delta v^n \end{pmatrix} \quad (27)$$

The operation is repeated for $n = 0, \dots$ until the change

$$\begin{pmatrix} \Delta i \\ \Delta v \end{pmatrix}$$

is significantly small.

3. Results

In this section the time-domain waveforms of a sinusoidal wave at various nodes are shown first using the time-stepping difference equation technique discussed in Section 2.2. Here, a single time-domain sinusoidal wave is divided into 16 discrete digitized points (17 points including both ends) in the analysis. Then, the detected sinusoidal wave, averaged over many cycles which corresponds to dc output of the electrically short dipole with the beam lead Schottky barrier diode, is given.

Figure 2 shows the time-domain waveform of the first sinusoidal wave at 100 MHz at various nodes. At node 1, an applied sinusoidal wave V_1 is shown with unit amplitude. At node 2, the detected sinusoidal wave, which is skewed or distorted due to nonlinearity of the diode, is shown. It is obvious that, at node 2, the detected voltage starts developing in the negative polarity, which eventually leads to dc negative output for the dipole with the diode.

Figure 3 shows the time-domain waveform of the dipole with the diode due to the 10th sinusoidal wave driving voltage. Again, a sinusoidal wave at node 1 is an applied driving voltage with unit amplitude. The detected time-domain waveform at node 2 after diode detection indicates a much more pronounced negative charge accumulation. Finally, an almost dc detected voltage starts appearing in the negative polarity after 10 cycle time average.

Figure 4 shows the time-domain waveforms of the dipole with the diode due to the 100th sinusoidal wave induced voltage. The negative charge accumulation is much more pronounced, and constant detected dc voltage appears after 100 cycle time average.

To arrive at the steady-state time average of these sinusoidal wave excitations using the time-stepping, finite difference equation technique along with the iteration method, 400 sinusoidal waves, which correspond to 6400 discrete points (16 points per one cycle), are applied successfully to compute 6400 discrete output voltages, which are then numerically time-averaged.

Figure 5 shows the detected dc voltage V_0 from the dipole with the diode as a function of induced voltage $V_i (= e_{inc} \times |h_e|$, where e_{inc} is the normal incident electric field and h_e is the dipole effective length). The detected dc voltages V_0 as a function of the induced voltage V_i are calculated using both the analytical technique given in Section 2.1 and the numerical technique described in Section 2.2. Generally, the agreement between the analytical and numerical solutions is very good. Below an induced voltage V_i of about 0.1 volt, the detected dc voltage V_0 is equal to square of the induced voltage V_i . On the other hand, above an induced voltage V_i of about 0.1 volt, the detected dc voltage V_0 is proportional to the induced voltage V_i . Thus, the diode detection is square-law at a small signal level, but becomes linear at a large signal level.

Figure 6 shows the transfer function of an electrically short dipole with the beam lead Schottky barrier diode as a function of frequency. Here, the transfer function is defined as a ratio of the detected dc voltage V_0 to the amplitude of the induced voltage $V_i (= e_{inc} \cdot h_e)$ expressed in dB when e_{inc} is equal to 1 V/m. Thus, the transfer function so

defined is for a detected dc voltage V_0 of several mV, which corresponds to a square-law signal level. As indicated in eq (2), the effective length of an electrically short dipole is independent of frequency. The transfer functions of the dipole with the diode are calculated both analytically and numerically. Except at frequencies below 10 KHz, the analytical and numerical transfer functions agree to within 8 dB, and are constant with frequency up to several gigahertz.

The discrepancy between the analytical and numerical results of the transfer function at the extremely low-frequency range could be due to insufficient time-averaging in the numerical solution; that is, although successive 400 sinusoidal waves are used in the numerical computation, the convergence at the extremely low-frequency range may still be poor.

The sharp cut-off (20 ~ 40 dB per octave) below 10 kHz in the transfer function predicted from both analytical and numerical results can be explained as follows. Figure 7 shows the detected time-domain sinusoidal waveforms at node 2 at various frequencies. It is clearly indicated that, for example the detected time-domain sinusoidal waveform at 1 kHz (whose amplitude is magnified by a factor of 10) is very similar to the original sine wave excitation, whereas the time-domain waveforms at 1 and 100 MHz are more skewed (or distorted) compared to the original sinusoidal wave. The detected time-domain sinusoidal waveform is less strongly skewed (or distorted) at lower frequencies than at higher frequencies. Since the skewness of the detected waveform and the rate of charge accumulation decrease in the lower frequency range below 10 kHz, so does the transfer function of the dipole with the diode, as shown in Fig. 6.

4. Conclusion

This paper introduces two independent techniques to analyze the electrically short dipole with a nonlinear load. The nonlinear load considered in this paper is a beam lead Schottky barrier diode. The analytical solution, given in Section 2.1 in terms of the Anger function of imaginary order and imaginary argument, was derived from the nonlinear differential equation for the Thévenin's equivalent circuit of the dipole with the diode. The numerical technique, explained in Section 2.2, is basically to solve nodal equations using the timestepping finite difference equations technique. The nonlinear resistance of a diode was treated by an iteration method.

Both analytical and numerical solutions basically agree very well. The transition from the square-law detection region to the linear detection region was observed as the induced voltage was varied. The transfer function of the electrically short dipole with a diode was also investigated. The decrease in the transfer function at frequencies below 10 kHz was explained through the time-domain sinusoidal waveforms obtained from the time-stepping, finite difference equation technique.

One of the advantages of using the analytical solution in terms of the Anger function of imaginary order and imaginary magnitude is that the solution is given in the closed form, and is very easy to evaluate. However, it is very difficult, or may be even impossible, to find the closed-form solution of a nonlinear differential equation for much more complicated models of an antenna and a diode, e.g., including a nonlinear capacitance, a linear inductance, as well as nonlinear and linear resistances, and a linear capacitance. In such cases, a time-stepping finite difference equation technique along with an iteration method provides an accurate time-domain solution for more general nodal equations.

The analysis of a linear antenna with a nonlinear load, in which a diode model consists of a parallel combination of a nonlinear resistance and a nonlinear junction capacitance along with a linear series inductance, a linear series resistance, and a linear package capacitance, has been carried out using a time-stepping, finite difference equation technique along with an iteration method and will be presented in the future.

Acknowledgments

The author wishes to express his appreciation to P. F. Wacker, N. S. Nahman, R. R. Bowman, S. M. Riad, C. K. S. Miller, and F. X. Ries for many stimulating discussions and suggestions in the preparation of this paper.

References

- [1] Wacker, P. F. and Bowman, R. R., Quantifying hazardous electromagnetic fields: Scientific basis and practical considerations, IEEE Trans. on Microwave Theory and Technique, MTT-19, No. 2, pp. 178-187, February 1971.
- [2] Wiener, N., Nonlinear Problems in Random Theory, Cambridge, Mass.: MIT Press, 1959.
- [3] Sarkar, T. K. and Weiner, D. D., Scattering analysis of nonlinearly loaded antennas, IEEE Trans on Antennas and Propagation, AP-24, No. 2 pp. 125-131, March 1976.
- [4] Schuman, H., Time-domain scattering from a nonlinearly loaded wire, IEEE Trans. on Antennas and Propagation, AP-22, pp. 611-613, July 1974.
- [5] Liu, T. K. and F. M. Tesche, Analysis of antennas and scatterers with nonlinear loads, IEEE Trans. on Antennas and Propagation, AP-24, No. 2, pp. 131-139, March 1976.
- [6] Lied, T. K., Tesche, F. M. and Deadrick, F. J., Transient excitation of an antenna with a nonlinear load: Numerical and experimental results, IEEE Trans. on Antennas and Propagation, AP-25, No. 4, pp. 539-541, July 1977.
- [7] Landt, J. A., A unified time-domain electromagnetic and circuit analysis code, 1977 USNC/URSI Meeting, Stanford, California, pp. 153, June 1977.
- [8] Kanda, M., The characteristics of broadband, isotropic, electric field and magnetic field probes, NBSIR 77-868, November 1977.
- [9] Calahan, D. A., Computer-aided Network Design, revised edition. New York: McGraw-Hill, Inc., 1972.

List of Figures

Figure 1. Thévenin's equivalent circuit of an electrically short dipole with a diode.

Figure 2. Time-domain waveform of 1st detected 100 MHz sinusoid.

Figure 3. Time-domain waveform of 10th detected 100 MHz sinusoid.

Figure 4. Time-domain waveform of 100th detected 100 MHz sinusoid.

Figure 5. Detector voltage response at 100 MHz.

Figure 6. Transfer function of an electrically short dipole with a nonlinear load.

Figure 7. Time-domain waveform of detected sinusoids at various frequencies.

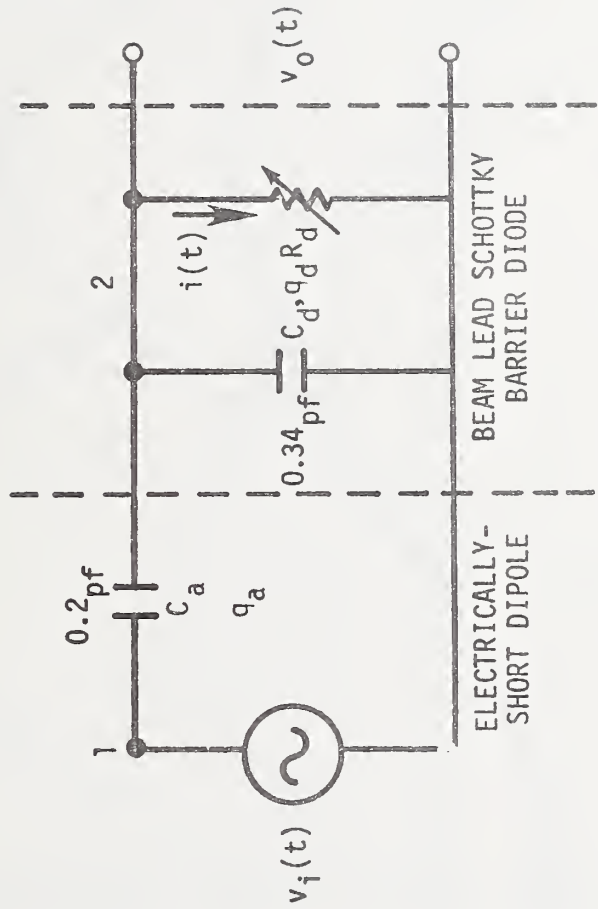


Figure 1. Thévenin's equivalent circuit of an electrically short dipole with a diode.

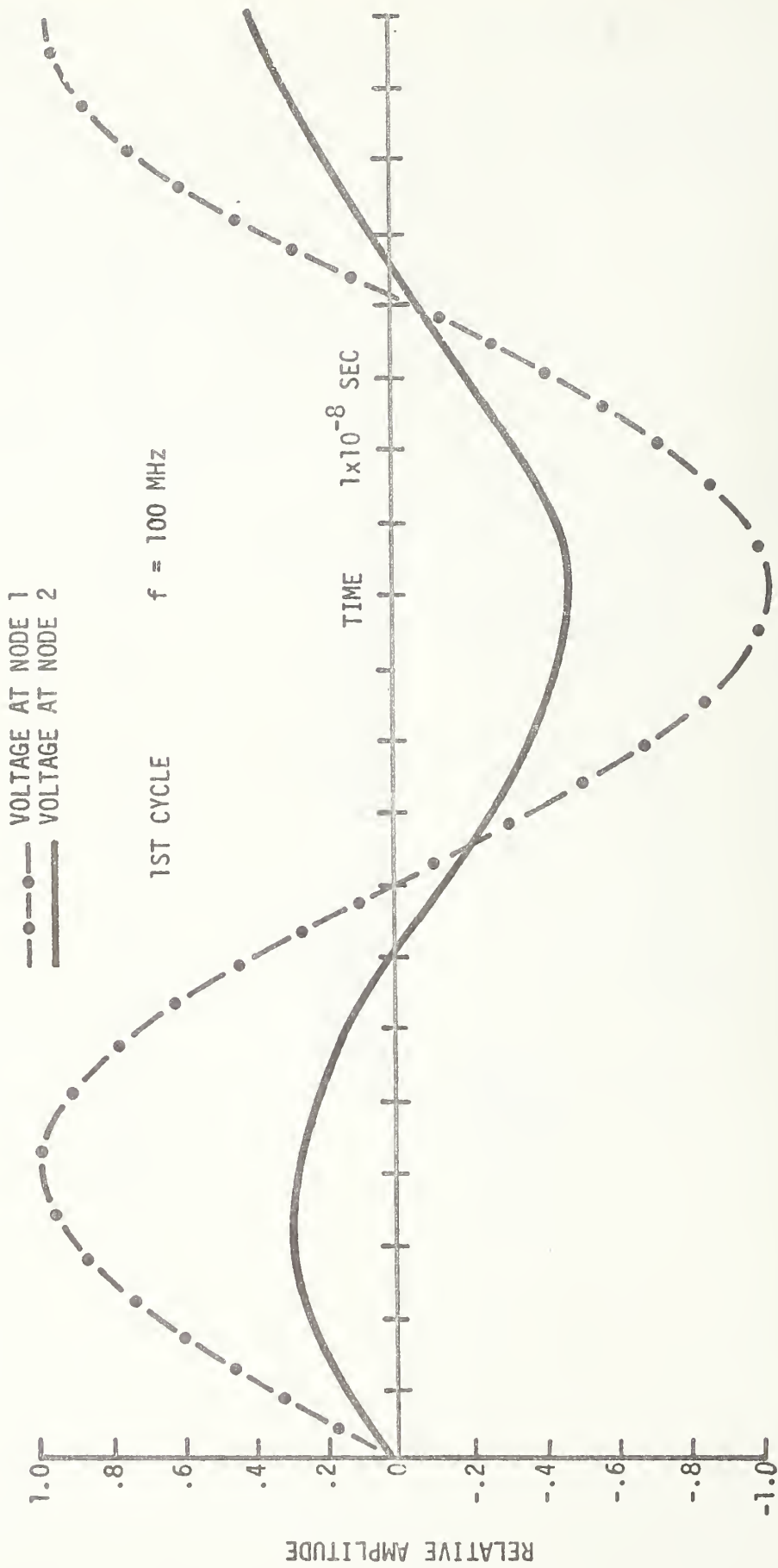


Figure 2. Time-domain waveform of 1st detected 100 MHz sinusoid.

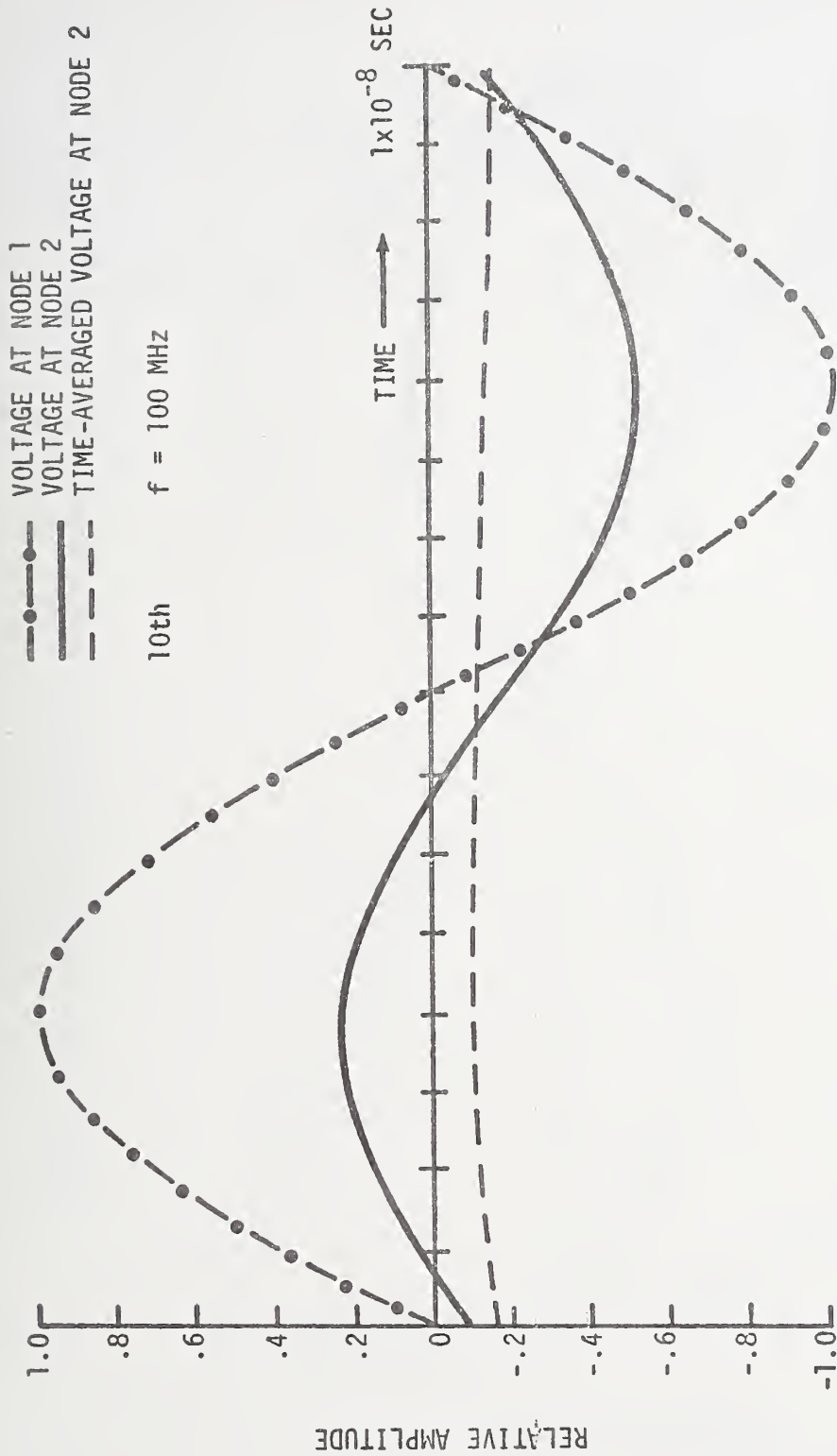


Figure 3. Time-domain waveform of 10th detected 100 MHz sinusoid.

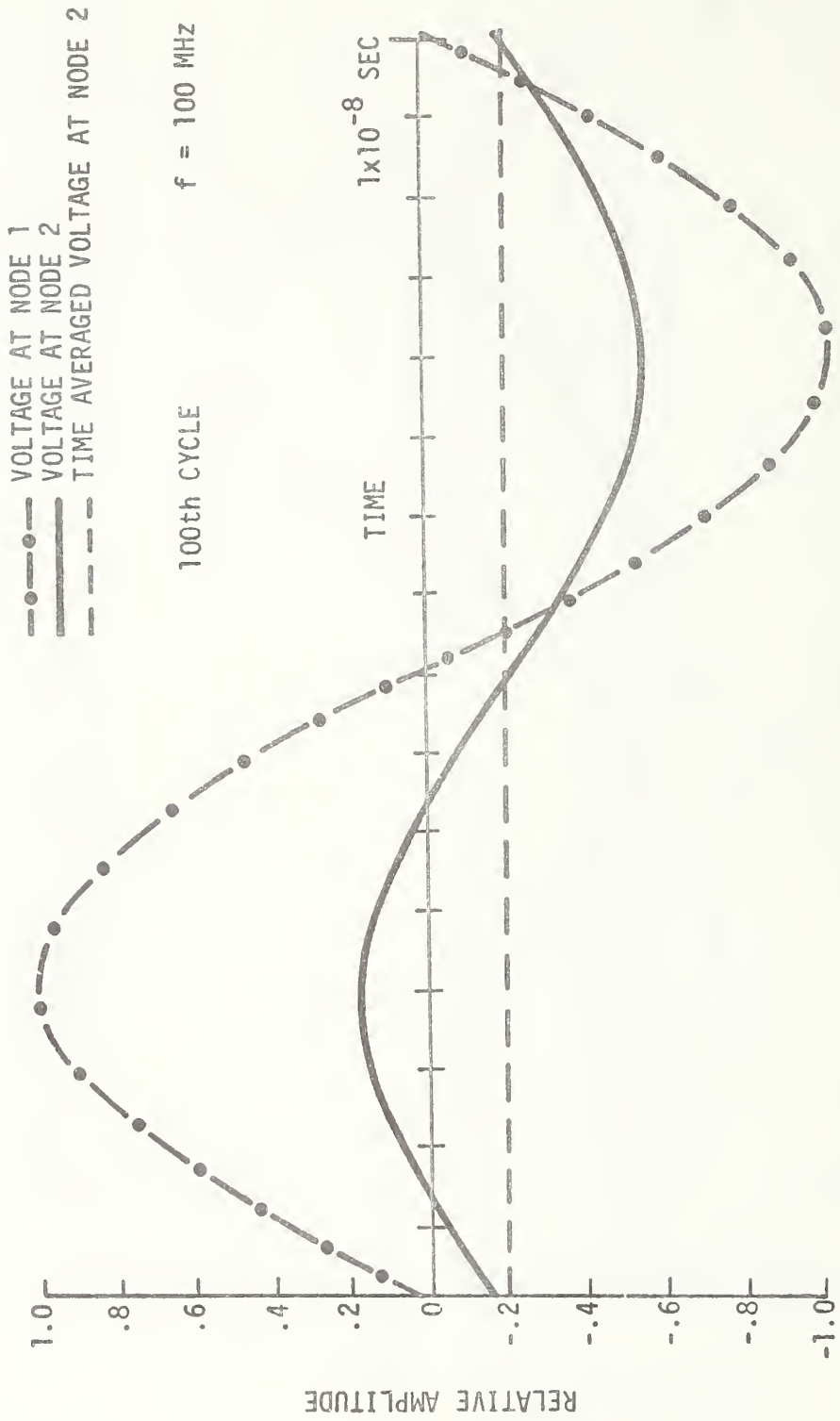


Figure 4. Time-domain waveform of 100th detected 100 MHz sinusoid.

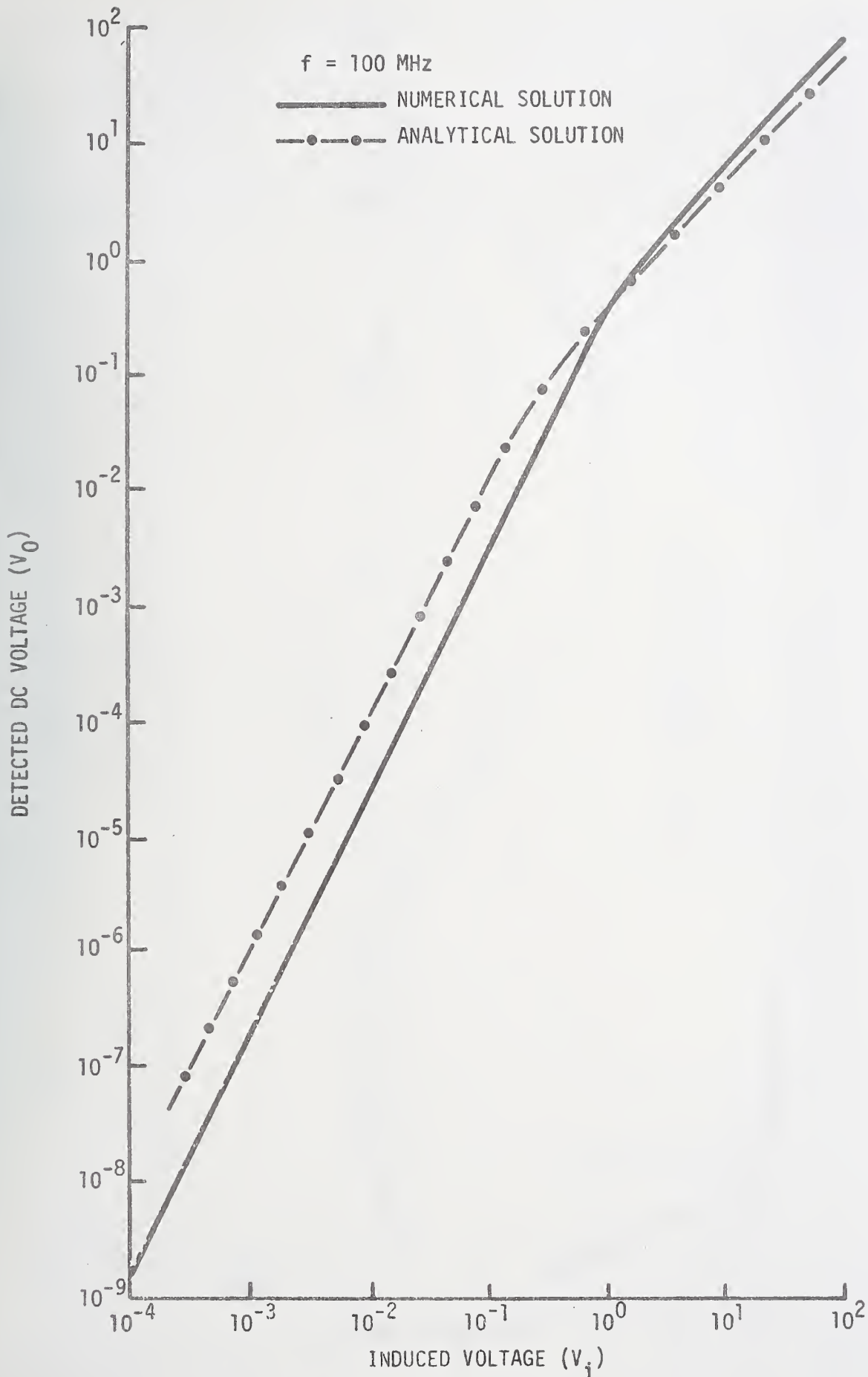


Figure 5. Detector voltage response at 100 MHz.

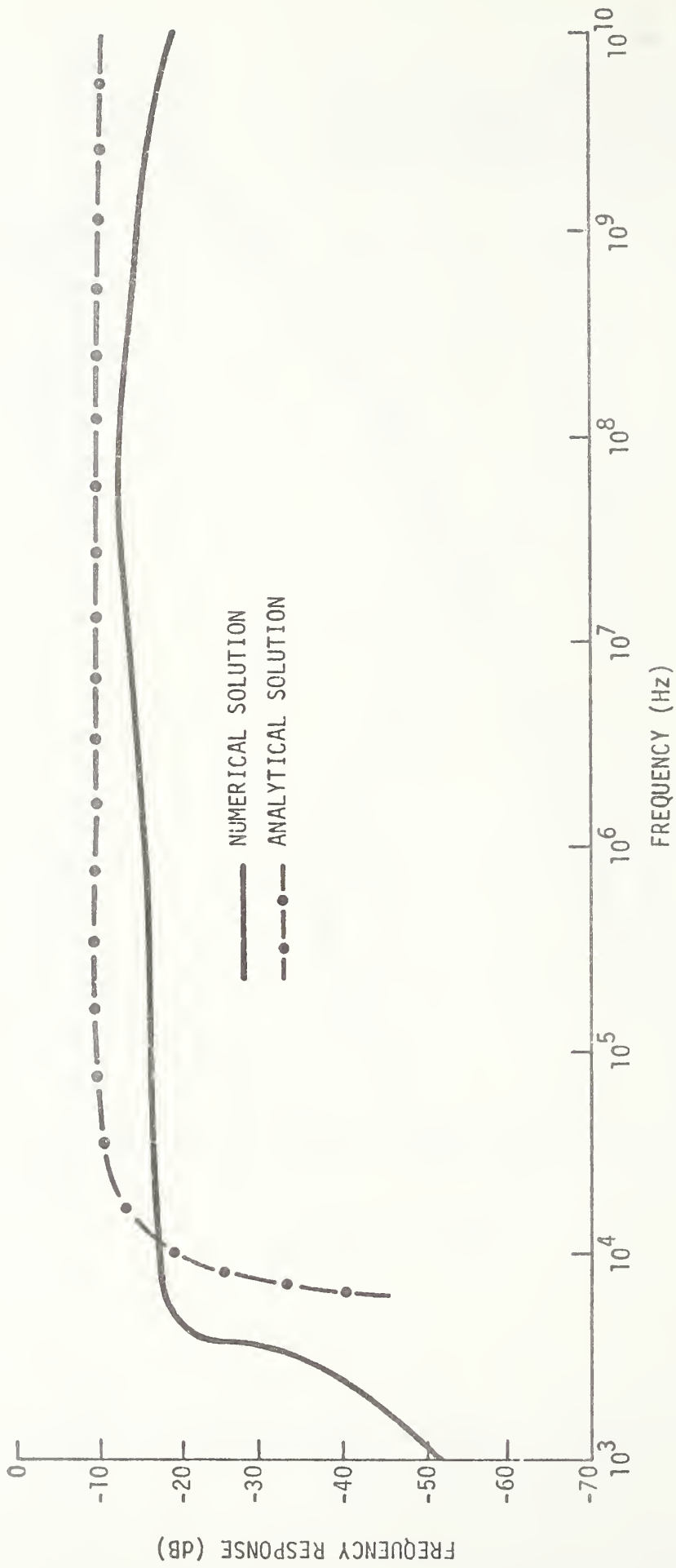


Figure 6. Transfer function of an electrically short dipole with a nonlinear load.

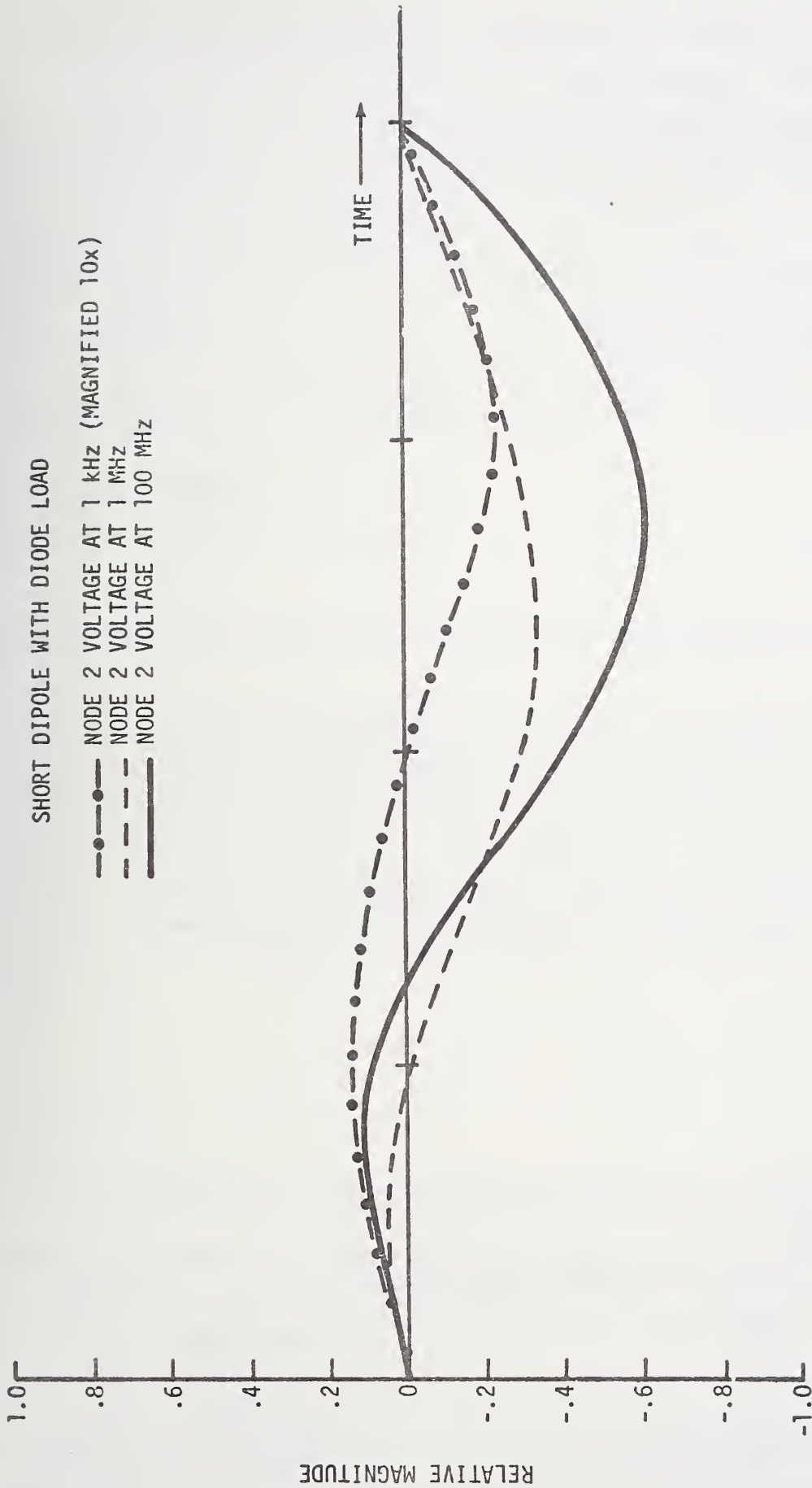


Figure 7. Time-domain waveform of detected sinusoids at various frequencies.

U.S. DEPT. OF COMM. BIBLIOGRAPHIC DATA SHEET		1. PUBLICATION OR REPORT NO. NBSIR 78-898	2. Gov't Accession No.	3. Recipient's Accession No.
4. TITLE AND SUBTITLE Analytical and Numerical Techniques for Analyzing an Electrically Short Dipole With a Nonlinear Load			5. Publication Date November 1978	
			6. Performing Organization Code 723.03	
7. AUTHOR(S) M. Kanda,			8. Performing Organ. Report No.	
9. PERFORMING ORGANIZATION NAME AND ADDRESS NATIONAL BUREAU OF STANDARDS DEPARTMENT OF COMMERCE Boulder, Colorado 80303			10. Project/Task/Work Unit No. 7233281	
			11. Contract/Grant No.	
12. Sponsoring Organization Name and Complete Address (Street, City, State, ZIP) Same as Item 9.			13. Type of Report & Period Covered	
			14. Sponsoring Agency Code	
15. SUPPLEMENTARY NOTES				
16. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.) An electrically short dipole with a nonlinear dipole load is analyzed theoretically using both the analytical and numerical techniques. The analytical solution is given in terms of the Anger function of imaginary order and imaginary argument, and is derived from the nonlinear differential equation for the Thévenin's equivalent circuit of a dipole with a diode. The numerical technique used was to solve the nodal equation using a time-stepping finite difference equation method. The nonlinear resistance of the diode is treated using an iteration technique. A comparison between the analytical and numerical solutions is given.				
17. KEY WORDS (six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons) Anger function; electrically-short dipole; iteration method; nonlinear differential equation; nonlinear load; time-stepping finite difference equation technique.				
18. AVAILABILITY <input checked="" type="checkbox"/> Unlimited <input type="checkbox"/> For Official Distribution. Do Not Release to NTIS <input type="checkbox"/> Order From Sup. of Doc., U.S. Government Printing Office Washington, D.C. 20402, SD Cat. No. C13 <input checked="" type="checkbox"/> Order From National Technical Information Service (NTIS) Springfield, Virginia 22151		19. SECURITY CLASS (THIS REPORT) UNCLASSIFIED		21. NO. OF PAGES 20
		20. SECURITY CLASS (THIS PAGE) UNCLASSIFIED		22. Price \$3.00



