3-1505

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NBSIR 78-1505

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U.S. DEPARTMENT OF COMMERCE, Juanita M. Kreps, *Secretary*

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LIST OF SYMBOLS

$E_U(\omega)$	Power spectral density of true velocity fluctuation.
$E_{U_i}(\omega)$	Power spectral density of indicated velocity fluctuation.
f	Frequency (Hz).
f_c	Characteristic frequency.
K	Slope of steady flow calibration curve.
L	Response length or distance constant of helicoid anemometer.
L_1	Estimate of alongwind integral length scale.
S	Rotation rate of anemometer rotor.
t	Time.
U	True instantaneous velocity.
$\overline{U}$	True mean velocity.
U_o	Velocity intercept of steady flow calibration curve.
U_i	Velocity indicated by helicoid anemometer.
$\overline{U}_i$	Mean indicated velocity.
u_c	Characteristic amplitude of true velocity fluctuation.
$u(t)$	True velocity fluctuation, normalized.
$u_i(t)$	Indicated velocity fluctuation, normalized.
ε	Normalized amplitude of sinusoidal velocity fluctuation.
ω	Circular frequency.
Ω	Reduced frequency.
τ_o	Time constant of helicoid anemometer.

LIST OF FIGURES

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ABSTRACT

An experimental study of the overspeeding error for helicoid anemometers in periodic air flows is described. The ranges of amplitude and frequency for which a simple nonlinear model for the dynamic response of such instruments remains valid are presented. It is shown that the model is valid for typical atmospheric applications of such instruments. A simple method is presented whereby the effects of inertial lag and nonlinearity may be taken into account in obtaining measurements of alongwind power spectra in the atmosphere.

Key Words: Air; anemometer; dynamic response; experimental; lag; unsteady flow.

1. INTRODUCTION

In November of 1975 a National Bureau of Standards Interagency Report, NBSIR 75-772, entitled "The Dynamic Response of Helicoid Anemometers" was prepared for the Federal Highway Administration. In that report certain questions were raised as to the effect that nonlinearities inherent in the response characteristics of helicoid anemometers might have upon the validity and accuracy of atmospheric turbulence spectra measured with these instruments. In particular, if the mathematical model presented in the above mentioned report were valid under typical atmospheric conditions, the linear transfer function commonly used to correct for the inertial averaging characteristics of these instruments would be inadequate in principle, and an improved method of correction based on the nonlinear model would be required to refine spectral data. The present study addresses this question in two parts. First, over what range of the basic parameters is the nonlinear model valid? And second, if the criterion for validity were satisfied for typical atmospheric conditions, how then should one apply corrections to the measured spectral data?

The range of applicability of the dynamic response equation developed in Reference (1) is ascertained by examining experimentally the overspeeding error encountered in a periodic flow field produced in the NBS Unsteady Wind Tunnel. A parametric study of the effect of mean speed, fluctuation amplitude, and frequency on the overspeeding error was conducted. The overspeeding error was selected to validate the nonlinear response equation because it is relatively easily measured, and its existence is due solely to the instrument nonlinearity. Furthmore, in the previous study the measured overspeeding error, as presented, exhibited a large amount of apparent scatter, and the overall trend of the data indicated a departure from the predicted values. Only a careful parametric study can reveal the reasons for this behavior.

Under the conditions of validity established experimentally, a correction procedure based on the model is presented in the present report.

2. BASIC LIMITATIONS ON THE RANGE OF APPLICABILITY

The basic dynamic response equation in (1) may be written

$$L \frac{dS}{dt} + (U + U_o)S = KU^2, \qquad (1)$$

where L is the response length (distance constant) characteristic of the instrument, S is the anemometer rotation rate, U is the instantaneous flow velocity, and K and U_o are empirically determined from the asymptotic steady flow calibration curve:

$$S \simeq K\,(U - U_o). \qquad (2)$$

Equation (1) is based on the assumption that under dynamical conditions the instantaneous departure from equilibrium is not too large. For helicoid anemometers this means that instantaneous local blade angles of attack must remain small. As the departure from equilibrium increases additional terms may be required in the Taylor expansion used to represent the driving torque in the derivation of Equation (1). In extreme cases local flow separation may occur on the propeller blades radically altering the value of torque coefficients in the expansion.

Intuitively, at low frequencies the anemometer is expected to follow the velocity fluctuations quite closely with departures from equilibrium remaining small. This is so even for amplitudes approaching 100% of the mean speed because at any instant anemometer lag is small, and the instantaneous rotation rate is never far from its equilibrium value where S(t) = KU(t). However, as the frequency increases the increasing inertial lag of the instrument causes departures from equilibrium which can be kept small only by correspondingly reducing the amplitude of the flow fluctuations. Hence, a reasonable criterion under which Equation (1) should remain applicable is that the product of frequency and amplitude (suitably normalized) must remain less than some critical value. In fact, a rigorous perturbation analysis of Equation (1) and its associated torque expansion shows that this is precisely the case. Physically, this condition amounts to restricting flow accelerations to sufficiently modest values that the anemometer speed of rotation can change at a comparable rate.

The restriction to small departures from equilibrium may be rigorously stated that $u_c\Omega$ must remain small compared to unity, where u_c is a measure of the velocity fluctuation amplitude relative to the mean velocity $\overline{U}$, and Ω is a reduced characteristic frequency given by

$$\Omega = \frac{2\pi f_c L}{\overline{U}} \tag{3}$$

where f_c is a frequency in Hz which is characteristic of the velocity fluctuation. Just how small $u_c\Omega$ must remain may be determined experimentally by conducting a parametric study of the overspeeding error at various mean speeds, frequencies, and amplitudes, and comparing the results to values predicted from the mathematical model.

3. EXPERIMENTAL PROCEDURES AND RESULTS

The experimental apparatus and test procedures for the measurement of overspeeding errors were substantially the same as those presented in Reference (1), with the following refinements to reduce measurement uncertainty. On a day to day basis, steady flow calibrations were performed to determine K and U_o for each set of overspeeding error measurements. Small changes in U_o affected the mean indicated velocity. In addition, changes in the cold resistance of the hot-wire used to measure the velocity fluctuations were monitored closely. These changes were due to temperature drift in the ambient air stream. The hot resistance of the wire was adjusted at frequent intervals to compensate for this drift in ambient temperature. With these refinements uncertainty in the true mean air speed determined from the hot-wire anemometer was reduced to ± 1%.

The helicoid anemometer was tested in the NBS Unsteady Wind Tunnel where essentially sinusoidal longitudinal velocity fluctuations of varying amplitude and frequency were superimposed on mean air speeds of 15, 20, 30, and 40, feet per second (4.6, 6.1, 9.1, and 12.2 m/s).

Neglecting bearing friction, the overspeeding error for sinusoidal velocity fluctuations, as derived in Reference (1), is given by

$$\frac{\overline{U}_i - \overline{U}}{\overline{U}} = \frac{\varepsilon^2}{2} \frac{\Omega^2}{1+\Omega^2} \tag{4}$$

where $\overline{U}_i$ is the mean indicated velocity defined by

$$\overline{U}_i = \frac{\overline{S}}{K} + U_o ,$$

and ε is the peak amplitude of the velocity fluctuation.

Data for the overspeeding error were acquired parametrically. For each mean velocity, the fundamental amplitude ε was varied while frequency was held constant. Thus for each set of data Ω was held constant. Frequencies of 0.5, 1.0, 2.0, 3.0, and 4.0 Hz were used, and values of ε ranged as high as 0.8. Equation (4) shows that for constant Ω the overspeeding error should vary linearly with $\varepsilon^2/2$.

Figures 1, 2, 3, and 4 present measured values of the overspeeding error as functions of $\varepsilon^2/2$ for four values of $\bar{U}$. The mean velocity is indicated in the legend to each figure. For each figure several sets of data are shown corresponding to the different frequencies for which data were acquired at each velocity. Data for successive frequencies are offset vertically for clarity of presentation. The solid straight line for each frequency represents the theoretical overspeeding error from Equation (4), as calculated from measured values of ε and Ω.

It is evident from these figures that at the lowest frequency (0.5 Hz) the theoretical curves based on Equation (4) are in good agreement with the measured overspeeding error. At higher frequencies it is apparent that the model breaks down for the reasons already discussed, and agreement between theory and experiment is limited to successively smaller values of $\varepsilon^2/2$ as the frequency increases. Thus, the inference drawn earlier, that the range of applicability for the model equation may be simply stated in terms of a maximum permissable value of $\varepsilon\Omega$, appears reasonable. An appropriate criterion, based on the data, is

$$\varepsilon\Omega \leq 0.4 \ . \qquad (5)$$

On each straight line in each of the four figures an asterisk is placed at the maximum value $\varepsilon^2/2$ for which the above condition is satisfied. In Figure 5 all the measured overspeeding error data satisfying this criterion are plotted as a function of the predicted values from Equation (4). It is inferred from these results that Equation (1) is valid for the helicoid anemometer within the experimental limits given by Expression (5).

4. APPLICABILITY TO TYPICAL ATMOSPHERIC CONDITIONS

An estimate of the frequency of the energy containing eddies present in the atmosphere may be obtained from the longitudinal scale and the mean wind speed at any given elevation. The scale may be conservatively taken to be of the same order as the elevation. Hence, at an elevation of 52 feet (15.8 meters), the approximate height of the deck of the Sitka, Alaska cable-stayed bridge (Reference 2), the longitudinal scale L_1 is on order of 15 meters. The characteristic frequency is then approximately

$$f_c \simeq \frac{\bar{U}}{L_1}$$

and the reduced frequency is simply

$$\Omega = 2\pi \frac{L}{L_1} \ .$$

A measure of the amplitude of alongwind velocity fluctuations may be obtained from typical values of the variance reported in Reference (2). Thus,

$$u_c \simeq 0.2$$

An estimate of the numerical value of the dimensionless group, $\varepsilon\Omega$, is

$$u_c\Omega \simeq 0.2 \ (2\pi\frac{L}{L_1}) \ .$$

For Gill helicoid anemometers of the type used in the present study, L = 1.07 m, and one finds

$$u_c\Omega \simeq 0.09 \ .$$

This estimate is nearly an order of magnitude smaller than the maximum value given in Expression (5). It is evident, therefore, that Equation (1) is indeed valid for typical atmospheric conditions.

5. PROCEDURES FOR CORRECTING MEASUREMENTS OF ATMOSPHERIC SPECTRA

The usual correction procedure for spectral measurements is based on a linear approximation to Equation (1). For illustrative purposes the effect of friction (U_o) shall be neglected here. Defining $U_i = \frac{S}{K}$ to be the indicated velocity, Equation (1) may be written.

$$L \frac{dU_i}{dt} + UU_i = U^2 \ . \tag{6}$$

The velocity field is of the form

$$U = \overline{U} \ (1 + u(t)), \tag{7}$$

where u(t) is a stationary random function of time with zero mean. The indicated velocity may be represented by

$$U_i = \overline{U}_i \ (1 + u_i(t)), \tag{8}$$

where from Equation (6), $u_i(t)$ satisfies the following equation to first order:

$$\tau_o \frac{du_i}{dt} + u_i = u(t) \tag{9}$$

where the time constant τ_o is given by $\tau_o = L/\overline{U}$. This linear approximation necessarily implies that $\underline{U}_i = \overline{U}$. The magnitude of the associated transfer function is simply

$$\frac{1}{\sqrt{1 + (\omega\tau_o)^2}}$$

where $\omega\tau_o = \Omega$, and ω is the circular frequency.

The power spectrum of U, $E_U(\omega)$, may be obtained quite simply from a measured spectrum of U_i, $E_{U_i}(\omega)$ by using the relation

$$E_U(\omega) = [\ 1 + (\omega\tau_o)^2]\ E_{U_i}(\omega). \tag{10}$$

An equivalent linear correction to the spectrum $E_U(\omega)$ can be computed from a record of $u_i(t)$ either digitally or by analog methods by simply performing the linear operations indicated in Equation (9) to obtain a record of u(t) from which $E_U(\omega)$ may then be obtained.

The latter technique may also be used to obtain an improved record of U(t) from $U_i(t)$ which takes into account the nonlinear response characteristics by performing the nonlinear operations indicated in Equation (6). Solving this equation for U(t),

$$U(t) = \frac{U_i(t)}{2}\left[1 + \sqrt{1 + \frac{4\ L}{U_i{}^2}\ \frac{dU_i}{dt}}\right] . \tag{11}$$

These operations on the measured signal $U_i(t)$ can be performed readily by analog or digital methods, and the spectrum $E_u(\omega)$ can be computed directly from the record of U(t). In this manner both the inertial lag and the instrument nonlinearity are taken into account. This approach has the added advantage that an estimate of τ_o is not required a priori since it is the distance constant L which appears explicitly in Equation (11).

6. CONCLUSIONS

A suitable mathematical model for helicoid anemometers has been validated experimentally in a periodic air flow. The model has been shown to be applicable to such instruments in a typical environment of atmospheric turbulence. A simple procedure to correct measurements of atmospheric alongwind spectra for both inertial lag and nonlinearity has been suggested. These results cannot be expected to apply to indefinitely high frequencies since turbulence scales on the order of rotor characteristic dimensions will be spatially averaged and the model presented does not take this into consideration. However, instruments of the type investigated in the course of this study may be used over the range of frequencies and scales of primary interest in the study of wind loading of such structures as cable-stayed suspension bridges.

7. REFERENCES

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2. C. L. Gerhardt and D. D. Nelson, Monitoring and Recording Wind Velocities, Directions, and Effects at Long-Span Bridge Sites, Report No. FHWA-RD-76-10, August 1975.

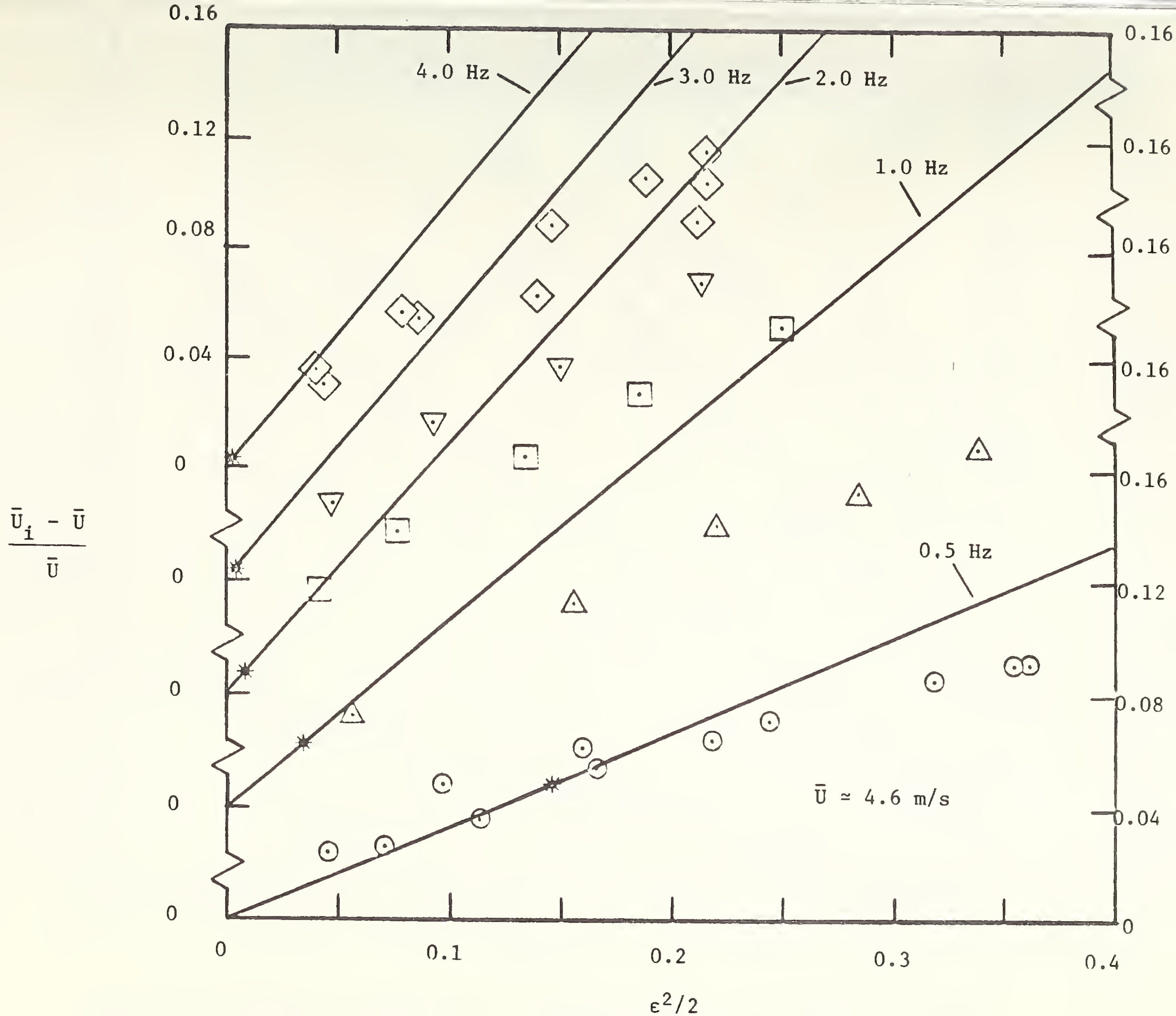

Figure 1. Variation of Overspeeding - Error with Amplitude and Frequency.

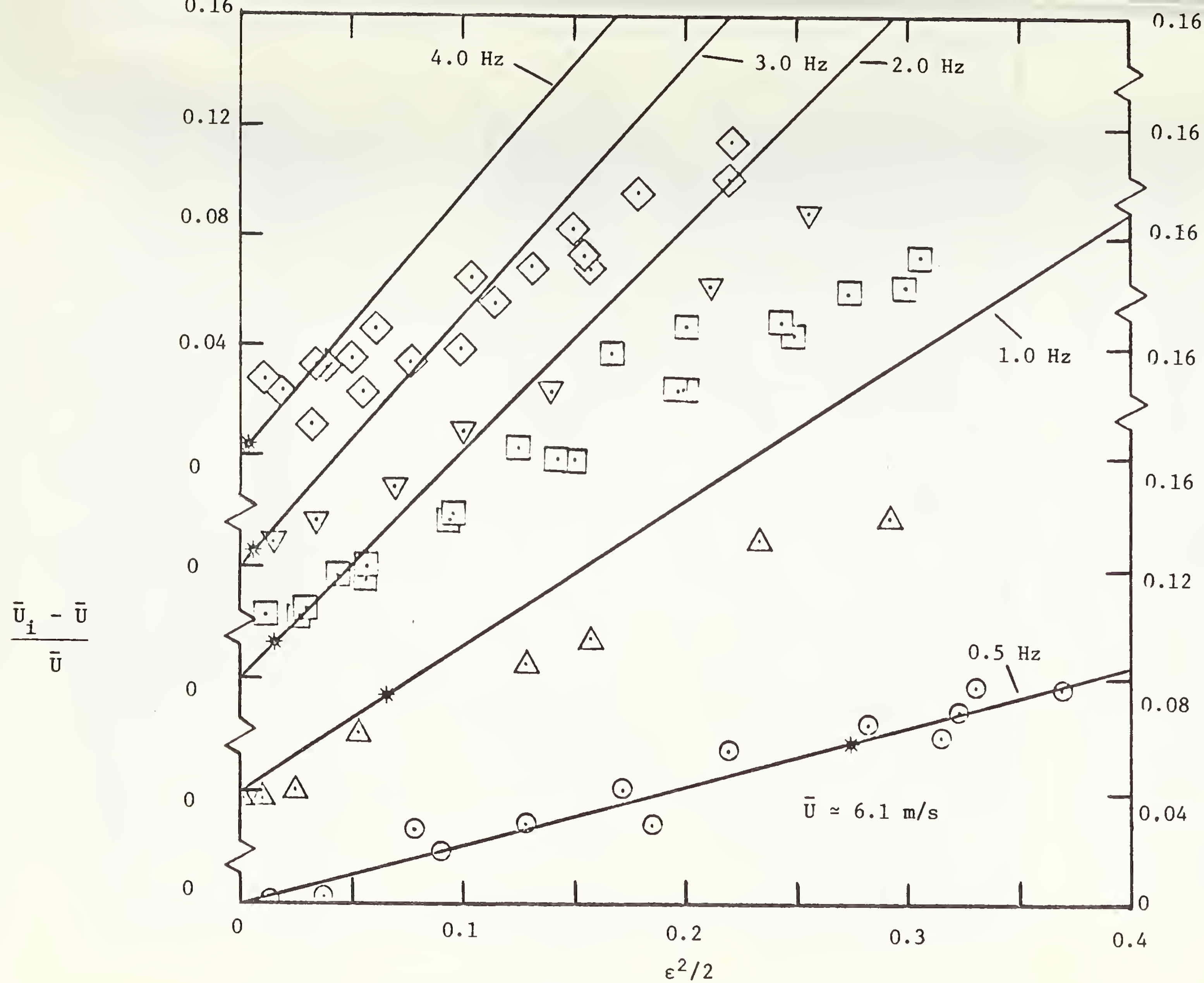

Figure 2. Variation of Overspeeding - Error with Amplitude and Frequency.

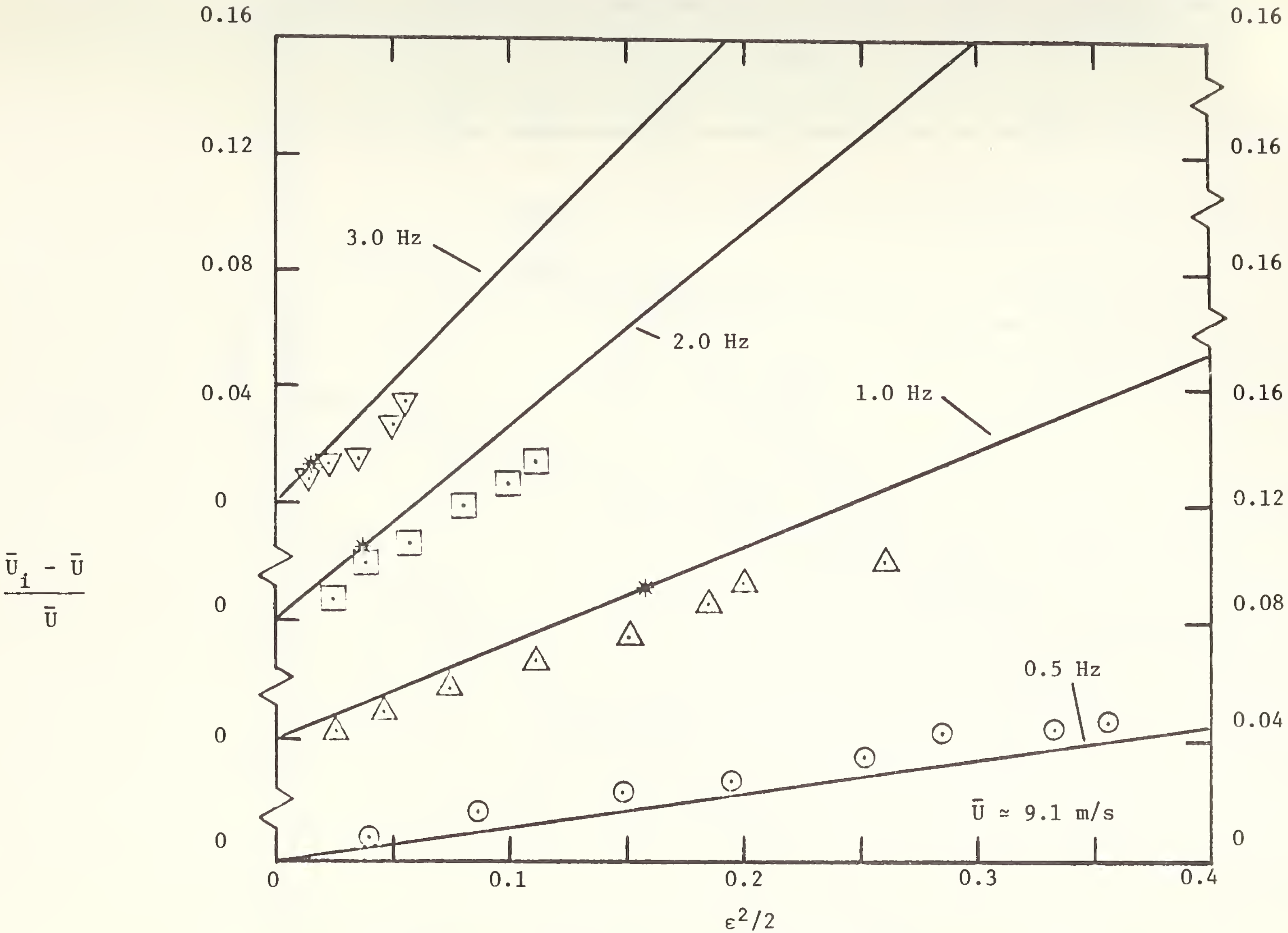

Figure 3. Variation of Overspeeding - Error with Amplitude and Frequency.

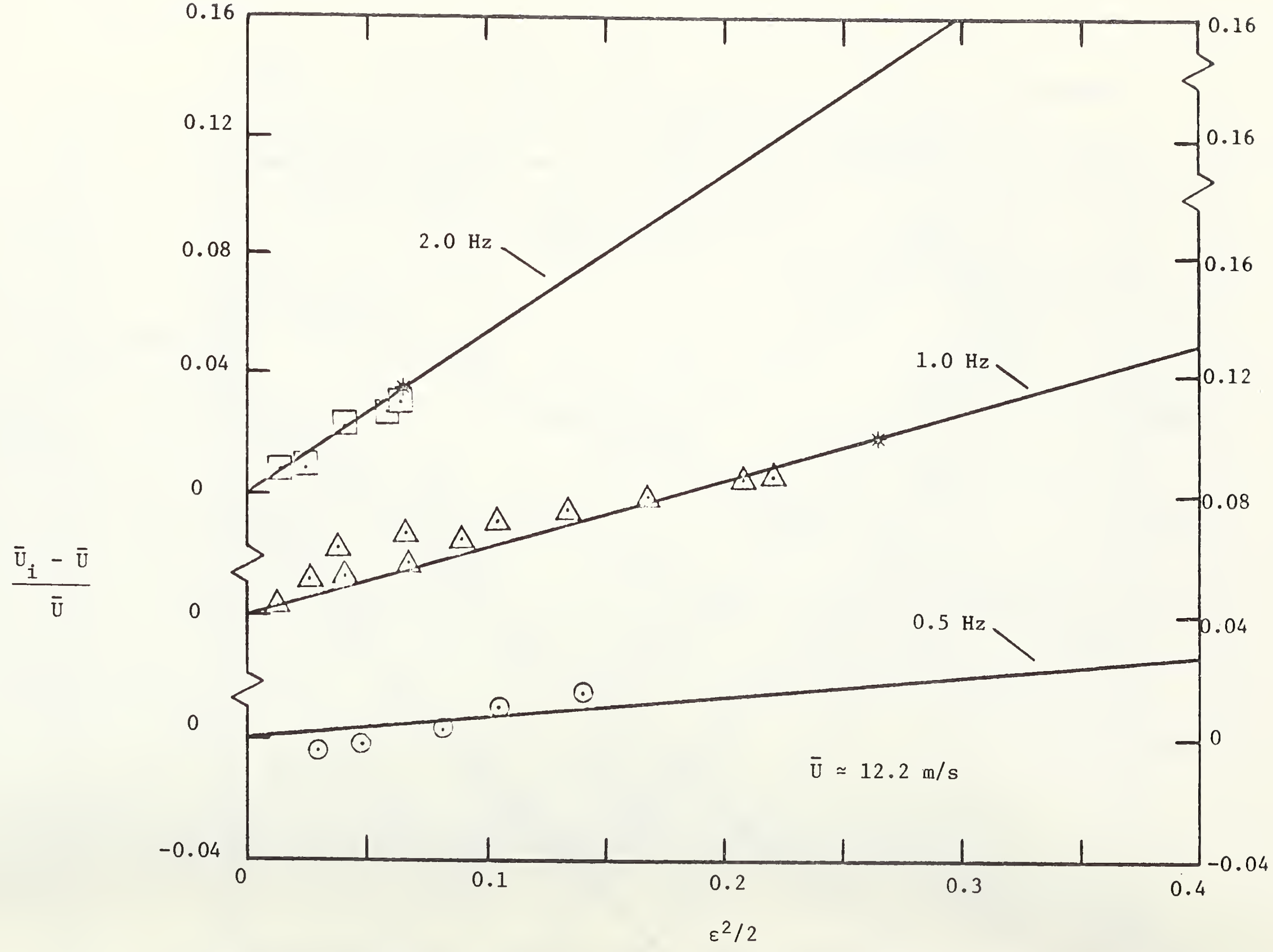

Figure 4. Variation of Overspeeding - Error with Amplitude and Frequency.

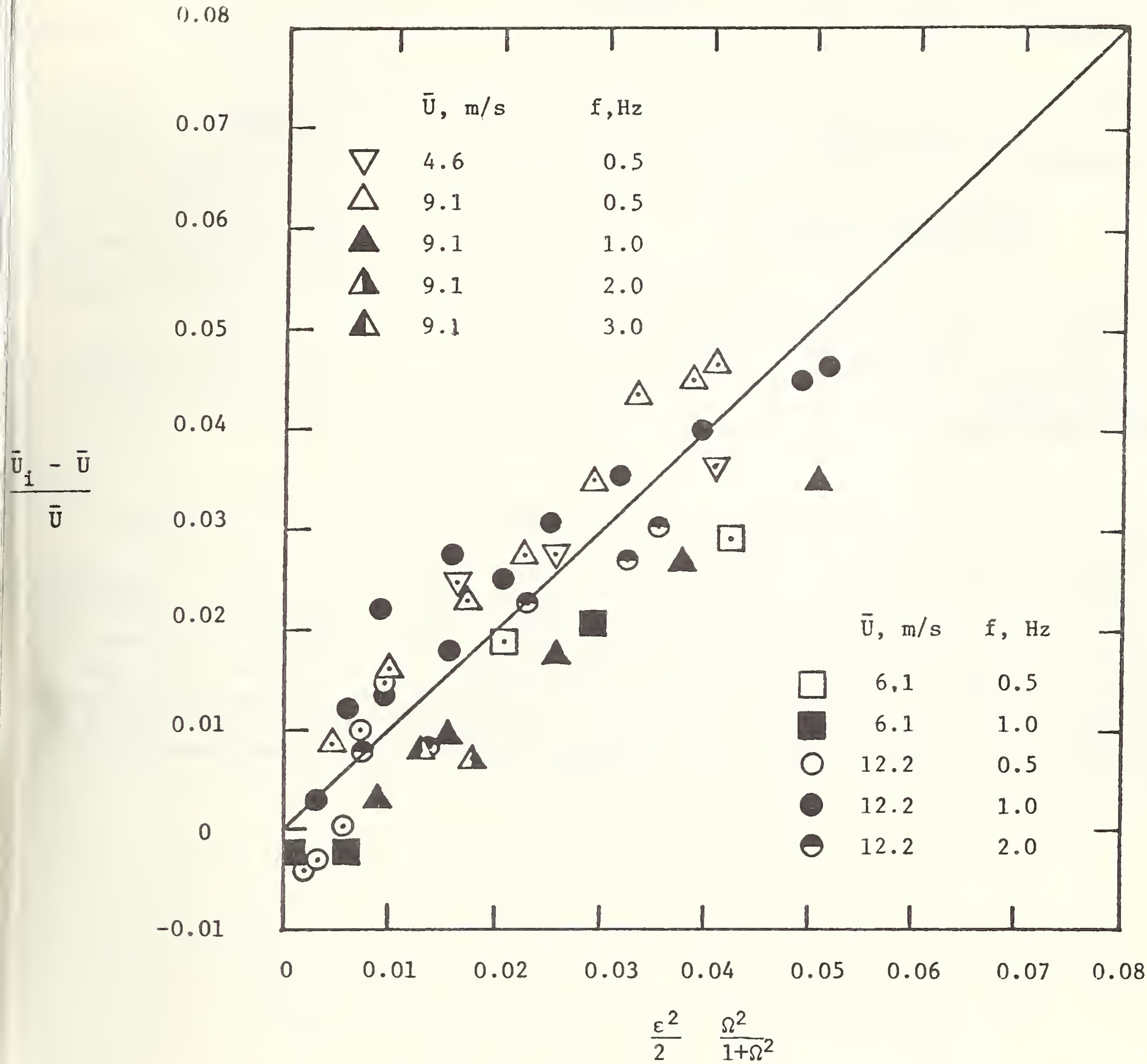

Figure 5. Comparison of Measured and Predicted Overspeeding - Error for $\varepsilon\Omega \leq 0.4$

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BIBLIOGRAPHIC DATA SHEET

U.S. DEPT. OF COMM. BIBLIOGRAPHIC DATA SHEET	1. PUBLICATION OR REPORT NO. NBSIR 78-1505	2. Gov't Accession No.	3. Recipient's Accession No.

4. TITLE AND SUBTITLE

CHARACTERISTICS OF HELICOID ANEMOMETERS

5. Publication Date

6. Performing Organization Code

7. AUTHOR(S)
J. M. McMichael and W. G. Cleveland

8. Performing Organ. Report No.
NBSIR 78 - 1505

9. PERFORMING ORGANIZATION NAME AND ADDRESS

NATIONAL BUREAU OF STANDARDS
DEPARTMENT OF COMMERCE
WASHINGTON, D.C. 20234

10. Project/Task/Work Unit No.
2130482

11. Contract/Grant No.
Purchase Order No. 5-3-0200

12. Sponsoring Organization Name and Complete Address *(Street, City, State, ZIP)*
Federal Highway Administration (DoT)
Office of Research
Structures and Applied Mechanics Division
Washington, D. C. 20590

13. Type of Report & Period Covered
Final

14. Sponsoring Agency Code

15. SUPPLEMENTARY NOTES

16. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

An experimental study of the overspeeding error for helicoid anemometers in periodic air flows is described. The ranges of amplitude and frequency for which a simple nonlinear model for the dynamic response of such instruments remains valid are presented. It is shown that the model is valid for typical atmospheric applications of such instruments. A simple method is presented whereby the effects of inertial lag and nonlinearity may be taken into account in obtaining measurements of alongwind power spectra in the atmosphere.

17. KEY WORDS *(six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons)*
Air; anemometer; helicoid anemometer; lag; overspeeding error; rotary anemometer; unsteady flow.

18. AVAILABILITY [X] Unlimited	19. SECURITY CLASS (THIS REPORT)	21. NO. OF PAGES
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USCOMM-DC 66680-P78