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Cost/Benefit Analysis of Automated Transit Information Systems

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Urban Mass Transportation Administration
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U.S. DEPARTMENT OF COMMERCE, Juanita M. Kreps, *Secretary*
Dr. Sidney Harman, *Under Secretary*
Jordan J. Baruch, *Assistant Secretary for Science and Technology*
NATIONAL BUREAU OF STANDARDS, Ernest Ambler, *Acting Director*

ABSTRACT

This report discusses the costs and benefits associated with automating the route-finding portion of a telephone transit information system that responds to telephone inquiries. The various costs of implementing such a system are categorized and compared with those of a manual system over an appropriate time span using a present value approach. A queuing model, described in the report, is used for computing manpower requirements of the two systems, manual and automated. Outputs of the queuing model for a wide range of input parameters are tabulated in an appendix. Benefits from automating transit information route-finding are discussed, and measures of performance improvement available as output from the queuing model are provided.

Key Words: Automation; cost/benefit; models; queuing; telephone systems; transit information; transportation.

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1. INTRODUCTION

Transit companies provide prospective riders with information about transit itineraries and service through the use of several different media: signs at transit stops, stations and in vehicles; printed schedules and maps (available in stores or transit facilities, or mailed to prospective riders); advertising messages in newspapers; and possibly a telephone information facility. It is this latter service, the one provided by the telephone transit information center, which is the subject of the present report. Our specific aim is to investigate the costs and benefits arising from automating such a service. Possible tradeoffs among the various media used to furnish transit information will not be analyzed; treatment of that broader topic would require an extensive marketing study, and the particular mix of media which best supplies information in a city is very likely to depend on particular characteristics of the transit system, transit riders, and alternative transportation modes available in that city.

In the typical telephone transit information center, transit system employees (whom we will call "operators") answer inquiries from prospective riders about schedules, routes and particular trip itineraries. Presently, the operators in most manual systems consult maps and schedules in piecing together the requested trip information. Operators must be quite familiar with both the transit system and the regional geography in order to locate a trip's origin and destination and to find appropriate routings for the trip between a transit stop near the origin and one near the destination. Automation of the route-finding function is under consideration, and this report is designed to aid in the evaluation of such proposed systems.

In an automated system the operator would ascertain from the caller the desired trip and would input this information to a computer, which would find an appropriate ("best") itinerary and report it back to the operator. The operator would then relay this itinerary to the caller. Although the possibility of further automating the response to the caller (through the use of an automated voice response device) has been proposed, it is not included in the analysis below, since several problems with its implementation remain unsolved. Thus the automated systems to be evaluated here have an operator to converse with the caller, to translate the request and to relay the answer; these systems include automation only of the route-finding portion of a call. Such an automated system will be compared to a similar manual system in which the route-finding function is performed by operators consulting hardcopy maps and schedules.

An important first step in weighing a (partially) automated transit information system against a manually oriented system is to survey the individual cost elements incurred by the two systems. Accordingly, the following sections will provide a framework for specifying and classifying in detail the various costs associated with such systems. Moreover, a general cost model (which employs a queuing analysis to generate appropriate manpower estimates) will be described for combining, over a given time horizon, the cost elements into an estimated total system cost. The cost differential between the systems can then be compared with the net benefits accruing from automation to obtain a final evaluation of the impact of automation on a transit information system.

The less mathematically oriented reader may wish to skip the detailed description of the queuing model in Section 3 and Appendix A, concentrating on the description of the cost elements in Section 2, the use of the queuing model output for estimating the number of operator positions in Section 4, the discussion of benefits in Section 5, and the tables in Appendix B.

2. COST ELEMENTS

The major elements of total cost are identified in Tables 1 and 2 for both an automated and a manual information system. The type of automation envisioned requires utilizing a computer code to provide point-to-point trip itineraries, under some appropriate criterion for a "best" trip or several alternative "good" trips. Several such criteria are discussed in some detail in a previous report [4]. Those portions of an incoming transit information request which involve comprehending the caller's question and providing the response are assumed here to remain under manual control.

In Tables 1 and 2 the cost elements are further classified according to whether they represent initial (capital) costs, recurring (annual) costs or both. In addition, those cost elements which depend on the number of transit information operators are identified. Detailed descriptions of the individual items that constitute these cost elements are provided in the sections that follow. The costs for an automated system are presented in Sections 2.1.1-2.1.11, while those for a manual system are described in Sections 2.2.1-2.2.6.

Note that although some categories of cost appear to be similar in the automated and manual systems, the costs associated with those categories may vary in magnitude between the two systems. For example, telephone and furniture costs may be higher in computer assisted systems in order to interface effectively with the terminals or operator activities. Costs of these apparently related categories may also vary significantly depending on the particular design of the information center.

TABLE 1

Cost Elements for Automated Information System *

Cost Description	Initial Cost	Recurring Cost	Dependent on Number of Information Operators
Computer Space Preparation	x		
Computer Hardware (Leased)		x	x
Software Development	x		
Computer Operation		x	
Terminals	x	x	x
- - - - - - - - - -			
Data Base Management		x	
Information Operator Personnel		x	x
Furniture	x	x	x
Telephones	x	x	x
Physical Plant Overhead	x	x	
Training of Operators	x	x	x

*Items below the dotted line appear in both Tables 1 and 2; those above, appear only in Table 1.

TABLE 2

Cost Elements for Manual Information System

Cost Description	Initial Cost	Recurring Cost	Dependent on Number of Information Operators
Data Base Management	x	x	
Information Operator Personnel		x	x
Furniture	x	x	x
Telephones	x	x	x
Physical Plant Overhead	x	x	
Training of Operators	x	x	x

2.1 Cost Elements for an Automated Information System

2.1.1 COMPUTER SPACE PREPARATION

Automation of the transit information facility is expected to require a computer dedicated to this task. (Use of an existing computer configuration would require appropriate modification of the costs included here, which do not consider shared computer usage.) Therefore, additional space and appropriate preparation of that space will be necessary in order to provide the proper environment for the new computer. In particular, the following items will contribute to the (initial) expense of computer space preparation and should be included in this cost element:

a) Routine site preparation
b) Special electrical preparation
c) Installation of air conditioning
d) Installation of sound conditioning
e) Installation of data communication cabling ducts
f) Flooring modification

2.1.2 HARDWARE

These costs refer to the required computer hardware, which is assumed to be obtained on lease. This assumption is used for convenience of reference since such leasing costs could be replaced by amortized value of purchase cost if equipment purchase is deemed more appropriate for a particular system. The annual leasing cost should be incremented by the amounts necessary for maintenance and repair (if not provided in the leasing contract).

To some extent the choice of hardware configuration is dependent on the number of information operators, inasmuch as sufficient computing power must be available to offer time-sharing service to the operator terminals without excessive wait time. Some peripheral equipment may be purchased, rather than leased as needed.

2.1.3 SOFTWARE

Software costs represent an initial outlay to provide operating system software and the applications software for algorithms that perform the point-to-point trip processing. In addition, one should include in this figure the one-time cost of creating for the computer a data base representing the transit network. A certain amount of supervisory assistance from the transit authority will be required in creating this data base, and therefore an appropriate amount for supervisors' salaries and fringe benefits should also be included. Programs for editing and updating the data base are also required. Additional programs to provide management information on system performance may be included in the system program package.

2.1.4 COMPUTER OPERATION

This item consists primarily of salaries for the computer system operators. The number of operators required per shift and the number of shifts per day and week will depend on the sophistication of the computer operating system, the number of hours per day during which the transit information center will be answering requests and whether or not updates are performed during the day or later at night. In addition, this figure should reflect electricity costs for the computer system and any additional air conditioning required. Another item falling under this heading (if not already included in hardware costs) is a contract assuring maintenance and service for the computer equipment.

2.1.5 TERMINALS

The one-time costs referred to here are expenditures for the initial purchase of terminals. The number of these purchased will depend on the number of operator positions required to meet the initial level of demand; the number of such positions can be estimated through the use of the queuing model. In addition, it is appropriate to include the cost of purchasing additional terminals, which are to be used for training, for maintenance and for performing updates. To the extent that demand increases (on an annual basis), additional operator positions will be needed to meet this demand and still maintain a desired level of service. Recurring costs for these additional positions will be reflected in the cost of additional terminals.

2.1.6 DATA BASE MANAGEMENT

This item refers to the cost of maintaining and updating the computer data base. In contrast to the activity of creating an appropriate data base (see Section 2.1.3), this aspect will involve a recurring cost (in addition to an initial programming expense), the magnitude of which depends on the frequency of updates and the overall complexity of the underlying transit network. It is possible that such updates could be entered through one of the information operator terminals during off-peak hours. If instantaneous corrections to schedules or routings are required, a separate spare terminal could be used to enter such changes.

2.1.7 INFORMATION OPERATOR PERSONNEL

An important factor in comparing automated and manual information systems is the level of staffing required for the transit information operators. This staffing level depends on the number of operator <u>positions</u> which are being used at any one time by the systems. In turn, the number of operator positions reflects the capacity of the system, in terms of the maximum number of calls which can be simultaneously handled. Since operator working shifts cannot be immediately and completely compatible with changes in the level of demand (number of requests for information) during the day, it is appropriate to assume that a constant number of operator positions are manned throughout the peak demand period. Indeed, it is at such times of maximum demand that the information system would most likely exhibit overloading or congestion and a high incidence of lost calls. It is the ability of an information system to cope under such circumstances that is a key issue in assessing how well the system is providing service.

In order to provide a consistent basis for comparing the automated and the manual systems during periods with a constant number of operator positions maintained, a queuing model (which is described in fuller detail in Section 3) is used to calculate relative manpower requirements for the two systems. This queuing model estimates the minimum number of operator positions required by each system to achieve a prescribed level of service. For example, one might specify that the percent of calls which encounter a busy signal during the period should not exceed 1%. Alternative measures for the "level of service" are: the expected number of lost calls (i.e., the average number of callers encountering a busy signal), the average number of persons "on hold" at any time, the average length of time a person who is not serviced immediately must wait before an operator is free, and the average time required for the caller to complete the transaction (including any time spent "on hold" together with the time spent communicating with the information operator). The queuing model employed here represents an acknowledged simplification of a complicated process, but it can provide a useful benchmark for estimating the relative manpower requirements of alternative information systems. The formal specification of this queuing model is given in Section 3, where the types of input required by the model as well as its underlying assumptions are discussed.

Once the number of operator positions s has been determined for a given time period (using the tables of Appendix B produced via the queuing model), the required number n of operators for the automated system can be calculated from: s , the ratio of demand in peak and non-peak periods, and the number of hours worked per week by an operator. The total cost contribution of operator salaries and fringe benefits is then simply n times the appropriate average salary plus benefit figure for a single operator. In addition, one should include salary and fringe benefit costs for any supervisory personnel required to coordinate the information operators.

2.1.8 FURNITURE

The one-time costs referred to here are expenditures for the initial furnishings of the transit information facility, such as desks and filing cabinets. The number of desks purchased initially will depend on the number of operator positions required to meet the initial level of demand; the number of such positions can be estimated through use of the queuing model. To the extent that demand increases (on an annual basis), additional operator positions will be needed to meet this demand and still maintain a desired level of service. Recurring costs for these additional positions will be reflected in the cost of extra desks and cabinets.

2.1.9 TELEPHONES

Initial costs will be incurred for installing operator telephones and for instituting an Automatic Call Distribution System (ACDS), if such a system is not already present. The ACDS is required for routing calls to the information system. Most probably the operator terminals will be connected directly into the mainframe computer system; if not, then additional fixed costs are incurred in providing data phone lines between the computer and the terminals. Recurring costs take the form of operating (rental) costs for telephone service to each of the operator positions. Such costs depend on the number of operator positions provided and thus would reflect any increase in the number of operator positions as a result of increased demand.

2.1.10 PHYSICAL PLANT OVERHEAD

These costs are those for preparing the site which houses the information operators and terminals (e.g., space partitioning) as well as undertaking routine electrical preparation, installing air conditioning and installing sound conditioning. Besides such initial expenditures, there will be a number of recurring expenditures for both the information operator room and the computer room: namely, the costs for rental of space, utilities, insurance and janitorial services. It is assumed here that the space provided for the information operators is sufficient to allow for any later expansion in the number of operator positions to meet increased demand.

2.1.11 TRAINING OF OPERATORS

It will be necessary to train the operators in using the terminal, inputing data about itinerary requests and interpreting output, but extensive training in city geography and available routes will not be required. Training emphasis in an automated system could focus more directly on improved communications skills, including how to elicit information more efficiently and how to articulate more clearly. Costs of training will depend on the operator turnover rate, the number of training hours required per operator, the need for special materials, and the cost of supervisors who conduct training.

2.2 Cost Elements for a Manual System

2.2.1 DATA BASE MANAGEMENT

Initial costs are incurred here in setting up the data base from which route and schedule information can be generated for the transit information operators. This cost will be reduced considerably if an appropriate data base already exists or if currently available schedules are already in a form appropriate for use by the information operators. In any event, recurring costs will be encountered in updating and maintaining such a data base for use in periodically providing up-to-date route and schedule information to the operators.

2.2.2 INFORMATION OPERATOR PERSONNEL

As in the case of the automated system, manpower requirements for the manual system can be estimated using the queuing model of Section 3. The major difference is that now one of the input parameters to this model is changed: namely, the parameter describing the average number of calls which can be serviced per operator per hour during a busy period. As described in Section 3.2, the difference between the automated and manual systems would be reflected by more rapid servicing of calls in the former system compared to the latter. That is, an automated system having fewer operator positions can achieve the same level of service as a manual one. Once the number of operator positions s required in the manual system to meet certain minimum performance levels has been determined, then the total number n of operators can be found as well as the total cost of such personnel. The cost of maintaining supervisory personnel, at their appropriate salary and benefit levels, should also be included in this cost element. A manual system may require more intensive use of supervisors than will an automated system, since the system requires greater use of judgement by the individual operators.

2.2.3 FURNITURE

A manual system requires certain furniture for the transit information operators, in particular desks large enough to accommodate the various maps and schedules required to answer calls effectively. Moreover, there is a need for certain general office equipment (e.g., filing cabinets) and special equipment (possibly map display cases). As the number of operator positions expands to meet demand, recurring annual costs for furnishing these additional positions will be incurred.

2.2.4 TELEPHONES

Initial costs are incurred for installing operator telephones as well as for instituting an ACDS if one were not already available.

Recurring costs will result from charges for ordinary telephone service. The total charge will increase with any increase in the number of operator positions (which entails an increase in the number of incoming lines).

2.2.5 PHYSICAL PLANT OVERHEAD

This cost element will include initial expenditures for preparation of the site housing the transit information operators, undertaking routine electrical preparation, installing air conditioning and installing sound conditioning. There are a number of recurring expenditures for the information operator room, including the rental cost of the space, utilities, insurance and janitorial services. Again, it is assumed that the space allocated for the information operators will be sufficient to absorb any later addition to the number of operator positions.

2.2.6 TRAINING OF OPERATORS

It is necessary to train new information system operators in city geography, in transit system operations, routes and schedules, and in communication. Usually operators receive a period of intensive training in these skills in a classroom and then serve an "apprenticeship" period answering transit information requests with a gradually decreasing level of supervision. The training and apprenticeship periods vary with the complexity of the transit system, the type of program, and the background of the information operators (for example, former bus drivers would need considerably less training in city geography or the transit system). Training costs include full operators' salaries for the period of classroom training, partial salaries (reflecting lower productivity) for the apprenticeship period, salaries of supervisory personnel and teachers for the time spent involved in training, and expenses for instructional equipment and materials. Total training costs are affected by operator turnover rate and the size of staff required.

2.3 Aggregation of Cost Elements

We defer to the next section the procedure for estimating the number of operator positions, a calculation which influences the magnitude of several cost elements. (Tables 1 and 2 indicate which cost elements are so influenced.) The issue discussed in this section is how to aggregate the various cost elements into a total cost figure, for either a manual or an automated information system.

The steps of this aggregation process can be illustrated by using the sample worksheets in Tables 3 and 4, which refer respectively to automated and to manual systems. Here the use of Table 3 is discussed in detail; however, a similar procedure also applies to Table 4.

TABLE 3

Sample Worksheet for Aggregating Automated System Costs

T = 10 TIME PERIOD = 1 YEAR

COST ELEMENT	INITIAL COST	RECURRING COSTS BY TIME PERIOD										PRESENT VALUE
		1	2	3	4	5	6	7	8	9	10	
Space Preparation												TC(1)
Hardware												TC(2)
Software												TC(3)
Computer Operation												TC(4)
Terminals												TC(5)
Data Base												TC(6)
Operator Personnel												TC(7)
Furniture												TC(8)
Telephones												TC(9)
Physical Plant												TC(10)
Training												TC(11)
	K(0)	K(1)	K(2)	K(3)	K(4)	K(5)	K(6)	K(7)	K(8)	K(9)	K(10)	TC(A)

TABLE 4

Sample Worksheet for Aggregating Manual System Costs

T = 10 TIME PERIOD = 1 YEAR

COST ELEMENT	INITIAL COST	RECURRING COSTS BY TIME PERIOD										PRESENT VALUE
		1	2	3	4	5	6	7	8	9	10	
Data Base												TC(1)
Operator Personnel												TC(2)
Furniture												TC(3)
Telephones												TC(4)
Physical Plant												TC(5)
Training												TC(6)
	K(0)	K(1)	K(2)	K(3)	K(4)	K(5)	K(6)	K(7)	K(8)	K(9)	K(10)	TC(M)

(1) Decide upon (based on estimated life cycle) the number T of time periods which will constitute the planning horizon and specify the length of the time period. For example, Table 3 was developed in terms of one-year periods (so all cost figures to be entered here are treated on an annual basis) and a ten-year planning horizon ($T = 10$).

(2) Prepare and enter estimates of each initial cost element, and for the corresponding cost elements recurring in every time period. Certain of these estimates will require projections for the number of operator positions in each time period; such projections can be found using the queuing model tables of Appendix B. See Section 4 for a description of the use of these tables. In addition, certain unit costs (e.g., operator personnel costs) will increase over time as a result of inflationary forces. For example, a 6% annual inflation factor might be assumed applicable for operator salaries. One simple way to account for such rises is to assume a constant "inflationary" factor of i percent per year over the time horizon. Then the unit cost in period $k + 1$ will equal that in period k multiplied by $1 + i$, and thus the total stream of costs over time can be successively calculated by beginning with period 1. For example, an inflation factor of 6% might not be uncommon for operator salaries and benefits, while a rate of 3% might apply to telephone costs; on the other hand, computer hardware costs may remain relatively constant over the time horizon.

(3) For every cost element, combine the initial costs and the recurring costs. One method of doing this is by converting the stream of recurring

costs into a present value, taking into account an acceptable annual discount rate (e.g., 9-10%). More specifically, if d is the discount rate, then a cost of C dollars in time period 1 is equivalent to a cost of $C/(1+d)$ dollars in time period 0, the present. In a similar way, C dollars in time period t is equivalent to $C(1+d)^{-t}$ dollars in time period 0. Therefore, the stream of recurring costs can all be converted into equivalent "period 0" costs and then combined with initial costs. The total (period 0) cost $TC(j)$ for cost element j is then

$$TC(j) = IC + \sum_{t=1}^{T} RC(t)[1+d]^{-t},$$

where IC denotes the initial cost of element j and $RC(t)$ is the recurring cost of element j in period $t=1,\ldots,T$. There are other ways of combining initial and recurring costs (see [12], for example). However, the present-value method seems most appropriate, since it allows one to compare total costs (capital, operations and maintenance) over a specified time period in terms of constant dollars. It is customary to convert costs to the "present value" to facilitate comparisons with other different investment opportunities. Also, by considering <u>total</u> (present value) costs, one can easily determine how many years will be required before two alternative systems are comparable (if at all) in terms of total cost.

(4) Compute the total system cost by summing the total cost elements found in Step (3). Then the total cost for the automated system, $TC(A)$, based on the eleven cost elements of Table 1, is given by

$$TC(A) = \sum_{j=1}^{11} TC(j) .$$

A similar procedure applies when calculating the total cost TC(M) for the manual system.

Thus the total or aggregated costs can be determined for the two systems over the selected time horizon. There are several possible uses for these quantities. First, they give an idea of the <u>magnitude</u> of the total cost for either system over the selected or planning horizon. These costs are given in terms of present dollars, and represent how much the proposed system would cost if all expenditures were made today. In addition, by considering the difference in total cost between the systems, one obtains a cost differential (in present dollars) which can be compared with the benefits accruing from automation. Section 5 discusses in more detail the nature of such benefits and how a comparison of benefits and costs can be made. Furthermore, as mentioned earlier, these cost figures can be obtained over different time horizons, and so it can be determined when (if at all) the costs of the two systems would become comparable.

3. QUEUING ANALYSIS

3.1 Queuing Model

The representation of the transit information process using a queuing model can be described by reference to Figure 1. The physical system consists of some number s of servers (corresponding to operators in service) and a queue of maximum length Q (corresponding to a fixed number of telephone hold positions). Arriving calls randomly enter the system at an average rate of λ per hour; if a server is free, an arriving call is serviced immediately, but otherwise the call joins the current queue. (If the queue is full, an arriving call receives a busy signal, i.e., it cannot enter the system and so the call is "lost".) While in the queue, a particular caller will renege--i.e., become tired of waiting and leave the system without service--according to a negative exponential probability distribution with parameter α . When a server becomes free, he/she services the next caller (assumed to be chosen from the front of the queue). The length of time required for service varies from call to call, and it is assumed to follow a negative exponential probability distribution with mean $1/\mu$. The parameter μ can also be interpreted as the rate (average number per hour) at which a busy server processes the calls.

Under the above assumptions, the system will attain an equilibrium condition, or steady state. The queuing calculations to be described will all be based on the analysis of this steady state. In particular,

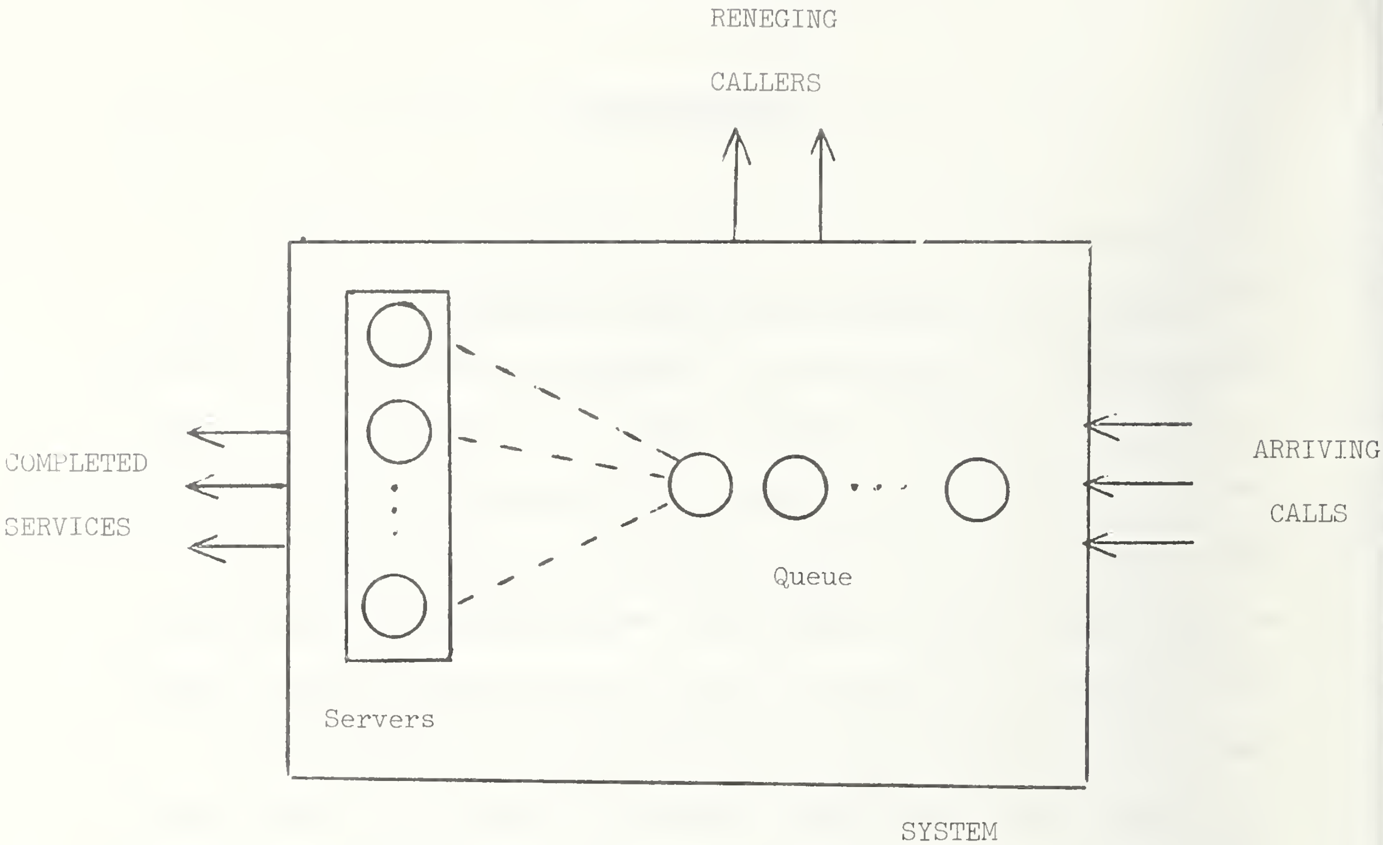

Figure 1

Queuing Model of Transit Information Process these calculations make it possible to estimate the probabilities P_m of finding m calls in the system at any time instant. These probabilities can be used, in turn, to calculate[1] important characteristics of the queue, namely:

(a) The percent of calls unable to enter the system because the queue is full (calls "turned away" by a busy signal)

(b) The average number of calls per hour unable to enter the system

(c) The average number of callers in the queue at any instant

(d) The average length of time a person who is not immediately serviced must wait in the queue

(e) The average length of time required for a call to pass through the system (i.e., the total call length time)

Several of the above outputs have been tabulated for a variety of different input specifications and are displayed in Appendix B. The use of such tables is described in Section 4.

3.2 Input Specifications

This section discusses the various input specifications needed for the queuing analysis and how the necessary parameters can be estimated. There are four basic types of input data that must be provided to the queuing model, and these are discussed in turn below.

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[1] The mathematical details of such calculations are described in Section 3.4.

The first type of input consists of projected call arrival rates λ_i for each of the years $i = 1,\ldots,T$ comprising the time horizon (e.g., the amortization period of the system). The quantities λ_i are measured in units of number of calls per hour, and they represent the average rate at which calls are attempting to enter the system. Since attention is really focused on the peak period during the day, it is probably reasonable to assume that over this period the arrival rate is constant. In order to estimate these quantities λ_i it would be useful to have data pertaining to the current year: more specifically, data on the number of calls attempting to enter the system per hour. Two difficulties may arise with currently available data, and may thus necessitate a special data collection effort.

First, existing data may only indicate the number of calls per week of operation. Such quantities can be converted into a peak hour rate by assuming, for example, that 60 percent of the calls arrive during the busiest eight hours of a day. Then an estimate for the peak hour arrival rate λ is

$$\lambda = (R/7) \times (.60/8) ,$$

where R is the measured number of calls per week. Off-peak rates may be calculated similarly. The factor 7 in the above expression is used to convert the weekly rate into a daily rate. In some cases a different factor (such as 5) might be more appropriate, depending on the number of days per week that the information facility operates.

A second issue that arises in using existing arrival rate data is that the estimated rates should include all calls <u>attempting</u> to enter the system, not just those calls which <u>succeed</u> in entering. In other words, the value for λ should include calls that receive a busy signal. However, the most widely available hardware measures the number of calls actually received. There are two possible approaches for dealing with this difficulty. First, one could assume that all callers receiving a busy signal will call back again at another time, and so the total number of calls received will ultimately be the same as the number of callers attempting to utilize the system. This approach suffers from the drawback that while the number of callers serviced will not be affected by this procedure, the total number of calls will be affected, as will the statistical distribution over time of arrivals, and so the input stream can no longer be guaranteed to follow strictly the Poisson process [3], assumed above. Moreover, a caller who finds a busy signal may not return the call during the same period; since the analysis treats periods within a day separately, such a caller would be effectively "lost" to our representation of the system.

An alternative approach is to estimate the proportion of calls lost (or assume a reasonable proportion for calls receiving a busy signal). The measured arrival rate during any period can then be "inflated" by this factor. The above method is only approximate. However, if a more accurate determination were required, then an iterative refinement process could be used to estimate this factor accurately. Namely, an initial guess for the factor is made, then the queuing model is employed (using data appropriate for current operations) to produce an estimate of the proportion of lost calls. This new estimate is used to modify the arrival rate and the model is again used, resulting in an improved estimate for the proportion. The procedure is iteratively applied until successive estimates agree, whereupon an appropriate estimate for the proportion p of lost calls will be available. The measured arrival rate λ^*, which excludes lost calls, can then be modified to produce an estimate λ, which <u>will</u> include lost calls; namely, $\lambda = \lambda^*/(1-p)$.

Alternatively, if one assumes that the reneging rate α is effectively zero, a mathematical expression can be derived which gives a good approximation to the true value λ, based on the observed arrival rate λ^*. This analytical expression (the origin and use of which is described further in Appendix A) approximates rather closely the actual relationship between λ and λ^* over the range $1 \leq s \leq 60$, where s denotes the number of servers (operator positions) in the system.

Finally, recall that estimates must be provided for λ_i in each of the years to be used in the analysis. If service-demand projections for future years are not available, it may not be unreasonable to assume that the call rates increase at some rate a% per year (or time interval) for the duration of the time horizon being considered.

The next type of input information needed for the queuing analysis consists of the <u>service rates</u> μ_i for each of the years. It is probable that these service rates do not change significantly from year to year, and so only their common value $\mu = \mu_i$ needs to be estimated. (If, however, there are strong reasons to believe that the service rate will change appreciably over time, then appropriate estimates μ_i should be provided for each year.) The required value μ describes the service rate per busy server (in units of number of calls serviced per hour). Since $1/\mu$ gives the average duration of a service in hours, it is probably easier to estimate this quantity than to estimate μ directly. Existing data on the average duration of time to service a call can then be used to give an estimate of $1/\mu$, and thereby a service rate μ_M for the manual system. In order to estimate the service rate μ_A for the automated system, it will be assumed (see [4] for justification) that the computerized system can reduce the duration of a call by 20% compared to a manual system. Therefore μ_A can be estimated using $\mu_A = 1.25\ \mu_M$. This relation was used in developing the tables in Section 4, but is not a basic requirement of our analysis method.

A third input parameter is the <u>reneging rate</u> α, also assumed to be constant from year to year. The value α measures in a sense the number of reneging calls per hour. More precisely, $1/\alpha$ represents the average length of time (in hours) a person will wait on hold before leaving without service.

The tables in Appendix B correspond to the case in which $\alpha = 0.0$ (that is, no reneging occurs). Similar tables can be provided for other values of α, but have not been included here. Indeed, the case $\alpha = 0.0$ is computationally simpler, the volume of tabular material becomes excessive if several values of α are used, and no data are available on the range of reasonable values for this parameter. Moreover, the system size figures resulting from the use of the tables for $\alpha = 0.0$ overestimate manpower requirements and thus provide a conservative overestimation of costs.

In addition to the input parameters described above, a value needs to be provided for the maximum length of queue (number of hold positions) that can be accommodated in the system. This value Q may be specified by the hardware of the Automatic Call Distribution System being used. For ease of display and use, the tables produced in Appendix B have as input the total number of lines $(Q + s)$ with Q varying usually between s and $2s$, the range found in [7] and used in [12].

3.3 Model Assumptions

Use of the queuing model described in the preceding sections requires that certain assumptions be fulfilled, if not exactly then at least approximately. Certain other simplifying assumptions have been made for the purpose of computational and expository convenience, and these can be changed without affecting the model's validity. In this section the intrinsic assumptions of the model (assumptions which must hold, at least approximately, for the model to be valid) are categorized under the following four components of the queuing system.

(1) Input Calls. It is assumed that the input calls form a Poisson process (see [3] for a mathematical description of this process). Such a process describes a stream of arrivals that occur in a "purely random" manner, i.e., the time until the next arrival is completely uninfluenced by when the last arrival occurred. In practice, it has

been found [2], [5] that the pattern of arrivals at a telephone system is quite closely approximated by a Poisson process. Accordingly, such a process would also seem to be appropriate for arrivals at a transit information facility. It should be noted that in a Poisson process, the arrival rate λ is assumed to remain constant over the period of interest. Since the present analysis concentrates on the period of peak demand for the information facility, this requirement of a constant arrival rate is likely to be fulfilled.

(2) Queue Discipline. It is supposed here that the queue operates on a "first come, first served" basis; that is, earlier arrivals to the system will always be served before later arrivals. Such an assumption will be satisfied for any reasonable policy of handling incoming calls. In particular, the use of ACDS will ensure that this is the case. Employing ACDS also has the advantage of allowing a pre-recorded message to be played to callers as they enter the system. This message can, for example, serve as a screening device for callers (routing callers to other transit property phone numbers if they are seeking other than itinerary information) or to alert callers as to the format for information requests.

(3) Service Times. The queuing model assumes that the service times (times for an operator to answer completely a request) are distributed according to a negative exponential distribution with parameter μ . This particular form of probability distribution (as well as the form of the input call distribution) is the "traditional" form that has been assumed in order to facilitate mathematical analysis. In fact, under these assumptions the steady-state probabilities for the queue can

be given in an explicit form (see Section 3.4). With other assumptions about the arrival and service time distributions, the analysis becomes more difficult, and it is unlikely that there are closed-form expressions for measures of system performance, such as queue length, waiting time and lost calls. While the arrival pattern of calls conforms quite closely to a Poisson process, the assumption of a negative exponential distribution for service times requires careful consideration. In general, a negative exponential distribution is appropriate when a large number of calls require short service times, and a smaller number of calls require longer service times [2]. In the present circumstances, this may only be approximately true, since there is a minimum service time for each call; for example, it does not appear possible for a call to require less than (say) 15 seconds of service time. By contrast, the negative exponential distribution would predict that short calls of (say) 0-15 seconds are quite probable. Perhaps a more accurate (but less tractable) assumption about service times would be that the length of a service <u>in excess</u> of, say, 15 seconds follows a negative exponential distribution.

Since the queuing analysis is to be used mainly for comparing the two systems, strict adherence of service times to a negative exponential form may not be necessary. In fact, the assumption of exponentially distributed service times provides in a sense a conservative approach to comparing the systems, since it tends to underestimate the contribution of an automated system. The reason for this is that the exponential distribution puts relatively more emphasis (higher probability) on calls of a short duration compared to calls requiring a longer service time.

But the automated system ought to perform better relative to the manual system when there are more time-consuming (i.e., more complicated) calls. Finally, it should be noted again that it is necessary to confine the queuing analysis to a period during which the parameter μ for the service time distribution will be (approximately) constant over the time span of interest.

(4) Reneging. The general queuing model assumes that the time an individual caller will wait before reneging follows an exponential distribution with parameter α. This assumption is imposed mainly for mathematical tractability, and verification would require the collection of data on actual customer behavior. Again the assumption of exponentially distributed reneging times implies that shorter reneging times are more likely than longer ones. Therefore, this situation will obtain in practice to the extent that early impatience of callers outweighs their patience. A convenient baseline for comparing the manual and automated systems is the case $\alpha = 0$: that is, when no reneging is "allowed" by the model. This case provides worst-case estimates for many of the quantities characterizing the level of service. For example, average waiting time will be longest when reneging is not allowed, since reneging callers reduce the waiting times of all callers following them in the queue.

3.4 Mathematical Description

This section will detail the mathematical calculations used to analyze a transit information facility. Specifically, the facility is modeled as a multiple server queue having a maximum queue size. Arrivals are assumed to be generated by a Poisson process, while service times and reneging times are assumed to be governed by an exponential distribution. For notation, we define

λ = arrival rate (number/hr),

μ = call service rate (number/hr),

α = reneging rate (number/hr),

s = number of servers,

Q = maximum queue size,

$M = s+Q$ = maximum number allowable in system,

$r = \lambda/\mu$.

Then the steady-state probability P_m of finding m callers in the system can be calculated using the recursive relations

$$P_m = (r/m)\, P_{m-1} \qquad \text{for} \qquad 1 \leq m \leq s \ ,$$

$$P_m = \frac{\lambda}{\mu s + (m-s)\alpha}\, P_{m-1} \qquad \text{for} \qquad s < m \leq M \ ,$$

and the fact that the probabilities must sum to 1:

$$\sum_{i=0}^{M} P_i = 1 \ .$$

The proportion of callers that find the system full (i.e., the proportion of lost calls) is just P_M , and the expected number of calls lost per hour is then λP_M . In addition, it is straightforward to calculate the expected number L_q of calls in the queue as

$$L_q = \sum_{i=s}^{M} (i-s)P_i \ .$$

Once L_q has been found, then the average waiting time $\overline{W}_q$ for a caller who does not find a server free on arrival can be determined [1]:

$$\overline{W}_q = \frac{L_q}{\lambda(r_o - P_M)} \ ,$$

where $r_o = \sum_{i=s}^{M} P_i$ is the probability that all servers are busy. Finally, the average length of time W spent by a caller in the system (including both queuing and service times) is given by

$$W = \frac{1}{\mu} + \frac{L_q}{\lambda(1-P_M)} \ .$$

It should be noted that the values for $\overline{W}_q$ and W include the queuing times for reneging callers, as well as callers who are ultimately served.

These calculated quantities (P_M, λP_M, L_q, $\overline{W}_q$ and W) can serve as measures of the level of service provided by the queuing system (transit information facility). Therefore, such measures can be used either individually or collectively to determine the minimum number of servers that will be required in order to meet certain service level standards. Further details on this procedure are given in the following section.

4. MANPOWER ESTIMATES USING THE QUEUING MODEL

Once the necessary inputs to the queuing model have been specified (Section 3.2), the mathematical calculations of Section 3.4 can be employed to determine values for various characteristics of a queuing system with those input parameters. The particular queue characteristics detailed in Section 3.1 provide measures of the amount of congestion in the system, with special emphasis on the number of lost calls, waiting time in the queue and total transaction time. If in addition certain minimum performance standards are prescribed for these measures, then estimates can be made for the number of operator positions required to achieve such standards.

For ease of application, tables have been prepared which give queue characteristics for transit information facilities with various input specifications. These tables are listed in Appendix B for the case $\alpha = 0.0$, according to arrival rate λ, service rate μ, total number of telephone lines L and number of servers s. From these latter two parameters one can calculate the maximum queue length $Q = L-s$. Three different queue characteristics are listed in each table. For example, the first table refers to an arrival rate $\lambda = 200$ calls/hour and a service rate of 15 calls per hour. The entries corresponding to 40 lines and 13 servers are

(a) number of hold positions = 27

(b) percentage of lost calls = 4%

(c) average waiting time = 286 seconds

(d) average time in the system = 500 seconds

If we require the proportion of lost calls to be at most 1%, then for the given specifications $s = 14$ operator positions will be required. If, in addition to this requirement, we insist on an average waiting time in the queue of no more than 100 seconds, then $s = 16$ operator positions will be needed.

A user of this model can choose any single criterion or set of criteria for minimum performance levels, and such choices will enable the determination of a required number of operator positions s. Such values for s can be determined for both the manual and automated systems (which will differ only in that service rates follow $\mu_A = 1.25\ \mu_M$), and these values can then be entered into the appropriate cost calculations described in Section 2. Since the difference in number of operator positions between the two systems is of major interest, the exact performance levels which are set may not be crucial. However, levels should be set which are reasonable in light of current operating policy and desired quality of service. As guidelines here, we suggest as typical using a maximum of 1% calls lost and a maximum waiting time of one to two minutes. The sensitivity of the number of operator positions to various design factors, such as the arrival rate, the service rate, and the number of hold lines should also be taken into account, so that variation from the design level will not adversely affect the system performance.

5. BENEFITS

The previous sections have described how to estimate and compare the costs of an automated and a manual transit information system. It is not at all clear that automation will necessarily cost less, over its useful lifetime, than a comparable manual operation. In addition, even if an optimal automated system would save money, in the long run other considerations such as union contracts or tight budgets may necessitate selection of a less efficient system, in which case the theoretical savings predicted by the cost model would not be attained. On the other hand, there are benefits associated with an automated transit information center other than direct cost savings; there are also potential disadvantages to this type of automation. This section explores these concepts in detail.

We will first describe, and quantify where possible (using the queuing model of Section 3), the system improvements resulting from automation. Next we will discuss the benefits from these improvements and note _who_ is benefited by each improvement, an important consideration since a transit company may be unwilling to fund automation if it does not perceive adequate benefit to the transit property. Also the public may be reluctant to see tax dollars used to pay for a system which does not provide much public benefit. Automation must either provide sufficient benefit to the transit company to underwrite its cost, or provide additional benefits to the public which justify public subsidy.

Although we will indicate methods for quantifying the levels of system improvements, we will not attempt to relate them directly to system costs by assigning dollar values to the benefits, since such values could differ widely from system to system and would tend to be arbitrary. Rather, for decision-making purposes, we prefer to provide the levels of system improvement; these then can be evaluated directly on their own separate merits.

5.1 Improvements Resulting from Automation

Table 5 provides a list of improvements resulting from a well-designed and implemented automation of the transit information center. They provide benefits both to the transit company and to transit system users. These include improved service and productivity resulting from the faster response of the computer, increased reliability and consistency because the computer always supplies the same answer to the same question, a reduction in the training required for operators, the ability to rapidly incorporate changes in the transit system into the data base, the capability of automatically gathering statistics about the operation of the information center, and the development of a data base which can be used for such other purposes as scheduling and automated printing of schedules.

These improvements, to be discussed more fully in the following sections, bring benefits to the transit property in the form of increased efficiency and goodwill, to the transit system user in the form of easier access to information and greater confidence in the response, and to society in general to the extent that easier access to transit information increases transit ridership. There are also benefits to society as a whole from the automation of a transit information facility, but

TABLE 5

Benefits from Automation

Improvements	Benefits	
	To Transit Company	To Users
Shorter service time	increased productivity	quicker response
Shorter wait time	better service	less time used
Fewer waiting	fewer lines	less frustration
Fewer lost calls	better service	less frustration, fewer recalls
Increased reliability	better service	more confidence
Increased consistency	better service	more confidence
Less operator training	cheaper, can use new people sooner	more confidence
Rapid response to changes	better service, flexibility	more confidence
Management Information System	better evaluation of performance	----
Data Base available for other users	increased efficiency and productivity	----

they are somewhat diffuse and difficult to measure and assess. They include reductions in road congestion, pollution and energy usage resulting from any increased patronage because of better availability of information on transit service. Improvements in information dissemination which lead to increased patronage may also result in expanded transit service instituted to meet the larger demand, thus providing better service to all users, both new and old customers. Improvements in the access to transit information may aid local business, such as the tourist industry or downtown stores, and might also attract new business. All of the benefits to society suggested here depend upon a demonstrated relationship between transit system patronage and the availability of transit information, a relationship which has not yet been proved. Thus, although we note these possible benefits to society as a whole, we make no attempt to quantify them, and they are not emphasized in this analysis. Transit information dissemination is a part of the broader subject of marketing mass transportation.

5.2 Benefits to Transit System Users and Transit Properties

Benefits to the transit system user are more tangible than those to society. They include a reduction in customer frustration in obtaining information on proposed trips: information is more accessible since the telephone is busy less often and the wait time is shorter. A second benefit is the time savings in obtaining information because of quicker response and shorter wait time. Finally, increased reproducibility and consistency of computer responses enhance public confidence and perhaps even reduce the need for confirmatory calls. All of these advantages make it much easier and less frustrating to obtain desired information concerning transit travel, allowing more people to have access to information and perhaps actually encouraging rather than discouraging potential riders to seek such information. More consistent answers, while not necessarily "better" than manually produced routes, may be perceived as better simply by virtue of their consistency.

Benefits to the transit company from automation are many. The first is increased productivity in a very labor-intensive operation. Since personnel salaries have risen faster than most other costs, companies are anxious to automate any activity which can be automated without overwhelming expense. A second benefit from a well-designed computerized system is ease of making changes to the data base. Either new sets of routes and schedules or temporary changes in them, as well as special services, can be incorporated easily and quickly, and old data can be readily discarded (by being overwritten with new). A third benefit is the improvement in customer relations because of the better service. When potential riders can obtain with relative ease needed information about desired trips, they have a better opinion of transit system service in general. Contributing also to the improved service is the increased reliability and consistency of responses. Callers will no longer be disconcerted by receiving a different routing if they get a different operator or call at a different time. The first response given is guaranteed to be the "best" route, according to the chosen criterion; the computer has considered, and rejected, many alternatives.

Another benefit to the transit company from automating its information facility is a reduction in training costs, since extensive training in city geography and transit routes is no longer necessary. Training for the operators of an automated center would emphasize dealing with callers, eliciting the information to specify information requests, articulating clearly and using methods of responding to the requests to aid the caller's retention of information. While these functions are important in a manual system also, training for an automated system can focus more specifically on them, and personnel recruiters can emphasize these characteristics in hiring operators.

Two ancillary benefits resulting from automation of the transit information center are (a) the availability of continuing statistics about the operation and (b) the availability of the data base for other uses within the transit company. The computer program package should include, at very little additional expense, programs to collect and print, upon demand, various statistics about the operation of the center, such as how many calls are answered at each station during specified intervals of time, the size of the hold queue (perhaps average, maximum and minimum or fraction of the time the queue is full), and the length of calls. In addition, statistics can be generated on the operation of the transit system, such as the trips requested, what areas (origins and destinations) are represented, the routes chosen, and the times of day requested. Such statistics can be used to evaluate performance of the operators, to examine causes of any problems, to suggest geographic areas in which to focus information campaigns, to provide a partial basis for establishing new routes or changing old ones, and to indicate new names or addresses to be added to the geographical location data base. Once the data base has been encoded for use by the transit information system, it is available for other uses such as scheduling of vehicles and crews, or automated printing of schedules. It is usually the case that the computerization of one activity leads naturally to automation of other related functions, and the availability of a common data base facilitates this process.

A final hoped-for benefit from automating the transit information center is increased patronage resulting from the greater accessibility of schedule and routing information. This is based on the assumption that some potential transit trips are not now being made (at least by mass transit) primarily because of lack of easy access to routing and schedule information, and that the improved accessibility of that information will encourage the trips to be made by mass transit. Although researchers have attempted to establish a positive relationship between the investment in improved telephone transit information provision and increased ridership, they have been unable to do so [7]. Thus we note increases in patronage as a potential benefit of automation, but do not focus on it as a prime justification for implementation.

5.3 Measuring the Benefits

As noted in the discussions above, many of the benefits or possible disbenefits of automation are difficult to measure because of the subjectivity of the assessment and the differences among transit properties and cities. We can, however, use the queuing model in Section 3 to quantify the degree of automation-induced improvement in such factors as the number of lost calls, the average waiting time in queue, the queue length and the time to service a request. The levels of improvement can then be compared to any cost differential which was calculated using the cost model of Section 2. Subjective assessments by the local transit people and any other interested parties (local Department of Transportation, citizen groups, etc.) can then be used in assessing the value of the improvements as compared with any incremental costs of the automated system over a manual system. If the automated system can be justified solely on the basis that it costs less than a manual system handling the same number of calls, quantification of the levels of performance improvement will not be as important in the evaluation as if the automated system is more costly, or the breakeven point is in the distant future, or the cost difference though favorable is smaller than the uncertainty in the values of the cost estimates.

5.4 Disadvantages to Transit System Users and Transit Properties

There are, however, some potential drawbacks for the transit system user from automation. The computer routing is based on one criterion (or at the most a small set of criteria), and some special requests may either have to be ignored or be answered manually. Examples of such requests are requests for routes which avoid certain areas of the city because of fear of crime, or for "triangle" routes in which the user wants to stop at intermediate points to run an errand. The operators could come to rely on the computer to the point that they will be unable to supply intelligent variances to the routes provided, for example advice to walk up two blocks to save an additional fare.

Easier access to the automated information system may increase demand on the telephone information system, even if improvements are made in other methods of information dissemination. (In a similar vein, the telephone company notes persisting calls to the information operator despite the availability of telephone books and even charges for the service.) Initial improvements in access to information may thus lead only to a newly saturated system with all the long waits of the old one, though more customers are served.

A final drawback to a computer-aided information system may be that the routings it provides depend on better on-time performance by transit vehicles than is actually attained. This may be somewhat alleviated by using minimum transfer times in computing the routes. However, the public may still perceive deviations from schedule more vividly when computerized transit information responses are very explicit. Some of these disadvantages, such as the improvement-induced demand, may also apply to an improved manual system. Although all of these (lack of flexibility in route selection, increased demand for telephone information, and heightened perception of system delays) are potential drawbacks to automation, we do not know how to measure their effects, and their magnitudes are likely to vary greatly from city to city and from transit property to transit property. Effective management may be able to minimize or eliminate some of these disadvantages. Potential benefits, however, appear to outweigh the possible disadvantages.

Automation of the transit information center may also have disadvantages to the transit company. First among these is that increasing the accessibility of transit information may increase the demand for that information. As more potential riders discover that telephone information service has improved, they may be encouraged to use the service more often for more types of trips. The result could be that the new automated system quickly becomes saturated, and service deteriorates to a level similar to that existing before automation. More people are being served, but they have to wait as long and face a busy signal as often as in the old manual system.

A second disadvantage of the automated system is that operators can tend to rely on the computer to supply answers to simple questions which, in the manual system, they might have answered "off the top of their heads". The result of such actions may be lengthening of the response to some short requests. Automation is expected to speed significantly the longer calls which require more complicated routes, but may increase the length of shorter calls, thus reducing the variation in call length, making it depend less on the complexity of the requested route and more on the ability of the caller to formulate his question.

Increased operator reliance on the computer will also be a great disadvantage during periods in which the computer is not operational. To take advantage of the reduced training costs available with an automated system, the operators will not receive the intensive instruction in city geography and the transit system that would be required to deal efficiently with queries in a manual setting. Even operators previously adept in the manual system can become so accustomed to using the computer that their skills in a manual operation become rusty.

Another possible effect of automating the transit information center is a greater public perception of delays because more patrons know at what time the vehicle is to arrive or depart. With information less accessible, fewer people know actual scheduled stop times, so the system's on-time performance (or misperformance) is not as widely known. When told specific departure or arrival times by the telephone information operator, riders may expect the system to adhere more precisely to the scheduled times than actually occurs. Wider dissemination of precise-sounding information about an inherently imprecise system may call attention to system variability.

A final difficulty which may arise from computerizing the transit information function is that changes in the on-line procedures or process (as opposed to those in the data base) may require altering a computer program, which can be time-consuming and expensive. The need for such changes can be minimized by careful system design, but all possibilities cannot be foreseen initially and some future program changes will probably be necessary.

Again it is believed that the benefits to the transit company from automation outweigh potential disadvantages, but no attempt has been made here to quantify the relative merits and disutilities involved because of their subjective nature and the differences among cities and transit properties.

5.5 Other Transit Information System Improvements

The cost-benefit analysis presented here has focused on a comparison between a manual and an automated transit information system, in which a computer is used to select a "best" route. Other modifications in methods of providing information are possible and should be considered by any transit property planning to upgrade its information service. Among these are several methods of improving the telephone information service, short of using a computer. They include the use of microfiche to speed retrieval of schedule and routing data, improvements in the workspace of the operators and the organization of their materials, the inclusion of a message to those waiting on hold aiding them in formulating their requests, and the channeling of incoming calls from particular originating zones to operators with special knowledge of the transit service in those zones. Besides considering improvements in the telephone information service, a transit property should also examine the broad spectrum of methods to enhance the provision of information. Examples include wider availability of printed schedule and routing information, more informative signs, and use of mail for specific requests or for dissemination of schedule and route updates or changes. All of these methods of improving the public's access and awareness of transit information should be studied in the context of the specific system being analyzed. This report has focused on evaluating a computer-aided telephone information service, because such a change represents a wide departure from previous attempts at improved service, but this focus is not meant to suggest that such an approach is either the only or the best plan for any particular situation.

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APPENDIX A

AN APPROXIMATION FOR THE RELATIONSHIP BETWEEN λ and λ^*

As noted in the text, care has to be exercised in describing arrival rates associated with the queuing system under study. The use of the queuing model to provide manpower estimates is based on obtaining an estimate λ of the rate at which calls attempt to enter the system. In practice, however, it is more likely that one is able to measure the rate λ^* at which calls actually enter the system (i.e., calls which are not turned away by encountering a busy signal).

The fundamental relationship between these two quantities is the equation

$$(1) \qquad \lambda^* = \lambda(1-P_M),$$

where P_M denotes the probability that the system will be found to be full (i.e., all hold positions are occupied) at any given instant of time. It is possible [5] to express P_M analytically in terms of the parameters $r = \lambda/\mu$, s and q (see Section 3.4 for their respective definitions) using

$$(2) \qquad P_M = \frac{r^{s+q}}{s^q \cdot s!} \Big/ \left\{ \sum_{i=0}^{s-1} \frac{r^i}{i!} + \frac{r^s}{s!} \sum_{j=0}^{q} \left(\frac{r}{s}\right)^j \right\} .$$

This expression is obtained under the assumption that reneging does not occur, i.e., $\alpha = 0$.

Given, therefore, λ (along with other parameters such as s, μ, q) it is possible using (1) and (2) to find λ^*. In fact, these equations provide a direct functional relationship of the form $\lambda^* = g(\lambda)$. However, it is really the inverse of this relationship that addresses the major issue here: namely, given λ^*, calculate the corresponding value of λ. There appears to be no simple analytical way to invert the given relationship $\lambda^* = g(\lambda)$ in order to obtain the relationship $\lambda = f(\lambda^*)$ that is sought. The object of this Appendix, then, is to present an approximation to this latter relationship that will be reasonably accurate over appropriate ranges for the parameter values.

To begin, it is useful to study the relationship $\lambda^* = g(\lambda)$ which is defined by (1) and (2). Certain observations relevant to this relationship can be readily established. First, as $\lambda \to 0$ it can be shown, using (1) and (2), that $g(\lambda) \sim \lambda$. In other words, for "small" values of λ, λ^* and λ will be nearly identical; intuitively, this is reasonable since for small λ, the turning-away of calls will be rare. Second, as $\lambda \to \infty$ it can be verified that $g(\lambda) \to s\mu$. That is, for very large arrival rates the system essentially becomes saturated, and the rate λ^* at which calls can enter the system approaches the maximum effective rate $s\mu$ at which calls (from s busy servers) can leave the system. Third, it can be verified that the function $g(\lambda)$ is a (strictly) monotone increasing function of λ, and $g(\lambda) \leq \lambda$ holds for all λ. The above observations show that the relationship expressing λ^* as a function of λ, $\lambda^* = g(\lambda)$, has the general form shown in Figure 2. This figure displays the horizontal asymptote at $\lambda^* = s\mu$ as well as the (dashed) line of equality $\lambda^* = \lambda$.

Consider now the functional value $g(s\mu)$ at $\lambda = s\mu$. Then, using (1) and (2), it is straightforward to deduce that

$$(3) \qquad g(s\mu) = s\mu[1-\{q + \phi(s)\}^{-1}],$$

where

$$(4) \qquad \phi(s) = \frac{s!}{s^s} \sum_{i=0}^{s} \frac{s^i}{i!} .$$

We first find an approximation to $\phi(s)$, as a function of s, and this will provide in turn an approximation to $g(s\mu)$.

An asymptotic expansion[1] of $\phi(s)$ as $s \to \infty$ can be obtained by using the Euler-Maclaurin formula [11]. Essentially this asymptotic expansion takes the form

$$\phi(s) \sim \beta_1 + \beta_2 s^{\beta_3} + \beta_4 s^{-\beta_5},$$

with specific positive values assigned to $\beta_1, \ldots, \beta_5$. This particular approximation is quite good as s becomes large, and is accurate for values of s even as small as 10. In order to obtain a best approximation over the range $s = 1,2,\ldots,60$ (which covers reasonable values for the number of servers), a nonlinear least squares fit was obtained using a computer routine[2] developed at NBS. This approximation $A(s)$ is of the specific form

$$(5) \qquad A(s) = .606 + 1.26\, s^{.499} + .133\, s^{-.251}$$

[1] We are indebted to Dr. F. J. Olver (University of Maryland, and NBS) for providing a derivation of this expansion.

[2] This data-fitting routine has been implemented by Dr. James Filliben of the NBS Statistical Engineering Laboratory. This implementation is based on a computer program of A. J. Miller (CSIRO, Sydney, Australia) and incorporates features of the Levenberg-Morrison-Marquardt method [8].

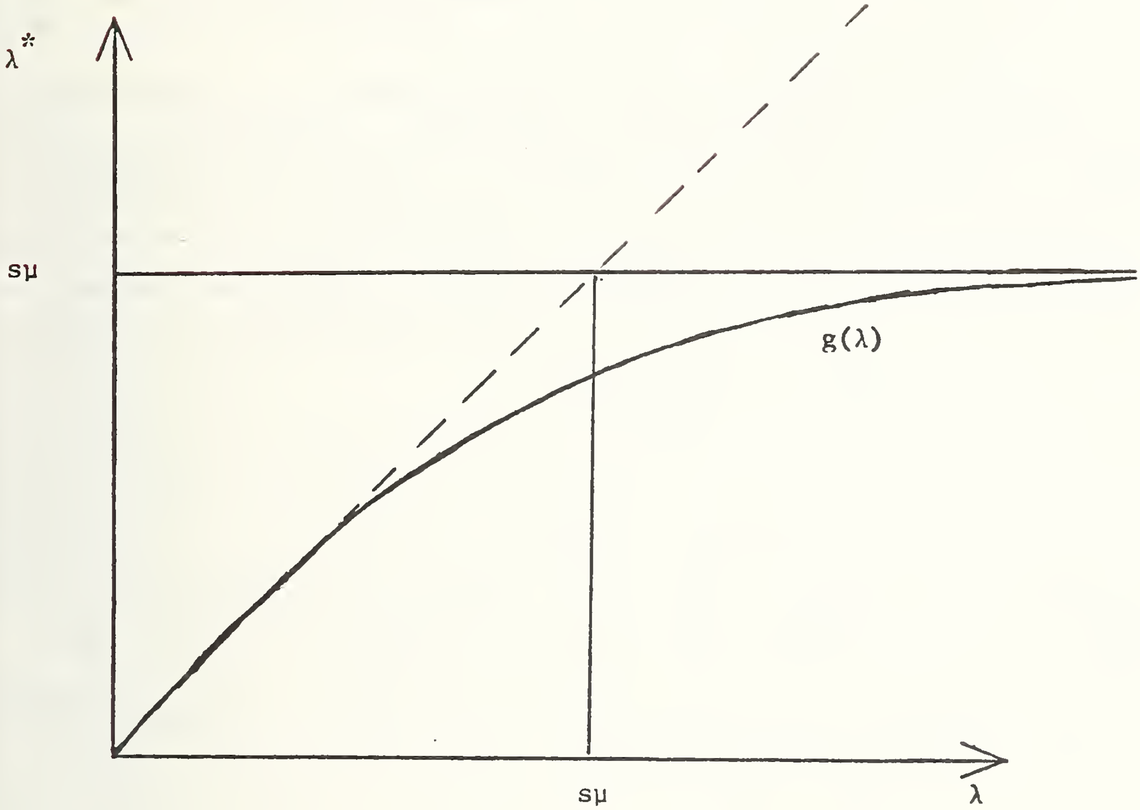

Figure 2

<u>The Functional Relationship $\lambda^* = g(\lambda)$</u>

and yields a residual standard deviation of 0.000013; such a small value for the residual standard deviation indicates an extremely good fit of (5) to the function (4) over the range of interest.

Given equation (5), it is now possible to tackle the original problem of approximating the relationship $\lambda = f(\lambda^*)$. What is required then is a function $h(\lambda^*)$ which is asymptotically equal to λ^* as $\lambda^* \to 0$, and which approaches ∞ as λ^* approaches $s\mu$ from below (see Figure 2). In addition, such a function could be made to have the ordinate $\lambda = s\mu$ when the abscissa is $\lambda^* = g(s\mu)$. Since the calculation of $g(s\mu)$ requires knowing $\phi(s)$ and since the latter is somewhat involved to calculate, we use instead the approximation $A(s)$ for $\phi(s)$. Accordingly, we require that $h(x)$ have the value $s\mu$ when $x = s\mu[1-\{q + A(s)\}^{-1}]$. One of the simplest functional forms for h that will meet the above requirements is given by

$$(6) \qquad h(x) = x + B(s,q)x^2/(s\mu-x),$$

where

$$(7) \qquad B(s,q) = [q - 1 + A(s)]^{-2} .$$

It is direct to show that $h(x)$ does in fact satisfy the three properties mentioned above. For example, it is clear from (6) that $h(x) \geq x$, as also required.

In order to illustrate this procedure, consider the case when $s=14$, $q=20$ and $\mu=38$. Direct calculations give

$$A(14) = 5.377,$$

$$B(14,20) = (24.377)^{-2} = 0.001683,$$

$$h(x) = x + \frac{0.001683x^2}{532-x} .$$

Suppose for example that one observes the value $\lambda^* = 518$. The last equation above can then be used with $x = 518$ to estimate the true arrival rate λ: namely, $\lambda \simeq h(518) = 550$. As a matter of fact, the true value of λ corresponding to $\lambda^* = 518$ is found to be $\lambda = 549$. As another example, when $\lambda^* = 472$ then $\lambda \simeq h(472) = 478$, while the true value is $\lambda = 475$.

In the above cases, the approximation provided by (5)-(7) gives quite close agreement to the true values for λ. One caveat must be borne in mind, however, when using this type of approximation to the $\lambda = f(\lambda^*)$ relationship. Namely, since $\lambda \to \infty$ as $\lambda^* \to s\mu$, the function $f(\lambda^*)$ becomes extremely steep in the vicinity of $s\mu$. That is to say, small changes in λ^* in this vicinity will produce disproportionately large changes in the corresponding value of λ. The approximation that has been given here is made to agree closely with the true $\lambda = f(\lambda^*)$ relation for values of λ that are not too large. It cannot, however, approximate to any reasonable accuracy the steep behavior of $f(\lambda^*)$ as λ^* becomes close to $s\mu$.

In practical terms, this means that the approximation defined by (5)-(7) should not be used when $\lambda^* \approx s\mu$. More specifically, it has been found that if $\lambda^* > (.95)s\mu$ -- i.e., if λ^* is within 5% of $s\mu$ -- the above approximation will not be accurate. As a matter of fact, if λ^* is really as close to $s\mu$ as this, then the true value $f(\lambda^*)$ is so sensitive to changes in λ^* that accurate estimation of λ is in principle very difficult. The reason is that if λ^* is sufficiently close to $s\mu$ then just the uncertainty in the measurement of λ^* is enough to create an extremely large uncertainty in the true value of λ. In such a case (which corresponds to an almost complete saturation of system capacity), the true value of λ may be unobtainable to any reasonable accuracy. Fortunately, this type of situation (near-complete saturation) is not expected to occur in actual applications.

To summarize, a reasonably accurate approximation (over the range of parameter values of interest) has been provided here. The basic procedure is as follows.

1. Determine appropriate numerical values for s, μ and q.

2. Calculate $A(s)$ using (5), and then $B(s,q)$ using (7).

3. Given any observed arrival rate λ^* for calls which successfully enter the system, compute $h(\lambda^*)$ using (6). The value $h(\lambda^*)$ then provides an estimate of the true arrival rate λ for calls attempting to enter the system.

Finally, it should be emphasized that the above approximation is based upon the assumption of no reneging in the queuing system. Unfortunately, it does not appear possible in any straightforward way to extend the procedure given here to the case with reneging: i.e., when $\alpha > 0$.

APPENDIX B

TABLES FOR USE IN ESTIMATING MANPOWER REQUIREMENTS

QUEUEING MODEL

TRUE ARRIVAL RATE (LAMBDA) = 200 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

		NUMBER OF SERVERS												
		8	9	10	11	12	13	14	15	16	17	18	19	20
	LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
	30	22	21	20	19	18	17	16	15	14	13	12	11	10
SERVICE		40	32	25	17	10	5	2	1	0	0	0	0	0
RATE		615	504	409	322	243	177	128	94	72	57	46	39	33
15		855	744	648	558	470	389	328	289	266	254	247	244	242
	40	32	31	30	29	28	27	26	25	24	23	22	21	20
SERVICE		40	32	25	17	10	4	1	0	0	0	0	0	0
RATE		915	771	648	532	410	286	185	121	85	64	51	42	35
15		1155	1011	888	771	643	500	378	307	272	256	248	244	242
	50	42	41	40	39	38	37	36	35	34	33	32	31	30
SERVICE		40	32	25	17	10	3	0	0	0	0	0	0	0
RATE		1214	1037	888	748	594	403	231	134	88	65	51	42	35
15		1455	1277	1128	988	830	618	418	315	274	256	248	244	242

		NUMBER OF SERVERS												
		5	6	7	8	9	10	11	12	13	14	15	16	17
	LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
	30	25	24	23	22	21	20	19	18	17	16	15	14	13
SERVICE		50	40	30	20	10	4	1	0	0	0	0	0	0
RATE		864	675	531	408	291	189	119	79	57	44	35	29	25
20		1044	855	711	587	461	339	256	214	196	187	183	181	180
	40	35	34	33	32	31	30	29	28	27	26	25	24	23
SERVICE		50	40	30	20	10	2	0	0	0	0	0	0	0
RATE		1224	974	788	630	464	279	148	87	59	44	35	29	25
20		1404	1155	968	810	639	428	278	219	197	187	183	181	180
	50	45	44	43	42	41	40	39	38	37	36	35	34	33
SERVICE		50	40	30	20	10	2	0	0	0	0	0	0	0
RATE		1584	1274	1045	855	651	369	164	89	59	44	35	29	25
20		1764	1455	1225	1035	828	518	291	220	197	187	183	181	180

QUEUEING MODEL

TRUE ARRIVAL RATE (LAMBDA) = 200 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

		NUMBER OF SERVERS												
		1	2	3	4	5	6	7	8	9	10	11	12	13
	LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
	20	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		85	70	55	40	25	11	3	1	0	0	0	0	0
RATE		2258	1054	647	435	292	183	108	67	45	33	26	21	18
30		2378	1174	767	555	410	290	200	153	133	125	122	120	120
	30	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		85	70	55	40	25	10	1	0	0	0	0	0	0
RATE		3458	1654	1047	735	528	341	169	83	50	35	27	22	18
30		3578	1774	1167	855	648	454	256	164	135	126	122	121	120
	40	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		85	70	55	40	25	10	0	0	0	0	0	0	0
RATE		4658	2254	1447	1035	768	519	218	88	51	35	27	22	18
30		4778	2374	1567	1155	888	635	302	167	136	126	122	121	120

		NUMBER OF SERVERS												
		1	2	3	4	5	6	7	8	9	10	11	12	13
	LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
	20	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		80	60	40	20	5	0	0	0	0	0	0	0	0
RATE		1687	780	465	280	143	72	42	29	22	17	14	12	11
40		1777	870	555	367	213	131	103	94	91	90	90	90	90
	30	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		80	60	40	20	3	0	0	0	0	0	0	0	0
RATE		2587	1229	765	496	233	85	44	29	22	17	14	12	11
40		2677	1320	855	586	302	139	104	95	91	90	90	90	90
	40	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		80	60	40	20	2	0	0	0	0	0	0	0	0
RATE		3487	1679	1064	720	323	88	44	29	22	17	14	12	11
40		3577	1770	1155	810	392	142	104	95	91	90	90	90	90

QUEUEING MODEL

TRUE ARRIVAL RATE (LAMBDA) = 200 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

		NUMBER OF SERVERS												
	LINES	1	2	3	4	5	6	7	8	9	10	11	12	13
	20	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		75	50	25	5	0	0	0	0	0	0	0	0	0
RATE		1344	612	339	152	64	35	23	17	14	11	10	8	7
50		1415	684	410	206	106	82	75	73	72	72	72	72	72
	30	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		75	50	25	3	0	0	0	0	0	0	0	0	0
RATE		2064	972	576	242	70	35	23	17	14	11	10	8	7
50		2135	1043	648	295	111	82	75	73	72	72	72	72	72
	40	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		75	50	25	2	0	0	0	0	0	0	0	0	0
RATE		2784	1332	816	333	71	35	23	17	14	11	10	8	7
50		2855	1403	888	385	111	82	75	73	72	72	72	72	72

		NUMBER OF SERVERS												
	LINES	1	2	3	4	5	6	7	8	9	10	11	12	13
	20	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		70	40	11	0	0	0	0	0	0	0	0	0	0
RATE		1114	495	228	76	35	22	16	12	10	8	7	6	6
60		1174	555	280	109	71	63	61	60	60	60	60	60	60
	30	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		70	40	10	0	0	0	0	0	0	0	0	0	0
RATE		1714	795	393	86	35	22	16	12	10	9	7	6	6
60		1774	855	449	116	71	63	61	60	60	60	60	60	60
	40	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		70	40	10	0	0	0	0	0	0	0	0	0	0
RATE		2314	1094	575	89	35	22	16	12	10	9	7	6	6
60		2374	1155	633	118	71	63	61	60	60	60	60	60	60

Q U E U E I N G   M O D E L

TRUE ARRIVAL RATE (LAMBDA) = 200 PER HOUR

VALUES IN EACH CELL ARE:
(1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

NUMBER OF SERVERS

	LINES	1	2	3	4	5	6	7	8	9	10	11	12	13
	20	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		65	30	3	0	0	0	0	0	0	0	0	0	0
RATE		949	403	134	44	23	16	12	9	8	7	6	5	5
70		1000	454	166	71	56	52	51	51	51	51	51	51	51
	30	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		65	30	1	0	0	0	0	0	0	0	0	0	0
RATE		1463	660	190	44	23	16	12	9	8	7	6	5	5
70		1515	711	219	71	56	52	51	51	51	51	51	51	51
	40	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		65	30	0	0	0	0	0	0	0	0	0	0	0
RATE		1978	917	235	44	23	16	12	9	8	7	6	5	5
70		2029	968	262	71	56	52	51	51	51	51	51	51	51

NUMBER OF SERVERS

	LINES	1	2	3	4	5	6	7	8	9	10	11	12	13
	20	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		60	20	0	0	0	0	0	0	0	0	0	0	0
RATE		825	322	77	29	17	12	9	8	6	5	5	4	4
80		870	365	98	54	47	45	45	45	45	45	45	45	45
	30	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		60	20	0	0	0	0	0	0	0	0	0	0	0
RATE		1274	541	87	29	17	12	9	8	6	5	5	4	.4
80		1320	585	105	54	47	45	45	45	45	45	45	45	45
	40	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		60	20	0	0	0	0	0	0	0	0	0	0	0
RATE		1725	765	89	29	17	12	9	8	6	5	5	4	4
80		1770	810	107	54	47	45	45	45	45	45	45	45	45

Q U E U E I N G   M O D E L

TRUE ARRIVAL RATE (LAMBDA) = 200 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

NUMBER OF SERVERS

	LINES	1	2	3	4	5	6	7	8	9	10	11	12	13
		--	--	--	--	--	--	--	--	--	--	--	--	--
	20	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		55	11	0	0	0	0	0	0	0	0	0	0	0
RATE		727	243	50	22	14	10	8	6	5	5	4	4	3
90		767	277	67	45	41	40	40	40	40	40	40	40	40
	30	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		55	10	0	0	0	0	0	0	0	0	0	0	0
RATE		1127	410	51	22	14	10	8	6	5	5	4	4	3
90		1167	447	68	45	41	40	40	40	40	40	40	40	40
	40	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		55	10	0	0	0	0	0	0	0	0	0	0	0
RATE		1527	594	51	22	14	10	8	6	5	5	4	4	3
90		1567	632	68	45	41	40	40	40	40	40	40	40	40

NUMBER OF SERVERS

	LINES	1	2	3	4	5	6	7	8	9	10	11	12	13
		--	--	--	--	--	--	--	--	--	--	--	--	--
	20	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		50	4	0	0	0	0	0	0	0	0	0	0	0
RATE		648	170	35	17	11	8	7	5	5	4	3	3	3
100		684	193	51	39	36	36	36	36	36	36	36	36	36
	30	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		50	3	0	0	0	0	0	0	0	0	0	0	0
RATE		1008	260	35	17	11	8	7	5	5	4	3	3	.3
100		1043	283	51	39	36	36	36	36	36	36	36	36	36
	40	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		50	2	0	0	0	0	0	0	0	0	0	0	0
RATE		1368	351	35	17	11	8	7	5	5	4	3	3	3
100		1403	373	51	39	36	36	36	36	36	36	36	36	36

Q U E U E I N G   M O D E L

TRUE ARRIVAL RATE (LAMBDA) = 300 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

	NUMBER OF SERVERS												
LINES	15	16	17	18	19	20	21	22	23	24	25	26	27
	--	--	--	--	--	--	--	--	--	--	--	--	--
40	25	24	23	22	21	20	19	18	17	16	15	14	13
SERVICE	25	20	15	10	6	3	2	1	0	0	0	0	0
RATE	352	301	252	205	162	125	97	76	61	50	42	36	31
15	592	540	489	436	385	339	304	279	264	254	248	245	243
50	35	34	33	32	31	30	29	28	27	26	25	24	23
SERVICE	25	20	15	10	5	2	1	0	0	0	0	0	0
RATE	512	450	388	321	251	185	133	97	73	57	47	39	34
15	752	690	627	557	478	398	334	293	270	257	249	245	243
60	45	44	43	42	41	40	39	38	37	36	35	34	33
SERVICE	25	20	15	10	5	2	0	0	0	0	0	0	0
RATE	672	600	527	446	349	245	161	108	77	59	47	39	34
15	912	840	767	684	579	457	358	300	272	257	250	245	243

	NUMBER OF SERVERS												
LINES	10	11	12	13	14	15	16	17	18	19	20	21	22
	--	--	--	--	--	--	--	--	--	--	--	--	--
30	20	19	18	17	16	15	14	13	12	11	10	9	8
SERVICE	33	26	20	13	8	4	2	1	0	0	0	0	0
RATE	324	266	214	168	127	95	72	56	44	36	30	26	22
20	504	446	392	341	293	253	224	206	194	188	184	182	181
40	30	29	28	27	26	25	24	23	22	21	20	19	18
SERVICE	33	26	20	13	7	3	1	0	0	0	0	0	0
RATE	504	429	360	291	220	155	107	75	55	43	35	29	25
20	684	609	540	469	390	311	252	218	199	190	185	183	181
50	40	39	38	37	36	35	34	33	32	31	30	29	28
SERVICE	33	26	20	13	7	2	0	0	0	0	0	0	0
RATE	684	593	510	424	324	215	132	84	59	44	35	29	25
20	864	773	690	603	497	371	273	223	201	190	185	183	181

Q U E U E I N G   M O D E L

TRUE ARRIVAL RATE (LAMBDA) = 300 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

	LINES	NUMBER OF SERVERS 5	6	7	8	9	10	11	12	13	14	15	16	17
		--	--	--	--	--	--	--	--	--	--	--	--	--
	30	25	24	23	22	21	20	19	18	17	16	15	14	13
SERVICE		50	40	30	20	10	4	1	0	0	0	0	0	0
RATE		576	450	354	272	194	125	79	52	38	29	23	19	17
30		696	570	474	391	307	226	171	143	130	125	122	121	120
	40	35	34	33	32	31	30	29	28	27	26	25	24	23
SERVICE		50	40	30	20	10	2	0	0	0	0	0	0	0
RATE		815	650	525	420	309	185	98	58	39	29	23	19	17
30		936	770	645	540	426	285	185	146	131	125	122	121	120
	50	45	44	43	42	41	40	39	38	37	36	35	34	33
SERVICE		50	40	30	20	10	2	0	0	0	0	0	0	0
RATE		1055	849	697	570	434	245	109	59	39	29	23	19	17
30		1176	970	817	690	552	345	194	146	131	125	122	121	120

	LINES	NUMBER OF SERVERS 2	3	4	5	6	7	8	9	10	11	12	13	14
		--	--	--	--	--	--	--	--	--	--	--	--	--
	20	18	17	16	15	14	13	12	11	10	9	8	7	6
SERVICE		73	60	46	33	20	9	3	1	0	0	0	0	0
RATE		793	490	334	234	159	102	64	42	30	23	18	15	12
40		883	580	424	324	246	180	134	110	99	93	91	90	90
	30	28	27	26	25	24	23	22	21	20	19	18	17	16
SERVICE		73	60	46	33	20	7	1	0	0	0	0	0	0
RATE		1243	789	559	414	301	191	101	55	35	25	19	16	13
40		1333	880	649	504	391	273	166	117	100	94	91	90	90
	40	38	37	36	35	34	33	32	31	30	29	28	27	26
SERVICE		73	60	46	33	20	7	0	0	0	0	0	0	0
RATE		1693	1089	784	594	450	292	127	58	35	25	19	16	13
40		1783	1180	874	684	540	376	190	119	101	94	91	90	90

QUEUEING MODEL

TRUE ARRIVAL RATE (LAMBDA) = 300 PER HOUR

VALUES IN EACH CELL ARE:
(1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

NUMBER OF SERVERS

	LINES	1	2	3	4	5	6	7	8	9	10	11	12	13
	20	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		83	66	50	33	17	5	1	0	0	0	0	0	0
RATE		1353	629	384	252	158	89	51	32	22	17	14	11	10
50		1425	702	456	324	227	147	101	83	76	73	72	72	72
	30	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		83	66	50	33	16	3	0	0	0	0	0	0	0
RATE		2073	990	624	432	291	149	64	35	23	17	14	11	10
50		2145	1062	696	504	362	206	111	84	76	73	72	72	72
	40	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		83	66	50	33	16	2	0	0	0	0	0	0	0
RATE		2793	1349	863	612	432	210	69	35	23	17	14	11	10
50		2865	1422	935	684	504	266	114	84	76	73	72	72	72

NUMBER OF SERVERS

	LINES	1	2	3	4	5	6	7	8	9	10	11	12	13
	20	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		80	60	40	20	5	0	0	0	0	0	0	0	0
RATE		1125	520	310	186	95	48	28	19	14	11	9	8	7
60		1185	580	370	244	142	87	69	63	61	60	60	60	60
	30	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		80	60	40	20	3	0	0	0	0	0	0	0	0
RATE		1725	819	510	331	155	56	29	19	14	11	9	8	7
60		1785	880	570	390	201	93	69	63	61	60	60	60	60
	40	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		80	60	40	20	2	0	0	0	0	0	0	0	0
RATE		2324	1119	709	480	215	59	29	19	14	11	9	8	7
60		2385	1180	770	540	261	94	69	63	61	60	60	60	60

Q U E U E I N G   M O D E L

TRUE ARRIVAL RATE (LAMBDA) = 300 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

NUMBER OF SERVERS

	LINES	1	2	3	4	5	6	7	8	9	10	11	12	13
		--	--	--	--	--	--	--	--	--	--	--	--	--
	20	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		76	53	30	9	1	0	0	0	0	0	0	0	0
RATE		961	440	252	127	55	28	18	13	10	8	7	6	5
70		1012	491	303	170	86	61	54	52	51	51	51	51	51
	30	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		76	53	30	7	0	0	0	0	0	0	0	0	0
RATE		1475	697	422	220	66	29	18	13	10	8	7	6	5
70		1527	748	474	266	95	62	54	52	51	51	51	51	51
	40	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		76	53	30	7	0	0	0	0	0	0	0	0	0
RATE		1990	954	594	324	70	29	18	13	10	8	7	6	5
70		2041	1006	645	372	98	62	54	52	51	51	51	51	51

NUMBER OF SERVERS

	LINES	1	2	3	4	5	6	7	8	9	10	11	12	13
		--	--	--	--	--	--	--	--	--	--	--	--	--
	20	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		73	46	20	2	0	0	0	0	0	0	0	0	0
RATE		838	379	200	80	34	19	13	10	8	7	6	5	4
80		883	424	244	109	60	49	46	45	45	45	45	45	45
	30	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		73	46	20	1	0	0	0	0	0	0	0	0	0
RATE		1288	604	345	112	35	19	13	10	8	7	6	5	4
80		1333	649	390	139	61	49	46	45	45	45	45	45	45
	40	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		73	46	20	0	0	0	0	0	0	0	0	0	0
RATE		1738	829	495	136	35	19	13	10	8	7	6	5	4
80		1783	874	540	160	61	49	46	45	45	45	45	45	45

QUEUEING MODEL

TRUE ARRIVAL RATE (LAMBDA) = 300 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

NUMBER OF SERVERS

SERVICE RATE	LINES	1	2	3	4	5	6	7	8	9	10	11	12	13
90	20	19	18	17	16	15	14	13	12	11	10	9	8	7
		70	40	11	0	0	0	0	0	0	0	0	0	0
		742	330	152	50	23	14	10	8	7	5	5	4	4
		782	370	186	72	47	42	40	40	40	40	40	40	40
90	30	29	28	27	26	25	24	23	22	21	20	19	18	17
		70	40	10	0	0	0	0	0	0	0	0	0	0
		1142	530	262	57	23	14	10	8	7	6	5	4	4
		1182	570	299	77	47	42	40	40	40	40	40	40	40
90	40	39	38	37	36	35	34	33	32	31	30	29	28	27
		70	40	10	0	0	0	0	0	0	0	0	0	0
		1542	729	383	59	23	14	10	8	7	6	5	4	4
		1582	770	422	79	47	42	40	40	40	40	40	40	40

NUMBER OF SERVERS

SERVICE RATE	LINES	1	2	3	4	5	6	7	8	9	10	11	12	13
100	20	19	18	17	16	15	14	13	12	11	10	9	8	7
		66	33	5	0	0	0	0	0	0	0	0	0	0
		665	288	108	34	17	11	8	7	5	5	4	3	3
		702	324	133	53	40	37	36	36	36	36	36	36	36
100	30	29	28	27	26	25	24	23	22	21	20	19	18	17
		66	33	3	0	0	0	0	0	0	0	0	0	0
		1026	468	168	35	17	11	8	7	5	5	4	4	.3
		1062	504	193	54	40	37	36	36	36	36	36	36	36
100	40	39	38	37	36	35	34	33	32	31	30	29	28	27
		66	33	2	0	0	0	0	0	0	0	0	0	0
		1385	648	228	35	17	11	9	7	5	5	4	4	3
		1422	684	252	54	40	37	36	36	36	36	36	36	36

QUEUEING MODEL

TRUE ARRIVAL RATE (LAMBDA) = 400 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

NUMBER OF SERVERS

	LINES	21	22	23	24	25	26	27	28	29	30	31	32	33
		--	--	--	--	--	--	--	--	--	--	--	--	--
	40	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		21	17	14	11	8	6	4	3	2	1	1	0	0
RATE		177	151	127	106	88	73	60	50	42	36	31	26	23
15		415	388	361	336	314	294	279	267	258	252	248	245	243
	50	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		21	17	13	10	7	4	2	1	0	0	0	0	0
RATE		289	255	221	187	155	126	101	82	66	55	46	39	34
15		529	494	459	422	384	348	316	292	274	262	254	249	245
	60	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		21	17	13	10	6	3	1	0	0	0	0	0	0
RATE		403	363	322	278	231	183	141	107	82	64	52	43	37
15		643	603	561	515	463	406	353	312	284	267	256	250	246

NUMBER OF SERVERS

	LINES	15	16	17	18	19	20	21	22	23	24	25	26	27
		--	--	--	--	--	--	--	--	--	--	--	--	--
	40	25	24	23	22	21	20	19	18	17	16	15	14	13
SERVICE		25	20	15	10	6	3	2	1	0	0	0	0	0
RATE		264	226	189	154	121	94	73	57	46	38	32	27	23
20		444	405	367	327	288	254	228	209	198	190	186	183	182
	50	35	34	33	32	31	30	29	28	27	26	25	24	23
SERVICE		25	20	15	10	5	2	1	0	0	0	0	0	0
RATE		384	337	291	241	188	139	100	72	55	43	35	29	25
20		564	517	470	418	358	298	250	220	202	192	187	184	182
	60	45	44	43	42	41	40	39	38	37	36	35	34	33
SERVICE		25	20	15	10	5	2	0	0	0	0	0	0	0
RATE		504	450	395	335	262	184	121	81	58	44	35	29	25
20		684	630	575	513	434	342	268	225	204	193	187	184	182

QUEUEING MODEL

TRUE ARRIVAL RATE (LAMBDA) = 400 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

SERVICE RATE	LINES	NUMBER OF SERVERS 8	9	10	11	12	13	14	15	16	17	18	19	20
		--	--	--	--	--	--	--	--	--	--	--	--	--
30	30	22	21	20	19	18	17	16	15	14	13	12	11	10
		40	32	25	17	10	5	2	1	0	0	0	0	0
		307	252	204	161	121	88	64	47	36	28	23	19	16
		427	372	324	279	235	194	164	144	133	127	123	122	121
30	40	32	31	30	29	28	27	26	25	24	23	22	21	20
		40	32	25	17	10	4	1	0	0	0	0	0	0
		457	385	324	266	205	143	92	60	42	32	25	21	17
		577	505	444	385	321	250	189	153	136	128	124	122	121
30	50	42	41	40	39	38	37	36	35	34	33	32	31	30
		40	32	25	17	10	3	0	0	0	0	0	0	0
		607	518	444	374	297	201	115	67	44	32	25	21	17
		727	638	564	494	415	309	209	157	137	128	124	122	121

SERVICE RATE	LINES	NUMBER OF SERVERS 5	6	7	8	9	10	11	12	13	14	15	16	17
		--	--	--	--	--	--	--	--	--	--	--	--	--
40	30	25	24	23	22	21	20	19	18	17	16	15	14	13
		50	40	30	20	10	4	1	0	0	0	0	0	0
		432	337	265	204	145	94	59	39	28	22	17	14	12
		522	427	355	293	230	169	128	107	98	93	91	90	90
40	40	35	34	33	32	31	30	29	28	27	26	25	24	23
		50	40	30	20	10	2	0	0	0	0	0	0	0
		612	487	394	315	232	139	74	43	29	22	17	14	12
		702	577	484	405	319	214	139	109	98	93	91	90	90
40	50	45	44	43	42	41	40	39	38	37	36	35	34	33
		50	40	30	20	10	2	0	0	0	0	0	0	0
		792	637	522	427	325	184	82	44	29	22	17	14	12
		882	727	612	517	414	259	145	110	98	93	91	90	90

Q U E U E I N G   M O D E L

TRUE ARRIVAL RATE (LAMBDA) = 400 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

		NUMBER OF SERVERS												
	LINES	3	4	5	6	7	8	9	10	11	12	13	14	15
		--	--	--	--	--	--	--	--	--	--	--	--	--
	30	27	26	25	24	23	22	21	20	19	18	17	16	15
SERVICE		62	50	37	25	12	3	0	0	0	0	0	0	0
RATE		633	450	336	252	176	103	56	34	23	17	14	11	10
50		705	522	408	324	245	162	107	85	77	74	73	72	72
	40	37	36	35	34	33	32	31	30	29	28	27	26	25
SERVICE		62	50	37	25	12	2	0	0	0	0	0	0	0
RATE		873	630	480	372	271	148	65	35	23	17	14	11	10
50		945	702	552	444	342	206	114	86	77	74	73	72	72
	50	47	46	45	44	43	42	41	40	39	38	37	36	35
SERVICE		62	50	37	25	12	2	0	0	0	0	0	0	0
RATE		1113	810	623	492	371	193	69	35	23	17	14	11	10
50		1185	882	696	564	443	251	117	86	77	74	73	72	72

		NUMBER OF SERVERS												
	LINES	1	2	3	4	5	6	7	8	9	10	11	12	13
		--	--	--	--	--	--	--	--	--	--	--	--	--
	20	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		85	70	55	40	25	11	3	1	0	0	0	0	0
RATE		1129	527	323	217	146	91	54	33	22	16	13	10	9
60		1189	587	383	277	205	145	100	76	66	62	61	60	60
	30	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		85	70	55	40	25	10	1	0	0	0	0	0	0
RATE		1729	827	523	367	264	170	84	41	25	17	13	11	9
60		1789	887	583	427	324	227	128	82	67	63	61	60	60
	40	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		85	70	55	40	25	10	0	0	0	0	0	0	0
RATE		2329	1127	723	517	384	259	109	44	25	17	13	11	9
60		2389	1187	783	577	444	317	151	83	68	63	61	60	60

Q U E U E I N G   M O D E L

TRUE ARRIVAL RATE (LAMBDA) = 400 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

NUMBER OF SERVERS

	LINES	1	2	3	4	5	6	7	8	9	10	11	12	13
	20	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		82	65	47	30	13	3	0	0	0	0	0	0	0
RATE		966	449	272	176	106	57	32	21	15	11	9	8	6
70		1017	500	323	227	153	95	67	57	53	52	51	51	51
	30	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		82	65	47	30	12	1	0	0	0	0	0	0	0
RATE		1480	706	443	304	194	87	38	22	15	11	9	8	7
70		1531	757	495	355	244	123	71	58	53	52	51	51	51
	40	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		82	65	47	30	12	0	0	0	0	0	0	0	0
RATE		1994	963	615	432	291	111	39	22	15	11	9	8	7
70		2046	1014	666	484	342	145	72	58	53	52	51	51	51

NUMBER OF SERVERS

	LINES	1	2	3	4	5	6	7	8	9	10	11	12	13
	20	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		80	60	40	20	5	0	0	0	0	0	0	0	0
RATE		843	390	232	140	71	36	21	14	11	8	7	6	5
80		888	435	277	183	106	65	51	47	45	45	45	45	45
	30	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		80	60	40	20	3	0	0	0	0	0	0	0	0
RATE		1293	614	382	248	116	42	22	14	11	8	7	6	5
80		1338	660	427	293	151	69	52	47	45	45	45	45	45
	40	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		80	60	40	20	2	0	0	0	0	0	0	0	0
RATE		1743	839	532	360	161	44	22	14	11	8	7	6	5
80		1788	885	577	405	196	71	52	47	45	45	45	45	45

Q U E U E I N G   M O D E L

TRUE ARRIVAL RATE (LAMBDA) = 400 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

		NUMBER OF SERVERS												
	LINES	1	2	3	4	5	6	7	8	9	10	11	12	13
	20	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		77	55	32	11	1	0	0	0	0	0	0	0	0
RATE		748	343	199	106	47	24	15	11	8	7	6	5	4
90		788	383	239	141	73	49	43	41	40	40	40	40	40
	30	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		77	55	32	10	0	0	0	0	0	0	0	0	0
RATE		1148	543	332	187	60	25	15	11	8	7	6	5	4
90		1188	583	372	225	84	50	43	41	40	40	40	40	40
	40	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		77	55	32	10	0	0	0	0	0	0	0	0	0
RATE		1548	743	465	278	67	25	15	11	8	7	6	5	4
90		1588	783	505	316	89	50	43	41	40	40	40	40	40

		NUMBER OF SERVERS												
	LINES	1	2	3	4	5	6	7	8	9	10	11	12	13
	20	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		75	50	25	5	0	0	0	0	0	0	0	0	0
RATE		672	306	169	76	32	17	11	8	7	5	5	4	3
100		707	342	205	103	53	41	37	36	36	36	36	36	36
	30	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		75	50	25	3	0	0	0	0	0	0	0	0	0
RATE		1032	486	288	121	35	17	11	8	7	5	5	4	3
100		1067	521	324	147	55	41	37	36	36	36	36	36	36
	40	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		75	50	25	2	0	0	0	0	0	0	0	0	0
RATE		1392	666	408	166	35	17	11	8	7	5	5	4	3
100		1427	701	444	192	55	41	37	36	36	36	36	36	36

QUEUEING MODEL

TRUE ARRIVAL RATE (LAMBDA) = 500 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

NUMBER OF SERVERS

SERVICE RATE	LINES	28	29	30	31	32	33	34	35	36	37	38	39	40
15	50	22	21	20	19	18	17	16	15	14	13	12	11	10
		16	13	10	8	6	4	3	2	1	1	0	0	0
		147	128	110	93	79	67	57	48	41	36	31	27	24
		385	364	343	323	305	289	276	266	258	253	248	246	244
15	60	32	31	30	29	28	27	26	25	24	23	22	21	20
		16	13	10	7	5	3	2	1	0	0	0	0	0
		230	204	178	152	128	106	87	72	60	50	43	37	32
		469	443	414	385	355	327	303	284	270	260	253	249	246
15	70	42	41	40	39	38	37	36	35	34	33	32	31	30
		16	13	10	7	4	2	1	0	0	0	0	0	0
		315	285	252	218	181	146	115	90	72	58	48	40	35
		555	524	491	453	410	367	329	299	278	264	255	250	246

NUMBER OF SERVERS

SERVICE RATE	LINES	20	21	22	23	24	25	26	27	28	29	30	31	32
20	40	20	19	18	17	16	15	14	13	12	11	10	9	8
		20	16	12	9	6	4	3	2	1	0	0	0	0
		146	124	103	85	70	57	47	39	33	28	24	21	18
		325	301	278	256	237	221	208	199	192	188	185	183	182
20	50	30	29	28	27	26	25	24	23	22	21	20	19	18
		20	16	12	8	5	3	1	0	0	0	0	0	0
		234	205	175	146	118	93	73	58	47	38	32	28	24
		414	384	353	320	286	255	230	212	200	192	188	184	183
20	60	40	39	38	37	36	35	34	33	32	31	30	29	28
		20	16	12	8	4	2	0	0	0	0	0	0	0
		324	289	253	213	170	129	95	71	54	43	35	29	25
		504	469	432	389	340	290	249	221	204	194	188	185	183

TRUE ARRIVAL RATE (LAMBDA) = 500 PER HOUR

VALUES IN EACH CELL ARE:
(1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

NUMBER OF SERVERS

	LINES	11	12	13	14	15	16	17	18	19	20	21	22	23
	30	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		34	28	22	16	11	7	4	2	1	0	0	0	0
RATE		186	154	126	101	79	61	47	37	29	24	20	17	14
30		306	274	245	217	192	170	152	140	132	127	124	122	121
	40	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		34	28	22	16	10	5	2	1	0	0	0	0	0
RATE		295	254	216	180	143	108	78	56	42	32	26	21	18
30		415	374	336	299	259	218	181	155	139	130	126	123	121
	50	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		34	28	22	16	10	4	1	0	0	0	0	0	0
RATE		404	354	308	264	215	159	107	70	47	35	27	22	18
30		524	474	428	383	333	271	208	165	142	131	126	123	121

NUMBER OF SERVERS

	LINES	7	8	9	10	11	12	13	14	15	16	17	18	19
	30	23	22	21	20	19	18	17	16	15	14	13	12	11
SERVICE		44	36	28	20	12	6	2	1	0	0	0	0	0
RATE		279	227	184	146	110	79	55	39	29	23	18	15	13
40		369	317	274	235	196	159	130	111	101	95	93	91	90
	40	33	32	31	30	29	28	27	26	25	24	23	22	21
SERVICE		44	36	28	20	12	5	1	0	0	0	0	0	0
RATE		407	340	284	234	183	128	80	52	34	25	19	16	13
40		497	430	374	324	271	209	153	119	103	96	93	91	90
	50	43	42	41	40	39	38	37	36	35	34	33	32	31
SERVICE		44	36	28	20	12	4	0	0	0	0	0	0	0
RATE		536	452	384	324	261	181	101	56	35	25	19	16	13
40		626	542	474	414	350	254	171	122	104	96	93	91	90

Q U E U E I N G   M O D E L

TRUE ARRIVAL RATE (LAMBDA) = 500 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

NUMBER OF SERVERS

	5	6	7	8	9	10	11	12	13	14	15	16	17
LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
30	25	24	23	22	21	20	19	18	17	16	15	14	13
SERVICE	50	40	30	20	10	4	1	0	0	0	0	0	0
RATE	345	270	212	163	116	75	47	31	22	17	14	11	10
50	417	342	284	234	184	135	102	85	78	75	73	72	72
40	35	34	33	32	31	30	29	28	27	26	25	24	23
SERVICE	50	40	30	20	10	2	0	0	0	0	0	0	0
RATE	489	390	315	252	185	111	59	34	23	17	14	11	10
50	561	462	387	324	255	171	111	87	78	75	73	72	72
50	45	44	43	42	41	40	39	38	37	36	35	34	33
SERVICE	50	40	30	20	10	2	0	0	0	0	0	0	0
RATE	633	509	418	342	260	147	65	35	23	17	14	11	10
50	705	582	490	414	331	207	116	88	78	75	73	72	72

NUMBER OF SERVERS

	3	4	5	6	7	8	9	10	11	12	13	14	15
LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
30	27	26	25	24	23	22	21	20	19	18	17	16	15
SERVICE	64	52	40	28	16	6	1	0	0	0	0	0	0
RATE	528	376	282	214	155	98	55	32	21	16	12	10	8
60	588	436	342	274	214	150	99	75	66	62	61	60	60
40	37	36	35	34	33	32	31	30	29	28	27	26	25
SERVICE	64	52	40	28	16	5	0	0	0	0	0	0	0
RATE	728	526	402	314	238	149	69	35	22	16	12	10	.8
60	788	586	462	374	298	202	111	77	66	62	61	60	60
50	47	46	45	44	43	42	41	40	39	38	37	36	35
SERVICE	64	52	40	28	16	4	0	0	0	0	0	0	0
RATE	928	676	521	414	323	204	77	35	22	16	12	10	8
60	988	736	582	474	383	258	118	77	66	62	61	60	60

QUEUEING MODEL

TRUE ARRIVAL RATE (LAMBDA) = 500 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

NUMBER OF SERVERS

	LINES	2	3	4	5	6	7	8	9	10	11	12	13	14
	20	18	17	16	15	14	13	12	11	10	9	8	7	6
SERVICE		72	58	44	30	16	6	2	0	0	0	0	0	0
RATE		452	279	189	131	86	53	33	22	16	12	10	8	7
70		504	330	240	182	134	96	71	60	55	53	52	51	51
	30	28	27	26	25	24	23	22	21	20	19	18	17	16
SERVICE		72	58	44	30	16	4	0	0	0	0	0	0	0
RATE		709	450	317	233	163	94	47	26	17	13	10	8	7
70		761	501	369	284	214	137	82	62	55	53	52	51	51
	40	38	37	36	35	34	33	32	31	30	29	28	27	26
SERVICE		72	58	44	30	16	3	0	0	0	0	0	0	0
RATE		967	621	446	336	247	138	54	27	17	13	10	8	7
70		1018	673	497	387	298	181	88	62	55	53	52	51	51

NUMBER OF SERVERS

	LINES	1	2	3	4	5	6	7	8	9	10	11	12	13
	20	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		84	68	52	36	20	7	2	0	0	0	0	0	0
RATE		846	394	241	160	103	61	35	22	15	11	9	7	6
80		891	439	286	205	147	98	67	53	48	46	45	45	45
	30	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		84	68	52	36	20	5	0	0	0	0	0	0	0
RATE		1296	619	391	272	189	108	48	25	16	11	9	7	6
80		1341	664	436	317	234	146	78	55	48	46	45	45	45
	40	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		84	68	52	36	20	5	0	0	0	0	0	0	0
RATE		1746	844	541	385	279	159	54	25	16	11	9	7	6
80		1791	889	586	430	324	199	83	55	48	46	45	45	45

QUEUEING MODEL

TRUE ARRIVAL RATE (LAMBDA) = 500 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

NUMBER OF SERVERS

	LINES	1	2	3	4	5	6	7	8	9	10	11	12	13
	20	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		82	64	46	28	11	2	0	0	0	0	0	0	0
RATE		751	348	211	135	79	41	23	15	11	8	7	6	5
90		791	388	251	174	114	70	50	44	41	40	40	40	40
	30	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		82	64	46	28	10	1	0	0	0	0	0	0	0
RATE		1151	548	344	234	143	60	27	16	11	8	7	6	5
90		1191	588	384	274	180	86	52	44	41	40	40	40	40
	40	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		82	64	46	28	10	0	0	0	0	0	0	0	0
RATE		1551	748	477	334	215	72	27	16	11	8	7	6	5
90		1591	788	517	374	253	97	53	44	41	40	40	40	40

NUMBER OF SERVERS

	LINES	1	2	3	4	5	6	7	8	9	10	11	12	13
	20	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		80	60	40	20	5	0	0	0	0	0	0	0	0
RATE		674	312	186	112	57	28	17	11	8	7	5	5	4
100		711	348	222	146	85	52	41	37	36	36	36	36	36
	30	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		80	60	40	20	3	0	0	0	0	0	0	0	0
RATE		1035	491	306	198	93	34	17	11	8	7	5	5	4
100		1071	528	342	234	121	55	41	38	36	36	36	36	36
	40	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		80	60	40	20	2	0	0	0	0	0	0	0	0
RATE		1394	671	426	288	129	35	17	11	8	7	5	5	4
100		1431	708	462	324	156	56	41	38	36	36	36	36	36

Q U E U E I N G   M O D E L

TRUE ARRIVAL RATE (LAMBDA) = 600 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

	NUMBER OF SERVERS												
	35	36	37	38	39	40	41	42	43	44	45	46	47
LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
60	25	24	23	22	21	20	19	18	17	16	15	14	13
SERVICE	12	10	8	6	4	3	2	1	1	0	0	0	0
RATE	129	113	98	85	73	62	54	46	40	35	31	27	24
15	367	349	331	314	299	285	274	265	258	253	249	246	244
70	35	34	33	32	31	30	29	28	27	26	25	24	23
SERVICE	12	10	7	5	3	2	1	0	0	0	0	0	0
RATE	194	173	151	130	110	92	77	65	55	47	40	35	31
15	433	410	386	361	337	314	294	279	267	259	253	249	246
80	45	44	43	42	41	40	39	38	37	36	35	34	33
SERVICE	12	10	7	5	3	2	1	0	0	0	0	0	0
RATE	261	236	209	180	150	122	99	79	64	53	44	38	33
15	500	475	445	413	377	343	313	290	273	262	255	250	246

	NUMBER OF SERVERS												
	25	26	27	28	29	30	31	32	33	34	35	36	37
LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
50	25	24	23	22	21	20	19	18	17	16	15	14	13
SERVICE	16	13	10	7	5	3	2	1	0	0	0	0	0
RATE	145	126	108	91	75	63	52	43	37	31	27	24	21
20	324	304	283	263	244	227	213	203	196	190	187	184	183
60	35	34	33	32	31	30	29	28	27	26	25	24	23
SERVICE	16	13	10	7	4	2	1	0	0	0	0	0	0
RATE	216	192	167	141	115	93	73	59	47	39	33	28	24
20	396	371	344	315	285	256	232	214	202	194	189	185	183
70	45	44	43	42	41	40	39	38	37	36	35	34	33
SERVICE	16	13	10	6	4	2	0	0	0	0	0	0	0
RATE	288	260	229	195	158	123	92	69	53	42	35	29	25
20	468	439	408	371	329	285	248	222	206	196	189	186	183

Q U E U E I N G   M O D E L

TRUE ARRIVAL RATE (LAMBDA) = 600 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

NUMBER OF SERVERS

		15	16	17	18	19	20	21	22	23	24	25	26	27
	LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
	40	25	24	23	22	21	20	19	18	17	16	15	14	13
SERVICE		25	20	15	10	6	3	2	1	0	0	0	0	0
RATE		176	150	126	102	81	62	48	38	30	25	21	18	15
30		296	270	244	218	192	169	152	139	132	127	124	122	121
	50	35	34	33	32	31	30	29	28	27	26	25	24	23
SERVICE		25	20	15	10	5	2	1	0	0	0	0	0	0
RATE		256	225	194	160	125	92	66	48	36	28	23	19	17
30		376	345	313	278	239	199	167	146	135	128	124	122	121
	60	45	44	43	42	41	40	39	38	37	36	35	34	33
SERVICE		25	20	15	10	5	2	0	0	0	0	0	0	0
RATE		336	300	263	223	174	122	80	54	38	29	23	19	17
30		456	420	383	342	289	228	179	150	136	128	125	122	121

NUMBER OF SERVERS

		10	11	12	13	14	15	16	17	18	19	20	21	22
	LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
	30	20	19	18	17	16	15	14	13	12	11	10	9	8
SERVICE		33	26	20	13	8	4	2	1	0	0	0	0	0
RATE		162	133	107	84	63	47	36	28	22	18	15	13	11
40		252	223	196	170	146	126	112	103	97	94	92	91	90
	40	30	29	28	27	26	25	24	23	22	21	20	19	18
SERVICE		33	26	20	13	7	3	1	0	0	0	0	0	0
RATE		252	214	180	145	110	77	53	37	27	21	17	14	12
40		342	304	270	234	195	155	126	109	99	95	92	91	90
	50	40	39	38	37	36	35	34	33	32	31	30	29	28
SERVICE		33	26	20	13	7	2	0	0	0	0	0	0	0
RATE		342	296	255	212	162	107	66	42	29	22	17	14	12
40		432	386	345	301	248	185	136	111	100	95	92	91	90

QUEUEING MODEL

TRUE ARRIVAL RATE (LAMBDA) = 600 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

NUMBER OF SERVERS

		7	8	9	10	11	12	13	14	15	16	17	18	19
	LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
	30	23	22	21	20	19	18	17	16	15	14	13	12	11
SERVICE		41	33	25	16	9	4	1	0	0	0	0	0	0
RATE		222	180	144	111	81	56	39	28	21	16	13	11	10
50		294	252	216	182	148	118	97	85	78	75	73	72	72
	40	33	32	31	30	29	28	27	26	25	24	23	22	21
SERVICE		41	33	25	16	8	3	0	0	0	0	0	0	0
RATE		325	270	224	180	134	87	52	33	23	17	14	11	10
50		397	342	296	252	203	148	107	87	79	75	73	72	72
	50	43	42	41	40	39	38	37	36	35	34	33	32	31
SERVICE		41	33	25	16	8	2	0	0	0	0	0	0	0
RATE		427	360	304	252	192	116	60	35	23	17	14	11	10
50		499	432	376	324	262	177	114	88	79	75	73	72	72

NUMBER OF SERVERS

		5	6	7	8	9	10	11	12	13	14	15	16	17
	LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
	30	25	24	23	22	21	20	19	18	17	16	15	14	13
SERVICE		50	40	30	20	10	4	1	0	0	0	0	0	0
RATE		288	225	177	136	97	62	39	26	19	14	11	9	8
60		348	285	237	195	153	113	85	71	65	62	61	60	60
	40	35	34	33	32	31	30	29	28	27	26	25	24	23
SERVICE		50	40	30	20	10	2	0	0	0	0	0	0	0
RATE		407	325	262	210	154	92	49	29	19	14	11	9	.8
60		468	385	322	270	213	142	92	73	65	62	61	60	60
	50	45	44	43	42	41	40	39	38	37	36	35	34	33
SERVICE		50	40	30	20	10	2	0	0	0	0	0	0	0
RATE		527	424	348	285	217	122	54	29	19	14	11	9	8
60		588	485	408	345	276	172	97	73	65	62	61	60	60

QUEUEING MODEL

TRUE ARRIVAL RATE (LAMBDA) = 600 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

	LINES	NUMBER OF SERVERS 3	4	5	6	7	8	9	10	11	12	13	14	15
	30	27	26	25	24	23	22	21	20	19	18	17	16	15
SERVICE		65	53	41	30	18	7	2	0	0	0	0	0	0
RATE		453	323	242	185	137	91	52	31	20	14	11	9	7
70		505	374	294	237	188	137	92	68	58	54	52	52	51
	40	37	36	35	34	33	32	31	30	29	28	27	26	25
SERVICE		65	53	41	30	18	7	1	0	0	0	0	0	0
RATE		625	451	345	271	210	141	69	34	21	14	11	9	7
70		676	503	397	322	261	189	107	70	58	54	52	52	51
	50	47	46	45	44	43	42	41	40	39	38	37	36	35
SERVICE		65	53	41	30	18	6	0	0	0	0	0	0	0
RATE		796	580	448	357	283	195	83	35	21	14	11	9	7
70		847	631	499	408	334	244	119	70	58	54	52	52	51

	LINES	NUMBER OF SERVERS 2	3	4	5	6	7	8	9	10	11	12	13	14
	20	18	17	16	15	14	13	12	11	10	9	8	7	6
SERVICE		73	60	46	33	20	9	3	1	0	0	0	0	0
RATE		396	245	167	117	79	51	32	21	15	11	9	7	6
80		441	290	212	162	123	90	67	55	49	46	45	45	45
	30	28	27	26	25	24	23	22	21	20	19	18	17	16
SERVICE		73	60	46	33	20	7	1	0	0	0	0	0	0
RATE		621	394	279	207	150	95	50	27	17	12	9	8	.6
80		666	440	324	252	195	136	83	58	50	47	45	45	45
	40	38	37	36	35	34	33	32	31	30	29	28	27	26
SERVICE		73	60	46	33	20	7	0	0	0	0	0	0	0
RATE		846	544	392	297	225	146	63	29	17	12	9	8	6
80		891	590	437	342	270	188	95	59	50	47	45	45	45

Q U E U E I N G   M O D E L

TRUE ARRIVAL RATE (LAMBDA) = 600 PER HOUR   VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

NUMBER OF SERVERS

	LINES	1	2	3	4	5	6	7	8	9	10	11	12	13
		--	--	--	--	--	--	--	--	--	--	--	--	--
	20	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		85	70	55	40	25	11	3	1	0	0	0	0	0
RATE		752	351	215	145	97	61	36	22	15	11	8	7	6
90		792	391	255	185	136	96	66	51	44	41	40	40	40
	30	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		85	70	55	40	25	10	1	0	0	0	0	0	0
RATE		1152	551	349	245	176	113	56	27	16	11	9	7	6
90		1192	591	389	285	216	151	85	54	45	42	40	40	40
	40	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		85	70	55	40	25	10	0	0	0	0	0	0	0
RATE		1552	751	482	344	256	173	72	29	17	11	9	7	6
90		1592	791	522	385	296	211	100	55	45	42	40	40	40

NUMBER OF SERVERS

	LINES	1	2	3	4	5	6	7	8	9	10	11	12	13
		--	--	--	--	--	--	--	--	--	--	--	--	--
	20	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		83	66	50	33	17	5	1	0	0	0	0	0	0
RATE		676	314	192	126	79	44	25	16	11	8	7	5	5
100		712	351	228	162	113	73	50	41	38	36	36	36	36
	30	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		83	66	50	33	16	3	0	0	0	0	0	0	0
RATE		1036	495	312	216	145	74	32	17	11	8	7	5	5
100		1072	531	348	252	181	103	55	42	38	36	36	36	36
	40	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		83	66	50	33	16	2	0	0	0	0	0	0	0
RATE		1396	674	431	306	216	105	34	17	11	8	7	5	5
100		1432	711	467	342	252	133	57	42	38	36	36	36	36

QUEUEING MODEL

TRUE ARRIVAL RATE (LAMBDA) = 700 PER HOUR

VALUES IN EACH CELL ARE:
(1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

		NUMBER OF SERVERS												
		41	42	43	44	45	46	47	48	49	50	51	52	53
	LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
	60	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		12	10	8	7	6	5	4	3	2	2	1	1	1
RATE		79	69	60	53	46	40	35	30	27	23	20	18	15
15		315	303	292	283	274	267	260	255	251	248	246	244	242
	70	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		12	10	8	6	5	3	2	1	1	1	0	0	0
RATE		131	117	103	91	79	68	59	51	45	39	35	31	27
15		369	354	338	322	307	294	282	272	264	258	253	249	246
	80	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		12	10	8	6	4	3	2	1	0	0	0	0	0
RATE		187	169	151	133	115	98	83	70	59	51	44	38	33
15		426	408	388	366	345	323	304	288	275	265	258	252	249

		NUMBER OF SERVERS												
		30	31	32	33	34	35	36	37	38	39	40	41	42
	LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
	50	20	19	18	17	16	15	14	13	12	11	10	9	8
SERVICE		14	11	9	7	5	4	3	2	1	1	1	0	0
RATE		89	77	66	56	48	41	35	30	26	22	19	17	15
20		267	253	240	227	217	207	200	194	190	187	185	183	182
	60	30	29	28	27	26	25	24	23	22	21	20	19	18
SERVICE		14	11	8	6	4	3	1	1	0	0	0	0	0
RATE		145	128	111	95	80	66	55	46	39	33	29	25	22
20		324	306	287	267	249	232	217	206	198	192	188	185	183
	70	40	39	38	37	36	35	34	33	32	31	30	29	28
SERVICE		14	11	8	6	3	2	1	0	0	0	0	0	0
RATE		204	183	161	137	114	92	74	59	48	39	33	28	24
20		384	362	338	312	284	256	233	216	204	195	190	186	184

QUEUEING MODEL

TRUE ARRIVAL RATE (LAMBDA) = 700 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

		NUMBER OF SERVERS												
	LINES	18	19	20	21	22	23	24	25	26	27	28	29	30
		--	--	--	--	--	--	--	--	--	--	--	--	--
	40	22	21	20	19	18	17	16	15	14	13	12	11	10
SERVICE		22	18	14	10	7	4	3	1	1	0	0	0	0
RATE		124	106	89	74	60	48	39	32	26	22	19	16	14
30		244	225	207	189	172	158	146	137	131	127	124	122	121
	50	32	31	30	29	28	27	26	25	24	23	22	21	20
SERVICE		22	18	14	10	6	3	1	0	0	0	0	0	0
RATE		190	168	145	122	99	77	59	45	36	29	23	20	17
30		310	288	265	240	213	186	164	147	136	130	126	123	122
	60	42	41	40	39	38	37	36	35	34	33	32	31	30
SERVICE		22	18	14	10	6	3	1	0	0	0	0	0	0
RATE		257	231	204	175	142	107	77	55	40	31	25	21	17
30		377	351	324	293	257	216	180	154	139	131	126	123	122

		NUMBER OF SERVERS												
	LINES	12	13	14	15	16	17	18	19	20	21	22	23	24
		--	--	--	--	--	--	--	--	--	--	--	--	--
	30	18	17	16	15	14	13	12	11	10	9	8	7	6
SERVICE		31	25	20	14	10	6	4	2	1	1	0	0	0
RATE		118	98	80	63	50	39	30	24	19	16	13	11	9
40		208	188	168	150	134	120	110	102	97	94	92	91	90
	40	28	27	26	25	24	23	22	21	20	19	18	17	16
SERVICE		31	25	20	14	9	5	2	1	0	0	0	0	0
RATE		193	166	141	117	92	70	51	38	29	23	18	15	13
40		283	256	231	206	178	151	128	112	103	97	94	92	91
	50	38	37	36	35	34	33	32	31	30	29	28	27	26
SERVICE		31	25	20	14	8	4	1	0	0	0	0	0	0
RATE		268	236	205	174	140	103	70	47	33	25	19	16	13
40		358	326	295	264	228	185	146	119	105	98	94	92	91

QUEUEING MODEL

TRUE ARRIVAL RATE (LAMBDA) = 700 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

NUMBER OF SERVERS

	LINES	9	10	11	12	13	14	15	16	17	18	19	20	21
		--	--	--	--	--	--	--	--	--	--	--	--	--
	30	21	20	19	18	17	16	15	14	13	12	11	10	9
SERVICE		35	28	21	14	8	4	2	1	0	0	0	0	0
RATE		153	126	101	79	59	43	32	24	19	15	12	10	9
50		225	198	173	148	125	106	92	83	78	75	73	72	72
	40	31	30	29	28	27	26	25	24	23	22	21	20	19
SERVICE		35	28	21	14	7	3	0	0	0	0	0	0	0
RATE		233	198	165	134	100	69	45	31	22	17	14	11	10
50		305	270	237	205	168	131	103	87	79	76	74	73	72
	50	41	40	39	38	37	36	35	34	33	32	31	30	29
SERVICE		35	28	21	14	7	2	0	0	0	0	0	0	0
RATE		313	270	231	192	147	95	55	34	23	17	14	11	10
50		385	342	303	264	216	156	111	89	80	76	74	73	72

NUMBER OF SERVERS

	LINES	6	7	8	9	10	11	12	13	14	15	16	17	18
		--	--	--	--	--	--	--	--	--	--	--	--	--
	30	24	23	22	21	20	19	18	17	16	15	14	13	12
SERVICE		48	40	31	22	14	7	3	1	0	0	0	0	0
RATE		229	184	148	118	89	63	43	30	21	16	13	10	9
60		289	244	208	177	148	118	93	77	68	64	62	61	60
	40	34	33	32	31	30	29	28	27	26	25	24	23	22
SERVICE		48	40	31	22	14	6	1	0	0	0	0	0	0
RATE		329	270	223	184	145	103	63	37	24	17	13	11	9
60		389	330	283	244	205	158	111	82	70	64	62	61	60
	50	44	43	42	41	40	39	38	37	36	35	34	33	32
SERVICE		48	40	31	22	14	6	1	0	0	0	0	0	0
RATE		429	355	298	250	204	146	80	41	25	17	13	11	9
60		489	415	358	310	264	203	128	85	70	64	62	61	60

QUEUEING MODEL

TRUE ARRIVAL RATE (LAMBDA) = 700 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

		NUMBER OF SERVERS												
		5	6	7	8	9	10	11	12	13	14	15	16	17
	LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
	30	25	24	23	22	21	20	19	18	17	16	15	14	13
SERVICE		50	40	30	20	10	4	1	0	0	0	0	0	0
RATE		246	192	151	116	83	53	34	22	16	12	10	8	7
70		298	244	203	167	131	97	73	61	56	53	52	51	51
	40	35	34	33	32	31	30	29	28	27	26	25	24	23
SERVICE		50	40	30	20	10	2	0	0	0	0	0	0	0
RATE		349	278	225	180	132	79	42	24	17	12	10	8	7
70		401	330	276	231	182	122	79	62	56	53	52	51	51
	50	45	44	43	42	41	40	39	38	37	36	35	34	33
SERVICE		50	40	30	20	10	2	0	0	0	0	0	0	0
RATE		452	364	298	244	186	105	46	25	17	12	10	8	7
70		504	415	350	295	236	148	83	62	56	53	52	51	51

		NUMBER OF SERVERS												
		3	4	5	6	7	8	9	10	11	12	13	14	15
	LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
	30	27	26	25	24	23	22	21	20	19	18	17	16	15
SERVICE		65	54	42	31	20	9	2	0	0	0	0	0	0
RATE		397	283	213	163	123	83	49	29	18	13	10	8	7
80		442	328	258	208	167	125	85	61	52	48	46	45	45
	40	37	36	35	34	33	32	31	30	29	28	27	26	25
SERVICE		65	54	42	31	20	8	1	0	0	0	0	0	0
RATE		547	395	302	238	186	130	68	33	19	13	10	8	7
80		592	440	348	283	231	173	103	64	52	48	46	45	45
	50	47	46	45	44	43	42	41	40	39	38	37	36	35
SERVICE		65	54	42	31	20	8	1	0	0	0	0	0	0
RATE		697	508	392	313	250	181	85	35	19	13	10	8	7
80		742	553	438	358	295	225	119	65	52	48	46	45	45

Q U E U E I N G   M O D E L

TRUE ARRIVAL RATE (LAMBDA) = 700 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

		NUMBER OF SERVERS												
	LINES	2	3	4	5	6	7	8	9	10	11	12	13	14
	20	18	17	16	15	14	13	12	11	10	9	8	7	6
SERVICE		74	61	48	35	23	12	4	1	0	0	0	0	0
RATE		353	218	149	105	73	48	30	20	14	10	8	7	5
90		393	258	189	145	112	84	63	50	44	42	40	40	40
	30	28	27	26	25	24	23	22	21	20	19	18	17	16
SERVICE		74	61	48	35	22	10	2	0	0	0	0	0	0
RATE		553	351	249	185	137	92	51	28	17	12	9	7	6
90		593	391	289	225	177	130	82	56	46	42	41	40	40
	40	38	37	36	35	34	33	32	31	30	29	28	27	26
SERVICE		74	61	48	35	22	10	1	0	0	0	0	0	0
RATE		753	484	349	265	204	143	70	31	17	12	9	7	6
90		793	524	389	305	244	181	100	58	46	42	41	40	40

		NUMBER OF SERVERS												
	LINES	2	3	4	5	6	7	8	9	10	11	12	13	14
	20	18	17	16	15	14	13	12	11	10	9	8	7	6
SERVICE		71	57	42	28	15	5	1	0	0	0	0	0	0
RATE		316	195	132	90	58	35	22	15	10	8	6	5	4
100		352	231	168	126	92	65	49	41	38	37	36	36	36
	30	28	27	26	25	24	23	22	21	20	19	18	17	16
SERVICE		71	57	42	28	14	3	0	0	0	0	0	0	0
RATE		496	314	222	162	111	61	30	17	11	8	7	5	5
100		532	351	258	198	146	90	54	42	38	37	36	36	36
	40	38	37	36	35	34	33	32	31	30	29	28	27	26
SERVICE		71	57	42	28	14	2	0	0	0	0	0	0	0
RATE		676	434	311	234	169	87	33	17	11	8	7	5	5
100		712	471	348	270	204	116	57	42	38	37	36	36	36

Q U E U E I N G   M O D E L

TRUE ARRIVAL RATE (LAMBDA) = 800 PER HOUR   VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

NUMBER OF SERVERS

	LINES	48	49	50	51	52	53	54	55	56	57	58	59	60
		--	--	--	--	--	--	--	--	--	--	--	--	--
	70	22	21	20	19	18	17	16	15	14	13	12	11	10
SERVICE		10	8	7	6	4	4	3	2	2	1	1	1	1
RATE		77	68	60	53	46	41	36	32	28	25	22	20	18
15		312	302	292	283	275	268	262	257	253	249	247	245	243
	80	32	31	30	29	28	27	26	25	24	23	22	21	20
SERVICE		10	8	6	5	4	3	2	1	1	0	0	0	0
RATE		120	108	96	84	74	65	56	49	43	38	34	30	27
15		358	344	330	316	302	290	279	270	263	257	253	249	247
	90	42	41	40	39	38	37	36	35	34	33	32	31	30
SERVICE		10	8	6	4	3	2	1	1	0	0	0	0	0
RATE		167	151	135	119	103	89	76	65	55	48	41	36	32
15		406	389	371	352	332	314	297	283	272	263	257	252	248

NUMBER OF SERVERS

	LINES	35	36	37	38	39	40	41	42	43	44	45	46	47
		--	--	--	--	--	--	--	--	--	--	--	--	--
	60	25	24	23	22	21	20	19	18	17	16	15	14	13
SERVICE		12	10	8	6	4	3	2	1	1	0	0	0	0
RATE		97	85	74	64	55	47	40	35	30	26	23	20	18
20		275	261	248	235	224	214	205	199	193	189	187	184	183
	70	35	34	33	32	31	30	29	28	27	26	25	24	23
SERVICE		12	10	7	5	3	2	1	0	0	0	0	0	0
RATE		145	129	113	97	83	69	58	48	41	35	30	26	23
20		324	308	290	271	252	235	221	209	200	194	190	186	184
	80	45	44	43	42	41	40	39	38	37	36	35	34	33
SERVICE		12	10	7	5	3	2	1	0	0	0	0	0	0
RATE		195	177	156	135	112	92	74	59	48	40	33	28	25
20		375	356	334	309	283	257	235	217	205	196	191	187	185

QUEUEING MODEL

TRUE ARRIVAL RATE (LAMBDA) = 800 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

NUMBER OF SERVERS

	LINES	21	22	23	24	25	26	27	28	29	30	31	32	33
		--	--	--	--	--	--	--	--	--	--	--	--	--
	40	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		21	17	14	11	8	6	4	3	2	1	1	0	0
RATE		88	75	63	53	44	36	30	25	21	18	15	13	11
30		207	194	180	168	157	147	139	133	129	126	124	122	121
	50	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		21	17	13	10	7	4	2	1	0	0	0	0	0
RATE		144	127	110	93	77	63	50	41	33	27	23	19	17
30		264	247	229	211	192	174	158	146	137	131	127	124	122
	60	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		21	17	13	10	6	3	1	0	0	0	0	0	0
RATE		201	181	161	139	115	91	70	53	41	32	26	21	18
30		321	301	280	257	231	203	176	156	142	133	128	125	123

NUMBER OF SERVERS

	LINES	15	16	17	18	19	20	21	22	23	24	25	26	27
		--	--	--	--	--	--	--	--	--	--	--	--	--
	40	25	24	23	22	21	20	19	18	17	16	15	14	13
SERVICE		25	20	15	10	6	3	2	1	0	0	0	0	0
RATE		132	113	94	77	60	47	36	28	23	19	16	13	11
40		222	202	183	163	144	127	114	104	99	95	93	91	91
	50	35	34	33	32	31	30	29	28	27	26	25	24	23
SERVICE		25	20	15	10	5	2	1	0	0	0	0	0	0
RATE		192	168	145	120	94	69	50	36	27	21	17	14	12
40		282	258	235	209	179	149	125	110	101	96	93	92	91
	60	45	44	43	42	41	40	39	38	37	36	35	34	33
SERVICE		25	20	15	10	5	2	0	0	0	0	0	0	0
RATE		252	225	197	167	131	92	60	40	29	22	17	14	12
40		342	315	287	256	217	171	134	112	102	96	93	92	91

QUEUEING MODEL

TRUE ARRIVAL RATE (LAMBDA) = 800 PER HOUR

VALUES IN EACH CELL ARE:
(1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

NUMBER OF SERVERS

	LINES	11	12	13	14	15	16	17	18	19	20	21	22	23
		--	--	--	--	--	--	--	--	--	--	--	--	--
	30	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		31	25	18	13	8	5	2	1	0	0	0	0	0
RATE		110	90	72	57	44	33	26	20	16	13	11	9	8
50		182	162	143	126	110	97	87	81	77	75	73	73	72
	40	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		31	25	18	12	7	3	1	0	0	0	0	0	0
RATE		175	150	126	102	77	56	39	28	21	17	14	11	10
50		247	222	197	172	145	119	99	86	79	76	74	73	72
	50	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		31	25	18	12	6	2	0	0	0	0	0	0	0
RATE		240	210	181	150	115	78	50	32	23	17	14	11	10
50		312	282	252	222	184	141	107	89	80	76	74	73	72

NUMBER OF SERVERS

	LINES	8	9	10	11	12	13	14	15	16	17	18	19	20
		--	--	--	--	--	--	--	--	--	--	--	--	--
	30	22	21	20	19	18	17	16	15	14	13	12	11	10
SERVICE		40	32	25	17	10	5	2	1	0	0	0	0	0
RATE		153	126	102	80	60	44	32	23	18	14	11	9	8
60		213	186	162	139	117	97	82	72	66	63	61	61	60
	40	32	31	30	29	28	27	26	25	24	23	22	21	20
SERVICE		40	32	25	17	10	4	1	0	0	0	0	0	0
RATE		228	192	162	133	102	71	46	30	21	16	12	10	8
60		288	252	222	192	160	125	94	76	68	64	62	61	60
	50	42	41	40	39	38	37	36	35	34	33	32	31	30
SERVICE		40	32	25	17	10	3	0	0	0	0	0	0	0
RATE		303	259	222	187	148	100	57	33	22	16	12	10	8
60		363	319	282	247	207	154	104	78	68	64	62	61	60

Q U E U E I N G   M O D E L

TRUE ARRIVAL RATE (LAMBDA) = 800 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

		NUMBER OF SERVERS												
		6	7	8	9	10	11	12	13	14	15	16	17	18
	LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
	30	24	23	22	21	20	19	18	17	16	15	14	13	12
SERVICE		47	38	30	21	12	6	2	0	0	0	0	0	0
RATE		196	157	126	99	74	52	35	24	17	13	10	9	7
70		247	208	177	150	123	97	76	64	57	54	53	52	51
	40	34	33	32	31	30	29	28	27	26	25	24	23	22
SERVICE		47	38	30	21	12	5	1	0	0	0	0	0	0
RATE		281	230	190	156	121	82	48	29	19	14	11	9	7
70		333	282	242	207	171	128	89	67	58	54	53	52	51
	50	44	43	42	41	40	39	38	37	36	35	34	33	32
SERVICE		47	38	30	21	12	4	0	0	0	0	0	0	0
RATE		367	304	255	213	170	115	59	31	19	14	11	9	7
70		419	355	306	264	221	162	98	69	58	54	53	52	51

		NUMBER OF SERVERS												
		5	6	7	8	9	10	11	12	13	14	15	16	17
	LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
	30	25	24	23	22	21	20	19	18	17	16	15	14	13
SERVICE		50	40	30	20	10	4	1	0	0	0	0	0	0
RATE		216	168	132	102	72	47	29	19	14	11	8	7	6
80		261	213	177	146	115	84	64	53	49	46	45	45	45
	40	35	34	33	32	31	30	29	28	27	26	25	24	23
SERVICE		50	40	30	20	10	2	0	0	0	0	0	0	0
RATE		306	243	197	157	116	69	37	21	14	11	8	7	6
80		351	288	242	202	159	107	69	54	49	46	45	45	45
	50	45	44	43	42	41	40	39	38	37	36	35	34	33
SERVICE		50	40	30	20	10	2	0	0	0	0	0	0	0
RATE		396	318	261	213	162	92	41	22	14	11	8	7	6
80		441	363	306	258	207	129	72	55	49	46	45	45	45

Q U E U E I N G   M O D E L

TRUE ARRIVAL RATE (LAMBDA) = 800 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

		NUMBER OF SERVERS												
		3	4	5	6	7	8	9	10	11	12	13	14	15
	LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
	30	27	26	25	24	23	22	21	20	19	18	17	16	15
SERVICE		66	55	43	32	21	10	3	0	0	0	0	0	0
RATE		353	251	189	146	110	77	46	27	17	12	9	7	6
90		393	291	229	186	150	114	79	56	47	43	41	40	40
	40	37	36	35	34	33	32	31	30	29	28	27	26	25
SERVICE		66	55	43	32	21	10	2	0	0	0	0	0	0
RATE		486	351	269	212	167	120	66	32	18	12	9	7	6
90		526	391	309	252	207	159	98	60	47	43	41	40	40
	50	47	46	45	44	43	42	41	40	39	38	37	36	35
SERVICE		66	55	43	32	21	10	1	0	0	0	0	0	0
RATE		619	451	349	279	224	167	85	34	18	12	9	7	6
90		659	491	389	319	264	206	117	61	47	43	41	40	40

		NUMBER OF SERVERS												
		3	4	5	6	7	8	9	10	11	12	13	14	15
	LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
	30	27	26	25	24	23	22	21	20	19	18	17	16	15
SERVICE		62	50	37	25	12	3	0	0	0	0	0	0	0
RATE		316	225	168	126	88	51	28	17	11	8	7	5	5
100		352	261	204	162	122	81	53	42	38	37	36	36	36
	40	37	36	35	34	33	32	31	30	29	28	27	26	25
SERVICE		62	50	37	25	12	2	0	0	0	0	0	0	0
RATE		436	315	240	186	135	74	32	17	11	8	7	5	5
100		472	351	276	222	171	103	57	43	38	37	36	36	36
	50	47	46	45	44	43	42	41	40	39	38	37	36	35
SERVICE		62	50	37	25	12	2	0	0	0	0	0	0	0
RATE		556	405	311	246	185	96	34	17	11	8	7	5	5
100		592	441	348	282	221	125	58	43	38	37	36	36	36

Q U E U E I N G   M O D E L

TRUE ARRIVAL RATE (LAMBDA) = 900 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

		NUMBER OF SERVERS												
		55	56	57	58	59	60	61	62	63	64	65	66	67
	LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
	80	25	24	23	22	21	20	19	18	17	16	15	14	13
SERVICE		8	7	6	5	4	3	2	2	1	1	1	0	0
RATE		75	67	59	53	47	41	37	33	29	26	24	21	19
15		310	300	291	283	275	268	262	257	253	250	248	246	244
	90	35	34	33	32	31	30	29	28	27	26	25	24	23
SERVICE		8	7	5	4	3	2	1	1	0	0	0	0	0
RATE		112	101	90	80	70	61	54	47	42	37	33	30	27
15		349	336	323	310	298	287	277	269	262	257	252	249	247
	100	45	44	43	42	41	40	39	38	37	36	35	34	33
SERVICE		8	6	5	4	2	1	1	0	0	0	0	0	0
RATE		152	138	123	108	94	81	70	60	52	45	39	34	31
15		390	375	358	340	323	306	291	279	269	261	256	251	248

		NUMBER OF SERVERS												
		40	41	42	43	44	45	46	47	48	49	50	51	52
	LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
	60	20	19	18	17	16	15	14	13	12	11	10	9	8
SERVICE		11	9	7	6	5	4	3	2	2	1	1	1	1
RATE		63	55	48	42	36	31	27	24	21	18	16	14	13
20		239	230	221	213	206	200	195	191	188	186	184	183	182
	70	30	29	28	27	26	25	24	23	22	21	20	19	18
SERVICE		11	9	7	5	4	2	2	1	0	0	0	0	0
RATE		103	91	80	69	60	52	44	38	33	29	26	23	20
20		281	268	255	242	230	219	210	202	196	192	188	186	184
	80	40	39	38	37	36	35	34	33	32	31	30	29	28
SERVICE		11	9	6	5	3	2	1	0	0	0	0	0	0
RATE		145	130	115	100	85	71	60	50	42	36	31	27	23
20		324	309	292	274	255	238	223	211	202	195	191	187	185

Q U E U E I N G   M O D E L

TRUE ARRIVAL RATE (LAMBDA) = 900 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

NUMBER OF SERVERS

		25	26	27	28	29	30	31	32	33	34	35	36	37
	LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
	50	25	24	23	22	21	20	19	18	17	16	15	14	13
SERVICE		16	13	10	7	5	3	2	1	0	0	0	0	0
RATE		97	84	72	60	50	41	34	29	24	21	18	16	14
30		216	202	189	175	162	151	142	135	130	127	124	123	122
	60	35	34	33	32	31	30	29	28	27	26	25	24	23
SERVICE		16	13	10	7	4	2	1	0	0	0	0	0	0
RATE		144	128	111	94	77	61	49	39	31	26	22	18	16
30		264	247	229	210	190	170	154	143	135	129	126	123	122
	70	45	44	43	42	41	40	39	38	37	36	35	34	33
SERVICE		16	13	10	6	4	2	0	0	0	0	0	0	0
RATE		192	173	153	130	105	81	61	46	35	28	23	19	17
30		312	293	272	247	219	190	165	148	137	130	126	124	122

NUMBER OF SERVERS

		17	18	19	20	21	22	23	24	25	26	27	28	29
	LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
	40	23	22	21	20	19	18	17	16	15	14	13	12	11
SERVICE		24	20	15	11	8	5	3	1	1	0	0	0	0
RATE		105	90	76	63	51	41	33	26	21	18	15	13	11
40		195	180	165	150	136	123	113	105	99	96	93	92	91
	50	33	32	31	30	29	28	27	26	25	24	23	22	21
SERVICE		24	20	15	11	7	4	2	0	0	0	0	0	0
RATE		158	140	121	103	83	65	49	37	29	23	18	15	13
40		248	230	211	191	169	147	128	113	104	98	95	93	91
	60	43	42	41	40	39	38	37	36	35	34	33	32	31
SERVICE		24	20	15	11	6	3	1	0	0	0	0	0	0
RATE		211	190	168	145	119	90	64	45	32	24	19	16	13
40		301	280	258	235	206	173	141	119	106	99	95	93	91

Q U E U E I N G   M O D E L

TRUE ARRIVAL RATE (LAMBDA) = 900 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

		NUMBER OF SERVERS												
	LINES	13	14	15	16	17	18	19	20	21	22	23	24	25
		--	--	--	--	--	--	--	--	--	--	--	--	--
	40	27	26	25	24	23	22	21	20	19	18	17	16	15
SERVICE		27	22	16	11	6	3	1	0	0	0	0	0	0
RATE		135	115	97	78	61	45	34	26	20	16	13	11	9
50		207	187	168	148	128	109	95	85	79	76	74	73	72
	50	37	36	35	34	33	32	31	30	29	28	27	26	25
SERVICE		27	22	16	11	6	2	0	0	0	0	0	0	0
RATE		190	167	144	119	92	65	44	31	22	17	14	11	10
50		262	239	216	190	160	129	103	88	80	76	74	73	72
	60	47	46	45	44	43	42	41	40	39	38	37	36	35
SERVICE		27	22	16	11	5	2	0	0	0	0	0	0	0
RATE		245	218	192	163	127	85	53	33	23	17	14	11	10
50		317	290	264	234	196	148	110	90	81	76	74	73	72

		NUMBER OF SERVERS												
	LINES	10	11	12	13	14	15	16	17	18	19	20	21	22
		--	--	--	--	--	--	--	--	--	--	--	--	--
	30	20	19	18	17	16	15	14	13	12	11	10	9	8
SERVICE		33	26	20	13	8	4	2	1	0	0	0	0	0
RATE		108	88	71	56	42	31	24	18	14	12	10	8	7
60		168	148	130	113	97	84	74	68	64	62	61	60	60
	40	30	29	28	27	26	25	24	23	22	21	20	19	18
SERVICE		33	26	20	13	7	3	1	0	0	0	0	0	0
RATE		168	143	120	97	73	51	35	25	18	14	11	9	.8
60		228	203	180	156	130	103	84	72	66	63	61	61	60
	50	40	39	38	37	36	35	34	33	32	31	30	29	28
SERVICE		33	26	20	13	7	2	0	0	0	0	0	0	0
RATE		228	197	170	141	108	71	44	28	19	14	11	9	8
60		288	257	230	201	165	123	91	74	67	63	61	61	60

QUEUEING MODEL

TRUE ARRIVAL RATE (LAMBDA) = 900 PER HOUR

VALUES IN EACH CELL ARE:
(1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

NUMBER OF SERVERS

		7	8	9	10	11	12	13	14	15	16	17	18	19
	LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
	30	23	22	21	20	19	19	17	16	15	14	13	12	11
SERVICE		45	37	30	22	14	8	3	1	0	0	0	0	0
RATE		160	130	106	85	65	48	34	24	18	14	11	9	7
70		211	182	158	136	115	95	78	66	59	55	53	52	52
	40	33	32	31	30	29	28	27	26	25	24	23	22	21
SERVICE		45	37	30	22	14	7	2	0	0	0	0	0	0
RATE		233	195	163	136	109	80	52	33	22	15	12	9	8
70		285	246	215	187	160	128	95	73	61	56	53	52	52
	50	43	42	41	40	39	38	37	36	35	34	33	32	31
SERVICE		45	37	30	22	14	7	1	0	0	0	0	0	0
RATE		307	259	220	187	155	115	70	38	23	16	12	9	8
70		358	310	272	239	206	164	112	77	62	56	53	52	52

NUMBER OF SERVERS

		6	7	8	9	10	11	12	13	14	15	16	17	18
	LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
	30	24	23	22	21	20	19	18	17	16	15	14	13	12
SERVICE		46	37	28	20	11	5	1	0	0	0	0	0	0
RATE		171	137	109	85	63	43	29	20	14	11	9	7	6
80		216	182	154	130	106	82	65	55	49	47	46	45	45
	40	34	33	32	31	30	29	28	27	26	25	24	23	22
SERVICE		46	37	28	20	11	4	0	0	0	0	0	0	0
RATE		246	201	166	135	103	67	39	23	16	11	9	7	6
80		291	246	211	180	147	107	73	57	50	47	46	45	45
	50	44	43	42	41	40	39	38	37	36	35	34	33	32
SERVICE		46	37	28	20	11	3	0	0	0	0	0	0	0
RATE		321	265	222	185	145	93	46	25	16	11	9	7	6
80		366	310	267	230	190	133	79	58	50	47	46	45	45

QUEUEING MODEL

TRUE ARRIVAL RATE (LAMBDA) = 900 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

		NUMBER OF SERVERS												
		5	6	7	8	9	10	11	12	13	14	15	16	17
	LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
	30	25	24	23	22	21	20	19	18	17	16	15	14	13
SERVICE		50	40	30	20	10	4	1	0	0	0	0	0	0
RATE		192	150	118	90	64	41	26	17	12	9	7	6	5
90		232	190	158	130	102	75	57	47	43	41	40	40	40
	40	35	34	33	32	31	30	29	28	27	26	25	24	23
SERVICE		50	40	30	20	10	2	0	0	0	0	0	0	0
RATE		271	216	175	140	103	61	32	19	13	9	7	6	5
90		312	256	215	180	142	95	61	48	43	41	40	40	40
	50	45	44	43	42	41	40	39	38	37	36	35	34	33
SERVICE		50	40	30	20	10	2	0	0	0	0	0	0	0
RATE		351	283	232	190	144	81	36	19	13	9	7	6	5
90		392	323	272	230	184	115	64	48	43	41	40	40	40

		NUMBER OF SERVERS												
		4	5	6	7	8	9	10	11	12	13	14	15	16
	LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
	30	26	25	24	23	22	21	20	19	18	17	16	15	14
SERVICE		55	44	33	22	11	3	0	0	0	0	0	0	0
RATE		226	171	132	100	71	44	26	16	11	8	7	5	5
100		262	207	168	136	105	73	52	43	39	37	36	36	36
	40	36	35	34	33	32	31	30	29	28	27	26	25	24
SERVICE		55	44	33	22	11	2	0	0	0	0	0	0	0
RATE		316	242	192	151	111	64	31	17	11	8	7	5	.5
100		352	279	228	187	146	93	56	43	39	37	36	36	36
	50	46	45	44	43	42	41	40	39	38	37	36	35	34
SERVICE		55	44	33	22	11	2	0	0	0	0	0	0	0
RATE		406	314	252	203	154	83	33	17	11	8	7	5	5
100		442	351	288	239	190	113	58	43	39	37	36	36	36

Q U E U E I N G   M O D E L

TRUE ARRIVAL RATE (LAMBDA) = 1000 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

NUMBER OF SERVERS

		61	62	63	64	65	66	67	68	69	70	71	72	73
LINES		--	--	--	--	--	--	--	--	--	--	--	--	--
	80	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		9	8	7	6	5	4	3	3	2	2	2	2	1
RATE		49	44	39	35	31	27	24	22	19	17	15	13	11
15		284	277	271	266	261	257	253	250	248	246	244	243	242
	90	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		8	7	6	5	4	3	2	2	1	1	1	0	0
RATE		81	73	65	59	52	47	42	37	33	30	27	24	22
15		317	308	299	290	282	275	268	263	258	254	251	248	246
	100	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		8	7	5	4	3	2	2	1	1	0	0	0	0
RATE		116	105	95	85	75	67	59	52	46	41	36	32	29
15		354	342	330	318	305	294	284	275	267	261	256	252	249

NUMBER OF SERVERS

		45	46	47	48	49	50	51	52	53	54	55	56	57
LINES		--	--	--	--	--	--	--	--	--	--	--	--	--
	70	25	24	23	22	21	20	19	18	17	16	15	14	13
SERVICE		10	8	6	5	4	3	2	1	1	1	0	0	0
RATE		71	63	56	49	43	37	33	29	25	23	20	18	16
20		248	239	229	221	213	206	200	195	191	188	186	184	183
	80	35	34	33	32	31	30	29	28	27	26	25	24	23
SERVICE		10	8	6	4	3	2	1	1	0	0	0	0	0
RATE		107	96	85	74	64	55	48	41	36	31	27	24	21
20		286	273	260	247	234	223	213	205	198	193	190	187	185
	90	45	44	43	42	41	40	39	38	37	36	35	34	33
SERVICE		10	8	6	4	3	2	1	0	0	0	0	0	0
RATE		145	131	117	102	87	73	61	51	43	36	31	27	24
20		324	310	293	276	258	240	225	213	203	196	191	188	185

QUEUEING MODEL

TRUE ARRIVAL RATE (LAMBDA) = 1000 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

NUMBER OF SERVERS

	28	29	30	31	32	33	34	35	36	37	38	39	40
LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
50	22	21	20	19	18	17	16	15	14	13	12	11	10
SERVICE	16	13	10	8	6	4	3	2	1	1	0	0	0
RATE	73	64	55	46	39	33	28	24	20	18	15	13	12
30	192	182	171	161	152	144	138	133	129	126	124	123	122
60	32	31	30	29	28	27	26	25	24	23	22	21	20
SERVICE	16	13	10	7	5	3	2	1	0	0	0	0	0
RATE	115	102	89	76	64	53	43	36	30	25	21	18	16
30	234	221	207	192	177	163	151	142	135	130	126	124	123
70	42	41	40	39	38	37	36	35	34	33	32	31	30
SERVICE	16	13	10	7	4	2	1	0	0	0	0	0	0
RATE	157	142	126	109	90	73	57	45	36	29	24	20	17
30	277	262	245	226	205	183	164	149	139	132	127	125	123

NUMBER OF SERVERS

	20	21	22	23	24	25	26	27	28	29	30	31	32
LINES	--	--	--	--	--	--	--	--	--	--	--	--	--
40	20	19	18	17	16	15	14	13	12	11	10	9	8
SERVICE	20	16	12	9	6	4	3	2	1	0	0	0	0
RATE	73	62	51	42	35	28	23	19	16	14	12	10	9
40	162	150	139	128	118	110	104	99	96	94	92	91	91
50	30	29	28	27	26	25	24	23	22	21	20	19	18
SERVICE	20	16	12	8	5	3	1	0	0	0	0	0	0
RATE	117	102	87	73	59	46	36	29	23	19	16	14	12
40	207	192	176	160	143	127	115	106	100	96	94	92	91
60	40	39	38	37	36	35	34	33	32	31	30	29	28
SERVICE	20	16	12	8	4	2	0	0	0	0	0	0	0
RATE	162	144	126	106	85	64	47	35	27	21	17	14	12
40	252	234	216	194	170	145	124	110	102	97	94	92	91

QUEUEING MODEL

TRUE ARRIVAL RATE (LAMBDA) = 1000 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

	LINES	NUMBER OF SERVERS 15	16	17	18	19	20	21	22	23	24	25	26	27
		--	--	--	--	--	--	--	--	--	--	--	--	--
	40	25	24	23	22	21	20	19	18	17	16	15	14	13
SERVICE		25	20	15	10	6	3	2	1	0	0	0	0	0
RATE		105	90	75	61	48	37	29	23	18	15	12	10	9
50		177	162	146	131	115	101	91	83	79	76	74	73	72
	50	35	34	33	32	31	30	29	28	27	26	25	24	23
SERVICE		25	20	15	10	5	2	1	0	0	0	0	0	0
RATE		153	135	116	96	75	55	40	29	22	17	14	11	10
50		225	207	188	167	143	119	100	88	81	77	74	73	72
	60	45	44	43	42	41	40	39	38	37	36	35	34	33
SERVICE		25	20	15	10	5	2	0	0	0	0	0	0	0
RATE		201	180	158	134	104	73	48	32	23	17	14	11	10
50		273	252	230	205	173	137	107	90	81	77	75	73	72

	LINES	NUMBER OF SERVERS 11	12	13	14	15	16	17	18	19	20	21	22	23
		--	--	--	--	--	--	--	--	--	--	--	--	--
	30	19	18	17	16	15	14	13	12	11	10	9	8	7
SERVICE		34	28	22	16	11	7	4	2	1	0	0	0	0
RATE		93	77	63	50	39	30	23	18	14	12	10	8	7
60		153	137	122	108	96	85	76	70	66	63	62	61	60
	40	29	28	27	26	25	24	23	22	21	20	19	18	17
SERVICE		34	28	22	16	10	5	2	1	0	0	0	0	0
RATE		147	127	108	90	71	54	39	28	21	16	13	10	9
60		207	187	168	149	129	109	90	77	69	65	63	61	60
	50	39	38	37	36	35	34	33	32	31	30	29	28	27
SERVICE		34	28	22	16	10	4	1	0	0	0	0	0	0
RATE		202	177	154	132	107	79	53	35	23	17	13	11	9
60		262	237	214	191	166	135	104	82	71	65	63	61	60

Q U E U E I N G   M O D E L

TRUE ARRIVAL RATE (LAMBDA) = 1000 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

NUMBER OF SERVERS

	LINES	9	10	11	12	13	14	15	16	17	18	19	20	21
		--	--	--	--	--	--	--	--	--	--	--	--	--
	30	21	20	19	18	17	16	15	14	13	12	11	10	9
SERVICE		37	30	23	16	10	5	2	1	0	0	0	0	0
RATE		110	90	73	58	44	32	24	18	14	11	9	8	6
70		161	142	124	108	92	78	67	60	56	54	53	52	51
	40	31	30	29	28	27	26	25	24	23	22	21	20	19
SERVICE		37	30	23	16	9	4	1	0	0	0	0	0	0
RATE		167	142	120	98	75	53	36	24	17	13	10	8	7
70		218	193	171	149	125	99	77	64	58	54	53	52	51
	50	41	40	39	38	37	36	35	34	33	32	31	30	29
SERVICE		37	30	23	16	9	3	0	0	0	0	0	0	0
RATE		224	193	166	140	110	75	45	27	18	13	10	8	7
70		275	245	218	191	161	121	86	66	58	55	53	52	51

NUMBER OF SERVERS

	LINES	7	8	9	10	11	12	13	14	15	16	17	18	19
		--	--	--	--	--	--	--	--	--	--	--	--	--
	30	23	22	21	20	19	18	17	16	15	14	13	12	11
SERVICE		44	36	28	20	12	6	2	1	0	0	0	0	0
RATE		139	113	92	73	55	39	27	19	14	11	9	7	6
80		184	158	137	117	98	79	65	55	50	47	46	45	45
	40	33	32	31	30	29	28	27	26	25	24	23	22	21
SERVICE		44	36	28	20	12	5	1	0	0	0	0	0	0
RATE		203	170	142	117	91	64	40	25	17	12	9	8	6
80		248	215	187	162	135	104	76	59	51	48	46	45	45
	50	43	42	41	40	39	38	37	36	35	34	33	32	31
SERVICE		44	36	28	20	12	4	0	0	0	0	0	0	0
RATE		268	226	192	162	130	90	50	28	17	12	9	8	6
80		313	271	237	207	175	132	85	61	52	48	46	45	45

Q U E U E I N G   M O D E L

TRUE ARRIVAL RATE (LAMBDA) = 1000 PER HOUR

VALUES IN EACH CELL ARE: (1) NUMBER OF HOLD POSITIONS
(2) PERCENTAGE OF LOST CALLS
(3) AVERAGE QUEUEING TIME (SECONDS)
(4) AVERAGE TIME IN SYSTEM (SECONDS)

	LINES	NUMBER OF SERVERS 6	7	8	9	10	11	12	13	14	15	16	17	18
		--	--	--	--	--	--	--	--	--	--	--	--	--
	30	24	23	22	21	20	19	18	17	16	15	14	13	12
SERVICE		46	37	28	19	10	4	1	0	0	0	0	0	0
RATE		152	121	97	75	55	37	24	17	12	9	7	6	5
90		192	161	137	115	92	71	56	48	43	41	40	40	40
	40	34	33	32	31	30	29	28	27	26	25	24	23	22
SERVICE		46	37	28	19	10	3	0	0	0	0	0	0	0
RATE		218	178	147	119	89	57	32	19	13	10	8	6	5
90		258	218	187	158	128	91	62	49	44	42	41	40	40
	50	44	43	42	41	40	39	38	37	36	35	34	33	32
SERVICE		46	37	28	19	10	2	0	0	0	0	0	0	0
RATE		285	235	197	163	126	77	37	20	13	10	8	6	5
90		325	275	237	203	165	111	66	50	44	42	41	40	40

	LINES	NUMBER OF SERVERS 5	6	7	8	9	10	11	12	13	14	15	16	17
		--	--	--	--	--	--	--	--	--	--	--	--	--
	30	25	24	23	22	21	20	19	18	17	16	15	14	13
SERVICE		50	40	30	20	10	4	1	0	0	0	0	0	0
RATE		172	135	106	81	58	37	23	15	11	8	7	5	5
100		208	171	142	117	92	67	51	42	39	37	36	36	36
	40	35	34	33	32	31	30	29	28	27	26	25	24	23
SERVICE		50	40	30	20	10	2	0	0	0	0	0	0	0
RATE		244	195	157	126	92	55	29	17	11	8	7	5	5
100		280	231	193	162	127	85	55	43	39	37	36	36	36
	50	45	44	43	42	41	40	39	38	37	36	35	34	33
SERVICE		50	40	30	20	10	2	0	0	0	0	0	0	0
RATE		316	254	209	171	130	73	32	17	11	8	7	5	5
100		352	291	245	207	165	103	58	44	39	37	36	36	36

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16. ABSTRACT (*A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.*)

This report discusses the costs and benefits associated with automating the route-finding portion of a telephone transit information system. The various costs of implementing such a system are categorized and compared with those of a manual system over an appropriate time span using a present value approach. A queuing model, described in the report, is used for computing manpower requirements of the two systems, manual and automated. Outputs of the queuing model for a wide range of input parameters are tabulated in an appendix. Benefits from automating transit information route-finding are discussed, and measures of performance improvement available as output from the queuing model are provided.

17. KEY WORDS (*six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons*)

Automation; cost/benefit; models; queuing; telephone systems; transit information; transportation.

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