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Equipment Maintenance for Energy Conservation

Joel Levy

Building Economics Section
Center for Building Technology
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National Bureau of Standards
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Final Report

U. S. DEPARTMENT OF COMMERCE
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PREFACE

"Example of Equipment Maintenance Model", Chapter 6 of this report, is the work of James Kao, an engineer in the Building Environment Division of CBT. Appreciation is also extended to other members of the NBS staff who helped with advice and encouragement while this work was in progress. This acknowledgement is addressed in particular to the members of the CBT staff who in addition reviewed the paper: Dr. Harold E. Marshall, Chief, Building Economics Section, and Robert E. Chapman, Stephen R. Petersen, and Dr. Stephen F. Weber, economists in the Building Economics Section. We thank Mr. Reid Hartsell of the Federal Energy Administration for helpful conversations. Thanks are also expressed to Debbie Dorman who typed the report.

ABSTRACT

A general model of equipment performance as a function of maintenance is developed that permits quantification of the optimal level of maintenance in terms of performance attainment and relative factor costs. The model formulation is that of a finite state, finite action Markovian decision process. The report supplies a listing for a program in BASIC of the policy improvement algorithm for finding a best policy. The model will help maintenance engineers, building managers and others responsible for making decisions concerning maintenance policies in selecting economically efficient levels of maintenance for elements of building service equipment. The report also contains an illustrative example applying the model to the maintenance of an air handling unit.

Key words: Dynamic programming; Economic analysis; Energy conservation; Equipment maintenance; Markov decision process; Policy improvement algorithm.

EXECUTIVE SUMMARY

Energy use in the United States relative to the availability of energy resources has reached such proportions that it is regarded as cause for national concern. An implication of the energy shortage is that the cost of energy resources relative to the cost of equipment maintenance has risen. Better maintained equipment will use less energy per unit output. Under these conditions it is commonly profitable to increase the level of maintenance of equipment used to deliver building services above the level used at the historical, lower cost of energy resources.

The primary purpose of this report is to provide households and firms with a means to reduce the operation and maintenance cost of their energy using equipment. A further purpose is to show how to analyze the energy conservation effect of the cost minimizing policies derived.

Economic evaluation and comparison of alternative maintenance policies requires consideration of factors beyond the immediate impact of individual maintenance actions like machine cleaning and lubrication or part replacement for greater efficiency. A complete costing of any policy requires that the consequences of such maintenance actions for the long term future performance of the equipment be taken into account.

To make an adequate comparison of the economic performance of different maintenance policies, technological and cost data are needed. It is also necessary to have a method for analyzing and evaluating the implications of these data for the present values of alternative maintenance policies.

In many cases future energy consumption by various units of equipment and the results of maintenance actions on their energy utilization are not known with certainty. The report presents a method for decision making to be applied in a stochastic environment in which only the probability distribution of these values are known.

The perspective of the firm or household faced with maintenance decisions is considered. The report derives policies that will minimize costs that these units can be expected to take into account. It is thus principally the perceived costs that firms and households have to pay for energy consumed and for equipment maintenance for which the report supplies an analysis.

The report contains listings of computer programs that will enable the person responsible for formulating maintenance policies to select a policy that will minimize the expected present value of future costs. This is the incentive that is expected to motivate him (or her) to implement an optimal policy.

There is no attempt in this report to assess possible inducements for energy conservation not inherent in the price mechanism. The report does, however, include a discussion of how to estimate the energy conservation effect at the micro-economic level (the firm or household) of an economically responsive maintenance policy by such a unit.

The method presented for deriving optimal equipment maintenance policies can be implemented with current computer hardware and software. An illustrative example is worked out within the report. The analysis and programs are developed for conditions in which it is assumed that relative prices will remain unchanged. However, the report also shows how to modify, for a case in which energy prices are expected to be increasing, the methods and computer programs supplied.

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1. INTRODUCTION

1.1 Background

The cost of energy resources relative to the cost of equipment maintenance has risen. Better maintained equipment will use less energy resources per unit output. Under these conditions it is sometimes profitable to increase the level of maintenance of equipment used to deliver building services above the level at the historical, lower cost of energy resources.

Economic evaluation and comparison of alternative maintenance policies requires consideration of factors additional to the immediate impact of actions like machine cleaning and lubrication or part replacement for greater efficiency. A complete costing of any policy requires that the consequences of such maintenance actions for the long term future performance of the equipment be taken into account.

To make an adequate comparison of the economic performance of different maintenance policies, we need technological and cost data as well as a method for analyzing and evaluating the implications of these data for the present values of the costs of alternative maintenance policies.

1.2 Purpose

The general purpose of this paper is to provide firms and households with a means for making more effective use of energy and maintenance resources. The steps used in the report to achieve this purpose are: (1) to explain and illustrate with an example a method for modeling the performance of building service equipment as a function of maintenance policies, (2) to demonstrate a format in which the technological and cost data concerning equipment performance and maintenance actions can be filed within a computer for use in deriving and evaluating maintenance policies, (3) to explain how to evaluate the expected present value of future costs, (4) to explain a method for finding a maintenance policy that minimizes the present value of future costs, and (5) to present the listing of a computer program in BASIC to achieve the optimization.

The paper is intended primarily for planners, decision makers, and researchers in the area of equipment maintenance. Based on their knowledge of the equipment to be serviced they can formulate its description and use the analysis and programs presented in the report to evaluate alternative policies and/or to find a policy that will minimize the present value of expected future costs.

1.3 Scope and Organization

The assumptions concerning the structural properties of the system being modeled are explained in Chapter 2. Basic technological and economic characteristics of Markov chain processes and some implications of these characteristics are discussed.

Chapter 3 describes stationary policies and shows how to find the expected present values of their costs. In Chapter 4 an optimization procedure for Markovian decision processes is explained. In Chapter 5 the effect of different fuel prices on costs and consequently the relative merits of different maintenance policies is described in terms of comparative statics. In this section it is also shown how to apply the policy evaluation routine and optimization method to the case of rising rather than constant energy prices.

Chapter 6 (by James Kao) is an application of the model of the previous sections to the maintenance problem for an air handling unit. Chapter 7 summarizes the paper briefly. It also suggests areas in the economics of equipment maintenance for energy conservation which require further research.

2. FINITE STATE-ACTION MARKOVIAN DECISION MODELS

This chapter gives some information on the model that will be used to enable firms and households to make more efficient maintenance decisions. First it describes the nature of the finite state stochastic processes under consideration. In particular a stationarity assumption for the processes is formulated. Then the sequential character of the decision making procedure that can be used is discussed. Stationary policies are defined. The section closes with the remark that future costs are discounted to get their present value. Thus different sequences of costs can be compared and ranked.

2.1 Markov property

By a system we mean any set of pieces of equipment and the technology governing the behavior of the equipment over time. Specific examples of systems are a heating system or an air conditioning system. A state is a possible condition of the system and a maintenance controller is someone who decides which of the alternative maintenance actions should be taken concerning the system. While more general models in roughly the same framework are possible, for simplicity we restrict consideration to a system that is observed at equally spaced time intervals. Upon examination the system is found to be in one of a given finite number of states. After observation the maintenance controller takes one of a finite number of specified actions.

Let Si, i=1, . . . D denote the possible states of the system and A^i_j, j=1, ... Ai denote the various actions that can be taken in state i. When the system is in state Si and action A^i_j is taken then two things occur as a result: (1) a cost whose expected value is R(Si, A^i_j)

is incurred, and (2) at the time the system is next observed it will be in each of the states, Sk, with probability Pi k (A_j^i).

Note that the cost immediately incurred, R(Si, A_j^i), depends only on the current state-action pair. In particular, both the costs incurred and the transition probabilities associated with each state-action pair do not depend on the calendar date at which the event occurrs, except as this is incorporated in the state description. Also they do not depend on the history of the system prior to the present, except as this history resulted in the system being in the current state.

The first of the above properties is called stationarity. The second is the Markovian property. Both are significant for developing decision procedures for such systems.

2.2 State-Action Pairs

This section presents additional details on the state-action description of the equipment maintenance model. The computer listing for a file of data describing a hypothetical piece of equipment is given. Stationary policies are then defined and a computer output illustrating policy specification is presented.

2.2.1 Description of State-Action Pairs

As indicated above in the Markov model of equipment maintenance the following are specified:

(a) a set of possible states of the system,

(b) for each state a set of possible maintenance actions, including possibly the instruction "do nothing",

(c) for each allowable state-action pair, the expected cost that will be incurred over the coming time interval,

(d) for each current state-action pair, the probability that the system will be in each of the possible states of the system at the start of the immediately following period.

Table 1 is a computer listing of the data specified for a hypothetical piece of equipment with nine possible states. For reference the hypothetical equipment will be called MAC9A.

Table 1. Listing for MAC9A

```
100  9             16
110  .12  .11  .11  .11  .11  .11  .11  .11  .11  10  1  1
1r0  .04  .12  .12  .12  .12  .12  .12  .12  .12  12  2  1
130  .12  .11  .11  .11  .11  .11  .11  .11  .11  20  2  2
140  0  0  .15  .15  .15  .15  .15  .15  .1  14  3  1
150  .12  .11  .11  .11  .11  .11  .11  .11  .11  20  3  2
160  0  0  0  .17  .17  .17  .17  .17  .15  16  4  1
170  .12  .11  .11  .11  .11  .11  .11  .11  .11  20  4  2
180  0  0  0  0  .2  .2  .2  .2  .2  18  5  1
190  .12  .11  .11  .11  .11  .11  .11  .11  .11  20  5  2
200  0  0  0  0  0  .25  .25  .25  .25  20  6  1
210  .12  .11  .11  .11  .11  .11  .11  .11  .11  20  6  2
220  0  0  0  0  0  0  .34  .33  .33  22  7  1
230  .12  .11  .11  .11  .11  .11  .11  .11  .11  20  7  2
240  0  0  0  0  0  0  0  .5  .5  24  8  1
250  .12  .11  .11  .11  .11  .11  .11  .11  .11  20  8  2
260  .12  .11  .11  .11  .11  .11  .11  .11  .11  25  9  1
```

The interpretation of the listing in Table 1 is as follows:

The number of possible states of the equipment is given by the first integer in line 100 while the second gives the number of state-action combinations possible.

(a) the next to last entry in each row after line 100 specifies by an integer the state of the system in accordance with a nomenclature conventionally specified for the equipment and the program,

(b) the last entry in each such row specifies by an integer a possible maintenance action when the system is in the state indicated according to (a): a given integer may have different interpretations when indicating actions in different states,

(c) the entry third from the last in each of these rows gives the expected cost in dollars over the next time interval for the state-action pair identified in accordance with (a) and (b), and

(d) the nine first entries in each row specify the probability that the system at the immediately following period will be in each of its nine possible states (entry one: probability that the system will be the state with label one, entry two: probability that the system will be in the state with label two...) given the current state-action pair identified by the last two entries in the row.

If, as in the notation of section 2.1, we denote the number of actions possible in state Si by Ai and the number of possible states by D, then the number of state-action combinations, i.e. the second number in line 100 of the above file, is $\sum_{i=1}^{D} Ai$.

2.2.2 Stationary Policies

A policy for a Markov decision process is a rule specifying for each state of the system the action that will be taken in that state. In general, the rule for selecting the action may depend on a number of factors such as the past history of the system, or calendar time. Sometimes the rule is in the form of specifying a random selection from several of the actions available. By a stationary policy is meant a rule for selecting the action to be taken in each of the states that does not depend on any other factor than the state of the system. A non-random stationary policy is specified when for each possible state of the system one of the possible actions associated with that state is selected as the action to be taken.

Consideration in this report will be restricted to non-random stationary policies. It has been shown elsewhere, that for the Markov programming model with a finite number of states and actions when costs and transition probabilities do not vary with time, the class of non-random stationary policies contains one that is optimal over the set of all possible policies.[1] If we find the best stationary policy we will thus have an optimal policy. Policy evaluation is effected by summing the discounted expected value of future costs.

In Table 2 we present a sample of the printout of two policies and the results of the policy evaluations. The information will be the vector of a policy and the values of the expected future costs of the policy discounted to the present. The index set of the vectors is the set of different states of the system. The evaluation is given for each state considered as an initial state.

The printout takes the form V(i, j) = C, where i takes on the integer values from 1 thru D, indexing the possible states of the system. The pair (i, j) indicates that in state i action j for that state will be taken. (Recall that a given integer may have different interpretations when indicating actions in different states.) C at V(i, j) is the expected value of discounted future cost when the system starts in state i and the policy (k, j(k)), k = 1, . . . D is used. The sample computer outputs given below refer to the nine-state piece of equipment of which the computer description is given in subsection 2.2.1. The outputs yield evaluations of two different policies applied to the maintenance of this equipment with a discount factor of .97. This is equivalent to a rate of interest per period of about 3.1%. But note that we have not specified the length of time that constitutes one period.

Policy A:
```
V( 1 , 1 )= 697.299
V( 2 , 1 )= 700.291
V( 3 , 1 )= 704.106
V( 4 , 1 )= 707.347
V( 5 , 1 )= 710.382
V( 6 , 1 )= 713.21
V( 7 , 1 )= 715.66
V( 8 , 1 )= 717.407
V( 9 , 1 )= 712.299
```

[1]See, for example, David Blackwell, "Discrete Dynamic Programming", Annals of Mathematical Statistics, Vol. 33, (1962), pp. 719-726.

Policy B:

V(1 , 1)=	588.983
V(2 , 1)=	591.68
V(3 , 1)=	594.652
V(4 , 1)=	597.496
V(5 , 2)=	598.983
V(6 , 2)=	598.983
V(7 , 2)=	598.983
V(8 , 2)=	598.983
V(9 , 1)=	603.983

Table 2. Policy Values

It can be seen that for each initial state discounted expected costs are smaller for Policy B than for Policy A.

Expressed in the same notation as in the previous paragraph, the total number of nonrandom stationary policies is $\prod_{i=1}^{D} Ai$. A method for selecting a policy to minimize expected present value of future costs will be described in chapter 4 "Finding an Optimal Policy."

2.3 Discounting future costs

If the rate of interest is i per unit time then future expenditures and receipts are converted to their present value by being multiplied by $(\frac{1}{1 + i})^m$, where m is the number of units of time in the future at which the expenditures or receipts occur. The number $\frac{1}{1 + i}$ is called the discount factor. It will be denoted A1. When discounted any bounded sequence of costs sums to a finite present value. The sum obtained after discounting offers a basis for comparing different time patterns of costs, and it is future costs summed in this manner that will be used to compare the performance of alternative maintenance policies.

3. FINDING COSTS OF STATIONARY POLICY

Using the formulation of the equipment maintenance problem as a Markov decision process, this section shows how to evaluate a given stationary policy. The process structure and available data are those described in the previous chapter. Expected present value of future costs is the evaluation criterion. For expository clarity we include a statement of the one state case as a basis for comparison. The chapter includes a program listing for computing the sum of expected discounted costs when the process generating the costs is a Markov chain.

3.1 Present Value of cost with one state

To elucidate the evaluation formula for a system in which there are several states we first express the formula for the present value of costs in the case in which there is only one possible state for the system.

The one state situation might be the case of a unit that is disposable and lasts only one period. The choice is whether to get for its one period use a more efficient, more expensive item or one that is cheaper but less energy efficient.

If we denote the discount factor by A1 (not to be confused with the use of the symbol for an action available in state S1), and if R denotes the cost that we expect to incur in each period, then the sum of expected discounted future costs is: $\sum_{n=o}^{\infty} (A1)^n R = (1-A1)^{-1} R.$

3.2 Present value of costs with several states

The formula above for present value of discounted costs as the sum of a convergent geometric progression generalizes to the case of a system governed by a stationary probability transition matrix. In the first two paragraphs we recall and focus part of the development of the previous section. The final paragraph gives the matrix formula analogous to that for a scalar given above and a program listing to effect the computation.

3.2.1 Single period costs

We are now dealing with a Markov decision process for which a policy has been specified. The development of the system is thus described by a Markov chain and, given the state of the system at any time, the expected costs that will be incurred at that time are also specified. The expected cost for a single period as a function of the state of the system can be conveniently represented as a (column) vector. Denote this vector by $\vec{R}$.

3.2.2 Transition probability matrix

Once a policy has been specified we can read off from the data for the system described in section 2.2.1 the transition matrix characterizing the Markov chain under the policy. Denote this matrix by M. The conditional probability that the system will be in state j in time period n given that at time o the system is in state i is the (i,j)-entry in the matrix M^n. The expected costs incurred at time n is by definition the sum of the products of the probability of being in state j at time n times the expected costs incurred in state j, where the sum is taken over all states. This cost, for each assumed initial state of the system, can be conveniently expressed in matrix notation:

$$M^n \cdot \vec{R}.$$

3.2.3 Formula for present value of costs

If we again use A1 to denote the discount factor it follows from the preceding paragraph that the expected present value of costs incurred over the future is for each given initial state of the system:

$$\vec{V} = \sum_{n=o}^{\infty} (A1)^n \cdot M^n \cdot \vec{R} = \left(\sum_{n=o}^{\infty} (A1)^n \cdot M^n\right) \vec{R}.$$

$(A1)^n M^n$ tends to 0 (the zero matrix) as n increases. So just as with a geometric progression of numbers, $\sum_{n=o}^{\infty} (A1)^n M^n = (I - A1 \cdot M)^{-1}$, and $\vec{V} = (I - A1 \cdot M)^{-1} \cdot \vec{R}$.

In Table 3 we give the listing for a subroutine in BASIC to implement the calculation described above. All matrices and vectors are dimensioned prior to calling the subroutine.

Parameter values of the following are also specified in the calling program:

(1) M is the Markov transition matrix associated with the specified policy,

(2) R is a column vector the entries of which are the expected costs over the next time interval associated with the specified policy,

(3) I is the identity matrix, and

(4) A1 is a scalar, the discount factor being used in determining the present value of future costs.

The entries in the matrices T, U, and S and the entries in the vector V are determined in the subroutine. T and U are intermediate storage locations. S is in effect $\sum_{n=o}^{\infty} (A1)^n M^n$ so that V is in the expected present value of future costs under the given policy discounted by the factor A1.

Table 3. Listing for Policy Evaluation Subroutine

```
50 MAT T=(A1)*M
60 MAT U=I-T
70 MAT S=INV(U)
80 MAT V=S*R
99 RETURN
100 END
```

4. FINDING AN OPTIMAL POLICY

This chapter describes the policy improvement iteration method for optimizing Markov decision processes. For expository clarity the case of one state is first explained in detail. The case of several states is then treated. A program listing for effecting the optimization is included.

4.1 One State and Several Actions

As a basis for describing policy improvement iteration for Markov decision processes, this section describes the way the optimization procedure would work for a one state situation. This procedure, which is a little stilted for the one state case, is then generalized to apply to any finite state-action case.

Reference is made to the discussion in section 3.1 above of policy evaluation. Since cost in a single period is a function of the action taken, we denote this cost as R (ai), with ai to indicate the action taken. The value of this policy, i.e. the sum of expected future discounted costs, is:

$$V(ai) = (1 - A1)^{-1} R(ai).$$

In considering an alternative policy, aj, compare the value of the old policy with R(aj), the one period cost of action aj, plus the value of the old policy discounted one period. In other words, use the following test quantity to make a decision between the old and the new policy:

$$T \equiv R\ (aj) + (A1) \cdot V\ (ai) - V\ (ai).$$

If $T < 0$, then policy aj has a smaller expected cost than policy ai. If $T \geq 0$, then policy aj does not have smaller expected cost than policy ai.

4.2 System with Several States and Several Actions

Since we are dealing with a finite state-action system, one could in principle evaluate all stationary nonrandom policies and select a best one. However, the number of such policies is $\prod_{i=1}^{D} Ai$. Thus, for example, if there are 10 states and 2 actions in each state the number of those policies is more than a thousand. We shall show how the optimization procedure described in section 4.1 generalizes to a system with several states.

Reference is made to chapters 2 and 3 for a discussion of stationary policies and their evaluation. Assume that a policy has been specified and evaluated. We now wish to see whether a better policy is available. A vector with components V(Si) gives the value of the specified policy. In considering whether or not there exists better alternative policies it is only necessary to examine the effect of changing the action in one state at a time. It was noted by Ron Howard[1] that if for each state no improvement in the value of the system can be obtained by changing the action in only that state then the policy being considered is optimal. Otherwise the new policy obtained by introducing the improving action in the one state and keeping the actions in the other states the same will result in a better policy. Furthermore, it is not necessary to evaluate the new policy to see whether the change in action in the one state will yield an improvement or not. Use of a test quantity that we will describe is sufficient.

The cost in a single period is a function of the state of the system and the action taken. Denote this cost as $R(Si, a_j^i)$. If a policy $\{a_j^i\}_{i=1}^{D}$ has been specified and we wish to examine whether introducing action a_k^i in place of action a_j^i for state si would lead to a policy with greater value, i.e. smaller cost, it is sufficient to examine the following test quantity:

---

[1] See Howard, Ronald A., Dynamic Programming and Markov Processes, (Technology Press, 1960).

$$E \equiv R(Si,\ a_k^i) + (A1) \cdot \sum_{j=1}^{D} Pij(a_k^i) \cdot V(Sj) - V(Si).$$

The terms A1, $R(Si,\ a_k^i)$, $V(Si)$ have already been defined. The symbol $Pij(a_k^i)$ denotes the probability that if the system is in state Si and action a_k^i is taken then the system will be in state sj at the start of the next period. If $E1 < 0$ then the policy with action a_k^i rather than a_j^i taken in state Si has smaller expected cost than the original one. If $E1 \geq 0$ then the new policy does not have smaller expected cost than the original one.

Using data described and formatted in subsection 2.2.1, the subroutine in Table 4 can be used to effect optimization based on the above considerations.

```
40 DIM Q(1,20),E(1,1)
50 C=0
60 FOR S1=1 TO D
110 MAT Q=ZER(1,D)
120 FOR K=1 TO L STEP 1
130 IF F(K,D+2)<>S1 THEN 270
140 FOR I=1 TO D STEP 1
150 Q(1,I)=F(K,I)
160 NEXT I
170 MAT E=Q*V
180 E1= F(K,D+1)+A1*E(1,1)-V(S1,1)
190 IF E1>-.00001 THEN 270
200 C=1
210 FOR I=1 TO D STEP 1
220 M(S1,I)=F(K,I)
230 NEXT I
240 R(S1,1)=F(K,D+1)
250 X(S1,1)=F(K,D+3)
260 CALL PEVAL1
270 NEXT K
280 NEXT S1
290 IF C=1 THEN 50
400 RETURN
450 END
```

Table 4. Listing for Policy Improvement Subroutine

The subroutine operates as follows:

(1) Starting with instruction 60 for each state in succession, it examines each of the actions available for that state to see whether the action yields a smaller value of the objective function than the action specified in the given policy.

(2) If an action does not yield a "sufficient" (see instruction 190 for the definition of sufficient) improvement of the cost function, the next possible action in the file for the given state is examined.

(3) If for a state an action yielding an improvement is found, this action is introduced into the policy (instructions 210 thru 250); the value of the new policy is computed (instruction 260 calls the subroutine listed in subsection 3.2.3 to do this); and search for still better actions continues (instruction 270).

(4) When all actions in a given state have been examined, the same procedure is followed for the next state (instruction 280).

(5) If during the steps (1) thru (4) a policy revision has taken place (this would be recorded in instruction 200), the search for improvement is repeated (instruction 290).

(6) If a search thru all actions of all states yields no "sufficient" improvement of the cost function, the program leaves the subroutine and returns to the main program.

Appendix I contains a listing of the subroutines discussed in the report and of the program used to call the subroutines and to print the optimal policy yielded by the computations.

It is conjectured that the policy finally selected by this routine will in most cases of practical interest be an optimal one. However, since in step (2) of the algorithm (corresponding to instruction 190 of the subroutine) a new policy is introduced only if the improvement of the cost function is greater than .00001 there exists the possibility that a potential improvement not larger than this would go undetected.

However, we can place an upper bound on the amount by which the policy finally decided on by the routine exceeds the minimum possible cost function. Set D(V) as the quantity used to test whether a sufficiently large improvement is possible or not. (The subroutine above uses D(V) = -.00001). Let V* be the value of a cost minimizing policy, and U a vector with components all 1. We can assert that the value V of a policy obtained by means of the subroutine listed above will satisfy the inequality

$V - V^* \leq - \frac{D(V)}{1-(A1)} \cdot U.$[1] In other words, $V^* \geq V + \frac{D(V)}{1-(A1)} \cdot U.$

---

[1]See Theorem 1, p. 167 of E.V. Denardo, "Contraction Mappings in the Theory Underlying Dynamic Programming", SIAM Review, Vol. 9, (1967).

5. MARKOVIAN MODELS OF EQUIPMENT MAINTENANCE

Chapter 2 described finite state-action Markovian decision models. Chapter 3 showed how to evaluate a policy in such a system, and chapter 4 showed how to find an optimal policy. Chapter 5 will apply the model to the problem of equipment maintenance. The trade-off between energy utilization and the use of other resources is brought into focus. The response to price signals is indicated through a study of comparative statics. The section also shows how to apply the policy evaluation routine and optimization method to a situation of rising energy prices.

5.1 Operation and Maintenance Cost per Period

Chapter 2, "Finite State-Action Markovian Decision Models", describes the basic data needed for a Markov programming model of an equipment maintenance problem. The chapter also presents a computer file format for keeping the data available for computer analysis in the mathematical programming operations needed to develop acceptable policies. A piece of equipment is described in the format mentioned by a table of numbers. The table consists of L rows, where L is the number of state-action pairs possible for the piece of equipment, and D+3 columns, where D is the number of possible states of the equipment. In each row corresponding to a state-action pair the entry in column D+1 is the expected cost over the current time period when the system is in the given state and the action specified in the given pair is taken. For the equipment maintenance model the costs incurred are of two kinds:

(1) cost of labor and parts for maintenance, and

(2) expected cost of the energy consumed.

Denote the cost in the current period for the state-action pair (i,k) by Ri(k). Then, Ri(k) = Mi(k) + P . Qi(k) where Qi(k) is the amount of energy (in physical units) that the equipment is expected to use during the period in state i if maintenance action k for that state is used, P is the price of energy and Mi(k) is the amount of other costs, expressed in dollars, if maintenance action k is applied in state i.

The program described in chapter 4, "Finding an Optimal Policy", and listed in Appendix I, generates in succession policies of lower cost until it no longer detects any possible cost reduction. In effect, within a certain range the more valuable the price indicator shows energy to be, the more will energy be conserved by policies to reduce cost.

5.2 Substitution of maintenance for energy resources.

In attempting to minimize cost in situations in which energy prices are higher it is reasonable to assume that policies employing additional maintenance in place of more energy will be used.[1] However, when a small change in energy prices leads to a change in maintenance policy, the cost reduction at the new price resulting from the policy change will be small (not exceeding the order of magnitude of the price change). The present value of a policy is a vector with a component defined for each of the states of the system as the starting state. The graphs of Figure 1 can be taken as of one given state.

---

[1]Arithmetic examples can be devised that will have each cost minimizing policy for a higher energy price entail larger average per period energy consumption. Thus it is not a mathematical consequence of the assumptions described in the previous chapters that, when the price of energy rises, energy utilization defined as expected energy use per period will not rise. These examples would illustrate why Paul Samuelson developes an argument to prove that such phenomena do not occur in the production function he is discussing. ("Although my intuition is poor enough in three dimensional space, I can assert with confidence on the basis of the above that raising any input's price while holding all remaining inputs' prices constant will definitely reduce the amount demanded of that input by the firm-i.e., $\frac{\partial Vi}{\partial Pi} < 0$," Paul Samuelson, "Maximum Principles in Analytical Economics", The American Economic Review, (1972), Vol. 62, No. 3, p. 253). Economic judgement suggests making the additional assumption formulated in the text.

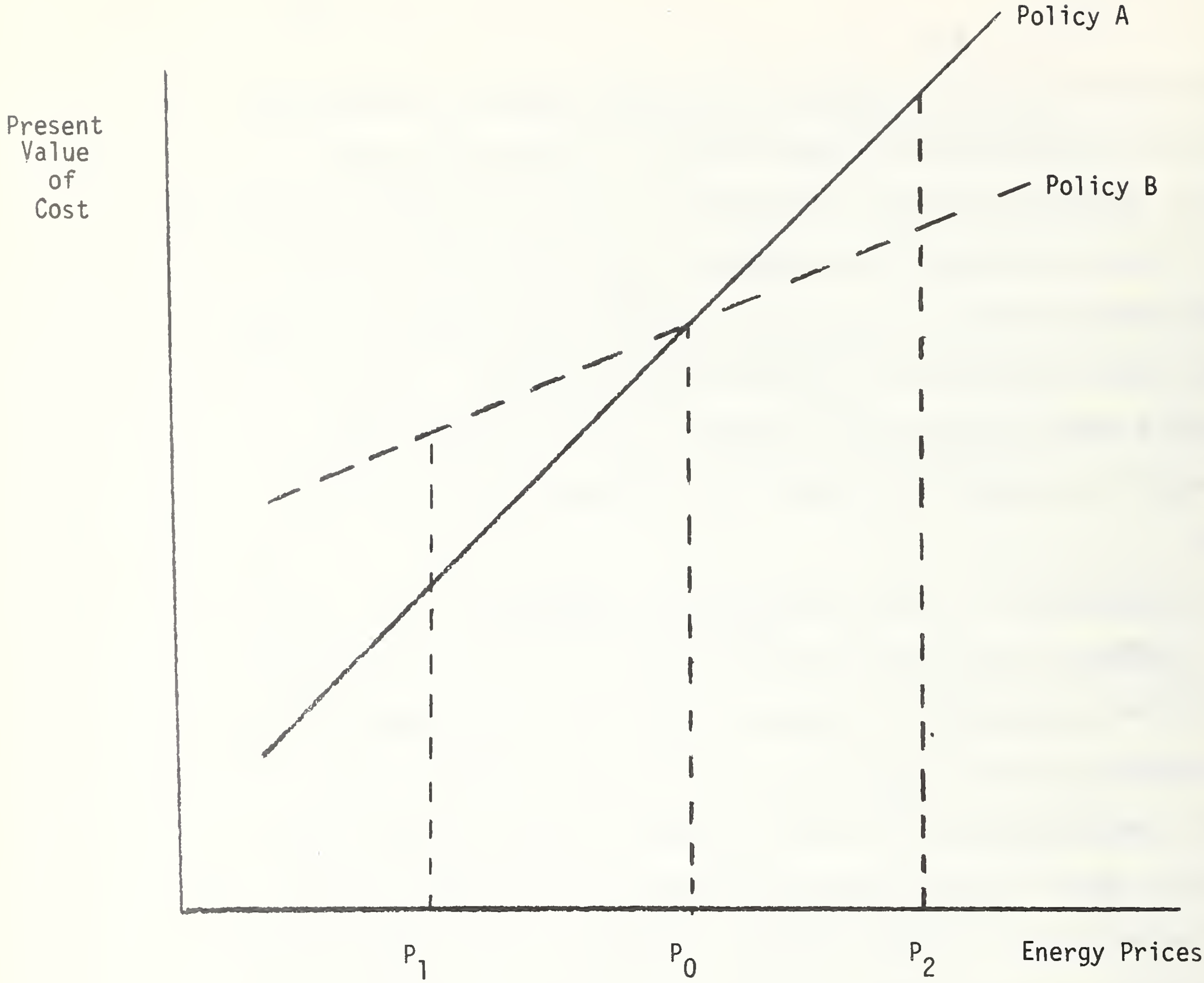

Figure 1. Policy costs as a function of energy prices

Figure 1 illustrates the cost effects of a rise in energy prices leading to a policy change. At price P_1, Policy A has lower cost. At price P_2, Policy B has lower cost. At price P_0 the costs of the two policies are the same. When P_1 and P_2 are both close to P_0 the difference in cost between the cost of Policy A and that of Policy B will not be large.

Suppose expected energy utilization under Policy B is smaller than expected energy utilization under Policy A. A change from Policy A to Policy B resulting from the desire to use the lower cost policy may save only a small amount in cost, but, it will also conserve energy and the energy conservation effect of the policy change needs separate evaluation.

To compare expected energy utilization under the policies appropriate respectively for energy prices P_1 and P_2, we run the policy selection program for both prices. For each policy (which is stationary) the resulting stationary Markov chain yields a stationary probability measure on the states of the system. Multiplying the expected energy utilization under the policy for each state by the probability of that state and summing yields the expected energy utilization per period for the given policy.

The subroutine STATV, the listing of which is given in Table 5, was written to help evaluate the expected energy utilization resulting from operating the equipment under a given maintenance policy. It calculates the probability measure on the states of the system associated with a given policy as we demonstrate in the next paragraph.

```
50 MAT U=I-M
60 FOR K=1 TO D
70 U(K,1)=1
80 NEXT K
90 MAT S=INV(U)
200 RETURN
250 END
```

Table 5. Listing for subroutine STATV

In the terminology of Kemeny and Snell, "An ergodic chain is characterized by the fact that it consists of a single ergodic class, that is, it is possible to go from every state to every other state."[1] Let M be the transition matrix of an ergodic chain, modified by the possible addition of some transient states (states to which one cannot go from every other state). If the total number of states in the chain is D then the matrix U ($\equiv$I-M) has rank D-1. The column vector $\vec{1}$ is linearly independent of the columns of U, since the fixed vector of M is orthogonal to each

[1] John G. Kemeny and J. Laurie Snell, Finite Markov Chains, (Van Nostand, 1960). pp. 99.

column of U. Substituting $\vec{1}$ for the first column of U and denoting the new matrix U* gives a matrix of rank D, for only the zero vector is orthogonal to all the columns of the resulting matrix. The first row of the inverse of U* is the fixed probability vector of M.[1]

5.3 Optimal Policy for Rising Energy Prices

The models for equipment maintenance discussed in this report up to now are all stationary dynamic programming models. The stationarity assumption in particular means that the models can be used to compare appropriate policies for different fuel prices. The discussion, however, does not immediately apply to the non-stationary situation in which fuel prices are expected not to remain constant, but to be increasing. This section shows the modifications that will permit the previous analysis to find a policy that will minimize the sum of future expected costs discounted to the present when fuel prices are increasing at a constant rate.

Let A1 denote the discount factor applicable to a dollar one period in the future to obtain its present value. Suppose that fuel prices are increasing at the rate r per period. Define $A2 = A1 \cdot (1+r)$. It is assumed that $A2 < 1$. Then the present value of a physical unit of fuel n periods in the future is: $(A2)^n \cdot P$, where P is the current price of fuel.

Suppose for the moment a policy is specified, i.e., for each possible state of the system an action has been identified as the one that will be taken in that state. Let $\vec{MC}$ be the vector of expected maintenance costs and $\vec{FC}$ the vector of expected energy costs at current prices, each over a single time period. Denote the transition matrix under the given policy by M.

---

[1]See Denardo, Eric V. "A Markov Decision Problem" in Mathematical Programming, edited by T.C. Hu (Academic Press, 1973).

If V is the present value of the future costs of the given policy, it can be expressed as:

$V = \vec{VM} + \vec{VF}$, where $\vec{VM}$ is the vector of the expected present value of maintenance costs and $\vec{VF}$ is the vector of the expected present value of energy costs. In terms of the data specified above:

$$\vec{VM} = (I - A1 \cdot M)^{-1} \cdot \vec{MC}, \text{ and}$$
$$\vec{VF} = (I - A2 \cdot M)^{-1} \cdot \vec{FC}.$$

The policy improvement routine can now be implemented for this case of increasing energy prices. Recall that, for each state, each action available for that state is examined to see whether it would yield a smaller value of the objective function than the action specified in the given policy or not. In testing action a_t^* of state i the quantity that should be examined is:

$$E1 \cong MC_i \ (a_t^*) + FC_i \ (a_t^*) + m_i \ (a_t^*) \ [(A1) \times \vec{VM} + (A2) \times \vec{VF}] - V(i).$$

The results of the test are applied as described above in section 4.2.

Note that, if it were of interest, a case of a decreasing factor price could be treated in an anlogous fashion.[1]

---

[1] I wish to thank Stephen R. Petersen for having suggested that the case of increasing energy prices be treated.

6. EXAMPLE OF EQUIPMENT MAINTENANCE MODEL

by

James Kao

This chapter applies the model described above to the maintenance of an air handling unit. The equipment is first described in engineering terms. A finite state description is next given and the possible actions in each state identified. The transition from state to state under alternative maintenance actions is discussed. Cost per period as a function of state-action is specified. Finally, an optimal policy for each of several energy prices is identified. In the spirit of presenting an example of the general optimization procedure, some remarks on the special structure of the example that would permit use of special methods to obtain a minimum cost policy are relegated to Appendix II at the end of the report.

6.1 Filter for Air Handling Unit

The air filters for building air filtration are well suited for illustrating the use of the equipment model to determine a cost minimizing service policy. For medium and high efficiency air filters, with the exception of electrostatic filters, there are usually air filter gauges installed to indicate the filter air resistance. The air resistance can be conveniently used to represent the "state" of the air filter. The state of the equipment will be discussed further in section 6.2.

Although the energy consumption due to air filter friction is small compared to the energy required for heating and cooling of the building, it may consume as much as one quarter of the entire power input of the fan motor for air distribution. The energy consumption of air filtration is especially high for areas requiring high air circulating rate, better air quality, and long operating hours. Spaces which may have these requirements are computer rooms, some areas in hospitals and laboratories, and some industrial plants.

To illustrate the equipment model, a 22,000 CFM, constant fan speed, air handling system is used. The air filters in the air handling unit are 8-cell, 80-85% efficient (ASHRAE Atomspheric Dust Spot Efficiency rating)[1] bag type with an initial friction of .35" water gauge pressure (WG). The filter bags are periodically replaced when they are loaded and the general practice is to follow the filter manufacturer's recommended replacement friction, which varies from approximately .8" to 1.0" WG for this type of filter, depending on the manufacturer.

6.2 States of Systems

The state of the system may be any variable condition of the system having a direct relationship with the energy consumption of the system. In many cases, the maintenance personnel's judgment must be relied on in determining the state of the system, although, it is preferred that some definitive indicators be used. For a steam-to-water heat exchanger, the water pressure drop of the water tubes may be used as the state to represent the cleanliness of the heat exchanger. In our example here, the air flow rate of the air handling unit or the air velocity at a certain point inside the unit may be used as the state, but the most convenient state can be expressed by the friction of the filter as displayed on the filter gauge. This pressure for our type of filters can be anywhere in the interval .35 to .95 inches WG. This interval is

---

[1] "Method for Testing Air Cleaning Devices Used in General Ventilation for Particulate Matter (ASHRAE Standard 52-68)" (American Society of Heating, Refrigeration and Air Conditioning Engineers, Inc., 1968).

divided into 12 sub-intervals of .05" WG each and each of the sub-intervals is consolidated into a single state for purposes of our approximate analysis.

S1 corresponds to $.35 \leq WG < .4$

S2 corresponds to $.40 \leq WG < .45$

etc.,

S12 corresponds to $.9 \leq WG$.

6.3 Possible Actions in Each State

For the type of air filters in the example, the possible actions in each state are the same, namely, replacing the filters or doing nothing. For the more complicated mechanical equipment, the possible actions may be many and may differ from state to state. For instance, the possible actions for a refrigeration machine may include the following: doing nothing, cleaning condenser tubes, cleaning evaporator tubes, overhauling compressor or any combination of these actions.

6.4 Transitions from State to State

It is not the intention of this example to discuss in detail the probability distribution of the states at the end of the time intervals. For most pieces of equipment used in buildings for there are probably not enough operating records on their deterioration rate to warrant a thorough probability distribution study. Sometimes judgment must be relied upon to determine the state transition. For air filters, the dirt loading is very much dependent on the outside air inlet location, the air quality around the building and the recirculated air quality which varies with the building occupants' activities. A filter life chart should be constructed by observing the filter gauge at periodic time intervals. The filter life chart can be used to help calculate the transition probabilities. In this illustrative example, we shall use a filter life curve of expected values such as the one shown in Figure 2.[1] It should be noted, due to the reasons given above, that the life curve of an actual air filter in a certain building may not duplicate this curve exactly. The time interval constituting one period is taken to be a month in this example.

Judgment, based on the operating person's experience, must be used in determining the possible state transition at the end of the time interval. For example, at the end of one month after a new set of filters are installed, we know that the resistance will not be below .35" WG, but will the resistance go beyond .45" WG? In other words, should we distribute the transition into two states (.35" WG and .40" WG) or three states (.35" WG, .40" WG and .45" WG)? If the judgment of the operating personnel is that the chances of being above .45" WG at the next inspection is nil, then, a two point distribution is selected. The transition probabilities can be computed by:

$$\mu_1 P_1 + \mu_2 P_2 = E$$

$$P_1 + P_2 = 1$$

$$P_1 \geq 0$$

$$P_2 \geq 0$$

where

μ = air friction at certain states

P = transition probability to these certain states

E = the expected air friction at the end of inspection period obtained from the life curve of Figure 2.

---

[1]Base curve from "Air Filtration: Resistance, Energy and Service Life," by Robert Avery, Heating/Piping/Air Conditioning, December 1973. Curve extended from .8" WG to 1.1" WG resistance by fitting a polynomial.

In our example, at the end of one month after the filters are replaced:

$$.35P_1 + .40P_2 = .375$$

$$P_1 + P_2 = 1$$

$$P_1 \geq 0$$

$$P_2 \geq 0$$

Therefore,

$$P_1 = .5$$

$$P_2 = .5$$

If it is decided that a three point distribution of the transition probability should be made, such as the transition from initial state 2 then we have

$$\mu_1 P_1 + \mu_2 P_2 + \mu_3 P_3 = E$$

$$P_1 + P_2 + P_3 = 1$$

$$P_1 \geq 0$$

$$P_2 \geq 0$$

$$P_3 \geq 0$$

or

$$.4\ P_1 + .45\ P_2 + .5\ P_3 = .46$$

$$P_1 + P_2 + P_3 = 1$$

$$P_1 \geq 0$$

$$P_2 \geq 0$$

$P_3 \geq 0$

The solution set of this system of inequalities is a line segment, and the midpoint of the segment is:

$P_1 = .1$

$P_2 = .6$

$P_3 = .3$

In the absence of better information on the exact probability distribution leading to the expected value recorded in the chart we have taken this as an estimate.

6.5 Cost Per Period as Function of State-Action

The cost per period mainly includes the cost of energy to operate the equipment and the cost of servicing the equipment. Ordinarily, the servicing cost consists of the labor cost and the material cost of replacing parts.

In the air filter example, the energy consumption of the air filters is approximated by taking the average of the fan power at the beginning and the end of the time interval multiplied by the number of hours of operation time during the inspection period of one month. The air flow rate used in the fan power computation is adjusted to reflect the changing system and fan characteristics due to filter dust loading. For each state, an air handling system characteristic curve representing that for the average air filter resistance in the time interval can be constructed to intersect with the fan characteristic curve to obtain the air flow rate of that state. The filter energy consumption is then computed by using the equation:

$$E = \frac{Q \times R \times T}{8510 \times e_m \times e_f}$$

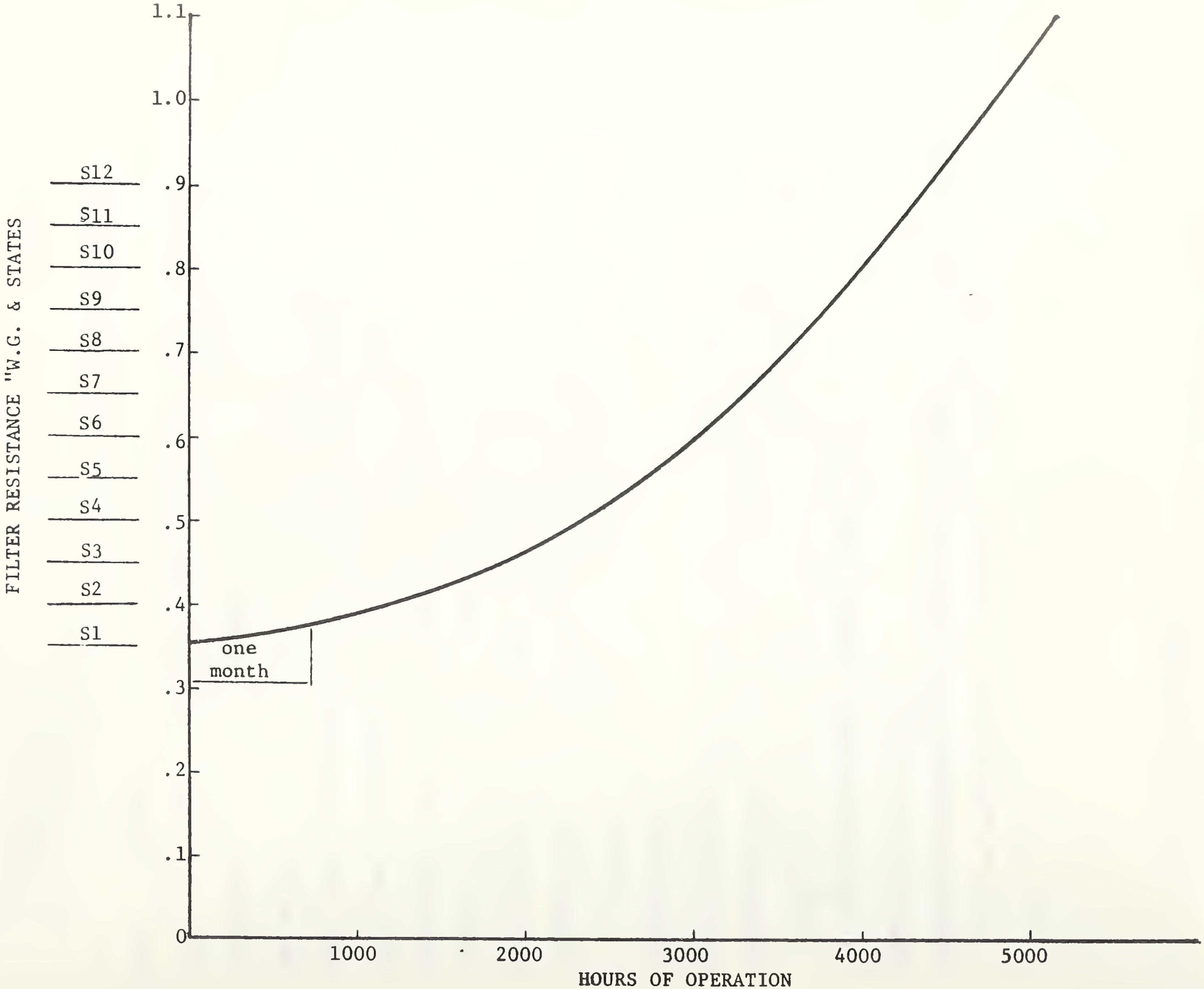

Figure 2. Sample Air Filter Life Curve

	States											
	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12
R, Filter Average Resistance, in WG	.37	.43	.49	.55	.62	.68	.73	.79	.84	.89	.95	1.00
Q, Air Flow Rate 1000 CFM	22.0	21.8	21.7	21.5	21.3	21.2	21.1	21.0	20.8	20.7	20.6	20.5
E, Filter Energy, KWH	1080	1244	1410	1569	1752	1913	2044	2201	2318	2444	2597	2720

Table 6 Filter Resistance, Air Flow Rate and Filter Energy Consumption

where

E = energy consumption for the time interval, KWH

R = average air filter resistance in the interval, in. WG

Q = air flow rate, Ft^3/min

T = air handling unit operating hours in the time interval, hr

e_m = motor effieiency

e_f = fan static efficiency

Table 6 shows the filter frictions, air flow rates and the filter energy consumption. The motor effiency was assumed to be .85 and the fan efficiency was assumed to be .75.

The cost of replacing the air filters include $320 for filters and $15 labor cost for one replacement.

The possible actions, transitions from state to state and the costs per period are listed in Table 7.

6.6 Cost Minimizing Policies

Cost minimizing policies obtained by the optimization procedure are shown in the computer output of Table 8. A discount factor of .99 and four prices for energy were used.

The printout takes the form V(i, j) = C, where i takes on the integer values from 1 thru 12, indexing the possible states of the system. If the j corresponding to a given i is 2, this indicates that the filter is to be replaced when the system is in that state. If the j corresponding to a given i is 1, this indicates that the filter is not replaced in that state. C at V(i, j) is the expected value of discounted future cost when the system starts in state i and the policy (k, j(k)), k = 1, . . . 12 is used.

Initial	FINAL												ACTION 1-NONE 2-REPLACE	ENERGY COST KWH	REPLACEMENT COST, $
	s-1	s-2	s-3	s-4	s-5	s-6	s-7	s-8	s-9	s-10	s-11	s-12			
s-1	.5	.5	0	0	0	0	0	0	0	0	0	0	1	1161	0
s-2	0	.1	.6	.3	0	0	0	0	0	0	0	0	1	1441	0
s-2	.5	.5	0	0	0	0	0	0	0	0	0	0	2	1161	335
s-3	0	0	0	.44	.53	.03	0	0	0	0	0	0	1	1676	0
s-3	.5	.5	0	0	0	0	0	0	0	0	0	0	2	1161	335
s-4	0	0	0	0	.1	.8	.1	0	0	0	0	0	1	1910	0
s-4	.5	.5	0	0	0	0	0	0	0	0	0	0	2	1161	335
s-5	0	0	0	0	0	.44	.53	.03	0	0	0	0	1	1991	0
s-5	.5	.5	0	0	0	0	0	0	0	0	0	0	2	1161	330
s-6	0	0	0	0	0	0	0	.1	.8	.1	0	0	1	2181	0
s-6	.5	.5	0	0	0	0	0	0	0	0	0	0	2	1161	335
s-7	0	0	0	0	0	0	0	0	.1	.6	.3	0	1	2340	0
s-7	.5	.5	0	0	0	0	0	0	0	0	0	0	2	1161	335
s-8	0	0	0	0	0	0	0	0	0	.03	.53	.44	1	2482	0
s-8	.5	.5	0	0	0	0	0	0	0	0	0	0	2	1161	335
s-9	0	0	0	0	0	0	0	0	0	0	.03	.97	1	2621	0
s-9	.5	.5	0	0	0	0	0	0	0	0	0	0	2	1161	335
s-10	0	0	0	0	.0	0	0	0	0	0	0	1	1	2659	0
s-10	.5	.5	0	0	0	0	0	0	0	0	0	0	2	1161	335
s-11	0	0	0	0	0	0	0	0	0	0	0	1	1	2720	0
s-11	.5	.5	0	0	0	0	0	0	0	0	0	0	2	1161	335
s-12	.5	.5	0	0	0	0	0	0	0	0	0	0	2	1161	335

Table 7 State-Action Information for Air Handling Unit.

For energy cost of $.025/KWH

```
V( 1 , 1 )= 9133.64
V( 2 , 1 )= 9259.52
V( 3 , 1 )= 9309.97
V( 4 , 1 )= 9349.1
V( 5 , 1 )= 9370.12
V( 6 , 1 )= 9397.26
V( 7 , 1 )= 9404.8
V( 8 , 1 )= 9421.16
V( 9 , 1 )= 9438.69
V( 10 , 1 )= 9440.43
V( 11 , 1 )= 9441.95
V( 12 , 2 )= 9468.64
```

For energy cost of $.05/KWH

```
V( 1 , 1 )= 13488.7
V( 2 , 1 )= 13643.9
V( 3 , 1 )= 13702.6
V( 4 , 1 )= 13743.3
V( 5 , 1 )= 13765.4
V( 6 , 1 )= 13786.7
V( 7 , 1 )= 13797.9
V( 8 , 1 )= 13808.2
V( 9 , 1 )= 13816.4
V( 10 , 1 )= 13818.4
V( 11 , 1 )= 13821.4
V( 12 , 2 )= 13823.7
```

For energy cost of $.075/KWH

```
V( 1 , 1 )= 17554.3
V( 2 , 1 )= 17733.
V( 3 , 1 )= 17797.
V( 4 , 1 )= 17838.1
V( 5 , 1 )= 17858.2
V( 6 , 1 )= 17874.
V( 7 , 1 )= 17885.9
V( 8 , 2 )= 17889.3
V( 9 , 2 )= 17889.3
V( 10 , 2 )= 17889.3
V( 11 , 2 )= 17889.3
V( 12 , 2 )= 17889.3
```

For energy cost of $.1/KWH

```
V( 1 , 1 )= 21388.5
V( 2 , 1 )= 21586.
V( 3 , 1 )= 21652.1
V( 4 , 1 )= 21695.4
V( 5 , 1 )= 21705.3
V( 6 , 2 )= 21723.5
V( 7 , 2 )= 21723.5
V( 8 , 2 )= 21723.5
V( 9 , 2 )= 21723.5
V( 10 , 2 )= 21723.5
V( 11 , 2 )= 21723.5
V( 12 , 2 )= 21723.5
```

Table 8 Optimal Policies for Different Energy Prices

At the energy costs of \$.025/KWH, \$.05/KWH, \$.075/KWH, and \$.1/KWH, the best policies are to replace the filters at S12, S12, S8, and S6 respectively. These results illustrate that at higher energy cost, it pays to replace the filters more frequently.

7. Summary and Suggestions for Further Research

This paper explains a method for modeling the performance of building service equipment as a function of maintenance policies. It shows how to use the model to optimize, by a correct choice of maintenance policies, the equipment performance with special attention to energy costs.

Technological and economic assumptions are explained. Chapter 3 discusses the class of policies that will be investigated and references literature showing that this class contains an optimal policy. Then an optimization procedure is explained. The comparative statics of maintenance policies for different fuel prices is examined; it is also shown how to apply the policy evaluation routine and optimization method to the case of rising energy prices. Chapter 6 is an application of the model of the previous sections to the maintenance problem for an air handling unit.

The paper formulates equipment maintenance for energy conservation as an economic problem and, subject to the assumptions explained in the report, solves the problem of optimization. But this is not an exhaustive study of the problem. The following tasks appear timely and significant:

(1) The analysis of this report is based on predetermined periodic examination of the equipment to ascertain its state. The appropriate examination procedure and its frequency should be the subject of investigation. Tradeoffs between better focused maintenance policies and less intensive and/or frequent equipment inspection will often be available.

(2) The data required (fully described in the report) to implement the model are not immediately available for many units of building service equipment. Therefore, a program for systematic collection and analysis of data concerning performance of equipment and the effects of maintenance actions on equipment performance should be undertaken.

(3) Appropriate statistical procedures for estimating parameters characterizing equipment performance should be investigated.

(4) Methods should be formulated for decision making with respect to maintenance policies on the basis of accumulating data with respect to structure and parameters of the model.

Appendix I. Listing for Programs

In this appendix we give a listing of each of the programs described in the text. In some cases we duplicate a listing already given in the main body of the report for the completeness of the appendix.

```
POELFR

10 A1=.99
20 DIM F(50,50)
30 DIM I(25,25),M(25,25),S(25,25),T(25,25),U(25,25)
40 DIM R(25,1),V(25,1),X(25,1)
50 FILES MAC9A
60 READ #1,D,L
65 MAT F=ZER(L,D+3)
70 MAT M=ZER(D,D)
80 MAT S=ZER (D,D)
90 MAT T=ZER(D,D)
100 MAT U=ZER(D,D)
110 MAT R=ZER(D,1)
120 MAT V=ZER(D,1)
130 MAT X=ZER(D,1)
140 MAT I=IDN(D,D)
150 MAT READ #1,F
160 CALL BLMM
170 CALL PEVAL1
180 FOR I=1 TO D
190        PRINT "V("I","X(I,1)")="V(I,1)
200 NEXT I
210 PRINT
220 CALL RTEST
230 FOR S=1 TO D
240        PRINT "V("S","X(S,1)")="V(S,1)
250 NEXT S
260 END
```

Table A1. Listing for Calling Program

Table A1 is the listing for POELFR which is a program to identify the discount factor being used [ln 10], to call up the analyzing subroutines, and to print the results of the analysis.

Table A2 is the computer listing of the data for a hypothetical piece of equipment. The interpretation of the listing in terms of the physical and economic characteristics of the equipment is given in subsection 2.2.1 of the report.

The subroutine BLMM is called in the program POELFR [ln 160]. A listing for this subroutine is given in Table A3. The purpose of this subroutine is to specify a starting policy preliminary to the policy improvement routine.

```
100  9            16
110  .12  .11  .11  .11  .11  .11  .11  .11  .11  10  1  1
120  .04  .12  .12  .12  .12  .12  .12  .12  .12  12  2  1
130  .12  .11  .11  .11  .11  .11  .11  .11  .11  20  2  2
140  0  0  .15  .15  .15  .15  .15  .15  .1  14  3  1
150  .12  .11  .11  .11  .11  .11  .11  .11  .11  20  3  2
160  0  0  0  .17  .17  .17  .17  .17  .15  16  4  1
170  .12  .11  .11  .11  .11  .11  .11  .11  .11  20  4  2
180  0  0  0  0  .2  .2  .2  .2  .2  18  5  1
190  .12  .11  .11  .11  .11  .11  .11  .11  .11  20  5  2
200  0  0  0  0  0  .25  .25  .25  .25  20  6  1
210  .12  .11  .11  .11  .11  .11  .11  .11  .11  20  6  2
220  0  0  0  0  0  0  .34  .33  .33  22  7  1
230  .12  .11  .11  .11  .11  .11  .11  .11  .11  20  7  2
240  0  0  0  0  0  0  0  .5  .5  24  8  1
250  .12  .11  .11  .11  .11  .11  .11  .11  .11  20  8  2
260  .12  .11  .11  .11  .11  .11  .11  .11  .11  25  9  1
```

Table A2. Listing for MAC9A.

```
BLMM

30 FOR I=1 TO D STEP 1
40 FOR K=1 TO L STEP 1
50 IF F(K,D+2)<>I THEN 120
60 FOR J=1 TO D STEP 1
70 M(I,J)=F(K,J)
80 NEXT J
90 X(I,1)=F(K,D+3)
100 R(I,1)=F(K,D+1)
110 GO TO 130
120 NEXT K
130 NEXT I
150 RETURN
200 END
```

Table A3. Listing for BLMM

Once an initial policy has been specified, the program calls subroutine PEVALI [ln 170] to evaluate the policy. The listing for PEVALI is given in Table A4. The operation of this subroutine is discussed in subsection 3.2.3 of the report.

The program POELFR then calls subroutine RTEST [ln 220], the policy improvement subroutine. The operation of this subroutine is explained in subsection 4.2. The listing for RTEST is given in Table A5.

In conclusion, Table A6 gives the listing for a subroutine, STATV, to calculate the fixed probability vector associated with a given policy. Justification of the algorithm used in STATV is given in subsection 5.2.

```
PEVAL1

50 MAT T=(A1)*M
60 MAT U=I-T
70 MAT S=INV(U)
80 MAT V=S*R
99 RETURN
100 END
```

Table A4. Listing for PEVALI

```
RTEST

40 DIM Q(1,20),E(1,1)
50 C=0
60 FOR S1=1 TO D
110 MAT Q=ZER(1,D)
120 FOR K=1 TO L STEP 1
130 IF F(K,D+2)<>S1 THEN 270
140 FOR I=1 TO D STEP 1
150 Q(1,I)=F(K,I)
160 NEXT I
170 MAT E=Q*V
180 E1= F(K,D+1)+A1*E(1,1)-V(S1,1)
190 IF E1>-.00001 THEN 270
200 C=1
210 FOR I=1 TO D STEP 1
220 M(S1,I)=F(K,I)
230 NEXT I
240 R(S1,1)=F(K,D+1)
250 X(S1,1)=F(K,D+3)
260 CALL PEVAL1
270 NEXT K
280 NEXT S1
290 IF C=1 THEN 50
400 RETURN
450 END
```

Table A5. Listing for RTEST

```
STATV

50 MAT U=I-M
60 FOR K=1 TO D
70 U(K,1)=1
80 NEXT K
90 MAT S=INV(U)
200 RETURN
250 END
```

Table A6. Listing for STATV

Appendix II. Control-Limit Maintenance Policies

The example presented in Chapter 6 of this report was discussed only in terms of the general Markov decision problem. However, the equipment described there has a characteristic structure that permits use of a special method to select a minimum cost policy from those called control-limit maintenance policies, a term explained below. Discussion of this special structure is relegated to an appendix because in general good maintenance policies will not be of the control-limit form.

Recall that the number of states in the model described in Chapter 6 is 12. One action is available in each of the states, state one and state twelve. There are two actions available in each of the other states. Based on the formula of Section 2.2 there are 2^{10} stationary policies possible. However, the argument used by Derman[1] shows that the set of control-limit policies contains a policy optimal over the set of all policies. In the case of the air handling unit described in Chapter 6 such policies are of the form:

for friction < k" WG, do not change the filter

for friction $\geq$ k" WG, change the filter.

The number of control-limit policies for the model above are 11 rather than 2^{10}.

Because the cost function we use is slightly more general than that used by Derman we reproduce his argument with the necessary adjustment. Let $M_i(k)$ denote maintenance cost in state i when action k is used and $P \cdot Q_i(k)$ the value of energy used in state i when action k is applied. (This is the notation of Section 5.1). For the air handling unit of

---

[1] Cyrus Derman, "An Optimal Replacement Rules When Changes of State are Markovian", pp. 201-210, in Mathematical Optimization Techniques, edited by Richard Bellman.

Chapter 6:

$M_i(k)$ does not depend on i

$P \cdot Q_i$ (1) is non-decreasing in i

$P \cdot Q_i$ (2) does not depend on i

as can be verified from Table 7, p. 33.

Under action 1, not replacing the filter, for each m the function

$$W_m(i) = \sum_{j=m}^{n} Pij$$

with domain i is non-decreasing.

Set $\phi(i,0) = 0$ for $i=1, \ldots, 11$

$\phi(12,0) = M_{12}(2)$.

After $\phi(i,N-1)$ has been defined procede by induction to define

$$\phi(1,N) = P \cdot Q_1(1) + (A1) \cdot \sum_{j=1}^{12} P1j\, \phi(j,N-1)$$

$$\phi(i,N) = \min \{P \cdot Q_i(1) + (A1) \cdot \sum_{j=1}^{12} Pij\, \phi(j,N-1),$$

$$M_i(2) + P \cdot Q_i(2) + (A1) \cdot \sum_{j=1}^{12} P1j\, \phi(j,N-1)\}$$

for $i = 2, \ldots, 11$, and set

$$\phi(12,N) = M_{12}(2) + P \cdot Q_{12}(2) + (A1) \sum_{j=1}^{12} P1j \cdot \phi(j,N-1)$$

Since $\phi(i,0)$ is non-decreasing as a function of i the same is true for every N.[1] $\phi(i) \equiv \lim_{N\to\infty} \phi(i,N)$ is also non-decreasing and $\phi(i)$ satisfies the functional equation

$$\phi(1) = P \cdot Q_1(1) + (A1) \sum_{j=1}^{12} P1j\, \phi(j)$$

---

[1]See Derman, op. cit. Lemma on p. 207.

$$\phi\ (12) = M_{12}\ (2) + P \cdot Q_{12}\ (2) + (A1) \sum_{j=1}^{12} P1j\ \phi\ (j)$$

$$\phi\ (i) = \min \{P \cdot Q_i\ (1) + (A1) \cdot \sum_{j=1}^{12} Pij\ \phi\ (j),$$

$$M_i\ (2) + P \cdot Q_i\ (2) + (A1) \cdot \sum_{j=1}^{12} P1j\ \phi\ (j)\ \}$$

This establishes that some optimal policy has the control-limit form.

Selected Bibliography

ASHRAE Standard 52-68. "Method for Testing Air Cleaning Devices Used in General Ventilation for Particulate Matter" (American Society of Heating, Refrigeration and Air Conditioning Engineers, Inc. 1968).

Avery, Robert "Air Filtration: Resistance, Energy and Service Life", Heating/Piping, Air Conditioning, December 1973.

Blackwell, David "Discrete Dynamic Programming" Annals of Mathematical Statistics Vol. 33, (1962), pp. 719-726.

Denardo, Eric V. "Contraction Mappings in the Theory Underlying Dynamic Programming" SIAM Review, Vol. 9, (1967) pp. 165-177.

Denardo, Eric V. "A Markov Decision Problem" in Mathematical Programming, edited by T.C. Hu and Stephen M. Robinson, pp. 33-68, (Academic Press, 1973).

Derman, Cyrus "On Optimal Replacement Rules When Changes of State Are Markovian" pp. 201-210, in Mathematical Optimization Techniques, edited by Richard Bellman (University of California Press, 1963).

Howard, Ronald A. Dynamic Programming and Markov Processes, (Technology Press, 1960).

Kemeny, John G., and Snell, J. Laurie Finite Markov Chains, (Van Nostrand, 1960).

Samuelson, Paul "Maximum Principles in Analytical Economics", The American Economic Review, Vol. 62, (1972), pp. 249-262.

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A general model of equipment performance as a function of maintenance is developed that permits quantification of the optimal level of maintenance in terms of performance attainment and relative factor costs. The model formulation is that of a finite state, finite action Markovian decision process. The report supplies a listing for a program in BASIC of the policy improvement algorithm for finding a best policy. The model will help maintenance engineers, building managers and/or others responsible for making decisions concerning maintenance policies in selecting economically efficient levels of maintenance for elements of building service equipment. The report also contains an illustrative example applying the model to the maintenance of an air handling unit.

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