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Surveillance Test Procedures

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Edited by: Jerry Keller

Institute for Basic Standards
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SURVEILLANCE TEST

PROCEDURES

H. E. Almer

Abstract

Surveillance tests are designed to monitor the values of mass standards between calibrations. Two types are described; both consist of comparisons of the weights of an ordered set of mass standards with each other. The differences found are compared with those computed from the reported mass values. Surveillance limits based on the precision of both the calibration and the surveillance test processes are computed. These limits are estimates of the departure of the measured differences from the expected, or predicted, differences as computed from the reported values. A larger change is considered significant. Additional measurements to identify individual weights which have changed are required when a given comparison indicates that the mass of one or more of the weights involved has changed. Buoyancy corrections are used to correct for the difference in the buoyant effect on weights of differing densities. Records document the surveillance test results, and control charts help detect trends. Judgments concerning recalibration can be made based on the constancy of the weights relative to the use requirements.

Key words: Apparent mass; buoyancy; buoyancy correction; change; comparison; difference; mass; records; set; surveillance limits; surveillance test; test interval; true mass; value; weighing design; weights.

CONTENTS

1. INTRODUCTION

Surveillance test procedures are designed to monitor the values of mass standards between calibrations. This is important because the problem of the continuing validity of the values contained in the report of calibration is always present, and especially so for those who look to others for calibration service. Surveillance test procedures, if properly implemented, so provide a means of detecting gross changes as soon as possible with a minimum expenditure of time and effort.

Two types of surveillance tests are described. The first type, designated Type I, uses a minimum number of measurements that involve all of the weights in the set. The second type, designated Type II, requires a larger number of measurements which are grouped so that they are a series of 3-1's weighing designs[1]. This method has some redundancy.

Included in the surveillance test procedures are methods of identifying any weights whose mass values may have changed since they were calibrated, and methods of correcting for the buoyant effect of the atmosphere.

2. MEASUREMENT PROCEDURES

A surveillance test consists of a series of comparisons of the weights of an ordered set of mass standards with each other, according to an appropriate weighing design, and comparing the differences in mass value found by these comparisons with those computed from the values contained in the report of calibration [1]*. Ideally a suitable known weight, other than one of the weights in the set being tested, is used as the standard on which the values found by the surveillance test are based. This also establishes whether or not the whole set has changed proportionally. For sets where the largest weight is one kilogram or less, the nominal value of the weight used as a standard should be that of the largest weight in the set. For example, a set whose largest weight is 100g is being tested. For this set, a 100g weight whose mass value is known would be suitable for use as a standard. For sets having weights greater than one kilogram, a suitable one kilogram weight may be used as the standard. For sets in the avoirdupois system having weights greater than one pound, a suitable one pound weight may be used as the standard. Generally the uncertainty of the mass value of a one kilogram, or a one pound standard, is less than the uncertainty of the value of a larger standard.

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[1] A title given to the three intercomparisons of three objects A, B, and C, namely the measurements of the differences A-B, A-C, and B-C.

* Figures in brackets refer to similarly numbered references at the end of this paper.

If a weight of the suggested denomination is not available, a suitable known weight of a different denomination, if available, may be used to establish whether or not the whole set has changed. The nominal value of this weight should be equal to that of one of the larger weights in the set being tested, say not less than 20g for a set beginning at 100g, or less than 100g for a set beginning at 1kg. Where the weight used as the standard has the same nominal value as the largest weight in the test set, up to one kilogram, the comparison between the standard and the largest weight of the set is a part of the surveillance test weighing design. Where the nominal value of the weight used to establish whether the whole set has changed is not the same as the largest weight of the test set, the comparison between it and the corresponding weight of the test set is a side measurement and not a part of the surveillance test weighing design.

Where a suitable known weight, other than the weights in the set being tested, is not available, the usual procedure is to base the values found by the surveillance test on the largest weight of the set under test, up to one kilogram. Weights larger than one kilogram may be based on the largest weight of the set. The weighings may be made by either the substitution or the transposition method of weighing [2].

In general, the capacities of the balances selected for surveillance tests should be the smallest available that will accommodate the maximum load to be placed on it. For example, when testing a set of weights ranging from 100g to 1mg, a balance having a capacity of from 100g to 200g would be used for loads from 100g to 20g, and a balance of 20g capacity for loads under 20g. If a balance of say 1g and 2g capacity were available, it would be used for the fractional weights.

2.1 Type I Surveillance Test

In a type I surveillance test, the first measurement is the comparison between the largest weight of the set and a summation of the next smaller weights, from the set, the sum of whose nominal values is equal to that of the largest weight. The next comparison would be between a selected weight from the summation, that is, the summation used in the first comparison, and another summation whose nominal value is equal to that of the selected weight.

This procedure of selecting a weight from each summation and comparing it with a summation of the next smaller weights is repeated until all of the weights of the set have been involved in a comparison. Any given comparison should involve the fewest weights that will permit all of the weights of the set to be included in the chain of comparisons.

If a suitable weight having the same nominal value as the largest weight of the set is available for use as a standard, then the first comparison would be between this weight and the largest weight of the set.

If, for example, a set of weights ranging from 100g to 1mg is to be tested using the Type I surveillance test procedures where another 100g weight is to be used as a standard, the ratios of the weights to each other are 5, 3, 2, 1. The first comparison would be:

$$100g - S100g = a_1$$

The second comparison would be:

$$100g - \Sigma 100g = a_2$$

where $\Sigma 100g = 50g + 30g + 20g$

The third comparison would be:

$$20g - \Sigma 20g = a_3$$

where $\Sigma 20g = 10g + 5g + 3g + 2g$

This procedure is continued until all of the weights have been compared.

In this example the last comparison would be:

$$3mg - \Sigma 3mg = a_n$$

where $\Sigma 3mg = 2mg + 1mg.$

The observed differences in mass values ($a_1, a_2, \ldots . a_n$) found by these comparisons are compared with the accepted differences, as computed from the reported values, to determine the degree of agreement between the observed and the accepted differences. If the agreement is within the limits for surveillance (see section 3) any indicated changes may be regarded as being insignificant, and the continuing validity of the reported values may be assumed. If the agreement between the observed and the accepted differences is not within the surveillance limits, the indicated changes should be regarded as significant, and the weights exhibiting a significant change should be recalibrated. When the result of a comparison indicates that one or more of the weights has changed significantly, additional measurements are made to identify the weight, or weights, that have changed.

2.2 Type II Surveillance Test

In a Type II surveillance test, the measurements of the first 3-1's weighing design series are between the largest weight of the set, another weight of the same nominal value, and a summation of the next smaller weights from the set also having the same nominal value as the largest weight of the set. The comparisons of the next 3-1's weighing would be between a selected weight from the summation, used in the first 3-1's series, and two other summations, of the next smaller weights, whose nominal values are the same as that of the selected weight. This procedure of selecting a weight from a summation and comparing it with other summations of the next smaller weights according to the 3-1's weighing design is repeated until all of the weights of the set have been involved in the comparisons.

For example, a set ranging from 100g to 1mg is to be tested using the Type II surveillance test procedures, where another 100g weight[1] is to be used as a standard. The ratios of the weights to each other are 5, 3, 2, and 1. The first series according to the 3-1's weighing design would be:

$$S100g - 100g = a_1$$

$$S100g - \Sigma 100g = a_2$$

$$100g - \Sigma 100g = a_3$$

where S100g is the standard

100g is the 100g of the set being tested

$\Sigma 100g = 50g + 30g + 20g$

[1] If a suitable known 100g weight is not available for use as a standard, the first series according to the 3-1's weighing design would be:

$$100g - 100g' = a_1$$

$$100g - \Sigma 100g - a_2$$

$$100g' - \Sigma 100g = a_3$$

where

100g' is any 100g weight, or a summation whose nominal value is 100g, used to fill the series $\Sigma 100g = 50g + 30g + 20g$. The other series remain as indicated.

The second series would be:

$$30g - \Sigma 30g_1 = a_1$$

$$30g - \Sigma 30g_2 = a_2$$

$$\Sigma 30g_1 - \Sigma 30g_2 = a_3$$

where $\Sigma 30g_1 = 20g + 10g$

$\Sigma 30g_2 = 20g + 5g + 3g + 2g$

This procedure is continued for each decade until all of the weights in the set have been compared. Unless the set contains an extra 1mg weight or another 1mg weight whose mass value is known is available, the 3-1's weighing design cannot be used for the last decade. Where the set has only one 1mg weight and another is not available to fill the series, the comparisons for the last decade are:

$$5mg - 3mg - 2mg = a_1$$

$$3mg - 2mg - 1mg = a_2$$

These two comparisons are treated as the comparisons in Type I surveillance test. Where the set has two 1mg weights, or another 1mg weight whose value is known, is available, the last series is:

$$3mg - \Sigma 3mg_1 = a_1$$

$$3mg - \Sigma 3mg_2 = a_2$$

$$\Sigma 3mg_1 - \Sigma 3mg_2 = a_3$$

where $\Sigma 3mg_1 = 2mg + 1mg_1$

$\Sigma 3mg_2 = 2mg + 1mg_2$

The $1mg_2$ may be either the second 1mg weight of the set or another 1mg weight whose mass value is known.

If a weight other than one of the same denomination as the largest weight in the set is used to establish whether or not all the weights of the set have changed proportionately, then some other known weight must be compared to a weight of the set or (e.g. in this case) a known 30g is compared with the 30g of the set.

$$30g - S30g = a$$

If this difference agrees with the expected difference as computed from the reported values of the two weights, within the surveillance limit, (see section 3), and the observed differences of the other comparisons are in agreement with the predicted differences, it may be assumed that the set as a whole has not changed significantly.

Because in most of the series used in a Type II surveillance test one of the weights is part of both summations used in a given series, the weighings are made by the substitution method of weighing. For example, in the series involving the 30g weight, $\Sigma 30g_1$, and $\Sigma 30g_2$, the 20g weight is part of both summations.

3. SURVEILLANCE LIMITS

3.1 Uncertainties of Each of the Summations from the Calibration Process Known [1], [3]

Ideally the surveillance limits are calculated from the standard deviations of the calibration process and surveillance test process as follows:

$$sl = U_c + 3\sigma_d \tag{1}$$

where U_c = uncertainty of calibration process

σ_d = standard deviation of one weighing of the surveillance test process

sl = surveillance limit

3.2 Uncertainties for Individuals but not Summations from the Calibration Process Known

Sometimes only the uncertainties associated with the mass values of the weights, as reported in the Calibration Report, are available for estimating the uncertainties of the summations. In this situation, an approximate estimate of the uncertainties is found by taking the square root of the sum of the squares of the uncertainties of the values of the weights in a given comparison [4].

Suppose that the comparison is between a selected weight, W_1, and a summation consisting of three weights, W_2, W_3 and W_4, whose nominal value is equal to that of the selected weight, W_1. The uncertainty of each value is U_1, U_2, U_3 and U_4 respectively.

An approximate estimate of the uncertainty, U_c, for these weights is:

$$U_c = \sqrt{U_1^2 + U_2^2 + U_3^2 + U_4^2} \qquad (2)$$

where U_c is the uncertainty of the calibration mass measurement process and

U_i is the uncertainty for the individual weights as reported on the Report of Calibration.

With this procedure, the expression for the surveillance limit is:

$$sl = U_c + 3\sigma_d \qquad (3)$$

where U_c is the uncertainty as defined above, and

sl and σ_d have the same meaning as in equation (1).

This process is equally applicable for any number of weights.

For most designs, this procedure gives a somewhat smaller uncertainty than the uncertainties from the calibration process.

3.2.1 Numerical Example

Assume that the following weights and their associated uncertainties are involved in the comparison 100g - Σ100g.

Weight	Uncertainty
100g	0.015
50g	0.011
30g	0.012
20g	0.010

$$U_c = \sqrt{0.015^2 + 0.011^2 + 0.012^2 + 0.010^2}$$

$$= \sqrt{0.00025 + 0.000121 + 0.000144 + 0.0001}$$

$$= \sqrt{0.00059}$$

$$= 0.024 \text{ mg}$$

This is an approximate estimate of the uncertainty of the calibration process for this comparison.

Now let us assume that the standard deviation of one weighing of the surveillance test process is 0.015 mg.

Then the surveillance limit, sl, is:

$$sl = 0.024 + 3(0.015)$$

$$sl = 0.024 + 0.045$$

$$= \underline{0.069 \text{ mg}}$$

4. Identifying the Weights Which Have Changed

If, in any comparison, the observed difference differs from the predicted value of the difference by more than the surveillance limits for that comparison, the weight, or weights, that have changed must be identified so that they can be recalibrated. The identity of the weights that have changed may be established by additional measurements. In general, these additional measurements are comparisons between the weights making up the summation that was compared with the selected weight.

Suppose, for example, that the observed difference of

$$20g - \Sigma 20g = a$$

where $\Sigma 20g = 10g + 5g + 3g + 2g$

differs from the predicted value of the difference by more than the surveillance limits. Assume, also, that the observed differences in the comparison in which the 20g weight was a part of the summation, $100g - \Sigma 100g$, and the comparison in which the 2g weight was the selected weight, $2g - \Sigma 2g$, are in good agreement with their predicted, or accepted, differences as computed from the reported values. This indicates that neither the 20g weight nor the 2g weight have changed significantly. The following measurements are made and their results analyzed to identify the weight, or weights, whose masses have changed:

$$10g - (5g + 3g + 2g) = a'$$

$$5g - (3g + 2g) = a''$$

$$3g - (2g + 1g) = a'''$$

4.1 Analysis of Measurement Results

If a' differs from the predicted value by more than the surveillance limits and a'' and a''' agree with the predicted value within the surveillance limits, it is probable that the 10g weight has changed. If both a' and a'' differ from the corresponding predicted values by more than the surveillance limits by about the same amount, numerically, but with opposite signs, and a''' agrees with the predicted value within the surveillance limit, it is probable that the 5g weight has changed. If a' and a'' differ from the corresponding predicted values by markedly difference amounts which are greater than the corresponding surveillance limits, and a''' agrees with the corresponding predicted value within the surveillance limit, it is probable that both the 10g and the 5g weights have changed.

If a', a'', and a''' all differ from the corresponding predicted values by more than the surveillance limits, but by about the same amount, it is probable that the 3g weight is the one that has changed. If a' and a'' differ from the corresponding predicted values by about the same amount, but a''' differs from the corresponding predicted value by a markedly different amount, it is probable that both the 5g weight and the 3g weights have changed.

If the results of all three measurements differ from the corresponding predicted values by more than the corresponding surveillance limits, by markedly different amounts, it is probable that all three weights have changed and may require recalibration.

If all three (a', a'', and a''') of the observed differences are in good agreement with the predicted differences, it is still possible that the weights involved in either of the comparisons

$$100g - \Sigma 100g = a_1 \quad \text{or} \quad 2g - \Sigma 2g = a_3$$

experienced compensating changes in mass, even though the agreement between the observed differences and the predicted differences were within the surveillance limits. However, this is an unlikely situation. But, if it does occur, the weights that have changed may be identified in the manner described for the comparison between the 20g and Σ20g weights, as may the weights involved in any measurements where the observed difference does not agree with the predicted difference within the surveillance limits.

In any event, if it is determined that several weights of a given set require recalibration (more than, say, three or four weights in a 100g to 1mg set, or more in a larger set) the entire set should be recalibrated.

4.2 Numerical Example

The following numerical example, using difference measurement 20g - Σ20g, discussed above, illustrates the procedure.

The observed value of the difference:

$$20g - \Sigma 20g = +0.084mg$$

The predicted value is +0.052mg. The surveillance limit is +0.028mg. The difference between the observed value and the predicted value is:

$$+0.084mg - 0.052mg = 0.032mg$$

This difference exceeds the surveillance limits and indicates that the mass of one or more of the weights involved has changed. Three weighings were made to determine which weight, or weights, have changed. The results of these measurements are:

	Observation		Observed Value of Difference	Predicted Value of Difference	Surveillance Limit
a'	10g - (5g + 3g + 2g)	=	-0.025 mg	-0.057 mg	0.024 mg
a''	5g - (3g + 2g)	=	-0.065 mg	-0.032 mg	0.018 mg
a'''	3g - (2g + 1g)	=	+0.031 mg	+0.034 mg	0.015 mg

Examining these results, we find that the agreement between the observed value and the predicted value for a''' is well within the surveillance limit, thus virtually ruling out any change in the masses of the 3g and 2g weights. But, the observed values for a' and a'' do not agree with the predicted values within the surveillance limits. Further, the observed values for both a' and a'' differ from the predicted values by about the same amount, but with opposite signs.

For a' -0.025 - (-0.057) = +0.032 mg

For a'' -0.065 - (-0.032) = -0.033 mg

Had it been only for a' that the observed value did not agree with the predicted value, within the surveillance limit, it would be logical to conclude that the mass of the 10g weight had changed. But, for both a' and a'', the observed values of the differences do not agree with the predicted values by about the same amount, numerically, though with opposite signs. Therefore, the conclusion is that the mass of the 5g weight has changed because it is involved in both a' and a'', while the 10g weight is involved only in a'. Further, the 5g weight is in opposed positions in the two equations.

5. BUOYANCY CORRECTIONS

Buoyancy corrections are used to account for the difference in the buoyant effect of the air on weights of differing densities [5]. In some instances it will be necessary to apply buoyancy corrections to the measured differences between weights in surveillance tests because the buoyant effect on the weights may mask real changes in their masses, or apparent changes in mass may be indicated when there is no change. This is true whether the computations of the results are made on the true mass or the apparent mass basis. In general, the buoyancy corrections computed on the true mass basis are numerically greater than buoyancy corrections computed on the apparent mass basis when weights having widely different densities are involved in a given comparison.

It is always good practice to compute, at least roughly, the magnitude of the correction to establish the order of magnitude with reference to the uncertainty of the surveillance test measurement [1]. If the correction is not significant, it can be ignored.

5.1 Buoyancy Corrections Computed on True Mass Basis

When the results of the surveillance test weighings are computed on the true mass basis, the expected differences being computed from the reported mass (true mass) values, the true mass buoyancy correction term, $\rho \Delta V$, for the measured difference may be derived from the weighing equation for the difference between two weights.

$$(M_C - \rho V_C)g - (M_D - \rho V_D)g = ag \qquad \text{weighing equation (1)}$$

where: M_C and M_D = the masses of weights C and D, respectively

V_C and V_D = the volumes of C and D, respectively, from the Report of Calibration

ρ = air density when weighing was made

a = the indicated difference in mass units

g = acceleration of gravity

The derivation of the buoyancy correction term, $\rho\Delta V$, for the true mass difference between the two masses C and D is:

$(M_C - \rho V_C)g - (M_D - \rho V_D)g = ag$	weighing equation	(1)
$M_C - \rho V_C - M_D + \rho V_D = a$	dividing by g	(2)
$M_C - M_D = a + \rho(V_C - V_D)$	transposing and collecting terms	(3)
$M_C - M_D = a + \rho\Delta V$	substituting ΔV for $(V_C - V_D)$	(4)

It is better to use the form of the buoyancy correction term, $\rho(V_C - V_D)$, in equation (3) above when computing the buoyancy correction because its sign is more readily apparent. The following example illustrates this.

The measured difference, a, between 2g and Σ2g is 0.0388mg.

Weight	Volume		
2 g		0.2564 cm^3	from Report of Calibration
1 g	0.12820 cm^3		" " " "
500 mg	0.03012 cm^3		" " " "
300 mg	0.01807 cm^3		" " " "
200 mg	0.01205 cm^3		" " " "
Σ2g		0.1884 cm^3	" " " "

$\rho = 1.17 \text{ mg/cm}^3$

The true mass difference:

$$2g - \Sigma 2g = +0.0388 + 1.17(0.2564 - 0.1884)$$

$$= +0.0388 + 0.0796 = +0.1184 \text{ mg}$$

If volumes are not listed on the Report of Calibration, they may be computed from:

$$\text{Volume} = \frac{\text{Mass}}{\text{Density}}$$

5.2 Buoyancy Corrections Computed on Apparent Mass Basis

When the results of the weighings are computed on the apparent mass[1] basis [5], the expected differences being computed from the reported apparent mass values, the apparent mass buoyancy correction term, $\Delta\rho\Delta V$, for the measured differences may be derived from the expression for finding the apparent mass when the true mass and the volume are known.

$$AM_W = M_W - \rho_n(V_W - V_R) \tag{5}$$

where

AM_W = apparent mass value of weight "W" versus the reference material (R)

M_W = mass (true mass) of weight "W"

ρ_n = density of normal air

V_W = volume of weight "W" at 20 °C

V_R = volume of equivalent mass of the reference material (R) at 20 °C

The derivation of the buoyancy correction term, $\Delta\rho\Delta V$, for the apparent mass difference between the weights C and D is:

$$AM_C = M_C - \rho_n(V_C - V_b) \tag{6}$$

$$AM_D = M_D - \rho_n(V_D - V_b) \tag{7}$$

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1 In the United States, the apparent mass is usually expressed as apparent mass versus normal brass in normal air. Normal brass is defined as brass having a density of 8.4 g/cm^3 at 0 °C and a coefficient of cubical expansion 0.000054 per degree C. Normal air is defined as air having a density of 1.2 mg/cm^3 at 20 °C.

$$AM_C - AM_D = M_C - \rho_n(V_C - V_b) - M_D + \rho_n(V_D - V_b) \quad \text{subtracting (8)}$$

$$AM_C - AM_D = M_C - M_D - \rho_n V_C + \rho_n V_b + \rho_n V_D - \rho_n V_b$$

$$= a + \rho(V_C - V_D) - \rho_n(V_C - V_D)$$

substituting $a + \rho(V_C - V_D)$ for $(M_C - M_D)$ (see equation (3)) (9)

$$AM_C - AM_D = a + (\rho - \rho_n)(V_C - V_D) \quad \text{combining terms} \quad (10)$$

$$= a + \Delta\rho\Delta V \quad \text{substituting } \Delta\rho\Delta V \text{ for } (\rho - \rho_n)(V_C - V_D) \quad (11)$$

where AM_C and AM_D = the apparent mass of weights C and D

M_C and M_D = the masses of weights C and D

V_C and V_D = the volumes of C and D, respectively, from the Report of Calibration

V_b = the volume of equivalent mass of normal brass, the reference material

ρ = the air density when the weighing was made

ρ_n = the density of normal air at 20 °C.

It is better to use the form in equation (10) above when computing the buoyancy correction term because its sign is more readily apparent.

The following example illustrates this:

The measured difference, a, between 2g and Σ2g is 0.0388 mg.

Weight	Volume		
2 g		$0.2564\ cm^3$	from Report of Calibration
1 g	$0.12820\ cm^3$		" " " "
500 mg	$0.03012\ cm^3$		" " " "
300 mg	$0.01807\ cm^3$		" " " "
200 mg	$0.01205\ cm^3$		" " " "
Σ2 g		$0.1884\ cm^3$	" " " "

$\rho = 1.17\ mg/cm^3$

$\rho_n = 1.20\ mg/cm^3$

The apparent mass difference

$$2g - \Sigma 2g = + 0.0388 + (1.17 - 1.20)(0.2564 - 0.1884)$$

$$= + 0.0388 + (-0.03)(0.0680)$$

$$= + 0.0388 - 0.0020$$

$$= + 0.0368 \text{ mg}$$

5.3 Application of Buoyancy Correction

The buoyancy correction terms derived above are correct when the mass difference and the volume difference of the weights are taken in the same direction. That is, if the difference between the masses of weights C and D is taken as $M_C - M_D$ then their volume difference must be taken as $V_C - V_D$ or the buoyancy correction will have the wrong sign.

If, when assigning a mass value to one of the two weights being compared with each other, the other weight being used as the standard, a buoyancy correction is used, it is essential that the correct sign be used for the buoyancy correction term.

5.3.1 Buoyancy Correction Application for True Mass

Consider the relationship

$$C - D = a + \rho(V_C - V_D) \qquad (1)$$

If D is the standard then

$$C = a + \rho(V_C - V_D) + D \qquad (2)$$

substituting for a, ρ, V_C, and V_D and D their values, we get the true mass value of C, provided the true mass value of D was used.

If C, the first weight in the difference, C - D, is the standard (this is the situation in many weighing designs) then,

$$-D = a + \rho(V_C - V_D) - C$$

and

$$D = -a - \rho(V_C - V_D) + C \qquad (3)$$

Substituting for a, ρ, V_C, V_D and C their values, we get the true mass value of D, provided the true mass value of C was used.

Note that the sign of the buoyancy correction term in (3) above is minus. This application is illustrated on the computation sheet for the 3-1's weighing design.

5.3.2 Buoyancy Correction Application for Apparent Mass

Consider the relationship

$$C - D = a + (\rho - \rho_n)(V_C - V_D) \tag{4}$$

If D is the standard, then

$$C = a + (\rho - \rho_n)(V_C - V_D) + D \tag{5}$$

Substituting for a, ρ, ρ_n, V_C, V_D, and D their values, we get the apparent mass value of C, provided the apparent mass value of D was used.

If C, the first weight in the difference, C - D, is the standard (this is the situation in many weighing designs) then,

$$-D = a + (\rho - \rho_n)(V_C - V_D) - C$$

and

$$D = -a - (\rho - \rho_n)(V_C - V_D) + C \tag{6}$$

Substituting for a, ρ, ρ_n, V_C, V_D and C their values, we get the apparent mass value of D, provided the apparent mass value of C was used.

Note that the sign of the buoyancy correction term in (6) above is minus. This application is illustrated on the computation sheets for the 3-1's weighing design as used in the example for the Type II surveillance test (see appendix 2).

6. RECORDS

Records are an essential part of any measurement program. In a surveillance test program, adequate records are necessary to document the continuing validity of the reported mass values and to realize the full value of the program. Such records may be simple, or elaborate, as long as they contain the information needed to document the claimed validity of the mass values. A notebook or card file should be maintained containing a description of the test system. This should include a statement of the procedures, a list of standards (if any) and weighing instruments, test intervals, and a tabulation of the accumulated results of tests. The records should also include the identity of the weights, the expected, or predicted, values of the differences measured as computed from the reported values, and the surveillance limits. The calibration report should be an integral part of the records. In addition, where the Type II Surveillance Test is used, the estimate of the standard deviation should be compared for each 3-1's series, compared with the long term estimate of the standard deviation and recorded. This information, combined with the original data sheets, forms an adequate record. A large operation may require a more elaborate record keeping system.

Control charts [3] similar to the one illustrated on page 18 are a useful addition to the surveillance test records. Control charts show more readily than tabulations whether a trend in the values of the differences being measured is developing. Such trends, when detected, can signal the need for recalibration before the values of the mass standards become invalid.

7.SURVEILLANCE TEST INTERVAL

The purpose of surveillance test procedures is to assure continuing validity of the values contained in the calibration report and to prevent, or at least minimize, the possibility of using the weights as standards when their reported values are no longer valid. But, when and how frequently should the surveillance test procedures be used in order to achieve this goal? Because of the many variables affecting the stability of the weights, such as the type of weights, the use to which the weights are put, the care they receive, etc., a categorical answer covering all situations cannot be given.

SURVEILLANCE TEST
CONTROL CHART

SET RANGE: 100g - 1g
CALIBRATION TEST NO. NBS 200390

COMPARISON	EXPECTED DIFF.	SURVEILLANCE LIMIT	MG
		+0.020	+.020
100g - S100g	-0.040		
		-0.100	-.100
		+0.004	+.00[illegible]
100g - Σ100g	-0.065		
		-0.134	-.13[illegible]
		+0.0796	+.07
20g - Σ20g	+0.0516		
		-0.0236	+.0[illegible]
		-0.049	+.0[illegible]
3g - Σ3g	+0.0340		
		-0.019	+.0

The following suggestions, where they are applicable, may serve as general guide lines for the use of surveillance tests and the interval between surveillance tests.

1. Immediately upon the receipt of a newly calibrated set of weights, comparisons should be made to verify the values reported.

2. If this is a set for which no history exists, the comparisons should be repeated monthly, or bimonthly until the degree of stability of the weights has been demonstrated.

3. Where sufficient information about a set of weights has been developed to predict their performance with some degree of certainty, this information may be used in determining the interval between surveillance tests.

4. If there has been an accident with the weights, such as dropping them on the floor, at least the weights involved in the accident should be given a surveillance test before being used as standards to be sure that their reported values are still valid.

5. If a facility performs a large number of calibrations, its procedures should provide "built-in" checks on standards and if the standards checked on are part of the set in question, the information developed from these "built-in" checks can be used to determine when a surveillance test is needed.

6. Where the number of calibrations performed is small, the standards may be given a surveillance test just prior to using the standards in the calibration of other weights.

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[4] Youden, W. J., Statistical Methods for Chemists, John Wiley & Sons, New York (1541).

[5] Nat. Bur. Stand. (U.S.), Handbook 77, Precision Measurement and Calibration, Volume III, Optics, Metrology and Radiation, Circular 3, pp. 671/53 to 683/65. This handbook is available for reference in most Government Depository Libraries throughout the United States.

APPENDIX I. WEIGHING DESIGNS FOR SURVEILLANCE TESTS

The weighing design used in a given surveillance test depends on the range of the set and ratio of the weights in the set to each other. Some suggested weighing designs for weight sets having 5,3,2,1; 5,2,2,1 and 5,2,1,1,Σ1 ratios are shown for various ranges. Other designs may be developed by using the principles outlines in section 2 for situations where the suggested weighing designs do not apply. The surveillance test weighing designs are shown with metric units of mass. But with a given design, customary units of mass can be substituted for the metric units, provided the ratios of the weights to each other are the same in both systems.

Weighing Designs for Type I Surveillance Test

Design 1

Set - Range: 1kg to 1mg   Ratio 5, 3, 2, 1

$$1kg - S1kg = a_1{*}$$
$$S1kg = \text{Standard } 1kg$$

$$1kg - \Sigma 1kg = a_2$$
$$\Sigma 1kg = 500g + 300g + 200g$$

$$200g - \Sigma 200g = a_3$$
$$\Sigma 200g = 100g + 50g + 30g + 20g$$

$$20g - \Sigma 20g = a_4$$
$$\Sigma 20g = 10g + 5g + 3g + 2g$$

$$2g - \Sigma 2g = a_5$$
$$\Sigma 2g = 1g + 500mg + 300mg + 200mg$$

$$200mg - \Sigma 200mg = a_6$$
$$\Sigma 200mg = 100mg + 50mg + 30mg + 20mg$$

$$20mg - \Sigma 20mg = a_7$$
$$\Sigma 20mg = 10mg + 5mg + 3mg + 2mg$$

$$3mg - \Sigma 3mg = a_8$$
$$\Sigma 3mg = 2mg + 1mg$$

* If a known 1kg weight suitable for use as a standard is not available, this "a" is omitted and 1kg - Σ1kg becomes the first "a", 200g - Σ200g = a_2, 20g - Σ20g = a_3, etc.

Design 2

Set - Range: 100g to 1mg Ratio: 5, 3, 2, 1

$$100g - S100g = a_1*$$
$$S100g = \text{Standard } 100g \text{ Weight}$$

$$100g - \Sigma 100g = a_2$$
$$\Sigma 100g = 50g + 30g + 20g$$

$$20g - \Sigma 20g = a_3$$
$$\Sigma 20g = 10g + 5g + 3g + 2g$$

$$2g - \Sigma 2g = a_4$$
$$\Sigma 2g = 1g + 500mg + 300mg + 200mg$$

$$200mg - \Sigma 200mg = a_5$$
$$\Sigma 200mg = 100mg + 50mg + 30mg + 20mg$$

$$20mg - \Sigma 20mg = a_6$$
$$\Sigma 20mg = 10mg + 5mg + 3mg + 2mg$$

$$3mg - \Sigma 3mg = a_7$$
$$3mg = 2mg + 1mg$$

For sets in which the smallest weight is 1g, the last "a" would be:

$$3g - \Sigma 3g = a$$
$$\Sigma 3g = 2g + 1g$$

* If a known 100g weight suitable for use as a standard is not available, this "a" is omitted and 100g - Σ100g becomes the first "a", 20g - Σ20g = a_2, etc.

Design 3

Set - Range: 1kg to 1mg   Ratio: 5, 2, 2, 1

$$1kg - S1kg = a_1{}^*$$

$$S1kg = \text{Standard } 1kg \text{ Weight}$$

$$1kg - \Sigma 1kg = a_2$$

$$\Sigma 1kg = 500g + 200g_1 + 200g_2 + 100g$$

$$100g - \Sigma 100g = a_3$$

$$\Sigma 100g = 50g + 20g_1 + 20g_2 + 10g$$

$$10g - \Sigma 10g = a_4$$

$$\Sigma 10g = 5g + 2g_1 + 2g_2 + 1g$$

$$1g - \Sigma 1g = a_5$$

$$\Sigma 1g = 500mg + 200mg_1 + 200mg_2 + 100mg$$

$$100mg - \Sigma 100mg = a_6$$

$$\Sigma 100mg = 50mg + 20mg_1 + 20mg_2 + 10mg$$

$$10mg - \Sigma 10mg = a_7$$

$$\Sigma 10mg = 5mg + 2mg_1 + 2mg_2 + 1mg$$

* If a known 1kg weight suitable for use as a standard is not available, this "a" is omitted and 1kg - Σ1kg becomes the first "a", and 100g - Σ100g = a_2, 10g - Σ10g = a_3, etc.

Design 4

Set - Range: 100g - 1mg Ratio: 5, 2, 2, 1

$$100g - S100g = a_1{}^*$$

$$S100g = \text{Standard } 100g \text{ Weight}$$

$$100g - \Sigma 100g = a_2$$

$$\Sigma 100g = 50g + 20g_1 + 20g_2 + 10g$$

$$10g - \Sigma 10g = a_3$$

$$\Sigma 10g = 5g + 2g_1 + 2g_2 + 1g$$

$$1g - \Sigma 1g = a_4$$

$$\Sigma 1g = 500mg + 200mg_1 + 200mg_2 + 100mg$$

$$100mg - \Sigma 100mg = a_5$$

$$\Sigma 100mg = 50mg + 20mg_1 + 20mg_2 + 10mg$$

$$10mg - \Sigma 10mg = a_6$$

$$\Sigma 10mg = 5mg + 2mg_1 + 2mg_2 + 1mg$$

For the set in which the smallest weight is 1g, the last "a" would be:

$$5g - \Sigma 5g = a$$

$$\Sigma 5g = 2g_1 + 2g_2 + 1g$$

* If a known 100g weight suitable for use as a standard is not available, this "a" is omitted and 100g - Σ100g becomes the first "a", and 10g - Σ10g = a_2, etc.

Design 5

Set - Range: 30kg - 1mg   Ratio: 5, 3, 2, 1

$$30kg - \Sigma 30kg = a_1$$
$$\Sigma 30kg = 20kg + 10kg$$

$$10kg - \Sigma 10kg = a_2$$
$$\Sigma 10kg = 5kg + 3kg + 2kg$$

$$2kg - \Sigma 2kg = a_3$$
$$\Sigma 2kg = 1kg_1 + 1kg_2$$

$$1kg_1 - \Sigma 1kg = a_4$$
$$\Sigma 1kg = 500g + 300g + 200g$$

$$200g - \Sigma 200g = a_5$$
$$\Sigma 200g = 100g + 50g + 30g + 20g$$

$$20g - \Sigma 20g = a_6$$
$$\Sigma 20g = 10g + 5g + 3g + 2g$$

$$2g - \Sigma 2g = a_7$$
$$\Sigma 2g = 1g + 500mg + 300mg + 200mg$$

$$200mg - \Sigma 200mg = a_8$$
$$\Sigma 200mg = 100mg + 50mg + 30mg + 20mg$$

$$20mg - \Sigma 20mg = a_9$$
$$\Sigma 20mg = 10mg + 5mg + 3mg + 2mg$$

$$2mg - \Sigma 2mg = a_{10}$$
$$\Sigma 2mg = 1mg_1 - 1mg_2$$

Design 6

Set - Range: 100g - 1mg   Ratio: 5, 2, 1, 1, Σ1

$$100g - S100g = a_1{*}$$
$$S100g = \text{Standard } 100g \text{ weight}$$

$$100g - \Sigma 100g = a_2$$
$$\Sigma 100g = 50g + 20g + 10g_1 + 10g_2 + \Sigma 10g$$

$$10g_1 - \Sigma 10g = a_3$$
$$\Sigma 10g = 5g + 2g + 1g_1 + 1g_2 + \Sigma 1g$$

$$1g_1 - \Sigma 1g = a_4$$
$$\Sigma 1g = 500mg + 200mg + 100mg_1 + 100mg_2 + \Sigma 100mg$$

$$100mg_1 - \Sigma 100mg = a_5$$
$$\Sigma 100mg = 50mg + 20mg + 10mg_1 + 10mg_2 + \Sigma 10mg$$

$$10mg_1 - \Sigma 10mg = a_6$$
$$\Sigma 10mg = 5mg + 2mg + 1mg_1 + 1mg_2 + 1mg_3$$

When comparing the unit of weight of a given decade of weights with summation of smaller weights from a weight set in which the ratio of the weights to each other 5, 2, 1, 1, 1, it is necessary to include all of the set's weights smaller than the unit weight to which summation is being compared. For example: in the comparison 100g - Σ100g, the Σ100g includes all of the weights in the set smaller than 100g; and in the comparison 10g - Σ10g the Σ10g includes all of the weights smaller than 10g and so on.

* If a known 100g weight suitable for use as a standard is not available, this "a" is omitted and 100g - Σ100g becomes the first "a" and 10g - Σ10g = a_2, etc.

Design 7

Set - Range: 1kg to 1mg Ratio: 5, 3, 2, 1

Series 1

$$\begin{aligned} S1kg - 1kg \quad\quad\quad &= a_1{}^* \\ S1kg \quad\quad - \Sigma 1kg &= a_2 \\ 1kg - \Sigma 1kg &= a_3 \end{aligned}$$

S1kg = Standard 1kg

Σ1kg = 500g + 300g + 200g

Series 2

$$\begin{aligned} 300g - \Sigma 300g_1 \quad\quad\quad &= a_1 \\ 300g \quad\quad - \Sigma 300g_2 &= a_2 \\ \Sigma 300g_1 - \Sigma 300g_2 &= a_3 \end{aligned}$$

$\Sigma 300g_1$ = 200g + 100g

$\Sigma 300g_2$ = 200g + 50g + 30g + 20g

Series 3

$$\begin{aligned} 30g - \Sigma 30g_1 \quad\quad\quad &= a_1 \\ 30g \quad\quad - \Sigma 30g_2 &= a_2 \\ \Sigma 30g_1 - \Sigma 30g_2 &= a_3 \end{aligned}$$

$\Sigma 30g_1$ = 20g + 10g

$\Sigma 30g_2$ = 20g + 5g + 3g + 2g

Series 4

$$\begin{aligned} 3g - \Sigma 3g_1 \quad\quad\quad &= a_1 \\ 3g \quad\quad - \Sigma 3g_2 &= a_2 \\ \Sigma 3g_1 - \Sigma 3g_2 &= a_3 \end{aligned}$$

$\Sigma 3g_1$ = 2g + 1g

$\Sigma 3g_2$ = 2g + 500mg + 300mg + 200mg

Series 5

$$\begin{aligned} 300mg - \Sigma 300mg_1 &= a_1 \\ 300mg - \Sigma 300mg_2 &= a_2 \\ \Sigma 300mg_1 - \Sigma 300mg_2 &= a_3 \end{aligned}$$

$$\Sigma 300mg_1 = 200mg + 100mg$$

$$\Sigma 300mg_2 = 200mg + 50mg + 30mg + 20mg$$

Series 6

$$\begin{aligned} 30mg - \Sigma 30mg_1 &= a_1 \\ 30mg - \Sigma 30mg_2 &= a_2 \\ \Sigma 30mg_1 - \Sigma 30mg_2 &= a_3 \end{aligned}$$

$$\Sigma 30mg_1 = 20mg + 10mg$$

$$\Sigma 30mg_2 = 20mg + 5mg + 3mg + 2mg$$

Series 7

$$\begin{aligned} 3mg - \Sigma 3mg_1 &= a_1 \\ 3mg - \Sigma 3mg_2 &= a_2 \\ \Sigma 3mg_1 - \Sigma 3mg_2 &= a_3 \end{aligned}$$

$$\Sigma 3mg_1 = 2mg + 1mg_1$$

$$\Sigma 3mg_2 = 2mg + 1mg_2{}^{**}$$

* If a known 1kg weight, suitable for use as a standard is not available, any 1kg or Σ1kg may be used to fill the series. Then the 1kg of the set is used as the standard and the first series of measurements is:

$$\begin{aligned} 1kg - 1kg' &= a_1 \\ 1kg - \Sigma 1kg &= a_2 \\ 1kg' - \Sigma 1kg &= a_3 \end{aligned}$$

where 1kg' is either the 1kg weight or the Σ1kg used to complete the series.

** The $1mg_2$ is an extra 1mg weight used to fill the last series.

Design 8

Series - Range: 100g to 1mg Ratio: 5, 3, 2, 1

Series 1

$$S100g - 100g = a_1{}^*$$
$$S100g - \Sigma 100g = a_2$$
$$100g - \Sigma 100g = a_3$$

$$\Sigma 100g = 50g + 30g + 20g$$

Series 2

$$30g - \Sigma 30g_1 = a_1$$
$$30g - \Sigma 30g_2 = a_2$$
$$\Sigma 30g_1 - \Sigma 30g_2 = a_3$$

$$\Sigma 30g_1 = 20g + 10g$$
$$\Sigma 30g_2 = 20g + 5g + 3g + 2g$$

Series 3

$$3g - \Sigma 3g_1 = a_1$$
$$3g - \Sigma 3g_2 = a_2$$
$$\Sigma 3g_1 - \Sigma 3g_2 = a_3$$

$$\Sigma 3g_1 = 2g + 1g$$
$$\Sigma 3g_2 = 2g + 500mg + 300mg + 200mg$$

Series 4

$$300mg - \Sigma 300mg_1 = a_1$$
$$300mg - \Sigma 300mg_2 = a_2$$
$$\Sigma 300mg_1 - \Sigma 300mg_2 = a_3$$

$$\Sigma 300mg_1 = 200mg + 100mg$$
$$\Sigma 300mg_2 = 200mg + 50mg + 30mg + 20mg$$

Series 5

$$30mg - \Sigma 30mg_1 = a_1$$
$$30mg - \Sigma 30mg_2 = a_2$$
$$\Sigma 30mg_1 - \Sigma 30mg_2 = a_3$$

$$\Sigma 30mg_1 = 20mg + 10mg$$
$$\Sigma 30mg_2 = 20mg + 5mg + 3mg + 2mg$$

Series 6

$$3mg - \Sigma 3mg_1 = a_1$$
$$3mg - \Sigma 3mg_2 = a_2$$
$$\Sigma 3mg_1 - \Sigma 3mg_2 = a_3$$

$$\Sigma 3mg_1 = 2mg + 1mg_1$$
$$\Sigma 3mg_2 = 2mg + 1mg_2\text{**}$$

* If a known 100g weight suitable for use as a standard is not available, any 100g weight or Σ100g weight may be used to fill the first series. Then the 100g of the set is used as the standard and the first series of measurement is:

$$100g - 100g' = a_1$$
$$100g - \Sigma 100g = a_2$$
$$100g' - \Sigma 100g = a_3$$

where 100g' is either the 100g or the Σ100g used to complete the series.

** The $1mg_2$ is an extra 1mg weight used to fill the last series.

Design 9

Set - Range: 1kg to 1mg Ratio: 5, 2, 2, 1

Series 1

$$\begin{aligned} S1kg - 1kg \quad &= a_1^* \\ S1kg \quad - \Sigma 1kg &= a_2 \\ 1kg - \Sigma 1kg &= a_3 \end{aligned}$$

$$\Sigma 1kg = 500g + 200g_1 + 200g_2 + 100g$$

Series 2

$$\begin{aligned} 200g_1 - 200g_2 \quad &= a_1 \\ 200g_1 \quad - \Sigma 200g &= a_2 \\ 200g_2 - \Sigma 200g &= a_3 \end{aligned}$$

$$\Sigma 200g = 100g + 50g + 20g_1 + 20g_2 + 10g$$

Series 3

$$\begin{aligned} 20g_1 - 20g_2 \quad &= a_1 \\ 20g_1 \quad - \Sigma 20g &= a_2 \\ 20g_2 - \Sigma 20g &= a_3 \end{aligned}$$

$$\Sigma 20g = 10g + 5g + 2g_1 + 2g_2 + 1g$$

Series 4

$$\begin{aligned} 2g_1 - 2g_2 \quad &= a_1 \\ 2g_1 \quad - \Sigma 2g &= a_2 \\ 2g_2 - \Sigma 2g &= a_3 \end{aligned}$$

$$\Sigma 2g = 1g + 500mg + 200mg_1 + 200mg_2 + 100mg$$

Series 5

$$
\begin{aligned}
200mg_1 - 200mg_2 \quad &= a_1 \\
200mg_1 \quad - \Sigma 200mg &= a_2 \\
200mg_2 - \Sigma 200mg &= a_3
\end{aligned}
$$

$$\Sigma 200mg = 100mg + 50mg + 20mg_1 + 20mg_2 + 10mg$$

Series 6

$$
\begin{aligned}
20mg_1 - 20mg_2 \quad &= a_1 \\
20mg_1 \quad - \Sigma 20mg &= a_2 \\
20mg_2 - \Sigma 20mg &= a_3
\end{aligned}
$$

$$\Sigma 20mg = 10mg + 5mg_1 + 2mg_1 + 2mg_2 + 1mg$$

Series 7

$$
\begin{aligned}
2mg_1 - 2mg_2 \quad &= a_1 \\
2mg_1 \quad - \Sigma 2mg &= a_2 \\
2mg_2 - \Sigma 2mg &= a_3
\end{aligned}
$$

$$\Sigma 2mg = 1mg_1 + 1mg_2{}^{**}$$

* If a known 1kg weight suitable for use as a standard is not available, any 1kg weight or Σ1kg weight may be used to fill the series. Then the 1kg of the set is used as the standard and the first series of measurements is:

$$
\begin{aligned}
1kg - 1kg' &= a_1 \\
1kg - \Sigma 1kg &= a_2 \\
1kg' - \Sigma 1kg &= a_3
\end{aligned}
$$

where 1kg' is either the 1kg weight or the Σ1kg used to complete the series.

** The $1mg_2$ is an extra 1mg weight used to fill the last series.

Design 10

Set - Range: 30kg to 1mg   Ratio: 5, 3, 2, 1

Series 1

$$30kg - \Sigma 30kg_1 = a_1$$
$$30kg - \Sigma 30kg_2 = a_2$$
$$\Sigma 30kg_1 - \Sigma 30kg_2 = a_3$$

$$\Sigma 30kg_1 = 20kg + 10kg$$
$$\Sigma 30kg_2 = 20kg + 5kg + 3kg + 2kg$$

Series 2

$$3kg - \Sigma 3kg_1 = a_1$$
$$3kg - \Sigma 3kg_2 = a_2$$
$$\Sigma 3kg_1 - \Sigma 3kg_2 = a_3$$

$$\Sigma 3kg_1 = 2kg + 1kg.$$
$$\Sigma 3kg_2 = 2kg + 1kg..$$

Series 3

$$1kg. - 1kg.. = a_1$$
$$1kg. - \Sigma 1kg = a_2$$
$$1kg.. - \Sigma 1kg = a_3$$

$$\Sigma 1kg = 500g + 300g + 200g$$

Series 4

$$300g - \Sigma 300g_1 = a_1$$
$$300g - \Sigma 300g_2 = a_2$$
$$\Sigma 300g_1 - \Sigma 300g_2 = a_3$$

$$\Sigma 300g_1 = 200g + 100g$$
$$\Sigma 300g_2 = 200g + 50g + 30g + 20g$$

Series 5

$$30g - \Sigma 30g_1 = a_1$$
$$30g - \Sigma 30g_2 = a_2$$
$$\Sigma 30g_1 - \Sigma 30g_2 = a_3$$

$$\Sigma 30g_1 = 20g + 10g$$
$$\Sigma 30g_2 = 20g + 5g + 3g + 2g$$

Series 6

$$3g - \Sigma 3g_1 = a_1$$
$$3g - \Sigma 3g_2 = a_2$$
$$\Sigma 3g_1 - \Sigma 3g_2 = a_3$$

$$\Sigma 3g_1 = 2g + 1g$$
$$\Sigma 3g_2 = 2g + 500mg + 300mg + 200mg$$

Series 7

$$300mg - \Sigma 300mg_1 = a_1$$
$$300mg - \Sigma 300mg_2 = a_2$$
$$\Sigma 300mg_1 - \Sigma 300mg_2 = a_3$$

$$\Sigma 300mg_1 = 200mg + 100mg$$
$$\Sigma 300mg_2 = 200mg + 50mg + 30mg + 20mg$$

Series 8

$$30mg - \Sigma 30mg_1 = a_1$$
$$30mg - \Sigma 30mg_2 = a_2$$
$$\Sigma 30mg_1 - \Sigma 30mg_2 = a_3$$

$$\Sigma 30mg_1 = 20mg + 10mg$$
$$\Sigma 30mg_2 = 20mg + 5mg + 3mg + 2mg$$

Series 9

$$3mg - \Sigma 3mg_1 = a_1$$
$$3mg - \Sigma 3mg_2 = a_2$$
$$\Sigma 3mg_1 - \Sigma 3mg_2 = a_3$$

$$\Sigma 3mg_1 = 2mg + 1mg_1$$
$$\Sigma 3mg_2 = 2mg + 1mg_2$$

Type I Surveillance Test Example

Example of a Type I surveillance test for a set of metric mass standards according to design 2, appendix 1. This set was calibrated by the National Bureau of Standards. The National Bureau of Standards Report of Calibration Test No. 200390 is reproduced on pages 45-48. The standards used (other than the set) are listed below together with their apparent mass corrections and uncertainties. The balances used and their standard deviation for one double substitution weighing are also listed. The double substitution weighing method is used. The standard deviation of the calibration mass measurement process is not known, so an estimate is computed from the reported uncertainties, as described in section 3.2, and used in computing the surveillance limits.

Standards	Apparent Mass Corr. (mg)	Volume (cm^3)	Uncertainty (mg)
S100g	- 0.019	12.822	0.015
h 10mg	+ 0.0450	0.0037	0.0006
h 5mg	+ 0.0045	0.0018	0.0005

Balance Laboratory Designation	Standard Deviation (mg)	Capacity (g)
H - 200	0.015	200g
M - 10	0.003	20g

Computation of Surveillance Limits (sl)

In the following, "U_s" will denote the uncertainty of the standard and "S.D." the standard deviation of the process:

For 100g - S100g

$$U_s = 0.015\text{mg}$$

$$\text{S.D.} = 0.015\text{mg}$$

$$\text{sl} = 0.015 + 3(0.015) = \underline{0.060\text{mg}}$$

In the following, "U_c" will denote the uncertainty (see Equation (2) Page 7, and "S.D." the standard deviation of the process:

For 100g - Σ100g

$$U_c = \sqrt{.015^2 + .011^2 + .012^2 + .010^2}$$

$$= \sqrt{.000225 + .000121 + .000144 + .0001}$$

$$= \sqrt{.00059}$$

$$U_c = 0.024\text{mg}$$

$$\text{S.D.} = 0.015\text{mg}$$

$$\text{sl} = 0.024 + 3(0.015) = \underline{0.069\text{mg}}$$

For 20g - Σ20g

$$U_c = \sqrt{.010^2 + .013^2 + .007^2 + .004^2 + .003^2}$$

$$= \sqrt{.0001 + .000169 + .000049 + .000016 + .000009}$$

$$= \sqrt{.000343}$$

$$U_c = 0.019\text{mg}$$

$$\text{S.D.} = 0.003\text{mg}$$

$$\text{sl} = 0.019 + 3(0.003) = \underline{0.028\text{mg}}$$

For $2g - \Sigma 2g$

$$U_c = \sqrt{.0032^2 + .0030^2 + .0016^2 + .0011^2 + .0008^2}$$

$$= \sqrt{.00001 + .000009 + .00000256 + .0000012 + .00000064}$$

$$= \sqrt{.00002341}$$

$U_c = 0.0048$

S.D. = 0.003mg

sl = 0.0048 + 3(0.003) = <u>0.014mg</u>

For $200mg - \Sigma 200mg$

$$U_c = \sqrt{.0008^2 + .0008^2 + .0005^2 + .0005^2 + .0005^2}$$

$$= \sqrt{.00000064 + .00000064 + .00000025 + .00000025 + .00000025}$$

$$= \sqrt{.00000203}$$

$U_c = 0.0014mg$

S.D. = 0.003mg

sl = 0.0014 + 3(0.003) = <u>0.010mg</u>

For $20mg - \Sigma 20mg$

$$U_c = \sqrt{.00042^2 + .00059^2 + .00049^2 + .00052^2 + .00045^2}$$

$$= \sqrt{.000000176 + .000000348 + .000000240 + .000000270 + .00000020}$$

$$= \sqrt{.000001234}$$

$U_c = 0.0011mg$

S.D. = 0.003mg

sl = 0.0011 + 3(0.003) = <u>0.010mg</u>

For 3mg - Σ3mg

$$U_c = \sqrt{.00052^2 + .00045^2 + .00059^2}$$

$$= \sqrt{.000000270 + .000000202 + .000000349}$$

$$= \sqrt{.00000082}$$

$$U_c = 0.00091\text{mg}$$

$$S.D. = 0.003\text{mg}$$

$$sl = 0.00091 + 3(0.003) = \underline{0.0099\text{mg}}$$

For the weighings made on the smaller balance, the uncertainty of the values of the weights is small compared to the standard deviation of that balance. Therefore, for all practical purposes, three times the standard deviation of the balance may be taken as the surveillance limit for these weighings.

Computation of Buoyancy Corrections

The buoyancy corrections, $\Delta\rho\Delta V$, are computed according to the procedure set forth in section 5.2, using the formula:

$$\text{buoyancy correction} = (\rho - \rho_n)(V_C - V_D)$$

Consider the weighing 100g - S100g = a

where ρ = 1.17mg/cm^3 air density at time of weighing

ρ_n = 1.20mg/cm^3 density of normal air

V_C = 12.821cm^3 volume of 100g weight of set under test, from Report of Calibration

V_D = 12.822cm^3 volume of S100g standard

$$\begin{aligned} \text{buoyancy correction} &= (1.17 - 1.20)(12.821 - 12.822) \\ &= (-0.03)(-0.001) \\ &= +0.00003\text{mg} \end{aligned}$$

This amount is insignificant compared to the surveillance limit of 0.060mg and may be ignored. Similarly, the differences in the volumes in the weighings 100g - Σ100g and 20g - Σ20g are small enough so that the buoyancy corrections are negligible. But, in the weighings 2g - Σ2g and 200mg - Σ200mg weights of differing densities are involved. Consequently, the volumes of the individual weight and the summation of weights are different.

The volumes are:

For the weighing 2g - Σ2g:

Weights			Volumes
2g			0.2564cm^3
	1g	0.1282cm^3	
	500mg	0.0301cm^3	
	300mg	0.0181cm^3	
	200mg	0.0120cm^3	
Σ2g			0.1884cm^3

The actual air density, ρ, is the same as for 100g - Σ100g.

$$\begin{aligned} \text{buoyancy correction} &= (1.17 - 1.20)(0.2564 - 0.1884) \\ &= (-0.03)(+.0680) \\ &= -0.0020\text{mg} \end{aligned}$$

This buoyancy correction, while relatively small compared to the surveillance limit, is not insignificant and must be applied to the measured difference between 2g and Σ2g.

For the weighing 200mg - Σ200mg:

Weights			Volumes
200mg			$0.01205 cm^3$
	100mg	$0.00602 cm^3$	
	50mg	$0.00301 cm^3$	
	30mg	$0.01111 cm^3$	
	20mg	$0.00741 cm^3$	
Σ200mg			$0.02755 cm^3$

$$\text{buoyancy correction} = (1.17 - 1.20)(0.01205 - 0.02755)$$
$$= (-0.03)(+0.01550)$$
$$= -0.00046\text{mg}$$

This buoyancy correction is small compared to the surveillance limits for this comparison and in most cases can be ignored.

For the weighing 20mg - Σ20mg:

Weights			Volumes
20mg			$0.00741 cm^3$
	10mg	$0.00371 cm^3$	
	5mg	$0.00185 cm^3$	
	3mg	$0.00111 cm^3$	
	2mg	$0.00074 cm^3$	
Σ20mg			$0.00741 cm^3$

The volumes of the two masses are equal, therefore the buoyancy correction is zero.

For the weighing 3mg - Σ3mg:

Weights			Volumes
3mg			$0.00111 cm^3$
	2mg	$0.00074 cm^3$	
	1mg	$0.00037 cm^3$	
Σ3mg			$0.00111 cm^3$

The volumes of the two masses are equal, therefore the buoyancy correction is zero.

Computation of Predicted or Expected Differences

The expected differences are computed from the reported values as follows: (see report on page 47).

For the weighing 100g - S100g:

Weights	Values 100g	Values S100g
100g	-0.058mg	
S100g		-0.019mg
Sums	-0.058mg	-0.019mg
Expected Diff. =	-0.039mg	

For the weighing 100g - Σ100g:

Weights	Values 100g	Values Σ100g
100g	-0.0589mg	
50g		-0.0133mg
30g		-0.0134mg
20g		+0.0330mg
Sums	-0.0589mg	+0.0063mg
Expected Diff. =	-0.065 mg	

For the weighing 20g - Σ20g:

Weights	Values 20g	Values Σ20g
20g	+0.0330mg	
10g		-0.0378mg
5g		-0.0065mg
3g		+0.0191mg
2g		+0.0066mg
Sums	+0.0330mg	-0.0186mg
Expected Diff. =	+0.0516mg	

For the weighing 2g - Σ2g:

Weights	Values 2g	Values Σ2g
2g	+0.0066mg	
1g		-0.0216mg
500mg		-0.0005mg
300mg		-0.0041mg
200mg		-0.0049mg
Sums	+0.0066mg	-0.0311mg
Expected Diff. =	+0.0377mg	

For the weighing 200mg - Σ200mg:

Weights	Values 200mg	Values Σ200mg
200mg	-0.0049mg	
100mg		+0.0008mg
50mg		+0.0074mg
30mg		-0.0049mg
20mg		-0.0020mg
Sums	-0.0049mg	+0.0013mg
Expected Diff. =	-0.0062mg	

For the weighing 20mg - Σ20mg:

Weights	Values 20mg	Values Σ20mg
20mg	-0.0020mg	
10mg		+0.0028mg
5mg		+0.0065mg
3mg		+0.0030mg
2mg		-0.0142mg
Sums	-0.0020mg	-0.0019mg
Expected Diff. =	-0.0001mg	

For the weighing 3mg - Σ3mg:

Weights	Values	
	3mg	Σ3mg
3mg	+0.0030mg	
2mg		-0.0142mg
1mg		+0.0091mg
Sums	+0.0030mg	-0.0051mg
Expected Diff. =	+0.0081mg	

SUMMARY

FOR CONVENIENCE, THE RESULTS OF THIS WORK ARE SUMMARIZED IN TABLES I AND II. THE VALUES ASSIGNED ARE WITH REFERENCE TO THE STANDARDS IDENTIFIED ON THE DATA SHEETS. THE UNCERTAINTY FIGURE IS AN EXPRESSION OF THE OVERALL UNCERTAINTY USING THREE STANDARD DEVIATIONS AS A LIMIT TO THE EFFECT OF RANDOM ERRORS OF THE MEASUREMENT ASSOCIATED WITH THE MEASUREMENT PROCESSES. THE MAGNITUDE OF SYSTEMATIC ERRORS FROM SOURCES OTHER THAN THE USE OF ACCEPTED VALUES FOR CERTAIN STARTING STANDARDS ARE CONSIDERED NEGLIGIBLE. IT SHOULD BE NOTED THAT THE MAGNITUDE OF THE UNCERTAINTY REFLECTS THE PERFORMANCE OF THE MEASUREMENT PROCESS USED TO ESTABLISH THESE VALUES. THE MASS UNIT, AS REALIZABLE IN ANOTHER MEASUREMENT PROCESS, WILL BE UNCERTAIN BY AN AMOUNT WHICH IS A COMBINATION OF THE UNCERTAINTY OF THIS PROCESS AND THE PROCESS IN WHICH THESE STANDARDS ARE USED.

THE ESTIMATED MASS VALUES LISTED IN TABLE I ARE BASED ON AN EXPLICIT TREATMENT OF DISPLACEMENT VOLUMES, E.G., 'TRUE MASS', 'MASS IN VACUO', MASS IN THE NEWTONIAN SENSE. THE DISPLACEMENT VOLUME ASSOCIATED WITH EACH VALUE IS LISTED AS WELL AS THE VOLUMETRIC COEFFICIENT OF EXPANSION. THESE VALUES SHOULD BE USED, TOGETHER WITH APPROPRIATE CORRECTION FOR THE BUOYANT EFFECTS OF THE ENVIRONMENT, TO ESTABLISH CONSISTENT MASS VALUES FOR OBJECTS WHICH DIFFER SIGNIFICANTLY IN DENSITY AND/OR FOR MEASUREMENTS WHICH MUST BE MADE IN DIFFERING ENVIRONMENTS. THE RELATION 1LB AVDP=.45359237KG IS USED AS REQUIRED.

THE ESTIMATED MASS VALUES LISTED IN TABLE II ARE BASED ON AN IMPLICIT TREATMENT OF DISPLACEMENT VOLUMES, E.G., 'APPARENT MASS', 'APPARENT MASS VERSUS BRASS', 'APPARENT MASS VERSUS DENSITY 8.0'. THE VALUES ARE LISTED AS CORRECTIONS TO BE APPLIED TO THE LISTED NOMINAL VALUE (A POSITIVE CORRECTION INDICATES THAT THE MASS IS LARGER THAN THE STATED NOMINAL VALUE BY THE AMOUNT OF THE CORRECTION). THESE VALUES ARE COMPUTED FROM THE VALUES BASED ON AN EXPLICIT TREATMENT OF DISPLACEMENT VOLUMES USING THE FOLLOWING DEFINING RELATIONS AND ARE UNCERTAIN BY THE AMOUNT SHOWN IN TABLE I.

THE ADJUSTMENT OF WEIGHTS TO MINIMIZE THE DEVIATION FROM NOMINAL ON THE BASIS OF 'NORMAL BRASS' (IN ACCORDANCE WITH COR. A BELOW) IS WIDESPREAD IN THIS COUNTRY AND IN MANY PARTS OF THE WORLD. VALUES STATED ON EITHER BASIS ARE INTERNALLY CONSISTENT AND DEFINITE. THERE IS, HOWEVER, A SYSTEMATIC DIFFERENCE BETWEEN THE VALUES ASSIGNED ON EACH BASIS, THE VALUE ON THE BASIS OF 'DENSITY 8.0' BEING 7 MICROGRAMS/GRAM LARGER THAN THE VALUE ON THE BASIS OF NORMAL BRASS. THIS SYSTEMATIC DIFFERENCE IS CLEARLY DETECTABLE ON MANY DIRECT READING BALANCES.

CORRECTION A - 'APPARENT MASS VERSUS BRASS' OR 'WEIGHT IN AIR AGAINST BRASS' IS DETERMINED BY A HYPOTHETICAL WEIGHING OF THE WEIGHT AT 20 CELSIUS IN AIR HAVING A DENSITY OF 1.2 MG/CM3, WITH A (NORMAL BRASS) STANDARD HAVING A DENSITY OF 8.4 G/CM3 AT 0 CELSIUS WHOSE COEFFICIENT OF VOLUMETRIC EXPANSION IS 0.000 054 PER DEGREE CELSIUS, AND WHOSE VALUE IS BASED ON ITS TRUE MASS OR WEIGHT IN VACUO.

COMPANY X
NEW YORK, NEW YORK
SET OF MASS STANDARDS 100G TO 1MG
TEST NUMBER 232.09/200390

1/30/70

CORRECTION B - 'APPARENT MASS VERSUS DENSITY 8.0' IS DETERMINED BY A HYPOTHETICAL WEIGHING OF THE WEIGHT, IN AIR HAVING A DENSITY OF 1.2 MG/CM3, WITH A STANDARD HAVING A DENSITY OF 8.0 G/CM3 AT 20 CELSIUS, AND WHOSE VALUE IS BASED ON ITS TRUE MASS OR WEIGHT IN AIR.

SAMPLE REPORT (CONTINUED)

COMPANY X
NEW YORK, NEW YORK
SET OF MASS STANDARDS 100G TO 1MG
TEST NUMBER 232.08/2[illegible]130[illegible]

1/3 / 7

TABLE I

ITEM	MASS (G)	UNCERTAINTY (G)	VOL AT 20 (CM3)	COEF OF EXP
100G	100.00132471	.00001596	12.82064	.000045
50G	50.00052848	.00001128	6.41032	.000045
30G	30.00031168	.00001166	3.84619	.000045
20G	20.00024971	.00000996	2.56413	.000045
10G	10.00007056	.00001287	1.28206	.000045
5G	5.00004767	.00000667	.64103	.000045
3G	3.00005156	.00000446	.38462	.000045
2G	2.00002831	.00000325	.25641	.000045
1G	.99998924	.00000296	.12820	.000045
500MG	.49996400	.00000155	.03012	.000020
300MG	.29997462	.00000177	.01807	.000020
200MG	.19998[illegible]	.00000079	.01205	.000020
100MG	.09999375	.00000076	.00602	.000020
50MG	.05000386	.00000054	.00301	.000020
30MG	.03000416	.00000054	.01111	.000069
20MG	.02000436	.00000044	.00741	.000069
10MG	.01000583	.00000059	.00371	.000069
5MG	.00500803	.00000049	.00195	.000069
3MG	.00300395	.00000052	.00111	.000069
2MG	.00198636	.00000045	.00074	.000069
1MG	.00100943	.00000059	.00037	.000069

SAMPLE REPORT
(continued)

COMPANY X
NEW YORK, NEW YORK
SET OF MASS STANDARDS 100G TO 1MG
TEST NUMBER 232.20/20:39

1/3./ 7

TABLE II

ITEM	COR.A (MG)	COR.B (MG)
100G	-.05893	.64003
50G	-.01333	.33615
30G	-.01342	.19627
20G	.03298	.17277
10G	-.03781	.03209
5G	-.00651	.02844
3G	.01935	.04002
2G	.00664	.02062
1G	-.02163	-.01461
500MG	-.00055	.00294
300MG	-.00409	-.00199
200MG	-.00495	-.00356
100MG	.00082	.00152
50MG	.00743	.00775
30MG	-.00488	-.00468
20MG	-.00198	-.00184
10MG	.00282	.00289
5MG	.00652	.00656
3MG	.00375	.00387
2MG	-.01424	-.01423
1MG	.00913	.00913

Temperature 23.5 °C

Humidity 40 %

Barometer 748.2

ρ = 1.17

FORM NBS-345.06 (6-1-61)

U.S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

SUBSTITUTION WEIGHING
Single Pan Damped Balances
OBSERVATION SHEET

Sheet 1

Unit (Wts.) g

Unit (Cr. & Diff.) mg

Observer	Balance	Date	Set	NBS Test No.
C.B.G.	H-200	10-12-72		200390

Load	Dial Setting	Scale Reading	Computations
100g	100.0g	2.34	a_1: −0.04
S 100g	"	2.38	−0.05
			$-0.04 \times \frac{10.045}{10.06} = -0.04$ mg
S 100g + h 10mg	"	12.44	
100g + h 10mg	"	12.39	Expected Diff. = −0.039 mg
100g	"	2.36	a_2: −0.07
Σ 100g	"	2.43	−0.07
			$-0.07 \times \frac{10.045}{10.05} = -0.07$ mg
" + h 10mg	"	12.48	
100g + h 10mg	"	12.41	Expected Diff = −0.065 mg
20g	20.0	2.52	a_3: +0.05
Σ 20g	"	2.47	+0.06
			$+0.06 \times \frac{10.045}{10.05} = +0.06$
" + h 10mg	"	12.51	
20g + h 10mg	.	12.57	Expected Diff. = +0.516
	.	.	a
	.	.	
	.	.	
	.	.	
	.	.	a
	.	.	
	.	.	
	.	.	
"	"	.	Obs. D_s = .
			M_s = .

Temperature 23.6 °C	FORM NBS-345.06 (6-1-61)	U.S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS	Sheet 2
Humidity 40 %	**SUBSTITUTION WEIGHING**		Unit (Wts.) g and mg
Barometer 748.0	Single Pan Damped Balances		Unit (Cr. & Diff.) mg
ρ = 1.17	OBSERVATION SHEET		

Observer	Balance	Date	Set	NBS Test No.
C.B.G	M-10	10-12-72		200390

Load	Dial Setting	Scale Reading	Computations	
2g	2.0g	0.342	+0.040 +0.041	a_4
Σ2g	"	0.302	$+0.040 \times \frac{5.0051}{5.005} = +0.040$ mg	
Σ2g + h 5mg	"	5.307	Buoyancy cr. −0.002 " Diff. +0.038 "	
2g + h 5mg	"	5.348	Expected Diff. = +0.0375 mg	
200mg	0.20	0.223	−0.007 −0.006	a_5
Σ200 mg	"	0.230	$-0.006 \times \frac{5.0054}{5.006} = -0.006$ mg	
Σ200 mg + h 5mg	"	5.236	Buoyancy cr. +0.0046 " Diff. −0.0014 "	
200 mg + h 5mg	"	5.230	Expected Diff. = −0.0062 "	
20 mg	0.02	0.228	−0.001 0.000	a_6
Σ20 mg	"	0.229	$0.000 \times \frac{5.0054}{5.006} = 0.000$ mg	
Σ20mg + h 5mg	"	5.235		
20mg + h 5mg	"	5.235	Expected Diff. = −0.0001 mg	
3 mg	0.00	3.338	+0.008 +0.010	a_7
Σ3 mg	"	3.330	$+0.009 \times \frac{5.0054}{5.005} = +0.009$ mg	
Σ3mg + h 5mg	"	8.335		
3 mg + h 5mg	"	8.345	Expected Diff. = +0.0081 mg	
				a
"	"		Obs. D_s = M_s =	

Type II Surveillance Test Example

This example of a Type II Surveillance Test is for a set of metric mass standards according to Design 7, appendix 1. This set was calibrated by the National Bureau of Standards and reported under National Bureau of Standards Report of Calibration, Test No. 200390. The report is reproduced on page 47. This is the same set which was used for the example of the Type I Surveillance Test. The only standards (other than the set under test) used in this example are the sensitivity weights listed below, with their apparent mass corrections and uncertainties. The balances used and their standard deviation for a double substitution weighing are also listed. The double substitution method of weighing is used. The standard deviation of the calibration mass measurement is not known, so an estimate is computed from the reported uncertainties of the weights being tested, as described in section 3.2, and used in computing the surveillance limits.

Sensitivity Weight	Apparent Mass Corr. (mg)	Uncertainty (mg)
h10mg	+ 0.0450	0.0006
h 5mg	+ 0.0045	0.0005

Balance (Laboratory Designation)	Standard Deviation (mg)	Capacity g
H-200	0.015	200g
M-10	0.003	20g

Computation of Surveillance Limit (sl)

In the following "U_c" will denote the uncertainty (see Equation (2) Page 7) and "S.D." the estimate of the standard deviation. σ_B denotes the standard deviation of the balance used.

For series 1: 100g, 100g', Σ100g

Weight	U_c(mg)
100g	0.015
100g'	UNKNOWN MASS USED TO FILL SERIES
50g	0.011
30g	0.012
20g	0.010

Standard deviation of balance H-200 = 0.015mg

$$U_c = \sqrt{.015^2 + .011^2 + .012^2 + .010^2}$$

$$= \sqrt{.000225 + .000121 + .000144 + .0001}$$

$$= \sqrt{.00059}$$

$$U_c = 0.024\text{mg}$$

$$\text{S.D.} = 3\sqrt{2/3}\,\sigma_B$$

$$= 3\sqrt{2/3}(.015)$$

$$= 3\sqrt{.66666}\ (.015)$$

$$= 3(.81649)(.015)$$

$$\text{S.D.} = 0.037\text{mg}$$

sl for Σ100g = 0.024 + 0.037 = <u>0.061mg</u>

Since 100g' is assumed to be an unknown weight, a surveillance limit for it cannot be computed.

For series 2: 30g, $\Sigma 30g_1$, $\Sigma 30g_2$

Weight	U_c (mg)
30g	0.012
20g	0.010
10g	0.013
5g	0.0067
3g	0.0045
2g	0.0032

Standard deviation of balance H-200 = 0.015mg

$$U_c = \sqrt{.012^2 + .010^2 + .013^2}$$

$$= \sqrt{.000144 + .0001 + .000169}$$

$$= \sqrt{.000413}$$

$$U_c = 0.020\text{mg}$$

S.D. = SAME AS IN SERIES 1 (0.037mg)

sl for $\Sigma 30g_1$ = 0.020 + 0.037 = <u>0.057mg</u>

$$U_c = \sqrt{.012^2 + .010^2 + .0067^2 + .0045^2 + .0032^2}$$

$$= \sqrt{.000144 + .0001 + .0000448 + .00002025 + .00001024}$$

$$= \sqrt{.000319}$$

$$U_c = 0.018\text{mg}$$

S.D. = SAME AS IN SERIES 1 (0.037mg)

sl for $\Sigma 30g_2$ = 0.018 + 0.037 = <u>0.055mg</u>

For series 3: 3g, $\Sigma 3g_1$, $\Sigma 3g_2$

Weight	U_c (mg)
3 g	.0045
2 g	.0032
1 g	.0030
500mg	.0016
300mg	.0011
200mg	.0008

Standard deviation of balance M-10 = 0.003mg

$$U_c = \sqrt{.0045^2 + .0032^2 + .0030^2}$$

$$= \sqrt{.00002025 + .00001024 + .000009}$$

$$= \sqrt{.00003949}$$

$$U_c = 0.0063\text{mg}$$

$$\text{S.D.} = 3\sqrt{2/3}\sigma_B$$

$$= 3\sqrt{2/3}(.003)$$

$$= 3\sqrt{.66666}\ (.003)$$

$$= 3(.81649)(.003)$$

$$\text{S.D.} = 0.0073\text{mg}$$

sl for $\Sigma 3g_1$ = 0.0063 + 0.0073 = <u>0.014mg</u>

$$U_c = \sqrt{.0045^2 + .0032^2 + .0016^2 + .0011^2 + .0008^2}$$

$$= \sqrt{.00002025 + .00001024 + .00000256 + .00000121 + .00000064}$$

$$= \sqrt{.0000349}$$

$$U_c = 0.0059\text{mg}$$

S.D. = SAME AS FOR $\Sigma 3g_1$(0.0073mg)

sl for $\Sigma 3g_2$ = 0.0059 + 0.0073 = <u>0.013mg</u>

For series 4: 300mg, $\Sigma 300mg_1$, $\Sigma 300mg_2$

Weight	U_c (mg)
300mg	0.0011
200mg	0.0008
100mg	0.0008
50mg	0.0005
30mg	0.0005
20mg	0.0005

Standard deviation of balance M-10 = 0.003mg

$$U_c = \sqrt{.0011^2 + .0008^2 + .0008^2}$$

$$= \sqrt{.00000121 + .00000064 + .00000064}$$

$$= \sqrt{.00000249}$$

$$U_c = 0.0016\text{mg}$$

S.D. = SAME AS IN SERIES 3 (0.0073mg)

sl for $\Sigma 300g_1$ = 0.0016 + 0.0073 = <u>0.0089mg</u>

$$U_c = \sqrt{.0011^2 + .0008^2 + .0005^2 + .0005^2 + .0005^2}$$

$$= \sqrt{.00000121 + .00000064 + .00000025 + .00000025 + .00000025}$$

$$= \sqrt{.0000026}$$

$$U_c = 0.0016\text{mg}$$

S.D. = SAME AS FOR $\Sigma 300mg_1$ (0.0073mg)

sl for $\Sigma 300mg_2$ = 0.0016 + 0.0073 = <u>0.0089mg</u>

For series 5: 30mg, $\Sigma 30mg_1$, $\Sigma 30mg_2$

Weight	U_c (mg)
30mg	0.00054
20mg	0.00046
10mg	0.00059
5mg	0.00049
3mg	0.00052
2mg	0.00045

Standard deviation for balance M-10 = 0.003mg

$$U_c = \sqrt{.00054^2 + .00046^2 + .00059^2}$$

$$= \sqrt{.000000291 + .0000002116 + .0000003481}$$

$$= \sqrt{.0000008513}$$

$$U_c = 0.00092\text{mg}$$

S.D. = SAME AS IN SERIES 4 (0.0073mg)

sl for $\Sigma 30mg_1$ = 0.00092 + 0.0073 = <u>0.0082mg</u>

$$U_c = \sqrt{.00054^2 + .00046^2 + .00049^2 + .00052^2 + .00045^2}$$

$$= \sqrt{.0000002916 + .0000002116 + .0000002401 + .0000002704 + .0000002025}$$

$$= \sqrt{.0000012162}$$

$$U_c = 0.0011\text{mg}$$

S.D. = SAME AS FOR $\Sigma 30mg_1$ (0.0073mg)

sl for $\Sigma 30mg_2$ = 0.0011 + 0.0073 = <u>0.0084mg</u>

For 5mg - Σ5mg:

Weight	U_c (mg)
5mg	0.00049
3mg	0.00052
2mg	0.00045

Standard Deviation of Balance M-10 = 0.003mg

$$U_c = \sqrt{.00049^2 + .00052^2 + .00045^2}$$

$$= \sqrt{.0000002401 \ \_ \ .0000002704 + .0000002025}$$

$$= \sqrt{.000000713}$$

$$U_c = 0.00084\text{mg}$$

$$\begin{aligned} sl &= .00084 + 3(.003) \\ &= .00084 + .009 = \underline{0.0098\text{mg}} \end{aligned}$$

For 3mg - Σ3mg

Weight	U_c (mg)
3mg	0.00052
2mg	0.00045
1mg	0.00059

Standard Deviation of Balance M-10 = 0.003mg

$$U_c = \sqrt{.00052^2 + .00045^2 + .00059^2}$$

$$= \sqrt{.0000002704 + .0000002025 + .0000003481}$$

$$= \sqrt{.000000821}$$

$$U_c = 0.00091\text{mg}$$

$$\begin{aligned} sl &= 0.00091 + 3(.003) \\ &= 0.00091 + .009 = \underline{0.0099\text{mg}} \end{aligned}$$

Buoyancy Corrections

The buoyancy corrections $\Delta\rho\Delta V$ are computed according to the procedure set forth in section 5.2 using the formula:

$$\text{Buoyancy Correction} = (\rho - \rho_n)(V_C - V_D)$$

In this example, only two of the comparisons, $3g - \Sigma 3g_2$ and $300mg - \Sigma 300mg$, are between weights having different densities. A buoyancy correction need be computed only for these two comparisons. All of the other comparisons are between weights having the same density, so their volume differences are virtually zero and the buoyancy corrections are also virtually zero.

The buoyancy correction for the comparison $3g - \Sigma 3g_2$, a_2 of series 3, is:

Weights			Volumes	
3g			$0.3846 cm^3$	from Report of Calibration
	2g	$0.2564 cm^3$		"
	500mg	$0.0301 cm^3$		"
	300mg	$0.0181 cm^3$		"
	200mg	$0.0120 cm^3$		"
$\Sigma 3g$			$0.3166 cm^3$	"

$\rho = 1.16 mg/cm^3$ Air density at time of weighing

$\rho_n = 1.20 mg/cm^3$ Normal air density

Buoyancy correction = (1.16 - 1.20)(0.3846 - 0.3166) = -0.0027mg

This is the figure entered on the 3-1's computation sheet under the $\Sigma 3g_2$ column on the $-\Delta\rho\Delta V$ line, Sheet 2, Series 3. Note that it is $-\Delta\rho\Delta V$ that is called for and the buoyancy correction is -0.0027mg, therefore, the buoyancy correction is entered as +0.0027mg. (See section 5.3.2).

The buoyancy correction for the comparison $300mg - \Sigma 300mg_2$, a_2 of series 4, is:

Weight			Volume	
300mg			$0.0181 cm^3$	From Report of Calibration
	200mg	$0.0120 cm^3$		"
	50mg	$0.0030 cm^3$		"
	30mg	$0.0111 cm^3$		"
	20mg	$0.0074 cm^3$		"
Σ300mg			$0.0335 cm^3$	"

$\rho = 1.16 mg/cm^3$ Air density at time of weighing

$\rho_n = 1.20 mg/cm^3$ Normal air density

Buoyancy Correction = $(1.16 - 1.20)(0.0181 - 0.0335) = +0.0006$mg

This is the figure entered on the 3-1's computation sheet under the $\Sigma 300mg_2$ column on the $-\Delta\rho\Delta V$ line, Sheet 2, Series 4. Note that it is $-\Delta\rho\Delta V$ that is called for and that the buoyancy correction is +0.0006mg, therefore the buoyancy correction is entered as - 0.0006 mg. (See section 5.3.2).

Computation of Predicted or Expected Values

The expected differences are computed from the reported values as follows: (See report on page 47).

Series 1: $\Sigma 100g$

Weight		Apparent Mass Value
50g		-0.0133mg
30g		-0.0134mg
20g		+0.0330mg
$\Sigma 100g$	Expected Value	+0.0063mg

Series 2: $\Sigma 30g_1$

20g		+0.0330mg
10g		-0.0378mg
$\Sigma 30g_1$	Expected Value	-0.0048mg

Series 2: $\Sigma 30g_2$

20g		+0.0330mg
5g		-0.0065mg
3g		+0.0191mg
2g		+0.0066mg
$\Sigma 30g_2$	Expected Value	+0.0522mg

Series 3: $\Sigma 3g_1$

2g		+0.0066mg
1g		-0.0216mg
$\Sigma 3g_1$	Expected Value	-0.0150mg

Series 3: $\Sigma 3g_2$

2g		+0.0066mg
500mg		-0.0005mg
300mg		-0.0041mg
200mg		-0.0049mg
$\Sigma 3g_2$	Expected Value	-0.0029mg

Series 4: $\Sigma 300mg_1$

Weight		Apparent Mass Value
200mg		-0.0049mg
100mg		+0.0008mg
$\Sigma 300mg_1$	Expected Value	-0.0041mg

Series 4: $\Sigma 300mg_2$

200mg		-0.0049mg
50mg		+0.0074mg
30mg		-0.0049mg
20mg		-0.0020mg
$\Sigma 300mg_2$	Expected Value	-0.0044mg

Series 5: $\Sigma 30mg_1$

20mg		-0.0020mg
10mg		+0.0028mg
$\Sigma 30mg_1$	Expected Value	+0.0008mg

Series 5: $\Sigma 30mg_2$

20mg		-0.0020mg
5mg		+0.0065mg
3mg		+0.0030mg
2mg		-0.0142mg
$\Sigma 30mg_2$	Expected Value	-0.0067mg

Series 6: For weighing 5mg - Σ5mg

Weight	Apparent Mass Value 5mg	Σ5mg
5mg	+0.0065mg	
3mg		+0.0030mg
2mg		-0.0142mg
Sums	+0.0065mg	-0.0112mg
Expected Difference		-0.0047mg

Series 6: For weighing 3mg - Σ3mg

	3mg	Σ3mg
3mg	+0.0030mg	
2mg		-0.0142mg
1mg		+0.0091mg
Sums	+0.0030mg	-0.0051mg
Expected Difference	-0.0021mg	

Temperature 24.1 °C	FORM NBS-345 .6 (6-1-61)	U.S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS	Sheet 1 Series 1
Humidity 40%	**SUBSTITUTION WEIGHING**		Unit (Wts.)
Barometer 747.2 mm	Single Pan Damped Balances		Unit (Cr. & Dill.)
ρ = 1.16 mg/cm³	OBSERVATION SHEET		

Observer S.V.O.	Balance H-200	Date 11-1-72	Set	NBS Test No. 200390

Load	Dial Setting	Scale Reading	Computations
100g	100.0 g	2.38	a_1: +0.37, +0.38; $+0.38 \times \frac{10.045}{10.05} = +0.38$ mg
100'g	"	2.01	
" + h 10 mg	"	12.06	
100g + h 10 mg	"	12.44	
100g	"	2.39	a_2: −0.08, −0.09; $-0.085 \times \frac{10.045}{10.06} = -0.08$
Σ 100g	"	2.47	
" + h 10 mg	"	12.53	
100g + h 10 mg	"	12.44	
100'g	"	2.03	a_3: −0.44, −0.46; $-0.45 \times \frac{10.045}{10.06} = -0.45$
Σ100g	"	2.47	
" + h 10 mg	"	12.53	
100'g + h 10 mg	"	12.07	
			a
			a

100'g = any 100g weight summation of weights whose nominal is 100g. Used to fill the series.

Σ 100g = 50g + 30g + 20g

Obs. D_s =
M_s =

Sheet *2 Series 1*

Date *11-1-72*

Std. 1, 1_1, 1_2

Std. 1	1_1	1_2	Observations
+	-		a_1
+		-	a_2
	+	-	a_3

K - A M Cor. Std. 1 = *- 0.058 mg*

	100g Std. 1	*100'g* 1_1	*Σ 100g* 1_2	Check
a_1 *+ 0.38*	0	-2	-1	-3
a_2 *- 0.08*	0	-1	-2	-3
a_3 *- 0.45*	0	1	-1	0
K *- 0.058*	3	3	3	9
Sum =	*-0.174*	*-1.304*	*+0.056*	Σ down Σ across
d =	3	3	3	
Sum/d	*- 0.058*	*-0.435*	*+0.019*	
$-\Delta\rho\Delta V$				
Est. True Mass Cor.				
App. Mass vs. Brass Cor.	*- 0.058 mg*	*- 0.435 mg*	*+0.019 mg*	
Accepted Cor. from Report	*- 0.058 mg*	***	*+0.007 mg*	

	Δ_1	Δ_2	Δ_3	Check
a_1 *+0.38*	1	-1	1	1
a_2 *- 0.08*	-1	1	-1	-1
a_3 *-0.45*	1	-1	1	1
Sum =	*+0.01*	*-0.01*	*+0.01*	Σ down Σ across
d =	3	3	3	
Sum/d =	*+ 0.0033*	*-0.0033*	*+0.0033*	

$S = \sqrt{(\Delta_1)^2 + (\Delta_2)^2 + (\Delta_3)^2}$ = *0.0057 mg*

** This is an unknown*

Temperature 24.1 °C	FORM NBS-345.06 (6-1-61) U.S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS	Sheet 1 Series 2
Humidity 40 %	**SUBSTITUTION WEIGHING**	Unit (Wts.)
Barometer 747.3 mm	**Single Pan Damped Balances**	Unit (Cr. & Dill.) 200390
ρ = 1.16 mg/cm³	**OBSERVATION SHEET**	

Observer	Balance	Date	Set	NBS Test No.
S.V.O.	H-200	11-1-72		200390

Load	Dial Setting	Scale Reading	Computations
30g	30.0g	2.30	a_1: 30g − Σ30g₁ =
Σ30g₁	"	2.33	−0.03
" + h 10 mg	"	12.39	$\frac{-0.01}{-0.02} \times \frac{10.045}{10.06} = -0.02$ mg
30g + h 10 mg	"	12.38	
30g	"	2.28	a_2: 30g − Σ30g₂ =
Σ30g₂	"	2.34	−0.06
" + h 10 mg	"	12.39	$\frac{-0.07}{-0.065} \times \frac{10.045}{10.06} = -0.06$ mg
30g + h 10 mg	"	12.32	
Σ30g₁	"	2.30	a_3: Σ30g₁ − Σ30g₂ =
Σ30g₂	"	2.37	−0.07
" + h 10 mg	"	12.43	$\frac{-0.08}{-0.075} \times \frac{10.045}{10.06} = -0.07$ mg
Σ30g₁ + h 10 mg	"	12.35	
	.	.	a
	.	.	
	.	.	
	.	.	
Σ30g₁ = 20g +	10g	.	a
Σ30g₂ = 20g +	5g +	3g + 2g	
	.	.	
	.	.	
"	"	.	Obs. D_s = ; M_s =

Sheet 2 Series 2

Date 11-1-72

Std. 1, 1_1, 1_2

Std. 1	1_1	1_2	Observations
+	-		a_1
+		-	a_2
	+	-	a_3

K - A M Cor. Std. 1 = - 0.013 mg

	30g Std. 1	Σ 30g₁ 1_1	Σ 30g₂ 1_2	Check
a_1 -0.02	0	-2	-1	-3
a_2 -0.06	0	-1	-2	-3
a_3 -0.07	0	1	-1	0
K -0.013	3	3	3	9
Sum =	-0.039	-0.009	+0.171	Σ down Σ across
d =	3	3	3	
Sum/d	-0.013	-0.003	+0.057	
$-\Delta\rho\Delta V$				
Est. True Mass Cor.				
App. Mass vs. Brass Cor.	-0.013 mg	-0.003 mg	+0.057 mg	
Accepted Cor. from Report	-0.013 mg	-0.005 mg	+0.053 mg	

	Δ_1	Δ_2	Δ_3	Check
a_1 -0.02	1	-1	1	1
a_2 -0.06	-1	1	-1	-1
a_3 -0.07	1	-1	1	1
Sum =	-0.03	+0.03	-0.03	Σ down Σ across
d =	3	3	3	
Sum/d =	-0.01	+0.01	-0.01	

$$S = \sqrt{(\Delta_1)^2 + (\Delta_2)^2 + (\Delta_3)^2} = \sqrt{3 \times 10^{-4}} = 0.17\ mg$$

Temperature 24.2 °C	FORM NBS-345 J6 (6-1-61)	U.S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS	Sheet 1 Series 3
Humidity 40 %	SUBSTITUTION WEIGHING		Unit (Wts.)
Barometer 747.2	Single Pan Damped Balances		Unit (Cr. & Dill.)
ρ = 1.16 mg/cm^3	OBSERVATION SHEET		

Observer	Balance	Date 11-1-72	Set	NBS Test No. 200390

Load	Dial Setting	Scale Reading	Computations	
3g	3.00g	0.340	$3g - \Sigma 3g_1 =$	a1
$\Sigma 3g_1$	"	0.301	+0.034 +0.037	
" + h 5mg	"	5.306	$+0.038 \times \frac{5.0045}{5.005} = +0.038$ mg	
3g + h 5mg	"	5.343		
3g	"	0.324	$3g - \Sigma 3g_2 =$	a2
$\Sigma 3g_2$	"	0.305	+0.019 +0.017	
" + h 5mg	"	5.310	$+0.018 \times \frac{5.0045}{5.005} = +0.018$	
3g + h 5mg	"	5.327		
$\Sigma 3g_1$	"	0.304	$\Sigma 3g_1 - \Sigma 3g_2 =$	a3
$\Sigma 3g_2$	"	0.319	−0.015 −0.014	
" + h 5mg	"	5.324	$-0.0145 \times \frac{5.0045}{5.005} = -0.014$	
$\Sigma 3g_2$ + h 5mg	"	5.310		
	.	.		a
	.	.		
	.	.		
	.	.		
$\Sigma 3g_1 = 2g + 1g$	.	.		a
$\Sigma 3g_2 = 2g + 500\,mg + 300\,mg + 200\,mg$	.	.		
	.	.		
	.	.		
"	"	.	Obs. D_s = . M_s = .	

Sheet 2 Series 3

Date 11-1-72

Std. 1, 1_1, 1_2

Std. 1	1_1	1_2	Observations
+	-		a_1
+		-	a_2
	+	-	a_3

K - A M Cor. Std. 1 = + 0.0191 mg

	3g Std. 1	$\Sigma 3g_1$ 1_1	$\Sigma 3g_2$ 1_2	Check
a_1 + 0.038 mg	0	-2	-1	-3
a_2 +0.018	0	-1	-2	-3
a_3 -0.014	0	1	-1	0
K +0.0191	3	3	3	9
Sum =	+0.0573	-0.0507	-0.0027	Σ down Σ across
d =	3	3	3	
Sum/d	+0.0191	-0.0139	-0.0009	
$-\Delta\rho\Delta V$	————	————	+0.0027	
Est. True Mass Cor.	————	————	————	
App. Mass vs. Brass Cor.	+0.0191 mg	-0.0139	+0.0018 mg	
Accepted Cor. from Report	+0.0191 mg	-0.0150	- 0.0027 mg	

	Δ_1	Δ_2	Δ_3	Check
a_1 +0.038	1	-1	1	1
a_2 +0.018	-1	1	-1	-1
a_3 -0.014	1	-1	1	1
Sum =	+0.006	-0.006	+0.006	Σ down Σ across
d =	3	3	3	
Sum/d =	+0.002	-0.002	-0.002	

$$S = \sqrt{(\Delta_1)^2 + (\Delta_2)^2 + (\Delta_3)^2} = \sqrt{12 \times 10^{-6}} = 0.0035 \text{ mg}$$

Temperature 24.2 °C	FORM NBS-345.C6 (6-1-61)	U.S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS	Sheet 1 Series 4
Humidity 40 %	**SUBSTITUTION WEIGHING**		Unit (Wts.)
Barometer 747.2 mm	**Single Pan Damped Balances**		Unit (Cr. & Dill.)
ρ = 1.16 mg/cm^3	**OBSERVATION SHEET**		

Observer	Balance	Date	Set	NBS Test No.
S.V.O.	M-10	11-1-72		200390

Load	Dial Setting	Scale Reading	Computations
300 mg	0.30	0.291	300 mg − Σ 300 mg₁ = (a₁)
Σ 300 mg₁	"	0.289	+ 0.002
" + h 5 mg	"	5.294	+ 0.001
300 mg + h 5 mg	"	5.295	+ 0.0015 × 5.0045 / 5.005 = + 0.001 mg
300 mg	"	0.297	300 mg − Σ 300 mg₂ = (a₂)
Σ 300 mg₂	"	0.296	+ 0.001
" + h 5 mg	"	5.300	− 0.001
300 mg + h 5 mg	"	5.299	0.000 × 5.0045 / 5.004 = 0.000 mg
Σ 300 mg₁	"	0.295	Σ 300 mg₁ − Σ 300 mg₂ = (a₃)
Σ 300 mg₂	"	0.294	+ 0.001
" + h 5 mg	"	5.299	+ 0.003
Σ 300 mg₁ + h 5 mg	"	5.232	+ 0.002 × 5.0045 / 5.005 = + 0.002 mg
	.	.	a
	.	.	
	.	.	
	.	.	
Σ 300 mg₁ = 200 mg + 100 mg			a
Σ 300 mg₂ = 200 mg + 50 mg + 30 mg + 20 mg			
	.	.	
	.	.	
"	"	.	Obs. D_s = M_s =

Sheet 2 Series 4

Date 11-2-72

Std. 1, 1_1, 1_2

Std. 1	1_1	1_2	Observations
+	-		a_1
+		-	a_2
	+	-	a_3

K - A M Cor. Std. 1 = - 0.0040 mg

	300 mg Std. 1	Σ 300 mg_1 1_1	Σ 300 mg_2 1_2	Check
a_1 +0.001	0	-2	-1	-3
a_2 0.000	0	-1	-2	-3
a_3 +0.002	0	1	-1	0
K -0.0040	3	3	3	9
Sum =	-0.0120	-0.0120	-0.0150	Σ down Σ across
d =	3	3	3	
Sum/d	-0.0040	-0.0040	-0.0050	
$-\Delta\rho\Delta V$			-0.0006	
Est. True Mass Cor.				
App. Mass vs. Brass Cor.	-0.0040 mg	-0.0040 mg	-0.0056 mg	
Accepted Cor. from Report	-0.0040 mg	-0.0041 mg	-0.0044 mg	

	Δ_1	Δ_2	Δ_3	Check
a_1 +0.001	1	-1	1	1
a_2 0.000	-1	1	-1	-1
a_3 +0.003	1	-1	1	1
Sum =	+0.003	-0.003	+0.003	Σ down Σ across
d =	3	3	3	
Sum/d =	+0.001	-0.001	+0.001	

$$S = \sqrt{(\Delta_1)^2 + (\Delta_2)^2 + (\Delta_3)^2} = \sqrt{3 \times 10^{-6}} = 0.0017 \text{ mg}$$

Temperature 24.2 °C	FORM NBS-345 (6-1-61)	U.S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS	Sheet 1 Series 5
Humidity 40 %	**SUBSTITUTION WEIGHING**		Unit (Wts.)
Barometer 747.1 mm	**Single Pan Damped Balances**		
ρ = 1.16 mg/cm^3	**OBSERVATION SHEET**		Unit (Cr. & Dill.)

Observer S.V.O.	Balance M-10	Date 11-1-72	Set	NBS Test No. 200390

Load	Dial Setting	Scale Reading	Computations
30 mg	0.03 g	0.285 mg	30 mg − Σ30 mg$_1$ = (a$_1$)
Σ30 mg$_1$	0.03	0.291	−0.006
" + h 5 mg	"	5.296	−0.004
30 mg + h 5 mg	"	5.292	$-0.005 \times \frac{5.0045}{5.005} = -0.005$ mg
30 mg	"	0.283	30 mg − Σ30 mg$_2$ = (a$_2$)
Σ30 mg$_2$	"	0.287	−0.004
" + h 5 mg	"	5.293	−0.003
30 mg + h 5 mg	"	5.290	$-0.0035 \times \frac{5.0045}{5.006} = -0.003$ mg
Σ30 mg$_1$	"	0.291	Σ30 mg$_1$ − Σ30 mg$_2$ = (a$_3$)
Σ30 mg$_2$	"	0.285	+0.006
" + h 5 mg	"	5.290	+0.005
Σ30 mg$_1$ + h 5 mg	"	5.295	$+0.0055 \times \frac{5.0045}{5.005} = +0.005$
			a
			a
			Obs. D_s =
			M_s =

Sheet 2 Series 5

Date 11-2-72

Std. 1, 1_1, 1_2

Std. 1	1_1	1_2	Observations
+	-		a_1
+		-	a_2
	+	-	a_3

K - A M Cor. Std. 1 = -0.0049 mg

	30 mg Std. 1	Σ30 mg, 1_1	Σ30 mg₂ 1_2	Check
a_1 -0.005	0	-2	-1	-3
a_2 -0.003	0	-1	-2	-3
a_3 +0.005	0	1	-1	0
K -0.0049	3	3	3	9
Sum =	-0.0147	+0.0033	-0.0087	Σ down Σ across
d =	3	3	3	
Sum/d	-0.0049	+0.0011	-0.0029	
$-\Delta\rho\Delta V$				
Est. True Mass Cor.				
App. Mass vs. Brass Cor.	-0.0049 mg	+0.0011 mg	-0.0029 mg	
Accepted Cor. from Report	-0.0049 mg	+0.0008 mg	-0.0067 mg	

	Δ_1	Δ_2	Δ_3	Check
a_1 -0.005	1	-1	1	1
a_2 -0.003	-1	1	-1	-1
a_3 +0.005	1	-1	1	1
Sum =	+0.003	-0.003	+0.003	Σ down Σ across
d =	3	3	3	
Sum/d =	+0.001	-0.001	+0.001	

$$S = \sqrt{(\Delta_1)^2 + (\Delta_2)^2 + (\Delta_3)^2} = \sqrt{3 \times 10^{-6}} = 0.0017 \text{ mg}$$

Temperature 24.3 °C	FORM NBS-343.06 (6-1-61) U.S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS	Sheet 1 Series 6
Humidity 40 %	**SUBSTITUTION WEIGHING**	Unit (Wts.)
Barometer 747.3 mm	**Single Pan Damped Balances**	Unit (Cr. & Dill.) 200390
ρ = 1.16 mg/cm³	**OBSERVATION SHEET**	

Observer S.V.O.	Balance M-10	Date 11-1-72	Set	NBS Test No.

Load	Dial Setting	Scale Reading	Computations
5 mg	0.00	0.257	a_1 5 mg − Σ 5 mg =
Σ 5 mg	"	0.239	+0.018 +0.016
" + h 5 mg	"	5.244	$+0.017 \times \frac{5.0045}{5.005} = +0.017$ mg
5 mg + h 5 mg	"	5.260	
3 mg	"	0.241	a_2 3 mg − Σ 3 mg =
Σ 3 mg	"	0.232	+0.004 +0.008
" + h 5 mg	"	5.238	$+0.0085 \times \frac{5.0045}{5.006} = +0.008$ mg
3 mg + h 5 mg	"	5.246	
	.	.	a
	.	.	
	.	.	
	.	.	
	.	.	a
	.	.	
	.	.	
	.	.	
	.	.	a
	.	.	
	.	.	
	.	.	
"	"	.	Obs. D_s = M_s =

NBS-114A (REV. 7-73)

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15. SUPPLEMENTARY NOTES

16. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

Surveillance tests are designed to monitor the values of mass standards between calibrations. Two types are described; both consist of comparisons of the weights of an ordered set of mass standards with each other. The differences found are compared with those computed from the reported mass values. Surveillance limits based on the precision of both the calibration and the surveillance test processes are computed. These limits are estimates of the departure of the measured differences from the expected, or predicted, differences as computed from the reported values. A larger change is considered significant. Additional measurements to identify individual weights which have changed are required when a given comparison indicates that the mass of one or more of the weights involved has changed. Buoyancy corrections are used to correct for the difference in the buoyant effect on weights of differing densities. Records document the surveillance test results, and control charts help detect trends. Judgments concerning recalibration can be made based on the constancy of the weights and the use requirements.

17. KEY WORDS *(six to twelve entries; alphabetical order, capitalize only the first letter of the first key word unless a proper name; separated by semicolons)* Apparent mass; buoyancy; buoyancy correction; change; comparison; difference; mass; records; set; surveillance limits; surveillance test; test interval; true mass; value; weighing design; weights.

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