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ERROR EQUATIONS USED IN THE NBS PRECISION G/T MEASUREMENT SYSTEM

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W.C. Daywitt

ABSTRACT

Equations presently being used in the precision NBS G/T measurement system are presented in this report. Included are the assumptions upon which these equations are based and sample calculations showing how the measurement errors vary with antenna elevation angle.

Key words: Error analysis; G/T; radio star; ratio precision measurements; satellite communications.

I. INTRODUCTION

This report summarizes the approximate error equations presently being used by NBS in its precision satellite earth-terminal G/T (system gain/system noise temperature) measurement system [1]. These equations are the result of a four-year evolution begun in 1972 with a G/T measurement [2] of the earth terminal located at Camp Roberts, California, using the G/T equations then available in the literature [3-8]. The equations presented here represent the final results of a comprehensive G/T error analysis. This analysis was preceded by a thorough study [9] of the literature concerning the use of Cassiopeia A in G/T measurements. A further study [10] of the errors involved in the absolute flux density measurement of Cassiopeia A was subsequently performed since the literature was found to be lacking in this respect; this study does not identify an absolute flux density value for Cassiopeia.

The equations to be presented are approximations of "exact" equations derived in the analysis. The requirements of high accuracy and the use of models naturally narrows the validity of the resulting equations, and although the range of validity of each equation will be clearly stated, it should be mentioned initially that most of the equations are appropriate to the 1 GHz to 10 GHz frequency region.

Most of the financial support for these studies has come from the U.S. Army Communication Command (ACOM) Headquarters, Fort Huachuca, Arizona, and is gratefully acknowledged.

II. THE G/T EQUATION

The radio star method [3,11,12] of measuring G/T involves measuring the ratio of two noise powers at some point in the IF portion of the earth-terminal receiving system (for a complete definition of the word "system" see reference [9]). These two powers W_1 and W_2 correspond respectively to powers received when pointing the earth-terminal antenna at and then adjacent to the calibrated radio star. The measured ratio Y is related to these two powers by

$$Y = \frac{W_1}{W_2}. \tag{1}$$

Since the earth-terminal system gain is high, noise from the NBS apparatus attached to the system to measure Y is insignificant, and therefore for all practical purposes eq. (1) is exact.

Assuming that the system has a linear power-transfer characteristic and that the noise in the system is stationary, both W_1 and W_2 obey an equation of the form

$$W = \int w(f)g(f)\, df \tag{2}$$

where f is the frequency, w(f) is the spectral power available at some reference point in the front end of the system, and g(f) is the power gain of the system from the reference point to the output where the ratio Y is measured. Both w(f) and g(f) are integrated over the limiting bandwidth of the system. Under these assumptions Y is related to G and T via the equation

$$Y = 1 + \frac{G}{T} \cdot \frac{\lambda^2 S k_1 \cdots k_7}{8\pi k} . \tag{3}$$

G (for a complete definition of G and T see reference [9]) is the gain (at the earth-terminal receive frequency) of the system from the far field of the antenna to the reference point; T is the system noise temperature (at the receive frequency) at the reference point; λ is the free-space wavelength corresponding to the earth-terminal receive frequency; S is the radio star flux density calibrated at the receive frequency; k is Boltzmann's constant; and k_1 through k_7 are correction factors defined to insure the equality in eq. (3).

All of the k factors (k_1 through k_7) involve integrations of the radio-star brightness distribution with various quantities over the power pattern of the antenna. However, if the power pattern were precisely known, G/T could be simply and accurately determined without the aid of a radio star. This is seldom the case. Furthermore, accurate brightness distributions are difficult to obtain. The usefulness of the radio star method is based upon three commonly present conditions: 1) for brightness distributions with angular extents less than the antenna HPBW, the smoothing effect of the antenna allows the distribution to be removed from the integral and replaced by a disk or gaussian distribution [8,10] and a flux density; 2) the real antenna power pattern can often be adequately approximated by a gaussian pattern inside the HPBW; and 3) the k factors are often close enough to unity to allow condition 2) to be used as a perturbation calculation of these factors. For an accurate radio-star determination of G/T from eq. (3) all three of these conditions must be met, and all three are assumed in the k-factor equations to follow.

III. Y-RATIO, WAVELENGTH, AND FLUX DENSITY

A. Y

The Y-ratio measurement is performed by an automated measurement system [14] built around one of the most accurate power bridges known, the NBS Type II self-balancing bridge [15]. This bridge is capable of measuring stable noise power ratios to an error no greater than 0.002 dB. The resulting Y-ratio system error is assumed to be no greater than 0.004 dB [15]. That is

$$\frac{\delta Y}{Y} = 0.1\% \ (0.004 \text{ dB}). \tag{4}$$

The corresponding error E(Y) in the determination of G/T from eq. (3) is

$$E(Y) = 0.1\left(\frac{Y}{Y - 1}\right)\%. \tag{5}$$

In order for an accurate determination of G/T to be made the measured Y ratio should be greater than approximately 0.25 dB. This means that the earth-terminal G/T must obey the inequality

$$\frac{G}{T} > \frac{2.3 \times 10^4 \cdot f^{2-\alpha} \cdot 10^{0.004 \text{ Csc } \theta}}{S_1} \tag{6}$$

where f is the receive frequency in GHz, α is the spectral index [8] of the radio star, θ is the antenna elevation angle at which the radio star is observed, and S_1 is the radio-star flux density at 1 GHz in flux units (fu) (1 fu is equivalent to 10^{-26} W/m^2/Hz).

B. $\underline{\lambda}$

The free-space wavelength λ appearing in eq. (3) is taken to be that wavelength corresponding to the received rf signal from the satellite even though, at the time of the G/T measurement, the earth-terminal antenna is not pointed at the satellite and is therefore not receiving the satellite signal. The G/T error $E(\lambda)$ resulting from an uncertainty $\delta\lambda$ in the value of λ is

$$E(\lambda) = \frac{2\delta f}{f} \% \tag{7}$$

where f is the receive frequency and δf is its corresponding uncertainty, and $\delta f/f$ is expressed in percent.

C. $\underline{S}$

The G/T error E(S) resulting from an uncertainty in the flux density S of the radio source used in the measurement is

$$E(S) = \frac{\delta S}{S} \% \tag{8}$$

where $\delta S/S$ is the uncertainty in the flux density expressed in %.

A number of strong extraterrestrial radio sources (all referred to as radio stars in this report) exist whose potential flux-density accuracy is good: they are the strong radio stars; the moon and possibly the sun. The source chosen for a particular measurement depends on the earth-terminal G/T (eq. (6)); and on the requirement that the HPBW of the earth-terminal antenna be greater than the angular extent of the source. The accuracy

$\delta S/S$ to which the stronger radio stars can be calibrated [10] with present-day-not-quite-state-of-the-art equipment is approximately 5% (3σ). The lower bound to the error [10] using the best equipment of the foreseeable future under ideal conditions is approximately 2% (3σ). Recent work [16] also indicates that the moon might be used as a standard source to a 9% accuracy. Where no other source is strong enough, even the sun might be used as a transfer standard if sufficiently monitored [17].

Considerable effort has been devoted to establishing the flux density of the stronger radio stars [4,8,18-27]. However, comparisons among various sets of observations are often complicated by a lack of agreement on absolute flux density scales, and, moreover, all the observations have not been of comparable accuracy. Sometimes the problems are compounded by confusing accuracy with precision. Often errors less than half the lower-bound error quoted (2%) in the preceding paragraph are reported. Therefore, as regards the radio stars, it is not clear from the literature which are the correct flux-density values to be used in the G/T measurements (currently NBS is using the values given by Kellerman et al. [4] with an assumed 4.46% accuracy). This situation is unacceptable where highly accurate flux data are needed, and NBS is hoping in the near future to calibrate some of the stronger radio stars in the neighborhood of 7.5 GHz with one of the well-characterized antennas at the Jet Propulsion Laboratories.

IV. ATMOSPHERIC TRANSMISSION CORRECTION FACTOR (k_1)

Since the calibrated radio star used in the G/T measurement is outside the earth's atmosphere, the effects of the atmosphere upon the source flux density must be taken into account. This effect appears in the atmospheric transmission correction factor k_1, which accounts for atmospheric absorption and scattering. The relatively high atmosphere transparency in the 1 GHz to 10 GHz microwave region [28] allows the effects of absorption and scattering to be separated with negligible error. That is

$$k_1 = k_g k_d k_r \qquad (9)$$

where k_g, k_d, and k_r are the correction factors respectively for absorption by atmospheric gases, diffusion by tropospheric irregularities, and refractive spreading by the continuous component of the atmospheric refractivity.

The corresponding G/T error $E(k_1)$ is thus the sum of the three component errors, $E(k_g)$, $E(k_d)$, and $E(k_r)$.

The effects [7] of clouds, fog, rain, hail, or snow on the attenuation cannot be accurately calculated on a single-measurement basis. A high level of accuracy therefore requires that the G/T measurements be performed in relatively clear weather, and no discussion of these effects will be presented here.

A. k_g

The atmospheric gases to be considered as important absorbers in the 0.1 GHz to 50 GHz frequency range are oxygen and water vapor [7]. For this range, calculations show that 99% of the attenuation of energy from outside the earth's atmosphere to the earth's surface takes place in the lower 10 km of the troposphere. The calculations reported here take into account the first 20 km (about 20% into the stratosphere [29]) of the atmosphere above the earth's surface with a resulting k_g error of less than 0.1% for neglecting the remaining atmosphere.

The oxygen and water vapor absorption coefficients (dB/km) used to calculate the atmospheric attenuation (dB) are taken from the VanVleck theory [7,30,31] of absorption. These coefficients are integrated along the ray path taken by the energy from the radio star through the atmosphere to the antenna on the earth's surface. The bending of this path by the atmosphere is accounted for by using a bi-exponential model [7,32] for the atmospheric refractivity. The coefficients are functions of the atmospheric temperature, dry-air pressure, and water vapor density, which in turn vary along the ray path. The vertical variations of these parameters are described by average altitude profiles [28] for various geographical locations and seasons.

The actual values of the atmospheric variables along the ray path are not known. What is done is to let the yearly average profiles at the location of the earth terminal describe a uniformly stratified atmosphere for a large radius (less than 200 km (124 miles) for elevation angles greater than 2° and a 10 km thick atmosphere) about the location. The atmospheric attenuation is then calculated for this ficticious atmosphere by using the profiles. The difference between the calculated and actual attenuations is assumed to be bracketed by an error equal to one half the difference between the attenuations calculated by using the maximum and minimum average seasonal profiles for the particular location of the earth terminal.

What results from the foregoing procedure when applied to the calculation of the zenith attenuation A_g is an equation of the form

$$A_g = A_1 + \rho A_2 \qquad (10)$$

where A_1 is the zenith attenuation (dB) due to oxygen, A_2 is the zenith attenuation coefficient (dB/gm/m^3) for water vapor absorption, and ρ is the antenna-level water-vapor density (gm/m^3).

The calculation of atmospheric attenuations along other-than-zenith ray paths can be performed sufficiently accurately (less than 0.3% error for elevation angles greater than 4°) by using an equation of the form

$$A_g(\theta) = A_g \cdot \mathrm{Csc}\ \theta \qquad (11)$$

where $A_g(\theta)$ is the attenuation for the ray path corresponding to the elevation angle θ.

Calculations show that for elevation angles greater than 2°, the correction factor k_g and the atmospheric transmission coefficient are equal to within 0.01 percent. Thus, using eq. (11),

$$k_g = 10^{-A_g \csc \theta/10} \tag{12}$$

to better than 0.31 percent.

The total error in k_g using eq. (12) includes the partial errors resulting from the following items: errors in A_1 and A_2; errors in ρ; errors in θ; errors caused by using the cosecant approximation of eq. (11); errors caused by using average temperature, pressure, density, and refractivity profiles; and finally errors in the altitude of the earth-terminal antenna above sea level.

For elevation angles above 4°, the G/T error $E(k_g)$ resulting from errors in k_g is

$$E(k_g) = 40(k_g^{-1} - 1)\% \tag{13}$$

B. k_d

Atmospheric turbulence causes a random variation of the atmospheric refractive index. These variations can be pictured as "closely packed blobs" [33] which act as scattering centers [34] to incident energy. In this report the attenuation due to this scattering will be called "diffusive" after Yohoi [35]. The effect of this type of attenuation on the antenna temperature resulting from radio noise from outer space is greatest for a point radio source, rapidly diminishing to zero for a source whose angular diameter D is greater than the antenna HPBW θ_H [35].

At the present time Yohoi's paper appears to be the only source reporting diffusive attenuation measurements for microwave energy from outer space. It can be deduced from his paper that the resulting transmission coefficient (taken to be identical with diffusive correction factor k_{d0} in this report) for power emanating from a point source takes the form

$$k_{d0} = 10^{-A_d \csc \theta/10} \tag{14}$$

at least for a limited frequency range and for elevation angles greater than 5°. The zenith attenuation A_d is found to be

$$A_d = 0.0011\ f^2 \tag{15}$$

where f is the frequency in GHz, and A_d is in decibels.

For antenna elevation angles greater than 5° the correction factor k_d for a non-point source is assumed to take the form

$$k_d = 1 - (1-k_{d0})e^{-0.467x^2} \tag{16}$$

where the exponential factor is taken from the results applicable to refractive attenuation (see the next section for a more complete description) and accounts for the finite source size. The variable x is given by

$$x = D/1.201\ \theta_H. \tag{17}$$

where D and θ_H are the source diameter and HPBW, respectively.

Equations (14), (15), and (16) are uncertain and probably crude approximations to the correct quantitative description of the diffusive transmission coefficient. Especially alarming is the rapid rise of the zenith attenuation (eq. (15)) with frequency, even when restricted to the 1 GHz to 10 GHz frequency region intended here. Nevertheless, eqs. (14) through (15) will be used in the present G/T measurements until a more complete description of k_d can be obtained.

The G/T error $E(k_d)$ caused by error in k_d is assumed to correspond to a 75% error in the diffusive loss and follows the form of eq. (13). That is

$$E(k_d) = 75(k_d^{-1} - 1)\%. \tag{18}$$

C. $\underline{k_r}$

As microwave energy traverses the troposphere from outside the atmosphere, its wavefront is bent downward and spread out as it travels to the antenna. This spreading reduces the flux density so that an antenna will intercept less energy than would be intercepted if the atmosphere were not present. The resulting "refractive" [35] attenuation, like its "diffusive" counterpart, vanishes as D/θ_H increases.

The refractive transmission correction factor k_r is defined here to be the ratio of 1) that portion of the antenna temperature due solely to energy from the extraterrestrial radio source with the atmosphere present and neglecting gaseous and diffusive losses to 2) the corresponding antenna temperature with the atmosphere absent. For a point source and a uniformly stratified atmosphere this ratio reduces to [35,36]

$$k_{r0} = \frac{\text{Cos}\ \theta}{\text{Cos}\ \theta'} \left(\frac{\partial\theta}{\partial\theta'}\right)_a \tag{19}$$

where θ and θ' are the antenna elevation angle when pointing at the source, and the true source elevation angle, respectively. For antenna elevation angles greater than 2°, θ and θ' are approximately related by

$$\theta - \theta' = B\ \text{Ctn}\ \theta - C\ \text{Ctn}^2\theta \tag{20}$$

where B and C are seasonal constants calculated for a particular geographical location by fitting eq. (20) at 2° and 20° elevation angle to the "exact" calculation [7,32]. The resulting error in the ray bending angle (θ-θ') is no larger than 15% from 2° to 90° elevation angles.

For a non-point source ($D/\theta_H \neq 0$) the following equation for k_r can be used with negligible error

$$k_r = 1 - (1-k_{r0})e^{-0.467x^2} \tag{21}$$

where x is defined in eq. (17). The derivation of eq. (21) assumes a gaussian main beam.

For antenna elevation angles greater than 3°, k_{r0} can be approximated by

$$k_{r0}^{-1} = 1 + \frac{\pi(B-2C \text{ Ctn } \theta)\text{Csc}^2\theta}{180 \times 60} . \tag{22}$$

The G/T error $E(k_r)$ due to errors in k_r is given by

$$E(k_r) = 18(k_r^{-1} - 1)\% \tag{23}$$

for antenna elevation angles greater than 3°, and for HPBW's determined to 3% or better.

V. STAR SHAPE CORRECTION FACTOR

The star shape correction factor k_2 accounts for the finite angular extent and shape of the radio star. It is defined [10] as the ratio of 1) the radio-source brightness distribution integrated over the power pattern of the antenna to 2) the brightness distribution integrated over a unity power pattern. This definition is the reciprocal of the correction factor appearing in the literature [3,8].

The approximate equations for k_2 given here assume a gaussian main beam for the antenna, and a radio-star diameter (in radians) much less than unity [8,10]:

$$\begin{aligned} k_2 &= \frac{1 - e^{-x^2}}{1.001x^2} \quad \text{(disk source)} \\ &= \left(1 + \frac{D_1^2}{\theta_H^2}\right)^{-1/2} \left(1 + \frac{D_2^2}{\theta_H^2}\right)^{-1/2} \quad \text{(Gaussian sources)} \end{aligned} \tag{24}$$

where x is defined in eq. (17), D is the equivalent angular diameter [10] of the disk source, and D_1 and D_2 are the equivalent [8] major and minor angular diameters of the elliptically symmetric gaussian sources. The correct procedure for obtaining the diameters is outlined by Kanda [10].

The G/T error $E(k_2)$ due to errors in k_2 for the disk source was found to be

$$E(k_2) = 6(1-k_2) + 0.1 \% \tag{25}$$

This equation assumes that radio star drift curves are used to measure θ_H to 3% or better, and that the D/θ_H ratio is less than or equal to 0.8 [10]. Equation (25) was derived for the radio source Cassiopeia A, but it is assumed for the present to apply to all sources used.

The work by Kanda [10] indicates the possibility of treating all sources of whatever shape (disk, gaussian, double point, etc.) in terms of an equivalent disk model with no significant increase in the G/T error. This possibility apparently stems from the smoothing effect the antenna has upon the source distribution for D/θ_H ratios less than unity.

VI. BANDWIDTH CORRECTION FACTOR

The G/T ratio is a single-frequency figure-of-merit referenced to the earth-terminal receive frequency, while the measurement represented by eq. (1) is the ratio of two noise powers whose spectra occupy the entire passband of the earth-terminal receiver. The conversion from this "passband" measurement to the single-frequency G/T ratio is accomplished by the bandwidth correction factor k_3 which accounts for the variation of all the frequency-dependent parameters of the system across the receiver passband.

The "exact" expression for k_3 involves integrals whose order of magnitudes are proportional either to: 1) the difference in system noise temperatures between when the earth-terminal antenna is pointed at the calibrated radio star and when it is pointed at the cold sky; or to 2) the product of the radio-star brightness temperature [37] and its own solid angle [37] divided by the antenna solid angle [37]. Which one of these integrals predominately determines the final form of k_3. For sources like Cassiopeia A where the temperature-difference integral predominates k_3 becomes

$$k_3 = 1 + (\Delta f/2f)^2 \qquad (26)$$

where Δf is the noise bandwidth [37] of the system passband, and f is the receive frequency to which the G/T is referenced. The assumptions leading to the second term in eq. (26) are: 1) that the system noise temperature difference is due entirely to cosmic radio noise [37]; 2) that the average cosmic background in which the calibrated radio star source is immersed is due to ionized hydrogen in addition to the 3-Kelvin blackbody noise [38] and that the ionized hydrogen emits radiation proportional to the reciprocal of the frequency squared from 1 GHz to 10 GHz; 3) that the cosmic region to which the antenna is pointed after pointing at the radio star emits only 3 Kelvin blackbody noise; 4) that the average background within one bandwidth of the radio source has an average noise temperature around 1 Kelvin; 5) that the system bandpass is roughly rectangular; 6) that Δf is less than 500 MHz; and 7) that the earth-terminal antenna gain is less than 70 decibels.

For small antenna-earth terminals only the sun or moon [39] emit sufficient radiation to allow a G/T measurement to be made (see eq. (6)). Equation (26) probably does not apply to these terminals.

The G/T error $E(k_3)$ due to errors in k_3 is assumed to be

$$E(k_3) = 50(\Delta f/2f)^2\%. \qquad (27)$$

VII. DIFFERENTIAL SYSTEM TEMPERATURE FACTOR

In a G/T measurement the earth-terminal antenna is first pointed at the cosmic radio source, and then a few degrees in azimuth to the side. The system [9] noise temperature is conceivably different for these two antenna positions, and this difference is accounted for in the differential system temperature factor k_4. This factor is given by

$$k_4 = 1 + \frac{0.004\ T_{AVE}\ \theta_H^2}{k_1 k_2 S} \tag{28}$$

where T_{AVE} is the average noise temperature of the cosmic background within one antenna beamwidth (the main lobe) surrounding the radio star, θ_H is the antenna HPBW in arc-minutes, k_1 and k_2 are correction factors previously described, and S is the radio source flux density in fu. The assumptions leading to eq. (28) are: that the system temperature difference is due solely to changes in the cosmic background between the two antenna positions and that the system temperature difference is equal to the average temperature T_{AVE} divided by the frequency in GHz squared.

The maximum G/T error $E(k_4)$ due to errors in k_4 is assumed for all the strong radio stars to be

$$E(k_4) = \frac{0.4\ T_{AVE}\ \theta_H^2}{k_1 k_2 S}\ \%. \tag{29}$$

VIII. ANTENNA POINTING CORRECTION FACTOR

During that part of the G/T measurement when the star is being used, the center of the earth-terminal antenna mainbeam and the center of the radio source should be coincident. Any angular spearation between these two centers will cause a reduction in the power entering the antenna. This reduction is accounted for by the antenna pointing correction factor k_5 given by

$$k_5 = 1 - \frac{2(\ell n\ 2)x^2(\varepsilon/100)^2}{e^{x^2} - 1} \tag{30}$$

where x has been previously defined, and ε is the pointing error in percent of the antenna HPBW.

The resulting G/T error $E(k_5)$ is taken to be

$$E(k_5) = \frac{0.014\ x^2\varepsilon^2}{e^{x^2} - 1}\ \%. \tag{31}$$

In the event that the pointing error is large (ε > 6%) a radio-source drift method [10] is available which will give k_5 to a 0.5% error or less.

IX. POLARIZATION CORRECTION FACTOR

Some of the stronger radio stars are known to possess a small component of linear polarization [19]. The reduction in the power received by the earth-terminal antenna resulting from a mismatch between this component and the antenna polarization is accounted for in the polarization correction factor k_6 [10]. It is assumed in this report that the antenna polarization is circular to within 3° of 45° [10]. This assumption allows k_6 to be taken as unity with a corresponding G/T error $E(k_6)$ no greater than 0.1%. That is

$$k_6 = 1 \tag{32}$$

and

$$E(k_6) = 0.1\%. \tag{33}$$

Equations (32) and (33) may not apply to radio star Taurus A.

X. SYSTEM RESPONSE CORRECTION FACTOR

Instabilities in the earth-terminal receiver gain and a long system response time can limit the accuracy of a single G/T measurement. However, with the present instrumentation the response-time effect is negligible, so that only the effects of gain instabilities have been incorporated into the system response correction factor k_7. This factor is taken to be

$$k_7 = 1 \tag{34}$$

with a single-measurement G/T error $E(k_7)$ given by

$$E(k_7) = \left(\frac{Y}{Y - 1}\right) \frac{\delta g}{g} \% \tag{35}$$

where Y is the ratio in eq. (1), and $\delta g/g$ is the percentage relative gain instability of the earth-terminal receiver.

Gain instabilities encountered in the G/T measurements performed at Camp Roberts [2], California, and at Fucino [40], Italy, were 0.45% and 1.27%, respectively. The resulting single-measurement error in k_7 was 4% and 16%, respectively. These large errors were reduced by measuring the G/T many times at a number of antenna elevation angles and fitting a smooth curve to the results. A more desirable method to reduce the error can be obtained by adding noise [41] through the side arm of a directional coupler into the front end of the earth-terminal receiver system, and monitoring changes in magnitude changes that occur in this added noise after it has been amplified by the system.

XI. RESULTS AND CONCLUSIONS

The use of the previous equations is illustrated in table I where the results of four sample calculations (one at each of the four antenna elevation angles, 5°, 15°, 30°, and 90°) are presented. The variables appearing in the table are those needed to determine the

G/T ratio from eq. (3). The values of the parameters used in the equations were: f, 7.5 GHz; $\delta f/f$, 0; Δf, 10 MHz; $\delta g/g$, 0.46%; θ_H, 9.2 arc minutes; ε, 5%; S_1, 3090 fu (Cassiopeia A); α, -0.765; D, 4.6 arc minutes; B, 1; C, 0.013; A_1, 0.038 dB; A_2, 0.00035 dB m^3/gm; and ρ, 8.5 gm/m^3. These values are typical of those which might be encountered in an earth terminal located in the Continental United States, having an antenna gain of 60 dB and a system temperature of 100 kelvins. The use of these values in eq. (6) indicates that a reasonably accurate G/T measurement could be expected for an earth terminal with a G/T as low as 33.4 dB.

The table shows that of the correction factors (k_1 through k_7) only k_g, k_d, k_r and k_2 are important, the others being close to unity magnitude. Excluding the rest (k_3 through k_7) will not cause a significant additional rms error. Of the "important" correction factors k_d and k_r have not previously appeared in the literature.

The table further illustrates the tendency of the total G/T error E(G/T) to decrease rapidly from low elevation angles, encountering a "knee" in the neighborhood of 15° elevation with a much reduced fall-off thereafter. The error below the "knee" at higher elevation angles compares very favorably with the G/T error encountered using the best of other methods than the radio-star method examined here. This condition strongly recommends the radio-star method of G/T measurement because other accurate methods require considerable skill and highly qualified personnel to perform.

The "E(G/T)-without E(S)" row indicates the possible accuracy in relative G/T measurements, where a highly accurate value for the radio-source flux density S is not needed. This kind of measurement is of value where only relative values of G/T are needed for quality comparison between various earth terminals.

Two sets of errors stand out in the "Resulting Error" columns of the table, those arriving from the flux density S and the diffusive correction factor k_d. The 4.46% G/T error due to the flux density is examined in an earlier report [10] from which it can be concluded that, at least for the present, it will be difficult to reduce this error further. It should be again pointed out that it is not clear from the literature which, of all the flux density values reported, is the best value to use in the G/T measurements.

The G/T error resulting from k_d reflects the fact that at the present time few investigations concerning the diffusive attenuation of microwave noise from radio stars have taken place. Some work along this line would probably reduce this error considerably.

The values of the parameters (D, D_1, and D_2) used to calculate the other "important" factor k_2 for the strong radio sources should be determined [10] by minimizing the difference beween 1) the k_2 calculated from a brightness distribution map of the source, and 2) the k_2 calculated using the source's equivalent shape. When this was done for the radio star Cassiopeia A treated as a uniformly bright disk of angular diameter D, D was determined [11] to be 4.6' in distinction to the 4.3' value appearing in the literature. The additional G/T error incurred by using 4.3' instead of 4.6' in eq. (24) is 1.1%, a value too large to ignore. Similar determinations for the other radio sources still need to be performed.

XII. ACKNOWLEDGMENTS

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Table I.

Variables in Eq. (3)	Magnitudes				Resulting G/T Error (Percent)			
	5°	15°	30°	90°	5°	15°	30°	90°
Y	1.242	1.293	1.306	1.313	0.51	0.44	0.43	0.42
λ (cm)	4.00	4.00	4.00	4.00	0.00	0.00	0.00	0.00
S (f.u.)	662	662	662	662	4.46 [11]	4.46	4.46	4.46
k_1	0.754	0.913	0.955	0.978	12.96	4.14	2.13	1.06
(k_g)	(0.897)	(0.964)	(0.981)	(0.991)	(4.59)	(1.49)	(0.76)	(0.38)
(k_d)	(0.861)	(0.951)	(0.974)	(0.987)	(12.11)	(3.86)	(1.99)	(0.99)
(k_r)	(0.976)	(0.996)	(0.999)	(1)	(0.44)	(0.07)	(0.02)	(0.00)
k_2	0.917	0.917	0.917	0.917	0.60	0.60	0.60	0.60
k_3	1	1	1	1	0.00	0.00	0.00	0.00
k_4	1.001	1.001	1.001	1.001	0.07	0.06	0.06	0.06
k_5	0.997	0.997	0.997	0.997	0.32	0.32	0.32	0.32
k_6	1	1	1	1	0.10	0.10	0.10	0.10
k_7	1	1	1	1	2.36	2.03	1.96	1.93
E(G/T) rms %					13.93	6.47	5.38	5.04
E(G/T) dB					0.57	0.27	0.23	0.21
E(G/T) rms % without E(S)					13.79	4.68	3.01	2.35%
E(G/T) dB without E(S)					0.56	0.20	0.13	0.10 dB

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16. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

Equations presently being used in the precision NBS G/T measurement system are presented in this report. Included are the assumptions upon which these equations are based and sample calculations showing how the measurement errors vary with antenna elevation angle.

17. KEY WORDS *(six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons)*

Error analysis; G/T; precision measurements; radio star; satellite communications.

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