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# The Effect of "Resource Impact Factors" on Energy Conservation Standards for Buildings

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Stephen F. Weber

Building Economics Section
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National Bureau of Standards
Washington, D. C. 20234

Sponsored by:

The Office of Conservation and Environment
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U.S. DEPARTMENT OF COMMERCE, Juanita M. Kreps, *Secretary*

Dr. Betsy Ancker-Johnson, *Assistant Secretary for Science and Technology*

NATIONAL BUREAU OF STANDARDS, Ernest Ambler, *Acting Director*

## PREFACE

This study was funded by the Federal Energy Administration (FEA) as part of a program being carried out by the National Bureau of Standards to develop energy conservation performance standards for the design of buildings. The legislative mandate for this program is in PL 94-385, Title III, "Energy Conservation Standards for New Buildings Act of 1976," which calls for the development and implementation of performance standards for new buildings to achieve the maximum practicable improvements in energy efficiency. The performance standards being developed under this program are expected to take energy prices into account. This study addresses the question of whether the energy prices used in determining the standards should be the actual market prices paid or a price which has been adjusted to reflect the social value of energy resources. FEA is developing such adjustment factors under the name "Resource Impact Factors" (RIF's). Moreover, the report assesses the effect of using RIF's on the rate of energy consumption allowed by the standard and on the economic efficiency of the standard.

The Building Economics Section of the Center for Building Technology, Institute for Applied Technology, National Bureau of Standards, has prepared this report to provide those involved in the development of energy conservation standards with some guidance in determining what factors are appropriate to be included in RIF's, and how the use of such RIF's would affect the standards being developed.

Dr. Harold E. Marshall, Chief of the Building Economics Section, and Stephen R. Petersen deserve special thanks for the time they spent discussing these issues, reading and commenting on the earlier drafts of this report, and providing the author with invaluable insights and ideas. The author is also grateful for the helpful comments and suggestions of the reviewers, Robert R. Jones, William L. Carroll, Dr. Douglas R. Shier, and Dr. Carl O. Muehlhause. Thanks are also due to Noreen Rigopoulos and Linda Sacchet, who typed the several drafts of this report, as well as to Reid Hartsell, the general technical representative for FEA. Responsibility for whatever errors and shortcomings may remain rests solely with the author.

ABSTRACT

This report addresses the question of the proper price for energy to be used in the development of optimum (i.e., cost-effective) energy conservation performance standards for buildings. This study finds that the appropriate price for energy is its social value, which should be determined through the development and application of Resource Impact Factors (RIF's). Some guidelines are provided for the formulation and development of RIF's. A simple life-cycle cost minimization model for determining the optimum conservation standard is employed to show how the use of RIF's would generally lower the maximum allowable energy consumption specified in the standard. Indeed, it is found that the higher the RIF value, the lower the energy consumption allowed by the standard, although this effect steadily diminishes as the RIF value increases. When a more general cost model with less restrictive assumptions is employed, the same inverse relationship appears between the energy consumption allowed by the standard and the RIF value. Finally, a geometric measure is derived for the net gain in economic efficiency that would result from using RIF's in developing energy conservation performance standards.

## EXECUTIVE SUMMARY

This report focuses on the role which the price of energy is expected to play in the development of optimum (i.e., cost-effective) energy conservation performance standards for new buildings. The maximum rate of energy consumption allowed under the optimum standard is based on a comparison of the value of the energy saved relative to the cost of complying with the standard over the life cycle of the building. Thus, the price used to value the energy saved has a significant effect on the resulting standard. If the standard is to be set at a level which is socially optimum, then it must be based on the social value of energy. Because of a range of price distorting factors in energy markets, however, it is unlikely that the actual prices paid for various energy types accurately reflect the true social values of these resources. The Federal Energy Administration (FEA) is developing a system of indices which could be used to adjust actual prices so that the true social values of energy resources would be better represented. One of the purposes of this report is to offer some guidelines for the development of these indices which are called "Resource Impact Factors" (RIF's). The other major purpose of this report is to assess the effects of using a system of RIF's in the development of energy conservation standards. To accomplish this latter goal, a comparison is made between a standard that is optimum from the private point of view (using the actual unadjusted prices paid for energy) and one that is optimum from the social point of view (using prices adjusted with RIF values). The comparision is made in terms of two implications of the standards: (1) the relative amounts of energy saved; and (2) the relative economic efficiencies of the standards.

The guidelines offered for developing RIF's fall into two categories. The first concerns the explicit method of formulation. One method of formulation, as a quantity multiplier, results in impact numbers which would serve well to compare alternative energy types. Because these impact numbers are not denominated in dollar terms, however, they would be inappropriate for determining the economic balance between energy and nonenergy resources. To achieve such a balance, a common basis is needed to allow comparison between the value of the energy saved and the cost of the resources expended to save it. The other method of formulation, as a price multiplier, provides this common denominator because the RIF value converts the actual price paid for the energy to its corresponding social price. Since this social price is denominated in dollars, it can be compared with the cost of the nonenergy resources.

The second category of guidelines relates to the factors which RIF's should take into account. These are the same factors that cause a divergence between the actual price and the social value of energy. These factors include the presence of unit taxes or of monopoly power, environmental effects of energy production or consumption, the desire for national economic independence, and the existence of price controls such as those on natural gas.

To assess the likely effect on energy consumption resulting from the use of RIF's, several forms of a life-cycle cost minimization model are employed. The present value of total life-cycle heating-related costs is expressed as a function of the thermal resistance of the building envelope. This cost expression is minimized to find the economically optimum (i.e., most cost-effective) level of thermal resistance. Because of the exactly inverse relationship between resistance and the rate of heat flow conducted through the building envelope, this model also yields a formula for the economically optimum rate of thermal transmission, which forms the basis for the optimum performance standard itself. Since the actual price paid for energy is used in this formula, the resulting rate of thermal transmission is considered privately optimum. Moreover, it is possible to introduce price multiplier RIF's into this formula in order to determine the socially optimum rate of transmission. Thus, the socially optimum rate (using RIF's) can be compared with the privately optimum rate (without RIF's). The comparison is made for the alternative forms of the model which result from varying the assumptions regarding the cost of resistance and effective degree days.

In general, it is found that the introduction of RIF's would have a restraining influence on the energy consumption ceiling allowed under the conservation performance standard, assuming the most probable range for the price multiplier RIF values: RIF > 1 (that is, the social value is greater than the actual price paid for energy). Using the example of natural gas, a rough estimate can be made of the additional energy savings resulting from the use of RIF's. A RIF value of 2.82 results from a comparison of the regulated interstate wellhead price of $1.42/MCF with the expected cost of synthetic gas from Western coal of about $4.00. Introducing this 2.82 figure into the expression for the socially optimum standard shows that the energy consumption ceiling allowed under the standard would be 40.5% lower with RIF's than without them. The model further shows that in general the higher the RIF value introduced (that is, the higher the ratio of social value to actual price) for an energy type, the lower the energy consumption ceiling allowed by the standard, although this effect steadily diminishes as the RIF value increases. When the more general forms of the cost model are employed with less restrictive assumptions regarding degree days and the cost of resistance, the same inverse relationship appears between the energy consumption ceiling and the RIF value.

Regarding the efficiency effect of RIF's, it turns out that the introduction of RIF's increases the economic efficiency of the resulting energy conservation standard. A standard developed without RIF's is in effect based on the actual private prices paid for energy, whereas to assure a socially optimum standard, the appropriate prices to use are the social values of the energy types (that is, the product of the private prices times the corresponding RIF values). Using the private rather than the social prices of energy will likely result in a standard that is not socially optimum. This deviation of the privately optimum from the socially optimum standard can be termed economically inefficient in the sense that an opportunity for a net economic gain to society is being foregone. That is, the use of a RIF value greater than one will generally lead to an additional reduction in energy consumption whose social value exceeds that of the additional thermal resistance required to achieve it. This net gain in economic efficiency (the social value of the additional energy savings less the cost of achieving them) which results from the use of RIF's is depicted graphically and a geometric measure of its magnitude is illustrated in the Appendix.

One important area requiring further research concerns the actual development of empirical estimates for RIF's. Those factors preventing the actual price from reflecting the social value need to be modeled more realistically and accurately. These models must be applied to derive empirical estimates for each type of price-distorting factor. Then, a method must be found to combine these individual factors into an overall RIF value.

The other important area for further research is that of the relationship of RIF's to building (rather than component) performance standards. This type of analysis would take into account the many energy-flow interactions which occur among the various components of a building, rather than study each component in isolation. Researchers at NBS are currently involved in developing such a model for determining optimum building performance standards. Once this model has been completed, the effect of using RIF's in determining socially optimum building performance standards can be assessed.

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# 1.0 INTRODUCTION

## 1.1 PURPOSE

The focus of this report is on the role of energy prices in the development of optimum (i.e., cost-effective) energy conservation standards for buildings. It is expected that the level of energy prices will influence the amount of energy allowed under such an optimum standard. This study assesses the effect on the conservation standard of using "adjusted" prices rather than actual market prices for energy. The method of adjustment would be to employ indices being developed by the Federal Energy Administration (FEA). These indices are called "Resource Impact Factors" (RIF's) and are intended to reflect the true social value of energy resources.

This report compares an energy conservation standard developed using actual market prices of energy with one developed using adjusted social prices of energy. Thus, a standard that is optimum in private profitability terms is compared with one that is optimum from a social perspective. The comparison is made in terms of two implications of the standards: (1) the relative amounts of energy saved, and (2) the relative economic efficiencies of the standards.

In addition, the report will discuss the manner in which RIF's should be specified in order to be useful in the development of optimum energy conservation standards for buildings. Moreover, there will be some discussion of the need for RIF's and of the various factors which will require quantification before usable RIF's are developed by FEA.

These factors are the main causes for the failure of actual market prices to reflect the true social values of energy resources.

## 1.2 ENERGY CONSERVATION STANDARDS FOR BUILDINGS

### 1.2.1 Types of Standards

There are three fundamental types of energy conservation standards which could be developed for buildings. The first is a prescriptive standard which might specify the kind and amount of insulating materials to be used in the walls and attic of a particular type of building. Another kind of standard is a component performance standard, which would only require that a component be able to meet a particular goal or target specification without regard to how that goal is achieved. An example of such a standard would be a requirement that the attic have an overall coefficient of heat transmission (or U value) of less than .06 Btu/hr./sq.ft./°F.[1] The third type is a building performance standard, which would specify an overall goal for the design energy consumption of the building, thus allowing the designer to make cost-reducing tradeoffs between building components in order to conform to the standard. For example, a building performance standard might require that the amount of energy needed to satisfy the annual heating and cooling load be less than 60,000 Btu per square foot of floor space. This is referred to as the allowable "annual energy budget" for heating and cooling. Another building performance standard might impose an upper limit on the annual energy consumption for domestic hot water.

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[1] Because of the present accepted practice in this country for building technology, customary U.S. units of measurement are used throughout this report. Relevant conversion factors from customary to metric (SI) units can be found in Table A.1 at the end of the Appendix.

### 1.2.2 Legislation

On August 14, 1976, the President signed into law the Energy Conservation and Production Act (P.L. 94-385). Title III of this law, Energy Conservation Standards for New Buildings Act of 1976, requires the Secretary of Housing and Urban Development to consult with the Federal Energy Administration, the National Bureau of Standards, and the General Services Administration in order to develop proposed performance standards for new residential and commercial buildings. Key elements of this law are: (1) the explicit specification of performance rather than prescriptive standards; (2) the requirement that the standards developed be analyzed in terms of their economic costs and benefits; (3) the stated goal of achieving the maximum practicable improvements in energy efficiency through reasonable conservation features; (4) the requirement that the standards take into account the climatic variations among different regions of the country; (5) the avowed purpose of encouraging greater use of nondepletable sources of energy; and (6) that the standard apply to the heating, cooling, ventilating, and domestic hot water loads of buildings.

The fact that performance standards are to be developed means that designers will be free to choose the most economical means of meeting the standards, given regional variations in the costs of building materials and labor. From the legislative hearings, it appears that building performance standards are the ultimate goal of the Act. These building performance standards will be developed by combining the results of anlaysis of individual components and their energy interdependencies. In most applications, manuals of accepted practice regarding individual building components will provide the guidelines for conformity with the standards.

The second key element of the Act, the need to consider costs and benefits, points out the importance of developing an economically balanced conservation standard. This requirement seems to suggest that the value of the total energy saved by the standard should at least cover the extra costs imposed by the standards. A further guideline for developing the standard may be implicit in this requirement. That is, that the standard be set so as to achieve the greatest possible net economic gain, taking into account both savings and costs. Such a standard could be considered economically optimum.

The goal of maximum practicable gains in energy efficiency requires that the standard provide for the most energy savings possible within the limits of practicability. This suggests that the operating constraint preventing an extremely stringent standard is the increased cost required for compliance. According to this interpretation, the standard would be made increasingly stringent as long as the additional energy savings still justify the extra costs. In this way, cost-effective energy conservation features would be introduced into the design of buildings. Likewise, such a cost-effective standard would directly address the expressed concern of Congress over excessive long-term operating costs due to inadequate energy conservation measures in new buildings.

The concern for cost effectiveness is also apparent in the fourth element of the Act, the requirement to consider climatic variations in developing the standard. The reason for considering these variations is so that the standard can be tailored to the specific heating and cooling loads of each region. The dollar-value benefit of the energy saved by one additional inch of attic insulation is greater in more severe climates. Thus, these higher levels of benefits will tend to justify the extra cost of complying with more stringent performance standards developed for more severe climates.

The fifth key element of the Act, the desire to encourage greater use of nondepletable resources, is especially relevant to this report. We will show how RIF's can be used in the development of performance standards which are optimum from the social point of view. Properly designed RIF's will take into account the policy preferences of society, which could easily include the goal of fostering the development and use of nondepletable resources.

The sixth key element concerns which energy loads will have standards. Those loads specifically mentioned in the Act are for heating, cooling, ventilating, and domestic hot water services. It is likely, however, that standards will be developed for illumination loads as well. Of these energy loads, heating and cooling are the ones most affected by the design of commercial and residential buildings. Hence, a wider range of trade-offs are possible between heating and cooling loads and building design decisions. For this reason, we have chosen to concentrate our analysis on the development of energy conservation performance standards for the heating and cooling of buildings.

### 1.2.3 ASHRAE Standard 90-75

An energy conservation standard for buildings has already been developed and is being proposed by the American Society of Heating, Refrigerating, and Air-Conditioning Engineers (ASHRAE). This is ASHRAE Standard 90-75 and is entitled, "Energy Conservation in New Building Design." This standard serves as the basis for energy conservation regulation in several states as well as for the formulation of energy conservation provisions of three model codes. For purposes of comparison it is worth noting how it differs from the standard called for by the Energy Conservation and Production Act.

#### 1.2.3.1 Contrast with Legislation

The major point of difference between these two standards regards the method of establishing the appropriate degree of stringency. ASHRAE 90-75 did not attempt to achieve the maximum practicable improvements in energy efficiency as called for by the Energy Conservation and Production Act. Indeed, the ASHRAE standard specifically states that it is presenting only minimum requirements.[1] Moreover, the degree of stringency embodied in the ASHRAE standard does not seem to be based on economic considerations. By contrast, as we have seen, the language of the Energy Act suggests that the standards developed should be based on a balancing of costs and benefits.

Another less significant point of difference between these two standards lies in the relative emphasis placed on component versus building performance. Most of the text of the ASHRAE standard uses the component performance approach. Sections 4 through 9 specify performance standards for the various building components such as the exterior

[1] ASHRAE 90-75: Energy Conservation in New Building Design (New York: ASHRAE, Inc., 1975), Section 4.2.1.

envelope, heating and cooling equipment, service water heating, and lighting. Only in the rather brief Section 10 is a systems analysis approach suggested as an acceptable alternative to the component standards. In other words, deviations from the specific component standards are allowed as long as the entire building's annual energy performance is shown to be superior to that of a comparable building which does conform to the component standards. The Energy Conservation and Production Act, on the other hand, has building performance standards as its ultimate goal, although these will be based on the analysis of individual components and will be mainly complied with through manuals of accepted practice.

1.2.3.2 Resource Utilization Factors (RUF's)

The proposed twelfth and final section of ASHRAE 90-75, "Annual Fuel and Energy Resource Determination," provides a method for calculating the equivalent resource quantities required to supply the energy needs of a building. The procedure involves the use of conversion coefficients, called Resource Utilization Factors (RUF's),[1] to be applied to the required energy output delivered to the building site. These RUF's are based on the total energy losses that result from processing, refining, transporting, converting, and delivering the energy from the point of extraction to the building site.

The rationale behind the development of RUF's was to treat separately those factors which are more readily quantifiable. This is because they are basically technical in nature. The less manageable factors, such as

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[1] RUF's are not to be confused with RIF's.

environmental, economic, and national security concerns, would be accounted for by developing a system of RIF's, an analogous kind of energy quantity multiplier. The format for annual fuel and energy calculations presented in Section 12 of ASHRAE 90-75 suggests that the RIF's are to be applied after and in addition to the RUF's, so that the total resource impact of each energy load can be determined.[1]

This distinction between RUF's and RIF's has several limitations. In the first place, RUF's do not include all of the energy losses which occur in making energy available to deliver a service to a building. The energy expended in extracting the resource from the ground is ignored, as is the energy lost in the final conversion at the building. At a more fundamental level, the use of RUF's is similar to applying the technique known as Net Energy Analysis. According to this technique, resources to be employed in the production of energy for consumer use should be so allocated as to maximize "net energy." In this context net energy means the amount of energy that remains for consumer use after the energy costs of finding, developing, extracting, upgrading, and delivering the energy have been paid. These energy costs are measured by the energy content of the resources (i.e., by the amount of energy required to produce them) which are employed in carrying out these processes. The basic shortcoming of this type of analysis is that it implies that all nonenergy resources are relevant

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[1]For further discussion of Section 12 of ASHRAE Standard 90-75 and its distinction between RIF's and RUF's, see Robert R. Jones, "Resource Impact Factor (RIF) Approach to Optimal Use of Energy Resources," ASHRAE Journal, Vol. 18, No. 10 (October 1976) pp. 15-18. Also, see Harry H. Phipps, "The RUF Concept as an Energy Measurement Tool," ASHRAE Journal, Vol. 18, No. 5 (May 1976) pp. 28-30.

only in the measure of the energy required to produce them. Thus, many important resources are either totally or partially ignored.[1]

Another difficulty with this distinction between RIF's and RUF's is that it involves a certain amount of double counting. This occurs because all of the elements included in RUF's also affect the economic cost (which is included in RIF's) of making the energy available to serve the building's needs. That is, the effective cost of delivering a million Btu's to a building project site could be reduced by an increase in refining efficiency (i.e., a decrease in energy losses at the refining stage).

## 1.3 APPROACH AND ORGANIZATION

In the next section we will develop the conceptual framework for determining the optimum energy conservation standard for the heating and cooling loads of buildings. The concept of life-cycle cost minimization will be discussed from the private and then the social points of view. The reasons why actual market prices may fail to reflect the social values of energy resources will be explained and illustrated. These reasons will serve to suggest ways to adjust energy prices so as to reflect social values.

In Section 3.0, RIF's will be applied to a simple model for determining optimum standards for the heating load characteristics of a building component. Alternative formulations for RIF's will first be

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[1]For a discussion of the inadequacies of Net Energy Analysis, see David A. Huettner, "Net Energy Analysis: An Economic Assessment," Science, Vol. 192 (April 9, 1976), pp. 101-104. For a brief overview of several approaches to energy analysis see Kenneth C. Hoffman, "Some Comments on Energy Accounting," paper presented at panel discussion Energy, Environment and Economics, 68th Annual Meeting, American Institute of Chemical Engineers, November 1975.

presented and then the cost minimization model will be developed. Then, a family of RIF values will be introduced into the model, the resulting optimum standards will be analyzed, and the limitations of the analysis will be discussed. In Section 4.0 several alternative forms of the cost model will be developed by varying the assumptions regarding climatic loads and the cost of energy conservation techniques. The effects of introducing a family of RIF's into these more general forms of the model will be analyzed.

Section 5.0 will present a general summary of the report and of its major findings. These conclusions concern: (1) the appropriate formulation of RIF's for use in developing an energy conservation standard; (2) the effects of using RIF's on the degree of stringency (allowable energy consumption rate) specified by the standard; and (3) the net gain in economic efficiency from using RIF's rather than unadjusted market prices for energy in the development of the standard. Finally, this gain in economic efficiency resulting from the use of RIF's will be graphically illustrated and analyzed in the Appendix.

## 2.0 CONCEPTUAL FRAMEWORK FOR AN OPTIMUM STANDARD

As we have seen in our discussion of the Energy Conservation and Production Act, the language of Title III calls for development of a conservation standard which is designed to achieve the maximum practicable improvements in energy efficiency. Moreover, the Act emphasizes the need to analyze the standard in terms of its costs and benefits. Thus, we have chosen to concentrate our analysis on the effects of RIF's on an <u>optimum</u> standard, that is, a standard which achieves the greatest possible net benefits (dollar savings minus costs).[1] Moreover, for purposes of illustration we will focus on setting a standard for the conservation of energy used only to heat and cool buildings.

There are two possible points of view to take in measuring net benefits: (1) that of a private investor or potential homebuyer; and (2) that of society or the public at large. In rare circumstances these two points of view lead to the same optimum. Often, what is optimum for a private consumer or producer may not be optimum for society as a whole. As we shall see, the latter is usually the case when dealing with energy. We will define more precisely what is meant by an optimum standard first from the private viewpoint and then from the social viewpoint.

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[1] If, on the other hand, the standard to be developed is not based on a comparison of costs and benefits, then there would be no way to introduce RIF's into the development process.

## 2.1 OPTIMUM STANDARDS FROM THE PRIVATE PERSPECTIVE

### 2.1.1 Life-Cycle Cost Minimization

To select an optimum energy conservation standard, one must consider all costs occurring over the life of the building which relate to the supply of heating and cooling services.[1] These would include such first-cost items as the insulation used in the building envelope, the heating and cooling equipment and the choice of storm windows. The recurring costs would depend on the annual fuel consumption required to achieve a given level of comfort. These recurring future costs must be discounted in order to be comparable with the first-cost items. Then, all these cost items are summed to get the total present value of the cost of heating and cooling the building over its life. Finally, this present value figure is minimized with respect to the entire range of choices for insulation levels, heating and cooling equipment, and storm windows. That is, the least-cost design combination of these components is selected.

#### 2.1.1.1 Alternative Choices for the Same Component

The simplest application of life-cycle cost minimization is for the case of a single building component with alternative levels of energy conservation. An example would be various levels of insulation in the

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[1] This discussion of minimization of life-cycle heating and cooling costs is intended only as a brief overview. For a more detailed development see Stephen R. Petersen, Retrofitting Existing Housing for Energy Conservation: An Economic Analysis (Building Science Series 64; Washington, D.C.: U.S. Department of Commerce, 1974), and "Economic Optimization in the Energy Conservation Design of Single-Family Housing," ASHRAE Transactions, Vol. 82, Part I, 1976.

attic. As the level of insulation is increased, the energy required to heat and cool the building decreases, but at a decreasing rate. This means that each successive unit of insulation yields less benefits in terms of saved energy than the preceding units. If the cost of each unit of insulation increases, remains constant, or at least does not decrease at a rate faster than that of the benefits, then that level of insulation which yields incremental benefits equal to its incremental costs will be the level which also minimizes the life-cycle costs of supplying heating and cooling services to the building.

2.1.1.2 Alternative Components

If the above minimization procedure is applied to all the components of a building, then the optimum energy conservation standard for that building will generally be found. One exception to this result is due to possible interdependencies between the various building components. Thus, for example, the efficiency of the heating equipment will have some effect on the optimal level of insulation in the walls and attic. When such interdependencies are significant, all possible combinations of levels of equipment efficiency and of insulation should ideally be examined to find the one with minimum life-cycle costs.[1]

An optimum building design developed with this minimum-cost method has the convenient characteristic that no more energy savings can be achieved by shifting resources from one component to another. In other words, greater energy savings are available only by increasing construction

[1] Currently, research is being conducted at the National Bureau of Standards to model these various interdependencies.

costs and the extra dollar savings would not justify those additional construction costs. This means that an economic balance has been achieved among the components in that the total energy-related construction budget cannot be reallocated differently among the components to reduce energy consumption.

#### 2.1.1.3 Alternative Buildings

An energy conservation standard that is to be optimum must also achieve an economic balance among all buildings governed by the standard. This requires that an alternative standard could not achieve lower energy expenditures for the same national energy-related construction budget by setting stricter specifications for some buildings and more lenient ones for others. Otherwise the alternative standard would be superior.

An important implication of this optimality requirement is that the climatic conditions of each building must be taken into account. Buildings located in severe climates have greater energy loads than elsewhere, and consequently the benefits available from an additional expenditure on energy-saving construction techniques are greater for such buildings. A standard which treats all climates alike by requiring the same overall coefficient of heat transmission (or U value), could always be improved by raising the maximum allowable U value for mild climates and lowering it for severe climates -- without increasing the nation's energy-related construction budget at all. Thus, the life-cycle cost minimization procedure will lead to a more stringent standard (in the sense of a lower allowable U value) for buildings in severe climates than for similar buildings in mild climates.[1]

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[1]It should be noted that if a cost-effective standard were stated in terms of an annual energy budget rather than a U value, the more severe climate would be permitted a higher annual energy budget. This is because the additional insulation called for under a cost-effective standard will not completely compensate for the greater heating load imposed by the severe climate. This result is demonstrated quantitatively (pp. 48-50).

### 2.1.2 Price Differentiation among Energy Types

Just as for differentiation among climatic conditions, a similar argument can be made for distinguishing among the effective prices[1] paid for different energy types. A building which uses a more costly energy form will yield greater benefits from an extra expenditure on energy-saving techniques because the value of the saved energy is greater. If a standard were developed which ignored the differences in the prices of energy types, then there would be room for improvement in benefits without even adding to the overall cost of compliance. One could simply design a new standard that would shift some of the energy-saving construction resources from buildings which use low-cost energy to those which use high-cost energy. The net dollar savings from the latter standard would be greater than those from the original standard which did not differentiate among the prices of energy types.

Thus, if the standard is to achieve the greatest possible net economic benefits (i.e., be truly optimum), it must take account of differences in energy prices. There is, moreover, a further relationship between optimality and energy prices. The type of energy prices used in the analysis determines the nature of the optimality achieved. The conceptual framework presented thus far has focused on the development of a standard which is optimum from the private building owner's point of view.

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[1]Effective price here means the number of dollars required to be spent to get one unit of usable heat, cooling power, or light added to the conditioned space.

That is, the standard was said to be based on the principle of minimizing the present value of all the privately incurred energy-related costs of the building over its entire life cycle. We now will modify this principle so that it will lead to a standard which is optimum from society's viewpoint.

## 2.2 OPTIMUM STANDARDS FROM THE SOCIAL PERSPECTIVE

The purpose of developing a national energy conservation standard is to encourage practices in the design of new buildings which are optimum for the nation. Because of certain distortions in the markets for energy resources, what is optimum for the nation may not be optimum for the individual prospective homeowner. That is, a standard which minimizes the life-cycle energy-related costs actually incurred by the homeowner will not necessarily minimize those costs when they are measured from society's point of view. This is because energy resources are valued differently by individuals and by society.

### 2.2.1 Private Prices do not Reflect Social Energy Values

Traditional economic theory of perfectly competitive markets shows that under very restrictive assumptions the prices paid for resources are equal to their true social values.[1] According to this theory, resource users are motivated to employ them up to the point at which the value produced by the last unit of the resource equals the

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[1] For example, see Richard H. Leftwich, The Price System and Resource Allocation (3rd ed.; New York: Holt, Rinehart, and Winston, 1966), Chapter 15, or Ian M.D. Little and James A. Mirrlees, Manual of Industrial Project Analysis in Developing Countries, Vol. II: Social Cost Benefit Analysis (Paris:Development Centre of the Organization for Economic Co-operation and Development, 1968), Chapter II.

price paid for it. Similarly, the suppliers of the resource will make it available up to the point at which the cost of supplying the last unit will just be covered by the price received for it. In this way resources are allocated most efficiently since no more surplus can be squeezed out of them. That is, to use a resource beyond the point dictated by its price would mean that the additional units employed would produce outputs worth less than the cost of the resources themselves. Thus, the resource price is the key which leads users to employ and suppliers to make available the optimum amount.

This ideal world of perfect competition in the supply and use of a resource is illustrated graphically in Figure 2.1. The line S represents the quantities of the resource that suppliers would be willing to make available at various selling prices in a given market and during a specific time period. The upward slope is used to show that as more of the resource is produced, the cost of producing each successive unit generally increases, which in turn requires a higher price to justify production. The line U, on the other hand, represents the quantities of the resource that users would be willing to buy at various prices in the same market and during the same time period. The downward slope indicates that as more of the resource is used, the value generated by each successive unit employed declines, which in turn reduces the price which users are willing to pay for additional units. At prices above P*, suppliers make more units available than users want and the resulting surplus drives the price down to P*. At prices below P*, suppliers make fewer units available

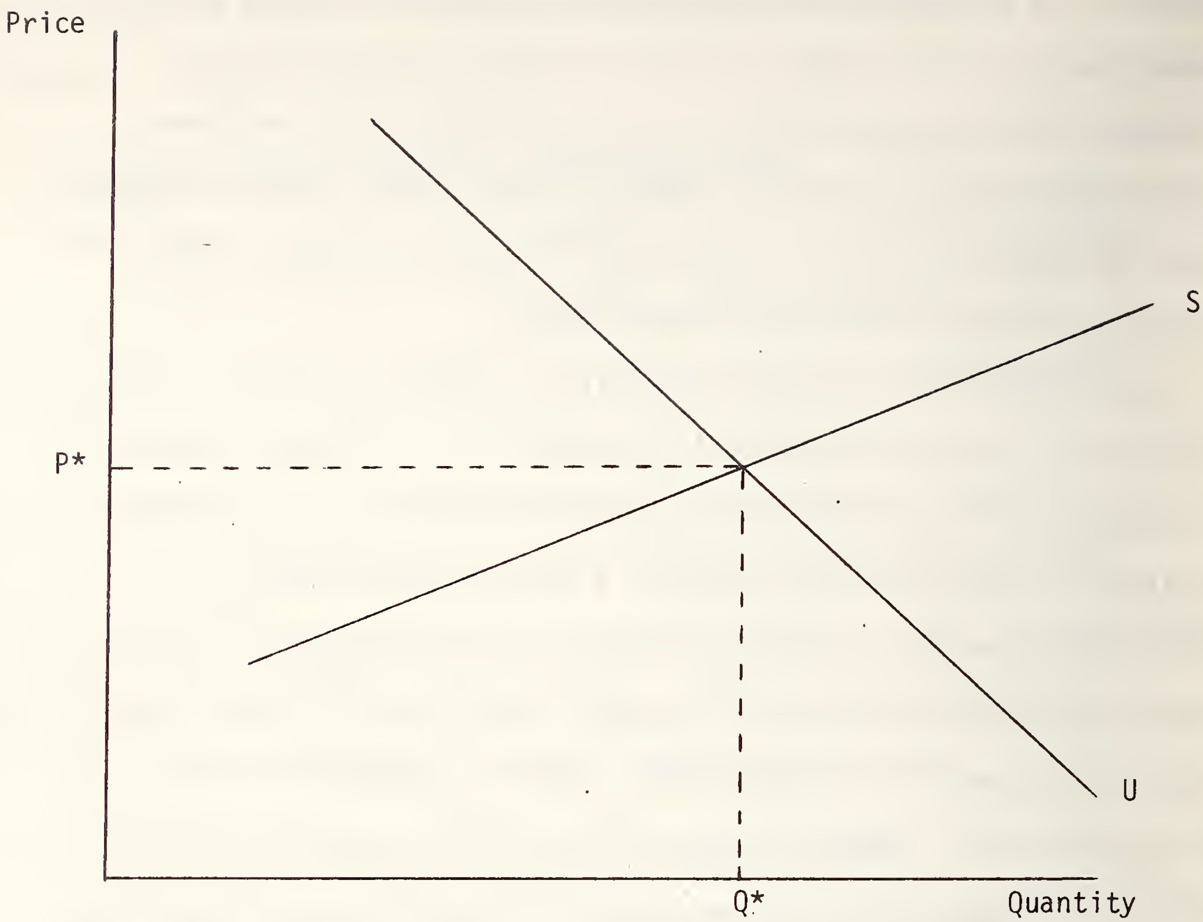


Figure 2.1

Competitive Market for a Resource

than users want and the resulting shortage drives the price up to P*. At P* there is neither shortage nor surplus and Q* is supplied and used. Users and suppliers are willing to continue buying and selling this equlibrium quantity at this equilibrium price. In addition to providing for an equilibrium in the sense that supply and demand are in balance, P* and Q* are optimum in the sense that the correct amount of the resource is being produced and used. That is, units of the resource used in excess of Q* would be employed to produce output whose value is less than the cost of supplying the resources (U is below S to the right of Q*). On the other hand, if resource units fewer than Q* are used, they would be employed to produce output valued above the cost of supplying the resources (U is above S to the left of Q*).

To operate so efficiently, however, the conditions of the perfectly competitive model must be satisfied for the market being considered. These conditions guarantee that the resulting equilibrium price that is visible to all participants in the market will indicate to the suppliers, on the one hand, the true value of additional units of the resource, and to the users, on the other hand, the true social cost of producing additional units. If the price is in any way prevented from performing this vital function, then the resource will not be supplied nor employed at optimum levels. There are a number of ways in which the price can fail to provide this essential information to the users and suppliers of the resource. We will now focus on several of the most important ones for energy resources.

2.2.1.1 Unit Taxes and Monopoly

One possible source of divergence between actual prices and social values or costs of energy resources is a tax per unit produced or sold.[1] Examples

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[1] A useful collection of articles on energy taxes and their implications can be found in Gerard M. Brannon (ed.), Studies in Energy Tax Policy (Cambridge, Mass.: Ballinger Publishing Company, 1975).

would be the utility taxes on the number of therms of gas or kilowatt-hours of electricity consumed, and the former oil import tax per barrel included in the per gallon price of home heating oil. Such taxes generally force the market equilibrium price to differ from the true social cost of supplying the resource.[1] This difference is illustrated in Figure 2.2, which is the same as Figure 2.1 except that the line $S_T$ has been added above and parallel to line S. The line $S_T$ represents actual costs of production plus the amount of the unit tax. This upward shift in the suppliers' requirements changes both the equilibrium quantity (from Q* to $Q_T$) and price (from P* to $P_T$). The effect on the price means that the actual market price ($P_T$) visible to and paid by the users exceeds the price (P*) which would exist if there were no tax and by an even greater amount exceeds the price ($P_C$) necessary to cover the actual costs of production.[2] This implies that users do not correctly perceive the true social cost of producing the resource, are forced to pay a higher price, and consequently under-utilize the resource.[3]

A similar situation of distorted market prices exists when there are so few firms competing in the market that the suppliers have a certain degree of monopoly power. In this case, suppliers in an attempt to maximize profits, will tend to restrict the quantity available such that the price paid per unit is greater than the true cost of producing

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[1] This is true unless the purpose of the tax is to offset negative environmental effects or excess profits, or to fulfill some other policy objective of the agency responsible for imposing the tax.

[2] The difference between $P_T$ and $P_C$ is given by the amount of the per unit tax.

[3] The same type of analysis applies to the case of subsidies, which can be treated as negative taxes. In this case, however, a subsidy per unit of production would cause the supply line to fall by the amount of the subsidy rather than to rise. As a result, users are able to purchase the resource at a price below its true cost of production, and consequently the resource is over-utilized.

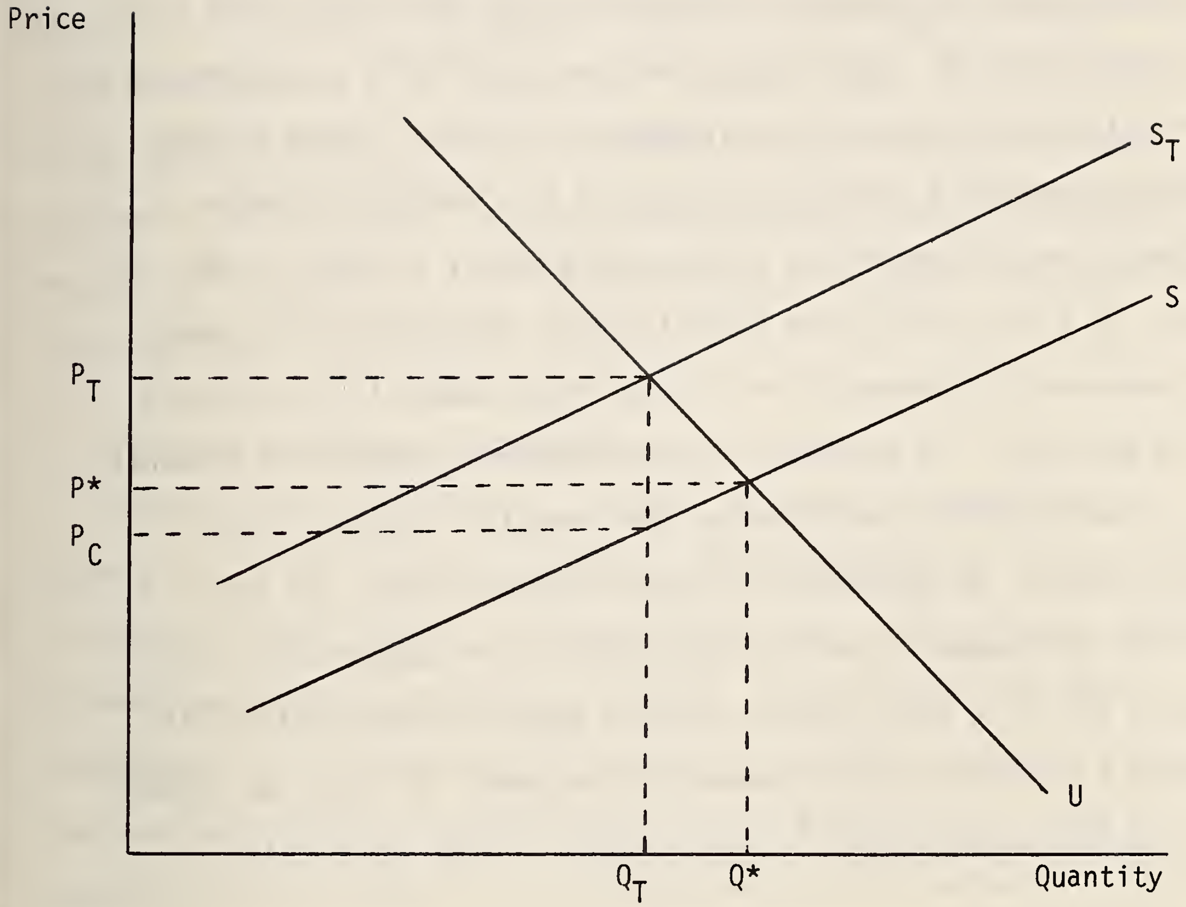


Figure 2.2

Effects of a Unit Tax

those units.[1] Here again, then, the decisions of resource users are not guided by the true cost of the resource.

2.2.1.2 Environmental Considerations

Another source of possible divergence between the actual price paid for a resource and the social cost of making it available is a group of factors called externalities or environmental effects.[2] These effects are present whenever a particular production or consumption process generates a cost or benefit for persons not directly involved in the process. As a result the price established by the suppliers and users of the resource will generally not reflect these external (or indirect) benefits and costs. In the case of energy resources most of the external effects take the form of costs rather than benefits.[3] As an illustration, consider the generation of electricity from coal. To the extent that environmental control legislation allows some residual air pollution, the price users pay for electric power will not reflect all of society's pollution costs of producing the power, but only the direct costs incurred by the utility.[4] This example is shown graphically in

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[1] See Edwin Mansfield, Microeconomics: Theory and Applications (New York: W. W. Norton & Company, 1970), Chapter 9, for a detailed explanation of the theory of pricing and output choice for monopolists.

[2] For two complete collections of articles on the economic theory of externalities, see Robert J. Staaf and Francis X. Tannian, Externalities: Theoretical Dimensions of Political Economy (New York: Dunellen Publishing Company, Inc. n.d.), and Steven A.Y. Lin (ed.), Theory and Measurement of Economic Externalities (New York: Academic Press, 1976).

[3] For a very detailed overview of alternative energy technologies and their environmental impacts, see Oklahoma University Science and Public Policy Program, Energy Alternatives: A Comparative Analysis (Springfield, Va: U.S. Dept. of Commerce, National Technical Information Service, 1975).

[4] Of course, if a tax were imposed which exactly offset these external costs, then the price (including the tax) paid would reflect all social costs of production.

Figure 2.3. The line $S_p$ indicates the costs incurred by utilities which must be covered by the price actually paid by users. The line $S_s$ is the sum of both the private costs and the external costs (i.e., the total social costs) caused by the air pollution. In the absence of an adequate pollution tax, the suppliers and users will agree on amount Q and exchange price P for each unit of electric power. This private price, however, is deceptively low in that it ignores the social costs incurred by the pollution. In fact, the total social costs of producing the last unit of power in this case would be $P_s$ and the amount by which that last unit is underpriced is $P_s$-P.

2.2.1.3 National Economic Independence

In support of various energy policies an appeal has been made in recent years to the need for national economic independence.[1] The appeal is based on concern over U.S. vulnerability to political influence through oil embargoes and unreasonable price rises. To the extent that energy independence is a national policy objective, it can be reasonably argued that each unit of energy consumed has an extra social cost in terms of increasing our requirement for oil imports and thus our vulnerability to embargoes. Indeed, when considering energy conservation standards that affect the energy use of buildings for many years to come, the argument becomes even more cogent. The more stringent the standard, the less our commitment to energy imports during the next 30 to 40 year life of buildings now being designed.

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[1] Uzi Arad, Barry J. Smernoff, and Haim ben-Shaher, American Security and the International Energy Situation, 4 vols. (Croton-on-Hudson, N.Y.; Hudson Institute, 1975) provides an overview of the energy independence issue.

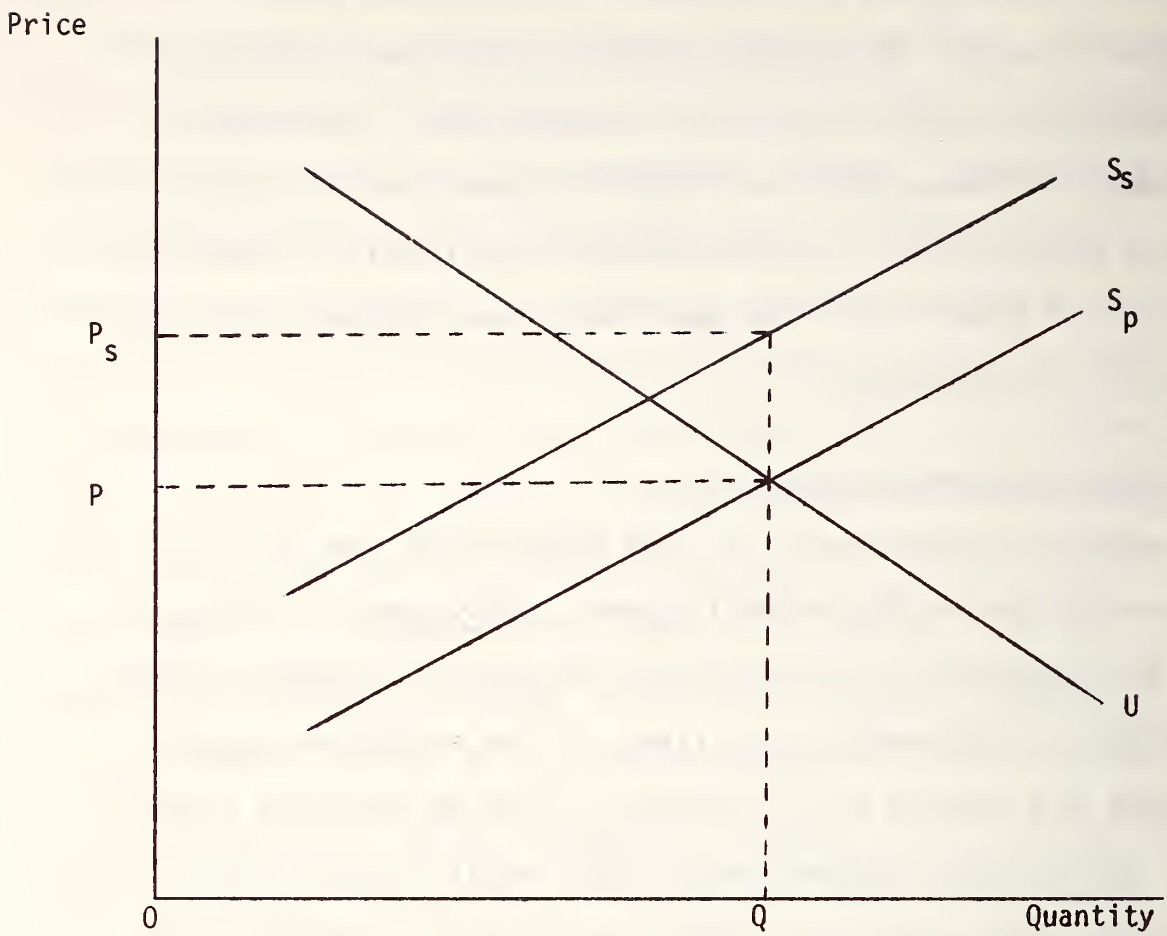


Figure 2.3

Social Costs of Coal-Fired Electric Power

This national independence goal has implications for the need to adjust energy prices similar to those of externalities discussed above. Just as the external cost of pollution causes the social price of electricity to be above the privately determined market price, so the cost to the nation's economic independence (if such is a public policy goal) makes the social cost of energy consumption exceed its private cost.

2.2.1.4 Price Controls

Deviations between the prices paid for resources and their social value can also come about through price controls. In this situation, the price actually paid is set by legislation or regulation and not by the forces of the market operating through the costs of supply and the benefits from use. The objective of such regulation may be to control excess profits in an industry with little competition or to make a vital resource affordable for low-income households. Thus, even in the absence of externalities, monopoly power, and taxes, the administered price will not necessarily be equal to the social value of the resource. This is illustrated for the case of natural gas in Figure 2.4. Because the regulated price, $P_r$, is set so far below the optimum price, $P^*$, the suppliers of natural gas are willing to make only $Q_r$ units available rather than the optimum quantity, $Q^*$. With so few units available, there are many unsatisfied high-value uses to which the gas could have been put. This is indicated by the fact that at $Q_r$, the premium users would be willing to pay for additional units over and above the regulated price, would be as much as $P_s-P_r$.

2.2.2 The Need to Adjust Prices to Reflect Social Values

The preceding discussion covers some of the ways in which the actual prices paid for energy resources may fail to reflect their full

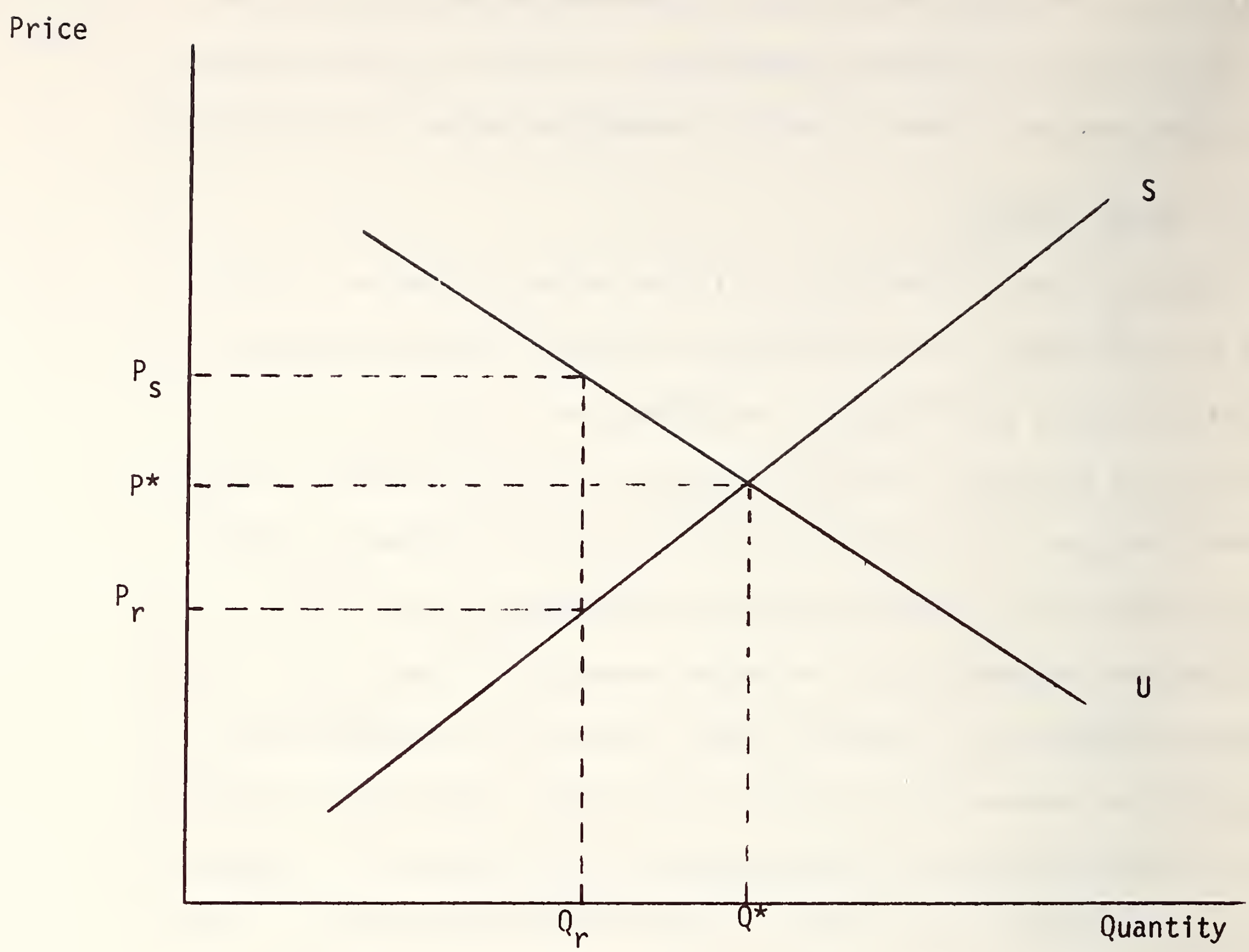


Figure 2.4

Regulation of the Natural Gas Price

social values. As such it offers some insight into the difficulty of using actual energy prices as a basis for energy-related policies such as the conservation standard for building design. Moreover, the items discussed provide at least a partial checklist of factors which should be considered in the development and actual quantification of RIF's. Before RIF's can be quantified, however, a further consideration will have to be made regarding possibly conflicting objectives which may exist among government agencies. For example, a goal of keeping the cost of vital home heating fuels within the budgets of low-income households may conflict with a goal of national energy independence. If RIF's are to reflect a unified national energy policy, such conflicts must first be resolved.

## 3.0 RIF's IN AN OPTIMUM STANDARD DETERMINATION MODEL

In this section we will apply a range of RIF's to a life-cycle cost minimization model for the determination of a socially optimum conservation standard. Two alternative ways of formulating RIF's will be presented and then the optimum standard determination model will be explained. These formulations will be used to introduce a family of RIF values explicitly into the model to compare the resulting energy standard with and without RIF's.

### 3.1 ALTERNATIVE FORMULATIONS FOR RIF's

RIF's may be formulated to be applied to energy resources in either of two fundamental ways. The first way would have RIF's formulated to be multiplied by the physical quantities of the energy resource used. According to the other formulation, RIF's would be applied directly to the actual price of the energy resource. Each of these formulations will be discussed in turn.

#### 3.1.1 Quantity Multiplier Formulation

Quantity multiplier RIF's would be formulated as coefficients to be multiplied times the particular energy load in question.[1] The annual energy load would be denominated in units of energy output ($10^6$ Btu's) required to meet the final energy consumption needs of the building. In the case of space heating, for example, this energy load would be the heating

[1] This quantity formulation is suggested in the definition of RIF under review by the ASHRAE Section 12 Panel: "A multiplier applied to the fuel and energy resources required by a building project to permit evaluation of the effect on non-renewable resources resulting from the selection of fuel and energy sources, giving consideration to time, location, economic environmental and national interest issues."

value of the energy output into the conditioned space that is necessary to maintain a desired comfort level. To specify the RIF coefficient one would have to define a unit of measurement for scaling the relative impacts of using the various energy types. Such a measurement unit could be arbitrarily specified as a "resource impact unit."[1] These impact units would attempt to measure such factors as the environmental and national security issues. Then the RIF for a particular energy type would be a coefficient dimensioned as the number of resource impact units per unit of output for that energy type. Table 3.1 presents some hypothetical quantity multiplier RIF's to illustrate how they might operate.

Table 3.1

Hypothetical Quantity Multiplier RIF's

| Energy Type | Units Needed[a] | Heating Load ($10^6$ Btu) | RIF | Impact |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4)=(2)x(3) |
| Nat. Gas (MCF) | 60 | 36 | 1.3 | 46.8 |
| Oil (Gal.) | 429 | 36 | 1.2 | 43.2 |
| Electric(kWh) | 10,548 | 36 | 2.0 | 72.0 |

[a]The number of physical units of input of the energy type needed to satisfy the given annual heating load of 36 x $10^6$ Btu. The heating system output of 36 x $10^6$ Btu was converted to corresponding input requirements by assuming the following energy contents and conversion efficiencies:

(1) Natural Gas: $10^6$ Btu/MCF and 60% efficiency
(2) Oil: 140,000 Btu/Gal. and 60% efficiency; and
(3) Electric: 3413 Btu/kWh and 100% efficiency.

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[1]Alternatively, the impact unit could be stated in terms of the number of equivalent Btu's, along the lines suggested for Resource Utilization Factors in the proposed ASHRAE Standard 90-75, Section 12. As indicated in Section 1.0 above, however, the Btu equivalence approach amounts to Net Energy Analysis with its noted shortcomings.

Column 1 shows the number of physical units of each alternative energy type required to satisfy the annual heating load of 36 x $10^6$ Btu specified for a particular building (e.g., 429 gallons of fuel oil). Column 2 gives the energy load in terms of the heating value of the energy output needed to maintain a desired degree of comfort (36 x $10^6$ Btu). Column 3 presents the RIF value corresponding to each energy resource type, measured in terms of impact units per $10^6$ Btu of energy output needed to meet the heating load (e.g., 1.2 impact units per $10^6$ Btu of heating output from oil). Finally, Column 4 shows the product of Columns 2 and 3, which represents the number of impact units required to meet the heating load by means of each energy type (e.g., 43.2 impact units for oil).

This quantity multiplier formulation is useful in that the resulting impact numbers permit comparison of the relative merits of using one energy type rather than another. Using these impact numbers, standards could be developed so as to achieve a relative balance among energy types with respect to the stringency of the standards. Thus, for a given energy conservation construction budget, a system of such standards could be designed so as to minimize the sum of the impact numbers for space heating.

The fundamental shortcoming of this formulation lies in its inability to establish the appropriate construction budget. Because the impact numbers are not denominated in dollars (the measurement unit of the construction costs), the formulation cannot be used to establish the appropriate economic balance between the amount of one energy type and the application of energy conservation features in a building design.

This quantity multiplier does not allow one to weigh the importance of using all types of energy resources as compared with energy-conserving resources. Consequently, it cannot be used to determine how much energy is worth being saved. Of course, this quantity multiplier could be converted into a value multiplier if a dollar value could be established for the resource impact unit chosen. Then, the energy and non-energy resources would have a common basis for comparison. In this case, however, the RIF would no longer be a quantity multiplier, but rather a value multiplier. This is equivalent to the price multiplier formulation which we will now discuss.

### 3.1.2 Price Multiplier Formulation

The other formulation possible for RIF's is as a multiplier to be applied to the actual price of each energy type. This multiplier would represent the ratio of the estimated social price to the actual price of the resource. As such, this factor would convert the private price or the actual price paid for the energy to its corresponding social price. In short, it would act as an adjustment factor which corrects existing actual prices for the various price-distorting effects (external effects, the need for economic independence, unit taxes, monopoly power, and price controls) which were discussed in Subsection 2.2.1. Table 3.2 offers some hypothetical price multiplier RIF's to illustrate this kind of formulation.

Table 3.2

Hypothetical Price Multiplier RIF's

| Energy Type | Actual Price ($/Unit) | Effective Price[a] ($/$10^6$Btu) | RIF | Social Price ($/$10^6$ Btu) |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4)=(2)x(3) |
| Nat. Gas (MCF) | 2.00 | 3.33 | 2.0 | 6.66 |
| Oil (Gal.) | .40 | 4.76 | 1.5 | 7.14 |
| Electric (kWh) | .03 | 8.79 | 1.1 | 9.67 |

[a]Energy contents and conversion efficiencies for each type are as follows:

(1) Natural Gas: $10^6$ Btu/MCF and 60% efficiency;
(2) Oil: 140,000 Btu/Gal. and 60% efficiency; and
(3) Electric: 3,413 Btu/kWh and 100% efficiency.

Column 1 gives the approximate prices actually paid by homeowners for each physical unit of the energy types and Column 2 uses energy contents and efficiencies to convert these to effective prices paid for one million Btu's delivered to the conditioned space. Column 3 lists hypothetical price multiplier RIF's for each type of energy and Column 4 gives the products of Columns 2 and 3 which represent the adjusted social prices of the energy types.

## 3.2 AN OPTIMUM STANDARD DETERMINATION MODEL

In this section we shall briefly summarize the life-cycle cost minimization model developed by Stephen Petersen to determine the economically optimum levels of thermal resistance in building components.[1]

[1]Stephen R. Petersen, "Economic Optimization in the Energy Conservation Design of Single-Family Housing," ASHRAE Transactions, Vol. 82, Part I, 1976.

Thermal resistance (R) is defined as the exact reciprocal of the overall coefficient of thermal transmission (U). The resistance of components of the building envelope is one of the primary determinants of heating and cooling energy used in buildings, especially of the residential and small commercial type. This model will be used to show how climate factors and energy prices can be considered directly in determining the extent to which energy conservation modifications should be made to the design of the building envelope. By determining the optimum level of resistance ($R_o$) for a particular component such as the attic floor, the model also determines the optimum coefficient of thermal transmission ($U_o$), because of the exactly inverse relationship between R and U. In speaking of the optimum standard we shall henceforth be referring to this optimum coefficient of thermal transmission ($U_o$). Thus, the more stringent the standard (i.e., the more energy conserving), the lower will be $U_o$.

### 3.2.1 Derivation of the Model

Considering only the heating load, the life-cycle costs (LCC) attributable to heat losses through one square foot of attic floor area for given heating equipment and climatic conditions can be represented by the following expression:

$$LCC = a + b \cdot R + (UPW \cdot 24 \cdot DD \cdot P_o)(1/R). \qquad (3.1)$$

The notation is to be interpreted in the following manner:

a = The fixed cost component of thermal resistance ($/sq. ft.).

b = The cost per additional unit of thermal resistance ($/sq. ft./R).

R = The number of units of thermal resistance on each square foot of attic floor area.

$$UPW = \sum_{n=1}^{N} \left[\frac{1+e}{1+d}\right]^n = \left[\frac{1+e}{d-e}\right] \cdot \left[1 - \left(\frac{1+e}{1+d}\right)^N\right],$$

where e is the annual rate of fuel price escalation in real terms, d is the real discount rate, and N is the number of years in the useful life of the building. For example, with a 10% real discount rate, a 3% rate of real fuel price escalation, and a 40 year life, UPW = 13.65. This uniform present worth (UPW) factor operates as a fixed coefficient to convert annual energy costs to their equivalent present value.

$$DD = \sum_{i=1}^{365} (B - \overline{T}_i),$$ for all $\overline{T}_i < B$, where $\overline{T}_i$ is the average

temperature for the ith day and B is the base temperature (usually 65° F), chosen as the cutoff point below which heat is needed from the heating system to keep the conditioned space at a target temperature (usually 70°). This summation measure is referred to as heating degree days and is commonly used to approximate the annual heating load imposed by a given climate. Multiplication by 24 simply converts degree days to degree hours.

$P_e$ = the effective price actually paid for one Btu of heating energy output delivered to the conditioned space, taking account of conversion efficiencies.

This expression for life-cycle costs can be minimized by partial differentiation with respect to R, setting the first derivative equal to zero and solving to find the optimum level of thermal resistance, $R_o$.

$$R_o = [UPW \cdot 24 \cdot DD \cdot P_e \cdot (1/b)]^{\frac{1}{2}} . \quad (3.2)$$

Since the rate of thermal transmission, U, is defined as the exact reciprocal of R, we have the optimum coefficient of thermal transmission, $U_o$, for a square foot of attic floor area:

$$U_o = \frac{1}{R_o} = \left[\frac{b}{UPW \cdot 24 \cdot DD \cdot P_e}\right]^{\frac{1}{2}} . \quad (3.3)$$

Although this formulation of the model is quite simple, it provides the most important elements needed to determine the optimum component performance standard for attics.

### 3.2.2 Significance of the Model

Several features of this expression for the optimum component performance standard are worthy of note. In the first place the optimum standard ($U_o$) is an increasing function of the marginal cost of adding resistance units (b), although it increases as the square root of this cost. This means that a doubling of the price of insulation would result in only a 41.4% increase in the maximum heat transmission rate allowed by the optimum standard, other things being equal.[1] This positive relationship has the intuitive appeal

[1] Letting $U_o'$ represent the standard after the doubling of the marginal cost of resistance, we have $U_o' = \sqrt{2} \cdot U_o = 1.414\ U_o$, which means an increase of 41.4%.

that the more costly insulation becomes, the more lenient will be the standard by allowing higher rates of thermal transmission, and consequently calling for lesser amounts of insulation.

Another significant aspect of this expression is that there is an inverse relationship between the number of degree days (DD) and the standard ($U_o$). This implies that buildings in more severe climates should be subject to a more stringent standard requiring a lower rate of thermal transmission. It should be noted that this inverse relationship is not linear but rather is based on the square root of degree days. Thus, a house in a climate with twice as many degree days would be subject to a standard requiring a 29.3% lower rate of thermal transmission.[1]

The final noteworthy feature of this expression for the optimum component performance standard is that, just as with degree days, $U_o$ also varies inversely as the square root of the price of energy ($P_e$). That is, a doubling of the price of energy calls for a 29.3% decrease in $U_o$, and consequently the same 29.3% decrease in the maximum energy consumption allowed by the optimum standard. The negative direction of this relationship makes sense in that an increased price of energy should lead to decreased consumption of it through the substitution of additional insulation. This last feature has a direct bearing on the effect of applying RIF's to the model.

## 3.3 APPLICATION OF PRICE MULTIPLIER RIF's

As it stands, the expression for $U_o$ developed above is based on the goal of cost minimization from the private point of view. To

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[1] Letting $U_o'$ represent the standard for the climate with twice as many degree days, we have $U_o' = U_o/\sqrt{2} = 0.707\ U_o$, which means a decrease of 29.3%.

adjust the expression to reflect the social viewpoint, it is necessary to replace the effective price paid for energy ($P_e$) with the social price of energy.[1] Using the price multiplier formulation shown in Table 3.2 above, which gives RIF's as the ratio of the social price to the effective price, all that needs to be done is to multiply the symbol $P_e$ in the expression for $U_o$ by the appropriate RIF of the fuel type being considered.[2] This will result in the following new expression for the optimum performance standard from the social point of view ($U_o^*$):

$$U_o^* = \left[ \frac{b}{UPW \cdot 24 \cdot DD \cdot P_e \cdot RIF} \right]^{\frac{1}{2}} . \tag{3.4}$$

Thus, the relationship of the socially optimum standard to the privately optimum standard can be established as follows:

---

[1] It might also be necessary to replace the marginal cost paid for insulation (b) by its appropriate social cost counterpart, if there is reason to believe that a significant divergence exists between its actual cost and the social cost. If such social cost estimates are developed, then they should be used in place of b in the formula.

[2] This formulation allows us to establish clearly the effect of RIF's on the optimum standard. It is conceivable, however, that the estimated social value of energy may not continue to be a constant (or even constantly growing) multiple of the actual price paid for energy over time. In this case, the proper approach to actually determining the socially optimum standard would be to replace the product $UPW \cdot P_e \cdot RIF$ in expression (3.4) with the summation, $\sum_{n=1}^{N} \frac{(P_s)_n}{(1+d)^n}$, where $(P_s)_n$ represents the estimated social value of the energy type for the nth year.

$$U_o^* = \left[ \frac{b}{UPW \cdot 24 \cdot DD \cdot P_e} \right]^{\frac{1}{2}} \cdot \left[ \frac{1}{RIF} \right]^{\frac{1}{2}} \text{, or}$$

$$U_o^* = U_o \cdot \left[ \frac{1}{RIF} \right]^{\frac{1}{2}} . \tag{3.5}$$

So, in general, it can be said that for a given privately optimum standard the socially optimum standard varies inversely with the square root of the RIF value.

In order to analyze this relationship properly, the range of RIF values must be established. A RIF value would always be positive but could be less than, equal to, or greater than one, depending on the relative importance of the various elements to be included in the derivation of RIF's. For example, if the price effects of unit taxes and monopoly power predominate, then the social price of the energy type could be lower than the actual price, making RIF less than one. If the actual price paid for one type of energy exactly reflected its social value, then the corresponding RIF would equal one. Finally, if factors such as external effects and price controls predominate, then RIF's will be greater than one.

Table 3.3 shows the relationship between particular RIF values and the resulting socially optimum standard expressed as a proportion of the privately optimum standard. As can be seen, the value of the RIF determines whether or not the use of RIF's leads to greater or less energy consumption. When the RIF is less than one (i.e., the social price is less than the actual price paid), then the introduction of RIF's would have the effect of raising the level of energy consumption

Table 3.3

Possible RIF Values with Corresponding Socially Optimum Standards

| RIF | $U_o^* = [1/RIF]^{\frac{1}{2}} \cdot U_o$ | |
|---|---|---|
| .50 | 1.414 $U_o$ | |
| .60 | 1.291 $U_o$ | |
| .70 | 1.195 $U_o$ | |
| .80 | 1.118 $U_o$ | |
| .90 | 1.054 $U_o$ | |
| 1.00 | 1.000 $U_o$ | (Equivalent to not using RIF's) |
| 1.10 | 0.953 $U_o$ | |
| 1.20 | 0.913 $U_o$ | |
| 1.30 | 0.877 $U_o$ | |
| 1.40 | 0.845 $U_o$ | |
| 1.50 | 0.816 $U_o$ | |
| 1.60 | 0.791 $U_o$ | |
| 1.70 | 0.767 $U_o$ | |
| 1.80 | 0.745 $U_o$ | |
| 1.90 | 0.725 $U_o$ | |
| 2.00 | 0.707 $U_o$ | |
| 2.25 | 0.667 $U_o$ | |
| 2.50 | 0.632 $U_o$ | |
| 2.75 | 0.603 $U_o$ | |
| 3.00 | 0.577 $U_o$ | |
| 4.00 | 0.500 $U_o$ | |
| 5.00 | 0.447 $U_o$ | |
| 10.00 | 0.316 $U_o$ | |
| 100.00 | 0.100 $U_o$ | |

allowed by the standard. This would mean less energy conservation with RIF's than without them. If the RIF equals one, then the introduction of RIF's would have no effect on energy consumption. In the more probable event that RIF's are greater than one, the effect of using them is to lower the allowable energy consumption and, thus, to increase the energy savings attributable to the standard. For example, the introduction of a RIF equal to 1.5 (meaning the social price is 50% higher than the actual price paid) would lower the maximum energy consumption allowed by the standard by 18.4%, to only 81.6% of what it would be without the RIF.

An observation can be made from Table 3.3 which holds generally, regardless of the value of the RIF applied. That is, as the RIF value increases, the corresponding energy consumption rate allowed under the socially optimum standard decreases relative to the privately optimum standard, and absolutely. This inverse relationship is also apparent from Figure 3.1 which merely plots the points given in Table 3.3. The general pattern of the points is downward sloping to the right which indicates that increases in the RIF value lead to decreases in $U_0^*$ as a proportion of $U_0$. A more rigorous mathematical demonstration of this observed inverse relationship requires evaluation of the sign of the first derivative of $U_0^*$ with respect to RIF. Thus, partial differentiation of expression (3.5) leads to the following:

$$\frac{\partial U_0^*}{\partial RIF} = [-0.5\ RIF^{-1.5}] \cdot U_0. \tag{3.6}$$

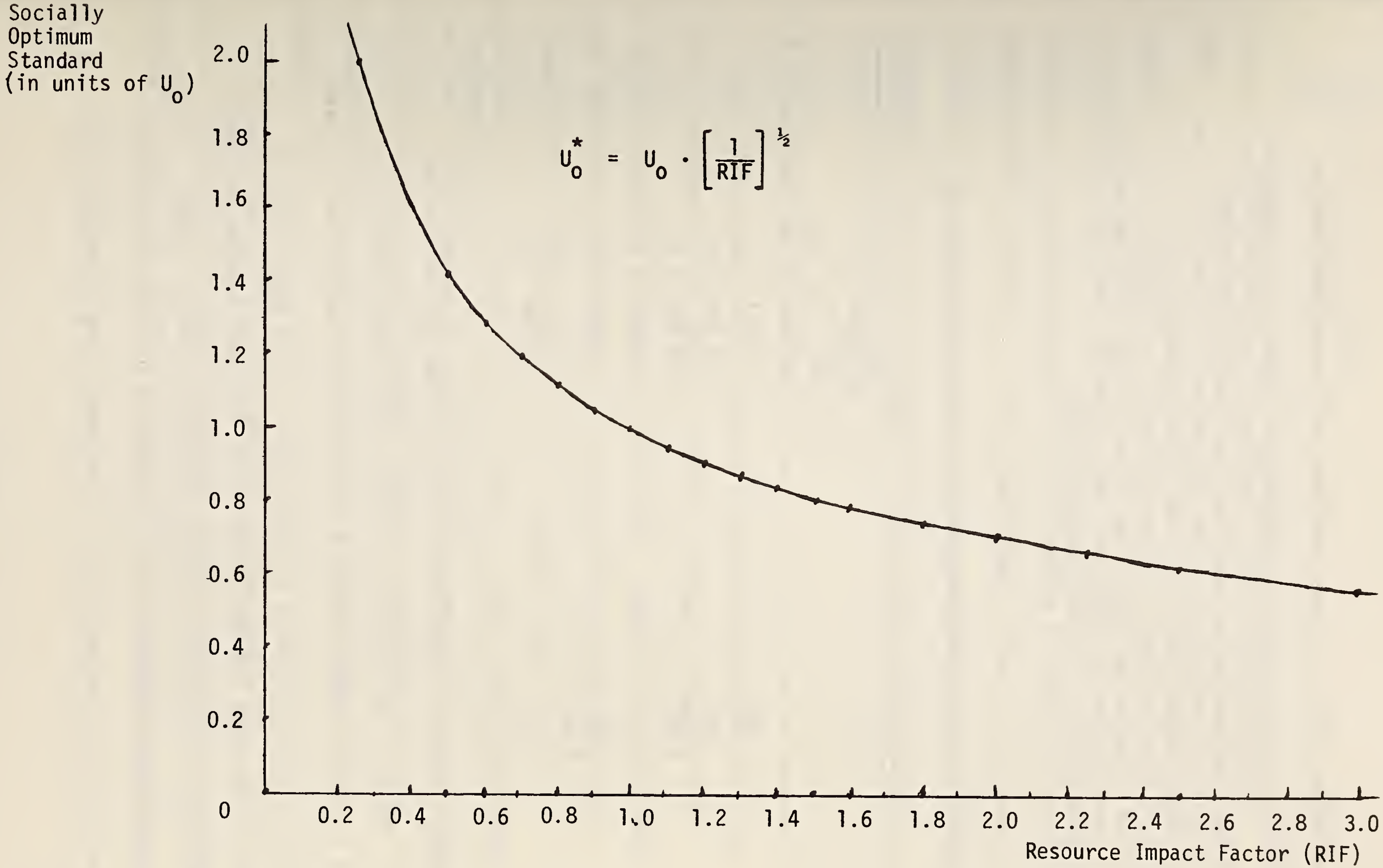


Figure 3.1

Plot of the Socially Optimum Standard (in units of $U_o$) as a Function of the RIF Value, for a Given Value of the Privately Optimum Standard ($U_o$)

Since the value of RIF is always positive, this expression must be negative for the entire range of possible RIF values. This negative value of the first derivative means that the inverse relationship (downward slope) between RIF values and $U_o^*$ holds everywhere.

Table 3.4 presents the calculated values of this first derivative measured in units of $U_o$ for selected RIF values.

Table 3.4

Selected RIF Values with Corresponding First Derivatives of $U_o^*$

| RIF | $\partial U_o^*/\partial RIF = [-0.5\ RIF^{-1.5}] \cdot U_o$ |
|---|---|
| .50 | $-1.414 \cdot U_o$ |
| .75 | $-0.770 \cdot U_o$ |
| 1.00 | $-0.500 \cdot U_o$ |
| 1.25 | $-0.358 \cdot U_o$ |
| 1.50 | $-0.272 \cdot U_o$ |
| 2.00 | $-0.177 \cdot U_o$ |
| 3.00 | $-0.096 \cdot U_o$ |

As indicated above, the signs of all the calculated derivatives are negative. Moreover, an additional pattern emerges from Table 3.4. Note that as the RIF values increase, the calculated derivatives decrease in absolute value but increase in actual value. This means that, as one moves to the right (increasing RIF's) on the curve plotted in Figure 3.1, the slope of the tangent to that curve gets steadily flatter. This can be demonstrated mathematically by inspecting the sign of the second derivative of $U_o^*$ with respect to RIF. Differentiating expression (3.6) we have

$$\frac{\partial^2 U_o^*}{\partial RIF^2} = U_o \, [0.75 \, RIF^{-2.5}]. \tag{3.7}$$

Since this second derivative is positive for all positive values of RIF, it means that the first derivative, which is negative, is always increasing (i.e., decreasing in absolute value). Thus, as the RIF value is increased by increments of equal size, the effect of each increase in reducing the energy consumption rate allowable by the socially optimum standard steadily decreases. The important implication of this result is that there are diminishing returns in terms of the energy savings attributable to the socially optimum standard from successive equal increments in the value of RIF's.

## 3.4 LIMITATIONS OF THE ANALYSIS

The foregoing analysis showed the energy-saving effects of using RIF's in determining the socially optimum performance standard for a building component such as the attic floor. Generally, it was found that the higher the RIF value, the lower would be the maximum energy consumption rate allowed by the socially optimum standard. It was also found that in the more likely situation of RIF values greater than one, the energy saved by the socially optimum standard is greater than that saved by the privately optimum standard. The analysis of the economic efficiency effects of using RIF's will be treated in the Appendix.

The model used in the above analysis has two general limitations: (1) it is suitable for studying component rather than building performance standards; and (2) it is based on some restrictive assumptions which may be violated under actual conditions. A model for determining optimum building performance standards is currently being developed at the National Bureau of Standards. When completed, the model will incorporate the many energy-flow interactions among the various building components. This will allow analysis of the economic trade-offs between interrelated components. Once this model is fully developed, the effect of using RIF's for determining building performance standards could be assessed.

Two restrictive assumptions underlying the model presented above limit its general applicability. The first regards the cost of each additional unit of resistance. It was assumed that the cost of resistance was a linear function of the level of resistance. That is, the marginal cost of each successive unit of resistance was assumed to be constant, regardless of the total level of resistance. This marginal cost component was represented by the constant parameter, b. It seems likely, however, that within the relevant range the marginal cost of resistance increases as more units are added. Secondly, it was assumed that the temperature base for calculating degree days was the same regardless of the level of resistance. This meant that DD could be treated as a fixed parameter, independent of the level of resistance. It will be shown that, as more resistance is added, the degree-day temperature base is likely to fall, resulting in fewer effective degree days for a given climate. In the next section a model is developed which takes account of these more realistic conditions.

# 4.0 THE OPTIMUM STANDARDS MODEL GENERALIZED

In this section we modify the optimum standards selection model of Section 3.0 by making alternative assumptions regarding the manner in which resistance costs and the number of degree days depend on the level of thermal resistance. We begin by maintaining our original assumption that the number of degree days is invariant and independent of the resistance level, while allowing resistance costs to assume several alternative functional forms. These modifications regarding resistance costs lead to the development of two new cases of the model which have different formulations for the rate of thermal transmission allowed by the optimum standard. These new formulations are each analyzed and compared in terms of the effect of using RIF's in the determination of the socially optimum standard. In the second subsection we consider the manner in which the number of degree days are likely vary as a function of the level of thermal resistance. This leads to a modification of the original model with degree days specified as a quadratic function of the resistance level. The implications of this formulation for the effect of using RIF's are discussed. In the final subsection, both cost and degree days are formulated as general functions of resistance without specifying particular functional forms. By making some reasonable assumptions about the first and second derivatives of these functions, some general conclusions are drawn regarding the relationship between the magnitude of the RIF values and the level of the socially optimum standard.

To facilitate the presentation of these alternate forms of the model, a slightly different, more condensed system of notation will be used throughout this section. As in Section 3.0, the objective of the model is to determine which level of thermal resistance is consistent with some minimum required indoor temperature for the least life-cycle cost. This optimum level of resistance is found by minimizing life-cycle heating-related costs (L) expressed as a function of thermal resistance (R). The model and explanations of the new notation are as follows:

Minimize with respect to R
$$L = K \cdot D \cdot (1/R) + C, \qquad (4.1)$$

where

$L$ = life-cycle heating-related costs of a unit of attic floor area,

$K = UPW \cdot P_e \cdot 24$,

$D$ = the number of effective degree days, which in one variation of the model is expressed as a function dependent on the level of thermal resistance,

$R$ = the thermal resistance of the attic floor, and

$C$ = the total construction costs of a unit of attic floor area expressed as a function of thermal resistance.

## 4.1 ALTERNATIVE COST FUNCTIONS

As stated above, the first group of model variations maintains the original assumption of degree days being fixed and independent of resistance, while allowing alternative assumptions regarding the manner in which construction costs (C) depend on the level of thermal resistance. In Section 3.0 we made the assumption that the per unit area construction costs of the attic portion of the building envelope are a linear function of the level of resistance. This assumption implies that each additional (or incremental) unit of resistance costs the same regardless of the level of total resistance.[1] In this subsection we will relax that linearity assumption to allow for the case of rising incremental costs first by a quadratic function and then by cubic and higher-order functions. For the sake of completeness, we begin by restating the original model of Section 3.0 using the new format given by expression (4.1) above.

### 4.1.1 Case I: A Linear Cost Function

Under the assumptions of linear costs and fixed degree days, the expression for life-cycle cost becomes

$$L = K \cdot D \cdot (1/R) + a + b \cdot R. \tag{4.2}$$

Differentiating with respect to R and setting the derivative equal to zero yields the following expression for the optimum resistance, $R_o$:

$$R_o = \left[\frac{K \cdot D}{b}\right]^{\frac{1}{2}} \quad . \tag{4.3}$$

This implies that the standard is given by

$$U_o = \left[\frac{b}{K \cdot D}\right]^{\frac{1}{2}} \quad . \tag{4.4}$$

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[1] In the terminology of economists, this is equivalent to assuming that the marginal cost of resistance is constant.

Substituting F $\cdot$ K for K, where F is the RIF value, we can establish the relationship between the socially optimum standard $U_o^*$, and the privately optimum one, $U_o$:

$$U_o^* = \left[\frac{b}{F \cdot K \cdot D}\right]^{\frac{1}{2}} = U_o \cdot \left[\frac{1}{F}\right]^{\frac{1}{2}} . \tag{4.5}$$

As was pointed out in Section 3.0, this means that as the RIF value increases, the rate of thermal transmission allowed by the socially optimum standard decreases (i.e., greater levels of thermal resistance could be needed to meet the standard), but at a steadily diminishing rate.

With this formulation it is also possible to compare the relative impacts of employing RIF's in different climates. For the entire range of heating loads represented by the number of degree days (D), the introduction of a given RIF value has the <u>same percentage</u> impact on the rate of thermal transmission allowed by the optimum standards. This is because the two different values of $U_o$ in equation (4.5) are being multiplied by the same value of $F^{-\frac{1}{2}}$, so that the percentage change is the same. By the same token, this implies that the absolute value of the impact on $U_o^*$ is greater for warmer climates than for colder ones, because the base values of $U_o$ are greater in the warmer climate.

This result is reversed, however, when one considers the relative impact of RIF's on the <u>annual amount</u> (rather than the rate per degree day) of energy consumption allowed by a socially optimum standard. A numerical example illustrates the point. Suppose the values of the parameters b, K, and D in expression (4.4) were such that $U_o$ equalled

0.10 in a particular region with a relatively warm climate. A colder climate with four times as many degree days would then require a $U_o$ equal to $0.10/\sqrt{4}$ (or one half as high a rate of transmission), for the same values of b and K. Let us denote the extra annual energy savings per square foot of attic attributable to the use of RIF's in the warmer climate by $\Delta S_w$ and those in the colder climate by $\Delta S_c$. Then we have

$$\begin{aligned} \Delta S_c &= (1-F^{-1/2}) \cdot (24) \cdot (4D) \cdot (0.10/\sqrt{4}), \text{ and} \\ \Delta S_w &= (1-F^{-1/2}) \cdot (24) \cdot (D) \cdot (0.10). \end{aligned} \tag{4.6}$$

Forming the ratio of the colder climate savings to the warmer climate savings, and cancelling like terms in the numerator and denominator, we have

$$\frac{\Delta S_c}{\Delta S_w} = 4/\sqrt{4} = \sqrt{4} = 2. \tag{4.7}$$

Thus in comparing two different climates, one with <u>four</u> times as many degree days as the other, the extra annual energy savings attributable to using RIF's are <u>twice</u> as great for the colder climate than for the warmer one. It is important to note that this result holds for all RIF values greater than one.[1] This result can be stated in a more general form. If a colder climate has n times

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[1]If F=1, then $\Delta S_c = \Delta S_w = 0$ and the ratio becomes meaningless. If $F < 1$, then $\Delta S_c$ and $\Delta S_w$ are both negative, which implies that application of the RIF would allow higher rather than lower levels of energy consumption.

as many degree days as a warmer climate, the extra annual energy savings attributable to using a positive RIF to develop the standard for the colder climate would be $\sqrt{n}$ times as much as that for the warmer climate.

### 4.1.2 Case II: A Quadratic Cost Function

We now abandon the linear cost assumption in order to investigate the case in which unit area construction costs of the attic floor are formulated as a quadratic function of thermal resistance. The general form of a quadratic cost function is as follows:

$$C = a + b \cdot R + c \cdot R^2, \tag{4.8}$$

where a, b, and c represent fixed parameters. This formulation implies that the marginal or incremental cost of adding a unit of thermal resistance is no longer constant but steadily changes as more resistance is added. Since the more probable situation is that the marginal cost increases (rather than decreases), the parameter c is assumed to be positive. Thus, the marginal cost rises at a constant rate equal to 2c.[1] Moreover, to rule out the unlikely situation of a negative marginal cost, we assume $b \geq 0$. In terms of geometry, these restrictions on b and c mean that we are confining our analysis to the upward sloping portion of a U-shaped quadratic curve, that is, one which is concave upward.

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[1] By taking the first derivative of (4.8) we find that marginal cost (MC) equals $b + 2c \cdot R$. Thus, MC rises with increasing R if c is positive, falls with increasing R if c is negative. Of course, if $c = 0$, then MC = b regardless of the value of R, which is equivalent to the linear cost case.

Under the assumption of fixed degree days and quadratic costs, the expression for life-cycle cost takes on the following form:

$$L = K \cdot D \cdot (1/R) + a + b \cdot R + c \cdot R^2. \tag{4.9}$$

Differentiating this expression with respect to R and setting the result equal to zero yields the following cubic equation:

$$2\,c \cdot R_0^3 + b \cdot R_0^2 - K \cdot D = 0. \tag{4.10}$$

The expression for the optimum level of thermal resistance, $R_0$, can be found by applying the formula for the solutions to a cubic equation.[1] Unfortunately, this results in an expression too cumbersome to be of any use in interpreting the relationship between RIF's and the resulting optimum resistance.

For the special case of b = 0, on the other hand, the explicit solution for $R_0$ can be easily derived and interpreted. The implication of assuming b = 0 is that marginal cost has no fixed component, but simply starts at zero and increases by 2c for every unit of resistance added.[2] Under this assumption, the following formula for $R_0$ can be derived from expression (4.10):

$$R_0 = \left[\frac{K \cdot D}{2c}\right]^{1/3} . \tag{4.11}$$

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[1] Murray R. Spiegel, Mathematical Handbook of Formulas and Tables (Schaum's Outline Series; New York: McGraw-Hill, Inc., 1968), p. 32.

[2] The geometric interpretation of b = 0 is that the MC straight line goes through the origin.

This implies that the maximum rate of thermal transmission allowed by the optimum standard is given by

$$U_o = \left[ \frac{2c}{K \cdot D} \right]^{1/3} . \qquad (4.12)$$

Substituting F · K for K, where F is the RIF value, we can establish the following relationship between the transmission rate allowed by the socially optimum standard, $U_o^*$, and that by the privately optimum standard, $U_o$:

$$U_o^* = \left[ \frac{2c}{K \cdot D} \right]^{1/3} \cdot \left[ \frac{1}{F} \right]^{1/3} = U_o \cdot \left[ \frac{1}{F} \right]^{1/3} . \qquad (4.13)$$

As with the linear cost case, this means that as the RIF value increases, $U_o^*$ decreases, but at a steadily diminishing rate. There is, however, a significant difference between the two cases. Note that for this quadratic cost case $U_o^*$ varies inversely as the cube root of F, whereas for the linear cost case, $U_o^*$ varies inversely as the square root of F.[1] For example, the effect of a RIF value of 2 is to reduce the maximum rate of transmission allowed under the quadratic cost standard by only 20.6%, whereas we saw in Section 3.0 that the effect of the same RIF value on the rate allowed under the linear cost standard would be a reduction of 29.3%. Thus, the higher-order cost function allows a smaller impact for RIF's on the rate of thermal transmission.

As with the linear cost case, it is possible to compare the relative impact of RIF's on the annual energy consumption allowed by the standard in different climates. A numerical example helps

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[1] See expression (4.5) above.

to illustrate the relative impacts. Suppose the values of the parameters c, K, and D in expression (4.13) were such that $U_o$ equalled 0.10 in a particular region with a relatively warm climate. Then, a much colder climate with four times as many degree days would require a $U_o$ equal to $0.10/4^{1/3}$ (or about 0.63 as high a rate of thermal transmission), for the same values of c and K. As before we can denote the extra annual energy savings per square foot of attic attributable to the use of RIF's in the warmer and colder climates by $\Delta S_w$ and $\Delta S_c$, respectively. Then we have

$$\begin{aligned} \Delta S_c &= (1 - F^{-1/3}) \cdot (24) \cdot (4D) \cdot (0.10/4^{1/3}), \text{ and} \\ \Delta S_w &= (1 - F^{-1/3}) \cdot (24) \cdot (D) \cdot (0.10). \end{aligned} \tag{4.14}$$

Thus, the ratio of the colder to the warmer climate savings becomes

$$\frac{\Delta S_c}{\Delta S_w} = 4/4^{1/3} = 4^{2/3} \cong 2.52. \tag{4.15}$$

Thus, we can conclude that for a climate with <u>four</u> times as many degree days as another, the extra annual energy savings attributable to RIF's would be about 2.52 times as great. Again note that this relationship holds for all RIF values greater than one.[1] In general, this relationship means that for a colder climate with n times as many degree days as a warmer climate, the extra annual energy savings due to using a positive RIF value would be $n^{2/3}$ times as much as for the warmer climate.

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[1] See footnote 1 on page 49 above.

### 4.1.3 Case III: Higher-Order Cost Functions

We now consider the implications of using cubic and even higher-order functions to represent the relationship between total construction costs and the level of thermal resistance. The general form of the functions we will analyze is

$$C = a + c \cdot R^{q} , \tag{4.16}$$

where the coefficient c is positive and the exponent, q, is a parameter (usually an integer) which specifies the order of the function. For $q \geq 3$, this form means the marginal cost of resistance not only rises but rises at an increasing rate as R increases. In general, for given values of a and c, both total construction costs, C, and marginal costs rise more quickly, the greater the value of q (the higher the order of the function).

Under the assumptions of fixed degree days and the cost function of expression (4.16), life-cycle costs become

$$L = K \cdot D \cdot (1/R) + a + c \cdot R^{q} . \tag{4.17}$$

Following the same procedure used for Cases I and II, we can derive the following expression for the maximum rate of thermal transmission allowed by the socially optimum standard:

$$U_0^{*} = \left[\frac{q \cdot c}{K \cdot D}\right]^{\frac{1}{q+1}} \cdot \left[\frac{1}{F}\right]^{\frac{1}{q+1}} = U_0 \cdot \left[\frac{1}{F}\right]^{\frac{1}{q+1}} . \tag{4.18}$$

This relationship means that in general the higher the order of the cost function, the smaller will be the impact of a given RIF value in reducing the maximum rate of thermal transmission allowed by the standard. For example, we saw in Case II with a quadratic cost function that a RIF value of 2 would reduce the maximum rate by 20.6%. With a cubic cost function (q=3), however, this reduction would be a mere 15.9%.

For the case of a cubic cost function, the relative impact of using RIF's on annual energy consumption in different climates can be established. It turns out that if a colder climate has n times as many degree days as a warmer climate, the ratio of extra energy savings in the colder climate to those in the warmer climate is equal to $n^{3/4}$. For example, a factor of four times as many degree days implies 2.83 times as much energy savings.

### 4.1.4 Comparison of Alternative Cost Functions

It is now appropriate to compare the alternative cost functions presented. They will first be compared in terms of the effects of various RIF values on the maximum rate of thermal transmission allowed by the socially optimum standard. Table 4.1 gives the ratio, $U_0^*/U_0$, for a range of RIF values and various cost functions. The ratios in the column entitled "linear" are the same as those in Table 3.3 of Section 3.0.

Several patterns emerge from an analysis of these ratios. In the first place, for all cost functions the ratio decreases as the RIF value increases, but does so at a diminishing rate.[1] Secondly,

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[1]This can also be shown in general by evaluating the signs of the first and second partial derivatives of $U_0^*$ with respect to F in expression (4.18). As expected, the first derivative is negative, indicating the inverse relationship, and the second derivative is positive, verifying the diminishing effect.

Table 4.1

Ratios of Socially Optimum to Privately Optimum Standards for Various RIF Values and Cost Functions[a]

| RIF Value | Cost Function | | | |
|---|---|---|---|---|
| | Linear (q=1) | Quadratic (q=2) | Cubic (q=3) | Quartic (q=4) |
| 1.00 | 1.000 | 1.000 | 1.000 | 1.000 |
| 1.25 | 0.894 | 0.928 | 0.946 | 0.956 |
| 1.50 | 0.817 | 0.874 | 0.904 | 0.922 |
| 1.75 | 0.756 | 0.830 | 0.869 | 0.894 |
| 2.00 | 0.707 | 0.794 | 0.841 | 0.871 |
| 2.25 | 0.667 | 0.763 | 0.817 | 0.850 |
| 2.50 | 0.633 | 0.737 | 0.795 | 0.833 |
| 2.75 | 0.603 | 0.714 | 0.777 | 0.817 |
| 3.00 | 0.577 | 0.693 | 0.760 | 0.803 |

[a]These ratios are calculated by the following relationship derived from expression (4.18):

$$U_o^*/U_o = F^{\frac{-1}{q+1}}$$

for any given RIF value as one moves from lower to higher-order cost functions (reading from left to right across one row of the table), the ratios get larger and closer to unity. This implies that as q increases, the effect of RIF's in lowering the maximum rate of transmission decreases. Thirdly, a unit change in the RIF value has less of an effect on the ratios,the higher the order of the cost function. For example, an increase in the RIF value from 2.00 to 2.25 yields a change of 0.040 for the linear cost case, while the same increase in the RIF leads to a change of only 0.031 for the quadratic cost case.

Another type of comparison can be made regarding the alternative cost functions. We can analyze the relative impact of using RIF's on annual energy consumption in different climates for all cost functions. If a colder climate has n times as many degree days as a warmer climate, the ratio of extra energy savings in the colder versus the warmer climate turns out to equal $n^{\frac{q}{q+1}}$, where q is the order of the cost function. Table 4.2 presents these ratios for a range of values of n and the various cost functions. For example, in a climate with twice as many degree days, RIF's would have a 41.4% greater impact on annual energy consumption with a linear cost function, but they would have a 58.7% greater impact with a quadratic cost function.

There are two general conclusions that can be drawn from these ratios. First, note that for any given value of n, the ratios increase as the order of the cost function (q) increases. This means that the energy-saving impact of having more degree days is greater for

Table 4.2

Ratios of Extra Energy Savings Due to RIF's in Colder Versus Warmer Climates, for Various Cost Functions[a]

| Degree-Day Multiple (n) | Cost Function | | | |
|---|---|---|---|---|
| | Linear (q=1) | Quadratic (q=2) | Cubic (q=3) | Quartic (q=4) |
| 1.00 | 1.000 | 1.000 | 1.000 | 1.000 |
| 1.50 | 1.225 | 1.310 | 1.355 | 1.383 |
| 2.00 | 1.414 | 1.587 | 1.682 | 1.741 |
| 2.50 | 1.581 | 1.842 | 1.988 | 2.081 |
| 3.00 | 1.732 | 2.080 | 2.280 | 2.408 |
| 4.00 | 2.000 | 2.520 | 2.828 | 3.031 |
| 5.00 | 2.236 | 2.924 | 3.344 | 3.624 |
| 6.00 | 2.450 | 3.302 | 3.834 | 4.193 |

[a]The ratios of the extra savings are calculated using the following relationship:

$$\frac{\Delta S_c}{\Delta S_w} = n^{\frac{q}{q+1}}$$

higher-order cost functions.[1] The second point is that for all values of q the ratios increase as n increases. This simply means that the greater the difference between the colder and warmer climates, the greater would be the relative energy-saving impact of using RIF's. Moreover, the effect of a one unit change in n decreases as n increases. This means that the same difference in degree days leads to extra annual energy savings due to RIF's which are greater when comparing two relatively warm climates than when comparing two relatively cold ones.

This concludes our analysis of alternative cost functions. There are a host of other forms possible (e.g., trigonometric, logarithmic, exponential). Most of these would not lead to an explicit expression for $U_o^*$. Moreover, the family of polynomial forms which have been analyzed adequately represents the probable way in which unit area construction costs of the attic floor are expected to vary with the level of thermal resistance. There is a shortcoming, however, which all these functional forms share. They assume that costs vary continuously. Because glass fiber batts or blankets come in a limited number of thicknesses, it would be more accurate to have these construction costs vary discretely as more resistance is added. For the loose fill type of insulation, on the other hand, the continuous assumption is quite realistic. Moreover, even for the case of limited sized batting, the continuous assumption is valid for establishing the general relationships between resistance

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[1] Note that this effect of increasing q gradually diminishes and approaches the limiting value of n as q gets arbitrarily large. That is,

$$\lim_{q \to \infty} n^{\frac{q}{q+1}} = n.$$

costs, energy costs, degree days and RIF's on the one hand, and the socially optimum energy conservation standard, on the other. When the actual calculations are made to develop the standard, the discrete nature of these costs can be taken into account.

## 4.2 DEGREE DAYS AS A FUNCTION OF RESISTANCE

Up to this point we have maintained our original assumption that the number of degree days (D) associated with the heating requirements of a building is invariant with respect to the level of resistance (R). In fact, it is expected that as more units of thermal resistance are added to the building envelope, the number of degree days effectively contributing to the heating load will decrease. It is further expected that as R is increased, the rate at which D falls will diminish. These expectations suggest that degree days could be adequately represented by a quadratic function of resistance.[1]

The reasoning leading to these expectations regarding the pattern of variation of degree days is best explained by distinguishing between the two links in the chain of causality from R to D: (1) an increase in resistance will cause the effective degree-day temperature base to decrease, (2) which, in turn, will lower the number of degree days in the heating season. Each of these links will be explained and illustrated in turn.

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[1]For the sake of completeness, the life-cycle model was also analyzed for the case of degree days assumed to be a linear function of resistance. This assumption leads to an expression for $R_0$ which would have a higher value than that which results from the fixed degree-day assumption. This result has the intuitive appeal that when the extra benefit of lowering the number of degree days is attributed to resistance, it makes sense to select a higher level of resistance than would otherwise have been chosen.

### 4.2.1 Degree-Day Temperature Base and Resistance

With regard to the first link, the effective degree-day temperature base is given by that critical temperature below which a supply of heat to the conditioned space is needed in order to maintain a given indoor temperature. This critical temperature is sometimes referred to as the balance point (B) and is generally less than that specified by the desired indoor winter temperature, because solar radiation and the normal activities of a household (lighting, hot water use, appliances, and body warmth) replace some of the heat being lost through the building envelope. Given an indoor temperature requirement and rates of solar gain and incidental internal heat generation, the lower the rate at which heat is being lost through the building envelope, the lower the balance point. This means that there is an inverse relationship between the balance point and the level of thermal resistance up to a limit determined by the rate of air infiltration in the building. The impact on the balance point of adding one unit of resistance, however, gradually diminishes as more units are added because the direct effect of more resistance on the rate of heat loss also diminishes. This implies that unit increases in resistance lead to reductions in the balance point but at a decreasing rate.

This inverse relationship between resistance and the balance point can be illustrated with a numerical example. Suppose the occupants of a building with 3000 sq. ft. of envelope area having an R-value of 5 set the thermostat at $70^{0}$ F. If the heat loss due to air infiltration is 200 Btu's per hour per degree of indoor-outdoor temperature difference, then an assumed balance point of $65^{0}$ F implies that spontaneous heat gains from solar radiation and household activities equal 4000 Btu's per hour. That is, if the outdoor temperature were exactly $65^{0}$ F, then the 4000 Btu's of spontaneous heat gains would just offset the heat losses due to conductance and infiltration so that the desired indoor temperature of $70^{0}$ F could be maintained without heat from the furnace. Now by successively adding 5 more units of resistance to the original resistance level of R-5, we can calculate the effective balance point for each new level of resistance. The results of these calculations are presented in Table 4.3. It is clear that as R is increased, B decreases. Moreover, note that the rate of decrease of B diminishes. For example, increasing R from 5 to 10 causes B to drop 3 degrees, whereas increasing R from 10 to 15 causes a drop of only 2 degrees. In fact, B approaches a theoretical limit of $50^{0}$ F, corresponding to an infinite level of resistance (no heat losses due to conduction).

### 4.2.2 Degree Days and the Degree-Day Base

The second link in the chain of causality between resistance and degree days deals with the way in which these reductions in the balance point affect the actual number of degree days in particular climatic areas. The pattern of this relationship is illustrated in Table 4.4 with data for several representative cities. Note that for all the cities as the balance point drops the number of degree days decreases but at a slightly diminishing rate. For example, in Washington the one-degree shift from $65^{0}$ to $64^{0}$ would cause a 213 degree-day decrease, whereas the one-degree drop from $64^{0}$ to $63^{0}$ would cause a 208 degree-day decrease. This pattern continues until the one-degree drop from $51^{0}$ to $50^{0}$ causes only a 150 degree-day decrease.

Table 4.3

Effective Balance Points for Various Levels of Thermal Resistance: Hypothetical Example[a]

| Thermal Resistance(R) | Balance Point ($^{o}F$) |
|---|---|
| 5 | 65.0 |
| 10 | 62.0 |
| 15 | 60.0 |
| 20 | 58.6 |
| 25 | 57.5 |
| 30 | 56.7 |
| 35 | 56.0 |
| 40 | 55.5 |
| 45 | 55.0 |
| 50 | 54.6 |
| 55 | 54.3 |
| 60 | 54.0 |
| . | . |
| . | . |
| . | . |
| $\infty$ (U=0) | 50.0 |

[a]The following assumptions underlie these calculations: (1) the envelope area (A) is 3000 sq. ft.; (2) the desired indoor temperature (T) is 70°F; (3) the spontaneous heat gain (H) from solar radiation and household activities is 4000 Btu's/hour; and (4) heat loss (I) due to air infiltration is 200 Btu/hour/degree of indoor-outdoor temperature difference. These parameter values are substituted into the following formula for calculating the balance point corresponding to each resistance level:

$$B = T - \left[\frac{H}{A/R+I}\right] .$$

Table 4.4

Degree Days Corresponding to Various Balance Points for Six Major U.S. Cities[a]

| Balance Point (°F) | City | | | | | |
|---|---|---|---|---|---|---|
| | Chicago | New York | Washington | San Francisco | Dallas | Los Angeles |
| 65 | 6509 | 5217 | 4667 | 3510 | 2552 | 1524 |
| 64 | 6250 | 4976 | 4454 | 3192 | 2391 | 1310 |
| 63 | 5998 | 4741 | 4246 | 2885 | 2236 | 1112 |
| 62 | 5752 | 4512 | 4044 | 2591 | 2087 | 938 |
| 61 | 5512 | 4289 | 3849 | 2310 | 1942 | 781 |
| 60 | 5278 | 4071 | 3659 | 2042 | 1803 | 639 |
| 59 | 5049 | 3859 | 3472 | 1789 | 1670 | 514 |
| 58 | 4825 | 3653 | 3290 | 1554 | 1541 | 402 |
| 57 | 4605 | 3453 | 3112 | 1331 | 1416 | 308 |
| 56 | 4389 | 3258 | 2939 | 1127 | 1296 | 224 |
| 55 | 4178 | 3067 | 2769 | 945 | 1180 | 151 |
| 54 | 3971 | 2880 | 2604 | 769 | 1069 | 97 |
| 53 | 3768 | 2698 | 2444 | 603 | 961 | 68 |
| 52 | 3566 | 2519 | 2288 | 455 | 857 | 44 |
| 51 | 3371 | 2343 | 2135 | 320 | 755 | 26 |
| 50 | 3178 | 2172 | 1985 | 193 | 656 | 10 |

[a]These data were made available through a private communication from Richard Erth, York Division of Borg-Warner Corporation, York, Pa. 17405. Monthly data were chosen to be representative of a ten-year historical pattern. Daily temperatures are based on the average of 24 hourly readings, rather than the midpoint of the high and low temperatures.

### 4.2.3 Case IV: A Quadratic Degree-Day Function

Now it is possible to bring these two links together to establish the relationship between degree days and the level of thermal resistance. By combining the data from Table 4.4 on Washington with that in Table 4.3,we can plot a curve representing degree days as a function of resistance. This curve is shown in Figure 4.1. As is apparent from the shape of the curve, degree days decrease as the level of resistance increases. Moreover, the rate of decrease in degree days gradually diminishes. The pattern of the relationship shown in Figure 4.1 can be represented reasonably well by expressing degree days as a quadratic function of thermal resistance:

$$D = d + e \cdot R + f \cdot R^2, \tag{4.19}$$

where d, the intercept is positive, f must also be positive to guarantee upward concave curvature, and e is negative because the slope is negative.

Now we can apply this assumption of quadratic degree days and maintain our original linear cost function to obtain the following expression for life-cycle cost:

$$L = K \cdot (d + e \cdot R + f \cdot R^2) \cdot (1/R) + a + b \cdot R. \tag{4.20}$$

Differentiating L with respect to R, solving for the optimum level of resistance, and inverting yields the following expression for the optimum rate of thermal transmission:

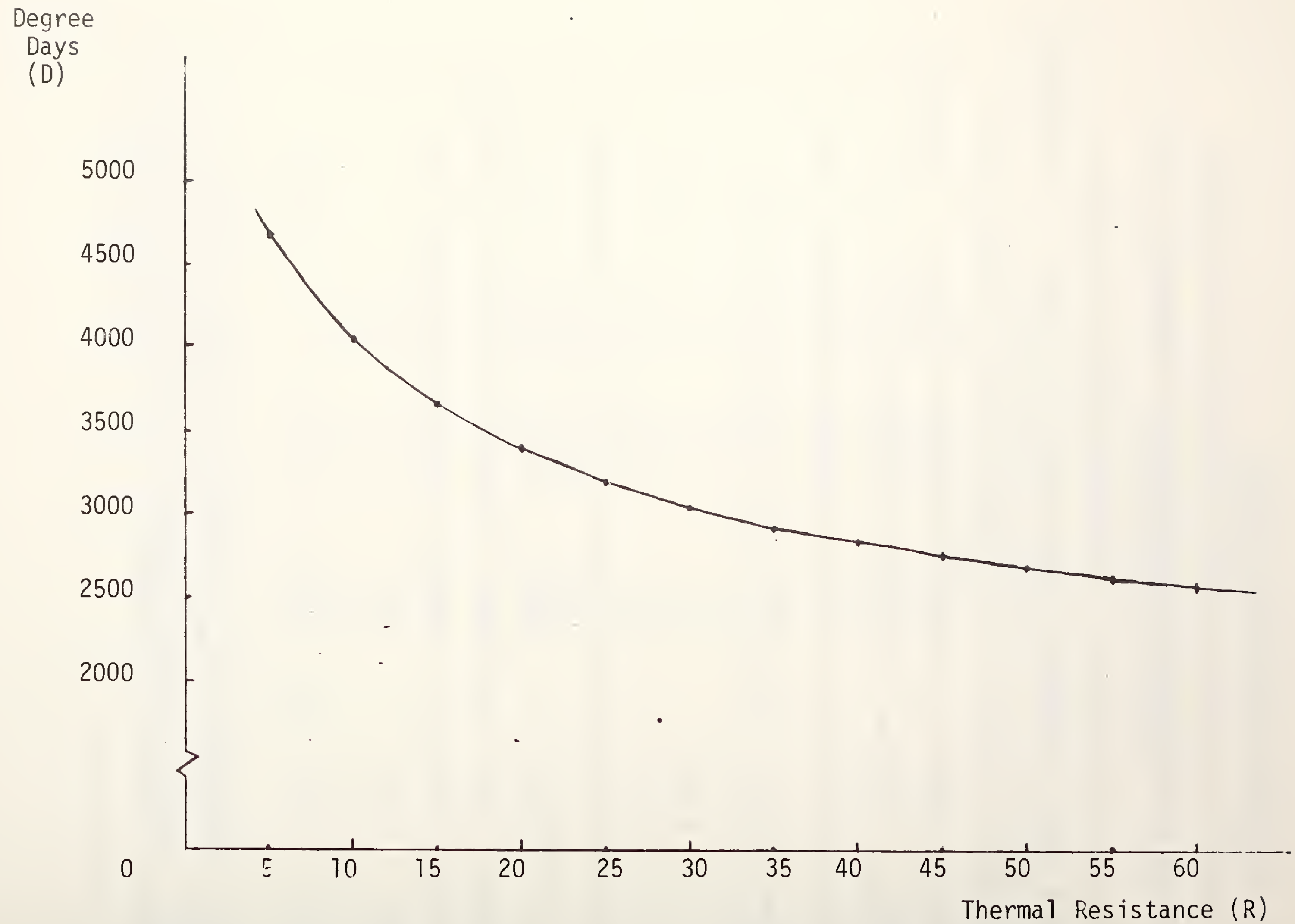


Figure 4.1

Degree Days as a Function of Thermal Resistance for a Typical House in Washington, D.C.

$$U_0 = \left[\frac{b + f \cdot K}{d \cdot K}\right]^{\frac{1}{2}} . \qquad (4.21)$$

When RIF's are introduced by substituting F · K for K in this expression, we obtain the maximum rate of transmission allowed by the socially optimum standard:

$$U_0^* = \left[\frac{b/F + f \cdot K}{d \cdot K}\right]^{\frac{1}{2}} . \qquad (4.22)$$

Unfortunately, expression (4.22) does not permit the separation of F which would be required in order to express the socially optimum transmission rate, $U_0^*$, as a function relative to the privately optimum rate, $U_0$. The best that can be done under these circumstances is to assess the general direction of the relationship between F and $U_0^*$ by evaluating the sign of the first derivative. Thus, we have

$$\frac{\partial U_0^*}{\partial F} = -1/2 \left[\frac{b/F + f \cdot K}{d \cdot K}\right]^{-\frac{1}{2}} \cdot \left[\frac{b}{d \cdot K}\right] \cdot \left[\frac{1}{F^2}\right] . \qquad (4.23)$$

Since it is assumed that additional units of resistance have a positive cost, we have $b > 0$. Moreover, since degree days are assumed to decrease at a decreasing rate, with increases in resistance, we have d and f both positive. Finally, since the only meaningful values of F and K are all positive, we know that the derivative is less than zero, which means that in general $U_0^*$ is a decreasing function of F. That is, the higher the value of the RIF, the lower will be the maximum rate of thermal transmission allowed by the socially optimum conservation standard. This is the same general conclusion we have seen with the previous formulations of the model. The higher RIF value implies a higher social value being imputed to energy, which in turn justifies more investment in energy conservation so as to lower the rate of energy consumption. On the other hand, whether $U_0^*$ decreases at a diminishing rate as it does in Cases I through III can only be determined by evaluating the sign of the second derivative. Unfortunately, it turns out that this sign cannot be unambiguously determined without actually specifying the magnitudes of the variables in expression (4.23).

## 4.3 Case V: THE GENERAL OPTIMUM STANDARDS MODEL

So far in Cases I through IV we have assumed alternative specific functional forms to represent the manner in which the level of thermal resistance may affect the building envelope construction costs on the one hand, and the number of degree days on the other hand. Rather than assuming particular algebraic forms for the cost and degree-day functions, we now employ general functional forms to represent these relationships. That is, we simply state that the number of degree days is a function of the level of resistance, D(R), and similarly that cost is a function of resistance, C(R). Thus, the expression for life-cycle cost becomes

$$L = K \cdot D(R) \cdot (1/R) + C(R). \qquad (4.24)$$

Moreover, it is reasonable to impose certain restrictions on the first and second derivatives of the functions D(R) and C(R). These restrictions are stated and briefly interpreted in Table 4.5.

Table 4.5

Restrictions on the First and Second Derivatives of D(R) and C(R)

| Restriction | Interpretation |
|---|---|
| 1. $D'(R) < 0$ | [D is a decreasing function of R] |
| 2. $D''(R) > 0$ | [D decreases at a decreasing rate] |
| 3. $C'(R) > 0$ | [C is an increasing function of R] |
| 4. $C''(R) \geq 0$ | [C increases at an increasing or constant rate] |

The rationale behind restrictions 1 and 2 regarding degree days was already discussed under Case IV (Subsection 4.2). Restriction 3 simply means that the addition of one unit of resistance increases rather than decreases the total amount spent on resistance (i.e., the marginal cost is positive). Restriction 4 implies that as more units of resistance are added, each successive unit costs more or the same as the previous one.

When the derivative of expression (4.24) with respect to R is set equal to zero, we obtain

$$K \cdot [R_o \cdot D'(R_o) - D(R_o)] + R_o^2 \cdot C'(R_o) = 0. \qquad (4.25)$$

As before, the RIF value can be introduced into this expression by substituting $F \cdot K$ in place of K. Although the resulting expression does not permit the explicit solution for the socially optimum level of resistance $R_o^*$, it is possible to determine the general direction of the effect of RIF's upon $R_o^*$ and in turn upon its inverse, $U_o^*$. By employing the implicit function rule of calculus,[1] we can find the derivative of $R_0^*$ with respect to F:

$$\frac{dR_0^*}{dF} = \frac{K[D(R_0^*) - R_0^* \cdot D'(R_0^*)]}{F \cdot K \cdot R_0^* \cdot D''(R_0^*) + 2R_0^* \cdot C'(R_0^*) + (R_0^*)^2 \cdot C''(R_0^*)} . \quad (4.26)$$

Using the restrictions specified in Table 4.5 and the fact that F, K, D(R), and $R_0^*$ are all positive, it can be seen that the entire expression is greater than zero. This means that the higher the RIF value introduced into the standard setting process, the greater the level of thermal resistance required to meet that standard.

This result can be restated in terms of the effect of RIF's on the maximum rate of thermal transmission allowed by the socially optimum standard. What is needed is the sign of the derivative of $U_0^*$ with respect to F.

Since in general U is the reciprocal of R, we have

$$U_0^* = 1/R_0^*. \quad (4.27)$$

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[1]The implicit function rule states that for an implicit function of two variables G(x,y) = 0,

$$\frac{dy}{dx} = - \frac{\partial G/\partial x}{\partial G/\partial y} .$$

See Jack R. Britton, Calculus (New York: Holt, Rinehart and Winston, 1961) p. 414.

Differentiating, we have

$$\frac{dU_o^*}{dR_o^*} = - 1/R_o^{*\,2} < 0. \tag{4.28}$$

By the chain rule,[1] we know that

$$\frac{dU_o^*}{dF} = \left[\frac{dU_o^*}{dR_o^*}\right] \cdot \left[\frac{dR_o^*}{dF}\right]. \tag{4.29}$$

From expression (4.28) the bracketed term on the left is negative and from our discussion of expression (4.26) the term on the right is positive. Thus, the product is negative and we can conclude that the use of higher RIF values will lead to the selection of a lower maximum rate of thermal transmission allowed by the socially optimum standard.

It would be possible to investigate the second derivative of $R_o^*$ with respect to F to discover whether the effect of RIF's diminishes or increases as higher RIF values are introduced. Unfortunately, however, the expression for the second derivative contains third-order derivatives of the degree-day and cost functions. Without actual empirical work, it would be meaningless to attempt to make assumptions regarding the signs

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[1]The chain rule for derivatives simply states that

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.$$

See Spiegel, Mathematical Handbook of Formulas and Tables, p. 53.

of these third-order derivatives. Hence, the sign of the second derivative of $R_0^*$ with respect to F could not be established.

Comparison of these results of the general model with those of the earlier, more specific model formulations reveals the same basic conclusion: the higher the RIF value used in deriving the socially optimum standard, the lower will be the corresponding maximum rate of thermal transmission allowed by that standard, *ceteribus paribus*. On the other hand, while for Cases I through III (fixed degree days) we were able to conclude that this effect of higher RIF values tends to diminish, we were unable to reach such a conclusion for Cases IV (quadratic degree days) and V (the general model).

## 5.0 SUMMARY, CONCLUSIONS AND FURTHER RESEARCH

This final section offers a brief summary of the report and presents the conclusions regarding the likely effects RIF's would have on the energy conservation performance standard. The section closes with several suggestions for further research.

### 5.1 SUMMARY

Our purpose has been to analyze the effects of using RIF's in the development of socially optimum energy conservation performance standards for buildings. We have discussed the various types of standards and the Energy Conservation and Production Act of 1976, which calls for the development of such a conservation performance standard. The language of this Act led us to concentrate our analysis on the effects of RIF's on an optimum standard, that is, a standard which reduces energy use to the extent that is economically justifiable while maintaining the habitability requirements of building occupants. The costs considered in such a standard include all expenditures occurring over the life of the building which relate to the energy uses governed by the standard --both first costs, such as for insulation or storm windows,and recurring annual energy costs. These cost items are put on a time-equivalent basis by discounting the future items to their present values. The process of minimizing these life-cycle costs implies a trade-off between expenditures for energy conservation and expenditures for energy. Thus, the price of energy used in the analysis is critical in determining the optimum standard.

Section 2.0 showed how the actual price paid for energy may not necessarily reflect its true value to society. Some of the possible reasons for a divergence between the actual price and the social value of energy were presented. These reasons include the presence of unit taxes or of monopoly power, environmental effects due to production or consumption, the desire for national economic independence, and the existence of price controls such as those on natural gas. Because of this divergence between actual energy prices and their social values, it is necessary to adjust the actual prices before using them to determine the socially optimum conservation standard. RIF's provide a method of making such an adjustment.

Section 3.0 began by discussing two alternative methods of formulating RIF's: (1) as a quantity multiplier; or (2) as a price or value multiplier. Then, a life-cycle cost minimization model was presented for determining the economically optimum level of thermal resistance in a building component such as the attic floor. Because of the exactly inverse relationship between resistance and the rate of thermal transmission, this model also determines the optimum rate of transmission which forms the basis for the optimum component performance standard itself. RIF's were introduced into this model so that the socially optimum standard (using RIF's) could be compared with the privately optimum standard (without RIF's). The relationship between the social and private standards was derived and analyzed. This analysis has several limitations. In the first place, it is addressed to component performance standards rather than building performance standards. As noted below in the suggestions for further research, the proper analysis of the effect of RIF's on building performance standards will be possible only after the energy-flow interactions between building components are adequately modeled. A second limitation is that this analysis focuses on a particular component, insulation in the building envelope. On the other hand, the same type of inverse relationship found to exist between the social value of energy and the optimum level of thermal resistance is expected to hold for the performance characteristics of other building components such as space heating and cooling equipment, illumination, and water heaters. Finally, for the particular component analyzed, the restrictive assumptions made regarding envelope construction costs and degree days may be violated under actual conditions.

Section 4.0 responded to this last limitation of the analysis by generalizing the cost minimization model. Four additional, more realistic sets of assumptions were developed and the corresponding formulas for the optimum standard were derived. Then, RIF's were introduced into these models and the effect of using RIF's was analyzed in terms of the amount of energy saved. The additional energy savings due to RIF's were also studied in relation to the number of degree days in the climate region.

## 5.2 CONCLUSIONS

The major conclusions of this report fall into three areas: (1) the appropriate method of formulating RIF's: (2) the potential effect

RIF's would have on energy consumption in buildings; and (3) the effect of using RIF's on the economic efficiency of the selected energy conservation standard. These conclusions are presented in turn.

### 5.2.1 Proper Formulation of RIF's

We conclude that the appropriate method of formulating RIF's is as a price multiplier rather than as a quantity multiplier. The application of a quantity multiplier RIF results in impact numbers which would serve well to compare alternative energy types. Because these impact numbers are not denominated in dollars, however, they would be unsuitable for determining the economic balance between energy and nonenergy resources. To achieve such a balance, a common basis for comparison is needed. The price multiplier formulation provides this common denominator because the RIF value converts the actual price paid for the energy to its corresponding social price. The resulting social price remains denominated in dollars and thus, is comparable with the values of nonenergy resources. Thus, with price multiplier RIF's an economic trade-off can be established between the proper amount of a particular type of energy and the appropriate level of insulation.

### 5.2.2 The Energy Consumption Effect

In general, it was found that the introduction of RIF's would have a restraining influence on the energy consumption of new buildings. The most probable situation is that price multiplier RIF values would be greater than one (that is, the social value of energy is most likely greater than the price paid for energy). This is because those factors which cause the social price to be higher than the actual price (environmental effects, price controls, and the national desire for economic independence) are likely to outweigh the other factors (unit taxes and monopoly power).

We can indicate the order of magnitude of the additional energy savings that might result from the use of RIF's in determining the standard for the case of natural gas. The current regulated price for interstate natural gas at the wellhead is $1.42/MCF. On the other hand, a rough approximation of the social cost of using natural gas can be derived from the fact that synthetic gas from Western coal is expected to cost about $4.00/MCF.[1] Thus, the corresponding RIF value would be $4.00/1.42 \approx 2.82$. Introducing this figure into expression (3.5) for the socially optimum standard, we find that the use of RIF's would lower the maximum rate of thermal transmission and hence, the annual energy consumption for space heating allowed by the standard by 40.5% from what they would have been without RIF's. A smaller reduction (29.2%) in the optimum rate of transmission results from the same RIF value of 2.82 when the cost function is assumed to be quadratic rather than linear. Finally, with a cubic cost function, the same RIF value leads to only a 22.8% reduction in the maximum rate of thermal transmission allowed by the socially optimum standard.

It was also found that the energy consumption effect of introducing RIF's depends on the severity of the climatic heating load. That is, the greater the number of degree days in an area, the greater will be the additional energy saved due to the introduction of RIF's. More

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[1]See Edward F. Renshaw, "A Cost Sharing Approach to the Conservation of Natural Gas," Public Utilities Fortnightly, Vol. 98, No. 1 (July 1, 1976), pp. 37-39.

specifically, for the linear cost model it was found that doubling the number of degree days leads to a 41.4% increase in the amount energy saved from RIF's. On the other hand, when envelope construction costs are assumed to be a quadratic function of the level of thermal resistance, it turns out that doubling the number of degree days leads to a 58.7% increase in RIF-induced energy savings. In general, this impact of degree days on the energy savings due to RIF's, steadily increases as the order of the cost function increases.

The ability to use a particular RIF value to find the corresponding percentage reduction in the energy consumption allowed by the standard is limited to Cases I through III (assuming fixed degree days) of the optimum standards determination model. For the other cases we are able to reach conclusions only regarding the general direction of the relationship between RIF values and the allowable energy consumption level. That is, for both the case in which degree days are assumed to be quadratic (Case IV) and the case in which degree days and costs were written as general functions of thermal resistance (Case V), we can conclude that the higher the RIF value used in determining the socially optimum standard, the lower (i.e., more restrictive) will be the maximum allowable energy consumption level specified in that standard. This makes intuitive sense because a higher RIF value implies a higher social value of energy, given the actual private price paid for it. It seems reasonable for society to recommend lower consumption levels for those resources which are valued more dearly.

5.2.3 The Economic Efficiency Effect

Our conclusion is that the use of RIF's will improve the economic efficiency (from the national standpoint) of an energy conservation standard based on the actual private prices paid for energy. As we have seen in Section 2.0, in order to reflect the national viewpoint, a standard should be based on the social values of energy types, which are equivalent to the product of the private prices times the corresponding price multiplier RIF values. Using the private rather than the social prices of energy results in an incorrect choice of the standard -- one that is not socially optimum. This deviation of the privately optimum from the socially optimum standard can be termed economically inefficient because an opportunity for a net economic gain to society is being foregone. That is, the use of a RIF value greater than one will lead to an additional reduction in energy consumption (over and above that of a privately optimum standard) whose social value exceeds that of the additional thermal resistance required to achieve it. This net gain in economic efficiency (value of additional energy savings less the cost of achieving them) which results from the use of RIF's can be depicted graphically.

The geometric measure of this gain in economic efficiency is developed and illustrated in the Appendix. In essence this measure is denominated in the same units as the annual energy consumption for space heating attributable to a square foot of the building envelope. The change in envelope construction costs due to the use of RIF's is converted to equivalent units of annual energy consumption by dividing by the social price of energy and the uniform present worth factor.[1] This equivalent value of construction costs can then be combined with the change in annual energy consumption due to the use of RIF's. In the case of a RIF value greater than one, the additional construction costs are subtracted from the additional annual energy savings to arrive at the net efficiency gain. In the case of a RIF value less than one, the additional energy consumption is subtracted from the additional construction cost savings to yield the net gain in economic efficiency.

## 5.3 SUGGESTIONS FOR FURTHER RESEARCH

There are two major areas related to RIF's in which further research is needed. The first and most obvious need is for the actual development of empirical estimates for RIF's. This report has suggested that the most useful formulation for RIF's is as price multipliers equivalent to the ratio of the social price of the energy type to its actual price. Section 2 briefly outlined the nature of those factors which cause the actual price paid for energy to deviate from its true social value. What remains to be done is: (1) to develop workable models which adequately reflect how each of those factors operate in reality; (2) to apply those models to derive empirical estimates for the magnitude and direction of each type of price-distorting factor; (3) to employ these empirical estimates to arrive at an overall numerical value for the social price of each energy type; and (4) to divide this social value by its corresponding actual price to obtain the price multiplier RIF value.

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[1]This makes annual energy consumption the *numeraire*.

The second area which requires further research is that of the relationship of RIF's to building performance standards. The analysis of Sections 3.0 and 4.0 was carried out using models to develop component performance standards for building envelope characteristics as they relate to space heating requirements. A full understanding of the relationship between RIF's and energy conservation performance standards will only be possible after the model for determining optimal building performance standards, which is currently being developed at NBS, has been completed. This model will incorporate the many energy-flow interactions which occur among the various components of a building. Thus, it will be possible to analyze the economic trade-offs between interrelated components. Moreover, the model will be able to take explicit account of the changes in the degree-day base which result from energy conservation techniques. This building performance standard model will be able to determine the socially optimum standard by basing the economic optimization routine on the social values of energy types. Thus, the effects of using RIF's will be determined by comparing the standard based on social values with one based on the actual private prices paid for the energy resources.

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## APPENDIX

### Economic Efficiency Effects of RIF's

As was mentioned in Section 5.0, the use of RIF's leads to a conservation standard with greater economic efficiency than the standard developed without RIF's. This is because the social value of the extra energy saved by using RIF's exceeds the additional cost required to save it. The purpose of this appendix is to present a geometric measure of this net gain in economic efficiency due to the use of RIF's.

Figure A.1 illustrates the physical trade-off possibilities between the level of thermal resistance (R) and the corresponding annual energy consumption (A) required to replace the heat lost through one square foot of attic floor surface for a given climate and desired indoor temperature level. The curve, TT', is drawn as an exact rectangular hyperbola because A is inversely proportional to R according to the following expression:

$$A = (DD \cdot 24)/R, \qquad (A.1)$$

where DD is a parameter representing the number of degree days which is determined by the given climate and indoor temperature. For convenience, DD is taken to be a fixed parameter, as was assumed for the model developed in Section 3.0. The alternative assumption of quadratic degree days discussed as Case IV in Section 4.0 would not change the basic analysis, but only slightly alter the position, slope, and curvature of the relationship. It should be noted that an infinite number of combinations of R and A are available which satisfy the indoor temperature requirements and climatic conditions

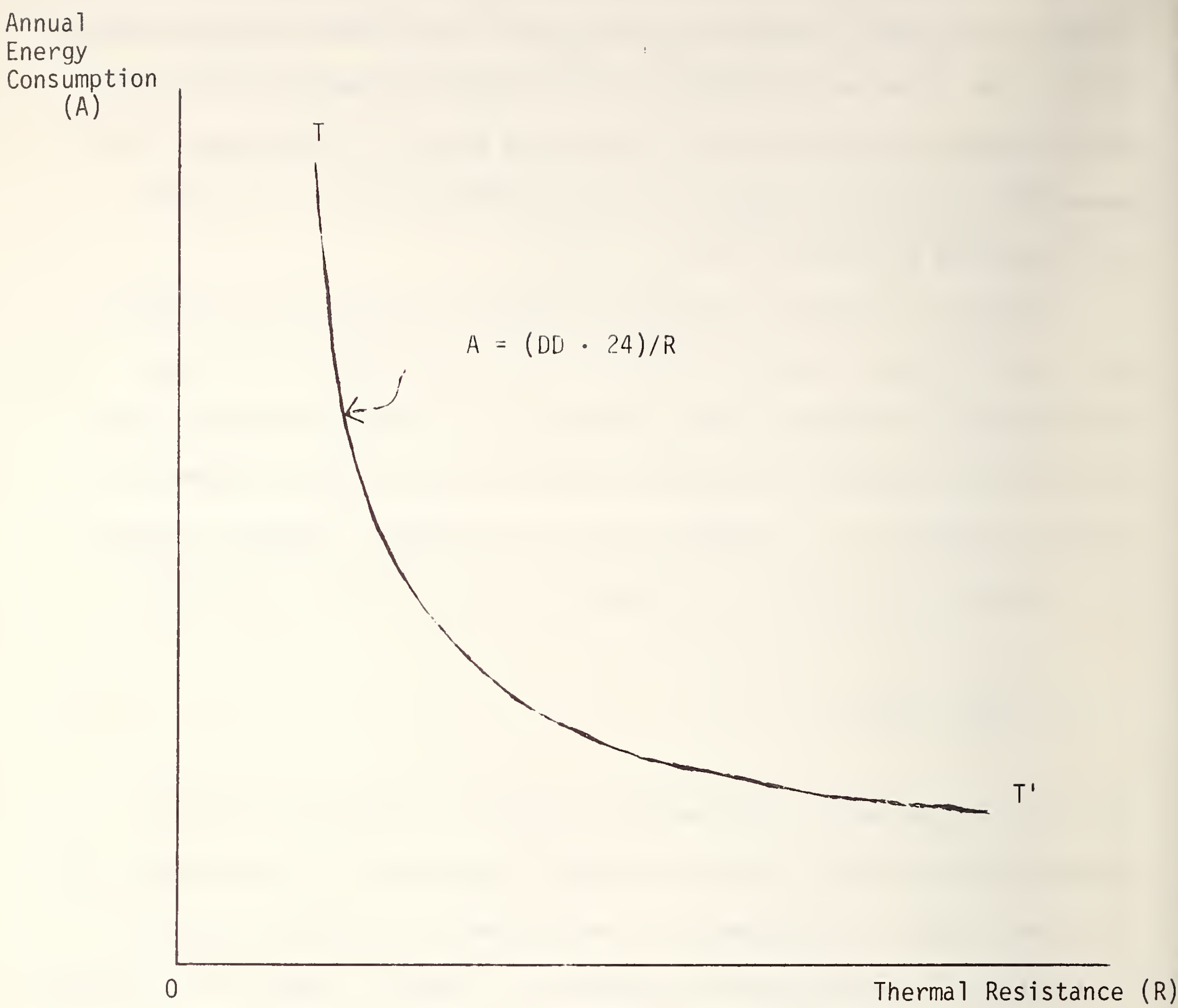


Figure A.1

Trade-off Curve between Energy Consumption and Thermal Resistance for Given Climate and Indoor Temperature specified. That is, one can achieve the desired indoor temperature by using energy at a high annual rate with little thermal resistance, or by using less energy with more resistance. Which of the many combinations of R and A should be chosen depends on the cost of energy relative to that of thermal resistance.

Figure A.2 has the same trade-off curve, TT', as shown above but introduces the concept of relative costs using the straight line, LL'.[1] This is called an isocost line because it represents all those possible combinations of A and R which lead to the same total present value of life-cycle heating-related costs, given the price of energy ($P_e$) and of resistance (b) and the uniform present worth factor (UPW). The algebraic formula for this line is given by

$$L = (P_e \cdot UPW) \cdot A + b \cdot R, \qquad (A.2)$$

where L is a parameter representing the total present value of life-cycle heating-related costs.[2] This equation can be rewritten to express A as an explicit function of R so that the intercept and slope can be clearly seen:

$$A = L/(P_e \cdot UPW) - [b/(P_e \cdot UPW)] \cdot R. \qquad (A.3)$$

By varying the value of the parameter L, the A-axis intercept is changed so that the isocost line can be lowered or raised. The particular isocost line drawn in Figure A.2 is just tangent to the trade-off curve TT',

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[1] If thermal resistance costs are assumed to be a quadratic or cubic rather than a linear function of the level of resistance, then LL' would be concave downward rather than a straight line.

[2] This formulation assumes there is no fixed cost component for thermal resistance. The inclusion of such fixed costs would unnecessarily complicate the argument without altering the conclusion.

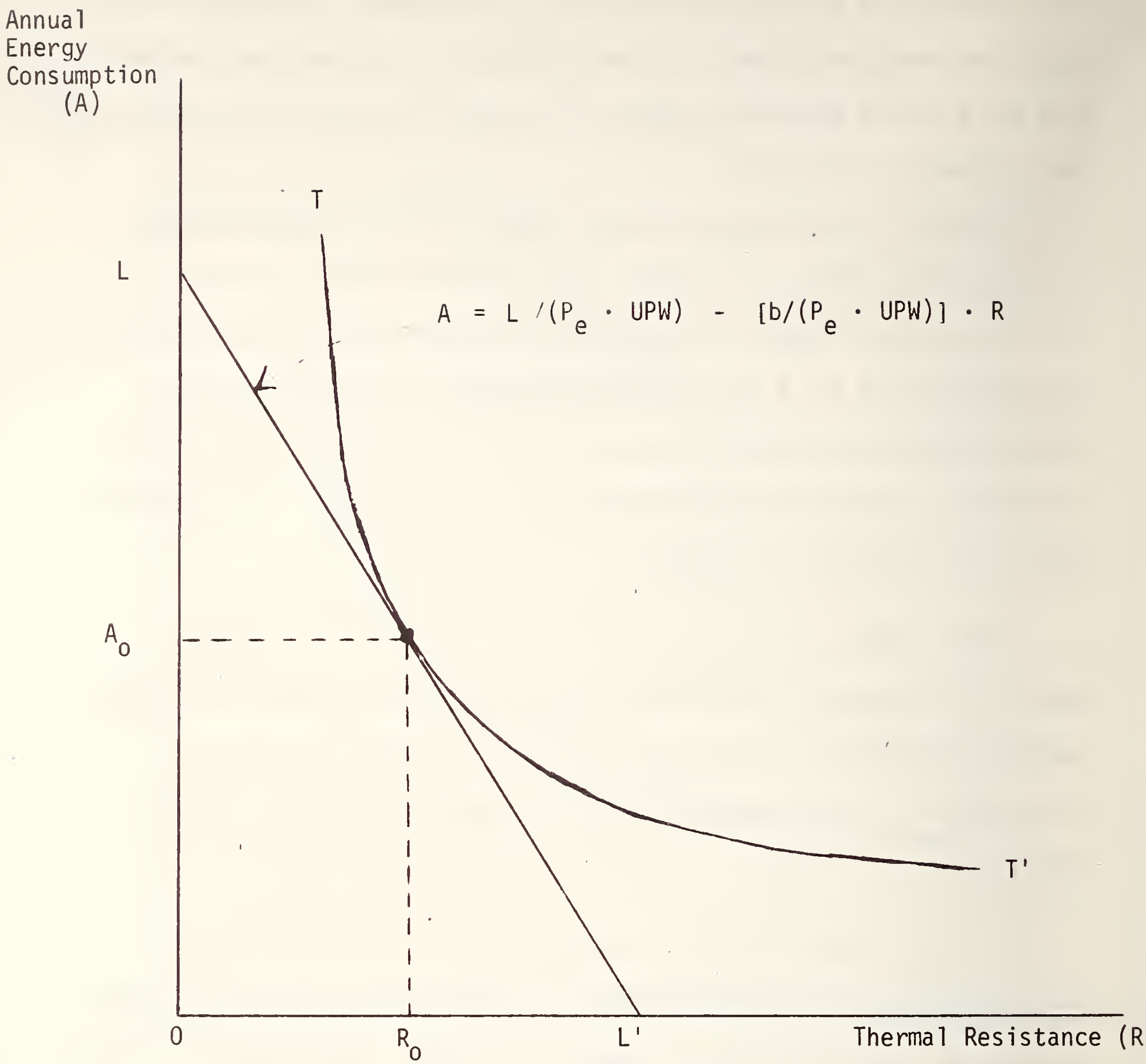


Figure A.2

Privately Optimum Energy Consumption Rate ($A_o$)

which means that the value of L chosen is the lowest possible life-cycle cost which will still satisfy the indoor temperature requirements and climatic conditions given by TT'. Hence, the values, $A_o$ and $R_o$, given by the point of tangency can be considered optimum for the particular ratio of relative prices given by the slope, $b/(P_e \cdot UPW)$. Since the actual price paid for energy, $P_e$, was used rather than the social price, $P_e \cdot RIF$, it can be said that $A_o$ represents the rate of annual energy consumption which would be embodied in the privately optimum conservation standard.

In Figure A.3, two more isocost lines have been added, both of which have the same slope, $b/(P_e \cdot RIF \cdot UPW)$, based on the social price of energy. The first line MM' is drawn to be tangent to the trade-off curve TT' so that the value $A_o^*$ at the point of tangency is interpreted as the socially optimum rate of annual energy consumption embodied in the standard. The second line NN' is drawn parallel to MM' by using the same slope, but it is made to go through the original point of tangency of isocost line LL' with trade-off curve TT'. This construction allows us to compare the life-cycle heating-related cost of the socially optimum standard with that of the privately optimum standard. The former is given by M and the latter by N, both found on the A axis. These cost figures are both denominated in units of annual energy consumption and represent the sum of the energy use plus the thermal resistance cost, with the latter being converted to equivalent energy units by division by the relative social price of energy.[1]

[1]This procedure means that annual energy consumption serves as the <u>numeraire</u>.

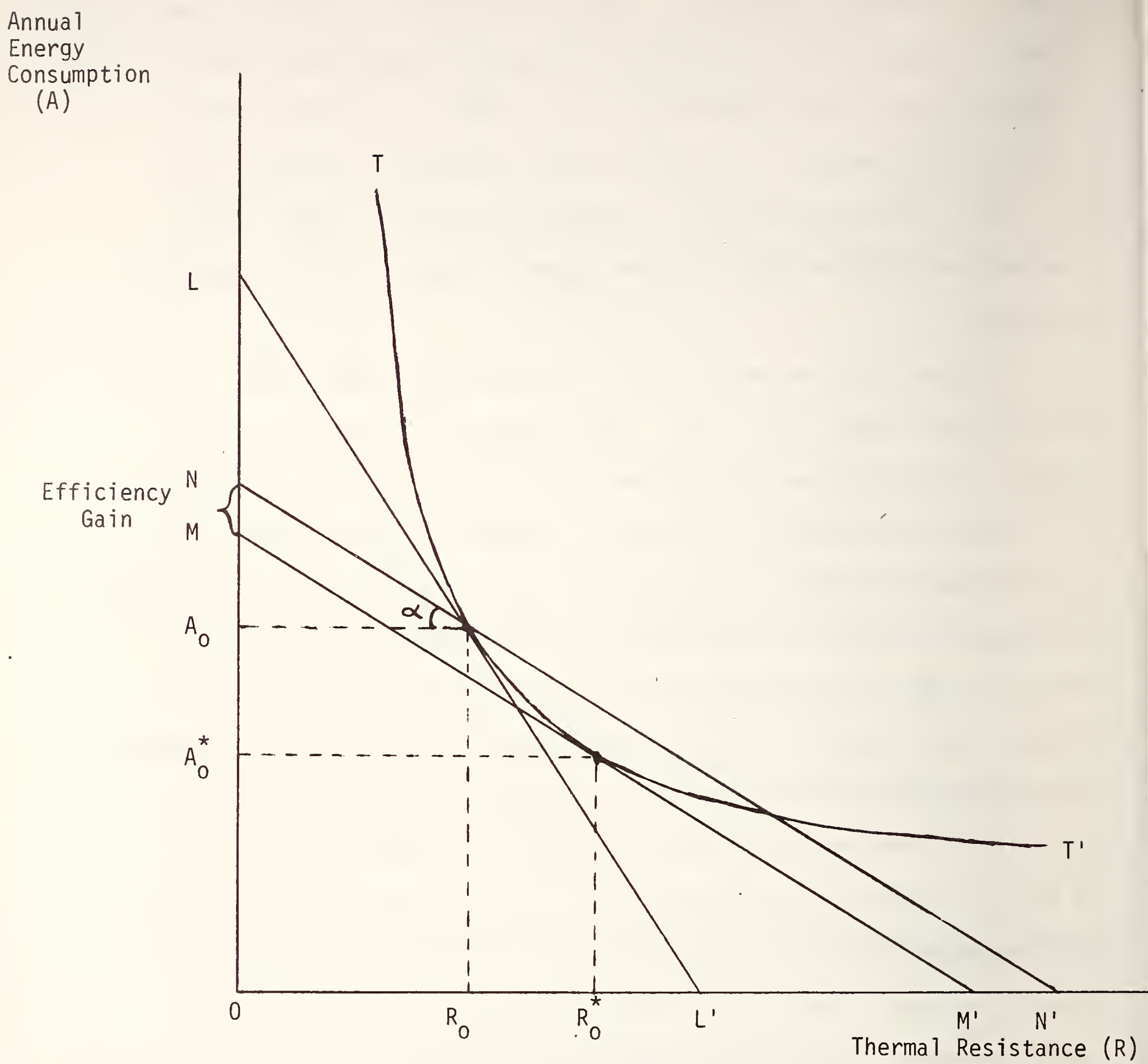


Figure A.3

Socially Optimum Energy Consumption Rate ($A_o^*$)
and the Net Efficiency Gain from Using RIF's (N-M)

The proof that N represents the life-cycle heating-related cost under the privately optimum standard is demonstrated in the following steps:

1. $A_o$ = the energy use under the standard.
2. $R_o$ = the resistance used under the standard.
3. To sum these we can convert the units of $R_o$ to their equivalent energy units using the relative price ratio based on the social price of energy. Thus, life-cycle cost measured in energy units equals $A_o + [b/(P_e \cdot RIF \cdot UPW)] \cdot R_o$.
4. Now the tangent of the angle $\alpha$ in Figure A.3 is equivalent to the absolute value of the slope of line NN', which we know to be $b/(P_e \cdot RIF \cdot UPW)$.
5. But by definition, $\tan \alpha = (N-A_o)/R_o$.
6. Thus, $N = A_o + (\tan \alpha) \cdot R_o = A_o + [b/(P_e \cdot RIF \cdot UPW)] \cdot R_o$, which combined with Step 3 shows that N represents life-cycle cost denominated in equivalent units of annual energy consumption.

Similarly, M can be shown to represent the life-cycle heating-related cost under the socially optimum standard. Thus, we can conclude that the net gain in economic efficiency is given by N - M, which represents the amount by which the social value of the extra energy saved by using RIF's exceeds the additional social cost required to save it.

It should be noted that MM' and NN' were drawn under the assumption that the RIF value is greater than one. If the RIF value for a particular fuel type happens to be less than one, then these lines would have a slope steeper than that of LL'. The same procedure should be followed for measuring the net gain in efficiency that would result from using RIF's.

As already noted, this analysis was carried out using a straight isocost line which is based on the assumption of linear resistance costs.

If quadratic or cubic costs are assumed, the isocost line becomes concave downward. Nevertheless, the optimum still occurs at the point of tangency between the isocost curve and the trade-off curve. In this case, the analysis of the net gain in efficiency would be based on the straight tangent line which goes through the socially optimum combination of A and R, rather than on the isocost curve itself. This is because the absolute value of the slope of that straight tangent line is given by the ratio of the marginal cost of thermal resistance (at the optimum point) to the social value of the energy saved.

Table A.1

Conversion Factors from Customary to Metric (SI) Units

| Physical Characteristic | To Convert From | To | Multiply By |
|---|---|---|---|
| Length | ft | m | $3.048 \times 10^{-1}$ |
| Area | $ft^2$ | $m^2$ | $9.290 \times 10^{-2}$ |
| Temperature | °F | °C | $t_C = (t_F-32)/1.8$ |
| Temperature difference | °F | °C | $5.556 \times 10^{-1}$ |
| Energy | Btu | J | $1.055 \times 10^3$ |
| U-value | $Btu/hr \cdot ft^2 \cdot °F$ | $W/m^2 °C$ | 5.678 |
| Thermal Resistance | $hr \cdot ft^2 \cdot °F/Btu$ | $m^2 °C/W$ | $1.761 \times 10^{-1}$ |

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15. SUPPLEMENTARY NOTES

16. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

This report addresses the question of the proper price for energy in the development of cost-effective energy conservation performance standards for buildings. This study finds that the appropriate price for energy is its social value, which should be determined through the development and application of Resource Impact Factors (RIF's). Some guidelines are provided for the formulation and development of RIF's. A simple life-cycle cost minimization model for determining the optimum conservation standard is employed to show how the use of RIF's would generally lower the maximum allowable energy consumption specified in the standard. Indeed, it is found that the higher the RIF value, the lower the energy consumption allowed by the standard, although this effect steadily diminishes as the RIF value increases. When a more general cost model with less restrictive assumptions is employed, the same inverse relationship appears between the energy consumption allowed by the standard and the RIF value. Finally, a geometric measure is derived for the net gain in economic efficiency that would result from using RIF's in developing energy conservation performance standards.

17. KEY WORDS *(six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons)*

Building economics; economics; economic efficiency; energy; energy conservation; life-cycle cost; optimization; performance standards; resources; resource impact factors; social optimum; standards

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