Deflection Performance Criteria for Floors

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Center for Building Technology
Institute for Applied Technology
National Bureau of Standards
Washington, D. C. 20234

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Final Report

Prepared for
Office of Policy Development and Research
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Abstract

Serviceability performance criteria for floor systems are discussed in terms of their static and dynamic components. Development of traditional static stiffness criteria is given along with a review of their strengths and weaknesses. Criteria for serviceable floors are presented from a vibration viewpoint and the derivation of an improved criterion is given. A new approach for future vibration criteria is described.

Key words: Deflection; dynamic; floor systems; human response; performance criteria; serviceability; static; vibration.
Notation

The following symbols are used in this report:

\[ \Delta = \text{Deflection} \]
\[ L = \text{Span} \]
\[ T = \text{Natural period of floor system} \]
\[ k = \text{Floor system stiffness} \]
\[ m = \text{Floor system mass} \]
\[ T_f = \text{Forced vibration time} \]
\[ f = \text{frequency} \]
\[ \nu = \text{Damping ratio} \]
\[ x = \text{Half-amplitude of peak-to-peak displacement} \]
\[ S_a, S_v, S = \text{Peak to peak amplitude acceleration, velocity and displacement, respectively} \]
\[ C_a, C_v, C_s = \text{Calibration constants} \]
\[ w = \text{Uniform static floor load} \]
\[ a = \text{Width of floor} \]
\[ E = \text{Elastic constant for floor} \]
\[ I = \text{Moment of inertia} \]
\[ P = \text{Static live load} \]
\[ d_v = \text{Equivalent vertical deflection} \]
\[ g = \text{Acceleration of gravity, 32.2 ft/sec}^2, 9.8 \text{ m/sec}^2 \]
\[ F(t) = \text{Random forcing function} \]
\[ G(t) = \text{Deterministic floor system characteristics} \]
\[ Y(t) = \text{Random floor response} \]
\[ D(t) = \text{Random human response to vibration} \]
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Deflection Performance Criteria for Floors

1. Introduction

This report covers the development of performance criteria for structural floor systems with the focus being on that part of the criteria which is related to serviceability. The report is based on a portion of the research done in a broader study (Structural Deflections [1]) at the National Bureau of Standards.

The design of floors for serviceability has been in existence for many years. A serviceable floor is one that meets the needs for which it is intended for everyday use. Seldom, if at all, does a serviceability design requirement equal a stress design requirement, i.e., a life safety requirement. Floors that are safe are not necessarily serviceable; however, floors that are serviceable generally are safe. Safety here refers to collapse and endangering human life.

Deflection criteria for the design of floors can appear in a hierarchy (figure 1) beginning with a performance statement. The examples in figure 1 converge from the generality of a performance statement to the specificity of technical engineering publications, research publications, and engineering texts and are the types of statements applicable to occupant comfort of floors. The differentiation made in the hierarchy given in figure 1 does not always appear in current practice. Performance statements occur in a specification without any other component of the hierarchy. This leaves the designer in a quandry as to what is "comfortable" or "uncomfortable". For an adequate design the entire heirarchy has to be followed through which requires more time, effort and professional skill than a mere check of deflection for an arbitrary superimposed uniform liveload, for example.

This report will show some principal examples of existing floor performance criteria, develop a new stage of floor performance criteria and then hypothesize an advanced stage of floor performance criteria. A complete state of the art and literature survey has been produced [1] in the broader study, thus no attempt will be made to survey the literature. New items will be given that have appeared after the publication of the literature survey. Even though the emphasis in this report is on floor deflections or vibrations, other horizontal framing systems such as roofs and stairways can be considered in the same category but their serviceability requirements must be carefully evaluated before they are related to floors. For instance, roofs may not be subject to the dynamic considerations of human comfort but should be required to not deflect excessively under static loading to prevent damage to partitions or ceilings below. Conversely, stairways or footbridges (especially connections between multistory structures) may be subjected to much more pronounced dynamic foot traffic than an average floor and thus dynamic design should be considered for different loadings than those used for floors.

2. Background

The performance of floor systems has been the recent concern of owners, builders, occupants, and the engineer. This concern has increased as demonstrated by the recent publications on the subject which have been essentially non-existent in the past [1,2,3,4]. Rigorous design procedures, higher allowable stresses and new construction materials result in effectively more flexible floor systems than have been constructed in the past. Traditionally, floor deflection considerations have been essentially ignored or "taken care of" by the present criterion which requires that a horizontal member shall not have a deflection greater than 1/360 of the span for a prescribed live load. More flexible floor systems are being introduced in the modern structure and they frequently show a lack of serviceability because of unsatisfactory deflection performance, i.e., vibration disturbance to occupants, rattling cupboards that set on the floor, vibration of ceiling covers.

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1/Numbers in brackets refer to literature references listed in Section 7 of this report.
(attached to floors), cracking, nonload bearing partition damage, etc. Some of these systems meet the present deflection criterion which indicates that the criteria is either inadequate or that it is not being applied appropriately.

The deflection problem can be broken down into its elements as is described in reference [1]. Figure 2 shows this breakdown which not only applies to the floor deflection problem but also to structural engineering problems in general; however, it will be used in this report with respect to the floor deflection problem.

Many different floor deflection criteria are possible by a thorough application of figure 2. However, the most pertinent appear to be:

The static system deflection should be controlled to limit

1) Human response to static deflection
2) Subsystem response to static deflection

The dynamic system deflection should be controlled to limit

1) Dynamic whole body vibration
2) Audible perception to motion
3) Dynamic visual perception to motion
4) Dynamic subsystem system response

Previously, the above have been encompassed in the panacea criterion of a deflection limit linearly related to the span of the floor as previously stated. It is obvious that with the introduction of the dimension of time for dynamic deflection and sensory perception simple criteria such as 1/360-of-the-span deflection limitation cannot be adequate for all cases.

Definition of dynamic and static deflections is not a simple matter. Generally speaking all deflections are dynamic, i.e., time dependent. Some approximations can be made which separate dynamic and static deflections for most practical purposes. A static load causing a static deflection is one which is slowly applied and released. Slowly refers to the duration of time for application and release of the load as compared to the natural period of the structure. If the ratio of time for load application and release to the natural period is large, the load and corresponding deflection can be considered static. Conversely, if this ratio is small, the load and corresponding deflection are dynamic and the dimension of time has to be considered. It also must be realized that in the dynamic case, inertial forces are significant and must be considered to create equilibrium. This report considers both static and dynamic floor deflection problems which are within the scope of these approximations for static and dynamic loads.

3. Traditional Floor Performance Criteria

As previously stated, criteria have to be separated into their static and dynamic components, before rationale can be applied to their derivation and use. The static and less complex problem will be considered first.

Existing serviceability criteria for structural floor systems use a static deflection limitation for both static and dynamic loads which has evolved from the experience obtained in using this type of criteria for over 150 years. These criteria are based on limiting the maximum deflection that can occur under a specified load in the floor system to insure that unpleasing asthetic conditions such as sagging, cracks, jammed doors, and undue loading of partitions does not occur. For most residential floor systems this means the maximum deflection in a joist is specified to prevent localized distortions, visible sagging and cracking. However, the most serious consequence of excessive floor deflection is the subsequent damage to other building components such as ceilings and partitions. Besides unsightly cracking in these components other effects like misalignment and malfunctioning of interior doors can occur. These deflection limitations were believed to indirectly also satisfy vibration serviceability criteria.
The historical development of these criteria have been traced by Russel [5] who credits the first recorded mention of such a limitation to Thomas Tredgold, an English civil engineer who suggested a maximum deflection of L/480 to prevent cracking in plastered ceilings where L is the span length of the floor. He also recognized the importance of stiffness in order to allow walking on a floor without vibrating objects in the room.

Subsequent development or improvement of the deflection limitation has been based on the experience gained from beams designed by such criteria. A later publication by Kidder [6] indicated that 1/360 of the span was not too much deflection to permit in floor joists since a floor is seldom subjected to its full load and then only for a short period of time. This recommended change recognized that serviceability specifications should be related to the service loads. The 1951 edition of Kidder-Parker's Architects and Builders Handbook indicates that a stiffness factor of a beam (Δ/L) based on experience should be 1/360 when the beam supported a plastered ceiling. It also suggests that because the deflection due to dead loads except for the plaster, has already taken place, the appropriate design standard should state: the deflection due to live load must not exceed 1/360 of the span in inches. In this statement of the criterion, the limitation is related to a specific situation, support of plastered ceilings, in order to prevent damage. As can be seen from this brief historical development of serviceability requirements, the development of the criterion has been toward one which prevents damage to structural components from static service loads. Implicitly it was believed from the onset of this criteria that this would also satisfy the needs of occupants for dynamic service loads on floor systems.

Present deflection limitations may take one of two forms, either a restriction on the maximum span to depth of beam ratio or maximum deflection under a given static loading condition. This type of criteria appears in wood, metal and concrete specifications. However, present deflection requirements even though applied to both static and dynamic loads may be sufficient for the static condition but not the dynamic one.

As pointed out by Clarke, Neville and Houghton-Evans [7] there is considerable confusion as to what the static deflection limit should be for a particular case. This is because while there is a good deal of literature on how to predict deflection, statements on allowable values are usually limited to a brief note that limits such as 1/360 are "customary" and have "worked well in the past". Also, there is little information available on which to judge the rationality of such rules.

It appears that the most appropriate criteria for static loads are still a restriction on the deflection under a given load or a restriction on the span to depth ratio. However, the shortcoming of current criteria is that the designer is not given a range of choices of deflection or span/depth allowable for a given situation. That is, he is given no guidance on what type of design will result, cracking but not readily visible, deflection that cause no distress to hung ceilings, but may damage partitions below or above, etc. The question also arises as who should determine deflection/span/depth ratios desirable for a given structure. This probably should be done on the level of the owner where he decides to tolerate certain deflections which give him an assigned risk of distress. The lower the risk, the higher the cost of the structure and vice versa. This approach has been proposed for lateral deflection requirements in tall structures subjected to wind loads [8].

Much more research would have to be completed before a quantification of static deflection risk can be made. Most risk assignments will have to be made on "good judgement" with the obvious trend of less deflection allowed corresponding to a decreased risk of an unserviceable floor system.

4. Improved Floor Performance Criteria

4.1 Vibration Criteria for Floor Performance

With the increased use of high strength materials, larger spans and generally more flexible construction, it has become apparent that the simple live load deflection or span to depth ratio limitations are not always sufficient to prevent annoying floor vibrations. As a result, modern standards require that due consideration be given to the design of floor sys-
items for vibration. Typically statements are often like the one found in the Specification for the Design, Fabrication and Erection of Structural Steel, AISC [9], which states,

"Beams and girders supporting large open floor areas free of partitions or other sources of damping, where transient vibrations due to pedestrian traffic might not be acceptable, shall be designed with due regard for vibration."

The Guide Criteria for the Evaluation of Operation BREAKTHROUGH Housing Systems [10] goes further by recommending that,

"transient vibrations induced by human activity should decay to 0.2 of their initial displacement-amplitude within a time not to exceed 1/2 second."

Also, steady-state vibration is to be isolated or, where this is not possible, a human perception curve (deflection amplitude versus frequency), based on the Lenzen modified curves [11] of Reither and Meister [12], should be satisfied. However it was recognized in the BREAKTHROUGH Guide Criteria that this criterion was tentative since further research was needed.

The International Organization for Standardization recently published a standard, Guide for the Evaluation of Human Exposure to Whole-Body Vibration, ISO 2631 [13]. This guide is written with a general approach for application to many vibratory environments. It is applicable to vibrations transmitted to the body as a whole through the supporting surface; such as the feet of a standing human or the buttocks of a seated human, and covers a frequency range of 1 to 80 Hz for steady state, periodic vibration, random vibration with a distributed frequency spectrum and continuous shock excitation when the energy is contained within the 1 to 80 Hz range. All allowable vibrations are given in terms of direction of transmission, frequency and rms (root-mean-square) acceleration with regard to satisfying the three general criteria of preserving comfort, working efficiency, and safety or health.

As can be seen from the brief discussion of the three standards, each is lacking with regard to consideration of all the vibration components which are necessary for a serviceable vibration design. The ISO Guide contains all of the essential components; however, much of it must be qualified through further research as there are insufficient data for all areas to be covered in floor design for vibration.

Consideration of dynamic floor deflections is a much more complex problem than the static case. In considering floor vibration criteria, the following items are to be considered:

1. Transient vibration
2. Steady-state vibration
3. Damping
4. Resonance
5. Type of occupancy
6. Type of structure
7. Structure location

Transient forcing functions are the most difficult to define in the floor vibration problem; yet are the most prevalent loads to occur. Transient vibration generally takes the form of non-stationary random vibration as classically defined in engineering mechanics. This is what makes it difficult to treat them as discrete and singly definable events. It is the consideration of this random nature that leads to the future criteria covered later in this report. However, for the development presented here transient forcing functions will be approximated as discrete events. Transient vibration has to be defined in terms of response of the structure. It has been observed [11] that if a floor has a response of 5 to 10 cycles then the initial response is the most perceptible and the vibration thereafter is not as perceptible. If this response is due to a single discrete event such as a single footfall, then multiple discrete events would appear as shown in figure 3, where T is the fundamental or lowest natural period of the structural system.

\[ T = 2\pi \sqrt{\frac{k}{m}} \]

\[ k = \text{floor system stiffness} \]

\[ m = \text{floor system mass} \]
Assuming that the human response is transient in nature (from Lenzen's observations) then at a minimum (where two footfalls are adjacent), the transients considered as footfalls would occur at intervals

\[ T_f + 10T \quad 0 < T_f < 2T \] (4.2)

where \( T_f \) represents the forced vibration phase of the footfall and \( 10T \) represents the duration of the free vibration phase. The primary assumption here is that the transient vibration is represented by the chain of damped responses as shown in figure 3 with the time characteristics given. Damping has to be defined with this approach or the transient chain could become effectively steady state if damping were too small. As previously stated, the assumption will be made that if a vibration damps out within 5 to 10 cycles the initial peak is responded to by the human and if greater than 10 cycles response is similar to that of a steady-state vibration.

"Damping out" is a vague area with respect to human response data, thus it will be defined here for the purposes of this development. Damping for this case will be assumed to be viscous and expressed in the form of the critical damping ratio and is given by

\[ \frac{x_n}{x_o} = e^{-2\pi \nu (1-\nu^2)^{1/2}} \]

or for \( \nu << 1 \)

\[ \frac{x_n}{x_o} = e^{-2\pi \nu} \] (4.3)

where

\[ x = \text{displacement amplitude} \]

\[ n=0,1,2,... \quad = \text{refer to successive deflection peaks increasing with time} \]

\[ \nu = \text{fraction at critical damping} \]

The displacement magnitude of equation 4.3 is applicable only to the free vibration portion of the response and not to the forced vibration part, thus any determination of damping has to be from the free part of the vibration. An estimate has to be made of the duration of the forcing function, say the footfall. A footfall [15] time history (figure 4) varies in duration from 50 to 100 msec (milliseconds). Most floors have a lowest natural frequency varying

\[ 3 < f_1 < 35 \text{ Hz} \quad [14, 16, 17] \]

or periods varying \( 30 < T_1 < 300 \) msec. It appears reasonable to assume for this development that the forcing function will not be acting on the floor after the first cycle of response. Thus, response after the first cycle will be the free portion of vibration. From the observation that the human responds only to the initial pulse of a transient if it damps out within 5 to 10 cycles, an assumption is made that "damping out" will be approximated by the tenth cycle damping to 25 percent of that of the second cycle, where the first cycle of free vibration is the second cycle of response. In terms of damping this means in equation 4.3

\[ \frac{x_9}{x_1} = 0.25 \]

substituting into equation 4.3

\[ 0.25 = e^{-2\pi \nu} \]

and solving for the fraction of critical damping

\[ \nu = 0.0276 \]

or 2.76% of critical damping.
Residential floors with partitions, furniture and carpeting have been observed to have critical damping ratio's varying \( 7 < \nu < 13 \) percent [16] whereas commercial building floors have been observed to have damping vary \( 1.9 < \nu < 4.9 \) percent [14]. This shows that damping is generally less than 10% and equation 4.3 for damping is within less than 1% error.

With the rationale that transient vibration is a chain of damped discrete responses, an equivalent static floor deflection limitation as a function of frequency will be developed. An empirical static deflection amplitude will be used as a measure of the dynamic component of motion for a person walking on a floor. The deflection limit to be developed is based on the concept that human response is proportional to the velocity of vibration and that residential wood-joist floors currently in service meet a deflection limit of \( 1/360 \) of the span for a uniformly distributed live load of 40 psf \((1.9 \text{ kN/m}^2)\), are acceptable with respect to comfort. As the development progresses the qualifications of these assumptions will be made.

4.2 Criteria for Human Perception to Vertical Vibrations

It has been shown by previous researchers [1,2,12,13,14] that human response to vertical vibration can be expressed in terms of acceleration, velocity or displacement as follows:

\[
\begin{align*}
S_a &= C_a f^2 \\
S_v &= C_v f \\
S &= C_s f
\end{align*}
\]  

Equations 4.4 represent a given level of human sensitivity to vibration varying from threshold to possible physical damage. This simplified representation of human response is not applicable to an unlimited frequency bandwidth. It has been shown that equations 4.4 are applicable to frequencies above 8 Hz and below 100 Hz [13]. However, for the observed lowest natural frequency of floors currently in service, it suffices for this development to use the approximations to human response represented by equations 4.4. Other references claim that equations 4.4 apply to frequency ranges from 1 to 80 Hz which still bounds almost all floors that are usually encountered in practice. It should be noted that equations 4.4 represent constant velocity human response criteria. Equations 4.4 will be used subsequently to relate human response to known floor characteristics.

4.3 Derivation of Mathematical Model for Floor Deflection

Two models will be used to develop an expression for equivalent static floor deflection: a uniformly loaded plate and a line loaded plate. The deflection at the center line of the span of an elastic plate with two opposite edges simply supported (figure 5a) is approximated by

\[
\Delta = \frac{5}{384} \frac{wal^4}{EI}
\]  

where \( w \) is expressed in force per unit area and \( a \) is the width of the plate parallel to the supported edges. The uniformly loaded plate (equation 4.5) represents the current model used in determining allowable floor deflections, i.e., the deflection shall not exceed a given amount generally based as a fraction of the span when it is subjected to a uniformly distributed live load.

The deflection at the centerline of the span of a line loaded plate with two opposite edges simply supported (figure 5b) is approximated by

\[
\Delta = \frac{PL^3}{48EI}
\]  

where \( P \) is distributed along a line at the center of the span over the width of the plate, \( a \). The line load model is introduced for the purpose of simulating static application of a
human at a position in the floor that causes maximum deflection. It is not likely that a human would represent a point load but rather a line load as presented in the model. A distribution length of the line load will be assumed to be the width of the floor activated by the human.

These two models (figure 5) will be combined to formulate a calibration equation for use in relating human response to static floor deflection. Solving equation 4.5 for EI results in

\[ EI = \frac{5\ waL^4}{384\ A} \]  

(4.7)

From equation 4.7, EI is calculated for a uniformly loaded plate with two opposite edges simply supported with a center line deflection that is equal to 1/360 of the span, i.e.,

\[ \Delta = \frac{L}{360} \]

then

\[ \frac{\Delta}{L} = \frac{1}{360} \]  

(4.8)

Substituting equation 4.8 into equation 4.7 results in

\[ EI = A\ waL^3 \]  

(4.9)

where

\[ A = \frac{5\times360}{384} \]

Let the line loaded plate (equation 4.6) have the same EI as the uniformly loaded plate in which the center line deflection is equal to 1/360. Substituting equation 4.9 into equation 4.6 results in,

\[ \Delta = \frac{P}{225wa} \]  

(4.10)

where the units of the respective variables are

P in F (force)

a in L (length)

w in F/L²

Equation 4.10 results in an expression for the deflection of a line loaded plate which has a stiffness measured by EI which is limited by a deflection determined from a uniformly distributed load. The majority of static deflection criteria require that the deflection not exceed L/360 as determined from a uniformly distributed live load of 40 psf (1.9 kN/m²) [1]. Substituting this live load into equation 4.10 results in

\[ \Delta = \frac{P}{9000a} \]  

(4.11)

where P and a are in lb and ft respectively and \( \Delta \) is in ft.

From the model represented by equation 4.6 it is assumed that an equivalent static human live load can be represented by a line load distributed over the width of the plate. Assuming a nominal human weight of 150 lb. and substituting into equation 4.11 results in

\[ \Delta = \frac{0.0166}{a} \]

where \( \Delta \) is in feet or for convenience,

\[ \Delta = \frac{0.20}{a} \]  

(4.12a)
where $\Delta$ is in inches and $a$ is in feet.

Also for the SI system of units

$$\Delta = \frac{1.55}{a} \quad (4.12b)$$

where $\Delta$ is in mm and $a$ is in meters.

Equations 4.12 are then the static calibration equations which will be correlated with the constant velocity criterion for human response.

4.4 Calibration of Human Response to Equivalent Static Deflections

It was previously established in Section 4.2 that the assumption of a constant velocity response for humans to vertical vibrations is reasonable for the frequency range that is encountered in floors currently in service. The constant velocity criterion as expressed by equation 4.4a,

$$S_a = C_a f$$

can be shown to be related to deflection if the assumption of steady state vibration is made.

$$S_a = 4\pi^2 f^2 S \quad (4.13)$$

where

- $S_a =$ peak to peak acceleration
- $S =$ peak to peak deflection

Solving equation 4.13 in terms of deflection and substituting $\Delta$ (half-amplitude deflection) results in a constant velocity criterion in terms of deflection.

$$\Delta = C \frac{1}{f} \quad (4.14)$$

which is similar to equation 4.4c.

The constant, $C$, of equation 4.14 will be used as a calibration constant to relate the dynamic deflection of the constant velocity criterion (4.4c) to the equivalent static deflection of equations 4.12. Equating equation 4.14 and 4.12a

$$C \frac{1}{f} = \frac{0.20}{a}$$

reduces to

$$C = 0.20 \frac{f}{a} \quad (4.15a)$$

where, $a$ is in feet and $f$ is in Hz.

Also for the SI system of units,

$$C = 1.55 \frac{f}{a} \quad (4.15b)$$

where, $a$ is in meters.

Considering that the majority of experience of acceptable floor performance is gained through wood joist systems, data obtained from these systems will be used to establish the calibration constant, $C$. Table 1 summarizes these data. In order to calculate $C$, consistent values of $a$ (the distribution width of the line load), the width of the plate, and the natural frequency need to be established. Data are generally only available for the lowest natural frequency (first mode of vibration) therefore this value will be used in the calculation of $C$. Without extensive study which is beyond the scope of this report, the active floor width is difficult to establish. For the purposes of this derivation it will be
assumed that the active floor width will be either the span of the floor or ten times the nominal joist spacing. The smaller value of these two will be used which follows from

\[
\begin{align*}
\Delta_{\min} & \rightarrow \Delta_{\min} & \text{by equation 4.12} \\
\Delta_{\min} & \rightarrow \Delta_{\max} & \text{by equation 4.14} \\
\Delta_{\min} & \rightarrow C_{\min} & \text{by equation 4.15}
\end{align*}
\]

The value of C (table 1) varies from 0.181 to 0.640 thus equation 4.14 is bounded by

\[
0.180 \frac{1}{f} < \Delta < 0.670 \frac{1}{f}
\]

or approximately

\[
\frac{2}{11} \frac{1}{f} < \Delta < \frac{2}{3} \frac{1}{f}
\]

which gives the boundary equations

\[
\begin{align*}
\Delta &= \frac{2}{11} \frac{1}{f} \\
\Delta &= \frac{2}{3} \frac{1}{f}
\end{align*}
\] (4.16a)

(4.16b)

where \(\Delta\) is in inches and \(f\) in Hz. Also for the SI system of units,

\[
\begin{align*}
\Delta &= 1.4 \frac{1}{f} \\
\Delta &= 5.0 \frac{1}{f}
\end{align*}
\] (4.16c)

(4.16d)

where \(\Delta\) is in mm and \(f\) is in Hz.

The data (table 1) are plotted on figure 6. The envelopes of equations 4.16 are also shown. Equations 4.16 bound all of the data which were selected to calculate the calibration coefficient, C. For comparative purposes the ISO reduced comfort boundary and fatigue decreased proficiency boundary both for the one minute duration vibration [13] and the Lenzen modified Reiher-Meister boundary [11] are also shown in figure 6. It should be noted that both the ISO and the Lenzen modified Reiher-Meister curves indicate that equations 4.16 establish a boundary of vibration perception that is definitely above that of a threshold perception. Lenzen refers to the range above his curve as strongly perceptible. The ISO refers to their perception range as fatigue-decreased proficiency boundary for one minute of vibration. A time factor is not necessarily attached to the Lenzen curve; however, Lenzen does infer that the curve applies to transient vibration because it was modified from the steady-state vibration curves originally established by Reiher-Meister [12]. It can be generally said from previous researchers' work [1] that as the vibration duration decreases, the human sensitivity to that vibration also decreases which infers that if the vibration duration were less than one minute, that the boundaries established by equations 4.16 would be in the approximate range of decreased proficiency according to ISO [13]. Without further extensive comparisons, calibrations and analysis, it will only be possible to represent a trend of response to floor vibrations by way of the use of equations 4.16. It can be said that equations 4.16 undoubtedly represent a range of flexibility of floors that are currently in use and that do have perceptible vibrations due to human traffic. However, due to the experience gained in these floors the level of vibration probably will cause some discomfort and human reaction but not to an unacceptable level.

4.5 Performance Criteria

Improved floor performance criteria have been developed in the previous sections which can be stated as follows:

(1) The stiffness of a floor supporting human activity in a building with all partitions in place shall be such that its maximum deflection under a 150-lb static
line load, distributed over a floor width not greater than the span or 10 times the joist spacing, whichever is smaller and applied at a location producing the greatest deflection, is within the following limit:

\[ d_v \leq \frac{2}{3f} \]

where \( d_v \) is in inches and \( f \) is the lowest natural frequency in units of Hz.

(2) A floor in the building carrying foot traffic and with surface coverings, partitions, furniture and any other items representative of normal occupancy in place, shall be adequately damped so that when subjected to a load of duration less than 1/f, there will be a reduction in the deflection amplitude in the 10th cycle of vibration to less than 1/4 of the amplitude in the 2nd cycle of vibration.

(3) Under the effect of steady-state vibrations and irregular vibrations exceeding 10/f seconds in duration or regularly repeated transient vibrations caused by service conditions:

a. Any floor supporting human activity installed in the building and with surface coverings, partitions, furnishings and other items representative of normal occupancy in place shall not exceed an acceleration of 0.004 g rms.

b. The natural frequency of any structural element or assembly shall be less than 0.7 times or greater than 2.0 times the frequency of any dynamic excitations to which it is exposed unless vibration isolation is provided.

These criteria address transient vibrations and damping in parts 1 and 2. Steady-state vibrations and resonance are covered in part 3. The maximum acceleration level specified for steady-state vibration and certain other forms of vibration excitation in part 3a is a constant acceleration criteria covering the frequency range anticipated for floors. This value corresponds approximately to the ISO Standard [13], 24-hour reduced comfort level in the 4 to 8 Hz range. It is intended that this acceleration level approximate a median perception level for most floors encountered. However, according to the more sophisticated approach of the ISO Standard [13], the 4 mg (millig) specification above 8 Hz is only an approximation and should be improved as more research data becomes available. The resonance specification given in part 3b is based on the standard deflection magnification versus frequency curve found in most engineering vibration texts [20]. This development is primarily with residences in mind and should be applicable to floor systems used for this type of occupancy. Further research is needed to account for types of occupancy and structural floor systems as well as location of the structure.

5. Future Floor Performance Criteria

Efforts to deal with the problem of perceptible floor vibrations has generally been through experimental techniques. The procedure has been to design the floor system using the traditional static stiffness concept and then attempt to assess its vibrational adequacy by questionable physical tests. A lack of knowledge about the proper forcing function to simulate human activity on floor systems and a similar deficiency in information on human response to this type of input has produced a concern about the validity of the results from this procedure. Furthermore, this type of experimental technique assumes that both human activity and occupant response can be characterized in a deterministic manner when realistically they both are random variables.

This realization that human activity on floor systems and occupant response to this activity are random variables becomes clear if one considers the character of these variables. For example, in walking, variations in weight, gait, heel-to-ball of foot contact and foot wear of individuals will all effect the instantaneous value of the dynamic loading produced by this type of activity. Hence, it is very difficult to predict the instantaneous value with any certainty for a given instant in time. An examination of the literature indicates that no definitive study has been made to determine the statistical data needed to characterize footfall as a random variable.
A thorough study of the nature of human response to vibration points out the many variables that are involved, such as vibration input where intensity, frequency, direction and duration must be considered; psychological influences in the form of mental state, motivation and experience; and physical influences from sound and sight. Considering this multiplicity of parameters, it is obvious that the only way to classify human response is also as a random variable.

Thus, the dynamic response of a floor system subjected to human activity represents a random physical phenomenon which cannot be described by an explicit mathematical relationship because each observation of the phenomenon will be unique. It should be pointed out that it is not the structural floor systems which are considered to be a random variable because, although there are several types of systems, their dynamic properties can from an engineering viewpoint, be described deterministically. Rather it is the forcing function (human activity) and the human response to this activity which must be considered as random processes; thus making the overall problem of human response to floor vibrations induced by human activity a random vibration problem. This can be more clearly visualized in functional form:

\[ F(t) \rightarrow G(t) \rightarrow Y(t) \rightarrow D(t) \] (5.1)

where \( F(t) \) is the random forcing function, \( G(t) \) is the deterministic floor system characteristics, \( Y(t) \) is the random response of the floor system and \( D(t) \) is the random human response to the floor vibration, \( Y(t) \).

Even though sufficient information is not presently available to describe the forcing function as a random variable it is possible to measure the random response of a floor system when subjected to human activity and compare it to human response.

A recent report, Correlation of Floor Vibration to Human Response [21], describes a random variable methodology for floor vibration evaluation. This methodology is based on physical testing and subsequent analysis and comparison of the dynamic response of floor systems to human activity.

Consequently a new approach to the problem of perceptible floor vibrations has been developed which leads to improved performance criteria for serviceability of floor systems.

A performance statement of the type presented in figure 1 has as a design standard a limitation on the vertical acceleration level that can occur under normal in-service conditions and a specification on the minimum amount of damping the floor system must possess. A basis for this type of performance statement has been presented in a standard proposed by Splittgerber [22] which provides vibration and shock limits for occupants of buildings.

Splittgerber's proposal is based on the ISO Guide for the Evaluation of Human Exposure to Whole-Body Vibration [13] which gives allowable root-mean-square (rms) acceleration at different frequencies of vibration. The ISO Guide is applicable to periodic (sinusoidal) vibration as well as narrow-band or wide-band random vibration within a frequency range from 1 Hz to 80 Hz. Allowable accelerations are specified in the Guide for the three general criteria of preserving comfort, working efficiency and safety or health.

Splittgerber proposes a modification, based on adjusting the ISO Guide reduced comfort limit, by applying a weighting factor to the allowable accelerations. Variation in the weighting factor account for the type of excitation (impulsive shock or vibration), occupancy (residential, office, workshop) and the time of day.

Figure 7 shows the ISO Guide [13] standard acceleration curve for the "reduced comfort level" with a correction applied to account for the environmental situation of private residences, hospitals, offices (curve A). This curve serves as the basis for Splittgerber's proposed standard and represents in his opinion good environmental standards for these occupancy types. Thus, for example, continuous or intermittent vibration levels which exceed those specified by curve A either during the day or night, would produce an increased number of complaints in hospitals. In residences, during the day, continuous or intermittent vibration levels could be increased by a factor of two for a minimum complaint level (curve B, figure 7). Splittgerber differentiates between intermittent vibration and impulsive shock in his standard by defining impulsive shock to be characterized by a rapid
build-up to a peak followed by decay while intermittent vibration may only last a few
seconds but is characterized by a build-up to a level which is maintained for several
cycles of vibration. Impulsive shock is typically produced by blasting, pile driving with
an impact procedure, or a human jumping onto a floor. Examples of intermittent vibrations
are traffic vibration, machinery starting up, humans walking across the floor, and modern
pile driving methods which use vibrating columns. Impulsive shock levels for residences
during the day could be increased by a factor of sixteen above curve A, if there are only
one to three occurrences per day (curve C, figure 7). Finally, for vibration during the
night, regardless of the type of excitation, the vibration level in residences can only be
increased above curve A by a factor of 1.41 (curve D, figure 7) without causing increased
complaints by occupants.

Plotted on figure 7 for purposes of comparison is Lenzen's [11] modification for transient
vibration of the original Reither-Meister work [12] on steady-state vibrations. Lenzen's
modification considers transient vibrations to be a single pulse while Splittgerber's
definition for intermittent vibration is a vibration which may last only a few seconds but
is characterized by a build-up to a level which is maintained for several cycles. Realisti-
cally, Lenzen's definition fits the vibration produced by a single footfall or someone
jumping with Splittgerber's definition fitting the vibration produced by walking or running.
The cross-hatched area on figure 7 denotes the slightly perceptible range for Lenzen's
modification which is again claimed to represent a minimum complaint level. Lenzen's
definition is more consistent with Splittgerber's impulsive shock criteria. It can be seen
from figure 7 that there is good agreement between these two definitions for \( f > 8 \text{ Hz} \) with
a divergence of the two below this frequency. The reason for this divergence remains
obscure with the information that is available at this time.

Plotted as a triangle on figure 7 is the rms acceleration of 0.95 mg at 16 Hz as determined
from a 40 second continuous walking test record. The procedure for obtaining the walking
record and its subsequent analysis is given in reference 21. The methodology in reference
15 considers the response of the floor as a non-stationary random process. As can be seen
from figure 7, the treatment of the walking record as an intermittent vibration, gives a
resulting rms acceleration slightly below the standard curve and well below the minimum
complaint level. The floor system tested was in-service and assumed to be acceptable in
the absence of major complaints from the occupants.

This indicates that Splittgerber's proposed standard may well be quite reasonable as a
basis for performance criteria for vibration of floor systems. However, considerably more
data from floors in-service must be obtained before a definite conclusion can be made.
Also, additional work needs to be done so that a designer can be given guidance on how to
apply this type of approach in the design stage of a given floor system.

6. Summary

Existing performance criteria for the serviceability design of structural floor systems use
a static deflection limitation for both static and dynamic loads which has evolved from the
experience obtained in using this type of criteria for over 150 years. These criteria are
based on limiting the maximum deflection that can occur under a specified load in the floor
system to prevent damage to the structure and unpleasing asthetic conditions. In addition,
it is generally believed that this limitation would also satisfy vibration serviceability
requirements. Present deflection limitations may take one of two forms, either a restric-
tion on the maximum span-to-depth ratio or maximum deflection under a given static loading
condition. The current deflection requirement for both static and dynamic service loads
appears to be sufficient for the static loads but not always for the dynamic ones. Another
shortcoming of current criteria is that the designer is given no guidance as to what level
of risk would result from using different deflection limitations.

With the increased use of high strength materials, larger spans and generally more flexible
construction, it has become apparent to the engineering community that the present criteria
does not always prevent perceptible floor vibrations. Present standards have recognized
this by the inclusion of statements which require that due consideration be given to the
dynamic characteristics of floor systems in the design of those floors which are subjected
to dynamic loading by pedestrian traffic. Three current vibration criteria are discussed and improved floor performance criteria for dynamic loading of floor systems are developed. These improved criteria treat transient vibration as a chain of damped discrete responses and give an equivalent static deflection limitation as a function of frequency. The relationship developed is calibrated so as to include floor systems currently in use and designed by the traditional stiffness procedure. These floors were generally considered to be acceptable by occupants, although some do exhibit perceptible vibrations when subjected to human traffic.

A new approach to the problem of perceptible floor vibrations is presented which could lead to future performance criteria for vibration serviceability of floor systems. This approach is based on the assumption that human activity which produces floor vibration and the occupant response to this vibration are random variables. A recent proposed ISO standard for Vibration and Shock Limits for Occupants of Buildings is discussed which would act as a basis for future performance criteria. Data for one residential in-service floor system subjected to a continuous walking test and analyzed as random data are compared with the standard and found to be below the minimum complaint level. However, more research is required before this new type of criterion can be fully developed and implemented.
7. References


7. Clarke, C.V., Neville, A.M., Houghton-Evans, W., "Deflection - Problems and Treatment in Various Countries," Deflections of Concrete Structures, SP-43, American Concrete Institute, 1974.


### TABLE I

| Source       | Descript. | Distribution | Lowest | | | |
|--------------|-----------|--------------|--------|--------|--------|
|              |           | width, a     | Nat. fre. | | |
|              |           | Span L | 10xjoist spacing | f | | |
|              |           | ft | ft | Hz | Hz/ft | in | |
| VPI [17]     | wood joist | 13.28 | 13.3 | 12-30 | .1801-.4519 | .015 |
| H1(FH1)4     | 20.5      | 10       | 13    | .260 | .020 |
| H2(FH2)5     | 13        | 20       | 24    | .369 | .015 |
| NBS [18]     | H3(FH3)5  | 12.5     | 13.3  | 26    | .416 | .016 |
| A1-4         | 14.5      | 13.3     | 23    | .396 | .015 |
|              | 10.25     | 13.3     | 22    | .429 | .020 |
| B             | 11.08     | 10       | 32    | .640 | .020 |
|              | 11.08     | 13.3     | 30    | .541 | .018 |
| C             | 13.08     | 43.4     | 21    | .321 | .015 |

*Calculated from $a_{min}$ (underscored)

Note: 1 ft = 0.3048 meters (exactly)
1 in = 25.4 mm (exactly)
PERFORMANCE STATEMENT
Floors shall not be annoying to occupants

GENERALIZED SPECIFICATION STANDARD
Serviceable floors shall not have excessive vertical accelerations

DESIGN STANDARD
Serviceable floors for residences shall not have accelerations exceeding \((x) \text{ gs rms}\) and shall have no less than \((y)\) percent of critical damping

DESIGN GUIDES
Methods of loading floors for floor response (in both the design stage and the field on in-service systems) and Methods of calculating floor response

DETAILED REFERENCES
Technical Engineering Publications
Research Publications
Engineering Texts

Figure 1. Example Heirachy of Criteria for Floors
Figure 2. Problem Statement, Structural Deflections
Figure 4. Footfall Time History (15)
Figure 6. Comparison of Human Response to Vertical Vibration, Deflection Vs. Frequency

- Fatigue Decreased Proficiency Boundary, 1 Min. (11)
- Reduced Comfort Boundary, 1 Min. (11)
- Equations 4.16a, 4.16c
- Equations 4.16b, 4.16d
- Strongly Perceptible Boundary (11)
Figure 7. Human Response to Vertical Vibration, Acceleration vs. Frequency
Serviceability performance criteria for floor systems are discussed in terms of their static and dynamic components. Development of traditional static stiffness criteria is given along with a review of their strengths and weaknesses. Criteria for serviceable floors are presented from a vibration viewpoint and the derivation of an improved criterion is given. A new approach for future vibration criteria is described.