NBSIR 75-775

Electrical Measurement of High Voltage Pulses in Diagnostic X-ray Units

Robert E. Hebner, Jr.

Electricity Division
Institute for Basic Standards
National Bureau of Standards

November 1, 1975

Interim Report

Food and Drug Administration Order FDA-1AG 224-74-6039
FDA Technical Consultant: Thomas Lee

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Bureau of Radiological Health
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U.S. DEPARTMENT OF COMMERCE, Rogers C.B. Morton, *Secretary*
James A. Baker, III, *Under Secretary*
Dr. Betsy Ancker-Johnson, *Assistant Secretary for Science and Technology*

NATIONAL BUREAU OF STANDARDS, Ernest Ambler, *Acting Director*

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I. Introduction

The purpose of this work was to develop a technique for the calibration of dividers used to measure high voltage pulses in diagnostic x-ray units. At present, three somewhat different techniques for the determination of the peak voltage in an x-ray unit are used.[1-9] The first is to infer the peak voltage level from the measurement of an appropriate parameter of the emitted x-ray spectrum. The second is to infer the peak voltage level from a measurement of the quantity of radiation transmitted through preselected thickness of an absorber. The final technique is to measure the peak voltage using a divider to provide a low voltage replica of the high voltage signal. The relative advantages of each of the methods have been discussed[4,7] and the consistency--under a limited range of voltage and time--of the three methods, when used to measure the same voltage waveform, has been determined. The scope of this work was limited solely to an assessment of calibration techniques for dividers currently in use.

The investigation focused on four areas. These were the divider ratio under direct voltage, the frequency dependence of the ratio, the voltage dependence of the ratio and the effect of self-heating on the device. Sixteen dividers were investigated and there was substantial variation among the devices tested. A few of the dividers were excellent under all test conditions. While most of the dividers had the expected ratio under direct voltage, some had unacceptably large variations of the divider ratio when the frequency of the applied voltage was varied. This could lead to a measurement error, particularly during relatively short pulses. No significant errors due to self-heating or voltage level were detected in the devices tested.

As a result of this work, a calibration procedure was developed. This procedure includes three different measurements. The first is a determination of the divider ratio under moderately high--nominally 25 kV--direct voltage. The second is a determination of the frequency response up to 10^4 Hz under low voltage. The final measurement is a determination of the variation of the divider ratio with changes of the applied voltage level.

This report provides measurement details and results of the tests discussed above. Section II provides technical background information and a consistent vocabulary with which the test results can be discussed. Section III provides a description of the measurements which were performed and some typical measurement results. Section IV summarizes the information gained from these tests. This report also contains two appendices. The first discusses conventional and electro-optical methods of measuring the high voltage pulses, while the second is a more detailed analysis of the feasibility of electro-optic measurement of these voltage pulses. Both of the appendices were previously circulated to interested parties but were not available for public distribution.

II. Preliminary Considerations

A basic model for the properties of a voltage measurement system has been used to describe pulse dividers.[10] In this model, the divider is represented by a matrix relating the output current and voltage to the input current and voltage. Mathematically this can be represented

$$\begin{bmatrix} V_{in}(\omega) \\ I_{in}(\omega) \end{bmatrix} = \begin{bmatrix} A_{11}(\omega) & A_{12}(\omega) \\ A_{21}(\omega) & A_{22}(\omega) \end{bmatrix} \begin{bmatrix} V_o(\omega) \\ I_o(\omega) \end{bmatrix} . \tag{1}$$

The voltage transfer function, $H(\omega)$, is defined for $I_o(\omega) = 0$, i.e.,

$$H(\omega) = V_o(\omega)/V_i(\omega)\Big|_{I_o(\omega) = 0} = 1/A_{11}(\omega) . \tag{2}$$

The divider ratio, N, is defined as

$$N = A_{11}(0) = \lim_{\omega \to 0} A_{11}(\omega) . \tag{3}$$

It should be noted that all of the matrix elements in Eq. 1 are, in general, complex numbers. If the elements, $A_{ij}(\omega)$, are known, the performance of the divider is well characterized. Measurement systems are designed so that the ratio of the input to the output voltage can be represented as a single real number. The divider ratio as defined by Eq. 3 has this property and is the quantity which has traditionally been used in the description of dividers used to measure the voltage in diagnostic x-ray units. For the divider ratio to be a valid quantity, however, it is necessary that $|A_{11}(\omega)|$ not differ significantly from N over the frequency range of interest. In this report, measurements of the frequency dependence are discussed.

A second point to consider is that for dividers used to measure the voltage pulses in diagnostic x-ray units it is desirable to minimize $I_{in}(\omega)$. In other words, because of the high output impedance of the high voltage source, the high voltage level is not independent of the current. To keep the current as small as possible, the resistance of the high impedance portion of the divider is typically of order $10^8\ \Omega$. Because the divider has extremely high resistance, the effects of stray capacitance will become significant at relatively low frequency. For example, a stray capacitance of 3 pF would present an impedance of only $10^8\ \Omega$ at 500 Hz. It is, therefore, necessary to consider the effects of stray and compensation capacitance when an assessment is made of the divider performance.

The experiments reported here were designed to determine the divider ratio as defined by Eq. 3. The element $A_{11}(0)$ was determined under direct voltage and $A_{11}(\omega)$ was measured at 50, 100, 500, 1000, 5000 and 10,000 Hz. To verify the linearity of the divider and therefore the applicability of Eq. 1, experiments were also performed to verify that $A_{11}(\omega)$ had no voltage dependence, i.e., that $A_{11}(\omega)$ is independent of $V_{in}(\omega)$, and that the errors due to self-heating were negligible, i.e., $A_{11}(\omega)$ is independent of the integral $\int I_{in} \cdot V_{in}\, dt$.

One of the measuring circuits developed to perform the necessary measurements was called an active divider. The design and construction of this device are discussed briefly in Section III. The operational behavior of this device can, however, be discussed in terms of the formalism presented above. As can be seen from Eqs. 1-3, the definition of the conventional divider ratio is predicated upon the assumption that $I_o(\omega) = 0$. In practice it is usually sufficient for $I_{in}(\omega) \gg I_o(\omega)$. This criterion puts conflicting demands upon the divider. As was noted earlier, it is necessary to minimize $I_{in}(\omega)$. To have a well defined divider ratio, however, it would be desirable to have a large $I_{in}(\omega)$ so that the divider ratio would be relatively independent of the input impedance of the measuring device attached to the output terminals of the divider. To accomplish these conflicting requirements, active divider circuits were constructed. An active divider is simply a conventional divider arranged so that the high impedance element of the divider serves as the input to an operational amplifier and the low impedance element serves as the amplifier feedback impedance. In terms of Eq. 1, this circuit configuration makes $A_{12}(\omega) = 0$ and $A_{22}(\omega) = 0$. Hence the voltage transfer ratio and the divider ratio are decoupled from the output current.

III. Performance Characteristics of Commercially Available Resistor Dividers

The performance of sixteen resistor dividers was evaluated. The evaluation consisted of experimentally determining operating parameters under four different conditions:

a) measurement of self-heating under high direct voltage

b) measurement of the divider ratio

c) determination of the frequency response for frequencies up to 10,000 Hz

d) determination of the magnitude of any variation in the high impedance element with a change in applied voltage.

The effect of self-heating was determined using a standard Wheatstone bridge circuit, Fig. 1. In this figure, R_x represents the high

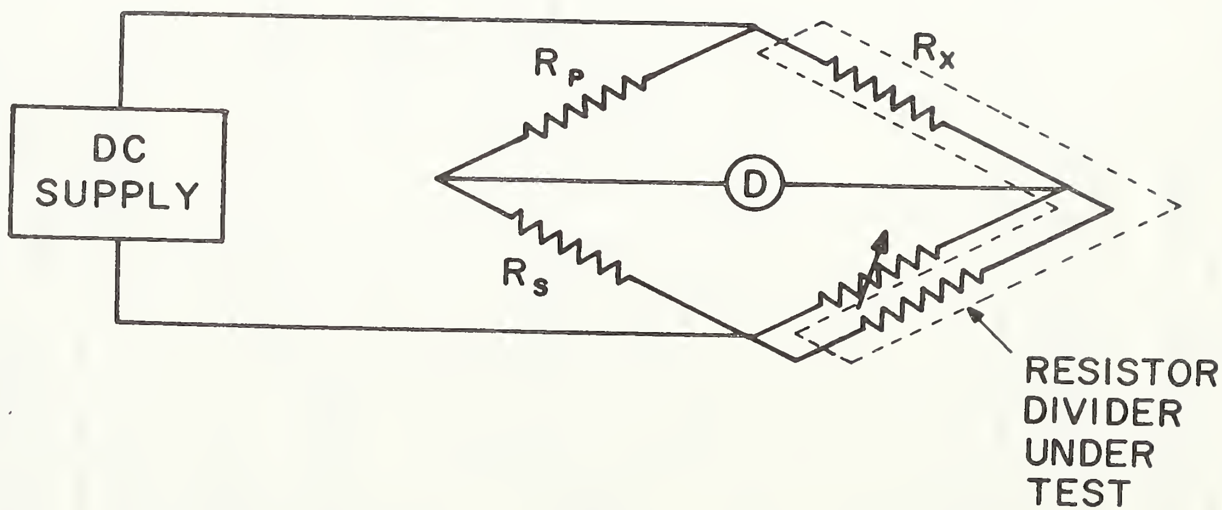

Fig. 1. Circuit used during the self-heating tests. The two resistors inside the dashed lines represent the divider under test. R_p and R_s are respectively a stable high voltage and a stable low voltage resistor.

impedance arm of the divider under test and the other high voltage element, R_p, is a well characterized, stable, high voltage resistor.[11] Measurement procedure was to apply a direct voltage of about 25 kV to the bridge. Every fifteen seconds the variable resistor was adjusted for a null indication on the detector. Using the value of the known resistors, the value of R_x was calculated. Typical values of the change of resistance with time are shown in Fig. 2. The two plots are for two different dividers of similar construction from the same manufacturer. Over a five minute interval the change in resistance was only about 80 ppm. This change is in most cases negligible because the self-heating of the x-ray tube would require the unit to be turned off before the variation of the resistance due to self-heating could become a factor. For this reason, more detailed investigations of the thermal characteristics of the divider were not undertaken.

Although testing of self-heating was performed at high voltage levels, the ratio under direct voltage was measured at a low applied voltage, nominally 125 V. Both the applied voltage and the voltage across the low impedance arm of the divider were measured using digital voltmeters. From these meter readings, the ratio was calculated.

The frequency dependence of the divider ratio was determined using the circuit shown in Fig. 3. The inductive voltage divider is a seven-decade device, the division ratio of which is characterized by an uncertainty of order one part per million of the full scale reading. The oscillator voltage is of order 10 V rms, so the output from the resistor divider is of order 10^{-2} - 10^{-3} V. The voltmeter was a vector voltmeter used in the differential input mode. The input impedance for each channel is rated at 10^8 Ω shunted by no more than 20 pF. To the measurement accuracy required, this input impedance is sufficiently large so that the measuring device is an insignificant perturbation on the remainder of the circuit. The vector voltmeter is used to monitor the magnitude of two components of the difference between the output voltages of the resistor and inductive dividers. The applied voltage also provided the reference frequency for the vector voltmeter. The meter was adjusted so that one of the measured components was in phase with the applied voltage. The other component was thus in quadrature. The voltage transfer function was determined by adjusting the inductive divider so that the "in phase" component of the applied voltage, as detected by the vector voltmeter, was zero. The ratio of the divider under test is read from the setting of the inductive divider. The magnitude of the quadrature component is measured. In all dividers tested, this voltage was sufficiently small that it was not a significant contribution to the magnitude of the voltage transfer function. To gain insight into the characteristics of this measurement system, it is helpful to analyze an approximate, simple representation of the resistor dividers used, Fig. 4.

The high resistance element is R_H and it is assumed that there is an effective parallel capacitance, C_H. The low impedance element is also modeled as a parallel combination of a capacitor, C, and a resistor, R. For this combination the voltage transfer function is given by the expression

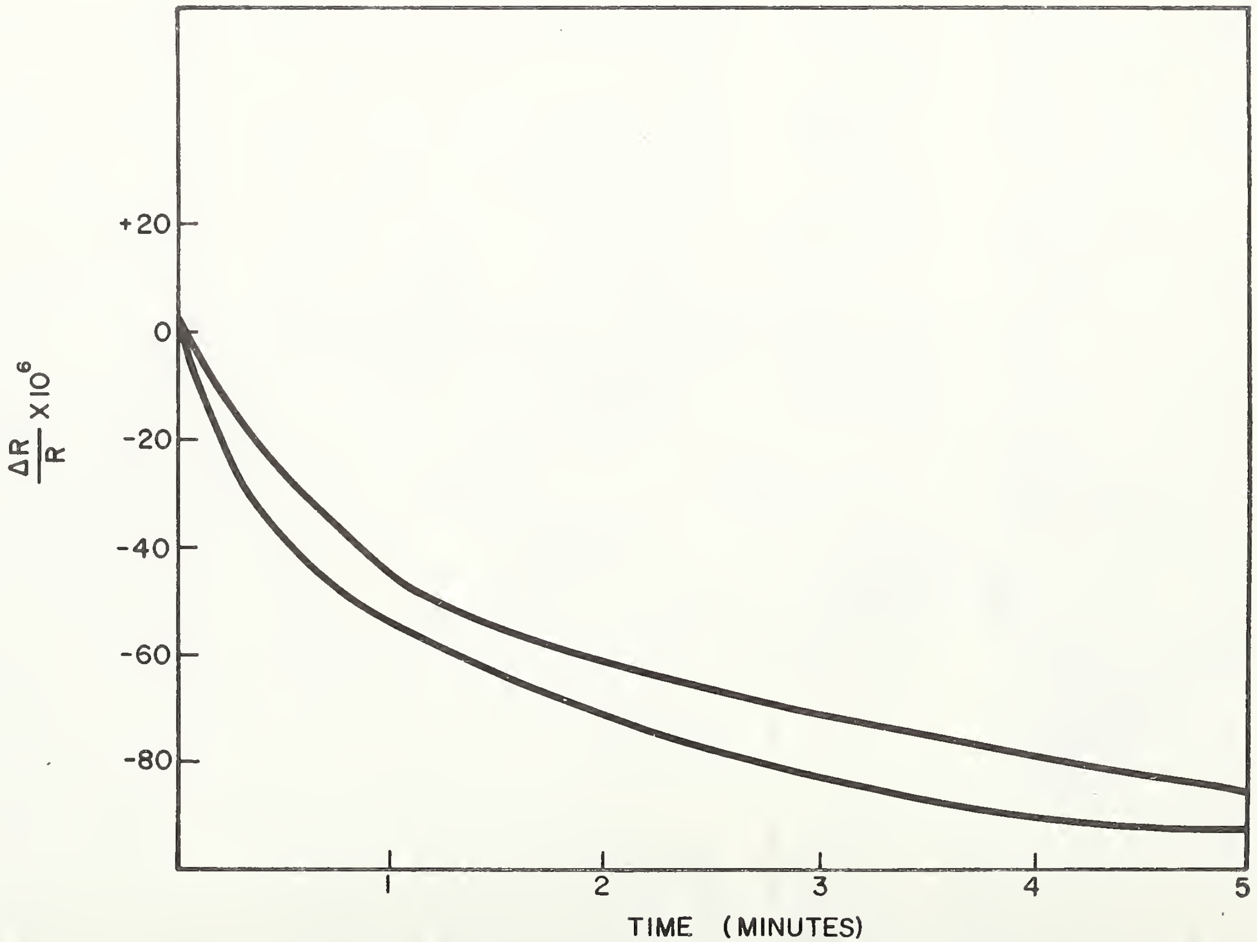

Fig. 2. The change in the value of the high resistance element of the divider under test at a nominal applied voltage of 25 kV.

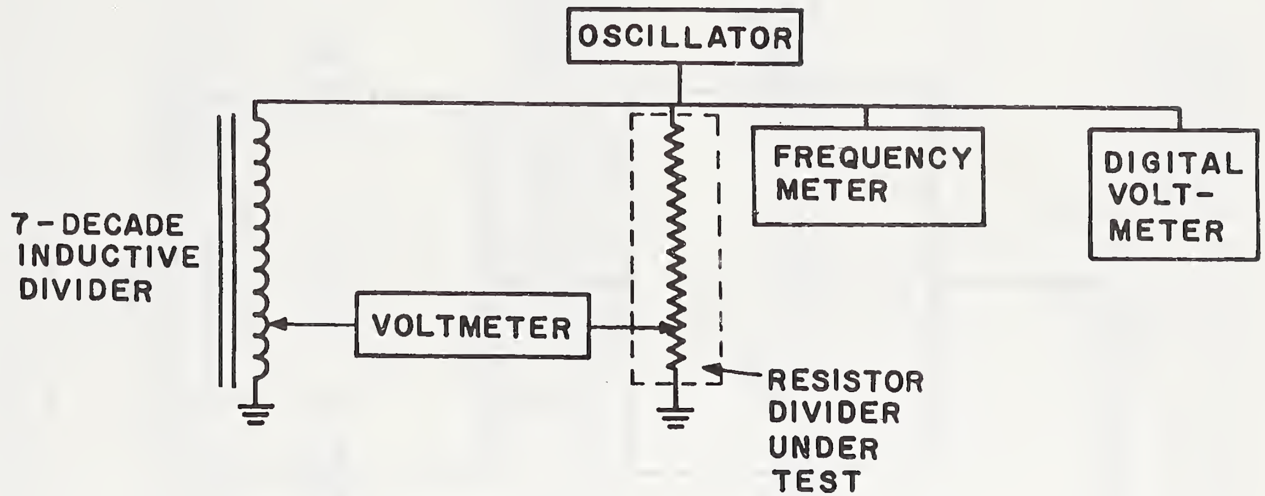

Fig. 3. Schematic representation of the apparatus used to determine the frequency dependence of the voltage transfer function of the divider under test.

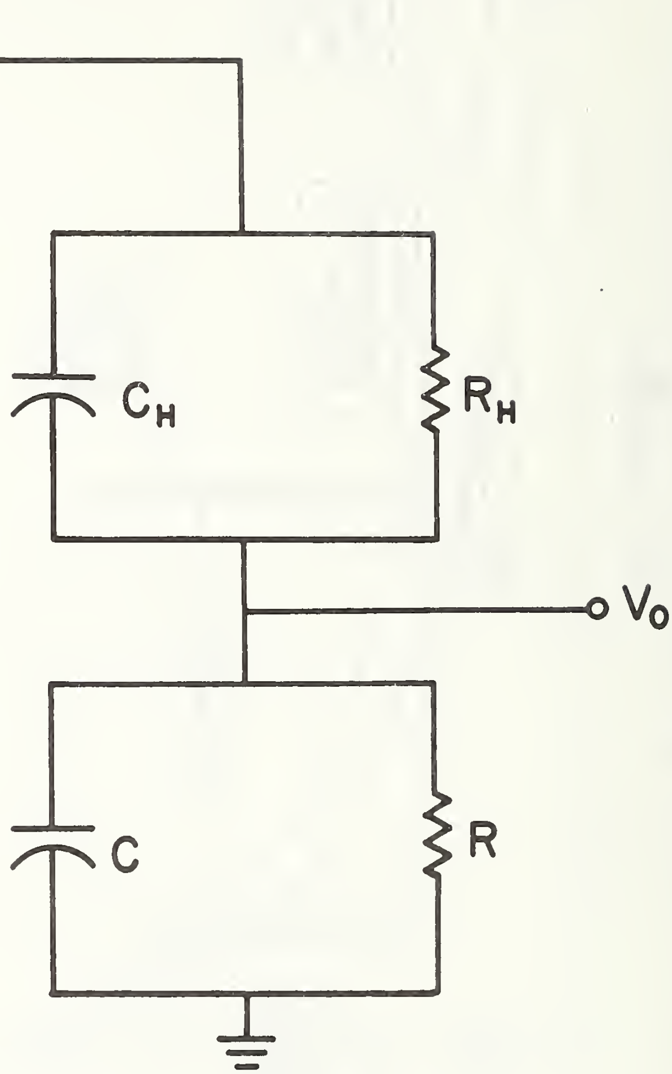

Fig. 4. Approximate equivalent circuit of dividers.

$$V_o/V = \frac{\frac{1}{R_H}\left(\frac{1}{R} + \frac{1}{R_H}\right) + \omega^2 C_H (C + C_H) + j\omega \left(\frac{C_H}{R} - \frac{C}{R_H}\right)}{\left(\frac{1}{R} + \frac{1}{R_H}\right)^2 + \omega^2 (C + C_H)^2}, \tag{4}$$

which is of the form,

$$V_o/V = \alpha + j\beta ,$$

where α represents the "in-phase" and β the quadrature component. For the measurements taken $\beta \neq 0$, that is

$$RC \neq R_H C_H , \tag{6}$$

or, verbally, the dividers were not perfectly compensated. It should be noted that in no case was β sufficiently large so that the quadrature component caused an error of 1%. This can be expressed mathematically by the following inequality

$$1 - \left(\frac{\alpha^2}{\alpha^2+\beta^2}\right)^{1/2} < 0.01 . \tag{7}$$

It should also be noted that in only a few of the dividers tested did ratio, as a function of frequency, have the functional form given in Eq. 4. To accurately account for the measured frequency response of the other dividers would require a more complex equivalent circuit.[12]

It is appropriate to emphasize that the definition of the divider ratio, Eq. 3, is consistent with accepted practice[4] of calibrating these dividers by determining the resistance ratio. From Eqs. 3 and 4,

$$N = \lim_{\omega \to 0} A_{11}(\omega) = (R_H + R)/R . \tag{8}$$

So in this case, the divider ratio is indeed the resistance ratio. The investigation of the frequency response was performed to verify that this ratio is valid over a sufficiently wide frequency range.

The measured values of $|A_{11}(\omega)|$ for a number of dividers at selected frequencies are presented in Table 1. The estimated uncertainty of the measurements of $|A_{11}(\omega)|$ was ±1% of the indicated value up to 5000 Hz and ±5% at 10,000 Hz. The quantity actually measured was the voltage transfer function--a complex quantity. To facilitate comparison with the divider ratio, the reciprocal of the magnitude of this quantity is presented in Table 1.

The final set of experiments was designed to provide an experimental value for any voltage dependence of the magnitude of high impedance element. The measurement circuit is shown schematically in Fig. 5. Because the dividers under test were not designed to withstand continuous full rated voltage, it was necessary to use pulsed high voltage to determine the voltage dependence. The circuitry employed a high voltage transformer powered from a variable autotransformer. A solid-state contactor between the two permitted the application of an integral number of half-cycles of the 60 Hz waveform to the input of the high voltage transformer.

The high voltage output of the transformer was measured using both the resistor divider under test and a high voltage divider having a compressed gas capacitor as the high voltage element. A compressed gas capacitor was chosen because this type of device has voltage induced variations of capacitance which are typically of order 50 ppm over the rated voltage range.[13] Tests on the individual capacitor used indicated that its variation was of this order.

The measurement procedure was to apply the same high voltage signal to the capacitor divider and the divider under test. The output voltages of the two peak detect and hold modules were then recorded for each setting of the high voltage over the range of interest. Fig. 6 shows a typical voltage signal from the capacitor divider and the response of the peak detect and hold circuit to that signal.

Using these measured voltages a least-squares linear fit was performed between the two low voltage outputs, that is, the coefficients in the expression

$$V_x = \alpha_o + \alpha_1 V_{CG} , \tag{11}$$

Table 1: Summary of the Frequency Dependence of $|A_{11}(\omega)|$ of the Devices Studied

DIVIDER	FREQUENCY	$\lvert A_{11}(\omega)\rvert$
A	0	952:1
	50	983:1
	100	1,030:1
	500	1,060:1
	1,000	1,070:1
	5,000	1,100:1
	10,000	1,100:1
B	0	9,970:1*
	50	9,960:1
	100	9,960:1
	500	10,000:1
	1,000	10,000:1
	5,000	9,760:1
	10,000	8,740:1
C	0	9,970:1
	50	9,970:1
	100	9,970:1
	500	10,000:1
	1,000	10,000:1
	5,000	9,300:1
	10,000	7,670:1
D	0	9,940:1
	50	9,940:1
	100	9,950:1
	500	9,980:1
	1,000	9,980:1
	5,000	9,260:1
	10,000	7,750:1
E	0	2,160:1
	50	2,090:1
	100	2,000:1
	500	1,900:1
	1,000	1,900:1
	5,000	1,900:1
	10,000	1,861:1
F	0	1,940:1
	50	1,940:1
	100	1,900:1
	500	1,880:1
	1,000	1,880:1
	5,000	1,880:1
	10,000	1,840:1

Table 1: (continued)

DIVIDER	FREQUENCY	$\lvert A_{11}(\omega)\rvert$
G	0	1,000:1
	50	1,010:1
	100	1,010:1
	500	1,160:1
	1,000	1,340:1
	5,000	853:1
	10,000	695:1
H	0	1,000:1
	50	1,010:1
	100	958:1
	500	1,150:1
	1,000	1,240:1
	5,000	825:1
	10,000	691:1
I	0	10,000:1*
	50	10,000:1
	100	10,000:1
	500	10,000:1
	1,000	10,000:1
	5,000	9,960:1
	10,000	8,810:1
J	0	952:1
	50	1,050:1
	100	1,140:1
	500	3,840:1
	1,000	3,430:1
	5,000	2,480:1
	10,000	2,240:1
K	0	---
	50	7,690:1*
	100	5,350:1*
	500	3,210:1*
	1,000	3,120:1*
	5,000	3,690:1*
	10,000	4,590:1*

*The uncertainty characteristic of these measurements was not determined. It is unlikely, however, that the uncertainty which characterizes these is significantly larger than the uncertainty in the other measurements.

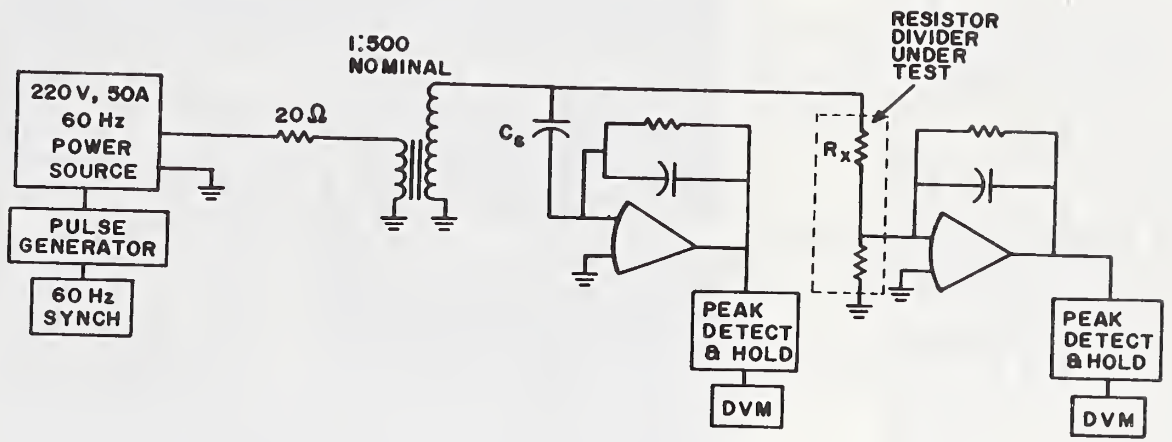

Fig. 5. Schematic representation of apparatus used to determine any voltage induced variation of the divider ratio.

were determined. In Eq. 11, V_x is the output voltage of the divider under test and V_{CG} is the output voltage of the capacitor divider. Typical results are shown in Table 2.

A final point concerning the measurement circuit should be mentioned. In order to match the output impedance of the divider under test to the measurement circuit, an active low side for the divider was constructed as was indicated schematically in Fig. 5. The low impedance of this active divider is the feedback impedance for an operational amplifier while the high impedance element of the divider under test serves to define the input current to the amplifier. The ratio of the applied voltage to the output voltage is the negative of the ratio of the high impedance to the feedback impedance. Because of the feedback configuration, the output voltage, and hence the voltage transfer function, is relatively independent of the current drawn by any instrumentation used to measure the low voltage signal.

IV. Conclusions

The investigation of these devices yielded substantial insight into the practical behavior of resistor dividers used to measure the voltage pulses in diagnostic x-ray units. This insight prompted some modification of the calibration procedure for subsequent dividers.

The most surprising result was that, to within ±0.5%, the divider ratios were voltage independent over the range of voltage from 10 volts to 75,000 volts. Because individual components in the divider are specified with somewhat larger voltage coefficients than would be reflected by this measured change, it is recommended that the investigation of the voltage dependence of the ratio be included in future calibrations. An additional consistency check that has been added is a direct voltage calibration at 25 kV. This will serve as a check on the operation of the voltage-dependence measurement circuit, Fig. 5.

Although most dividers were adequately compensated, some had unacceptably large variations of the voltage transfer function with a change in frequency. With the limited number of devices tested, there was no identifiable correlation between divider manufacturer or divider design and the frequency variation. It seems likely that the unsatisfactory frequency response was a result of abuse, poor quality control, or user modification rather than a flaw in design.

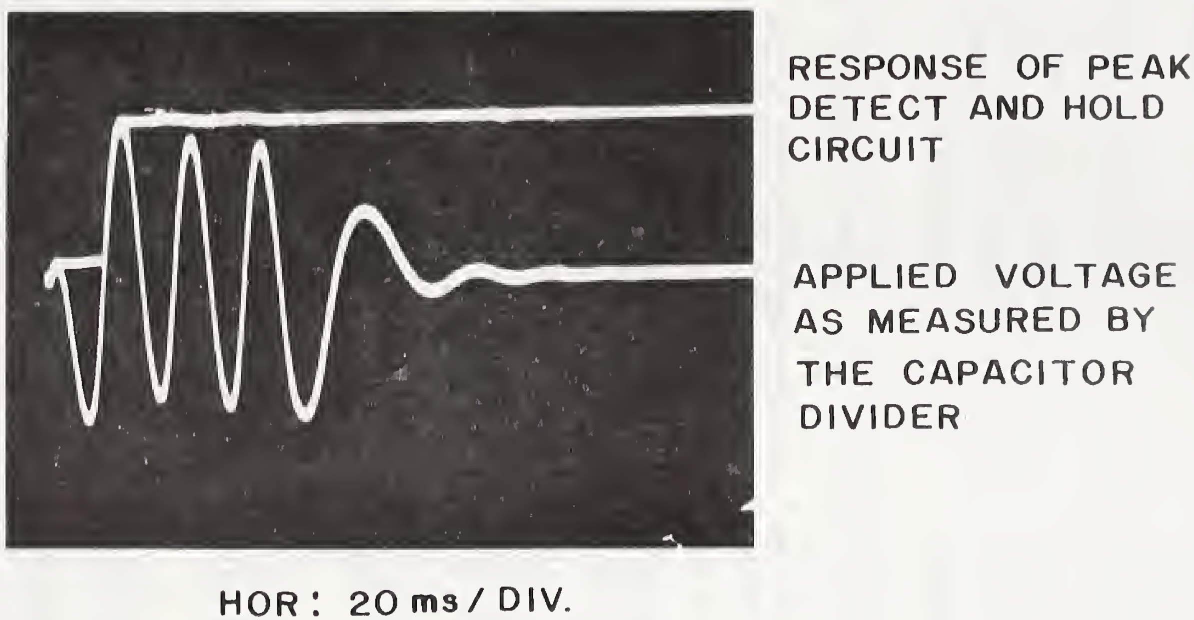

Fig. 6. Low voltage output of the capacitor divider and the response of the peak detect and hold circuit.

Table 2. Summary of the results of the determination of the variation of the high impedance element with a change in applied voltage.

V_x (volts) measured	V_{CG} (volts) measured	V_{CG} (volts) predicted
10.30	1.120	1.123
12.17	1.324	1.324
13.56	1.474	1.473
15.23	1.656	1.653
17.92	1.952	1.942
19.83	2.142	2.148
21.86	2.363	2.366
23.88	2.584	2.583
26.57	2.875	2.873
25.08	2.713	2.712
23.60	2.551	2.553
21.72	2.346	2.351
19.94	2.153	2.160
17.91	1.948	1.941
15.84	1.723	1.718
13.95	1.517	1.515
11.78	1.280	1.282
9.429	1.025	1.029

Coefficients of the fit:

α_o = 0.015 with a standard deviation of 0.004

α_1 = 0.1076 with a standard deviation of 0.0002

V. References

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VI. Acknowledgments

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APPENDIX I

"Electrical Measurement of the Pulsed Voltage in Diagnostic X-ray Units"

The material in this appendix served as the basis for a talk presented at a topical meeting on: "Measurement of kV_p of Diagnostic X-ray Units with Emphasis on Ardran and Crookes Technique", Michael Reese Hospital, Chicago, Illinois, December 1, 1974.

Electrical Measurement of the Pulsed Voltage in Diagnostic X-ray Units

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Abstract

A pulsed-voltage measurement system based on the electro-optic Kerr effect has been used to determine the magnitude of the high voltage pulses in medical x-ray machines. The purpose of this work is to provide a calibration, under operating conditions, for other voltage measurement devices. Among the advantages afforded by this approach are minimization of operator error, optical coupling of the high voltage circuitry to the detection circuitry, the ability to measure the potential difference between two points neither of which are grounded and the fact that a Kerr cell presents a very small burden to the pulse generator.

The dissemination of the calibration is accomplished through a recently developed active divider which incorporates a conventional bleeder bank (resistor divider) as the high impedance component. The use of active circuitry in the low voltage portion of the divider permits construction of a device with negligibly small output impedance. For this reason, errors due to the input impedance of the recording device and due to the impedance of connecting cables are minimized.

Preliminary operating characteristics of both the Kerr system and the active divider are presented.

Introduction

The aim of this work is to develop, to evaluate, and to disseminate techniques for the electrical measurement of high voltage pulses with an uncertainty no greater than ±1%. Two measurement methods have been developed. One is based on the electro-optic Kerr effect and the other on the use of active components to improve the measurement accuracy using a conventional divider. The remainder of this presentation will be devoted to a discussion of the measurement principles involved in these approaches including a brief discussion of the advantages and disadvantages of each technique.

Kerr Effect

To perform a measurement using the electro-optic Kerr effect, it is necessary to have a substance containing molecules with an electrical anisotropy. When a voltage is applied, these molecules will tend to align with the resulting field. The voltage measurement is made by measuring the degree of alignment. Because the molecules tend to align with the field, the refractive index of the medium in a direction parallel to the applied field is different from that in the direction perpendicular to it. To determine this difference a light beam linearly polarized at an angle of 45° to the applied field is used. This results in two equal components, one parallel to the field, one perpendicular to the field. As these two components pass through this substance, that component parallel to the field travels at a different speed from that perpendicular to the applied field. Upon passage through the substance, there is a phase shift between the two components of the light beam. The phase difference is given by

$$\phi = 2\pi \int_0^L BE^2 (x,y,t)\, dz \tag{1}$$

where B is the Kerr coefficient of the material.
If it is assumed that parallel plate geometry is used, Eq. (1) can be rewritten

$$\phi = 2\pi\, BV^2\, \ell'/d^2 , \tag{2}$$

where V is the applied voltage, d the plate spacing and ℓ' is an effective plate length which includes phase shift due to fringing fields at the electrode edges. This phase shift can be detected as a change in transmittance of the Kerr system if the light beam is then passed through a polarizer oriented at an angle of 90° to direction of original polarization. If the phase shift is given by Eq. (1), the transmittance of the system, I/I_m, is[1,2]

$$I/I_m = \sin^2 [1/2\, \pi\, (V/V_m)^2] \tag{3}$$

In this equation, V is the applied voltage, a time dependent parameter, and V_m, called the cell constant is defined,

$$V_m = d/(2B\ell')^{1/2} . \tag{4}$$

As the voltage changes, the transmittance through the system changes and Eq. (3) is the relationship between the applied voltage and the optical parameter measured. This equation is graphed in Fig. 1. When no voltage is applied there is zero transmittance through the system. As voltage is increased, a maxima is reached; as voltage is further increased, the transmittance passes through successive maximas and minimas. One point to be emphasized, which is a strength and in some cases a weakness of the Kerr effect, is that this is nonlinear. At higher field strengths, or higher applied voltage, the amount of voltage change necessary to get a 100% change in transmittance becomes smaller.

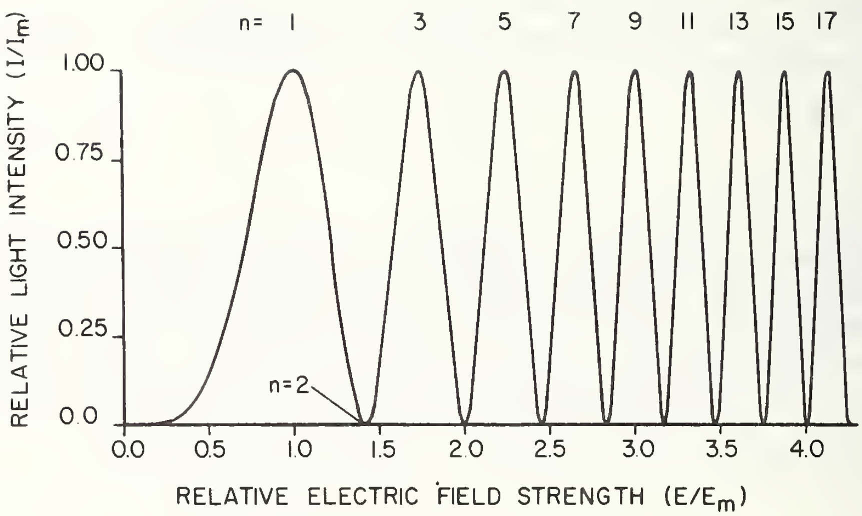

Fig. 1. Graphical representation of the relationship between the transmittance of the optical system and the applied electric field.

Because the sensitivity increases with increasing voltage, the Kerr effect is an attractive method for high voltage measurements. A schematic of the apparatus is shown in Fig. 2. The light source is usually a laser. The light beam impinges on a properly oriented polarizer and passes through a parallel-plate capacitor. This capacitor is a glass cell containing nitrobenzene. The light beam then passes through an analyzer and into the photodetector. The indicated ground connection is optional. The system measures the potential difference between the plates and does not require reference to ground. A typical observation of electric field, at an instant during the voltage pulse, is shown in Fig. 3. This figure shows a large central area of uniform transmittance indicating an approximately uniform field between plates. The fringes on either side are due to the edge fields at the sides of the electrodes. To perform a simple measurement of the voltage, a detector is placed at the center so that it records the transmittance as a function of time as shown in Figs. 4 and 5. Fig. 4 shows the simultaneous divider measurement, upper trace, and the Kerr measurement, lower trace, of the high voltage in a single phase x-ray unit. It can be seen from the Kerr measurement that the two peaks differ in magnitude, the first peak being slightly larger than the second.

A similar measurement made on a three phase unit is shown in Fig. 5. The salient feature of this photograph is the enhancement of the ripple in the applied high voltage that is evident in the Kerr response. Because the Kerr response sensitivity increases at higher voltages, the system responds preferentially to the ripple and not as identifiably to the average voltage level.

It is useful to summarize the primary advantages and disadvantages of using the Kerr effect to measure the pulsed voltage in medical x-ray units. This summary is presented in Table 1.

Table 1. The advantages and disadvantages of using the Kerr effect for the measurement of the pulsed high voltage in medical x-ray units.

Advantages	Disadvantages
High impedance	Skilled personnel
No reference to ground	Not commercially available
Optical coupling	Optimized for shorter pulses
Independent errors	

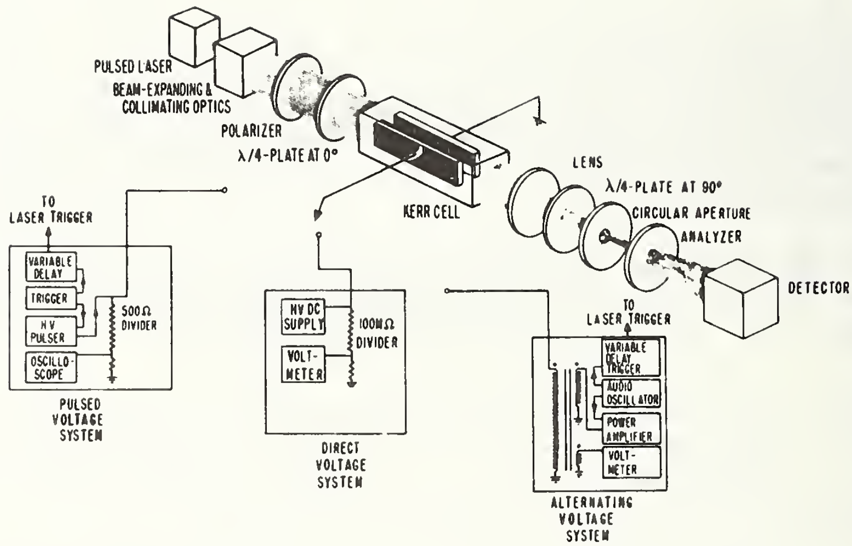

Fig. 2. Schematic diagram of both optical and electrical aspects of a measurement system based on the electro-optic Kerr effect.

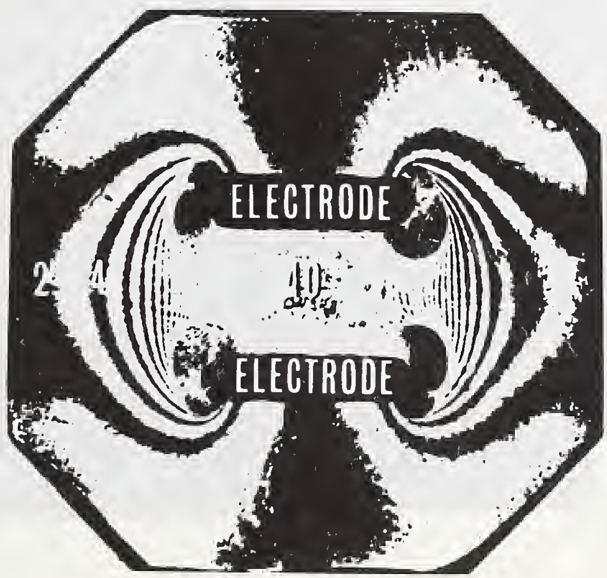

Fig. 3. Transmittance of a Kerr cell (parallel plate geometry) at an instant under the influence of a high voltage impulse. The numbers on the figure indicate ϕ/π (Eq. 1) for the spatial region in which each is located.

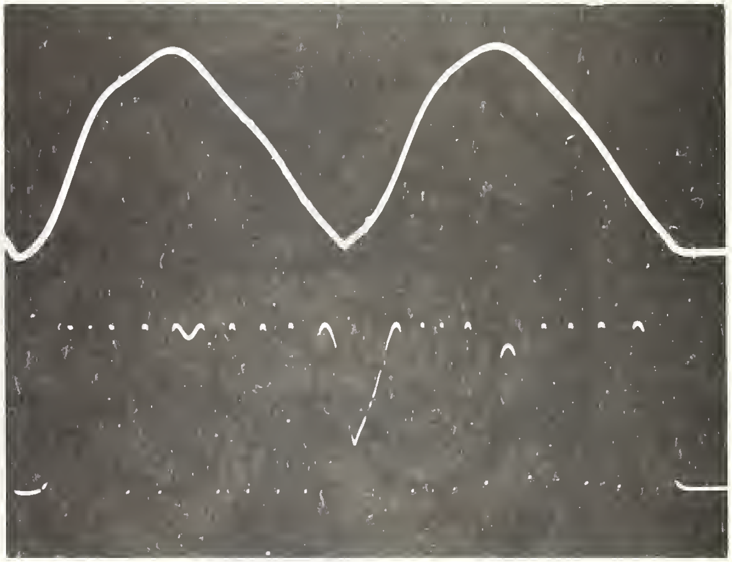

Fig. 4. Divider measurement (upper trace) and Kerr measurement (lower trace) of the high voltage in a single phase x-ray unit.

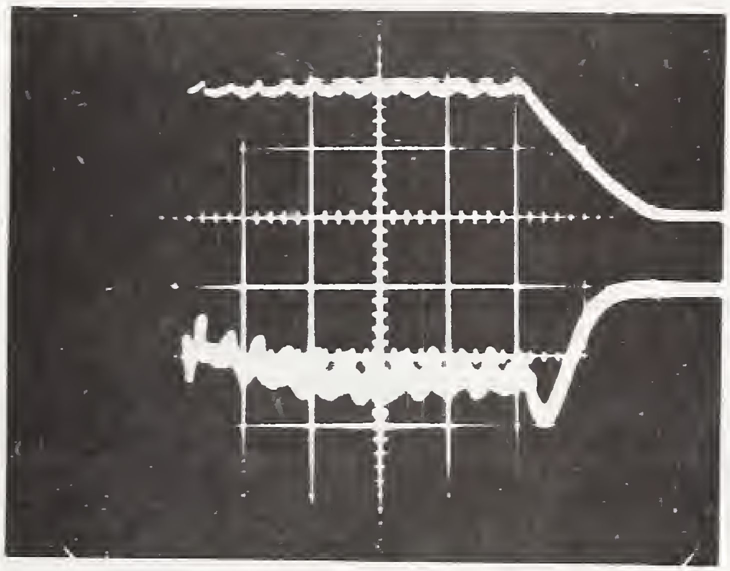

Fig. 5. Divider measurement (upper trace) and Kerr measurement (lower trace) of the high voltage in a three phase x-ray unit. Note that the ripple is enhanced in the Kerr system measurement.

The first advantage listed is the high impedance of the Kerr system. A Kerr cell is a capacitor with a capacitance of approximately 100 pF. This is the same order of magnitude as the stray capacitance in the x-ray unit. The voltage measurement device is therefore a negligible addition to the load seen by the voltage generator.

Kerr systems, as presently constructed, are essentially parallel plate capacitors. As can be seen from Eq. (1), the quantity measured is the electric field between the plates. The Kerr system then requires no reference to ground. Direct measurements of the voltage between the anode and cathode in a medical x-ray unit have been made.

Optical coupling between the high voltage circuitry and the measurement device offers the advantages both of decreased shock hazard and of the minimization of ground loop errors.

Finally the errors inherent in Kerr system measurements are different from the errors that have been identified in the measurement of high voltage using a divider. This is significant in calibration work because the applied voltage can be measured using two independent methods. If the values obtained by the two methods disagree by more than their combined uncertainties, it is likely that there is an error in one or the other. If, conversely, the two agree, it is highly unlikely that they are both in error by the same amount. In this respect the system is self-checking, thus minimizing the effect of operator error.

The use of the Kerr system is not a panacea. Among the disadvantages are that the Kerr system requires skilled personnel. It requires additional training beyond that necessary to operate an oscilloscope or a voltmeter to perform accurate measurements using a system based on the Kerr effect.

A second disadvantage is that these systems are not now commercially available. Most systems in operation today were constructed by the user and lack the user convenience features, the documentation and the availability of repair service that would accompany a commercial unit.

The final drawback is that the Kerr systems have been optimized for shorter pulses--pulses with characteristic times in the microsecond range rather than the millisecond and longer pulses that characterize medical x-ray units.

Active Divider

Fig. 6 indicates schematically a conventional measurement system using a divider. It consists of a divider with its high and low impedance segments, a low voltage measuring device and an interconnecting cable. A typical unit would consist of a commercially available bleeder-bank connected to an oscilloscope through a length of coaxial cable. The measured frequency response of such a system is shown in Fig. 7. The

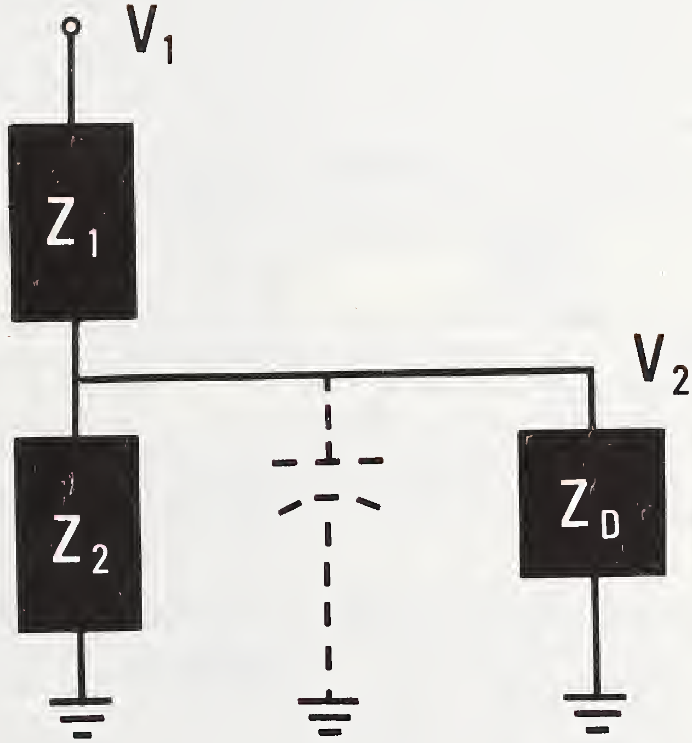

Fig. 6. Schematic representation of a conventional voltage measurement system based on a divider. The high impedance arm of the divider is Z_1, while Z_2 is the low impedance arm. The capacitor indicated by the dashed lines represents the effect of interconnecting cables and the input impedance of the voltage measuring device is Z_D. Note that these last two impedances are in parallel with the divider low side.

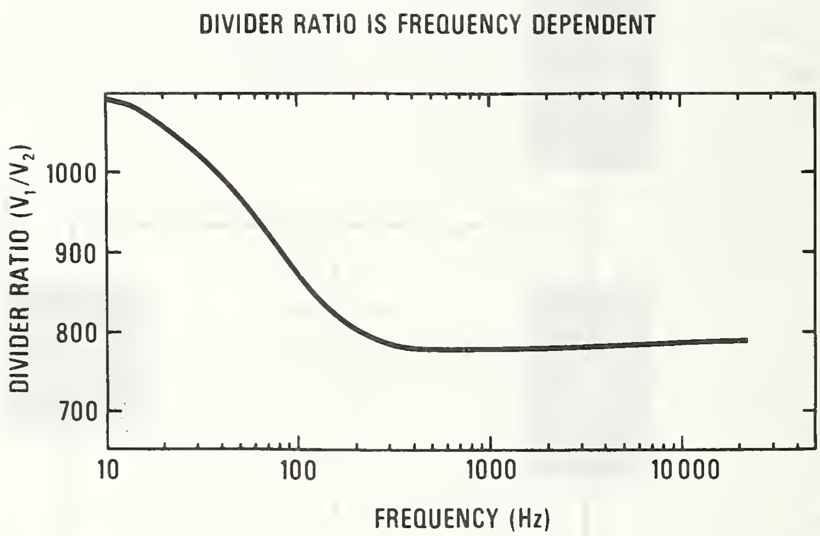

Fig. 7. The measured--at low voltage--frequency response of a commercially available divider.

details of the ratio with respect to frequency are, of course, characteristic of the particular measurement system. Other work has shown that variations of the ratio of the magnitude shown here are characteristic of a large variety of measurement systems based on bleeder banks. It is in principle possible to compensate the divider so both the high impedance segment and the low impedance segment have the same RC time constant. This approach frequently requires, however, that the low voltage measuring device and the interconnecting cable be dedicated to a specific bleeder bank as their resistance and capacitance to ground contribute to the net low-side impedance of the divider.

The technique employed here uses an operational amplifier to maintain the recognized advantages of a compensated divider while minimizing the effect of cables and the measuring device on the divider ratio. As indicated in Fig. 8, the low impedance side of the divider serves as the feedback impedance for the operational amplifier while the high impedance segment serves to define the input current. The ratio of the applied voltage to the output voltage is the negative of the ratio of the high impedance to the feedback impedance. The cable capacitance and the input impedance of the measuring device have no effect on the voltage ratio.

Fig. 9 demonstrates the validity of this approach. The upper photograph shows the step-response of the measurement system whose frequency response is plotted in Fig. 7. Using the same bleeder bank, the same cable, the same oscilloscope but an active rather than passive low-side, one obtains the step response shown in the lower photograph. In this case, the step response is faithfully reproduced indicating a system with a flat response over a sufficiently wide frequency range to reproduce the voltage pulses.

Some of the advantages and drawbacks of this approach are shown in Table II. The advantages include low output impedance so that the impedance of the cables and measurement instrumentation introduces negligible error in the voltage measurement. The divider's input impedance is determined by the bleeder bank. Thus, no unconventional components are required in the high voltage circuit. The final advantage is that the active low side uses commercially available components, i.e., resistors, capacitors, and operational amplifiers.

As with all measurement techniques there are disadvantages. First the reliability of this device for this specific application is unproven. Similar systems have, however, proven reliable for the measurement of 60 Hz voltages up to 200 kV. The second disadvantage is that to the author's knowledge, an active low side is not commercially available.

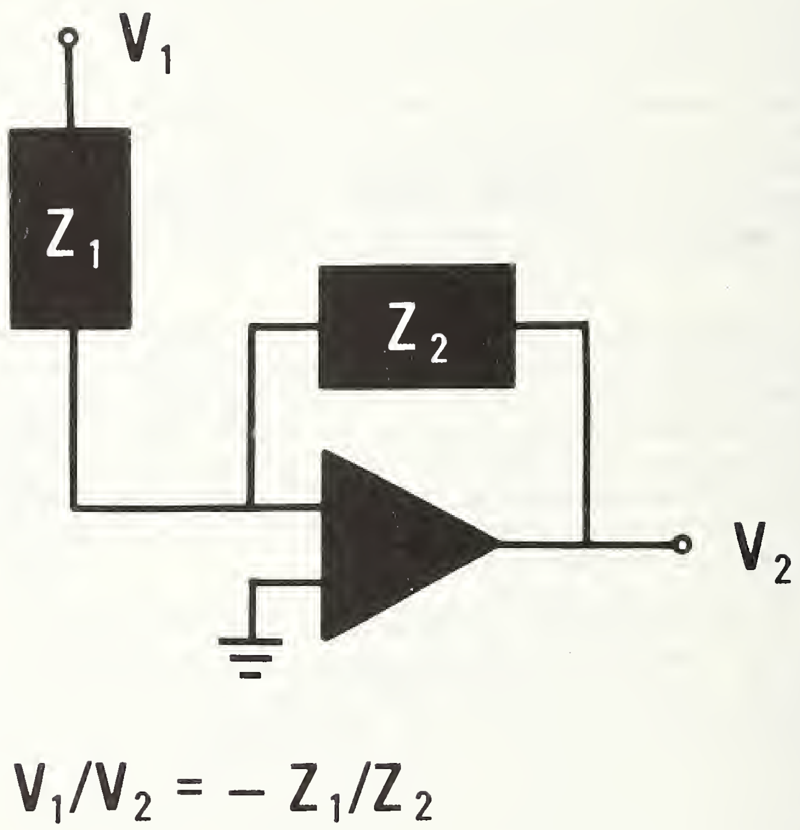

Fig. 8. Schematic representation of an active divider. Note that the divider low side, Z_2, serves as the feedback impedance for an operational amplifier and is no longer in parallel with any measuring devices.

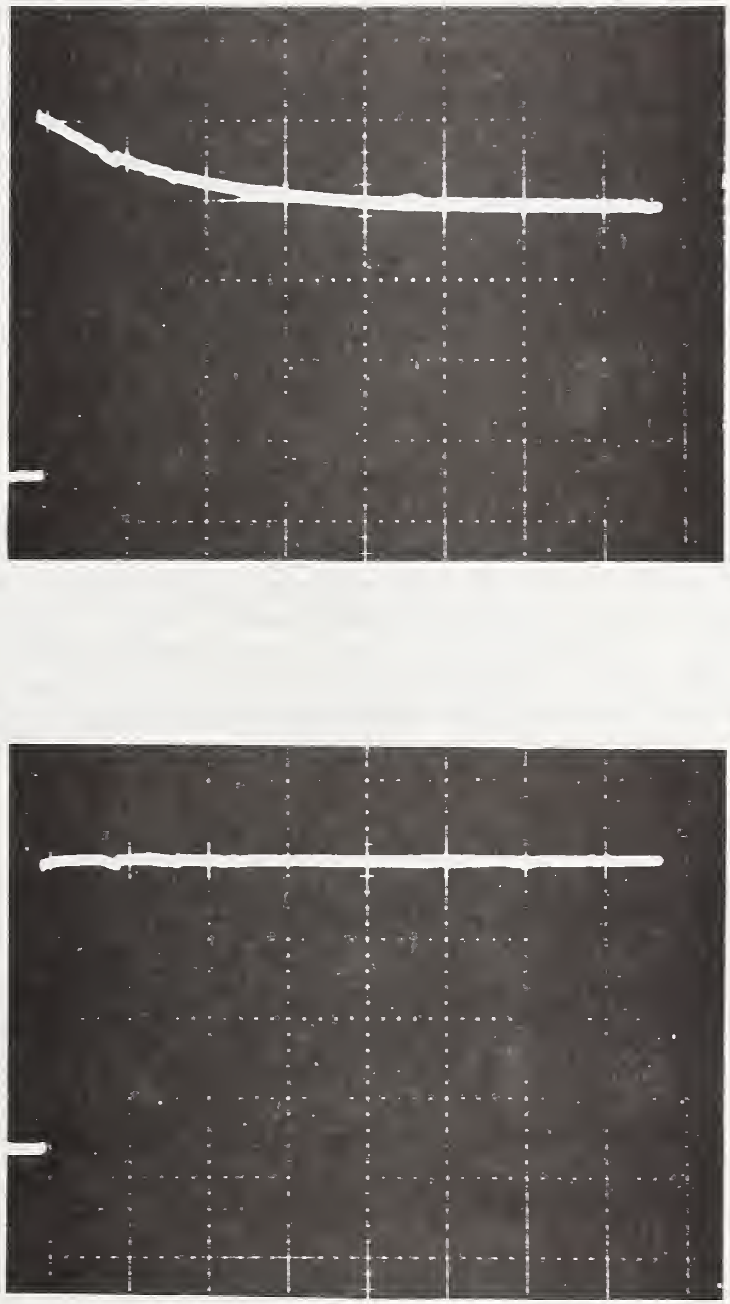

Fig. 9. The upper photo is the step response of a commercially available bleeder bank (vertical scale: V_2 in arbitrary units; horizontal scale: time with units 1 ms/division). Note that there is an overshoot as is predicted by the frequency response measurements in Fig. 7. The lower photo shows the response of the same system when an active low side is used.

Table 2. The advantages and disadvantages of using an active divider for the measurement of the pulsed high voltage in medical x-ray units.

Advantages	Disadvantages
Low output impedance	Reliability unproven
High input impedance	Not commercially available
Uses conventional components	

Summary

Two approaches have been used to electrically measure the high voltage pulses in medical x-ray units. The first is an optically coupled measurement based on the electro-optic Kerr effect. The other is an active divider which uses operational amplifiers to improve the voltage measurement using a conventional bleeder bank. Both of these approaches are capable of performing voltage measurements with an uncertainty no larger than ±1%. The present work centers on the more critical evaluation of how well the measurement techniques can be transferred from our laboratory to the people who are interested in using them.

References

1) R. E. Hebner and E. C. Cassidy, "Measurement of 60 Hz Voltages Using the Kerr Effect" Rev. Sci. Instru., Vol. 43, pp. 1839-1841, 1972.

2) E. C. Cassidy, R. E. Hebner, R. J. Sojka and M. Zahn, "Kerr Effect Studies of an Insulating Liquid Under Varied High-Voltage Conditions" IEEE Trans. Elec. Insul., Vol. EI-9, pp. 43-56, 1974.

APPENDIX II

Feasibility of Using the Electro-optic Kerr Effect for the Measurement of the High Voltage Pulses in Diagnostic X-ray Units

This feasibility study was prepared for the Bureau of Radiological Health and served as the basis for some of the experimental work reported in Appendix I.

Introduction

During the past several years, the Electricity Division of the National Bureau of Standards has been active in the development of high-voltage pulse measurements based on the electro-optic Kerr effect. As a result of this work the electro-optic Kerr effect is used, in conjunction with more conventional techniques, in the calibration of dividers used to measure high voltage pulses. The majority of this work has been directed toward the measurement of nearly rectangular pulses with a risetime of approximately 1 μs, a duration of less than 50 μs and a peak value of up to 300 kV. At the present time, work is continuing both at NBS and in other governmental and industrial laboratories to extend this work to higher voltages and to both longer and shorter pulses.

The primary advantages that have been identified are described in the following paragraphs.

The first advantage is that sources of error in Kerr measurements are different from those in divider measurements. This is significant in calibration work because the applied voltage can then be measured by two independent methods. If values obtained from the two methods disagree by more than their combined uncertainties, it is likely that there is an error in one or the other measurements. If, conversely, the two agree, it is highly unlikely that they are both in error by the same amount. In this respect, the system is self-checking thus minimizing the effects of operator error.

Secondly, the detection circuitry is optically connected to the high voltage circuitry. In addition to reducing ground current error, which falls under the first advantage, this buffers sensitive detection circuitry from the energy contained in the high voltage circuit. This permits the use of a variety of high speed solid-state devices for detection and data handling. These devices have low tolerance to high voltage or high current pulses and therefore they have not been generally accepted in connection with conventional pulse measurement techniques.

A third advantage is that voltage measurements using a Kerr cell do not have to be referenced to ground.[1] This technique is thus ideally suited to the measurements of large potential differences between two points both of which are at high voltage.

The final advantage is that the Kerr cell is a capacitor with a total capacitance typically in the range from less than one hundred picofarads to a few hundred picofarads. This means that the Kerr cell represents a burden which is at worst as small as and typically much smaller than that represented by a conventional divider.[1] Under some conditions, e.g., when a high voltage source with high output impedance provides a pulse to a high impedance load, the Kerr cell provides minimum perturbation on the high voltage pulse shape.

The two primary disadvantages are, first, that the existing systems have been designed and constructed to optimize response over a fairly narrow range of pulse shapes so to apply these techniques to a significantly different pulse requires further development. And second, pulsed-voltage measurement systems based on the electro-optic Kerr effect are not, as yet, commercially available. Therefore the technical support, warranty, etc., usually associated with commercial products are also not available.

The purpose of this study is to estimate, on the basis of existing development work, the accuracy expected of a Kerr-effect pulse-voltage measurement system when it is used to measure two different types of pulsed high voltage. The first (type A) is a train of pulses made up of an integral number of half-cycles of a 60 Hz sine wave in the form typical of the unfiltered output of a full wave rectifier. The second (type B) is an approximately rectangular pulse of risetime less than 1 ms and duration of 1 ms to 6 s.

Results from preliminary studies show a worst case error of approximately 10% for the first few half-cycles of a type A pulse and an uncertainty of order 1% for any remaining half-cycles of a type A pulse and for type B pulses. These estimates are of course preliminary and may not reflect the uncertainty characteristic of the final device. The method of arriving at these uncertainty estimates and proposed techniques for reducing the uncertainties constitute the remainder of this report.

Theory

In this section the basic principles of the Kerr effect are reviewed. This section is intended to provide a brief introduction to the vocabulary and equations necessary for a discussion of the proposed measurement system. A schematic of a Kerr system is shown in Fig. 1.

The voltage measurement is based on the fact that molecules with an electrical anisotropy, e.g., non-zero dipole moment, will tend to align with an applied electric field. The degree of alignment is then a measure of the applied field. In order to measure the degree of molecular alignment it is further necessary that the index of refraction of the medium have one value, $n_{\parallel}$, in a direction parallel to the applied field and a different value, $n_{\perp}$, in a direction perpendicular to the field. The quantity $n_{\parallel} - n_{\perp}$ is given by

$$n_{\parallel} - n_{\perp} = \lambda B E^2 . \tag{1}$$

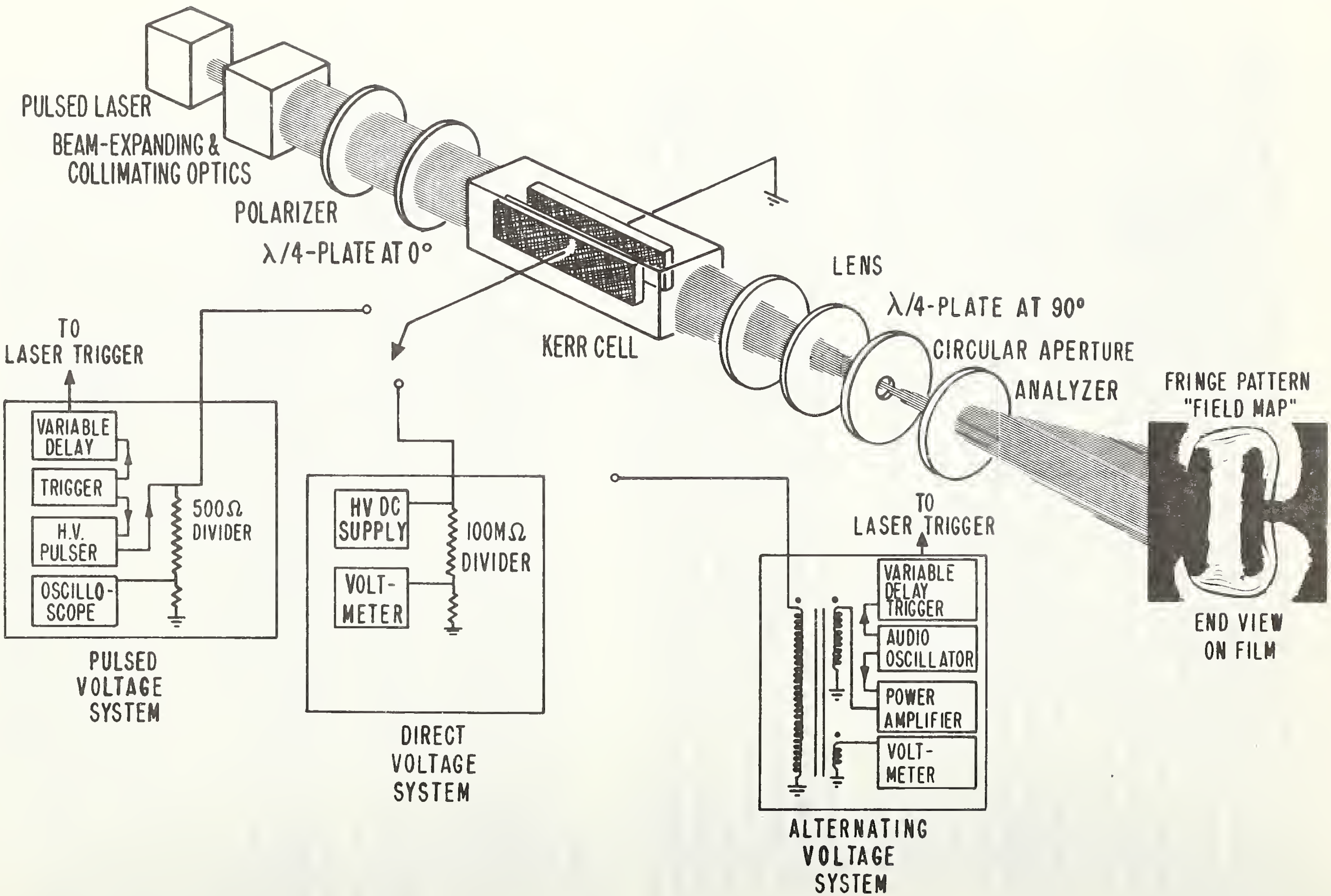

Fig. 1. Schematic diagram of the apparatus used to measure the electric field distributions in nitrobenzene. Typical Kerr effect fringe pattern is shown on the right of the drawing. The systems used to supply and measure the applied voltage are also shown.

In this expression, B is the Kerr constant of the material, λ is the wavelength of the incident light and E denotes the applied field. In the course of this discussion, it is necessary to consider two different electric fields. One is the field applied to the plates of the cell and this will be denoted by the symbol, E. The other is the electric field of the light beam. This field will be denoted by the symbol, F. For the remainder of this discussion a coordinate system will be used in which the light beam propagates in the z-direction and the applied field E is in the x-direction.

Eq. 1 can then be rewritten

$$n_x - n_y = \lambda B E^2 \ . \tag{2}$$

In general, the electric field of the light beam can be written in terms of two component polarizations

$$F_x = F_{ox} \sin (kz + \omega t)$$

$$F_y = F_{oy} \sin (kz + \omega t + \phi_y) \tag{3}$$

where ϕ_y is the phase difference between F_x and F_y. For linearly polarized light ϕ_y is zero.

To predict the response, the phase shift between F_x and F_y as the beam propagates through the system must be calculated.

The light is not shifted in frequency as it propagates through the cell, but the wavelength in the cell differs for the two polarizations:

$$k_x = n_x k$$

$$k_y = n_y k$$

where k is the free-space wavenumber, $k = \frac{2\pi}{\lambda}$. So, the phase shift, $\Delta\phi$, for linearly polarized light after propagation a distance Δz is given by

$$\begin{aligned} \Delta\phi &= (n_x k \Delta z - \omega t) - (n_y k \Delta z - \omega t) \\ &= k\,(n_x - n_y)\,\Delta z \end{aligned} \tag{4}$$

because we are to detect both beams at the same time, t. The total phase shift after propagating a distance L is given by

$$\phi = \frac{2\pi}{\lambda} \int_0^L (n_x - n_y)\, dz \; . \tag{5}$$

Using Eqs. 2 and 5, we then obtain

$$\phi = (2\pi/\lambda) \int_0^L \lambda B E^2 dz \; .$$

Making explicit possible x, y, z and t dependences of E, we obtain

$$\phi(x,y,t) = 2\pi \int_0^L B E^2\,(x,y,z,t)\, dz \; . \tag{6}$$

To use the cell as a voltage measuring device, we will assume that the light beam is an unexpanded laser beam, diameter ≃ 2mm, which propagates in the z-direction in the center of the cell. Previous experimental results have shown that the error in locating the central region is negligible with respect to other errors, so Eq. (6) can be simplified

$$\phi(t) = 2\pi \int_0^L B E^2\,(z,t)\, dz \; . \tag{7}$$

In order to use the electro-optic Kerr effect for voltage measurement, it is necessary to further manipulate Eq. 7. The aim is to calculate the transmittance of our entire optical system in terms of $\phi(t)$. A very convenient method of doing this is to use the Mueller matrices[2] of the components in the optical path. It is assumed that the initial light beam is unpolarized

$$[\Lambda_i] = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \; .$$

This beam then impinges upon a polarizer oriented at 45° to the y-axis. The matrix for this polarizer is

$$[M_1] = \tfrac{1}{2} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The beam then passes through a 90° retarder oriented at 0° with respect to the y-axis. The matrix for this retarder is

$$[M_2] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

The Kerr cell can then be represented as a retarder which causes a phase shift ϕ and is oriented at an angle β with respect to the y-axis. The appropriate matrix for this type device is

$$[M_3] =$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\beta + \sin^2 2\beta \cos\phi & \cos 2\beta \sin 2\beta\,(1-\cos\phi) & \sin 2\beta \sin\phi \\ 0 & \sin 2\beta \cos 2\beta\,(1-\cos\phi) & \sin^2 2\beta + \cos^2 2\beta \cos\phi & -\cos 2\beta \sin\phi \\ 0 & -\sin 2\beta \sin\phi & \cos 2\beta \sin\phi & \cos\phi \end{bmatrix}$$

The beam then passes through a 90° retarder oriented at 90°. This matrix is

$$[M_4] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Finally the beam exits through a second polarizer oriented at -45° to the y-axis

$$[M_5] = \tfrac{1}{2} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The output light beam, $[\Lambda_o]$, is then given by the equation

$$[\Lambda_o] = [M_5]\,[M_4]\,[M_3]\,[M_2]\,[M_1]\,[\Lambda_i]\ . \tag{8}$$

Performing the indicated multiplication, one obtains

$$[\Lambda_o] = \tfrac{1}{2} \begin{bmatrix} \sin^2 \tfrac{1}{2}\phi \\ 0 \\ -\sin^2 \tfrac{1}{2}\phi \\ 0 \end{bmatrix}$$

The implication of this matrix is that if the source intensity is I_o, the intensity of the beam upon passing through the complete optical system is given by the relationship

$$I = \tfrac{1}{2} I_o \sin^2(\tfrac{1}{2}\phi). \tag{9}$$

Furthermore the emergent light beam is linearly polarized at an angle of $-45°$. In the development of Eq. 9, it was assumed there was no attenuation. We can relax this requirement somewhat by rewriting (9) in terms of the maximum transmitted irradiance, I_m. That is

$$I = I_m \sin^2(\tfrac{1}{2}\phi) .$$

This equation relates the transmittance of the system, I/I_m to the phase shift caused by the electro-optic Kerr effect, ϕ . It now remains to relate the phase shift to the applied voltage. To do this, Eq. 7 is rewritten

$$\phi(t) = 2\pi \int_o^L BE^2 (z,t)\, dz = 2\pi B\ell' \frac{V^2}{d^2} \tag{10}$$

where d is the plate spacing, V is the applied voltage, ℓ' is the effective length of the plates, and B is the Kerr constant of the liquid. The effective length of the plates is defined as the length of the plates of an ideal parallel plate capacitor, i.e., no fringing fields, which would cause the same phase shift as the Kerr cell if the same voltage were applied. The quantity ℓ' is not a separately measurable quantity but to within the accuracy necessary depends only on the geometry of the Kerr cell. It is convenient to define the quantity, V_m, where

$$V_m = \frac{d}{\sqrt{2B\ell'}} \quad . \tag{11}$$

Then,

$$\phi = \pi \left(\frac{V}{V_m}\right)^2 \quad .$$

Inserting this into Eq. 10, yields

$$I/I_m = \sin^2 [\tfrac{1}{2}\pi (V/V_m)^2] \tag{12}$$

which is the desired relationship between the transmittance of the Kerr system and the applied voltage. Eq. (12) explicitly indicates the physical significance of the quantity V_m. It can be seen that when $V = V_m$, $I/I_m = 1$. So, V_m is the smallest applied voltage necessary to cause a maximum in the transmittance of the optical system. Calibration of a Kerr system consists in applying a known voltage, measuring (I/I_m), and using Eq. 12 to calculate V_m. Once V_m is known, Eq. 12 is used in the measurement of unknown voltages.

It should also be noted that for a central path, path dimensions small compared to both the width and spacing of the plates, the Kerr cell can be considered to be an ideal linear retarder with the fast axis at 90° and an arbitrary phase shift ϕ. If in Eq. 8, $[M_3]$ is simplified to reflect this condition and if $[M_2]$ and $[M_4]$ are removed, the same value for $[\Lambda_o]$ is obtained. Physically this means that if the light beam is confined to a central path there is no need for the 90° retarders ($\lambda/4$ plates) in the optical system. The Kerr constant, and therefore the cell constant, depends both on the wavelength of the incident light and on the temperature of the birefringent medium. Because the light source to be used is a laser, the wavelength dependence is significant only when one is using different types of lasers. The cell constant does, however, change approximately 0.5%/°C over the range of typical room temperatures. The cell temperature must, therefore, be monitored.

In suppressing the spatial dependence of the transmittance in Eq. 2, two important effects have been ignored. The first is the effect of fringing fields at the edges of the electrodes.[4] Previous work has shown that this effect will make a negligible contribution to the error if the light path is constrained to a small central region between the plates.[5] The second effect is that the electric field is distorted by the presence of space charge in the birefringent liquid.[3] It will be shown in the following sections of this report that confining the light path to this same central region should also reduce the error resulting from space charge distortion.

Using the information that has been presented in this section, it is possible to design a Kerr system with predictable response. To make explicit the steps in the design of a Kerr system, a sample calculation follows.

Sample Calculation

a) Assume the following parameters

V_p = Maximum Voltage = 300 kV

t_{HV} = High Voltage Pulse Risetime = 10^{-6} s

f_D = Detector Bandwidth = 5×10^7 Hz

b) As indicated in Fig. 3, an increase in V implies that the transmittance passes through a series of minima and maxima. The number of detectable maxima, n, is given approximately by the product of the detector bandwidth and the high voltage pulse risetime, i.e.,

$$n = t_{HV} \cdot f_D = 50$$

In addition,

$$n = \left(\frac{V_p}{V_m}\right)^2 \quad ,$$

so

$$V_m = \frac{300 \text{ kV}}{\sqrt{50}} = 42.4 \text{ kV} \quad .$$

This is the desired value of V_m to insure that the maximum applied voltage can be measured with the given detection system. It is now necessary to choose cell dimensions so that the cell will have the desired value of V_m.

c) As nitrobenzene is the most commonly used Kerr fluid, it is assumed in the remaining discussion that it is the liquid to be used. To insure that there is no possibility of electrical breakdown in the cell, it is sufficient to design so that the field strength remains below 100 kV/cm. This requires a plate spacing, d, of 3 cm.

The necessary plate length, ℓ, can then be determined from Eq. 11, if for the purposes of design it is assumed that $\ell = \ell'$. From Eq. 11,

$$\ell \simeq \ell' = d^2/2BV_m^{\,2}$$

$$= (3 \times 10^{-2}\text{m}/42.4 \times 10^3\text{V})^2/(2)(3.2 \times 10^{-12}\text{m}/\text{V}^2)$$

$$= 7.8 \text{ cm}$$

d) To insure that the field is uniform in the central region, it is necessary that the plate width be greater than the plate spacing. Appropriate plate dimensions would then be 8 cm length, 4 cm width, 1 cm thick and 3 cm spacing.

Table I lists the dimensions and operating voltage range of several cells constructed at NBS. It can be seen from this table that those cells which were successfully operated at 300 kV have dimensions similar to those calculated above.

Accuracy Expected in the Measurement of Type A Pulses

To estimate the accuracy of our measurement of type A (full wave rectified, 60 Hz) pulses, there are two sources of information. The first is to compare with the accuracy attainable under 60 Hz alternating voltage. The second source of data is the preliminary experiments performed in cooperation with and in the laboratories of BRH. In these studies actual x-ray machine voltages of the desired form were measured using a Kerr system.

Because there has been a substantial amount of data and analysis at 60 Hz, this report will first consider that situation. The information from this work will then be applied to the data obtained at BRH. An example of the accuracy of voltage measurement at 60 Hz is shown in Fig. 2.[6] In this figure the cell constant (voltage necessary for first maximum) is plotted against the applied voltage (number of light maxima in one half cycle). In principle this plot should be a horizontal straight line. There is however a systematic decrease, with a change of about 1% as the field strength increased from about 10 kV/cm to about 40 kV/cm. Partially to increase the precision of this work a more exhaustive study[3] of the low frequency behavior of nitrobenzene was undertaken. This information is summarized in reference 3 and provides much of the basis for this discussion.

To gain an insight into Fig. 2 it is helpful to consider the data presented in Fig. 14 of reference 3. That data was from a Kerr cell in which the plate spacing was 0.5 cm. The figure therefore provides a plot of the electric field as a function of position between the

Table I. Descriptive Data for Electrical-Measurement Kerr Cells

Electrodes						Operating Range
Geometry	Spacing (cm)	Dimensions (cm)			Material	(kV)
	d	ℓ	w	t		(kV)
(a)	0.5	10	0.5	0.3	Ni	0 - 50
(a)	1.0	10	1.5	0.3	Ni	0 - 100
(a)	1.5	7.5	1.5	0.5	Ni	0 - 150
(a)	2.8	12	2	0.7	Ni	0 - 300
(a)	2.9	6	4	1	Ni	0 - 300
(a)	3.0	30	4	1	Al	0 - 300
(b)	3.7	15	OD=0.6,	ID=4.3	St. Steel	0 - 70
(c)	3.0	8.6	Diam = 2.5		Ti	0 - 300
(a)*	0.5	12	2	0.3	St. Steel	0 - 50
(a)+	0.5	12	4	0.5	St. Steel	0 - 50
(a)+	0.5	24	4	0.5	St. Steel	0 - 50
(a)	Adjustable	8	8	1.3	Ti	0 - 300

(a) Parallel Plates, (b) Coaxial Cylinders, (c) Parallel Rods

* Glass-blasted electrodes

+ Electro-polished

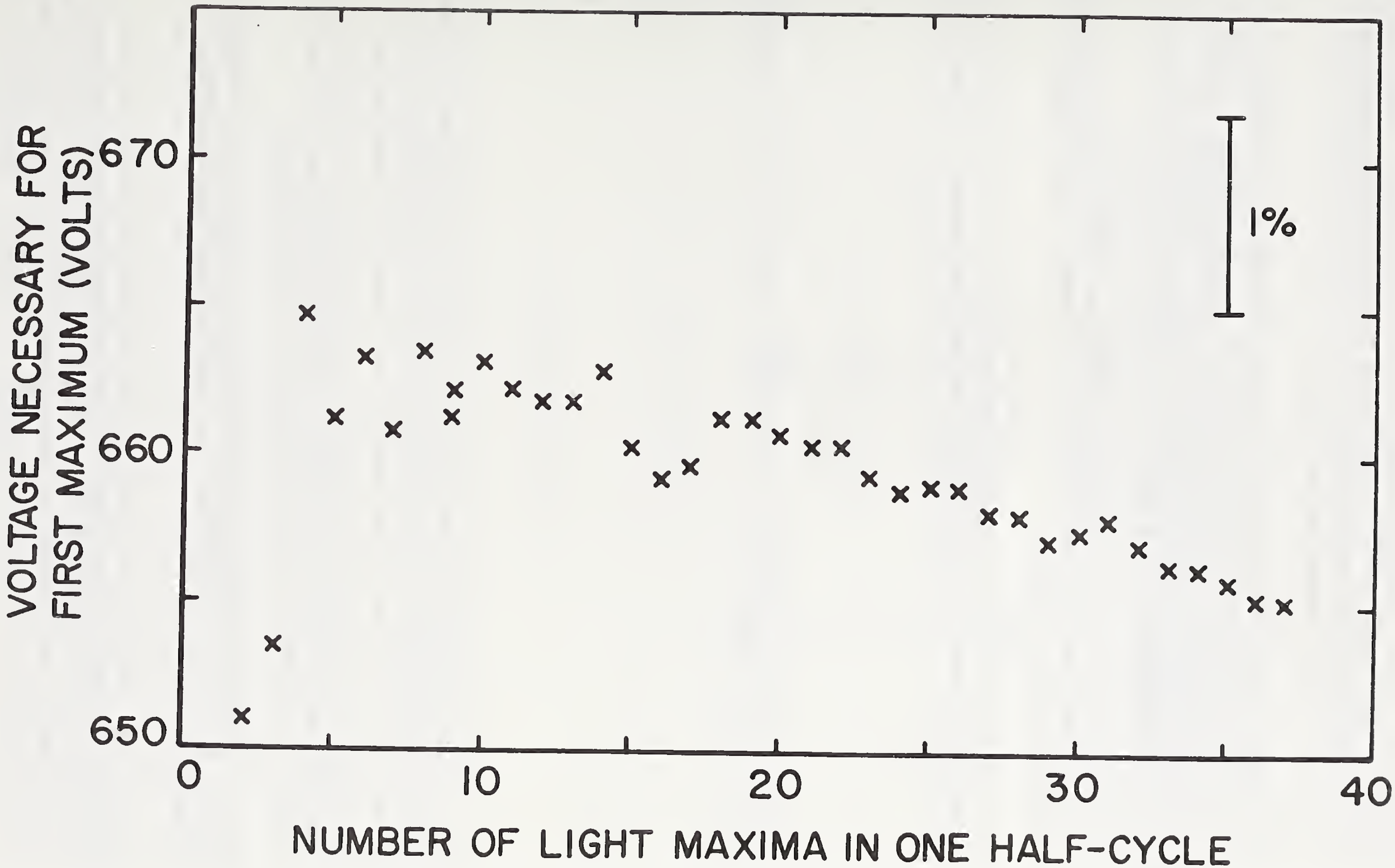

Fig. 2. Determination of the voltage necessary to obtain the first transmission maximum. This value should be independent of the applied voltage, but it does show a systematic decrease which is attributed to a non-uniform field distribution between the plates.

plates for field strengths ranging from 20-60 kV/cm. Performing a linear, least-squares fit to the data points one can obtain values for the slope of the best straight line through the data points. These values are summarized in Table II.

The primary observations that are made concerning this data are

1) The electric field distribution is approximately linear.

2) The slope of the distribution passes through a maximum at approximately 30 kV/cm.

In addition, Fig. 12 in reference 3 indicates that the slope of the plot of the electric field distribution remains approximately constant during the entire cycle.

These observations are significant because the aim of this work is to measure the potential difference between the plates. By definition, the magnitude, V, of the potential difference is given by

$$V = \int_0^d E(x)\, dx .$$

In this case the integral is an integral from one of the parallel plates to the other. If

$$E = A\, x + B,$$

then

$$V = d\left(A\left(\frac{d}{2}\right) + B\right) .$$

For a linear field then the potential difference between the plates is merely the value of the electric field at the center of the cell, $\frac{d}{2}$, multiplied by the plate spacing, d. From this observation, a second advantage of confining the light to a central path is identified. Not only are the effects of fringing fields minimized but, if the field is linear, the effects of space charge are also minimized.

Fig. 2 shows that under 60 Hz alternating voltage these assumptions hold to a sufficient precision to permit measurement uncertainty of order ±1%.

Let us now consider the data obtained from measurements of x-ray machine voltages in the light of the information developed thus far. The data to be considered are four measurements, each with the same nominal value for the applied voltage but the duration of the pulse train being one, two, four and eight half-cycles. The result of these

Table II. Slope, ρ_r, of the plate of the relative electrical field strength, E/E_m, vs. the relative position between the plates, x/d is presented at various values of the applied field strength. The data were taken at the positive maximum of the applied voltage.

Field Strength (kV/cm)	Slope (ρ_r)
20	1.24
30	1.71
40	1.44
50	1.20
60	1.10

measurements is presented in Table III. It can be seen from this table that there is a significant variation in the peak voltage within each set of half-cycles. This variation can be attributed to either of two factors or more likely a combination of both.

The first possibility is that the amplitude fluctuations are real. This is possible because the high voltage circuit consists essentially of a step-up transformer supplying a load which is a parallel combination of a resistor and a capacitor. An appropriate equivalent circuit is an ideal step-up transformer, powering a series RLC circuit. The output voltage is assumed to be the voltage across the capacitor.

The equation relating the input and output voltage is then

$$nV_i = LC \frac{d^2V_o}{dt^2} + RC \frac{dV_o}{dt} + V_o \ ,$$

where V_i and V_o are the input and output voltages, n is the transformer ratio, and R, L and C are the appropriate values for the resistance, inductance and capacitance in the equivalent circuit. If V_i, R, L, and C were known it would, of course, be possible to solve this equation for the output voltage. Those quantities were not determined at the time of the voltage measurement; but it is sufficient for the purposes of this document to consider the form of the solution when the input is zero for $t < 0$ and sinusoidal with constant amplitude for $t > 0$. In this case, the output voltage is of the form

$$V_o = V_p \sin(\omega t - \theta) + A e^{-\alpha t} \sin(\beta t + \delta) \ .$$

For times $t \lesssim 1/\alpha$ the peak output voltage can be either greater or less than the steady-state peak output voltage. This effect may contribute to the fluctuations in the peak values recorded in Table II.

The second possibility is that α is small compared to any relevant temporal variation of the applied voltage. In this case all peaks in the output pulse train would be of the same magnitude and the fluctuations in the measurements would be due to errors in the Kerr-effect voltage measurement-system. This would be true if the time for the space charge distribution to attain its steady-state value is greater than approximately 8 ms. Assuming that this is true leads to the error estimate of 10%. If this is the case, there are two approaches which should reduce this error. The first is to maximize the purity of the nitrobenzene. It is well documented (see reference 3 and references therein) that particulate and chemical impurities in the nitrobenzene will reduce the resistivity of nitrobenzene by orders of magnitude. Of particular importance is the removal of all

Table III. Data obtained using a Kerr system to measure the high voltage in an x-ray machine having a type A pulse.

Pulse Duration (half cycles)	Half-cycle	$(V/V_m)_{peak}$
1	1st	5.052
2	1st	4.964
	2nd	4.500
4	1st	4.752
	2nd	4.498
	3rd	4.305
	4th	4.332
8	1st	*
	2nd	*
	3rd	4.329
	4th	4.359
	5th	4.323
	6th	4.338
	7th	*
	8th	*

*The photograph was out of focus at the edges so the initial and final pulses are not resolved.

particulate impurities as these should be slower than ionic impurities in reaching equilibrium.

If this approach is unsuccessful, there are other liquids acceptable for use in Kerr cells which are less susceptible to impurities than nitrobenzene. The prime candidate for this application would be α-dichloronaphalene. Its Kerr constant is approximately an order of magnitude smaller than that of nitrobenzene but it has been used successfully at NBS under direct, 60 Hz and pulsed high voltage.

Accuracy Expected under Type B Pulses

The accuracy estimates for measurements of Type B pulses is again derived from two sources. The first source is the long history of the operation of Kerr cells under direct voltage. It is well documented that measurement uncertainties of order ±1% can be obtained for cells used to measure direct voltage. The measurements have traditionally been made using an expanded beam so that the spatial distribution of the electric field is recorded. In the same manner as discussed in the previous section, the plots in Figure 7 of reference 3 suggest that accurate measurements can be made using a light beam confined to a central path. Direct voltage conditions are expected to be representative of the situation when the type B pulse has a duration of some few seconds.

In addition to measurements under direct voltage, measurements have been taken where an electric field of strength 20-25 kV/cm was applied to the Kerr fluid with a risetime of about 1 ms and arbitrarily long duration. Measurements taken during the rise of this pulse are shown in Figs. 3-6. The steady-state field distribution was identical to that at 1.564 ms. In each of these figures the Kerr fringes are shown in the circular photographs on the left. The lower trace on the oscillograph on the right shows the voltage pulse. The upper trace shows the time when the pulsed laser (light pulse duration $\approx$ 6 μs) was triggered. To within the sensitivity of this experiment, approximately ±5%, the Kerr response followed the applied voltage.

Conclusions

Although Kerr cells have been successfully used for the measurement of microsecond duration pulses for a number of years, very little work has been directed toward accurate measurement of pulses from millisecond to second duration. Preliminary studies have shown that a cell which accurately measures short pulses measures the voltage waveforms of interest here with an uncertainty of no more than ±10%. Measurements performed with direct, low frequency alternating and step voltages indicate that with a carefully constructed cell, uncertainties of order ±1% should be attainable.

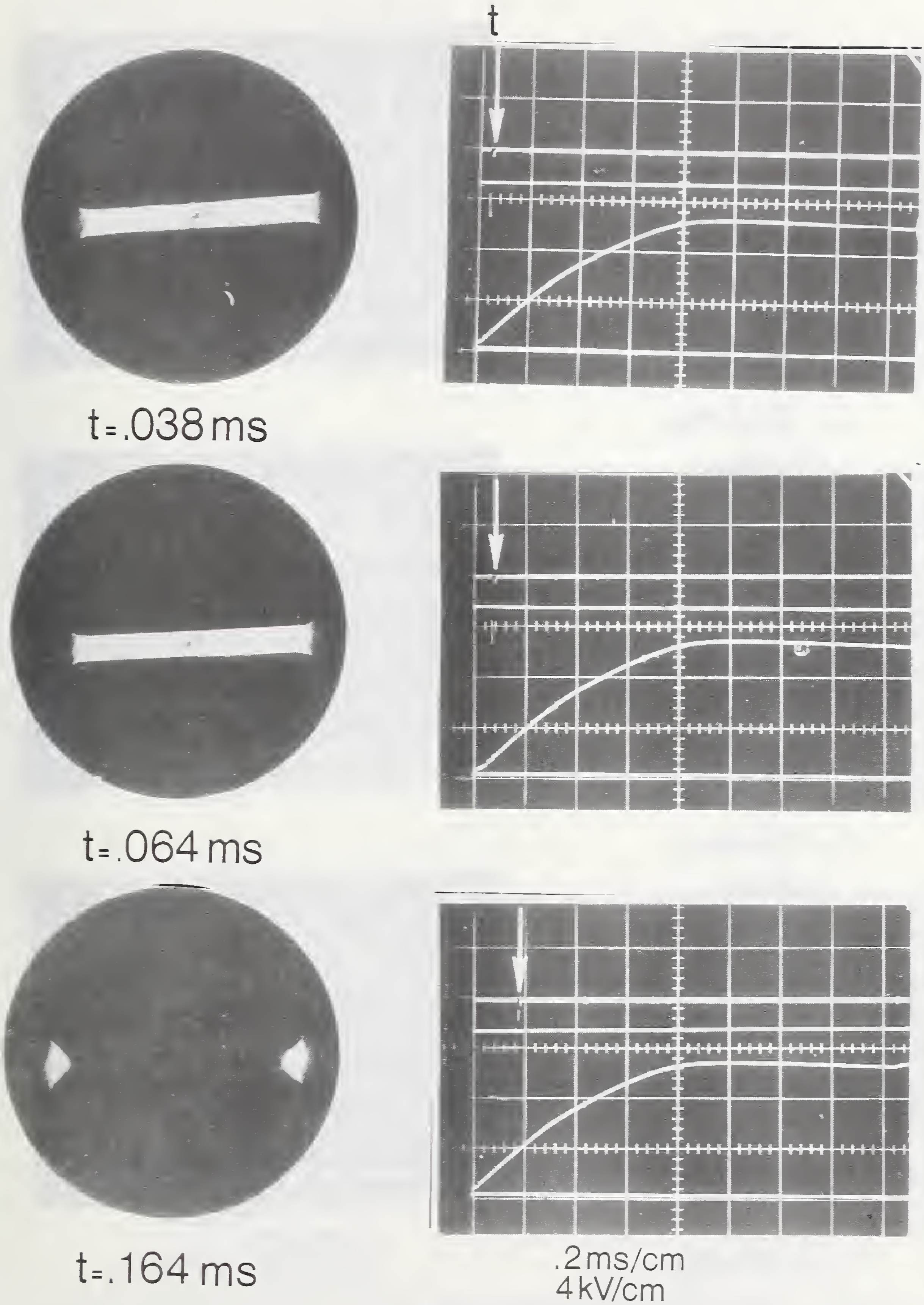

Fig. 3. Kerr fringe pattern photographs taken at various instants during a voltage pulse with a risetime of approximately 1 ms. The photographs on the right show the voltage and laser light pulses.

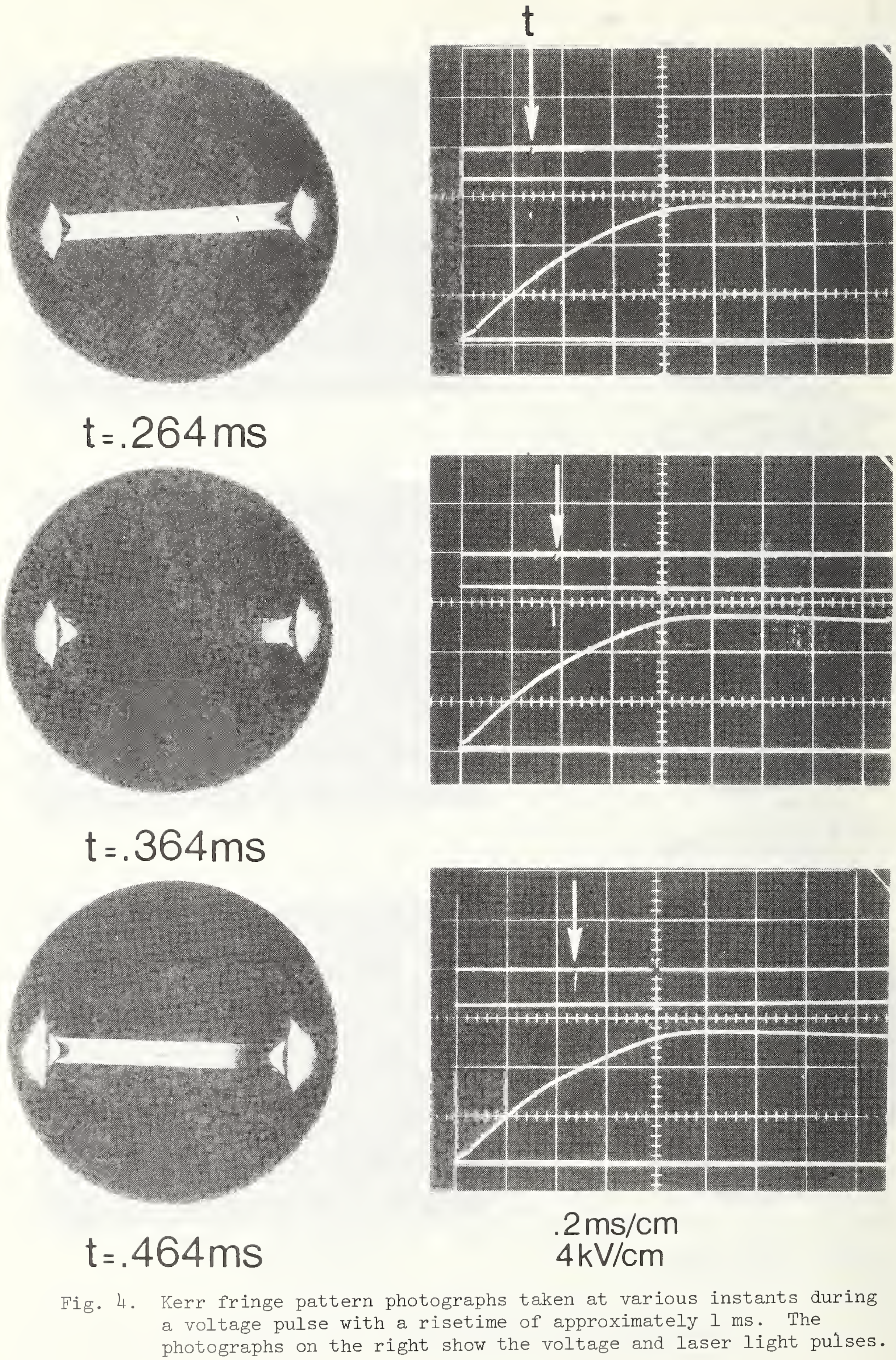

Fig. 4. Kerr fringe pattern photographs taken at various instants during a voltage pulse with a risetime of approximately 1 ms. The photographs on the right show the voltage and laser light pulses.

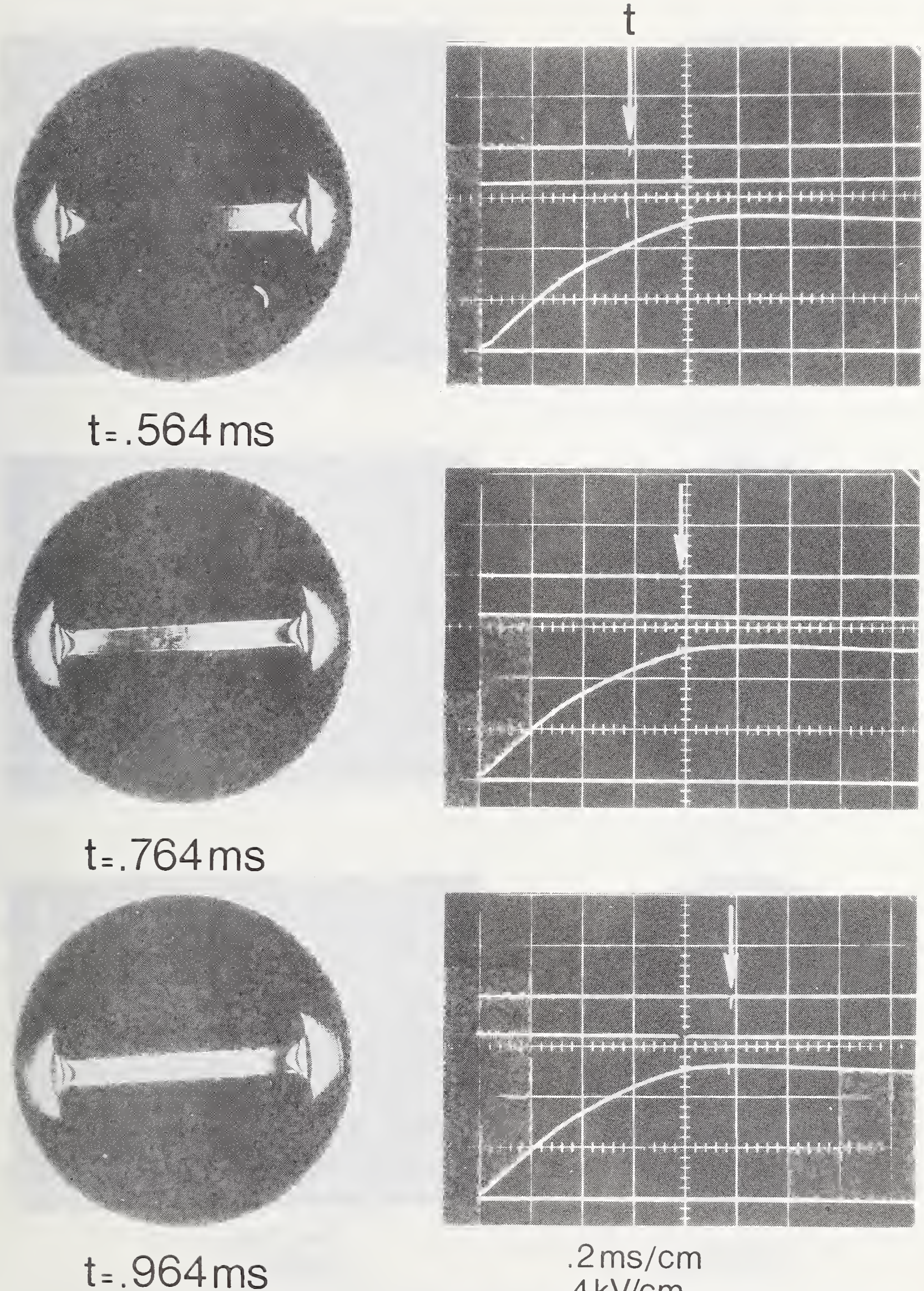

Fig. 5. Kerr fringe pattern photographs taken at various instants during a voltage pulse with a risetime of approximately 1 ms. The photographs on the right show the voltage and laser light pulses.

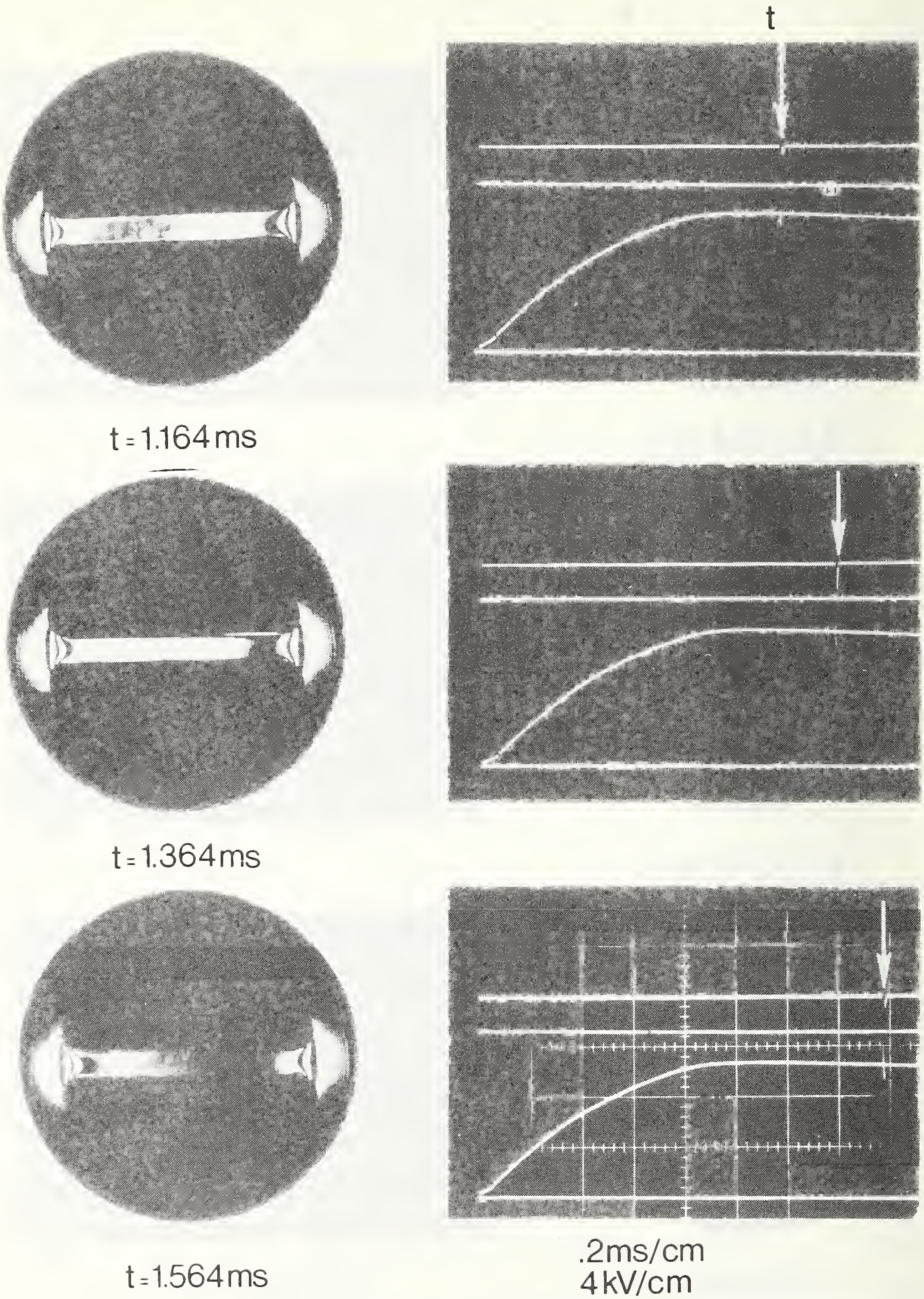

Fig. 6. Kerr fringe pattern photographs taken at various instants during a voltage pulse with a risetime of approximately 1 ms. The photographs on the right show the voltage and laser light pulses.

It is not anticipated that a peak applied voltage of 200 kV should cause any particular problems. All work performed to date indicates that for applied voltages in the range of 10-300 kV all effects have been field rather than voltage dependent. Scaling has therefore always been successful.

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15. SUPPLEMENTARY NOTES

16. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

The report describes a method of calibrating dividers used to measure high voltage pulses in diagnostic x-ray units. The experimental development emphasized four areas. These were the divider ratio under direct voltage, the frequency dependence of the ratio, the voltage dependence of the ratio and the effect of self-heating on the device. The results of measurements on approximately fifteen different dividers are summarized.

In addition, this report contains two appendices. The first discusses conventional and electro-optical methods of measuring the high voltage pulses, while the second presents a more detailed analysis of the feasibility of electro-optical measurement of these pulses.

17. KEY WORDS *(six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons)*

Calibration; electrical measurements; electro-optics; frequency response; high voltage; Kerr effect; operational amplifier; pulses; radiation; safety; x-ray generators; x-rays

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