Note on Simplified Estimators for Type I Extreme-Value Distribution

Julius Lieblein

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National Bureau of Standards
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U.S. DEPARTMENT OF COMMERCE, Frederick B. Dent, Secretary
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Methods for extreme-value analysis (for the Type I extreme-value distribution) that have optimum properties involve up to 20 quantities (depending on sample size) whose values are known to 6 decimal places. The present note shows how to modify these to much simpler values involving 2 decimal places that are more convenient to use yet sacrifice very little of the optimum features.

Key words: Simplified estimators; linear unbiased estimators; bias; efficiency; extreme values; Type I distribution; statistics.

1. Introduction

An NBSIR by the writer [1] described the occurrence and nature of the Type I extreme-value distribution and presented estimates of the two parameters of this distribution for various ranges of sample sizes from very small to very large. It was explained that for any sample size there exists a BLUE--best linear unbiased estimator--with optimum properties. These estimators are linear functions of the sample order statistics--observations arranged in ascending order. The coefficients of such estimators were given to sample size \( n = 16 \), and are known to \( n = 20 \), to six decimal places.

For rapid and convenient use, it seems desirable to try to replace the more exact six-decimal coefficients by much simpler values, with two or even one decimal place or significant figure. It is the purpose of this note to show how to obtain such "simplified estimators" with properties almost as good as the more exact best ones. For this it will first be necessary to present the expected value and variance of linear forms, as related to the extreme-value distribution.

The linear (order statistics) estimators of the parameters \( u, b \) are:

\[
\begin{align*}
\hat{u} &= \sum_{i=1}^{n} a_i x_i \\
\hat{b} &= \sum_{i=1}^{n} b_i x_i
\end{align*}
\]

or

\[
\begin{bmatrix}
\hat{u} \\
\hat{b}
\end{bmatrix} = \begin{bmatrix} a_1 & \ldots & a_n \end{bmatrix} \begin{bmatrix} x_1 \\
\vdots \\
x_n \end{bmatrix} = C' x,
\]

where

\[
C = \begin{bmatrix} a_1 & \ldots & a_n \\
b_1 & \ldots & b_n \end{bmatrix}
\]

and

\[
C' = \begin{bmatrix} a_1 & \ldots & a_n \\
b_1 & \ldots & b_n \end{bmatrix}^{-1}.
\]
where C is the nx2 matrix of coefficients (prime denotes transpose), x is the n-rowed vector of the n observations, after arrangement in ascending order (order statistics) i.e.,

\[ x_1 \leq x_2 \leq \cdots \leq x_n \]

Before ordering the x's are independent observations from the Type I extreme value distribution

\[ \frac{-\left(x-u\right)}{b} \]
\[ \text{Prob. } \{X \leq x\} = e^{-e^{-\left(-\infty < x < \infty\right)}} \]

The expected values of the estimators (1a) are given by:

\[ E(\hat{c}) = C'E(x) \text{.} \] (3)

In order to proceed, the random variables \( x_i \) must be expressed so as to exhibit the parameters explicitly. This is done by writing each \( x_i \) as:

\[ x_i = u + by_i, \quad i = 1, \ldots, n, \] (4)

where the \( y_i \) are the n order statistics from the "reduced", parameter-free distribution (corresponding to the standardized distribution in the normal-distribution case)

\[ \frac{-y}{-e} \]
\[ \text{Prob. } \{Y \leq y\} = e^{-e^{-\left(-\infty < y < \infty\right)}} \] (5)

Eq. (3) then becomes:

\[ E(\hat{c}) = C' \left( u_1 + bE(y) \right) \]

\[ = C' \begin{bmatrix} 1 & Ey_1 \\ \vdots & \vdots \\ 1 & Ey_n \end{bmatrix} \begin{bmatrix} u \\ b \end{bmatrix} = C' \cdot c. \] (6)
where \( 1 \) is the nx1 vector of 1's, \( E(y) \) is the nx1 vector of the known expected values of the order statistics, \( y_i; e \) is the nx2 matrix with column 1, \( E(y) \), and \( c \) is the 2x1 column vector of the parameters \( u, b \).

For unbiasedness, the expected values of the estimators (6) must equal the parameters. The conditions for this from (6) are:

\[
E(\hat{c}) = c, \text{ or } (C'e - I_2) c = 0, \text{ i.e., } \tag{7}
\]

\[
\sum_{i=1}^{n} a_i = 1, \quad \sum_{i=1}^{n} a_i E y_i = 0 \tag{7a}
\]

\[
\sum_{i=1}^{n} b_i = 0, \quad \sum_{i=1}^{n} b_i E y_i = 1 \tag{7b}
\]

The BLUE are unbiased, and unique, being "best", by definition and calculation. Therefore any alteration such as simplified estimators would result in bias. However, the variance may be less, since we are no longer restricted to the class of unbiased estimators. The measure of goodness of the estimator must then be modified to include the bias; it becomes the mean square error of the estimator about the parameter estimated, not about its expected value as is the case with the variance, i.e.

\[
\text{MSE}(\hat{u}) = E(\hat{u} - u)^2 = E[(\hat{u} - \hat{E}u) + (\hat{E}u - u)]^2
\]

\[
= E(\hat{u} - \hat{E}u)^2 + (\hat{E}u - u)^2
\]

\[
= \text{VARIANCE} (\hat{u}) + \text{BIAS} (\hat{u})^2 \tag{8}
\]

the middle term on expanding the square vanishing because it is a multiple (namely, \( (\hat{E}u - u) \)) of:

\[
E(\hat{u} - \hat{E}u) = \hat{u} - \hat{E}u = 0.
\]

For unbiased estimators, MSE and variance are the same.

2. **Bias**

For biased estimators, the bias is given by, in place of (7a) and (7b), \( (C'e - I_2) \) (see (7)), i.e.

\[
bias(\hat{u}) = \left( \sum_{i=1}^{n} a_i - 1 \right) u + \left( \sum_{i=1}^{n} a_i E y_i \right) b \tag{9a}
\]

\[
bias(\hat{b}) = \left( \sum_{i=1}^{n} b_i \right) u + \left( \sum_{i=1}^{n} b_i E y_i - 1 \right) b \tag{9b}
\]
The MSE is thus, in general, a quadratic function of the two unknown parameters \(u\) and \(b\) and so presents a difficult situation. To make it more tractable and reach definite results, we make adjustments, which will usually be small, in the coefficients of the simplified estimator, so that the parameter \(u\) will not appear in the bias.

3. Simplified Estimators

a. Construction

The simplified estimators are summarized in Table 1 for \(n = 10\). The first column gives the coefficients of the BLUE estimators for \(u\) and for \(b\). Col. (2) gives the BLUE coefficients rounded to 2 decimal places. The \(a\)'s add to 0.99 instead of 1.00 as would be necessary in (9a) for the \(u\)-term to disappear, so a slight adjustment is made that would least affect a coefficient—in this case, \(a_3\) is increased by the minute amount, 0.0012, which permits rounding to .01 more and so raise the total to 1.00 (Col. (3)). Also, it turns out that the \(b\)'s add to 0.00 so no adjustment is necessary there. Another type of rounding is to 2 significant figures instead of 2 places, and this time adjustment is necessary in both an \(a\)- and a \(b\)-coefficient, Cols. (4) and (5). The next 4 versions, Cols. (6) to (9), are formed similarly on the basis of 1 decimal place and 1 significant figure.

b. Variance, MSE and Efficiency Ratio

Using the "propagation of error" formula for variance of a linear form (see [2]),

\[
\text{var}(L'x) = L'V(x)L, \quad (10)
\]

where \(L\) and \(x\) are \(n\)-rowed column vectors of coefficients and variables, respectively, and \(V(x)\) is the \(nxn\) matrix of variances and covariances of the \(x\)'s, we have

\[
\text{var}(\hat{u}) = a'Va = (a'va)\ b^2 \quad (11)
\]

\[
\text{var}(\hat{b}) = b'_o\ Vb_o = (b'_o\ vb_o)\ b^2
\]

where by (4),

\[
V(x) = v(y)\ b^2, \quad (12)
\]

with \(v(y)\) the \(nxn\) variance-covariance matrix of the reduced extreme value order statistics, \(y_i\), and the arguments \(x\) and \(y\) are suppressed for convenience; the quantities \(a\) and \(b\) are the \(n\)-rowed vectors of the coefficients \(a_i\) and \(b_i\) respectively. (The subscript "o" is used to avoid confusion with the parameter \(b\)).
### Table 1. BLUE and Simplified 2- and 1-Figure Estimators for Parameters of Type I Extreme-Value Distribution

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(*) denotes adjusted coefficients. (**) denotes that adjustment is not necessary.
From (11) and relations such as (8), we have:

\[
\text{MSE}(\hat{u}) = \left( a'va + \left( \sum_{i=1}^{n} a_{1Ey_{i}}\right)^2 \right) b^2, \tag{13a}
\]

\[
\text{MSE}(\hat{b}) = \left( b'vb + \left( \sum_{i=1}^{n} b_{1Ey_{i}}\right)^2 \right) b^2. \tag{13b}
\]

Calculation of bias, variance, MSE were carried out by use of OMNITAB on the NBS 1108. A copy of the program is attached, and can be readily modified to give results for any other sample sizes where the BLUE coefficients are known; at present they are known* for sample sizes up to \( n = 20 \). They are shown in Table 2 for \( n = 10 \). The 4 adjusted estimators (Col. (1)) are those in Table 1, Cols. (3,5,7,9) as indicated. Bias (Col. (2)) is in terms of \( b \) only, as shown, since the term in \( u \) has been suppressed through the adjustment. Variance and mean square error, in terms of \( b^2 \), are shown in Cols. (3) and (4). Col. (5) gives the "efficiency ratio", which shows how the "efficiency" measure MSE compares with that of the "best", BLUE. (A ratio greater than 1 means BLUE is more efficient, and \textit{vice versa}.)

For example, when the estimator is simplified and adjusted to 2 decimal places as described above ("2D"), the efficiency is virtually the same for the estimator of the parameter \( u \), and only about 1/2% worse (larger MSE) as shown in the fourth line of Column (5) in Table 2. For two significant figures (the third estimator), the results are virtually the same as for two places. If the estimator is altered still more drastically, to 1 figure—whether decimal or significant—the efficiency becomes worse, as might be expected. Similar remarks apply to the amount of bias, being virtually nil with two-figure estimators, and more appreciable with one-figure estimators.

These results make plausible the following statement, for sample sizes that are not too small, say 6 or more:

Two-figure coefficients (whether 2 decimal places or 2 significant figures), for estimators of the two parameters of the Type I extreme-value distribution, can yield practically as good efficiency as is obtainable by BLUE.

\*See reference for paper by White [1].
Table 2. Bias and Efficiency of Simplified Estimators Adjusted so Bias Depends only on b, not u, n = 10 (upper line relates to u, lower line to b)

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<th>(var)/b^2</th>
<th>MSE/b^2</th>
<th>Efficiency Ratio</th>
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*Column numbers refer to estimators in Table 1.
REFERENCES


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<td>HEAD 4/OMX 10X10 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HEAD 5/COLS 4-12 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>READ 14117</td>
<td>0.2228670</td>
<td>-0.14729729</td>
</tr>
<tr>
<td>0.1623064</td>
<td>-0.0911563</td>
<td>0.0219764</td>
</tr>
<tr>
<td>0.1338452</td>
<td>-0.0192100</td>
<td></td>
</tr>
<tr>
<td>0.1178644</td>
<td>0.0271794</td>
<td></td>
</tr>
<tr>
<td>0.0956359</td>
<td>0.0486710</td>
<td></td>
</tr>
<tr>
<td>0.0861718</td>
<td>0.0697498</td>
<td></td>
</tr>
<tr>
<td>0.0698876</td>
<td>0.07702719</td>
<td></td>
</tr>
<tr>
<td>0.054930</td>
<td>0.0827706</td>
<td></td>
</tr>
<tr>
<td>0.0417478</td>
<td>0.0835515</td>
<td></td>
</tr>
<tr>
<td>0.0789290</td>
<td>0.0779399</td>
<td></td>
</tr>
<tr>
<td>DEFINE 1,0 IN COL 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HEAD 14/INP A 14,15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HEAD 16/INP CVM21-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$INPUT CHECKS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SUM 2 PUT IN COL 150</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
MMOVE 1,150 IX1 12,2 $SUM 0,5. IN 12,2
DEFINE 5.772156649 INTO 14,2 $10 GAMMA SHOULD = SUM IN 12,2*(CHECK NO.1)
SMANPROPERTIES 1,3 10X10 151
MMOVE 11,151 IX1 16,2 $SUM COV IN 16,2
DEFINE 16.49394066 INTO 18,2 $10*(PI-50./6) IN 18,2 SHOULD =SUM COV (CK NO 2)
SQUARE *PI* TO COL 13 $P1 50. IN 13
DIV COL 13 BY 6.0 TO COL 13*PI 50/6 IN COL 13
HEAD 13/CONST P150/6
MTRANSPOSE E IN 1,1 10X2 1,18 $E*(2X10) IN COLS 18-27
MMULT E* 1,18 2X10 BY C 1,14 10,2 1,3 vans. $E*C 38,39 WITH 1 2X2*(CHECK NO 3)
RESET 32
PRINT 1***13 150
RESET 10
PRINT 14***17

TITLE: COMPUTATION OF BLUE, AND CHECKS - BIAS, VAR, EFFIC'Y

$COMPUTATION OF BLUE, VAR, AND CKS VS. INPUT
MINV IN V 1,3 10X10 1,28 $IV-INV IN 28-37

******** SINGULAR POINT FOUND IN INVERTED MATRIX IS 1-45
******** SINGULAR POINT FOUND IN INVERTED MATRIX IS 1-45
MINV 1,45 2X2 1,47 $FV-1E-1 = $COV BLUE COMPD IN 52,53

******** SINGULAR POINT FOUND IN INVERTED MATRIX IS 1-45
******** SINGULAR POINT FOUND IN INVERTED MATRIX IS 1-45

MSUB 1,52 2X2 MINUS 1,16 2X2 TO 4,52 $COV BLUE COMPD (INPT) SHOULD = 0
MMULT 1,52 2X2 BY C 1,18 2X10 TO 1,40
MMULT 1,40 2X10 BY V-INV 1,28 10X10 TO 1,40
MTRANSPOSE 1,40 2X10 TO 1,40 $COMPD C (BLUE) IN 40,41 10X2

$CHECK COMPD = INPUT BLUE
MSUB 1,40 10X2 MINUS INPT C 1,14 10X2 TO 1,50 $DIFF'CE SHOULD = 0

$ EFFICIENCY
RESFT 2
DEFINE 0.110866 INTO 54 $CRLA U IN 54
DEFINE 0.060793 INTO 55 $CRLA A IN 55
DIV 54 BY 52 56 $EFF(U BLUE-COMPD) IN 56
DIV 55 BY 53 57 $EFF(A BLUE-COMPD) IN 57

$ BIAS COMPD BLUE
MMULT E* 1,18 2X10 BY C 1,14 10X2 TO 1,3A $E*C(BIAS) BLUE-COMPD IN 38,39

$PROPAGATION OF ERROR COV FOR INPT BLUE, COMPD BLUE
MINV 1,15 V IN 1,3 10X10 X 15 C IN 1,14 10X2 TO 1,5A $CVC=COV(PRGN INPT)58-59
MSUB 1,5A 2X2 MINUS 1,16 2X2 TO 1,60 PRGD COV BLUE-INPT COV SHOULD=0
MINV 1,5A V IN 1,3 10X10 X 15 COMP IN 1,40 10X2 TO 5,58PRGN COMP TO 58-59
MSUB 5,58 2X2 MINUS 1,16 2X2 TO 5,60 PRGD COV COMP EST-INPT COV SHOULD=0

$HEADS AND PRINTING
HEAD 1A/F*(2X10) 1A
LIST OF COMMANDS, DATA AND DIAGNOSTICS

HEAD 19/COLs 18-27
HEAD 28/V-INV(10X10)
HEAD 29/COLs 29-37
HEAD 38/C,CROOTAS
HEAD 39/BLUF CMPD 39
HEAD 40/CK VS INP 40
HEAD 41/UT 41
HEAD 50/CK(COMP),51
HEAD 51/-INPT CIO,52
HEAD 52/COMP COV BUL
HEAD 53/AND CK,52-53
HEAD 54/CRLA U 54
HEAD 55/CRLA R 55
HEAD 56/EFF(CBLUE),156
HEAD 57/EFF(BBLUE),157
HEAD 58/PROPG CV,158
HEAD 59/INP,CMPRUL59
HEAD 60/ZERO DIFF 60
HEAD 61/PR-INV COV,61
RESET 10
PRINT 1A****59
RESET 71
PRINT 40 41 50 51
RESET 5
PRINT 57****57

RESET A
HEAD 150/SUM 0.5,150
HEAD 151/MPROPTIES,1
PRINT 68****61
RESET 31
PRINT 151

TITLE1 SIMPLIFIED ESTIMATORS ROUNDED, ADJ. TO 20, 25, 10, 15

TITLE3 INPUT

READ 62 63 70 71 64 65 72 73
0.22  0.35  0.2  -0.3  0.22  -0.35  0.2  -0.4
0.14  -0.09  0.2  -0.1  0.16  -0.09  0.2  -0.1
0.13  -0.02  0.1  -0.0  0.14  -0.02  0.1  -0.0
0.11  0.02  0.1  0.0  0.11  0.02  0.1  0.0
0.10  0.05  0.1  0.0  0.10  0.05  0.1  0.0
0.08  0.07  0.1  0.1  0.08  0.07  0.1  0.1
0.07  0.08  0.1  0.1  0.07  0.08  0.1  0.1
0.05  0.08  0.1  0.1  0.05  0.08  0.1  0.1
0.04  0.08  0.0  0.1  0.04  0.08  0.0  0.1
TITLE 3 BIASES

MMULT E+1,18 2X10 BY CFIGHT FSTS) 1,62 10x16 13,62E+C(B)ROWS 13,14 COLS 62-77
MIDENTITY IN 16,62 2X2 51-SUB-2 IN ROWS 16,17 COLS 62,63
DUPLICATE INTO 8 TIMES ARRAY IN 16,62 2X2 TO 16,62
ATRANSPOSE ARRAY IN 16,62 16X2 TO 16,62 12(8X) 2X16 IN ROWS 16,17 COLS 62-77
MSUB E+C 13,62 2X16 MINUS IDENTITY'S 16,62 2X16 TO 20,62BIASES IN ROW 21, 62-77
ARISE ARRAY 21,62 1X16 TO 20 INTO 22,62BVIASISQ IN ROW 22,COL 62-77
AMULT ARRAY 22,62 1X16 BY 14,62 1X16 TO 22,62BVIASISQ,0 ALTERNAT,ROW 22, 62-77
AMULT ARRAY 22,62 1X16 BY 17,62 1X16 T23,6250V(BIASISQ,0 ALTERNAT,RUN 23, 62-77

TITLE 3 VARIANCES (PROPAGATION FORMULA) AND MSE'S

M*X|AX) A IS V 1,3 10X10, XCRM,2X2N,1.62 10X2 24,626PROPV=C+VCR(R20)24-25(62,63)
M*X|AX) A IS V 1,3 10X10, XCRM,2X2N,1.64 10X2 24,646PROPV=C+VCR(R22)24-25 (64,65)
M*X|AX) A IS V 1,3 10X10, XCTWOS,1.66 10X2 24,666PROPV=C+VCR(R25)24-25 (66,77)
M*X|AX) A IS V 1,3 10X10, XCTWOS,1.69 10X2 24,696PROPV=C+VCR(R29)24-25 (69,79)
M*X|AX) A IS V 1,3 10X10, XMLONEN,1.70 10X2 24,706PROPV=C+VCR(R30)24-25 (70,71)
M*X|AX) A IS V 1,3 10X10, XMLONEN,1.72 10X2 24,726PROPV=C+VCR(R31)24,25 (72,73)
M*X|AX) A IS V 1,3 10X10, XMLONEN,1.74 10X2 24,746PROPV=C+VCR(R32)24,25 (74,75)
M*X|AX) A IS V 1,3 10X10, XMLONEN,1.77 10X2 24,776PROPV=C+VCR(R33)24,25 (77,78)
AMULT 1'S 16,62 2X16 BY VAR 24,62 2X16 TO 28,62,62SALT, 0'S IN VAX MATRIXROWS 28,29
AADD 22,62 2X16 TO 28,62 2X16 TO 32,62,62SSEIS IN ROWS 32,33 COLS 62-77

TITLE 3 'EFFICIENCY-RATIOS'

DUPLICATE R TIMES 1,16 2X2 TO 34,62
ATRANSPOSE 36,62 1X2 TO 36,62 5 INPT COV HTX IN ROWS 36,37 COLS 62-77
ADIVIDE MSE 32,62 2X16 BY 36,62 2X16 TO 41,626EFFY-RATIOS IN 41,62 COLS 62-77
RESET 44
PRINT 62**77
STOP

NATIONAL BUREAU OF STANDARDS, WASHINGTON, D. C. 20234
OMNITAB II VERSION 5.05 JULY 3, 1974

*** CATSNU IN VERSION 5.05 *** END-OF-FILE. MARK WILL BE PUT ON THE END
OF THE CALCOP Tape PER EACH OMNITAB RUN. WHEREAS BEFORE AN END-OF-
FILE APPEARED AFTER EACH PLOT. TWO NEW STATISTICAL INSTRUCTIONS,
Methods for extreme-value analysis (for the Type I extreme-value distribution) that have optimum properties involve up to 20 quantities (depending on sample size) whose values are known to 6 decimal places. The present note shows how to modify these to much simpler values involving 2 decimal places that are more convenient to use yet sacrifice very little of the optimum features.