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Note on Simplified Estimators for Type I Extreme-Value Distribution

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Technical Analysis Division
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Washington, D. C. 20234

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Final Report

U.S. DEPARTMENT OF COMMERCE
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Methods for extreme-value analysis (for the Type I extreme-value distribution) that have optimum properties involve up to 20 quantities (depending on sample size) whose values are known to 6 decimal places. The present note shows how to modify these to much simpler values involving 2 decimal places that are more convenient to use yet sacrifice very little of the optimum features.

Key words: Simplified estimators; linear unbiased estimators; bias; efficiency; extreme values; Type I distribution; statistics.

1. Introduction

An NBSIR by the writer [1] described the occurrence and nature of the Type I extreme-value distribution and presented estimates of the two parameters of this distribution for various ranges of sample sizes from very small to very large. It was explained that for any sample size there exists a BLUE--best linear unbiased estimator--with optimum properties. These estimators are linear functions of the sample order statistics--observations arranged in ascending order. The coefficients of such estimators were given to sample size n = 16, and are known to n = 20, to six decimal places.

For rapid and convenient use, it seems desirable to try to replace the more exact six-decimal coefficients by much simpler values, with two or even one decimal place or significant figure. It is the purpose of this note to show how to obtain such "simplified estimators" with properties almost as good as the more exact best ones. For this it will first be necessary to present the expected value and variance of linear forms, as related to the extreme-value distribution.

The linear (order statistics) estimators of the parameters u, b are:

$$\hat{u} = \sum_{i=1}^{n} a_i x_i \qquad (1)$$

$$\hat{b} = \sum_{i=1}^{n} b_i x_i$$

or

$$\hat{c} = \begin{bmatrix} \hat{u} \\ \hat{b} \end{bmatrix} = \begin{bmatrix} a_1 & \cdots & a_n \\ b_1 & \cdots & b_n \end{bmatrix} \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} = C' \, x, \qquad (1a)$$

where C is the nx2 matrix of coefficients (prime denotes transpose), x is the n-rowed vector of the n observations, after arrangement in ascending order (order statistics) i.e.,

$$x_1 \leq x_2 \leq \ldots \leq x_n$$

Before ordering the x's are independent observations from the Type I extreme value distribution

$$\text{Prob. } \{X \leq x\} = e^{-e^{-(x-u)/b}}, \qquad \begin{matrix} -\infty < x < \infty \\ -\infty < u < \infty \\ 0 < b < \infty \end{matrix} \tag{2}$$

The expected values of the estimators (1a) are given by:

$$E(\hat{c}) = C'E(x). \tag{3}$$

In order to proceed, the random variables x_i must be expressed so as to exhibit the parameters explicitly. This is done by writing each x_i as:

$$x_i = u + by_i, \quad i = 1, \ldots, n, \tag{4}$$

where the y_i are the n order statistics from the "reduced", parameter-free distribution (corresponding to the standardized distribution in the normal-distribution case)

$$\text{Prob. } \{Y \leq y\} = e^{-e^{-y}} \qquad -\infty < y < \infty. \tag{5}$$

Eq. (3) then becomes:

$$E(\hat{c}) = C' \, (u\underline{1} + bE(y))$$

$$= C' \begin{pmatrix} 1 & Ey_1 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ 1 & Ey_n \end{pmatrix} \begin{pmatrix} u \\ b \end{pmatrix} = C' \, \underline{e} \, c, \tag{6}$$

where $\underline{1}$ is the nx1 vector of 1's, E(y) is the nx1 vector of the known expected values of the order statistics, y_i; $\underline{e}$ is the nx2 matrix with column $\underline{1}$, E(y), and c is the 2x1 column vector of the parameters u,b.

For unbiasedness, the expected values of the estimators (6) must equal the parameters. The conditions for this from (6) are:

$$E(\hat{c}) = c, \text{ or } (C'\underline{e} - I_2)\, c = 0, \text{ i.e.,} \tag{7}$$

$$\sum_{i=1}^{n} a_i = 1, \quad \sum_{i=1}^{n} a_i Ey_i = 0 \tag{7a}$$

$$\sum_{i=1}^{n} b_i = 0, \quad \sum_{i=1}^{n} b_i Ey_i = 1 \tag{7b}$$

The BLUE are unbiased, and unique, being "best", by definition and calculation. Therefore any alteration such as simplified estimators would result in bias. However, the variance may be less, since we are no longer restricted to the class of unbiased estimators. The measure of goodness of the estimator must then be modified to include the bias; it becomes the <u>mean square error of the estimator about the parameter estimated, not about its expected</u> value as is the case with the variance, i.e.

$$\begin{aligned} MSE(\hat{u}) &= E(\hat{u} - u)^2 = E[(\hat{u} - E\hat{u}) + (E\hat{u} - u)]^2 \\ &= E(\hat{u} - E\hat{u})^2 + (E\hat{u} - u)^2 \\ &= \text{VARIANCE } (\hat{u}) + [\text{BIAS } (\hat{u})]^2 \end{aligned} \tag{8}$$

the middle term on expanding the square vanishing because it is a multiple (namely, $(E\hat{u} - u)$) of:

$$E(\hat{u} - E\hat{u}) = E\hat{u} - E\hat{u} = 0.$$

For unbiased estimators, MSE and variance are the same.

2. <u>Bias</u>

For biased estimators, the bias is given by, in place of (7a) and (7b), $(C'e - I_2)$ (see (7)), i.e.

$$\text{bias}(\hat{u}) = (\sum_{i=1}^{n} a_i - 1)u + (\sum_{i=1}^{n} a_i Ey_i)\, b \tag{9a}$$

$$\text{bias}(\hat{b}) = (\sum_{i=1}^{n} b_i)u + (\sum_{i=1}^{n} b_i Ey_i - 1)\, b \tag{9b}$$

The MSE is thus, in general, a quadratic function of the two unknown parameters u and b and so presents a difficult situation. To make it more tractable and reach definite results, we make adjustments, which will usually be small, in the coefficients of the simplified estimator, so that the parameter u will not appear in the bias.

3. Simplified Estimators

a. Construction

The simplified estimators are summarized in Table 1 for n = 10. The first column gives the coefficients of the BLUE estimators for u and for b. Col. (2) gives the BLUE coefficients rounded to 2 decimal places. The a's add to 0.99 instead of 1.00 as would be necessary in (9a) for the u-term to disappear, so a slight adjustment is made that would least affect a coefficient—in this case, a_3 is increased by the minute amount, 0.0012, which permits rounding to .01 more and so raise the total to 1.00 (Col. (3)). Also, it turns out that the b's add to 0.00 so no adjustment is necessary there. Another type of rounding is to 2 significant figures instead of 2 places, and this time adjustment is necessary in both an a- and a b-coefficient, Cols. (4) and (5). The next 4 versions, Cols. (6) to (9), are formed similarly on the basis of 1 decimal place and 1 significant figure.

b. Variance, MSE and Efficiency Ratio

Using the "propagation of error" formula for variance of a linear form (see [2]),

$$\mathrm{var}(L'x) = L'V(x)L, \tag{10}$$

where L and x are n-rowed column vectors of coefficients and variables, respectively, and V(x) is the nxn matrix of variances and covariances of the x's, we have

$$\mathrm{var}(\hat{u}) = a'Va = (a'va)\, b^2 \tag{11}$$

$$\mathrm{var}(\hat{b}) = b'_o V b_o = (b'_o v b_o)\, b^2$$

where by (4),

$$V(x) = v(y)\ \ b^2, \tag{12}$$

with v(y) the nxn variance-covariance matrix of the reduced extreme value order statistics, y_i, and the arguments x and y are suppressed for convenience; the quantities a and b_o are the n-rowed vectors of the coefficients a_i and b_i respectively. (The subscript "o" is used to avoid confusion with the parameter b).

Table 1. BLUE and Simplified 2- and 1-Figure Estimators for Parameters of Type I Extreme-Value Distribution

	BLUE	SIMPLIFIED ESTIMATORS							
		2 Dec. Places		2 Sign. Figs.		1 Dec. Place		1 Sign. Fig.	
		Rounded	Adjusted	Rounded	Adjusted	Rounded	Adjusted	Rounded	Adjusted
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				a_i					
Coefficients for Estimators of Parameter u	.2228670	.22	.22	.22	.22	.2	.2	.2	.2
	.1623082	.16	.16	.16	.16	.2	.2	.2	.2
	.1338452	.13	.14*	.13	.14*	.1	.1	.1	.1
	.1128684	.11	.11	.11	.11	.1	.1	.1	.1
	.0956359	.10	.10	.10	.10	.1	.1	.1	.1
	.0806178	.08	.08	.081	.081	.1	.1	.08	.09*
	.0669876	.07	.07	.067	.067	.1	.1	.07	.07
	.0541930	.05	.05	.054	.054	.1	.1	.05	.06*
	.0417478	.04	.04	.042	.039*	.0	.0	.04	.05*
	.0289290	.03	.03	.029	.029	.0	.0	.03	.03
SUM	.9999999	.99	1.00	.993	1.000	1.0 **	1.0	.97	1.00
				b_i					
Coefficients for Estimators of Parameter b	-.3478297	-.35	-.35	-.35	-.35	-.3	-.4*	-.3	-.3
	-.0911583	-.09	-.09	-.091	-.091	-.1	-.1	-.09	-.09
	-.0192100	-.02	-.02	-.019	-.019	-.0	-.0	-.02	-.02
	.0221794	.02	.02	.022	.022	.0	.0	.02	.02
	.0486710	.05	.05	.049	.050*	.0	.0	.05	.05
	.0660648	.07	.07	.066	.066	.1	.1	.07	.05*
	.0770208	.08	.08	.077	.077	.1	.1	.08	.06*
	.0827706	.08	.08	.083	.083	.1	.1	.08	.08
	.0835515	.08	.08	.084	.084	.1	.1	.08	.08
	.0779399	.08	.08	.078	.078	.1	.1	.08	.07*
SUM	.0000000	.00	0.00 **	-.001	.000	.1	.0	.05	.00

(*) denotes adjusted coefficients. (**) denotes that adjustment is not necessary.

From (11) and relations such as (8), we have:

$$\mathrm{MSE}(\hat{u}) = \left[a'va + \left(\sum_{i=1}^{n} a_i Ey_i \right)^2 \right] b^2 \qquad (13a)$$

$$\mathrm{MSE}(\hat{b}) = \left[b'_o v b_o + \left(\sum_{i=1}^{n} b_i Ey_i \right)^2 \right] b^2 \qquad (13b)$$

Calculation of bias, variance, MSE were carried out by use of OMNITAB on the NBS 1108. A copy of the program is attached, and can be readily modified to give results for any other sample sizes where the BLUE coefficients are known; at present they are known* for sample sizes up to n = 20. They are shown in Table 2 for n = 10. The 4 adjusted estimators (Col. (1)) are those in Table 1, Cols. (3,5,7,9) as indicated. Bias (Col. (2)) is in terms of b only, as shown, since the term in u has been suppressed through the adjustment. Variance and mean square error, in terms of b^2, are shown in Cols. (3) and (4). Col. (5) gives the "efficiency ratio", which shows how the "efficiency" measure MSE compares with that of the "best", BLUE. (A ratio greater than 1 means BLUE is more efficient, and vice versa.)

For example, when the estimator is simplified and adjusted to 2 decimal places as described above ("2D"), the efficiency is virtually the same for the estimator of the parameter u, and only about 1/2% worse (larger MSE) as shown in the fourth line of Column (5) in Table 2. For two significant figures (the third estimator), the results are virtually the same as for two places. If the estimator is altered still more drastically, to 1 figure—whether decimal or significant—the efficiency becomes worse, as might be expected. Similar remarks apply to the amount of bias, being virtually mil with two-figure estimators, and more appreciable with one-figure estimators.

These results make plausible the following statement, for sample sizes that are not too small, say 6 or more:

> Two-figure coefficients (whether 2 decimal places or 2 significant figures), for estimators of the two parameters of the Type I extreme-value distribution, can yield practically as good efficiency as is obtainable by BLUE.

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*See reference for paper by White [1].

Table 2. Bias and Efficiency of Simplified Estimators Adjusted so Bias Depends only on b, not u, n = 10 (upper line relates to u, lower line to b)

Estimator (1)	(bias)/b (2)	(var)/b^2 (3)	(MSE/b^2 (4)	Efficiency Ratio $= \left[\frac{MSE/b^2}{var(BLUE)/b^2}\right]$ (5)
BLUE	0 0	0.112973 .071573	0.112973 .071573	1 1
Simplified Est.				
Adj. to 2 D (Col. (3))*	0.000458 .002725	.113002 .071973	.113002 .071980	1.000256 1.005686
Adj. to 1 D (Col. (5))	-.050827 .192893	.113345 .102266	.115928 .139472	1.026157 1.948668
Adj. to 2 S (Col. (7))	-.001240 .003570	.112926 .072085	.112927 .072097	0.999593 1.007321
Adj. to 1 S (Col. (9))	.044711 -.103800	.117207 .057531	.119205 .068305	1.055164 .954340

*Column numbers refer to estimators in Table 1.

REFERENCES

1. Lieblein, J. "Efficient Methods of Extreme-Value Methodology," NBSIR 74-602, October 1974.

2. "Generalized Propagation of Error Using a New Approach," Proceedings 11th Annual Meeting, Institute of Nuclear Materials Management, May 25-27, 1970, sec. 2.1, p. 192.

USCOMM-NBS-DC

LIST OF COMMANDS, DATA AND DIAGNOSTICS

```
]=@@@@@CC@*CNBS   @CJ@CJOMNITAB
DIM  70X170
TITLE1 DATA INPUT, ARRANGEMENT, AND INPUT CHECKS NOS. 1,2 AND 3
READ   2***7
-0.9998741   0.08571435   0.10319122   0.07893158   0.06603102   0.05785359
-0.5845581   0.           0.07439614   0.11470661   0.09635424   0.08463345
-0.2836893   0.           0.           0.07595876   0.12812176   0.11281706
-0.0120439   0.           0.           0.           0.08290706   0.14640959
 0.2574495   0.           0.           0.           0.           0.09478920
 0.5436122   0.           0.           0.           0.           0.
 0.8680818   0.           0.           0.           0.           0.
 1.2671822   0.           0.           0.           0.           0.
 1.8261956   0.           0.           0.           0.           0.
 2.8798008   0.           0.           0.           0.           0.
READ    8***12
0.05212940   0.04785812   0.04452558   0.04183846   0.03961629
0.07638738   0.07021184   0.06538028   0.06147607   0.05824178
0.10199708   0.09386503   0.08748531   0.08231892   0.07803153
0.13261946   0.12221472   0.11402737   0.10738105   0.10185461
0.17211269   0.15887529   0.14842203   0.13991198   0.13281900
0.11342756   0.20985667   0.19637486   0.18536001   0.17615189
0.           0.14369416   0.26954309   0.25489482   0.24260060
0.           0.           0.19850770   0.37650181   0.35918745
0.           0.           0.           0.32292931   0.61875615
0.           0.           0.           0.           0.82246703
MMOVE  1,3  10X10  12,3 $PRESERVE INPT V-TRIANG IN COLS 3-12
MTRANSPOSE  1,3  10X10  23,3
MADD  1,3  10X10  23,3  10X10  1,3 $ FULL INPT CV OS COLS 3-12
HEAD 1/ONES VCTR 1
HEAD 2/E(OS)10X1 2
HEAD 3/INPT V-COV 3
HEAD 4/OSMX 10X10 4
HEAD 5/COLS 4-12  5
READ  14***17
0.2228670    -0.3478297   0.1129729    0.0219764
0.1623082    -0.0911583   0.0219764    0.0715730
0.1338452    -0.0192100
0.1128684     0.0221794
0.0956359     0.0486710
0.0806178     0.0660648
0.0669876     0.0770208
0.0541930     0.0827706
0.0417478     0.0835515
0.0289290     0.0779399
DEFINE  1.0  IN  COL 1
HEAD 14/INP A 14,15
HEAD 16/INP CVM21-2

$INPUT CHECKS
SUM  2  PUT IN COL  150
```

```
MMOVE  1,150  IXI  12,2  $SUM O.S.. IN 12,2
DEFINE 5.772156649 INTO 14,2 $10 GAMMA SHOULD = SUM IN 12,2.(CHECK NO. 1)
SMPROPERTIES  1,3  10X10  151
MMOVE 11,151 IXI  16,2  $SUM COV IN 16,2
DEFINE 16.44934066 INTO 18,2 $10(PI-SQ./6) IN 18,2.SHOULD =SUM COV (CK NO 2)
SQUARE .PI. TO COL 13 $PI SQ. IN 13
DIV COL 13 BY 6.0 TO COL 13$PI SQ/6 IN COL 13
HEAD 13/CONST PISQ/6
MTRANSPOSE E IN 1,1 10X2  1,18 $E'(2X10) IN COLS 18-27
MMULT E' 1,18 2X10 BY C 1,14 10,2 1,38$CF. E'C 38,39 WITH I 2X2.(CHECK NO 3)
RESET  32
PRINT  1...13   150
RESET  10
PRINT  14...17


TITLE1 COMPUTATION OF BLUE,AND CHECKS - BIAS, VAR, EFFIC'Y

$COMPUTATION OF BLUE, VAR, AND CKS VS. INPUT
MINVERT V 1,3 10X10 1,28 $V-INVERSE IN 28-37
  +++++++++++++++++++ SMALLEST ERROR BOUND ON INVERTED MATRIX IS   .5-04   ++++
M(X'AX) A IS V-INV IN 1,28 10X10 X IS F IN 1,1 10X2 TO 1,52$E'V-IE IN 52,53
MINVERT 1,52 2X2 1,52 $(E'V-IE)-1 = COV BLUE COMPD IN 52,53
  +++++++++++++++++++ SMALLEST ERROR BOUND ON INVERTED MATRIX IS   .2-08   ++++
MSUB  1,52 2X2 MINUS 1,16 2X2 TO 4,52 $COV BLUE (COMPD-INPT) SHOULD = 0
MMULT  1,52 2X2 BY E' 1,18 2X10 TO 1,40
MMULT 1,40 2X10 BY V-INV 1,28 10X10 TO 1,40
MTRANSPOSE  1,40 2X10 TO 1,40 $COMPD C (BLUE) IN 40,41 10X2

$CHECK COMPTD = INPUT BLUE
MSUB  1,40 10X2 MINUS INPT C 1,14 10X2 TO 1,50 $DIFF'CE SHOULD = 0

$ EFFICIENCY
RESFT  2
DEFINF  0.110866  INTO 54  $CRLB U IN 54
DEFINE  0.060793  INTO  55  $CRLB B IN 55
DIV  54  BY  52  56  $ EFF(U BLUE-COMPD) IN 56
DIV  55  BY  53  57  $ EFF(B BLUE-COMPD) IN 57

$ BIAS  COMPD BLUE
MMULT E' 1,18 2X10 BY C 1,14 10X2 TO 1,38 $E'C(BIAS) BLUE-COMPD IN 38,39

$PROPAGATION OF ERROR COV FOR INPT BLUE, COMPD BLUE
M(X'AX) A IS V IN 1,3 10X10  X IS C IN 1,14 10X2 TO 1,58$C'VC=COV(PRGN INP)58-59
MSUB   1,58 2X2 MINUS 1,16 2X2 TO 1,60$PRGD COV BLUE-INPT COV SHOULD=0
M(X'AX) A IS V IN 1,3 10X10 X IS CCOMP IN 1,40 10X2 TO 5,58$PRGN COMP TO 58-59
MSUB   5,58 2X2 MINUS 1,16 2X2 TO 5,60$PRGD COV COMP EST-INPT COV SHOULD=0

$HEADS AND PRINTING
HEAD 18/F'(2X10) 18
```

LIST OF COMMANDS, DATA AND DIAGNOSTICS

```
HEAD 19/COLS 18-27
HEAD 28/V-INV(10X10)
HEAD 29/COLS 28-37
HEAD 38/E'C,BIAS38
HEAD 39/BLUF CMPD 39
HEAD 40/CK VS INP 40
HEAD 41/UT  41
HEAD 50/CK(COMP,51
HEAD 51/-INPT C10,52
HEAD 52/COMP COV BLU
HEAD 53/AND CK,52-53
HEAD 54/CRLB U 54
HEAD 55/CRLB B 55
HEAD 56/EFF(UBLUE)56
HEAD 57/EFF(BBLUE)57
HEAD 58/PROPG CVI,58
HEAD 59/INP,CMPBLU59
HEAD 60/ZERO DIFF 60
HEAD 61/PR-INP COV61
RESET  10
PRINT  18***39
RESET  21
PRINT  40  41   50  51
RESET  5
PRINT  52***57

RESET  8
HEAD 150/SUM O.S.150
HEAD 151/MPROPTIES,1
PRINT  58***61

RESET  31
PRINT  151

TITLE1 SIMPLIFIED ESTIMATORS ROUNDED,ADJ. TO 2D,2S,1D,1S

TITLE3 INPUT

READ  62 63  70 71  64 65  72 73
0.22   -0.35   0.2   -0.3   0.22   -0.35   0.2   -0.4
0.16   -0.09   0.2   -0.1   0.16   -0.09   0.2   -0.1
0.13   -0.02   0.1   -0.0   0.14   -0.02   0.1   -0.0
0.11    0.02   0.1    0.0   0.11    0.02   0.1    0.0
0.10    0.05   0.1    0.0   0.10    0.05   0.1    0.0
0.08    0.07   0.1    0.1   0.08    0.07   0.1    0.1
0.07    0.08   0.1    0.1   0.07    0.08   0.1    0.1
0.05    0.08   0.1    0.1   0.05    0.08   0.1    0.1
0.04    0.08   0.0    0.1   0.04    0.08   0.0    0.1
```

```
0.03     0.08      0.0       0.1       0.03      0.08      0.0       0.1
READ  66 67  74 75  68 69  76 77
0.22     -0.35     0.2       -0.3      0.22      -0.35     0.2       -0.3
0.16     -0.091    0.2    -0.09   0.16    -0.091    0.2    -0.09
0.13     -0.019    0.1       -0.02               0.14      -0.019    0.1 -0.02
0.11     0.022     0.1     0.02    0.11     0.022      0.1       0.02
0.10     0.049     0.1       0.05     0.10      0.050      0.1      0.05
0.081     0.066     0.08      0.07      0.081     0.066     0.09      0.05
0.067     0.077     0.07      0.08      0.067     0.077     0.07      0.06
0.054     0.083     0.05      0.08      0.054     0.083     0.06      0.08
0.042     0.084     0.04      0.08      0.039     0.084     0.05      0.08
0.029     0.078     0.03      0.08      0.029     0.078       0.03      0.07
HEAD 62/C R2D 62,63
HEAD 64/C 2DADJ64,65
HEAD 66/C R2S 66,67
HEAD 68/C 2SADJ68,69
HEAD 70/C R 1FIGUR70
HEAD 71/4 SETS 70-78

TITLE3  BIASES
MMULT E' 1,18 2X10 BY C(EIGHT ESTS) 1,62 10X16  13,62$E'C(8)ROWS 13,14 COLS62-77
MIDENTITY IN 16,62 2X2 $I-SUB-2 IN ROWS 16,17 COLS 62,63
DUPLICATE ITWO 8 TIMES ARRAY IN 16,62  2X2  TO  16,62
ATRANSPOSE ARRAY IN 16,62 16X2 TO 16,62 $I2(8X) 2X16 IN ROWS 16,17 COLS 62-77
MSUB E'C 13,62 2X16 MINUS IDENTY'S 16,62 2X16 TO 20,62$BIASES IN ROW 21, 62-77
ARAISE ARRAY 21,62 1X16 TO 2.0 INTO 22,62$(VIAS)SQ IN ROW 22,COLS 62-77
AMULT ARRAY 22,62 1X16 BY 16,62 1X16 TO 22,62$(BIAS)SQ,0 ALTERNAT,ROW22, 62-77
AMULT ARRAY 22,62 1X16 BY 17,62 1X16 TI 23,62 $0,(BIAS)SQ ALTERN,ROW 23, 62-77

TITLE3 VARIANCES (PROPAGATION FORMULA) AND MSE'S
M(X'AX) A IS V 1,3 10X10,XCRTWOD 1,62 10X2 24,62$PROPV=C'VC(R2D)24-25(62,63)
M(X'AX) A IS V 1,3 10X10,XCRTWOADJ 1,64 10X2 24,64$PROPVAR(C2ADJ)24-25 (64-65)
M(X'AX) A IS V 1,3 10X10,XCRTWOS   1,66 10X2 24,66$PROPVAR(C R2S)24-25 (66,77
M(X'AX) A IS V 1,3 10X10,XCTWOSADJ   1,68 10X2 24,68$PROPVAR(C2SADJ)24,25(68,69)
M(X'AX) A IS V 1,3 10X10,XCRONED   1,70 10X2 24,70$PROPVAR(C R1D)24,25 (70,71)
M(X'AX) A IS V 1,3 10X10,XCRONEDADJ  1,72 10X2 24,72$PROPVAR(C 1DADJ)24,25(72,73
M(X'AX! A IS V 1,3 10X10,XCPONES  1,74 10X2 24,74$PROPVAR(CR1S) 24,25 (74,75)
M(X'AX) A IS V 1,3 10X10,XCPONESADJ 1,76 10X2 24,76$PROPVAR(C1SADJ) 24,25(76,77)
AMULT I'S 16,62 2X16 BY VAR 24,62 2X16 T! 28,62$ALT. 0'S IN VAR MATRIXROWS 28,29
AADD 22,62 2X16 TO 28,62 2X16 TO 32,62 $MSE(S IN ROWS 32,33 COLS 62-77

TITLE3 'EFFICIENCY-RATIOS'
DUPLICATE 8 TIMES 1,16 2X2 TO 36,62
ATRANSPOSE 36,62 16X2 TO 36,62 $ INPT COV MTX IN ROWS 36,37  COLS 62-77
ADIVIDE MSE 32,62 2X16 BY 36,62 2X16 TO 41,62$(EFFY-RATIO( IN 41,42 COLS 62-77
RESET 44
PRINT  62...77
STOP

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        OMNITAB II VERSION 5.05 JULY 3, 1974

... "ATSNU IN VERSION 5.05 ... END-OF-FILE MARK WILL BE PUT ON THE END
OF THE CALCOMP TAPE PER EACH OMNITAB RUN,  WHEREAS BEFORE AN END-OF-
FILE APPEARED AFTER EACH PLOT.  TWO NEW STATISTICAL INSTRUCTIONS,
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15. SUPPLEMENTARY NOTES

16. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

Methods for extreme-value analysis (for the Type I extreme-value distribution) that have optimum properties involve up to 20 quantities (depending on sample size) whose values are known to 6 decimal places. The present note shows how to modify these to much simpler values involving 2 decimal places that are more convenient to use yet sacrifice very little of the optimum features.

17. KEY WORDS *(six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons)*

Simplified estimators; linear unbiased estimators; bias; efficiency; extreme values; Type I distribution; statistics.

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