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An In-Line Density and Viscosity Sensor

L. O. Olsen and F. W. Ruegg

Fluid Meters Section
Mechanics Division
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ABSTRACT

In-line density and viscosity sensors for liquids are developed to utilize measurements of differential pressure across a nozzle and a coiled capillary tube respectively, with known flowrates through each provided by a flow generator. Theory and principles of operation and instructions for calibration and use of the sensors are discussed, along with design consideration for the sensors and associated equipment. A calibration of the sensors demonstrated that viscosity and density each could be measured with a computed standard deviation of one percent. Viscosity was varied over the range of about 1 to 11 centistokes whereas density of the fluids used was near 0.8 g cm^{-3}. Application of well known similarity considerations is used to make the results applicable to other liquid densities, provided influence of other liquid properties (high vapor pressure, for instance) does not interfere.

1. INTRODUCTION

This is a report of work sponsored by the Department of Defense, Calibration Coordination Group under projects 71-58 and 72-60 of DOD/CCG, Flow Project. The corresponding National Bureau of Standards project numbers are 2130464 and 2130465. The objective of the work under project 71-58 was to develop a small in-line density sensor, range of 0.6 to 1.6 $g.cm^{-3}$, and having an electrical output for use with density-sensitive flowmeters. Under project 72-60 the objective was to develop an in-line viscosity sensor having a range of kinematic viscosities from 0.5 to 50 centistokes. An uncertainty of 0.5 and 1.0 percent was desired for measurement of density and viscosity respectively. Work on the development of both sensors was performed at the same time because of the shared common equipment. This report covers the theory and principles of operation, design of the sensors and associated equipment, and instructions for its calibration and use. A positive displacement flow generator is used to provide known flows through the sensors for viscosity and density. Measurements of differential pressure in each, together with the known flow, give the viscosity and density.

2. THEORY AND RESULTS

2.1 Viscosity Sensor-Theory. The early independent experimental work of Hagen in 1839 and Poiseuille in 1840 resulted in the empirical relation that the quantity of liquid which flows through a small, straight tube in a given time is proportional to the differential pressure causing flow, to the fourth power of the inside diameter of the tube and inversely to its length and inversely to the viscosity of the liquid. It was hoped that the proportionality between pressure and viscosity could be used to determine viscosity through measurement of the differential pressure across a coiled capillary tube. However, the data showed that a small correction to the proportionality relation is needed.

The proportionality relationship can be derived theoretically for laminar flow in a length of straight pipe of circular cross section. The flow is laminar in character (Reynolds number R less than 2300) and is assumed to be steady. The fluid moves through the pipe and the differential pressure over the length L of the tube required to generate this flow is (p_1-p_2). The distribution of velocity can be shown to be in the form of a paraboloid of revolution, and the volume rate of flow Q can be shown to be given by

$$Q = \pi(p_1-p_2)d^4/(128\mu L) \qquad (1)$$

where μ is the viscosity of the flowing fluid, d is the inside diameter and L the length of pipe. A complete table of nomenclature and dimensional units used herein is listed in Table 1. The Centimeter-Gram-Second (CGS) system of units are used herein rather than the International System of Units (SI) primarily because both poise and stokes, the most commonly used values of viscosity and kinematic viscosity, are based on CGS units.

When equation 1 is solved for μ we find

$$\mu = \pi d^4(p_1-p_2)/(128QL). \qquad (2)$$

Thus, the viscosity of the flowing liquid is directly proportional to the differential pressure across the capillary tube for a given flow Q in a straight tube. Measurements of Q, (p_1-p_2) and μ will be used to calibrate the sensor constructed of coiled capillary tubes.

2.2 Viscosity Sensor - Results. Equation 2 is based on laminar flow in straight tubes for all flows having Reynolds numbers less than about 2300. Because of the long length of tubing required in the viscosity sensor in this instrument the tubing was wound in the form of a coil having a diameter D. C. M. White [1] reports that a secondary flow is generated in the laminar flow in a coiled tube thereby causing a deviation from Equation 2 under certain flow conditions. The reported experiments further demonstrated that the correlating parameter for the new relationship should be taken as the quantity $R(d/D)^{1/2}$, where D is the diameter of the coil of tubing.

The change (increase) of flow resistance due to curvature of the capillary is demonstrated by the results in figure 1. These data were obtained using a coiled capillary with a nominal d equal to 0.142, D equal to 25.4 and L equal to 594 cm, and with 3 liquids whose viscosities are listed in the figure. Viscosity was measured with a Cannon-Fenske viscometer with an uncertainty estimated as up to 0.5 percent. The curve represents results by C.M. White. The data observed in our experiments are plotted using a value d_c = 0.160 cm in computations of values of the abscissae to bring them into agreement with the curve. This procedure was used because the effective internal diameter of the capillary d_c was not determined with flow observations using the tube uncoiled. As Q approaches zero, $\Delta p d_c{}^4/(\mu QL)$ will approach the Hagen-Poiseuille constant of $128/\pi$ (instead of 27.6 with use of d = 0.142 cm.) Deviation of the points from the line gave a standard deviation σ of about 1 percent for $R(d_c/D)^{1/2} \geq 17$. The precision depends on the precision of observations of the three quantities Δp, μ, and Q, while in use the precision of observations of μ would be better because observations of only two quantities Δp and Q would be needed. Data for a fourth liquid (ν = 6.4 centistokes) have been omitted from figure 1 because of disagreement with the data for the other three liquids. For this fourth liquid all values of $R(d_c/D)^{1/2}$ were in the range from 2 to 7, with values of $\Delta p d^4/(\mu QL)$ in the range from 0.91 to 0.97 times 27.6.

The increase of flow resistance due to curvature of the capillary, with the consequent Reynolds number dependence as illustrated in figure 1, is considered too great, at values of $R(d_c/D)^{1/2}$ above 15, for use with the viscometer.

[1] C. M. White. Streamline flow through curved pipes. Proc Roy Soc A, 123, 645, (1929)

Experiments with the second coil (nominal d, D and L equal 0.465, 25.4,1073 cm, respectively) were undertaken to determine the influence of the curvature, negligible as reported by C. M. White [1], when $R(d/D)^{1/2} \leq 15$. Data using this coil are presented in figure 2, using an "effective" value of $d_c = 0.473$ cm as derived from the intercept value of $\Delta p d^4/(\mu QL) = 38.08$. The straight line through the data points is a least squares line representing the equation

$$\Delta p d^4/(38.08\mu QL) = 1+0.00298\ R(d_c/D)^{1/2} \quad (3)$$

or

$$\pi \Delta p {d_c}^4/(128\mu QL) = 1+0.00298\ R(d_c/D)^{1/2} \quad (4)$$

for the case $R(d_c/D)^{1/2} \leq 15$, with a computed standard deviation of 1.0 percent. C. M. White's data for 3 capillary coils are represented by dashed lines labeled with values of $(d_c/D)^{1/2}$ as a parameter. He observed that for two of three coils Δp was essentially independent of R when $R(d_c/D^{1/2} \leq 10$.

It is possible that lack of steady flow may cause a Reynolds number dependence as demonstrated by our data. Both coils had adequate lengths of tubing (36 and 54 times d for the smaller and larger diameters, respectively) ahead of the pressure taps for an entrant or stilling length. These stilling lengths were not curved as possibly would have been desirable. Therefore, pending further investigation, it seems that our data should be used as a basis for the viscometer if the present coil is the design basis.

Although the Reynolds number dependence is smaller than demonstrated in figure 1, it is large enough to prevent use of the Hagen-Poiseuille relationship, equation (2), without modification when the capillary is coiled. Equation (4) can be solved for μ to give

$$\mu = \pi \Delta p {d_c}^4/(128QL) - [0.00298(4)\rho Q/\pi d_c](d_c/D)^{1/2},\ R(d_c/D)^{1/2} \leq 15 \quad (5)$$

in which R has been expressed as the quantity $4\rho Q/(\pi d_c \mu)$. Equation (5) cannot be used to compute μ from the observations without knowledge of ρ. However a "guessed" value of ρ can be used to derive μ because the 2nd term on the R.H.S. of (5) is a small correction to the first term. This value of μ along with the observations with the densitometer can be used to derive a more accurate value of ρ and then a more accurate μ.

Using the criterion that $R(d_c/D)^{1/2} < 15$, and expressing R as $4Q/(\pi d_c \nu)$, it can be seen that the capillary tubes should be sized for the fluid with the smallest viscosity. Fluid with larger viscosity would give a smaller value of R at the same values of Q and d_c. Two tubes appear to be necessary to cover the range of viscosities from 0.5 to 50 centistokes. A tube of small diameter for the range 0.5 to 5 centistokes, and the larger for 5 to 50 centistokes.

2.3. Density Sensor - Theory. The density sensor described in this report is based on the principle that, for a given flow Q, the pressure drop Δp_n across a fixed constriction such as a nozzle or orifice is porportional to the density ρ of the fluid flowing through the constriction of area A and to a coefficient of discharge K. The well known equation for flow through the orifice,

$$Q = KA\sqrt{2\Delta p_n/\rho}, \tag{6}$$

can be solved for ρ to get

$$\rho = 2K^2A^2\Delta p_n/Q^2. \tag{7}$$

In the case of a given restriction and a fixed volumetric rate of flow Eq. 7 can be written

$$\rho = C_1K^2\Delta p_n \tag{8}$$

where C_1 is a constant depending on the cross sectional area of the throat and the volumetric rate of flow ($C_1 = 2A^2/Q^2$). Thus, the fluid density ρ is directly proportional to the pressure drop across the restriction and the square of the coefficient of discharge. Unfortunately K is not a constant but rather is a function of the Reynolds number R_n, which can be stated as the relationship

$$K = f(R_n/K^2) \tag{9}$$

where $R_n = d_n\rho V_n/\mu$ is based on the diameter of the throat of the nozzle d_n. Q/A can be substituted for V_n and ρ/K^2 can be used from (7) to derive that

$$K = (2\ d_nA)f(\Delta p_n/\mu Q). \tag{10}$$

where $A = \pi d_n{}^2/4$.

As is usually the case Q will be set at some constant value (depending on the speed of the flow generator) and using a wide range of liquids of different densities and viscosities, we can plot the relationship f between measured values of K and $\Delta p_n/\mu$. For various flowrates K vs. $\Delta p_n/(\mu Q)$ is the relation to be plotted. As $\Delta p_n/g$ was measured as the product of density ρ and height h of the fluid in the manometer, $(\Delta p_n/g\mu Q)$ will be plotted.

2.4 Density Sensor - Results. Figure 3 is a plot of K vs. $(d_n Ag) \Delta p_n/(g\mu Q)$, for a nozzle having a diameter of 0.053 cm, obtained by flowing 9 liquids having a range of viscosities from about 1 to 10 centistokes and flowrates from 0.8 to 3.7 cm^3s^{-1}. Liquid density, however, was not varied significantly from .8 g cm^{-3}. In these experiments Q was varied so as to establish the curve with fewer liquids of different viscosities.

A continuous curve of K vs. $\Delta p_n/(g\mu Q)$ was not obtained when different liquids were used in the apparatus. Discontinuities in the curve of about one percent of K between liquids 12 and 15 and about one percent between liquids 19 and 20 are observed. An expected uncertainty of about 0.5 percent in the measured values of μ cannot explain the discontinuities and thus greater accuracy is needed in the measurement of the pressure drop across the nozzle, in the measurement of the rate of flow of the liquid, or in measurement of ρ used in the tests.

The data in figure 3 appears to be reasonably typical for the coefficient of discharge vs. Reynolds number relationships exhibited for nozzles. These relationships have been correlated by Benedict [2] with the result that the coefficient can be best expressed as a power series in the logarithm of the Reynolds number. A least squares fit for the data in Fig. 3 gives for this function

$$K = -2.6051 + 2.3243 (\log \Delta p/g\mu Q) - 0.51474 (\log \Delta p/g\mu Q)^2 + .03871 (\log \Delta p/(g\mu Q)^3 \qquad (11)$$

with a computed standard deviation of 0.5 percent of K. This uncertainty of 0.5 percent represents an uncertainty in ρ of 1 percent because ρ is proportional to K^2 (see equation 7). It is believed that some improvement in the apparatus, such as in the lead screw, piston seal, and pressure sensor can reduce the uncertainty to about 0.5 percent.

Note that the well - known similarity procedure used above can apply to liquids of density different than used provided that other phenomena such as heat transfer and vaporization in the nozzle do not interfere.

2.5 Summary of Theory and Results. Measurements of Δp and Q in the viscometer, along with use of other specified and known quantities in equation 5, gives the fluid viscosity μ with a computed σ of 1 percent. Equation (5) is applicable when $R(d_c/D)^{1/2} \leq 15$, where R is the Reynolds number $4 \rho Q/(\pi d_c \mu)$. As ρ is presumably unknown, a "guessed" value of ρ probably can be used to compute μ with sufficient accuracy. A criterion for this is discussed next.

The measured value of μ, and measurements of Δp_n and Q in the densitometer along with use of other specified and known quantities in equations (11) for K and (7) for ρ, gives the density with a computed σ of 1 percent. Measured ρ should be compared to the previous "guessed" ρ to be sure of

[2] R. P. Benedict. Most probable discharge coefficients for ASME flow nozzles. Trans of the ASME, 734, (1966)

sufficient accuracy for μ. If a 0.5 percent error in μ can be accepted as reasonable, equation (5) can be used to show that the relationship

$$\frac{0.012\Delta\rho Q}{\pi d_c \mu} \left(\frac{d_c}{D}\right)^{1/2} \leq 0.005$$

must be satisfied, in which $\Delta\rho$ is the difference between measured and guessed values of ρ. An error of 0.5 percent in μ will not cause significant error in ρ due to the Reynolds number dependence, except where K depends strongly on Reynolds number, such as near K=0.85.

Improvements in the apparatus and in the measurements of the properties of the calibration liquids probably would lead to an accuracy for the apparatus greater than demonstrated.

3. EXPERIMENTAL APPARATUS

The viscosity sensor described in this report is based on the principle of using a known rate of flow through both sensors. The apparatus is shown schematically in Figure 4 and consists of a positive displacement flow generator driven with a variable speed motor and a precision machine screw. Although the motor has variable speed capability it is operated at a constant speed. The speed is monitored with a frequency meter actuated by a shaft encoder on the motor shaft. The constant motor speed drives the piston of the flow generator at a constant linear speed thereby generating a constant flow through the capillary tube. The differential pressure Δp across the tube is measured with a pre-stressed diaphragm differential pressure transducer and an electronic readout having both analog and BCD output. The flow generator is equipped with limit switches which not only limit the piston travel but also reverse direction of rotation of the drive motor and operate the 4-way solenoid valve as follows. With the piston traveling from left to right, liquid from the supply flows through the 4-way valve from P to Cyl. #1 to fill the cylinder. Simultaneously, liquid is forced from the cylinder through the 4-way valve (Cyl. #2 - R), into the capillary tube and back to supply. When the limit switch at the right is actuated, the direction of rotation is reversed and the solenoid valve connects P to Cyl. #2 and Cyl. #1 to R. Except for a brief period at each end of the piston travel the flow through the capillary tube is continuous and constant. The duration of the piston travel between limit switches is sufficient to establish equilibrium conditions of flow so that the differential pressure transducer measures and displays a constant value of Δp.

4. DESIGN

4.1 <u>Viscometer Coils</u>. Let the ID of the large coil be 0.4648 cm with D of 25.4 cm and a maximum $(d/D)^{1/2}R$ of 15; thus, the maximum allowable value of R is 110.9. For these conditions and using a liquid having a viscosity of 6.64 centistokes, $Q_{max} = 2.67\ cm^3 s^{-1}$. Let us set a minimum pressure drop of 10 cm of mercury (133,320 dynes cm^{-2}) for a liquid having a kinematic viscosity of 5 centistokes.

Rearranging Eq. 1 to solve for L

$$L \simeq \pi(p_1-p_2)d_c{}^4/(128\mu Q)$$

and since $\mu = \nu\rho$ and using $\rho = 0.8$ g cm^{-3}

$$L \simeq \pi \times 133{,}320 \times .4648^4/(128 \times .05 \times .8 \times 2.67) = 1430 \text{ cm.}$$

Thus, a coil with a developed length of 1430 cm will produce differential pressures of 10 and 100 cm of mercury for liquids having viscosities of approximately 5 and 50 centistokes respectively. These values are approximately proportional to those shown in Figure 2 which are for a coil having a developed length of 1073 cm.

It was necessary to design a second coil for liquids of viscosities less than 5 centistokes. Let d be 0.10 cm, D = 25.4 cm, and again limit $(d/D)^{1/2}R$ to 15. The maximum value of R is seen to be 239. If we design for a liquid having a viscosity of 1 centistoke, Q = 0.1877 $cm^3 \cdot s^{-1}$. For a liquid of 1 centistoke let the pressure drop be 20 cm of mercury (26640 dynes$\cdot$cm^{-2}). Solving for L as before we find

$$L = \frac{3.1416 \times 266640 \times 0.10^4}{128 \times 0.01 \times 0.8 \times 0.1877} = 436 \text{ cm.}$$

This coil having a developed length of 436 cm should develop pressure drops of 10 and 100 cm of mercury for liquids having viscosities of 0.5 and 5 centistokes respectively.

4.2 <u>Nozzle for Density Determinations</u>. We can determine the nozzle diameter for density measurements from Eq. 6 and an arbitrary flow of 2.67 cm^3s^{-1}, the same as that required for the first viscous flow coil. At K of about 0.90 (see Fig. 3), a maximum density of 1.6 g cm^{-3}, a flow of 2.67 cm^3s^{-1}, and a pressure drop of 100 cm of mercury (1,33,200) dynes cm^{-2}), d_n = .054 cm. The convergent (inlet) part of the nozzle should be a circular arc (in cross-section) with a radius equal to roughly twice the throat diameter.

4.3 <u>Flow Generator</u>. Now that the sizes of the two laminar flow coils and the nozzle have been determined and the maximum flowrate has been established, we can design the flow generator. At the maximum flow of 2.67 cm^3s^{-1} let the travel time be 90 seconds; this will allow at least 45 seconds during which time the flow will be steady and the pressure drops will be constant.

Let the inside diameter of the cylinder be 2 inches (5.08 cm) and the drive shaft be 0.5 in (1.27 cm); the effective cross sectional area is then 19.001 cm^2. For a maximum flow of 90 second duration a linear displacement of 12.64 cm or 4.978 inches is necessary. If we design the lead screw with 20 threads per inch, 100 revolutions will be required for full travel and at maximum flow a speed of about 67 RPM. Ignoring friction the drive motor should have a theoretical torque of about 0.5 inch pounds; however, to allow for other line losses, friction etc., the drive motor should deliver at least 3 inch pounds at 67 RPM.

4.4 The entire apparatus shown schematically in Fig. 4 consists of the flow generator, the two coils, nozzle, valving and tubing. Other major components are listed below:

1. Reversing drive unit and controls. Minimum torque 3 inch pounds. (explosion proof) Not shown in Fig. 4.

2. Limit switches, two (explosion proof). Not shown in Fig. 4.

3. Four-way valve, solenoid operated (explosion proof).

4. Pressure gauge - Range 0-100 cm of mercury transducer to be explosion proof.

5. Electronic readout for above transducer. Model to be compatible with transducer.

6. Frequency meter. Not shown in Fig. 4.

7. Shaft encoder for drive unit. Designed for minimum frequency of 10,000 hertz at maximum RPM. Not shown in Fig. 4.

All components of the apparatus shown in Fig. 4 must be of explosion proof design. Instrumentation and other remotely located components not designed to be used in a hazardous environment, should be connected to the apparatus with cables, etc. encased within tubing free of leaks.

5. OPERATION AND CALIBRATION OF INSTRUMENT

5.1 Operation. The flow generator and assembly should be located as close to the pipeline as possible so as to minimize any possible difference in temperature between the liquid flowing in the pipeline and that through the instrument. If no openings are available in the pipeline, two holes shall be tapped for 3/8 inch tubing fittings. The upstream opening shall be connected to P (the supply for the assembly) and R (the return) shall be connected at least one pipe diameter downstream.

After these connections are made and before pressure is applied to the line close all valves on the assembly except the equalizer valve on the Δp transducer. When flow has been established open valves C_1, C_2, H and R. To bleed some of the air from the system open each of the four bleed valves from left to right until liquid flows from the bleed connection which should be piped to an open top collection vessel. This should remove all air and vapor from the lines, however, air and vapor will be trapped in both ends of the flow generator. To remove this air start the flow generator and operate at about maximum speed. WARNING: Equalizer valve must be open to prevent excessive differential pressure across the transducer during bleeding operations. With the piston of the generator moving from left to right open the bleed valve on the right end of the generator and leave open until liquid flows from bleed connection or until just before reversal of direction. After reversal, open bleed on left end. Repeat this procedure until no air or vapor passes from bleed connection.

The instrument is now prepared for the making of measurements.

1. Adjust the motor speed. There will be two set constant speeds. The lower will be used in conjunction with the smaller diameter coil C_1-C_1 for low viscosity measurements. The higher speed will be used for both the nozzle and coil C_2-C_2 which is used for liquids having a viscosity in excess of about five centistokes.

2. At the low speed close valves C_2, C_2, N and N. Repeat the bleeding operation through all four bleed valves, close the equalizer valve, observe the output of the Δp cell on the electronic manometer.

3. For liquids of higher viscosity, open valves C_2 and C_2 and close valves C_1 and C_1. Adjust motor speed to the higher value, open equilizer valve, repeat bleeding operation, close equalizer valve and observe output of electronic manometer.

4. For density determinations, open valves N, close valves C_2 and C_1, adjust speed to higher value, open equilizer valve, bleed through all four bleed valves, close equalizer valve and observe output of electronic manometer.

5. Caution: Electronic manometer output should be observed when it has become steady after about the mid-point of piston travel.

5.2 Calibration of Viscosity Sensor. As shown in Eq. 5 viscosity μ is approximately proportional to the pressure drop across the capillary tube. Coil C_1-C_1 was designed for liquid having viscosities ranging from 0.5 to 5 centistokes for which the pressure drops across the differential pressure transducer should be about 10 to 100 cm of mercury respectively. To calibrate this coil select a liquid having a viscosity of approximately 2 centistokes and its value should be known with sufficient accuracy. This liquid can be in a container at an elevation several feet above the instrument assembly. Connect both P and R to the container preferably some distance apart. Precaution: Drain instrument assembly then flush thoroughly with test liquid. Do not allow flushing liquid to return to container. After instrument is flushed connect R to container, bleed as described in previous section. Then close equilizer valve. Adjust motor speed so that

$$\mu \simeq h/2500,$$

where differential pressure h is measured in cm of mercury. In the above example where $\nu = 2$ centistokes (0.024 poise), the speed should be adjusted so that h = 60 cm of mercury (Q near 0.188 cm^3s^{-1}). It may be preferable to round off the speed to some convenient number as measured on the frequency meter.

The above example does not account for the Reynolds number dependence as illustrated in equation 5, and therefore will give only approximate results for other fluids or rates. For the general case equation (5) should be used as

$$\mu = C_2 \frac{h}{Q} - C_3 \rho Q$$

with constants C_2 and C_3 to be determined by calibration. This can be done by running with a fluid of known viscosity and density at two known rates Q which gives two simultaneous equations to solve for C_2 and C_3. The rates Q should be selected such that $R(d_c/D)^{1/2} \leq 15$. If Q is not accurately known it can be held constant while data for h is taken with two fluids each with known but different viscosities. Constant C_2 and C_3 also can be solved using these data.

Coil C_2-C_2 was designed to cover the range of viscosities from 5 to 50 centistokes and to develop differential pressures of 10 and 100 cm of mercury respectively. Again, select a test liquid having a midrange viscosity of about 22 centipoise. After thorough flushing and bleeding (do not contaminate test liquid with flushing liquid) adjust the speed so that.

$$\mu \simeq 0.004h$$

where again h is measured in cm of mercury. In the above example where $\mu = 22$ centipoise (0.22 poise), $\Delta p = 55$ cm of mercury. Here again, it may be more convenient to round off the indicated frequency to some convenient value. For the more general case (other fluids and rates) repeat the application of equation 5 as outlined above.

Although it is not necessary it is believed that this speed should be held to the same value for the density measurements to be described.

5.3 <u>Calibration of Density Sensor</u>. As shown in the section on theory of the density sensor it will first be necessary to establish the relation between the measured quantities

$$K \text{ vs. } \Delta p_n/\mu,$$

to establish a curve similar to that of Figure 3. K is determined by Eq. 7 with measured quantities Q, A, Δp_n, and ρ, and μ can be determined either by operation of the calibrated viscosity sensor or by other suitable viscometers. Both ρ and μ must be known with sufficient accuracy. Q is fixed by the speed of the flow generator and preferably is the same as that used with coil C_2-C_2. It will be necessary to use a wide range of test liquids of known viscosities and densities to cover the entire range of K which is determined by the range of viscosities and densities. This is necessary so that extrapolation of the curve will not be required to determine K from measurement of Δp_n and μ.

Once this curve is established it will be possible to determine density ρ as follows:

1. Determine μ by experiments in either C_1-C_1 or C_2-C_2 as required.

2. At the predetermined constant speed, measure Δp_n across the nozzle.

3. Determine K from $\Delta p_n/\mu$ and the calibration curve.

4. Solve for ρ from Eq. 8 where

$$\rho = C_1 K^2 \Delta p_n$$

Should the density and/or the viscosity of the liquid flowing in the pipe undergo a change, at least two minutes will be required for this change to be shown in the output of the electronic manometer even at the higher speed of the flow generator. If the change should occur at midstroke of the piston the entering liquid will form a mixture with that fluid already in the cylinder.

6. LIST OF FIGURES

Table 1

Symbol	Definition	Dimensional Units
A	Cross sectional area of throat	cm^2
C	Constant	None
D	Diameter of coil	cm
d_n	Throat diameter of nozzle	cm
d	Nominal internal diameter of capillary tube	cm
$\Delta p = p_1 - p_2$	Differential pressure	$dynes \cdot cm^{-2}$
g	Acceleration of gravity	$cm \cdot s^{-2}$
h	Height of liquid column	cm
L	Length of tube between pressure taps	cm
μ	Viscosity, poise	$g \cdot cm^{-1} \cdot s^{-1}$
p	Pressure	$dynes \cdot cm^{-2}$
Q	Volumetric rate of flow	$cm^3 \cdot s^{-1}$
ρ	Density of fluid	$g \cdot cm^{-3}$
R	Reynolds number, $\rho Vd/\mu$	None
ν	Kinematic viscosity, μ/ρ, stokes	$cm^2 \cdot s^{-1}$
V	Average velocity, fluid	$cm \cdot s^{-1}$
d_c	Effective internal diameter of capillary tube	cm
σ	Computed standard deviation	percent
K	Coefficient of discharge	None

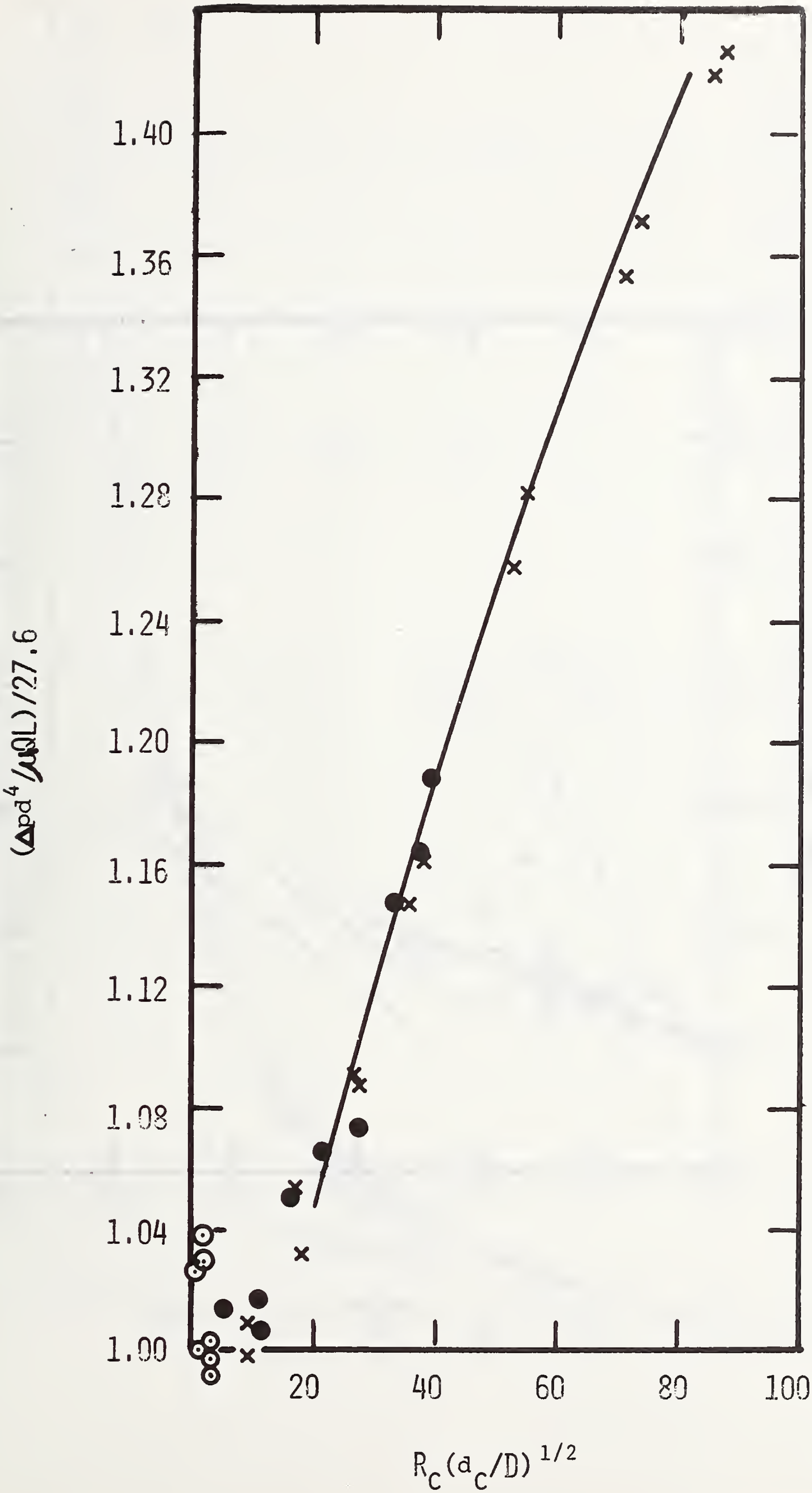
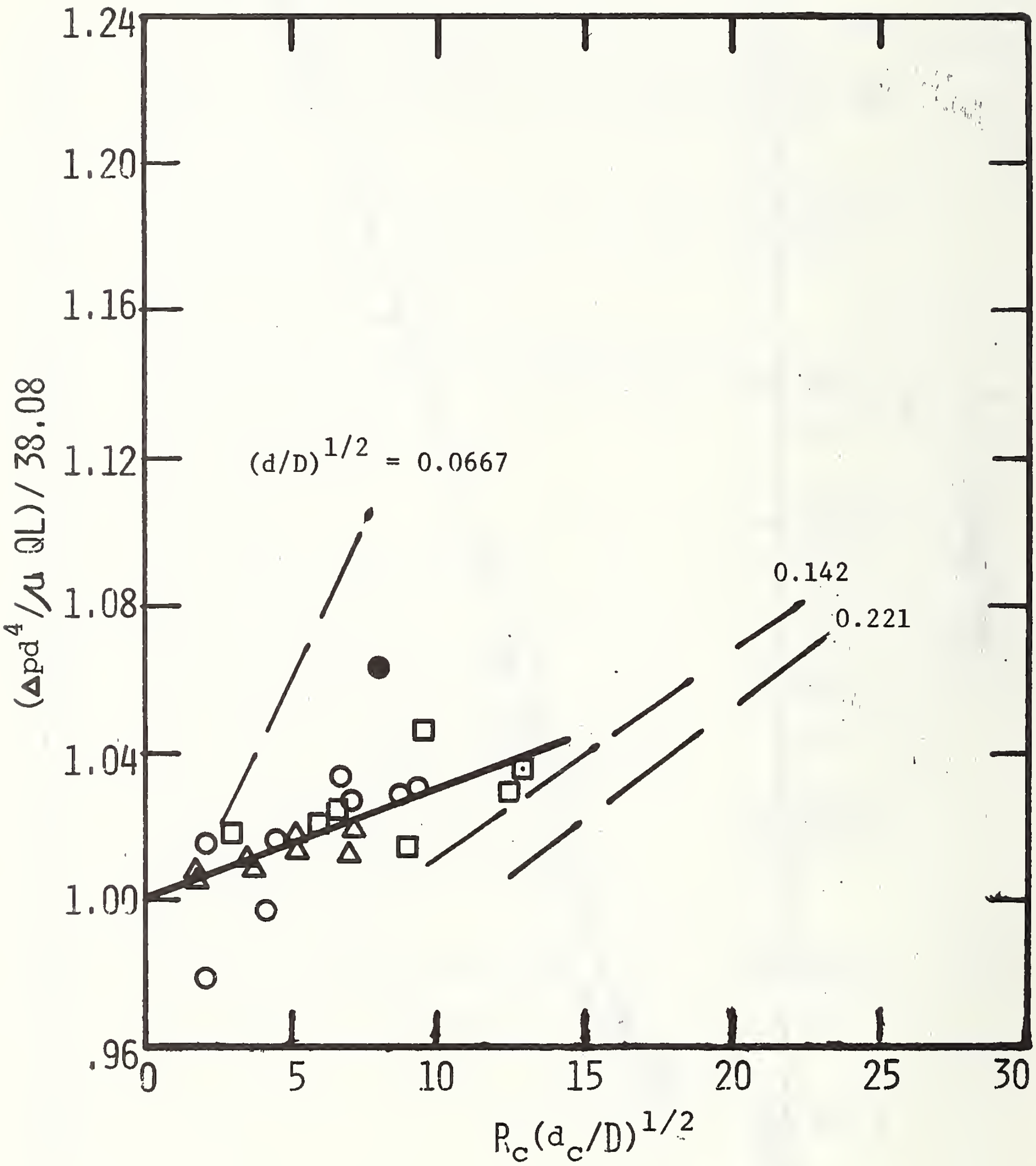
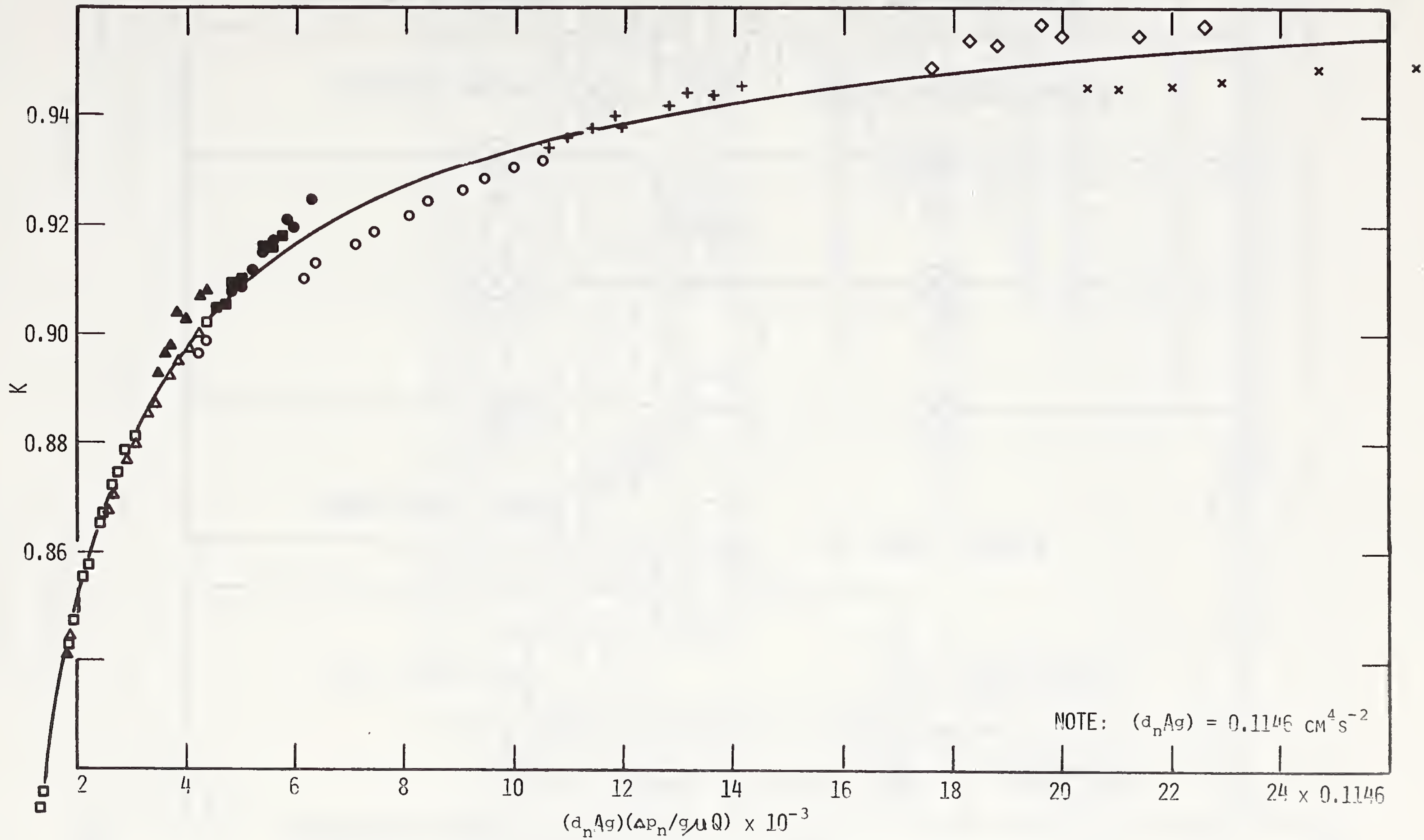

1.40
1.36
1.32
1.28
1.24
1.20
1.16
1.12
1.08
1.04
1.00
20
40
60
80
100
$(\Delta p d^4/\mu Q L)/27.6$
$R_c(d_c/D)^{1/2}$

Figure 1



1.24
1.20
1.16
1.12
1.08
1.04
1.00
.96
$(\Delta p d^4/\mu QL)/38.08$
$(d/D)^{1/2} = 0.0667$
0.142
0.221
0
5
10
15
20
25
30
$R_c(d_c/D)^{1/2}$

Figure 2



K
0.86
0.88
0.90
0.92
0.94
2
4
6
8
10
12
14
16
18
20
22
24 x 0.1146
$(d_n Ag)(\Delta p_n / g \mu Q) \times 10^{-3}$
NOTE: $(d_n Ag) = 0.1146\ cm^4 s^{-2}$

Figure 3

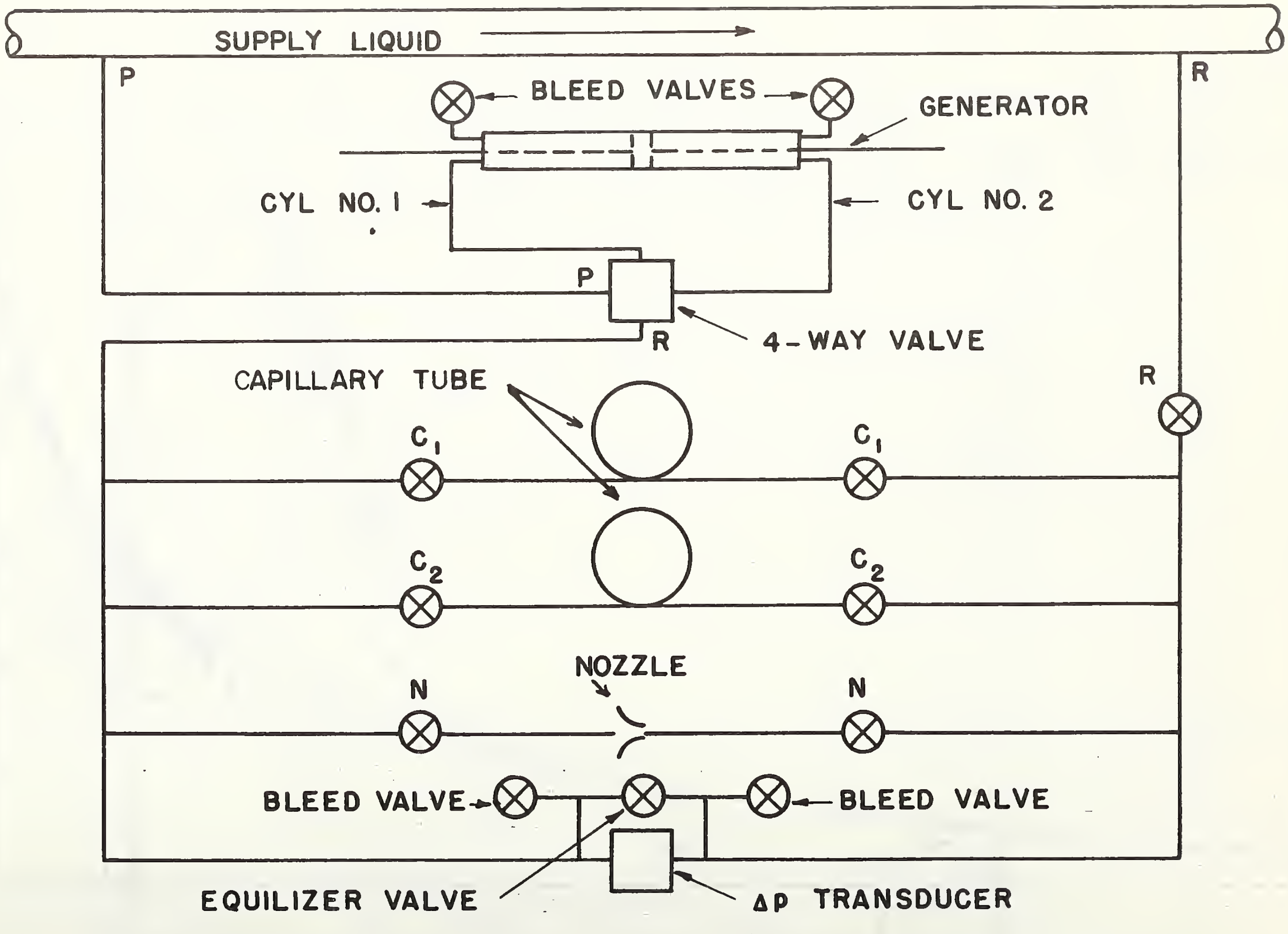

SUPPLY LIQUID
P
R
BLEED VALVES
GENERATOR
CYL NO. 1
CYL NO. 2
P
R
4-WAY VALVE
R
CAPILLARY TUBE
C_1
C_1
C_2
C_2
NOZZLE
N
N
BLEED VALVE
BLEED VALVE
EQUILIZER VALVE
Δp TRANSDUCER

FIGURE 4

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16. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

In-line density and viscosity sensors for liquids are developed to utilize measurements of differential pressure across a nozzle and a coiled capillary tube respectively, with known flowrates through each provided by a flow generator. Theory and principles of operation and instructions for calibration and use of the sensors are discussed, along with design consideration for the sensors and associated equipment. A calibration of the sensors demonstrated that viscosity and density each could be measured with a computed standard deviation of one percent. Viscosity was varied over the range of about 1 to 11 centistokes whereas density of the fluids used was near 0.8 g cm^{-3}. Application of well known similarity considerations is used to make the results applicable to other liquid densities, provided influence of other liquid properties (high vapor pressure, for instance) does not interfere.

17. KEY WORDS *(six to twelve entries; alphabetical order, capitalize only the first letter of the first key word unless a proper name; separated by semicolons)*

Capillary tubes, density sensor, flow nozzle, laminar flow, liquid properties, viscosity sensor.

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