# Torsional Buckling of Composite Cylindrical Shells 

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## Engineering Mechanics Section

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Prepared for
National Aeronautics and Space Administration
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Hampton, Virginia 23365

U. S. DEPARTMENT OF COMMERCE, Frederick B. Dent, Secretary

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of these materials in complex structures requires a thorough knowledge of the behavior of these materials when subjected to various mechanical loadings. Considerable data already exist on the tensile and compressive behavior of these materials, whereas shear data are not abundant. The need for shear data prompted experimental investigations (refs. 3 and 4) of the torsional behavior of thin-walled cylindrical shells laminated with unidirectional composites and aluminum cylindrical shells reinforced with unidirectional composites.

The primary purpose of the investigation reported herein was to investigate the elastic torsional buckling strength of all-composite and composite-reinforced metal shells in which the composite laminate is not unidirectional. Test results on 39 specimens are reported. Boron/epoxy composite specimens in three lengths and two diameters, and graphite/epoxy composite specimens in two lengths and two diameters were tested. In addition, three configurations of composite-reinforced.al uminum-alloy and titanium-alloy specimens were studied. The effects of stacking sequence, direction of loading, and linear scaling of specimen dimensions were studied and an investigation of a stronger or "more optimum" stacking sequence was analytically determined and subsequently investigated experimentally.

This investigation was conducted at the National Bureau of Standards under the sponsorship and with the financial assistance of the National Aeronautics and Space Administration, Langley Research Center.

## 2. NOMENCLATURE

### 2.1 Units

The units for physical quantities used in this report are given in both the U. S. Customary Units and the International System of Units (SI). Measurements and calculations were made in U. S. Customary Units. Conversion factors relating the two systems are presented in Appendix A3 of reference 5.

### 2.2 Designations

The system for designating specimens which identifies both the specimen material and the specimen length is given in table 1. In the following tables a sample number is appended to this designation.

The stacking sequence or ply orientation for a laminate is designated by $\left(\alpha_{1}, \alpha_{2}, \alpha_{3},--\right)$ where the $\alpha_{1}$ is the ply angle of the outermost lamina, $\alpha_{2}$ is the ply angle of the second lamina, etc. The subscript "s" indicates cases where the stacking sequence is symmetric about the mid-thickness of the shell. Positive ply angle is measured clockwise from a generator line on the cylindrical
surface as shown in figure 1.

## 3. ANALYSIS

### 3.1 Torsional Buckling

The computer program "Buckling of Generally Orthotropic Cylinders," which was developed by Chao (ref. 6), was used to predict the buckling torques for each specimen. These were compared with the experimental results. In the program, buckling loads are calculated for a multilayered cylindrical specimen loaded in combinations of torsion, axial compression and radial pressure. Chao's theory treats orthotropic layers of composite material whose principal material axes can be oriented in any direction. Specimen buckling is analyzed using Timoshenko's general equations of equilibrium (ref. 7). Using the concept of reduced flexural rigidity, the bending and membrane forces are uncoupled. The specimen is then assumed to have orthotropic el astic properties. The program seeks the solution with the lowest buckling torque by iterating on the number of circumferential buckling waves.

### 3.2 Design of an Optimum Laminate

Wu investigated the torsional buckling strength of all-composite four-ply cylindrical shells using two specific stacking sequences (ref. 8, fig. 27). Using this as a point of departure, several similar curves were produced using the Chao analysis in an effort to determine an "optimum" four-ply laminate configuration for both all-composite and composite-reinforced-metal cylinders. A comparison of some of these curves, including the stacking sequences chosen by $W u^{*}$, is shown in figure 2. Similar curves for boron/epoxy-reinforcedtitanium cylinders are shown in figure 3.

## 4. SPECIMENS

The experimental program consisted of room-temperature torsional testing of 39 cylindrical shells designed to fail by elastic torsional buckling. Replicate samples were tested for each specimen configuration. Table 2 presents the geometry, laminate thicknesses, filament volume fractions, and stacking sequence for each of the individual specimens. Composite specimens were tested in three lengths and two diameters, and reinforced-metal specimens were
*The buckling torques of these stacking sequences are computed for a negative loading according to Chao's sign convention.
tested in one length and one diameter. The dimensions for the allcomposite specimens wese chosen such that two diameter-to-thickness ( $D / t$ ) ratios and three length-to-radius ( $L / r$ ) ratios were tested.

### 4.1 Composite Materials

The boron/epoxy material used in this investigation was purchased in 3.0-in ( $7.6-\mathrm{cm}$ ) wide tape. The tape contained $0.004-\mathrm{in}$ ( 0.01 cm ) diameter filaments pre-impregnated with a $350^{\circ} \mathrm{F}\left(180^{\circ} \mathrm{C}\right)$-curing epoxy resin system. This material was supplied with a 0.001-in ( $0.003-\mathrm{cm}$ ) fiberglass scrim cloth to facilitate handling.

The graphite/epoxy material used was a high-modulus graphite fiber pre-impregnated with a $350^{\circ} \mathrm{F}$ ( $180^{\circ} \mathrm{C}$ )-curing epoxy resin system. The pre-impregnated material was supplied by the manufacturer in the form of $3.0-\mathrm{in}(7.6-\mathrm{cm})$ wide tape.

All uncured resin materials were stored at $0^{\circ} \mathrm{F}\left(-18{ }^{\circ} \mathrm{C}\right)$ prior to use.

### 4.2 Composite-Reinforced-Aluminum-Alloy Specimens

The aluminum-alloy cylinders, which were to be reinforced with composite material, were machined from a single lot of 6061T6 seamless drawn tubing. These cylinders had a nominal wall thickness of 0.022 in ( 0.056 cm ) and a nominal outer diameter of 6.0 in ( 15 cm ). The specimens tested had a gage length of 10.0 in ( 25.4 cm ). The thickness of the composite reinforcement was approximately equal to the thickness of the aluminum-alloy cylinder. The dimensions of these specimens are shown in table 2.

### 4.3 Composite-Reinforced-Titanium-Alloy Specimens

The titanium-alloy cylinders, which were to be reinforced with composite material, were rolled from 6Al-4V titanium-alloy sheet and butt welded along the seam. The finished cylinders had a nominal wall thickness of 0.025 in ( 0.064 cm ), a nominal outer diameter of 6.0 in ( 15 cm ), and a gage length of 10.0 in ( 25.4 $\mathrm{cm})$. The dimensions of these specimens are shown in table 2 .

### 4.4 Material Properties

Longitudinal elastic modulus ( $E_{11}$ ), transverse elastic modulus $\left(E_{12}\right)$ and Poisson's ratio ( $v_{12}$ ) were determined for all materials used in this program. The preparation of and testing procedure for these specimens are described in the appendix to reference 3. The in-plane shear modulus ( $G_{12}$ ) was determined for each material as the average value obtained from torsional tests on the 0 -degree unidirectional composite specimens and the all-metal specimens tested in this program and in the program reported in refs. 3 and 4. The average material properties measured on the aluminum-alloy,
titanium-alloy, boron/epoxy, and graphite/epoxy materials are shown in table 3 .

The values of el astic moduli for the composite materials, $E_{11}, E_{12}$, and $G_{12}$, were corrected for thickness variation when they were used in the analysis. A rule-of-mixtures equation,

$$
E_{k}=\frac{t_{c} E_{c}+\left(t_{k}-t_{c}\right) E_{m}}{t_{k}}
$$

was used where

$$
\begin{aligned}
& \mathrm{E}_{\mathrm{k}}=\text { the corrected modulus, } \\
& \mathrm{t}_{\mathrm{c}}=\text { nominal thickness of the composite, } \\
& \mathrm{t}_{\mathrm{k}}=\text { measured thickness of the composite, } \\
& \mathrm{E}_{\mathrm{c}}=\text { measured modulus of the composite }
\end{aligned}
$$

$$
\text { and } \quad E_{m}=\text { nominal modulus of the epoxy matrix material. }
$$

### 4.5 Specimen Fabrication

Boron/epoxy and graphite/epoxy-composite cylinders were fabricated in a manner similar to that described in refs. 3 and 9. The composite material was laid-up over a mandrel and covered by one ply of treated release cloth and the number of fiberglass bleeder plies required to absorb the excess of epoxy resin from the curing sample. Pressure was applied by enclosing the layup in FEP heat-shrinkable fluorocarbon tubing. A thermocouple was embedded in the laminate and the specimen was cured using the manufacturer's suggested time-temperature cycle.

The aluminum-alloy and titanium-alloy cylinders which were reinforced on the outer surface with composites were chemically cleaned and primed immediately prior to the layup process. A single layer of film-epoxy adhesive was applied to the metal cylinder followed by the required number of plies of prepreg material, glass release cloth, bleeder, and heat-shrinkable tubing. With the boron/epoxy prepreg, it was necessary to add an additional layer of 0.001-in ( $0.003-\mathrm{cm}$ ) fiberglass scrim cloth between the film-epoxy adhesive and the first ply of boron/epoxy in order to balance the total laminate. The composite layup techniques for these cylinders were similar to those used for the all-composite specimens, except that the metal cylinder served as a non-removable mandrel.

## 5. TESTING PROCEDURE

### 5.1 Instrumentation

Each of the cylindrical test specimens was instrumented on the outer surface of the shell at the midlength. Five $45^{\circ}$ rosette foil strain gages having a gage length of 0.25 in ( 0.63 cm ) were equally spaced around the specimens. These were oriented to measure strains in the axial direction and at $\pm 45^{\circ}$ to the axial direction. Several specimens were tested with additional strain gages in an attempt to detect the earliest indication of buckling and to check the uniformity of strain distribution.

### 5.2 End Fixtures

The end fixtures used for the all-composite specimens are shown in figure $4 a$. They are close-fitting end plugs which are bonded to the inner surface of the cylinder with an epoxy adhesive. The end fixtures used to test the composite-reinforced-metal specimens are shown in figure 4b. In these fixtures, both the inner and outer surfaces of the cylinders are bonded into the end fixtures.

### 5.3 Torsional Testing Procedure

Thirty-three specimens were tested in the Engineering Mechanics Laboratory of the National Bureau of Standards in a 40,000-lbfin ( $4,500-\mathrm{N}-\mathrm{m}$ ) capacity torsional testing machine (ref. 10). Six other specimens were tested at the NASA-Langley Research Center, Structures and Materials Laboratory. These included two 40-in (101.6cm ) long all-composite specimens tested in a $60,000-1 \mathrm{bf-in}$ (6,800-$\mathrm{N}-\mathrm{m}$ ) capacity torsional testing machine and four composite-reinforced metal specimens tested in torsion in a $100,000-1 b f-i n(11,300-N-$ $\mathrm{m})$ capacity triaxial testing machine. A typical test setup in each of these machines is shown in figure 6. Torque was transmitted from the testing machine heads to the specimen fixtures by means of rectangular keys inserted through slots in the testing machine heads and the specimen end fixtures. All tests were performed at room temperature and at a uniform twisting rate of $0.005 \mathrm{rad} / \mathrm{min}$.

During the torsional tests performed at the National Bureau of Standards, strain measurements were made at discrete values of torque. The magnitude of the torque increment between successive sets of strain readings was determined by the specimen behavior during the previous increment. During tests performed at the NASALangley Research Center, torque and strain were recorded at a virtually continuous rate in NASA-Langley's central digital-data recording facility.

All specimens were first loaded with a negative (counterclockwise) torque (fig. l). This direction of twist caused a tensile stress in the filaments of cylinders fabricated with positive ply-orientation
angles. Several specimens were loaded in both the positive and negative directions to determine the effects of the direction of twist on the buckling strength.

## 6. TEST RESULTS

### 6.1 Elastic Behavior of Specimens

The elastic properties of the torsional specimens tested are shown in table 4. This table gives the effective values of $\mathrm{E}_{11}$ and $G_{12}$ which were predicted by the 1 aminate analysis of the Chao program and measured during tests. The experimental moduli were determined from a least squares fit to the data of the first six load increments applied in the linear elastic range of each test.

### 6.2 Shear Stress-Shear Strain Curves

The shear stress-shear strain curves from all specimens tested are shown in figure 7. The shear strains were averaged from the measurements made at the mid-length of the specimens with the five rosette gages. Curves for aluminum-alloy specimens similar to the $20-i n(50.8-\mathrm{cm})$ specimjns reported here are given in reference 3.

### 6.3 Buckling Torque

The maximum measured torques for the specimens tested in this investigation are given in table 5. These represent experimental buckling torques except where noted. Included in this table are the values of the buckling torque predicted by the Chao analysis and the ratio (correlation factor) of the maximum measured torque to the torque predicted by the analysis. The elastic material properties used in the analysis to predict the buckling torques are those presented in table 3 corrected for thickness variations.

Photographs of typical buckled specimens are shown in figure 8.

## 7. DISCUSSION

### 7.1 Elastic Buckling

The ability of the Chao theory to predict the buckling load of composite cylinders loaded in torsion can be estimated by analysis of the data given in table 5. For the specimens which fail by elastic buckling, the correlation factor between analysis and experiment is .81 with a standard deviation of 13 percent. This is approximately the same degree of agreement as that generally attributed to Donnell's treatment of isotropic cylinders (ref. 11). It is interesting to
note that the analysis appears to be in better agreement with experiments for boron/epoxy specimens than for graphite/epoxy specimens. An evaluation of the effects of fabrication variables can be made by comparing the measured buckling torques of replicate specimens. The standard deviation of the differences in measured buckling torques between replicate specimens was five percent for all specimens tested which failed elastically. The difference between 13 percent and 5 percent reflects the "modeling error" in using the prediction equation. There appears to be a tendency for higher strength specimens fo fail by mechanisms other than elastic buckling.

### 7.2 Effect of Direction of Twist

The effect of direction of twist in specimens with unidirectional, off-axis layups is shown by the results on specimens BL03, (ref. 4, Table 4) BL05, BL06 and BL07 (table 5). Specimens BL03 (Predicted Buckling Torque, $3,480 \mathrm{lbf-in}$ ( $393 \mathrm{~N}-\mathrm{m}$ ), Experimental Buckling Torque 3, $400 \mathrm{lbf-in}(384 \mathrm{~N}-\mathrm{m})$ ) and BL07 with the +45-degree ply orientation failed by elastic buckling near the predicted value. However, the two specimens with -45-degree orientation, where principle tensile stresses were normal to the fibers, failed in the matrix or matrixfiber interface at torques well below the expected elastic buckling torques.

A more complete understanding of the effects of direction of twist and stacking sequence is obtained from the behavior of specimens B01 through B04 and C04, C06, C07 and C08. These specimens were symmetric, balanced $\pm 45$-degree laminates. They were twisted in both the positive and negative directions. In all cases, the direction of twist which loaded the outer ply fibers in compression resulted in the higher buckling torques (figs. 9 and l0). In general, these torques were approximately twice the torques of the specimens twisted in the direction in which the fibers in the outer ply were loaded in tension.

### 7.3 Linear Scaling

The effect of a linear scaling of cylinder dimensions was examined. Specimens B05 and B06 were half as long, half as thick and had one-half the diameter of specimens BL08 and BL09. This scaling resulted in the same $\mathrm{L} / \mathrm{r}$ and $\mathrm{D} / \mathrm{t}$ ratios. As predicted by the analysis, and as can be seen in table 6, the buckling shear stresses for the two sizes tested are nearly equal for both sizes of cylinder (fig. ll). Similar results were expected on specimen pairs C09, C10 and CLO1, CLO2. However, specimens CLO1 and CLO2 failed at loads well below the expected buckling torques. These failures were catastrophic without evidence of prior buckling. These premature failures may have been caused by local irregularities in the cylinder wall. Such irregularities could have occurred during the fabrication of the shell.

Specimens were also tested in which the effect of scaling only the specimen length was examined (fig. 12). These specimens were fabricated with a nominal $6-i n(15-\mathrm{cm})$ diameter and 4 ply laminate thickness in three lengths. As can be seen from table 2, specimens BO1 and BO2 were 10 in ( 25.4 cm ) long, specimens BL10 and BL1l were 20 in ( 50.8 cm ) long, while specimens BLLO1 and BLL02 were 40 in ( 101.6 cm ) long.

### 7.4 Optimum Layup

Using, the results of the computer study shown in figure 2, specimens were fabricated and tested with a stacking sequence of ( $-82.5,30,20,-82.5$ ) degrees. As predicted by the analysis and as can be seen in table 7 , the buckling torque for the optimum layup is twice that for a $(-45,+45)_{s}$ degree layup and seven times the buckling torque for a $(0,0)$ degree layup.

### 7.5 Composite-Reinforced-Metal Specimens

Tests were performed on specimens in which composite reinforcement, in accordance with the "optimum" ply stacking sequence discussed above, had been applied to the outer surface of thin-walled metal cylinders. This was a further evaluation of a concept for the economic use of composite materials which had been explored previously (refs. 3 and 4). These results are shown in table 8. The buckling strengths of the equivalent-weight aluminum-alloy and titanium-alloy cylinders have been calculated. The aluminum-alloy cylinder having a thickness of $0.038 \mathrm{in}(0.096 \mathrm{~cm})$, a length of $10 \mathrm{in} \mathrm{( } 25.4 \mathrm{~cm}$ ) and a diameter of 6 in ( 15 cm ) has a predicted buckling strength of $40,200 \mathrm{lbf}-$ in ( $4,500 \mathrm{~N}-\mathrm{m}$ ). The similar titanium alloy cylinder, with a thickness of 0.034 in ( 0.076 cm ) has a predicted buckling strength of 50,100 lbf-in ( $5,700 \mathrm{~N}-\mathrm{m}$ ). The analysis predicts a considerable strength-to-weight savings for a reinforced-metal specimen over the equivalentweight metal specimen. The actual strength-to-weight savings is somewhat less than predicted, due to the relatively low load at which the composite material debonded from the metal or endcap failure occurred, for the specimens tested. The results of the tests on specimens Cll and Cl2 indicate that the ultimate shear strain of the graphite material in this ply configuration is about 0.0150 . At failure, the shear strains in specimen AC03 were about 0.0075 . The buckling failure loads, therefore, would be expected to be somewhat higher and the strength-to-weight savings would improve. Nevertheless, the strength-to-weight ratios which were obtained are larger than those obtained with unidirectionally-reinforcedmetal specimens (ref. 3).

### 7.6 Post-Buckling Failure Behavior

As the specimen wall is loaded in torsion near the buckling load, it begins to assume the buckled wave pattern particular to that specimen configuration. At initiation of buckling and during post-buckling loading it was observed that the failure behavior of the two composite materials was different. The specimens fabricated with boron/epoxy continued to carry load in the post-buckling regime, as indicated by curves of figure 7. Deep el astic buckles, as shown in figures $8 \mathrm{~d}, \mathrm{e}, \mathrm{f}$ and h can be twisted into the boron/epoxy specimens without permanent damage. Specimens fabricated with graphite/ epoxy, however, failed catastrophically soon after the maximum torque was reached. An example of this type of failure is shown in figures 8 c and g .

## 8. CONCLUSIONS

Based on the investigation of elastic torsional buckling in thin-walled-composite cylindrical shells reported herein, the following conclusions are drawn:

1. An available computer analysis has been exercised extensively and its usefulness has been established in predicting buckling torques for several laminate configurations. The effect of uncoupling the bending and extension forces for the thin 1 aminate, which was assumed by the analysis, has been experimentally tested and the validity of the assumption verified by the resulting agreement between analysis and experiment. Correlation factors in excess of 0.80 for experiment in rel ation to analysis were not uncommon for these tests. There appears to be a modeling error of approximately 8 percent between the analysis and experiment.
2. Torsional buckling strengths which differ by as much as a factor of two may result from reversing the direction of twist of a thin-walled-composite cylinder. This is of potential importance in applications where reversals of loading may occur and in selection of a lay-up configuration for a single direction of torque.
3. The buckling results obtained by reversing the stacking sequence in a laminated thin-walled cylinder loaded in torsion have been shown to be equivalent to the buckling results obtained by reversing the direction of twist on the original stacking sequence.
4. Shear stress-shear strain curves computed from the results of tests on large composite, and composite-reinforced-metal torsional specimens, and the principal elastic moduli ( $\mathrm{E}_{11}$ ), and in-plane shear moduli ( $G_{12}$ ) computed from these curves, have been given.
5. A 2:1 linear scaling of cylinder dimensions, while maintaining constant $\mathrm{L} / \mathrm{r}$ and $\mathrm{D} / \mathrm{t}$ ratios, resulted in buckling torques which differed by about an order of magnitude, while the shear stresses at failure were approximately equal.
6. An "optimum" stacking sequence which produced significant increases in the predicted and measured buckling loads for a four-ply cylinder was determined. This results in considerable increases in the strength-to-weight ratio over other sequences examined herein and in reference 3 .
7. The "optimum" stacking sequence was al so used for al uminum-alloy and titanium-alloy cylinders reinforced on their outer surfaces with composite materials. This sequence resulted in significant increases in the strength-to-weight ratio over several other ply sequences tested. There is a tendency for these higher strength specimens to fail by mechanisms other than elastic buckling.
8. Boron/epoxy cylinders which fail by elastic torsional buckling have considerably more postbuckling strength than similar cylinders fabricated from graphite/epoxy.

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Table 1 - Specimen Designations

| Designation | Material | Specimen length |  |
| :---: | :---: | :---: | :---: |
|  |  | in | cm |
| T | titanium alloy | 10.0 | 25.4 |
| B | boron/epoxy | 10.0 | 25.4 |
| BL | boron/epoxy | 20.0 | 50.8 |
| BLL | boron/epoxy | 40.0 | 101.6 |
| C | graphite/epoxy | 10.0 | 25.4 |
| CL | graphite/epoxy | 20.0 | 50.8 |
| AC | graphite/epoxy reinforced aluminum alloy | 10.0 | 25.4 |
| TB | boron/epoxy-reinforced titanium alloy | 10.0 | 25.4 |
| TC | graphite/epoxy-reinforced titanium alloy | 10.0 | 25.4 |

Table 2 - Dimensions of Torsional Specimens

|  | Specimen length |  | Specimen diameter |  | Metal thickness |  | Composite thickness |  | Nominal $\mathrm{L} / \mathrm{r}$ | Nominal D/t | $\begin{gathered} \text { Ply } \\ \text { orientation } \\ \hline \end{gathered}$ | Filament volume fraction |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | in | cm | in | cm | in | cm | in | cm |  |  | degrees | percent |
| All-metal Specimens |  |  |  |  |  |  |  |  |  |  |  |  |
| T01 | 10.0 | 25.4 | 5.990 | 15.21 | 0.023 | 0.058 | --- | --- | 3.3 | 300 | --- | --- |
| T02 | 10.0 | 25.4 | 5.994 | 15.22 | . 026 | . 066 | --- | --- | 3.3 | 300 | --- | --- |
| Unidirectional Composite Specimens |  |  |  |  |  |  |  |  |  |  |  |  |
| в07 | 10.0 | 25.4 | 5.964 | 15.15 | --- | --- | . 022 | . 056 | 3.3 | 300 | $(0,0) \mathrm{s}$ | 47.4 |
| B08 | 10.0 | 25.4 | 5.970 | 15.16 | --- | --- | . 022 | . 056 | 3.3 | 300 | $(0,0) \mathrm{s}$ | 48.2 |
| BL07 | 20.0 | 50.8 | 6.016 | 15.28 | --- | --- | . 023 | . 058 | 6.7 | 300 | $(45,45) \mathrm{s}$ | 45.0 |
| BL05 | 20.0 | 50.8 | 6.000 | 15.24 | --- | --- | . 022 | . 056 | 6.7 | 300 | (-45, -45)s | 46.5 |
| BL06 | 20.0 | 50.8 | 6.066 | 15.41 | --- | --- | . 021 | . 053 | 6.7 | 300 | $(-45,-45)$ s | 48.2 |
| c01 | 10.0 | 25.4 | 6.003 | 15.25 | --- | --- | . 025 | . 064 | 3.3 | 300 | $(0,0) s$ | 51.4 |
| C05 | 10.0 | 25.4 | 6.016 | 15.28 | --- | --- | . 024 | . 061 | 3.3 | 300 | $(0,0) \mathrm{s}$ | 52.2 |
| Effect of Stacking Sequence |  |  |  |  |  |  |  |  |  |  |  |  |
| B01 | 10.0 | 25.4 | 6.008 | 15.26 | --- | --- | 0.022 | 0.056 |  | 300 | $(-45,+45) \mathrm{s}$ | 47.4 |
| B02 | 10.0 | 25.4 | 6.008 | 15.26 | --- | --- | . 022 | . 056 | 3.3 | 300 | $(-45,+45) \mathrm{s}$ | 47.4 |
| B03 | 10.0 | 25.4 | 6.012 | 15.27 | --- | --- | . 022 | . 056 | 3.3 | 300 | ( $+45,-45$ ) s | 46.5 |
| B04 | 10.0 | 25.4 | 6.000 | 15.24 | --- | --- | . 022 | . 056 | 3.3 | 300 | $(+45,-45) \mathrm{s}$ | 46.5 |
| C04 | 10.0 | 25.4 | 6.001 | 15.24 | --- | --- | . 023 | . 058 | 3.3 | 300 | ( $+45,-45$ ) s | 53.9 |
| C08 | 10.0 | 25.4 | 5.988 | 15.21 | --- | --- | . 024 | . 061 | 3.3 | 300 | ( $+45,-45$ ) s | 48.8 |
| C06 | 10.0 | 25.4 | 6.016 | 15.28 | --- | --- | . 024 | . 061 | 3.3 | 300 | $(-45,+45) \mathrm{s}$ | 53.9 |
| C07 | 10.0 | 25.4 | 6.002 | 15.25 | --- | --- | . 024 | . 061 | 3.3 | 300 | $(-45,+45) \mathrm{s}$ | 52.2 |
| Effect of Linear Scaling |  |  |  |  |  |  |  |  |  |  |  |  |
| BLLO1 | 40.0 | 101.6 | 6.056 | 15.38 | --- | --- | 0.023 | 0.058 | 13.3 | 300 | ( $-45,+45$ ) s | 45.0 |
| BLL02 | 40.0 | 101.6 | 6.039 | 15.34 | --- | --- | . 024 | . 061 | 13.3 | 300 | $(-45,+45) \mathrm{s}$ | 44.3 |

Table 2 - Dimensions of Torsional Specimens

| Specimen No. | Specimen length |  | Specimen diameter |  | Metal thickness |  | Composite thickness |  | $\begin{gathered} \text { Nominal } \\ \mathrm{L} / \mathrm{r} \\ \hline \end{gathered}$ | $\begin{gathered} \text { Nominal } \\ \mathrm{D} / \mathrm{t} \\ \hline \end{gathered}$ | $\begin{gathered} \text { Ply } \\ \text { orientation } \\ \hline \end{gathered}$ | Filament volume fraction |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | in | cm | in | cm | in | cm | in | cm |  |  | degrees | percent |
| Effect of Linear Scaling - continued |  |  |  |  |  |  |  |  |  |  |  |  |
| BL10 | 20.0 | 50.8 | 5.992 | 15.22 | --- | --- | 0.023 | 0.058 | 6.7 | 300 | $(-45,+45) \mathrm{s}$ | 45.0 |
| BLII | 20.0 | 50.8 | 6.074 | 15.43 | --- | --- | . 023 | . 058 | 6.7 | 300 | $(-45,+45) \mathrm{s}$ | 46.5 |
| B05 | 10.0 | 25.4 | 3.030 | 7.696 | --- | --- | . 022 | . 056 | 6.7 | 150 | $(+45,-45) \mathrm{s}$ | 46.5 |
| B06 | 10.0 | 25.4 | 3.028 | 7.691 | --- | --- | . 022 | . 056 | 6.7 | 150 | $(+45,-45) \mathrm{s}$ | 46.5 |
| BL08 | 20.0 | 50.8 | 6.048 | 15.36 | --- | --- | . 046 | . 117 | 6.7 | 150 | $(-45,-45,+45,+45) \mathrm{s}$ | 45.7 |
| BL09 | 20.0 | 50.8 | 6.036 | 15.33 | --- | --- | . 044 | . 112 | 6.7 | 150 | $(-45,-45,-45,+45) s(b)$ | 47.4 |
| C09 | 10.0 | 25.4 | 3.036 | 7.711 | --- | --- | . 025 | . 064 | 6.7 | 150 | $(-45,+45) s$ | 50.5 |
| C10 | 10.0 | 25.4 | 2.992 | 7.600 | --- | --- | . 024 | . 061 | 6.7 | 150 | (-45, +45) s | 53.0 |
| CLO1 | 20.0 | 50.8 | 6.108 | 15.51 | --- | --- | . 049 | . 125 | 6.7 | 150 | $(-45,-45,+45,+45)$ s | 50.5 |
| CLO2 | 20.0 | 50.8 | 6.093 | 15.48 | --- | --- | . 048 | . 122 | 6.7 | 150 | $(-45,-45,+45,+45) \mathrm{s}$ | 50.5 |
| Optimum Angle Specimens |  |  |  |  |  |  |  |  |  |  |  |  |
| B09 | 10.0 | 25.4 | 6.013 | 15.27 | --- | --- | 0.023 | 0.058 | 3.3 | 300 | $(-82.5,20,30,-82.5)$ (c) | ) 45.0 |
| B10 | 10.0 | 25.4 | 5.959 | 15.14 | --- | --- | . 023 | . 058 | 3.3 | 300 | $(-82.5,30,20,-82.5)$ | 46.5 |
| Cll | 10.0 | 25.4 | 5.996 | 15.23 | --- | --- | . 024 | . 061 | 3.3 | 300 | (-82.5, 30, 20, -82.5) | 48.8 |
| C12 | 10.0 | 25.4 | 5.976 | 15.18 | --- | --- | . 024 | . 061 | 3.3 | 300 | $(-82.5,30,20,-82.5)$ | 50.5 |
| Composite - Reinforced Metal Specimens |  |  |  |  |  |  |  |  |  |  |  |  |
| ACO2 | 10.0 | 25.4 | 5.967 | 15.16 | 0.023 | 0.058 | 0.032 (a) | 0.081 | 3.3 | 120 | $(-82.5,30,20,-82.5)$ |  |
| ACO3 | 10.0 | 25.4 | 5.964 | 15.15 | . 022 | . 056 | . 032 (a) | . 081 | 3.3 | 120 | (-82.5, 30, 20, -82.5) |  |

$$
+2+2
$$

Table 2 - Dimensions of Torsional Specimens
page 3

| $\begin{gathered} \text { Specimen } \\ \text { No. } \\ \hline \end{gathered}$ | Specimen length |  | Specimen diameter |  | Metal thickness |  | Composite thickness |  | $\begin{gathered} \text { Nominal } \\ \mathrm{L} / \mathrm{r} \\ \hline \end{gathered}$ | $\begin{gathered} \text { Nominal } \\ \mathrm{D} / \mathrm{t} \\ \hline \end{gathered}$ | $\begin{gathered} \text { Ply } \\ \text { orientation } \\ \hline \end{gathered}$ | Filament volume fraction |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | in | cm | in | cm | in | cm | in | cm |  |  | degrees | percent |
| Composite - Reinforced Metal Specimens - continued |  |  |  |  |  |  |  |  |  |  |  |  |
| TB01 | 10.0 | 25.4 | 5.995 | 15.23 | 0.027 | 0.068 | 0.028 (a) | 0.071 | 3.3 | 120 | (-82.5, 30, 20, -82.5) |  |
| TB02 | 10.0 | 25.4 | 5.996 | 15.23 | . 025 | . 064 | . 029 (a) | . 074 | 3.3 | 120 | $(-82.5,30,20,-82.5)$ |  |
| TCO2 | 10.0 | 25.4 | 6.002 | 15.24 | . 024 | . 061 | . 032 (a) | . 081 | 3.3 | 120 |  |  |
| TCO3 | 10.0 | 25.4 | 6.001 | 15.24 | . 025 | . 064 | . 031 (a) | . 079 | 3.3 | 120 | $(-82.5,30,20,-82.5)$ $(-82.5,30,20,-82.5)$ |  |

[^1]Table 3 - Average Material Properties

| Material | Elastic modulus in thedirection of |  |  |  | In-plane shear modulus |  | Principal <br> in-plane <br> Poisson's <br> ratio |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Major |  | Mino | xis |  |  |  |
|  | lbf/in ${ }^{2}$ | $\mathrm{N} / \mathrm{m}^{2}$ | lbf/in ${ }^{2}$ | $\mathrm{N} / \mathrm{m}^{2}$ | lbf/in ${ }^{2}$ | $\mathrm{N} / \mathrm{m}^{2}$ |  |
| Aluminum alloy | $10.2 \times 10^{6}$ | $70 \times 10^{9}$ | $10.2 \times 10^{\circ}$ | $70 \times 10^{9}$ | $3.9 \times 10^{6}$ | $27 \times 10^{9}$ | 0.33 |
| Titanium alloy | 16.0 | 110 | 16.0 | 110 | 6.3 | 43 | . 33 |
| Boron/epoxy | 31.0 | 219 | 2.5 | 17 | 0.59 | 4.1 | . 21 |
| Graphite/epoxy | 30.9 | 213 | 1.1 | 7.6 | 0.53 | 3.7 | . 28 |


| $\begin{aligned} & \text { Tube } \\ & \text { No. } \end{aligned}$ | $\begin{gathered} \text { Ply } \\ \text { Orientation } \end{gathered}$ | ${ }^{\mathrm{E}_{11}}$ | heor. <br> Effectiv Elastic Mo | $\text { dulus } E_{11}$ |  | $\mathrm{G}_{12}$ | heor. <br> Effecti Shear Mod | $\begin{array}{ll} \text { ulus } & G_{12} \\ \hline \end{array}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | degrees | 1bf/in ${ }^{2}$ | $\mathrm{N} / \mathrm{m}^{2}$ | 1bf/in ${ }^{2}$ | $\mathrm{N} / \mathrm{m}^{2}$ | $1 \mathrm{bf} / \mathrm{in}^{2}$ | $\mathrm{N} / \mathrm{m}^{2}$ | 1bf/in2 | $\mathrm{N} / \mathrm{m}^{2}$ |
| All Metal Specimens |  |  |  |  |  |  |  |  |  |
| T01 | -- | $16.0 \times 10^{6}$ | $11.0 \times 10^{10}$ | $16.5 \times 10^{6}$ | $11.4 \times 10^{10}$ | $6.2 \times 10^{6}$ | $4.3 \times 10^{10}$ | $6.3 \times 10^{6}$ | $4.3 \times 10^{10}$ |
| T02 | -- | 16.0 | 11.0 | 17.6 | 12.1 | 6.2 | 4.3 | 6.3 | 4.3 |
| Unidirectional Composite Specimens |  |  |  |  |  |  |  |  |  |
| B07 | $(0,0){ }_{s}$ | 27.4 | 18.9 | 28.0 | 19.3 | 0.58 | 0.40 | 0.65 | 0.45 |
| B08 | $(0,0) \mathrm{s}$ | 27.4 | 18.9 | 28.0 | 19.3 | 0.58 | 0.40 | 0.66 | 0.46 |
| BL07 | $(+45,+45)_{s}$ | 1.8 | 1.2 | 1.8 | 1.2 | 1.9 | 1.3 | 1.7 | 1.1 |
| BL05 | $(-45,-45)_{s}$ | 1.8 | 1.2 | 1.5 | 1.0 | 1.9 | 1.3 | 1.6 | 1.1 |
| BL06 | $(-45,-45)_{8}$ | 1.8 | 1.2 | 1.7 | 1.1 | 2.0 | 1.4 | 1.6 | 1.1 |
| C01 | $(0,0) s$ | 25.4 | 17.5 | 19.5 | 13.4 | 0.53 | 0.36 | 0.51 | 0.35 |
| C05 | $(0,0) s$ | 26.6 | 18.3 | 22.3 | 15.4 | 0.53 | 0.36 | 0.55 | 0.38 |
| Effect of Stacking Sequence |  |  |  |  |  |  |  |  |  |
| B01 | $(-45,+45) \mathrm{s}$ | 2.2 | 1.5 | 2.4 | 1.6 | 7.2 | 5.0 | 7.2 | 5.0 |
| B02 | $(-45,+45) \mathrm{s}$ | 2.2 | 1.5 | 2.4 | 1.6 | 7.2 | 5.0 | 6.8 | 4.7 |
| B03 | $(+45,-45){ }_{\text {s }}$ | 2.2 | 1.5 | 2.0 | 1.4 | 7.1 | 4.9 | 6.1 | 4.2 |
| B04 | $(+45,-45){ }_{\text {s }}$ | 2.2 | 1.5 | 2.0 | 1.4 | 7.1 | 4.9 | 6.4 | 4.4 |
| C04 | $(+45,-45) s$ | 2.0 | 1.4 | 2.0 | 1.4 | 7.1 | 4.9 | 5.4 | 3.7 |
| C08 | $(+45,-45){ }_{\text {s }}$ | 2.0 | 1.4 | 1.8 | 1.2 | 6.8 | 4.7 | 5.7 | 3.9 |
| C06 | $(-45,+45) s$ | 2.0 | 1.4 | 1.9 | 1.3 | 6.9 | 4.8 | 5.6 | 3.9 |
| C07 | $(-45,+45) s$ | 2.0 | 1.4 | 1.8 | 1.2 | 6.8 | 4.7 | 5.9 | 4.1 |


| Table 4 - Elastic Properties of Torsional Specimens |  |  |  |  |  |  |  |  | page 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { Tube } \\ & \text { No. } \end{aligned}$ |  | Effective <br> Elastic Modulus |  |  |  | Effective <br> $\mathrm{G}_{12}$ theor. ${ }^{\text {Shear Modulus }} \mathrm{G}_{1}$ |  |  |  |
|  | Ply Orientation |  |  |  |  |  |
|  | degrees | $\mathrm{lbf} / \mathrm{in}^{2}$ | $\mathrm{N} / \mathrm{m}^{2}$ | $\overline{\mathrm{lbf} / \mathrm{ln}^{2}}$ | $\mathrm{N} / \mathrm{m}^{2}$ |  |  |  | $1 \mathrm{bf} / \mathrm{in}^{2}$ | $\mathrm{N} / \mathrm{m}^{2}$ | $\mathrm{lbf} / \mathrm{in}^{2}$ | $\mathrm{N} / \mathrm{m}^{2}$ |
| Effect of Linear Scaling |  |  |  |  |  |  |  |  |  |
| BLL01 | $(-45,+45)_{s}$ | $2.1 \times 10^{6}$ | $1.4 \times 10^{6}$ | -- | -- | $6.8 \times 10^{6}$ | $4.7 \times 10^{10}$ | $6.0 \times 10^{6}$ | $1 \times 10^{10}$ |
| BLL02 | $(-45,+45)_{s}$ | 2.1 | 1.4 | -- | -- | 6.7 | 4.6 | 6.7 | 4.6 |
| BL10 | $(-45,+45)_{s}$ | 2.1 | 1.4 | $2.2 \times 10^{6}$ | $1.5 \times 10^{6}$ | 6.8 | 4.7 | 6.4 | 4.4 |
| BL11 | $(-45,+45)_{s}$ | 2.2 | 1.5 | 2.1 | 1.4 | 7.1 | 4.9 | 6.4 | 4.4 |
| B05 | $(+45,-45){ }_{\text {s }}$ | 2.2 | 1.5 | 1.9 | 1.2 | 7.1 | 4.9 | 6.9 | 4.8 |
| B06 | $(+45,-45){ }_{s}$ | 2.2 | 1.5 | 2.1 | 1.4 | 7.1 | 4.9 | 7.1 | 4.9 |
| BL08 | $(-45,-45,+45,+45)_{s}$ | 2.2 | 1.5 | 2.2 | 1.5 | 7.0 | 4.8 | 6.9 | 4.8 |
| BL09 | $(-45,-45,-45,+45)_{s}$ | 2.1 | 1.4 | 2.1 | 1.4 | 5.9 | 4.1 | 5.6 | 3.9 |
| C09 | $(-45,+45) s$ | 2.0 | 1.4 | 1.7 | 1.2 | 6.6 | 4.6 | 5.7 | 3.9 |
| C10 | $(-45,+45)_{s}$ | 2.0 | 1.4 | 1.9 | 1.3 | 6.8 | 4.7 | 6.1 | 4.2 |
| CL01 | $(-45,-45,+45,+45)_{s}$ | 2.0 | 1.4 | 1.9 | 1.3 | 6.7 | 4.6 | 6.7 | 4.6 |
| CLO2 | $(-45,-45,+45,+45)_{s}$ | 2.0 | 1.4 | -- | -- | 6.8 | 4.7 | 5.6 | 3.9 |
| Optimum Angle Specimens |  |  |  |  |  |  |  |  |  |
| B09 | (-82.5, 20, 30, -82.5) | 3.9 | 2.7 | 3.6 | 2.5 | 1.0 | 0.69 | 0.91 | 0.63 |
| B10 | (-82.5, 30, 20, -82.5) | 4.0 | 2.8 | 3.9 | 2.7 | 1.0 | 0.69 | 1.1 | 0.76 |
| C11 | (-82.5, 30, 20, -82.5) | 3.5 | 2.4 | 3.2 | 2.2 | 0.87 | 0.60 | 0.97 | 0.67 |
| C12 | (-82.5, 30, 20, -82.5) | 3.2 | 2.2 | 3.2 | 2.2 | 0.87 | 0.60 | 0.88 | 0.61 |

Table 4 - Elastic Properties of Torsional Specimens

| Tube No. | $\begin{gathered} \text { Ply } \\ \text { Orientation } \end{gathered}$ | $\mathrm{E}_{11}$ | Effec Elastic heor. | ulus $\mathrm{E}_{11}$ |  | $\mathrm{G}_{12}$ | $\qquad$ | ve ulus $\mathrm{G}_{12}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | degrees | 1bf/in ${ }^{2}$ | $\mathrm{N} / \mathrm{m}^{2}$ | $1 \mathrm{bf} / \mathrm{in}^{2}$ | $\mathrm{N} / \mathrm{m}^{2}$ | 1bf/in ${ }^{2}$ | $\mathrm{N} / \mathrm{m}^{2}$ | 1bf/in ${ }^{2}$ | $\mathrm{N} / \mathrm{m}^{2}$ |
| Composite - Reinforced Metal Specimens |  |  |  |  |  |  |  |  |  |
| AC02 | $(-82.5,30,20,-82.5)$ | $7.5 \times 10^{6}$ | $5.2 \times 10^{6}$ | $6.5 \times 10^{6}$ | $4.5 \times 10^{10}$ | $2.4 \times 10^{6}$ | $1.6 \times 10^{10}$ | $2.3 \times 10^{6}$ | $1.6 \times 10^{10}$ |
| AC03 | (-82.5, 30, 20, -82.5) | 7.5 | 5.2 | 7.2 | 5.0 | 2.4 | 1.6 | 2.2 | 1.5 |
| TB01 | $(-82.5,30,20,-82.5)$ | 11.3 | 7.8 | 12.4 | 8.5 | 4.0 | 2.8 | 3.6 | 2.5 |
| TB02 | $(-82.5,30,20,-82.5)$ | 11.0 | 7.6 | 12.0 | 8.3 | 3.8 | 2.6 | 3.6 | 2.5 |
| TC02 | $(-82.5,30,20,-82.5)$ | 10.4 | 7.2 | 9.8 | 6.8 | 3.6 | 2.5 | 3.2 | 2.2 |
| TC03 | $(-82.5,30,20,-82.5)$ | -- | - | -- | -- | 3.7 | 2.6 | 3.4 | 2.3 |


Table 5 - Buckling Torques of Torsional Specimens



 Predicted $\qquad$


300
300
300
300
150
150
150
150
150
150
150
앙ㅇㅇㅇㅇ
13.33
13.33
6.7
6.7
6.7
6.7
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6.7
6.7
6.7
6.7
6.7
3.3
3.3

3.3
3.3
Table 5 - Buckling Torques of Torsional Specimens

| Specimen No. | $\begin{gathered} \text { Ply } \\ \text { orientation } \end{gathered}$ | $\begin{gathered} \text { Nominal } \\ \mathrm{L} / \mathrm{r} \end{gathered}$ | $\begin{gathered} \text { Nominal } \\ \mathrm{D} / \mathrm{t} \end{gathered}$ | Loading direction | Predicted buckling torque |  | Maximu measured | torque | Correlation factor |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | degrees |  |  |  | lbfoin | $\mathrm{N}, \mathrm{m}$ | 1bf-in | $\mathrm{N}-\mathrm{m}$ |  |
| Composite - Reinforced Metal Specimens |  |  |  |  |  |  |  |  |  |
| ACO2 | $(-82.5,30,20,-82.5)$ | 3.3 | 120 | Neg | 85800 | 9694 | 31250 (c) | 3531 (c) | 0.36 (c) |
| AC03 | (-82.5, 30, 20, -82.5) | 3.3 | 120 | Neg | 85700 | 9683 | 43000 (c) | 4745 (c) | . 49 (c) |
| TB01 | $(-82.5,30,20,-82.5)$ | 3.3 | 120 | Neg | 118000 | 13332 | 86100 | 9728 | . 73 |
| TB02 | (-82.5, 30, 20, -82.5) | 3.3 | 120 | Neg | 111000 | 12541 | 93500 (c) | 10564 (c) | . 84 (c) |
| TCO2 | $(-82.5,30,20,-82.5)$ | 3.3 | 120 | Neg | 112000 | 12654 | 63100 | 7129 | . 56 |
| TCO3 | (-82.5, 30, 20, -82.5) | 3.3 | 120 | Neg | 115000 | 12993 | 75600 (c) | 8542 (c) | . 66 (c) |

[^2]Table 6 - Effects of Stacking Sequence and Loading Direction

| Tube <br> No. | $\begin{gathered} \text { Ply } \\ \text { orientation } \\ \hline \end{gathered}$ | Loading direction |  |  | Experimental |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Predicted buckling torque |  | Maximum |  | Shear stress |  |
|  |  |  |  |  | measur | torque |  |  |
|  | degrees |  | lbf-in | $\mathrm{N}-\mathrm{m}$ | 1 bf -in | $\mathrm{N}-\mathrm{m}$ | $\mathrm{lbf} / \mathrm{in}^{2}$ | $\mathrm{N} / \mathrm{m}^{2}$ |
| B05 | (+45, -45)s | Neg | 2270 | 256 | 1750 | 198 | 5470 | $37.7 \times 10^{6}$ |
|  |  | Pos | 4180 | 472 | 3500 | 395 | 10950 | 75.5 |
| B06 | ( $+45,-45$ )s | Neg | 2270 | 256 | 1960 | 221 | 6100 | 42.1 |
| BL08 | (-45, -45, +45, +45)s | Neg | 34930 | 3946 | 30550 | 3452 | 11660 | 80.4 |
|  |  | Pos | 18930 | 2139 | 16700 | 1887 | 6380 | 44.0 |
| BLO9 | (-45, -45, -45, +45)s | Neg | 33130 | 3743 | 30700 | 3468 | 12140 | 83.7 |
|  |  | Pos | 14240 | 1609 | 13450 | 1520 | 5320 | 36.7 |
| $\mathrm{CO9}$ | $(-45,+45) \mathrm{s}$ | Neg | 5120 | 578 | 3540 | 151 | 9940 | 68.5 |
|  |  | Pos | 2500 | 282 | 1970 | 223 | 5530 | 38.1 |
| C10 | $(-45,+45) \mathrm{s}$ | Neg | 4700 | 531 | 3560 | 402 | 10320 | 71.2 |
|  |  | Pos | 2300 | 260 | 1870 | 211 | 5420 | 37.4 |
| CL01 | (-45, -45, +45, +45)s | Neg | 39520 | 4465 | 24500 | 2768 | 8540 | 58.9 |
| CL02 | $(-45,-45,+45,+45) \mathrm{s}$ | Neg | 37780 | 4268 | 22300 | 2519 | 7960 | 54.9 |

Table 7 - Buckling Strengths of Optimum Ply Angle Specimens

| Tube No. | $\begin{gathered} \text { Ply } \\ \text { orientation } \end{gathered}$ | Loading direction | Predicted buckling torque |  | Experimental |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | Maximum |  | Shear stress |  |
|  |  |  |  |  | measur | torque |  |  |
|  | degrees |  | 1 bf -in | $\mathrm{N}-\mathrm{m}$ | lbf-in | $\mathrm{N}-\mathrm{m}$ | $\mathrm{lbf} / \mathrm{in}^{2}$ | $\mathrm{N} / \mathrm{m}^{2}$ |
| B09 | (-82.5, 20, 30, -82.5) | Neg | 21500 | 2429 | 17000 | 1921 | 13000 | $89.6 \times 10^{6}$ |
| B10 | (-82.5, 30, 20, -82.5) | Neg | 20400 | 2305 | 18000 | 2034 | 14300 | 98.6 |
|  |  | Pos | 7260 | 820 | 5400 | 610 | 4280 | 29.5 |
| C11 | (-82.5, 30, 20, -82.5) | Neg | 22200 | 2508 | 13600 | 1536 | 10400 | 71.7 |
| C12 | (-82.5, 30, 20, -82.5) | Neg | 23100 | 2610 | 12450 | 1407 | 9170 | 63.2 |

Table 8 - Buckling Strengths of Composite-Reinforced Metal Specimens

|  |  |  |  |  | Experimental |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Tube No. | $\begin{gathered} \text { Ply } \\ \text { orientation } \end{gathered}$ | Loading direction | Predicted buckling torque |  | Maximum sured torque |  | Shear stress |  |
|  | degrees |  | 1bf-in | $\mathrm{N}-\mathrm{m}$ | lbf-in | $\mathrm{N}-\mathrm{m}$ | $1 \mathrm{bf} / \mathrm{in}^{2}$ | $\mathrm{N} / \mathrm{m}^{2}$ |
| AC02 | $(-82.5,30,20,-82.5)$ | Neg | 85800 | 9694 | 31250 (a) | 3531 (a) | 10200 | $70.3 \times 10^{6}$ |
| AC03 | $(-82.5,30,20,-82.5)$ | Neg | 85700 | 9683 | 42000 (a) | 4745 (a) | 14200 | 97.9 |
| TB01 | $(-82.5,30,20,-82,5)$ | Neg | 118000 | 13332 | 86100 | 9728 | 27700 | 191.0 |
| TB02 | $(-82.5,30,20,-82.5)$ | Neg | 111000 | 12541 | 93500 (a) | 10564 (a) | 30700 | 211.7 |
| TC02 | $(-82.5,30,20,-82.5)$ | Neg | 112000 | 12651 | 63100 | 7129 | 20300 | 140.0 |
| TC03 | $(-82.5,30,20,-82.5)$ | Neg | 115000 | 12993 | 75600 (a) | 8542 (a) | 23900 | 164.8 |



FIG. I- SHELL COORDINATE SYSTEM





FIG. 40 END FIXTURES FOR ALL-COMPOSITE TORSION SPECIMENS. ALL DIMENSIONS ARE GIVEN IN INCHES (I IN. = 2.54 CM )


NOTES: A NOMINAL 6.OO IN.
B NOMINAL 3.02 IN. $\qquad$ THESE NOMINAL DIAMETERS WILL VARY WITH THE SAMPLE TESTED

FIG. 4b END FIXTURES FOR COMPOSITE-REINFORCED METAL TORSION SPECIMENS. ALL DEMENSIONS ARE GIVEN IN INCHES ( 1 IN. $=2.54$ CM.)


FIG. 5 COMPRESSION TEST SET-UP


FIG. $6 a 40,000$ Ibf-in TORSION TEST SET-UP


FIG. 6b 60,000 Ibf-in TORSION TEST SET-UP


FIG. 6 C 100,000 $\mathrm{Ibf}-$-in TORSION TEST SET-UP


FIG. 7a- SHEAR STRESS - SHEAR STRAIN CURVES FOR ALL METAL IORSION SPECIMENS


FIG. 7b - SHEAR STRESS - SHEAR STRAIN CURVES FOR UNIDIRECTIONAL COMPOSITE SPECIMENS
(


FIG. 7C-SHEAR STRESS - SHEAR STRAIN CURVES FOR UNIDIRECTIONAL COMPOSITE SPECIMENS

FIG. 7d-SHEAR STRESS - SHEAR STRAIN CURVES FOR UNIDIRECTIONAL COMPOSITE SPECIMENS














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\times
$$
-


FIG. 8b SPECIMEN B IO LOADED NEGATIVE


FIG. 8d SPECIMEN BOG LOADED NEGATIVE


FIG.8c SPECIMEN COS
FIG. 8 TYPICAL BUCKLED SPECIMENS




FIG. 8 e SPECIMEN BO3 LOADED NEGATIVE
 FIG. 8 g SPECIMEN COT LOADED NEGATIVE
FIG. 8 TYPICAL BUCKLED SPECIMENS

FIG. 9- SHEAR STRESS - SHEAR STRAIN CURVES SHOWING EFFECT OF REVERSING LOADING




U.S. DEPT. OF COMM. BIBLIOGRAPHIC DATA SHEET
4. TITLE: AND SUBTITLE

Torsional Buckling of Composite Cylindrical She11s
7. AUTIIOR(S)
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9. PERFORMING ORGANIZATION NAME AND ADDRESS

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No.

NBSIR 74-572
3. Reripient's Accession No
5. Publication Datc

September 1974
6. Performing Organization Code
8. Performing Organ. Report No. NBSIR 74-572
10. Project/Task/Work Unit No. 2130445
11. Contract/Grant No.

L-48, 826
13. Type of Report \& Period Covered

Final
14. Sponsoring Agency Code
15. SUPPI.EMEN TARY NOTES
16. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)

The elastic buckling strength has been determined for thin-walled composite and composite-reinforced-metal cylindrical shells. Tests were performed on boron/epoxy and graphite/epoxy-all-composite specimens, on boron/epoxy-reinforced-titanium specimens and on boron/epoxy and graphite/epoxy-reinforced aluminum specimens. Cylinders were tested with several unidirectional-ply and cross-ply layups.

The results of the tests were compared with the buckling strengths predicted by the torsional buckling analysis of Chao. For the cylinders which fail by buckling, the experimental buckling torques were approximately 81 percent of the torques predicted by the analysis.

The experimental results of tests on 39 specimens are presented. Torsional buckling strengths which differ by as much as a factor of two may result from reversing the direction of twist of a thin-walled cross-ply composite cylinder. This has been shown to be equivalent to reversing the stacking sequence of the laminate. This is of potential importance in applications where reversals of loading may occur. An "optimum" stacking sequence which produced significant increases in the predicted and measured buckling loads was determined. Cylinders fabricated with this stacking sequence exhibit considerable increases in the strength-to-weight ratio over other sequences examined.
17. KEY WORDS (six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons)

Aircraft structures; buckling; composite materials; metal reinforcement; stability; stacking sequence; thin shells; torsion.
18. AVAILABILITY $X$ Unlimited
$\square$ For Official Distribution. Do Not Release to NTIS

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$\square$ Order From National Technical Information Service (NTIS) Springfield, Virginia 22ISI

| 19. SECURITY CLASS <br> (THIS REPORT) <br> UNCL ASSIFIED | 21. NO. OF PAGES |
| :--- | :--- |
| 20. SECURITY CLASS <br> (THIS PAGE) <br> UNCLASSIFIED) | 22. Price |


[^0]:    Prepared for
    National Aeronautics and Space Administration
    Langley Research Center
    Hampton, Virginia 23365

[^1]:    (a) Includes $0.005-\mathrm{in}(0.013-\mathrm{cm})$ thickness of film-epoxy adhesive.
    (b) The analysis indicated that this specimen stacking sequence was equally as strong as the balanced configuration of Specimen blo8.
    (c) This stacking sequence is the result of a fabrication error.

[^2]:    (a) Matrix tension failure.
    (b) Catastrophic failure without evidence of prior buckling.
    (c) End cap failure; buckling did not occur.

[^3]:    FIG. $7 r$ - SHEAR STRESS-SHEAR STRAIN CURVES FOR COMPOSITE REINFORCED METAL

