MATHEMATICAL FOUNDATIONS

CITY GAMES
Price of City Games
Computer Files Includes
Related Manuals
CITY GAMES
MATHEMATICAL FOUNDATIONS

Written by John E. Moriarty

Technical Analysis Division
Institute for Applied Technology
National Bureau of Standards
Washington, D.C. 20234

November 1973
Final Report

Sponsored by
National Technical Information Service
5285 Port Royal Road
Springfield, Virginia 22151

U. S. Department of Commerce, Frederick B. Dent, Secretary
National Bureau of Standards, Richard W. Roberts, Director
Acknowledgement

We wish to acknowledge the significant contributions made by Mr. S. Guiland of Johns Hopkins University for his tireless effort in proof-reading the draft of this text. Also the efforts of Mrs. R. Ciufolo, Mrs. L. Groves and Miss C. Naas for their typing and editing of this manuscript is appreciated.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. PRIVATE CAPITAL ALLOCATION</td>
<td>8</td>
</tr>
<tr>
<td>A. Choice of Investment</td>
<td>8</td>
</tr>
<tr>
<td>B. Types of Investment</td>
<td>10</td>
</tr>
<tr>
<td>C. Generalized Private Capital Allocation Equation</td>
<td>14</td>
</tr>
<tr>
<td>D. Residential Housing</td>
<td>16</td>
</tr>
<tr>
<td>E. Commercial</td>
<td>20</td>
</tr>
<tr>
<td>F. Industrial</td>
<td>23</td>
</tr>
<tr>
<td>III. GOVERNMENT CAPITAL</td>
<td>26</td>
</tr>
<tr>
<td>A. Government Decision Objectives</td>
<td>27</td>
</tr>
<tr>
<td>B. Departmental Rates</td>
<td>28</td>
</tr>
<tr>
<td>1. Ratio for Education</td>
<td>28</td>
</tr>
<tr>
<td>2. Ratio for Adult Education</td>
<td>29</td>
</tr>
<tr>
<td>3. Ratio for Municipal Services</td>
<td>29</td>
</tr>
<tr>
<td>4. Ratio for Welfare Payments</td>
<td>30</td>
</tr>
<tr>
<td>5. Ratio for Utilities</td>
<td>30</td>
</tr>
<tr>
<td>6. Ratio for Transportation</td>
<td>30</td>
</tr>
<tr>
<td>C. Budget Allocation</td>
<td>31</td>
</tr>
<tr>
<td>IV. TIME ALLOCATION</td>
<td>32</td>
</tr>
<tr>
<td>A. Allocations</td>
<td>33</td>
</tr>
<tr>
<td>1. Part-Time Work ($r_w$)</td>
<td>34</td>
</tr>
<tr>
<td>2. Free Training and Adult Education ($\gamma A_F$)</td>
<td>35</td>
</tr>
<tr>
<td>3. Paid Training and Education ($\gamma A_P$)</td>
<td>35</td>
</tr>
<tr>
<td>4. Political Activities ($r_{PC}$)</td>
<td>36</td>
</tr>
<tr>
<td>5. Recreational Activities ($\gamma R_C$)</td>
<td>36</td>
</tr>
<tr>
<td>B. Decision to Invest Time</td>
<td>37</td>
</tr>
<tr>
<td>V. PUBLIC AND PRIVATE INFLUENCE</td>
<td>39</td>
</tr>
<tr>
<td>A. Voting</td>
<td>40</td>
</tr>
<tr>
<td>B. Allocation of Power</td>
<td>41</td>
</tr>
<tr>
<td>C. Power Objective</td>
<td>43</td>
</tr>
<tr>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>--------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>VI. OPERATION OF THE MODEL</td>
<td>45</td>
</tr>
<tr>
<td>A. Business Goods and Services</td>
<td>45</td>
</tr>
<tr>
<td>1. Demand</td>
<td>45</td>
</tr>
<tr>
<td>2. Supply</td>
<td>45</td>
</tr>
<tr>
<td>3. Price</td>
<td>46</td>
</tr>
<tr>
<td>4. New Prices</td>
<td>46</td>
</tr>
<tr>
<td>B. Employment</td>
<td>47</td>
</tr>
<tr>
<td>1. Demand for Jobs</td>
<td>47</td>
</tr>
<tr>
<td>2. Supply of Workers</td>
<td>47</td>
</tr>
<tr>
<td>3. Matching</td>
<td>48</td>
</tr>
<tr>
<td>4. Part-Time Employment</td>
<td>49</td>
</tr>
<tr>
<td>5. Salaries</td>
<td>49</td>
</tr>
<tr>
<td>C. Land Use</td>
<td>51</td>
</tr>
<tr>
<td>1. Residential</td>
<td>52</td>
</tr>
<tr>
<td>2. Industrial</td>
<td>53</td>
</tr>
<tr>
<td>3. Commercial</td>
<td>53</td>
</tr>
<tr>
<td>4. Government Facilities</td>
<td>54</td>
</tr>
<tr>
<td>D. Land Value</td>
<td>54</td>
</tr>
<tr>
<td>E. Rent</td>
<td>55</td>
</tr>
<tr>
<td>F. Location of a Commercial Site</td>
<td>56</td>
</tr>
<tr>
<td>G. Migration</td>
<td>58</td>
</tr>
<tr>
<td>H. School Assignments</td>
<td>60</td>
</tr>
<tr>
<td>I. Transportation</td>
<td>62</td>
</tr>
<tr>
<td>J. Value of Structure and Capital Equipment</td>
<td>64</td>
</tr>
<tr>
<td>K. Water</td>
<td>65</td>
</tr>
<tr>
<td>VII. POST SCRIPT</td>
<td>68</td>
</tr>
<tr>
<td>A. Glossary of Terms</td>
<td>69</td>
</tr>
<tr>
<td>B. List of Symbols</td>
<td>76</td>
</tr>
</tbody>
</table>
I. INTRODUCTION

The CITY models are operational simulation games in which the participants make economic, government and social decisions affecting a hypothetical metropolitan area. Through the use of a computer, the simulated urban system responds to the participant's decisions as any real city would. Each player is assigned to a team which shares an economic, government or social role. The interrelated decisions made by teams will guide the way the simulated city changes in composition and size.

The simulation approach to cities offers the players an opportunity not only to make decisions but to implement them as well. They receive a feedback from their actions and see the effects from other forces that are constantly at work altering the effectiveness of the player's decisions. Players therefore have a learning experience on how to deal with a changing environment. The round-by-round play gives the players the necessary experience in selecting the type of analysis to move them towards their objectives while the allocation of their time and Game resources is a critical determinant of the success they hope to achieve. As the Game progresses, players learn to increase their involvement in the management of the environment while at the same time learning about the relationships between business and society.

The CITY model is really a series of Games. CITY I is the smallest of the Games consisting of the Economic and Government decision makers and operates on an IBM 1131 computer. It is primarily designed for land use and business decisions with the Government Departments acting more in the role of provider of services rather than regulatory agents. TELECITY or CITY II is CITY I with a partially operative social sector and operating on a Univac 1108 Exec 8 from a terminal. CITY III and CITY IV contain the Economic and Government decision makers and further expand CITY I by including the Social decision makers. The primary difference between CITY III and CITY IV is that CITY III is run on a Univac 1108 computer and CITY IV is run on an IBM 360/370 operating system. CITY IV was further expanded to include the water supply system known as the "water module."

When the "water module" is used with CITY IV, the model is then known as
the River Basin Model.

To simplify the description of the theory of these modules, this manual is written for the four module game. The equations for the other games can easily be obtained by eliminating the descriptors that do not apply to the particular game of interest. The theoretical structure of all models are consistent and relevant only within the range from a central city area to a regional configuration. The description of the module components and equations are meant to show the scope of decisions including those by the users and by the programs and algorithms of the computer model.

All models have associated with them Players', Directors' and Operators' manuals which should be used in conjunction with this manual. The numerous details and particulars of each Game are described in the other manuals whereas this manual describes the theoretical base for all Games in the series.

To orientate the reader to this manual, it must be emphasized that all models are based on a grid matrix to locate and tabulate the larger number of interrelationships and to locate specific parcels of land or highways. Within each square or grid, numerous restrictions and/or data are imposed by each module. If a player and/or computer change is proposed for a particular location in the matrix, all conditions must be met or the proposal will not be accepted by the computer. The computer therefore acts as a large accounting system with minor simulation roles played in external relationships to the theoretical city such as the business cycle or export sales. The players serve as a decision making body which tries to optimize their decisions for the purpose of achieving individual objectives. This optimizing process might maximize individual goals but when placed in competition for a finite number of available resources, it might well be suboptimal for the entire city. It is within this framework that learning occurs. The numerous interrelationships must be negotiated before any system proposal can be obtained to improve the city as a whole. The computer will consider all proposed changes and allow only those changes which meet all constraints for a particular grid. In general, this process is expressed in mathematical form as:
\[ D = \sum_{i=1}^{n} \sum_{m} d_{m} + \sum_{i=1}^{n} \sum_{j=1}^{m} d_{h_{ij}} S_{ij} \]

where

- \( S_{ij} \) = one-zero variable
  - 1 - the decision by player \( j \) at location \( i \) is valid
  - 0 - invalid decision

\( D \) = Total number of actual decisions made by all individuals at all grid locations accepted by the computer as valid

\( d_{m} \) = Decisions made by algorithms designed into the model for all grid locations \( i=1,2...n \)

\( d_{h_{ij}} \) = Decisions made by the players summed across all players at all locations.

The total number of possible decisions, say \( TD \), will always be \( TD \geq D \) since conflicting proposals can occur for the same grid or land parcel. For instance, the social players might propose a park for a particular land parcel and the economic sector might propose a building on the same site. The computer would then reject the two proposals as conflicting and \( TD > D \).

One of the objects of the game is to make \( TD \rightarrow D \) which implies negotiation and well informed players.

As mentioned earlier, the decision process is constrained by resources available to the players and to the computer. All games view resources in two general categories, fixed and liquid. The total resources (TR) of a particular system can be viewed as the sum of all fixed and liquid resources in the system. The fixed resources (FR) include water, land, people, buildings, roads, networks (utilities, transportation, communication, etc.), capital equipment and the related maintenance items necessary to keep in operation. In contrast, liquid resources (LR) are those items which can easily be changed in the short run such as taxes, wages, sales, social attitudes and the like. The total resources of a system are merely:

\[ TR = \sum_{p=1}^{n} FR_{p} + \sum_{i=1}^{m} LR_{i} \]
where

\[ p = \text{parcels or grid locations} \quad \text{and} \]
\[ i = \text{entities of LR} \]

Since player and computer decisions are always constrained by the total resources available, the uniqueness of the model lies in the method for relating a wide variety of specific resources to each other in a highly interactive fashion. To capture the spirit of these interactions, all mathematical descriptions will take the form of a definition of the resources to be allocated or the receptors to handle the resource, and an objective function describing the rationale for the allocation. The receptors are usually balanced by a matching function using supply and demand relations. For example, if (TR) represents the total resources as described earlier, realistically every resource available would not be changed in a single round of the game. This is particularly true for the change in FR. Land owners may not want to alter all of the property holdings or the government might be satisfied with prior decisions and not want extensive changes to the "status quo." To express this idea of partial resource allocation (PR), as a mathematical function we write:

\[ PR = \alpha CP + \beta CG + \gamma T + \epsilon P \]

where

- \( PR \) is the partial resource allocation
- \( CP \) is private liquid or semi-liquid capital
- \( CG \) is public liquid or semi-liquid capital
- \( T \) is available time
- \( P \) is the sum of all public and private influence
- \( \alpha, \beta, \gamma, \epsilon \) are components which equate all of the resources to a least common denominator
The objective function for PR is conceived of as balancing the possible resource allocations such that there is a tendency to maximize the total resource base of the system. This partial resource objective function (PRO) is expressed as:

\[ \text{PRO} = \phi_c + \sum_{i=1}^{n} \text{PR}_i \]

where

- PRO is the Partial Resource Objective Function
- \( \phi_c \) is added to represent those decisions which are made on a seemingly random basis
- \( \text{PR}_i \) is the specific partial resource \( i \)

Since the player input equations are normally optimizing or satisfying a rate of return or benefit ratio from the allocation of a finite set of resources, it follows that the player objective function is merely:

\[ \max_{i=1}^{n} \sum \text{PR}_i \quad \text{from above.} \]

This objective function represents all players by summing over all \( \text{PR}_i \)'s. Each individual \( \text{PR}_i \) is taken to be of the form:

\[ \text{PR}_i = \frac{\hat{B}_i - C_i}{V_i} \]

where

- \( \text{PR}_i \) is the partial resource allocation of specific partial resource \( i \)
- \( \hat{B}_i \) is the estimated benefit from project \( i \) in the form of dollars earned, service provided, expenditures saved, etc.
- \( C_i \) is the known expenditures such as current expenditures for services, etc.
is the total value of the project

In all cases there must be some procedure established for choosing among the many possible projects and this choice may be either the optimizing or satisfying assumptions. Associated with each PR, is a project cost $C_i$. There is a limitation of the values of PR that will be acceptable and a limitation that the sum of the costs ($C_i$) will not exceed the available resources as measured in money, time or influence. Note that PR is indeed a rate of return.

The computer equations are designed to balance or allocate supply and demand. The general computer demand equation is:

$$DC = \theta_1 (X_{x_1, x_2}, Y_{y_1, y_2}, ...)$$

where

$DC$ = demand (computer)

$X$ is some set of local system demanders such as population groups, businesses, etc.

$x_n$ are descriptors of the demanders such as class for population units and land use type for business, etc.

$Y$ is some set of local system decisions such as time allocations for population units, operation level for business, etc.

$y_n$ are descriptors of the decisions such as parcel location for population groups, maintenance levels for businesses, etc.

$\theta_1$ is a quantifier or scale for the units of demand.

In the same manner the computer model equation for supply is written:

$$SC = \theta_2 (W_{w_1, w_2}, Z_{z_1, z_2}, ...)$$
where

\[
\begin{align*}
SC & = \text{supply (computer)} \\
W & = \text{some set of local system suppliers such as housing units, employers, stores, etc.} \\
w_n & = \text{the fixed descriptors of these suppliers in the form of housing types, workers per level, design capacity, etc.} \\
Z & = \text{some set of local system decisions such as rents and quality of housing, salary level for employees, prices at stores, etc.} \\
z_n & = \text{the descriptors of the decisions such as locations for residences, businesses and stores, etc.} \\
\theta_2 & = \text{is a quantifier or scale for the units of supply.}
\end{align*}
\]

Throughout this manual, these general form equations will be used to describe the various game parts. General equations are given in all cases and where necessary specific equations are detailed to clarify the general equations.

The reader of this manual is advised to use a Players, Directors and Operators manual to clarify details which may not be fully explained in this manual. If the program is used as a reference, slight differences will occur in the variable names. Some of the symbols used in the main program are also used in subroutines with different meanings. To protect the continuity of this manual, duplicate symbols found in the program have been changed in this manual so that one definition is assigned to each variable. In all cases the program symbols are used.
II. PRIVATE CAPITAL ALLOCATION

In the general equation of partial resource allocation (PR), the term CP was introduced and defined as private liquid or semi-liquid capital. It was further defined that all capital resources would not be used during a particular game play which in turn led to the partial resource allocation concept. Assuming this concept to be realistic, it follows that any consideration of the total money stock available in a round for allocation may be defined as:

\[ MS = \sum_{i=1}^{n} (S_i + dI_i + (aB - dC)_i) \]

where

- \( MS \) is the total money stock available in the system for allocation in a given round
- \( n \) is the total number of players who have money
- \( S_i \) is the available savings on private liquid capital for player \( i \)
- \( dI_i \) is the present value of the capital investment of player \( i \)
- \( (aB-dC)_i \) is the net amount of investment capital available after all costs of borrowing are taken into account for player \( i \)
- \( aB \) is the amount of money borrowed
- \( dC \) is the present value of the cost of borrowing

A. Choice of Investment

When the cost of making an investment becomes a larger and larger part of the money supply available, an investor is more apt to rank the net rates of return for his investment and choose those that will give him a greater rate of return. In terms of probability one might say that the probability that the net rate of return chosen is the one which gives the maximum rate of return tends towards one as the available investment money
supply diminishes to zero or the cost of the specific investment chosen increases to the money supply. Expressed mathematically:

\[ p(\bar{N} = N_{\text{max}} | MS \to 0; K_i \to MS) = 1 \]

where

- \( p \) is the probability function
- \( \bar{N} \) is the net rate of return on the chosen investments
- \( N_{\text{max}} \) is the maximum rate of return
- \( MS \) is the money supply
- \( K_i \) is the total cost of player i's series of investments composed of the actual cost of the investment and the cost of obtaining investment information.

The probability statement can be further defined if one lets \( N_i = r_i - C_i \) where \( N_i \) is the net rate of return for i's investment series, \( r_i \) is the estimated rate of return from i's investment series, and \( C_i \) is the cost of using money on i's investment series.

It then follows that \( N_{\text{max}} \geq N_i \) and if \( \mu \) is the error bound in estimating the net rate of return, then \( N_i - \mu \geq 0 \).

Since all liquid capital is assumed to at least earn a rate of return equal to the current rate of interest, it is assumed that the rational decision maker, i, would tend to order his investment series so that the first capital used would be borrowed as long as

\[ (aB - dC)_i \leq r_i \]

That is, the net amount of investment capital available after all costs of borrowing are taken into consideration must be less than or equal to the estimated rate of return in dollars. When it becomes no longer profitable to borrow, i.e. \( (aB - dC)_i > r_i \), the saving \( S_i \) and then the present value of invested capital \( dI_i \) would be considered. The invested
capital is only considered when the series of investments from the previous round \((t - 1)\) yields a greater present value than that of a rearrangement of the investment series in the present round, which is expressed as

\[
dI_{i(t-1)} < dI_{it}
\]

B. Types of Investments

Investors might wish to invest for long range potential or to make quick profits for a single round. In each of these cases, different objectives and methods for analyzing the investments must be considered.

In the long run, the investor is primarily concerned with the average rate of return to be expected for each investment. As long as the average expected value of the investment choice is greater than the cost, investment will take place. Decisions in this mode range from ones which consider any investment that is profitable on the average to more careful choices which can be ranked from the most to least profitable.

To clarify the above process and relate the process to the computer, one must first visualize the various players and then investment process independent of the computer. An individual player has limited resources to invest and will not always invest all resources in any given round. For instance, a player might be saving for a future investment or be content with his past investments and not want to invest at all during a current round.

Consider each player who has investment potential on a single parcel. A player might be from the Economics, Government or Social sectors and the investment potential will be biased toward the individual player's point of view. When individual investment is viewed this way, one can reason that the player would consider his investment associated with some probability of obtaining the benefits as the player visualizes and some probability that either no investment is made on the parcel or that a proposed investment will be invalid for violations of zoning or the
like. Viewed this way, one can construct a player investment equation to express the expected value of the net rate of return on all investments for a specific parcel. This can be expressed as:

\[ E\tilde{N} = \frac{\sum_{i=1}^{n} B_i - DK_i}{V_i} \]

where

- \( E\tilde{N} \) is the expected value of the net rate of return on all investments possible on a particular parcel
- \( n \) is the number of potential investors for the parcel
- \( \hat{B}_i \) is the estimated benefit of the ith player to invest on the parcel
- \( dK_i \) is the cost of the investment distributed over time
- \( V_i \) is the total value of the investment series to player i

In this context, \( E\tilde{N} \) becomes a random variable in terms of the investment potential for a single parcel built upon an individual's probability assessment summed over all players who have potential to invest in the particular parcel. This is another way of expressing the thought process of individual players before submitting their decisions to the computer. One can further theorize that there is a maximum expectation possible on a given parcel (\( E\tilde{N}_{\text{max}} \)) and that \( E\tilde{N}_{\text{max}} > 0 \) for all parcels. The probability restrictions imposed by the money stock given earlier as:

\[ P(\tilde{N} = N_{\text{max}} | MS \rightarrow 0; K_i \rightarrow MS) \approx 1 \]

limits the value of \( E\tilde{N}_{\text{max}} \) to

\[ \lim_{\tilde{N} \rightarrow 0} E\tilde{N}_{\text{max}} = E\tilde{N} \text{ providing} \]

\[ V > dk V K_i \]
is based on the concept of marginal analysis where the greatest rate of return is sought for each land use at a specific location. Again, the decision possibilities range from situations where any positive rate of return is acceptable to when choices are carefully selected by ranked maximum rates of returns. Expressed mathematically, this becomes:

$$\hat{E}_N = \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{\hat{B}_{ij} - dK_{ij}}{V_{ij}}$$

where

- $\hat{E}_N$ is the expected value of the rate of return from individual investments made at the margin for all parcels
- $\hat{B}_{ij}$ is the estimated benefit
- $dK_{ij}$ is the cost of the investments at all locations distributed over time
- $V_{ij}$ is the total value of the investments at all locations
- $n$ is the number of potential investors for a particular parcel
- $m$ is the number of parcels available for investment

It then follows that $\hat{E}_N_{\text{max}} \geq \hat{E}_N > 0$ then $\lim_{\hat{N} \to 0} \hat{E}_N_{\text{max}} = \hat{E}_N$

providing that

$$V_{ij} \geq dK_{ij} \forall K_{ij}$$

Having considered the player investment in detail, one can now develop this process in terms of the general computer equations. At any particular time period, the possibility that the net rate of return is at any point on the long or short continuums can be expressed as:

$$\bar{N} = \sum_{i=1}^{n} N_i P_i \quad P_i = P(N = N_i)$$

where

- $\bar{N}$ is the net rate of return on the chosen investment
\( P_i \) is the probability function

It then follows from the definition of expected value that:

\[
\hat{N} = \sum_{i=1}^{n} d_i P_i = \hat{E}N
\]

where

\( d_i \) is a specific decision by player \( i \)

Hence, \( \hat{E}N \) can be viewed as the expectation of a particular parcel and \( \hat{E}N \) given earlier can be viewed as an estimator for total investment in a particular round. The computer simply views the equation:

\[
\hat{E}N = \sum_{i=1}^{n} \sum_{j=1}^{m} B_{ij} - dK_{ij}
\]

as

\[
\hat{E}N_j = \sum_{p=1}^{m_j} \frac{\hat{B}_p - \hat{C}_p}{p}
\]

where

\( \hat{E}N_j \) is the expected rate of return for an investment of a given type

\( \hat{B}_p, \hat{C}_p, V_p \) as defined before for a particular parcel

\( m_j \) is the number of parcels of a given type

The next chapter deals with the various types of investments available to the players. It should be noted that when a player makes his investment decisions, he places these decisions on a coding form for computer processing. The computer in turn acts only as an accountant and does not judge the merits of any investment. If the investment is possible under the particular game constraints, it will be allowed. The statistics published by the
computer for total investments are merely the summation of investments successfully made during a particular round.

C. Generalized Private Capital Allocation Equation

Private capital is invested in fixed assets through three general categories: residential housing, commercial activities, and industrial plant and equipment. Since each of these categories can be subdivided in types of land development and no two types of land development can occupy the same parcel of land, we can write the general equation of a particular type of land use summed over all parcels for the whole system. Thus

\[ \tilde{N}_j = \frac{m_j}{\sum_{p=1}^{n_{ij}} \left( \frac{B_p - C_p}{V_p} \right)} \]

where

- \( \tilde{N}_j \) is the expected rate of return for all investments of a particular type \( j \) in the whole system
- \( \hat{B}_p \) is the benefit or income for a particular parcel's land use
- \( C_p \) is the cost discounted over time for a particular parcel's land use
- \( V_p \) is the value of the investment on a particular parcel
- \( m_j \) is the total number of parcels of a given type of land use investment

This equation then provides the expected rate of return throughout the system for a particular type of investment. The following is a detailed analysis of this type of capital investment in terms of the expected rate of return.

The three general categories of fixed assets are subdivided as follows:
1. Residential Housing
   a) Single Family Dwelling
   b) Duplex and Garden Apartments
   c) Multiple Units and High Rise Apartments

2. Commercial Activities
   a) Personal Goods
   b) Personal Services
   c) Business Goods
   d) Business Services

3. Industrial Plant and Equipment
   a) Manufacturing
      1) Furniture and Lumber
      2) Stone, Clay and Glass
      3) Primary Metals
      4) Fabricated Metals
      5) Non-Electrical Machinery
      6) Electrical Machinery
      7) Transportation Equipment
      8) Food
      9) Textiles, Apparel and Leather
     10) Paper
     11) Chemical, Plastics and Rubber
   b) Non-Manufacturing
      1) National Services
      2) Construction Industry

The balance of this section deals with the development of the equations for these general categories. The industrial category will be treated as two categories: Manufacturing and Non-Manufacturing.

In each of the three major categories, we must develop equations for income, expenditures and value in order to apply $\tilde{E}_j$. 
D. Residential Housing

The income for a given residential property is the product of the amount of space used and the rent per space unit for the property. This is expressed as:

\[ \hat{B}_p = R_p \cdot S_p \]

where

- \( R_p \) is the rent per space unit
- \( S_p \) is the number of space units occupied

Let

\[ S_p = \sum_{c} T_{p_c} \cdot K_c \]

where

- \( T_{p_c} \) is the number of population units of class \( c \) on property
- \( K_c \) is a conversion factor for the class

Since we are summing over all available classes, i.e., high, middle and low income classes in each parcel of land, \( S_p \) may be substituted into the equation \( \hat{B}_p = R_p \cdot S_p \). When this substitution is made, you get:

\[ \hat{B}_p = R_p (\sum_c T_{p_c} \cdot K_c) \]

And summing over all parcels we get total income

\[ \sum_{p=1}^{m} \hat{B}_p = \sum_{p=1}^{m} R_p (\sum_{c} T_{p_c} \cdot K_c) \]

The expenditures for a property \((C_p)\) are found by summing for each property the utility, tax, maintenance and water costs. This can be expressed for a single parcel as:
\[ C_p = UT_p + TX_p + MT_p + WTR_p \]

where

- \( UT_p \) is the parcel utility expenditure
- \( TX_p \) is the total parcel tax expenditure
- \( MT_p \) is the maintenance cost
- \( WTR_p \) is the water cost

\( TX_p \), the total parcel tax expenditure, is the sum of three taxes: property, income and sales taxes.

The residence's maintenance expenditure is the sum of its expenditures for personal goods and personal services. The number of consumption units required for maintenance is a function of the total percent depreciation, the level of the residence, and the number of PG and PS units required for each percent depreciation. Total percent depreciation is the sum of normal depreciation and depreciation due to other factors. \( D_T = D_N + D_0 \). \( D_N \) is obtained from the Master Tables,\(^1\) and \( D_0 \) is due to overuse and abuse of the residence. This is usually provided on the residence output. Note that the Master Table also provides the amount of PG and PS units required for each 1% of total residence depreciation. Let \( M_{PG} \) and \( M_{PS} \) be the amount required for each percent depreciation. Thus with a level of residence at three, the amount PG and PS units required are:

\[ D_T \cdot M_{PG} \cdot 3 \quad \text{in PG units of consumption} \]
\[ D_T \cdot M_{PS} \cdot 3 \quad \text{in PS units of consumption} \]

The price that the residence owner pays depends on where he purchases PG and PS and this is obtained from the Commercial Detail Output.

Since we are interested in the cost for all parcels, \( C_p \) is merely summed over all parcels to get:

\(^1\)RMB Economic Sector Manual P. 129.
Finally we must calculate the value \( V_p \) to complete \( N_j \). The value of a property is the sum of the value of the land occupied and the value of the structure.

The value of the occupied land depends on the level of development of the three types of residential development. The term levels refers to level of development and not the condition of the structure. It is important to remind ourselves that when we refer to a level 1 (e.g., RBL or a duplex type of dwelling at one level of development) we are in aggregate terms and for this example we are really talking of a type of housing which provides dwellings for six to twelve thousand people distributed throughout the parcel or square mile. In other words we are not talking about a single duplex building but instead we are referring to a large group of buildings aggregated by type. Since no two types could occupy the same parcel even among housing, we cannot have a RA and RB type in the same parcel. In addition, since we know that each of the three types of housing only occupies two percent of the land at a level one development, we may develop each type to a maximum of level fifty which would occupy one hundred percent of the land. A RB developed to a level of fifty would provide housing for fifty times the population density of 300,000 to 600,000 persons. Thus the product of level development, the space required for each type of housing and the unit market value of the land on the parcel is the value of the occupied land. This value of occupied land can be expressed as:

\[
LND_p = \sum_{p=1}^{m} \sum_{j=1}^{C} (U_{p}^j + T_{X_p} + M_{p}^j + WTR_{p}^j)
\]

where

- \( LND_p \) is the value of the occupied land on parcel \( p \)
- \( L_p \) is the number of levels of property development
- \( S_h \) is space in units of percentage of the unit value of land required for a single level of development of type \( h \) (\( h = RA, RB, RC \))

where RA, RB, RC refer to the types of single, duplex or multiple dwellings
\( U_p \) is the unit value of land at parcel \( p \) (e.g., LND \( \text{p} \) for an RB3 = \( 3 \cdot \frac{2}{100} \cdot 635,000 = \$38,100 \))

The structure value for a given property is the original cost of a level of the structure (type specified) times the value ratio times the number of levels on the parcel. The structure value may be expressed as:

\[
BLDG_p = L_p \cdot O_h \cdot VR_p
\]

where

- \( BLDG_p \) is the structure value of parcel \( p \)
- \( L_p \) is the number of levels of property development
- \( O_h \) is the original cost of a level one development of a structure of type \( h \)
- \( VR_p \) is the value ratio of building on parcel \( p \)

As earlier stated, \( V_p \) for a particular parcel is equal to the sum of the value of land and buildings or

\[
V_p = LND_p + BLDG_p
\]

\[
V_p = L_p \cdot S_h \cdot U_p + L_p \cdot O_h \cdot VR_p
\]

\[
V_p = L_p (S_h \cdot U_p + O_h \cdot VR_p)
\]

Summing over all parcels of land we have

\[
\sum_{p=1}^{n} V_j = \sum_{p=1}^{n} L_p (S_h \cdot U_p + O_h \cdot VR_p)
\]

Finally substituting into \( \tilde{N}_j \) we have:
E. **Commercial**

The income for any commercial establishment is simply the product of the number of units sold and the price per unit or:

\[ \hat{B}_p = \sum_{p=1}^{m_j} U_p \cdot P_p \]

where

- \( U_p \) is the units sold
- \( P_p \) is the price per unit

As before, summing over all parcels results in:

\[ \sum_{p=1}^{m_j} \hat{B}_p = \sum_{p=1}^{m_j} U_p \cdot P_p \]

The expenditures for a given commercial property are simply the sum of the utility, tax, maintenance, water, salary, transportation, and goods and service costs. This is written as:

\[ C_p = U_T + T_X + M_T + W_T + S_L + T_R + G_S \]

where

- \( C_p, U_T, T_X, M_T \) and \( W_T \) are defined as before
- \( S_L \) are salaries paid
- \( T_R \) are transportation costs
- \( G_S \) is the cost of goods and services

It is important to note that the price is set by the seller or owner of the commercial plant. The buyer who requests the number of units he
needs is granted the units if the seller can supply them. The commercial establishment is either business commercial (BG and BS) or personal commercial (PG and PS). BG and BS purchase their needed supplies from the outside systems whereas PG and PS purchase their supplies from the local BG and BS establishments. The customers of BG and BS are either private (i.e., PG, PS and Industrial establishments) or public (i.e., Municipal Services Highway Department, etc.).

BG and BS spend money for service charges, $G_S_p$, which represent the purchases from the outside system. On the other hand PG and PS spend money for business goods and business services which are together referred to as $G_S_p$. This $G_S_p$ is the sum of goods and services that are required for the PG and PS to operate. In both cases for BG, BS and PG, PS, $G_S_p$ is directly related to the number of capacity units sold. Hence the model assumes that the commercial establishments only purchase as much as is required for them to produce the number of capacity units sold. This relationship is found in the Master Table for commercial business. The production of 1 CU (capacity unit) of PG requires the purchase of .037 CU of BG and .017 CU of BS.

Transportation costs incurred by commercial business are dependent upon the number of capacity units purchased, the type of buyer and seller, the distance travelled to the destination and the type of roads used. Regardless of the distance travelled, a business pays a base cost to travel to a destination. Thus the total transportation cost can be expressed as follows.

$$TR_p = (C_U_p \cdot BTC) + \{C_U_p \cdot BTC \cdot L \cdot [(4 \cdot NT_p) - RT_p]\}$$

where

$C_U_p$ is the number of capacity units consumed

$BTC$ is the base transportation cost per unit consumed

thus $(C_U_p \cdot BTC)$ is the base transportation cost to business

and $L$ is the length of a parcel side in miles for this model

$NT_p$ is the number of parcel sides travelled along the least cost route between origin and destination

21
RT_p is the sum of the road types traversed along the least cost route

BTC is the transportation cost per unit per mile travelled and it is numerically the same as BTC

hence \((4 \cdot NT_p) - RT_p\) is the weighted sum of parcel sides travelled from origin to destination along least cost route. The parcel sides or roads are weighted according to the road quality; the best road being of type 3 and the worst being of type 1. Thus four parcel sides of type 2 produce a weighted sum of \((16 - 8) = 8\) parcel units. Hence if each parcel side is 1 mile long, then the weighted distance travelled is 8 mile units and the cost with a BTC of $400 per mile per unit for 3000 units is:

\[
TR_p = (3000 \cdot 400) + (3000 \cdot 400 \cdot 8) = 10,800,000
\]

Note BTC and B̂TC are obtained from the Commercial Master Table.

Again summing over all parcels of land you have:

\[
\sum_{p=1}^{m} \sum_{p=1}^{j} C_p = \sum_{p=1}^{m} \sum_{p=1}^{j} (UT_p + TX_p + MT_p + WTR_p + SL_p + TR_p + GS_p)
\]

Finally the value of the property is calculated in the same manner as housing or

\[
V_p = L_p (S_h \cdot U_p + O_h \cdot VR_p)
\]

or summed over all parcels:

\[
\sum_{p=1}^{m} \sum_{p=1}^{j} V_p = \sum_{p=1}^{m} \sum_{p=1}^{j} L_p (S_h \cdot U_p + O_h \cdot VR_p)
\]

Substituting the values of B_p, C_p and V_p into the general formula for \(E\) we have for commercial investment:
\[
\tilde{E}_j^* = \sum_{p=1}^{m_j} \left( U_{S_p} \cdot P_{p_p} - \sum_{p=1}^{m_j} (U_T + T_X_p + M_T + W_T + S_L_p + T_R + G_S_p) \right)
\]

\[
\sum_{p=1}^{m_j} I_p \left( S_h \cdot U_p + O_h \cdot V_R \right)
\]

F. **Industrial**

The income for a given industrial property is the product of the price per unit (industry specific) and the number of units produced on the properties. This is defined as:

\[
\hat{R}_p = P_m \cdot U_P
\]

where

- \( P_m \) is the industry specific price
- \( U_P \) is the number of units produced

\( U_P \) for all industrial property is at a maximum of 1000 units per level. The actual number produced depends on the employment effect and the value ratio of the property.

Thus

\[
U_P = EE_p \cdot V_R
\]

where

- \( EE \) is the employment effect

and

- \( VR \) is the value ratio of the property, i.e., 90/100 or .9

The value ratio is determined by the amount of depreciation that occurred during a given round but it is limited by the level at which maintenance is set. So that if the maintenance level is set at 70\% then the value ratio cannot fall below this level of 70\%. In fact, maintenance is automatically done each year to keep the property at this value ratio.

The employment effect, \( EE \), is the product of the maximum employment effect, 1000, and the fraction of the total number of employees hired to
the total number of employees required.

\[ EE = EE_{\text{max}} \cdot EH/ER \]

where

- \( EE_{\text{max}} \) are the maximum employment effect
- \( EH \) are the employees hired
- \( ER \) are the employees required

\( P_m \), the price per unit of industrial product is determined by the national business cycle price relative and this normal price per unit for the industrial type.

Thus

\[ P_m = \text{NBCPR} \cdot NP_m \]

NBCPR is the national business cycle price relative which is generated by the outside system i.e., the computer. Its range is .90 to 1.12. In each round, the current NBCPR is printed under the Transactions with the National Economy.

\( NP_m \) is normal price per unit of the industrial type sold and is obtained from the Master Table for Industrial Establishments for each type.

Summing over all industries properties of a type:

\[ m_j \cdot \frac{\sum_{p=1}^{m_j} \hat{B}_p}{\sum_{p=1}^{m_j} P_m} \cdot UP_p = \sum_{p=1}^{m_j} P_m \cdot \frac{\sum_{p=1}^{m_j} UP_p}{m_j} \]

The expenditures for an industrial property are the same set as those for commercial properties (\( UT_p, TX_p, MT_p, NTR_p, TR_p, SL_p, GS_p \)) plus terminal usage costs (\( TM_p \)) and expenses connected with pollution control (\( PTN_p \)).

As before the expenditures are found by summing these values for each parcel and then summing for all parcels to get:
\[
\sum_{p=1}^{m_j} C_p = \sum_{p=1}^{m_j} (U_{p} + TX_{p} + MT_{p} + WTR_{p} + TR_{p} + SL_{p} + GS_{p} + TM_{p} + PTN_{p})
\]

The value of industrial property is the same formula as for commercial property, namely:

\[
\sum_{p=1}^{m_j} V_p = \sum_{p=1}^{m_j} L_p (S_{h, p} \cdot U_p + O_{h, p} \cdot VR_p)
\]

Substituting these values into \( EN_j \), we have:

\[
EN_j = \frac{\sum_{p=1}^{m_j} U_p - \sum_{p=1}^{m_j} (U_{p} + TX_{p} + MT_{p} + WTR_{p} + TR_{p} + SL_{p} + GS_{p} + TM_{p} + PTN_{p})}{\sum_{p=1}^{m_j} L_p (S_{h, p} \cdot U_p + O_{h, p} \cdot VR_p)}
\]
III. GOVERNMENT CAPITAL

The main objective for the government is to maximize total benefits to the system by the allocation of funds among the various departments. Before we can consider an allocation scheme, we must first define the sources of income available to a governmental jurisdiction in the game. The government income is derived from property taxes, income taxes, sales taxes, auto expenditure taxes, bonds (2 year short-run and 25 year long-run), charges for services and transfers of funds, i.e., fines, bribes, etc.. It should be noted that aid from state and federal sources are not treated as income but rather as a reduction in the cost of the local projects they support.

Having considered the sources of income, we can now express the supply of allocatable funds for a jurisdiction \( (MG_j) \) as:

\[
MG_j = \sum_{i=1}^{4} (TX_{i,j}) + B_j + C_j + TR_j
\]

where

\[
\sum_{i=1}^{4} (TX_{i,j}) = TX_1 + TX_2 + TX_3 + TX_4
\]

and

- \( TX_1 \) are property taxes
- \( TX_2 \) are income taxes
- \( TX_3 \) are sales taxes
- \( TX_4 \) are auto expenditure taxes
- \( B_j \) are bonds (long and short)
- \( C_j \) are charges for services
- \( TR_j \) are transfers from all other sources including the outside system
A. **Government Decision Objectives**

The main objective for the government is to maximize total benefits to the system through the allocation of funds among the various departments. If one considers the money supply available for each jurisdiction \((MG_j)\), a limit is obviously placed on the supply of money available for distribution. Each governmental department places a demand on this supply which suggests a balanced budget to be one where supply equals demand. To quantify this idea, consider for each department, or \(r_d\) which is the ratio of demand to supply for services provided by department \(d\), or:

\[
r_d = \frac{D_d}{S_d}
\]

where

- \(D_d\) is the demand for departmental services
- \(S_d\) is the supply of services by department \(d\)

Since these services are directly related to the money available, one can talk about services as well as money. For instance, to reduce the student-teacher ratio you must either build a new school or hire more teachers since it's not realistic to reduce the pupils unless there is heavy out migration. Both of these solutions reduce taxable income which directly relates services available to money available for services in each governmental department.

The objective for each department is to bring \(r_d\) to 1.00 from either side for a balanced budget. If \(r_d > 1.00\), then \(D_d > S_d\) and there is not a sufficient supply of services from department \(d\). Conversely, if \(r_d < 1.00\) then \(D_d < S_d\) and the supply of services from department \(d\) is in excess of the demand for those services and a wasteful condition exists.

If one now assumes that the overall government ratio is a function of all department ratios, then it is possible to determine an overall set of \(\Delta r_d\) based on estimated changes in supply and demand, \(\Delta r_d\), which will bring the overall government ratio to 1.00. In other words, the
total demand for government services and the supply of all government services are balanced.

\[ R_d = \frac{\sum_{d=1}^{n} w_d r_d}{\sum_{d=1}^{n} w_d} \]

where

- \( r_d \) is a departmental ratio
- \( w_d \) is a weighing factor

In this case, \( R_d = 1.00 \) when all \( r_d = 1.00 \), or when tradeoffs (as determined by the \( w_d \)) balance. When \( R_d > 1.00 \), the total demand for services exceeds the supply and when \( R_d < 1.00 \), the supply exceeds the demand and a wasteful condition exists.

Hence the objective function is to bring \( R_d \) as close as possible to 1.00 with the constraints of a finite amount of allocatable funds \( MG_j \) and that all individual departments meet as much of their estimated demands without exceeding the amount of money required to finance their decisions.

B. **Departmental Ratios**

1. The ratio for education \( (I_{sc}) \) can be expressed in terms of supply and demand as:

\[ I_{sc} = \frac{\sum_{c=1}^{n} P_c \cdot SP_c}{\sum_{p=1}^{m} SC_p \cdot VR_p \cdot T_p} \]
where

\[ P_c \] is the population by class

\[ SP_c \] are the students by class

\[ SC_p \] is the school facility by parcels

\[ VR_p \] is the value ratio

\[ T_p \] is the capacity as a function of teacher mix

2. The ratio for adult education \((I_{AE})\) can be expressed as:

\[ I_{AE} = \frac{D_{AE}}{S_{AE}} = \frac{\sum_{c=1}^{n} \sum_{p=1}^{m} EF_{cp}}{\sum_{p=1}^{m} Ph_p + Pm_p} \]

where

\[ EF_{cp} \] is the time allocated to free adult education by parcel and class

\[ Ph_p \] are the PH's hired

\[ Pm_p \] are the PM's hired

3. The ratio for municipal services \((I_{ms})\) is:

\[ I_{ms} = \frac{D_{ms}}{S_{ms}} = \frac{\sum_{p=1}^{m} LU_p \cdot L_p \cdot MU}{\sum_{p=1}^{m} MS_p \cdot VR_p \cdot T} \]

where

\[ LU_p \] is the land use

\[ L_p \] is the number of levels on property
MU is the MS units demanded per level of land use
MS_p is the number of facilities per parcel
VR_p is the value ratio per parcel
T is capacity as a function of worker mix

4. The ratio for welfare payments \((I_{WL})\) is:

\[
I_{WL} = \frac{\text{Actual WL payments}}{\text{Desired or estimated WL payments}}
\]

5. The ratio for utilities \((I_{UT})\) is:

\[
I_{UT} = \frac{D_{UT}}{S_{UT}} = \frac{\sum_{p=1}^{m} LU_p \cdot L_p \cdot UU_p}{\sum_{p=1}^{m} UT_p \cdot DZ_p}
\]

where

UU equals UT units demanded per level of land
UT_p is the utility of a plant facility on a parcel
DZ is the optimal operational capacity of a utility plant
LU_p is the land use
L_p is the number of levels of development on property

6. The ratio for transportation \((I_{TR})\) is:

\[
I_{TR} = \frac{D_{TR}}{S_{TR}} = \frac{(\alpha \beta \gamma) \sum_{i=1}^{n} PW_i + \beta \sum_{p=1}^{m} \sum_{i=1}^{n} (PS_i + L_i)_p}{LRC}
\]
where

\( \alpha \) is the average travel time

\( \beta \) is the average cost

\( \gamma \) is the average inconvenience

\( \alpha \beta \gamma \) is a measure of the mean ecological distance

\( PW_i \) are workers

\( PS_i \) are shops

\( L_i \) is the inter-business transport

\( P \) is parcel

\( LRC \) is the lowest real cost. This cost is at a minimum when

\( \alpha = \alpha_p, \beta = \beta_p \) and \( \gamma = 0 \)

C. **Budget Allocation**

The total budget to be allocated to each department must be the entire supply in order to avoid wastefulness. This decision format for allocation can be expressed as:

\[
\text{MG}_j = \max \sum_{\lambda=1}^{n} (IG_\lambda + \zeta + P_i)
\]

s.t. \( IG_\lambda \geq 1 \)

where

\( IG_\lambda \) are the specific services indexes for each department

\( \zeta \) is a random factor for deciding expenditures

\( P_i \) is the sum of all public and private influence which affect allocation decisions

Since the objective is clearly to spend all available funds, i.e., \( IG_\lambda \geq 1 \), the tendency will be to allocate the government capital resources to departments with the highest ratios. This is the reason for optimizing with a maximum.
IV. TIME ALLOCATION

The social sector directly influences and controls the government through its voting power and the economic sector through its ability to boycott. In addition, it has the ability to allocate the leisure time of small aggregates of workers which produce complex interactive effects throughout the entire model. Leisure time is that time not spent in full-time work, eating, sleeping and personal care. It is divided into 100 units which can be allocated to part-time work, adult education, political activity or recreation. In addition, the time used to travel to and from work is subtracted from the leisure time. A further reduction of leisure time occurs due to sickness as a result of the high health index at a workers resident location. With this in mind we would describe the total amount of allocatable time, $T$, as:

$$T = T_u \cdot b_{P_k}$$

where

$T$ is the total amount of real time units of leisure time used in the entire system

$T_u$ is the specific amount of time per unit

$b_{P_k}$ is the total number of people in the system by socio-economic class weighted by the real value of their time units ($b$)

In theory, this model is sufficient, but in practice it takes into account such detail that the model would become extremely complicated.
To avoid this difficulty, define the real time available as:

\[ T = (T_N + T_A)bPK \]

where

- \( T_N \) is the fixed amount of time that is more or less available and falls outside short-run discretionary allocations
- \( T_A \) is the amount of real time actually available to be allocated in the short-run

A. Allocations

Let \( T_N \) be defined as before and

\[ T_N = T_T + T_Z + T_I \]

where

- \( T_T \) is the time spent in traveling to work
- \( T_Z \) is time lost to illness
- \( T_I \) is time spent in involuntary activities

Using the above definitions, \( T_A \) can then be considered as resource improving uses of time which can be controlled as time allocations.

Let

\[ T_A = T_W + T_{AP} + T_{AF} + T_{PA} + T_R \]
where

\[ T_w \] is time spent in a part-time job

\[ T_{AP} \] is time spent in paid training and education programs

\[ T_{AF} \] is time spent in free training and education programs provided by the local school system

\[ T_{PA} \] is time spent in political activities

\[ T_R \] is time spent in recreational pursuits

Associated with each of the pursuits in \( T_A \) is a rate of return for the time invested given by:

\[ 1. \quad r_w + \gamma_1 \left( \frac{SAL_c}{N_c} \right) \]

where

\( r_w \) is the rate of return from part-time work

\( SAL_c \) is the full-time salary offered to the class \( c \)

\( N_c \) is the relative value of part-time work to class \( c \)

\( \gamma_1 \) is a function designed to make the absolute value of \( r_w \) relative to other time pursuits for comparison and is defined as:

\[ \gamma_1 = f \left( \frac{\sum_{c=1}^{m} P_c}{m \sum_{c=1}^{m} \text{JOB}_{pt_c}}, \text{IND.} \right) \]

where

\( P_c \) is the population units by class wishing to allocate time units to part-time jobs.
JOB<sub>pt</sub> are part-time jobs offered by all employees for a class c (of class M & L)

IND. is an indeterminate quantity such as propensity, likes, dislikes, etc.

2. \( \gamma_{AF_c} = \gamma_2(\Delta EL_c) \)

where

\( \gamma_{AF_c} \) is the rate of return for free training and adult education

\( \Delta EL_c \) is the change in education level for class c

\( \gamma_2 \) is a function designed to make the absolute value of \( \gamma_{AP_c} \) relative to other time pursuits for comparison and is defined as:

\[
\gamma_2 = f \left\{ \sum_{j=1}^{m} \frac{P_{ML}}{P_{ML}}, \text{IND} \right\}
\]

where

\( P_{ML} \) is the number of population units wanting to allocate time to free training and education

\( FAE_i \) is the supply of free education and training programs provided by the local school systems

IND. is an indeterminate quantity such as prosperity, likes, dislikes, etc.

3. \( \gamma_{AP_c} = \gamma_3 (\Delta EL_c - PR_c) \)

where

\( \gamma_{AP_c} \) is the rate of return for paid training and education
$\Delta E L_c$ is the change in educational level for class $c$

$PR_c$ is the cost of spending one-time unit in private education and training

$\gamma_3$ is a function designed to make the absolute value of $r_{AFC}$ relative to other time pursuits for comparison and is defined as:

$$\gamma_3 = f(C_{AE}, \text{IND.})$$

where

$C_{AE}$ is the cost of private training and education

IND. is an indeterminate quantity such as propensity, likes, dislikes, etc.

4. $r_{PC} = \gamma_4 \Delta VT_c$

where

$r_{PC}$ is the rate of return for political activities

$\Delta VT_c$ is the increase in voter registration for class $c$

$\gamma_4$ is a function designed to make the absolute value of $r_{PC}$ relative to other time pursuits for comparison and is defined as:

$$\gamma_4 = f(r_{PC}, \text{IND.})$$

where

$r_{PC}$ is the expected return from political power

IND. is an indeterminate quantity such as propensity, likes, dislikes, etc.

5. $\gamma R_c = \gamma_5 \Delta PS_c$
where

\[ \gamma_{RC} \] is the rate of return for recreational activities

\[ \Delta PS_c \] is the decrease in personal dissatisfaction due to recreation for class c

\[ \gamma_5 \] is a function designed to make the absolute value of \( \gamma_{RC} \) relative to other time pursuits for comparison and is defined as:

\[
\gamma_5 = f \left\{ \frac{\sum_{j=1}^{m} P_{RT,j}}{\sum_{i=1}^{n} \sum_{j=1}^{n} PLA_i}, \text{IND} \right\}
\]

where

\[ P_{RT,j} \] is the number of population units wanting to allocate recreational time

\[ PLA_i \] is the supply of park land and other recreational facilities usually provided by the planning and zoning department

IND. is an indeterminate quantity

B. Decision to Invest Time

Let \( \tilde{T} \) be the pursuit one choses to invest time in given some return rate from \( T \). Then:

\[
\tilde{T} = T_w \quad \text{if} \quad r_w > r_{APC}, r_{AFC}, r_{PC}, r_{RC}
\]

\[
\tilde{T} = T_{AP} \quad \text{if} \quad r_{APC} > r_w, r_{AFC}, r_{PC}, r_{RC}
\]

\[
\tilde{T} = T_{AF} \quad \text{if} \quad r_{AFC} > r_w, r_{APC}, r_{PC}, r_{RC}
\]

\[
\tilde{T} = T_P \quad \text{if} \quad r_P > r_w, r_{APC}, r_{AFC}, r_{RC}
\]

\[
\tilde{T} = T_R \quad \text{if} \quad r_{RC} > r_w, r_{APC}, r_{AFC}, r_{RC}
\]

Further, if \( \tilde{T} \) is the return rate from \( T \) and \( B \) is a benefit from all time allocatable investments, then:
\[ B = \sum_{i=1}^{n} r_i T_i \]

where

\[ T_i \in \{ T_w, T_{AP}, T_{AF}, T_p, T_k \} \]

An investor in allocatable time will tend towards maximizing \( B \) by finding the best mix of time investments according to:

\[ \max B = \max \sum_{i=1}^{n} r_i T_i \]
V. PUBLIC AND PRIVATE INFLUENCE

Public and private influence is by far the most difficult to capture in quantitative form. It is desirable to include this so called "power" into the game since voting, economic suasion, and influence peddling in the form of bribes and boycotts are all part of a real city. For the purposes of the game, this quantity must be in the standard form of supply and demand recognizing that government officials vote on issues and private sector members vote for government officials. This dual role can be readily seen to apply in the economic and influence roles as well. It is within this dual functional idea, that this model is built. One could, for instance, claim that public influence is related to legal cases it wins, the percentage of expenditures for influencing voter groups and commercial enterprises and the moral suasion of the incumbents. On the other hand, private power can also take a large number of forms. We could include voting power, lobbying, migration, bribes and numerous other influences such as protests that influence government decisions.

The ratios for economic power were described in Chapters II and III and have the built-in bias of determination for public and private goals. It is therefore convenient to express total power as:

\[ P = \sum_{i=1}^{n} P_{pi} + P_{gi} \]

where

- \( P \) is the total power in the system at a particular point in time
- \( P_{pi} \) is the private power, both social and economic for all individuals
- \( P_{gi} \) is the public power for all individuals

To develop this particular model, we must develop equations for voting, a scheme for allocating power and a supply and demand ratio in
terms of money to measure this influence so that it will be compatible with other functions of the partial resource allocation defined earlier as:

\[
PR = \alpha CP + \beta CG + \delta T + \varepsilon P
\]

In other words, \( \varepsilon \) must be defined explicitly as well as \( P \).

A. **Voting**

Voting can be described as:

\[
\text{POP}_V = \sum_{k=1}^{m} \lambda_k \sum_{i=1}^{n} P_{ki} C
\]

where

- \( \text{POP}_V \) is the total number of people who vote
- \( \lambda_k \) is the propensity of each class to vote
- \( P_{ki} \) is the number of people who actually vote
- \( C \) is a random factor

Since the above equation is difficult to use in its present form, we can consider the probability that a voter will vote for a particular candidate or issue. When this is done, \( P_k \) then becomes:

\[
P_k = \sum_{i=1}^{n} \delta_i p_i
\]

where

- \( i \) is an individual member of a class
- \( \delta_i \) is the propensity of that individual to vote
- \( p_i \) is the probability that \( i \) will vote

In other words, given the inclination to vote (\( \delta_i \)), there is a probability that he will vote (\( p_i \)).
The above statements can be further described as when \( i \) is an individual, then \( \forall i \) and \( 0 \leq p_i \leq 1 \). Now if we let \( g_i \) be the probability that a vote will be cast for an alternative \( j \) on an issue, then \( 0 \leq g_i \leq 1 \). When this is true, \( k(p_i, g_i) \) becomes the probability that \( i \) will vote for \( j \) in class \( k \) and \( \text{POP}_V \) then becomes:

\[
(\text{POP}_V)_j = \sum_{k=1}^{m} \lambda_k \left[ \sum_{i=1}^{n} (\delta_i p_i g_i) \right] k
\]

and finally for all issues we have

\[
\sum_{j=1}^{m} (\text{POP}_V)_j = \sum_{j=1}^{m} \left\{ \sum_{k=1}^{m} \lambda_k \left[ \sum_{i=1}^{n} (\delta_i p_i g_i) \right] k \right\} j
\]

Based on the above equation, one can now define \( C \), given as a random factor, as influence or the abstract quantity that sway voters by emotional or financial means at the last minute on issues and intended voting desires.

B. Allocation of Power

The objective sought in influencing is to allocate that maximum amount of influence in the public and private sectors. This allocation can be expressed as:

\[
P = \max \sum_{i=1}^{n} (p_i + g_i)
\]

subject to:

\[
\frac{p_i}{g_i}, \frac{p_i}{g_i} > 0
\]

It is not reasonable to assume that a democratic government can exist without some influence from both the government and private sectors.
Based on the assumption that power exists in both sectors, one can then look at the distribution of this power between the sectors and the incremental cost associated to change this power ratio. With public and private power always in existence, we must consider incremental changes in costs for this ratio. If influence is viewed in purely capitalistic terms, there will always be a supply and demand for influence. In this format, one can now refer to ratio's in the same manner as the government ratio's of Chapter III.

Let $\phi_p$ be defined as the incremental cost to change the ratio $r_p$ when

$$ r_p = \frac{D_p}{S_p} $$

in terms of supply and demand.

Now if $r_p = \frac{D_p}{S_p}$ we can solve for $S_p$ and find that $S_p = \frac{D_p}{r_p}$. The change in $S_p$ required to meet new conditions will become:

$$ \Delta S_p = S'_p - S_p = \frac{D'_p}{r'_p} - \frac{D_p}{r_p} $$

when $S'_p$, $r'_p$ and $D'_p$ are the new conditions.

Rearranging the terms in $\Delta S_p$:

$$ \Delta S_p = \frac{D'_p}{r'_p} \frac{r_p - D_p}{r'_p} $$

and noting that

$$ \Delta r_p = r'_p - r_p \quad \text{or} \quad r'_p = r_p + \Delta r_p $$

$$ \Delta D_p = D'_p - D_p \quad \text{or} \quad D'_p = D_p + \Delta D_p $$

42
we can substitute these values into $\Delta S_p$ and get:

$$\Delta S_p = \frac{r_p (D_p + \Delta D_p) - D_p (r_p + \Delta r_p)}{r_p (r_p + \Delta r_p)}$$

Expanding:

$$\Delta S_p = \frac{r_p D_p + r_p \Delta D_p - r_p D_p + D_p \Delta r_p}{r_p^2 + r_p \Delta r_p}$$

Collecting terms:

$$\Delta S_p = \frac{r_p \Delta D_p + D_p \Delta r_p}{r_p^2 + r_p \Delta r_p}$$

Since the cost per unit of supply is known for each sector and is $C_p$, then the incremental cost to change the ratio becomes:

$$\phi_p = C_p \Delta S_p = \frac{r_p \Delta D_p C_p + D_p \Delta r_p C_p}{r_p^2 + r_p \Delta r_p}$$

This latter equation might be viewed as the incremental cost to change the power base.

C. Power Objective

The power base can now be defined as:

$$P = V + E_p + \text{IN}$$

where

- $V$ are votes
- $E_p$ is the economic power
IN is influence

This equation may be re-written as:

\[ P = \phi \sum_{j=1}^{m} (POP_j) + E_P + \text{IN} \]

where

IN is defined in terms of money persuasion and the first term is the incremental cost of swaying elections.
VI. OPERATION OF THE MODEL

The previous chapter related to the development of the partial resource allocation for the general system. In the game, the variables for the fixed and liquid resources must be related to each other to integrate all city functions. This process is accomplished through a series of operating programs plus the decision choices of the actual players. The sections of this chapter will describe the major operating programs and the calculations they perform.

A. Business Goods and Services

1. The total demand for business goods and business services is given by:

$$ TD_{(BS/BG)} = \sum_{i=1}^{n} K_{i}LU_{i} + C_{pg}PG + B_{ps}PS + \sum_{i=1}^{n} Z_{i}(DEP_{i} + MT_{i} - VR_{i}) + G $$

where

- $TD_{(BS/BG)}$ is the total demand for business goods and services
- $LU_{i}$ is the land use at a particular site $i$
- $PG$ are sales of personal goods
- $PS$ are sales of personal services
- $L_{i}$ are the number of levels at location $i$
- $K_{i}, C_{pg}, B_{ps}, Z_{i}$ are constants

2. The supply in the commercial sector is dependent upon level, employment and value ratio given by:

$$ TS = DC \sum_{i-1}^{n} L_{i}VR_{i} (1 - \frac{\sum_{k=1}^{m} E_{k}}{E^{*}}) $$
where

\(TS\) is the total supply

\(DC\) is design capacity

\(L_i\) are the number of levels at location \(i\)

\(VR_i\) is the value ratio at \(i\)

\(E^*_{ki}\) is the number of employees at \(i\) by class \(k\)

\(E^*\) is the total of employees needed at that location

3. Price is a function of the total demand and total supply curves and is given by:

\[
\bar{p} = \frac{\sum_{i=1}^{n} P_i \,(t-1)}{n}
\]

where

\(\bar{p}\) is the average price paid last year

\(P_i\) is the price of goods or service for the last round \((t-1)\)

\(n\) is the number of \(i\) goods or \(i\) services rendered last round

4. The new prices for goods or services are dependent on the change in demand and the change in supply for the next round.

This may be expressed as:

\[
P = R \left[ \frac{\Delta Po - \Delta Ps}{2} \right] \bar{p}
\]

where

\(P\) is the new price for goods or services

\(\Delta Po\) is the change in price if supply is kept constant
ΔPs is the change in price if demand is kept constant

R is a functional multiplier

P is the average price paid last year

B. Employment

The employment process matches the people who are looking for jobs with the employers who are trying to obtain workers for needed skill levels.

1. Demand for Jobs

The demand for jobs is a function of the amount of new industry in the system, the expansion of government services, the displacement of workers from firms going out of business or site location changes of existing firms. In the game, the demand is also caused by jobs not filled from the previous round.

This demand for jobs is:

\[ J_{jk} = NP_k + NG_k - J_k + V_k \]

where

- \( J_{jk} \) is the total demand for jobs by each income class
- \( NP_k \) is the amount of new industry constructed during the previous time period
- \( NG_k \) are the new government positions
- \( J_k \) are jobs created due to employee displacement
- \( V_k \) are job openings from the previous round.

2. Supply of Workers

The supply of workers is given by:
\[ p_{jk} = p_{sk} + p_{nk} + pl_k + p_{xk} \]

where

- \( p_{jk} \) is the total worker supply by socio-economic class
- \( p_{sk} \) are workers who desire to move by skill category
- \( p_{nk} \) are new residents in the system
- \( pl_k \) are people who have moved in the system or have otherwise vacated their original jobs
- \( p_{xk} \) are the remaining unemployed persons

3. **Matching**

The process of matching the supply and demand for workers is accomplished by locating workers by skill and educational level to fill the demand, whereas the workers are interested in obtaining positions which maximize that income and/or achieve their satisfying objectives.

To accomplish this matching process, the supply (\( p_{jk} \)) is ranked by educational level, the most skilled first in the form:

\[ p_{jkp} \max(S_{epk} + KS_{ep} - T_{epp}) \]

if

\[
\sum_{k=1}^{m} p_k > \sum_{k=1}^{m} j_k
\]

then

\[ p_{jkp} = \max \left[ S_{ep'}(k-1) + KS_{ep'}(k-1) - T_{epp'} \right] \]

where

- \( p_{jkp} \) is the supply of jobs by parcel and class
- \( S_{ep'k} \) is the salary offered on parcel position
$T_{epp}$ is the transportation cost to travel from home to work

$K$ is the percent change in job attractiveness to entice worker to change positions, i.e., salary, etc.

This process continues for each class until

- $P_k$ matched with $J_k$ are employed
- $P_k$ matched with $J_{k-1}$ are unemployed
- $P_k$ not matched are unemployed

4. **Part-Time Employment**

Part-time jobs must be matched against workers who desire part-time work. The matching is performed in the same manner as for full-time work, namely:

$$SPT_k = \sum_{i=1}^{n} T_e_i \cdot Z$$

where

$$Z = \sum_{k=1}^{m} P_k$$

and

- $SPT_k$ is the supply of part-time jobs by parcel
- $T_e_i$ is the transportation cost per individual
- $Z$ is the sum of available jobs by parcel

5. **Salaries**

The salary that can be offered at a given location varies with transportation costs, unemployment and salaries offered at other locations. Salary may be computed according to the formula:
\[ S \Phi = \frac{S_{BZ_k} \left[ T_{\phi i} \right]}{T_i} \]

where

\[ S = \sum_{j=1}^{m} S_j E_j \]

\[ U = \frac{m}{Z_k} \sum_{j=1}^{m} E_j \]

\[ \Phi = \frac{m}{Z_k} \sum_{i=1}^{m} T_i \]

and upon substitution

\[ S \Phi = \frac{S_{BZ_k} \left[ S_j E_j \right]}{Z_k} \]

\[ \frac{1}{T_i} \sum_{j=1}^{m} E_j \]

\[ \frac{1}{Z_k} \sum_{i=1}^{m} T_i \]

\[ \frac{1}{Z_k} \sum_{j=1}^{m} E_j \]

\[ \frac{1}{Z_k} \sum_{i=1}^{m} T_i \]

\[ \frac{1}{Z_k} \sum_{j=1}^{m} E_j \]

\[ \frac{1}{Z_k} \sum_{i=1}^{m} T_i \]

\[ \frac{1}{Z_k} \sum_{j=1}^{m} E_j \]

\[ \frac{1}{Z_k} \sum_{i=1}^{m} T_i \]

\[ \frac{1}{Z_k} \sum_{j=1}^{m} E_j \]

\[ \frac{1}{Z_k} \sum_{i=1}^{m} T_i \]

50
\[ S_j = \frac{B \sum_{j=1}^{n} E_j S_j \sum_{i=1}^{m} T_i}{Z_k \sum_{j=1}^{m} \sum_{i=1}^{n} T_{ij}} \]

where

- \( S_j \) are the total salaries
- \( E_j \) are the number of employees at \( j \)
- \( S_j \) is the salary per worker at \( j \)
- \( T_i \) is the transportation cost for work \( i \)
- \( Z_k \) is the total labor force by class
- \( B \) is the coefficient of unemployment

C. Land Use

In the previous section concerning the allocation of private capital sources, we assumed that an allocation could be made for a particular land use type. In general, if \( L_{ij} \) is the cost of locating a specific land use in a particular location, then:

\[ \text{Land Costs} \approx \sum_{i=1}^{n} \min L_{ij} \]

subject to:

- Formal Zoning
- Presence of Utilities
- Topographical Features

and

\[ Y_d \geq B_c + D + LC \]
where

\( Y_d \) is the discounted income

\( B_c \) is the structure cost

\( D \) is the demolition condition

\( LC \) is the land cost

One can further require that

\[ MS_L > B_c + D + LC \]

and

\[ G_y = k_i \]

where

\( MS_L \) is the money supply available for land

\( G_y \) is a constant gross income per land use

Within this framework a model for specific land uses can be developed.

1. **Residential**

Residential land use can be modeled as:

\[ L_R \approx \sum_{i=1}^{n} \min L_{ij} = \alpha_1 E_{I_i} + \alpha_2 L_{C_i} + \alpha_3 \bar{T}_i \]

where

\( E_{I_i} \) is an environmental index for a parcel

\( L_{C_i} \) is the parcel land cost

\( \bar{T}_i \) is the average transportation cost

\( \alpha_1, \alpha_2, \alpha_3 \) are coefficients to relate the variables to each other
2. **Industrial**

Industrial land use is:

\[ L_I = \sum_{i=1}^{n} \min L_{ij} = \beta_1 T_i + \Lambda \]

where

\[ \Lambda = \beta_2 L_{C_i} + \beta_3 C_{H_i} + \beta_4 T_i + \beta_5 W_i + \beta_6 M_{S_i} \]

and

- \( T_i \) is the average transportation cost
- \( L_{C_i} \) is the parcel land cost
- \( C_{H_i} \) are the utility charges
- \( T_i \) are the taxes
- \( W_i \) are the water charges
- \( M_{S_i} \) are the municipal service charges
- \( \beta_i \) are coefficients to relate the variables to each other

3. **Commercial**

The commercial land use model is:

\[ L_C = \sum_{i=1}^{n} \min L_{ij} = \gamma_1 A_i + \gamma_2 B_i \]

where

- \( A_i \) is the same as industrial
- \( B_i = \max \sum_{i=1}^{n} C_{ij} \)
B₁ is the best relative competitive advantage for a new location given all existing land uses

γ₁,γ₂ are coefficients to relate the variables to each other

4. Government Facilities

\[ Lg = \max B = \sum_{i=1}^{n} \sum_{j=1}^{m} N_{ij} \]

given:

The existing government land uses \( (Lg_{1t}) \)

Location of all other land uses \( (LU_i) \)

Location of all people by class \( (P_{ik}) \)

and

\( N_{ij} \) are any government services to be located in a specific jurisdiction

D. Land Value

Land value is a function of the proximity of the particular parcel of land to various facilities and the presence of certain amenities. In equation form this becomes:

\[ LV = BV + \epsilon_1 T_T + \epsilon_2 R + \epsilon_3 E + Z + \epsilon_4 T_R \]

where

\( LV \) is the land value of a particular parcel

\( BV \) is the base value which is usually the agricultural value or approximating the construction value
$T_T$ is the distance to the nearest terminal

$R$ is the distance to the nearest residence

$E$ is the distance to the nearest employment opportunity

$Z$ is the presence of zoning by type

$T_R$ is road access by type of road

E. Rent

The rent for a particular dwelling can be calculated according to the following formula:

$$R_f = (\bar{R}) \left( \frac{BQI_f}{\bar{Q}I} \right) \left( \frac{\gamma E\bar{I}}{EI_f} \right) (\bar{I}_c)$$

where

$$\bar{R} = \frac{\sum_{i=1}^{n} R_i S_i}{\sum_{i=1}^{n} S_i}$$

$$\bar{Q}I = \frac{\sum_{i=1}^{n} QI_i}{n}$$

$$\bar{E}I = \frac{\sum_{i=1}^{n} EI_i}{n}$$
\[ \bar{\mathcal{R}} = \frac{\sum_{i=1}^{n} S_i}{n} \]

\[ \bar{\mathcal{Q}} = \frac{\sum_{i=1}^{n} Z_i}{n} \]

and

- \( \bar{\mathcal{R}} \) is the average rent over system
- \( R_i \) is rent per space unit
- \( S_i \) is space units rented
- \( QI_i \) is quality of housing index
- \( EI_i \) is environmental index
- \( \bar{QI} \) is average quality of housing index
- \( \bar{EI} \) is average environmental index
- \( \bar{\mathcal{Q}} \) is average occupancy
- \( Z_i \) is total space units available at parcel i
- \( QI_f \) is the quality housing index for a particular dwelling
- \( EI_f \) is the environmental index for a particular dwelling

F. Location of a Commercial Site

In order to locate any non-residential income producing property, one must consider:

1. Cost-distance to the nearest terminal
2. Cost-distance to suppliers of goods and services
3. Cost of treating water
4. Suitability of site for development which includes zoning, utilities and space.
To construct this model, matrices must be constructed whose cells reflect the status of each site. Let the matrix $T$ contain for each cell the cost-distance from each parcel to the nearest terminal. If the parcel is suitable, enter the value, otherwise the cell has a value of 0. Construct a matrix $S$ such that each cell contains the cost, price and transportation cost from the nearest supplier of goods and services. If the cost value is suitable, enter this value in the cell, otherwise enter 0. In a similar fashion construct $W$ and $X$ where each suitable cell in $W$ contains the water costs (or 0 if unsuitable) and each cell in $X$ contains the taxes on the property (or 0 if unsuited).

Combining these matrixes:

$$K = T + S + W + X$$

and $j$ is the total cost associated with each cell $j$.

Let $i$ represent a cell where the products of the activity which we are locating are consumed. For each $i$, there exist the following known quantities:

- $C_i$: the current cost per unit of transporting goods from point of sale
- $U_i$: the number of units consumed by $i$

Now for any potential site $j$, ($K_j > 0$) calculate $W_{ij}$ which is the cost per unit of transporting goods and/or services from point of sale $j$ to consumer $i$. The sale for $j$ is given by:

$$S_j = \sum_{k=1}^{n} U_k$$

where

$$W_{ij} < C_i$$

and $n$ is the number of cases where $W_{ij} < C_i$.

The gross income is:
\[ I_j = P * S_j = P \sum_{k=1}^{n} U_k \]

and profit by

\[ \phi_j = I_j - K_j = P \sum_{k=1}^{n} U_k - K_j \]

Finally calculate \( \phi_j \) for all \( j \) where \( K_j > 0 \) and the site selected, \( S \) is given by:

\[ S = j \exists \phi_j \text{ is maximum} \]

G. Migration

People move in and out of the urban systems on the bases of opportunities available to them. They move within the urban system or ultimately out of the area on the basis of the ability of the local system to satisfy given needs. One measure of population movement is the net population change of the urban system. This change can be represented as:

\[ \Delta P_T = N_{G_T} + I_{N_T} - O_{UT_T} \]

where

- \( \Delta P_T \) is the total population change in a specific time period
- \( N_{G_T} \) is the natural change (births - deaths)
- \( I_{N_T} \) is in-migration
- \( O_{UT_T} \) is out-migration

Further,

\[ N_{G_T} = K_k P_k(T-1) \quad \text{or} \quad N_{G_T} = \sum_{k=1}^{n} K_k P_k(T-1) \]

58
where the natural growth rate is a function of the total population segregated by socio-economic class.

\[ \text{IN}_T = \alpha J_{k(T-1)} \quad \text{or} \quad \text{IN}_T = \sum_{k=1}^{n} \alpha_k J_{k(T-1)} \]

where in-migration is largely a function of the number of jobs available in the system in the previous time period for each socio-economic class.

\[ \text{OUT}_{T_k} = \sum_{k=1}^{n} (\text{MV-MS})_k + \alpha U_{dk} + \beta U_{rk} \]

where

\[ \text{MV} = \sum_{i=1}^{n} \alpha P_{ki} + \sum_{i=1}^{n} D_i R_i \]

and

\[ \text{MS} \]

is the number of people who migrate

are movers who stay within the local system

is the percent of the population by socio-economic class

are people who are displaced by the razing of residences

is a random number of people who move

is the percent of under employed

is the percent of unemployed

\[ \text{MS} \quad \text{can be further described as:} \]

\[ \text{MS} = \text{EI}_{\max} - \sum_{i=1}^{n} \text{EI}_i > 0 \]

where

is the maximum available environmental index available to residences
The present environmental index for a particular parcel or

\[ EI_i = PI + R_R + cRQ + dMS + eSc + gTx \]

where

- \( EI \) is the environmental index
- \( PI \) is the water pollution index
- \( R_R \) is the rent
- \( cRQ \) is the residence quality
- \( dMS \) are movers who stay within the local system
- \( eSc \) are the residential services
- \( gTx \) is the tax rate or welfare level

**H. School Assignments**

The school model considers the needs of the various socio-economic classes and compares these needs to the service levels of the schools. When there is more service available than the local district needs, the model parallels the concept of excess capacity in the plant or factory. Conversely, when there is not enough capacity in the public system, the shortages are assigned to the private sector, i.e., private education.

The process of school assignments or establishing school districts can be done in terms of supply and demand. In this light, several objective functions are possible such as minimizing cost, maximizing service along with containment within political boundaries, neighborhood boundaries and so forth. The simplest way to model is based on school districts. Let \( D_s \) be the total student demand by socio-economic class on all school districts then:

\[ D_s = \sum_{k=1}^{n} \alpha P_{k\text{sd}} \]
where

\[ \alpha \] is the number of students per population class

\[ P_{ksd} \] is the population by class by school district

Since the objective is to let \( D_s = S_s \) where \( S_s \) are the total services that meet the total demand, we must consider the effective supply of each school within a district and the demand on that school. In other words \( D_L / K_L < 1 \) when \( K_L = n \) and \( D_L \) is the demand on school \( L \) and \( K_L \) is the effective supply or capacity of school \( L \) given the quality thresholds of cost and socio-economic class for \( n \) students. The definition of \( K_L \) becomes:

\[
K_L = \sum_{i=1}^{n} C_{dsd} R_{ki}
\]

where

\[ C_{dsd} \] is the physical capacity of the school plant in a specific school district

\[ R_{ki} \] is the index for the quality of the school

For a second school, say \( m \), let \( D_m / K_m < 1 \) and \( K_m = m \) in the same manner as before and then determine:

\[
K_m = \sum_{i=1}^{m} C_{dsd} R_{ki} - D_L
\]

This process is repeated for all schools in the system and each \( K \) is summed to determine \( S_s \). If

\[
\sum_{j=1}^{m} K_j < S_s
\]

then excess capacity exists and of course if
this surplus must go to the private sector or the public school system must be expanded. In terms of cost, \( S_L = (P_{PLT}, K_L, T_L \) where for school \( L \), the cost of the physical plant \( P_{PLT} \), the cost of maintaining the quality threshold in teachers, supplies, etc., and the transportation costs for bussing must be minimized. This is performed by arranging the students within school districts to minimize the total cost of all schools in the system. If additional constraints are placed upon this objective such as segregation ratio, the model simply requires new school districts which do not minimize the costs due to excess bussing but do comply with ratio definitions.

I. Transportation

Transportation represents movements of people and goods to jobs and shopping locations. There are four modes of transportation considered in the journey to work. Namely car, bus, rapid rail and walking. The cost is measured in terms of time, money and inconvenience (ecological distance). All work travel is calculated at peak hours. The traveling to and from commercial and industrial sites for non-employment purposes are calculated at off-peak hours and only consider car or truck and walking as alternative modes.

Since we are interested in minimizing travel costs, we first must consider \( T_{ew} \) or the transportation to jobs. Let

\[
T_{ew} = \sum_{p_1=1}^{n} \min E_{p_1p_2}
\]

where

\( E_{p_1p_2} \) is the ecological distance between points \( P_1 \) and \( P_2 \)

also let
\[ E_i = A_i T_i + M_i + C(t-1)_i \]

where

- \( i \) equals trips
- \( A \) is the dollar value of time \( T \)
- \( M \) is the money costs of travel
- \( C \) is dynamically adjusted congestion factor for iterative trips, i.e., historical value

If \( T_s \) is the transportation to shopping and commercial centers, then let

\[ T_s = \sum_{P_1=1}^{n} \min C_{P_1 P_2} \]

where

- \( C \) is the money cost of travel between points \( P_1 \) and \( P_2 \). Let \( M_{T_i} \) be the money cost of a specific trip such that

\[ M_{T_i} = \min (H + B + C + W) \]

where

- \( H \) is highway
- \( B \) is bus
- \( C \) is rapid rail
- \( W \) is walk

Finally the total transportation cost \( T \) is

\[ T = \left( \sum_{i=1}^{n} E_i + T_s \right) \sum_{i=1}^{n} M_{T_i} \]

where the cost to work is
\[ T_w = T_{ew} \sum_{i=1}^{n} E_i \sum_{i=1}^{n} M_{T_i} \]

and the cost for shopping is

\[ T_{\text{shop}} = T_s \sum_{i=1}^{n} M_{T_i} \]

The travel for industrial or commercial for non-employment purposes is, of course, found in \( T_s \) with rapid rail eliminated as a possibility in \( M_{T_i} \).

J. Value of Structure and Capital Equipment

The fixed capital assets are assumed to decrease in value or depreciate each time period. Expenditures to repair or modernize the asset will negate the depreciation at the time period the improvement was made and the depreciation will begin again for the asset's new value.

The value of an asset at time \( t + 1 \) is:

\[ V_{t+1} = NV_t - DL_{t+1} NC + M_{t+1} \]

subject to

\[ V_{t+1} \leq NC \]

where

- \( NC \) is the original cost
- \( NV_t \) is the present value in a given time period
- \( DL_{t+1} \) is the depreciation during a given time period in percent of original cost
- \( M_{t+1} \) is the maintenance expenditure made during a time period.

Depreciation or land use can be determined as:
\[ \text{DL}_{p\lambda} = \sum_{\lambda=1}^{n} [d_{\lambda e} \text{MS}_{dmp} + d_{2e\lambda p} + d_{3e} \text{FL}_{KP} + d_{4e} \text{FR}_{dup} + d_{5e} U_p] \]

where

- \( \text{DL}_{p\lambda} \) is the total depreciation for each land use on each parcel in the system
- \( d_{\lambda e} \text{MS}_{dmp} \) is the municipal service quality index per parcel by land use
- \( d_{2e\lambda p} \) is the coefficient of time by land use type
- \( d_{3e} \text{FL}_{KP} \) is the depreciation of land use due to floods
- \( d_{4e} \text{FR}_{dup} \) is the depreciation due to inadequate fire protection
- \( d_{5e} U_p \) is the depreciation due to inadequate utility capacity

**K. Water**

The water sector is designed to replicate resources of surface water, rivers, lakes and run-off. By relating the supply of water of a given quality to the demand for that quality of water, the need for water supplies is adjusted by importing water and upgrading the quality of available water.

The demand \( (D_d) \) is given by:

\[ D_d = \sum_{p=1}^{n} L_p (WC) + \sum_{p=1}^{n} (P_{KP}) (WC_{KR}) + L_{SW} (We) \]

where

- \( D_d \) is the demand for water in a given water district
- \( L_p \) is land use on a parcel
- \( WC \) is water consumption
- \( P_{KP} \) is the population of a given class or a parcel
$L_{SW}$ is the imported water

$W_{e}$ is the water quality

The supply $(S_d)$ is given as:

$$S_d = LV_t (TR_d) \begin{bmatrix} A_p \\ D_p \end{bmatrix}$$

where

- $S_d$ is the supply of water in a district
- $LV_t$ is the land value of a particular parcel
- $TR_d$ is the transportation value to a given terminal
- $A_p$ is available water on a parcel
- $D_p$ is the total demand on a parcel

The total water supply is then found as:

$$T_W = \sum_{d=1}^{m} S_d + \sum_{p=1}^{m} W_{sp} + S_o$$

where

- $W_{sp}$ is surface water
- $S_o$ outside purchase water

If the demand is greater than the supply, there is a tendency to buy outside water such that:

$$\text{local } RW_p = A(D_p / S_p)$$

where

$A =$ cost of bringing the present quality of water up to the desired quality
If the quality of water is less than some standard, or if

$$RW_p > 1$$

then

$$S_o - D_p = S_p$$
VII. POST SCRIPT

As stated in the Introduction, further clarification of the terms, game procedures and examples of computer outputs will be found in the Computer Operator's, Director's and Player's manuals associated with each game. The primary purpose of this manual is to give the serious reader a detailed understanding of the mathematical models associated with the games and not the computer programming techniques utilized to make the models operational.

It is hoped that the user might utilize some of the models in his own programs or modify them for his own needs. Each major model can be considered and analyzed apart from the actual CITY game. The programming becomes complex only when all the models are integrated within a single game, such as CITY IV. It is then necessary to account for the inter-relations between models as well as the model solutions themselves.

To further help the reader, a Glossary of Terms and a List of Symbols is included in this Chapter. Only those terms which are unique to the Game or complicated in their definition are listed. Terms such as School Assignments or Transportation are defined in the text in their usual sense and have not been included in the Glossary.

Everywhere possible, the actual computer symbols are used. A few symbol changes have been made in this manual where a symbol in a main program might be the same as a symbol used in a sub-routine or function.
A. Glossary of Terms

1. Average Rate of Return - The expectation over the long run for each investment made by an investor. Since some investment might not be profitable in the first few years and extremely profitable in the last years, a more representative yield on the investment would be the average return over the entire life of the investment.

2. Budget Allocations - In the Game context, the Government has a fixed amount of money at the beginning of each round. This money, acquired from debt and income, must be distributed to each governmental department in its entirety to avoid wastefulness. The proportionate distribution to each department becomes the budget for that round.

3. CITY I - A two module game consisting of the Economic and Government sectors played on an IBM 1131 computer. Manuals are available for purchase describing the details of this game at the National Technical Information Service (NTIS).

4. CITY II - This game is essentially CITY I with a partially operative Social Sector and operates on a Univac 1108, Exec. 8 computer from a terminal. There are presently no manuals available for sale for this game but arrangements can be made through NTIS for acquiring a set.

5. CITY III - A three module game consisting of the Economic, Government and Social sectors played on a Univac 1108, Exec. 8 computer. Manuals for this game are available for purchase from NTIS.

6. CITY IV - CITY IV is for the most part identical to CITY III and is written for an IBM 360/70 computer. Manuals for this game are available for purchase from NTIS.
7. City Model - The City models are operational simulation games in which the participants or players make decisions that affect a hypothetical metropolitan area built into the computer. CITY I, II, III, IV and the River Basin are different versions of the City Model.

8. Commercial Expenditures - These are the sum of the utility, tax, maintenance, water, salary, transportation, and goods and service costs.

9. Commercial Income - The income for any commercial establishment is the product of the number of units sold and the price per unit.

10. Computer Model Equations - These are the generalized computer equations for supply and demand. The format of the general equations is followed throughout the models.

11. Decision-Making Bodies - In the gaming context, the decision makers are the players. When one or more players are assigned a role, such as the School Department, the players are then known as a decision-making body.

12. Departmental Ratios - Ratios that relate supply and demand to education, adult education, municipal services, welfare payments, utilities and transportation.

13. Director's Manual - A City Games manual written expressly for the Game Director. The manual contains the technical details for running a game including the computer commands and options open to the Director for various gaming plays. Each game has its own unique Director's manual.
14. Expected Value - A player investment criteria used to express the average value of the net rate of return on all investments made specific land parcel.

15. Fixed Resources - These resources include water, land, people, buildings, roads, networks (utilities, transportation, communication, etc.), capital equipment and the related maintenance items necessary to keep them in operation.

16. Government Decision Objective - To maximize the total benefits to the system through the allocation of funds among the various departments.

17. Grid Matrix - In all models, a grid or checker-board arrangement is used. Each square or grid is used to identify and locate the numerous interrelationships, specific parcels of land and roads. The matrix then becomes the entire checker-board or the sum of all the individual grids.

18. Housing Income - The income for a given residential property is the product of the amount of space used and the rent per space unit for the property.

19. Industrial Expenditures - These are the sum of the utility, tax, maintenance, water, salary, transportation, goods and services, and terminal usage costs and the expenses connected with pollution control.

20. Industrial Property Income - The income for a given industrial property is the product of the price per unit (industry specific) and the number of units produced on the properties.

21. Industrial Value Ratio - The value ratio is determined by the amount of depreciation that occurred during a given round limited by the level at which maintenance is set.
22. Investment Capital - In the game, investment capital or money that a player has to invest. The rational is that the net amount of investment capital available after all costs of borrowing are taken into consideration must be less than or equal to the estimated rate of return in dollars.

23. Investment Series - In the game, investment series refers to all of the investments made by a player in a given round. For example, if in round one a player invested $25,000 in various investments, this would be known as his investment series for round one.

24. Liquid Resources - Those items which can easily be changed in the short-run such as taxes, wages, sales, social attitudes and the like.

25. Matching - This is the process of locating workers by skill and educational level to fill the demand from the job market. The workers who are located must then be interested in the job offer because it will maximize their income and/or achieve their satisfying objectives.

26. Migration - People move in and out of the urban system on the bases of opportunities available to them. The game accounts for this in and out migration as well as the natural birth-death growth to consider the net population changes of the Urban System.

27. Modules - The computer program considers each major game function separately. For example, the Economic Sector is a module in terms of the computer programming. CITY III is a three module game consisting of the Economic module, the Government module and the Social module. These modules are referred to in the gaming instructions as Sectors. The distinction is made
so that if one refers to a sector, the game play and the players are referred to the Players' manual. The module terminology has meaning to the computer operator and the Game Director.

28. Money Stock - This term refers to the total money supply in the system at any given round of play.

29. Objective Function - This is the mathematical expression of the goals or standards set by each sector and the sub-division within a sector, such as the School Department's goals, for each game play.


31. Partial Resource Allocation - In a game play, it is not realistic to assume that every resource available would be changed. A resource committed in a prior game play may still be the best commitment for the player. The player would only invest part of his remaining resources in the current round. The concept of not investing or altering all resources during a particular round is known as a partial resource allocation.

32. Player's Manual - A City Games manual written expressly for the player. The manual contains directions, explanations and exhibits to guide the player through a game play. Each game has its own unique Players' Manual.

33. Private Capital - Private capital or money is invested in fixed assets through three general categories: residential housing, commercial activities and industrial plant and equipment.
34. Private Liquid Capital - Liquid Resources were defined as those resources which could easily be changed such as taxes, wages, sales and the like. Private capital is non-government money invested by the private citizens and therefore Private Liquid Capital refers to the liquid capital of private investors.

35. Property Expenditures - The expenditures for a property are found by summing for each property the utilities, taxes, maintenance and water costs.

36. Public and Private Influence - This refers to the so called "power" or "political power" exerted by individuals or groups to gain an objective. The game considers voting, economic suasion, and influence peddling in the form of bribes and boycotts as the tools for a power base.

37. Rate of Return - The amount of money earned from an investment divided by the investment results in a percentage figure or rate of return. If a principal sum is invested and the net return from the investment is 10%, then the rate of return on the investment is 10% if a single payment or the rate can be in terms of compound interest over time.

38. River Basin Model - This game is CITY IV with the water module attached. It is the largest of the present series of games consisting of four modules.

39. Semi-Liquid Capital - Semi-liquid capital is another term meaning private liquid capital. The term arises from the definition of partial resource allocation and supplies that only some of the private liquid capital will be reinvested during a particular game round or play.
40. Structure Value - The structure value for a given property is the original cost of a level of the structure (type specified) times the value ratio times the number of levels on the parcel.

41. Time Allocation - The ability to allocate the leisure time of small aggregates of workers in the Social Sector. This time allotment produces complex interactive effects throughout the entire model and can become a significant source of worker "power."

42. Total Resources - The total resources of a particular system can be viewed as the sum of all fixed and liquid resources in the system.

43. Water Module - This module is added to the CITY IV game and considers the water distribution problems within the entire river basin rather than just the metropolitan area of the particular game in play.
### B. List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Page(s)</th>
<th>Symbol</th>
<th>Page(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>63,66</td>
<td>C</td>
<td>40,41,63</td>
</tr>
<tr>
<td>aB</td>
<td>8</td>
<td>(C_{AE})</td>
<td>36</td>
</tr>
<tr>
<td>((aB-dC)_i)</td>
<td>8</td>
<td>(C_{dsd})</td>
<td>61</td>
</tr>
<tr>
<td>(A_i)</td>
<td>53</td>
<td>(C_{ij})</td>
<td>53</td>
</tr>
<tr>
<td>(A_P)</td>
<td>66</td>
<td>(C_j)</td>
<td>26</td>
</tr>
<tr>
<td>B</td>
<td>37,51,54,63</td>
<td>(C_P)</td>
<td>13,14,16,43</td>
</tr>
<tr>
<td>B(_c)</td>
<td>52</td>
<td>(C_{pg})</td>
<td>45</td>
</tr>
<tr>
<td>BG</td>
<td>21</td>
<td>(C_{P_1P_2})</td>
<td>63</td>
</tr>
<tr>
<td>(B_i)</td>
<td>53,54</td>
<td>(cRQ)</td>
<td>60</td>
</tr>
<tr>
<td>(\hat{B}_i)</td>
<td>5,11</td>
<td>(C_U)</td>
<td>21</td>
</tr>
<tr>
<td>(\hat{B}_{ij})</td>
<td>12</td>
<td>(C_{U_P})</td>
<td>21</td>
</tr>
<tr>
<td>B(_j)</td>
<td>26</td>
<td>D</td>
<td>3,52</td>
</tr>
<tr>
<td>BLDG(_p)</td>
<td>19</td>
<td>(D_{AE})</td>
<td>29</td>
</tr>
<tr>
<td>(\hat{B}_P)</td>
<td>13,14</td>
<td>DC</td>
<td>6,46</td>
</tr>
<tr>
<td>bPk</td>
<td>32</td>
<td>dC</td>
<td>8</td>
</tr>
<tr>
<td>B(_{ps})</td>
<td>45</td>
<td>(D_d)</td>
<td>27,65</td>
</tr>
<tr>
<td>BQI(_f)</td>
<td>55</td>
<td>(d_{h_{ij}})</td>
<td>3</td>
</tr>
<tr>
<td>BS</td>
<td>21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BTC</td>
<td>21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\hat{BTC})</td>
<td>22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BV</td>
<td>54</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Symbol</td>
<td>Page(s)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------------</td>
<td>---------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_i$</td>
<td>59</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_i$</td>
<td>13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_{ie MS} dmsp$</td>
<td>65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$dI_i$</td>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$dI_{it}$</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$dI_i(t-1)$</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$dK$</td>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$dK_i$</td>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$dK_{ij}$</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_L$</td>
<td>61</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$DL_{PL}$</td>
<td>65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$DL_{t+1}$</td>
<td>64</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$dm_i$</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_{ms}$</td>
<td>29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$dMS$</td>
<td>60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_N$</td>
<td>17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_0$</td>
<td>17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_p$</td>
<td>42,66</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_p'$</td>
<td>42</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_s$</td>
<td>60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_{sc}$</td>
<td>28</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_T$</td>
<td>17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_{TR}$</td>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_{UT}$</td>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$DZ$</td>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_{2e p}$</td>
<td>65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_{3e FL,KP}$</td>
<td>65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_{4e FR_{dup}}$</td>
<td>65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_{5e U}$</td>
<td>65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E$</td>
<td>55</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E^*$</td>
<td>46</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$EE$</td>
<td>23</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$EE_{max}$</td>
<td>24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$EF_{cp}$</td>
<td>29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$EH$</td>
<td>24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$EI$</td>
<td>55,60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_i$</td>
<td>56</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$EI_{max}$</td>
<td>59</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$EI_f$</td>
<td>56</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$EI_{i}$</td>
<td>52,56,60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_j$</td>
<td>51</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_{ki}$</td>
<td>46</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$EL_c$</td>
<td>35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E\bar{N}$</td>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E\bar{N}_{max}$</td>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Symbol</td>
<td>Page(s)</td>
<td>Symbol</td>
<td>Page(s)</td>
</tr>
<tr>
<td>--------------</td>
<td>---------</td>
<td>--------------</td>
<td>---------</td>
</tr>
<tr>
<td>EN</td>
<td>12</td>
<td>I_{ms}</td>
<td>29</td>
</tr>
<tr>
<td>EN_{max}</td>
<td>12</td>
<td>IN</td>
<td>43</td>
</tr>
<tr>
<td>EN_{j}</td>
<td>13</td>
<td>IND.</td>
<td>35</td>
</tr>
<tr>
<td>E_{p}</td>
<td>44</td>
<td>I_{T}</td>
<td>58</td>
</tr>
<tr>
<td>E_{p_1p_2}</td>
<td>62</td>
<td>I_{sc}</td>
<td>28</td>
</tr>
<tr>
<td>ER</td>
<td>24</td>
<td>I_{TR}</td>
<td>30</td>
</tr>
<tr>
<td>eSc</td>
<td>60</td>
<td>I_{UT}</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>I_{WL}</td>
<td>30</td>
</tr>
<tr>
<td>FAE_{i}</td>
<td>35</td>
<td>J</td>
<td>3</td>
</tr>
<tr>
<td>FR</td>
<td>3</td>
<td>J_{jk}</td>
<td>47</td>
</tr>
<tr>
<td>FR_{p}</td>
<td>3</td>
<td>J_{k}</td>
<td>47,59</td>
</tr>
<tr>
<td>g_{i}</td>
<td>41</td>
<td>J_{k(T-1)}</td>
<td>59</td>
</tr>
<tr>
<td>GS_{p}</td>
<td>20</td>
<td>JOB_{ptc}</td>
<td>35</td>
</tr>
<tr>
<td>gTx</td>
<td>60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G_{y}</td>
<td>52</td>
<td>K</td>
<td>49,57</td>
</tr>
<tr>
<td>H</td>
<td>63</td>
<td>K_{c}</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>K_{i}</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>K_{ij}</td>
<td>12</td>
</tr>
<tr>
<td>i</td>
<td>3,4,9,40,57,63</td>
<td>K_{j}</td>
<td>57</td>
</tr>
<tr>
<td>I_{AE}</td>
<td>29</td>
<td>K_{k}</td>
<td>58</td>
</tr>
<tr>
<td>IG_{x}</td>
<td>31</td>
<td>K_{L}</td>
<td>61</td>
</tr>
<tr>
<td>I_{j}</td>
<td>58</td>
<td>K_{L_{i}}</td>
<td>45</td>
</tr>
<tr>
<td>Symbol</td>
<td>Page(s)</td>
<td>Symbol</td>
<td>Page(s)</td>
</tr>
<tr>
<td>--------</td>
<td>---------</td>
<td>--------</td>
<td>---------</td>
</tr>
<tr>
<td>L</td>
<td>21,62</td>
<td>M_{G_j}</td>
<td>26,31</td>
</tr>
<tr>
<td>LC</td>
<td>52</td>
<td>m_j</td>
<td>13,14</td>
</tr>
<tr>
<td>L_c</td>
<td>53</td>
<td>M_{P_G}</td>
<td>17</td>
</tr>
<tr>
<td>L_{C_i}</td>
<td>52</td>
<td>M_{P_S}</td>
<td>17</td>
</tr>
<tr>
<td>Lg</td>
<td>54</td>
<td>M_S</td>
<td>8,9,59</td>
</tr>
<tr>
<td>L_{git}</td>
<td>54</td>
<td>M_{S_i}</td>
<td>53</td>
</tr>
<tr>
<td>L_I</td>
<td>53</td>
<td>M_{S_L}</td>
<td>52</td>
</tr>
<tr>
<td>L_i</td>
<td>31,45,46</td>
<td>M_{S_P}</td>
<td>30</td>
</tr>
<tr>
<td>L_{ij}</td>
<td>51</td>
<td>M_{T_i}</td>
<td>63</td>
</tr>
<tr>
<td>L_{ND_p}</td>
<td>18</td>
<td>M_{T_{p+1}}</td>
<td>64</td>
</tr>
<tr>
<td>L_P</td>
<td>18,19,29,30,65</td>
<td>M_{T_P}</td>
<td>17</td>
</tr>
<tr>
<td>LR</td>
<td>3</td>
<td>M_J</td>
<td>30</td>
</tr>
<tr>
<td>L_{R}</td>
<td>52</td>
<td>M_V</td>
<td>59</td>
</tr>
<tr>
<td>LRC</td>
<td>31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L_{R_i}</td>
<td>3</td>
<td>n</td>
<td>8,11,12,46</td>
</tr>
<tr>
<td>L_{SW}</td>
<td>66</td>
<td>\tilde{N}</td>
<td>9</td>
</tr>
<tr>
<td>L_{U_i}</td>
<td>45,54</td>
<td>N_{\text{max}}</td>
<td>9</td>
</tr>
<tr>
<td>L_{U_p}</td>
<td>29,30</td>
<td>NBCPR</td>
<td>24</td>
</tr>
<tr>
<td>LV</td>
<td>54</td>
<td>N_C</td>
<td>64</td>
</tr>
<tr>
<td>L_{V_t}</td>
<td>66</td>
<td>N_c</td>
<td>34</td>
</tr>
<tr>
<td>M</td>
<td>63</td>
<td>N_{G_k}</td>
<td>47</td>
</tr>
<tr>
<td>m</td>
<td>12</td>
<td>N_{G_{T_k}}</td>
<td>58</td>
</tr>
<tr>
<td></td>
<td></td>
<td>N_{G_T}</td>
<td>58</td>
</tr>
<tr>
<td>Symbol</td>
<td>Page(s)</td>
<td>Symbol</td>
<td>Page(s)</td>
</tr>
<tr>
<td>--------</td>
<td>---------</td>
<td>--------</td>
<td>---------</td>
</tr>
<tr>
<td>$N_i$</td>
<td>9</td>
<td>$p_{ik}$</td>
<td>54</td>
</tr>
<tr>
<td>$N_{ij}$</td>
<td>54</td>
<td>$p_{jk}$</td>
<td>48</td>
</tr>
<tr>
<td>$\tilde{N}_j$</td>
<td>14</td>
<td>$p_{jkp}$</td>
<td>48</td>
</tr>
<tr>
<td>$NP_k$</td>
<td>47</td>
<td>$p_k$</td>
<td>40,49</td>
</tr>
<tr>
<td>$NP_m$</td>
<td>24</td>
<td>$p_{KP}$</td>
<td>65</td>
</tr>
<tr>
<td>$NT_p$</td>
<td>21</td>
<td>$p_{ksd}$</td>
<td>61</td>
</tr>
<tr>
<td>$NV_t$</td>
<td>64</td>
<td>$p_{K(T-1)}$</td>
<td>59</td>
</tr>
<tr>
<td>$O_h$</td>
<td>19</td>
<td>$PLA_i$</td>
<td>37</td>
</tr>
<tr>
<td>OUT_T</td>
<td>58</td>
<td>$PL_K$</td>
<td>48</td>
</tr>
<tr>
<td>OUT_TK</td>
<td>59</td>
<td>$PM$</td>
<td>29</td>
</tr>
<tr>
<td>$p$</td>
<td>4,39,46</td>
<td>$p_m$</td>
<td>23,24</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>46</td>
<td>$p_{MLj}$</td>
<td>35</td>
</tr>
<tr>
<td>$P_c$</td>
<td>29,34</td>
<td>$p_m$</td>
<td>29</td>
</tr>
<tr>
<td>PG</td>
<td>17,21,45</td>
<td>$p_{nk}$</td>
<td>48</td>
</tr>
<tr>
<td>$p_{gi}$</td>
<td>39</td>
<td>Po</td>
<td>46</td>
</tr>
<tr>
<td>PH</td>
<td>29</td>
<td>POP_V</td>
<td>40</td>
</tr>
<tr>
<td>$p_{P}$</td>
<td>29</td>
<td>$p_p$</td>
<td>20</td>
</tr>
<tr>
<td>$p_{pi}$</td>
<td>39</td>
<td>$p_{pi}$</td>
<td>39</td>
</tr>
<tr>
<td>$p_{plT}$</td>
<td>62</td>
<td>PR</td>
<td>4,40</td>
</tr>
<tr>
<td>$P_i$</td>
<td>13,31,46</td>
<td>$PR_c$</td>
<td>36</td>
</tr>
<tr>
<td>$P_i$</td>
<td>40</td>
<td>$PR_i$</td>
<td>5</td>
</tr>
<tr>
<td>PRO</td>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### List of Symbols (Continued)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Page(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{RT_j}$</td>
<td>37</td>
</tr>
<tr>
<td>$PS$</td>
<td>17, 21, 45</td>
</tr>
<tr>
<td>$P_s$</td>
<td>47</td>
</tr>
<tr>
<td>$PS_i$</td>
<td>31</td>
</tr>
<tr>
<td>$P_{sk}$</td>
<td>48</td>
</tr>
<tr>
<td>$PTN_p$</td>
<td>24</td>
</tr>
<tr>
<td>$P_T$</td>
<td>58</td>
</tr>
<tr>
<td>$PW_i$</td>
<td>31</td>
</tr>
<tr>
<td>$P_{xk}$</td>
<td>48</td>
</tr>
<tr>
<td>$\bar{Q}I$</td>
<td>56</td>
</tr>
<tr>
<td>$QI_f$</td>
<td>56</td>
</tr>
<tr>
<td>$QI_i$</td>
<td>56</td>
</tr>
<tr>
<td>$R$</td>
<td>47, 55</td>
</tr>
<tr>
<td>$\tilde{R}$</td>
<td>56</td>
</tr>
<tr>
<td>$\bar{r}$</td>
<td>37</td>
</tr>
<tr>
<td>$RA$</td>
<td>18</td>
</tr>
<tr>
<td>$RB$</td>
<td>18</td>
</tr>
<tr>
<td>$R_d$</td>
<td>28</td>
</tr>
<tr>
<td>$r_d$</td>
<td>28</td>
</tr>
<tr>
<td>$R_f$</td>
<td>55</td>
</tr>
<tr>
<td>$R_i$</td>
<td>56, 59</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Page(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_i$</td>
<td>9</td>
</tr>
<tr>
<td>$\bar{r_i}$</td>
<td>38</td>
</tr>
<tr>
<td>$R_{ki}$</td>
<td>61</td>
</tr>
<tr>
<td>$r^2_p$</td>
<td>43</td>
</tr>
<tr>
<td>$R_p$</td>
<td>16</td>
</tr>
<tr>
<td>$r_p$</td>
<td>42</td>
</tr>
<tr>
<td>$r_{PC}$</td>
<td>36</td>
</tr>
<tr>
<td>$r_{P_p}$</td>
<td>36</td>
</tr>
<tr>
<td>$R_R$</td>
<td>60</td>
</tr>
<tr>
<td>$RT_p$</td>
<td>22</td>
</tr>
<tr>
<td>$r_w$</td>
<td>34</td>
</tr>
<tr>
<td>$RW_p$</td>
<td>66</td>
</tr>
<tr>
<td>$S$</td>
<td>57, 58</td>
</tr>
<tr>
<td>$S_{AE}$</td>
<td>29</td>
</tr>
<tr>
<td>$SAL_c$</td>
<td>34</td>
</tr>
<tr>
<td>$SC$</td>
<td>7</td>
</tr>
<tr>
<td>$SC_p$</td>
<td>29</td>
</tr>
<tr>
<td>$S_d$</td>
<td>27, 66</td>
</tr>
<tr>
<td>$S_{epk}$</td>
<td>48</td>
</tr>
<tr>
<td>$S_h$</td>
<td>18</td>
</tr>
<tr>
<td>$S_i$</td>
<td>8, 56</td>
</tr>
<tr>
<td>$S_{ij}$</td>
<td>3</td>
</tr>
<tr>
<td>Symbol</td>
<td>Page(s)</td>
</tr>
<tr>
<td>-------</td>
<td>--------</td>
</tr>
<tr>
<td>$S_j$</td>
<td>51,57</td>
</tr>
<tr>
<td>$S_L$</td>
<td>62</td>
</tr>
<tr>
<td>$SL_p$</td>
<td>20</td>
</tr>
<tr>
<td>$S_{ms}$</td>
<td>29</td>
</tr>
<tr>
<td>$S_o$</td>
<td>66</td>
</tr>
<tr>
<td>$S'_p$</td>
<td>42</td>
</tr>
<tr>
<td>$S_p$</td>
<td>16,42</td>
</tr>
<tr>
<td>$SP_c$</td>
<td>29</td>
</tr>
<tr>
<td>$SPT_k$</td>
<td>49</td>
</tr>
<tr>
<td>$S_s$</td>
<td>61</td>
</tr>
<tr>
<td>$S_{sc}$</td>
<td>28</td>
</tr>
<tr>
<td>$S_{TR}$</td>
<td>30</td>
</tr>
<tr>
<td>$S_{UT}$</td>
<td>30</td>
</tr>
<tr>
<td>$S_\phi$</td>
<td>51</td>
</tr>
<tr>
<td>$T$</td>
<td>4,30,32,57,63</td>
</tr>
<tr>
<td>$\sim T$</td>
<td>37</td>
</tr>
<tr>
<td>$t+1$</td>
<td>64</td>
</tr>
<tr>
<td>$T_A$</td>
<td>32</td>
</tr>
<tr>
<td>$T_{AF}$</td>
<td>34</td>
</tr>
<tr>
<td>$T_{AP}$</td>
<td>34</td>
</tr>
<tr>
<td>TD</td>
<td>3</td>
</tr>
<tr>
<td>TD$_{(BS/BG)}$</td>
<td>45</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Page(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Te$_i$</td>
<td>49</td>
</tr>
<tr>
<td>T$_{epp'}$</td>
<td>49</td>
</tr>
<tr>
<td>T$_{ew}$</td>
<td>62</td>
</tr>
<tr>
<td>T$_{I}$</td>
<td>33</td>
</tr>
<tr>
<td>T$_{i}$</td>
<td>38,51,53</td>
</tr>
<tr>
<td>$\hat{T}_i$</td>
<td>52,53</td>
</tr>
<tr>
<td>T$_{L}$</td>
<td>62</td>
</tr>
<tr>
<td>TM$_p$</td>
<td>24</td>
</tr>
<tr>
<td>T$_N$</td>
<td>32</td>
</tr>
<tr>
<td>T$_p$</td>
<td>29</td>
</tr>
<tr>
<td>T$_{PA}$</td>
<td>34</td>
</tr>
<tr>
<td>T$_{pc}$</td>
<td>16</td>
</tr>
<tr>
<td>T$_R$</td>
<td>34,55</td>
</tr>
<tr>
<td>TR</td>
<td>3</td>
</tr>
<tr>
<td>TR$_d$</td>
<td>66</td>
</tr>
<tr>
<td>TR$_{j}$</td>
<td>26</td>
</tr>
<tr>
<td>TR$_R$</td>
<td>20,22</td>
</tr>
<tr>
<td>TS</td>
<td>46</td>
</tr>
<tr>
<td>T$_s$</td>
<td>63</td>
</tr>
<tr>
<td>T$_{shop}$</td>
<td>64</td>
</tr>
<tr>
<td>T$_T$</td>
<td>32,55</td>
</tr>
<tr>
<td>Tu</td>
<td>32</td>
</tr>
</tbody>
</table>

82
### List of Symbols (Continued)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Page(s)</th>
<th>Symbol</th>
<th>Page(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{W}$</td>
<td>66</td>
<td>$V_{R_p}$</td>
<td>19,29,30</td>
</tr>
<tr>
<td>$T_{w}$</td>
<td>34,64</td>
<td>$W$</td>
<td>7,57,63</td>
</tr>
<tr>
<td>$T_{X_1}$</td>
<td>26</td>
<td>$W_{C}$</td>
<td>65</td>
</tr>
<tr>
<td>$T_{X_2}$</td>
<td>26</td>
<td>$W_d$</td>
<td>28</td>
</tr>
<tr>
<td>$T_{X_3}$</td>
<td>26</td>
<td>$W_e$</td>
<td>66</td>
</tr>
<tr>
<td>$T_{X_4}$</td>
<td>26</td>
<td>$W_i$</td>
<td>53</td>
</tr>
<tr>
<td>$T_{X_p}$</td>
<td>17</td>
<td>$W_{ij}$</td>
<td>57</td>
</tr>
<tr>
<td>$T_{Z}$</td>
<td>33</td>
<td>$w_n$</td>
<td>7</td>
</tr>
<tr>
<td>$U_i$</td>
<td>57</td>
<td>$W_{sp}$</td>
<td>66</td>
</tr>
<tr>
<td>$U_p$</td>
<td>19</td>
<td>$W_{TR_p}$</td>
<td>17</td>
</tr>
<tr>
<td>$U_{P_p}$</td>
<td>23</td>
<td>$X$</td>
<td>6,57</td>
</tr>
<tr>
<td>$U_{S_p}$</td>
<td>20</td>
<td>$x_n$</td>
<td>6</td>
</tr>
<tr>
<td>$U_{T_p}$</td>
<td>17,30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$U_{U}$</td>
<td>30</td>
<td>$Y$</td>
<td>6</td>
</tr>
<tr>
<td>$V$</td>
<td>11,43,64</td>
<td>$Y_d$</td>
<td>52</td>
</tr>
<tr>
<td>$V_i$</td>
<td>6,11</td>
<td>$y_n$</td>
<td>6</td>
</tr>
<tr>
<td>$V_{ij}$</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_k$</td>
<td>47</td>
<td>$Z$</td>
<td>7,49,55</td>
</tr>
<tr>
<td>$V_p$</td>
<td>13,14,19</td>
<td>$Z_i$</td>
<td>56</td>
</tr>
<tr>
<td>$V_{R}$</td>
<td>23</td>
<td>$Z_k$</td>
<td>51</td>
</tr>
<tr>
<td>$V_{R_i}$</td>
<td>46</td>
<td>$Z_{\ell_i}$</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$z_n$</td>
<td>7</td>
</tr>
<tr>
<td>Symbol</td>
<td>Page(s)</td>
<td>Symbol</td>
<td>Page(s)</td>
</tr>
<tr>
<td>------------</td>
<td>---------</td>
<td>------------</td>
<td>---------</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>4,31,61</td>
<td>$\Delta D_p$</td>
<td>42</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>52</td>
<td>$\Delta E L_c$</td>
<td>35</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>52</td>
<td>$\Delta P_o$</td>
<td>46</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>52</td>
<td>$\Delta P_s$</td>
<td>47</td>
</tr>
<tr>
<td>$(\alpha, \beta, \gamma)$</td>
<td>31</td>
<td>$\Delta P_{T_c}$</td>
<td>58</td>
</tr>
<tr>
<td>$\alpha_{CP}$</td>
<td>40</td>
<td>$\Delta r_p$</td>
<td>42</td>
</tr>
<tr>
<td>$\alpha_{d_k}$</td>
<td>59</td>
<td>$\Delta S_p$</td>
<td>42</td>
</tr>
<tr>
<td>$\alpha_k$</td>
<td>59</td>
<td>$\Delta V T_c$</td>
<td>36</td>
</tr>
<tr>
<td>$\alpha_{P_{ki}}$</td>
<td>59</td>
<td>$\delta_i$</td>
<td>40</td>
</tr>
<tr>
<td>$\alpha_{U_{dk}}$</td>
<td>59</td>
<td>$\delta T$</td>
<td>40</td>
</tr>
<tr>
<td>$\beta$</td>
<td>4,31</td>
<td>$\varepsilon$</td>
<td>4</td>
</tr>
<tr>
<td>$\beta_{CG}$</td>
<td>40</td>
<td>$\epsilon_1$</td>
<td>54</td>
</tr>
<tr>
<td>$\beta_{U_{rk}}$</td>
<td>59</td>
<td>$\epsilon_2$</td>
<td>54</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>4,31</td>
<td>$\epsilon_3$</td>
<td>54</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>34,54</td>
<td>$\epsilon_4$</td>
<td>54</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>35,54</td>
<td>$\epsilon_P$</td>
<td>40</td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>36</td>
<td>$\zeta$</td>
<td>31</td>
</tr>
<tr>
<td>$\gamma_4$</td>
<td>36</td>
<td>$\theta_1$</td>
<td>6</td>
</tr>
<tr>
<td>$\gamma_5$</td>
<td>36</td>
<td>$\theta_2$</td>
<td>7</td>
</tr>
<tr>
<td>$\gamma_{A F_c}$</td>
<td>35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_{A P_c}$</td>
<td>35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_{R_c}$</td>
<td>36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Symbol</td>
<td>Page(s)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>---------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_k$</td>
<td>40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>42</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_c$</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{\phi}_c$</td>
<td>56</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_j$</td>
<td>58</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The exchange announces its annual outsider applications in its annual report. The following tests have been conducted in the test.

The outside interest rate averaged 5.3% over the past year, and our government bonds were selling well at 3.1%. The Chamber's annual canvass of corporations in the area shows a 6.4% average return on conservative investments in the national economy and an excellent return, averaging 0.5% on growth stocks (speculative investments) which vary widely from year to year.

Mr. Bigg coated with a deceptively heavy label. He is certainly aware of the price index movements at the beginning of a major upswing. He also reported a very favorable "trade balance" for the year.

He sold $400,000 worth of goods, chiefly heavy industrial, (half) and light industrial, (half) to the national economy. Purchases from outside were only $2.3 billion, far less, and that was chiefly the federal corporation lands. Our regional and business goods and services have been keeping us well with the world on pg. 3.

The city is facing a major problem in rising costs. It is estimated that $150,000 worth of goods, chiefly heavy industrial, (half) and light industrial, (half) to the national economy. Purchases from outside were only $2.3 billion, far less, and that was chiefly the federal corporation lands. Our regional and business goods and services have been keeping us well with the world on pg. 3.

The city is facing a major problem in rising costs. It is estimated that $150,000 worth of goods, chiefly heavy industrial, (half) and light industrial, (half) to the national economy. Purchases from outside were only $2.3 billion, far less, and that was chiefly the federal corporation lands. Our regional and business goods and services have been keeping us well with the world on pg. 3.
### CITY GAMES MATHEMATICAL FOUNDATIONS

#### Author(s)
Mr. John E. Moriaty

#### Performing Organization Name and Address
National Bureau of Standards
Department of Commerce
Washington, D. C. 20234

#### Sponsoring Organization Name and Address
National Technical Information Service
5285 Port Royal Road
Springfield, Virginia 22151

#### Abstract
The CITY models are operational simulation games in which the participants make economic, government and social decisions affecting a hypothetical metropolitan area. Through the use of a computer, the simulated urban system responds to the participant’s decisions as any real city would. To simplify the description of the theory of the CITY GAMES modules, this manual is written for the four module game. The equations for the other games can easily be obtained by eliminating the description that do not apply to the particular game of interest. The theoretical structure of models is consistent and relevant only within the range from a central city area to regional configuration. The description of the module components and equations are meant to show the scope of decisions including those by the user and by the programs and algorithms of the computer model.

#### Key Words and Document Analysis

<table>
<thead>
<tr>
<th>17a. Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td>* Computers</td>
</tr>
<tr>
<td>* Simulation</td>
</tr>
<tr>
<td>* Mathematics</td>
</tr>
<tr>
<td>* Equations</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>17b. Identifiers/Open-Ended Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>* Mathematical foundations</td>
</tr>
<tr>
<td>* Computer Programs</td>
</tr>
<tr>
<td>* Computer simulation</td>
</tr>
<tr>
<td>Simulation module</td>
</tr>
</tbody>
</table>

This manual is available only with the lease of the CITY I or CITY IV Games systems, COM-74-10701 and COM-74-10706.