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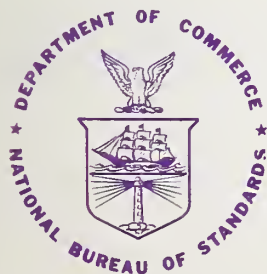München, Germany

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I. Introduction

It has been stressed many times that nucleus-nucleus collisions at high energies are worth investigating both theoretically and experimentally [1-6] because virtually nothing is known about this exotic field and, at the same time, the theoretical models one may envisage give several simple and definite predictions. The present note discusses one such model which was originally proposed for hadron-nucleus scattering [7] but can be straightforwardly generalized to nucleus-nucleus collisions. The main purpose of this note is to spell out this generalization and to give the formulae for the total cross sections, diffractive production cross sections, and inclusive single particle distributions in multiparticle production in nucleus-nucleus collisions. One of the important conclusions of this analysis is that the measurements of the relations between various total cross sections proposed in refs. [3-6] could test the model of ref. [7] .

2. The model.

We shall work in the c.m. system of the two nuclei containing A and B nucleons. The S matrix for such a collision is, according to the model of ref. [7],

$$
S_{AB} = \prod_j^A \prod_\ell^B S_{j\ell} = \prod_j^A \prod_\ell^B \left[1 - \Gamma_{j\ell} + i\, T_{j\ell} \right]
$$

(2.1)

where $S_{j\ell}$ is the S matrix for the /j-th nucleon from A colliding with the ℓ-th nucleon from B. As in [7], the "absorptive" part, $\Gamma_{j\ell}$, which describes both elastic scattering and diffractive production, is generated by the driving term $T_{j\ell}$ responsible for non-diffractive multiple production processes $\left(T_{j\ell} = T_{j\ell}^\dagger \right)$, and the unitarity of $S_{j\ell}$ gives

$$
2\Gamma_{j\ell} = \Gamma_{j\ell}^2 + T_{j\ell}^2 \cong T_{j\ell}^2 .
$$

(2.2)

Again, following the approach of ref. [7], we accept that the production processes (in fact all possible processes) are completely coherent over the two nuclei and this fact is incorporated into our description in a relativistically invariant manner by making the operators of Eqs. (2.1) and (2.2) dependent only on the transverse degrees of freedom. In the present simple version we shall use only the impact parameter

and the transverse distances of the individual nucleons from
the axis of cylindrical symmetry of the collision as these
transverse degrees of freedom.

One should also keep in mind that, henceforth, we shall
deal only with sum rules in which the momentum transfer
between the two nuclei (more precisely: between the two groups
of particles which emerge from the collision and go to the
left and to the right in the c.m. system) is integrated over
and the sum performed over all possible excitations (of nuclei
and of their components: nucleons) . Cross sections for
specific processes are obtained, in the present approach,
through the introduction of sutable projection operators into
such sums and integrations. Note that in this approach one
completely avoids dealing with the longitudinal momentum
transfers in the process of multiple collisions: for example,
as can be seen in Sec. C on inclusive cross sections, the
dependence on the longitudinal momenta comes only through
the final expression, (2.6) , where they are accounted for
in the "elementary" hadron-nucleon inclusive cross section,
$d\bar{\sigma}_i^{(N)}(p_i)$.

Assuming the same properties of the individual
matrices as in ref. $[7]$ and going through algebra analogous
to that used in the case of hadron-nucleus collisions $[7]$,
we readily obtain:

A. Elastic scattering and coherent diffractive production.
The amplitude for both processes at the impact parameter

b reads (to lowest order in diffractive production)

$$\langle n; \phi_i^{(A)} \phi_i^{(B)} | \left\{ 1 - \prod_{j=1}^{A} \prod_{\ell=1}^{B} \left[1 - \Gamma_{j\ell} (\underline{b} - \underline{s}_j^{(A)} + \underline{s}_\ell^{(B)}) \right] \right\} | 0; \phi_i^{(A)} \phi_i^{(B)} \rangle$$

$$= \langle \phi_i^{(A)} \phi_i^{(B)} | \left\{ 1 - \prod_{j=1}^{A} \prod_{\ell=1}^{B} \left[1 - \gamma_{j\ell} (\underline{b} - \underline{s}_j^{(A)} + \underline{s}_\ell^{(B)}) \right] \right\} | \phi_i^{(A)} \phi_i^{(B)} \rangle \, \delta_{no}$$

$$+ \varepsilon \sum_{j=1}^{A} \sum_{\ell=1}^{B} \langle \phi_i^{(A)} \phi_i^{(B)} | \prod_{k \neq j}^{A} \prod_{r \neq \ell}^{B} \left[1 - \gamma_{kr} (\underline{b} - \underline{s}_k^{(A)} + \underline{s}_\ell^{(B)}) \right] | \phi_i^{(A)} \phi_i^{(B)} \rangle \, \langle n | d_{j\ell} | 0 \rangle .$$

$$(2.3)$$

In this formula the absorptive part of the matrix $S_{j\ell}$ was split into the elastic and diffractive inelastic parts

$$\Gamma_{j\ell} = \gamma_{j\ell} + \varepsilon d_{j\ell}$$

where $\gamma_{j\ell}$ is diagonal (this is the "profile" of the Glauber model) and the off-diagonal correction responsible for diffractive production is assumed to be small. In ref. [7] such a form for $\Gamma_{j\ell}'$ was explicitely obtained from the bremsstrahlung model of multi-particle production [8]. It was also suggested by the general considerations of ref. [9]. In Eq. (2.3) the states are labelled by the nuclear wave functions of both nuclei $(\phi^{(A)}, \phi^{(B)})$ and by the produced

particles, schematically denoted (n) . The first part of the
r.h.s. of (2.3) gives the well known nucleus-nucleus high
energy elastic scattering amplitude $[1]$. Note that/second
the
part of the r.h.s. of (2.3) , which gives diffractive
production, is proportional to AB, which means that the cross
sections have a $A^2 B^2$ dependence. Hence it would seem that
diffractive production in nucleus-nucleus collisions may
contribute more significantly to, e.g., one particle inclusive
distributions (see below) than in the case of hadron-nucleus
collisions, where $B = 1$.

B. The total cross sections. Similarly, as in $[7]$, after
employing closure $\sum_n \sum_{ff'} |n; \phi_f^{(A)} \phi_{f'}^{(B)}\rangle\langle \phi_f^{(A)} \phi_{f'}^{(B)}; n| = 1$
and integrating over the momentum transfer between the two
nuclei, we obtain the following formula for the total nucleus-
nucleus cross section:

$$\sigma_T = \int d^2 b \langle 0; \phi_i^{(A)} \phi_i^{(B)}| \left(1 - S_{AB}^\dagger(\underline{b}; \underline{s}^{(A)}, \underline{s}^{(B)})\right)\left(1 - S_{AB}(\underline{b}; \underline{s}^A, \underline{s}^B)\right)$$
$$\hookrightarrow |0; \phi_i^{(A)} \phi_i^{(B)}\rangle$$

$$= 2 \int d^2 b \langle 0; \phi_i^{(A)} \phi_i^{(B)}| \left[1 - Re\, S_{AB}(\underline{b}; \underline{s}^{(A)}, \underline{s}^{(B)})\right]|0; \phi_i^{(A)} \phi_i^{(B)}\rangle$$

$$(2.4)$$

where we employed unitarity: $S_{AB}^\dagger S_{AB} = 1$. Again, as in
$[7]$, the total cross section is, to zeroth order in ϵ ,

the same as that given by the Glauber model, refs. [1,3,4],
and all the relations between the total cross sections
proposed in refs. [3,4,5,6], when compared with experiment,
will provide a test for the multiparticle production model
proposed in ref [7] and used here.

C. Inclusive single particle distribution in multiparticle
production in nucleus-nucleus collisions. We again follow
the same algebra as in [7]:

$$d\bar{\sigma}_1^{(AB)}(p_1) = \frac{dp_1}{E_1} \sum_n n \int d\sigma_n(p_1,\dots p_n) \frac{dp_2}{E_2} \cdots \frac{dp_n}{E_n}$$

$$= \frac{dp_1}{E_1} \int d^2 b \langle 0; \phi_1^{(A)} \phi_1^{(B)} | [1 - S_{AB}^+(b; A^{(A)}, A^{(B)})]$$

$$\cdot a^+(p_1)a(p_1)[1 - S_{AB}(b; A^{(A)}, A^{(B)})] | 0; \phi_1^{(A)} \phi_1^{(B)} \rangle.$$

$$(2.5)$$

The commutation relations $[a(p_1), S_{ij}]$ were worked
out in [7] for the case of the hadronic bremsstrahlung model.
When we employ them, we again obtain the result that the
"diagonal" terms give the leading contribution:

$$d\bar{\sigma}_1^{(AB)}(p_1) = AB \, d\bar{\sigma}_1^{(N)}(p_1) \tag{2.6}$$

and the non-diagonal contributions appear to be small.

The same qualifications made regarding Eq. (11) of ref. $\left[7\right]$, viz., that corrections are to be expected in the very forward direction, can be made here. However, we would like to emphasize the following point. In performing the algebraic operations which lead to (2.6) or Eq. (11) of ref. $\left[7\right]$ one neglects energy-momentum conservation in $\hat{\Omega}_j$, which <u>results in suppressing diffractive production</u>. Hence neither Eq. (2.6) nor Eq. (11) of ref. $\left[7\right]$ contain contributions to $d\bar{\sigma}_1^{(A)}(\rho_1)$ coming from diffractive production. In the case of hadron-nucleus interactions this simplification may still be not too bad, but, as has already been pointed out in this note, in the case of nucleus-nucleus collisions it is more serious. It would seem that one could estimate the relevant corrections starting directly from Eq. (2.3) . In any case one should expect the relation (2.6) to fail in the extreme forward and backward directions, which are populated by products of diffractive processes.

In concluding, one may say that one can literally take over the model of ref. $\left[7\right]$ and extend it trivially to the case of nucleus-nucleus scattering just as/the Glauber model for hadron-nucleus elastic scattering can be extended to nucleus-nucleus elastic scattering.

References

1. W. Czyż and L.C. Maximon, Ann.Phys. (N.Y.) $\underline{52}$, 59 (1969).

2. H.H. Heckman, High Energy Heavy Ions: A New Area for Physics Research, 5th Int. Conf. on High-Energy Physics and Nuclear Structure, Uppsala, June 1973.

3. P.M. Fishbane and J.S. Trefil, Phys. Rev. Letters $\underline{32}$, 396 (1974).

4. V. Franco, Phys. Rev. Letters $\underline{32}$, 911 (1974).

5. S. Barshay, C.B. Dover and J.P. Vary, The Validity of the Factorization Hypothesis for Nucleus-Nucleus Cross Sections at High Energies, Brookhaven preprint BNL 18811, April, 1974.

6. S. Barshay, C.B. Dover and J.P. Vary, Nucleus-Nucleus Cross Sections and the Validity of the Factorization Hypothesis at Intermediate and High Energies, Brookhaven preprint BNL 18972, May, 1974.

7. A. Białas and W. Czyż, Short-time behavior of hadronic interactions and the hadron-nucleus production processes, Max-Planck Institute for Physics and Astrophysics preprint, to be published in Physics Letters.

8. A. Białas and A. Kotański, Acta Physica Polonica $\underline{B4}$, 659 (1973).

9. W. Czyż, Phys. Rev. $\underline{D8}$, 3219 (1973).

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16. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)

A model for high energy nucleus-nucleus collisions is presented. Formulae are given for the total cross sections, diffractive productive cross sections, and inclusive single particle distributions in multiparticle production in nucleus-nucleus collisions. It is noted that measurement of the relations between a few nucleus-nucleus total cross sections could be used to test this model.

17. KEY WORDS (six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons)

Diffractive production cross sections; glauber model; high energy hadron interactions; inclusive single particle distributions; nucleus-nucleus collisions; total hadron cross sections.

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