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Evaluation of Structural Properties of Masonry in Existing Buildings

S. G. Fattal, and L. E. Cattaneo

Center for Building Technology
Institute for Applied Technology
National Bureau of Standards
Washington, D. C. 20234

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Final Report

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U. S. DEPARTMENT OF COMMERCE, Frederick B. Dent, Secretary
NATIONAL BUREAU OF STANDARDS, Richard W. Roberts, Director

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List of Symbols

A	Cross sectional area
A_f	Area of flanges (frame columns) of concrete infilled frame
A_o	Total (vertical) area of wall openings
A_t	Area of transformed section of wall of more than one material
A_v	Area (vertical) of wall without openings
A_w	Cross sectional area of web (masonry wall) of infilled frame assembly
a	Coefficient greater than unity reflecting apparent increase in flexural compressive strength
C	Numerical constant related to fixity conditions at top and base of wall
C_m	Coefficient in moment amplification equation to account for the influence of loading configuration
c	Distance from centroidal axis to outermost tensile surface
c_1, c_2	Distances from centroidal axis to outermost fibers in maximum compression
E	Modulus of elasticity in compression normal to bed joint
E_b	Modulus of elasticity of brick masonry
E_c	Modulus of elasticity of concrete block masonry
E_1, E_2	Modulus of elasticity in direction normal to bed joint of masonry on sides 1 and 2, respectively, of the centroidal axis
e_{k1}, e_{k2}	Kern eccentricities from centroidal axis of cross section in directions 1 and 2, respectively
f	Form factor dependent on geometry of cross section
f_m	Compressive stress in masonry (allowable or calculated)
f_v	Nominal uniform shear stress on cross section
$\bar{f}_m$	Calculated average compressive stress on masonry cross section
f'_m	Compressive strength of masonry
f'_t	Tensile strength of masonry in flexure
f'_{t1}, f'_{t2}	Flexural tensile strength of masonry on sides 1 and 2, respectively, of centroidal axis.
f'_v	Shear strength of masonry
G	Modulus of rigidity (shear modulus)
G_b	Modulus of rigidity of brick masonry
G_c	Modulus of ridigity of concrete block masonry

g	Gage length, or, gravitational acceleration
g_1, g_2	Coefficients used in moment capacity equations for out-of-plane flexure of masonry walls
h	Height of masonry, or, incremental length of discretized cantilever beam model of a building
I	Moment of inertia about a major principal axis of cross section
I_f	Moment of inertia contributed by concrete columns of infilled frame assembly
I_w	Moment of inertia contributed by solid masonry filler wall of infilled frame assembly
k	Effective height coefficient dependent on wall top and bottom fixity conditions
L	Horizontal length of masonry wall, or, length of cantilever beam model of a building
M	Internal resultant moment on wall cross section
M_{ex}	In-plane moment capacity of wall about major principal axis
M_{ey}	Out-of-plane moment capacity of wall about minor principal axis
M_{e1}, M_{e2}	Out-of-plane moment capacity of masonry cross section producing maximum compressive stress in outer fibers on sides 1 and 2, respectively
M_{k1}, M_{k2}	Out-of-plane moment capacity of masonry cross section under compressive kern loading, producing maximum compressive stress in outer fibers on sides 1 and 2, respectively
M_s	Internal moment on a section induced by transverse loads and/or eccentrically applied compressive loads
M_x	Internal resultant (in-plane) moment on wall cross section
M_y	Internal resultant (out-of-plane) moment on wall cross section
M_1	Lesser of the two moments at the top and bottom of a wall
M_2	Greater of the two moments at the top and bottom of a wall
m	Distributed mass per unit of length, or, ratio of compressive elastic moduli of two materials comprising a composite section
P	Compressive force on cross section
P_{k1}, P_{k2}	Compressive load capacity of masonry wall under load applied at kern eccentricities e_{k1} and e_{k2}, respectively
P_o	Axial load capacity of masonry wall
P_u	Test load at failure of masonry specimens
p	Circular natural frequency of a beam
Q	Ratio of maximum static deflection due to shearing deformation to that due to flexural deformation in a beam

q	Distributed load on a beam
R	Out-of-plane flexural stiffness of masonry wall
r	Strength reduction factor, or, ratio of net area of mortar contact to gross area of concrete block masonry
S_o	Vertical (surface) area of wall openings
S_w	Vertical (surface) area of wall without openings
T	Fundamental period of vibration
t	Thickness
u	Coefficient indicative of the influence of compression on in-plane shear strength of masonry
V	Shear force parallel to bed joints, in plane of wall
v	Coefficient of variation
v'	Average racking shear strength of masonry in the presence of axial compressive load
W	Total weight of a building
W_b	Weight of bracket assembly in eccentric flexure test
W_m	Weight of masonry specimen above cracking plane in eccentric flexure test
w	Width of small test specimen
X	Distance from centroid of cross section to point of load application in eccentric flexure test
X_b	Distance from centroid of cross section to center of gravity of bracket assembly in eccentric flexure test
α	Factor which accounts for moment amplification produced by eccentric compressive load acting on transverse displacement of wall section
γ	Average shear strain
Δ	Change in gage length in small test specimen, or, in-plane horizontal deflection of top of wall relative to its base
Δ_f	Maximum static deflection of beam due to flexural deformation
Δ_v	Maximum static deflection of beam due to shear deformation
ε_m	Average compressive strain in masonry
θ	Slope of deflected beam

SI Conversion Units

In view of present accepted practice in this technological area, U. S. customary units of measurement have been used throughout this report. It should be noted that the U. S. is a signatory to the General Conference on Weights and Measures which gave official status to the metric SI system of units in 1960. Readers interested in making use of the coherent system of SI units will find conversion factors in ASTM Standard Metric Practice Guide, ASTM Designation E 380-72 (available from American Society for Testing and Materials, 1916 Race Street, Philadelphia, Pennsylvania 19103). Conversion factors for units used in this paper are:

<u>Length</u>

1 in = 0.0245* metre
1 ft = 0.3048* metre

<u>Area</u>

1 in^2 = 6.4516* x 10^{-4} $metre^2$
1 ft^2 = 9.2903 x 10^{-2} $metre^2$

<u>Force</u>

1 lb (lbf) = 4.448 newton
1 kip = 4448 newton

<u>Pressure, Stress</u>

1 psi = 6895 pascal
1 psf = 47.88 pascal

<u>Moment</u>

1 lbf-ft = 1.3558 newton-metre
1 lbf-in = 0.1130 newton-metre

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*Exact Value

Evaluation of Structural Properties of Masonry In Existing Buildings*

S. G. Fattal and L. E. Cattaneo

The current state of knowledge on the structural behavior of masonry is synthesized to develop a methodology for the evaluation of the load capacity of masonry walls in existing buildings. A procedure is described for direct sampling and testing of specimens removed from masonry walls of buildings to determine their strength in shear, flexure and compression, and to measure their load-deformation characteristics. A documentation of strength and stiffness properties obtained from available test data is included to provide an alternate source of information on masonry of comparable construction. Sample calculations of masonry building analysis for seismic forces are given in Appendices A and B.

Key Words: Analysis; compressive strength; deflection; flexural strength; masonry walls; racking strength; seismic loading; shear strength; shear wall; stiffness.

1. Introduction and Objective

After the disastrous San Fernando earthquake 1970, the Veterans Administration began implementing a program for the evaluation of VA hospital buildings in accordance with seismic design requirements developed by the VA Earthquake and Wind Forces Committee. This report relates to that task by prescribing procedures for evaluating the strength and stiffness of unreinforced masonry walls in existing buildings.

An initial literature survey indicated a scarcity of technical documentation dealing with old masonry construction. Among possible methods of data acquisition on masonry properties considered during the course of this investigation, the direct method of sampling and testing small specimens removed from existing masonry construction was found to be the most feasible. Information acquired in this manner will be more representative of the masonry in the actual construction than that acquired from tests on new specimens of similar construction or test logs taken during construction. Other techniques such as non-destructive testing by ultra-sonic devices, are relatively recent developments which have not found widespread commercial application.

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The direct test method recommends three types of prism tests as a means of acquiring the basic strength parameters of the masonry which they represent. Procedures are described for sample extraction in the field, transportation and test set-up including appropriate instrumentation for deformation measurements and interpretation of results.

This report also compiles and interprets test data on specimens which represent some of the common types of masonry construction, using, as its source, the results from three different experimental studies conducted at the National Bureau of Standards [4,50,65].* Since this information is derived from tests of new specimens built under controlled environmental conditions, it will be less representative of existing masonry properties than that obtained from sampled test specimens.

2. Scope

The report is organized into three main sections. Section 3 begins with an introductory background information on common types of masonry constituents and wall systems which are classified according to type of construction or function in a building. This is followed by a documentation and discussion of available test data on masonry strength in shear, compression and flexure. For comparison, the tables also include allowable stress values recommended by seven different codes and standards, three of which are foreign. The last part of Section 3 specifies sectional properties of masonry walls to be used for stress and stiffness calculations.

Section 4 presents sampling and test methods of masonry specimens removed from existing construction. The three types of tests pertain to the evaluation of masonry strength in flexural tension, shear and compression. Interpretation of test results is discussed in Section 4.7.

Section 5 describes limit states of masonry walls under combined loading conditions. Interaction relationships are discussed for ultimate strength under combined compression and flexure, compression and shear, and under simultaneous compression, shear and flexure. The load-deflection relationships of shear walls are given in Section 5.5.

Appendix A consists of a numerical example employing the methodology for seismic investigation of typical masonry structures. Appendix B describes approximate methods for the evaluation of the natural period of a building. Appendix C describes the test setup for data listed in Section 3.

*Numbers in brackets indicate references listed in the Bibliography.

3. Properties of Masonry

3.1 Types of Masonry Constituents and Wall Systems

Common types of units used in masonry construction are clay and sand-lime brick, concrete block and structural clay tile. Other products such as natural building stone and adobe block are also utilized, although to a much lesser extent.

Building and facing bricks are available in a variety of rectangular sizes. The units are classified as hollow if the core area exceeds 25 per cent of the gross cross-sectional area, otherwise they are considered as solid. Detailed descriptions and classification of various brick units can be found in ASTM designations C55, C62, C73, C216 and C652.

Concrete block masonry units are made of standard or lightweight concrete aggregate. The hollow units which have more than 25 per ce nt coring are used for load-bearing as well as nonload-bearing masonry application including non-structural partition walls, while solid blocks, with less than 25 per cent core area are used in load-bearing type construction.

Detailed classifications of concrete masonry are found in ASTM degignations C90, C129, C140, C145, and C331. Illustrations of various common shapes and sizes of units, reproduced from reference [24], are shown in figure 3.1. Clay tile masonry units which are similar to brick in composition are available in a variety of sizes and sectional configuration and are generally characterized by relatively thin webs. The hollow construction offers savings in material and weight and provides good insulation. Classifications of structural clay tiles are found in ASTM designations C34, C56, and C112. Common shapes of clay tile units, reproduced from reference [26], are illustrated in figure 3.2.

In conventional practice, masonry units are identified by their nominal diminsions. For instance, the actual dimensions of an 8x8x16-in concrete block are 7-5/8x7-5/8x15-5/8 in. Hollow tile masonry units are laid with the cores, either horizontal or vertical (figure 3.2). In the text of this report, masonry units are identified by their nominal dimensions while calculation of sectional properties for strength and stiffness determination are based on the actual dimensions of the units.

Masonry units are laid in mortar which acts as their binding agent. Full mortar bedding is usually employed between courses of masonry built with solid units. For hollow units, use of face shell mortar bedding is a common practice. As strength properties of masonry walls are usually governed by the type of mortar used, the latter is specified when strength values are prescribed (see tables 3.1 to 3.3).

Prevailing types of mortar are cement-lime-sand and masonry cement-sand mortar. Different proportions of the constituents in these types will produce different strength properties. Standard mortar designations, namely, types M, S, N, O and K are used for the appropriate mortar identification according to the range of constituent proportions, by volume, specified in ASTM designation C270. Types M, S, and N are commonly used for structural masonry applications. Types M or S are specified for high flexural strength requirements. Various other specifications for mortar and mortar ingredients are found in ASTM designations C5, C91, C109, C110, C144, C150, C157, and C207.

Masonry walls are classified according to type of construction and intended use in a building. A few of the common ones are identified here for purposes of convenient reference. A <u>single-wythe wall</u> has one masonry unit in its thickness. A <u>multi-wythe wall</u> has several masonry units in its thickness. In a <u>composite wall</u> construction, at least one of the wythes is built with masonry units and/or mortar dissimilar from those in the neighboring wythes. Multi-wythe walls without cavity are laid contiguously with the spaces between the wythes, called <u>collar joints</u>, filled with mortar or grout. To insure monolithic action of the assembly, additional bonding is effected by the use of <u>header units</u> or <u>metal ties</u> laid horizontally in bed mortar across the wythes at periodic intervals throughout the height of the wall. A <u>cavity wall</u> is identified by a continuous vertical air space between any two adjacent wythes and by metal ties laid as in composite wall construction and connecting the two wall sections flanking this space. <u>Reinforced masonry walls</u> built with solid units are reinforced by placing steel bars, vertically and/or longitudinally, as needed, in the space between consecutive wythes and by grouting this space. In hollow block walls, vertical reinforcement may be placed as needed, through the hollow cores which are then grouted.

The following types of masonry walls are identified by their intended function in a building. A <u>load bearing wall</u> supports the vertical loads above it in addition to its own weight, with or without the aid of a vertical load-carrying space frame. A <u>non-load bearing wall</u> supports no vertical loads other than its own weight. A <u>shear wall</u> resists planar forces which are primarily induced by exterior horizontal loads acting on a building. A <u>curtain wall</u> is a non-load bearing wall, built outside the building frame

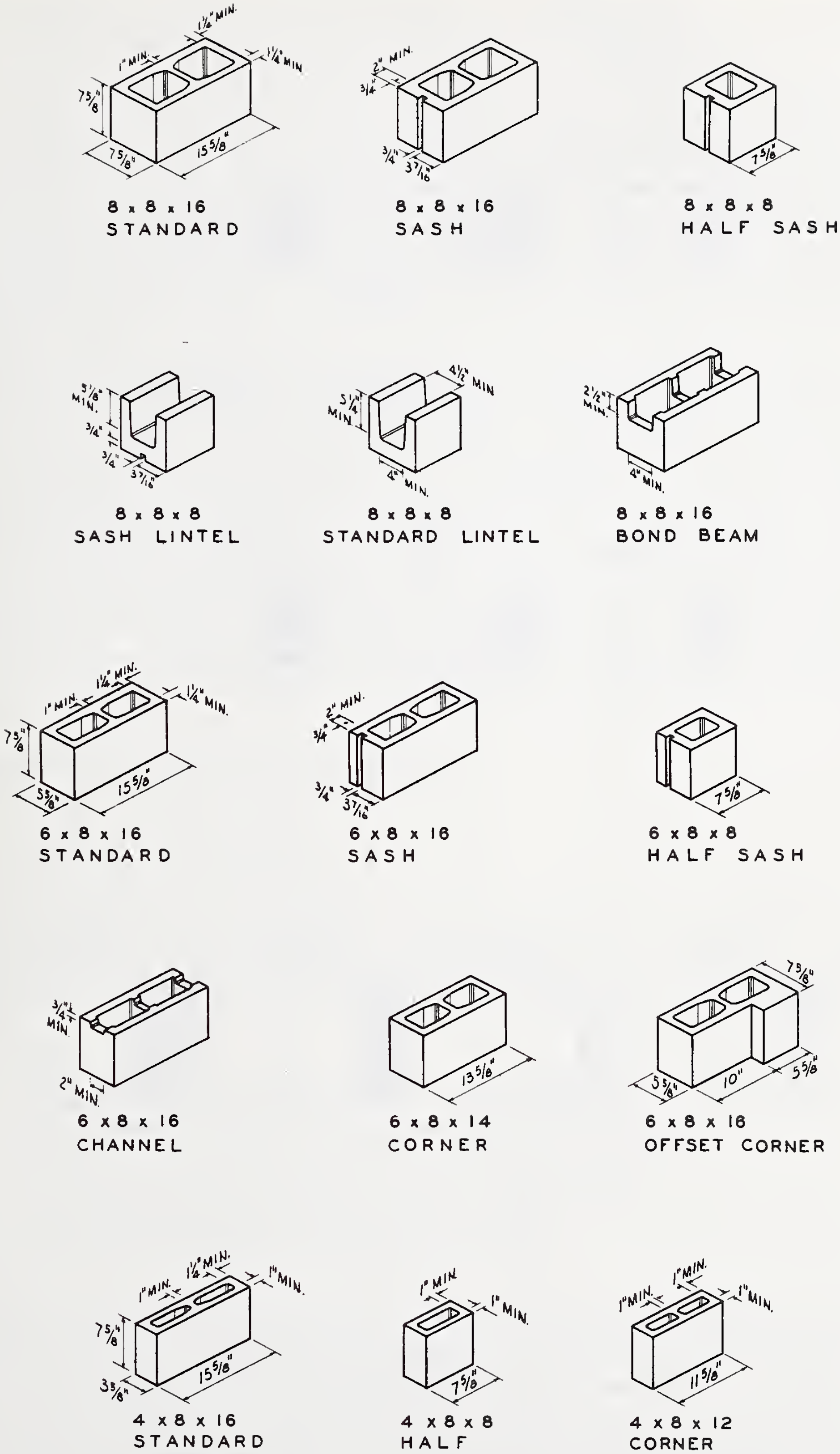


Figure 3.1 - Concrete masonry units.

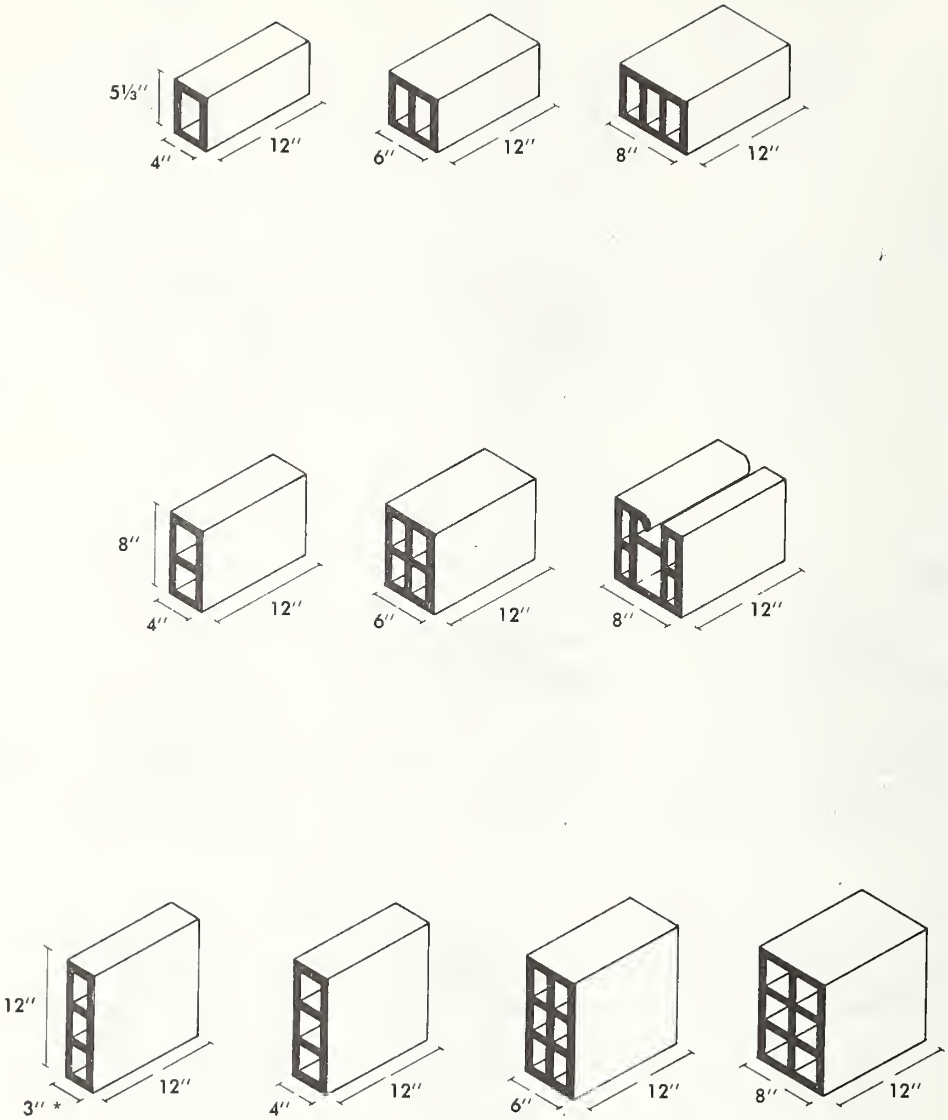


Figure 3.2 - Clay tile masonry units.

and not entirely supported at each floor. A panel wall is a non-load bearing exterior wall supported at each story level. A partition is a non-load bearing interior space divider which will function as a shear wall unless isolated along three edges from the rest of the structure. Veneer is the exterior masonry layer of a two-layer wall system, connected to the interior layer and/or to the primary load-supporting structure by horizontal ties. Veneer is generally designed to be non-load bearing. A pier is a masonry wall segment flanked by two adjacent openings or by an opening and the vertical edge of the wall. A lintel is a wall segment above an opening.

A masonry filler wall or infill wall designates a wall fully enclosed within a structural frame or bounded between two columns and wholly supported at each story level. Filler walls are generally designed to be non-load bearing although they may participate in supporting a portion of the gravity loads depending on the construction sequence used in the field. Because of their confinement and high in-plane lateral stiffness relative to the surrounding frame, filler walls function as shear walls by absorbing the major portion of the horizontal thrust on a building before cracking. The presence of filler walls significantly alters the structural behavior and response of the primary frame under earthquake loads.

3.2 Evaluation of structural Properties

3.2.1 Sources of Information

To investigate the capacity of masonry elements in existing buildings a knowledge about their structural properties must be acquired. The following are possible sources of such information listed in the order of decreasing reliability.

1. Data from samples of actual construction.
2. Test logs of samples taken during construction.
3. Available data from comparable construction.
4. Data from new specimens of the same material construction.
5. Assumed properties from code tables.

Information acquired from source 1 would be much more desirable than information from any of the other sources listed above since the masonry samples represent actual conditions in the existing structures at the time of the survey. However, the reliability of results will depend upon the method employed in the removal of samples and the implementation of the appropriate specifications for testing procedures. Consequently, Section 4 has been dedicated in its entirety to the detailed discussion of a methodology describing field sampling procedures from actual construction, test methods, and interpretation of test results to evaluate strength and stiffness properties of masonry.

Information from test logs of samples taken during actual construction will probably seldom be available. Likewise, available test data on old masonry is too limited to be of practical significance for purposes of this study. By contrast, a substantial amount of experimental research data on new masonry construction exists to justify its consideration as a possible source for predicting the structural behavior of masonry walls in existing buildings. To assist the user in his search of documented test data on masonry, a complehensive list of selected references have been included in the Bibliography.

In Section 3.2.2 and 3.2.3, available data from recent masonry tests conducted at the National Bureau of Standards have been compiled and interpreted in order to provide a supplementary source of information on the procedure that can be used to reduce and synthesize experimental data on masonry from existing publications.

3.2.2 Available Test Data

This section is devoted to the documentation of strength and stiffness properties obtained from available test data for masonry comparable in construction to masonry in existing structures. The objective is to provide an alternate source of information to assist in approximate and reasonably conservative evaluation of properties. This information is compiled in tables 3.1 to 3.5 which contain entries of the following masonry propertiies derived from selected documents referenced in the text.

f'_m = compressive strength
f'_v = shear strength
f'_t = tensile strength in flexure
E = modulus of elasticity
G = modulus of rigidity (shear modulus)

The format of the first three tables permits comparisons, in each case, (Table 3.1: Compression; Table 3.2: Flexural Tension; Table 3.3: Racking Shear) of the given property as determined by tests, both of small specimens and large specimens (when available) of various types of masonry construction. Other pertinent physical data are also tabulated together with values of maximum allowable stresses as determined by seven different recognized design recommendations or codes (listed under column headings J[45], K[46], L[13], M[48], N[47], O[25], and P[23] in the tables). Table 3.4 gives a summary of allowable stresses in masonry recommended by the same codes and standards.

The source of strength properties listed in tables 3.1 to 3.3 is a bank of data obtained from tests conducted at the National Bureau of Standards and reported in three different publications [4, 50, 65]. These data are interpreted in the following section.

A summary of background information on the specimens and testing procedures is given in Appendix C for ready reference. For a thorough description of the test setups the reader should consult the specified references mentioned in the text.

3.2.3 Interpretation of Available Data

The entries in Tables 3.1 to 3.3 are samplings of experimentally determined strengths of brick and concrete block masonry, together with values of maximum allowable stresses included by various masonry standards building codes. To assist in the judicious use of this data as an alternate source of information on masonry, some additional explanation is provided in this and the next sections.

In the calculation of cross sectional area of hollow concrete block, distinction is made between gross area, net solid area, and net area of mortar contact. Representative values [4, 65] of net solid areas are 52% for 8x8x16-in two-cell hollow concrete block and 72% for 4x8x16-in three-cell hollow concrete block. The same references indicate that mortar contact area in the 8x8x16-in block is in the range of 38 to 46% of the gross area; and in the 4x8x16-in block, approximately 67% or more.

Note that in tables 3.1 and 3.2 the mortar used in masonry specimens under Item 1 was classified as Type N, approximately, in order to limit tabulation of mortar types to ASTM C270 designations. As indicated in ASTM C270, a given type of mortar includes different combinations of materials and their proportions. This method of classification is herein adopted for simple reference to various building codes which use the same classification.

a. Compressive Strength Properties (Table 3.1)

Determination of compressive strength (f'_m) is far from being a widely standardized procedure. Methods of test recommended by various organizations

[13, 23, 25, 45, 46, 47, 48] and ASTM Designation E447, differ for the same type of masonry as well as for different types. Methods employing small specimens recommended by various sources prescribe different height-to-thickness ratios (h/t) of specimens as the standard parameter (correction factor = 1.0) for correcting tested strength of specimens of other (h/t) ratios. Compressive strengths in columns F and H are tabulated as recorded by the referenced investigator for the size specimen shown in columns E and G. Applications of any (h/t) correction factor to values in column F are reflected, when applicable, in the values of columns J through P.

Allowable stresses in columns J through P are, in general, obtained from the various sources (UBC, etc.) using recommended relationships between allowable compressive stress (f_m), and ultimate compressive strength (f'_m). This relationship is usually given as a table or in mathematical form. In some instances tabular correlation is given directly between strength of the masonry units used and allowable masonry compressive strength. Such tabulations usually give separate values of allowable stress for masonry containing different types of mortars, and for different conditions of workmanship (i.e., with or without inspection).

Whenever small specimen compressive strength was available (listed in column F), allowable stresses were derived using the respective procedures of each organization (columns J thorugh P). It is to be noted that, in some cases, values of allowable compressive stress recommended by a given organization are intended for only one particular kind of masonry (e.g. brick masonry only, column K; or concrete block masonry only, column L). When small specimen strength was not available, derived allowable stresses were based on assumed masonry strength obtained from the particular organization's table which correlated assumed compressive strengths with strength of units and types of mortar.

Due to scatter in the data of Table 3.1 a detailed treatment of the values appears impractical. However, an overall appraisal of the itemized strengths and allowable stresses leads to some inferences. It appears that, whether derived from available prism strengths or from tabular "assumed" strengths based on a knowledge of masonry unit and mortar, the more conservative allowable stresses (excluding U.B.C.) are about 1/4 to 1/5 of the masonry wall strengths.

This fact is reflected in the often encountered ($0.2f'_m$) used as an alternate expression for calculating the allowable compressive stress in some of the cited codes and seems to strengthen the correlation of these quantities. Lacking more specific values of (f'_m), Table 3.1 could provide reasonably close values of compressive strength of types of masonry construction comparable to those being examined. Alternately, if some information about the unit and the mortar is available, (a conservative assumption could be made of the mortar type) an "assumed" (f'_m) could be selected from a suitable code tabulation as in BIA [46]; or, if this procedure leads to an allowable stress value, as in UBC [45], it could be projected to (f'_m) by application of a conservative factor (a value of 4 is suggested).

b. Flexural Tensile Strength Properties (Table 3.2)

Flexural tensile strength normal to the bed joints is relatively low in unreinforced masonry. It is dependent on the bond between mortar and units, and easily affected by poor workmanship and other causes of bond disturbance (cracks). Since the prescriptive codes and standards on masonry [13,23,25, 45,46,47,48] are based on working stress design, tensile strength values are generally specified by reference to commentaries or other supporting research documentation [70,80]. Recommended maximum allowable tensile stresses in flexure are presented by all of the organizations named, usually as specific values, dependent on types of mortar and workmanship.

Added to the features of tensile strength mentioned above (low value, sensitivity of bond to workmanship, development of bond cracks), the data presented in Table 3.2 include anomalies which raise doubt: (a) wide range of experimental strengths for equal specimens, (b) proximity of experimental strengths to allowable stresses, and (c) experimental values less than allowable values. These facts rule against putting too much reliance on the limited tensile strength data. The cored brick in items 1 and 2 show a somewhat higher strength, although not conclusively. It is to be noted that the Australian and British codes [23, 25] recommend that, in general, no reliance be placed on masonry tensile strength. However, attention is directed to other circumstances which affect the apparent tensile strength of masonry, such as the influence of vertical loads on the transverse flexural strength of walls.

c. Shear Strength Properties (Table 3.3)

Racking shear strength of masonry is also comparatively low and frequently dependent on the bond between mortar and units. Test Method ASTM E-72 includes a procedure for determining the racking shear strength of an 8-ft square panel (a method which does not lend itself to economy or production). Other methods which show promise are discussed in Section 4. For allowable shear stresses, the same organizations make reference to experimentally determined shear strength in recommending maximum allowable shear stress values as a function of (f'_m) or as specific values dependent on types of masonry unit, mortar, and workmanship. Based on the background data available so far (BIA Commentary [46]) allowable shear stresses for brick represent approximately a safety factor of 4.

To the extent to which it can be applied, lower experimental values from Table 3.3 are proposed as shear strengths for comparable types of masonry. For other brick masonry it is suggested that it be identified conservatively by unit and mortar in the BIA Standard to establish an assumed (f'_m), and that the shear strength be taken as $2\sqrt{f'_m}$. It is also suggested that other concrete block masonry be likewise identified for selection of an allowable shear stress in the NCMA Standard (A.C.I. Standard and Canadian Code are similar); a conservative assumption that these allowable shear stresses also contain a safety factor of at least 2 would provide interim shearing strengths taken as twice the allowable shear stress. As in flexure, the apparent racking strength may be increased by superimposed vertical loads.

d. Modulus of Elasticity E, and Modulus of Rigidity G, (Table 3.5)

There are no standard procedures for the experimental evaluation of elastic properties of masonry. The modulus of elasticity may be determined from measurements obtained from compression tests of masonry prisms while the modulus of rigidity (or shear modulus) may be obtained from measurements of diagonal deformations in racking test specimens. These and other test methods and procedures are discussed in Section 4.

In the absence of sufficient data (in the references of tables 3.1 to 3.3) to give a satisfactory indication of modulus values or, as a substitute of tests of field samples (section 4), reference is made to the information in table 3.5 of various organizations. With the exception of the Australian values, consistency of the recommended relationships for E and for G is noteworthy, particularly since their sources represent interests in different types of masonry (brick only, concrete block only, and, brick or block). Plummer [63] points out that for a given group of brick data alone the equation $E = 1000\ f'_m$ passes through a range bounded by $E = 1200\ f'_m$ and $E = 700\ f'_m$; the observed values of E varied from 2,652,000 psi to 473,000 psi and (f'_m) values between 2800 psi and 600 psi (not respectively). With such dispersion and the desire to be conservative kept in mind, the equations $E = 1000\ f'_m$

and $G = 400 f'_m$ subject to the inspection limitations prescribed (Sect. 3.2.4), are suggested for adoption.

3.2.4 Strength Reduction Factors

The strength reduction factors discussed in this section are applicable to strength estimates obtained from sources other than direct tests of sampled specimens removed from existing buildings. The strength reduction factors for the latter case are specified in Section 4.7.5.

a. Workmanship

One of the major factors influencing the strength of masonry elements in a structure is the quality of workmanship exercised at the time of construction. In recognition of the importance of quality control in the field, various codes (UBC, BIA, NCMA, etc.) make a clear distinction between inspected and uninspected construction by prescribing different allowable values for masonry design. Sometimes different values are also prescribed for the elastic moduli, as in UBC and BIA (table 3.5) and for lateral support requirements of walls, as in BIA. Values of compressive stress reduction factors specified for masonry construction without inspection range from about 0.67 by BIA to 0.50 by UBC and NCMA.

For the purpose of evaluating the strength of masonry walls in existing structures it is proposed that a reduction factor of 2/3 be introduced for construction without inspection, to be applied as a multiplier to the mean values of the strength properties (f'_m), (f'_v) and (f'_t) obtained from sources other than tests of specimens directly obtained from the existing structure under investigation. In cases where no information is available about inspection, the masonry should be assumed to have been constructed without inspection.

b. Variability

Another important factor inherent in masonry construction is the variability of strength exhibited between test specimens of seemingly "identical" construction. Depending on the type of test, type of masonry and size of specimen used, the scatter of test results may be considerable even for tests conducted under controlled laboratory conditions. In an actual construction where the controlled environmental conditions of a laboratory are absent, a wider scatter may reasonably be expected in the strength of masonry walls built with the same constituents.

In the absence of any documentation for the quantitative prescription of strength variability of masonry construction according to type, a variability factor of 2/3 is tentatively proposed for use as a reduction factor to be applied as a multiplier to the mean values of strength properties available from sources (e.g.: tables 3.1 to 3.3) other than direct tests of specimens obtained from the existing structure under investigation, to account for the effect of variability.

c. Size of Specimen

There is a difference in masonry strength attributable to size that should be recognized when interpreting available test data as a source of such information. Usually small prisms will develop greater strength than full-scale walls. Test experience indicates the difference may be as much as 50 per cent for flexure specimens and between 10 to 30 percent for shear and compression specimens, depending primarily on the size of test prisms used.

In the absence of a better quantitative documentation of test strength as a function of size, a reduction factor of 2/3 is proposed to be applied as a multiplier to the mean values of strength properties derived from prism tests available from sources other than direct tests of specimens obtained from existing masonry contruction, to account for size effects. Data from full-scale wall tests need not be so reduced. Attention is drawn to the fact that tables 3.1 to 3.3 compile test data for prisms as well as large-scale specimens.

d. Peak Loading History

Exposure to cyclic or peak loads induced by earthquakes and other natural disasters could adversely effect the masonry strength in existing buildings. Site measurements recorded after an earthquake frequently show an increase in the natural period of a building which tends to become more compliant as a result of internal structural damage. Such damage is not always obvious enough to detect by visual means, nor is it possible to assess total level of damage attributable to cumulative exposure to past disaster loads.

In recognition of the detrimental effect of disaster loads on masonry, it is proposed that a reduction factor of 2/3 be applied as a multiplier to the mean values of masonry strength data derived from sources other than direct testing of specimens removed from existing construction.

The total reduction factor to be applied as a multiplier to mean masonry strength property values, is the product of applicable reduction factors specified in items (a) through (d) above.

3.3 Sectional Properties

3.3.1 Introduction

Sectional properties are used in combination with elastic constants to determine the distribution of lateral and gravity loads to the appropriate elements in a building and to convert element forces to stress values for comparison with available capacity. The sectional properties of masonry are specified in accordance with loading condition (in-plane or out-of-plane), type of masonry units (hollow or solid), type of construction (single wythe, double wythe or cavity), confinement condition (with or without frame enclosure), and configuration of openings.

3.3.2 Walls Without Openings, In-Plane Loads

Sectional properties will be specified by reference to the wall shown in figure (3.3). For the specified in-plane loads the sectional properties are defined in terms of length L and thickness t as follows:

$$A = Lt \tag{3.1}$$

$$I = \frac{tL^3}{12} \tag{3.2}$$

where

A = cross sectional area

I = moment of inertia

t = effective thickness of the wall as defined below

For single-wythe solid masonry unit construction, t is the actual wall thickness. For two-wythe masonry construction consisting of solid units of identical material composition, t is the sum of the thicknesses of the two units plus the thickness of the collar joint. For single wythe hollow concrete block construction it is governed by the net area of mortar bedding. Representative thickness values are: 40% of the gross thickness for 8x8x16-in and 6x8x16-in 2-cell hollow block, 67% of the gross thickness for 4x8x16-in 3-cell hollow concrete block [4, 65, 75]. For two-wythe hollow block construction of the same material com-position, t is the sum of computed thicknesses of the two blocks plus the thickness of the collar joint.

For two-wythe construction of masonry units of dissimilar composition the principle of transformed section may be applied to obtain the equivalent thickness t. For instance, the effective thickness t of a solid brick wall equivalent to a wall consisting of a brick wythe and a hollow concrete block wythe is obtained from the equations:

$$t = t_b + rt_c \, (E_c/E_b) \tag{3.3}$$

$$t = t_b + rt_c \, (G_c/G_b) \tag{3.4}$$

where

t_b = thickness of brick unit
t_c = gross thickness of concrete block unit
r = ratio of net area of mortar contact to gross area of concrete block
E_c/E_b = ratio of block-to-brick elastic moduli
G_c/G_b = ratio of block-to-brick shear moduli

Equations (3.3) and (3.4) apply in calculations involving in-plane flexural and shearing deformations, respectively. For cavity wall construction of masonry units of dissimilar composition the effective thickness is determined according to the method used for two-wythe construction of masonry units of dissimilar composition if loading and boundary conditions induce identical deformations in both wythes. Otherwise the thickness of each wythe is determined separately assuming no interaction exists between the two wythes. Cavity walls of identical material composition are treated in the same manner.

3.3.3 Walls With Openings, In-Plane Loads

A masonry wall with openings is characterized by different sectional properties throughout its height. Depending on the type of calculation involved, a distinction will be made in the method for determining the effective thickness t.

a. For calculations used in determining the distribution of seismic shear forces on a building, the equivalent thickness of a wall without openings as described in section 3.3.2 is further reduced by a factor which is the ratio of the net (area of openings deducted) to gross wall areas. In equation form,

$$t = t_s \left(1 - \frac{Ao}{A}\right) \tag{3.5}$$

where:

t = equivalent thickness of wall with openings

t_s = effective thickness of the same wall without openings (s=solid)

A_o = total (vertical) area of openings

A = gross (vertical) surface area of solid wall.

b. For stress and stiffness calculations of individual walls the equivalent thickness is calculated according to the procedures described in section 3.3.2. Depending on the geometry and arrangement of the openings , the wall is divided into a number of piers (as illustrated in figure 3.5). The wall is then analyzed according to principles of equilibrium, deformation compatibility and constitutive relations applied to the individual piers.

3.3.4 Walls Without Openings, Out-of-Plane Loads

Sectional properties will be specified by reference to figure 3.1. The sectional properties of walls of single or multi-wythe integral construction having the same material composition and built with solid masonry units is determined according to the equations:

$$A = Lt \tag{3.6}$$

$$I = \frac{Lt^3}{12} \tag{3.7}$$

where:

A = area of horizontal cross section of wall

I = moment of inertia of horizontal cross section of wall about minor principal axis

L = width of the wall cross-section

t = sum of thicknesses of individual wythes

For single wythe construction of hollow sectional configuration equation (3.6) applies except t designates the equivalent thickness as determined by the net mortar bed area (Section 3.3.2). The moment of inertia may be conservatively (but closely) estimated by considering only the face shell areas to be effective in flexure. In equation form,

$$I = \frac{L}{12}\left[t^3 - (t - 2t_f)^3\right] \tag{3.8}$$

where:

t = out-to-out thickness of masonry unit

t_f = thickness of one face shell or flange.

For two-wythe construction of hollow masonry units of the same composition, the area is the sum of the areas of the individual wythes as specified above. The centroid of the section and the moment of inertia about this centroid may be conservatively estimated by considering only the face shell areas of the individual wythes to be effective in flexure.

For two-wythe construction of masonry units of dissimilar composition the area may be determined according to the principles of transformed sections. In equation form,

$$A_t = A_1 + A_2 \ (E_2/E_1) \tag{3.9}$$

where:

A_t = area of the transformed section of wall equivalent in composition to material 1

A_1 = area of material 1

A_2 = area of material 2

E_2/E_1 = ratio of elastic moduli of material 2 to material 1

The moment of inertia is determined on the basis of the same transformed section. However, care must be exercised to preserve the relative positions of different sections within the actual (gross) thickness of the wall (transformation applies to sectional widths). For cavity walls the sectional properties of each individual wythe is determined as above, on the basis of uncoupled action.

3.3.5 Walls With Openings, Out-of-Plane Loads

Openings will change the net sectional configuration at various levels through the height of the wall. The sectional properties are determined at each such level in the manner specified for walls without openings. This will allow the wall to be treated as a non-prismatic beam spanning in the vertical direction and loaded transversely. The procedure for calculating critical flexural stresses is used in the sample problem in Appendix A.

3.3.6 Filler Walls, In-Plane Loads

To assess the contribution of masonry filler walls to the sectional properties of concrete infilled frames subjected to in-plane loads the effective wall thickness obtained by the procedures described in subsections 3.3.2 and 3.3.3 is further modified by the ratio of masonry-to-concrete elastic moduli if the infill wall is mechanically connected to the surrounding frame in a manner that will insure integral action of the assembly. The result is an I-shaped figure the sectional properties of which can be readily obtained (figure 3.4).

To account for the influence of openings in filler walls, the sectional properties of the transformed I-section are reduced as follows:

$$A = A_f + A_w \left(1 - \frac{S_o}{S_w}\right) \qquad (3.10)$$

$$I = I_f + I_w \left(1 - \frac{S_o}{S_w}\right) \qquad (3.11)$$

where:

A_f = area of flanges (frame columns)
A_w = transformed area of solid web
S_o = vertical (surface) area of wall openings
S_w = vertical (surface) area of solid wall
I_f = the portion of moment of inertia contributed by the concrete columns
I_w = the portion of moment of inertia contributed by the transformed solid web

Equations (3.10) and (3.11) are considered to be sufficiently accurate for calculating the distribution of lateral seismic forces in a structure. For stress and stiffness calculations the frame-wall assembly is divided into several piers according to its geometry and location of openings as shown in figure 3.4.

For the case where the infill wall is not mechanically attached to the surrounding frame, sectional properties of the uncracked infill wall are determined as in Sections 3.3.2 and 3.3.3, by treating it independently of the frame.

3.3.7 Filler Walls, Out-of-Plane Loads

The procedures prescribed in sections 3.3.4 and 3.3.5 are also applicable to filler walls. The need to transform the wall section into equivalent concrete does not arise in this case since the only function of the frame is to provide a certain amount of rotational constraint at the frame-wall interface.

	A	B	C	D	E	F	C	H	J	K	L	M	N	O	P	R
ITEM No.	Type of Construction	Type of Unit	Compr. Strength of Unit psi	Type of Mortar	Small Specimen Size	Compress. Strength of Small Specimen	Large Specimen Size	Compress. Strength of Large Specimen psi	U B C Allowable Compressive Stress psi	BIA Allowable Compr. Stress psi	N C M A Allowable Compr. Stress psi	ACI-531 Allowable Compr. Stress psi	Canada N B C Allow. Compr. Stress psi	British Code Allow. Compr. Stress psi	Austral. Code Allow. Stress psi	Reference
1	1- wythe 3.63" brick	Cored Brick	14,480 [1]	S	5-course[2] Prism	5400[1]	2' x 8'	3218-3139[1]	225[1]	983[1]	---	---	1229[3]	667[1]	859[1]	[65]
2	1-wythe 3.75" Brick	Cored Brick	20,660[1]	N	5-course[2] Prism	4870-3010[1]	---	---	200-140[1]	886-548[1]	---	---	1108-685[3]	696[1]	506[1]	[65]
3	1-wythe 8" Hollow Block	Hollow Block	1100[1]	N	3-Block High, Prism	440-400[1] 840-770[3]	4' x 8'	886-808[4]	140[4]	---	203-185[3]	169-154[3]	228-208[3]	189[3]	---	[65]
4	1-wythe 8" Solid Block	Solid Block	3370	N	3-Block High Prism	1790-1560	4' x 8'	1649-1353[3]	140[1]	---	430-374[3]	358-312[1]	430-374[3]	290[1]	---	[65]
5	1-wythe 4" Hollow Block	Hollow Block	1530[1]	N	3-Block High, Prism	1020-860[1] 1410-1190[3]	---	---	140[4]	---	340-287[3]	283-239[3]	214[3]	203[1]	---	[65]
6	4"-2"-4" Cavity Block-Block	Hollow Block	1530[1]	N	---	cf Item 5	4' x 8'	1104-1035[4]	50[4]	---	340-287[3]	283-239[3]	214[3]	203[1]	---	[65]
7	4"-2"-4" Cavity Brick Block	cf Items 2, 5	cf Items 2, 5	N	---	cf Items 2, 5	4' x 8'	1229[4]	50[4]	---	340-287[3]	283-239[3]	214[3]	203[1]	---	[65]
8	8" Composite Brick-Block	cf Items 2, 5	cf Items 2, 5	N	---	cf Items 2, 5	4' x 8'	1587-1365[4]	140[4]	---	340-287[3]	283-239[3]	214[3]	203[1]	---	[65]
9	1-wythe 8" Hollow Block	Hollow Block	1200[1]	N	---		4' x 8'	1200-1000[4]	140[4]	---	250-220[3]	200[3]	214[3]	189[1]	---	[4]
10	1-wythe 8" Hollow Block	Hollow Block	1200[1]	S	---		4' x 8'	1240-1160[4]	140[4]	---	400-310[3]	270[3]	304[3]	203[1]	---	[4]
11	8" Composite Brick-Block	Solid Brick, Hollow Block	16,100 1240[1]	N	---		4' x 8'	800-700[1]	140[4]	---	220-175[3]	175[3]	180[3]	189[1]	---	[4]
12	8" Composite Brick-Block	Solid Brick, Hollow Block	16,100 1240[1]	S	---		4' x 8'	1020-850[1]	140[4]	---	310-230[3]	230[3]	259[3]	203[1]	---	[4]

1 Gross Area
2 Stacked Bond
3 Net Cross-Sectional Area
4 Net Mortar Contact Area

Table 3.1 - Masonry compressive strength properties.

	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	R
Item No.	Type of Construction	Type of Unit	Compress Strength of Unit psi	Type of Mortar	Small Specimen Size	Flexure Strength of Small Specimen psi	Large Specimen Size	Flexure Strength of Large Specimen psi	U B C Allowable Flexural Tensile Stress psi	BIA Allow. Flex. Tensile Stress psi	N C M A Allow. Flex. Tensile Stress psi.	ACI-531 Allow Flex. Tensile Stress psi	Canada N B C Flex. Tensile Stress psi	British Code Allow. Flex. Tensile Stress psi	Australian Code Allow. Flex. Tensile Stress psi	Reference
1	1-wythe 3.63" Brick	Cored Brick	14,480[1]	S	7-course[2] Prism	35[1]	4' x 8'	50[1]	20-10[1]	36-24[1]	---	---	36[3]	10-0	10-0	[65]
2	1-wythe 3.75"Brick	Cored Brick	20,660[1]	N	7-course[2] Prism	54[1]	---	---	15-7.5[1]	28-19[1]	---	---	28[3]	10-0	10-0	[65]
3	1-wythe 8" Hollow Block	Hollow Block	1100[1]	N	2-Block High, Prism	9[3]	4' x 8'	6[4]	10-5[4]	---	16[4]	16[4]	16[4]	10-0	---	[65]
4	1-wythe 8" Solid-Block	Solid Block	3370	N	2-Block High, Prism	25[3]	4' x 8'	16-14[3]	12-6[1]	---	27[1]	27[1]	28[4]	10-0	---	[65]
5	1-wythe 4" Hollow-Block	Hollow Block	1530[1]	N	2-Block High, Prism	27[3]	---	---	10-5[4]	---	16[4]	16[4]	16[4]	10-0	---	[65]
6	4"-2"-4" Cavity Block-Block	Hollow Block	1530[1]	N	---	cf Item 5	4' x 8'	24-22[4]	10-5[4]	---	16[4]	16[4]	16[4]	10-0	---	[65]
7	8" Composite Brick-Block	cf Items 2, 5	cf Items 2, 5	N	---	cf Items 2, 5	4' x 8'	30[5]	10-5[4]	---	16[4]	16[4]	16[4]	10-0	---	[65]
8	1-wythe 8" Hollow Block	Hollow Block	1200[1]	N	2-Block High, Prism	24-6[1] 34-8[3]	4' x 8'	25-10[1] 35-14[3]	10-5[4]	---	16[4]	16[4]	16[4]	10-0	---	[4]
9	1-wythe 8" Hollow Block	Hollow Block	1200[1]	S	2-Block High, Prism	52-12[1] 74-17[3]	4' x 8'	47-14[1] 66-20[3]	10-5[4]	---	23[4]	23[4]	23[4]	10-0	---	[4]
10	8" Composite Brick Block	Solid Brick Hollow Block	16,100 1240[1]	N	16" or 24" High Compos. Prism	81-39[1]	4' x 8'	49-21[1]	10-5[4]	---	16[4]	16[4]	16[4]	10-0	---	[4]
11	8" Composite Brick-Block	Solid Brick, Hollow Block	16,100 1240[1]	S	16" or 24" High Compos. Prism	94-53[1]	4' x 8'	61-27[1]	10-5[4]	---	23[4]	23[4]	23[4]	10-0	---	[4]

[1] Gross Area
[2] Stacked Bond
[3] Net Cross-Sectional Area
[4] Net Mortar Contact Area
[5] Based on I of Transformed Section

Table 3.2 - Masonry flexural tensile strength properties.

Item No.	A Type of Construction	B Type of Unit	C Compress. Strength of Unit psi	D Type of Mortar	E Small Specimen Size	F Shear Strength of Small Specimen	G Large Specimen Size	H Shear Strength of Large Specimen psi	J U B C Allowable Shear Stress psi	K BIA Allow. Shear Stress psi	L N C M A Allow. Shear Stress psi	M ACI-531 Allow. Shear Stress psi	N Canada N B C Allow. Shear Stress psi	O British Code Allow. Shear Stress psi	P Australian Code Allow. Shear Stress psi.	R Reference
1	1-wythe, 8" Hollow-Block	Hollow Block	1200[1]	N	---	---	8' x	111-63[4]	-5[4]	---	23[4]	23[4]	23[4]	15	---	[4]
2	1-wythe, 8" Hollow-Block	Hollow Block	1200[1]	S	---	---	8' x 8'	137-111	12-6[4]	---	34[4]	34[4]	34[4]	15	---	[4]
3	8" Composite Brick-Block	Solid Brick, Hollow Block	16,100 1240[1]	N	---	---	8' x 8'	>102-6	10-5[4]	---	23[4]	23[4]	23[4]	15	---	[4]
4	8" Composite Brick Block	Solid Brick, Hollow Block	16,100 1240[1]	S	---	---	8' x 8'	>102-7[4]	12-6[4]	---	34[4]	34[4]	34[4]	15	---	[4]
5	1-wythe 8" Hollow Block	Hollow Block	1100[1]	N	32" x 32"	78[4]	8' x 8'	84[4]	10-5[4]	---	23[4]	23[4]	23[4]	15	---	[50]
6	1-wythe 3.63" Brick	C. d Brick	14,480[1]	N	4' x 4'	193-179[1]	8' x 8'	219-210[1]	15-7.5[1]	28-23[1]	---	---	50[4]	15	15	[50]

[1] Gross Area

[4] Net Mortar Contact Area

Table 3.3 - Masonry shear strength properties.

Table 3.4 - Summary of code recommended allowable stresses in masonry.

Source	Material Referenced	Axial Compressive psi	Flexural Tensile psi	Shear psi
UBC	Brick	250-100	20-7.5	20-7.5
	Block	175-50	12-5	12-5
BIA	Brick	$0.2\ f'_m$	36-19	$80 - 0.5\sqrt{f'_m}$
NCMA	Block	$0.2\ f'_m$	39-16	34-23
ACI	Block	$0.2\ f'_m$	39-16	34-23
CANADA	Brick	$0.25\ f'_m$	36-28	$50 - \sqrt{f'_m}$
	Block	$0.2\ f'_m$	36-16	34-23
BRITAIN	Brick	900-43	10-0	15
	Block	725-72		
AUSTRALIA	Brick	$0.75\ f'_m$	10-0	15

Table 3.5 - Code recommended values of modulus of elasticity (E) and modulus of rigidity (G).

Source	Material Referenced	E, psi		G, psi	
		Inspection	No Inspect.	Inspection	No Inspect.
UBC	Brick or Block	$1000f'_m$ $\leq 3 \times 10^6$	$500f'_m$ $\leq 1.5 \times 10^6$	$400f'_m$ $\leq 1.2 \times 10^6$	$200f'_m$ $\leq 0.6 \times 10^6$
BIA	Brick	$1000f'_m$ $\leq 3 \times 10^6$	$1000f'_m$ $\leq 2 \times 10^6$	$400f'_m$ $\leq 1.2 \times 10^6$	$400f'_m$ $\leq 0.8 \times 10^6$
NCMA	Block	$1000f'_m$ $\leq 3 \times 10^6$		$400f'_m$ $\leq 1.2 \times 10^6$	
ACI	Block	$1000f'_m$ $\leq 2 \times 10^6$		$400f'_m$ $\leq 0.8 \times 10^6$	
CANADA	Brick or Block	$1000f'_m$ $\leq 3 \times 10^6$		$400f'_m$ $\leq 1.2 \times 10^6$	
BRITAIN	Brick or Block	$4000f_b$ (allow.)*		------	
AUSTRALIA	Brick	$562\ f'_m$ $\leq 2.4 \times 10^6$		$225f'_m$ $\leq 1 \times 10^6$	

$^*f_b$ = code basic (i.e., allowable) compressive stress

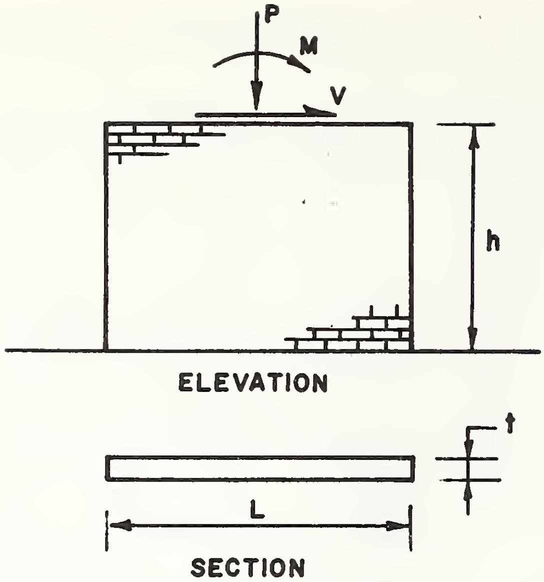


Figure 3.3 - Masonry wall without openings.

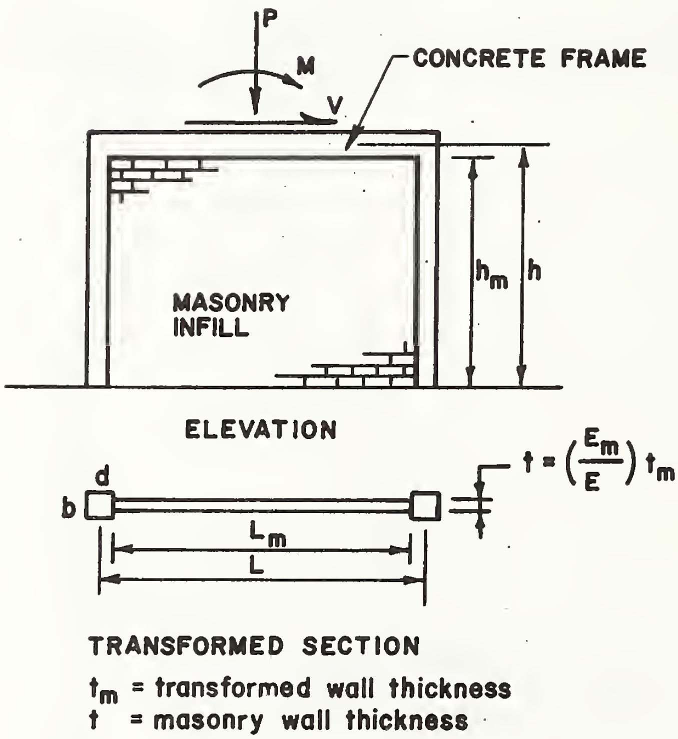


Figure 3.4 - Infilled frame without openings.

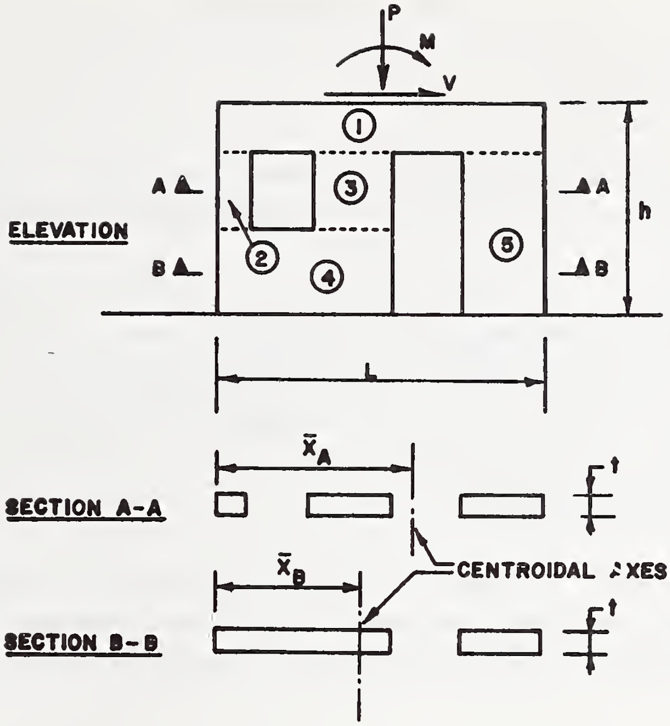


Figure 3.5 - Masonry wall with openings.

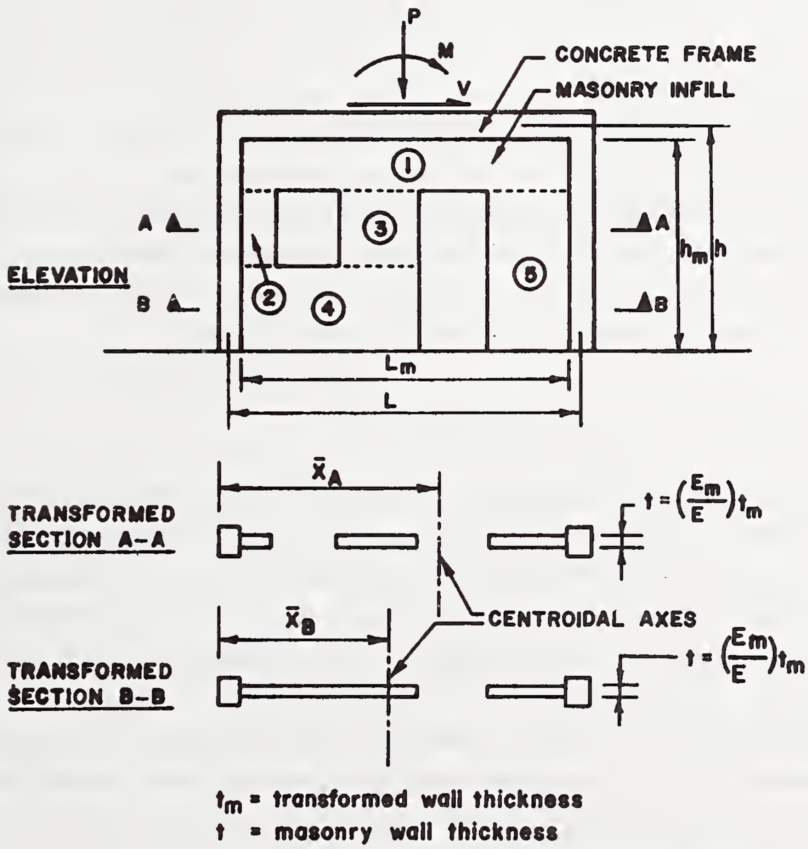


Figure 3.6 - Infilled frame with openings.

4. Test Methods

4.1 Scope

The structural investigation of masonry buildings under seismic loads discussed in Appendix A makes use of the following properties of masonry wall sections:

f'_m = strength under axial compression
f'_v = shear strength under diagonal compression
f'_t = tensile strength under out-of-plane flexure
E = modulus of elasticity
and G = modulus of rigidity

The direct manner of acquiring this information on masonry in existing structures is to remove wall samples and to conduct appropriate laboratory tests using specimens prepared from these samples.

In Sections 4.2 to 4.6, procedures are prescribed for sample extraction and transportation, specimen preparation and execution of tests in a laboratory. Section 4.7 describes the interpretation of test results to determine compressive, shear and flexural strength as well as load-deformation properties. Guidelines for the implementation of the testing program are discussed in Section 4.8. With regard to standard test methods which relate to the types of tests herein prescribed, the following sources are cited: ASTM Designation E447-72, Standard Methods of Tests for Compressive Strength of Masonry Assemblages, ASTM Designation E-518*, Standard Method of Test for Flexural Bond Strength of Masonry, and ASTM Designation E-519*, Standard Method of Test for Diagonal Tension (Shear) in Masonry Assemblages.

4.2 Types of Tests and Specimen Dimensions

In order to obtain representative test values of masonry properties, some codes require that a test shall consist of not less than 5 specimens, while others require a minimum of 3 specimens. It is recommended that at least 3 specimens be used in each of the 3 different types of tests (compression, shear, flexure) for a given type of masonry in an existing building.

The optimum size of a test specimen is the smallest size that will yield results representative of in-situ wall strength. The prescribed sizes of

*Publication by ASTM pending.

test specimens are principally governed by considerations of the combined effect of cutting, handling and actual specimen size on the masonry strength. The following requirements are listed to assist in selecting dimensions of specimens for use in the three types of tests.

4.2.1 Compression Specimens (Figure 4.1a)

The axial compression test is used to evaluate the compressive strength (f'_m) and the modulus of elasticity (E) of the masonry in the direction normal to the mortar bed. The tests should be conducted in triplicate for each type of masonry comprising the wall cross section. For instance, a composite wall section consisting of a 4-in brick wythe and an 8-in hollow block wythe will require separate testing of specimens of each wythe prepared from the composite samples by cutting through the collar joint and chiseling off excess surface mortar. In cases where header joints are unavoidable between two adjacent wythes of the same composition, the assembly should be tested as a unit. The general dimensional requirements for the test specimens are as follows:

1. Width of specimen (w) should contain not less than one whole unit in the bottom and top courses and should not be less than thickness (t) of specimen.

2. Height of specimen (h) should be not less than 12 inches.

3. Height of specimen (h) should contain not less than 3 whole courses plus the minimum whole number more to make h/t equal to or greater than 3.

4.2.2 Shear Specimens (Figure 4.1b)

Shear specimens should be tested by compression applied along a diagonal axis within the centroidal plane of the cross section. In order to avoid fabricating specially fitted loading shoes for different specimens of random height-to-width ratios, all specimens should be cut square. The diagonal compression test will be used to evaluate the shearing strength (f'_v) and the modulus of rigidity (G) of the masonry. The criteria for testing multi-wythe wall sections are those prescribed for the compression specimens. However, a specimen of composite construction (dissimilar wythes) in which header units are unavoidable, may be tested as a unit if it conforms to the sectional configuration and type of construction of the masonry which it is intended to represent, and to the dimensional requirements herein specified. The general dimensional requirements for the test specimen are as follows:

1. Height of specimen (h) should contain a whole number of, but not less than, 3 courses.
2. Height of specimen (h) should be not less than 12 inches.
3. The height-to-thickness ratio (h/t) should be not less than 2.
4. The width of specimen should be equal to not less than 2 masonry units.
5. The width should be established with respect to the vertical joint pattern in such a way that one pair of diagonally opposite corners of the specimen contain whole units or the largest possible fractions thereof.

4.2.3 Flexure Specimens

Since under seismic excitation walls may flex in either direction, a total of 6 tests will be necessary to evaluate flexure bond strength at the two opposite outer fibers of the wall. As an alternate option, only 3 specimens may be tested by determining only the flexural strength at the outer fibers corresponding to the exterior face of the wall in the structure if it can be reasonably ascertained that the exterior face will develop less tensile strength as a result of exposure to the generally more severe environmental condition.

For multi-wythe construction, test specimens of the two outermost wythes can possibly be obtained from a single sample of the full wall cross section by further careful cutting through the collar joint mortar in the laboratory. Since flexural strength is particularly sensitive to adverse conditions in bed joints, special care must be exercised to obtain samples free of any such defects. Visual probing of both surfaces of the wall, preferably with the aid of a magnifying glass, helps detection of surface cracks in bed mortar or at bonded interfaces.

Generally, the thickness of flexure test specimens should be limited to the thickness of a single wythe. The two outermost wythes of a multi-wythe sample should be detached and tested separately in a manner that will induce tension in the fibers corresponding to the two surfaces of the wall. This recommendation is prompted by the following considerations: (1) flexural strength calculations of a single wythe specimen do not require knowledge of an elastic modular ratio, otherwise needed to transform a section of dissimilar masonry units, and (2) reduction of thickness permits the use of considerably smaller test specimens without violating the minimum (h/t) requirements, resulting in a corresponding reduction in the number of cut-out samples and in the number of standard fixtures required for the tests.

Sometimes a situation is encountered that requires headers between two adjacent wythes of the same composition to be included in the samples. In such exceptional cases the two-wythe assembly may be tested as a unit to avoid the necessity of cutting through the header units and causing possible damage to the rest of the sample. Two types of loading options are provided for the flexure tests; specimens may be tested as horizontal beams with the transverse loads applied vertically, or, they may be tested in the vertical position and loaded in a manner that will induce equal and opposite couples at the ends. The general dimensional requirements for the flexure specimens are given below according to loading type.

a. Transversely loaded specimens (Figure 4.1c)

1. Width of specimen (w) should contain not less than 2 whole units in the bottom and top courses, plus the minimum whole number more needed to make width (w) equal to or greater than thickness (t) of specimen.
2. Height (span length) of specimen (h) should contain not less than 2 whole courses plus the minimum whole number more to make (h/t) equal to or greater than 4 (plus allowance for span overhang).
3. The specimen should extend at least 3/4 in beyond the simple supports at each end.

b. Eccentrically loaded specimens (Figure 4.1d)

1. Brick masonry specimens should be at least five courses high and preferably two units wide (figure 4.2). One-unit wide specimens having whole units at the top and at the bottom may be used if it can be demonstrated that test results are not significantly altered by such reduction in width. This may be accomplished by comparison of results obtained from exploratory tests using specimens of both sizes.

2. Concrete block and clay tile masonry specimens should be at least one unit wide and three courses high having whole units at the top and at the bottom.

4.3 Sampling and Transportation

4.3.1 General Considerations

Locations of samples for the preparation of replicate test specimens should be well dispersed with the aim of achieving wide representation. However, each set consisting of 3 specimens for the 3 different types of tests should come from the same vicinity in order to correlate different property values. The necessity for duplicate sets of test specimens of seemingly identical types of masonry walls in different parts of a particular structure should be governed by consideration of the significance of differences in the types of mortar, units, and in the ages of the respective walls. Consideration should be given to the structural safety of the building by confining sample extraction to regions of low stress intensity and by using appropriate shoring of the voided portions of the wall. For a cluster of buildings of approximately the same age, of comparable size and geometric layout, and of the same type of masonry wall construction and sectional configuration, the number of replicate sets of test specimens (1 set = 3 test types x 3 replicates = 9 specimens) should be comparable to the accepted norm for new masonry construction, which is about one set per 5000 sq. ft. of vertical wall area.

Wall samples should be obtained from areas of sound masonry construction without defects. By careful visual inspection of both surfaces it should be ascertained that samples represent wall areas which are free of cracks and of unduly deteriorated mortar. Other defects such as spalled masonry units and broken out mortar should also be avoided. Because of their greater exposure to weathering agents, parapets and other free-standing exterior walls should be particularily suspect of such adverse conditions.

The dimensional requirements of samples taken from composite walls are usually governed by the economic necessity of obtaining the required number of test specimens using the least number of cutout samples. Samples and test specimens of walls need not necessarily be of the same size. Samples should be of the same thickness as the walls from which they are extracted. Any cutting through the collar joints of multiple-wythe samples as called for in the preparation of test specimens should be conducted in the laboratory. Removal of larger samples for the purpose of providing multiple test specimens will reduce the likelihood of possible damage caused by the cutting operations at the site and will permit additional cutting to be conducted in the laboratory under conditions of maximum control. Removal of samples having twice the width (and/or height) of test specimens will be generally governed by handling and transportation requirements and considerations of structural safety of the existing building.

4.3.2 Equipment and Cutting Procedures

Samples should be extracted from the wall with a saw having a diamond or silicon-carbide cutting edge capable of cutting samples without excessive heating or shock. Preferably, the saw should be capable of cutting completely through the wall thickness from one surface. This may be accomplished by using a circular saw mounted on a fixture which can be securely attached to the wall. If cutting must be done from both surfaces of the wall, a positioning pilot hole, off to one side, should be drilled through the wall for referencing the sample outline on both surfaces. As cutting progresses, the sample should be stabilized with wedges or through-the-wall clamps to prevent fracturing. Upon having cut the sample free of the wall, it may be necessary to chisel away some of the surrounding wall to facilitate removal of the sample. Field sampling equipment is available from commercial sources which also provide instructional guidance in their use.

If the sample is to be used as a source for a specimen of smaller thickness, such further extracton should be conducted at a suitably equipped laboratory or stone cutting plant. Prior to cutting through a collar joint, the masonry on both sides of the surface to be cut should be securely held by clamps bearing against the vertical edges normal to the bed joints. Cut surfaces should be trimmed of excess mortar so that specimen dimensions are determined by masonry unit surfaces.

4.3.3 Transportation

Transportation of samples from their source should be accomplished with care to avoid damage by vibration or shock. Samples should be crated for transportation in a vertical position (i.e., as they existed in the wall), fully supported on their bases and clamped or wedged in a direction prependicular to their horizontal (bed) joints. Further aid can be derived by effective use of vibration absorbent materials or devices used to cushion the crates and by lashing the crates to the vehicle. Handling and transporting a sample in a horizontal position should be avoided.

4.4 Preparation of Specimens

4.4.1 General Requirements

Specimens which were obtained from areas of sound masonry construction might have been damaged in the cutting and transportation process. All specimens should be examined visually as carefully as possible to detect cracks, spalls, undercuts or other damage which might be detrimental to test results. If a specimen is found to contain such impairments, it should be replaced with an undamaged one.

Specimens which are considered acceptable for testing should have their load bearing surfaces capped with high-strength gypsum plaster prior to testing. This is done in order to distribute test loads uniformly and to prevent load concentrations which might be caused by projections or by general lack of planeness of the load bearing surface. Guidance in this procedure can be obtained from the following ASTM Designations: E447, Compressive Strength of Masonry Assemblages; C67, Sampling and Testing Brick and Structural Clay Tile; and C140, Sampling and Testing Concrete Masonry Units, whichever relates most appropriately to the type of masonry in the specimen at hand. Nevertheless the following general recommendations are made for preparing all specimens.

Bearing surfaces of these specimens and portions adjoining them should be brushed free of dust and loose particles, then coated with shellac (to prevent absorption of water from plastic gypsum mortar) and be allowed to dry. Casting surfaces to be placed against gypsum mortar should be lightly coated with oil to facilitate their removal. The average thickness of the hardened gypsum cap should not exceed 1/8 in. Since caps cannot be properly patched after setting, imperfect caps should be removed and replaced with new ones but without damaging the specimen. Caps should be made of special high-strength gypsum mixed with just enough water to form an easily troweled paste. When ready for test, the capping gypsum should develop a compressive strength of at least 5000 psi, tested as 2-in cubes aged in the same manner as the caps. Proper blending of gypsum with a minimal amount of water is more easily achieved by slowly sprinkling the gypsum into water unaccompanied by stirring.

4.4.2 Compression Specimens (Figure 4.1a)

Compression specimens should be capped over their complete top and bottom bearing surfaces. The bottom surface is capped by lifting the specimen and pressing it down into plastic gypsum mortar spread on a level, oiled casting surface. Removal of extruded mortar will assist in checking thickness of the cap. The weight of the specimen may cause it to sink instead of being pressed down. It is important that the vertical axis of the specimen be kept perpendicular to the capping surface and that the operation be done in a single motion. Tilting and rocking adjustments of the specimen are likely to destroy full bearing of the cap on the specimen. If the specimen is too heavy for lifting by hand, it can be clamped or bolted between two horizontal 2 by 4's nominal, and handled mechanically (e.g. by hoist or fork lift).

After the cap has hardened enough to permit separation of the casting surface and capped specimen without damage to the cap, the capping operation should be repeated for the top bearing surface. It is recommended that this be done by overturning the specimen and capping the opposite bearing surface in the same manner. If, however, it is impractical or hazardous to the specimen to attempt overturning, the following alternative method is recommended. Without waiting for the bottom cap to be uncovered, plastic gypsum mortar may be spread over the top bearing surface in excess of the desired thickness. The second oiled capping plate can then be carefully lowered onto the plastic gypsum with the aid of a carpenter's level. This top capping plate should be sufficiently heavy to allow gravity to assist in its placement; additional weight should be superimposed simultaneously if necessary. The same precautions against any disturbing adjustments which might destroy the bearing contact between specimen, gypsum and plate must be observed. Caps are to be formed approximately parallel to each other and perpendicular to the specimen axis. After caps are sufficiently hardened to permit handling of the specimen, capping plates should be removed and the specimen temporarily supported in a way which permits circulation of air to assist drying of the caps.

4.4.3 Shear Specimens (Figure 4.1b)

Shear specimens (which are cut square - cf. Section 4.2.2) should be prepared for testing in diagonal compression. This requires preparation of 2 diagonally opposite corners to serve as bearing points for the test load. The compressive in-plane load is directed, through steel loading shoes, along the diagonal of the specimen which joins the two prepared load bearing corners. This manner of loading eliminates the need for a separate hold-down force to prevent rotation of the specimen (cf. ASTM Method E-72).

The steel loading shoes may be one of the two types shown in Figure 4.3. The open end type, (a), lends itself to use in laterally placed pairs at both corners of a specimen to provide bearing over the entire thickness of larger specimens without fabricating overly large shoes. Because of the wide range of specimen sizes that might be encountered, no specific dimensions are given, but the length of bearing of the shoe is especially important. The length of shoe bearing along the perimeter of the specimen should be enough to prevent excessive bearing stress. Experience indicates that a shoe bearing length, on one side, equal to approximately 1/8-th the length of the side of the specimen is satisfactory. If the shoes are longer than needed for the specimen at hand, the recommended bearing length can be achieved by using temporary steel liner plates of the shorter length. Shoes should be of welded construction fabricated from 1/2-in (or heavier) steel plate, designed with sufficient braces to prevent distortion under load.

The position of the specimen for capping the loading corners and for testing, is obtained by rotating it (in-plane) through 45° from its natural orientation in the building wall. With the diagonal which joins the loading corners maintained perpendicular to a level surface, the loading corners should be bedded in high-strength gypsum in the loading shoes. Hollow cores of the loading corner masonry units which will rest within the shoes should previously be filled solid with the capping gypsum. These operations generally follow the manner described for capping compression specimens (Section 4.4.2) using the steel shoes instead of the plane capping plates. Care should be exercised to prevent the gypsum from constraining the face surfaces of the corners in the loading shoes.

4.4.4 Transversely Loaded Flexure Specimens (Figure 4.1c)

Flexure specimens should be prepared for testing with the specimen's vertical axis in a horizontal position. In order to reduce load concentrations and to provide hard and smooth bearing surfaces for load and reaction rollers, cold rolled steel bar stock should be embedded flatwise in high-strength gypsum plaster on the specimen at the required locations. The length of the bars should be equal to the specimen width and it is recommended that a bar cross section of 1/2" x 1-1/2" be used (for adaptability to specimens of most masonry unit sizes).

In order to avoid the necessity for specially fabricated articulated loading equipment which compensates for lack of planeness in the specimen, the following procedure is recommended: On a plane level surface place two of the bars directly opposite and parallel to each other and spaced at a center to center distance equal to the span length chosen for the specimen. The upper surfaces of the bars should contain several keying recesses (e.g. shallow, slightly inclined drill holes, approx. 1/2" diam.) to provide mechanical bond between the bars and bedding plaster while assembling the test. After oiling the upper surfaces and recesses of the bars (to facilitate cleaning for re-use) place a suitable quantity of plastic gypsum on top of the bars. Lower the specimen, centered over the bars in the horizontal test position, and with a single motion imbed the specimen in the gypsum firmly against the bars as nearly level as possible. It may be necessary to gain more space (for manual or mechanical handling) by placing the bars on separate supports above the level reference surface beforehand. If this is done, it is important that the separate elevators maintain and transfer the level reference surface reliably. Equal steel blocks are recommended for this purpose. When the gypsum has hardened sufficiently, carefully overturn the specimen, with bars attached, for similar preparation of the opposite surface.

The procedure to be followed is the same except that the bars are to be spaced at a distance equal to 1/2 the span length of the specimen. This will result in the bars being attached to the opposite surface at the 1/4-point test loading positions of the specimen span length. The specimen must be carefully turned over once more to its original horizontal position to be made ready for test. Each pair of bars (and, by attachment, the respective surface of the specimen) will then have bearing surfaces in a single plane, thus compensating for any warpage in the specimen which might be detrimental to test results. The upper and lower planes of bearing are not necessarily parallel; this will be offset by a spherically seated loading head discussed in Section 4.5.

4.4.5 Eccentrically Loaded Flexure Specimens (Figure 4.1d)

Since loading is transmitted to these specimens through the side clamping action of the brackets gypsum capping is not necessary.

4.5 Test Apparatus

4.5.1 Testing Machine

The testing machine used for exerting and measuring loads applied to the specimens should be of sufficient load capacity and conform to the requirements of Section 16, 17 and 18 of ASTM Designation E-4, Verification of

Testing Machines. It must be power operated in order to apply load continuously, rather than intermittently, and should have an adjustable loading rate control which will provide the rate required in Section 4.6. The space provided in the machine for specimens should be large enough to accommodate test assemblies with attached displacement gages in a readable position if gages must be read visually. Alternative remote observations are discussed in Section 4.5.2.

The upper crosshead of the testing machine must be equipped with a spherically seated bearing block which can be adjusted so that its contact surface is made parallel to the upper load bearing surface of the test assembly at the beginning of the test. The spherical seat should be lockable to prevent slipping when used on unstable test assemblies (e.g., those requiring the use of rollers). Selection of diameter of the spherical bearing block, and of size of auxiliary loading equipment, (such as bearing plates, rollers, I-beams, etc.) should be made on the basis of accepted engineering design. Undue deformation of loading equipment should be avoided to prevent undesirable loading conditions which would be detrimental to test results.

4.5.2 Deformation Gages

Instruments for measuring deformation of masonry under load are used in the compression and shear tests. These measurements are used in calculations discussed in Section 4.7.4 to determine the elastic moduli of the masonry. Contraction or extension of the masonry over a given gage length should be measured with a dial micrometer having dial graduations of 0.001 inch or a Linear Variable Differential Transformer (LVDT) having an equivalent least count. The spindle-actuated rack and pinion type dial micrometer is read visually; the transformer coil, silding core type of LVDT is used in combination with auxiliary electronic equipment for read-out. LVDT readings can be made remotely with a visually read voltmeter (preferably digital) or with automatic recording equipment if available. Gage length deformations should be measured and recorded to the nearest estimated 0.001 inch (i.e. 1/10 the least count).

Mounting of instruments on specimens is done expediently with supporting brackets attached to the masonry units by hot-melt adhesive. Adhesive in small cartridge form is dispensed easily by electrically heated pistol style applicators. Mounting hardware of aluminum, rolled or extruded sections and tubing are recommended for lightness of weight.

To illustrate, deformation in a given gage length can be measured by a dial micrometer gage attached to the specimen by a bracket at one end of the gage length (Figures 4.1a and b). The detecting spindle of the gage should be parallel to and point toward the opposite end of the surface gage-line. At this same opposite end, a light, stiff tube or rod is pin-mounted at one end with the swivel pin perpendicular to the mounting surface. The tube is positioned parallel to the surface gage line with the free end of the tube in contact with the end of the gage spindle (connected if necessary; details are discussed for particular tests in Section 4.6). It is usually necessary to provide a slide guide bracket for the tube at the end away from the swivel.

If, instead, an LVDT is used in the above illustration, it is recommended that the transformer coil be mounted on the tube at the end away from the swivel; and that one end of the sliding core be attached to a bracket mounted at the location which had been occupied by the dial gage.

Loose fits between gage assembly parts can be overcome, and contact maintained, by stretched rubber bands.

4.6 Testing Procedures

4.6.1 Compression Test (Figure 4.1a)

Specimens should be tested with the centroid of their bearing surfaces aligned vertically with the center of thrust of the spherically seated bearing block. If necessary, a top bearing plate and other hardware may be used to uniformly distribute the load from the spherically seated loading head.

Four deformation gages of the type described in Section 4.5 should be attached for measuring axial contraction of the specimen. These compressometers are mounted over four corresponding vertical gage lengths which are each near a different corner of the specimen on the faces that were parallel to the wall surface. It is recommended that the four equal gage lengths be chosen to extend between the midheights of the bottom and top courses of masonry units. In this way deformations can be observed occurring over an equal whole number of courses and joints.

When gage assemblies are mounted over vertical gage lengths as in this test, it is recommended that the dial micrometer (or LVDT core) be bracket-mounted at the lower end (Figure 4.1). The tube (or rod) extension of the gage, thus being suspended from the upper end, will tend to be more stable in a hanging position.

The gage length and initial readings of the gages (with no load on the specimen) should be recorded. As the spherically seated block is brought to bear on the test assembly, the moving portion should be slightly rotated to help obtain uniform seating.

Load should be applied to the specimen continuously at a uniform rate of from 1/3 to 1/4-th the estimated expected maximum load per minute. Loading may be stopped briefly at equal load increments to record load and gage readings. Observation intervals should be at a convenient load increment value of approximatley, but no more then, 1/15 the expected maximum load.

This will provide approximately 10 sets of observations to derive 2/3 of the load-deformation relationship. At 2/3 the expected maximum load (or sooner if sudden failure appears imminent) the deforamtion gages should be removed and the load continuously increased at the specified rate until the maximum load that can be applied to the specimen is determined.

4.6.2 Shear Test (Figure 4.1b)

Since shear tests are to be conducted by application of compressive load along a vertically positioned diagonal axis of the specimen, such tests should follow the same loading procedure (where applicable) as described for the compression test (Section 4.6.1).

With the specimen properly positioned in the testing machine, four deformation gages, of the type described in Section 4.5.2, should be attached (2 on each face) for measuring the average horizontal extension and vertical contraction along the diagonals of the specimen under load (cf. Figure 4.1b). The four deformation gages are mounted over the gage lengths on the four face diagonals of the specimen. These gage lengths should be equal and symmetrical about the intersections of the face diagonals. Their length is determined by the intersection of the diagonals with the mid-heights of the uppermost and lowermost courses of the specimen; or, that same length reduced by a reasonable clearance from the loading shoes to avoid local disturbance of the mounting brackets.

If a composite specimen must be tested as a unit, the necessary transformed area calculations for location of load application points at the centroid of the section can be made using the values of elastic moduli obtained from preceeding compression tests and equation (3.9). Proper positioning of the specimen in the testing machine will minimize the effect of out-of-plane flexure on test results. Large differences in deformations recorded by the two vertically-oriented diagonal gages at opposite faces of the specimen are an indication of improper positioning.

4.6.3 Flexure Test, Transverse Loading (Figure 4.1c)

With the surface selected to receive tensile stress placed underneath (Section 4.4.4), the specimen should be positioned horizontally in the testing machine so that the center of thrust of the spherically seated loading head is aligned with the centroid of the load bearing plates previously attached to the specimen.

The specimen should be supported across its full width on two rollers, one placed under the mid-width of each bottom plate to establish the span length. Temporary chocks may be necessary to prevent rolling. Similarly, a roller should be placed on each plate over the 1/4-span loading positions, together with any distributing hardware (e.g., I-beams) needed to transfer load applied by the testing machine symmetrically and uniformly to the two 1/4-span loading rollers. Since flexural test loads will be relatively small, weight of equipment superimposed on the specimen should be measured for later addition to observed applied loads.

By hand rotation, the spherically seated loading head should be adjusted as nearly parallel as possible to the contact surface of the loading hardware. Contact should be made gently by applying a small load with the testing machine and locking the spherical block. The load should be continuously increased at a uniform rate until the maximum load the specimen can withstand is determined. Rate of loading should be sufficient to cause failure in 1 to 2 minutes. No deformation gages are used in this test.

4.6.4 Flexure Test, Eccentric Loading (Figures 4.4, 4.5, 4.6)

This alternate test method to determine flexure bond strength of existing masonry walls is essentially patterned after the testing procedure for concrete masonry block assemblies described in ASTM Desingation E149-66, Standard Method of Test for Bond Strength of Mortar to Masonry Units. The adaptation of the ASTM procedure to test sampled specimens of concrete block, and brick as well as other types of masonry construction requires the fabrication and use of special apparatus consisting of brackets for clamping the specimens at top and bottom and lever arm attachments for application of eccentric loads.

A test set-up, using the ASTM E149 apparatus for testing concrete block masonry specimens is shown in figure 4.4 and shop drawings of the fixtures appear in ASTM E149. This equipment is dimensioned to accommodate the size of specimens made of large masonry units such as concrete block and clay tile.

The primary intent of the test method is to induce equal and opposite couples at the ends of the specimen, flexing it in single curvature. By making the lever arms sufficiently long the effect of axial load is kept to a minimum and a condition of pure flexure is closely simulated. For this purpose, the length of the lever arm should be designed in a manner that would keep the stress which is attributable to total axial load (applied load, plus appropriate weight of hardware and specimen) within 30 percent of the maximum calculated flexural stress attributable to the total end moment (the product of applied load and lever arm, plus the product of the weight of upper hardware attachment and the distance of its centroid from the centroidal axis of the cross section of the test specimen).

An apparatus similar to that used for ASTM E149 testing has been introduced by the Brick Institute of America, BIA (previously Structural Clay Products Institute, SCPI), for testing small brick masonry specimens in flexure. For background information reference is made to the following publications:

1. Research Report Number 9, Compressive, Transverse and Racking Strength Tests of Four-Inch Brick Walls, Structural Clay Products Research Foundation, Geneva, Illinois, August 1965.
2. Progress Report No. 1, Small Scale Testing, Structural Clay Products Research Foundation, Geneva, Illinois, October 1964.

A typical test set-up using a six-course stacked bond brick specimen is shown in figure 4.5. Figure 4.6 shows photographs of the top and bottom bracket assemblies of the BIA equipment which was used in a recent experimental program conducted at the National Bureau of Standards. This type of equipment can be fabricated to accomodate one-unit-wide specimens as in figures 4.5 and 4.6, and two-unit-wide specimens as in figure 5.5 of Report No. 9 cited above.

In general, testing procedures for masonry specimens should be in accordance with those described in ASTM E149. Load should be applied at a rate sufficient to cause failure in 1 to 2 minutes. No deformation gages are used in this test.

4.7 Interpretation

Sections 4.7.1 to 4.7.4 discuss procedures for deriving masonry strength properties from individual test results, including use of load-deformation histories to calculate the elastic constants. In section 4.7.4 these values are further modified to account for variability and other inherent factors. For the purpose of correlating calculations of sectional properties of test specimens and of walls, frequent reference is made to Section 3.3 in which length of wall (L) corresponds to width of specimen (w). For single-wythe specimens or for multi-wythe specimens of the same composition, (A) is the horizontal cross sectional area of the specimen calculated on a gross or net basis as prescribed in Section 3.3.

4.7.1 Compression

For a specimen of single wythe construction, or multiple wythes of the same material, tested with the load over the geometric centroid

$$f'_m = \frac{P_u}{A} \tag{4.1}$$

where:

- f'_m = compressive strength
- P_u = compressive load at failure
- A = appropriate (gross or net) cross sectional area

Intermediate values of stress (f_m), at observed test loads $P < P_u$, are calculated as

$$f_m = \frac{P}{A} \tag{4.2}$$

Corresponding values of average compressive strain measured by 4 compressometers (cf. Section 4.6.1) are calculated as

$$\varepsilon_m = \frac{\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4}{4g} \tag{4.3}$$

where:

- ε_m = average compressive strain
- Δ_i = change in length of i-th gage
- g = compressometer gage length

Values of compressive stress (f_m) and corresponding strain (ε_m) are plotted to develop a stress-strain diagram (Figure 4.7). At a level of ($f'_m/2$), a value of the tangent modulus of elasticity (E) is graphically determined. This value of (E), modified according to the provisions of section 4.7.5, will be used in the transformed area calculations, where necessary, and in seismic resistance evaluations.

4.7.2 Shear

Barring premature compressive crushing of the masonry at the diagonally loaded corners, shear specimens will fail along the loaded diagonal by shear cracking in the mortar joints, or by tensile cracking of the units, or by a combination of both. These different types of cracking patterns will be primarily governed by the relative strengths of the masonry constituents. High-strength mortar used with low-strength masonry units will cause cracking to occur through the masonry units while low-strength mortar with high-strength masonry units will produce cracking along the mortar joints.

For a shear specimen of single wythe construction, or multiple wythes of the same material, tested with the diagonal compressive load in the centroidal plane of the specimen thickness (t), the shear strength of masonry corresponding to the mode of failure characterized by diagonal cracking through the mortar is calculated as

$$f'_v = \frac{0.707\ P_u}{A}\ (1-u) \tag{4.4}$$

where:

f'_v = masonry shear strength

P_u = diagonal compressive load at failure

A = appropriate (gross or net) cross sectional area parallel to bed joints

u = coefficient representing influence of compression of shear strength

The results of various racking tests conducted at NBS indicate a value of coefficient (u) which varies between 0.3 and 0.5. The use of the lower value in eq. (4.4) will give

$$f'_v = 0.5\ \frac{P_u}{A} \tag{4.5}$$

This simplified equation should be used to calculate the shear strength of masonry from test results regardless of the observed failure mode. It is noted that the use of the smaller frictional coefficient overestimates shear capacity at zero axial load but conservatively predicts this capacity under a combination of axial load and diagonal compression; a situation most frequently encountered in practice. It is further noted that in cases where specimens fail by diagonal cracking through the masonry units, eq. (4.5) conservatively predicts a nominal shear capacity which is less than the actual shear capacity by an indeterminate amount.

Intermediate values of average shear stress (f_v) at observed test loads (P) are calculated as

$$f_v = 0.707 \frac{P}{A} \tag{4.6}$$

Values of average shear strain which correspond to the above stresses and which are determined by means of the four diagonal deformation gages of equal length (cf. Section 4.6.2) are calculated as

$$\gamma = \frac{\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4}{2g} \tag{4.7}$$

where:

γ = average shear strain

Δ_i = change (+ or −) in gage length no. i

g = length of one gage

It should be noted that the absolute values of length change are used in the above expression although, under load, the two vertical gage lengths (Figure 4.1b) are contracted and the two horizontal gage lengths are extended. the 1/2 factor, is used in eq. (4.7) to obtain average strain on 2 opposite faces of the specimen since approximate average shear strain on one face is given by

$$\gamma = \frac{\Delta_V + \Delta_H}{g} \tag{4.8}$$

where subscripts (V) and (H) designate vertical and horizontal directions, respectively. It is also emphasized that the 4 gages must be of equal length for these calculations. Values of shear stress (f_v) and corresponding strain (γ) are plotted to obtain a stress-strain diagram (similar to that for compression in Figure 4.7). At a level of ($f_v'/2$), a value of tangent modulus of rigidity (G) is determined graphically. This value of (G), modified in accordance with the provisions of section 4.7.5, will be used in the seismic resistance evaluations. For composite specimens eqs. (4.5) to (4.8) still apply except that (A) should be replaced by the area of the transformed section (A_t) given by eq. (3.9).

4.7.3 Flexure, Transverse Loading

Calculations for flexural tests involve only the determination of the ultimate flexural (tensile) strength by the equation

$$f_t' = \frac{P_u}{2} \cdot \frac{h}{4} \cdot \frac{c}{I} \tag{4.9}$$

where:

f_t' = flexural tensile strength

P_u = total test load at failure, (Figure 4.1c)

h = flexural test span length

c = distance from centroidal axis to outermost tensile surface

I = net moment of inertia of mortar bed joint about centroid of cross section

As mentioned in section 3.3, it is to be emphasized that in flexure of hollow masonry construction, only the net mortar contact area is to be considered effective.

4.7.4 Flexure, Eccentric Loading

Taking all forces into account, the tensile strength in flexure should be calculated from the following expression:

$$f_t' = (XP_u + X_b W_b) \cdot \frac{c}{I} - \frac{(P_u + W_b + W_m)}{A} \tag{4.10}$$

where:

f_t' = tensile strength in flexure

P_u = applied load at failure

W_b = weight of top bracket assembly

W_m = weight of the portion of masonry specimen above the observed plane of cracking

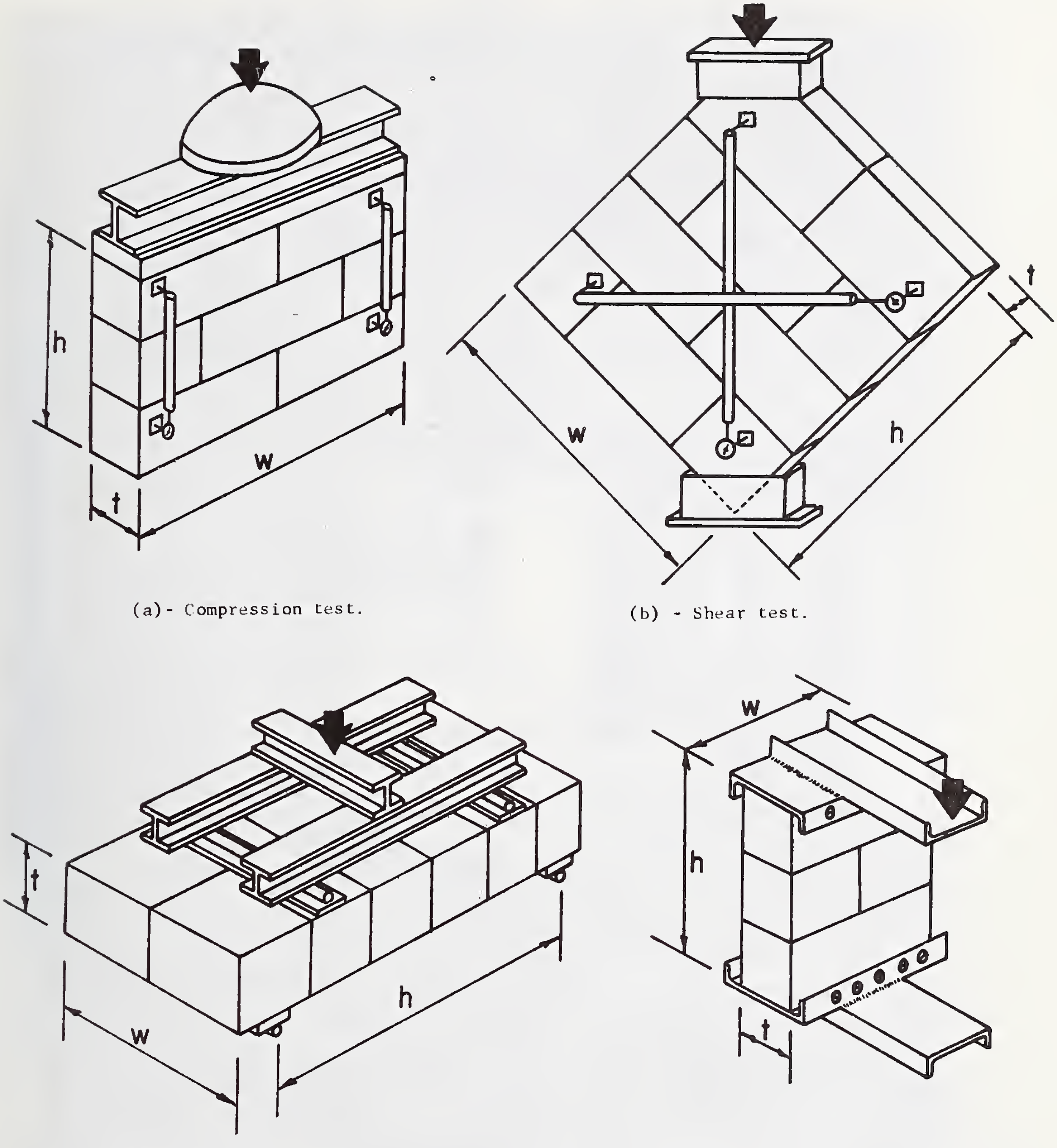
X = distance from centroid of cross section to point of application of load P_u

X_b = distance from centroid of cross section to center of gravity of top bracket assembly

A = net cross-sectional area of mortar bed joint

I = net moment of inertia of mortar bed joint about centroid of cross section

c = distance from centroid of cross section to extreme fiber.



(a)- Compression test.

(b) - Shear test.

(c) - Flexure test (transverse loading).

(d) - Flexure test (eccentric loading)

Figure 4.1 - Methods of testing masonry specimens.

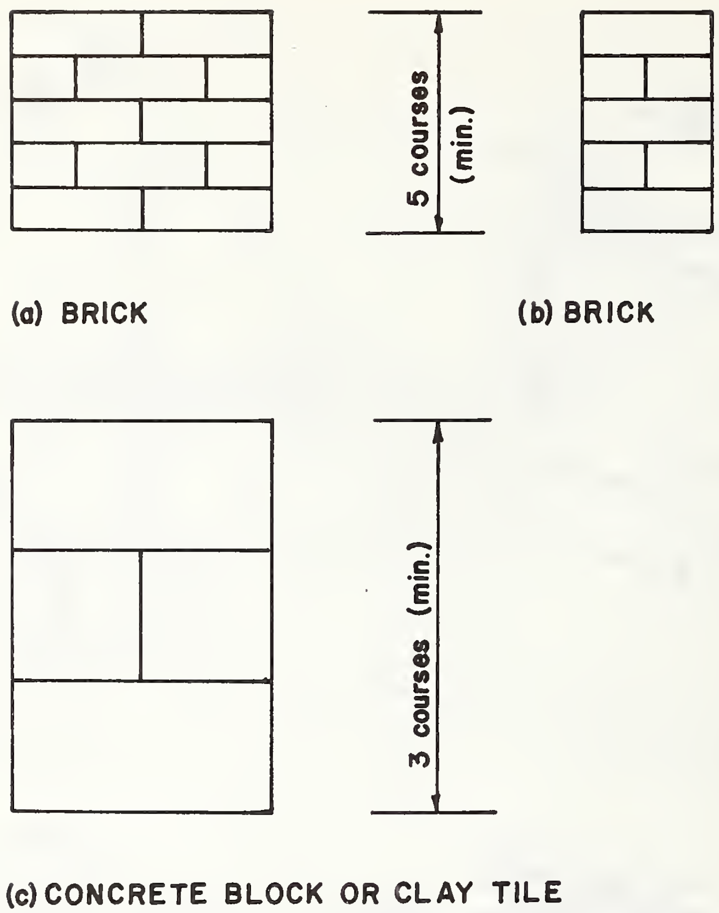


Figure 4.2 - Flexure test specimens for eccentric loading.

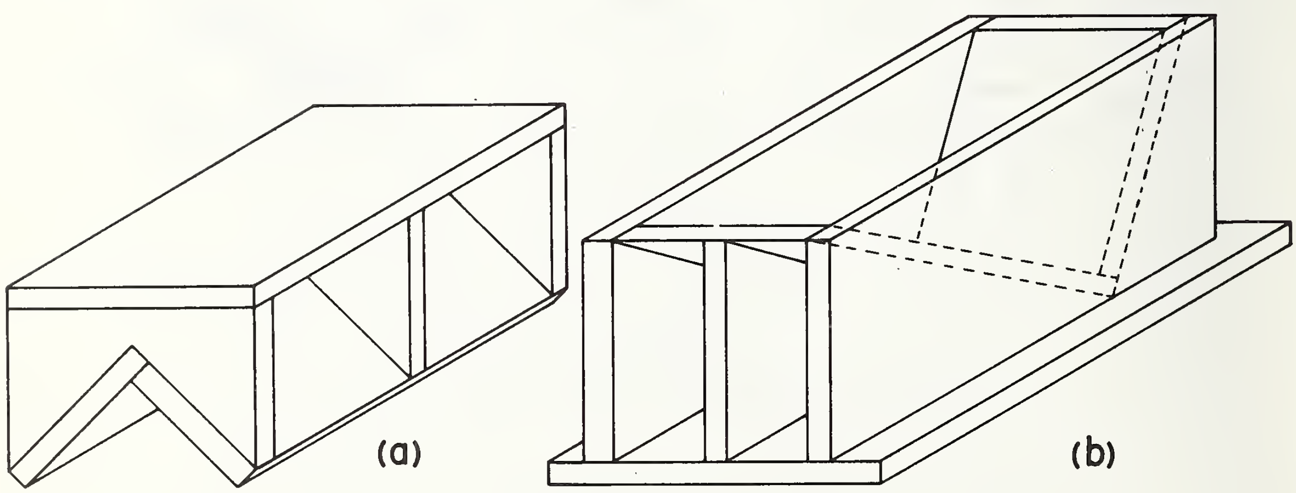


Figure 4.3 - Shear test loading shoes.

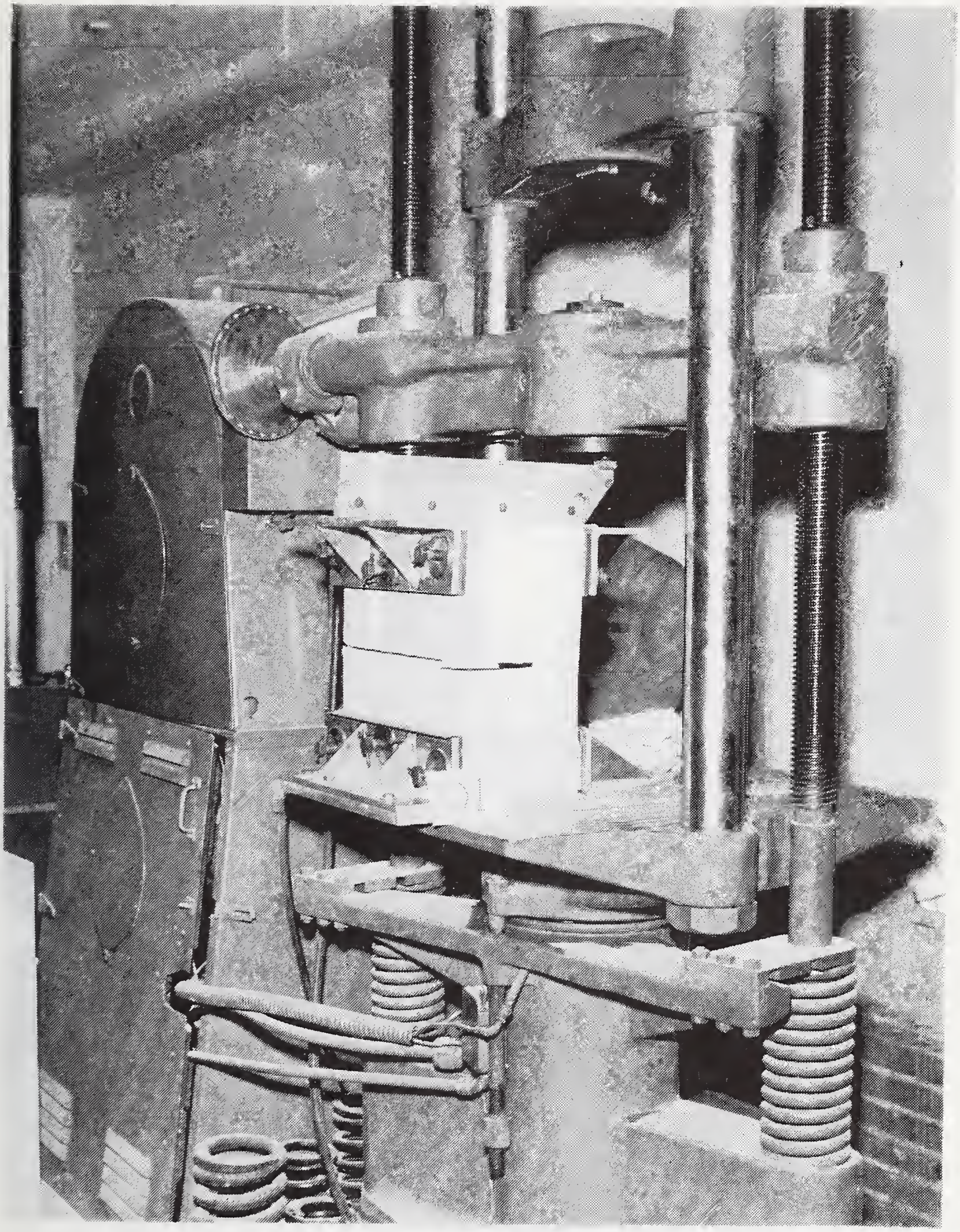


Figure 4.4 - Eccentrically loaded flexure test setup for concrete block prisms.

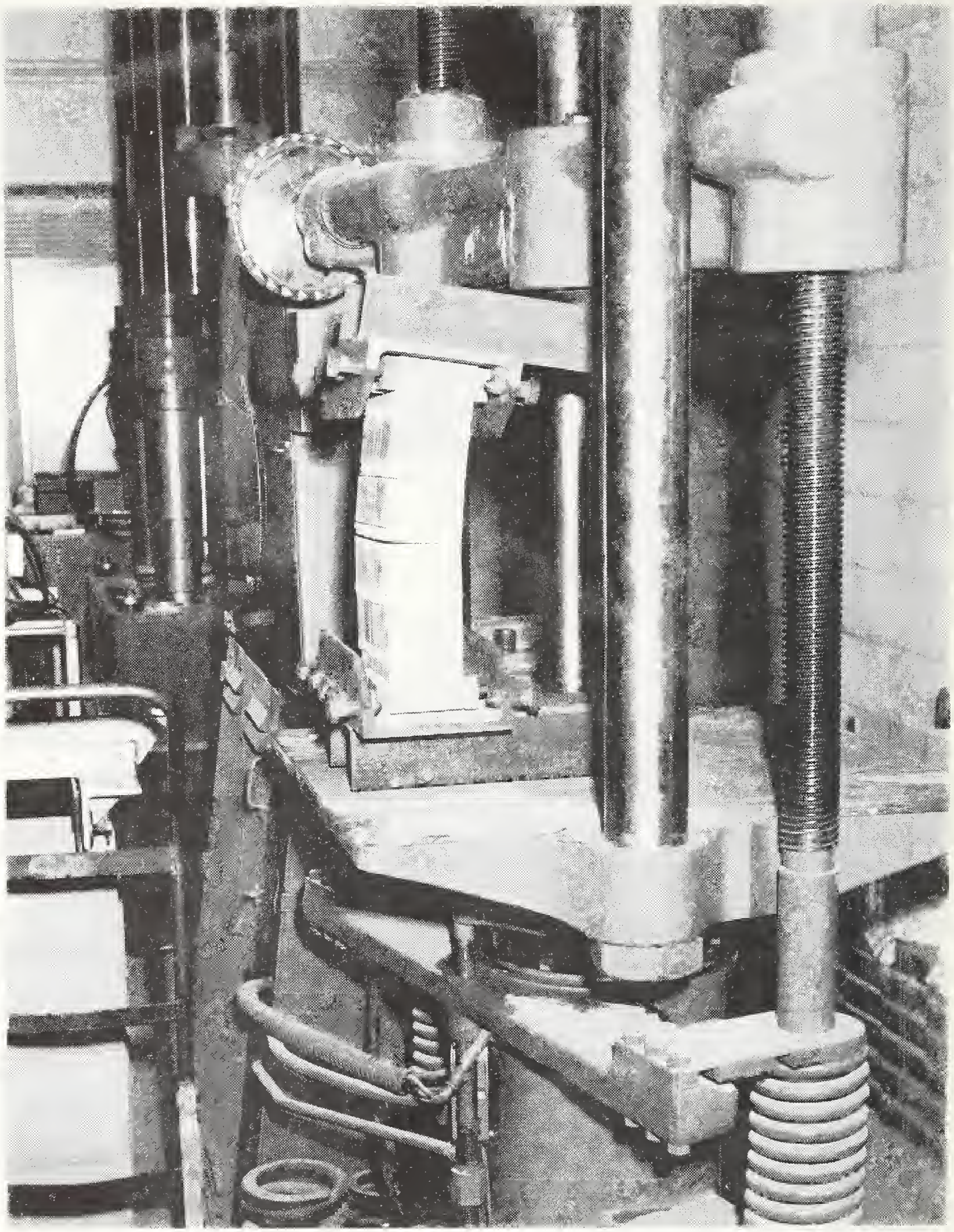
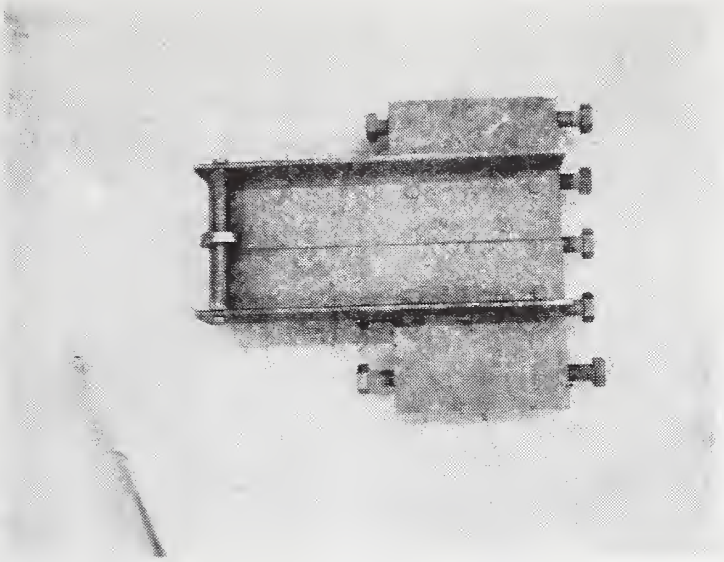
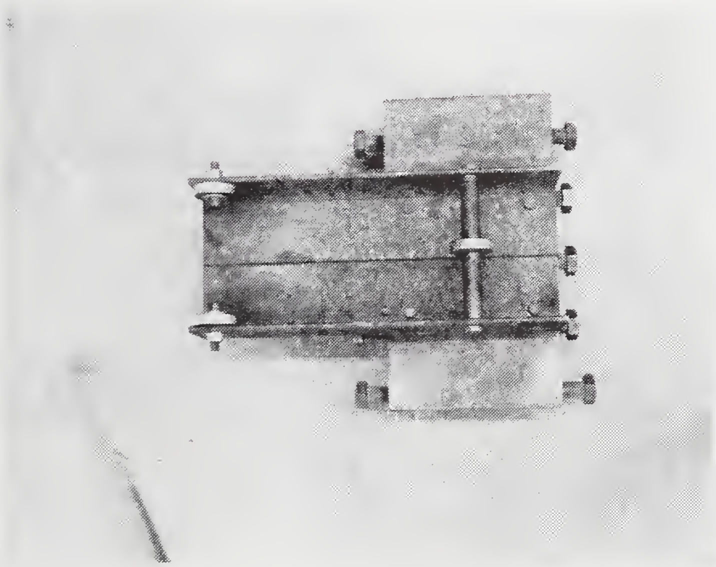


Figure 4.5 - Eccentrically loaded flexure test setup for brick prisms.



UPPER BRACKET



LOWER BRACKET

Figure 4.6 - End brackets for eccentrically loaded flexure test of brick prisms.

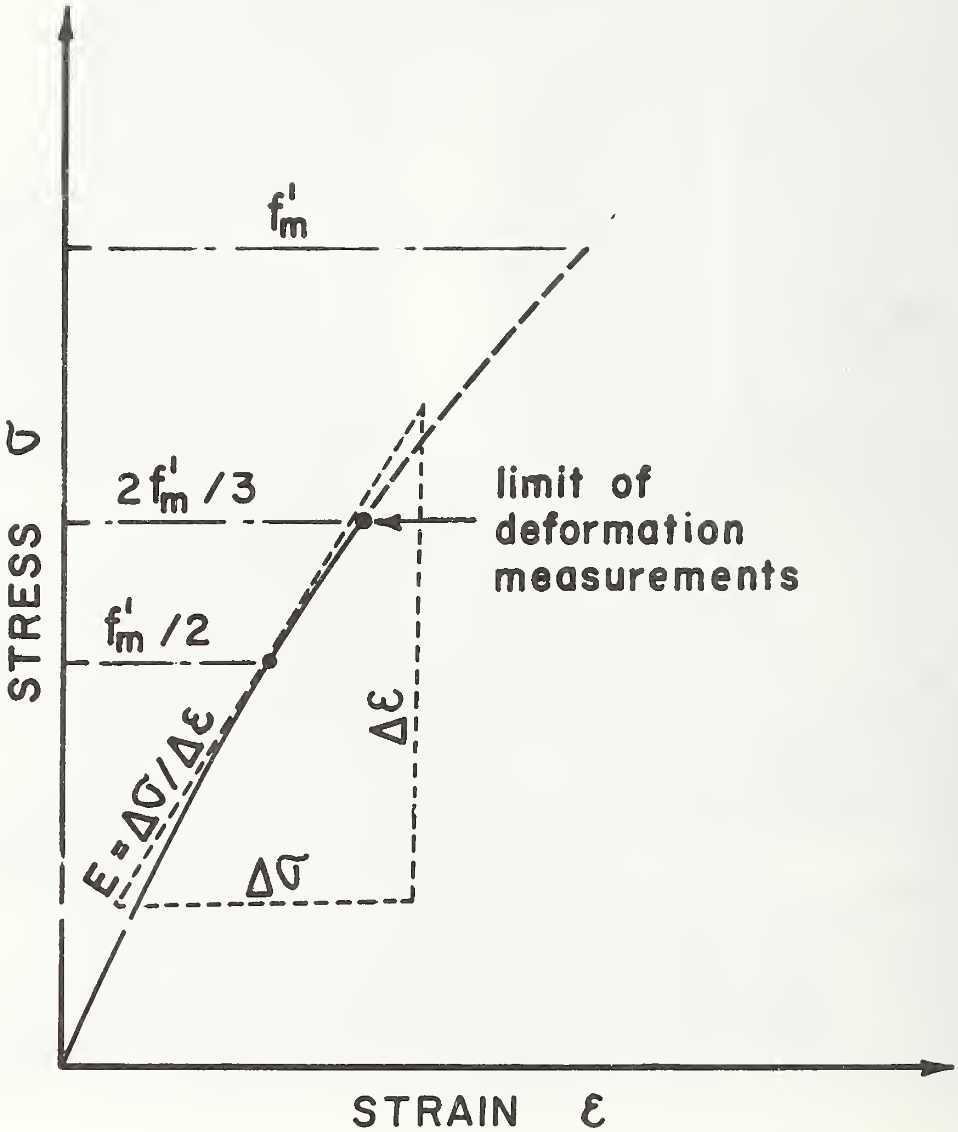


Figure 4.7 - Stress-strain diagram.

4.7.5 Capacity Reduction

All strength and stiffness values derived from direct tests of sampled specimens in accordance with the procedures discussed in Section 4 should be reduced to account for the aggregate effect on masonry properties of: (1) variability, (2) specimen size and (3) past earthquake exposure. The reductions should be made as follows:

(a) Strength (f'_m, f'_v, or f'_t)

$$(0 \leq v \leq 0.50): \quad x' = \frac{\bar{x}}{3}$$

$$(v > 0.50): \quad x' = \frac{\bar{x}}{3}\,[1-1.5(v - 0.50)] \qquad (4.11)$$

(b) Stiffness (E or G)

$$x' = \frac{2\bar{x}}{3} \qquad (4.12)$$

where: $\bar{x}$ = arithmetic mean of a strength or stiffness property derived from replicate tests

x' = maximum strength or stiffness property to be used in structural investigation of masonry buildings

v = coefficient of variation expressed as a decimal fraction ($0 \leq v \leq 1.00$) and calculated from

$$v = \frac{1}{\bar{x}} \quad \frac{\Sigma\,(x - \bar{x})^2}{n - 1} \qquad (4.13)$$

where: x = an individual measurement

n = total number of replicate specimens or measurements of a property

4.8 Implementation

4.8.1 Sampling and Testing

The extent of implementation of the testing procedures discussed in the foregoing sections will depend on the feasibility of preparing undamaged specimens from existing masonry construction and on the relative importance of a specific test information in the seismic analysis of a particular building. In both instances, the course of action to be taken will depend on prior knowledge about the structure at hand and the experience gained while field sampling is in progress. Nonetheless, some general guidelines are provided for situations that can be anticipated.

Compression testing requirements may be relaxed on the basis of an initial seismic investigation which will indicate that the masonry in the structure under consideration has an adequate margin of safety against failure by compression under the incremental loads induced by lateral and vertical acceleration. This situation is likely to be encountered in buildings in which the ratio of height to least horizontal dimension is small.

The scope of compression tests may be relaxed by reducing the number of replicate tests to that which will be required to make an independent assessment of the modulus of elasticity (E) on the basis of load-deformation measurements. Such a reduction will be justified because, in general, the elastic constants need not be evaluated with the same degree of accuracy as the strength properties. It is noted, for instance, that the modulus of elasticity prescribed by various masonry codes and standards (UBC, BIA and NCMA) is derived from the compressive strength (f'_m), rather than from independent test measurements.

The existence of tensile or bond strength in the bed joint mortar of masonry wall elements of a building is the necessary precondition for the flexure tests. In addition, the bond strength should be high enough to permit the extraction of a sufficient number of undamaged test specimens without two high an attrition rate. In existing masonry buildings, initial bond, if any, might have been partially or wholly destroyed by exposure to past disaster loads and as a result of a progressive deterioration of the mortar caused by environmental agents. A typical example of the latter case is the degradation of mortar types with high lime content (such as were commonly used in old masonry buildings) through exposure to moisture.

In some exceptional instances, it might be possible to detect poor mortar condition by visual inspection or by manual probing in the field and thus eliminate the need for flexure tests. It is more likely, however, that such decisions will have to be deferred until field sampling indicates a considerable level of damage in the samples extracted from the walls. As an approximate guideline, it is suggested that an attrition rate of 50 percent or greater, precluding accidental attrition, be used as grounds for suspension of further sampling. In such an event, no tensile strength should be assigned to the masonry under consideration ($f'_t = 0$).

These guidelines may also be used to determine the feasibility of conducting the shear tests of the masonry of a particular structure. In the event that such shear tests are eliminated, no base shear strength should be assigned to the masonry under consideration ($f'_v = 0$). In this case, the average shear strength (v'), of masonry under axial compression is evaluated using the second term on the right hand side of equation (5.18).

4.8.2 Field Inspection

A documentation of the actual condition of the masonry in an existing building could provide useful and valuable information for seismic evaluation. Such information may be conveniently compiled by the field crew assigned to the tasks related to the removal of wall samples for testing. A check list will provide guidance on the relevant items to be considered in the survey. Description of cracks, spalls and other visually identifiable defects in the masonry may be effectively related by appropriate sketches on elevation profiles and a list of correspondingly numbered commentaries. The use of contract drawings, whenever available, will be most expedient for such purposes. The contract documents may also be helpful in identifying the types of mortar and masonry used and other pertinent data on masonry specifications.

5. Strength of Masonry Walls

5.1 Introduction

The procedure for predicting the racking strength of masonry walls described in Section 5.3 is based primarily upon the findings of a series of racking tests conducted at the National Bureau of Standards [50]. A major objective of these experiments was to study the influence of compressive loads on in-plane shear capacity of walls. A total of 73 full-scale walls of 8-in hollow concrete block and 4-in brick masonry were tested. In addition, a large number of small-scale specimens of similar construction were tested to investigate the correlation between wall strength and small specimen strength. The sizes of full-scale specimens and the types of racking tests are displayed in Figure 5.3. The specimens and test setups for some of the racking specimens are discussed in Sections C.5 and C.6 of Appendix C.

In another series of tests conducted recently at the National Bureau of Standards, a considerable amount of data was compiled on out-of-plane (vertical) flexural strength of masonry walls of various types of construction in the presence of vertical compressive loads. The test results were synthesized to develop analytical procedures for the prediction of wall strength under compression and flexure. These experimental findings and the analytical formulations for the proposed methodology, related in a series of separate publications [52, 64, 65, 75], constitute the basis of the flexure-compression interaction relations discussed in Section 5.2.

5.2 Flexure-Compression Interaction

The moment capacity of short masonry walls under compression and out-of-plane bending can be reasonably predicted from equilibrium considerations of the cracked or uncracked section assuming a linear stress distribution on the cross section at failure. Using this approach, Yokel et al. [65] proposed approximate analytical procedures for evaluating masonry strength which was found to be in good agreement with experimental results. The validity of this approach was further verified by a substantial body of related data obtained from subsequent tests [52, 64, 75]. The governing relationships are given here without derivation as an aid to the investigation of masonry walls under combined axial load and out-of-plane flexure.

Assuming masonry has no tensile strength, the approximate flexure-compression interaction relationships for the general case of an asymmetrical transformed section (masonry units of dissimilar composition) as shown in figure 5.1, are given by the following equations:

For a cracked section

$$
\begin{aligned}
M_{e1} &= Pc_1 \left(1 - g_1 \frac{P}{aP_o}\right) \\
M_{e2} &= Pc_2 \left(1 - g_2 \frac{P}{aP_o}\right) \\
g_1 &= \frac{aP_o}{P_{k1}} \left(1 - \frac{e_{k1}}{c_1}\right) \\
g_2 &= \frac{aP_o}{P_{k2}} \left(1 - \frac{e_{k2}}{c_2}\right) \qquad (5.1) \\
P_{k1} &= \frac{aP_o}{1 + \frac{c_1}{c_2}} \\
P_{k2} &= \frac{aP_o}{1 + \frac{c_2}{c_1}} \\
e_{k1} &= \frac{I}{Ac_2} \\
e_{k2} &= \frac{I}{Ac_1} \\
P_o &= rAf'_m
\end{aligned}
$$

where I = moment of inertia of net transformed section about its centroidal axis

A = net area of transformed section

c_1, c_2 = distances from centroidal axis of transformed section to outermost fibers in maximum compression

f'_m = axial compressive strength of the weaker masonry element in the composite section

P_o = axial load capacity of masonry

P = compressive force on the cross section

e_{k1}, e_{k2} = kern eccentricities from centroid of transformed section in directions 1 and 2, respectively

P_{k1}, P_{k2} = compressive load capacity of masonry applied at kern eccentricities e_{k1} and e_{k2}, respectively

M_{e1}, M_{e2} = moment capacity of masonry corresponding to maximum compressive stress in outer fibers on sides 1 and 2 respectively

r = strength reduction factor (see Sections 3.2.4 and 4.7.5)

a = flexural strength coefficient defined below.

Flexural strength coefficient (a) is a factor greater than unity to account for an experimentally observed increase in the apparent compressive strength of masonry from (f'_m) under axial compression to (af'_m) under combined flexure and compression. The expressions for (P_{k1}) and (P_{k2}) in eqs. (5.1) are based on the simplifying assumption that the ratio of the elastic moduli of the two materials comprising the composite section is the same as the ratio of the flexural compressive strengths. In addition, the expressions for (M_{e1}) and (M_{e2}) in eqs. (5.1) are approximate when the load is applied at an eccentricity greater than the kern eccentricity. However, for a solid rectangular section of non-composite masonry, the (g_i) terms appearing in eqs (5.1) reduce to the values of 4/3 and the expression for (M_e) is no longer approximate.

A note of explanation is needed with regard to the compressive strength (f'_m). For a composite construction, such as a brick-block wall assembly, the axial compressive strengths of the two wythes may have different values. The lower of these two values defines (f'_m) to be used in these equations.

For an uncracked section

$$M_{e1} = P_{k1}e_{k1}\,\frac{aP_o - P}{aP_o - P_{k1}}$$

$$M_{e2} = P_{k2}e_{k2}\,\frac{aP_o - P}{aP_o - P_{k2}} \qquad (5.2)$$

The cracking line which separates the uncracked and cracked regions is defined by the equations

$$M_{k1} = M_{k2} = P_{k1}e_{k1} = P_{k2}e_{k2} \tag{5.3}$$

Equations (5.1) are applicable in the regions $(P \leq P_{k1})$ and $(P \leq P_{k2})$ and equations (5.2) are applicable in the regions $(P \geq P_{k1})$ and $(P \geq P_{k2})$. Figure (5.2) shows an interaction diagram reproduced from reference [65] for an asymmetrical composite section of 4-in brick and 4-in hollow block assuming $a = 1$. Note that the (M_e) curve for a cracked section obtained from eq. (5.1) agrees reasonably well with the solid curve developed from cracked section theory.

Equations (5.1) were derived on the basis of zero tensile strength of masonry. For large (M/P) ratios, failure occurs when the maximum flexural tensile strength of the specimen is developed. Assuming a non-zero tensile strength for masonry the expressions for the cracking lines become

$$M_{e1} = \frac{rf'_{t2}I}{c_2} + Pe_{k1}$$

$$M_{e2} = \frac{rf'_{t1}I}{mc_1} + Pe_{k2} \tag{5.4}$$

$$m = \frac{E_1}{E_2}$$

where

f'_{t1}, f'_{t2} = unreduced flexural tensile strength of masonry on sides 1 and 2, respectively.

E_1, E_2 = modulus of elasticity in direction normal to bed joint of masonry on sides 1 and 2, respectively.

The moment capacity for the cracked section is the larger of the two values determined from eqs. (5.1) and (5.4).

For a symmetrical section, the interaction equations become considerably simpler. Thus, for a cracked section

$$M_e = Pc\,(1 - g\,\frac{P}{aP_o})$$

$$g = 2(1 - \frac{1}{Ac^2})$$

$$P_k = \frac{aP_o}{2} \qquad (5.5)$$

$$e_k = \frac{I}{Ac}$$

$$P_e = rAf'_m$$

and, for an uncracked section,

$$M_e = \frac{I}{Ac}\,(aP_o - P) \qquad (5.6)$$

The cracking line is defined by

$$M_k = P_k e_k \qquad (5.7)$$

or, in the case where $f'_t > 0$, by

$$M_e = \frac{rf'_t I}{c} + Pe_k \qquad (5.8)$$

The moment capacity for the cracked section is the larger of the two values obtained from eqs. (5.5) and (5.8).

To evaluate masonry strength by means of eqs. (5.1) to (5.8) a value should be assigned to coefficient (a). The values derived from the test data varied within a range of 1.25 to 1.65 depending on the type of masonry construction. Significantly, it is noted that flexural compressive stress allowed by several masonry codes for working stress design (BIA, NCMA, UBC), is 60 to 65 percent greater than the allowable stress under axial compression. For investigating the capacity of masonry walls in existing buildings an arbitrary but generally conservative value of a = 1.3 is suggested, together with the necessary condition (because a > 1) that axial load P, existing singly or in combination with a moment M on a given cross section, should not exceed the axial load capacity (P_o) of the masonry wall,

$$P \leq P_o \qquad (5.9)$$

Equation (5.9) must be satisfied before eqs. (5.1) to (5.8) can be used to determine the moment capacity of the section assiciated with (P). In the case where ($P > P_o$), axial capacity of the section is exceeded, and no further investigation is needed.

In the case of slender walls, (P) should not exceed the buckling load capacity (P_c) of the wall,

$$P \leq P_c \qquad (5.10)$$

Equations (5.9) and (5.10) must be independently satisfied if eqs. (5.1) to (5.8) are to be used for moment capacity determination. The buckling load is calculated as follows:

$$P_c = \frac{r\pi^2 R}{(kh)^2} \qquad (5.11)$$

$$R = EI \left(0.20 + \frac{rP}{P_o}\right) \leq 0.70 \qquad (5.12)$$

where (R) is the flexural stiffness, (k) is the effective height coefficient dependent on top and bottom fixity conditions and (h) is the clear unsupported height of the wall. A value of k = 2.0 for cantilever walls such as parapets and k = 1.0 for all other cases may be conservatively assumed. Eq. (5.12) is taken from Ref. [65] where it was used to correlate analytically predicted strength of slender walls with test results.

Equations (5.1) to (5.8) do not take into account moment capacity reduction attributed to slenderness effects. Consequently, to determine the adequacy of a section, the calculated moment capacity (M_e), should be compared to the resultant internal moment (M) on that section. In the presence of an axial load (P), the calculations of moment (M) become quite involved. In an effort to simplify these calculations the following approximate procedure similar in concept to those used by concrete and steel building codes and standards, is introduced.

$$M = \alpha M_s \qquad (5.13)$$

$$\alpha = \frac{C_m}{1 - \frac{P_o}{P_c}} \geq 1.0 \qquad (5.14)$$

where (M_s) is the internal moment on the section induced by transverse loads and/or eccentrically applied compressive loads, and (α) is a factor to account for the moment produced by the compressive load acting on the transverse displacement of that section. Coefficient (C_m) is taken as

$$C_m = 0.6 + 0.4 \frac{M_1}{M_2} \tag{5.15}$$

but not less than 0.4 for walls other than cantilevers in which the maximum moment (designated by the positive symbol M_2) occurs at one end. The moment at the other end, (M_1), is taken as positive if the member is bent in single curvature, negative if bent in double curvature. For all other cases (C_m) is taken as 1.0.

5.3 Compression-Shear Interaction

The results of the NBS racking tests [50] are shown in figures 5.4 and 5.5. Both graphs express the average shear stress at failure versus the average compressive stress. The average shear stress (f'_v) is obtained by dividing the horizontal racking force component at failure by the net horizontal cross-sectional area of the masonry wall. The average compressive stress $(\bar{f}_m)$ is obtained by dividing the vertical load by the same area. The average measured compressive and tensile bond strengths were 4150 psi and 79 psi, respectively, for the brick masonry, and 1200 psi and 30 psi, respectively, for the concrete block masonry based on a net area assumed equal to 52 percent of the gross area. The mortar used was of type N specified in ASTM Designation C270 and consisted of one part of masonry cement and three parts of masonry sand in the case of concrete block specimens, and of one part of portland cement, one part of type S lime (ASTM Designation C207) and four and one-half parts of masonry sand, in the case of brick specimens, all contituents being proportioned by volume. The mortar cubes developed an average compressive strength of 580 psi.

To interpret racking test results, the distinction is made between (a) failure by shear cracking of the mortar joint along the loaded diagonal, (b) failure by tensile cracking through the masonry units along the loaded diagonal and, (c) compressive failure by crushing of the masonry near the toe. The different failure modes were observed to depend on wall geometry, loading configuration and type of masonry. The 8x8-ft walls, tested in accordance with the ASTM E72 method, as in figure 5.3(d), and the 16x16-in prisms under diagonal loading, failed by diagonal shear cracking. The diagonally loaded 4x4-ft brick walls, as shown in figure 5.3(e), failed in a like manner under small or no edge loads. Greater edge loads produced diagonal tensile cracking through the brick units. The 4x4-ft concrete block walls were not tested in this manner. The horizontal driving force (H) in the NBS type tests, as in figure 5.3(a) to (c), induced failure by toe crushing with the exception of 4x8-ft brick walls which failed in shear.

In the plots of figures 5.4 and 5.5, attention is drawn to the fact that results of square specimens without edge loads fall on a 45-degree line through the origin. In the presence of edge loading, the plotted results appear to describe approximately a straight line failure envelope either through the mortar or through the masonry units at failure. The results of tests conducted elsewhere [29, 56, 59] tend to corroborate this behavior.

Assuming a linear relationship between the average shear strength (f'_v) and the average axial stress ($\bar{f}'_m$), the racking strength of a masonry wall as goverened by failure initiated by shear cracking may be reasonably approximated by a linear relationship of the type,

$$v' = f'_v + u\,\bar{f}'_m \qquad (5.16)$$

in which (f'_v) represents the average shearing strength of masonry without axial load, and (u) is a coefficient representing the influence of compressive load on shear strength (Section 4.7.2) and may be evaluated from curves fitted to the results of compression-shear tests (figures 5.4 and 5.5). For the 4x8-ft horizontally loaded and 4x4-ft diagonally loaded brick specimens a value of u = .40 was observed. This compared with u = .55 for the 8x8-ft diagonally tested hollow block walls (figure 5.4). In the absence of experimentally determined values of coefficient (u) for field samples removed from actual construction the following expression should yield generally conservative estimates of shear strength,

$$v' = f'_v + 0.3\,\bar{f}_m \qquad (5.17)$$

where,

v' = average shear strength of masonry in the presence of axial load

With an appropriate reduction factor (r), calculated as in Section (3.2.4) or Section (4.7.5), depending on the source of information on masonry strength, the lower bound value of shear strength is given by

$$v' = r\,f'_v + 0.3\bar{f}_m \qquad (5.18)$$

Equation (5.17) should give reasonably conservative estimates of shear strength when failure is triggered by shear or tension cracking along the diagonal. As noted earlier, the NBS type tests, displayed in figure 5.3(a) to (c), produced compressive cracking near the toe of the specimen under high edge loads. In figures 5.4 and 5.5, these points are plotted below the shear failure envelope which is approximately represented by equation (5.16). It is therefore necessary to compare the maximum compressive stress in the wall under high edge loading conditions against the compressive strength of masonry under combined in-plane flexure and direct compression. The interaction equations (5.5) to (5.9) may be used judiciously for this purpose keeping in mind that the sectional properties now relate to the major principal axis and that approximations introduced by linear stress distribution theory are rapidly amplified with decreasing (h/L) ratio because of deep beam action. However, on the basis of studies on the behavior of deep beams [78], the equations in Section 5.2 are judged to be adequate within the practical range of (h/L) ratios ($0.5 < h/L < 2$).

5.4 Flexure-Shear-Compression Interaction

Very little is known on the behavior of masonry walls under the action of loads which produces simultaneous biaxial bending, axial compression and in-plane shear. A simple rational (albeit conservative) approach for use in the seismic investigation of low or medium rise buildings which are of concern to this study is given below.

Assuming the presence of out-of-plane flexure does not significantly influence the racking shear strength, equation (5.18) is used independently to determine whether the wall is capable of resisting the induced shearing stresses without diagonal rupture. The wall is then checked for the condition of combined compression and biaxial bending by the following linear interaction relationship,

$$\frac{M_x}{M_{ex}} + \frac{M_y}{M_{ey}} \leq 1 \qquad (5.19)$$

where subscripts (x) and (y) designate major and minor principal axes. Thus, (M_y) is the same moment as (M) given in eq. (5.13), (M_x) is the internal (in-plane) moment, (M_{ey}) is the moment capacity about the minor principal axis calculated by the equations in Section 5.2, and (M_{ex}) is the planar moment capacity calculated as noted in Section 5.3. Attention is once more drawn to the fact that, as in the case of out-of-plane bending, eqs. (5.9) and (5.10) must also be satisfied.

5.5 Load-Deflection Relationships for Shear Walls

The load-deflection relationships used in the sample proglem of Appendix A are based on principles of mechanics of materials assuming the constitutive relationships of masonry to be linear. The distribution of earthquake-induced story shears to the individual walls is in proportion to their in-plane stiffnesses which are determined from the following equation,

$$\Delta = f \frac{Vh}{AG} + C \frac{Vh^3}{EI} \tag{5.20}$$

where:

- Δ = in-plane horizontal deflection at top of wall relative to its base
- V = horizontal shear force in plane of wall
- h,L = height and length of wall, respectively
- A = horizontal cross sectional area of wall
- I = moment of inertia about major principal axis of horizontal cross section of wall
- E,G = elastic and shear moduli of masonry material, respectively
- f = form factor related to geometry of wall cross section
- C = numerical constant related to fixity conditions at top and base of wall

For the condition relevant to the problem of Appendix A, it is assumed that $G = 0.4E$ [13, 45, 46], $f = 1.2$ [54], and $C = 1/12$ (rotational fixity at top and base). Substitution in eq. (5.20) gives,

$$\Delta = \frac{V}{Et} \left(\frac{3h}{A/t} + \frac{h^3}{12I/t}\right) \tag{5.21}$$

from which stiffness (k) is calculated as,

$$k = \frac{V}{\Delta} = \frac{Et}{\frac{3h}{A/t} + \frac{h^3}{12I/t}} \tag{5.22}$$

A plot of (k/Et) vs. (h/L) for a rectangular section ($A=Lt$, $I=tL^3/12$) is given in figure 5.6.

6. Summary and Conclusions

Methods have been prescribed for the evaluation of the strength and stiffness of masonry wall elements in existing buildings by removal and testing of small rectangular segments. The procedures describe methods of extraction and transportation of wall samples, preparation of specimens, instrumentation for deformation measurements, execution of tests and interpretation of results. The basic properties sought are the compressive

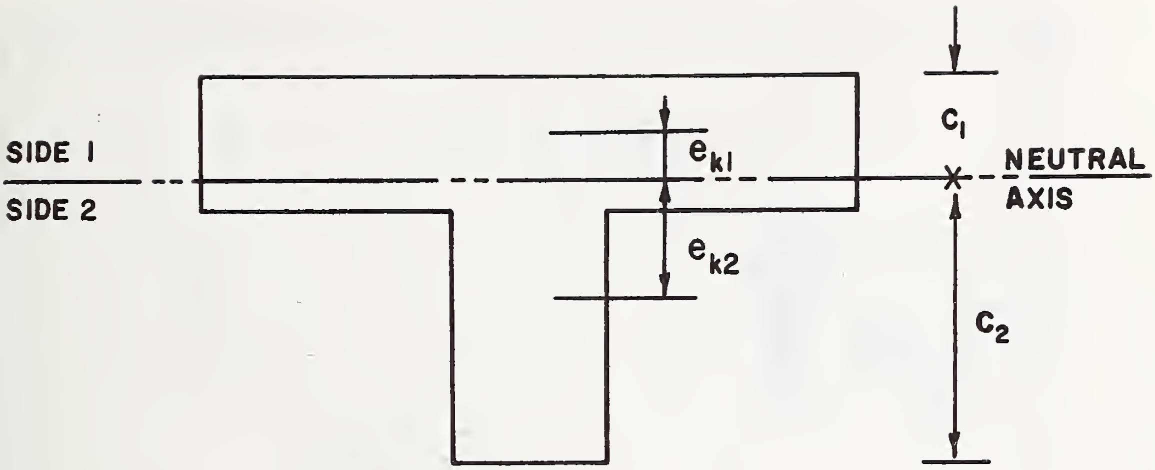


Figure 5.1 - Asymmetrical section.

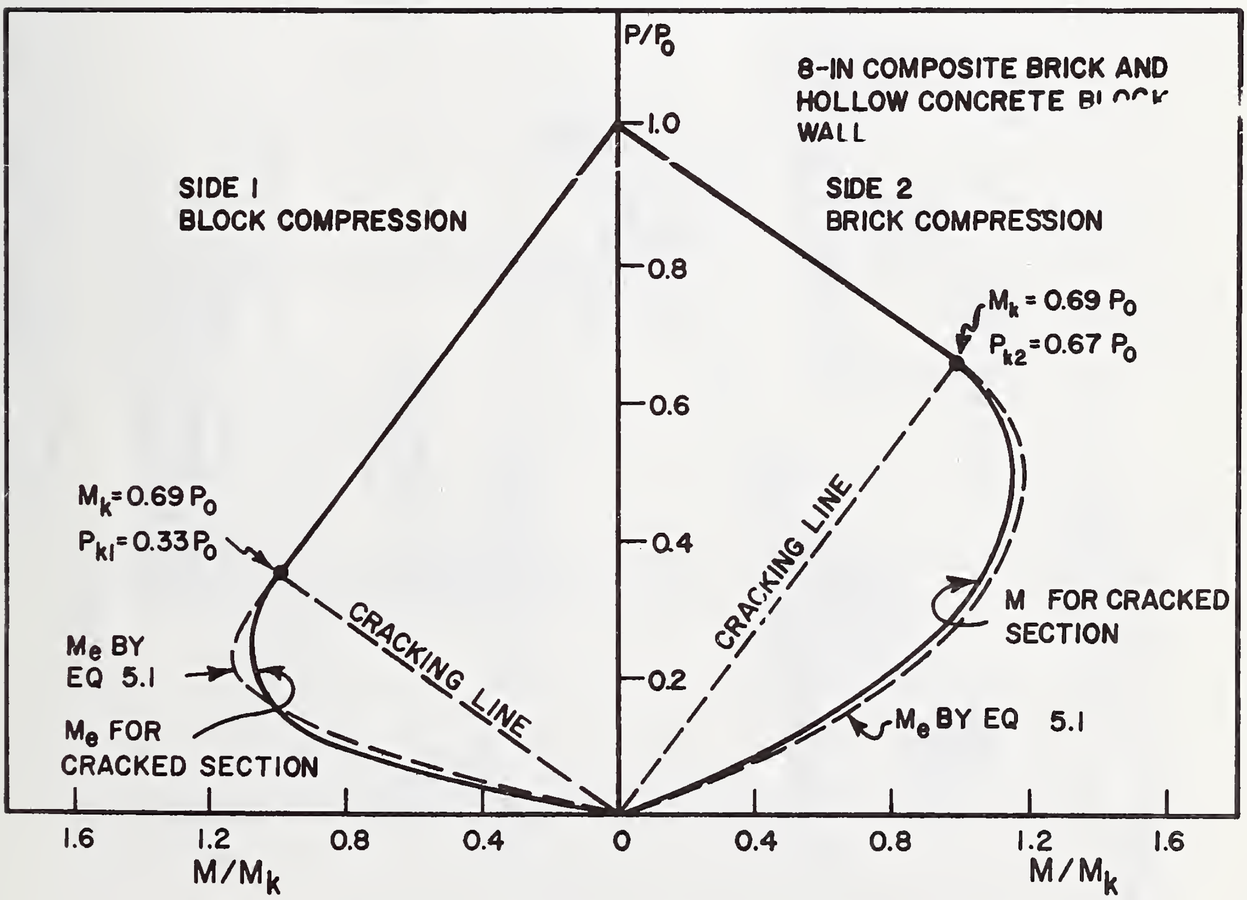


Figure 5.2 - Cross sectional moment capacity of asymmetrical section.

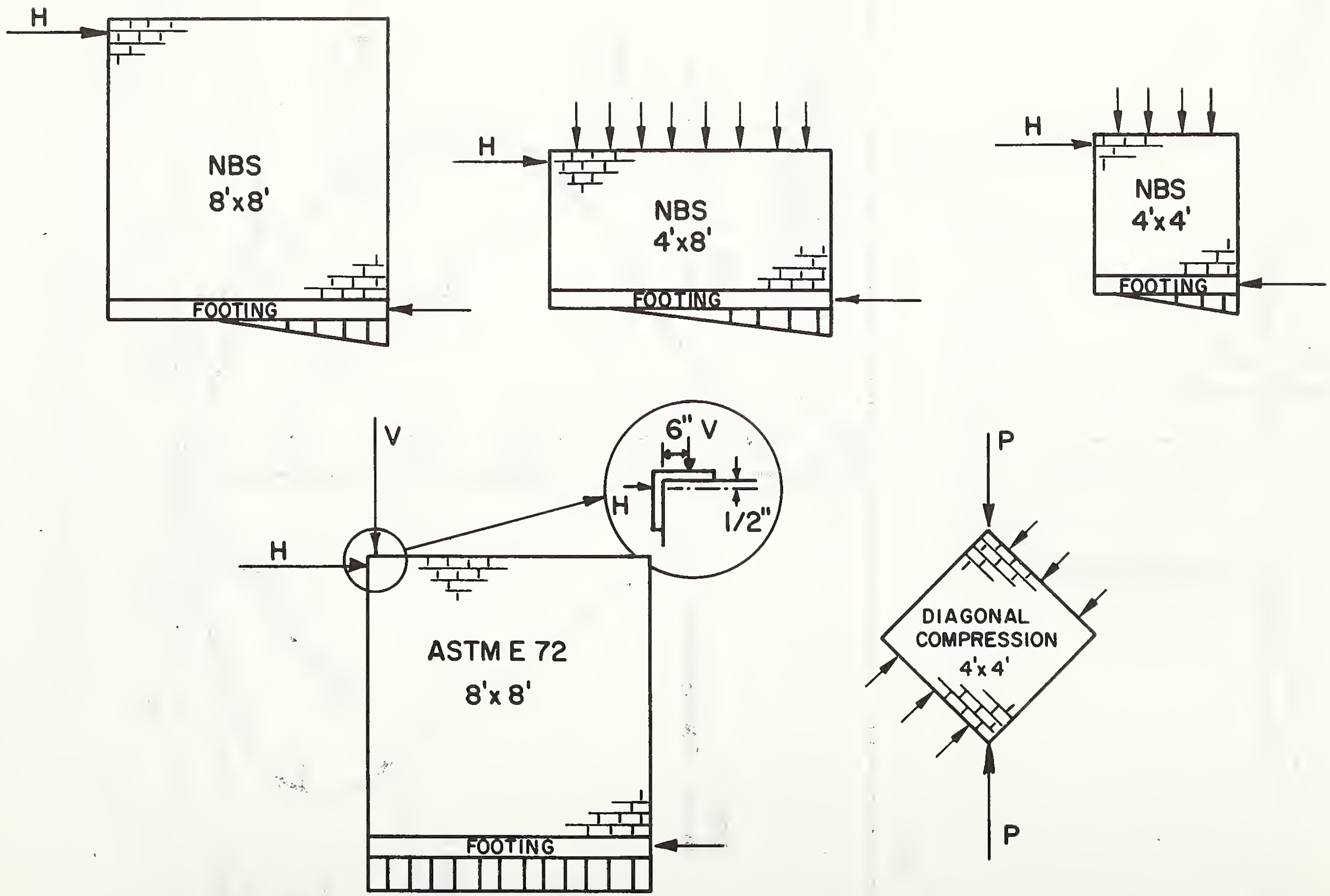


Figure 5.3 - Types of racking tests.

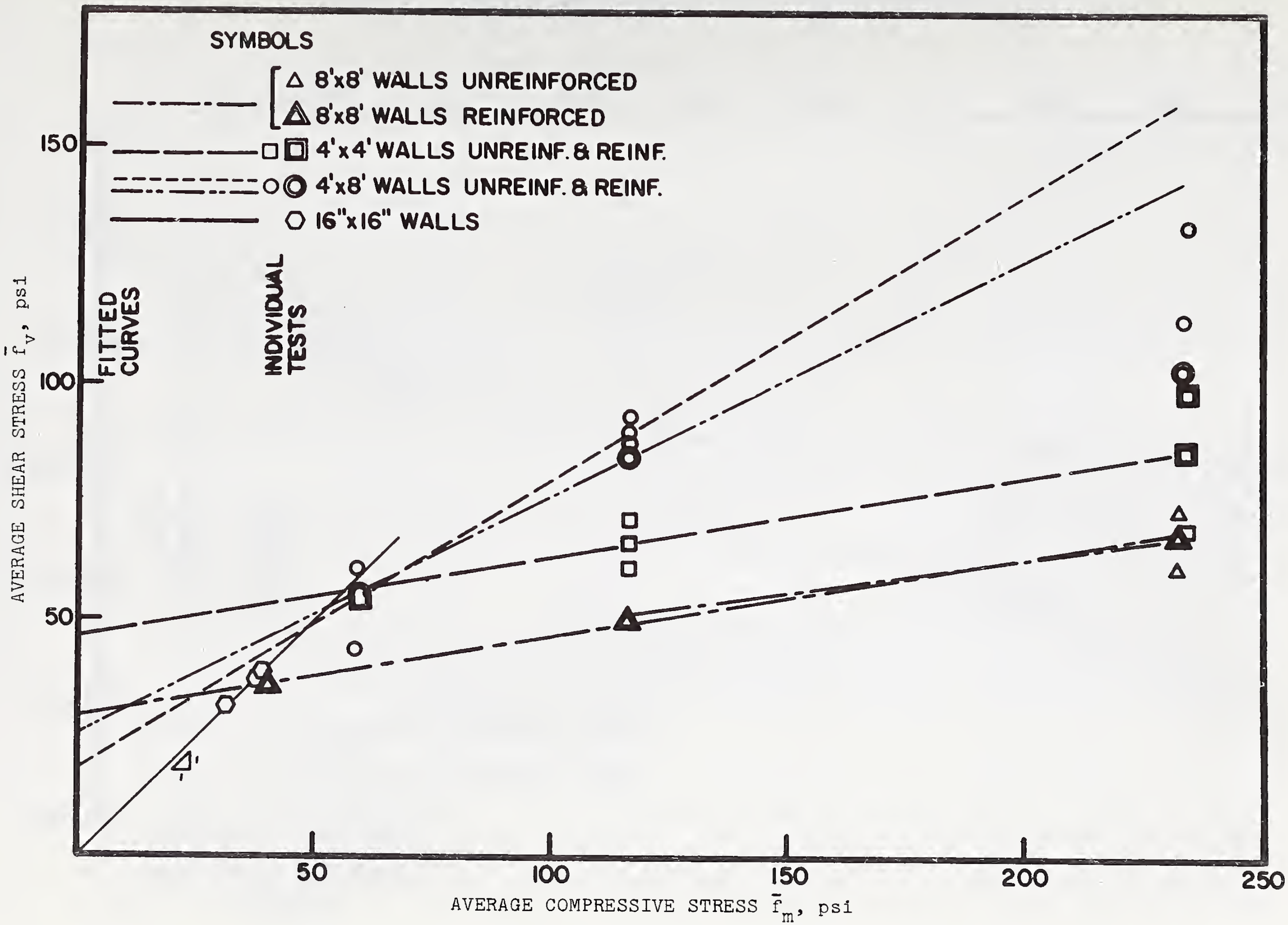


Figure 5.4 - Racking test results of concrete block masonry specimens.

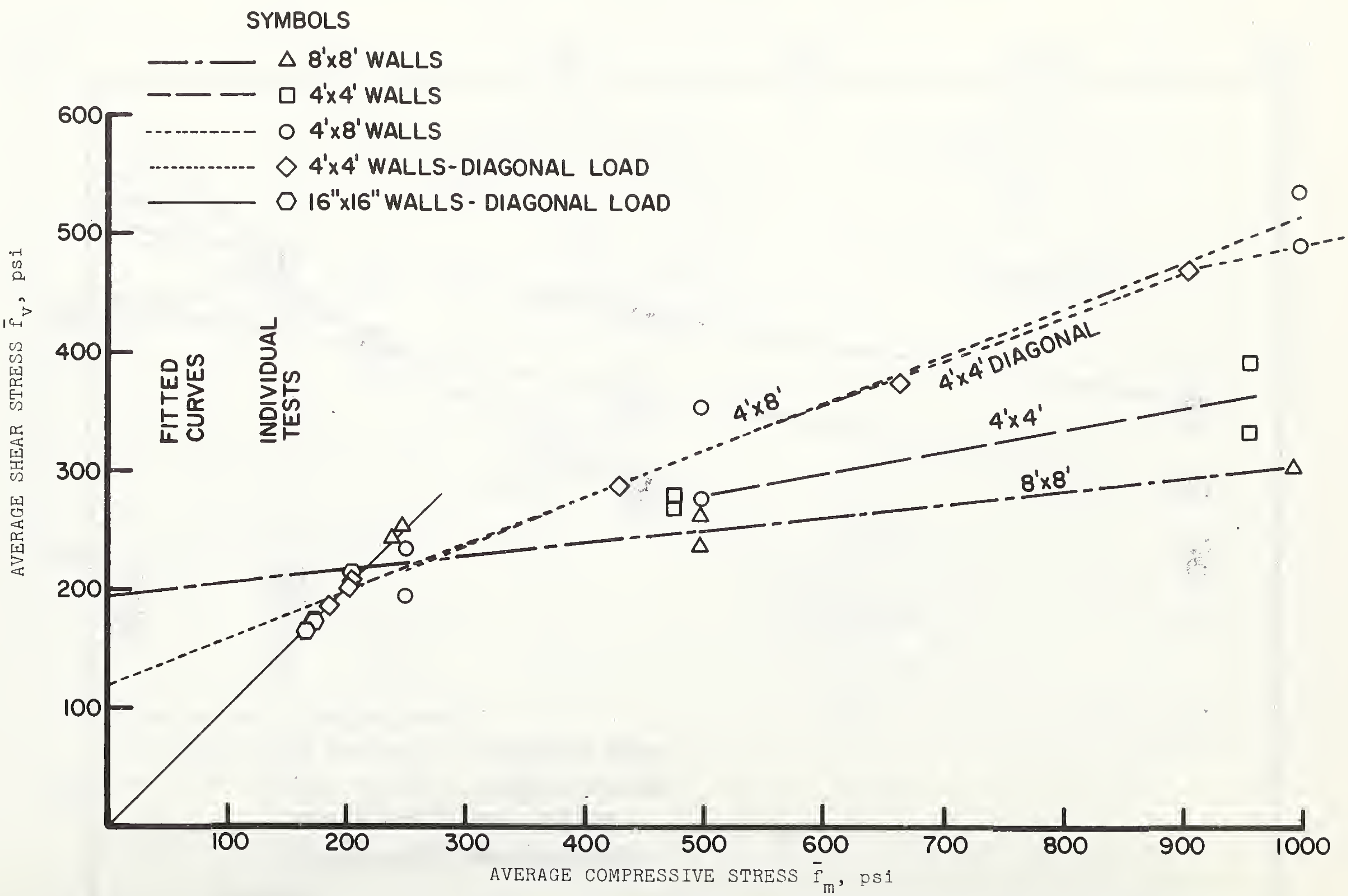


Figure 5.5 - Racking test results of brick masonry specimens.

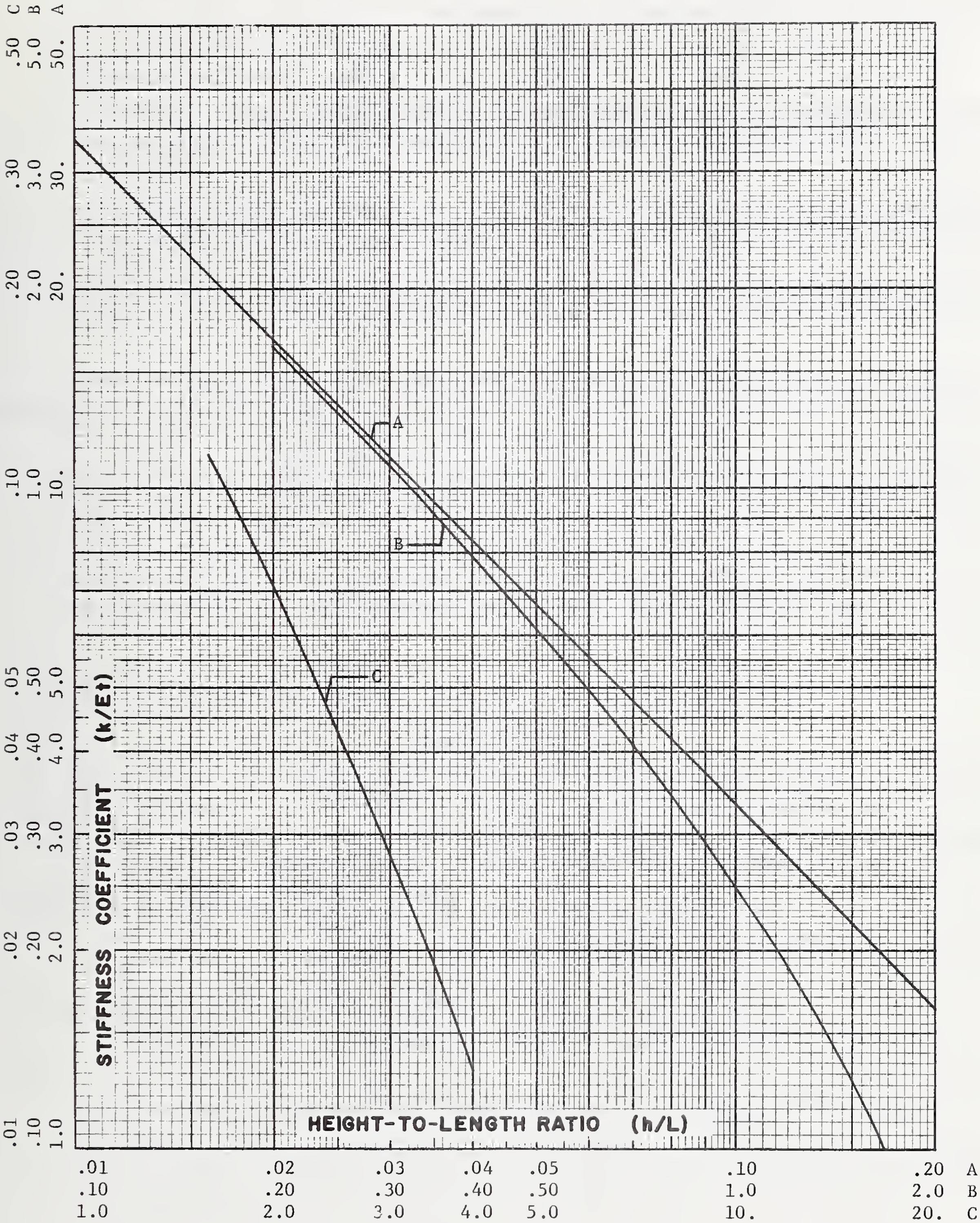


Figure 5.6 - Stiffness vs. h/L ratio for shear walls.

strength, racking shear strength, flexural bond strength and stress-strain relationships in shear and axial compression.

The sectional capacity of a masonry wall has been specified by means of interactive relationships among axial compression, shear and flexure, using the basic strength and stiffness parameters derived from tests.

Supplementary information on the basic strength properties of brick and concrete block specimens derived from available test data is compiled to provide an indication of strength of comparable masonry construction under controlled environmental conditions and good workmanship.

The direct test approach described in this report has been proposed after having studied possible alternate methods of evaluating masonry properties in buildings which have been in service for various periods of time, and is believed to be both practical and comprehensive. Among the more radical methods, the use of ultra-sonic devices for non-destructive testing appears to have good potential for structural application. Other promising research areas are the use of spectroscopic or x-ray analysis and hardness tests on mortars to develop criteria for predicting tensile and shear strength of masonry.

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APPENDIX A

Seismic Investigation of a Masonry Building

A.1 Three-Story Building with Brick Masonry Bearing Wall and Rigid Floor

The intent of the following calculations is to provide a numerical demonstration of a procedure for the analysis of a masonry building of moderate height, under the action of lateral forces of seismic origin. The criteria for calculating base and story seismic shear forces and the load factors used in this analysis have been selected arbitrarily only for the purpose of the stated objective. In practice, such criteria will need to be specified in accordance with the seismic provisions of the appropriate building regulatory agencies.

Figure (A.1) shows a three-story building with 12-in exterior brick masonry bearing walls and 8-in hollow concrete block interior walls as indicated. Interior concrete columns are at locations 6, 7, 10 and 11. Concrete girders are located along 5-7 and 9-12. The concrete joist floor spanning in the y-direction is treated as a rigid diaphragm. The distributed dead loads are 80 psf for the roof and third floor, and 73 psf for the second floor. For simplicity, a distributed live load equal to 75% of dead load is assumed. Masonry weights are 120 psf and 55 psf of wall area for exterior and interior walls, respectively. The estimated ultimate compressive strength is f'_m = 1200 psi for concrete block and f'_m = 3600 psi for brick masonry (table 3.1). The respective elastic moduli for brick and concrete block masonry are assumed as E_b = 3000 ksi and E_c = 1200 ksi. The respective shear moduli are assumed as G_b = 1200 ksi and G_c = 480 ksi. In accordance with Section 3.2.3c, shear strength is calculated as 120 psi and 70 psi, for the brick and concrete block masonry, respectively, assuming $f'_v = 0.2 \sqrt{f'_m}$.

The validity of the approach herein used to analyze the building itself, is dependent on two primary assumptions: (a) the wall system is capable of integral action in flexure induced by lateral forces, and (b) the floors are rigid in their own plane. Assumption (a) is analogous to the condition of vertical continuity, through adequate connections, between intersecting wall elements. If the connections between abutting wall elements in a particular building cannot be relied upon to offer total continuity, the building may be conservatively analyzed for the two limiting conditions; the first, assuming complete continuity, the second, assuming no vertical continuity. The strength of individual wall elements can then be compared to the most critical stress condition resulting from either assumption.

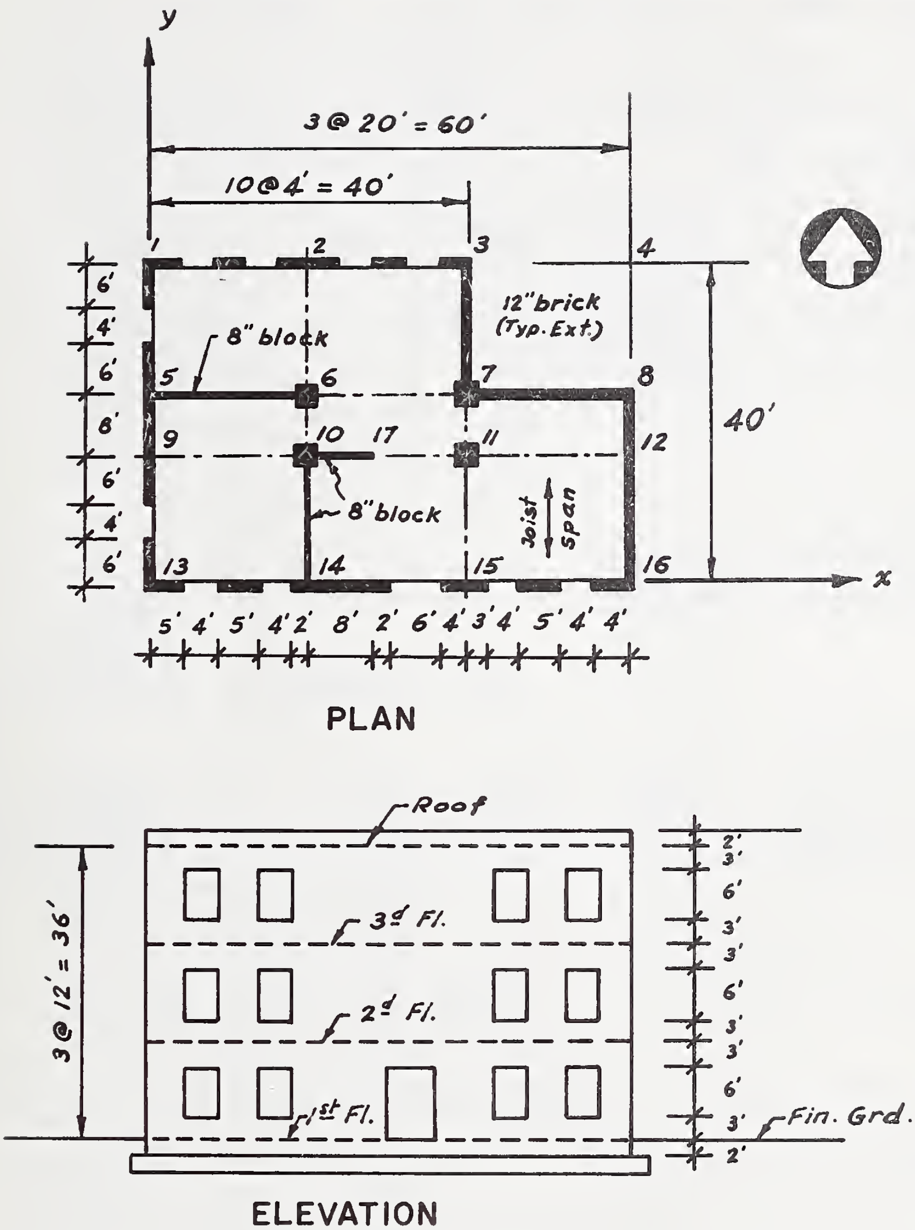


Figure A.1 - Three-story brick masonry building of bearing wall construction.

a. Lumped Weights at Floor Levels

Net area factor (Sect. 3.3.3) is calculated as:

1^{st} Story:	0.878
2^{d} & 3^{rd} stories:	0.900

Weights (kip):

Roof:	$[(60)(40) - (20)(16)](.08)$	=	167
Parapet:	$(200)(2)(.120)$	=	48
Ext. Walls:	$(200)(6)(.120)(.900)$	=	128
Int. Walls:	$(44)(6)(.055)$	=	15
	Total at Roof	=	358^k
3^{rd} Floor:		=	167
Ext. Walls:	$(2)(128)$	=	256
Int. Walls:	$(2)(15)$	=	30
	Total 3^{d} Floor	=	453^k
2^{nd} Floor:	$(167)(73)/80$	=	152
Ext. Walls:	$(128)[1 + (.878/.900)]$	=	253
Int. Walls:			30
	Total 2^{nd} Floor	=	435^k
	Tot. Bldg. Wt.	=	1246^k

b. Sectional Properties

Units: Length = ft., Area = ft^2, Mom. of Inertia = ft^4

Wall Section	Thick. Red. Factor	Gross Thick.	Effect. Thick.	Length	Net Area	x	y
1-3	0.8(1)	1.0	0.8	40	32	20	40
5-6	0.16(2)	.67	.107	20	2	10	24
7-8	1.0	1.0	1.0	20	20	50	24
10-17	.16	.67	.107	8	1	24	16
13-16	.88	1.0	.88	60	52.8	30	0
(13-16)	(.81)	(1.0)	(.81)	(60)	(48.6)	(30)	(0)
1-13	.90	1.0	.90	40	36	0	20
10-14	.16	.67	.107	16	1.7	20	8
3-7	1.0	1.0	1.0	16	16	40	32
8-16	1.0	1.0	1.0	24	24	60	12
Σ 2d or 3d stories					185.5		
(Σ 1st story)					(181.3)		

Wall Section	Ax	Ax^2	I_{oy}	Ay	Ay^2	I_{ox}
1-3	640	12800	4267	1280	51200	-
5-6	20	200	67	48	1152	-
7-8	1000	50000	667	480	11520	-
10-17	24	576	5	16	256	-
13-16	1584	47520	15840	0	0	-
(13-16)	(1458)	(43740)	(14580)	(0)	(0)	(-)
1-13	0	0	-	720	14400	4800
10-14	34	680	-	14	112	36
3-7	640	25600	-	512	16384	341
8-16	1440	86400	-	288	3456	1152
Σ 2d or 3d story	5382	223776	20846	3358	98480	6329
(Σ 1st story)	(5256)	(220000)	(19586)	(3358)	(98480)	(6329)

Notes :

(1) Reduction made according to percentage of net (openings deducted) to gross vertical wall area (sect. 3.2).

(2) Net area is 40% of gross area (Section 3.3.2). Net area is multiplied by $E_c/E_b = 1200/3000 = .4$ to obtain net equivalent transformed brick area. Therefore, area reduction factor is $= (.4)(.4) = .16$.

Centroids:

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \begin{cases} 5382/185.5 = 29.0 \text{ ft. } 2^{d} \text{ or } 3^{d} \text{ story} \\ 5256/181.3 = 29.0 \text{ ft. } 1^{st} \text{ story} \end{cases}$$

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \begin{cases} 3353/185.5 = 18.1 \text{ ft. } 2^{d} \text{ or } 3^{d} \text{ story} \\ 3353/181.3 = 18.5 \text{ ft. } 1^{st} \text{ story} \end{cases}$$

Centroidal moments of inertia

$$I_x = I_{ox} + Ay^2 - A\bar{y}^2$$

$$= \begin{cases} 6329 + 98480 - 185.5(18.1)^2 = 44,040 \text{ ft}^4 \ (2^{d} \text{ or } 3^{d} \text{ story}) \\ 6329 + 98480 - 181.3(18.5)^2 = 42,760 \text{ ft}^4 \ (1^{st} \text{ story}) \end{cases}$$

$$I_y = I_{oy} + Ax^2 - A\bar{x}^2$$

$$= \begin{cases} 20846 + 223776 - 185.5(29)^2 = 88,620 \text{ ft}^4 \ (2^{d} \text{ or } 3^{d} \text{ Story}) \\ 19586 + 220000 - 181.3(29)^2 = 87,110 \text{ ft}^4 \ (1^{st} \text{ story}) \end{cases}$$

c. Fundamental Period

For demonstration purposes the fundamental period of the building in the orthogonal coordinate directions will be calculated using four different methods: (a) prismatic beam equation, (b) numerical integration, (c) the 1973 UBC formula and (d) in accordance with the V.A. recommendation.

Shear wall areas in coordinate directions

(a) parallel to x-axis:

$$A_x = A_{1\text{-}3} + A_{5\text{-}6} + A_{7\text{-}8} + A_{10\text{-}17} + A_{13\text{-}16}$$

$$= 32 + 2 + 20 + 1 + (52.8 \text{ or } 48.6)$$

$$= 107.8 \text{ ft}^2;\quad 2^{d} \text{ or } 3^{d} \text{ story}$$
$$= 103.6 \text{ ft}^2,\quad 1^{st} \text{ story}$$

(b) parallel to y-axis:

$$A_y = A_{1\text{-}13} + A_{10\text{-}14} + A_{3\text{-}7} + A_{8\text{-}16}$$

$$= 36 + 1.7 + 16 + 24 = 77.7 \text{ ft}^2, \text{ all stories}$$

Since in this case $G_c/G_b = E_c/E_b = .25$, the same transformed sections apply. Also, since the shape of the cross section is similar to a box, $f = 1.0$. Then,

$$R = \frac{EIf}{GA} = (1.0)\frac{E}{G}\cdot\frac{I}{A} = (1.0)(2.5)\frac{I}{A} = 2.5\left(\frac{I}{A}\right)$$

$$R_y = \frac{2.5\, I_x}{A_y} = \frac{2.5}{77.7}\begin{Bmatrix}44040\\42760\end{Bmatrix} = \begin{matrix}1417 \text{ ft}^2\ (2^{d} \text{ or } 3^{d} \text{ story})\\ 1376 \text{ ft}^2\ (1^{st} \text{ story})\end{matrix}$$

$$R_x = \frac{2.5\, I_y}{A_x} = 2.5\begin{Bmatrix}88620/107.8\\87110/103.6\end{Bmatrix} = \begin{matrix}2055 \text{ ft}^2\ (2^{d} \text{ or } 3^{d} \text{ story})\\ 2102 \text{ ft}^2\ (1^{st} \text{ story})\end{matrix}$$

Prismatic beam model (Appendix B.2):

Since sectional properties and lumped masses are fairly constant throughout the height of the building, average values are used with eq. (B.3) for the prismatic beam model. The height of the building is taken as 36 ft.

$$Q_y = \frac{4}{h^2}\left(\frac{EIf}{GA}\right)_x = \frac{4R_y}{h^2} = \frac{4}{(36)^2}\cdot\frac{2(1417)+1376}{3} = 4.33$$

$$Q_x = \frac{4R_x}{h^2} = \frac{4}{(36)^2}\cdot\frac{2(2055)+2102}{3} = 6.39$$

$$T_y = 1.79h^2\left[(1+Q_y)\frac{W}{hgEI_x}\right]^{1/2}$$

or,

$$T_y = (1.79)(36)^2\left[\frac{(1+4.33)(1246/36)}{32(3000\times144)(43610)}\right]^{1/2} = \underline{.041} \text{ sec.}$$

likewise,

$$T_x = (1.79)(36)^2\left[\frac{(1+6.39)(1246/36)}{32(3000\times144)(38120)}\right]^{1/2} = \underline{.034} \text{ sec.}$$

(Note: R & I used above are weighed ave. values)

UBC Eq. (14-3):

Use weighed average plan dimensions

$$D_y = \frac{(16)(40)}{60} + 24 = 35'$$

$$D_x = \frac{(24)(20)}{40} + 40 = 52'$$

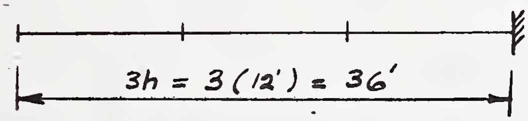
$$T_y = \frac{.05h}{\sqrt{D_y}} = \frac{(.05)(36)}{\sqrt{35}} = \underline{.304} \text{ sec.}$$

$$T_x = \frac{.05h}{\sqrt{D_x}} = \frac{(.05)(36)}{\sqrt{52}} = \underline{.250} \text{ sec.}$$

Numerical Integration (Appendix B.3):

To avoid large tabular values, units will be expressed in terms of $\bar{m} = 358/g$ for mass (value at roof node), EI_x, and EI_y for flexural rigidities.

Y-DIRECTION (Fig. A.1)

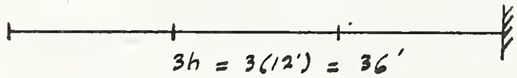


Function								Units
$\bar{m}$	1.0		1.27		1.22			mh
EI_x		1		1		1		EI_x
R_y		1400		1400		1400		
y_1	5.7		3.0		1.0		0	Assumed
$\bar{q}$ (distr. ld.)	5.7		3.81		1.22			mhp^2
V		5.7		9.51		10.73		mhp^2
M/EI_x	0		5.7		15.27		26.0	mh^2p^2
$\bar{s}$ (sect. B.3)	.79		6.02		15.37		11.16	mh^3p^2/EI_x
θ_m		32.55		26.53		11.16		mh^3p^2/EI_x
y_m	70.24		37.69		11.16		0	mh^4p^2/EI_x
$\Delta\theta_v = \frac{R_y V}{EI_x} \cdot \frac{h^2}{12}$		55.42		92.46		104.32		mh^3p^2/EI_x
θ_v		252.2		196.8		104.3		mh^3p^2/EI_x
Δy_v	252.2		196.8		104.3			mh^4p^2/EI_x
y_v	553.3		301.1		104.3			mh^4p^2/EI_x
$y_2 = y_m + y_v$	624.		339.		115.0			mh^4p^2/EI_x
$p^2 = y_1/y_2$	.0091		.0088		.0087			EI_x/mh^4

$$T_y = 2\pi\left(\frac{\Sigma y_2^2}{\Sigma y_1 y_2}\right)^{1/2} = 2\pi\,(10.5)\,h^2\sqrt{\frac{m}{EI}}$$

$$\text{or } T_y = 2\pi\,(10.5)(12)^2\left[\frac{358/12}{32\,(3000\times 144)(43610)}\right]^{1/2} = \underline{\underline{0.067}}\ \text{sec.}$$

X - DIRECTION (Fig. A.1)



$3h = 3(12') = 36'$

Function								Units
$\bar{m}$	1.0		1.27		1.22			mh
EI_y		1		1		≈ 1		EI_y
R_x		2070		2070		2070		
y_1	5.5		3.0		1.0			Assumed
$\bar{q}$	5.5		3.81		1.22			mhp^2
V		5.5		9.31		10.53		mhp^2
M/EI_y	0		5.5		14.81		25.34	mh^2p^2
$\bar{s}$	.76		5.81		14.91		10.86	mh^3p^2/EI_y
θ_m		31.58		25.77		10.86		mh^3p^2/EI_y
y_m	68.21		36.63		10.86			mh^4p^2/EI_y
$\Delta\theta_v = \frac{R_x V}{EI_y} \cdot \frac{h^2}{12}$		79.1		133.8		151.4		mh^3p^2/EI_y
θ_v		364.3		285.2		151.4		mh^3p^2/EI_y
y_v	801.		436.6		151.4			mh^4p^2/EI_y
$y_2 = y_m + y_v$	869.2		473.2		162.3			mh^4p^2/EI_y
$p^2 = y_1/y_2$	.00637		.00634		.00616			

$$T_x = 2\pi (12.57)(12^2)\left[\frac{358/12}{32(3000\times 144)(88120)}\right]^{1/2}$$

or $T_x = \underline{\underline{0.056}}$ sec.

V.A. Manual [80] :

$T_x = T_y = 0.05N = .05(3) = \underline{\underline{0.150}}$ sec.

d. Distribution of Seismic Forces & Overturning Moments

Calculate according to VA Manual, Method I [80]
Assume $A_{max} = 0.057$ (from site evaluation study)
$DAF = 3$, $\alpha = 2/3$
$S_a = (DAF)A_{max} = (3)(.057) = .171$
Base shear: $V_x = V_y = \alpha S_a W = (2/3)(.171)(1246) = \underline{\underline{142^k}}$

For the sake of clarity, different base shear values will be assumed for x and y directions in subsequent calculations as follows:

$$V_x = \underline{\underline{148^k}}, \quad V_y = \underline{\underline{138^k}}$$

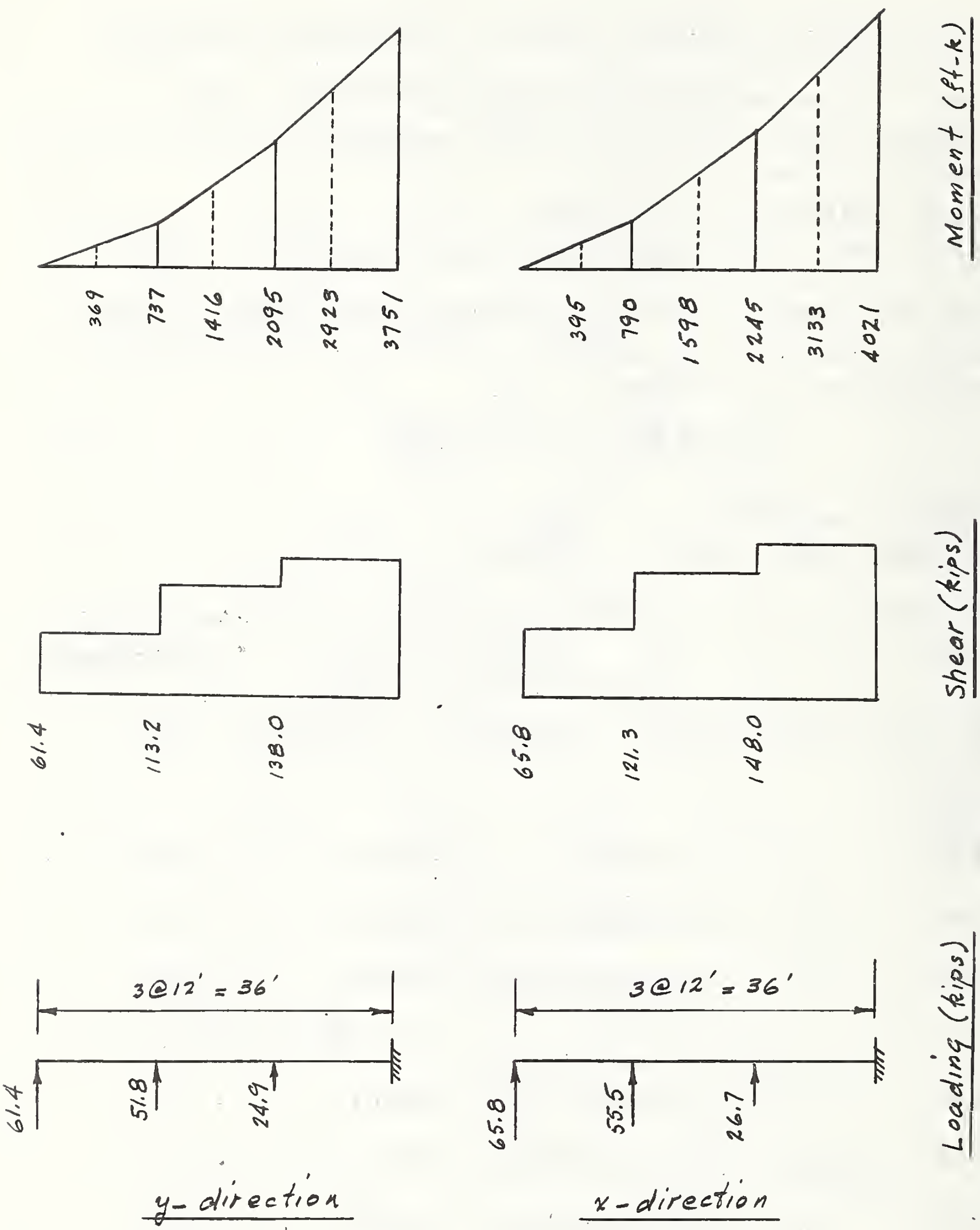
$h_n/D_y = 36/35 < 3.0$, $\therefore F_{ty} = 0$
$h_n/D_x = 36/52 < 3.0$, $\therefore F_{tx} = 0$

At any level j,
$$\left.\begin{aligned} F_{jx} &= V_x w_j h_j / \sum_{i=1}^{n} w_i h_i \\ F_{jy} &= V_y w_j h_j / \sum_{i=1}^{n} w_i h_i \end{aligned}\right]$$
(UBC Eq. 14-5, & VA manual)

$$\sum_1^n w_i h_i = (435 + 453 \times 2 + 358 \times 3)12 = 28,980 \text{ ft-k}$$

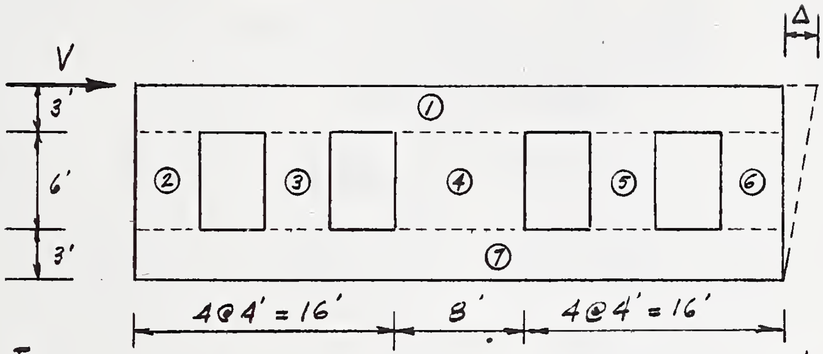
Floor

2^{nd} :	$F_{1y} = (138)(435)(12)/28980$	=	24.9^k
3^{rd} :	$F_{2y} = (138)(435)(24)/28980$	=	51.8^k
Roof :	$F_{3y} = (138)(358)(36)/28980$	=	$\underline{61.4^k}$
	Σ	=	138.1^k ✓
2^{nd} :	$F_{1x} = (148)(435)(12)/28980$	=	26.7^k
3^{rd} :	$F_{2x} = (148)(435)(24)/28980$	=	55.6^k
Roof :	$F_{3x} = (148)(358)(36)/28980$	=	$\underline{65.8^k}$
	Σ	=	148.0^k ✓


y-direction
x-direction
3@12' = 36'
3@12' = 36'
61.4
51.8
24.9
65.8
55.5
26.7
Loading (kips)
61.4
113.2
138.0
65.8
121.3
148.0
Shear (kips)
369
737
1416
2095
2923
3751
395
790
1598
2245
3133
4021
Moment (ft-k)

e. Analysis of Walls

North Wall (1-3)



I	A
$I_1 = I_7 = t(40)^3/12 = 5333t \text{ ft}^4$	$40t \text{ ft}^2$
$I_2 = I_3 = I_5 = I_6 = t(4)^3/12 = 5.3t \text{ ft}^4$	$4t \text{ ft}^2$
$I_4 = t(8)^3/12 = 42.7 \text{ ft}^4$	$8t \text{ ft}^2$

$$\Delta = \frac{Vh}{AG/f} + \frac{Vh^3}{cEI}, \quad G/E = .4, \quad f = 1.2$$

$$\Delta_1 = \Delta_7 = \frac{V(3)}{(40t)(.4E)/1.2} + (\text{neglect}) = .225\frac{V}{Et}$$

$$\Delta_2 = \Delta_3 = \Delta_5 = \Delta_6 = \frac{V_2(6)}{(4t)(.4E)/1.2} + \frac{V_2(6)^3}{(12)(E)(5.3t)}$$

$$= (4.5 + 3.4)\frac{V_2}{Et} = 7.9\frac{V_2}{Et}$$

$$\Delta_4 = \frac{V_4(6)}{(4t)(.4E)/1.2} + \frac{V_4(6)^3}{12E(42.7t)} = (4.5 + .4)\frac{V_4}{Et} = 4.9\frac{V_4}{Et}$$

V is distributed to piers 2 to 6 according to their lateral stiffnesses

$$k_2 = k_3 = k_5 = k_6 = \frac{V_2}{\Delta_2} = Et/7.9 = .126\,Et$$

$$k_4 = V_4/\Delta_4 = Et/4.9 = .204\,Et$$

$$\Sigma k_{2-6} = [(4)(.126) + .204]\,Et = .708\,Et$$

$$V_2 = V_3 = V_5 = V_6 = \frac{k_2}{\Sigma k_{1-6}} \cdot V = \frac{.126}{.708} V = .178 V$$

$$V_4 = \frac{k_4}{\Sigma k_{1-6}} \cdot V = \frac{.204}{.708} V = .288 V$$

$$4V_2 + V_4 \stackrel{?}{=} V$$

$$4(.178V) + .288V = 1.000 V \qquad \text{(checks)}$$

$$\Delta_2 = 7.9 \frac{V_2}{Et} = \frac{1}{Et}(7.9 \times .178V) = 1.41 \frac{V}{Et}$$

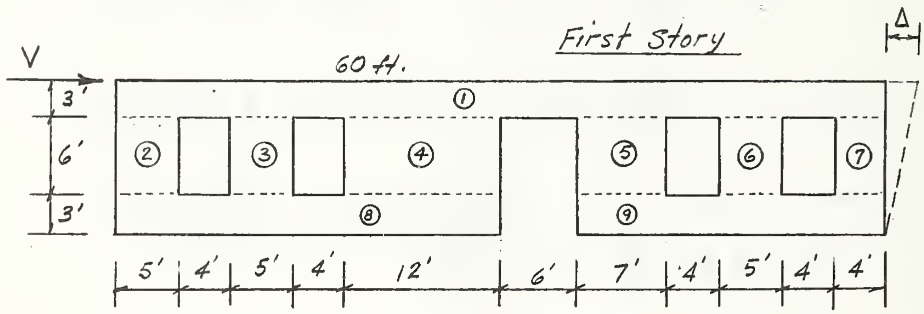
$$\Delta_4 = \frac{1}{Et}(4.9 \times .288V) = 1.41 \frac{V}{Et} \quad \text{(checks)}$$

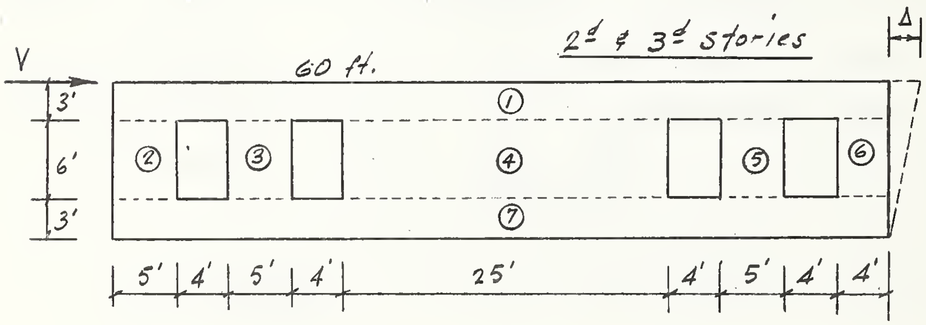
$$\Delta = \Delta_1 + \Delta_2 + \Delta_7 = (2 \times .227 + 1.41)\frac{V}{Et} = 1.864 \frac{V}{Et}$$

$$\therefore \; k_{1-3} = V/\Delta = Et/1.864 = \underline{.536\, Et}$$

w/ $Et = 432{,}000$ kpf, $\qquad k_{1-3} = \underline{231{,}550}$ kpf

South Wall (13-16)





$$\Delta = \frac{V}{Et}\left(\frac{3h}{A/t} + \frac{Ch^3}{I/t}\right) \qquad \text{(see Eq. 5.21)}$$

First Story Wall (units in feet and powers of feet)

Pier No.	h	c	A/t	I/t	$C_1 = 3h/A/t$	$C_2 = Ch^3/I/t$	$Et\Delta_n = (C_1+C_2)V_n$	V_n	$\Delta_n\left(\frac{Et}{V}\right)$
1	3	—	60	—	.15	—	.15 V	V	.150
2	6	1/12	5	10.4	3.6	1.73	5.33 V_2	.114 V	.608
3	6	1/12	5	10.4	3.6	1.73	5.33 V_3	.114 V	.608
4	6	1/12	12	144.	1.5	.125	1.63 V_4	.373 V	.608
5	6	1/12	7	28.6	2.57	.629	3.20 V_5	.199 V	.636
6	6	1/12	5	10.4	3.6	1.73	5.33 V_6	.119 V	.636
7	6	1/12	4	5.33	4.5	3.38	7.88 V_7	.082 V	.640
8	3	—	30	—	.3	—	.3 V_8	.601 V	.180
9	3	—	24	—	.375	—	.38 V_9	.400 V	.152

2d or 3d Story Wall

1	3	—	60	—	.15	—	.15 V	V	.150
2	6	1/12	5	10.4	3.6	1.73	5.33 V_2	.090 V	.480
3	6	1/12	5	10.4	3.6	1.73	5.33 V_3	.090 V	.480
4	6	1/12	25	1302	.72	.01	.72 V_4	.666 V	.480
5	6	1/12	5	10.4	3.6	1.73	5.33 V_5	.090 V	.480
6	6	1/12	4	5.33	4.5	3.38	7.88 V_6	.061 V	.480
7	3	—	60	—	.15	—	.15 V	V	.150

Last two column entries are determined from equilibrium and deflection compatibility as follows:

Compatibility:

$\Delta_2 = \Delta_3, \quad 5.33 V_2 = 5.33 V_3, \quad V_3 = V_2 \quad \ldots\ldots\ldots (1)$

$\Delta_2 = \Delta_4, \quad 5.33 V_2 = 1.63 V_4, \quad V_4 = 3.27 V_2 \quad \ldots\ldots (2)$

$\Delta_5 = \Delta_6$, $\quad 3.20 V_5 = 5.33 V_6$, $\quad V_6 = .60 V_5$ .... (3)

$\Delta_5 = \Delta_7$, $\quad 3.20 V_5 = 7.88 V_7$, $\quad V_7 = .41 V_5$ .... (4)

$\Delta_8 + \Delta_2 = \Delta_9 + \Delta_5$, $\quad .3 V_8 + 5.33 V_2 = .38 V_9 + 3.20 V_5$ ..... (5)

Equilibrium :

$V_5 + V_6 + V_7 = V_9$, $\quad V_5 (1 + .6 + .41) = V_9$, $\quad V_9 = 2.01 V_5$ ...... (6)

$V_2 + V_3 + V_4 = V_8$, $\quad V_2 (1 + 1 + 3.27) = V_8$, $\quad V_8 = 5.27 V_2$ ..... (7)

$V_8 + V_9 = V$ ..... (8)

Solution of eqs. (5) through (9) and back substitution yield the entries in the V_n column of first story wall. Calculations for second story wall will not be shown here. The results are tabulated above.

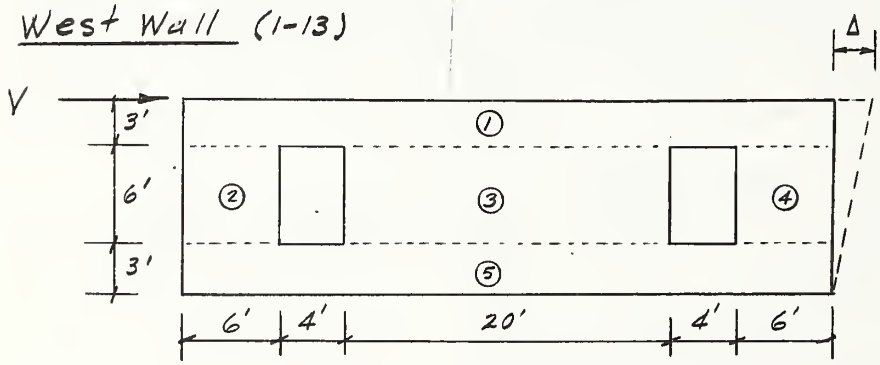
Stiffness : story

$k_{13-16} = V/\Delta = V/(\Delta_1 + \Delta_2 + \Delta_8) = Et/.938 = \underline{1.006 Et}$ (1^{st})

$k_{13-16} = V/\Delta = V/(\Delta_1 + \Delta_2 + \Delta_7) = Et/.780 = \underline{1.282 Et}$ $(2^d, 3^d)$

or, $k_{13-16} = \underline{432,590}$ kpf $(1^{st}$ story), $\underline{553,824}$ kpf $(2^d \& 3^d$ stories)

West Wall (1-13)



(units in feet & powers of feet)

Pier NO.	h	c	A/t	I/t	$c_1 = 3h/A/t$	$c_2 = ch^3/I/t$	$Et\Delta_n = (c_1+c_2)V_n$	V_n	$\Delta_n\left(\frac{Et}{V}\right)$
1	3	–	40	—	.225	—	.225V	V	.225
2	6	1/12	6	18	3	1	$4.V_2$	.155V	.620
3	6	–	20	—	.9	—	$.9V_3$	.689V	.620
4	6	1/12	6	18	3	1	$4.V_4$	.155V	.620
5	3	–	40	—	.225	—	.225V	V	.225

$$k_{1-13} = V/\Delta = V/(\Delta_1+\Delta_2+\Delta_5) = \underline{0.935\ Et} = \underline{403{,}920\ kpf}$$

Wall (3-7)

$$k = \frac{V}{\Delta} = Et\Big/\left(\frac{3h}{A/t} + \frac{h^3}{cI/t}\right)$$

$$k_{3-7} = Et\Big/\left(\frac{3\times12}{16} + \frac{12^3}{12(16^3/12)}\right) = \frac{Et}{2.25+.421} = \frac{.374\ Et}{\underline{161{,}570\ kpf}}$$

Wall (7-8)

$$k_{7-8} = Et\Big/\left[\frac{3\times12}{20} + \frac{12^3}{12(20^3/12)}\right] = \frac{Et}{1.8+.216} = \frac{.496\ Et}{\underline{214{,}270\ kpf}}$$

Wall (8-16)

$$k_{8-16} = Et\Big/\left[\frac{3\times12}{24} + \frac{12^3}{12(24^3/12)}\right] = \frac{Et}{1.5+.125} = \frac{.615\ Et}{\underline{265{,}680\ kpf}}$$

Wall (5-6)

$$k_{5-6} = Et(.16)^*\Big/\left[\frac{30\times12}{20} + \frac{12^3}{12(20^3/12)}\right] = \frac{.16Et}{1.8+.216} = \frac{.079\ Et}{\underline{34{,}128\ kpf}}$$

Wall (10-17)

$$k_{10-17} = .16\ Et\Big/\left[\frac{3\times12}{8} + \frac{12^3}{12(8^3/12)}\right] = \frac{.16Et}{4.5+3.375} = \frac{.020\ Et}{\underline{8{,}640\ kpf}}$$

Wall (10-14)

$$k_{10-14} = .16Et\Big/\left[\frac{3\times12}{16} + \frac{12^3}{12(16^3/12)}\right] = \frac{.16\ Et}{2.25+.422} = \frac{.0584\ Et}{\underline{25{,}230\ kpf}}$$

* t = Equivalent thickness (see sect. d).

The analysis of walls maybe simplified considerably by using the stiffness chart of figure 5.6. To demonstrate its use two walls will be analyzed.

North Wall (1-3)

$(h/L)_1 = 3/40 = .075$, $\quad k_1 = 4.45\,Et$

$\Delta_1 = V_1/k_1 = V/4.45Et = .2247\,V/Et$

$\Delta_7 = V_7/k_7 = .2247\,V/Et$

V is distributed to piers 2 to 6 in proportion to stiffness,

$(h/L)_{2,3,5,6} = 6/4 = 1.5$, $\quad k_{2,3,5,6} = .1275\,V/Et$

$(h/L)_4 = 6/8 = .75$, $\quad k_4 = .372\,V/Et$

$\Sigma k = 4(.1275) + .372 = .882\,V/Et$

$V_{2,3,5,6} = (.1275/.882)V = .1445\,V$

$V_4 = (.372/.882)V = .4220\,V$

$\Sigma V = [4(.1445) + .422]V = 1.0\,V$ ✓

Δ_2 to Δ_6 are all equal

$V_4/k_4 = 1.1344\,V/Et$

$\Delta = \Delta_1 + \Delta_4 + \Delta_7 = (.2247 \times 2 + 1.1344)V/Et = 1.584\,V/Et$

$k_{1-3} = V/\Delta = Et/1.584 = \underline{.631\,Et}$

$Et = 432{,}000^{ksf} \times 1.0' = 432{,}000$ kpf

$\therefore k_{1-3} = 432{,}000\,(.631) = \underline{272{,}590}$ kpf

South Wall (13-16)

$(h/L)_1 = 3/60 = .05, \qquad k_1 = 6.64\ Et$

$(h/L)_8 = 3/30 = .10, \qquad k_8 = 3.34\ Et$

$(h/L)_9 = 3/24 = .125, \qquad k_9 = 1.75\ Et$

$$V_8 = \frac{k_8}{k_8 + k_9} V = \frac{3.34}{5.09} V = .6562\ V$$

$\Delta_1 = V/k_1 = .1506\ V/Et$

$\Delta_8 = V_8/k_8 = (.6562/3.34)(V/Et) = .1965\ V/Et$

V_8 is distributed to piers 2 & 4 according to their stiffness,

$(h/L)_{2,3} = 6/5 = 1.2, \qquad k_{2-3} = .187\ Et$

$(h/L)_4 = 6/12 = .5, \qquad k_4 = .615\ Et$

$\Sigma k = .989\ Et$

$V_2 = (.187/.989)(.6562\ V) = .1241\ V$

$\Delta_2 = V_2/k_2 = (.1241/.187)(V/Et) = .6636\ V/Et$

$\Delta = \Delta_8 + \Delta_2 + \Delta_1 = (.1965 + .6636 + .1506)\ V/Et = 1.011\ V/Et$

$k_{13-16} = V/\Delta = \underline{.9894\ Et}$

$= (.9894)(432{,}000) = \underline{427{,}420}\ kpf$

f. Mass Centroids

Roof Diaphragm and 3rd Story Walls

Designation	Operation	W (kip)	x (ft)	y (ft)	Wx (ft-k)	Wy (ft-k)
Parapet 1-3	.120(2)(40)	9.6	20	40	192	384
" 3-7	.24 (16)	3.84	40	32	153.6	122.88
" 7-8	" (20)	4.8	50	24	240	115.2
" 8-16	" (24)	5.76	60	12	345.6	69.12
" 1-13	" (40)	9.6	0	20	0	192
" 13-16	" (60)	14.4	30	0	432	0
Roof 1-4-16-13	.08 (40×60)	192	30	20	5760	3840
" 3-4-7-8	−.08 (20×16)	−25.6	50	32	−1280	−819.2
Wall 1-3	.12(6)(40)(.8)*	23.04	20	40	460.8	921.6
3-7	.72 (16)(1.0)	11.52	40	32	460.8	368.64
7-8	" (20)(1)	14.4	50	24	720	345.6
8-16	" (24)(1)	17.28	60	12	1036.8	207.36
13-16	" (60)(.867)	37.45	30	0	1123.6	0
1-13	" (40)(.9)	25.92	0	20	0	518.4
5-6	.055(6)(20)(1)	6.6	10	24	66	158.4
10-17	.33 (8)(1)	2.64	24	16	63.36	42.24
10-14	" (16)(1)	5.28	20	8	105.6	42.24
	Σ =	358.5	27.56	18.15	9880.2	6508.5

* Factors to account for percent solid wall (openings out)

3rd story diaphragm

Slab 1-4-16-13	same as roof	192			5760	3840
" 3-4-7-8	"	−25.6			−1280	−819.2
Wall 1-3	Twice as above	46.08			921.6	1843.2
3-7	"	23.04			921.6	737.28
7-8	"	28.8			1440	691.2
8-16	"	34.56			2073.6	414.72
13-16	"	74.9			2247.2	0
1-13	"	51.84			0	1036.8
5-6	"	13.2			132	316.8
10-17	"	5.28			126.72	84.48
10-14	"	10.56			211.2	84.48
	Σ =	454.7	27.61	18.10	12554	8230

Second Story Diaphragm

Designation	Operation	W (kip)	x (ft)	y (ft)	Wx (ft-k)	Wy (ft-k)
Slab 1-4-16-13	Same as roof	192			5760	3840
" 3-4-7-8	"	-25.6			-1280	-819.2
Wall 1-3	Twice as roof	46.08			921.6	1843.2
3-7	"	23.04			921.6	737.28
7-8	"	28.8			1440	691.2
8-16	"	34.56			2073.6	414.72
13-16	.12(12)(60)(.824)	71.19	30	0	2135.8	0
1-13	Twice as roof	51.84			0	1036.8
5-6	"	13.2			132	316.8
10-17	"	5.28			126.72	84.48
10-14	"	10.56			211.2	84.48
	Σ	451	27.59	18.25	12443	8230

g. *Torsional Analysis and Shears in Walls*
(units : feet, kips)

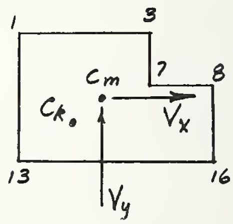
Third Story Walls and Roof Diaphragm

DIR.	Wall	k	x	y	kx	ky	d	kd^2	$\frac{k}{\Sigma k}\cdot V$	$\frac{kd}{\Sigma kd^2}\cdot M_T$	Accid. Torque	Wall V
y-direction	1-13	.935	0	–	0	–	-26.7	666.6	28.8	N.C.	∓2.2	31.
	3-7	.374	40	–	15.	–	13.3	66.2	11.6	"	±.4	12.
	8-16	.615	60	–	36.9	–	33.7	682.	18.9	"	±1.8	20.7
	10-14	.07	20	–	1.4	–	-6.7	3.1	2.1	"	0	2.1
	Σ	1.994			53.3				61.4		0	
x-direction	1-3	.536	–	40	–	21.5	25.25	341.7	14.6	1.4	±1.2	16.
	7-8	.496	–	24	–	11.9	9.25	42.4	13.6	.5	±.4	14.1
	13-16	1.282	–	0	–	0	-14.75	278.9	35.0	-2.0	∓1.7	36.7
	5-6	.079	–	24	–	1.9	9.25	6.8	2.1	.1	±.1	2.2
	10-17	.02	–	16	–	.32	1.25	0	.5	0	0	.5
	Σ	2.413				35.6		2087	65.8			

$x_m = 27.56'$, $\quad x_k = 53.3/1.994 = 26.73'$

$y_m = 18.15'$, $\quad y_k = 35.6/2.413 = 14.75'$

$V_y = 61.4^k$, $\quad V_x = 65.8^k$



$M_{Ty} = F_{3y}(x_m - x_k) = 61.4(.83) = 51^{'k}$ $(\div 2087 = .0244)$

$M_{Tx} = F_{3x}(y_k - y_m) = 65.8(-3.4) = -223.7^{'k}$ $(\div 2087 = -.107)$

Accidental Torques (2314 g 1970 UBC):

$M_{Ty} = 61.4(.05)(60) = \pm 184.2^{'k}$ $(\div 2087 = .088)$ Governs

$M_{Tx} = 65.8(.05)(40) = \pm 131.6^{'k}$ $(\div 2087 = .063$ N.C.$)$

Second Story Walls and Third Floor Diaphragms

	Wall	k	x	y	kx	ky	d	kd^2	$\frac{k}{\Sigma k}V$	$\frac{kd}{\Sigma kd^2}M_T$	Accid. Torque	Wall V
y - direction	1-13								53.1	N.C.	∓4.3	57.4
	3-7								21.2	"	±.8	22.
	8-16								34.9	"	±3.5	38.4
	10-14								4.0	"	0	4.0
	Σ								113.2		0✓	
x - direction	1-3								26.9	2.7	±2.3	29.6
	7-8								25.0	.9	±.8	25.9
	13-16								64.4	-3.8	∓3.2	67.6
	5-6								4.0	.2	±.1	4.2
	10-17								1.0	0	0	1.0
	Σ	Same as 3^{d} story values							121.3	0✓	0✓	

$x_m = 27.61'$, $x_k = 26.73'$

$y_m = 18.10'$, $y_k = 14.75'$

$V_y = 61.4 + 51.8 = 113.2^k \quad (\div 1.994 = 56.77)$

$V_x = 65.8 + 55.5 = 121.3^k \quad (\div 2.413 = 50.27)$

$M_{Ty} = 51.0 + F_{2y}(x_m - x_k) = 51 + 51.8(.88) = 96.6^{'k} (\div 2020 = .048)$

$M_{Tx} = -223.7 + F_{2x}(y_k - y_m) = -223.7 - 55.5(3.35) = -410.^{'k} (\div 2020 = -.203)$

Accidental Torques:

$M_{Ty} = 113.2(.05 \times 60) = \pm 339.6 \quad (\div 2020 = \underline{.168} \text{ Governs})$

$M_{Tx} = 121.3(.05 \times .40) = $ N.C.

First Story Walls and Second Floor Diaphragm

	Wall	k	x	y	kx	ky	d	kd^2	$\frac{k}{\Sigma k}V$	$\frac{kd}{\Sigma kd^2}M_T$	Accid. Torque	Wall Shear
y-direction	1-13	.935	0	–	0	–	-26.7	666.6	64.8	N.C.	∓5.2	70.
	3-7	.374	40	–	15	–	13.3	66.2	25.9	"	±1.0	26.9
	8-16	.615	60	–	36.9	–	33.3	682.	42.6	"	±4.2	46.8
	10-14	.07	20	–	1.4	–	-6.7	3.1	4.8	"	0	4.8
	Σ	1.994			53.3				138.1✓		0✓	
x-direction	1-3	.536	–	40	–	21.5	23.36	292.5	37.1	3.1	±2.6	40.2
	7-8	.496	–	24	–	11.9	7.36	26.9	34.3	.9	±.7	35.2
	13-16	1.006	–	0	–	0	-16.64	278.6	69.7	-4.1	∓3.4	73.1
	5-6	.079	–	24	–	1.9	7.36	4.3	5.5	.1	±.1	5.6
	10-17	.02	–	16	–	.3	-.64	0	1.4	0	0	1.4
	Σ	21.37				35.6		2020	148.✓	0✓	0✓	

$x_m = 28.58'$, $\quad x_k = 53.3/1.994 = 26.7'$

$y_m = 19.88'$, $\quad y_k = 35.6/21.37 = 16.64'$

$M_{Ty} = 96.6 + F_{1x}(x_m - x_k) = 96.6 + 24.9(1.84) = 142.4^{1k} (\div 2020 = .07)$

$M_{Tx} = -410.2 + F_{1x}(y_k - y_m) = -410. - 26.7(3.24) = -496.7^{1k} (\div 2020 = -.246)$

$V_y = 113.2 + 24.9 = 138.1^{k} \ (\div 1.994 = 69.26)$

$V_x = 121.3 + 26.7 = 148.0^{k} \ (\div 2.137 = 69.26)$

Accidental Torques:

$M_{Ty} = 138.1\,(.05 \times 60) = 414.3^{1k} \ (\div 2020 = \underline{.205}$ Governs$)$

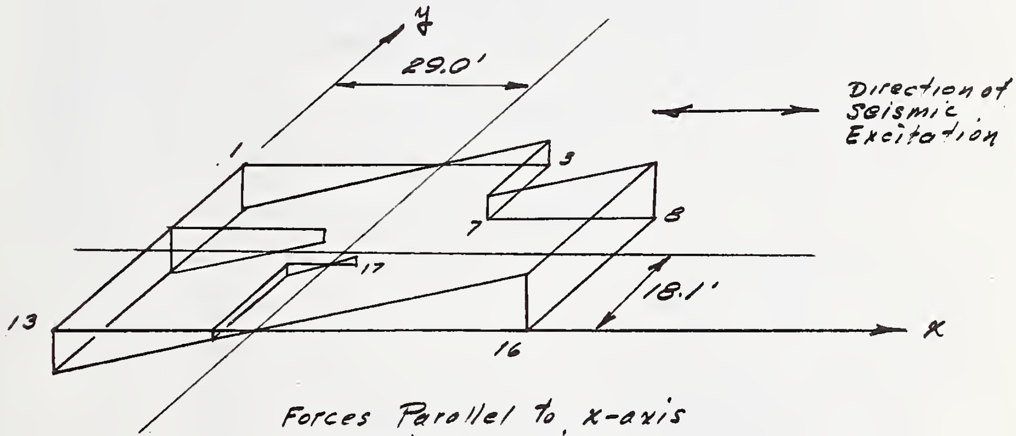
$M_{Tx} = 148.\,(.05 \times 40) = 296. \ (\div 2020 = .146$ N.C.$)$

h. Stresses Produced by Overturning Moments

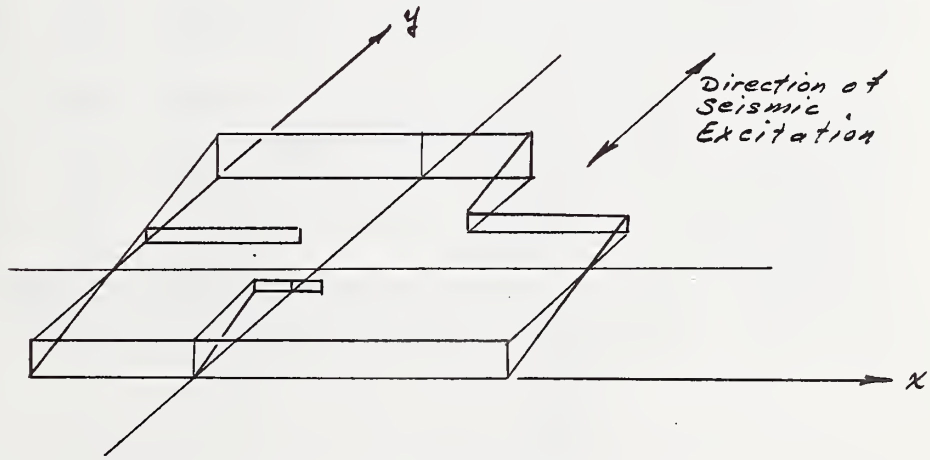
From section (b), $I_x = 43{,}610\ ft^4$ (all stories)

$I_y = 88{,}120\ ft^4$ (assumed for all stories)

From moment diagrams of section (d) and flexure formula $\sigma = Mc/I$, the distributed loads intensities are computed as shown schematically below. It is noted that all distributed loads are reversible.



Forces Parallel to x-axis
(y-axis bending)



Forces Parallel to y-axis
(x-axis bending)

Average moments taken at mid-heights of respective stories will be used in subsequent analysis.

3d story: $M_x/I_x = 369/43610 = .0085$ ksf, $M_y/I_y = 395/88120 = .0045$ ksf

2d Story: $M_x/I_x = 1416/$ " $= .0325$ ksf, $M_y/I_y = 1416/$ " $= .0161$ ksf

1st Story: $M_x/I_x = 2923/$ " $= .067$ ksf, $M_y/I_y = 3133/$ " $= .0356$ ksf

Point	x (ft)	y (ft)	$M_x y/I_x$ (ksf)			$M_y x/I_y$ (ksf)			
			3d story	2d st.	1st st.	3d st.	2d st.	1st st.	
1	29	21.9	.186	.712	1.467	.131	.467	1.032	
2	9	"	"	"	"	.041	.145	.320	
3	11	"	"	"	"	.050	.177	.392	
4	31	"	"	"	"	.140	.500	1.104	
5	29	5.9	.050	.192	.395				
6	9	"	"	"	"				Ditto
7	11	"	"	"	"				
8	31	"	"	"	"				
9	29	2.1	.018	.068	.141				
10	9	"	"	"	"				
11	11	"	"	"	"				Ditto
12	31	"	"	"	"				
13	29	18.1	.014	.588	1.213				
14	9	"	"	"	"				
15	11	"	"	"	"				Ditto
16	31	"	"	"	"				
17	1	2.1	.018	.068	.141	.005	.016	.036	

i stress Analysis of Walls

Wall 1-3

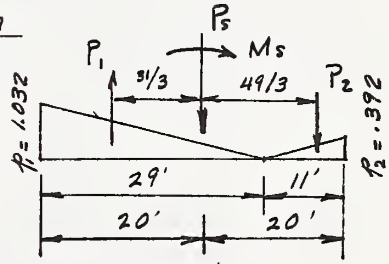
Gravity Loads

$P = L\,\Sigma w_D$

Location	operation	w_D (kpf)	Σw_D (kpf)	P_D (k)	$w_L = .75 w_D$ (kpf)	Σw_L (kpf)	P_L (k)
Roof	.08 (8)	.640			.480		
Parapet	.12 (2)	.240					
Wall (sect. f)	23.04 /40	.576	1.456	58.2		.480	19.2
3d Fl.	.073 (8)	.584			.438		
Wall	.576 (2)	1.152	3.192	127.7		.918	36.7
2d Fl.		.584			.438		
Wall		1.152	4.928	197.1		1.356	54.2

Seismic Loads in x-direction

First Story:



Effective thickness $t = .8'$

$$P_1 = (\mp 1.032)(29/2)(.8) = \mp 11.97$$

$$P_2 = (\pm .392)(11/2)(.8) = \pm 1.72$$

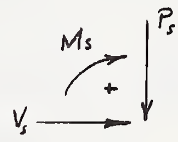
$$P_s = \mp 10.25^k$$

$$M_s = \pm\left[11.97(31/3) + 1.72(49/3)\right] = \pm 151.8^{'k}$$

Second Story:

$$P_s = \mp 10.25(.467/1.032) = 4.64^k$$

$$M_s = \pm 151.8(\quad " \quad) = 68.7^{'k}$$



SIGNS

Third Story:

$$P_s = \mp 10.25(.131/1.032) = 1.30^k$$

$$M_s = \pm 151.8(\quad " \quad) = 19.3^{'k}$$

$$V_s = \pm \begin{Bmatrix} 16.0^k - 3^d \text{ story} \\ 29.6^k - 2^d \text{ story} \\ 40.2^k - 1\text{st story} \end{Bmatrix}$$

(see sect. g)

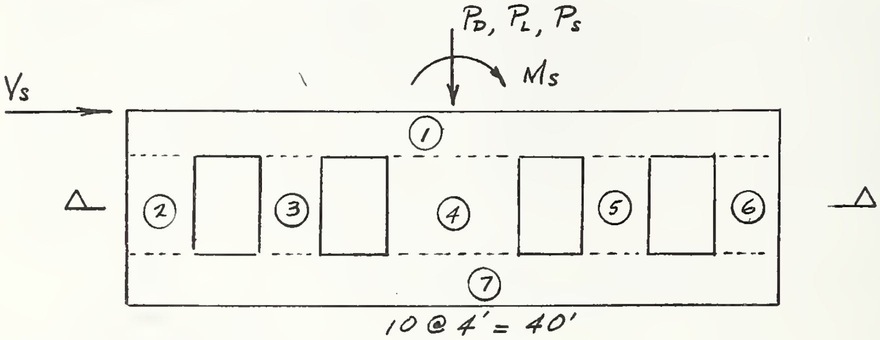
Seismic Loads in y-direction

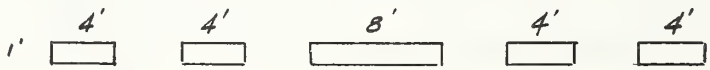
$M_s = 0$ for all floors

$$P_s = \pm \begin{Bmatrix} .186 \\ .712 \\ 1.467 \end{Bmatrix} (.8)(40) = \pm \begin{Bmatrix} 6.0 \\ 22.8 \\ 46.9 \end{Bmatrix} \text{kips @} \begin{Bmatrix} 3^{d} \text{ story} \\ 2^{d} \text{ "} \\ 1^{st} \text{ "} \end{Bmatrix}$$

$$V_s = \mp \begin{Bmatrix} 1.4 \\ 2.7 \\ 3.1 \end{Bmatrix} \text{kips @} \begin{Bmatrix} 3^{d} \\ 2^{d} \\ 1^{st} \end{Bmatrix} \text{stories} \quad \text{(see sect. g)}$$

Check Pier Stresses (x-direction)





Pier stresses due to V_s (see sect. e)

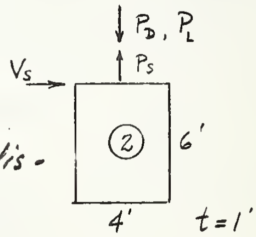
$$V_{2,3,5,6} = .1445\, V_s = .1445 \begin{Bmatrix} 16.0 \\ 29.6 \\ 40.2 \end{Bmatrix} = \begin{Bmatrix} 2.3^K \\ 4.3^K \\ 5.8^K \end{Bmatrix} \begin{matrix} 3^{d} \\ 2^{d} \\ 1^{st} \end{matrix}$$

$$V_4 = .422\, V_s = .422 \{ do \} = \begin{Bmatrix} 6.8^K \\ 12.5^K \\ 17.0^K \end{Bmatrix} \begin{matrix} 3^{d} \\ 2^{d} \\ 1^{st} \end{matrix}$$

Let I_p = moment of inertia of section through piers

$$I_p = 4\left(\frac{1 \times 4^3}{12}\right) + \frac{1(8)^3}{12} + 2\left[4(1)(18)^2 + 4(1)(10)^2\right] = 3456 \text{ ft}^4$$

Investigation of Pier 2



$A = 48 \times 12 = 576 \text{ in}^2$

Assuming concentric vertical lds. are distributed to the piers in proportion to their areas, Pier 2 takes $\frac{1}{6}$th of total load.

$$P_D = \frac{1}{6}\begin{Bmatrix}58.2\\127.7\\197.1\end{Bmatrix} = \begin{Bmatrix}9.7\\21.3\\32.8\end{Bmatrix} \text{ kips}$$

$$P_L = \frac{1}{6}\begin{Bmatrix}19.2\\36.7\\54.2\end{Bmatrix} = \begin{Bmatrix}3.2\\6.1\\9.1\end{Bmatrix} \text{ kips}$$

$$P_s' = -\frac{1}{6}\begin{Bmatrix}1.30\\4.64\\10.25\end{Bmatrix} = -\begin{Bmatrix}.22\\.77\\1.71\end{Bmatrix} \text{ kips}$$

$$P_s'' = -Lt\,\frac{Mc_2}{I_p} = -\frac{(18)(4)(1)}{3456}\begin{Bmatrix}19.3\\68.7\\151.8\end{Bmatrix} = -\begin{Bmatrix}.40\\1.43\\3.16\end{Bmatrix} \text{ kips}$$

$$P_s = P_s' + P_s'' = -\begin{Bmatrix}.62\\2.20\\4.87\end{Bmatrix}$$

Load factor criteria for ultimate design are to be specified by VA. The following criteria will be herein considered for illustration purposes only:

$$U = .75\,(1.4D + 1.7L) + S$$
$$U = .9D + S$$

Then,

$$P_{min} = .9P_D - |P_s| = .9\begin{Bmatrix}9.7\\21.3\\32.8\end{Bmatrix} - \begin{Bmatrix}.62\\2.2\\4.87\end{Bmatrix} = \begin{Bmatrix}8.11\\16.97\\24.65\end{Bmatrix} \text{ kips}$$

$$P_{max} = .75(1.4)\begin{Bmatrix}9.7\\21.3\\32.8\end{Bmatrix} + .75(1.7)\begin{Bmatrix}3.2\\6.1\\9.1\end{Bmatrix} + \begin{Bmatrix}.6\\2.2\\4.9\end{Bmatrix} = \begin{Bmatrix}14.9\\37.4\\50.9\end{Bmatrix} \text{ kips}$$

$$v = \frac{V_s}{A} = \frac{1}{576}\begin{Bmatrix}2300\\4300\\5800\end{Bmatrix} = \begin{Bmatrix}4.0\\7.5\\10.1\end{Bmatrix} \text{ psi}$$

$$f_{m\,(min)} = \frac{P_{min}}{A} = \frac{1}{576}\begin{Bmatrix}8{,}110\\16{,}970\\24{,}650\end{Bmatrix} = \begin{Bmatrix}14.1\\29.5\\42.8\end{Bmatrix} \text{ psi}$$

Strength reduction factor:

$$r = \underbrace{(1.0)}_{\text{Inspected construction}}\underbrace{(2/3)}_{\text{scatter}}\underbrace{(2/3)}_{\text{size effects}}\underbrace{(2/3)}_{\text{past E.Q. exposure}} = 8/27 = \underline{.30}$$

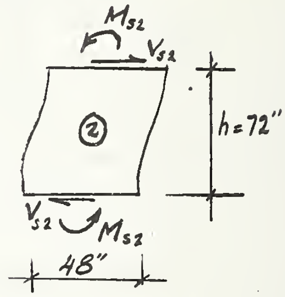
$f'_v = 120$ psi, then, in accordance w/ eq. (5.18),

$v' = r(f'_v + .3 f_m)$ or,

$$v' = .30\left(120 + .3\begin{Bmatrix}14.1\\29.5\\42.8\end{Bmatrix}\right) = \begin{Bmatrix}37.3\\38.7\\39.8\end{Bmatrix}\text{ psi} \gg \begin{Bmatrix}4.0\\7.5\\10.1\end{Bmatrix}\text{ psi}$$

∴ Pier 2 is adequate in racking shear

<u>Compression w/ in-plane flexure</u>



$$I/c = (48)^2(12)/6 = 4608 \text{ in}^3$$

$$M_{s2} = \frac{V_{s2}h}{2} = \frac{72}{2}\begin{Bmatrix}2300\\4300\\5800\end{Bmatrix} = \begin{Bmatrix}82,800\\154,800\\208,800\end{Bmatrix}\text{ in-lb}$$

$$f_{m\,(min)} = \frac{P_{min}}{A} - \frac{M_{s2}c_2}{I_2}$$

$$= \begin{Bmatrix}14.1\\29.5\\42.8\end{Bmatrix} - \frac{1}{4608}\begin{Bmatrix}82,800\\154,800\\208,800\end{Bmatrix} = -\begin{Bmatrix}3.7\\4.1\\2.5\end{Bmatrix}\text{ psi (all tension)}$$

$$f_{m\,(max)} = \frac{P_{max}}{A} + \frac{M_{s2}c_2}{I_2}$$

$$= \frac{1}{576}\begin{Bmatrix}14900\\32400\\50900\end{Bmatrix} + \frac{1}{4608}\begin{Bmatrix}82,800\\154,800\\208,800\end{Bmatrix} = \begin{Bmatrix}44\\90\\134\end{Bmatrix}\text{ psi}$$

$f_{m\,(max)}$ above was calculated on basis of uncracked section. If tensionless material is assumed for masonry, then moment capacity should be checked in accord w/ eq. (5.5) for cracked section. Thus,

$c = 24''$, $e_k = 48/6 = 8''$, $a = 1.3$, $r = .30$

$$P_k = \frac{aP_o}{2} = \frac{raAf'_m}{2} = \frac{(.3)(1.3)(576)(3.6)}{2} = 405^k$$

$$g = 2\left(1 - \frac{I}{Ac^2}\right) = 1.33$$

$$M_e = Pc\left(1 - \frac{gP}{aP_o}\right) = 24\left(1 - \frac{1.33}{2\times 405}\cdot\begin{Bmatrix}8.11\\16.97\\24.65\end{Bmatrix}\right)\begin{Bmatrix}8,110\\16,970\\24,650\end{Bmatrix} = \begin{Bmatrix}192,100\\395,800\\567,900\end{Bmatrix}\text{ in-lb}$$

Since these are > than M_{s2} values, section is adequate in in-plane flexure.

Out-of-plane flexure: (Y-direction)

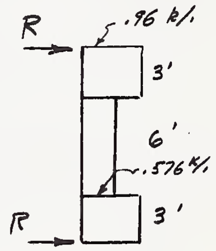
$V_2 = .1445\, V_s = .1445 \begin{Bmatrix} 1.4 \\ 2.7 \\ 3.1 \end{Bmatrix} = \begin{Bmatrix} .2 \\ .4 \\ .5 \end{Bmatrix}$ kips (neglect)

Transverse force coeff. according to Table 2, VA Manual [80].

$C_p = .20, \quad W_p = 2(23.04) = 46.08^k$ (sect. f)

$Z = 1.0$ (zone 3)

$F_p = Z C_p W_p = .2(46.08) = 9.2^k$



$A_{net} = .8(40 \times 12) = 384\ ft^2$

$w_p = F_p / A_n = 9.2/384 = .024$ ksf

$.024(40') = .96$ k/.
$.024(24) = .576$ k/.

$R = F_p/2 = 9.2/2 = 4.6^k$

$$M = \frac{.96(12)^2}{8} - \frac{.576(3)^2}{2} = 14.7^{\prime k}$$

On Pier 2, $M_2 = 14.7(4'/40') = 1.47^{\prime k} = 17.63$ in-k

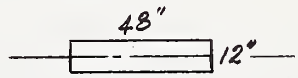
(note: wall is conservatively assumed s.s. at T & B only)

Check Pier 2 @ first floor level:

$P_D = 32.8^k, \quad P_L = 9.1^k, \quad P_S = 1.47^{ksf}(4' \times .8') = \pm 4.7^k$

$P_{max} = .75(1.4P_D + 1.7P_L) + P_S = 50.7^k$

$P_{min} = .9P_D - P_S = 24.8^k$



Using eqs. (5.5),

$c = 6'', \quad e_k = \frac{t}{6} = 12/6 = 2'', \quad a = 1.3, \quad r = .30$

$$P_k = \frac{aP_o'}{2} = \frac{aAf_m'}{2} = 405^k \text{ (as before)}$$

$$M_e = Pc\left(1 - \frac{gP}{aP_o}\right) = 24.8(6)\left(1 - \frac{4}{3} \cdot \frac{24.8}{2 \times 405}\right) = 6 \text{ in-kip} < 17.63$$

$\therefore$ out-of-plane flexural moment capacity of pier 2 is exceeded.

APPENDIX B

Approximate Methods for Evaluating Natural Period of Vibration of a Building

B.1 General

Earthquake regulations prescribed by design codes make use of the natural period of a structure for seismic force calculations. Two simple methods for the approximate calculation of this period are described in the following sections for expedient use by the analyst. The cantilever beam formula lends itself to buildings of uniform stiffness, geometry and mass distribution. The numerical integration method is more general and admits buildings of irregular configuration.

B.2 Cantilever Beam Method

The prismatic cantilever beam is the simplest model used for the idealization of multi-story structures. The fundamental period in this case is given by the closed form equation,

$$T = 1.79\, L^2 \sqrt{\frac{m}{EI}} \qquad \text{(B.1)}$$

where (L) is the length, (E) is the elastic modulus, (I) is the moment of inertia, (m) is the distributed mass per unit length and (T) is the fundamental period of the beam. This idealization requires the determination of a uniformly distributed mass value for the building and the appropriate evaluation of constants (E), (G), (I), and (A). In equation (B.1), (L) may be considered to be the above-ground height of the structure. The uniform mass may be obtained from the total weight (W) of the structure as follows,

$$m = \frac{W}{gL} \qquad \text{(B.2)}$$

where (g) is the gravitational acceleration. The elastic modulus of the most common material in the structure may be used as a base modulus (E) of the system. The other materials are appropriately transformed at various floor levels according to modular ratios to give moment of inertia values whith are averaged out for use in equation (B.1). The calculation of sectional properties requires certain prior idealizations of individual wall dimensions to account for the effect of openings in these walls. For the purpose of predicting the fundamental period it will be sufficiently accurate to smear in the openings according to eqs. (3.10) and (3.11).

Equation (B.1) only considers the effect of flexural deformations. In a short beam (high $\sqrt{I/A}/L$ ratio), the effect of shearing deformations and rotatory inertia will be significant and should be considered when modelling a building of small height-to-width ratio. To account for these effects equation (B.1) is modified by introducing an approximate factor [71] as follows,

$$T = 1.79\ L^2\ [(1 + Q)\ \frac{m}{EI}]^{1/2} \qquad (B.3)$$

where (Q) is the ratio of the maximum static deflection due to shearing deformation to that due to flexural deformation at the end of a uniformly loaded prismatic cantilever beam. Denoting the two deflections by (Δ_f) and (Δ_v), the distributed load by (q), and the form factor by (f), (Section 5.5), the expression for (Q) is derived as follows,

$$\Delta_v = \frac{fqL^2}{2GA}$$

$$\Delta_f = \frac{qL^4}{8EI}$$

then,

$$Q = \frac{4}{L^2} \cdot \frac{EI}{GA/f} \qquad (B.4)$$

In evaluating the above parameters it should be kept in mind that "immovable" or "fixed" live loads and non-structural attachments contribute to the mass of the system. In addition, non-structural elements such as partition walls, contribute to the lateral stiffness of the system whenever they are supported along more than one edge. Consideration of such refinements is subject to judgment in view of the approximation inherent in the beam model.

B.3 Numerical Integration

Certain structures cannot be adequately idealized by prismatic beam models. Buildings of irregular shape or with setbacks fall in this category. For such cases, the structure in question may be idealized by a non-prismatic beam having a discrete number of segments with lumped (story) masses at its nodes, and analyzed numerically. Among several good numerical techniques devised to analyze a discretized beam, Newmark's numerical integration method readily suggests itself [62]. The method lends itself to manual calculation (as opposed to computer processing) and is applicable with almose equal ease to problems in which segment lengths, sectional properties, elastic constants or lumped mass can be treated as variables. It can also be used to calculate higher natural frequencies. A concise treatment of the method follows.

According to ordinary beam theory, the relationships between distributed loads (q), shear forve (V), moment (M), slope (Θ) and deflection (y) in a beam are given in the equations,

$$V = \int q dx$$
$$M = \int V dx$$
$$\Theta = \int \frac{M}{EI} dx + \int \frac{f}{AG} q dx \qquad (B.5)$$
$$y = \int \Theta \; dx$$

The effect of shearing distortions is considered in the second term on the right hand side of the third equation above.

For a beam divided into segments of finite length interval (h), (as opposed to infinitesimal length dx), the approximate relationships corresponding to equations (B.5) are given by,

$$V \simeq h\Sigma\bar{q}$$
$$M \simeq h\Sigma V$$
$$\Theta \simeq h(\Sigma\frac{M}{EI} + \Sigma\frac{f\bar{q}}{AG}) \qquad (B.6)$$
$$y \simeq h\Sigma\Theta$$

where the quantities under the summation sign (Σ) represent either equivalent nodal values (q, M/EI, fq/AG and y), or constants between two consecutive nodes (V and Θ).

For dissimilar segment lengths, equations (B.6) are modified by including variable length (h) under the summation symbol. Thus,

$$V \simeq \Sigma h\bar{q}$$
$$M \simeq \Sigma hV$$
$$\Theta \simeq \Sigma\frac{Mh}{EI} + \Sigma\frac{fgh}{GA} \qquad (B.7)$$
$$y \simeq \Sigma h\Theta$$

The deflection of a beam due to a distributed load (q) can be computed by successive numerical integration using equation (B.6). To simplify numerical calculations, another approximation is introduced whereby distributed functions (such as, q, or M/EI) are converted into their nodal equivalent concentrated values (ch. 3, Ref. [62]). If the particular function is designated by (s), its nodal equivalent ($\bar{s}$), in the case of equal segments, is given by the equation,

$$\bar{s}_i = \frac{h}{12} (s_{i-1} + 10s_i + s_{i+1}) \qquad (B.8)$$

for an interior node (i), and

$$\bar{s}_o = \frac{h}{24} (7s_o - 6s_1 - s_2)$$

and $\qquad$ (B.9)

$$\bar{s}_n = \frac{h}{24} (7s_n + 6s_{n-1} - s_{n-2})$$

for end nodes (o) and (n), respectively.

Similarly, in the case of unequal segments,

$$\bar{s}_i = (1/6) [h_i (2s_i + s_{i-1}) + h_{i+1}(2s_i + s_{i+1})] \qquad (B.10)$$

for interior node i, and

$$\bar{s}_o = \frac{h_1}{6} (2s_o + s_1)$$

and $\qquad$ (B.11)

$$\bar{s}_n = \frac{h_n}{6} (2s_n + s_{n-1})$$

for end nodes (o) and (n), respectively.

To determine the natural frequency by numerical integration, a deflected shape is first assumed. For periodic motion, the inertial force ($-\bar{m}a$) is equal to ($\bar{m}_i p^2 y_i$) where ($\bar{m}_i$) and (y_i) are the lumped mass and the assumed displacement at any node (i), and (p) is the circular frequency. Using numerical integration, a new deflected shape expressed in terms of (p^2) is obtained. Equating computed nodal deflections to the assumed values, (p^2) is determined at each node. If the assumed shape is close to the modal shape, the computed frequencies at the various nodes will be in close agreement, otherwise, a second cycle is used in which the assumed deflections are the computed deflections of the first cycle. Due to rapid convergence of the process, one or at most two cycles should yield sufficiently accurate results if good judgement is used in selecting the initial displacements.

B.4 Fundamental Period of a Beam by Numerical Integration

The fundamental period of a prismatic and of a non-prismatic beam are calculated below by the numerical integration procedure described in Section B.3. Since notation form is retained for some of the parameters, units have been omitted in these calculations.

Prismatic Beam.

Mass per unit length (m), flexural rigidity (EI) and shear rigidity (GA/f) are treated as constants along the length of the beam, with G = 0.4E, f = 1.2 and EI = 36 (GA/f). The rectangular cross section has a width of 5 and a depth of 12. The four segments are of equal length h = 60. Also, A = (5)(12) = 60, I = $(5)(12)^3/12$ = 720.

The tables below indicate two cycles to yield sufficiently close values of (p^2) at the nodes. A linear least squares fit gives [62],

$$p^2 = \frac{\Sigma y_2 y_3}{\Sigma y_3^2} = .0432 \frac{EI}{mh^4}$$

which yields, upon substitution of 4h = L, the fundamental period,

$$T = \frac{2\pi}{p} = 30.2h^2\sqrt{\frac{m}{EI}} = 1.89L^2\sqrt{\frac{m}{EI}}$$

This is in good agreement with the closed-form solution (Eq. B.1), the higher estimate being due to the inclusion of shearing distortion effects and approximations involved in the numerical integration procedure. Neglecting shearing distortions,

$$T = 2\pi\left[\frac{\Sigma y_m^2}{\Sigma y_2 y_m}\right]^{1/2} = 1.83L^2 \sqrt{\frac{m}{EI}}$$

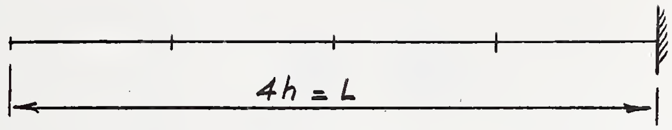
Non-prismatic Beam.

In this beam the distributed mass (m), flexural rigidity (EI) and shear rigidity (GA/f) are assumed to vary along the beam, with the nodal values as indicated in the top three lines of the numerical integration tables below. A least squares fit to the results of the second cycle gives the fundamental period,

$$T = 2\pi\left[\frac{\Sigma y_3^2}{\Sigma y_2 y_3}\right]^{1/2} = 1.54L^2 \sqrt{\frac{m}{EI}}$$

The contribution of shear deformations turned out to be negligible due to the slenderness of the beam selected for this example. In actual structures, shearing deformations are often more dominant as indicated by the numerical example in Appendix A.

Prismatic Beam



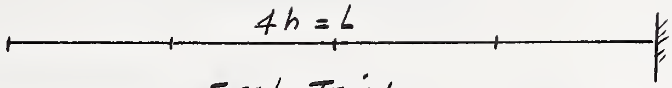
First Trial

Function										Units
$\bar{m}$	.5		1.0		1.0		1.0			mh
y_1	16		9		4		1		0	
$\bar{q} = \bar{m}p^2y_1$	8		9		4		1		0	mhp^2
$V = h\Sigma\bar{q}$		8		17		21		22		〃
$M = h\Sigma V$	0		8		25		46		68	mh^2p^2
M/EI	0		8		25		46		68	mh^2p^2/EI
$\bar{\varphi}$	.96		8.75		25.3		46.1		30.3	mh^3p^2/EI
$\theta_m = h\Sigma\bar{\varphi}$	111.4	110.4		101.7		76.4		30.3		〃
$y_m = h\Sigma\theta_m$	318.9		208.4		106.7		30.3		0	mh^4p^2/EI
$\Delta\theta_v = V/GA/f$		8		17		21		22		$mhp^2/\frac{GA}{f}$
$\theta_v = \Sigma\Delta\theta_v$		68		60		43		22		〃
$\Delta y_v = h\theta_v$	68		60		43		22		0	$mh^2p^2/\frac{GA}{f}$
$y_v' = \Sigma\Delta y_v$	193		125		65		22		0	〃
$y_v = 36y_v'/60^2$	1.9		1.3		.6		.2		0	mh^4p^2/EI
$y_2 = y_m + y_v$	320.8		209.7		107.3		30.5		0	〃
$p^2(y_1/y_2)$	.0499		.0429		.0373		.0328			EI/mh^4

Second Trial

Function						Units
$\bar{m}$	.5	1.0	1.0	1.0	1.0	mh
y_2	10.5	6.9	3.5	1.0	0	mhp^2
$\bar{q}$	5.25	6.9	3.5	1.0	0	mhp^2
V	5.25	12.15	15.65	16.65		mhp^2
M/EI	0	5.25	17.4	33.05	49.7	mh^2p^2/EI
$\bar{\varphi}$	.59	5.82	17.69	33.13	22.03	mh^3p^2/EI
θ_m	78.67	72.85	55.16	22.03		$m \cdot h^3p^2/EI$
y_m	228.7	150.	77.2	22	0	mh^4p^2/EI
$\Delta\theta_v = .1V/EI$	.525	1.215	1.565	1.665		mh^3p^2/EI
Δy_v	4.97	4.445	3.23	1.665	0	mh^4p^2/EI
y_v	14.32	9.34	4.9	1.67	0	"
$y_3 = y_m + y_v$	243.	159.4	82.1	23.7	0	"
$p^2 = y_2/y_3$	.0432	.0433	.0426	.0422	0	EI/mh^4
$p^2\ (y_2/y_m)$	.0459	.0460	.0453	.0454		"

Non-Prismatic Beam



First Trial

Function										Units
$\bar{m}$	.5		1.1		1.2		1.3			mh
EI		1.1		1.25		1.4		1.55		
$R = EI/\frac{GA}{f}$		34.5		33.		32.		30.5		
y_1	16		9		4		1		0	
$\bar{q}$	8		9.9		4.8		1.3		0	mhp^2
V		8		17.9		22.7		24.0		"
M	0		8		25.9		48.6		72.6	mh^2p^2
M/EI	0		6.67		19.92		32.4		45.38	mh^2p^2/EI
$\bar{\varphi}$	.84		7.21		19.85		32.44		19.97	mh^3p^2/EI
θ_m	80.42	79.58		72.37		52.41		19.97		"
y_m	224.3		144.7		72.4		20.0		0	mh^4p^2/EI
$\Delta\theta_v = RV/EI$		276		590.7		726.4		732		mh^2/EI
θ_v		2325.1		2049.1		1458.4		732		mh^2p^2/EI
Δy_v	2325.1		2049.1		1458.4		732		0	"
y_v'	6564.6		4239.5		2190.4		732		0	"
$y_v = y_v'/60^2$	1.8		1.2		.6		.2			mh^4p^2/EI
$y_2 = y_v + y_m$	226.1		145.9		73.		20.2			"
$p^2 = y_1/y$	.0708		.0617		.0548		.0495			EI/mh^4

Second Trial

Function						Units
y_2	11.2	7.22	3.61	1	0	
$\bar{q}$	5.6	7.94	4.33	1.3	0	mhp^2
V	5.6	13.54	17.87	19.17		mhp^2
M	0	5.6	19.14	37.01	56.18	mh^2p^2
M/EI	0	4.67	14.70	24.67	35.11	mh^2p^2/EI
$\bar{\varphi}$	.75	5.12	14.7	24.7	15.36	mh^3p^2/EI
θ_m	59.88	54.76	40.06	15.36		mh^3p^2/EI
y_m	170.6	110.18	55.42	15.36	0	mh^4p^2/EI
$\Delta\theta_v$	193.2	446.8	572	585		mhp^2/EI
Δy_v	1797	1604	1157	585	0	mh^2p^2/EI
y_v'	5143	3346	1742	585	0	"
y_v	1.43	.93	.48	.16	0	mh^4p^2/EI
$y_3 = y_m + y_v$	172.0	111.11	55.9	15.52	0	"
$p^2 = y_2/y_3$	.0651	.0650	.646	.0644		EI/mh^4

APPENDIX C

Description of Specimens and Test Setup for Data Listed in Tables (3.1) to (3.3)

C.1 Small Compression Specimens (table 3.1)

Tests of small compression specimens described in reference [65] employed prisms of the following description: Compression tests were conducted on 3-block high and 5-brick high prisms. The block prisms were constructed in stacked bond. The brick prisms were constructed in running bond with a whole unit in the first, third and fifth courses and two half units in the second and fourth courses. The prisms constructed of 8x8x16-in hollow concrete units contained only face-shell mortar bedding. Full bed joints were used in fabricating prisms made of 8x8x16-in solid and 4x8x16-in hollow concrete block. The brick prisms were constructed with full head and bed mortar joints. The height of the brick prisms was 12.8 in and their height-to-thickness ratio was 3.5.

The small compression prisms in reference [65] were subjected to axial compressive loads in a universal testing machine. Most of the specimens were capped at top and bottom with high-strength plaster but some were tested with fiberboard instead of plaster to provide support constraints similar to those of large wall specimens. Loads were applied through steel bearing plates and a spherical loading head which allowed rotation at the top (simulated pinned support condition).

C.2 Large Compression Specimens (table 3.1)

Wall panel compression specimens described in reference [65] were constructed in running bond and were nominally 4 ft wide and 8 ft high. Thickness and cross section of the panels depended on the type of masonry units and type of construction used. The brick used in these walls were cored; for such brick, gross cross sectional areas were used in stress calculations. Bricks were laid with full bed and head mortar joints.

Hollow block walls consisted of 8x8x16-in whole 2-cell units and half units that were obtained by cutting kerf block; the bottom course contained a half unit at each end. Bed and head joint mortar was applied only to the face shells except, at the outside edges of the walls, bed mortar was applied to the exposed webs.

Solid concrete block walls were laid in the same manner as the 8-in hollow block walls except that 8x8x16-in 100% solid block were used. Full bed and head mortar joints were used in constructing these walls.

The 4-2-4-in thick block-block cavity walls contained 4-in 3-cell hollow concrete block in both wythes. Full bed and head mortar was applied, and head joints of opposite wythes were staggered. The wythes were bonded with metal ties in accordance with ANSI Standard A41.1 [81].

The cavity walls containing brick were made with a facing of brick and a backing of 3-cell 4x8x16-in hollow concrete block. Full bed and head mortar was applied in the brick facing wythe and face shell mortar was applied in the concrete block backing wythe as described previously.

In the 8-in composite wall panels the facing was made of brick and the backing of 4-in hollow block. Bonding consisted of a brick header course in every seventh brick course. Full head and bed joints were used in the brick facing and block backing. The collar joint between wythes was filled as nearly as possible by pargeting the back of the brick facing and slush-filling the remaining gap as each course of block was laid.

Compression testing of wall panels in reference [65] was accomplished in a universal testing machine. The load was applied to the wall concentrically through a loading beam, a 1-in square steel bar centered along the mid-thickness of the wall and a steel plate that covered the top area of the wall.

Fiberboard was used as bedment material at the top end of the wall and under a steel carrying channel in which the bottom course of the wall was laid in mortar during construction. Generally, the compression testing of the large wall panels followed the method of ASTM E72.

Construction of the compression wall panels of reference [4] was essentially the same as that described above for corresponding types of construction in reference [65]. However, materials used in the tests of these two references were from different sources and exhibited somewhat different properties.

ASTM Method E-72 was closely followed in the compression testing of wall panels in reference [4]. Testing was performed in a hydraulic universal testing machine. The steel carrying channel (in which the wall was built with its bottom course bedded in mortar) was set in high-strength plaster on the platen over the top of the wall and a 1-inch square steel bar transmitted the load from a loading beam to the top bearing plate along a line parallel to, and at a distance of 1/3 the wall thickness from the inside face of the wall.

C.3 Small Flexure Specimens (table 3.2)

Flexure tests were carried out on various small prisms reported in reference [65] to measure bond or tensile strength at bed joints. Two-block high concrete block prisms, laid in stacked bond, were made of both hollow and solid 8x8x16-in block and 4x8x16-in hollow block, and were constructed in the same manner as the prisms used for the compression tests. Flexure tests of concrete block prisms were conducted in accordance with ASTM E149-66. The test setup is shown in figure 4.4 and described in Section 4.

Flexure tests were also conducted on 7-course brick prisms tested as beams with the 8-in dimension of the brick horizontal. These prisms were constructed in stacked bond with full bed mortar joints, and were test loaded transversely at the third points over a 16-in clear span.

Flexure tests of 2-block high prisms reported in reference [4] were also performed by the method described in ASTM E149. Composite assemblies were tested in the same manner but were constructed differently. A prism tested with the brick facing in tension was 16-in long, 16-in high and 8-in thick. Such assemblies consisted of 6 courses of brick facing in running bond and 2 units of backing block, in stacked bond. Assemblies tested with the concrete backing in tension were 24 in high and contained three block courses in stacked bond and nine brick courses in running bond.

C.4 Large Flexure Specimens (table 3.2)

Wall panel specimens for flexure tests reported in reference [65] were of the same nominal size (4x8 ft) and construction as the corresponding types of wall panel compression specimens. Flexural testing of these wall panels was similar to the method employing air bag loading described in ASTM Method E72. However, these walls were tested in a normal vertical standing position. The transverse load was applied uniformly by a polyvinyl air bag covering the width and span of the panel. A steel reaction frame provided support for the air bag on one side of the specimen. On the opposite side of the wall, upper and lower horizontal reaction bars were spaced approximately 7 ft apart and attached to another supporting frame. The two frames were bolted together and the air bag was inflated with compressed air. The reaction bars were faced with teflon and leather.

Flexure test wall panel specimens reported in reference [4] were similar in construction to the corresponding types of compression wall panels described under C.2 above. However, the height of these flexure walls was actually 104 in to permit flexural loading over a 7 1/2-ft span. Flexural testing procedures were patterned after those of ASTM Method E72. The flexural load test apparatus consisted of a structural steel frame fitted with lateral hydraulic loading ram and reaction rollers to flex the wall in an upright vertical position. Walls were tested in flexure by application of lateral live loads to the quarter points of a simply supported 90-in vertical span.

C.5 Small Racking Shear Specimens (table 3.3)

Small prisms tested in shear are reported in reference [50]. The 32 x 32-in hollow block prisms were built in running bond using face shell mortar bedding. Two diagonally opposite end cores in the top and bottom course, respectively, were filled with mortar to prevent crushing of the masonry by concentrated test loads applied at these locations. The intermediate size 4x4-ft specimens used in reference [50] for racking shear tests, were built in running bond using full bed and head mortar joints. Both of these types of prisms were tested in the manner shown in figure 4.1(b) and 5.3(e). The steel shoes at two diagonally opposite loaded corners were set in plaster and were of the type shown in figure 4.3(a). The vertical compressive load was applied by a conventional testing machine.

C.6 Large Racking Shear Specimens (table 3.3)

Wall panels used for racking tests reported in reference [4] were 8 ft high and 8 ft long. They were constructed in a manner similar to that used for compression wall specimens of corresponding types of construction. Cored spaces in the blocks, and other cavities in the masonry, at two diagonally opposite corners of the walls which received a concentrated load during the racking tests were filled solidly with mortar to prevent local failure of the masonry during the tests. The racking test apparatus was in the form of a yoke consisting of two steel side bars, one on each side of the wall, connected at the ends to steel shoes placed at diagonally opposite corners of the wall. The upper end of the yoke was fitted with a hydraulic ram to apply a compressive load along the wall diagonal between the shoes. Shoes were set in mortar (at time of construction) or in plaster (at time of yoke placement).

Wall panel racking specimens reported in reference [50] were laid in running bond and were 8 ft high and 8 ft long. Hollow concrete block walls contained 8x8x16-in two-cell units. Half-unit kerf blocks were used at the end of alternate courses. Face shell mortar was applied in the manner described in Section C.2. Cores at loading corners of the test walls were filled with mortar to prevent local crushing. Brick wall panel specimens were constructed with full bed and head mortar joints. The racking tests of these wall panels were performed essentially according to ASTM Method E72 as shown in figure 5.3(d). One main exception was the substitution of vertical hold-down tie rods by a vertically positioned hydraulic ram at the loading corner, reacting against a structural frame attached to the laboratory test floor.

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16. ABSTRACT (*A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.*)

The current state of knowledge on the structural behavior of masonry is synthesized to develop a methodology for the evaluation of the load capacity of masonry walls in existing buildings. A procedure is described for direct sampling and testing of specimens removed from masonry walls of buildings to determine their strength in shear, flexure and compression; and to measure their load-deformation characteristics. A documentation of strength and stiffness properties obtained from available test data is included to provide an alternate source of information on masonry of comparable construction. Sample calculations of masonry building analysis for seismic forces are given in Appendices A and B.

17. KEY WORDS (*six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons*)

Analysis; compressive strength; deflection; design; flexural strength; masonry walls; racking strength; seismic loading; shear strength; shear wall; stiffness

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