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WAVELENGTH OF A SLOTTED RECTANGULAR LINE CONTAINING TWO DIELECTRICS

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The titled electromagnetic wave property is obtained approximately for a rectangular slab line with two dielectrics. The perturbing dielectric is a thin sheet set on the center conductor and slotted to permit travel of the probe when the line is used as a slotted line. The purpose is to measure an unknown dielectric filling most of the line, but perturbed by the thin sheet.

Key words: Capacitance; dielectric measurement; slab line; slotted line.

1. Introduction

A rectangular line has been designed by Ramon L. Jesch of the NBS Electromagnetics Division for use as a slotted section. The center conductor is rectangular shaped (fig. 1), and a dielectric strip sets on one boundary of the center conductor to protect the traveling probe when dirt fills the remainder of the cross section.

The purpose of this work is to find the propagation constant of the line containing the dielectric strip while the remainder of the line is filled with another dielectric, such as dirt, the dielectric constant of which is to be determined from a weighted average dielectric constant of the two materials.

Section 2 of this report gives the dependence of the phase velocity on the capacitance and hence on the dielectric filling. Section 3 describes the method of finding the capacitance by computer approximation and gives results for certain dielectric fillings. Section 4 compares the theoretically computed wavelength with measurements.

2. Propagation Constant as a Function of Line Capacitance

The lumped circuit representation of transmission line characteristics uses a series inductance, L, and a shunt capacitance, C, each per unit length [1]. The characteristic impedance of a uniform line is

$$Z = (L/C)^{\frac{1}{2}}, \tag{1}$$

and the propagation velocity is

$$v = 1/(LC)^{\frac{1}{2}}. \tag{2}$$

For a TEM mode in vacuum, v = c, the velocity of light. When a uniform dielectric of relative permittivity $\varepsilon^* = \varepsilon/\varepsilon_o = \varepsilon' - j\varepsilon''$ fills the line, then $C = \varepsilon^* C_o$ where C_o is the vacuum capacitance, and the velocity v is

$$v = c/\varepsilon'^{\frac{1}{2}}. \tag{3}$$

When the dielectric space is inhomogeneously filled, an exact solution is difficult. Therefore, we approximate by assuming that eqs. (1) to (3) still apply when the electrostatic capacitance is found with two dielectrics present. This is a good approximation when the width, 1 inch, multiplied by $\varepsilon'^{\frac{1}{2}}$ is small compared to the wavelength, i.e. when the line is below cutoff, transversely, and the volume of the perturbation is small. (A correct treatment would involve hybrid modes instead of the assumed TEM mode.)

Losses were not treated.* It is believed that the method given below, that of using a filling factor to calculate the perturbation of the velocity by the small strip, is applicable even for damp soil with $\varepsilon''/\varepsilon'$ up to 0.2 to 0.3.

3. Capacitance of the Line

The capacitance C and charge q per unit length of the cross section, and the voltage V are related by the expression

$$C = q/V, \tag{4}$$

and assuming $V = 1$, $C = q$. A method of solving electrostatic field problems is given in reference [2], and a computer program for two dimensional problems is available [3]. The program was modified by multiplying the charge on each segment of the boundary by ε', thus giving the true charge. Since $C = C_o\varepsilon'$ the computed charge is also taken as a measure of the average dielectric constant.

The transmission line (fig. 1) has a plane of symmetry (fig. 2). The program was run for one half of the total cross section using this symmetry. Running half the problem introduced a small error that was removed by an approximate method.

*With high loss the complex propagation constant should be used, whether or not a dielectric perturbation strip is involved.

The error arose because the charge on the center conductor increased (as given by the program) to a higher than correct value, at and near the intersections of the symmetry plane with the center conductor. Within the scope of the present work the program itself could not be corrected. As shown in figure 3, the charges on segments 1 to 6 and 55 to 60 were replaced by an extrapolated curve. The extra charge eliminated by this method amounted to 4.9% of the total, for the air filled line. Similar extrapolations were made with the various dielectrics inserted. The computed charge on the center conductor, proportional to ε' and C, is given in table I for various conditions.

A useful viewpoint is to assume that the capacitance is proportional to a weighted average dielectric constant given by

$$\varepsilon_{AV} = g\varepsilon_1' + (1-g)\varepsilon_2'. \tag{5}$$

With this notation eq. (3) becomes

$$v = c/\varepsilon_{AV}^{\frac{1}{2}}. \tag{6}$$

In eq. (5) g is the so-called filling factor for ε_1', where ε_1' is a perturbation of the main dielectric filling ε_2'. One might expect g to be nearly constant; however, g as computed from the results in table I and eq. (5) changes from approximately 0.04 when $\varepsilon_2 = 1$, to 0.067 when $\varepsilon_2 = 18$. Interpolation of g will allow estimation of ε_{AV} for values of ε_2 other than those given in the table.

The equation for the wavelength of the line with its dielectrics is

$$\lambda_{AV} = v/f = c/(f\ \varepsilon_{AV}^{\frac{1}{2}}). \tag{7}$$

The equation for the measured dielectric quantity is

$$\varepsilon_{AV} = (c/f)^2/\lambda_{AV}^2. \tag{8}$$

We consider two examples of using the results of table I.

Frequency is Measured

If the frequency f is measured then the TEM mode wavelength in vacuum is

$$\lambda_o = c/f. \tag{9}$$

Suppose the wavelength with unknown ε_2 is $\lambda_{AV} = \lambda_o/3$. Equation (8) gives

$$\varepsilon_{AV} = 9.$$

Interpolating in table I we choose g = .063. Using these values in eq. (5) gives

$$\varepsilon_2' = 9.43.$$

Frequency is Not Measured

If the frequency is not measured then λ_o must be obtained from the wavelength of the empty line with $\varepsilon_1' = 2.6$, $\varepsilon_2' = 1$. For this case g = 0.041. From eqs. (7) and (9)

$$\lambda_{AV,air} = \lambda_o/\varepsilon_{AV}^{\frac{1}{2}},$$

where $\varepsilon_{AV} = 0.0406 \times 2.6 + 0.959 = 1.0646$, giving

$$\lambda_o = \lambda_{AV,air} \times 1.032,$$

which is to be used for c/f in eq. (8).

In summary, the measurement of an unknown dielectric material requires the following:

1. Find the free space wavelength λ_o either from the frequency or from $\lambda_{AV,air} \times 1.032$.

2. Measure λ_{AV} with the unknown material and find ε_{AV} from eq. (8).

3. Interpolate a value of g from table I and convert from ε_{AV} to ε_2' using eq. (5).

4. Comparison with Experiment

The measured λ_{AV} of the air filled perturbed line was 3% less than the free space wavelength [4], i.e.,

$$\lambda_o/\lambda_{AV,air} = 1.031.$$

The predicted value from the computer work above is

$$\lambda_o/\lambda_{AV,air} = 1.032.$$

The prediction is in good agreement with experiment. If the effect of the slot in the dielectric strip were taken into account, the predicted ratio would be slightly lower.

The line was carefully filled with nylon, the dielectric constant of which had been measured as 3.08 using a coaxial holder. The present slab line gave $\varepsilon_{AV} = 3.09$ and using $g = 0.056$, ε_2' is 3.12.

An earlier experimental model had a narrower dielectric strip which with an air sample reduced the wavelength by 2%, compared to a theoretical prediction of 2.1%.

5. Error Discussion

We have assumed a quasi static model to relate the velocity to the effective dielectric constant of the two materials, and a filling factor model to obtain the effective dielectric constant of the two materials. It is believed that these are valid models when the filling factor, g, is as small as was found here, $g \simeq 0.05$. The error limits of either assumption have not been determined, and cannot be determined theoretically within the scope of this investigation.

The limits may perhaps be estimated. Experimentally we obtained $\lambda/\lambda_{AV} = 1.031$ and theoretically we obtained 1.032. The fractional error of the perturbation of λ equals that of g, i.e.,

$$\Delta(\Delta\lambda)/\Delta\lambda = \Delta g/g,$$

which is 3% in the above comparison. Thus the experimental evidence in one case suggests that g and $\Delta\lambda$ are in error by few percent. The error in the correction, $\Delta\varepsilon'$, i.e., $\Delta(\Delta\varepsilon')$ would be twice as many percent.

For $\varepsilon_2' = 10$, $\Delta\varepsilon = -0.472$ and $\Delta(\Delta\varepsilon)$ would be $0.03 \times 2 \times 0.47 \simeq \pm 0.03$, out of $\varepsilon' = 10$, i.e., a final error in the measured ε' of 0.3% due to a 3% error is g. It is likely that g is accurate to ± 10%, thus contributing approximately up to ± 1% errors in the final determination of ε_2'.

6. Conclusions

A quasi-static model has been developed that predicts the wavelength and average dielectric constant of an inhomogeneously filled slab line, to a good approximation. A method is given for calculating the dielectric constant of an unknown material from the measured wavelength, as perturbed by the described dielectric strip.

The analysis may also be applied to the characteristic impedance in eq. (1).

7. Acknowledgments

We thank Mr. Ramon Jesch for bringing this problem to our attention, and for his suggestions, and Dr. J. Brian Davies for references to the computer program.

References

[1] Chen, T.S., Determination of the capacitance, inductance and characteristic impedance of rectangular lines, IEEE Trans. on Microwave Theory Tech., Vol. MTT-9, pp. 510-519, Sept. 1960.

[2] Harrington, R.F., K. Pontoppidan, P. Abrahamsen, and N.C. Albertson, Computation of Laplacian potentials by an equivalent source method, Proc. IEE (London), Vol. 116, pp. 1715-1720, Oct. 1969.

[3] Pontoppidan, K., A Fortran IV subroutine for equivalent source solutions of electrostatic and magnetostatic problems, Report P 163, March 1971, Lab of Em Theory, Technical Univ. of Denmark, Lyngby. (Note: Several printing errors in this program must be corrected.)

[4] Jesch, R.L., Slab slotted line, Letter Report, June 4, 1973 (unpublished report).

Table I. Charge in arbitrary units on the center conductor held at 1 unit of potential and resulting ε_{AV} and g. The dielectric strip has dielectric constant ε_1 and the remainder of the space has dielectric constant ε_2.

ε_1'	ε_2'	Charge (Capacitance) as Computed	Charge (Capacitance) corrected[a]	ε_{AV}[c]	g
1.0	1.0	0.7613	0.7239	1.	
2.6	1.0	0.8328	0.7710	1.065	0.041
3.08	3.08	2.345[b]	2.230[b]	3.08	
2.6	3.08	2.318	2.210	3.053	0.056
5.0	5.0	3.806	3.620	5.0	
2.6	5.0	3.665	3.515	4.856	0.060
10.0	10.0	7.613	7.239	10.0	
2.6	10.0	7.145	6.897	9.528	0.064
18.0	18.0	13.703	13.021	18.0	
2.6	18.0	12.694	12.286	16.97	0.066

(a) Corrected for apparent error in program.

(b) Charge with uniform dielectric, $\varepsilon_1 = \varepsilon_2$, is always ε times the charge found for the vacuum case.

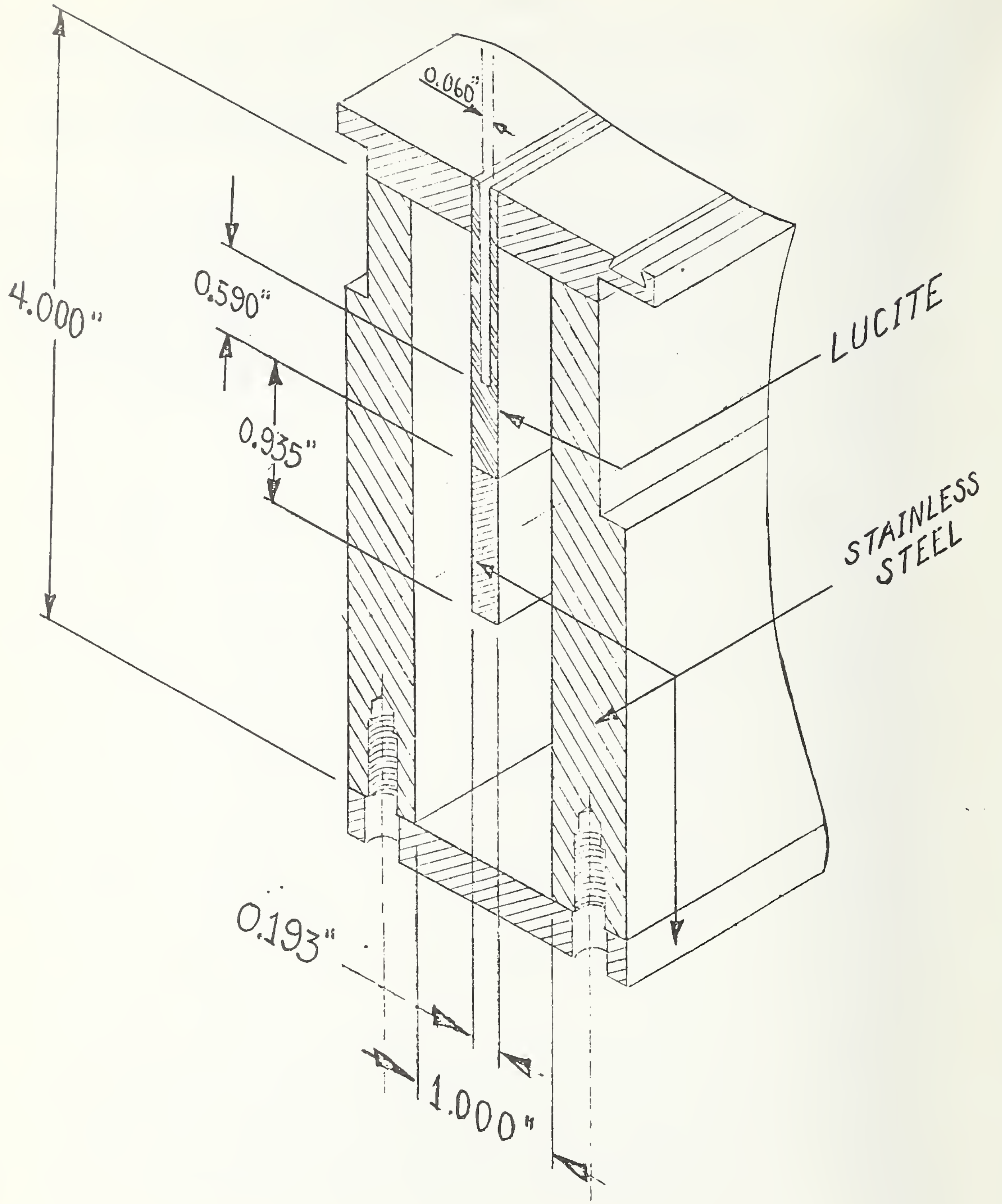
(c) ε_{AV} = corrected charge divided by 0.7239.

Figure Captions

Figure 1. Cross section dimensions of the slab slotted line.

Figure 2. Cross section showing the plane of symmetry, the image space, and the numbering sequence for the segments on the center conductor.

Figure 3. Histogram plots of the computed charge $\div\ \varepsilon'$ on each segment of the center conductor. Near the first and sixtieth segments the charge erroneously increased, as computed, and was replaced by the extrapolated curves. The increase at the corners is of course correct. The segments vary in width; smaller segments were used from 1 to 9, smaller yet from 10 to 12, and other variations were employed.



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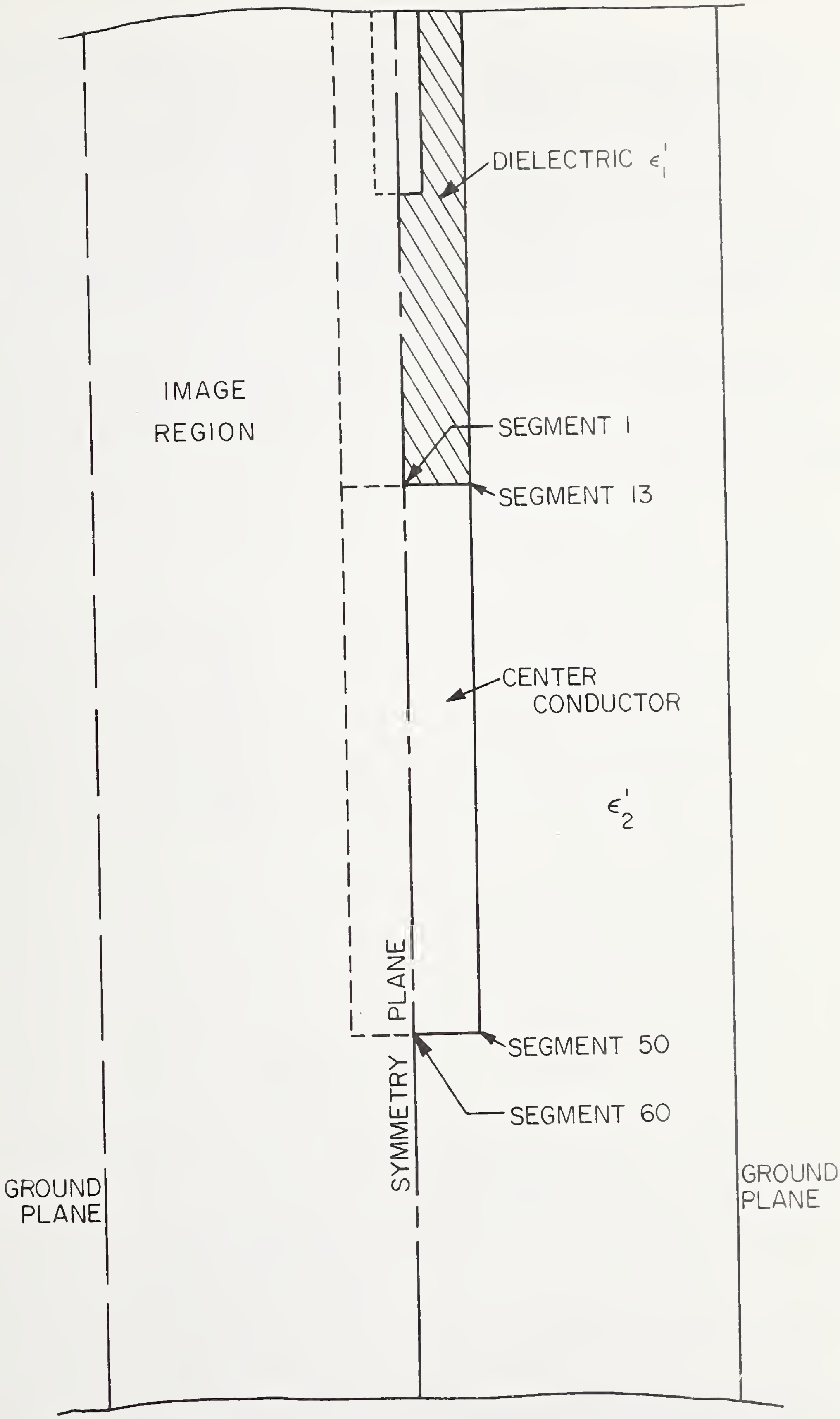

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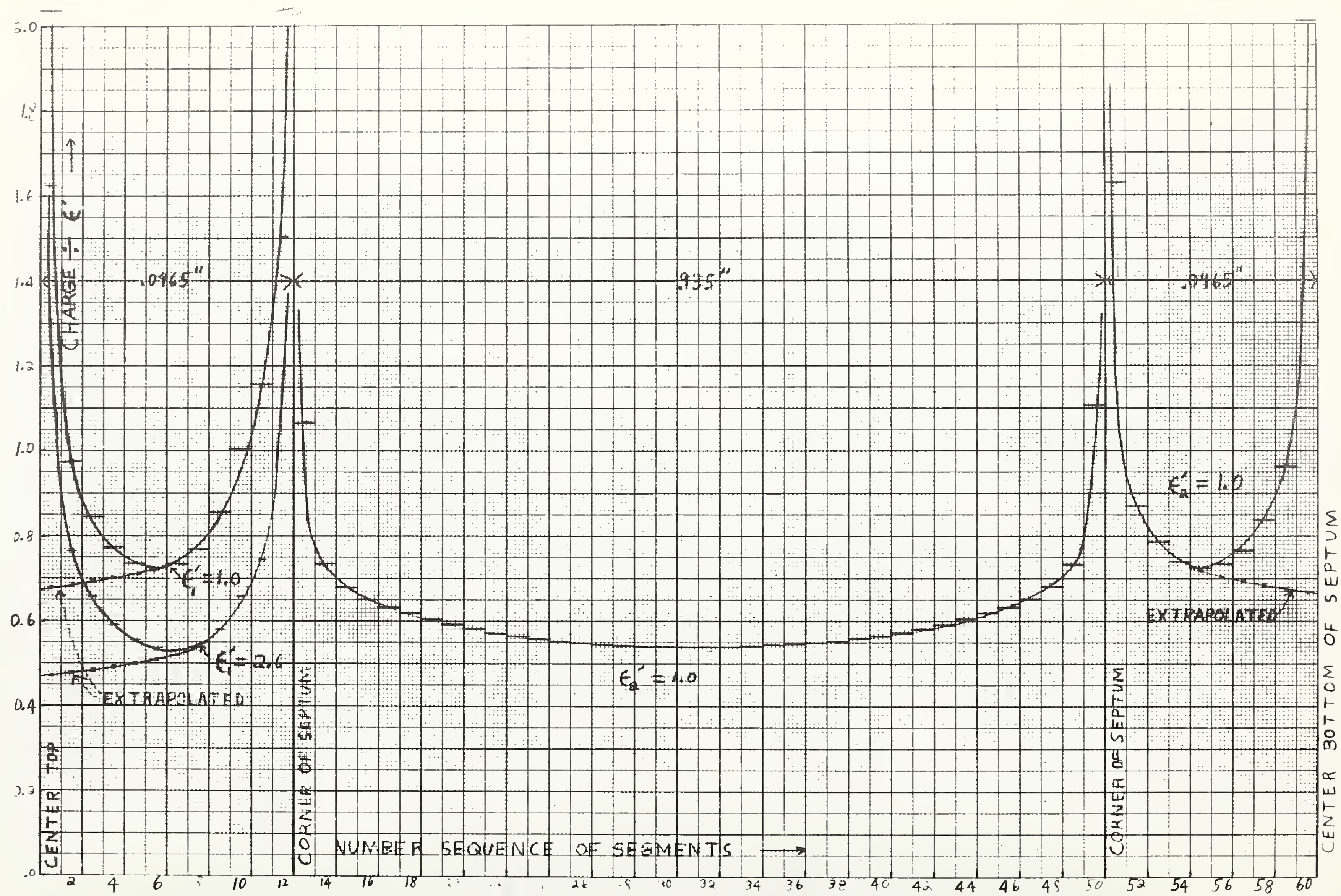

Figure 3. Histogram plots of the computed charge ÷ ε′ on each segment of the center conductor. Near the first and sixtieth segments the charge erroneously increased, as computed, and was replaced by the extrapolated curves. The increase at the corners is of course correct. The segments vary in width; smaller segments were used from 1 to 9, smaller yet from 10 to 12, and other variations were employed.

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17. KEY WORDS (Alphabetical order, separated by semicolons)

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