ELECTROMAGNETIC SCATTERING BY A THIN WIRE

WITH CONTINUOUS IMPEDANCE LOADING--

PART II: TIME DOMAIN ANALYSIS.

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PREFACE

This is the second part of a two-part work on electromagnetic scattering by a thin wire with continuous impedance loading. In the first part [1] we have solved the problem in the frequency domain. Since the frequency domain and the time domain analysis are related to each other mathematically by the Fourier transform, the transient scattering of a time pulse by a thin wire can be synthesized from the results obtained for the scattering of plane waves via the Fourier transform.
# CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>PREFACE</td>
<td>iii</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>1</td>
</tr>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. FREQUENCY DOMAIN AND TIME DOMAIN</td>
<td>3</td>
</tr>
<tr>
<td>III. NUMERICAL RESULTS</td>
<td>6</td>
</tr>
<tr>
<td>IV. INCIDENT FIELD OF ARBITRARY TIME DEPENDENCE</td>
<td>8</td>
</tr>
<tr>
<td>V. CONCLUSION</td>
<td>9</td>
</tr>
<tr>
<td>ACKNOWLEDGMENT</td>
<td>10</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>11</td>
</tr>
<tr>
<td>ILLUSTRATIONS</td>
<td>13</td>
</tr>
<tr>
<td>APPENDIX A - COMPUTER PROGRAM</td>
<td>23</td>
</tr>
</tbody>
</table>
ELECTROMAGNETIC SCATTERING BY A THIN WIRE WITH CONTINUOUS IMPEDANCE LOADING -- PART II: TIME DOMAIN ANALYSIS

Tommy C. Tong*

ABSTRACT

The scattering of electromagnetic pulses by a thin wire with continuous impedance loading is investigated theoretically. The basic approach used is the numerical Fourier transform of the frequency response. The incident pulse is assumed a truncated periodic Gaussian pulse. Numerical results for the transient current distribution and the transient scattered field in different directions of observation are presented. It is found that the shapes of the transient current distribution on the wire and of the scattered field can be controlled by loading the wire with some impedance.

Key Words: Electromagnetic scattering; Impedance loading; Pulses; Thin wire.

I. INTRODUCTION

In recent years, the time domain analysis of field problems has been receiving increasing attention partly because of the demand for generation of high power pulses and partly because of the fact that radar interrogation is essentially a transient process. In the case of wire structures, substantial work [2,3,4] has been devoted to the study of transient radiation of a dipole antenna. But very little work [5,6] has been done on investigating the

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Transient scattering characteristics of the dipole, and apparently none when the dipole is modified with some kind of impedance loading.

The present study investigates the scattering of electromagnetic pulses by a thin wire with continuous loading. In general, there are two different approaches to obtain the time domain response to an electromagnetic scattering problem. The first approach makes use of numerically evaluated frequency domain solutions and the Fourier transform. The second approach [7,8] is one which originates from a strictly time domain viewpoint, and requires solving the time dependent differential or integral equation directly.

The first approach is preferred in the present investigation, because mathematically it is easier to solve a time independent differential or integral equation than a time dependent one. Moreover, recent development of the fast numerical algorithm has made this approach more appropriate for this study.

Here we assume that the incident pulse is a Gaussian distribution, but any band-limited real time function can be used as well.
II. FREQUENCY DOMAIN AND TIME DOMAIN

A. Incident Pulse

As shown in Fig. 1, the thin wire \((a/L \ll 1)\) which is loaded symmetrically about \(z=0\) with a continuous impedance function \(Z(z)\) is illuminated by a truncated Gaussian pulse varying periodically at a frequency \(\omega_c = 2\pi/T\) with \(T\) being the period. The Gaussian whose pulse width is \(\tau\) \((T \geq 4\tau)\) is given by (Fig. 2(a))

\[
p(t) = \exp(-t^2/2\sigma^2), \quad |t| \leq T/2
\]

where \(\sigma\) is a time constant.

The frequency spectrum of a truncated single pulse is (Fig. 2(b))

\[
P(\omega) = \int_{-\infty}^{\infty} p(t) \, e^{-j\omega t} \, dt
\]

\[
= \sigma \sqrt{2\pi} \exp(-\sigma^2 \omega^2/2).
\]

Since \(p(t)\) is a real time function we must have

\[
P(\omega) = P^*(-\omega)
\]

where * denotes the complex conjugate.

B. Scattered Field

In the frequency domain the scattered field is given by

\[
E_\theta(\omega) = \frac{j\omega \eta_0}{4\pi r} e^{-jr} \sin \theta \int_{-L/2}^{L/2} I(z',\omega) e^{-jkz'\cos \theta} \, dz'
\]

where \(\eta_0\) is the intrinsic impedance of free space, and \(I(z,\omega)\) the current distribution for an incident plane wave of unit amplitude.
Then, the scattered field in the time domain for a periodic pulse can be obtained by superposition (since the system is linear) thus

\[ E^p_\theta(t) = \frac{\eta_0}{4\pi r} \sum_{\omega_i=-\infty}^{\infty} j\omega_i P(\omega_i) \int_{-L/2}^{L/2} \sin \theta I(z',\omega_i) \]

\[ \times e^{-j\frac{\omega_i}{c} z' \cos \theta} e^{-j\frac{j\omega_i r}{c} \frac{j\omega_i t}{dz'} dz} \]  

(5)

where \( \omega_i = \frac{2\pi i}{T} \), \( i = \pm 1, \pm 2, \ldots \),

which after terminating to a finite number of terms \( 2N + 1 \) is approximately given by

\[ E^p_\theta(t) \approx \frac{\eta_0}{4\pi r} \sum_{\omega_i=-N}^{N} j\omega_i P(\omega_i) \int_{-L/2}^{L/2} \sin \theta I(z',\omega_i) \]

\[ \times e^{-j\frac{\omega_i}{c} z' \cos \theta} e^{-j\frac{j\omega_i r}{c} \frac{j\omega_i t}{dz'} dz} \]  

(6)

Using Eq. (3) and the relation

\[ I(z',-\omega_i) = I^*(z',\omega_i), \]

we obtain

\[ E^p_\theta(t') \approx \frac{\eta_0}{4\pi r} \sum_{\omega_i=1}^{N} j\omega_i P(\omega_i) \int_{-L/2}^{L/2} \sin \theta I(z',\omega_i) \]

\[ \times e^{-j\frac{\omega_i}{c} z' \cos \theta} e^{j\omega_i t'} dz' \]  

(7)

+ complex conjugate term

where \( t' = t-r/c \) is the retarded time, and \( c \) the velocity of propagation.
In the frequency domain the scattered field due to the pulse is given by

\[ E_\theta^P(\omega) = \int_{-\infty}^{\infty} E_\theta^P(t') e^{-j\omega t'} dt' \]

\[
= \frac{\eta_0}{4\pi r} \int_{-\infty}^{\infty} \left\{ \sum_{\omega_i=1}^{N} j\omega_i P(\omega_i) \int_{-L/2}^{L/2} \sin \theta I(z', \omega_i) \right. \\
\times e^{\frac{j\omega_i}{c} z' \cos \theta j\omega_i t'} \\
\left. e^{j\omega_i t' + c c} e^{-j\omega t'} dt' \right\} e^{-j\omega t'} dt' \]  

(8)

\[
= \frac{\eta_0}{2\pi r} \Re \int_{-\infty}^{\infty} \left\{ \sum_{\omega_i=1}^{N} j\omega_i P(\omega_i) \int_{-L/2}^{L/2} \sin \theta I(z', \omega_i) \right. \\
\times e^{\frac{j\omega_i}{c} z' \cos \theta j\omega_i t'} \\
\left. e^{j\omega_i t' + c c} e^{-j\omega t'} dt' \right\} e^{-j\omega t'} dt' \]  

(9)

where \( c c \) represents the complex conjugate term.

C. The Surface Current

In a similar manner, the transient current induced on the wire is given by

\[ I(z,t) = \sum_{\omega_i=-\infty}^{\infty} I(z, \omega_i) P(\omega_i) e^{j\omega_i t}. \]  

(10)
III. NUMERICAL RESULTS

Fig. 3 shows the transient current at \( z = L/2 \) for a wire with \( L = 0.125 \lambda \) and \( a = 0.005 \lambda \) at the fundamental frequency for two different impedance loadings. The width and the period of an incident pulse are equal to 10 ns and 40 ns respectively. The Gaussian pulse is truncated when the amplitude is equal to 1% of the maximum amplitude. It is seen that there are marked differences between these two current distributions. If we take a Fourier analysis of them, we would find that the one corresponding to no loading \((Z = 0)\) is rich in the second harmonic while the one corresponding to loading is rich in the third harmonic. This indicates that it is possible to control the harmonic content of the transient current response by loading the wire with some impedance.

The transient scattered fields for different angles of observation for the same wire and same incident pulse are shown in Figs. 4-6. The kind of impedance loading used here is distributed capacitive. Properties of antennas with such loading were first described by Hallén [9] who made a monopole antenna in the form of a row of small conducting cylinders, between which dielectric discs of increasing thickness toward the antenna end were inserted. In this manner the antenna had a variable capacitive impedance which increased toward the end. It is seen that in each case the response with such loading decays at a faster rate than that without loading. Perhaps it should be pointed out that the shape of the...
scattered field in the time domain is closely related to the first derivative of the incident pulse, because $E_\theta(\omega)$ has a $j\omega$ factor (Eq. (4)).

Figs. 7-9 show the scattered field versus the retarded time for a wire with longer length ($L = 0.75\lambda$ and $a = 0.01\lambda$). Besides the continuous capacitive loading, a nonreflecting resistive loading is also investigated. This kind of loading was used by Wu and King [10] in antenna analysis. It is seen that except for $\theta=90^\circ$, the shapes of the scattered fields with or without loading differ significantly from those we saw previously for shorter wires. This difference can be understood if we realize that when the wire is illuminated by a pulse, a transient current is induced on the wire. This current travels along the wire at the speed of light, and when it encounters the ends of the wire, it will be reflected back. This process will continue until the current dies down. The longer the wire, the longer the time the current takes. Since the electrical length of the antenna is proportional to the frequency, the contributions due to the different harmonics would not add in phase in the far field. The case for $\theta=90^\circ$ is a special one, because the phase factor $e^{-j\omega t/c} \cos \theta z'$ in Eq. (4) is equal to unity under this circumstance. Note that the scattered field for the nonreflecting loading is not shown in Fig. 9 for $\theta=90^\circ$, since it is very close to that for the no loading case.
The effect of reducing the pulse duration is illustrated in Fig. 10. As the pulse width reduces, higher harmonics will become more important. For the numerical calculations we used ten harmonics \((N = \pm 10)\) for the longer pulse \((\tau = 10\text{ ns})\) and twenty two harmonics \((N = \pm 22)\) for the shorter pulse \((\tau = 5\text{ ns})\). In each case the period of the pulse is 40 ns.

IV. INCIDENT FIELD OF ARBITRARY TIME DEPENDENCE

For a linear system, the superposition theorem says that the time response of the scattered field from a dipole, illuminated by a plane wave of arbitrary time dependence \(y(t)\) is given by the convolution

\[
E_\theta(t) = \int_0^t y(t-\tau)h(\tau) \, d\tau
\]

(11)

where \(h(\tau)\) is the impulse response of the dipole and is related to \(E_\theta(\omega)\), the frequency response, by

\[
h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E_\theta(\omega) \, e^{j\omega t} \, d\omega.
\]

(12)

Therefore, once we get the impulse response we have solved the time domain problem entirely. Numerically, the impulse response is difficult to obtain because its frequency spectrum is constant. In other words, the delta function is not a band limited function. But there are various approximate functions that can be used for the delta function. However, we will not pursue this matter further in this study.
V. CONCLUSION

The scattering of electromagnetic pulses by a thin wire with continuous impedance loading has been investigated using the numerical Fourier transform of the frequency response. Numerical results show that the shapes of the induced current and the scattered field in the time domain can be modified by using the impedance loading technique.
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REFERENCES


Fig. 1. Geometry of the Thin Wire
Fig. 2. (a) Gaussian Pulse (b) Frequency Spectrum of a Gaussian Pulse.
Fig. 3. Transient Current at z=L/2 on the wire.

\[ L = 0.125\lambda, \ a = 0.005\lambda, \ \tau = 10\text{ns} \]

- \( Z = 0 \)
- \( Z = -j \frac{25}{\pi a} \exp \left( \frac{4z}{L} - 1 \right) \)
L = 0.125\lambda, \ a = 0.005\lambda, \ \tau = 10\text{ns}
\theta = 40^\circ
\begin{align*}
Z = 0 \\
Z = -j \frac{25}{\pi a} \exp \left( \frac{4z}{L} - 1 \right)
\end{align*}

Fig. 4. Scattered Field vs. Retarded Time.
$L = 0.125\lambda$, $a = 0.005\lambda$, $\tau = 10\text{ns}$

$\theta = 70^\circ$

$Z = 0$

$Z = -j \frac{25}{\pi a} \exp \left( \frac{4z}{L} - 1 \right)$

Fig. 5. Scattered Field vs. Retarded Time.
Fig. 6. Scattered Field vs. Retarded Time.
Fig. 7. Scattered Field vs. Retarded Time.
$L = 0.75\lambda, a = 0.01\lambda, \theta = 70^\circ, \tau = 10\text{ns}$

$Z = \frac{25}{L} \exp \left( \frac{4Z}{L} - 1 \right)$

Fig. 8. Scattered Field vs. Retarded Time.
Fig. 9. Scattered Field vs. Retarded Time.
\[ L = 0.125\lambda, \quad a = 0.005\lambda, \quad \tau = 5\text{ns} \]

\[ Z = -j \frac{25}{\pi a} \exp \left( \frac{4z}{L} - 1 \right) \]

- \( \theta = 90^\circ \)
- \( \theta = 70^\circ \)
- \( \theta = 40^\circ \)

Fig. 10. Scattered Field vs. Retarded Time.
APPENDIX A - COMPUTER PROGRAM

Subroutine FORTID performs the fast Fourier transform on a one-dimensional array of complex numbers A. It assumes the real parts of the elements of A are stored in the odd-indexed locations and the imaginary parts in the even-indexed ones. Suppose it is desired to evaluate

\[ F(\lambda) = \int \exp(-j2\pi\lambda x) f(x) \, dx \]

where \( f \) has limited support

\[ f(x) = 0 \quad |x| > a. \]

Then \( F(\lambda) \) is approximated by

\[ F_m \approx \frac{1}{\sqrt{N}} \sum_{n=-N/2}^{N/2-1} \exp(jmn/N) f_n, \quad -N/2 \leq m \leq N/2 \]

where \( f_n = f(ns) \) and \( F_m = F(m/T) \). The range of sampling \( T \) and the period of sampling \( s \) are related to the number of samples, \( N \), by

\[ T/s = N = 2^p, \quad p \text{ is an integer.} \]
PROGRAM FOURIER
DIMENSION A(256)
COMPLEX U1
DO 1 I=1,256
A(I)=0.
1 CONTINUE
U1=(6.53490E-003, 1.57600E-001)*(5.40757E-001,2.43945E-004)*1.*
1*(0.,1.)
A(131)=REAL(U1)
A(132)=AIMAG(U1)
A(127)=A(131)
A(128)=-A(132)
U1=(2.26778E-001, 9.20442E-001)*(5.29174E-001,4.50751E-004)*2.*
1*(0.,1.)
A(133)=REAL(U1)
A(134)=AIMAG(U1)
A(125)=A(133)
A(126)=-A(134)
U1=(2.09957E+000, 2.06968E+000)*(5.10402E-001,5.88935E-004)*3.*
1*(0.,1.)
A(135)=REAL(U1)
A(136)=AIMAG(U1)
A(123)=A(135)
A(124)=-A(136)
U1=(4.28172E+000,-1.63352E-001)*(4.85205E-001,6.37457E-004)*4.*
1*(0.,1.)
A(137)=REAL(U1)
A(138)=AIMAG(U1)
A(121)=A(137)
A(122)=-A(138)
U1=(3.46354E+000,-1.78828E+000)*(4.54582E-001,5.88934E-004)*5.*
1*(0.,1.)
A(139)=REAL(U1)
A(140)=AIMAG(U1)
A(119)=A(139)
A(120)=-A(140)
U1=(2.75311E+000,-2.16372E+000)*(4.19705E-001,4.50751E-004)*6.*
1*(0.,1.)
A(141)=REAL(U1)
A(142)=AIMAG(U1)
A(117)=A(141)
A(118)=-A(142)
U1=(2.35476E+000,-2.26216E+000)*(3.81846E-001,2.43946E-004)*7.*
1*(0.,1.)
A(143)=REAL(U1)
A(144)=AIMAG(U1)
A(115)=A(143)
A(116)=-A(144)
U1=(2.08824E+000,-2.31764E+000)*(3.42304E-001,4.64752E-010)*8.*
1*(0.,1.)
A(145)=REAL(U1)
A(146)=AIMAG(U1)
A(113) = A(145)
A(114) = -A(146)
1*(0, 1)
A(147) = REAL(U1)
A(148) = AIMAG(U1)
A(111) = A(147)
A(112) = -A(148)
U1 = (1.80356E+000, -2.52007E+000) * (2.63087E-001, -4.50749E-004) * 10.
1*(0, 1)
A(149) = REAL(U1)
A(150) = AIMAG(U1)
A(109) = A(149)
A(110) = -A(150)
U1 = (1.67960E+000, -2.69670E+000) * (2.25546E-001, -5.88932E-004) * 11.
1*(0, 1)
A(151) = REAL(U1)
A(152) = AIMAG(U1)
A(107) = A(151)
A(108) = -A(152)
U1 = (1.53167E+000, -2.92654E+000) * (1.90506E-001, -6.37456E-004) * 12.
1*(0, 1)
A(153) = REAL(U1)
A(154) = AIMAG(U1)
A(105) = A(153)
A(106) = -A(154)
U1 = (1.32086E+000, -3.19377E+000) * (1.58549E-001, -6.37456E-004) * 13.
1*(0, 1)
A(155) = REAL(U1)
A(156) = AIMAG(U1)
A(103) = A(155)
A(104) = -A(156)
U1 = (1.05350E+000, -3.48155E+000) * (1.30038E-001, -4.50750E-004) * 14.
1*(0, 1)
A(157) = REAL(U1)
A(158) = AIMAG(U1)
A(101) = A(157)
A(102) = -A(158)
U1 = (7.06912E-001, -3.73998E+000) * (1.05134E-001, -2.43944E-004) * 15.
1*(0, 1)
A(159) = REAL(U1)
A(160) = AIMAG(U1)
A(99) = A(159)
A(100) = -A(160)
1*(0, 1)
A(161) = REAL(U1)
A(162) = AIMAG(U1)
A(97) = A(161)
A(98) = -A(162)
U1 = (-1.01943E-001, -4.00139E+000) * (6.59205E-002, 2.43944E-004) * 17.
1*(0, 1)
A(163) = REAL(U1)
A(164) = AIMAG(U1)
A(95) = A(163)
A(96) = -A(164)
U1 = (-4.55164E-001, -4.01742E+000) * (5.11691E-002, 4.50750E-004) * 18.
1*(0., 1.)
A(165) = REAL(U1)
A(166) = AIMAG(U1)
A(93) = A(165)
A(94) = -A(166)
U1 = (-7.52346E-001, -4.00571E+000) * (3.92166E-002, 5.88933E-004) * 19.
1*(0., 1.)
A(167) = REAL(U1)
A(168) = AIMAG(U1)
A(89) = A(167)
A(92) = -A(168)
U1 = (-1.01354E+000, -3.99996E+000) * (2.96826E-002, 6.37456E-004) * 20.
1*(0., 1.)
A(169) = REAL(U1)
A(170) = AIMAG(U1)
A(89) = A(169)
A(90) = -A(170)
U1 = (-1.27136E+000, -4.01266E+000) * (2.21831E-002, 5.88933E-004) * 21.
1*(0., 1.)
A(171) = REAL(U1)
A(172) = AIMAG(U1)
A(87) = A(171)
A(88) = -A(172)
U1 = (-1.5684E+000, -4.03862E+000) * (1.63552E-002, 4.50750E-004) * 22.
1*(0., 1.)
A(173) = REAL(U1)
A(174) = AIMAG(U1)
A(85) = A(173)
A(86) = -A(174)
PRINT 3, (I, A(I), I = 84, 175)
CALL FORTID(A, 128, 1)
PRINT 3, (I, A(I), I = 1, 256)
3 FORMAT (I5, E20.5)
STOP
END
SUBROUTINE FORTID(A, N, IFS)
C A = 1D ARRAY, SIZE 2*N
C N = 2**P, NO. OF COMPLEX ELEMENTS, P AN INTEGER
C IFS = -1, FOR FOURIER ANALYSIS
C IFS = 1, FOR FOURIER SYNTHESIS
DIMENSION A(256)
NN = 2*N
FN = N
PI = 3.14159265
C SUPPLY PHASE TO A TO SHIFT THE TRANSFORM TO THE CENTER
DO 1 J = 1, N
JJ = J - 1
L = MOD(JJ, 2)
A(2*J) = ((-1.)**L) * A(2*J) / SQRT(FN)
A(2*J-1) = ((-1.)**L) * A(2*J-1) / SQRT(FN)
1 CONTINUE
C PERFORM BINARY SORT
J=1
DO 34 I=1,NN,2
IF (I-J) 2,3,3
2 TEMPR=A(J)
TEMPI=A(J+1)
A(J)=A(I)
A(J+1)=A(I+1)
A(I)=TEMPR
A(I+1)=TEMPI
3 M=NN/2
4 IF (J-M) 6,6,5
5 J=J-M
M=M/2
IF (M-2) 6,4,4
6 J=J+M
34 CONTINUE
C START MAIN LOOP
MMAX=2
7 IF (MMAX-NN) 8,11,11
8 ISTEP=2*MMAX
THETA=2.*PI/(IFS*MMAX)
SINTH=SIN(THETA/2.)*
WSTPR=-2.*SINTH*SINTH
WSTPI=SIN(THETA)
WR=1.
WI=0.
DO 10 M=1,MMAX,2
DO 9 I=M,NN,ISTEP
J=I+MMAX
TEMPR=WR*A(J)-WI*A(J+1)
TEMPI=WR*A(J+1)+WI*A(J)
A(J)=A(I)-TEMPR
A(J+1)=A(I+1)-TEMPI
A(I)=A(I)+TEMPR
A(I+1)=A(I+1)+TEMPI
9 CONTINUE
TEMPR=WR
WR=WR*WSTPR-WI*WSTPI+WR
WI=WI*WSTPR+TEMPR*WSTPI+WI
10 CONTINUE
MMAX=ISTEP
GO TO 7
C END OF TRANSFORM
C CORRECT PHASE ON TRANSFORM DUE TO SHIFTED A
11 DO 12 J=1,N
JJ=J-1
L=MOD(JJ,2)
IF (L .EQ. 0) GO TO 12
JR=2*J-1
JI=JR+1
A(JR)=-A(JR)
A(JI)=-A(JI)
12 CONTINUE
RETURN
END
SCOPE
'LOAD
'RUN 10 1500
"
The scattering of electromagnetic pulses by a thin wire with continuous impedance loading is investigated theoretically. The basic approach used is the numerical Fourier transform of the frequency response. The incident pulse is assumed a truncated periodic Gaussian pulse. Numerical results for the transient current distribution and the transient scattered field in different directions of observation are presented. It is found that the shapes of the transient current distribution on the wire and of the scattered field can be controlled by loading the wire with some impedance.