NBSIR 73-272

On Characterizing Master Involute Profiles

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ILLUSTRATIONS

ON CHARACTERIZING MASTER INVOLUTE PROFILES

by

Edgar G. Erber and B. Nelson Norden

Abstract

The involute profile plays an important role in gear metrology. It is necessary to quantify that profile on involute masters so that an accurate form for gear teeth may be transferred to the gear teeth from the master profile. The outlining of the procedures for calibration of a master involute profile on a two coordinate axis measuring machine is the function of this paper.

I. INTRODUCTION

The curve called an involute plays a vital role in gear metrology. The involute fulfills the requirements of gear teeth for transmitting smooth (uniform) angular motion.[1] It is a familiar curve since it can be described by the end of a taut string as it is wound upon or unwound from a fixed cylindrical surface. Figure 1 shows the basic elements of a gear with the corresponding definitions of each term.

For metrological purposes, an alternative definition of the involute profile is useful. Figure 2 illustrates the principle of

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[1] "The Involute Curve and Involute Gearing," Published by The Fellows Gear Shaper Company, 1969.

involute generation from a base circle. The base circle diameter is equivalent to the base circle diameter of a gear. To construct the involute profile from this base circle:

(a) the circumference of the circle is divided into a number of segments,

(b) from each division point a tangent is constructed

(c) the length of each corresponding arc is set off along each tangent, and

(d) the resultant curve through each terminal point is drawn.

From Figure 2 we see:

$$\text{Arc } BC = (\text{Radius of Base Circle} \times (\theta)$$

where θ = angle COB (in radians) and is normally called the angle of roll.

Since Arc BC = $\overline{AB}$ then

$$AB = (\text{Radius of Base Circle}) \times (\theta)$$

This simple equation permits master involute profiles to be generated or calibrated by comparison of the measured $\overline{AB}$ to that calculated by the above equation.

II. PROCEDURES

The calibration procedure is derived directly from the fact that a line normal to the involute profile is tangent to the base circle. Figure 3 illustrates the measurement process from these basic principles. If an indicator measuring lateral displacement, is mounted at a height above the center of the base circle equal to the base circle radius and the involute profile is rotated through an angle B about an axis concentric with the base circle center, then

$$\Delta \text{ (indicator change)} = \text{(radius of base circle)} \times \text{(B)}.$$

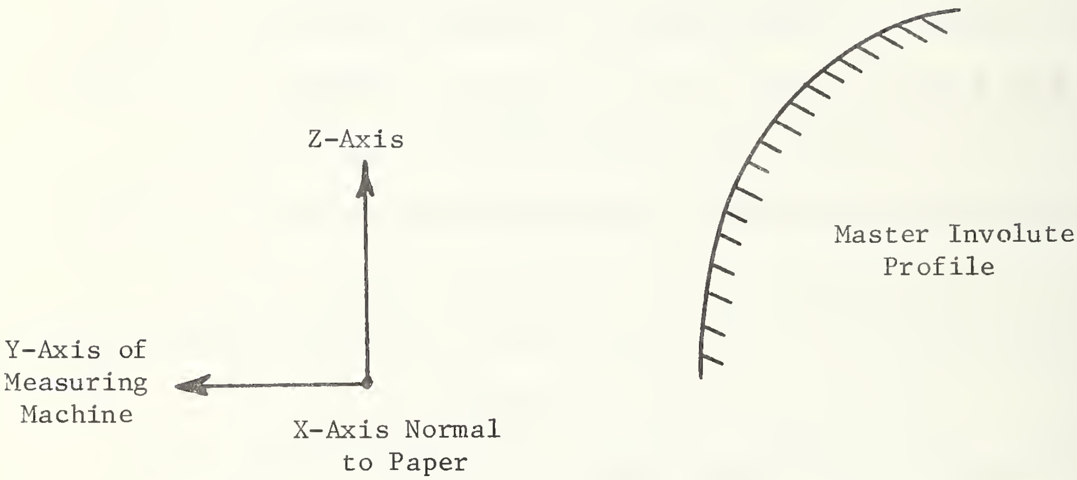
If the process is repeated for an incremental B, a complete profile for the actual curve is obtained.

In our non-perfect real world, the method described above has to be modified somewhat and the following discussion is the process realized at NBS for measurement of the most common form of involute master. Figure 4 is a photograph of a typical involute master which is used to qualify involute profile measuring instruments. The true involute surface is the surface characterized. The instrument used for measuring is a two-axis (x,y) measuring machine where the third axis (Z') is a precision spindle. The manual vertical movement of the spindle housing will be considered the Z axis in the measurement process.

(1) Initially a reference axis for the base circle must be established on the measuring machine before a meaningful measurement can be made. A precision 1.4 inch diameter test cylinder (arbor), which has been produced with tight tolerances on roundness (10 micro-inches) and taper (5 microinches/inch), is used to define that reference axis (Figures 5 and 6). The cylinder (arbor) is mounted at one end on a rotary table and on a precision adjustable cone point at the other end. By successive adjustments of the adjustable cone point, the vertical and horizontal planes of the reference axis are aligned to within a value limited only by the quality of the arbor itself (approximately

0.000020 inch). To insure that measurements are made normal to the reference axis, the spindle is rotated through a small angle in the x-y plane (Figure 6) until a maximum reading is indicated on the meter. At this position, the operator is assued that the "sensitive" axis of the probe is normal to the reference axis. The spindle of the measuring machine is then locked into position.

(2) The coordinate system which is used is as follows:



The involute master is then examined for wear and/or burrs and roundness tracings of the centers for mounting are recorded to assure that the quality is sufficient for measurement (Figure 7).

To provide a suitable reference from which to quantify these master profiles:

(1) the internal mounting centers on the involute master and the external mounting centers used to hold the involute should be made to a deviation (TIR) from roundness not in excess of thirty micro-inches, and

(2) the surface finish of the cylindrical reference surface shall be approximately seven microinches AA or better (arithmetic average).

If one considers the fact that the external mounting centers for holding the involute master are probably not round, then one can obtain a better appreciation of how difficult it is to define a reference axis for the involute master. If the number of lobes on both the internal and external centers are the same; then the condition is reached where the high areas of the external mounting centers may fit into the low areas of the internal centers on the involute master or vice versa. For the opposite case, the high areas of the external centers will contact the high areas of the internal centers. If the number of lobes in both centers is not the same, then various positions of the reference involute master will result when rotated.

(3) After one is convinced the involute master is of sufficient quality to warrant measurement, the test cylinder is withdrawn and the involute master is placed within the mounting centers (Figure 8). One can ascertain the degree of parallelism between the face and the reference axis by tramming the surface along the y-axis (Figure 9).

(4) The steps to establish the correct base radius is composed of: (a) establishing a reference zero (i.e. locating the X-Y axis of the involute mounting axis) by moving the spindle housing in the vertical Z direction while the probe is in contact with the involute mounting cylinder (Figure 10), and (b) raising the spindle housing on the measuring machine in the positive Z direction the required distance for the base radius by

using steel gage blocks (Figure 10).

(5) The rotary table is then rotated CCW (Figure 11) and the probe brought into contact with the machined part of the involute master. Note: most involute profiles have a six degree run-out and measurement should not be started until the master has been rotated approximately six degrees beyond the initial point where the probe contacts the surface. Only after this rotation does the true involute profile begin.

(6) The probe is only used as a nulling device (Figure 12) and displacements are obtained directly from the y-axis scale of the measuring machine which has a least count of 20 microinches with 10 microinch interpolation capability. The first reading is, of course, the reference reading.

(7) The slide of the measuring machine is then moved back to permit the involute master to be rotated by the rotary table through an appropriate angle (usually four degrees to give approximately ten data points). The y-slide is then brought forward to again bring the probe into contact with the surface at a new point. The process is repeated until the end of the true involute profile is reached (Figure 13, 14, 15).

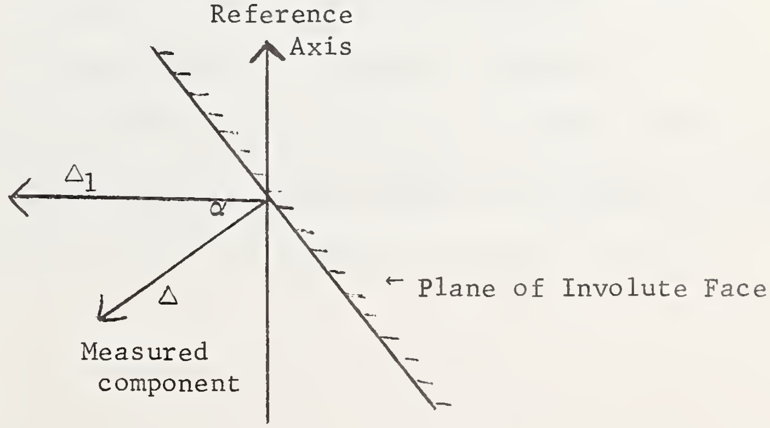
(8) The involute profile may be checked for non-parallelism to the reference axis. After steps 5-7 have been performed, the x-slide of the measuring machine can be translated some small amount ($\sim$0.1 inch) and the above procedures repeated (Figure 16).

III. EXAMPLE OF CALIBRATION

An example of an involute calibration follows: the data is from a test on an involute master having a base radius of 0.500 inch. Three sets of readings were recorded with the x-axis displaced 0.100 inch between each set of readings. By using the initial center reading as

- Readings -					
Left		Center		Right	
Rotary Table	Y-Axis	Rotary Table	Y-Axis	Rotary Table	Y-Axis
316° 8' 13"	5.53065	316° 8' 13"	5.53059	316° 8' 13"	5.53052
321° 8' 13"	5.57434	321° 8' 13"	5.57430	321° 8' 13"	5.57422
326° 8' 13"	5.61800	326° 8' 13"	5.61795	326° 8' 13"	5.61787
331° 8' 13"	5.66162	331° 8' 13"	5.66157	331° 8' 13"	5.66149
336° 8' 13"	5.70527	336° 8' 13"	5.70522	336° 8' 13"	5.70515
341° 8' 13"	5.74889	341° 8' 13"	5.74886	341° 8' 13"	5.74878
346° 8' 13"	5.79255	346° 8' 13"	5.79251	346° 8' 13"	5.79242
351° 8' 13"	5.83619	351° 8' 13"	5.83614	351° 8' 13"	5.83606
356° 8' 13"	5.87984	356° 8' 13"	5.87979	356° 8' 13"	5.87971

the reference reading, the plane of the involute face is not parallel to the reference axis (since 5.53059 ≠ 5.53065 ≠ 5.53052) and one must determine whether this affects measurement of the true involute profile.



The probe is set up to measure $\triangle_1$ (because of maximizing on the test cylinder to read normal displacements to the reference axis) and we see

$$(\text{Measured component } \triangle) = (\triangle_1) \times (\cos \alpha)$$

when the plane of the involute face is tilted at angle α to the reference axis. From the previous table of readings, we find an average α of 2.1 minutes and since the cosine of this angle is very close to unity, this introduces no appreciable error in the measurement process.

The deviations of the involute surface from a true involute are computed by comparing the measured values to the calculated nominal values. In this Case $\triangle$, which represents the linear distance imparted when the involute profile is rotated through an angle θ, is equal to the base radius multiplied by θ, or for $\theta = 5$ degrees

$$\triangle = (0.500 \text{ inch}) \times (5°) \times \left(\frac{\pi}{180°}\right)$$

$$\triangle = 0.043633 \text{ inch.}$$

From this, the profile errors may be calculated (Figure 17) and plotted (Figure 18) as the angle of roll, the abscissa, and the deviations from nominal the ordinate. For example at $\theta = 5$ degrees, the theoretical value of the involute should be

$$\text{Reference Reading} + \triangle = \text{New Value}$$

$$5.53059 + 0.043633 = 5.574223 \text{ inches.}$$

And since our reading at 321° 8' 13" was 5.57430, we find the true profile is 77 microinches greater than theory predicts.

IV. SOURCES OF ERROR

In any measurement process there are errors which affect the ultimate accuracy of the calibration. It is useful to classify these errors into (1) random observational errors, and (2) what can be defined as <u>model ambiguity</u>. The definition of the latter term is that error which arises because the object being measured has not been perfectly made, i.e. the difference between the embodiment of the concept in the material object to that embodiment in the model. For example, if one measures the diameter of a cylinder which is badly out-of-round, then that measured diameter is undefined to the extent that the object (out-of-round cylinder) differs from the model (perfectly round cylinder).

<u>Model Ambiquity Factors</u>:

(1) Because the character of the involute axis will be determined by the quality of the centering devices, the estimated uncertainty from this model ambiguity will be on the same order as the roundness of the centers.

(2) The amount of surface roughness also contributes to the total uncertainty. It is very difficult to locate and relocate a reference surface if there is a large degree of waviness on the surface.

(3) The actual involute master will have the involute profile generated with a base radius which is not exactly the value which is stated for that particular master. For example, if the base radius is actually 0.500050 inch then over a 40° angle of roll, the actual $\triangle$ is 35 microinches greater than that measured $\triangle$ on the assumption

that the base radius is exactly 0.500000 inch.*

Measurement Errors:

(1) The stylus probe tip can be a potential source of error. If the probe is a knife-edge, care must be taken to insure the edge is on the vertical tangent to the base circle. If the involute profile is incorrect or has a small high spot, the probe will contact the highest point on the profile, thus resulting in an erroneous profile. A spherical probe is preferable but again reasonable care must be taken to position on the vertical tangent to the base circle.

(2) The sources of observational errors involve the setting of the rotary table used to rotate the involute profile and the ability to repeat a reading on the axis of the measuring machine. The observed standard deviation of repeated readings on the measuring machine is approximately 10 microinches. The rotary table is accurate to one second which corresponds to a 2.4 microinch uncertainty when the base radius is 0.500 inch.

These random errors combined with estimations of error due to model ambiguities indicate that it is difficult to quantify an involute profile to better than $\pm$ 0.000050 inch in most cases.

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*The defining of the base radius of an involute master to be 0.500000 inch inherently means that any errors in the form are to be referenced to that base radius. This does not mean that the actual base radius needs to be known, but that this is one of the model ambiguities in the measurement process.

V. SUMMARY

The procedures and problems inherent in the calibration of master involute profiles have been discussed. A reasonably well-equipped metrology lab can apply the same basic principles for the calibration of their involute masters.

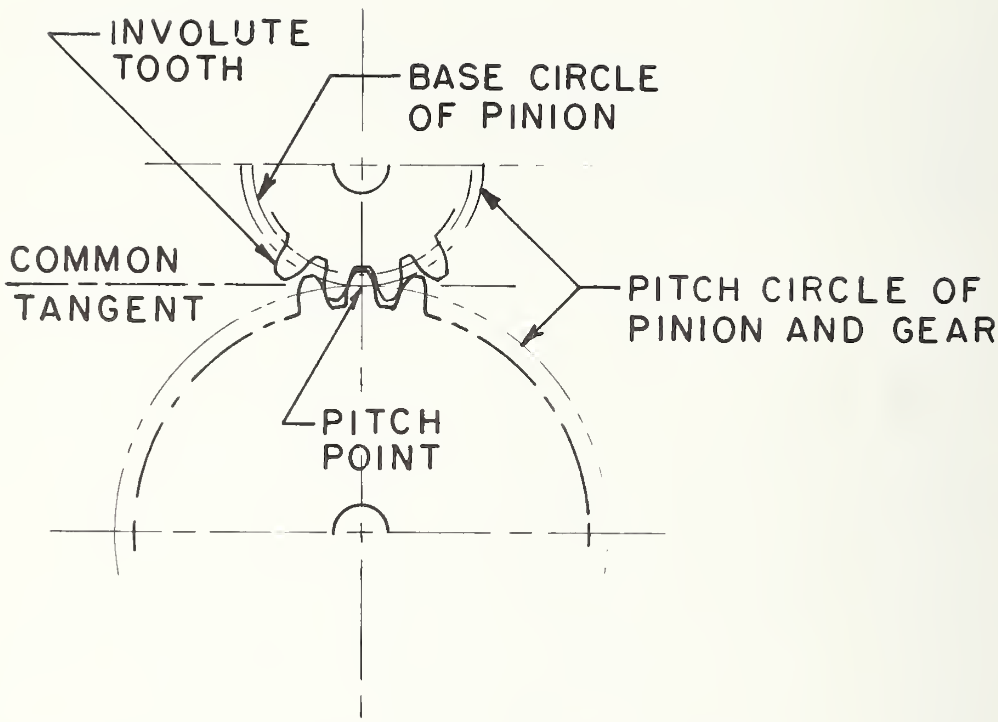

FIGURE 1. Illustration of Basic Gear Elements

Definitions:

<u>Involute Teeth</u> of gears are those in which the active portion of the profile is the involute of a circle.

<u>Base Circle</u> is the circle from which the involute tooth profiles are derived.

<u>Pitch Circle</u> is the imaginary circle which rolls without slipping with a pitch circle of a mating gear.

<u>Pitch Point</u> is the point of tangency of two pitch circles and is on the line of centers.

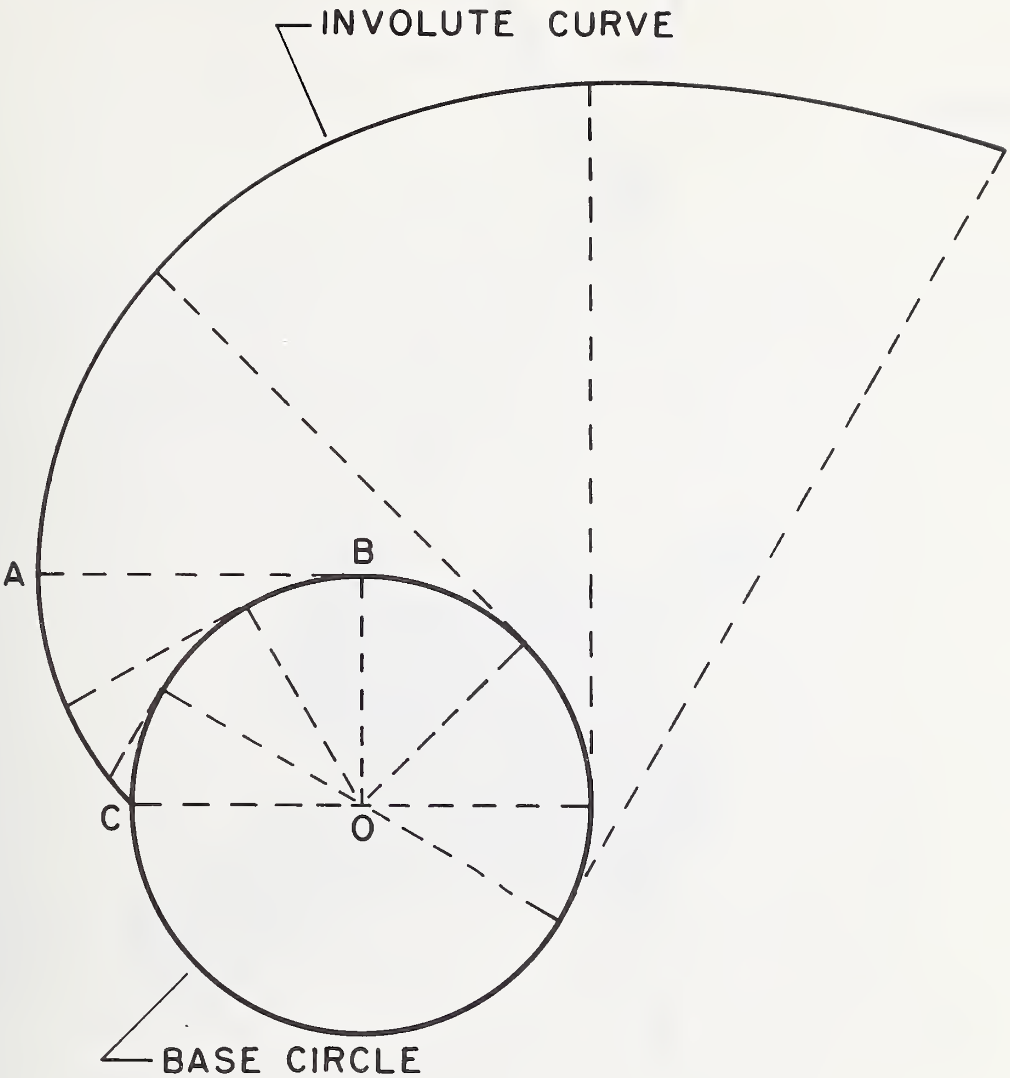

FIGURE 2. Illustration of the generation of the involue profile. The tangent to point B is extended and the length of line AB is equal to the arc BC on the base circle. Arc BC = $(\text{Radius})_{\text{Base Circle}} \times (\theta)_{\text{included angle}}$.

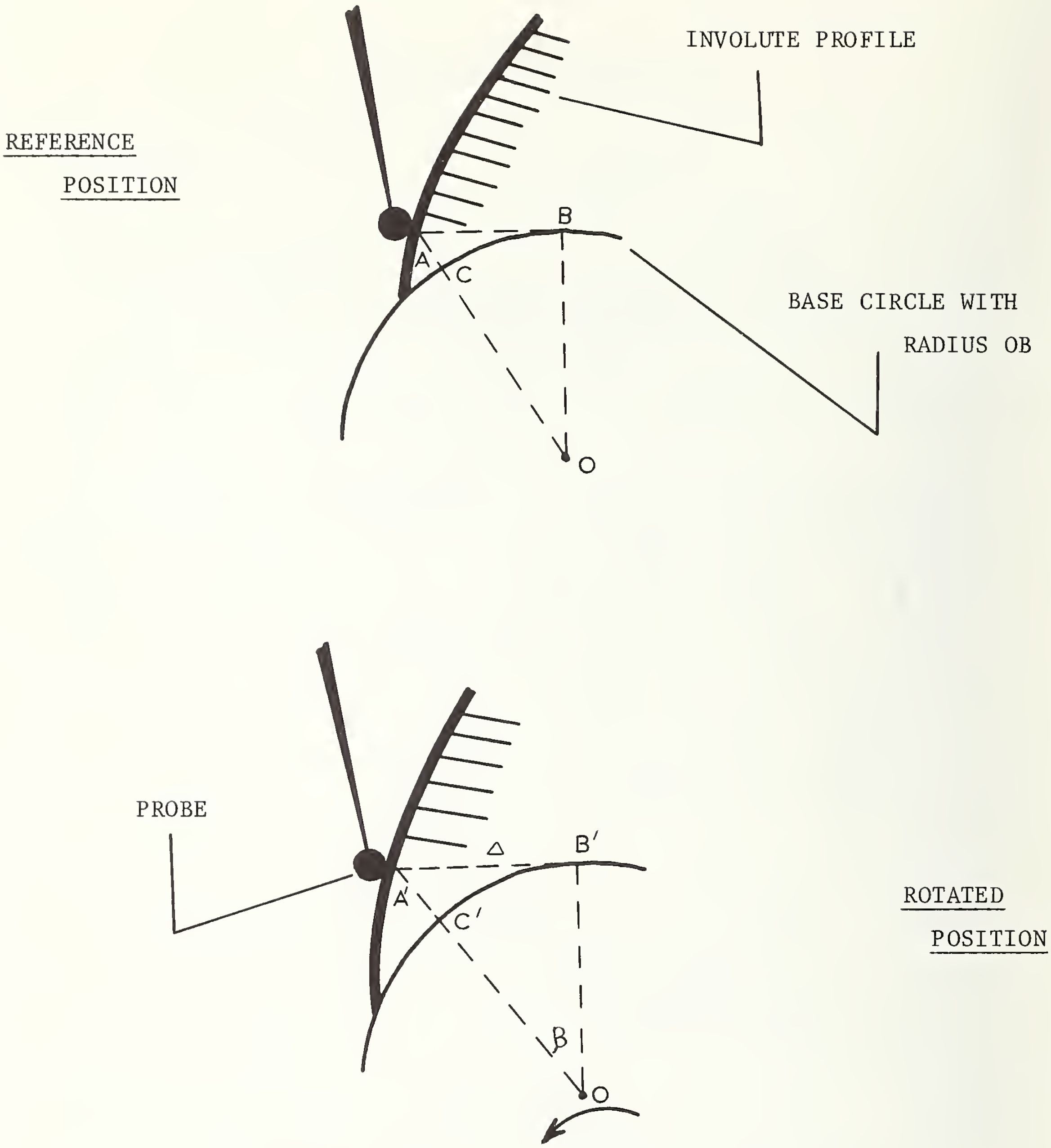

FIGURE 3. Illustration of the measurement of an involute profile. A reference reading is first obtained and then the involute master is rotated through an angle, β, which produces

$$\Delta = (\text{Base Radius}) \times (\beta)$$

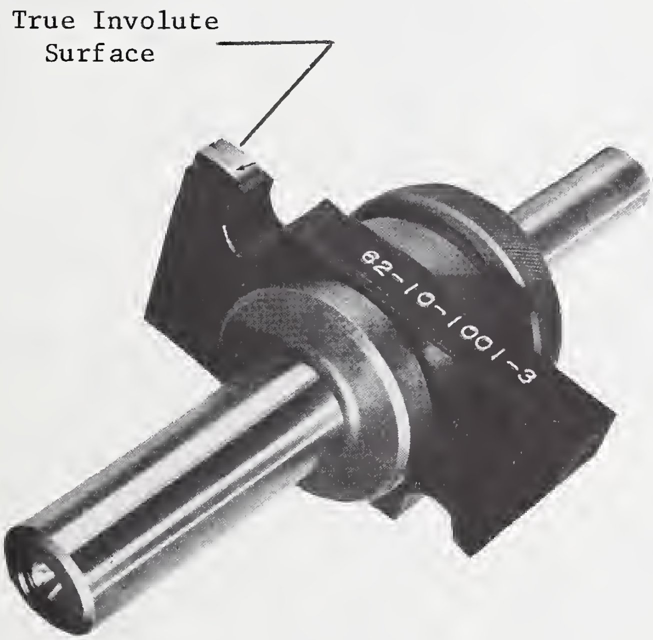

FIGURE 4. Involute Master

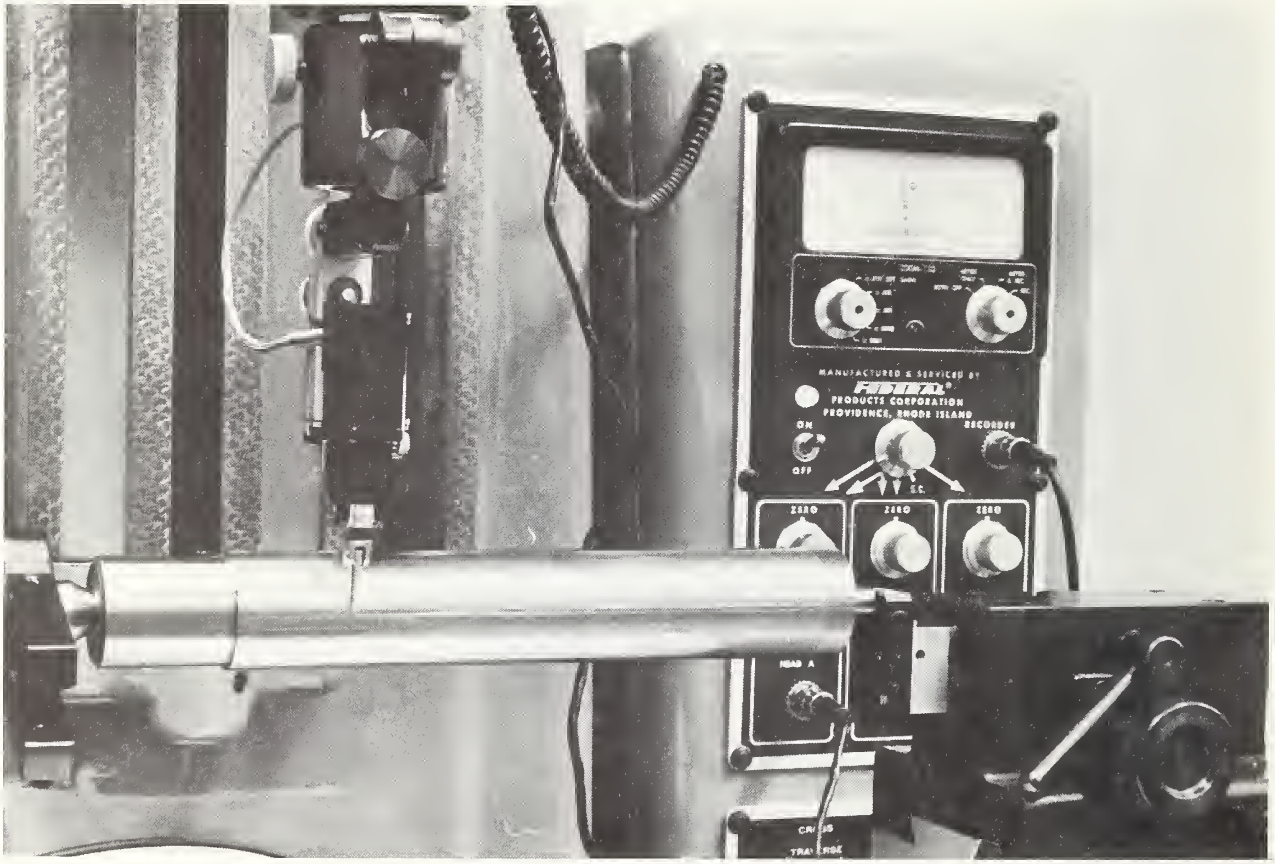

FIGURE 5. Establishing Reference Axis on Measuring Machine

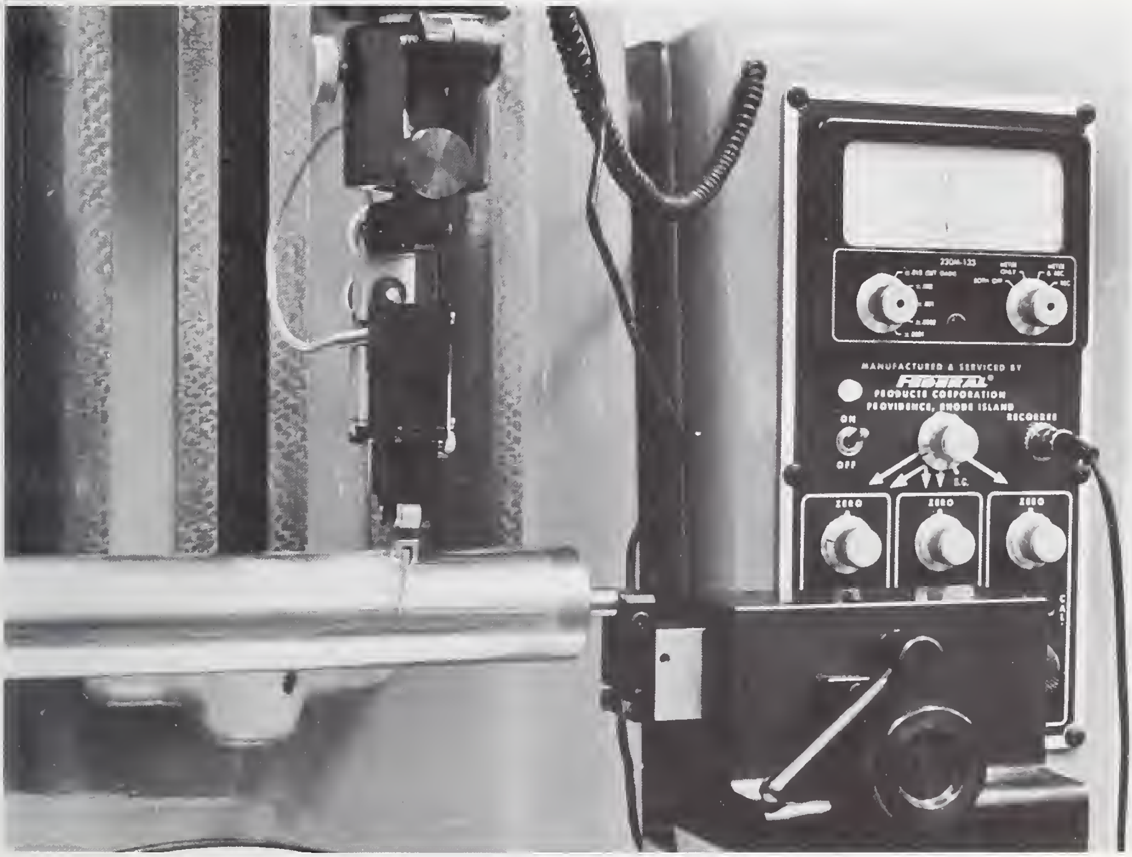

FIGURE 6. Establishing Reference Axis on Measuring Machine

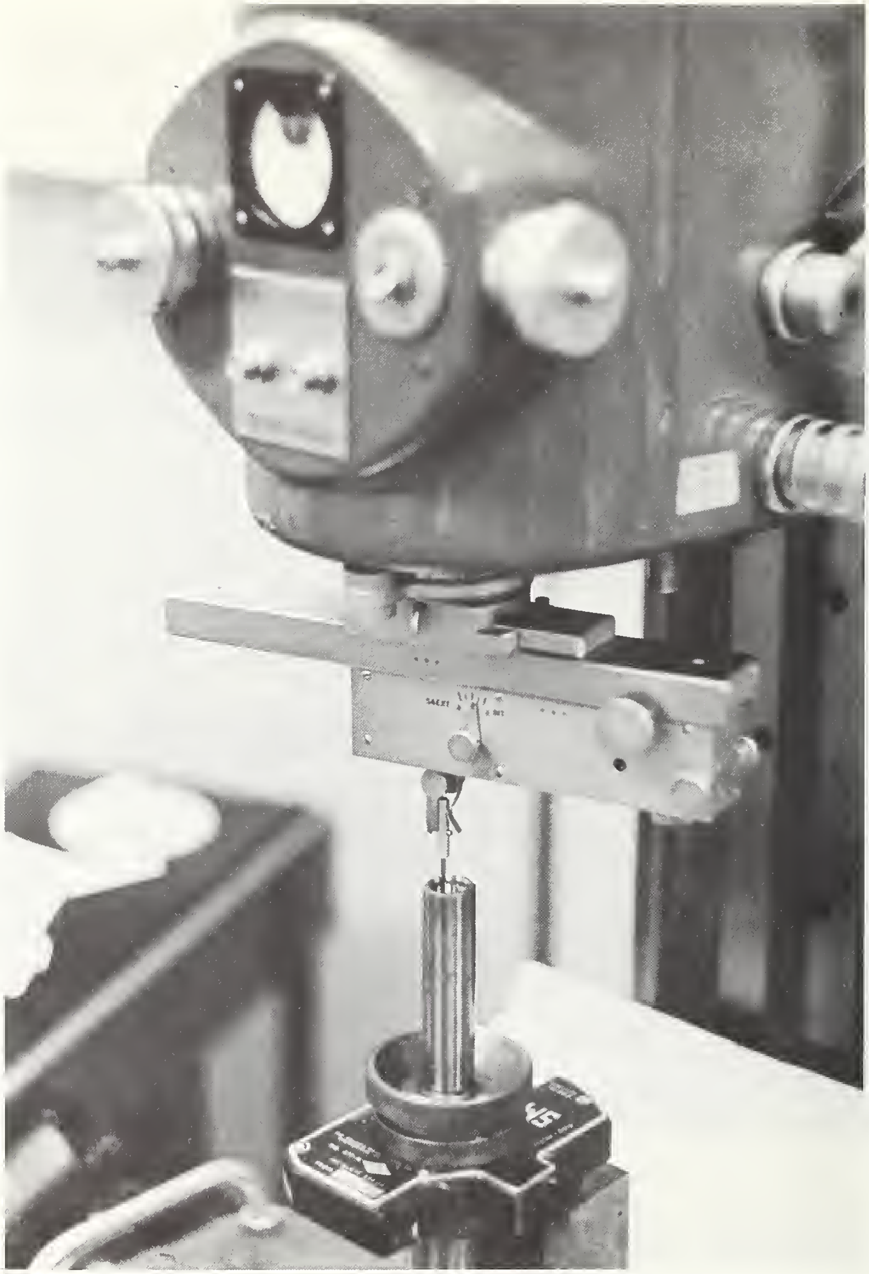

FIGURE 7. Checking Involute Master for Roundness

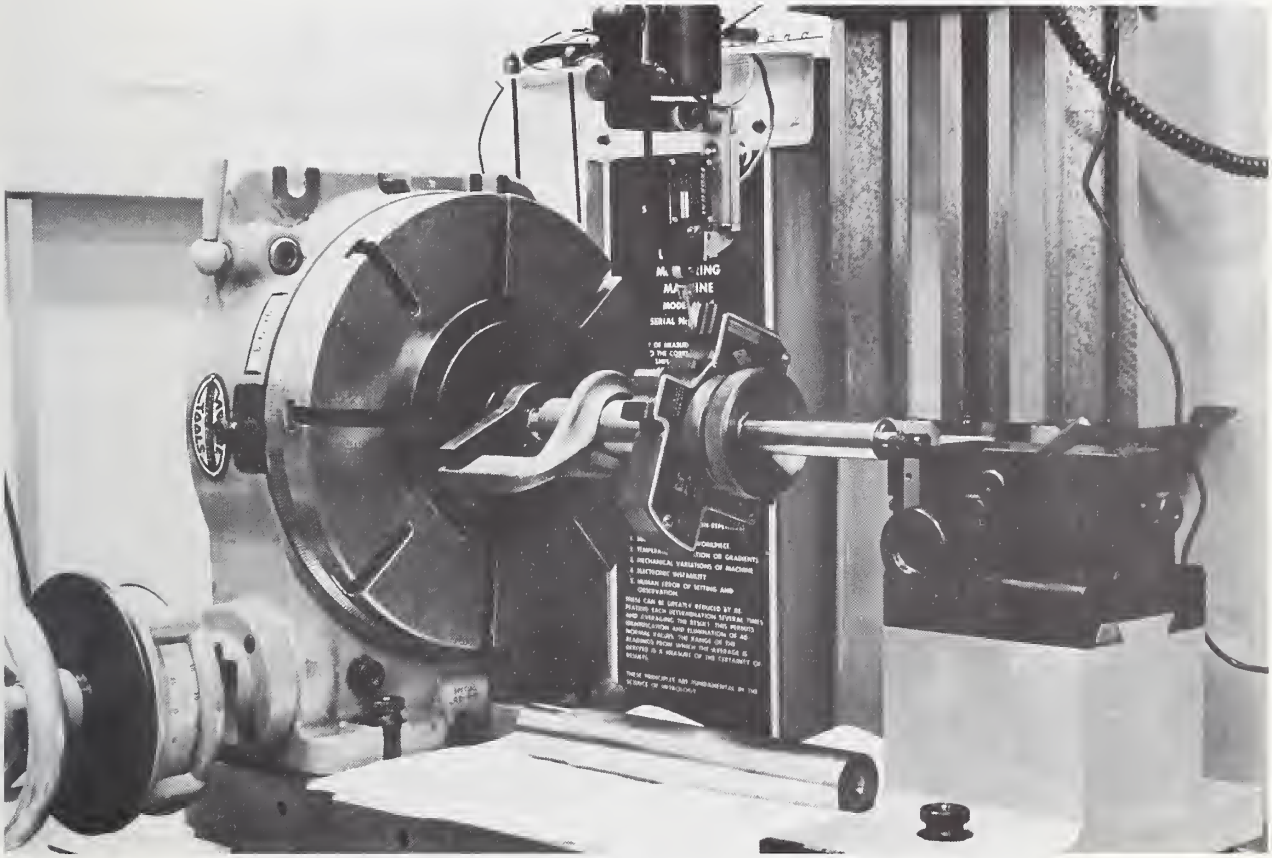

FIGURE 8. Installation of Master Involute into Measuring Machine

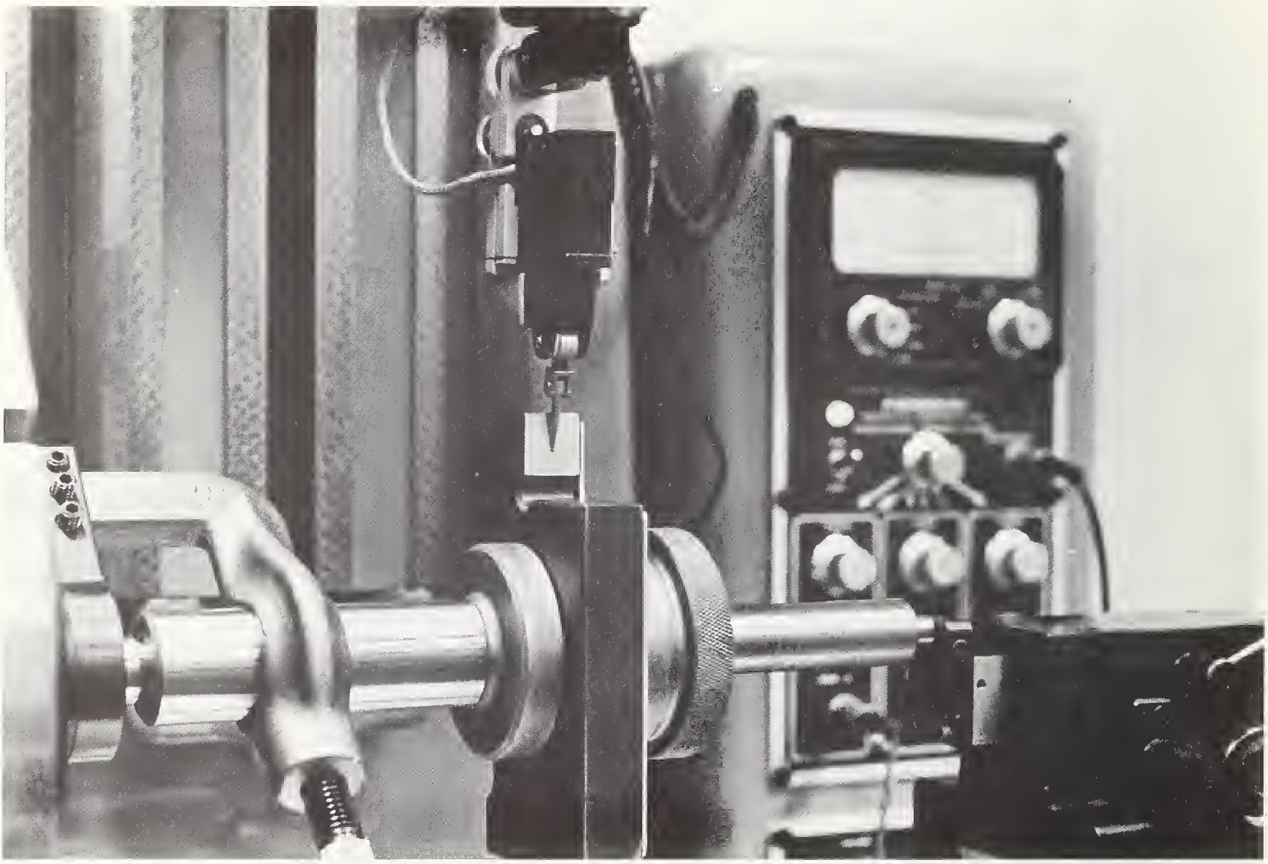

FIGURE 9. Check for Parallelism between Involute Axis and Reference Axis of Measuring Machine

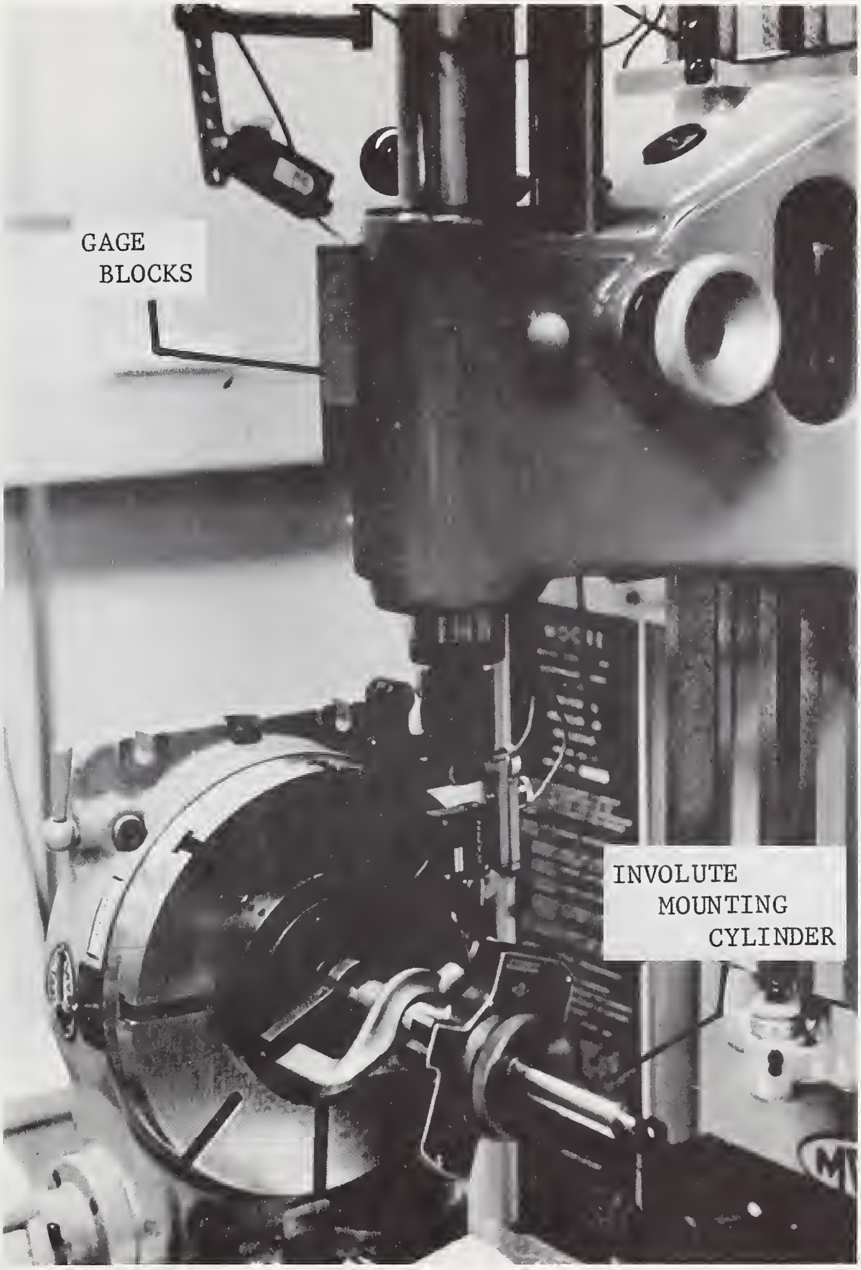

FIGURE 10. Establishing Base Radius Distance on Involute Master

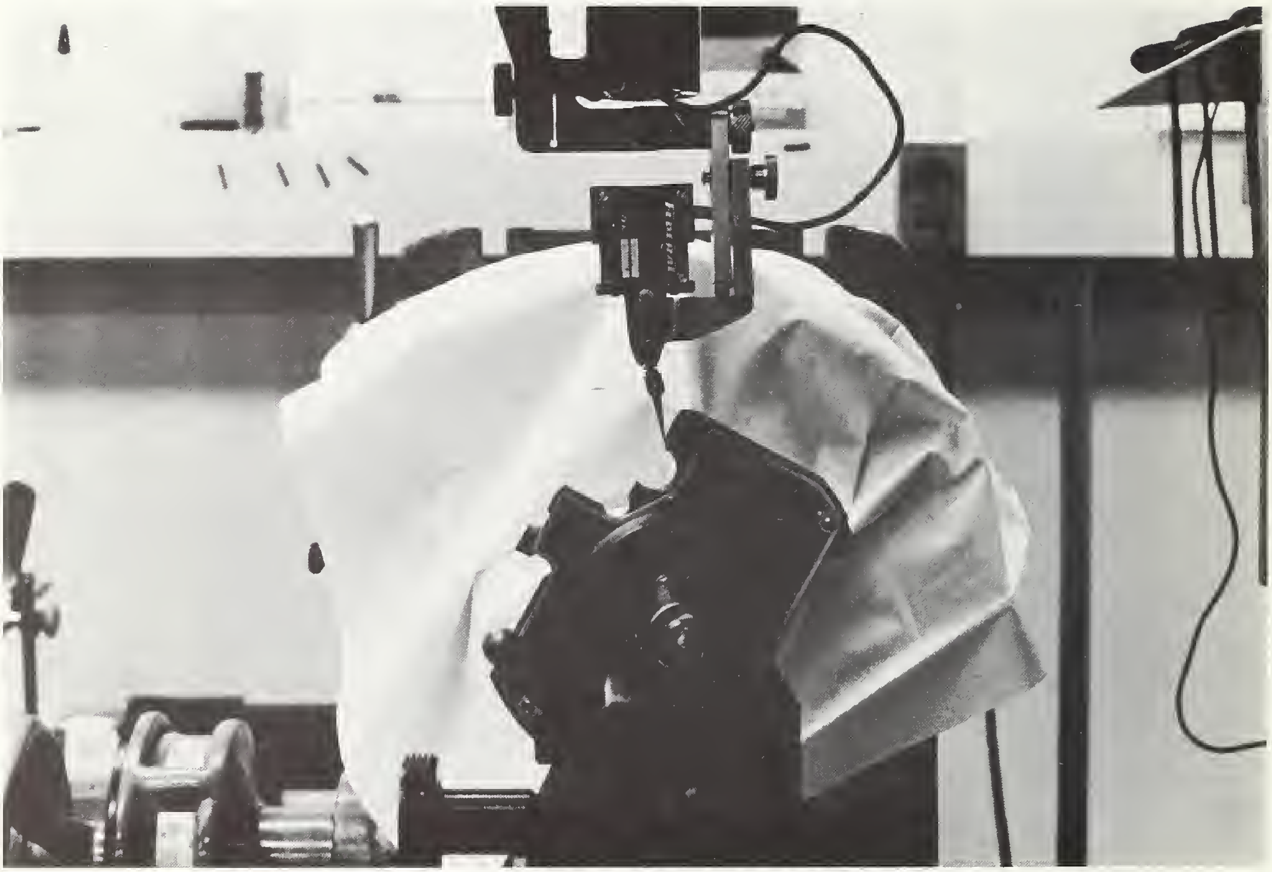

FIGURE 11. Bringing Probe into Contact with Involute Master

FIGURE 12. Probe used only as Nulling Device

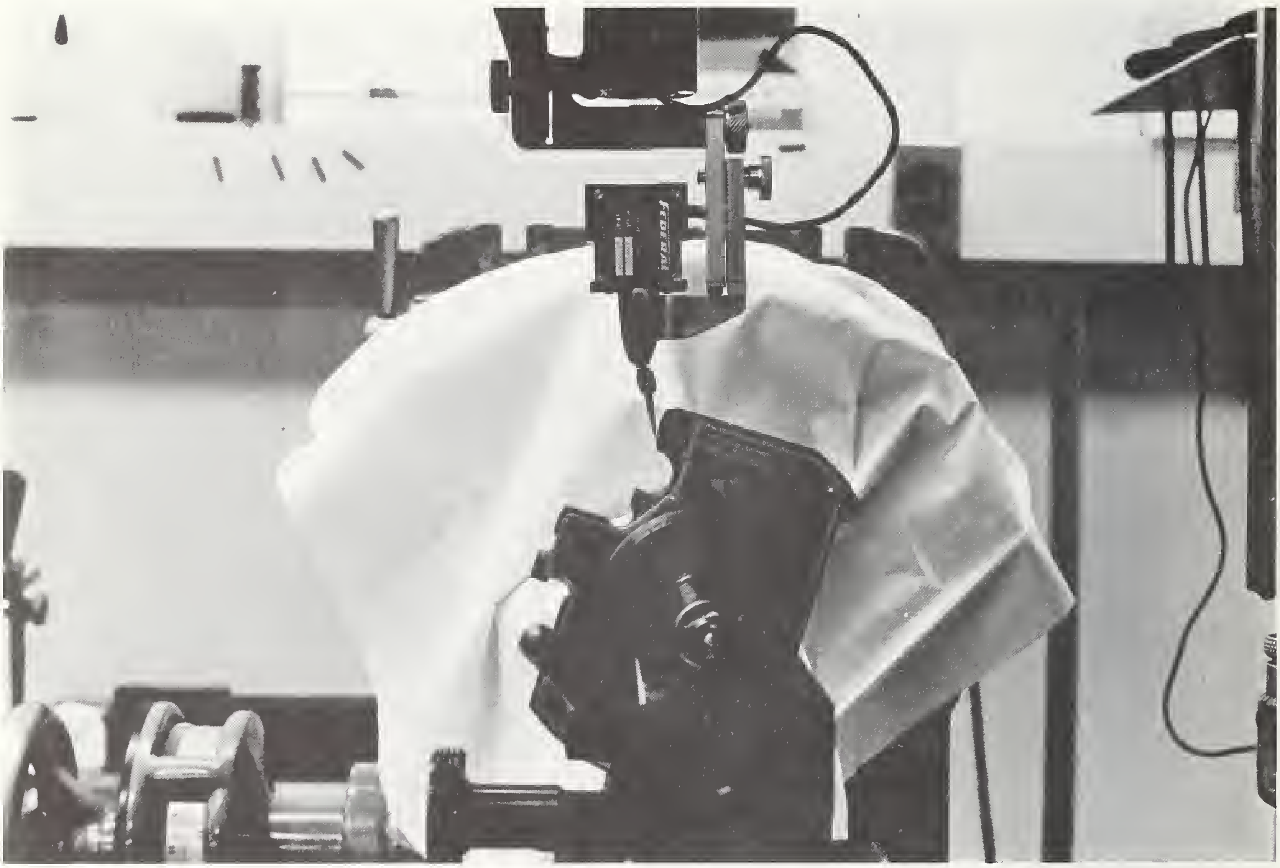

FIGURE 13. Rotation of Involute Master

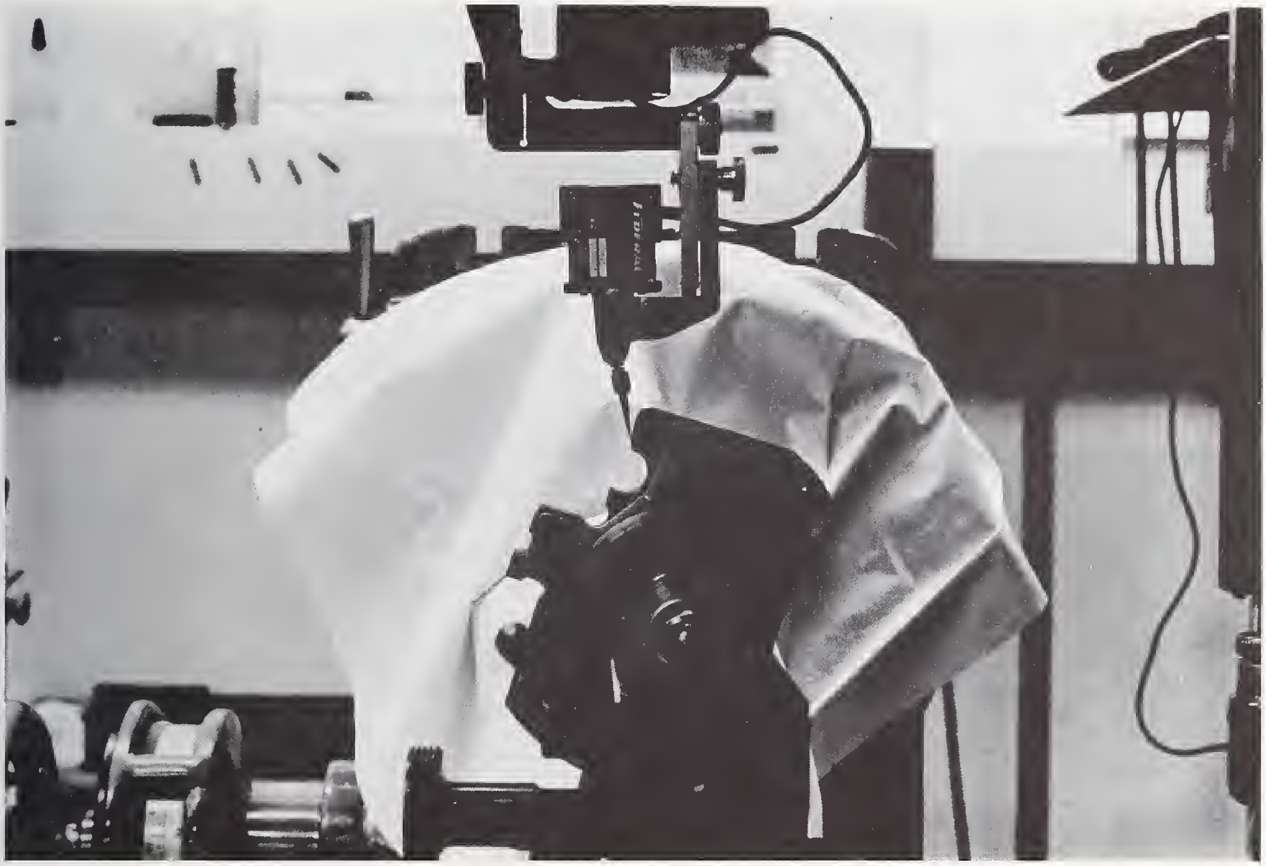

FIGURE 14. Rotation of Involute Master

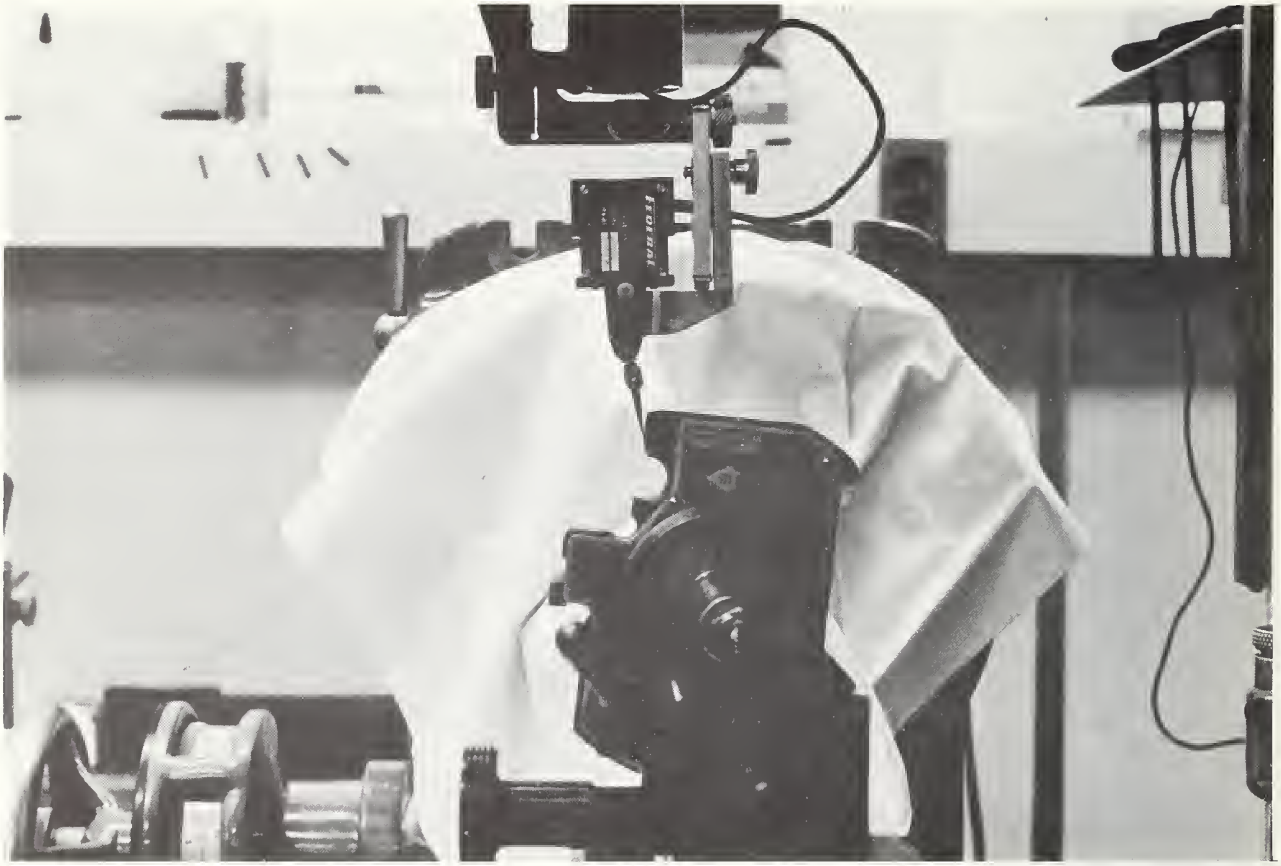

FIGURE 15. Rotation of Involute Master

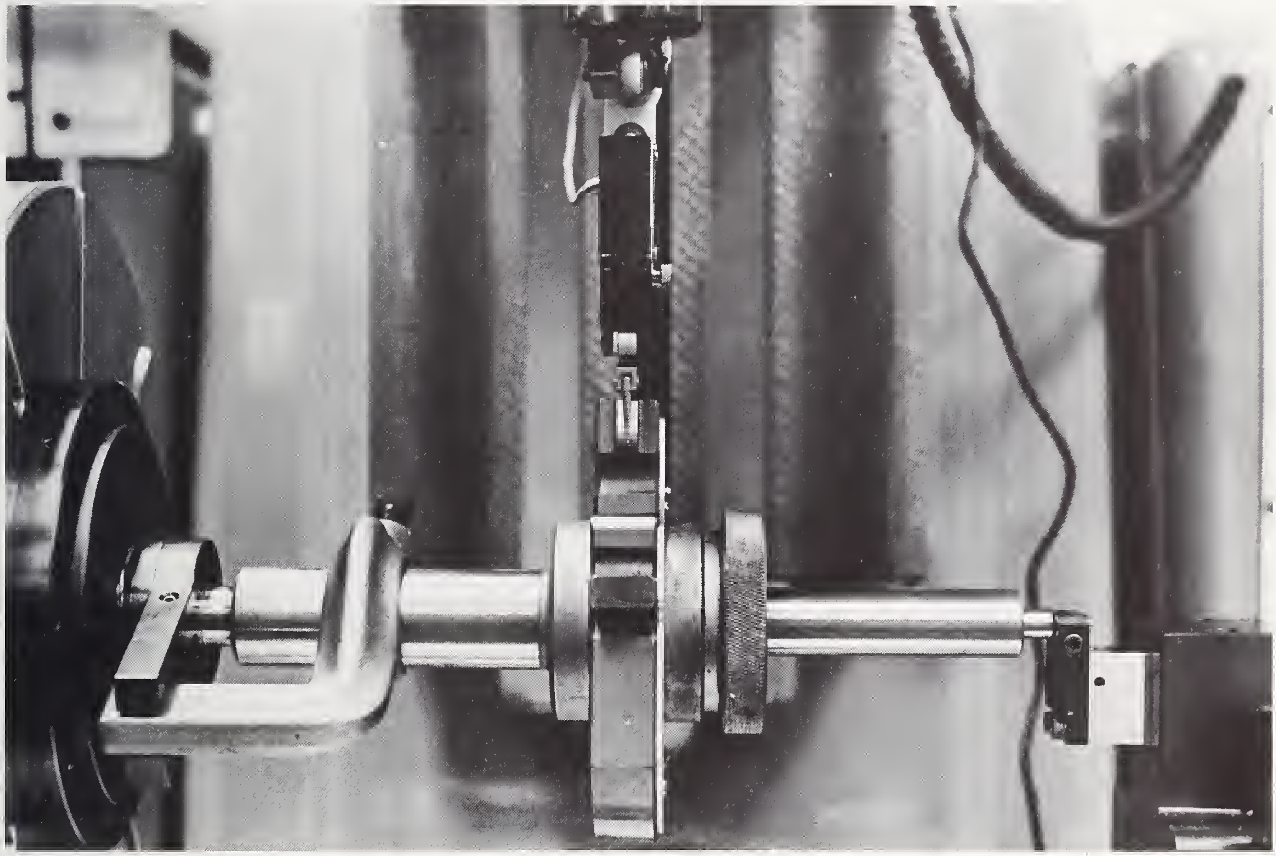

FIGURE 16. X-Axis of Machine Moved to Tram Surface

REDUCTION OF INVOLUTE DATA

$$\Delta = (\text{Radius})(\theta) = (0.500)(5)\left(\frac{\pi}{180}\right) = 0.043633 \text{ in.}$$

Angle of Roll Degrees	Observed Reading Inches	Theory Inches	Obs-Theory 10^{-6} Inches
0	5.53059	5.530590	0
5	5.57430	5.574223	+77
10	5.61795	5.617856	+94
15	5.66157	5.661489	+81
20	5.70522	5.705122	+98
25	5.74886	5.748755	+105
30	5.79251	5.792388	+122
35	5.83614	5.836021	+119
40	5.87979	5.879654	+136

FIGURE 17

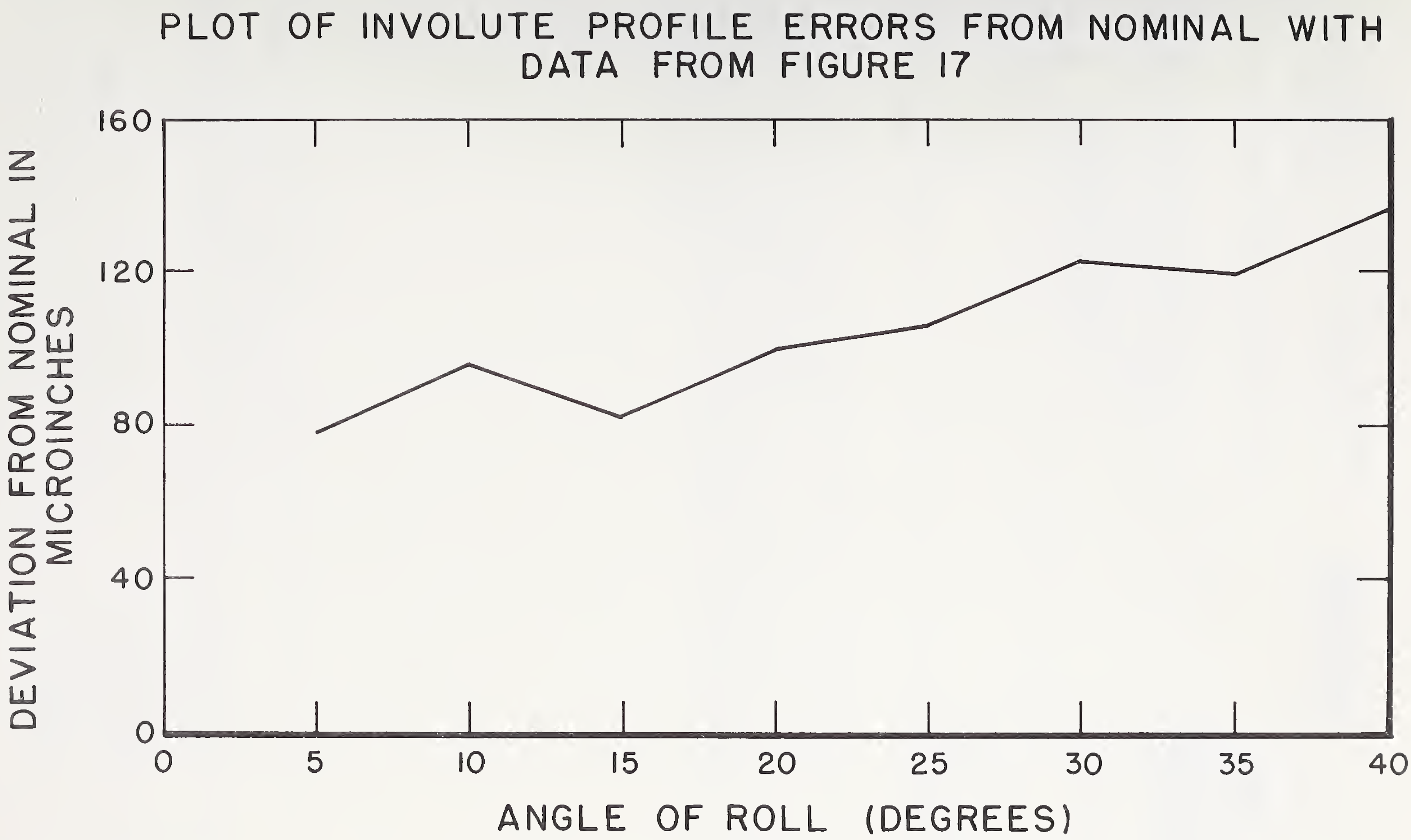
PLOT OF INVOLUTE PROFILE ERRORS FROM NOMINAL WITH DATA FROM FIGURE 17
DEVIATION FROM NOMINAL IN MICROINCHES
160
120
80
40
0
0
5
10
15
20
25
30
35
40
ANGLE OF ROLL (DEGREES)

FIGURE 18

FORM NBS-114A (1-71)

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15. SUPPLEMENTARY NOTES

16. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)

The involute profile plays an important role in gear metrology. It is necessary to quantify that profile on involute masters so that an accurate form for gear teeth may be transferred from the master profile. The procedures for calibration of a master involute profile on a two coordinate axis measuring machine is the function of this paper.

17. KEY WORDS (Alphabetical order, separated by semicolons)

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